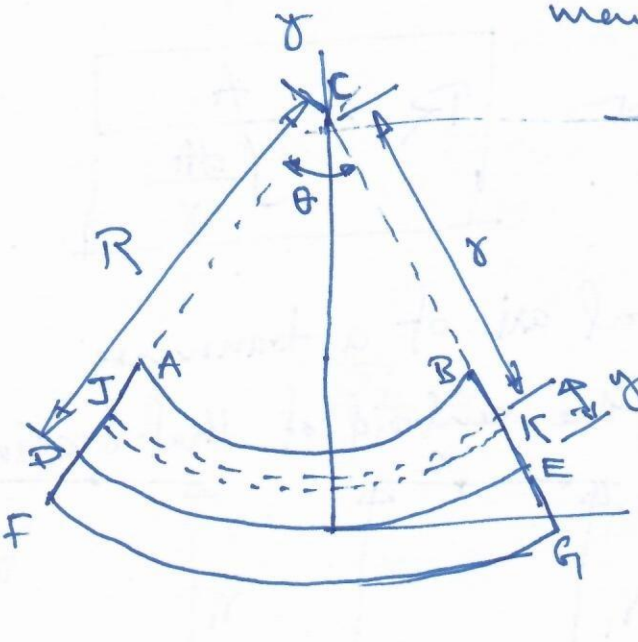
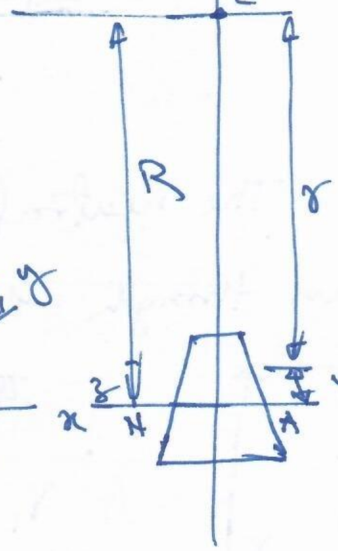
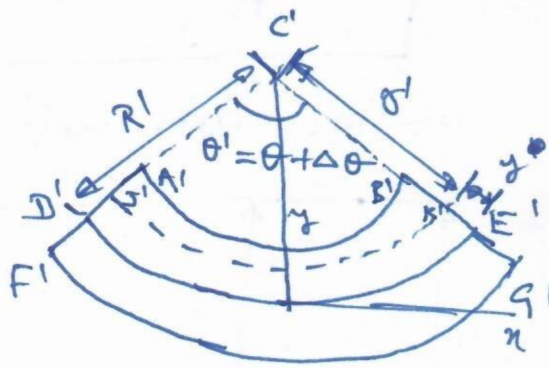
Bending of Curved Beams:

Assumptions:
(a) Uniform cross section
(b) Section possessing a plane of symmetry
(c) Stresses are below proportional limit

Curved member: When the radius of curvature & the dimensions of the cross section of the member are of the same order.







$R\theta = R'\theta'$

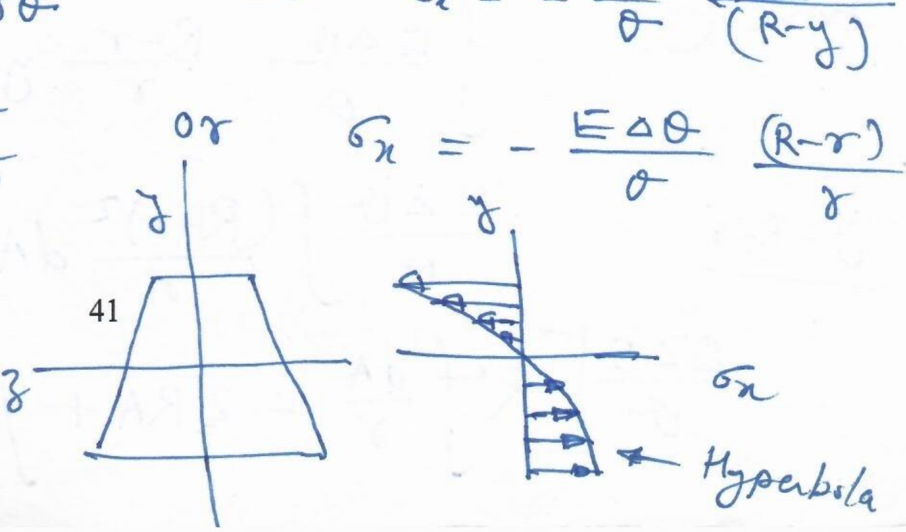
Deformation of $JK \Rightarrow \delta = \gamma'\theta' - \gamma\theta$

$\gamma = R - y \quad , \gamma' = R' - y \quad \Rightarrow \quad \delta = (R' - y)\theta' - (R - y)\theta$

$$\delta = -y\Delta\theta$$

$$\varepsilon_x = \frac{\delta}{\gamma\theta} = -\frac{y\Delta\theta}{\gamma\theta} \quad \Rightarrow \quad \varepsilon_x = -\frac{\Delta\theta}{\theta}\frac{y}{(R-y)}$$

$$\sigma_x = -\frac{E\Delta\theta}{\theta}\frac{y}{R-y} \quad \text{or} \quad \sigma_x = -\frac{E\Delta\theta}{\theta}\frac{(R-\gamma)}{\gamma}$$

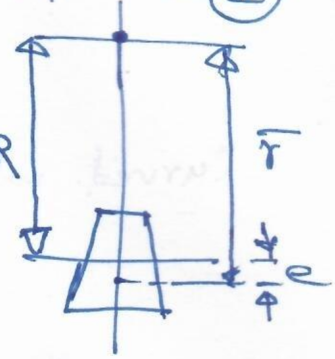


Location of neutral surface:

$$\int \sigma_x \, dA = 0 \quad \text{—(1)} \quad \& \quad \int(-y\,\sigma_x \, dA) = M \quad \text{—(2)}$$

from (1)

$$-\int \frac{E\Delta\theta}{\theta}\,\frac{R-r}{r}\, dA = 0$$

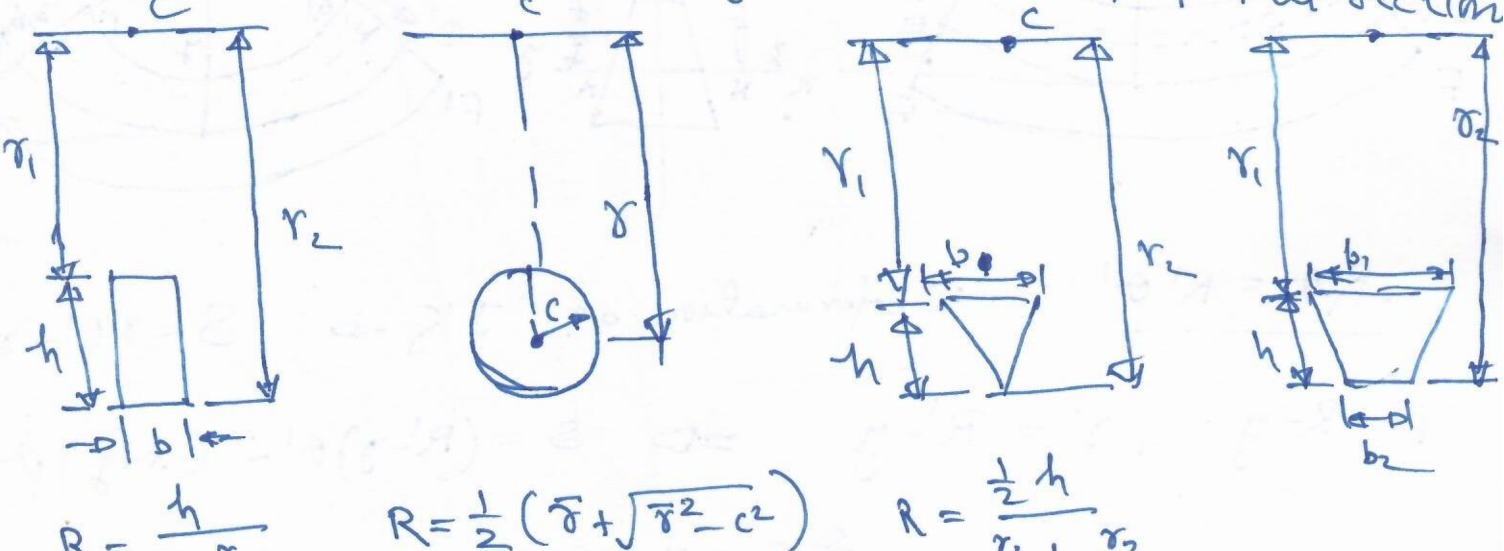


$$\int \frac{R-r}{r}\, dA = 0$$

$$R\int \frac{dA}{r} - \int dA = 0 \Rightarrow \boxed{R = \frac{A}{\int \frac{dA}{r}}}$$

$$\boxed{\bar{r} = \frac{1}{A}\int r\, dA}$$

The neutral axis of a transverse section does not pass through the centroid of that section



$$R = \frac{h}{\ln\frac{r_2}{r_1}} \qquad R = \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right) \qquad R = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1}$$

from (2)

$$\int \frac{E\Delta\theta}{\theta}\,\frac{R-r}{r}\, y\, dA = M$$

$y = R-r$

$$\frac{E\Delta\theta}{\theta}\int \frac{(R-r)^2}{r}\, dA = M$$

$$\frac{E\Delta\theta}{\theta}\left[R^2\int\frac{dA}{r} - 2RA + \int r\, dA\right] = M$$

$$R\int \frac{dA}{r} = A \quad \& \quad \int r\,dA = \bar{r}A$$

$$\frac{E\Delta\theta}{\theta} = \frac{M}{A(\bar{r}-R)}$$

$\Delta\theta > 0$ for $M > 0$

$\bar{r} - R > 0$ ~~for~~ or $R < \bar{r}$

$$\bar{r} - R = e$$

$$\frac{E\Delta\theta}{\theta} = \frac{M}{Ae}$$

$$\sigma_x = -\frac{My}{Ae(R-y)}$$

$$\sigma_x = \frac{M(r-R)}{Aer}$$

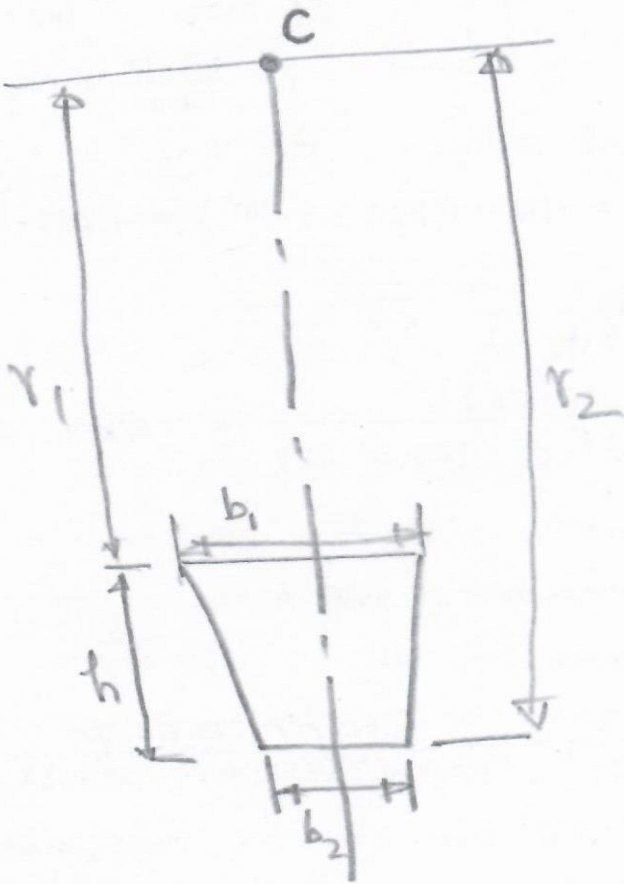
Change in curvature of neutral surface.

$$\frac{1}{R'} = \frac{1}{R}\frac{\theta'}{\theta}$$

$$\theta' = \theta + \Delta\theta$$

$$\frac{1}{R'} = \frac{1}{R}\left(1 + \frac{\Delta\theta}{\theta}\right) = \frac{1}{R}\left(1 + \frac{M}{EAe}\right)$$

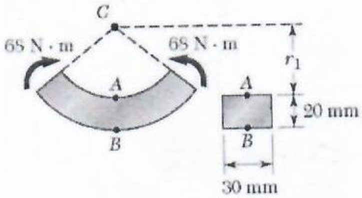
$$\frac{1}{R'} - \frac{1}{R} = \frac{M}{EAeR}$$



$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

Problem 4.156

4.156 For the curved bar and loading shown, determine the stress point A when (a) $r_1 = 30$ mm, (b) $r_1 = 50$ mm.



$h = 20\text{mm} \qquad A = (30)(20) = 600 \text{ mm}^2$

(a) $r_1 = 30\text{mm} \qquad r_2 = 50\text{mm}$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{20}{\ln\frac{5}{3}} = 39.15 \text{ mm.}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 40\text{mm.}$$

$$e = \bar{r} - R = 0.85\text{mm.}$$

$y_A = 39.15 - 30 = 9.15$ mm $\qquad r_A = r_1 = 30$ mm.

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(68)(0.00915)}{(600\times10^{-6})(0.00085)(0.03)} = -4.07 \text{ MPa}$$

-4.07 MPa ◄

(b) $r_1 = 50$ mm. $\quad r_2 = 70$ mm

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{20}{\ln\frac{7}{5}} = 59.44 \text{ mm.}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 60 \text{ mm.}$$

$$e = \bar{r} - R = 0.56 \text{ mm.}$$

$y_A = 59.44 - 50 = 9.44$ mm $\qquad r_A = r_1 = 50$ mm

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(68)(0.00944)}{(600\times10^{-6})(0.00056)(0.05)} = -38.2 \text{ MPa}$$

-38.2 MPa ◄