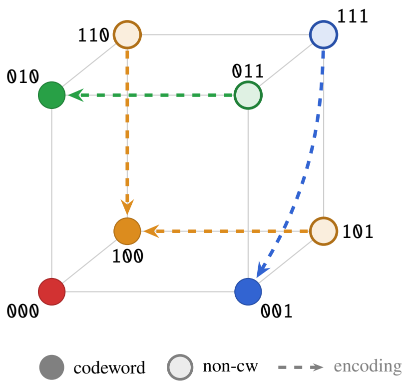
Example: An Optimal (n=3, M=4) Code for Bernoulli(0.3)

(a) Encoding on the Hamming Cube $\{0, 1\}^3$



(b) Encoding Table

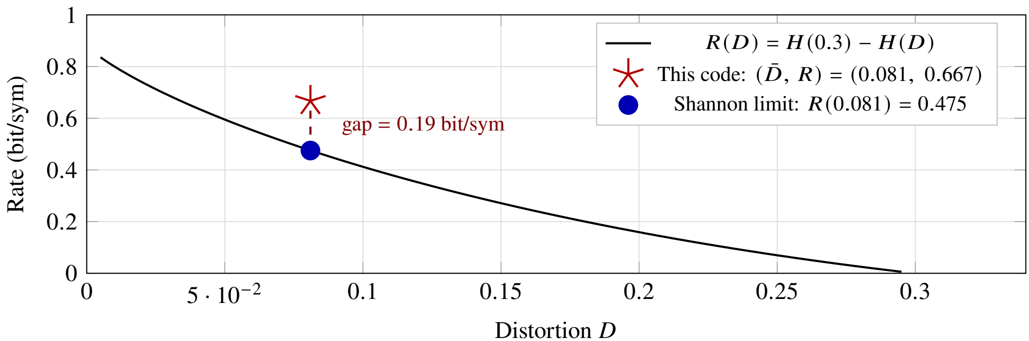
Source x^3	$p(x^3)$	Codeword $\hat{c}$	Output	d/n
000	0.343	000	00	0
001	0.147	001	01	0
010	0.147	010	10	0
100	0.147	100	11	0
011	0.063	010	10	1/3
101	0.063	100	11	1/3
110	0.063	100	11	1/3
111	0.027	001	01	2/3

The encoder transmits $\log_2 4 = 2$ bits per block of $n = 3$ source symbols.

$$R = \frac{1}{n} \log_2 M = \frac{2}{3} \approx 0.667 \text{ bit/sym}$$

$$\bar{D} = \sum_{x^3} p(x^3) \frac{d(x^3, \hat{c})}{n} = 0.081$$

(c) Comparison with the Shannon Limit



This finite code operates 0.19 bit/sym above the Shannon limit — Section 6 quantifies how this gap shrinks as the block length n grows.