Logarithm

Definition:-

If $a^x = y$ then then 'x' is called as logarithm of 'y' to the base 'a' and it is written as $x = \log_a y$

For Example,

If $2^3 = 8$ then $3 = \log_2 8$

Note:-

1) Exponent form:- $a^x = y$

2) Logarithmic Form:- $x = \log_a y$

Ex: Write the following in logarithmic form

a) $5^3 = 125$

Solution: - Logarithmic Form: $3 = \log_5 125$

Ex: Write the following in Exponential form

b) $\log_3 81 = 4$

Solution: - Exponential form: $3^4 = 81$

Exercise

1) Write the following in Logarithmic form.

a) $4^5 = 1024$ b) $5^{-2} = \frac{1}{25}$ c) $0.001 = 10^{-3}$

2) Write the following in Exponential form.

a) $\log_3 27 = 3$ b) $\log_4\left(\frac{1}{16}\right) = -2$ c) $\log_{0.01}(0.0001) = 2$

<u>Answer</u>

1) a) $5 = \log_4 1024$ b) $-2 = \log_5\left(\frac{1}{25}\right)$ c) $-3 = \log_{10}(0.001)$

2) a) $3^3 = 27$ b) $4^{-2} = \frac{1}{16}$ c) $(0.01)^2 = 0.0001$

Types of Logarithm

1) Natural Logarithm: - If base of logarithm is Napier's number 'e', then it is called as 'Natural Logarithm'.

E.g. $\log_e 7 = \log 7 = \ln 7$

2) Common Logarithm:- If base of logarithm is '10', then it is called as 'Common Logarithm'.

E.g. $\log_{10} 3$, $\log_{10} 5$

3) General Logarithm:- If base of logarithm is any number except 'e' and '10', then it is called as 'General Logarithm'

E.g. $\log_4 5$, $\log_3 9$

Laws of Logarithm

Standard Law

1) If $a^0 = 1$ **then** $\log_a 1 = 0$**, for any** $a \neq 0$

2) If $a^1 = a$ **then** $\log_a a = 1$**, for any** $a \neq 0$**,** $a \succ 1$

E.g. $\log_3 3 = 1$, $\log_8 8 = 1$

General Law

3) Addition Rule:- $\log_a m + \log_a n = \log_a (m \times n)$

E.g. $\log_2 3 + \log_2 5 = \log_2 (3 \times 5) = \log_2 15$, $\log_3 (4 \times 5) = \log_3 4 + \log_3 5$

4) Subtraction Rule:- $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

E.g. $\log_2 9 - \log_2 3 = \log_2 \left(\frac{9}{3}\right) = \log_2 3$, $\log_2 \left(\frac{3}{5}\right) = \log_2 3 - \log_2 5$

5) If m = 1, in 4th rule

➡ $\log_a 1 - \log_a n = \log_a \left(\frac{1}{n}\right)$

➡ $-\log_a n = \log_a \left(\frac{1}{n}\right)$ (As $\log_a 1 = 0$)

Hence, $-\log_a n = \log_a \left(\frac{1}{n}\right)$

E.g. $-\log_2 5 = \log_2 \left(\frac{1}{5}\right)$, $\log_3 \left(\frac{1}{4}\right) = -\log_3 4$

6) Multiple of logarithm:- $n.\log_a m = \log_a (m^n)$

E.g. $4\log_2 3 = \log_2 (3^4) = \log_2 81$

$\log_5 (3^2) = 2\log_5 3$

7) Change of Base Property:

$$\Rightarrow \quad \log_n m = \frac{\log_a m}{\log_a n}$$, where ‘a’ is any convenient base

E.g. $\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2} = \dfrac{\log_4 3}{\log_4 2} = \dfrac{\log 3}{\log 2}$

8) If a = m in 7th law,

$$\Rightarrow \quad \log_n m = \frac{\log_m m}{\log_m n} = \frac{1}{\log_m n}$$, (As $\log_m m = 1$)

Hence, $\log_n m = \dfrac{1}{\log_m n}$

E.g. $\log_2 3 = \dfrac{1}{\log_3 2}$

$\dfrac{1}{\log_5 4} = \log_4 5$

Solved Example

1) Evaluate Value of following logarithm, using law of logarithm

a) $\log_{81} 3$ b) $\log_{10}\left(\sqrt[3]{1000}\right)$

Solution:- a) Let assume that, $\log_{81} 3 = x$

Express this equation in exponent form

$$\Rightarrow \quad 81^x = 3$$

$$\Rightarrow \quad \left(3^4\right)^x = 3^1$$ (Use $81 = 3^4$)

$\Rightarrow \quad 3^{4x} = 3^1 \qquad$ (As $(a^m)^n = a^{m\times n}$)

As we know, If $a^x = a^y \Rightarrow x = y$

$\Rightarrow \quad 4x = 1$

$\Rightarrow \quad x = \frac{1}{4}$ Or $\log_{81} 3 = \frac{1}{4}$

Solution: - b) Let assume that, $\log_{10}\left(\sqrt[3]{1000}\right) = x$

In exponent form, $10^x = \sqrt[3]{1000}$

$\Rightarrow \quad 10^x = 10^1 \qquad$ (Since, $\sqrt[3]{1000} = 10$)

$\Rightarrow \quad x = 1$ **Or** $\log_{10}\left(\sqrt[3]{1000}\right) = 1$

2) Simplify following logarithmic expression & find value of it.

a) $\log\left(\frac{145}{8}\right) - 3\log\left(\frac{3}{2}\right) + \log\left(\frac{54}{29}\right)$

Solution:-

$\log\left(\frac{145}{8}\right) - 3\log\left(\frac{3}{2}\right) + \log\left(\frac{54}{29}\right) = \log\left(\frac{145}{8}\right) - \log\left(\frac{3}{2}\right)^3 + \log\left(\frac{54}{29}\right)$ (using, $n.\log_a m = \log_a\left(m^n\right)$)

$= \log\left(\frac{145}{8}\right) - \log\left(\frac{27}{8}\right) + \log\left(\frac{54}{29}\right)$

$= \log\left(\frac{145}{8} / \frac{27}{8}\right) + \log\left(\frac{54}{29}\right)$ (using, $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$)

$= \log\left(\frac{145}{27}\right) + \log\left(\frac{54}{29}\right)$

$= \log\left(\frac{145}{27} \times \frac{54}{29}\right)$ (using, $\log_a m + \log_a n = \log_a(m \times n)$)

$= \log(5 \times 2)$

$= \log(10)$

b) $\frac{1}{\log_5 10} + \frac{1}{\log_{20} 10}$

Solution:-

$\frac{1}{\log_5 10} + \frac{1}{\log_{20} 10} = \log_{10} 5 + \log_{10} 20$ (using, $\frac{1}{\log_m n} = \log_n m$)

$= \log_{10}(5 \times 20)$ (using, $\log_a m + \log_a n = \log_a(m \times n)$)

$= \log_{10}(100)$

$= \log_{10}(10^2)$

$= 2.\log_{10}(10)$ (using, $\log_a(m^n) = n.\log_a(m)$)

$= 2 \times 1$ (As $\log_a a = 1$)

$\frac{1}{\log_5 10} + \frac{1}{\log_{20} 10}$ **= 2**

3) Find Value of ‘x’ from following logarithmic equations.

a) $\log_2(x-3) = 3$ b) $\log_3(x-4) + \log_3(x-2) = 1$

Solution:- a) $\log_2(x-3) = 3$

$(x-3) = 2^3$ (In exponential form)

$$(x-3)=8$$

$$x=8+3$$

$$x=11$$

Solution:-b) $\log_3(x-4)+\log_3(x-2)=1$

$$\log_3\{(x-4)\times(x-2)\}=1$$

$$\log_3\{x^2-6x+8\}=1$$

$$x^2-6x+8=3^1$$

$$x^2-6x+8-3=0$$

$$x^2-6x+5=0$$

$$(x-5)(x-1)=0$$

Either $(x-5)=0$ or $(x-1)=0$

Values are, $x=5,1$

4) Prove that, $\dfrac{1}{\log_2 8}+\dfrac{1}{\log_{64} 8}+\dfrac{1}{\log_4 8}=3$

Proof:- L.H.S. = $\dfrac{1}{\log_2 8}+\dfrac{1}{\log_{64} 8}+\dfrac{1}{\log_4 8}$

= $\log_8 2+\log_8 64+\log_8 4$ (using, $\dfrac{1}{\log_m n}=\log_n m$)

= $\log_8(2\times 64\times 4)$ (using, $\log_a m+\log_a n=\log_a(m\times n)$)

$= \log_8(512)$

$= \log_8(8^3)$

$= 3.\log_8(8)$ (using, $\log_a(m^n) = n.\log_a(m)$)

$= 3\times 1$ (As $\log_a a = 1$**)**

L.H.S. **= 3 =** R.H.S. Hence, result is proved.

Exercise

1) Evaluate value of following logarithms.

a) $\log_3 243$ b) $\log_3 81$ c) $\log_5 625$ d) $\log_{12}(2\sqrt{3})^5$

2) Simplify following logarithmic expression & find value of it**.**

a) $\log_2 14 - \log_2 7$ b) $2\log\left(\frac{16}{15}\right) + \log\left(\frac{25}{24}\right) - \log\left(\frac{32}{27}\right)$

3) Find Value of 'x' from following logarithmic equations.

a) $\log_3(x+4) = 4$ b) $\log(x+3) + \log(x-3) = \log 27$

4) Prove that, $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ac} abc} = 2$

Answers

1) a) 5 b) 4 c) $\frac{5}{2}$

2) a) 1 b) 0

3) a) $x = 77$ b) $x = \pm 6$

Logarithm Introduction video link: https://youtu.be/Z5myJ8dg_rM

Logarithm Evaluation video link: https://youtu.be/eTWCARmrzJ0

Log & Exponential conversion link: https://youtu.be/Obch1OP5QyA

Laws of logarithm video 1 link: https://youtu.be/PupNgv49_WY

Laws of logarithm video 2 links: https://youtu.be/TMmxKZaCqe0