Partial Fraction

Polynomial: - A mathematical expression having various powers of 'x' like $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots\dots\dots + a_{n-1}x + a_n$ is called as polynomial in 'x'.

Where, $a_0, a_1, a_2, \dots\dots\dots a_n$ are called as coefficient of powers of 'x'.

e.g. $x^3 + 2x^2 - 5x + 3,\ 2x^2 + 3x + 5,\ x^4 - 27$

Degree of polynomial: It is highest power of expression in the polynomial.

e.g. $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots\dots\dots + a_{n-1}x + a_n$ is polynomial of degree 'n'.

$x^3 + 2x^2 - 5x + 3$ is polynomial of degree '3' or cubic polynomial.

$2x^2 + 3x + 5$ is polynomial of degree '2' or quadratic polynomial.

$3x - 4$ is polynomial of degree '1' or linear polynomial.

Fraction: It is a division of two polynomial expressions.

If $P(x)$ and $Q(x)$ are two polynomial then,

$$Fraction = \frac{P(x)}{Q(x)}$$

Types of fraction

1) Proper fraction: A $Fraction = \frac{P(x)}{Q(x)}$ is called as Proper fraction if degree of P(x) is less than degree of Q(x).

i.e. degree of $P(x)$ < degree of $Q(x)$

For Example: $\frac{x^2 + 3x - 1}{x^3 - 2x^2 + 3x + 6}$, $\frac{4}{x^2 - 9}$, $\frac{2x^2 + 1}{(x+1)(x-1)(2x+3)}$ are proper fraction.

2) Improper fraction: A $Fraction = \frac{P(x)}{Q(x)}$ is called as Improper fraction if degree of P(x) is greater than or equal degree of Q(x).

i.e. degree of $P(x) \geq$ degree of $Q(x)$

For Example: $\frac{x^3-1}{x^2+2x+3}, \frac{x^4}{x^2-1}, \frac{x^2+2x+1}{x^2-5x+6}$ are improper fraction.

Useful Standard Factorization formulae

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^4 - b^4 = (a - b)(a + b)(a^2 - ab + b^2)$$

Partial Fraction: It is a process of expressing one fraction into sum (addition) of two or more simple fraction.

e.g. $\frac{3}{5} = \frac{1}{5} + \frac{2}{5}, \frac{7}{9} = \frac{2}{9} + \frac{5}{9}$ is partial fraction of fraction of number.

Here we consider partial fraction of fraction of polynomial.

<u>Partial fraction of Proper fraction</u>

Type 1: When denominator contain different linear factors.

Resolution step of partial fraction

$$\frac{P(x)}{(ax+b)(cx+d)(ex+f)} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{ex+f}$$

For example: $$\frac{x^2+1}{(x+3)(x-3)(2x+1)} = \frac{A}{x+3} + \frac{B}{x-3} + \frac{C}{2x+1}$$

$$\frac{1}{x^2-4} = \frac{1}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$$

Note:

1) No. of term in partial fraction is equal to number of linear factors in denominator.

2) In every term of partial fraction numerator is constant and denominator has linear factor.

3) To find partial fraction, evaluate value of unknown constant A. B. C…..etc.

For Example: 1)

Resolve $\frac{7x-25}{(x-3)(x-4)}$ into partial fractions.

Solution:

$$\frac{7x-25}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4} \text{-------------------(1)}$$

Multiplying both sides by L.C.M. i.e., $(x-3)(x-4)$, we get

$$7x - 25 = A(x-4) + B(x-3) \text{ -------------- } (2)$$

Put $x - 4 = 0, \Rightarrow x = 4$ in equation (2)

$7(4) - 25 = A(4-4) + B(4-3)$

$28 - 25 = 0 + B(1)$

$B = 3$

Put $x - 3 = 0 \Rightarrow x = 3$ in equation (2)

$7(3) - 25 = A(3-4) + B(3-3)$

$21 - 25 = A(-1) + 0$

$-4 = -A$

$A = 4$

Hence the required partial fractions are

$$\frac{7x-25}{(x-3)(x-4)} = \frac{4}{x-3} + \frac{3}{x-4}$$

2)

Resolve into partial fraction: $\frac{1}{x^2 - 1}$

(Hint: Use $(x^2 - 1^2) = (x-1)(x+1)$)

Solutios: $\frac{1}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$

$1 = A(x+1) + B(x-1) \qquad (1)$

Put $x - 1 = 0, \Rightarrow x = 1$ in equation (1)

$1 = A(1+1) + B(1-1) \Rightarrow A = \frac{1}{2}$

Put $x + 1 = 0, \Rightarrow x = -1$ in equation (1)

$1 = A(-1+1) + B(-1-1)$

$1 = -2B, \Rightarrow B = \frac{1}{2}$

$$\frac{1}{x^2 - 1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

Partial fraction Type 1 link: https://youtu.be/HZTv4zCgEnA

3)

Resolve into partial fraction $\dfrac{8x-8}{x^3-2x^2-8x}$

Solution: $\dfrac{8x-8}{x^3-2x^2-8x}=\dfrac{8x-8}{x(x^2-2x-8)}=\dfrac{8x-8}{x(x-4)(x+2)}$

Let $\dfrac{8x-8}{x^3-2x^2-8x}=\dfrac{A}{x}+\dfrac{B}{x-4}+\dfrac{C}{x+2}$

Multiplying both sides by L.C.M. i.e., $x(x-4)(x+2)$

$$8x-8=A(x-4)(x+2)+Bx(x+2)+Cx(x-4)$$

(I)

Put $x=0$ in equation (I), we have

$$8(0)-8=A(0-4)(0+2)+B(0)(0+2)+C(0)(0-4)$$

$$-8=-8A+0+0$$

$\Rightarrow$ $A=1$

Put $x-4=0$ $\Rightarrow$ $x=4$ in Equation (I), we have

$$8(4)-8=B(4)(4+2)$$

$$32-8=24B$$

$$24=24B$$

$\Rightarrow$ $B=1$

Put $x+2=0 \Rightarrow x=-2$ in Eq. (I), we have

$$8(-2)-8=C(-2)(-2-4)$$

$$-16-8=C(-2)(-6)$$

$$-24=12C$$

$\Rightarrow$ $C=-2$

Hence the required partial fractions

$$\frac{8x-8}{x^3-2x^2-8x}=\frac{1}{x}-\frac{1}{x-4}-\frac{2}{x+2}$$

Exercise

Resolve into partial fraction:

Q.1 $\dfrac{2x+3}{(x-2)(x+5)}$

Q.2 $\dfrac{2x+5}{x^2+5x+6}$

Q.3 $\dfrac{3x^2-2x-5}{(x-2)(x+2)(x+3)}$

Q.4 $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$

Q.5 $\dfrac{x}{(x-a)(x-b)(x-c)}$

Q.6 $\dfrac{1}{(1-ax)(1-bx)(1-cx)}$

Q.7 $\dfrac{2x^3-x^2+1}{(x+3)(x-1)(x+5)}$

Q.8 $\dfrac{1}{(1-x)(1-2x)(1-3x)}$

Q.9 $\dfrac{6x+27}{4x^3-9x}$

Q.10 $\dfrac{9x^2-9x+6}{(x-1)(2x-1)(x+2)}$

Q.11 $\dfrac{x^4}{(x-1)(x-2)(x-3)}$

Q.12 $\dfrac{2x^3+x^2-x-3}{x(x-1)(2x+3)}$

Answer

Q.1 $\dfrac{1}{x-2}+\dfrac{1}{x+5}$

Q.2 $\dfrac{1}{x+2}+\dfrac{1}{x+3}$

Q.3 $\dfrac{3}{20(x-2)}-\dfrac{11}{4(x-2)}+\dfrac{28}{5(x+3)}$

Q.4 $1+\dfrac{3}{x-4}-\dfrac{24}{x-5}+\dfrac{30}{x-6}$

Q.5 $\dfrac{a}{(a-b)(a-c)(x-a)}+\dfrac{b}{(b-a)(b-c)(x-b)}+\dfrac{c}{(c-b)(c-a)(x-c)}$

Q.6 $\dfrac{a^2}{(a-b)(a-c)(1-ax)}+\dfrac{b^2}{(b-a)(b-c)(1-bx)}+\dfrac{c^2}{(c-b)(c-a)(1-cx)}$

Q.7 $2+\frac{31}{4(x+3)}+\frac{1}{12(x-1)}-\frac{137}{6(x+5)}$

Q.8 $\frac{1}{2(1-x)}-\frac{4}{(1-2x)}+\frac{9}{2(1-3x)}$

Q.9 $\frac{3}{x}+\frac{4}{2x-3}+\frac{2}{2x+3}$

Q.10 $\frac{2}{x-1}-\frac{3}{2x-1}+\frac{4}{x+12}$

Q.11 $x+6+\frac{1}{2(x-1)}-\frac{16}{x-2}+\frac{81}{2(x-3)}$

Q.12 $1+\frac{1}{x}-\frac{1}{5(x-1)}-\frac{8}{5(2x+3)}$

Type 2: When denominator contain repeated linear factors.

Resolution step of partial fraction

$$\frac{P(x)}{(ax+b)^r}=\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}+\ldots\ldots\ldots\ \ldots+\frac{const}{(ax+b)^r}$$

For Example: $\frac{3x-2}{(x+2)^3}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}+\frac{C}{(x+2)^3}$

$$\frac{x^2+2x}{(2x+1)(x-1)^2}=\frac{A}{2x+1}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$$

Note:

1) No. of term in partial fraction is equal to number of repetition of linear factors in denominator.

2) In every term of partial fraction numerator is constant and denominator power of linear factor get increased term by term.

3) To find partial fraction, evaluate value of unknown constant A. B. C…..etc.

Example

1)

Resolve into partial fractions: $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}$

Solution:

$$\frac{x^2-3x+1}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$$

Multiplying both sides by L.C.M. i.e., $(x-1)^2(x-2)$, we get

$$x^2-3x+1=A(x-1)(x-2)+B(x-2)+C(x-1)^2 \quad (I)$$

Putting $x-1=0 \Rightarrow x=1$ in (I), then

$$(1)^2-3(1)+1=B(1-2)$$
$$1-3+1=-B$$
$$-1=-B$$
$$\Rightarrow \quad B=1$$

Putting $x-2=0 \Rightarrow x=2$ in (I), then

$$(2)^2-3(2)+1=C(2-1)^2$$
$$4-6+1=C(1)^2$$
$$\Rightarrow \quad -1=C$$

Now $x^2-3x+1=A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$

Comparing the co-efficient of like powers of x on both sides, we get

$$A+C=1$$
$$A=1-C$$
$$=1-(-1)$$
$$=1+1=2$$
$$\Rightarrow \quad A=2$$

Hence the required partial fractions are

$$\frac{x^2-3x+1}{(x-1)^2(x-2)}=\frac{2}{x-1}+\frac{1}{(x-1)^2}+\frac{1}{x-2}$$

Partial fraction of Type 2 link: https://youtu.be/5l_wM1XQMLY

2)

Resolve into partial fractions $\dfrac{4+7x}{(2+3x)(1+x)^2}$

Solution:

$$\frac{4+7x}{(2+3x)(1+x)^2}=\frac{A}{2+3x}+\frac{B}{1+x}+\frac{C}{(1+x)^2}$$

Multiplying both sides by L.C.M. i.e., $(2+3x)(1+x)^2$

We get $\quad 4+7x = A(1+x)^2 + B(2+3x)(1+x) + C(2+3x)$ …. (I)

Put $2+3x=0 \quad \Rightarrow \quad x=-\frac{2}{3}$ in (I)

Then $\quad 4+7\left(-\frac{2}{3}\right)=A\left(1-\frac{2}{3}\right)^2$

$$4-\frac{14}{3}=A\left(-\frac{1}{3}\right)^2$$

$$-\frac{2}{3}=\frac{1}{9}A$$

$\Rightarrow \quad A=\frac{-2}{3}x\frac{9}{1}=-6$

$A=-6$

Put $\quad 1+x=0 \quad \Rightarrow \quad x=-1$ in eq. (I), we get

$4+7(-1)=C(2-3)$

$4-7=C(-1)$

$-3=-C$

$\Rightarrow \quad C=3$

Put $x = 1$ in equation (I), we get

$$4+7(1)=A(1+1)^2+B(2+3(1))(1+1)+C(2+3(1))$$

$$11=4A+10B+5C$$

$$11=4(-6)+10B+5(3)$$

$$11=-24+10B+15$$

$$11+24-15=10B$$

$$20=10B$$

$$B=2$$

Hence, partial fraction is

$$\frac{4+7x}{(2+3x)(1+x)^2}=\frac{-6}{2+3x}+\frac{2}{1+x}+\frac{3}{(1+x)^2}$$

Exercise

Resolve into partial fraction:

Q.1 $\dfrac{x+4}{(x-2)^2(x+1)}$ Q2. $\dfrac{1}{(x+1)(x^2-1)}$

Q.3 $\dfrac{4x^3}{(x+1)^2(x^2-1)}$ Q.4 $\dfrac{2x+1}{(x+3)(x-1)(x+2)^2}$

Q.5 $\dfrac{6x^2-11x-32}{(x+6)(x+1)^2}$ Q.6 $\dfrac{x^2-x-3}{(x-1)^3}$

Q.7 $\dfrac{5x^2+36x-27}{x^4-6x^3+9x^2}$ Q.8 $\dfrac{4x^2-13x}{(x+3)(x-2)^2}$

Q.9 $\dfrac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}$ (Hint: Put $\sin\theta=x$) (Answer: $\dfrac{-1}{\sin\theta+2}+\dfrac{2}{\sin\theta+3}$)

Answer

Q.1 $-\frac{1}{3(x-2)}+\frac{2}{(x-2)^2}+\frac{1}{3(x+1)}$

Q.2 $\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$

Q.3 $\frac{1}{2(x-1)}+\frac{7}{2(x+1)}-\frac{5}{(x+1)^2}+\frac{2}{(x+1)^3}$

Q.4 $\frac{5}{4(x+3)}+\frac{1}{12(x-1)}-\frac{4}{3(x+2)}+\frac{1}{(x+2)^2}$

Q.5 $\frac{10}{x+6}-\frac{4}{x+1}-\frac{3}{(x-1)^2}$

Q.6 $\frac{1}{x-1}+\frac{1}{(x-1)^2}-\frac{3}{(x-1)^3}$

Q.7 $\frac{2}{x}-\frac{3}{x^2}-\frac{2}{(x-3)}+\frac{14}{(x-3)^2}$

Q.8 $\frac{3}{x+3}+\frac{1}{x-2}-\frac{2}{(x-2)^2}$

Type 3: When denominator contain Irreducible quadratic factors.

Irreducible $\longrightarrow$ cannot factories into linear factor

Irreducible Quadratic factor: A Quadratic polynomial ax^2+bx+c is called as irreducible polynomial if $b^2-4ac<0$.

For example: x^2+x+1 is irreducible because $a=1, b=1, c=1$

$$b^2-4ac=(1)^2-4(1)(1)=1-4=-3<0$$

Note: - 1) If $b^2-4ac\geq 0$ then polynomial is reducible (factories into linear factor).

2) Any quadratic factor in the form (x^2+a^2) is always irreducible factor.

Resolution step of partial fraction

$$\frac{P(x)}{(ax^2+bx+c)}=\frac{Ax+B}{ax^2+bx+c}$$

For Example: 1) $\frac{3x-11}{(x+3)(x^2+x+1)}=\frac{A}{(x+3)}+\frac{(Bx+C)}{(x^2+x+1)}$

2) $\frac{x^2+2x-3}{(x+1)^2(x^2+4)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{Cx+D}{(x^2+4)}$

<u>Solved Example</u>

Resolve into partial fractions $\frac{9x-7}{(x+3)(x^2+1)}$

Solution:

$$\frac{9x-7}{(x+3)(x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{x^2+1}$$

Multiplying both sides by L.C.M. i.e., $(x+3)(x^2+1)$, we get

$$9x-7=A(x^2+1)+(Bx+C)(x+3) \qquad \text{(I)}$$

Put $x+3=0 \Rightarrow x=-3$ in Eq. (I), we have

$$9(-3)-7=A((-3)^2+1)+(B(-3)+C)(-3+3)$$

$$-27-7=10A+0$$

$$A=-\frac{34}{10} \Rightarrow \boxed{A=-\frac{17}{5}}$$

$$9x-7=A(x^2+1)+B(x^2+3x)+C(x+3)$$

Comparing the co-efficient of like powers of x on both sides

$$A+B=0$$

$$3B+C=9$$

Putting value of A in Eq. (i)

$$-\frac{17}{5}+B=0 \Rightarrow \boxed{B=\frac{17}{5}}$$

From Eq. (iii)

$$C=9-3B \quad =9-3\left(\frac{17}{4}\right)$$

$$=9-\frac{51}{5} \Rightarrow \boxed{C=-\frac{6}{5}}$$

Hence, required partial fraction is

$$\frac{9x-7}{(x+3)(x^2+1)}=\frac{\frac{-17}{5}}{(x+3)}+\frac{\left(\frac{17}{5}x-\frac{6}{5}\right)}{(x^2+1)}$$

Exercise

Resolve into partial fraction:

Q.1 $\dfrac{x^2+3x-1}{(x-2)(x^2+5)}$ Q.2 $\dfrac{x^2-x+2}{(x+1)(x^2+3)}$

Q.3 $\dfrac{3x+7}{(x+3)(x^2+1)}$ Q.4 $\dfrac{1}{(x^3+1)}$

Answer

Q.1 $\dfrac{1}{x-2}+\dfrac{3}{x^2+5}$ Q.2 $\dfrac{1}{x+1}-\dfrac{1}{x^2+3}$

Q.3 $-\dfrac{1}{5(x+3)}+\dfrac{x+12}{5(x^2+1)}$ Q.4 $\dfrac{1}{3(x+1)}-\dfrac{(x-2)}{3(x^2-x+1)}$

Partial fraction of Type 3 link: https://youtu.be/_YMG5mQTAOg

Partial fraction of Improper fraction

A $Fraction = \frac{P(x)}{Q(x)}$ is called as Improper fraction if degree of P(x) is greater than or equal degree of Q(x).

i.e. degree of $P(x) \geq$ degree of $Q(x)$

For Example: $\frac{x^3-1}{x^2+2x+3}, \frac{x^4}{x^2-1}, \frac{x^2+2x+1}{x^2-5x+6}$ are improper fraction.

Actual Division Method

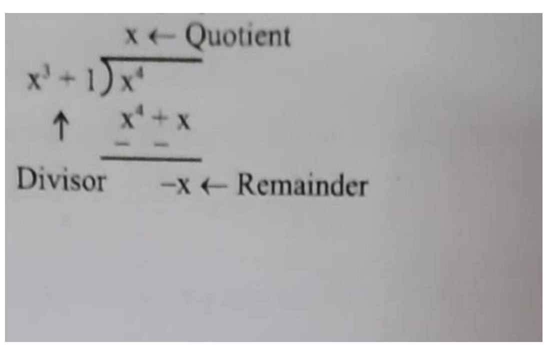
Using this method given improper fraction is expressed into proper fraction. Then this proper fraction is resolved into partial fraction using above method

$$\text{Improper Fraction} = Division + \frac{\text{Re}mainder}{Divisor}$$

$$= Q + \frac{R}{D} \qquad \text{(Q = Quotient or Division)}$$

Note: Fraction $\frac{R}{D}$ is always proper type fraction.

For Example: $\frac{x^4}{x^3+1}$ **Actual Division**



$$\frac{x^4}{x^3+1} = x + \frac{-x}{x^3+1} = x - \frac{x}{x^3+1} \quad \text{---------} \quad (1)$$

Improper Fraction Proper Fraction

Here $\quad \dfrac{x}{x^3+1} = \dfrac{x}{(x+1)(x^2-x+1)}$

Let $\quad \dfrac{x}{(x+1)(x^2-x+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-x+1} \quad \ldots (2)$

$$= \frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}$$

This gives, $x = A(x^2-x+1)+(Bx+C)(x+1)$ which is true for all values of $x \in R$.

When $x = -1$, $\quad -1 = A(1+1+1)+0$

$$-1 = 3A \qquad \therefore \boxed{A = -\frac{1}{3}}$$

When $x = 0$, $A = -\dfrac{1}{3}$

$$0 = -\frac{1}{3}(0-0+1)+(0+C)(0+1)$$

$$\therefore \quad 0 = -\frac{1}{3} + C \qquad \therefore \boxed{C = \frac{1}{3}}$$

When $x = 1$, $A = -\dfrac{1}{3}$, $C = \dfrac{1}{3}$

$$1 = -\frac{1}{3}(1-1+1)+\left(B+\frac{1}{3}\right)(1+1)$$

$$1 = -\frac{1}{3} + 2B + \frac{2}{3}$$

$\therefore$ $$2B = 1 + \frac{1}{3} - \frac{2}{3} = \frac{3+1-2}{3}$$

$\therefore$ $$2B = \frac{2}{3}$$ $\therefore$ $\boxed{B = \frac{1}{3}}$

Substituting for A, B, C in (2), we get

$$\frac{x}{x^3+1} = \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2 - x + 1}$$

$$= \frac{1}{3}\left[\frac{x+1}{x^2-x+1} - \frac{1}{x+1}\right] \quad \dots (3)$$

Substituting (3) in (1), we get the required partial fractions as :

$$\frac{x^4}{x^3+1} = x - \frac{1}{3}\left[\frac{x+1}{x^2-x+1} - \frac{1}{x+1}\right].$$

Improper fraction method link: https://youtu.be/Mke71wzJIGY

Example 2): $\dfrac{x^3+2}{x^2-1}$

$$\begin{array}{r l} & x \leftarrow \text{Quotient} \\ x^2-1 \overline{\big)} & x^3+2 \\ \uparrow & x^3 - x \\ \text{Divisor} & \underline{-\quad +} \\ & x+2 \leftarrow \text{Remainder} \end{array}$$

$$\therefore \quad \frac{x^3+2}{x^2-1} = Q + \frac{R}{D} = x + \underbrace{\frac{x+2}{x^2-1}}_{\text{Proper fraction}} \qquad \ldots (1)$$

where $\dfrac{x+2}{x^2-1} = \dfrac{x+2}{(x-1)(x+1)}$ in which denominator consists of distinct linear factors

Let $\dfrac{x+2}{(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$$= \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

Multiplying both sides by $(x-1)(x+1)$, we get,

$$x+2 = A(x+1)+B(x-1) \qquad \ldots (1) \text{ for all } x \in R$$

When $x = 1$, from (1)

$$3 = A(2) \qquad \therefore \boxed{A = \frac{3}{2}}$$

When $x = -1$, from (1)

$$1 = B(-2) \qquad \therefore \boxed{B = -\frac{1}{2}}$$

$$\therefore \quad \frac{x+2}{x^2-1} = \frac{\frac{3}{2}}{x-1} - \frac{\frac{1}{2}}{x+1}$$

$$= \frac{1}{2}\left[\frac{3}{x-1} - \frac{1}{x+1}\right] \qquad \ldots (2)$$

Using result (2) in (1),

$$\frac{x^3+2}{x^2-1} = x + \frac{1}{2}\left[\frac{3}{x-1} - \frac{1}{x+1}\right]$$

This is required partial fraction.

Exercise

Resolve following fraction into partial fraction

a) $\dfrac{x^3-1}{(x+1)(x+2)}$ b) $\dfrac{x^4}{x^2-1}$ c) $\dfrac{x^2-2x+2}{x^2+2x-3}$

Answer

a) $x-3+\dfrac{9}{x+2}-\dfrac{2}{x+1}$ b) $x^2+1-\dfrac{\frac{1}{2}}{x+1}+\dfrac{\frac{1}{2}}{x-1}$ c) $1+\dfrac{\frac{1}{4}}{x-1}-\dfrac{\frac{17}{4}}{x+3}$