TRIGONOMETRY

Important Formulae

Basic Identity:-

1) $\sin\theta = \dfrac{1}{\cos ec\theta}$ 2) $\cos\theta = \dfrac{1}{\sec\theta}$ 3) $\tan\theta = \dfrac{1}{\cot\theta}$

4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

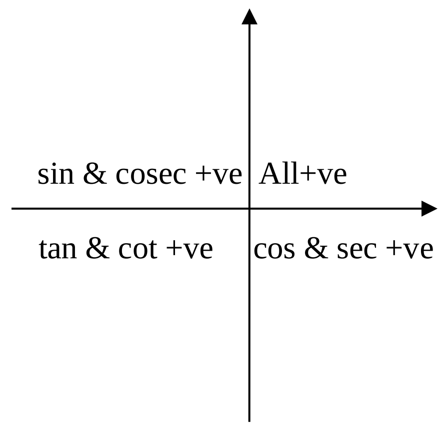
Fundamental Identity:-

1) $\sin^2\theta + \cos^2\theta = 1$

2) $1 + \tan^2\theta = \sec^2\theta$

3) $1 + \cot^2\theta = \cos ec^2\theta$

Sign Convention:-



Standard Value Table:-

$\theta \rightarrow$	0^0	$30^0 \left(\frac{\pi}{6}\right)$	$45^0 \left(\frac{\pi}{4}\right)$	$60^0 \left(\frac{\pi}{3}\right)$	$90^0 \left(\frac{\pi}{2}\right)$	$180^0(\pi)$	$360^0(2\pi)$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	1

cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	∞

Compound Angle:- It is a angle obtained by addition or subtraction of given angle.

If A & B are any angle then A+B, A-B are compound angle.

Compound Angle Formulae:-

1) $\sin(A+B) = \sin A.\cos B + \cos A.\sin B$

2) $\sin(A-B) = \sin A.\cos B - \cos A.\sin B$

3) $\cos(A+B) = \cos A.\cos B - \sin A.\sin B$

4) $\cos(A-B) = \cos A.\cos B + \sin A.\sin B$

5) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$

6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A.\tan B}$

Example

1) Without using calculator, find the value of

a) sin 15° b) cos 75°

Solution:

a) As 15° = 45° – 30°

$\therefore$ sin 15 = sin (45° – 30°)

Using formula sin (A – B) = sin A · cos B – cos A sin B

$= \sin 45° \cdot \cos 30° - \cos 45° \cdot \sin 30°$

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$ and $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$

$$= \left(\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}.\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\sin 15 = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

b) As $75° = 45° + 30°$

$\therefore\ \cos 75° = \cos(45° + 30°)$

By using Formula $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$$= \cos45° \cdot \cos30° - \sin45° \cdot \sin30°$$

$$= \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\cos 15 = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

2) If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ Find $\tan(A + B)$

Solution : Given that $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

By compound formula, $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \, . \tan B}$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

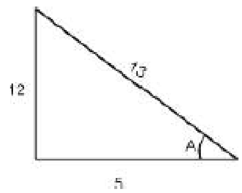
$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}}$$

$$= \frac{\frac{3+2}{6}}{\frac{6-1}{6}} = \frac{5}{5}$$

$$\tan(A + B) = 1$$

4) If $\angle A$ and $\angle B$ are both obtuse angles and $\sin A = \frac{12}{13}$ and $\cos B = \frac{-4}{5}$. Find $\sin(A + B)$.

► **Solution :** Given $\sin A = \frac{12}{13}$



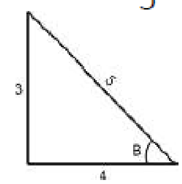
$\cos A = \frac{5}{13}$

$\therefore$ A is obtuse (More than 90º and less than 180º)

A is the second quadrant, cos A is negative.

$\cos A = \frac{-5}{13}$

Given $\cos B = \frac{-4}{5}$



$\sin B = \frac{3}{5}$

$\therefore$ B is obtuse (More than 90º and less than 180º)

B is the second quadrant, sin B is Positive.

$\sin B = \frac{3}{5}$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \left(\frac{12}{13}\right) \times \left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right) \times \left(\frac{3}{5}\right)$$

$$= \frac{-48}{65} + \frac{-15}{65}$$

$$= \frac{-48-15}{65}$$

$$\sin(A + B) = \frac{-63}{65}$$

Assignment: - 1

1) If $\cos A = \frac{-3}{5}$ and $\sin B = \frac{20}{29}$, where A & B are angles in third & second quadrant respectively. Find tan(A+B).

2) If A & B are obtuse angle such that, $\sin A = \frac{5}{13}$ and $\cos B = \frac{-4}{5}$. Find tan(A+B).

3) If $A + B = \frac{\pi}{4}$ .show that $(1 + \tan A)(1 + \tan B) = 2$

4) If $\tan(x + y) = \frac{3}{4}$ and $\tan(x - y) = \frac{8}{15}$ .show that $\tan 2x = \frac{77}{36}$.

5) Prove that, $\frac{\cot A - \cot 2A}{\cot A + \cot 2A} = \frac{\sin A}{\sin 3A}$

6) Show that, $\sin(A + B).\sin(A - B) = \sin^2 A - \sin^2 B$

7) Prove that, $\frac{\sin(A - B)}{\cos A.\cos B} + \frac{\sin(B - C)}{\cos B.\cos C} + \frac{\sin(C - A)}{\cos C.\cos A} = 0$

8) If $\cos A = \frac{1}{7}$, $\cos B = \frac{13}{14}$, A and B are positive acute angles. Find sin(A+B), cos(A+B).

9) Prove that, $\frac{\sin(A - B)}{\cos(A - B)} = \frac{\cot A + \cot B}{1 + \cot A.\cot B}$

10) If $\tan(x + y) = \frac{1}{2}$ and $\tan(x - y) = \frac{1}{3}$ then, find 1) tan2x 2) tan2y

<u>Allied Angle</u>

Definition:- If the sum or difference of the two angle is either zero or integral multiple of 90^0 i.e. $\frac{n\pi}{2}$, where 'n' is integer, then these two angles are called as 'allied angle'.

E.g. if θ is any angle then its allied angles are $-\theta, \left(\frac{\pi}{2} - \theta\right), \left(\frac{\pi}{2} + \theta\right), (\pi - \theta), (\pi + \theta), - - -$ etc.

Useful Formulae:- 1) $\sin(-\theta) = -\sin\theta$ 2) $\cos(-\theta) = \cos\theta$

3) $\sin(n\pi) = 0$ 4) $\cos(n\pi) = 1$, if 'n' is even.

= -1, if 'n' is odd.

5) $\sin\left(\frac{n\pi}{2} \pm \theta\right) = \pm \sin\theta$, if 'n' is even integer (sign is depend on quadrant)

$= \pm \cos\theta$, if 'n' is odd integer.

6) $\cos\left(\frac{n\pi}{2} \pm \theta\right) = \pm \cos\theta$, if 'n' is even integer

$= \pm \sin\theta$, if 'n' is odd integer

Significance:- Any angle of large measure can be express in term of standard angle.

For example, $570^0 = 6 \times 90^0 + 30^0 = \frac{6\pi}{2} + \frac{\pi}{6}$

1) Without using calculator, find the value of

i) $\sin(-765°)$ ii) $\sec(3660°)$

Solution:

i) $\sin(-765°)$

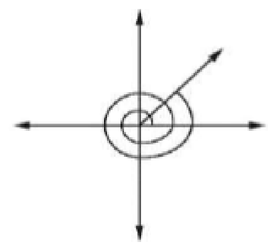
$\sin(-\theta) = -\sin\theta$

$= -\sin(765°)$

$= -\sin(8 \times 90° + 45°)$

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}}$



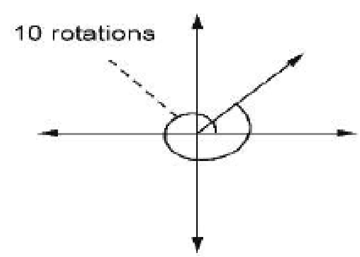
∴ 765° lies in first quadrant, where sin is +ve
n = 8 (even) function remains same.

ii) $\sec(3660°)$

$= \sec(40 \times 90° + 60°)$

$= +\sec 60°$

$= 2$



3660 lies in I^{st} quadrant, where sec. is positive.
n = 40 (even) function remains same.

2)Without using calculator, find the value of
sin (420°) cos (390°) + cos (– 300°) sin (– 330)

Solution: Given sin 420° cos 390° + cos (– 300°) sin (– 330°)

$$\sin(420^\circ) = \sin(4 \times 90^\circ + 60^\circ)$$
$$= \sin 60^\circ$$
$$= \frac{\sqrt{3}}{2}$$

$$\cos(390^\circ) = \cos(360^\circ + 30^\circ)$$
$$= \cos(4 \times 90^\circ + 30^\circ)$$
$$= \cos 30^\circ$$
$$= \frac{\sqrt{3}}{2}$$

$$\cos(-300^\circ) = \cos(300^\circ)$$
$$= \cos(3 \times 90^\circ + 30^\circ)$$
$$= \sin 30^\circ$$
$$= \frac{1}{2}$$

$$\sin(-330^\circ) = -\sin(330^\circ)$$
$$= -\sin(360^\circ - 30^\circ)$$
$$= -\sin(4 \times 90^\circ - 30^\circ)$$
$$= -(-\sin 30^\circ)$$
$$= \sin 30^\circ$$
$$= \frac{1}{2}$$

Given sin 420° cos 390° + cos (– 300°) sin (– 330°)

$$= \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2} + \frac{1}{2}\,\frac{1}{2}$$
$$= \frac{3}{4} + \frac{1}{4}$$
$$= \frac{3+1}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

Assignment: - 2

1) Find value of a) $\sin\left(120^0\right)$ b) $\cos\left(-390^0\right)$ c) $\sec\left(3660^0\right)$

2) Without using calculator, find value of : $\cos\left(570^0\right).\sin\left(510^0\right)+\sin\left(-330^0\right).\cos\left(-390^0\right)$

3) Prove that, $\dfrac{\tan 420^0+\tan 300^0}{1-\tan 420^0.\tan 660^0}=0$

4) Without using calculator, prove that: $\sin 240^0\cos 390^0+\cos\left(-300^0\right)\sin\left(-330^0\right)=1$

5) Using compound angle formulae, find value of $\sin 75^0, \sec 75^0$ & $\tan 120^0$.

6) Without using calculator, find value of :

a) $\cos 15^0$ b) $\sin 75^0$ c) $\sin(-135^0)$ d) $\sec(-660^0)$

7) Without using calculator, find value of :

$\sin(-690^0).\cos(300^0)+\cos(750^0).\sin(240^0)$

Multiple angle formulae

Definition:- If θ is any angle then $2\theta, 3\theta, 4\theta, -----etc$ are called 'multiple angle'

Useful Formulae: - **(Multiple angle)**

1) $\sin 2\theta = 2\sin\theta.\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

2) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1-2\sin^2\theta = 2\cos^2\theta - 1 = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(From this, $1+\cos 2\theta = 2\cos^2\theta$, $1-\cos 2\theta = 2\sin^2\theta$)

3) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

4) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

5) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

6) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Half Angle formulae

Trigonometric ratios of any angle in the form of half angle

1) $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)$

$$\sin\theta = \frac{\tan\theta}{1+\tan^2\theta}$$

2) $\cos\theta = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$

$$\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\cos\theta = \frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$$

3) $\tan\theta = \dfrac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}$

Example

1) If $\sin A = 0.4$, Find $\sin 3A$.

Solution : Given $\sin A = 0.4$

$$\begin{aligned} \text{Now, } \sin 3A &= 3\sin A - 4\sin^3 A \\ &= 3(0.4) - 4(0.4)^3 \\ &= 1.2 - 0.256 \\ &= 0.944 \end{aligned}$$

$$\therefore \quad \sin 3A = 0.944$$

2) If $\cos A = \frac{1}{2}$, find the value of $\cos 3A$.

Solution: Given that $\cos A = \frac{1}{2}$

Now, $\cos 3A = 4\cos^3 A - 3\cos A$

$$= 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)$$

$$= 4\left(\frac{1}{8}\right) - \frac{3}{2}$$

$$= \frac{1}{2} - \frac{3}{2}$$

$$= \frac{-2}{2}$$

$\therefore \cos 3A = -1$

3) Prove that : $\dfrac{\sin 4\theta + \sin 2\theta}{1 + \cos 2\theta + \cos 4\theta} = \tan 2\theta$

Solution: L.H.S. $= \dfrac{\sin 4\theta + \sin 2\theta}{1 + \cos 2\theta + \cos 4\theta}$

Use Formula $\sin 4\theta = 2\sin 2\theta \cos 2\theta$

And $1 + \cos 4\theta = 2\cos^2 2\theta$

$$\text{L.H.S.} = \frac{2\sin 2\theta \cos 2\theta + \sin 2\theta}{2\cos^2 2\theta + \cos 2\theta}$$

$$= \frac{\sin 2\theta(2\cos 2\theta + 1)}{\cos 2\theta\,(2\cos 2\theta + 1)}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

= R.H.S.

<u>Example on Half angle formulae</u>

1) If $\tan\left(\frac{A}{2}\right) = \frac{1}{\sqrt{3}}$, find the value of $cosA$.

Solution: We know that

$$cosA. = \frac{1-\tan^2\left(\frac{A}{2}\right)}{1+\tan^2\left(\frac{A}{2}\right)}$$

$$= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$$

$$= \frac{2}{4}$$

$$cosA = \frac{1}{2}$$

2) Prove that $\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1-\tan\left(\frac{\theta}{2}\right)}{1+\tan\left(\frac{\theta}{2}\right)}$

Solution: L.H.S. $= \dfrac{\cos\theta}{1-\sin\theta}$

$$\cos\theta = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$$

$$1 = \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)$$

$$\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)$$

$$= \frac{\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right) - 2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)}$$

$$= \frac{\left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)\left(\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right)}{\left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)^2}$$

$$= \frac{\left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)\left(\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right)}{\left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)\left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)}$$

$$= \frac{\left(\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right)}{\left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right)}$$

Divide numerator and denominator by $\cos\left(\frac{\theta}{2}\right)$

$$= \frac{\dfrac{\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}}{\dfrac{\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}}$$

$$= \frac{\dfrac{\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} + \dfrac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}}{\dfrac{\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} - \dfrac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}}$$

$$\therefore \frac{\cos\theta}{1-\sin\theta} = \frac{1-\tan\left(\frac{\theta}{2}\right)}{1+\tan\left(\frac{\theta}{2}\right)} = \text{R.H.S.}$$

Assignment: - 3

1) If $\sin A = \frac{1}{2}$, find $\sin 3A$.

2) Prove that, $\dfrac{\sin 4\theta + \sin 2\theta}{1+\cos 4\theta + \cos 2\theta} = \tan 2\theta$

3) Prove that, $\dfrac{1+\sin\theta - \cos\theta}{1+\sin\theta + \cos\theta} = \tan\left(\dfrac{\theta}{2}\right)$

4) Prove that, $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

5) Prove that, $\dfrac{1-\cos 2\theta - \sin\theta}{\sin 2\theta - \cos\theta} = \tan\theta$

6) If $A = 30^0$, verify that a) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$ b) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

7) Prove that, a) $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$ b) $\dfrac{\sin 4A}{\sin A} = 8\cos^3 A - 4\cos A$

c) $\sqrt{\dfrac{1-\sin 2\theta}{1+\sin 2\theta}} = \tan\left(\dfrac{\pi}{4}-\theta\right)$ d) $\cot 2\theta + \cos ec 2\theta = \cot\theta$

Factorization & Defactorization Formulae

1) Factorization Formulae:- a) $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right).\cos\left(\dfrac{C-D}{2}\right)$

b) $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right).\sin\left(\dfrac{C-D}{2}\right)$

c) $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right).\cos\left(\dfrac{C-D}{2}\right)$

d) $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right).\sin\left(\dfrac{D-C}{2}\right)$

Note:- This formulae are used to convert addition or subtraction into multiplication.

E.g. $\sin 2x + \sin 3x = 2\sin\left(\dfrac{5x}{2}\right).\cos\left(\dfrac{x}{2}\right)$

DE factorization Formulae:-1) $2SinA.CosB = Sin(A+B) + Sin(A-B)$

2) $2CosA.SinB = Sin(A+B) - Sin(A-B)$

3) $2CosA.CosB = Cos(A+B) + Cos(A-B)$

4) $2SinA.SinB = Cos(A-B) - Cos(A+B)$

Remark:- DE factorization formulae are used to convert product(multiplication) in to sum or difference.

Example

1) Evaluate $2\cos 75° \cdot \cos 15°$

Solution:

$$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$2\cos 75° \cdot \cos 15° = \cos(75° + 15°) + \cos(75° - 15°)$$

$$\Rightarrow 2\cos 75° \cdot \cos 15° = \cos(90°) + \cos(60°)$$

$$\Rightarrow 2\cos 75° \cdot \cos 15° = 0 + \frac{1}{2}$$

$$\Rightarrow 2\cos 75° \cdot \cos 15° = \frac{1}{2}$$

2. Prove that: $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \tan 3A \,.\, \sin A$

Solution:

L.H.S. $= \frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A}$

$$= \frac{(\cos 6A + \cos 2A) + 2\cos 4A}{(\cos 5A + \cos A) + 2\cos 3A}$$

$$= \frac{2\cos\left(\frac{6A + 2A}{2}\right) \cdot \cos\left(\frac{6A - 2A}{2}\right) + 2\cos 4A}{2\cos\left(\frac{5A + A}{2}\right) \cdot \cos\left(\frac{5A - A}{2}\right) + 2\cos 3A}$$

$$= \frac{2\cos 4A \cdot \cos 2A + 2\cos 4A}{2\cos 3A \cdot \cos 2A + 2\cos 3A}$$

$$= \frac{2\cos 4A(\cos 2A + 1)}{2\cos 3A(\cos 2A + 1)}$$

$$= \frac{\cos 4A}{\cos 3A} = \frac{\cos(3A + A)}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos A - \sin 3A \cdot \sin A}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos A}{\cos 3A} - \frac{\sin 3A \cdot \sin A}{\cos 3A}$$

$$= \cos A - \frac{\sin 3A}{\cos 3A} \cdot \sin A$$

$$= \cos A - \tan 3A \cdot \sin A = \text{R.H.S.}$$

3) *Prove that* $\dfrac{\sin 8\theta \cdot \cos\theta - \cos 3\theta \cdot \sin 6\theta}{\cos 2\theta \cdot \cos\theta - \sin 3\theta \cdot \sin 4\theta} = \tan 2\theta$

Multiply the Numerator & denominator by2

$$\text{L.H.S} = \frac{2\sin 8\theta \cdot \cos\theta - 2\sin 6\theta \cdot \cos 3\theta}{2\cos 2\theta \cdot \cos\theta - 2\sin 4\theta \cdot \sin 3\theta}$$

$$= \frac{[\sin(8\theta+\theta) + \sin(8\theta-\theta)] - [\sin(6\theta+3\theta) + \sin(6\theta-3\theta)]}{[\cos(2\theta+\theta) + \cos(2\theta-\theta)] - [\cos(4\theta-3\theta) - \cos(4\theta+3\theta)]}$$

$$= \frac{[\sin(9\theta) + \sin(7\theta)] - [\sin(9\theta) + \sin(3\theta)]}{[\cos(3\theta) + \cos(\theta)] - [\cos(\theta) - \cos(7\theta)]}$$

$$= \frac{\sin(9\theta) + \sin(7\theta) - \sin(9\theta) - \sin(3\theta)}{\cos(3\theta) + \cos(\theta) - \cos(\theta) + \cos(7\theta)}$$

$$= \frac{\sin(7\theta) - \sin(3\theta)}{\cos(7\theta) + \cos(3\theta)}$$

$$= \frac{2\cos\left(\frac{7\theta + 3\theta}{2}\right) \cdot \sin\left(\frac{7\theta - 3\theta}{2}\right)}{2\cos\left(\frac{7\theta + 3\theta}{2}\right) \cdot \cos\left(\frac{7\theta - 3\theta}{2}\right)}$$

$$= \frac{2\cos 5\theta \cdot \sin 2\theta}{2\cos 5\theta \cdot \cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

$$= \text{R.H.S}$$

4) Prove that $\sin 20^0 \cdot \sin 40^0 \cdot \sin 60^0 \cdot \sin 80^0 = \frac{3}{16}$

Solution: L.H.S.= $\sin 20^0 \cdot \sin 40^0 \cdot \sin 60^0 \cdot \sin 80^0$

$$=\frac{1}{2}\{2\sin 40^0 \cdot \sin 20^0\} \cdot \frac{\sqrt{3}}{2} \cdot \sin(90^0 - 10^0)$$

$$=\frac{1}{2}\{\cos(40^0 - 20^0) - \cos(40^0 + 20^0)\} \cdot \frac{\sqrt{3}}{2} \cdot \cos 10^0$$

$$=\frac{\sqrt{3}}{4}\{\cos 20^0 - \cos 60^0\}\cos 10^0$$

$$=\frac{\sqrt{3}}{4}\left\{\cos 20^0 - \frac{1}{2}\right\}\cos 10^0$$

$$=\frac{\sqrt{3}}{4}\left\{\frac{2\cos 20^0 - 1}{2}\right\} \cdot \cos 10^0$$

$$=\frac{\sqrt{3}}{8}\{2\cos 20^0 \cos 10^0 - \cos 10^0\}$$

$$=\frac{\sqrt{3}}{8}\{\cos(20^0 + 10^0) + \cos(20^0 - 10^0) - \cos 10^0\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 30^0 + \cos 10^0 - \cos 10^0\}$$

$$=\frac{\sqrt{3}}{8} \cdot \cos 30^0 = \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16} = \text{R.H.S}$$

Assignment: - 4

1) Prove that, $\dfrac{\sin 3A - \sin A}{\cos 3A + \cos A} = \tan A$

2) Prove that, $\dfrac{\cos 3x - \cos 11x}{\sin 11x - \sin 3x} = \tan 7x$

3) Prove that, $\dfrac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \cos 2A - \cot 5A.\sin 2A$

4) In any ΔABC, prove that: $\sin 2A + \sin 2B - \sin 2C = 4\cos A.\cos B.\cos C$

5) Prove that $\dfrac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \tan 3A.\sin A$