Homework 3

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1 Knights and Knaves

One day a traveller was wandering around the island of Knights and Knaves, when he encountered two local inhabitants ,P and Q. The traveller asked: "Is any of you a knave?".P replied: "At least one of us is a knave".

- We can not tell what p and q are because we only have one condition, at the moment all we know is that at least one of them must be a knave; in order to be able to tell what the other person is.

Later on, the traveller met two other locals, A and B. He asked whether either of them is a knight, A replied: "If B is a knave, then I am a knave too".

- A must be a Knave and B must be a Knight.

2 Logical Identities

Simplify the following propositions. Show all steps of your solutions.
1.$\neg(p \to (q \to p))$
$\neg(\neg p \vee (q \to p))$
$(p \vee (q \vee p))$
$p \wedge (q \wedge p)$
$q \wedge p$

2.$\neg((p \vee q) \to (q \wedge p))$
$\neg(\neg(p \wedge q) \vee (q \vee p))$
$(p \wedge q) \wedge \neg(q \vee p)$
$(p \wedge q) \wedge (\neg q \wedge \neg p)$
$p \wedge q \wedge \neg p \wedge \neg q$
$(p \wedge \neg p) \wedge (q \wedge \neg q)$
$F \wedge F$
F

3 Logical Equivalences

1. p $\rightarrow (q \rightarrow r) and (p \wedge q) \rightarrow r$
$\neg p \vee (q \rightarrow r) and \neg (p \wedge q) \vee r$
$\neg p \vee (\neg q \vee r) and (\neg p \vee \neg q) \vee r$
$\neg p \vee \neg q \vee r and\ \neg p \vee \neg q \vee r$
The expressions are equal

2. p $\rightarrow (q \rightarrow r) and (p \rightarrow q) \rightarrow r$
$\neg p \vee (q \rightarrow r) and (\neg p \vee q) \rightarrow r$
$\neg p \vee (\neg q \vee r) and \neg (\neg p \vee q) \vee r$
$\neg p \vee \neg q \vee r and$p $\wedge \neg q \vee r$
The expressions aren't equal

4 Logical Consequence

1. The inference is valid because Jimmy is smart, he must also be rich.
2. The inference is not valid because simply because continents could be surrounded by water and aren't classified as islands.