1. Units and Measurements

1.1. Physical Quantities

The quantities which can be measured are called physical quantities

Physical quantities
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1. Fundamental physical quantities
- do not depend on any other quantity for their measurements
- there are 7 fundamental quantities

2. Derived physical quantities
- depend on one or more fundamental quantities for their measurements
- there are many derived quantities

1] Fundamental Physical Quantities

They do not depend on any other quantity for their measurement. Seven fundamental quantities are as follows:

	Fundamental quantity	Unit	Symbol
1.	Length	meter	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Amount of substance	mole	mol
6.	Electric current	ampere	A
7.	Luminous intensity	candela	cd

Fundamental Units

The units of fundamental quantities are called fundamental units

eg. m, kg, A etc

Supplementary Physical Quantities

There are 2 supplementary quantities & their units are called supplementary units

	Supplementary Physical quantity	Unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

2) Derived Physical Quantities

Derived Physical Quantities depend on one or more physical quantities for their measurements (fundamental as well as derived)

eg. Volume, density, velocity

Volume depends on length, breadth, height

density depends on mass & volm

velocity depends on distance & time.

Derived Units

The units of derived quantities which depend on fundamental units for their measurements are called derived units

eg. m/s^2, m^3, kg/m^3

Velocity is a derived quantity

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

Unit of velocity is m/s
i.e unit of velocity depends on unit of distance (m) & unit of time (s).

Derived quantities & their units

Derived Physical quantity	Unit	Symbol
1. Area	square meter	m^2
2. Volume	cubic meter	m^3
3. velocity	meter/second	m/s
4. acceleration	meter/square second	m/s^2
5. Force	newton	N
6. Pressure	newton/square meter	N/m^2
7. Work/Energy	joule	J
8. Power	watt	W
9. Capacitance	farad	F
10 Inductance	henry	H
11. Momentum	kilogram.meter/second	kg m/s
12 Surface tension	newton/meter	N/m
13 Density	kilogram/cubic meter	kg/m^3
14 Electric charge	Coulomb	C
15. Electric resistance	ohm	Ω
16 Electric Potential	volt	V
17 Magnetic flux	weber	Wb
18 Specific heat	joule per kilogram degree kelvin	$J/kg°K$
19. Thermal conductivity	watt/meter degree Kelvin	$W/m°K$
20. angular velocity	radian per second	rad/s
21. angular accn	radian per square second	rad/s^2

22.	Viscosity	Newton second per square meter	Ns/m^2
23.	Electric field	volt per meter	V/m
24	Magnetic flux density	tesla	Wb/m^2 or T
25	Magnetic field strength	ampere per meter	A/m
26.	Frequency	Hertz	Hz

System of Units

Unit:
The reference standard used for measurement of a physical quantity is called unit
eg. mass of a body = 3 kg
Here kg is the standard used for measurement.

Different System of units

1) CGS system
centimeter - gram - second system
length - centimeter
mass - gram
time - second

2) MKS system
meter - kilogram - second system
length - meter
mass - kilogram
time - second

3) FPS system

Foot-pound-second system

length - foot

mass - pound

time - second

This system is not widely used

Use of above 3 systems of units became inconvenient for exchanging information across different parts of the world.

So a common system of units called Systeme Internationale d' Units (SI) units was accepted.

4) SI units

This is universally accepted system of units.

Almost all the units which we use are SI units. These are accepted all over the world.

1.2 Scalar and Vector Physical Quantities

Scalar Quantities

The physical quantities which have only magnitude & which do not depend on direction are called scalar quantities

eg. speed, distance, time, mass

Vector Quantities

The physical quantities which have magnitude as well as direction are called vector quantities

eg. velocity, acceleration, force, displacement.
Vector quantities are denoted by arrow over the symbol of unit ($\vec{v}$, $\vec{a}$, $\vec{F}$)

1.3 Dimensions, dimensional formula, applications of dimensional analysis; correctness of physical eqn, conversion factor for interconversion of units in different system of units.

Dimensions:
The powers to which fundamental units of mass, length, time etc must be raised to obtain the unit of a given physical quantity.

Dimensional formula
An expression which gives relation between derived units & fundamental units in terms of dimensions is called dimensional formula.
eg. Dimensional equation for speed.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{[L^1M^0T^0]}{[L^0M^0T^1]}$$

$$[S] = [L^1M^0T^{-1}]$$

Application of dimensional formula analysis:

1. To find correctness of physical equation
The dimensions of all the terms on the two sides of a physical equation must be same.

This is called "Principle of homogeneity of dimensions".

eg. $s = ut + \frac{1}{2}at^2$

$s = [L^1M^0T^0]$
$ut = [L^1M^0T^0]$
$at^2 = [L^1M^0T^0]$

$\frac{1}{2}$ has no dimensions

As all the terms have same dimensions the equation is dimensionally correct.

2. Conversion from one system to another
To find conversion factor between the units of force
i.e newton in SI system to dyne in CGS system

let 1 newton = x dyne

$$[L_1^1 M_1^1 T_1^{-2}] = x\,[L_2^1 M_2^1 T_2^{-2}]$$

$$x = \frac{[L_1^1 M_1^1 T_1^{-2}]}{[L_2^1 M_2^1 T_2^{-2}]}$$

Suffix 1 indicates SI system
Suffix 2 indicates CGS system

$$x = \left(\frac{m}{cm}\right)^1 \left(\frac{kg}{g}\right)^1 \left(\frac{s}{s}\right)^{-2}$$

$$= \left(\frac{10^2\,cm}{cm}\right)^1 \left(\frac{10^3\,g}{g}\right)^1 (1)^{-2}$$

$$= 10^2 \cdot 10^3$$

$$x = 10^5$$

$$1\,N = 10^5 \text{ dyne}$$

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3. Establish relationship between related physical quantities

$$T = k l^x g^y \quad \text{--- (1)}$$

K = constant

$[T] = [L^0 M^0 T^1]$

$[l] = [L^1 M^0 T^0]$

$[g] = [L^1 M^0 T^{-2}]$

Eqn (1) becomes

$$[L^0 M^0 T^1] = K [L^1 M^0 T^0]^x [L^1 M^0 T^{-2}]^y$$

$$[L^0 M^0 T^1] = K [L^{x+y} M^0 T^{-2y}]$$

$0 = x + y$ & $1 = -2y$

$x = +\frac{1}{2}$ $y = -\frac{1}{2}$

$$T = k l^{1/2} g^{-1/2}$$

$$T = k\sqrt{\frac{l}{g}} \quad (K = 2\pi)$$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

* Dimensional constants

Constants which possess dimensions

eg. Planck's constant (h)

Dimensional Variables

Physical quantities that possess dimensions but do not have fixed value are called dimensional variables

eg. displacement, force, velocity

Dimensionless Quantities

The physical quantities which do not possess dimensions are called dimensionless quantities eg. angle, strain

1.4 Errors, types of errors: instrumental, systematic & random error, estimation of errors: absolute, relative & percentage error, significant figures

Errors

Defects in measurement of physical quantity can lead to errors & mistakes

The measured value of a physical quantity is usually different from its true value.

Uncertainty in the measurement of a physical quantity is called error.

Types of error

1) Instrumental Errors

These are caused due to faulty construction of calibration of instruments

eg. if thermometer is not graduated properly the temperature measured by such a thermometer will differ from its value

How to minimise instrumental error?

By taking same measurement with different accurated instruments.

2) Systematic errors (persistent errors)

Occur due to defective setting of an instrument.

eg. Pointer of an ammeter is not pivoted exactly at zero of the scale it will

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not point to zero when no current is passing through it.

3) Random Errors:

Errors may occur due to different factors like change in temperature, pressure or fluctuation in voltage while the expt is being performed.

Such errors cannot be eliminated but can be minimized by

1) Taking large magnitude of quantity to be measured.

2) Taking large no. of readings & calculating their mean values.

3) Using an instrument whose least count is as small as possible.

* Least count error

It is associated with resolution (scale of the instrument). When the least count is not sufficiently small this error occurs.

Maximum possible error = least count error.

What is least count?

Smallest value that can be measured by an instrument.

eg. Ruler (scale) can measure length which is 0.1 cm. As this is smallest length that can be measured with a ruler it is called least count of ruler.

* Accidental errors

These errors give very high or very low results.

Measurements involving these errors are not added in calculation.

* Personal error (Human error)
  Due to fault of an observer taking readings
  eg error made in counting number of oscillations while measuring period of simple pendulum.

Calculating errors in measurement
  The difference between true value & measured value of a physical quantity is called error which may be positive or negative.
  let $R_1, R_2, R_3 \dots R_n$ be measured values of physical quantities in several measurements then their mean is considered as true value of that physical quantity.
  True value $R_0 = R_{mean}$

$$R_0 = \frac{R_1 + R_2 + R_3 + \dots R_n}{n}$$

Absolute error
  The magnitude of the difference between mean value & each individual value is called absolute error

$$|\Delta R_1| = |R_0 - R_1|$$
$$|\Delta R_2| = |R_0 - R_2|$$
$$\vdots$$
$$|\Delta R_n| = |R_0 - R_n|$$

Mean absolute error
The arithmetic mean of the absolute errors is known as mean absolute error

$$\Delta R_{mean} = \frac{|\Delta R_1| + |\Delta R_2| + \dots + |\Delta R_n|}{n}$$

Relative error
The ratio of mean absolute error to mean value

$$\text{Relative error} = \frac{\text{mean absolute error}}{\text{mean value}}$$

$$\text{Relative error} = \frac{\Delta R_{mean}}{R_{mean}} = \frac{\Delta R_{mean}}{R_0}$$

Percentage error
Relative error multiplied by 100 is called percentage error.

$$\text{Percentage error} = \frac{\Delta R_{mean}}{R_{mean}} \times 100$$

$$\%\ \text{error} = \frac{\Delta R_{mean}}{R_0} \times 100$$

Combination of errors
1) If a quantity z is expressed as
$z = x + y$ or $z = x - y$
then $\Delta z = \Delta x + \Delta y$
When two quantities are added or subtracted the absolute error in the final result is the sum of absolute errors in individual quantities.

2) If z is expressed as

$z = x \times y$ or $z = \frac{x}{y}$

Then maximum fractional error in z

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

When two quantities are multiplied or divided by the relative error in the result is sum of relative errors in the multipliers

3) If $z = x^m y^n p^l$ then maximum fractional error is given by,

$$\frac{\Delta z}{z} = m + n\frac{\Delta y}{y} + l\frac{\Delta p}{p}$$

Significant figures

It can be defined as a figure which is of some significance but it does not necessarily denote a certainty.

eg. if reading with vernier calliper is 2.53 cm the 2 & 5 are reliable digits & 3 is uncertain digit.

Thus measured value has 3 significant figures.

Rules for determining the number of significant figures:

1) All non-zero digits significant
2) All zeroes between non-zero digits are significant eg. 2003
3) All zeros to the right side of last non-zero digit are not significant in a number without decimal point.

4) All zeros to the right side of decimal point & to left side of non-zero digit are not significant.
5) All zero's to the right side of decimal point & to right side of non-zero digit are significant.
6) In addition & subtraction, least decimal place among the values operated must be retained in the final result.
7) In multiplication & division the result should be expressed with the least number of significant figures.
8) If a number is greater than 1 without any decimal, trailing zeros are not significant, if number is with decimal trailing zeros are significant.

Rounding off the number.
Process of omitting the insignificant digits & retaining only desired number of significant digits including required modification to last significant digit is called rounding off the number.

Rounding
While rounding to a certain place value all digits to the right to that place are dropped.
If first dropped digit is 0, 1, 2, 3, 4 then the least significant digit kept is not changed.
This is called rounding down

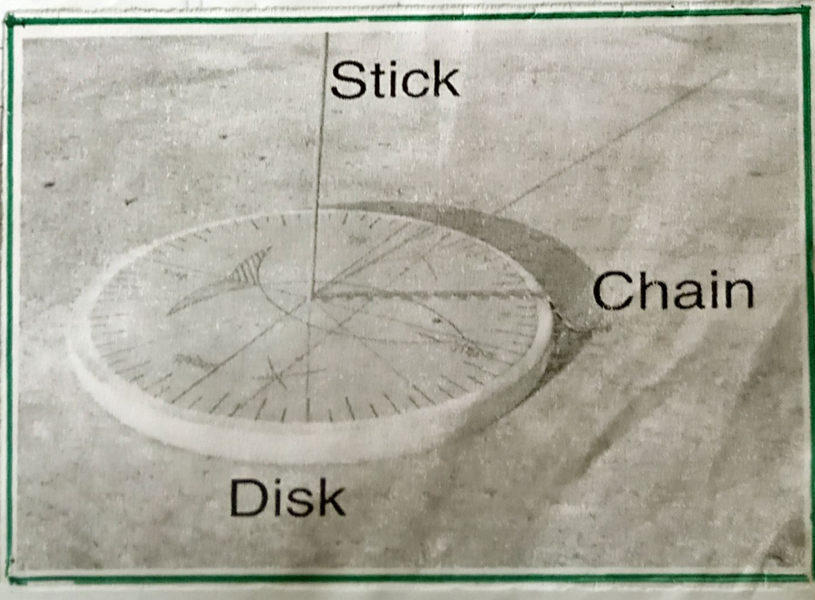
If first dropped digit is 5, 6, 7, 8 or 9 then least significant digit kept is increased by 1. This is called rounding up.
eg. rounding down 5.384 → 5.38
rounding up 5.3851 → 5.39

Significant figures

eg. 11.080 = 5
0.0196 = 3
200.0109 = 7
0.0005300 = 4
1.6×10^{-19} = 2

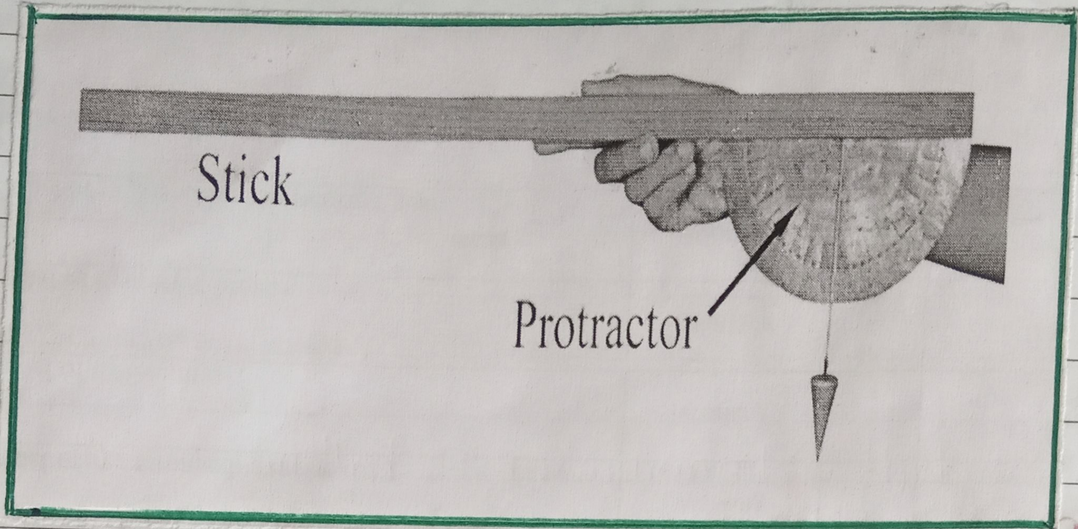
1.5. Ancient astronomical instruments : Chakra, Dhanuryantra, Yasti and Phalaka yantra

→ Chakra:
This instrument is a type of protractor used for angular marking of land & angular positioning of cities. It was used to measure time & some astrological parameters. It is also known as charkra yantra or disk machine



Dhanu Yantra

Dhanu is a semicircular disk with angular graduations & a stick pivoted at the center, it is a type of protractor with a plumb bod arrangement.



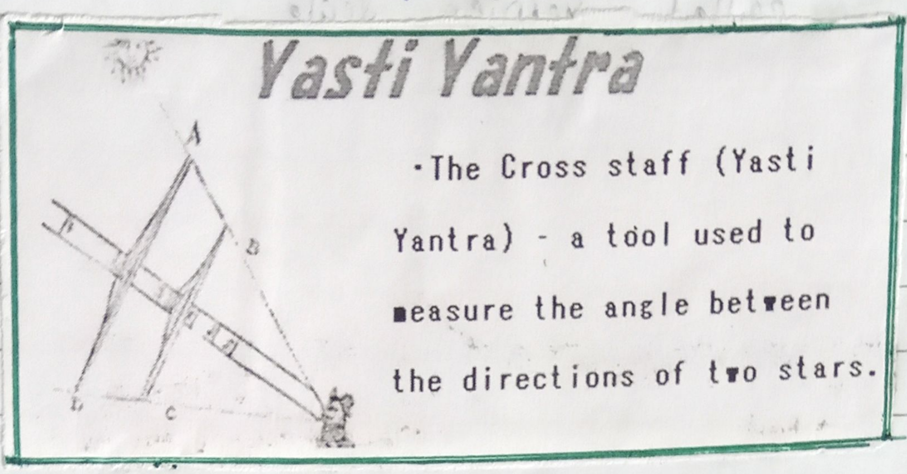
Yasti Yantra

Stick machine known as Yasti yantra of Bhaskaracharya. It was used to calculate the height of tree & terrestrial objects.

This machine has been also described by mathematicians like Lalla, Shripati & others but Bhaskaracharya has developed his unique method to calculate the height of terristrial objects like trees & mountains.

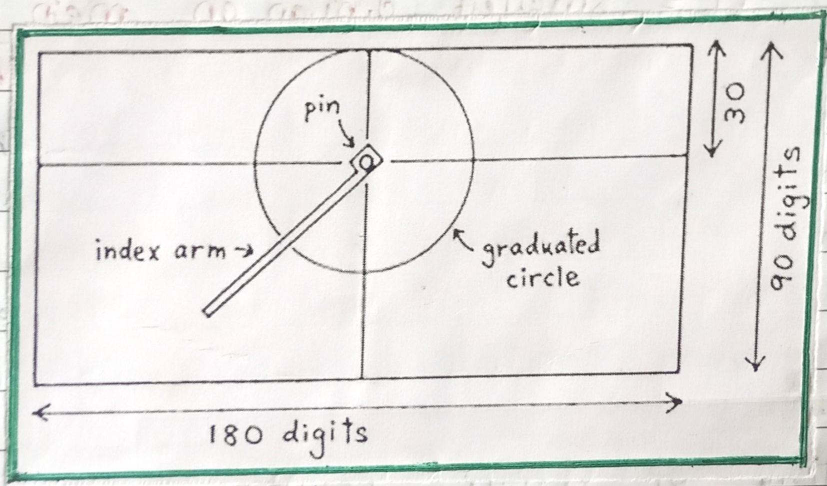
This machine has a stick pivoted to a board. To use this machine one has to focus the top & bottom of the object & draw two lines on the board. With these two lines two triangles are contructed & based on their ~~position~~ proportion height of the object is calculated. There is no need to measure the distance between the object

& observer. This instrument was also used for land survey.



Phalaka Yantra

A device called Phalaka Yantra was used to determine time from sun's altitude. It was invented by mathematician & astronomer Bhaskara I. It consisted of rectangular board with a pin & an index arm.



Applications of vernier callipers & screw gauge

Vernier Caltipers

Vernier Caltiper is an instrument used to measure dimensions such as inner diameter, outer diameter, height, depth, thickness of an object. Vernier Caliper consists of main scale calibrated in centimeters & millimeters on lower edge of the sliding window 10 equal division are callibrated in 9 mm length. This scale is called vernier scale.

The smallest measurement that can be taken using measuring instrument is called its least count.

$$LC = \frac{\text{smallest division on main scale (m)}}{\text{total no. of divisions on vernier scale (n)}}$$

For the measurement of the objects having dimensions less than 1 mm or to measure the dimensions of curved surfaces, normal scales cannot be used. Such kind of measurements are possible using instruments like vernier caliper.

Application of vernier calipers:

1. Measuring length

In manufacturing precise measurements are required for production of parts & components. Vernier calipers are used to measure the length of objects with great precision.

2. Measuring width & thickness
For precise measurements in electronic components or in measurement of metal sheet we use vernier caliper to measure width & thickness.

3. Measuring inside & outside diameter (inner & outer diameter)
In the production of bearings or other mechanical components we require precise measurement of inner & outer diameter. So we use vernier calliper for this purpose.

4. Checking the depth of holes:
In the production of threaded holes for bolts it is important to know the depth of a hole so vernier calipers are used to check depth of holes.

5. Checking the thickness of materials
In the production of packaging material vernier calipers can be used to check thickness of the materials such as paper, plastic & metal sheets.

Because of all above uses vernier calipers are used in education sector, science labs, medical industry, steel sector, aerospace sector.

Micrometer Screw-gauge
In industry there is need to measure thickness, diameter of objects with utmost precision. Micrometer screw gauge is used to measure dimensions of objects less than 0.[illegible] that cannot be measured by other instrume[illegible]
Micrometer consists of two scales i.e ma[illegible] scale & circular scale.

Pitch (p): It is the distance between two consecutive threads of the screw. The screw moves forward or backward through a distance equal to its pitch when one complete rotation is given to it.

$$L.C = \frac{\text{pitch of the screw (p)}}{\text{no. of divisions on circular scale (n)}}$$

Applications of screw gauge:
1) With an accuracy of 0.001 cm, a screw gauge is used to measure the diameter of circular objects (wires).
2) The radius of wires & other circular objects can be found using a screw gauge.
3) A screw gauge is also used to determine the thickness of a piece of paper.
4) Metal sheet & glass slab thickness can be identified using a screw gauge.
5) Uniform thickness of any shape can be found using a screw gauge.

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* Requirement of good unit :

1) It should be easily available
2) It should be invariable (should not change with space & time).
3) It should be universally accepted.
4) It should be reproducible & not perishable

Dimensions of some derived quantities

1.	Area	$A = L^2$	$[L^2M^0T^0]$
2.	Volume	$V = L^3$	$[L^3M^0T^0]$
3.	Density	$d = \frac{M}{V}$	$[L^{-3}M^1T^0]$
4.	Velocity	$v = \frac{s}{t}$	$[L^1M^0T^{-1}]$
5.	Acceleration	$a = \frac{v}{t}$	$[L^1M^0T^{-2}]$
6	Momentum	$p = mv$	$[L^1M^1T^{-1}]$
7.	Force	$F = ma$	$[L^1M^1T^{-2}]$
8	Impulse	$J = Ft$	$[L^1M^1T^{-1}]$
9.	Work	$W = Fs$	$[L^2M^1T^{-2}]$
10.	Kinetic energy	$KE = \frac{1}{2}mv^2$	$[L^2M^1T^{-2}]$

11. Potential energy $PE = mgh$ $[L^2 M^1 T^{-2}]$

12. Power $P = \frac{W}{t}$ $[L^2 M^1 T^{-3}]$

13. Pressure $P = \frac{F}{A}$ $[L^{-1} M^1 T^{-2}]$