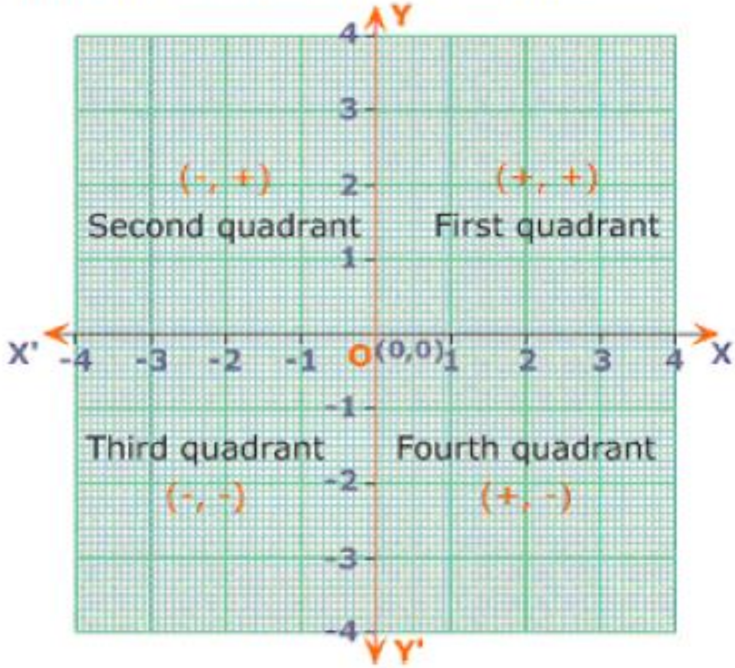
Co-ordinate Geometry (Revision)

Important Terms and Concepts



" If a pair of perpendicular lines XOX' and YOY' intersects at O, then these lines are called the **co–ordinate axes**". The axes divide the plane into four quadrants.
The plane containing the axes is called the **Cartesian plane.**

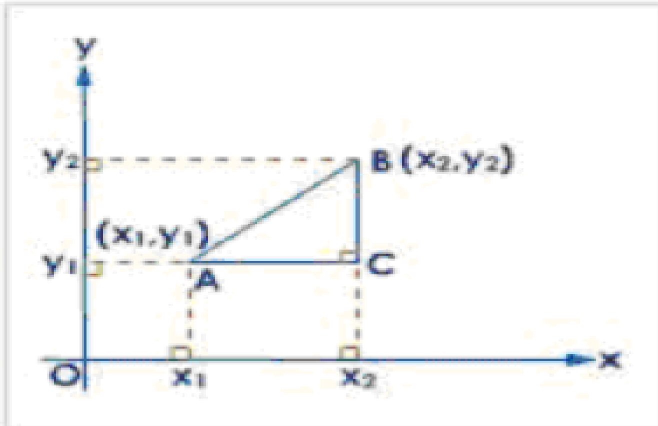
The lines XOX' and YOY' are usually drawn horizontally and vertically as shown in the figure, and are known as x–axis and y–axis respectively.
O, the point of intersection of the axes is called the **origin**.

Values of x are measured from O along the x–axis and are called **abscissae**. Along OX, x has positive values while OX', x has negative values.
Similarly, the values of y are measured from O along the y–axis and are called **ordinate**. Along OY, y has positive values while OY', y has negative values.

The ordered pair containing the abscissa and the ordinate of a point is called the **coordinates of the point**.

Distance Formula:

To find the distance two points A (x_1, y_1) and B(x_2, y_2)



$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

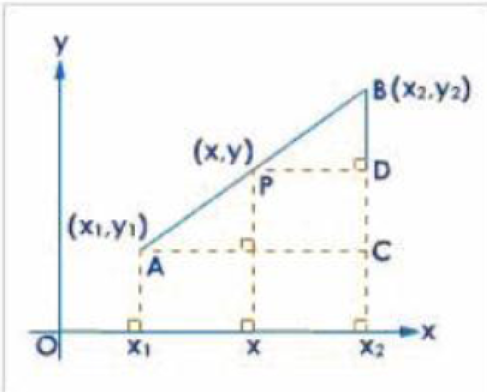
For Example

For example, distance between the points (6, – 4) and (3, 0) is

$$\sqrt{(3-6)^2 + (0+4)^2} = \sqrt{9+16} = 5 \text{ units.}$$

Section Formula:

To find the coordinates of a point which divides the line segment joining two given points in a given ratio (internally)



Let P(x, y) divide the join of A(x_1, y_1) and B(x_2, y_2) in the ratio m : n

Thus coordinate of P are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

For example, the coordinates of the point which divides the line segment joining A (1, –3) and B (–3, 9) internally, in the ratio 1: 3 are given by $x=\frac{1.(-3)+3.1}{1+3}=0$

and $y=\frac{1.9+3.(-3)}{1+3}=0.$

Find the coordinates of the point which divides the join of (– 1, 7) and (4, – 3) in the ratio 2:3.

Answer:

Let P(*x*, *y*) be the required point. Using the section formula, we obtain

$$x=\frac{2\times4+3\times(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

$$y=\frac{2\times(-3)+3\times7}{2+3}=\frac{-6+21}{5}=\frac{15}{5}=3$$

Therefore, the point is (1, 3).

Mid–Point Formula:

If P is the mid–point of AB, then m=n,

∴ The ratio becomes 1:1

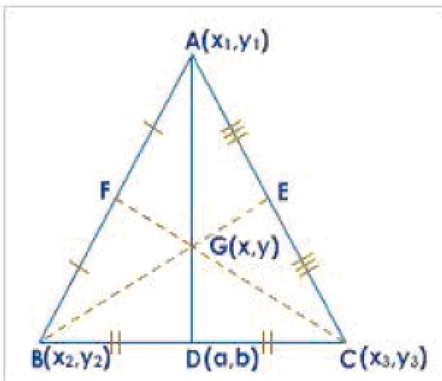
$\therefore x = \frac{mx_2+nx_1}{m+n} = \frac{x_2+x_1}{1+1} = \frac{x_1+x_2}{2}$

Similarly, we get $y = \frac{y_1+y_2}{2}$

Thus coordinates of point are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Centroid of a Triangle

Centroid is the point of intersection of three medians. It is the point of intersection of a median AG: GD is 2: 1



∴ Coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Properties of the Centroid of Triangle

The important properties of the centroid of a triangle are:

- The centroid of a triangle is located at the intersecting point of all three medians of a triangle
- It is considered one of the three points of concurrency in a triangle, i.e., incenter, circumcenter, centroid
- The centroid is positioned inside a triangle
- At the point of intersection (centroid), each median in a triangle is divided in the ratio of 2: 1

Determine the centroid of a triangle whose vertices are (5,3), (6,1) and (7,8).

Solution

Given parameters are,

$(x_1, y_1) = (5,3)$

$(x_2, y_2) = (6,1)$

$(x_3, y_3) = (7,8)$

The centroid formula is given by

$C = [(x_1 + x_2 + x_3)/ 3, (y_1 + y_2 + y_3)/ 3)$

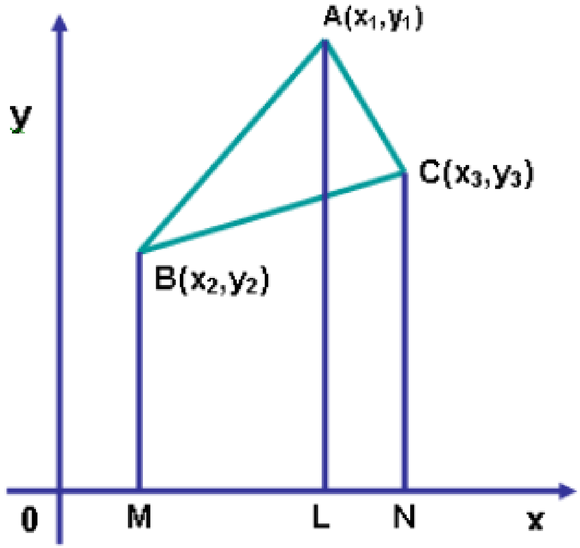
$C = [(5 + 6 + 7) / 3, (3 + 1 + 8) / 3]$

$C = (18 / 3, 12 / 3)$

$C = (6, 4)$

Area of the Triangle

To find the area of triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3).



<u>Area of Triangle ABC is given by,</u>

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)]$$

Note:

1. If the points A, B and C we take in the anticlockwise direction, then the area will be positive. If the points we take in clockwise direction the area will be negative.

   So we always take the absolute value of the area calculated.

   Area of triangle

   $$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)]$$

2. If the area of a triangle is zero, then the three points are collinear.

Find the area of the triangle whose vertices are:

(i) (2, 3), (− 1, 0), (2, − 4) (ii) (− 5, − 1), (3, − 5), (5, 2)

Answer:

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of the given triangle} = \frac{1}{2}\left[2\{0-(-4)\} + (-1)\{(-4)-(3)\} + 2(3-0)\right]$$

$$= \frac{1}{2}\{8+7+6\}$$

$$= \frac{21}{2} \text{ square units}$$

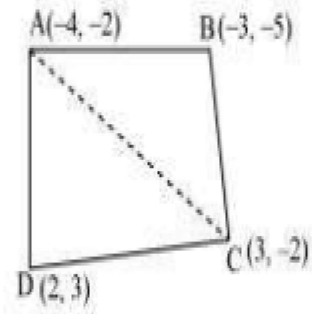
(ii) $$\text{Area of the given triangle} = \frac{1}{2}\left[(-5)\{(-5)-(2)\} + 3(2-(-1)) + 5\{-1-(-5)\}\right]$$

$$= \frac{1}{2}\{35+9+20\}$$

$$= 32 \text{ square units}$$

Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3)

Answer:



Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\left\{(-5)-(-2)\right\} + (-3)\left\{(-2)-(-2)\right\} + 3\left\{(-2)-(-5)\right\}\right]$$

$$= \frac{1}{2}(12+0+9) = \frac{21}{2}\text{square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\left\{(-2)-(3)\right\} + 3\left\{(3)-(-2)\right\} + 2\left\{(-2)-(-2)\right\}\right]$$

$$= \frac{1}{2}\left\{20+15+0\right\} = \frac{35}{2}\text{ square units}$$

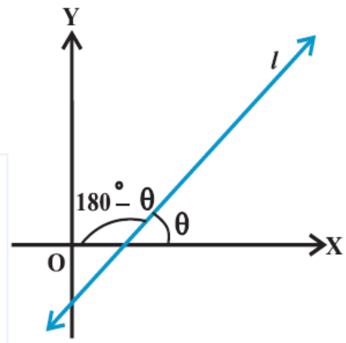
$$\text{Area of } \square ABCD = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2} + \frac{35}{2}\right)\text{square units} = 28\text{ square units}$$

Straight Line

Slope of line:

Definition If θ is the inclination of a line l, then $\tan\theta$ is called the *slope* or *gradient* of the line l.



The slope of a line is denoted by m.

Thus, $m = \tan\theta$, $\theta \neq 90^\circ$

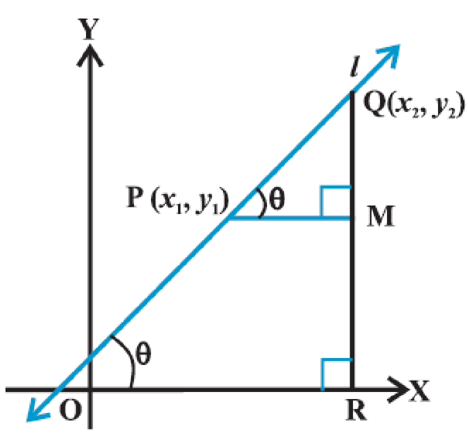
Note: 1) For X-axis, $\theta = 0^0$ **,** $\tan 0^0 = 0$

Slope of X-axis or line parallel to X-axis = 0

2) For Y-axis, $\theta = 90^0$ **,** $\tan 90^0 = \infty$

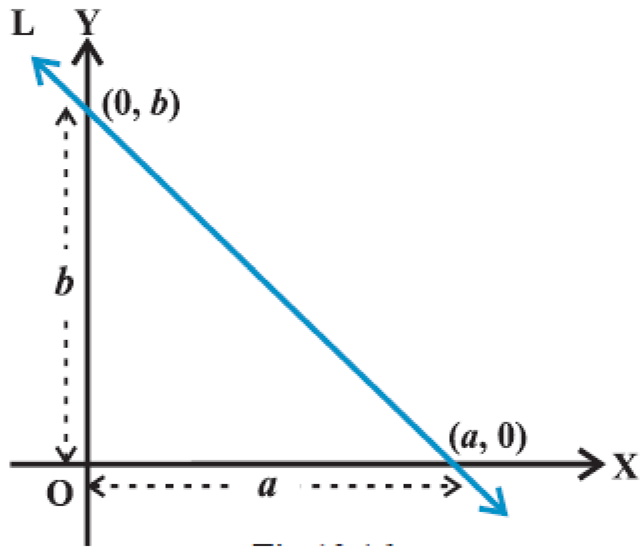
Slope of Y-axis or line parallel to Y-axis = ∞

Two point form of slope:



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Intercept of line on coordinate axis:

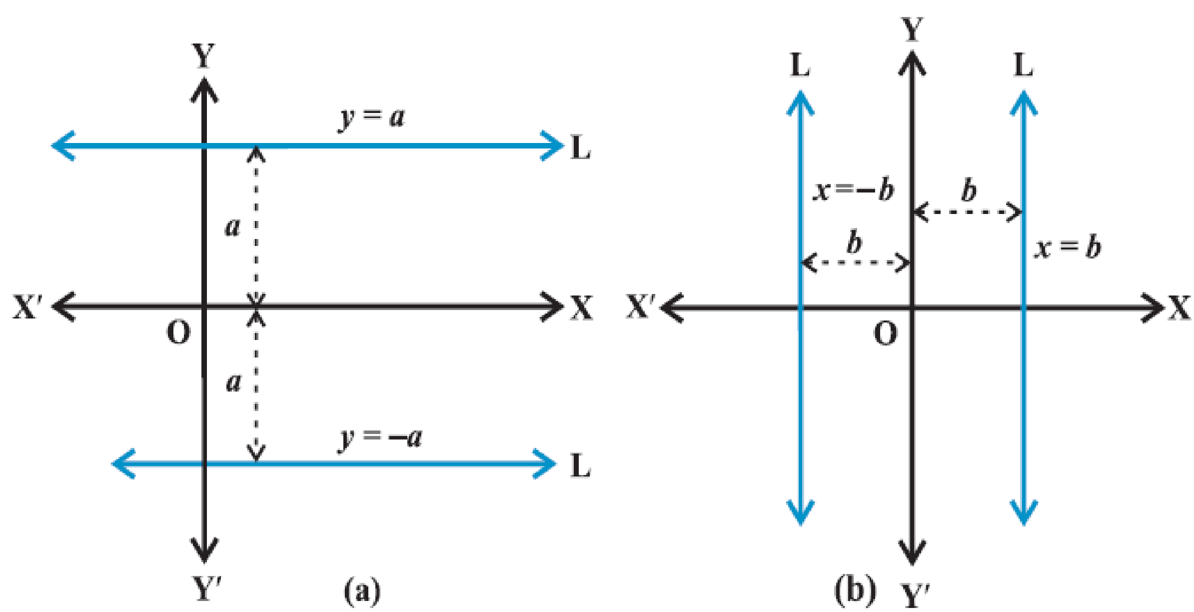


X- intercept = d(OA) = a,   Y-intercept = d(OB) = b.

Note: For X-intercept, put y = 0 in equation of straight line & for Y-intercept put x = 0 in equation of line.

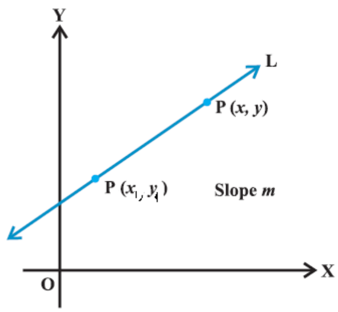
Different form of equation of line:

1) Line parallel to coordinate axis:



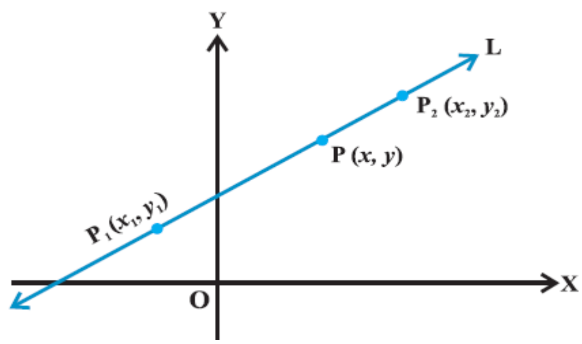
2) <u>Slope-Origin form:</u> If line passes through origin o and having slope 'm' then, equation of line is $y = mx$.

3) <u>Slope-point form:</u>



Equation of line is, $y - y_1 = m(x - x_1)$

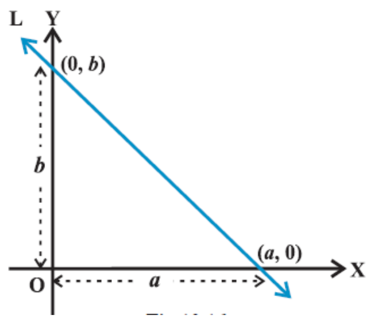
4) <u>Two point form:</u>



Equation of line is, $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1}$

5) <u>Slope-intercept form:</u> If 'm' is slope of line and 'b' is Y-intercept made by line on Y-axis. Then equation of line is $y = mx + b$.

6) **Two intercept form:**



Equation of line is, $\frac{x}{a}+\frac{y}{b}=1$.

Solved Example

Find the equation of line passing through (3, – 4) and having slope $\frac{3}{2}$.

Solution : Given Slope = (m) = $\frac{3}{2}$

and $(x_1, y_1) = (3, -4)$

By using equation of line in slope-point form.

$$y - y_1 = m(x - x_1)$$

$\therefore$ $y - (-4) = \quad (x - 3)$

$\therefore$ $2(y + 4) = 3(x - 3)$

$\therefore$ $2y + 8 = 3x - 9$

$\therefore$ $3x - 2y - 9 - 8 = 0$

$\therefore$ $3x - 2y - 17 = 0$

Find the equation of line passing through (1, 7) and having slope 2 units.

Solution :

Given slope = m = 2

and $(x_1, y_1) = (1, 7)$

By using equation of line in slope-point form.

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 2(x - 1)$$

$$y - 7 = 2x - 2$$

$$\therefore \quad 2x - 2 - y + 7 = 0$$

$$\therefore \quad \mathbf{2x - y + 5 = 0}$$

Find the equation of straight line passes through the points (3, 5) and (4, 6).

Solution :

Equation of line is, $\dfrac{y - y_1}{y_1 - y_2} = \dfrac{x - x_1}{x_1 - x_2}$

$$\therefore \quad \frac{y - 5}{5 - 6} = \frac{x - 3}{3 - 4}$$

$$\therefore \quad \frac{y - 5}{-1} = \frac{x - 3}{-1}$$

$$\therefore \quad x - y + 2 = 0$$

Find the equation of straight line passes through the points $(-4, 6)$ and $(8, -3)$.

Solution : Let the given point $A(x_1, y_1) = (-4, 6)$ and $B(x_2, y_2) = (8, -3)$.

By using equation of line in two point form

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\therefore \quad \frac{y-6}{6-(-3)} = \frac{x-(-4)}{(-4-8)}$$

$$\therefore \quad \frac{y-6}{9} = \frac{x+4}{-12}$$

$$\therefore \quad -12(y-6) = 9(x+4)$$

$$\therefore \quad -12y + 72 = 9x + 36$$

$$\therefore \quad 9x + 12y + 36 - 72 = 0$$

$$\therefore \quad 9x + 12y - 36 = 0$$

$$\therefore \quad 3x + 4y - 12 = 0$$

Find the equation of the line whose x-intercept is 3 and y intercept is 4.

Solution : Given x-intercept = a = 3

y-intercept = b = 4

By using equation of line in two intercept form.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\therefore \quad \frac{x}{3} + \frac{y}{4} = 1$$

$$\therefore \quad \frac{4x+3y}{12} = 1$$

$$\therefore \quad 4x + 3y = 12$$

$$\therefore \quad 4x + 3y - 12 = 0$$

Find the equation of the line whose x-intercept is 10 and y intercept is 3.

Solution : Given x-intercept = a = 10

y-intercept = b = 3

By using equation of line in two intercept form.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\therefore \quad \frac{x}{10} + \frac{y}{3} = 1$$

$$\therefore \quad \frac{3x + 10y}{30} = 1$$

$$\therefore \quad 3x + 10y = 30$$

$$\therefore \quad 3x + 10y - 30 = 0$$

General Equation of straight line

Every equation of line is finally expressed in the form $Ax + By + C = 0$. This form is called as general equation of straight line.

From general equation of line slope and intercept obtained by,

Slope = $m = -\frac{A}{B}$

X-intercept = $a = -\frac{C}{A}$

Y-intercept = $b = -\frac{C}{B}$

(Note: For x-intercept, put y = 0 in equation of line & for y-intercept, put x = 0 in equation of line)

Angle between two lines

If ϕ is angle between two line. Let 'm_1' and 'm_2' be slope of two lines. Then angle between two lines is obtained by formula,

$$\tan\phi = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$$

Or

$$\phi = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$$

1) Parallel line condition: $m_1 = m_2$

2) Perpendicular line condition: $m_1.m_2 = -1$

Solved Example

1)Show that the lines $2x + 3y - 5 = 0$ and $4x + 6y - 1 = 0$ are parallel.

Solution:

Let $L_1 : 2x + 3y - 5 = 0$

$\therefore$ Slope of L_1 is $m_1 = \frac{-2}{3}$

And $L_2 : 4x + 6y - 1 = 0$

$\therefore$ Slope of L_2 is $m_2 = \frac{-4}{6} = \frac{-2}{3}$

$\therefore m_1 = m_2$

$\therefore$ Given lines are parallel

2)Prove that lines $3x + 4y + 7 = 0$ and $28x - 21y + 50 = 0$ are perpendicular to each other.

Solution :

Let $L_1 : 3x + 4y + 7 = 0$

$\therefore$ Slope of L_1 is $m_1 = \frac{-3}{4}$

and $L_2 : 28x - 21y + 50 = 0$

$\therefore$ Slope of L_2 is $m_2 = \frac{-28}{-21} = \frac{4}{3}$

$m_1 \cdot m_2 = \frac{-3}{4} \cdot \frac{4}{3} = -1$

$\therefore$ Given lines are perpendicular

3)Find the acute angle between the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$.

Solution : Given equation of lines

$L_1 : 3x - 2y + 4 = 0$

$\therefore$ Slope $m_1 = \frac{-3}{-2} = \frac{3}{2}$

$L_2 : 2x - 3y - 7 = 0$

$\therefore$ Slope $m_2 = \frac{-2}{-3} = \frac{2}{3}$

Let 'ϕ' be the acute angle between the lines

Then $\tan\phi = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2}\,\frac{2}{3}\right)}\right|$$

$$= \left|\frac{\frac{9-4}{6}}{1+1}\right|$$

$$= \left|\frac{\frac{5}{6}}{2}\right|$$

$$= \left|\frac{5}{12}\right|$$

$$\tan\phi = \frac{5}{12}$$

$$\phi = \tan^{-1}\left(\frac{5}{12}\right)$$

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4) Find the acute angle between the lines $3x - y = 4$ and $2x + y = 3$

Solution: Given equation of lines

$$L_1 : 3x - y = 4$$

$$L_1 : 3x - y - 4 = 0$$

$$\therefore \text{Slope } m_1 = \frac{-3}{-1} = 3$$

$$L_2 : 2x + y - 3 = 0$$

$$\therefore \text{Slope } m_2 = \frac{-2}{1} = -2$$

The acute angle between the line is

$$\phi = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \tan^{-1}\left|\frac{3-(-2)}{1+(3)(-2)}\right|$$

$$= \tan^{-1}\left|\frac{5}{-5}\right|$$

$$= \tan^{-1}|-1|$$

$$= \tan^{-1}(1)$$

$$\phi = 45°$$

OR

$$\phi = \frac{\pi}{4}$$

Example Find the angle between the lines $y-\sqrt{3}x-5=0$ and $\sqrt{3}y-x+6=0$.

Solution Given lines are

$$y-\sqrt{3}x-5=0 \text{ or } y=\sqrt{3}x+5 \qquad \dots (1)$$

and

$$\sqrt{3}y-x+6=0 \text{ or } y=\frac{1}{\sqrt{3}}x-2\sqrt{3} \qquad \dots (2)$$

Slope of line (1) is $m_1 = \sqrt{3}$ and slope of line (2) is $m_2 = \dfrac{1}{\sqrt{3}}$.

The acute angle (say) ϕ between two lines is given by

$$\tan\phi = \left|\frac{m_2-m_1}{1+m_1m_2}\right| \qquad \dots (3)$$

Putting the values of m_1 and m_2 in (3), we get

$$\tan\phi = \left|\frac{\frac{1}{\sqrt{3}}-\sqrt{3}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}\right| = \left|\frac{1-3}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}} \qquad \therefore \phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

which gives $\phi = 30°$.

Example If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Solution We know that the acute angle between two lines with slopes m_1 and m_2 is given by

$$\tan\phi = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \qquad \text{... (1)}$$

Let $m_1 = \frac{1}{2}$, $m_2 = m$ and $\phi = \frac{\pi}{4}$. $\left(\frac{\pi}{4} = 45^\circ\right)$

Now, putting these values in (1), we get

$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right| \quad \text{or} \quad 1 = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right|,$$

which gives
$$\frac{m - \frac{1}{2}}{1 + \frac{1}{2}\ m} = 1$$

Therefore $m = 3$

Example Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x.

Solution Slope of the line through the points (– 2, 6) and (4, 8) is

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

Slope of the line through the points (8, 12) and (x, 24) is

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since two lines are perpendicular,
$m_1\, m_2 = -1$, which gives

$$\frac{1}{3} \times \frac{12}{x-8} = -1 \quad \text{or} \quad x = 4.$$

Example Find the equation of a line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point (1, – 2).

Solution Given line $x - 2y + 3 = 0$ can be written as

$$y = \frac{1}{2}x + \frac{3}{2} \qquad \text{...(1)}$$

Slope of the line (1) is $m_1 = \frac{1}{2}$. Therefore, slope of the line perpendicular to line (1) is

$$m_2 = -\frac{1}{m_1} = -2$$

Equation of the line with slope – 2 and passing through the point (1, – 2) is

$$y - (-2) = -2(x-1) \quad \text{or} \quad y = -2x,$$

which is the required equation.

Perpendicular Distance between Point and Line:

If $P(x_1, y_1)$ is any point and $Ax + By + C = 0$ is a line, the perpendicular distance of a point P from the line is given by $\left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$.

Solved Example

1) Find the length of the perpendicular on the line $3x + 4y - 5 = 0$ from the point (3, 4)

Solution: Given line is $3x + 4y - 5 = 0$

Here $A = 3$, $B = 4$, $C = -5$

Also, $P(x_1, y_1) = (3, 4)$

Length of perpendicular from $P(x_1, y_1)$ to the line $Ax + By + C = 0$ is given by

$$P = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$$

$$= \left|\frac{3(3) + 4(4) - 5}{\sqrt{(3)^2+(4)^2}}\right|$$

$$= \left|\frac{9 + 16 - 5}{\sqrt{9+16}}\right| = \left|\frac{20}{\sqrt{25}}\right| = \left|\frac{20}{5}\right|$$

P = 4 Units

2) Find the length of perpendicular from the point (– 3, – 4) on the line $4(x + 2) = 3(y - 4)$.

Solution : Given line is

$4(x + 2) = 3(y - 4)$

$4x + 8 = 3y - 12$

$4x - 3y + 8 + 12 = 0$

$4x - 3y + 20 = 0$

Here $A = 4$, $B = -3$, $C = 20$

Also, $P(x_1, y_1) = (-3, -4)$

Now length of perpendicular from $P(x_1, y_1)$ to the line $Ax + By + C = 0$ is given by

$$P = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$$

$$= \left|\frac{4(-3) + (-3)(-4) + 20}{\sqrt{(4)^2+(-3)^2}}\right|$$

$$= \left|\frac{-12 + 12 + 20}{\sqrt{16+9}}\right| = \left|\frac{20}{\sqrt{25}}\right| = \left|\frac{20}{5}\right|$$

P = 4 Units

3) If length of perpendicular from (5, 4) on the straight line 2x + y + K = 0 is $4\sqrt{5}$units. Find the value of K.

Solution :Given line is

$$2x + y + K = 0$$

Here $A = 2,\ B = 1,\ C = K$

Also, $P(x_1, y_1) = (5, 4)$ and Given $P = 4\sqrt{5}$

Now length of perpendicular from $P(x_1, y_1)$ to the line $Ax + By + C = 0$ is given by

$$P = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$$

$$4\sqrt{5} = \left|\frac{2(5) + 1(4) + k}{\sqrt{(2)^2+(1)^2}}\right|$$

$$4\sqrt{5} = \left|\frac{10 + 4 + k}{\sqrt{5}}\right|$$

$$4\sqrt{5} = \left|\frac{14 + k}{\sqrt{5}}\right|$$

$$4\sqrt{5}\,.\,\sqrt{5} = |14 + k|$$

$$20 = |14 + k|$$

$\therefore\quad 20 = 14 + K \quad \text{OR} \quad -20 = 14 + K$

$\therefore\quad 20 - 14 = K \quad \text{OR} \quad -20 - 14 = K$

$\therefore\quad K = 6 \qquad \text{OR} \quad K = -34$

Perpendicular Distance between Two Parallel Lines:

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$.

Solved Example

1) Find the distance between the parallel lines $3x - y + 7 = 0$ and $3x - y + 16 = 0$.

Solution : Given lines are

$3x - y + 7 = 0$, $3x - y + 16 = 0$

$a = 3$, $b = -1$, $c_1 = 7$, $c_2 = 16$

Distance between two parallel lines is

$$\therefore \quad d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$$

$$= \left|\frac{16 - 7}{\sqrt{3^2+(-1)^2}}\right|$$

$$= \left|\frac{9}{\sqrt{10}}\right|$$

$$d = \frac{9}{\sqrt{10}} \text{ Units}$$

2) Find the distance between the lines, $3x + 4y + 5 = 0$ and $6x + 8y = 25$.

Solution :

Given lines are

$3x + 4y + 5 = 0$

$\therefore 2(3x + 4y + 5) = 2 \times 0$

$\therefore 6x + 8y + 10 = 0$

$6x + 8y = 25$

$\therefore 6x + 8y - 25 = 0$

Here $a = 6$, $b = 8$, $c_1 = 10$, $c_2 = -25$

Distance between two parallel lines is

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$$

$$\therefore \quad = \left|\frac{-25 - 10}{\sqrt{(6)^2 + (8)^2}}\right|$$

$$\therefore \quad = \left|\frac{-35}{\sqrt{36+64}}\right|$$

$$\therefore \quad = \left|\frac{-35}{\sqrt{100}}\right|$$

$$\therefore \quad = \frac{35}{10}$$

$$\therefore \quad d = \frac{7}{2} \text{ Units}$$

3) Find the distance between the parallel lines $4x + 3y + 2 = 0$ and $4x + 3y - 9 = 0$.

Solution : Given lines are

$4x + 3y + 2 = 0$ and $4x + 3y - 9 = 0$

Here $a = 4$, $b = 3$, $c_1 = 2$, $c_2 = -9$

Distance between two parallel lines is

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right|$$

$$= \left|\frac{-9 - 2}{\sqrt{(4)^2+(3)^2}}\right|$$

$$= \left|\frac{-11}{\sqrt{16+9}}\right|$$

$$= \left|\frac{-11}{\sqrt{25}}\right|$$

$$= \left|\frac{-11}{5}\right|$$

$$d = \frac{11}{5} \text{ Units}$$

Exercise

1) Find the slope of a line through points (- 1, - 2) and (- 3, 8).
2) Find the equation of line passing through (2, 5) and having slope $-4/5$.
3) Find the equation of straight line passes through the points (2, 3) and (1, - 1).
4) Find the equation of the line whose x-intercept is -3 and y intercept is 4.
5) find the value of m, If the two lines $3mx - 2my - 10 = 0$ and $(5m + 2) x - 4my - 28 = 0$ are parallel.

6) Find the slope of $3x - 4y= 24$.
7) Find the acute angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.
8) Find the angle between the line $3x-4y=420$ and $4x +3y =420$.
9) Find the length of perpendicular from the origin to the straight line $4x +3y-2 =0$.

10) Find the distance between the parallel lines $3x + 2y +6 =0$, $9x +6y- 7=0$.
11) Find the distance between the parallel lines $y = 2x + 4$, $3y =6x -5$.