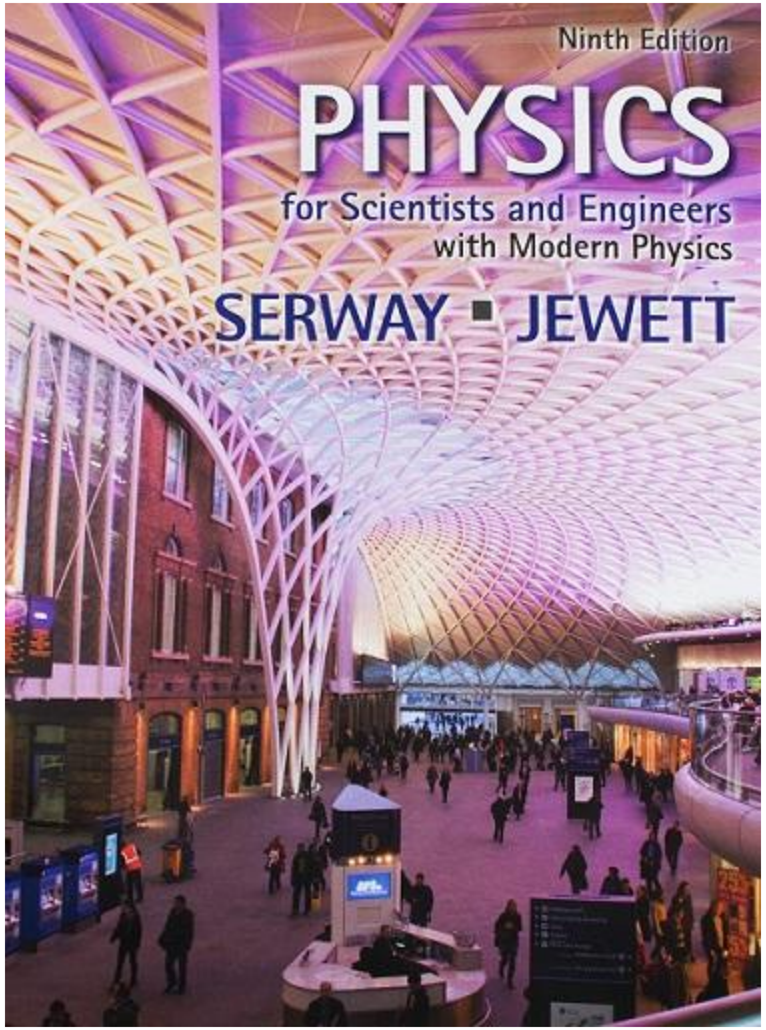
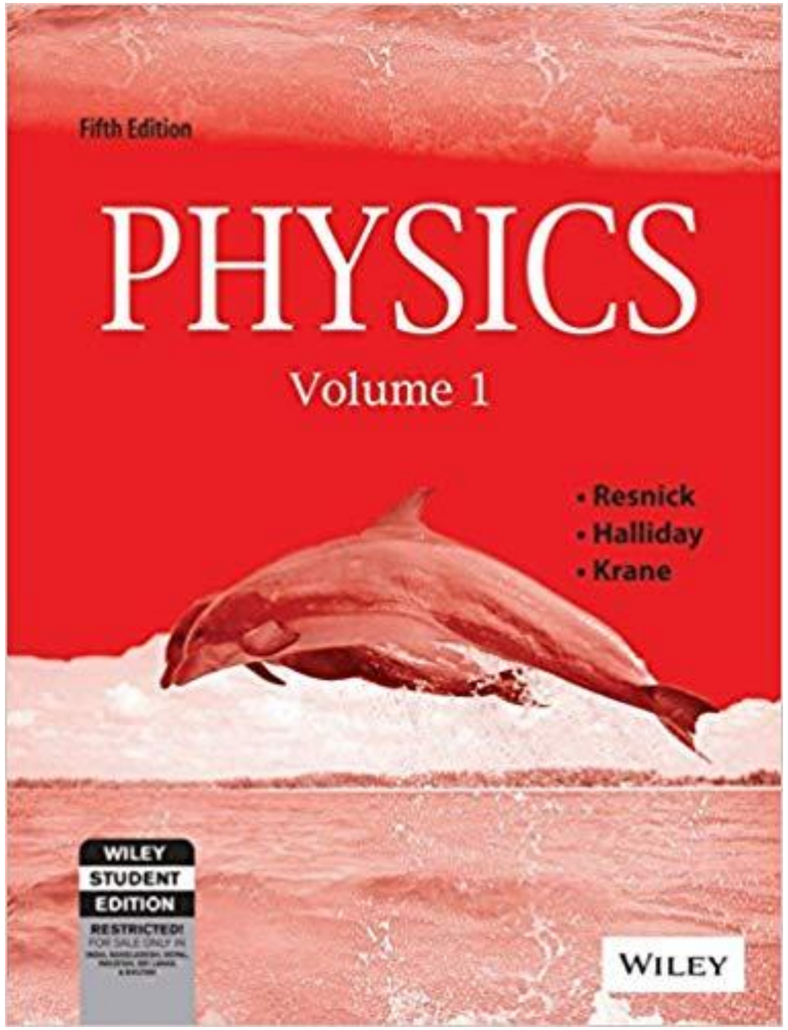
PHYSICS

General Physics I (PHYS 101)





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INTERFERENCE

Course Outline

- Coherent Sources
- Constructive and Destructive Interference
- Intensity Distribution in Double-Slit Interference Pattern
- Young's Double-Slit Experiment
- Interference in Thin Film Due to Reflected Light
- Newton's Rings

Physical Optics

Light as a Wave

- The first convincing ***wave theory for light*** was in **1678 by Dutch physicist Christian Huygens**.
- It nicely explained reflection and refraction in terms of waves and gave physical meaning to the index of refraction.
- Light is fundamentally a *wave*, and in some situations we have to consider its wave properties explicitly.

Physical Optics

- If two or more light waves of the same frequency overlap at a point, the total effect depends on the *phases* of the waves as well as their amplitudes. The resulting patterns of light are a result of the *wave* nature of light and cannot be understood on the basis of rays. Optical effects that depend on the wave nature of light are grouped under the heading **physical optics** .
- **T**he study of interference, diffraction, and polarization of light – Physical Optics or Wave Optics
- These phenomena cannot be adequately explained with the ray optics. When we treat **light as waves** rather than as rays, it leads to a satisfying description of such phenomena.

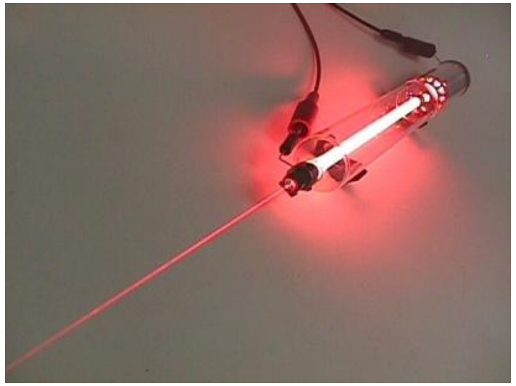
Monochromatic c Light

- **Monochromatic light** is light with a single frequency or single wavelength..
- In optics, **sinusoidal waves** are characteristic of **monochromatic light.**

The common sources of light *do not* emit monochromatic (single-wavelength) light.
Example: **Incandescent light bulbs** and **flames** emit a continuous distribution of wavelengths.

- Laser → The most nearly monochromatic source that is available at present

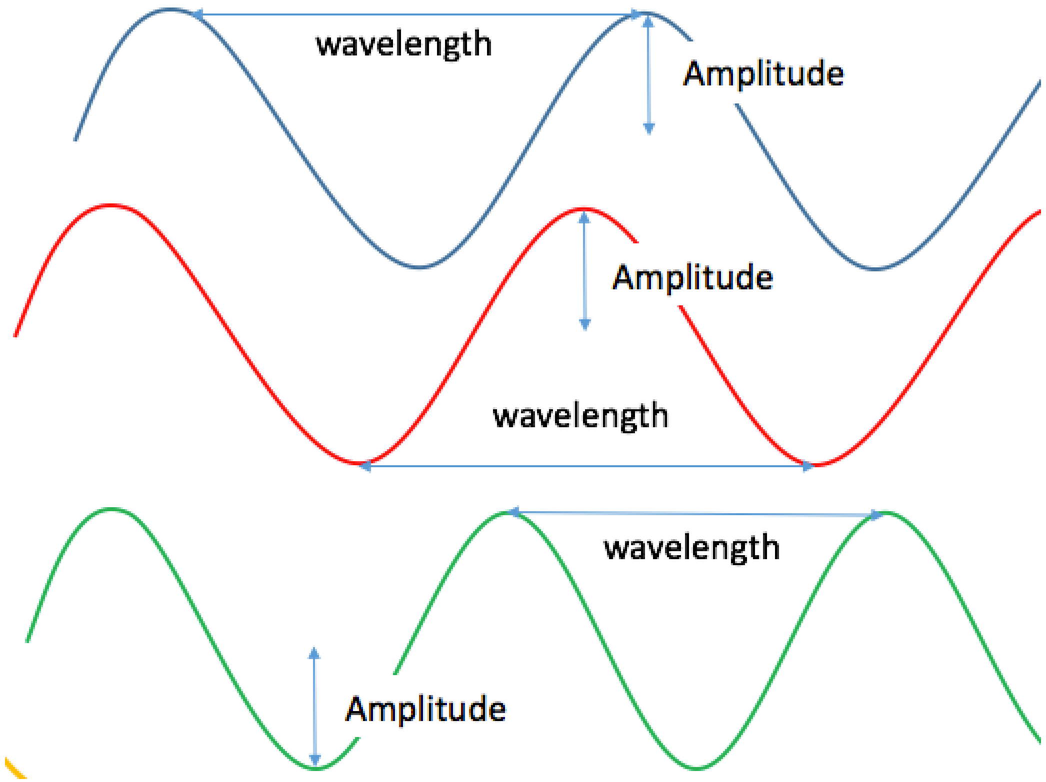
Example:
Helium–neon laser, which emits **red light at 632.8 nm** with a wavelength range of the order of $\pm$ 0.000001.

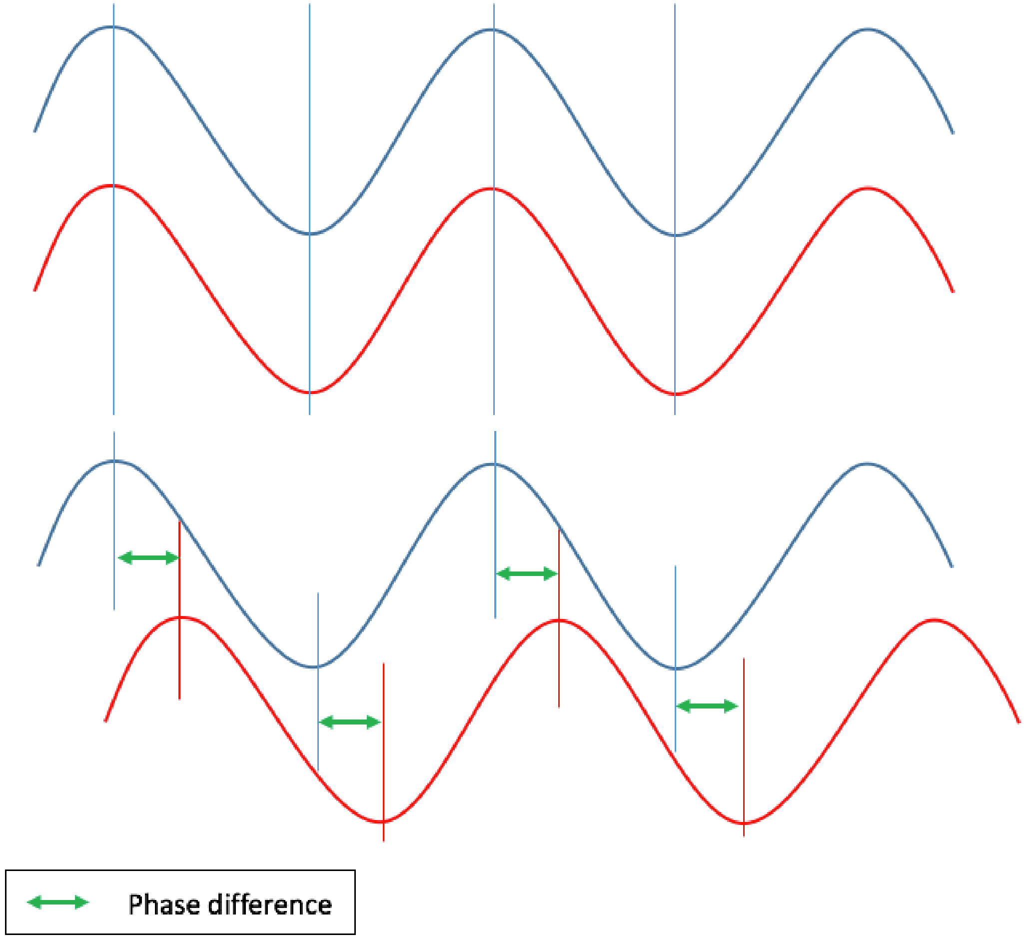


Coherence

Coherence

- **Coherence** is a definite, unchanging phase relationship between two waves.
- **Coherence**, a fixed relationship between the phase of waves in a beam of radiation of a single frequency. Two beams of light are coherent when the phase difference between their waves is constant; they are noncoherent if there is a random or changing phase relationship.





INTERFERENCE

Coherent Sources

INTERFERENCE

Coherent Sources

- **Coherent sources** are those, which emit continuous waves of the same frequency or wavelength which are always in phase with each other or have a constant phase difference.
- A common method for producing two coherent light sources is to use one monochromatic source (He-Ne Laser) to illuminate a barrier containing two small openings (usually in the shape of slits). The light emerging from the two slits is coherent because a single source produces the original light beam and the two slits serve only to separate the original beam into two parts.
- Two side-by-side loudspeakers driven by a single amplifier → coherent Sources: emit coherent sound waves
- **Coherent Waves** → The phase relationship between the two waves does not change with time.
- Two independent sources of light cannot be coherent.
- Light emitted from an **incandescent lightbulb** is incoherent because the light consists of waves of different wavelengths and they do not maintain a constant phase relationship.

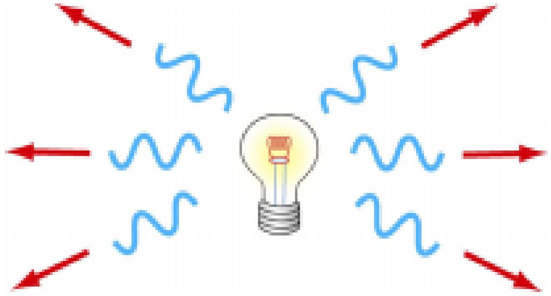


Figure 1 Incoherent Light Source

Interference

Interference

- The term **interference** refers to any situation in which two or more waves overlap in space. When this occurs, the total wave at any point at any instant of time is governed by the **principle of superposition.** This principle also applies to electromagnetic waves and is the most important principle in all of physical optics.
- The **principle of superposition of waves** states that :

> When two or more waves are incident on the same point, the total displacement at that point is equal to the vector sum of the displacements of the individual waves.
>
> If $\vec{y}_1, \vec{y}_2, \ldots$ be the displacements due to different waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \ldots$

- When otherwise identical waves from two sources overlap at a point in space, the combined wave intensity at that point can be greater or less than the intensity of either of the two waves. We call this effect **interference**. The interference can be either **constructive**, when the net intensity is greater than the individual intensities, or **destructive**, when the net intensity is less than the individual intensities.
- **Coherence is a necessary condition for interference to occur.**
- Although we deal only with light waves in this chapter, all other kinds of waves (such as sound waves and water waves) also can experience interference .

Optical Interference – Interference of Light Waves

Nature has long used optical interference for coloring.

Figure I-1
The blue of the top surface of a *Morpho butterfly wing* is due to optical interference and shifts in color as your viewing perspective changes.
(Philippe Colombi/PhotoDisc//Getty Images)

Figure I-2
The colors seen in soap bubble is a result of interference between light reflected from the front and back surfaces of a thin film of oil or soap solution

Optical Interference – Interference of Light Waves

Nature has long used optical interference for coloring.

Figure I-3
The colors in many of a hummingbird's feathers are not due to pigment. The iridescence that makes the brilliant colors that often appear on the bird's throat and belly is due to an interference effect caused by structures in the feathers. The colors will vary with the viewing angle.
(Dec Hogan/ Shutterstock.com)

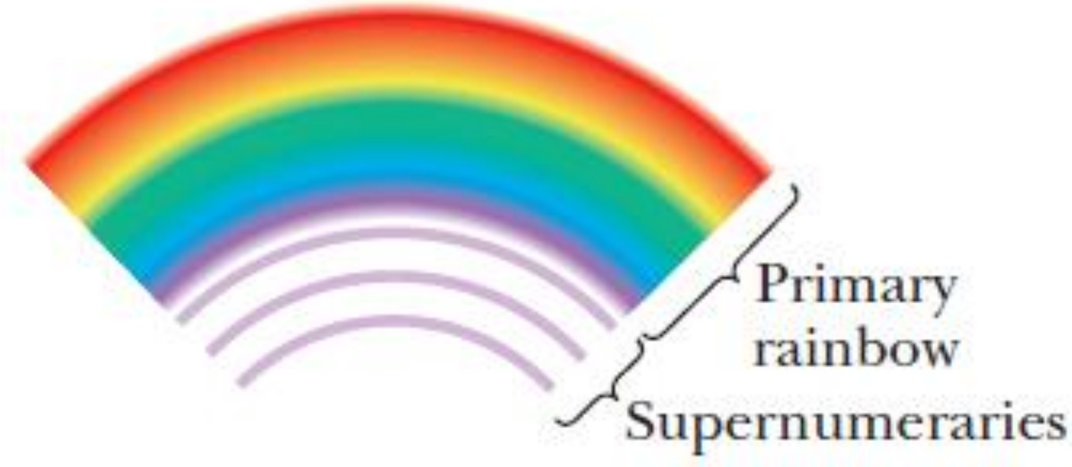


Figure I-4
A primary rainbow and the faint supernumeraries (dimmer colored arcs)below it are due to optical interference.

Interference of Light

- When a source of light emits energy, the distribution of energy is uniform. But when two coherent sources of light held close to each other, the distribution of energy in the surrounding medium is not uniform, but bright and dark regions are observed.

The phenomenon of non-uniform distribution of energy in the medium due to superposition of two light waves from two coherent sources is called interference of light.

- **Conditions for Sustained Interference of Light:**
  - ✓ The light sources must be coherent.
  - ✓ The sources should be monochromatic.
  - ✓ The coherent sources must be very close to each other as the wavelength of light is very small.

- **Why doesn't the light from the two head lights of distant car produce an interference pattern?**

→The two head-lights of a car cannot be coherent. The phase difference between the light waves emitted from the two head-lights of car keeps on changing continuously. So no interference pattern is produced.

Young's Interference Experiment

- In **1801, Thomas Young** experimentally proved that light is a wave, contrary to what most other scientists then thought. He did so by demonstrating that light undergoes interference, as do water waves, sound waves, and waves of all other types. In addition, he was able to measure the average wavelength of sunlight; his value, 570 nm, is impressively close to the modern accepted value of 555 nm.
- Figure Y-1 gives the basic arrangement of Young's experiment.

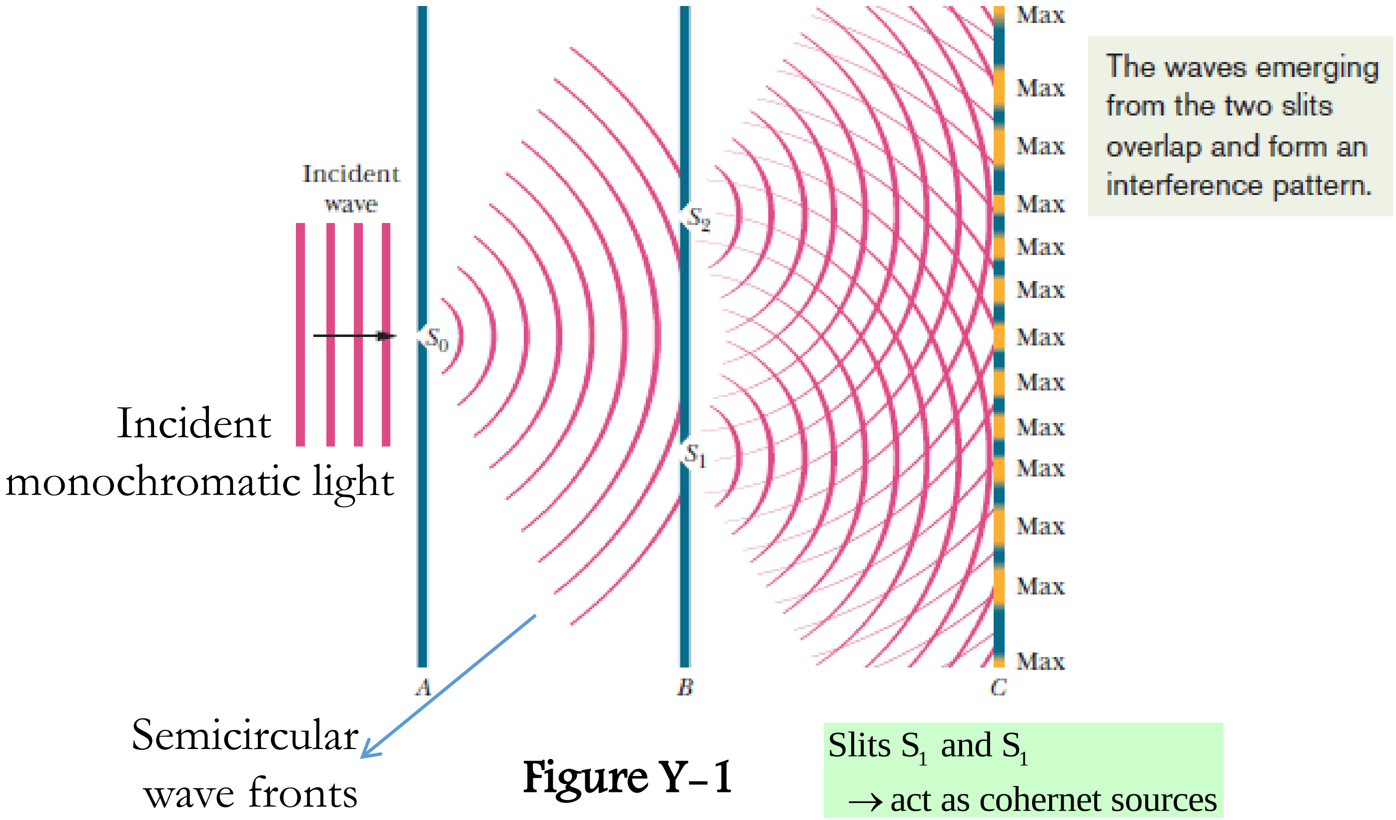


Figure Y–1

Slits S_1 and S_1
→ act as cohernet sources

Figure Y –2
A photograph of the interference pattern

Constructive Interference & Destructive Interference

- Figure CD-1 shows the Interference of light waves passing through two slits.

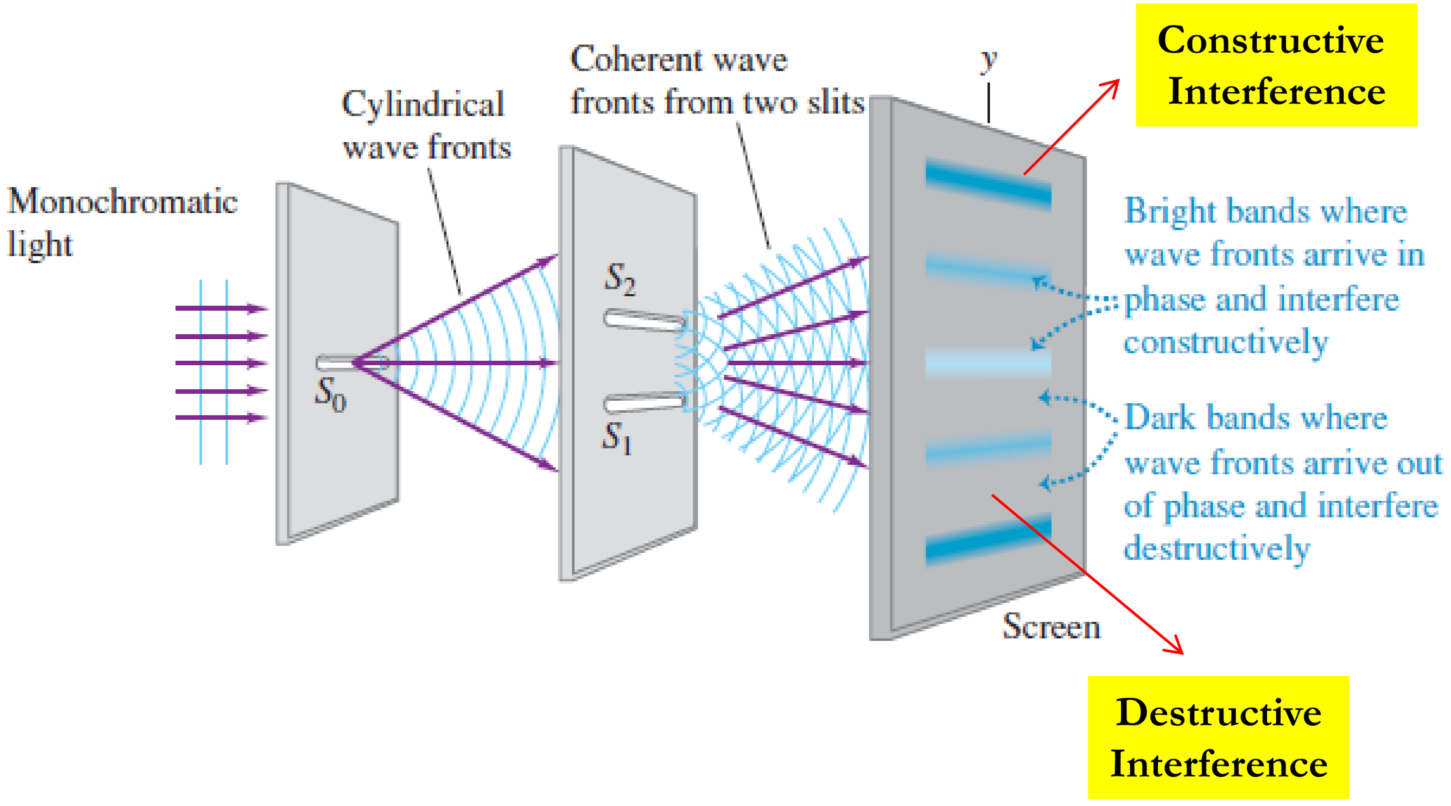


Figure CD-1
Young's experiment to show interference of light passing through two slits.
A pattern of bright and dark areas appears on the screen.

- Figure CD-2 is a photograph of interference pattern produced on the surface of a water tank by two vibrating sources.

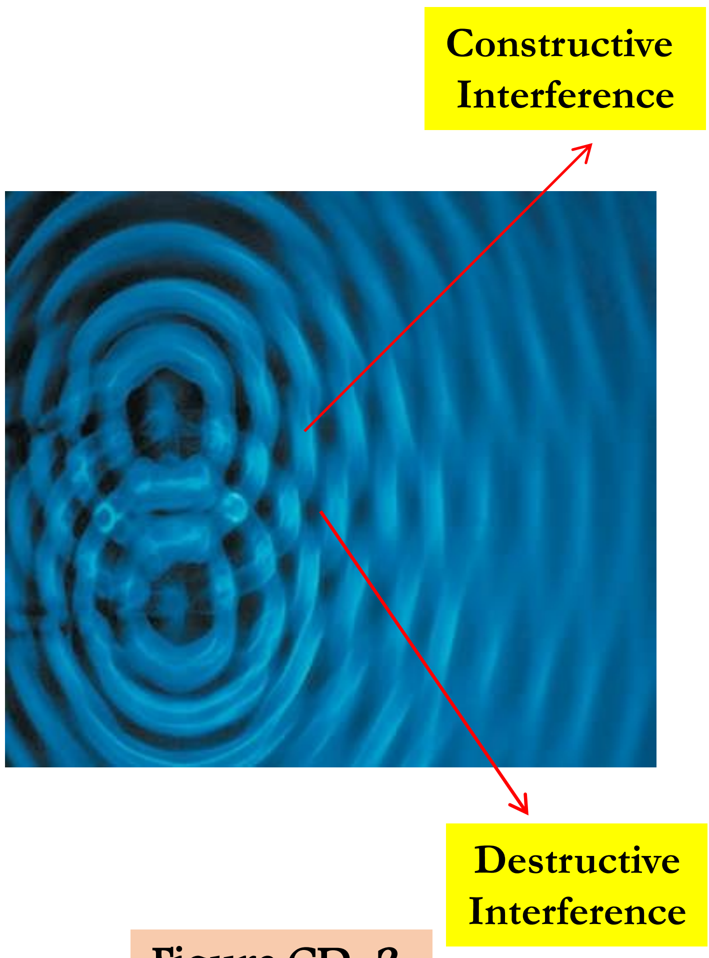


Figure CD-2

Constructive Interference & Destructive Interference

INTERFERENCE

Constructive Interference

- When two light waves superpose with each other in such a way that the crest of one wave falls on the crest of the second wave, and trough of one wave falls on the trough of the second wave, then the resultant wave has larger amplitude and it is called constructive interference.
- Conditions for Constructive Interference:

  ✓ Phase Difference:

  $$\phi = n(2\pi) \qquad \text{where } n = 0, \pm 1, \pm 2, \cdots$$

  ✓ Path Difference:

  $$x = n\lambda \qquad \text{where } n = 0, \pm 1, \mp 2, \cdots$$

Destructive Interference

- When two light waves superpose with each other in such a way that the crest of one wave coincides the trough of the second wave, then the amplitude of resultant wave becomes zero and it is called destructive interference.
- Conditions for Destructive Interference:

  ✓ Phase Difference:

  $$\phi = (2n+1)\pi \qquad \text{where } n = 0, \pm 1, \pm 2, \cdots$$

  ✓ Path Difference:

  $$x = (2n+1)\frac{\lambda}{2} \qquad \text{where } n = 0, \pm 1, \mp 2, \cdots$$

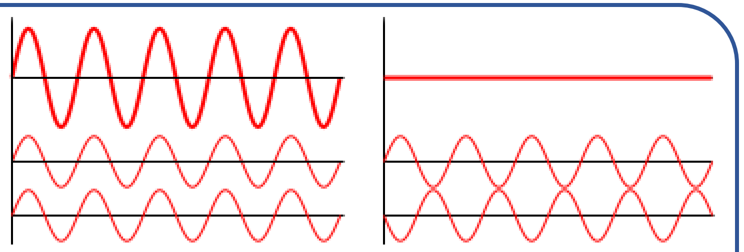
Phase Difference & Path Difference

$$\text{Phase Difference} = \frac{2\pi}{\lambda} \times \text{Path Difference}$$

$$\phi = \frac{2\pi}{\lambda} \times x$$

Constructive Interference & Destructive Interference

- In constructive interference two waves of light reinforce each other and bright fringe is formed on the screen.
- In destructive interference two cancel the effect of each other and dark fringe is formed on the screen.

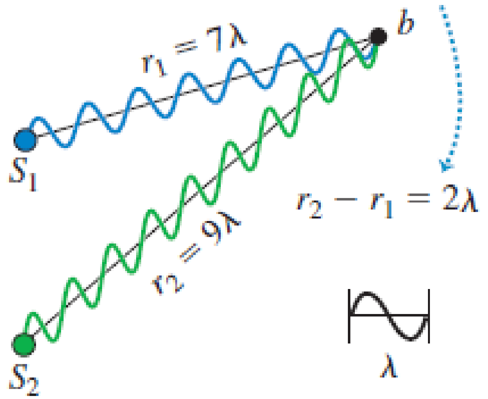


INTERFERENCE

Constructive Interference

Waves interfere constructively if their path lengths differ by an integral number of wavelengths:

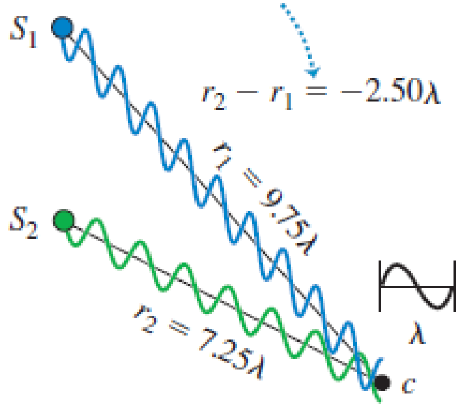
$$r_2 - r_1 = n\lambda$$



Destructive Interference

Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths:

$$r_2 - r_1 = \left(n + \frac{1}{2}\right)\lambda$$



Intensity in Interference Patterns

Intensity Distribution of the Double-Slit Interference Pattern

INTERFERENCE

- Suppose the two slits S_1 and S_2 represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ω and are in phase. The total magnitude of the electric field at point *P* on the screen in Figure ID-1 is the superposition of the two waves.
- Let us assume that the electric field components of the two waves in Figure ID-1 vary with time at point *P* as

$$E_1 = E_0 \sin \omega t$$

and $$E_2 = E_0 \sin(\omega t + \phi)$$

The phase difference between two waves can change if the waves travel paths of different lengths.

where ω (=2πυ) is the angular frequency of the waves and ϕ is the phase difference between them.

- Using the Superposition principle, the expression for the magnitude of the resultant electric field at point P is given by

$$E_P = E_1 + E_2 = E_0\left[\sin \omega t + \sin(\omega t + \phi)\right] = E_0 2 \sin\left[\frac{\omega t + (\omega t + \phi)}{2}\right]\cos\left[\frac{\omega t - (\omega t + \phi)}{2}\right]$$

$$= 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right)\cos\left(-\frac{\phi}{2}\right)$$

$$\therefore\ E_P = 2E_0 \cos\left(\frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$$

Assumption :
- slits $\rightarrow$ narrow
- $D \gg d$

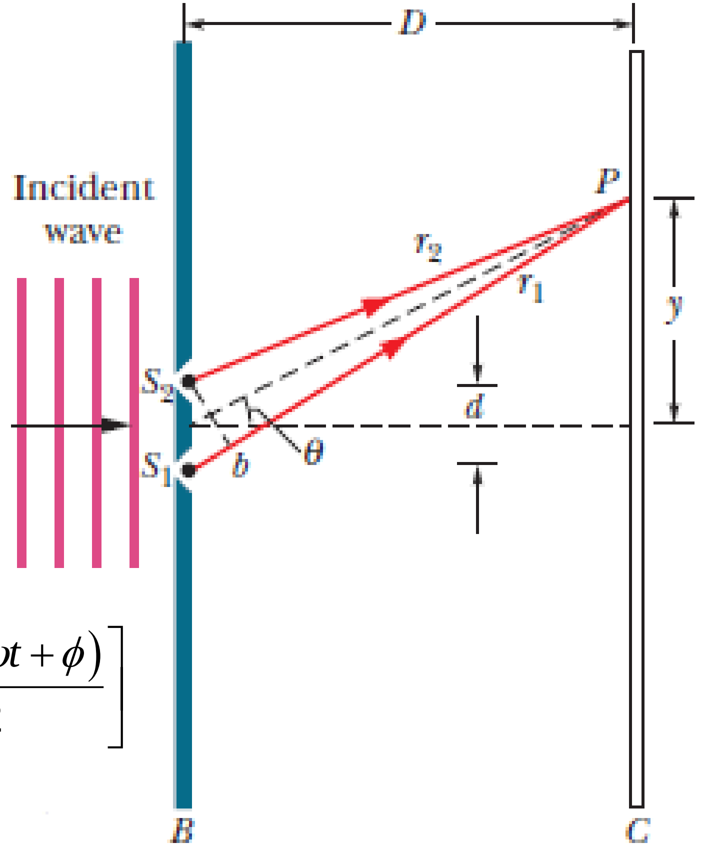


Figure ID-1

Intensity in Interference Patterns

INTERFERENCE

Intensity Distribution of the Double-Slit Interference Pattern

- The magnitude of the resultant electric field at point P:

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right) \qquad \text{..........} \quad (1)$$

  This result indicates that the electric field at point P has the same frequency ω as the light at the slits but that the amplitude of the field is multiplied by the factor $2E_0 \cos\left(\frac{\phi}{2}\right)$.

- Equation (1) can be written as

$$E_P = E_\theta \sin\left(\omega t + \beta\right)$$

  where the phase β is $\beta = \frac{1}{2}\phi$ and the amplitude is $E_\theta = 2E_0 \cos\beta$.

  The amplitude E_θ of the resultant wave disturbance, which determines the intensity of the interference fringes, depends on β, which in turn depends on the value of θ, that is, on the location of point P in Figure ID-1.

- The intensity I of an electromagnetic wave is proportional to the square of its electric field amplitude.

- If I_θ is the intensity of the resultant wave at P, and I_0 is the intensity that each single wave acting alone would produce, then $\frac{I_\theta}{I_0} = \left(\frac{E_\theta}{E_0}\right)^2 = \frac{4E_0^{\,2}\cos^2\beta}{E_0^{\,2}}$

$$\therefore \quad I_\theta = 4I_0 \cos^2\left(\frac{\phi}{2}\right) \qquad \text{..........} \quad (A)$$

Intensity in Interference Patterns

Intensity Distribution of the Double-Slit Interference Pattern

- **Constructive Interference**:

From the equation (1), it follows that the intensity of resultant wave at point P will be maximum $(I_\theta)_{max} = 4I_0$ if

$$\cos^2\left(\frac{\phi}{2}\right) = 1 \qquad \text{or,} \quad \phi = 0, 2\pi, 4\pi, \ldots$$

$$\text{or,} \quad \phi = n(2\pi) \qquad \text{where } n = 0, 1, 2, \ldots$$

It is the condition for constructive interference between the two light waves in terms of phase difference between two waves.

Condition for constructive interference in terms of path difference between two waves

$$x = n\lambda \qquad \text{where } n = 0, \pm 1, \mp 2, \cdots$$

For constructive interference to occur at P, the path difference for the two sources must be an integral multiple of the wavelength λ.

- **Destructive Interference**:

From the equation (1), it follows that the intensity of resultant wave at point P will be minimum $(I_\theta)_{min} = 0$, if

$$\cos^2\left(\frac{\phi}{2}\right) = 0 \qquad \text{or,} \quad \phi = \pi, 3\pi, 5\pi \ldots$$

$$\text{or,} \quad \phi = (2n+1)\pi \qquad \text{where } n = 0, 1, 2, \ldots$$

It is the condition for destructive interference between the two light waves in terms of phase difference between two waves.

Condition for destructive interference in terms of path difference between two waves

$$x = (2n+1)\frac{\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda \qquad \text{where } n = 0, \pm 1, \mp 2, \cdots$$

For constructive interference to occur at P, the path difference for the two sources must be a half-integral number of wavelength λ.

Intensity in Interference Patterns

Intensity Distribution of the Double-Slit Interference Pattern

- The intensity of the double slit interference pattern is given by

$$\boxed{I_\theta = 4I_0 \cos^2\left(\frac{\phi}{2}\right)} \qquad .... \quad (A)$$

- Figure ID-2, which is a plot of Eq. (A), shows the intensity of double-slit interference patterns as a function of the phase difference f between the waves at the screen.

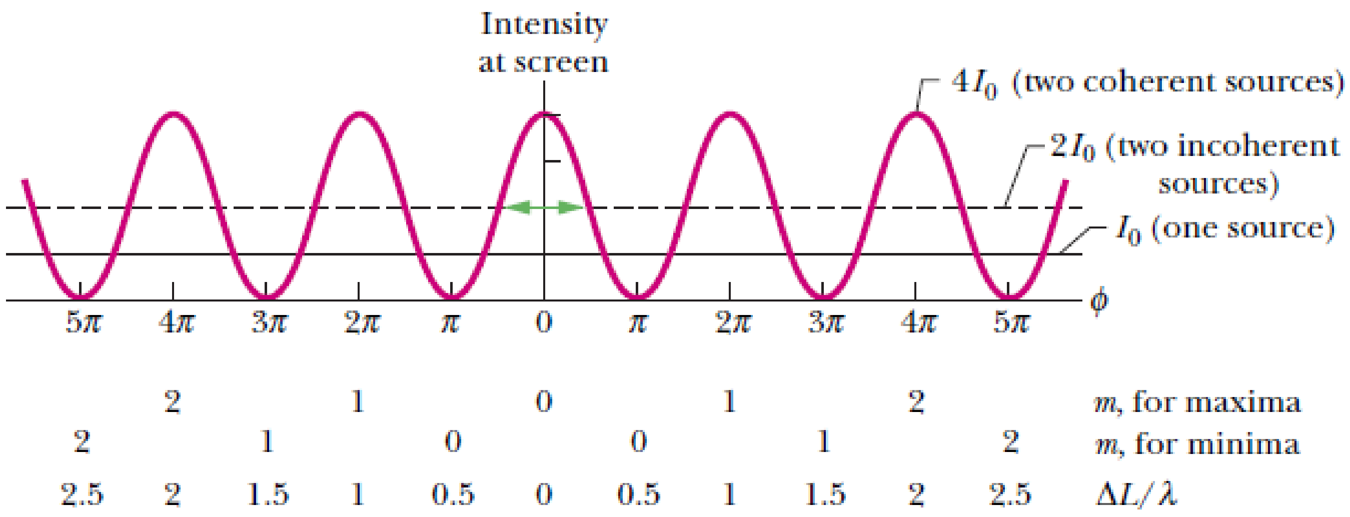


Figure ID-1

The phenomenon of interference is in agreement with the law of conservation of energy.

- The horizontal solid line is I_0 ; this describes the (uniform) intensity pattern on the screen if one of the slits is covered up.
- If the two sources were incoherent, the intensity would be uniform over the screen and would be $2I_0$, indicated by the horizontal dashed line.
- For coherent sources we expect the energy to be merely redistributed over the screen, because energy is neither created nor destroyed by the interference process. Thus the average intensity in the interference pattern should be $2I_0$, as for incoherent sources.

INTERFERENCE

Intensity in Interference Patterns

Intensity Distribution of the Double-Slit Interference Pattern

INTERFERENCE

- The intensity of the resultant wave at P when two light waves of amplitudes E_{01} and E_{02} and phase difference ϕ are superimposed is given by

$$I_\theta = (E_{01})^2 + (E_{02})^2 + 2E_{01}E_{02}\cos\phi = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\phi$$

- Ratio of maximum and minimum intensity at the point P:

$$\frac{(I_\theta)_{max}}{(I_\theta)_{min}} = \left(\frac{E_{01}+E_{02}}{E_{01}-E_{02}}\right)^2 = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

(Q)

Consider interference between waves from two sources of intensities I_0 and $4I_0$. **Find intensities at point where phase difference is (i) π , (ii) $\pi/2$.**

Ans: (i) $5I_0$ **(ii)** I_0

Young's Interference Experiment

Young's Double-Slit Experiment

- Interference in light waves from two sources was first demonstrated by Thomas Young in 1801.
- Young's experiment provided the first conclusive proof of the wave nature of light and also provided the first direct measurement of the wavelength of light.
- Figure Y-1 shows geometric construction for describing Young's double-slit experiment (two-dimensional nature)
- Consider a plane wave of monochromatic light of wavelength λ is incident on the barrier containing two very narrow slits S_1 and S_2.

  Two slits act as two coherent sources and are separated by a distance *d*.
- When a viewing screen is placed at a perpendicular distance D from the barrier containing two slits, an interference pattern consisting of alternating bright and dark fringes are observed on the screen.
- At a point O on the screen, waves from S_1 and S_2 travel equal distances and arrive in phase. The two waves undergo constructive interference and a bright fringe is observed at O. This is called **central bright fringe.**

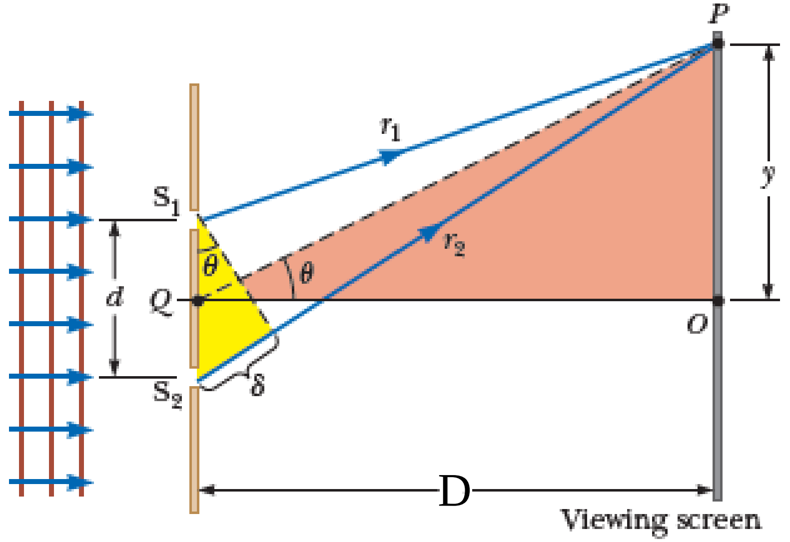


Figure Y-1

Assumption :

- slits → narrow
- D ≫ d

Young's Double-Slit Experiment

- Consider a point P on the screen at a distance y_n from O. Waves from two slits reach at the point P after travelling distances S_1P and S_2P respectively and will be in phase or out of phase depending upon the path $S_2P - S_1P = S_2N = \delta$.

  Now in practice S_1S_2 is very small, and QP is very much larger than S_1S_2 . Thus S_1N meets OP at right angles.

  It follows that $\angle PQO = \angle S_2S_1N = \theta(\text{say})$

- From right angled triangle ΔS_2S_1N ,

$$\sin\theta = \frac{S_2N}{S_1S_2} = \frac{\delta}{d} \qquad \text{..........} \quad (1)$$

From right angled triangle ΔPOQ

$$\tan\theta = \frac{PO}{QO} = \frac{y_n}{D} \qquad \text{..........} \quad (2)$$

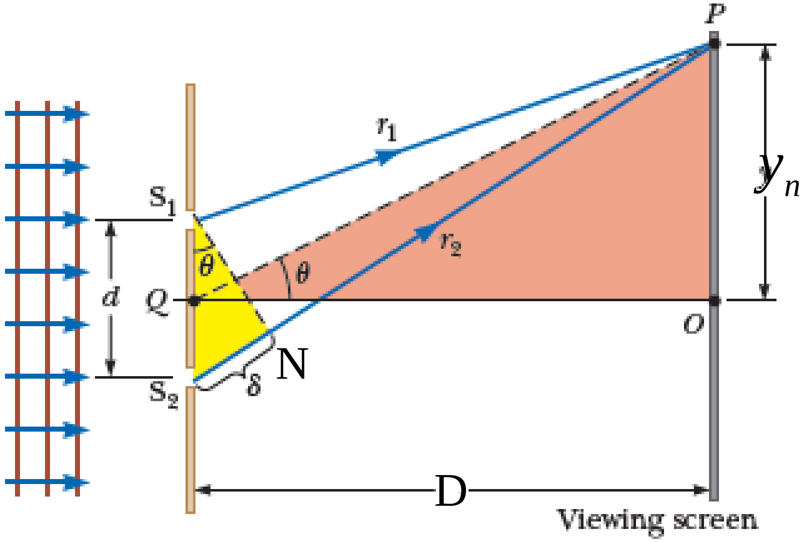


Figure Y-1

Assumption :
- slits $\rightarrow$ narrow
- $D \gg d$

INTERFERENCE

Young's Double-Slit Experiment

- Since θ *is* very small, about 0.5^0 for 20 fringes when D is 1 meter, $\tan\theta \approx \sin\theta$.

So, $$\frac{y_n}{\mathrm{D}} = \frac{\delta}{d}$$

$$\therefore \text{Path difference, } \delta = \frac{y_n d}{\mathrm{D}} \quad .... \quad (3)$$

For Bright Fringe

If δ is either zero or some integer multiple of the wavelength, the two waves are in phase at point P and constructive interference results. Therefore, the condition for bright fringes, or constructive interference, at point P is

$$\delta = n\lambda \qquad \text{where } n = 0, \pm 1, \mp 2, \cdots$$

or, $$\frac{y_n d}{\mathrm{D}} = n\lambda$$

$$\therefore \quad y_n = \frac{n\lambda \mathrm{D}}{d}$$

This gives the position of various bright fringe measured along the screen from O.

Young's Double-Slit Experiment

For Dark Fringe

If δ is an odd multiple of λ/2, the two waves arriving at point P are 180° out of phase and give rise to destructive interference. Therefore, the condition for dark fringes, or destructive interference, at point P is

$$\delta = (2n+1)\frac{\lambda}{2} \qquad \text{where } n = 0, \pm 1, \mp 2, \cdots$$

$$\text{or, } \frac{y_n d}{\mathrm{D}} = (2n+1)\frac{\lambda}{2}$$

$$\therefore \quad y_n = \frac{(2n+1)}{2}\frac{\lambda \mathrm{D}}{d}$$

This gives the position of various dark fringe measured along the screen from O.

Fringe Width (β)

The distance between any two consecutive bright fringes or dark fringes is known as fringe width.

Distance between two consecutive bright fringes:

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda \mathrm{D}}{d} - \frac{n\lambda \mathrm{D}}{d} \qquad \therefore \quad \beta = \frac{\lambda \mathrm{D}}{d}$$

Interference in Thin Film

- Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.
- Figure ITF-1 shows a diagram and Figure ITF-2 shows a photograph showing interference of light reflected from a thin film.

Interference between rays reflected from the two surfaces of a thin film

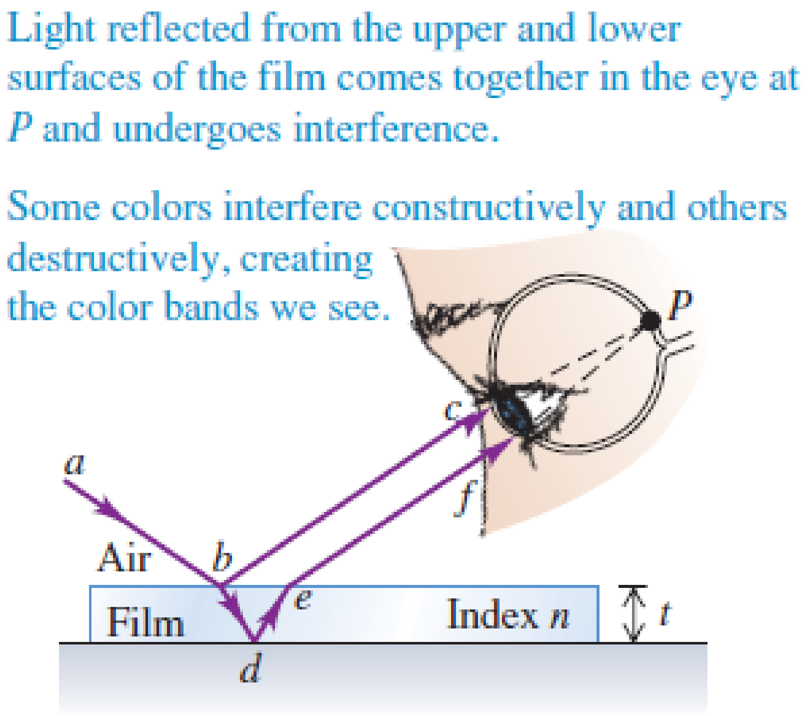


Figure ITF-1

The rainbow fringes of an oil slick on water

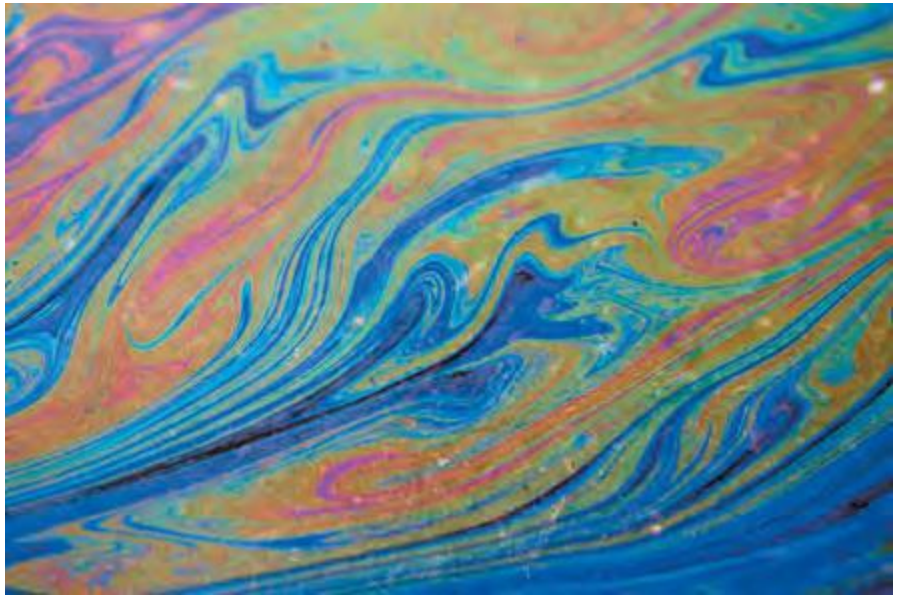


Figure ITF-2

Interference in Thin Film

INTERFERENCE

- The colors that we see when sunlight falls on a soap bubble, an oil slick, or a ruby-throated hummingbird are caused by the interference of light waves reflected from the front and back surfaces of thin transparent films. The film thickness is typically of the order of magnitude of the wavelength of light.
- Thin films deposited on optical components, such as camera lenses, can reduce reflection and enhance the intensity of the transmitted light. Thin coatings on windows can enhance the reflectivity for infrared radiation while having less effect on the visible radiation. In this way it is possible to reduce the heating effect of sunlight on a building.
- **Figure ITF-3 shows a transparent film of uniform thickness d illuminated by monochromatic light of wavelength A from a point source S.**

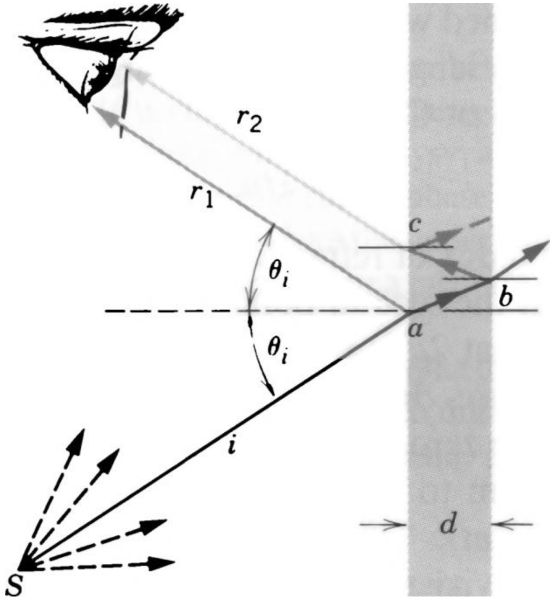


Figure ITF-3

A thin film is viewed by light reflected from a source S. Waves reflected from the front and back surfaces enter the eye as shown, and the intensity of the resultant light wave is determined by the phase difference between the combining waves. The medium on either side of the film is assumed to be air.

Figure ITF-4 shows the Light paths through a thin film.

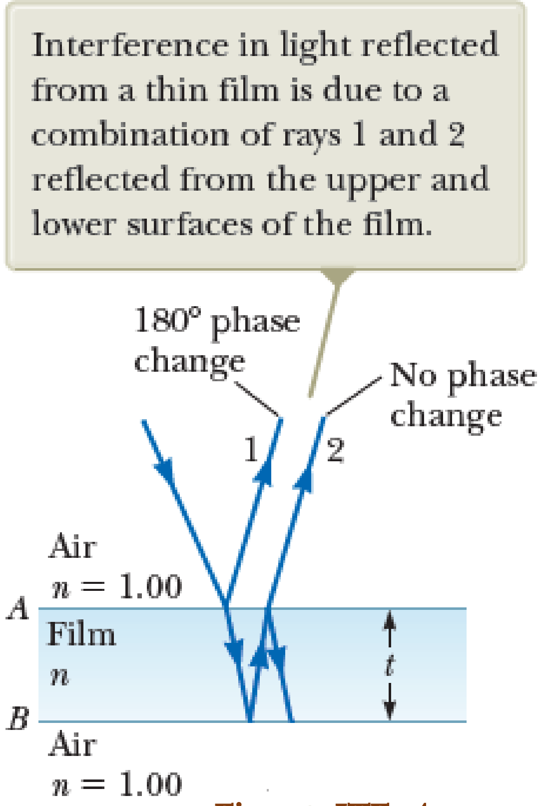
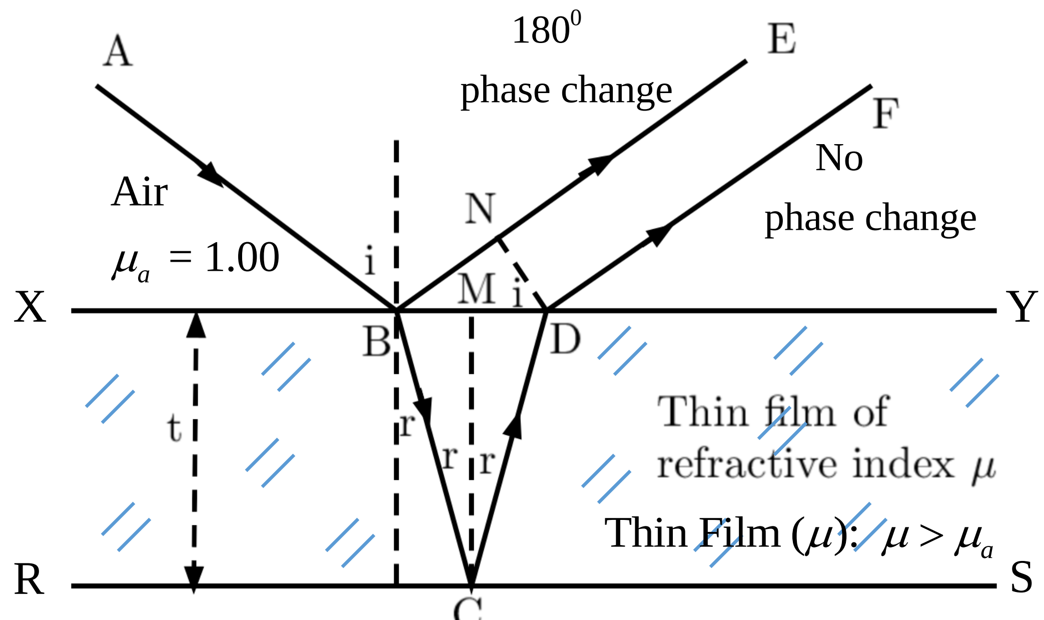


Figure ITF-4

Interference in Thin Film

Interference in Thin Film Due to Reflected Light

- Consider a thin parallel-sided film of thickness t and refractive index μ.



DN ⊥ BE
&
CM ⊥ BD

Figure ITF-5

A monochromatic ray of light AB incident on surface XY at an angle 'i' is partly reflected along BE and partly refracted into the medium along BC, making an angle of refraction 'r'. At C part of it is reflected along CD and finally emerges out along DF, which is parallel to BE.

The optical path difference between the two rays BE and BCDF is

$$x_0 = \mu(\text{BC+CD}) - \mu_a(\text{BN})$$

From the electromagnetic theory, it is revealed that when a ray of light is reflected by the denser medium, then it suffers a phase difference of π and corresponding path difference of $\frac{\lambda}{2}$.

The true path difference between the waves emerging from B and D is

$$\boxed{x_t = \mu(\text{BC+CD}) - \left(\text{BN} + \frac{\lambda}{2}\right)}$$

INTERFERENCE

Interference in Thin Film

Interference in Thin Film Due to Reflected Light

Path Difference :

$$x_t = \mu(\mathrm{BC}+\mathrm{CD}) - \left(\mathrm{BN} + \frac{\lambda}{2}\right)$$

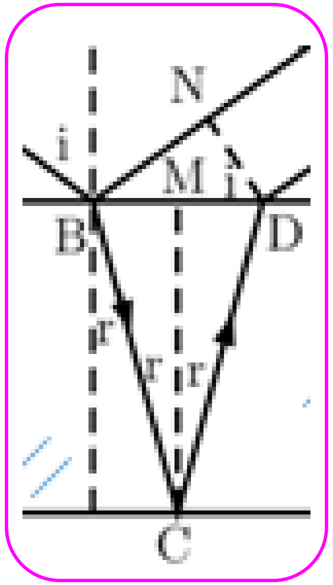
$$= \mu(2\mathrm{BC}) - \mathrm{BD}\sin i - \frac{\lambda}{2} \qquad \left\{ \begin{array}{l} \because \Delta\mathrm{BMC} \cong \Delta\mathrm{DMC} \Rightarrow \mathrm{BC} = \mathrm{CD} \\ \text{\& In right anlgled trianlge } \Delta\mathrm{BND},\ \sin i = \dfrac{\mathrm{BN}}{\mathrm{BD}} \end{array} \right\}$$

$$= 2\mu\left(\frac{t}{\cos \mathrm{r}}\right) - 2\mathrm{BM}\left(\mu \sin\mathrm{r}\right) - \frac{\lambda}{2} \qquad \left\{ \begin{array}{l} \because \Delta\mathrm{BMC} \cong \Delta\mathrm{DMC} \Rightarrow \mathrm{BM} = \mathrm{MD} \\ \text{\&}\ \ \mu = \dfrac{\sin i}{\sin r} \end{array} \right\}$$

$$= 2\mu\left(\frac{t}{\cos \mathrm{r}}\right) - 2\left(t \tan \mathrm{r}\right)\left(\mu \sin\mathrm{r}\right) - \frac{\lambda}{2} \qquad \left\{ \because \text{In right anlgled trianlge } \Delta\mathrm{BND},\ \tan\mathrm{r} = \frac{\mathrm{BM}}{\mathrm{MC}} = \frac{\mathrm{BM}}{t} \right\}$$

$$= \frac{2\mu t}{\cos \mathrm{r}} - \frac{2\mu t \sin^2 \mathrm{r}}{\cos \mathrm{r}} - \frac{\lambda}{2} \qquad = \frac{2\mu t}{\cos \mathrm{r}}\left(1 - \sin^2 \mathrm{r}\right) - \frac{\lambda}{2}$$

$$\therefore \quad \boxed{x_t = 2\mu t \cos r - \frac{\lambda}{2}}$$



INTERFERENCE

Interference in Thin Film Due to Reflected Light

Bright Fringes :

$$\text{path difference } (x_t) = n\lambda \qquad \text{where } n = 0,1,2,...$$

$$\text{or,}\quad 2\mu t\cos r - \frac{\lambda}{2} = n\lambda$$

$$\therefore\quad \boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$$

Dark Fringes :

$$\text{path difference } (x_t) = (2n+1)\frac{\lambda}{2} \qquad \text{where } n = 0,1,2,...$$

$$\text{or,}\quad 2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore\quad 2\mu t\cos r = (n+1)\lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore (n+1) can as well be replaced by nλ for simplicity in expression.

$$\boxed{\therefore\quad 2\mu t\cos r = n\lambda}$$

Interference in Thin Film

Interference in Thin Film Due to Reflected Light

- Consider a thin parallel-sided film of thickness t and refractive index μ.

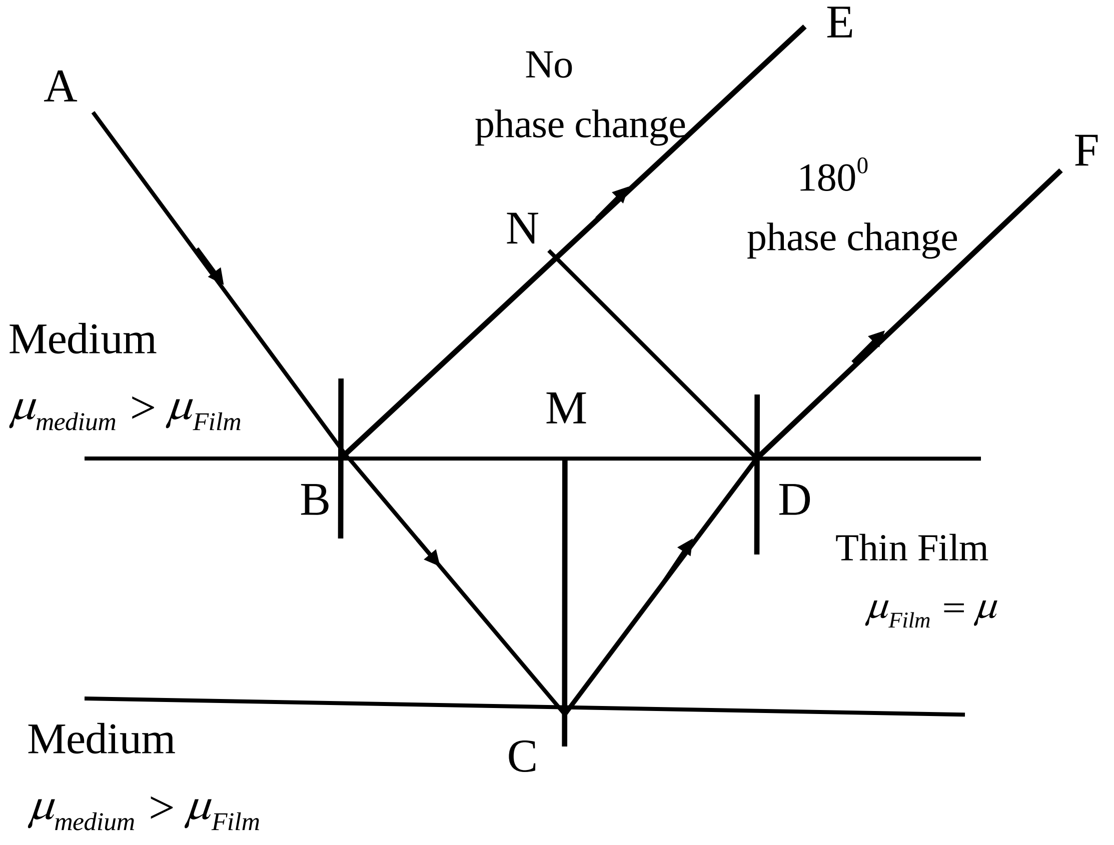


Figure ITF-6

True path difference between the relfected light from upper and lower surfaces of thin film

$$x_t = \left\{ \mu(\text{BC+CD}) + \frac{\lambda}{2} \right\} - \text{BN}$$

Condition of Constructive Interference

$$\boxed{2\mu t\cos r = (2n-1)\frac{\lambda}{2}}$$

Condition of Constructive Interference

$$\boxed{2\mu t\cos r = n\lambda}$$

INTERFERENCE

Interference in Thin Film

Interference in Thin Film Due to Reflected Light

- Consider a thin parallel-sided film of thickness t and refractive index μ.

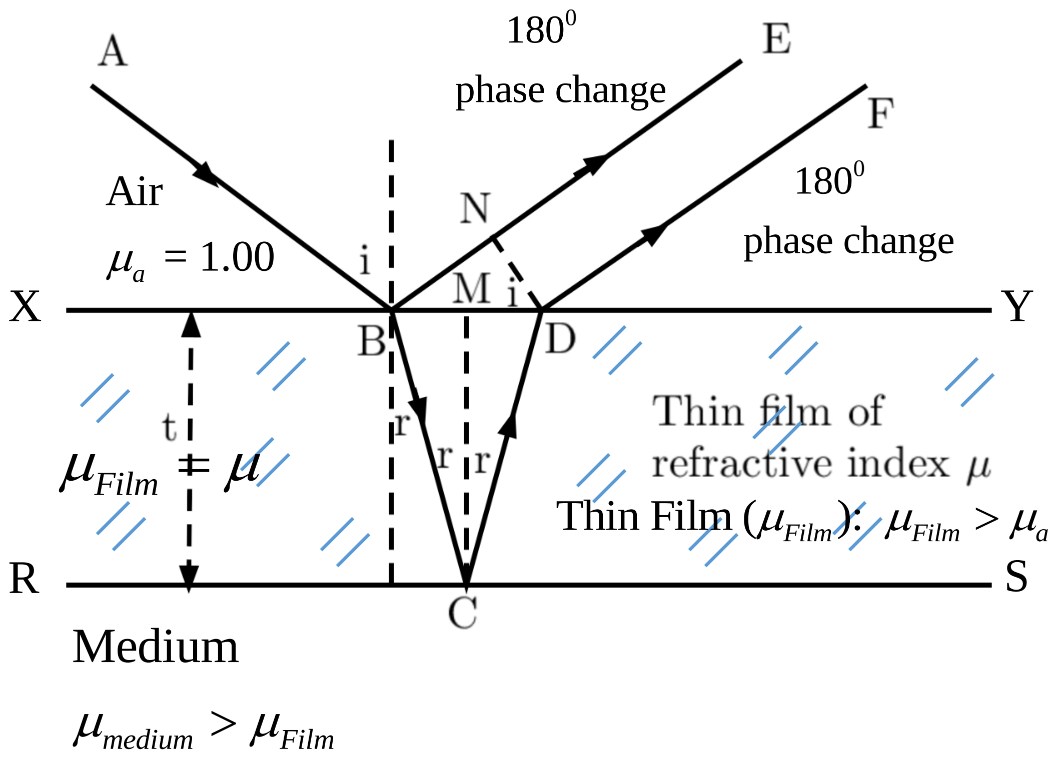


Figure ITF-7

True path difference between the relfected light from upper and lower surfaces of thin film

$$x_t = \left\{\mu(\mathrm{BC{+}CD}) + \frac{\lambda}{2}\right\} - \left(\mathrm{BN} + \frac{\lambda}{2}\right)$$

Condition of Constructive Interference

$$\boxed{2\mu t\cos r = n\lambda} \qquad \text{where } n = 0,1,2,\cdots$$

Condition of Constructive Interference

$$\boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}} \qquad \text{where } n = 0,1,2,\cdots$$

INTERFERENCE

Interference in Thin Film

Lenses are often coated with thin films of transparent substances such as MgF_2 (n = 1.38) to reduce the reflection from the glass surface. How thick a coating is needed to produce a minimum reflection at the center of the visible spectrum (A = 550 nm)?

- We assume that the light strikes the lens at near normal incidence (θ) is exaggerated for clarity in Figure IP-1, and we seek destructive interference between rays r_1 and r_2 .
- Condition for Destructive Interference:

$$2nt = \left(n + \frac{1}{2}\right)\lambda \qquad n = 0,1,2,\cdots \quad \text{(minima)}$$

Solving for t and putting *m = 0* we obtain

$$t = \frac{\left(n + \frac{1}{2}\right)\lambda}{2n} = \frac{\lambda}{4n} = 100 \text{ nm}$$

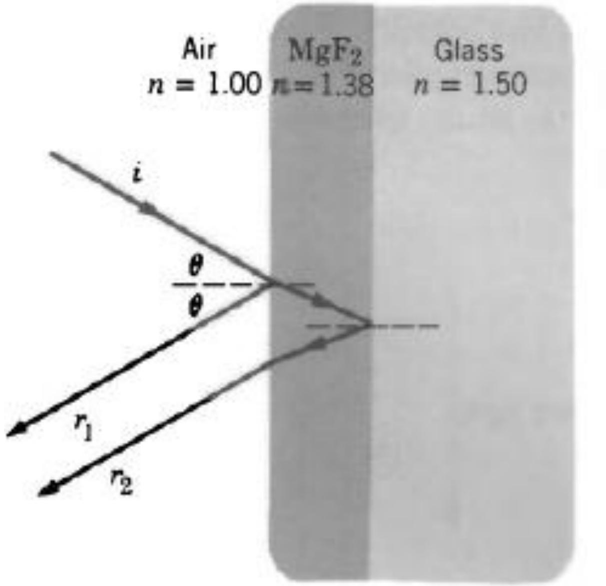


Figure IP-1

Newton's Rings

- Another method for observing interference in light waves is to place a plano-convex lens on top of a flat glass surface as shown in Figure NR-1. With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some nonzero value at point *P*. If the radius of curvature R of the lens is much greater than the distance r and the system is viewed from above, a pattern of light and dark rings is observed as shown in Figure NR-2. These circular fringes, discovered by Newton, are called **Newton's rings.**

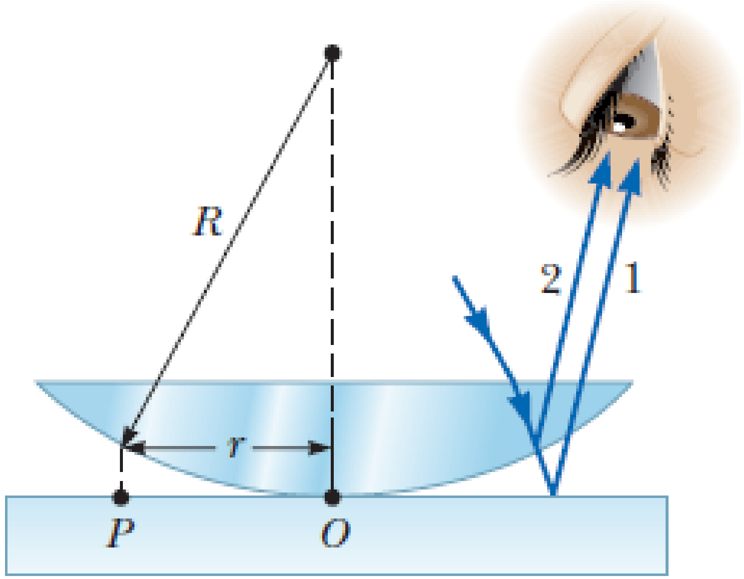


Figure NR-1

The combination of rays reflected from the flat plate and the curved lens surface gives rise to an interference pattern known as Newton's rings.

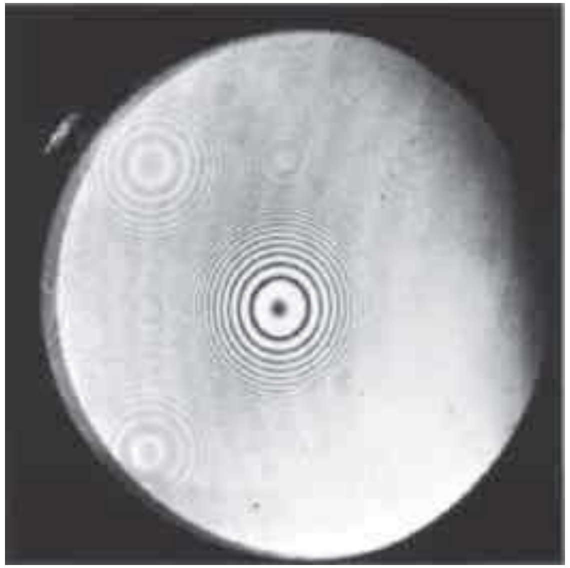
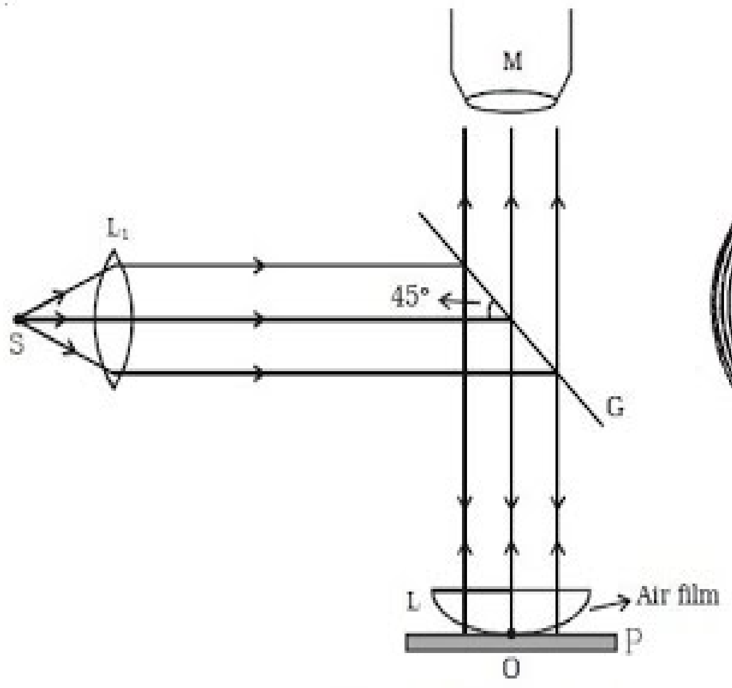


Figure NR-2

Photograph of Newton's rings.

Newton's Rings

- **Circular interference fringes** can be produced by enclosing a very thin film of air or any other transparent medium of varying thickness between a plano-convex lens of a larger radius of curvature and a glass plate. Such fringes were first obtained by Newton and are known as **Newton's Rings**.
- **Newton's rings** are formed due to the interference between the light waves reflected from the top and bottom surfaces of air film between the plano-convex lens and glass sheet.
- The fringes are circular because the air film is symmetric about the point of contact of the plano-convex lens with glass plate.



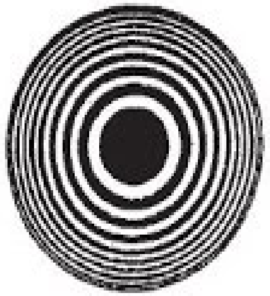


Figure N-1

Suppose a monochromatic source of light S is kept at the focus of convex lens L_1 so that the light beam on passing through it becomes parallel and strikes the plane glass plate G, placed at an angle of 45° to the horizontal. The beam now travels downwards and strikes the plano-convex lens L as shown in Figure N-1. The plano-convex lens is placed on a plane glass plate P with its curved surface touching the glass plate P. Since there is a thin air film in between the lens and plane glass plate, interference takes place between the light rays reflected from the lower surface of plano-convex lens and upper surface of plane glass plate P. The interference fringes consist of dark and bright concentric rings, known as Newton's rings and these are seen through a traveling microscope kept vertically above the lens L as shown in Figure N-1.

Newton's Rings

- Let R be the radius of curvature of lens L, t be the thickness of thin air film and r be the radius of n^{th} dark or bright ring as shown in Figure N-2.

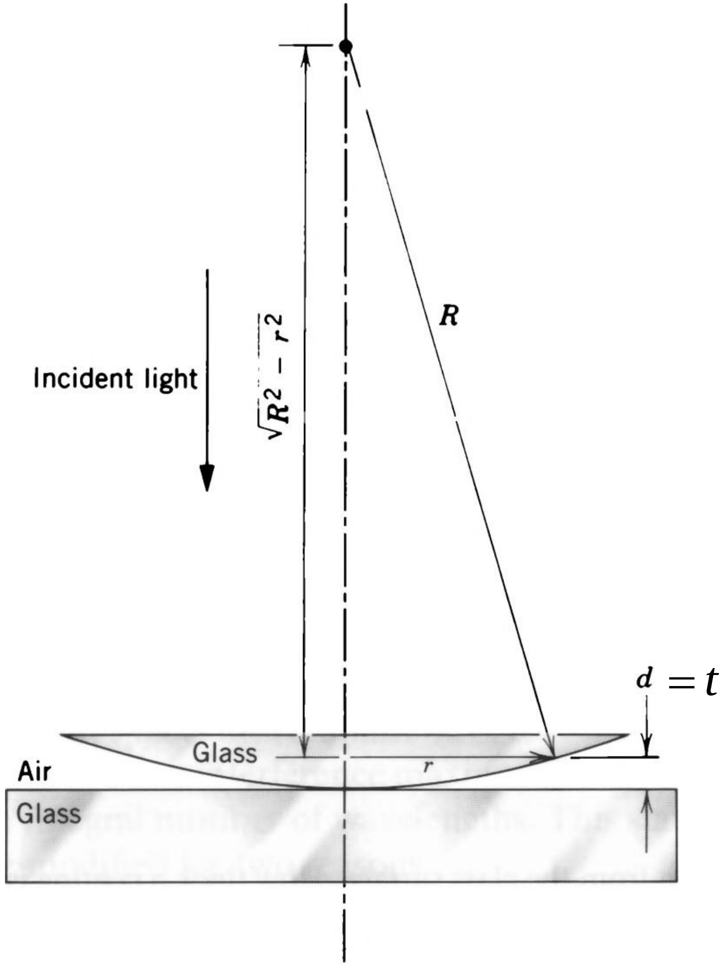


Figure N-2

From Figure N-2

$$t = R - \sqrt{R^2 - r^2}$$

or, $$t = R - \left[R^2\left\{1 - \frac{r^2}{R^2}\right\}\right]^{\frac{1}{2}}$$

or, $$t = R - R\left[1 - \frac{r^2}{R^2}\right]^{\frac{1}{2}}$$

or, $$t = R - R\left[1 - \frac{1}{2}\frac{r^2}{R^2}\right] \qquad \left(\because \frac{r}{R} << 1\right)$$

{Using binomial theorem and neglecting higher powers terms}

or, $$t = R - R + \frac{r^2}{2R}$$

$$\therefore \quad \boxed{2t = \frac{r^2}{R}} \quad \text{....................} \quad (1)$$

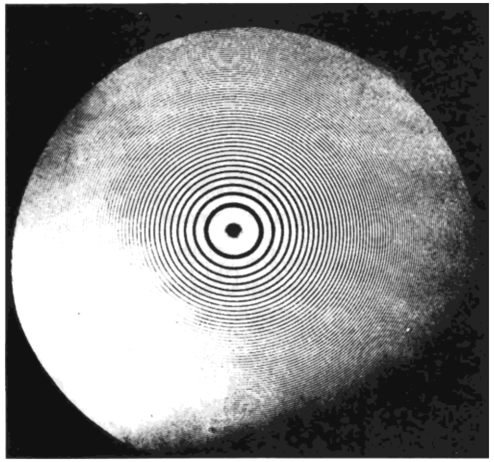


Figure N-3
Newton's Rings

Newton's Rings

INTERFERENCE

- **For Bright Fringes,** We have

$$2\mu t\cos\theta \;=\; (2n-1)\frac{\lambda}{2}$$

For air, $\mu = 1$ and for small θ, $\cos\theta \approx 1$

So

$$2t = (2n-1)\frac{\lambda}{2} \quad .... \quad (2)$$

From equations (1) and (2), we get

$$\frac{r^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\therefore \qquad r = \sqrt{(2n-1)\frac{\lambda R}{2}}$$

This relation gives the radius of n^{th} bright ring from center.

For diameter of nth bright ring

$$\boxed{D_n = 2r = 2\sqrt{(2n-1)\frac{\lambda R}{2}}}$$

- **For Dark Fringes,** We have

$$2\mu t\cos\theta \;=\; n\lambda$$

For air, $\mu = 1$ and for small θ, $\cos\theta \approx 1$

So

$$2t = n\lambda \quad .... \quad (3)$$

From equations (1) and (3), we get

$$\frac{r^2}{R} = n\lambda$$

$$\therefore \qquad r = \sqrt{n\lambda R}$$

For diameter of nth dark ring

$$\boxed{D_n = 2\sqrt{n\lambda R}}$$

This relation gives the radius of n^{th} bright ring from center.

When n=1, then $D_n = 0 \Rightarrow$ The centre of the ring system is dark.

Newton's Rings

Fringe width in Newton's rings is not equally spaced and it goes on decreasing with the increasing number of fringes:

- **Diameter of n^{th} dark ring is** $D_n = 2\sqrt{n\lambda R}$

So,

Diameter of 1^{st} dark ring is $D_1 = 2\sqrt{\lambda R}$.

& Diameter of 4^{th} dark ring is $D_1 = 2\sqrt{4\lambda R} = 4\sqrt{\lambda R}$.

$$\therefore \; D_4 - D_1 = 4\sqrt{\lambda R} - 2\sqrt{\lambda R} = 2\sqrt{\lambda R} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Diameter of 9^{th} dark ring is $D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$..

Diameter of 16^{th} dark ring is $D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R}$.

$$\therefore \; D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Equations (4) and (5) show that fringe width in Newton's rings are not equally spaced and it goes on decreasing with the increasing number of fringes.

That means if we go far from center, rings are found to be closely packed

Determination of Wavelength of Sodium Light by Newton's Rings Method:

- **Diameter of n^{th} dark ring is** $D_n = 2\sqrt{n\lambda R}$.

Squaring, we get

$$D_n^2 = 4n\lambda R \qquad .... \qquad (6)$$

For m$^{\text{th}}$ dark ring $(m > n)$

$$D_m^2 = 4m\lambda R \qquad .... \qquad (7)$$

Subtracting equation (6) from equation (7) yields

$$D_m^2 - D_n^2 = 4m\lambda R - 4n\lambda R = 4(m-n)\lambda R$$

$$\therefore \qquad \boxed{\lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}}$$

This relation is used to measure the wave length of sodium light.

Newton's Rings

Determination of Refractive Index of Transparent Liquid by Newton's Rings Method:

- The experimental set up for determination of refractive index of transparent liquid μ by Newton's rings method is as shown in Figure N-4.

- First, we measure diameter of n^{th} dark ring without liquid and then measure m^{th} dark ring ($m > n$) without liquid.

  Now,

  For diameter of n^{th} and m^{th} $(m > n)$ dark ring without liquid,

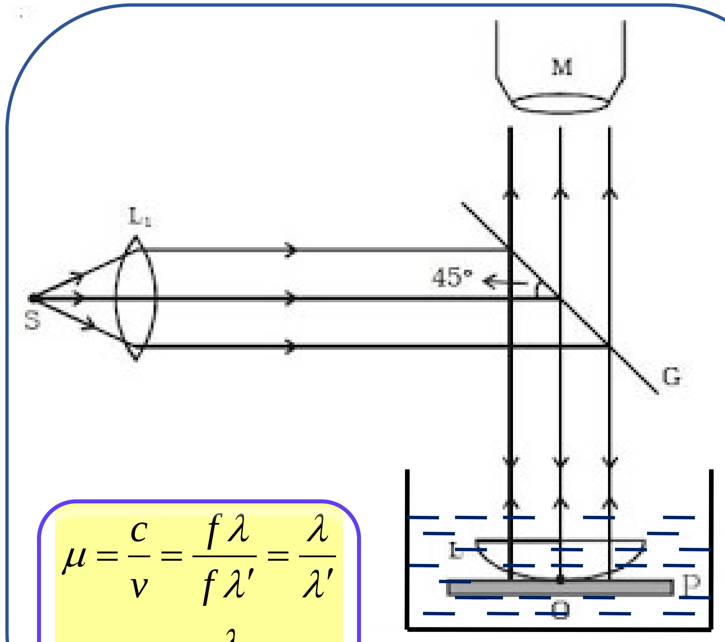
$$\boxed{D_m^2 - D_n^2 = 4(m-n)\lambda R} \qquad \text{....................} \qquad (8)$$

- Then, we measure diameter of n^{th} dark ring with liquid and then measure m^{th} dark ring ($m > n$) with liquid.

  Now,

  For diameter of n^{th} and m^{th} $(m > n)$ dark ring with liquid,

$$\boxed{D_m'^2 - D_n'^2 = \frac{4(m-n)\lambda R}{\mu}} \qquad \text{....................} \qquad (9)$$



$$\mu = \frac{c}{v} = \frac{f\lambda}{f\lambda'} = \frac{\lambda}{\lambda'}$$

$$\Rightarrow \lambda' = \frac{\lambda}{\mu}$$

Figure N-4

- From Eq.(8) and Eq. (9), we get $D_m'^2 - D_n'^2 = \dfrac{D_m^2 - D_n^2}{\mu}$ $\quad \therefore \quad \boxed{\mu = \dfrac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2}}$

This is the required relation for the measurement of refractive index of given transparent liquid.

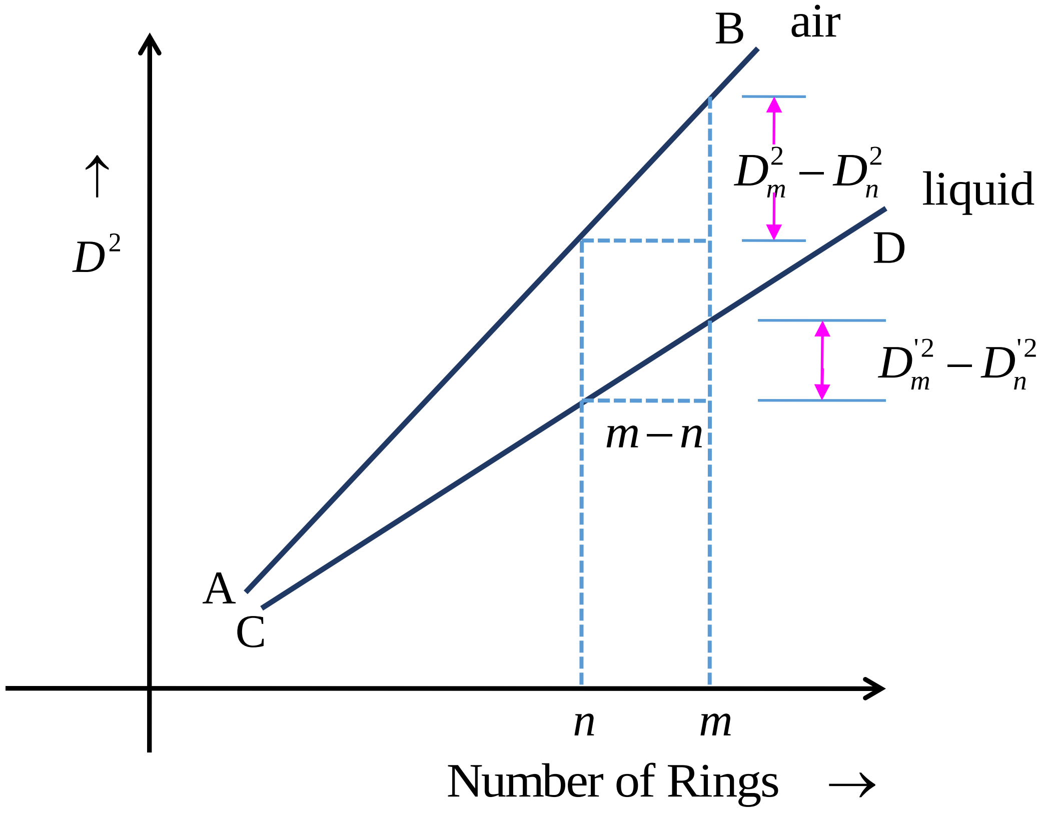
Determination of Refractive Index of Transparent Liquid by Newton's Rings Method:

- Refractive index of transparent liquid:

$$\mu = \frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2}$$

$$= \frac{\dfrac{D_m^2 - D_n^2}{m-n}}{\dfrac{D_m'^2 - D_n'^2}{m-n}}$$

$$\therefore \boxed{\mu = \frac{\text{slope of line AB}}{\text{slope of line CD}}}$$



References

1. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics

2. **Halliday and Resnick**, Fundamental of Physics

3. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 1, Fourth Edition

4. **Hugh D. Young, Roger A. Freedman,** University Physics with Modern Physics, 13TH Edition


Thank you