Assignment : 3

Q1 $L = \{a^n b^m \mid n \geq 1, m \geq 0 \text{ \& } n \text{ is even}\}$

a) V, Σ, P, S

Terminals $\Sigma = \{a, b\}$

Non Terminals $V = \{S, A\}$

Start symbol S

Production P

Productions :

$S \rightarrow aaS \mid aaA$

$A \rightarrow bA \mid \epsilon$

$S \rightarrow EB$ (even a's any b)
$E \rightarrow aa$ (2 a's atleast)
$E \rightarrow aaE$ (maintain even a
$B \rightarrow bB$ (maintain one b's & continues
$B \rightarrow \epsilon$ zeroes b's

S generates blocks of aa (ensuring even no of a's)

b) Leftmost & Rightmost Grammar

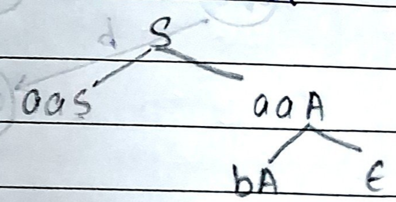
"aaaabb"

$S \rightarrow aaS \mid aaA$

$A \rightarrow bA \mid \epsilon$

Leftmost : always expand leftmost non terminal

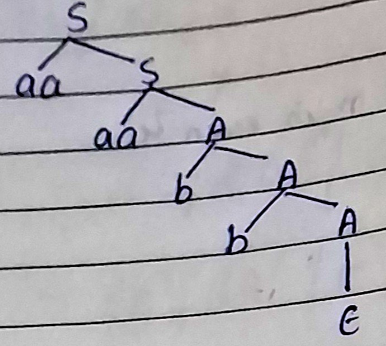
$S \rightarrow aaS$ ($S \rightarrow aaA$)
$S \rightarrow aaaaA$ ($A \rightarrow bA$)
$S \rightarrow aaaabA$ ($A \rightarrow bA$)
$S \rightarrow aaaabbA$ ($A \rightarrow \epsilon$)
$S \rightarrow aaaabb\epsilon$
$\Rightarrow S \rightarrow aaaabb$



Rightmost : always expand rightmost non terminal

$S \rightarrow aaS$ ($S \rightarrow aaA$)
$S \rightarrow aaaaA$ ($A \rightarrow bA$)
$S \rightarrow aaaabA$ ($A \rightarrow bA$)
$S \rightarrow aaaabbA$ ($A \rightarrow \epsilon$)
$S \rightarrow aaaabb$

c) Derivation Tree



d) Regularity / context freeness proof

Regular expression

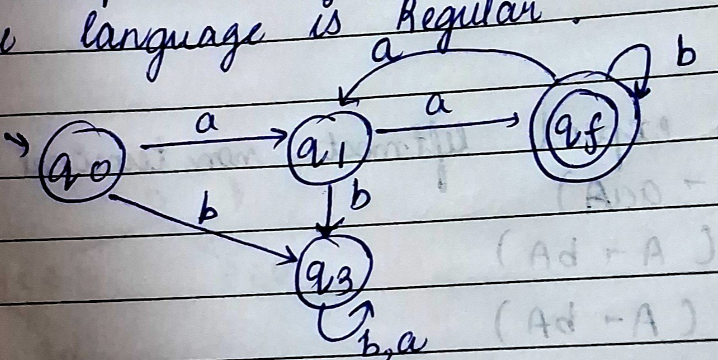
$(aa)^+b^*$

String with positive even no of of a's (aa, aaaa --)

followed by zero or more no of b's

Every Regular Grammar is context free Grammar.

An NFA / DFA can be built to accept this. Hence the language is Regular.



Q2 Ambiguity Analysis

$S \rightarrow aSbS \mid \epsilon$

a) "aabb"

Parse Tree 1 (leftmost derivation)

$S \rightarrow aSbS$ (Expand first S)

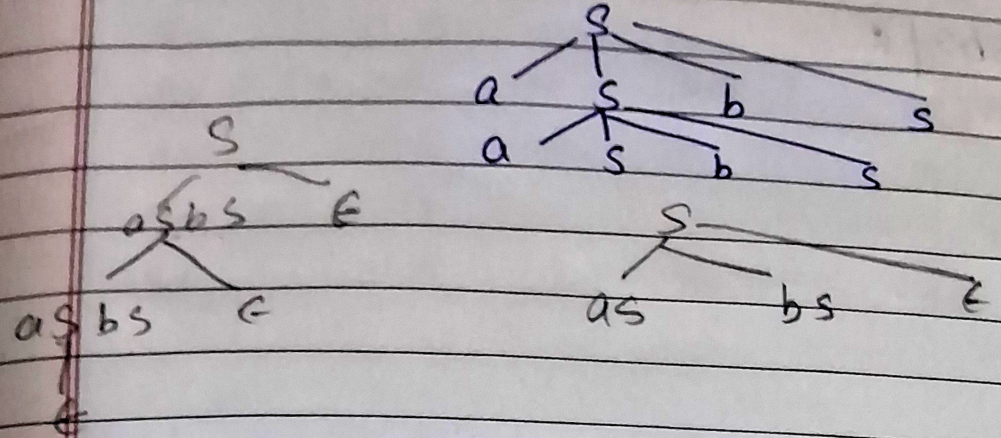
$S \rightarrow aaSbSbS$ (Expand first $S \rightarrow \epsilon$)

$S \rightarrow aa\epsilon bSbS$ (" 2^{nd} $S \rightarrow \epsilon$)

$S \rightarrow aa\epsilon b\epsilon bS$ (" 3^{rd} $S \rightarrow \epsilon$)

$S \rightarrow aa\epsilon b\epsilon b\epsilon$

$S \rightarrow aabb$



Parse tree 2 (rightmost derivation)

$S \rightarrow aSbS$

$S \rightarrow aSb$ $(S \rightarrow \epsilon)$

$S \rightarrow aSb$ $(S \rightarrow aSbS)$

$S \rightarrow aaSbSb$ (expand 2nd $S \rightarrow \epsilon$)

$S \rightarrow aaSb\epsilon b$ (expand 1st $S \rightarrow \epsilon$)

$S \rightarrow aa\epsilon b\epsilon b$

$S \rightarrow aabb$

S

b) A Grammar G is ambiguous if there exists atleast one string $w \in L(G)$ that has 2 or more distinct derivation le/rig trees (or parse trees) & in this case both LMD & RMD have same result hence it is ambiguous

Q3

a) PDA $L = \{a^n b^n \mid n \geq 0\}$ by final state

$(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

(Q): $Q = \{q_0, q_1, q_2\}$

q_0 - initial state (pushing a's)

q_1 - popping a's for every 'b'

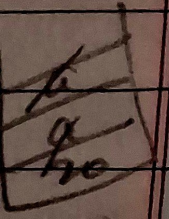
{ab, aabb

q_2 - Final state (accepts empty stack)

Input alphabet $\Sigma = \{a, b\}$

Stack alphabet $\Gamma = \{a, Z_0\}$

Z_0: stack bottom marker



Transition for (δ)

Initial state (q_0)

Initial stack symbol (Z_0)

Final state $F = \{q_2\}$

b)

State	Input	Stack top	Next state	Stack operation
q_0	a	z_0	q_0	az_0
q_0	a	a	q_0	aa
q_0	b	a	q_1	ϵ
q_0	ϵ	z_0	q_2	z_0
q_1	b	a	q_1	ϵ
q_1	ϵ	z_0	q_2	z_0

c) $\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

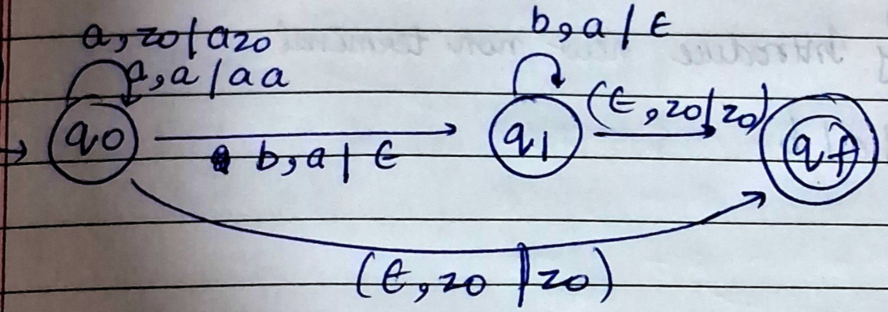
$\delta(q_0, b, a) = (q_1, \cancel{aa}\ \epsilon)$

$\delta(q_0, \epsilon, z_0) = (q_2, z_0)$ $\quad q_1 \quad \epsilon \quad z_0 \quad q_f = q_f, z_0$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

d)



Q4 CFG G: $S \to aA \mid bB$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

a) Eliminate ϵ production & unit production

$A \to \epsilon$

$B \to \epsilon$

Nullable variables $N = \{A, B\}$

New productions

$S \to aA \Rightarrow S \to a$	$S \to bB \Rightarrow S \to b$
$A \to aA \Rightarrow A \to a$	$B \to bB \Rightarrow B \to b$

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The Grammar without epsilon productions
$S \to aA \mid bB \mid a \mid b$
$A \to aA \mid a$
$B \to bB \mid b$

2) Eliminate unit production
There is no unit production

b) CNF
S1 Eliminate terminals on RHS with non terminals :
$A \to aA$
$B \to bB$
$S \to aA$
$S \to bB$

They have mixed terminals & non terminals
$T_a \to a$
$T_b \to b$ } Introduce new non terminal

$S \to T_a A \mid T_b B \mid a \mid b$
$A \to T_a A \mid T_a$
$B \to T_b B \mid b$

S2 Limit RHS length to two non terminals :
$A \to BC$ or $A \to a$
$S \to T_a A \mid T_b B \mid a \mid b$
$A \to T_a A \mid a$
$B \to T_b B \mid b$
$T_a \to a$
$T_b \to b$

$S \to T_a A \mid T_b B$
$A \to T_a A$
$B \to T_b B$ } $A \to BC$

$S \to a \mid b$
$A \to a$
$B \to b$ } $A \to a$

c) "aab"

G: $S \to aA \mid bB$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

$\therefore$ $S \to aA \mid bB$

$S \to aA \quad (A \to aA)$

$S \to aaA \quad (A \to \epsilon)$

$S \to aa$

From CNF

$S \to T_aA \mid T_bB \mid a \mid b$

$A \to T_aA \mid a$

$B \to T_bB \mid b$

$T_a \to a$

$T_b \to b$

$S \to T_aA \quad (A \to \cancel{T_aA}\ a)$

$S \to T_a \cancel{T_a}\ a \quad (T_a \to a)$

$S \to aa$

a) CNF (chomsky normal form)

1) Binary structure ($A \to BC$)
CNF guarantee that every internal node in a parse tree has exactly 2 children. This structure allows highly efficient parsing methods

2) Fixed Derivation length
For any string w derivation using a CNF always takes $2n-1$ steps.

3) Terminal rule $A \to a$
CNF require terminal directly generated from non terminal. This seperates the handeling of terminal input symbols from non terminal part often beneficial for stack operations

6) $L(G) = \{a^m b^n \mid m > 0 \;\&\; n \geq 0\}$
One or more 'a' : $A \to a \mid aA$
Zero or more 'b' : $B \to \epsilon \mid bB$
A followed by B

$V = \{S, A, B\}$

$\Sigma = \{a, b\}$

P:

$S \to AB$

$A \to aA \mid a \quad (a^m, m \geq 1)$

$B \to bB \mid b \quad (b^n, n \geq 0)$

7) $S \to aS \mid aSbS \mid c$

Let's check with acbc

D1

$S \to aSbS \quad (S \to c)$

$\to acbS \quad (S \to c)$

$\to acbc$

D2

$S \to aS$

$S \to aaSbSc$

$S \to aacbc$

D3

$S \to aS$

$S \to ac$

$S : ac \neq aacbc$

Yes the grammar is ambigious

Non ambigious Grammar

$S \to c \mid aSbS$

$S \to c \mid aS \mid SbS$

8) $S \to A$

$A \to B$

$B \to a$

Unit productions : $S \to A$

$A \to B$

1) $S \to A$: S can derive whatever A derives

$A \to B$

$A \to a \quad (B \to a)$

New Rules $S \to B$, $S \to a$

2) $A \to B$: A can derive whatever B derives

$B \to a$

New Rules $A \to a$

3) $S \to B$ (New unit production)
S can derive whatever B derives
$B \to a$
New Rule $S \to a$ (already added)

$S \to a$
$A \to a$
$B \to a$

9) $S \to A$
$A \to aB$
$B \to c$

Useless productions
(A production is useless if its non terminal cannot derive any terminal string

1) Non generating productions (Can't derive a terminal string)
$B \to c \therefore c$
$A \to aB \therefore ac$
$S \to A \therefore ac$
All are useful.

2) Non Reachable productions (can't be reached by S)
$$\left.\begin{array}{l} S \to A \\ A \to aB \\ B \to c \end{array}\right\} \text{all are reachable}$$

Hence Grammar is same as original

10) CFG to CNF

$S \rightarrow a \mid aA \mid B$
$A \rightarrow aBB \mid \epsilon$
$B \rightarrow Aa \mid b$

1) Eliminate ϵ productions
$A \rightarrow \epsilon$
$S \rightarrow aA \therefore S \rightarrow a$ (already exists)
$B \rightarrow Aa \therefore B \rightarrow a$

G1 without ϵ-productions
$S \rightarrow a \mid aA \mid B$
$A \rightarrow aBB$
$B \rightarrow Aa \mid b \mid a$

2) Eliminate unit productions
$S \rightarrow B$
$B \rightarrow Aa \mid b$

G2:
$S \rightarrow a \mid aA \mid Aa \mid b$
$A \rightarrow aBB$
$B \rightarrow Aa \mid b \mid a$

3) Eliminate useless productions (non generating & non reachable)
No useless productions

4) Convert to CNF
Terminals on RHS with non terminals

$S \longrightarrow aA$
$S \longrightarrow Aa$
$A \rightarrow aBB$
$B \rightarrow Aa$

$T_a \longrightarrow a$
$T_b \longrightarrow b$

$S \rightarrow T_aA$
$S \rightarrow AT_a$
$A \rightarrow T_aBB$
$B \rightarrow AT_a$

$S \longrightarrow a \mid T_aA \mid AT_a \mid b$
$A \rightarrow T_aBB$
$B \rightarrow AT_a$

RHS with length > 2 non terminals

$A \rightarrow T_aBB$

Introduce $C \rightarrow T_aB$

$A \rightarrow CB$
$C \rightarrow T_aB$
$B \rightarrow AT_a \mid b \mid a$

Final Grammar

$S \longrightarrow a \mid T_aA \mid AT_a \mid b$

$A \longrightarrow CB$
$C \longrightarrow T_aB$
$B \longrightarrow AT_a \mid a \mid b$
$T_a \longrightarrow a$