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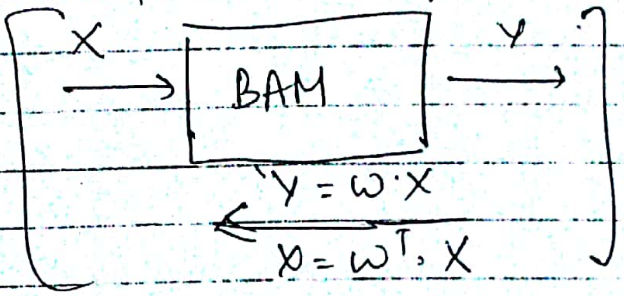
Bidirectional Associative Memory (BAM)

1. It is a two - layer network
2. Information flows from i/p to o/p → W

   and o/p to i/p → W'

Q. Consider a BAM network (with bipolar vectors) to map two simple letters (size of the letter → 5 X 3) to the following bipolar target (5/3 pattern) pattern for:



Pattern for (E) — Target (−1, 1)

```
X X X
X . .
X X X
X . .
X X X
```

Pattern for (H) — (1, 1)

```
X . X
X . X
X X X
X . X
X . X
```

1. Find the weight matrix with input pattern 'E' and 'H'.

2. Obtain the response of NET as 'E' and 'H' as input

3. Apply noisy symbol (i.e. faulty pattern) and check the response of the net.

Sol^n: I

x → +ve (+1)

• → -ve (−1)

Input pattern for E

$$E = [\,1\ 1\ 1 \mid 1\ {-1}\ {-1} \mid 1\ 1\ 1 \mid 1\ {-1}\ {-1} \mid 1\ 1\ 1\,]$$

target (−1, 1)

I/P pattern for H

$$H = [\,1\ {-1}\ 1 \mid 1\ {-1}\ 1 \mid 1\ 1\ 1 \mid 1\ {-1}\ 1 \mid 1\ {-1}\ 1\,]$$

target (+1, 1)

Weight matrix for pattern E : (W_1)

$$W_1 = E^T \cdot T$$

$$\underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{15 \times 1} \underbrace{\begin{bmatrix} -1 & 1 \end{bmatrix}}_{1 \times 2} = \underbrace{\begin{bmatrix} -1 & 1 \\ -1 & 1 \\ -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -1 & 1 \\ -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}}_{15 \times 2}$$

$$w_2 = H^T t$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}_{15 \times 1} \begin{bmatrix} 1 & 1 \end{bmatrix}_{1 \times 2} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 1 & 1 \\ 1 & 1 \\ -1 & -1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ -1 & -1 \\ 1 & 1 \\ 1 & 1 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}_{15 \times 2}$$

Overall weight $(w) = w_1 + w_2$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \\ 0 & 2 \\ 0 & 2 \\ 0 & -2 \\ 2 & 0 \\ 0 & 2 \\ 0 & 2 \\ 0 & 2 \\ 0 & 2 \\ 0 & 2 \\ 2 & 0 \\ 0 & 2 \\ -2 & 0 \\ 0 & 2 \end{bmatrix}_{15 \times 2}$$

$$Y = XW$$

Pattern E (input in bipolar form)

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 & -1 & -1 & | & 1 & 1 & 1 & | & 1 & -1 & -1 & | & 1 & 1 & 1 \end{bmatrix}_{1 \times 15} \begin{bmatrix} W \end{bmatrix}_{15 \times 2}$$

$$= \begin{bmatrix} -8 & 32 \end{bmatrix}_{1 \times 2}$$

∴ Applying bipolar step A.F.

$Y = [-1, 1] \rightarrow$ target of E

Pattern H.

$Y = HW$

$= [H][W]$

$= [8 \quad 22]$

$A.F. = [-1 \quad 1]$

(II) For false input, write reverse transpose

$W^T = [\ \ - \ \ ]_{2 \times 15}$

$X = YW^T$

~~Pattern of target E, target (-1, 1)~~

$X = [-1 \quad 1]_{1 \times 2}$ W^T (2×15)

Resultant order $= (1 \times 15)$

$$[-1 \ 1]\left[\begin{array}{ccc|ccc|ccc|ccc|ccc} 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 & 0 \\ 2 & 0 & 2 & 2 & -2 & 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 0 & 2 \end{array}\right]$$

$$= [2 \ -2 \ 2 \mid 2 \ -2 \ 2 \mid 2 \ 2 \ 2 \mid 2 \ -2 \ 2 \mid 2 \ 2 \ 2]$$

$$= \left[1\ -1\ 1 \mid 1\ -1\ 1 \mid 1\ 1\ 1 \mid 1\ -1\ 1 \mid 1\ 1\ 1 \right]$$

(not same as E or H)

→ Now target is (0, 1) for same Y. find input pattern

$$W = \begin{bmatrix} 0 & 2 \\ -2 & 0 \\ 0 & 0 \\ 0 & 2 \\ 0 & -2 \\ 2 & 0 \\ 0 & 2 \\ 0 & 2 \\ 0 & 2 \\ 0 & 2 \\ 0 & -2 \\ 2 & 0 \\ 0 & 2 \\ -2 & 0 \\ 0 & 2 \end{bmatrix} \quad 15 \times 2$$

Now,

$$X = \underset{\text{(target)}}{t} \cdot W^T$$

$$= \underset{1\times 2}{\begin{bmatrix} 0 & 1 \end{bmatrix}} \underset{2 \times 15}{\left[\begin{array}{ccc|ccc|ccc|ccc|ccc} 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 & 0 \\ 2 & 0 & 2 & 2 & -2 & 0 & 2 & 2 & 2 & 2 & -2 & 0 & 2 & 0 & 2 \end{array} \right]}$$

$$= \left[2\ 0\ 2 \mid 2\ -2\ 0 \mid 2\ 2\ 2 \mid 2\ -2\ 0 \mid 2\ 0\ 2 \right]$$

$$= \left[1\ 0\ 1 \mid 1\ -1\ 0 \mid 1\ 1\ 1 \mid 1\ -1\ 0 \mid 1\ 0\ 1 \right]$$

For C

C(1, -1) V(-1, -1)
A(1, 1) W = (1, -1)

```
x  x  x
x  •  •
x  •  •
x  •  •
x  x  x
```

$$C = [1\ 1\ 1 \mid 1\ -1\ -1 \mid 1\ -1\ -1 \mid 1\ -1\ -1 \mid 1\ 1\ 1]_{1\times 15}$$

$$w_1 = C^T \cdot t$$

$$= \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}_{15\times 1} \underbrace{\begin{bmatrix} 1 & -1 \end{bmatrix}}_{1\times 2} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}}_{15\times 2}$$

For U

$$\begin{array}{ccc} X & \cdot & X \\ X & \cdot & X \\ X & \cdot & X \\ X & \cdot & X \\ X & X & X \end{array}$$

$$U = \left[1\ {-1}\ 1 \mid 1\ {-1}\ 1 \mid 1\ {-1}\ 1 \mid 1\ {-1}\ 1 \mid 1\ 1\ 1 \right]$$

$$w_2 = U^T . t$$

$$w_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$$

Total $w = w_1 + w_2$.

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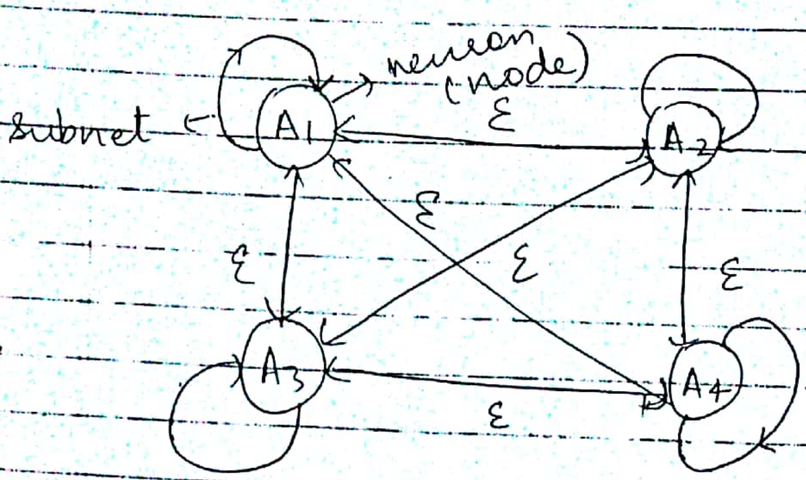
MAXNET (fixed weight network)

Architecture of Maxnet



1. Weight → fixed
update

Activation values

$$f(x) = \begin{cases} x, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Procedure

Step 1: Initialize activations and weights. Set $0 < \varepsilon < \frac{1}{n}$ (no. of nodes)

where ε → weight

$a_j(0)$ is the input to node A_j

$$w_{ij} = \begin{cases} 1, & \text{if } i = j \\ \varepsilon, & \text{if } i \neq j \end{cases}$$

Step 2: While stopping condition is false, do steps 3 to 5.

Step 3 Update the activation of each node for j = 1 to m.

$$a_j(new) = f\left[a_j(old) - \varepsilon \sum_{k \neq j} a_k(old)\right]$$

Step 4 Save activations for use in next iteration

$a_j(old) = a_j(new)$

for j = 1 to m

Step 5 Test for stopping condition (If more than one node has a non zero activation continue, otherwise stop).

Q. Construct a MAXNET with 4 neurons and inhibitory weights ($\varepsilon = 0.2$) Initial activations are:

First iteration

$a_1(0) = 0.3$, $a_2(0) = 0.5$, $a_3(0) = 0.7$, $a_4(0) = 0.9$

(carry out 5 iterations to get the final value)

Sol^n: Step 1

for the 4 nodes

Initial values (0.3, 0.5, 0.7, 0.9)

j = 1 to 4 (4 nodes)

updated values

$$a_1(1) = f\left[a_1(old) - 0.2 \sum_{k \neq j}^{m} a_k(0)\right]$$

$$= f\left[0.3 - 0.2(0.5 + 0.7 + 0.9)\right]$$

$$= f(-0.12) = 0 \ (\wedge \ r)$$

$a_1(1) = 0$

$$a_j(new) = f\left[a_j(old) - \varepsilon \sum_{k \neq j}^{m} a_k(old)\right]$$

$$= f[0.5 - 0.2(0.3 + 0.7 + 0.9)]$$

$$= f(0.12)$$

$a_2(1) = 0.12$

$$a_3(1) = f[0.7 - 0.2(0.3 + 0.5 + 0.9)]$$

$$= f[0.36]$$

$$= 0.36$$

$$a_4(1) = f[0.9 - 0.2(0.3 + 0.5 + 0.7)]$$

$$= f(0.6)$$

$$= 0.6$$

$a_1(0) = 0$
$a_2(0) = 0.12$
$a_3(0) = 0.36$
$a_4(0) = 0.60$

2nd iteration

$$a_1(1) = f[0 - 0.2(0.12 + 0.36 + 0.6)]$$

$$= f(-0.216)$$

$$= 0$$

$$a_2(1) = f[0.12 - 0.2(0.36 + 0.6)]$$

$$= f(-0.072)$$

$$= 0$$

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Answer (at the end of 5th iteration): $a_1(5)=0$, $a_2(5)=0$, $a_3(5)=0$

$a_4(5) = 0.4232$

$a_3(1) = f[0.36 - 0.2(0.12+0.6)]$

$= f(0.216)$

$= 0.216$

$a_4(1) = f[0.6 - 0.2(0.12+0.36)]$

$= f(0.504)$

$= 0.504$

$a_1(1) = 0$
$a_2(1) = 0$
$a_3(1) = 0.216$
$a_4(1) = 0.514$

Iteration 3

$a_1(2) = [-0.2(0.216+0.504)]$

$= 0$

$a_2(2) = [-0.2(\quad)]$

$= 0$

$a_3(2) = [0.216 - 0.2(0.504)]$

$= f(0.1152)$

$= 0.1152$

$a_4(2) = f[0.504 - 0.2(0.216)]$

$= f(0.4608)$

$= 0.468$

Iteration 4

$a_1(3) = 0$
$a_2(3) = 0$
$a_3(3) = f(0.1152 - 0.2(0.4608))$
$= f(0.02304)$
$= 0.02304$
$a_4(3) = f(0.4608 - 0.2(0.1152))$
$= 0.4377$

Iteration 5

$a_1(4) = 0$
$a_2(4) = 0$
$a_3(4) = f(0.02304 - 0.2(0.437))$
$= f(-0.06)$
$= 0$

$a_4(4) = f(0.43776 - 0.2(0.02304))$
$= f(0.43315)$
$= 0.4332$

	$a_1(1)$	$a_2(1)$	$a_3(1)$	$a_4(1)$
I	0	0.12	0.216	0.6
II	0	0	0.216	0.504
III	0	0	0.1152	0.4608
IV	0	0	0.023	0.4378
V	0	0	0	0.4332

$\underbrace{0.4332}_{\text{dominant value}}$

∴ 4th node is dominant

→ Input in 4th node is transmitted and other nodes' information will be suppressed

Q. Consider the activation of Max-net with 4 neurons and a positive weight when the given initial activations are:

$a_1(0) = 0.2$, $a_2(0) = 0.4$, $a_3(0) = 0.6$
$a_4(0) = 0.8$

Sol. First iteration

$a_1(1) = f[0.2 - 0.2(0.4 + 0.6 + 0.8)]$

$= f(-0.16)$

$= 0$

$a_2(1) = f[0.4 - 0.2(0.2 + 0.6 + 0.8)]$

$= f(0.08)$

$= 0.08$

$a_3(1) = f[0.6 - 0.2(0.2 + 0.4 + 0.8)]$

$= f(0.32)$

$= 0.32$

$a_4(1) = f[0.8 - 0.2(0.4 + 0.6 + 0.2)]$

$= f(0.56)$

$= 0.56$

Iteration 2

$a_1(1) = 0 - 0.2(\cancel{\ })$

$= 0$

$a_2(1) = 0.08 - 0.2(0.32 + 0.56)$

$= f(-0.10k)$

$= 0$

$a_3(1) = f(0.32 - 0.2(0.56 + 0.08)$

$= f(0.192) = 0.192$

$a_4(1) = f(0.56 - 0.2(0.08 + 0.32)$

$= f(0.48) = 0.48$

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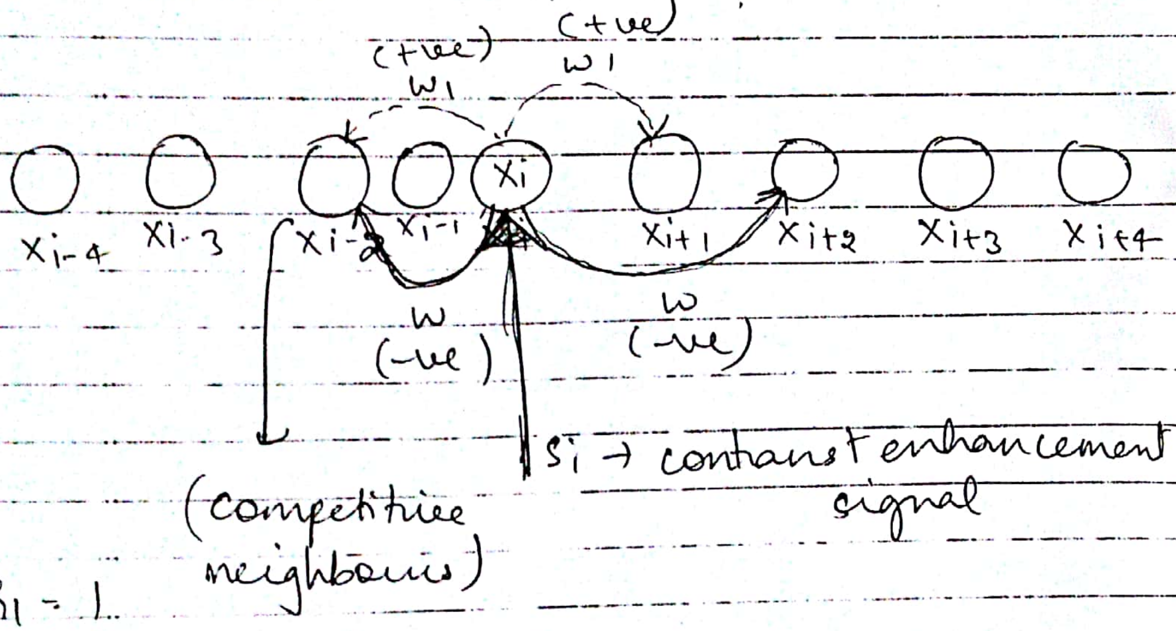
Mexican Hat Network

Architecture (cooperative neighbours)



$s_1 = 1$

$s_2 = 2$

Ques. A mexican-hat network consists of 7 input units. The net is initially activated by the i/p signals

[0 0.4 0.7 1 0.7 0.4 0]

The activation function of the neuron is given as:

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } 0 \le x \le 3 \end{cases}$$

Assume, $s_1 = 1$ and $s_2 = 2$ and

$C_1 = w_1 = 0.7$

$C_2 = w_2 = -0.3$

(Perform 3 iterations)

Solⁿ: Iteration 1

$t = 0$

Step 1

$$\begin{array}{cccccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ X = & [0 & 0.4 & 0.7 & 1 & 0.7 & 0.4 & 0] \end{array}$$

$$(X_{old}) = [0 \quad 0.4 \quad 0.7 \quad 1 \quad 0.7 \quad 0.4 \quad 0]$$

Step 2 Obtaining formula for activation function (updating signal values)

(for X_1)

$$X = [\underset{1}{0} \quad \underset{2}{0.4} \quad \underset{3}{0.7} \quad \underset{4}{1} \quad \underset{5}{0.7} \quad \underset{6}{0.4} \quad \underset{7}{0}]$$

+ve, −ve

s_i (self)

$$X_1 = 0.7 X_{old_1} \text{ (self)}$$
$$+ 0.7 X_{old_2} - 0.3 X_{old_3}$$

(+ve weight) (−ve weight)

For X_2 (2nd neuron in consideration)

$$X = [0 \quad 0.4 \quad 0.7 \quad 1 \quad 0.7 \quad 0.4 \quad 0]$$

+ve, +ve, −ve

s_i (self)

$$X_2 = 0.7 X_{old_1} + \underset{\text{(self)}}{0.7 X_{old_2}} + 0.7 X_{old_3} - 0.3 X_{old_4}$$

for X_3

$$X = [0 \quad 0.4 \quad 0.7 \quad 1 \quad 0.7 \quad 0.4 \quad 0].$$

(w) −ve, +ve (w), +ve (w), −ve (w)

s_i

$$X_3 = -0.3 X_{old_1} + 0.7 X_{old_2} + 0.7 X_{old_3} + 0.7 X_{old_4}$$
$$- 0.3 X_{old_5}$$

8)

For X_4

$X = [0 \quad 0.4 \quad 0.7 \quad 1 \quad 0.7 \quad 0.4 \quad 0]$

$X_4 = (-0.3\, X_{old_2} + 0.7\, X_{old_3} + 0.7\, X_{old_4} + 0.7\, X_{old_5} - 0.3\, X_{old_6})$

$X_5 = (-0.3\, X_{old_3} + 0.7\, X_{old_4} + 0.7\, X_{old_5} + 0.7\, X_{old_6} - 0.3\, X_{old_7})$

$X_6 = (-0.3\, X_{old_4} + 0.7\, X_{old_5} + 0.7\, X_{old_6} + 0.7\, X_{old_7})$

$X_7 = (-0.3\, X_{old_5} + 0.7\, X_{old_6} + 0.7\, X_{old_7})$

$$X = \begin{bmatrix} 0 & 0.4 & 0.7 & 1 & 0.7 & 0.4 & 0 \end{bmatrix}$$

$$\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ X_{old_1} & X_{old_2} & X_{old_3} & X_{old_4} & X_{old_5} & X_{old_6} & X_{old_7} \end{array}$$

Substituting these values in the 7 eq^n's

$X_{new} = [0.07 \quad 0.47 \quad 1.26 \quad 1.44 \quad 1.26 \quad 0.47 \quad 0.07]$

$A.F. = [\quad\quad]$

$X_{old} = [\quad\quad]$

Iteration 2 → same eq^n's new values

Perform 3 iterations

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Answer after 1st iteration

$X = [0.07 \quad 0.47 \quad 1.26 \quad 1.44 \quad 1.26 \quad 0.47 \quad 0.07]$

Iteration 2

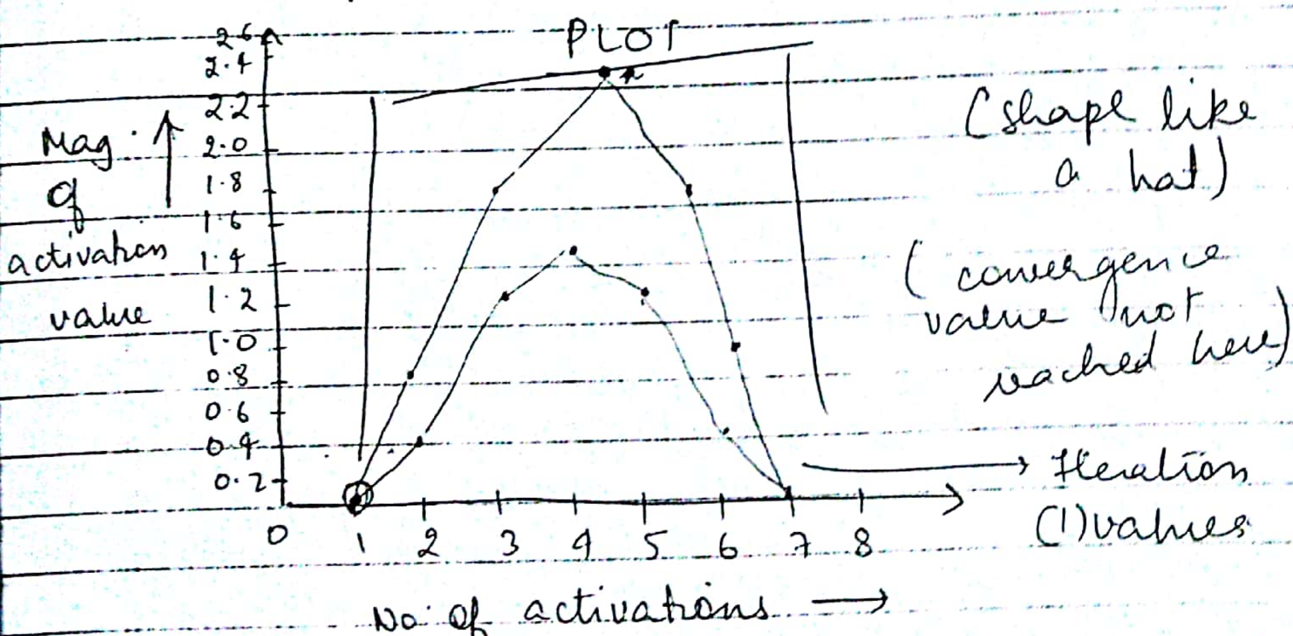
$X_{old} = [0.07 \quad 0.47 \quad 1.26 \quad 1.44 \quad 1.26 \quad 0.47 \quad 0.07]$

Repeating the same steps for iteration 2 (as for the 1st one)
(same eq^{n}s $\rightarrow$ new values)

End of iteration 2 (ans)

$X = [0 \quad 0.828 \quad 1.82 \quad 2.49 \quad 1.82 \quad 0.828 \quad 0]$

Now, plotting the pattern 'X'



Q. Construct a Mexican-hat net with 7 units with A.F. given as:

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } 0 \le x \le 2 \\ 2, & \text{if } 2 < x \end{cases}$$

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Stop the network performance if the iterations of contrast enhancement exceeds 2. The external signal is given as:

$$X = [0 \quad 0.3 \quad 0.7 \quad 1 \quad 0.7 \quad 0.3 \quad 0]$$

$R_1 = 1$ $\qquad C_1 = 0.8$

$R_2 = 2$ $\qquad C_2 = -0.4$

For X_1

+ve -ve

$$X = [\underline{0} \quad 0.3 \quad 0.7 \quad 1 \quad 0.7 \quad 0.3 \quad 0]$$

↑ S_i (self)

$$X_1 = 0.8 X_{old_1} + 0.8 X_{old_2} - 0.4 X_{old_3}$$

For X_2

+ve +ve -ve

$$X = [0 \quad \underline{0.3} \quad 0.7 \quad 1 \quad 0.7 \quad 0.3 \quad 0]$$

↑ S_i

$$X_2 = 0.8 X_{old_1} + 0.8 X_{old_2} + 0.8 X_{old_3} - 0.4 X_{old_4}$$

For X_3

-ve +ve +ve -ve

$$X = [0 \quad 0.3 \quad \underline{0.7} \quad 1 \quad 0.7 \quad 0.3 \quad 0]$$

↑ S_i

$$X_3 = -0.4 X_{old_1} + 0.8 X_{old_2} + 0.8 X_{old_3} + 0.8 X_{old_4} - 0.4 X_{old_5}$$

For X_4

-ve +ve +ve -ve

$$X = [0 \quad 0.3 \quad 0.7 \quad \underline{1} \quad 0.7 \quad 0.3 \quad 0]$$

$$X_4 = [-0.4 X_{old_2} + 0.8 X_{old_3} + 0.8 X_{old_4} + 0.8 X_{old_5} - 0.4 X_{old_6}]$$

Answers.

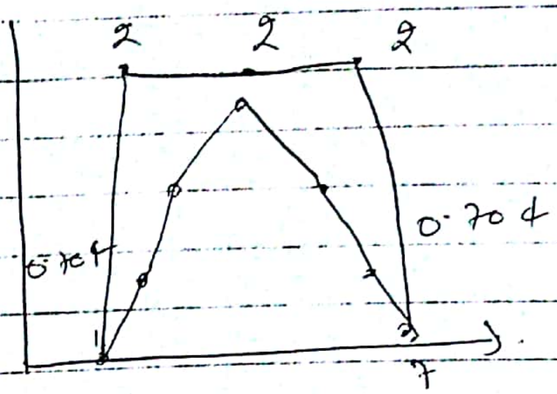
At the end of 1st iteration:

$$X = [0 \quad 0.4 \quad 1.32 \quad 1.68 \quad 1.32 \quad 0.4 \quad 0]$$

At the end of 2nd iteration:

$$X = [0 \quad 0.704 \quad 2 \quad 2 \quad 2 \quad 0.704 \quad 0]$$

Plot



(Perform max. to max 3 iterations in the exam, not more)

Q. Construct Max-net with 4 neurons and inhibitory weights, $\epsilon = 0.4$. The initial activations are given as:

$a_1(0) = 0.5$, $a_2(0) = 0.7$, $a_3(0) = 0.9$
$a_4(0) = 0.99$

Ans. $a_1(1) = f[0.5 - 0.4(0.7 + 0.9 + 0.99)]$

$= f[-0.536] = 0$

$a_2(1) = f[0.7 - 0.4(0.5 + 0.9 + 0.99)]$

$= f[-0.256] = 0$

$a_3(1) = f[0.9 - 0.4(0.5 + 0.7 + 0.99)]$

$= f[0.024] = 0.024$

$a_4(1) = f[0.99 - 0.4(0.5 + 0.7 + 0.9)]$

$= f[0.15] = 0.15$

$a_1(1) = 0$
$a_2(1) = 0$
$a_3(1) = 0.029$
$a_4(1) = 0.15$

2nd iteration

$a_1(1) = 0$, $a_2(1) = 0$, $a_3(1) = 0.021$
$a_4(1) = 0.15$

$a_1(2) = f[0 - 0.4(0 + 0.029 + 0.15)]$

$= f[-0.0696] = 0$

$a_2(2) = f[0 - 0.4(0 + 0.029 + 0.15)]$
$= 0$

$a_3(2) = f[0.029 - 0.4(0 + 0 + 0.15)]$

$= f[-0.036] = 0.$

$a_4(2) = f[0.15 - 0.4(0 + 0 + 0.024)]$

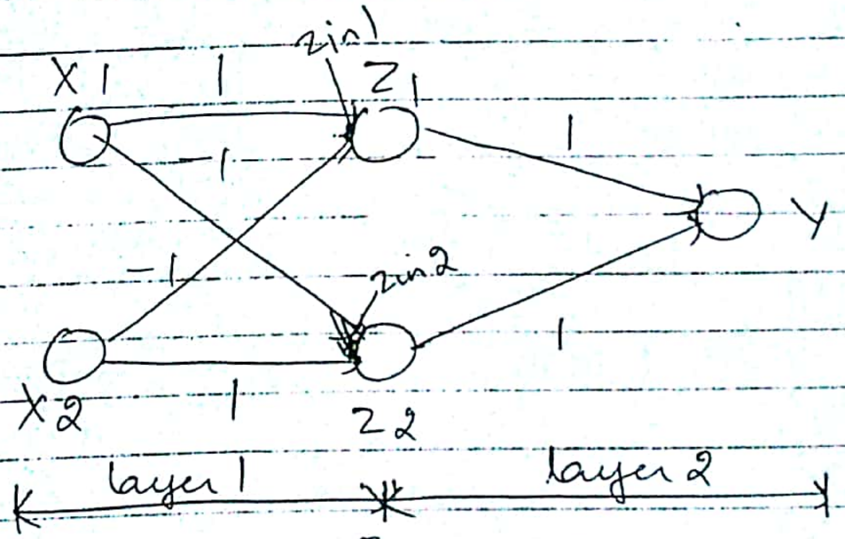
$= f(0.1404) = 0.1404$

13/2/18 CAT-I (Question)

Q2 Realise the X-OR function using Mc Culloch Pitts neuron.

Soln: Truth Table for XOR

x_1	x_2	y
1	1	0
1	0	1
0	1	1
0	0	0



x_1 XOR x_2 $= [(x_1$ AND NOT $x_2)$ OR $(x_2$ AND NOT $x_1)]$

$= [Z_1$ OR $Z_2]$

where,

$Z_1 \rightarrow x_1$ AND NOT x_2

$Z_2 \rightarrow x_2$ AND NOT x_1

Activation function

$$Z_1 = (Z_{in1}) = \begin{cases} 1, & \text{if } Z_{in1} \geq 1 \\ 0, & \text{if } Z_{in1} < 1 \end{cases}$$

$Z_2 = (z_{in2}) = \begin{cases} 1, & \text{if } z_{in2} \geq 1 \\ 0, & \text{if } z_{in2} < 1 \end{cases}$

$\boxed{Z_1 = X_1 \text{ AND NOT } X_2}$ (after applying Z_1 A.F)

X_1	X_2	z_{in1}	~~z_{in2}~~
1	1	0	0
1	0	1	1
0	1	-1	0
0	0	0	0

$z_{in1} = X_1 \underset{(w_1)}{(1)} + X_2 \underset{(w_2)}{(-1)}$

$z_{in2} = X_1 \underset{(w_1)}{(-1)} + X_2 \underset{(w_2)}{(1)}$

For z_{in1}

1st comb. $z_{in1} = 1(1) + 1(-1) = 1 - 1 = 0$.

2nd $z_{in1} = 1(1) + 0(-1) = 1$

3rd $z_{in1} = (0) \cdot -1 = -1$

4th $z_{in1} = 0 + 0 = 0$.

X_1	X_2	z_{in2}	$Z_2 \rightarrow$ A.F
1	1	0	0
1	0	-1	0
0	1	1	1
0	0	0	0

z_{in2} (for all 4 combinations)

1st $z_{in2} = 1(-1) + 1(1) = -1+1 = 0$

2nd $z_{in2} = 1(-1) + 0 = -1$

3rd $z_{in2} = 0 + (1) = 1$.

4th. $z_{in2} = 0 + 0 = 0$.

Outputs at the end of layer 1 i.e: z_1 and z_2 are known now:

→ $f(y_{in}) = y = \begin{cases} 1, & \text{if } y_{in} \geq 1 \\ 0, & \text{if } y_{in} < 1 \end{cases}$

(Activation function for y)

z_1	z_2	y_{in}	y
0	0	0	0
1	0	1	1
0	1	1	1
0	0	0	0

(Table corresponding to the 2nd layer)

$$\overset{z_1(w_1)}{y_{in} = z_1(1)} + \overset{z_2(w_2)}{z_2(1)}$$

$$y_{in} = z_1 + z_2$$

(applying for all the 4 comb. of z_1 & z_2)

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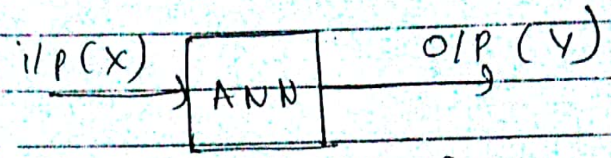
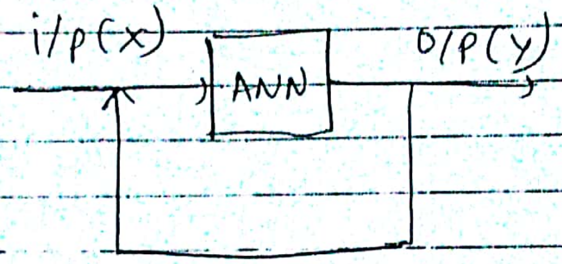
Feedback Network



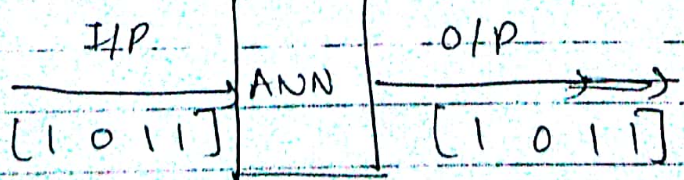
Feed forward flow



Feedback flow

Q. Consider a vector (1 0 1 1) to be stored in a net. Test a Hopfield net with mistakes in the first and the fourth component of the stored vector

Solⁿ Vector with missing data = (0 0 1 0)
1st 4th



(input same as the o/p, actually vice versa)

Convert original and new vectors to bipolar form.

Original → $[1 \quad -1 \quad 1 \quad 1]$

New → $[-1 \quad -1 \quad 1 \quad -1]$

Find out the weight matrix:

$$W = S^T . S$$

where

$S \rightarrow$ vector to be stored in ANN

$$W_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}_{4\times 1} \begin{bmatrix} 1 & -1 & 1 & 1 \end{bmatrix}_{1\times 4}$$

$$W_1 = \begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}_{4\times 4}$$

(extra)

$$W_2 = S^T . S$$

$$W_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

$$W = W_1 + W_2 = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

For Ist component $\quad I/p = O/p$

$$Y_{in_1} = X_1 + \sum_{j}^{n=4} Y_j \underbrace{W_{j1}}_{\text{column 1 of } W_1}$$

$X = [0 \quad 0 \quad 1 \quad 0]$
(take test sample in binary form)

$$Y_{in1} = X_1 + Y_1 W_{11} + Y_2 W_{21} + Y_3 W_{31} + Y_4 W_{41}$$

$$= 0 + 0(1) + 0(-1) + 1(1) + (1))$$
$$= 1$$

A.F

$Y = f(Y_{in}) = 1 \rightarrow$ shows the original value of the mistake i.e. 1

Vector becomes $\rightarrow [1 \quad 0 \quad 1 \quad 0]$

IV th component.

$$Y_{in4} = X_4 + \sum_{j}^{n=4} Y_j W_{j4}$$

$$Y_{in4} = X_4 + Y_1 W_{14} + Y_2 W_{24} + Y_3 W_{34} + Y_4 W_{44}$$

$$= 0 + 0 + 0 + 1 + 0$$
$$= 1$$

A.F. $\quad Y = f(Y_{in}) = 1.$

Vector $\rightarrow [1 \quad 0 \quad 1 \quad 1]$

We need to verify for 2nd & 3rd component also because of feedback (i.e. the original input)

IInd component

$$Y_{in2} = X_2 + Y_1 W_{12} + Y_2 W_{22} + Y_2 W_{22} + Y_4 W_{42}.$$

$= 0 + 0 + 0 - 1 + 0$

$= -1$

$\underline{\underline{AF = 0}}$

<u>IIIrd</u>

$Y_{in3} = X_3 + Y_1 W_{13} + Y_2 W_{23} + Y_3 W_{33} + Y_4 W_{43}$

$= 1 + 0 + 0 + 1 + 0$

$= 2$

$Y_{in3} = X_3 + \Sigma$

$= 1 + 0(1) + 0(-1) + 1(1) + 0(1)$

$= 1 + 0 + 0 + 1 + 0$

$= \underline{\underline{2}}$

$AF = \underline{\underline{1}}$

| 1 | 0 1 | 1 |

1 $\cancel{0}$ 0 1 $\cancel{0}$

$\underline{\underline{01}}$

<u>Q:</u> Design a Hopfield network for 4 bit bipolar patterns. The training patterns are

Ist $\rightarrow$ $[1 \quad 1 \quad -1 \quad -1]$

IInd $\rightarrow$ $[-1 \quad 1 \quad -1 \quad 1]$

IIIrd $\rightarrow$ $[-1 \quad -1 \quad -1 \quad 1]$

Find the weight matrix and the energy for the 3 i/p samples. Also determine the pattern to which the sample $S = [-1 \quad 1 \quad -1 \quad -1]$ associates

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Sol^n.
$$I \rightarrow [1 \ \ 1 \ \ -1 \ \ -1]$$
$$II \rightarrow [-1 \ \ 1 \ \ -1 \ \ 1]$$
$$III \rightarrow [-1 \ \ -1 \ \ -1 \ \ 1]$$

$$X = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$
(input matrix)

$$W = X^t . X$$

$$W = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}_{4\times 3} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}_{3 \times 4}$$

$$= \begin{bmatrix} 3 & 1 & 1 & -3 \\ 1 & 3 & -1 & -1 \\ 1 & -1 & 3 & -1 \\ -3 & -1 & -1 & 3 \end{bmatrix}$$

Calculating energy of the network for each pattern

(i). Energy for pattern 1

$$S_1 = [1 \ \ 1 \ \ -1 \ \ -1]$$

Formula :

$$E = -\frac{1}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} x_i w_{ij} x_i^t$$

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$$E_1 = -0.5 \underbrace{\begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}}_{S} \times W \times \underbrace{\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}}_{S_1^T}$$

$$E_1 = -0.5 \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 3+1-1+3 \\ 1+3+1+1 \\ 1-1-3+1 \\ -3-1+1-3 \end{bmatrix}$$

$$= -0.5 \underset{1\times 4}{\begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} 6 \\ 6 \\ -2 \\ -6 \end{bmatrix}}$$

$$= -0.5 \left[6+6+2+6 \right]$$

$$= \frac{-1}{2} \times 20 = \boxed{-10}$$

$$E_2 = -0.5 \begin{bmatrix} -1 & 1 & -1 & 1 \end{bmatrix} \times W \times \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$= -0.5 \left[6+6+2+6 \right]$$

$$\boxed{E_2 = -10}$$

$$E_3 = -0.5 \begin{bmatrix} -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -8 \\ -4 \\ -4 \\ 8 \end{bmatrix}$$

$$= -0.5 [24]$$

$$= -12$$

Energy of pattern S $= -0.5 \begin{bmatrix} -1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ -4 \\ 0 \end{bmatrix}$

$E = -0.5[0 + 4 + 4 + 0]$

$= -0.5 \times 8$

$= -4$

S sample can be associated with S_1 sample as it has the nearest energy to S.

Justification:

Using asynchronous updation method

Net i/p = S X W

$$= \underset{1\times 4}{\begin{bmatrix} -1 & 1 & -1 & -1 \end{bmatrix}} \underset{4\times 4}{\begin{bmatrix} 3 & 1 & 1 & -3 \\ 1 & 3 & -1 & -1 \\ 1 & -1 & 3 & -1 \\ -3 & -1 & -1 & 3 \end{bmatrix}}$$

$= [0 \ \ 4 \ \ -4 \ \ 0]$

$= [0 \ \ 1 \ \ -1 \ \ 0]$

$$\boxed{A \cdot F \cdot = \begin{cases} +1, & y_{in} \geq 0 \\ -1, & y_{in} \leq 0 \end{cases}}$$ memorise

After applying A·F.

$\Rightarrow [-1 \ \ 1 \ \ -1 \ \ -1]$

(Same as S)

Comparing S with S_1, S_2 and S_3

$S_1 = [1 \ \ 1 \ \ -1 \ \ -1]$

$S = [-1 \ \ 1 \ \ -1 \ \ -1]$

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$S_2 = [-1 \ 1 \ 1 \ 1]$

(15 marks Ques.)

$S = [-1 \ 1 \ -1 \ -1]$ 1 variation

$S_2 = [-1 \ -1 \ -1 \ 1]$

$S = [-1 \ 1 \ -1 \ -1]$ 2 variation (S rejected)

Q. Consider a vector (1 1 0 0) to be stored in a net. Test a Hopfield net with mistakes in the 2nd component of the stored vector.

Sol^n Original → (1 1 0 0)

New → (1 0 0 0)

⇒ (1 1 −1 −1)

(1 −1 −1 −1)

$W = S^T \cdot S$

$$W_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} [1 \ 1 \ -1 \ -1] = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

4×4

not needed

$$W_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} [1 \ -1 \ -1 \ -1] = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$W = W_1 + W_2 = \begin{bmatrix} 2 & 0 & -2 & -2 \\ 0 & 2 & 0 & 0 \\ -2 & 0 & 2 & 2 \\ -2 & 0 & 2 & 2 \end{bmatrix}$$

For IInd component $[1\ 0\ 0\ 0]$

$$y_{in2} = x_2 + \sum_{j=1}^{4} y_j w_{j2}$$

$$y_{in2} = x_2 + y_1 w_{12} + y_2 w_{22} + y_3 w_{32} + y_4 w_{42}$$

$$= 0 + 1 + 0 + 0 + 0$$
$$= 1$$

A.F. $y_2 = f(y_{in2}) = 1$

Ist

$$y_{in1} = x_1 + y_1 w_{11} + y_2 w_{21} + y_3 w_{31} + y_4 w_{41}$$

$$= 1 + 1 + 0 + 0 = 2$$

$$y_1 = f(y_{in1}) = 1$$

III

$$y_{in3} = x_3 + y_1 w_{13} + y_2 w_{23} + y_3 w_{33} + y_4 w_{43}$$

$$= 0 + 1(-1) + 0 + 0 + 0$$
$$= -1$$

$$y_2 = 0$$

IV

$$y_{in4} = x_4 + y_1 w_{14} + y_2 w_{24} + y_3 w_{34} + y_4 w_{44}$$

$$= 0 + 1(-1) + 0 + 0 = -1$$

$$y_4 = 0.$$

Vector ⇒ [1 1 0 0]

22/2/18 **Counter Propagation Network (CPN)**

It is a multilayer network with i/p, clustering (group of neurons) and o/p unit

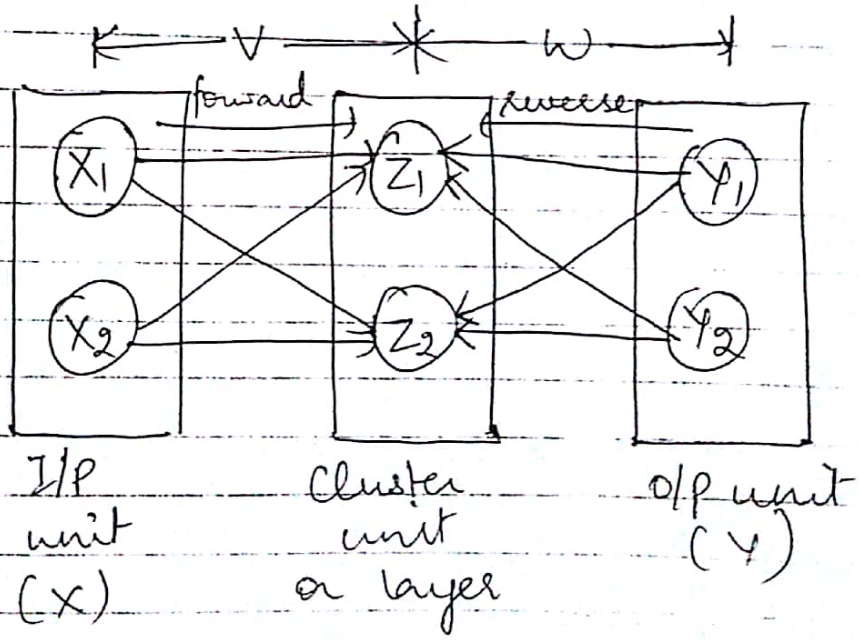
Applications :
1. Pattern Association
2. Data Compression
3. Signal enhancement

etc.

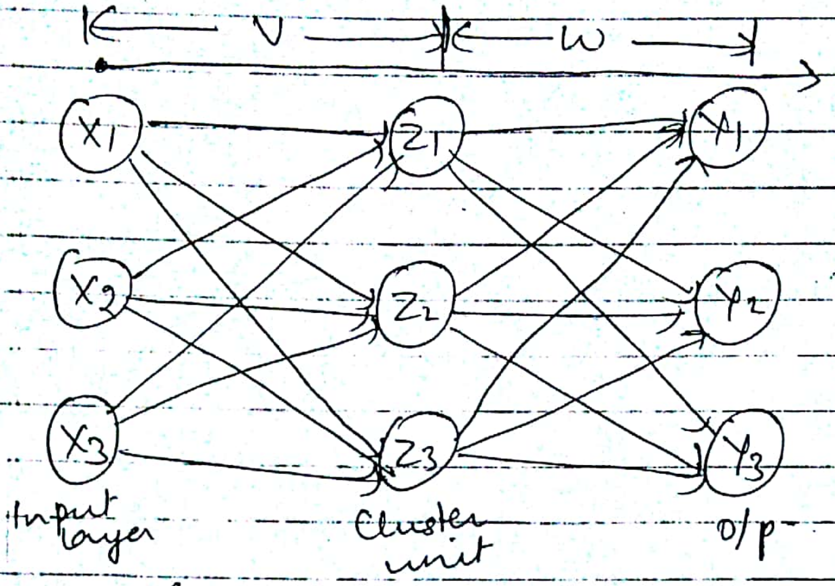
Types :

1. Fully counter propagation network (Full - CPN)
2. Forward only counter propagation network (Forward only - CPN)

Architecture : (Full CPN)

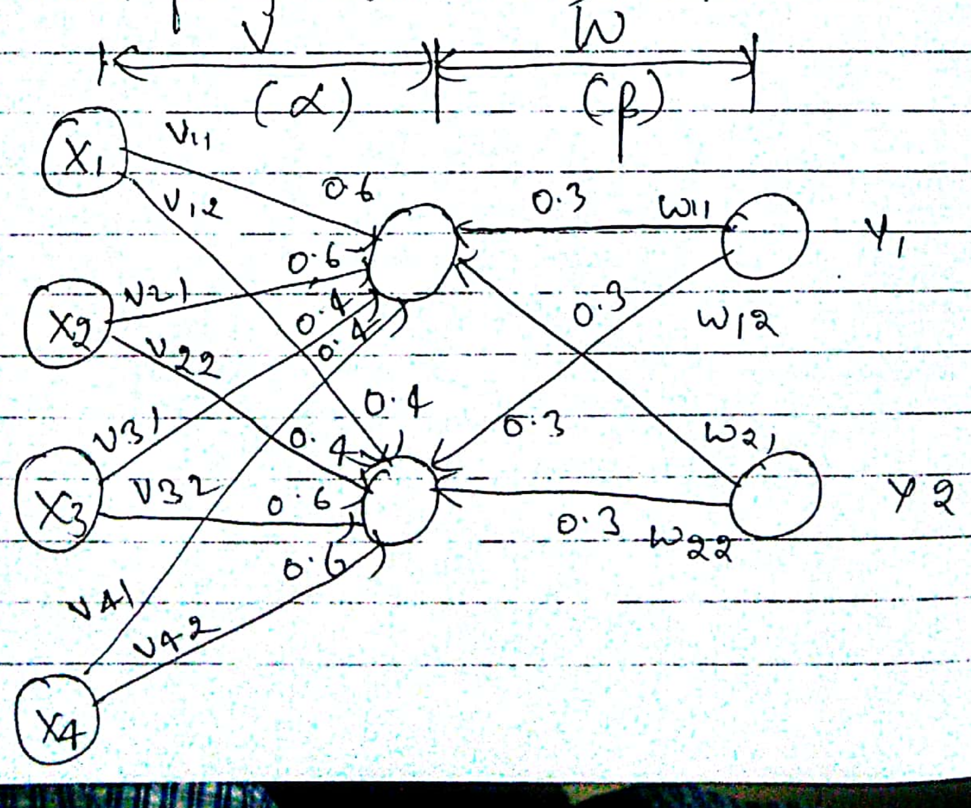


Architecture for forward only CPN



Q. Consider a following full CPN using the input pair X = (1 0 1 0) and output Y = (1 0). Perform the first phase of training (1 step only). Find the activations of the clustered layer units and obtain the weights using a learning rate of 0.2 (α for forward and β for reverse).

Soln.



Solⁿ:

Step 1: Initialize weights

$$V = \begin{array}{c|cc} & z_1 & z_2 \\ \hline x_1 & 0.6 & 0.4 \\ x_2 & 0.6 & 0.4 \\ x_3 & 0.4 & 0.6 \\ x_4 & 0.4 & 0.6 \end{array}$$

$$W = \begin{array}{c|cc} & z_1 & z_2 \\ \hline y_1 & 0.3 & 0.3 \\ y_2 & 0.3 & 0.3 \end{array}$$

Given $\alpha = 0.2$, $\beta = 0.3$

Input vector = $(1\ 0\ 1\ 0) \Rightarrow X$

Corresponding target = $(1\ 0) \Rightarrow Y$

Computing the winner unit using distance method:

$$D(j) = \sum_{i}^{4 \to \text{no. of neurons in i/p layer}} (x_i - v_{ij})^2 + \sum_{k}^{2 \to \text{no. of neurons in o/p unit}} (y_k - w_{jk})^2$$

$D(j)$ → neuron no. in the cluster unit

$\boxed{j = 1}$

$$D(1) = (1-0.6)^2 + (0-0.6)^2 + (1-0.4)^2 + (0-0.4)^2 + (1-0.3)^2 + (0-0.3)^2$$

(distance z_1)

$$= \underline{1.62}$$

$D(2)$ (distance z_2) $= (1-0.4)^2 + (0-0.4)^2 + (1-0.6)^2 + (0-0.6)^2 + (1-0.3)^2 + (0-0.3)^2 = \underline{1.40}$

$D(1) = z_1$
$D(2) = z_2$

Winner unit $\rightarrow z_1$
(implement $j = 1$)

<u>Weight updation on the winner unit</u>
<u>$J = 1$</u>

<u>General formula</u>

$\rightarrow v_{ij}(new) = v_{ij}(old) + \alpha(x_i - v_{ij}(old))$

$i = 1$ to n (here $n = 4$)

$\rightarrow w_{kj}(new) = w_{kj}(old) + \beta(y_i - w_{kj}(old))$

$i = 1$ to k (here $k = 2$)

So,

$v_{11}(new) = v_{11}(old) + \alpha(x_1 - v_{11}(old))$

$= 0.6 + 0.2(1 - 0.6)$
$= 0.68$

$v_{21}(new) = v_{21}(old) + \alpha(x_2 - v_{21}(old)$

$= 0.6 + 0.2(0 - 0.6)$
$= 0.48$

$V_{31}(new) = V_{31}(old) + \alpha(X_3 - V_{31}(old))$
$= 0.4 + 0.2(1 - 0.4)$
$= 0.52$

$V_{41}(new) = V_{41}(old) + \alpha(X_4 - V_{41}(old))$
$= 0.4 + 0.2(0 - 0.4)$
$= 0.32$

$W_{11}(new) = w_{11}(old) + \beta(Y_1 - w_{11}\,old)$
$= 0.3 + 0.2(1 - 0.3)$
$= 0.44$

$w_{21}(new) = w_{21}(old) + \beta(Y_2 - w_{21}\,old)$
$= 0.3 + 0.2(0 - 0.3)$
$= 0.24$

Thus, 1 step of operation (1 iteration) of the first phase of training is done.

Therefore the updated weights are:

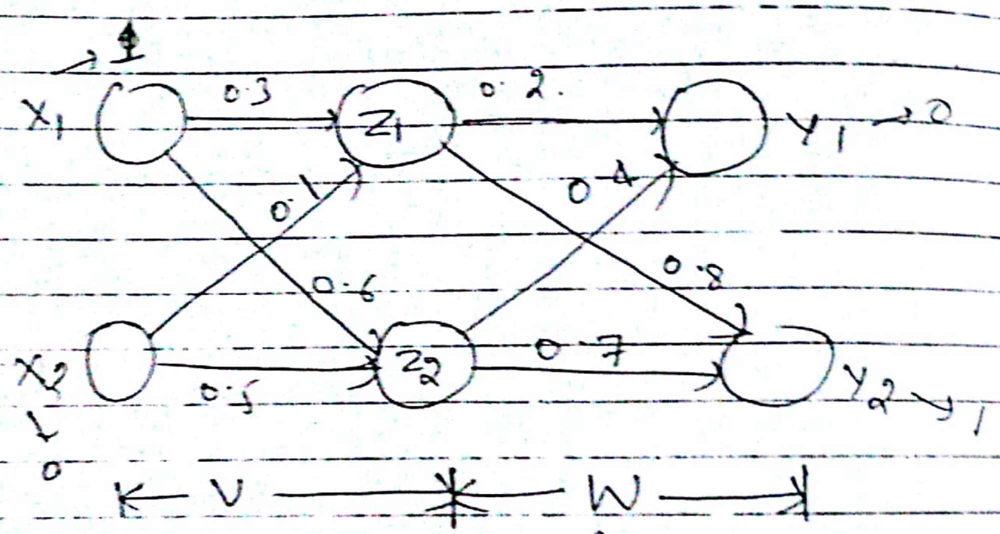
$$V = \begin{array}{c|cc} & z_1 & z_2 \\ x_1 & 0.68 & 0.4 \\ x_2 & 0.48 & 0.4 \\ x_3 & 0.52 & 0.6 \\ x_4 & 0.32 & 0.6 \end{array} \qquad W = \begin{array}{c|cc} & z_1 & z_2 \\ Y_1 & 0.44 & 0.3 \\ Y_2 & 0.24 & 0.3 \end{array}$$

updated column   updated column.

P-2 Forward only CPN

i/p pair $X = (1 \;\; 0)$ $Y = (0 \;\; 1)$

$\alpha = 0.5$, $\beta = 0.1$



$\alpha = 0.5$ $\beta = 0.1$

Initialize weights

$$V = \begin{array}{c} \\ X_1 \\ X_2 \end{array} \begin{array}{c} \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 \\ 0.1 & 0.5 \end{bmatrix} \end{array}, \quad W = \begin{array}{c} \\ Z_1 \\ Z_2 \end{array} \begin{array}{c} \begin{array}{cc} Y_1 & Y_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.7 \end{bmatrix} \end{array}$$

Begin training of the first phase (I layer)

Finding winning cluster unit

Z_1 $\boxed{D(1)} = (1 - 0.3)^2 + (0 - 0.1)^2$

distance unit no

$= 0.5$

Z_2 $\boxed{D(2)} = (1 - 0.6)^2 + (0 - 0.5)^2$

$= 0.41$

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$\therefore\ z_2 < z_1$

z_2 is the winner unit

Updating the weights

$\boxed{z_2}$

$v_{12}(\text{new}) = 0.6 + 0.5(1 - 0.6)$
$= 0.8$

$v_{22}(\text{new}) = 0.5 + 0.5(0 - 0.5)$
$= 0.25$

Updated winner matrix

$$
\begin{array}{cc}
 & \begin{array}{cc} z_1 & z_2 \end{array} \\
V = \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.3 & 0.8 \\ 0.1 & 0.25 \end{bmatrix}
\end{array}
$$

$z_2 \rightarrow$ winner unit

old new

v and w also update

Beginning | second phase of training;

Given, $X = (1 \quad 0)$ and $Y = (0 \quad 1)$

Calculate winner cluster unit

$z_1 \rightarrow D(1) = (1-0.3)^2 + (0-0.1)^2 = 0.5$

$z_2 \rightarrow D(2) = (1-0.8)^2 + (0-0.25)^2 = 0.1025$

$\boxed{z_2 < z_1}$

$\therefore\ z_2$ is the winner unit

$v \rightarrow v_{12}, v_{22}$

$w \rightarrow w_{21}, w_{22}$.

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Updating the weights,

$v_{12}(new) = 0.8 + 0.5(1-0.8) = 0.9$

$v_{22}(new) = 0.25 + 0.5(0-0.25) = 0.125$

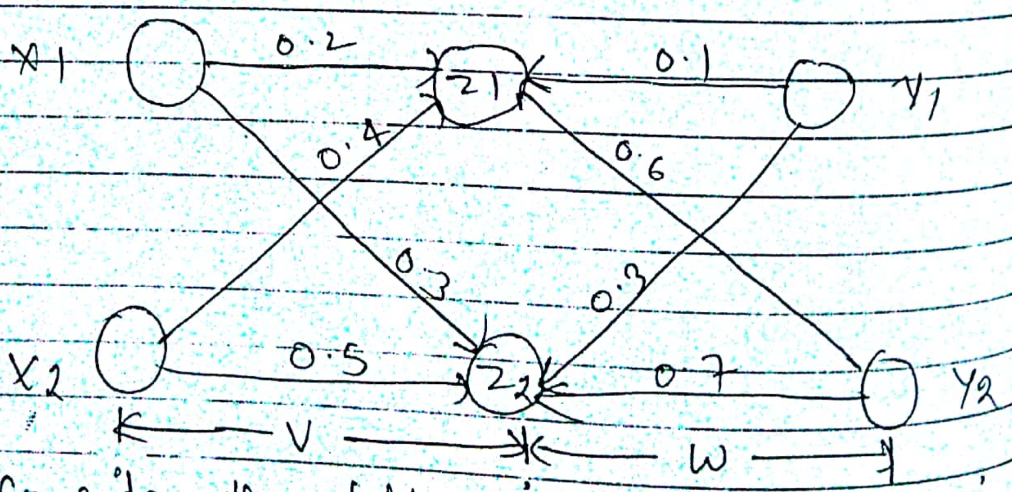
$w_{21}(new) = 0.4 + 0.1(0-0.4) = 0.36$

$w_{22}(new) = 0.7 + 0.1(1-0.7) = 0.73$

$$V = \begin{array}{c|c} & \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.3 & 0.9 \\ 0.1 & 0.125 \end{bmatrix} \end{array} \rightarrow \text{new}$$

$$W = \begin{array}{c|c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} z_1 \\ z_2 \end{array} & \begin{bmatrix} 0.2 & 0.8 \\ 0.36 & 0.73 \end{bmatrix} \end{array} \rightarrow \text{winner}$$

Q. $x = (1, 1)$ $\alpha = 0.3$
$y = (0, 1)$ $\beta = 0.3$



Consider the following full CPN using input pair $x = (1, 1)$ and $y = (0, 1)$

$Y \rightarrow$ o/p

$X \rightarrow$ i/p

Perform one phase of training (1 iteration / 1 step). Find the activation of the cluster layer units and update the weights using learning rate of 0.3

Sol^n: Initialize weights

$$V = \begin{array}{c c} & \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{bmatrix} \end{array}, \quad W = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} z_1 \\ z_2 \end{array} & \begin{bmatrix} 0.1 & 0.6 \\ 0.3 & 0.7 \end{bmatrix} \end{array}$$

Beginning training of the first phase

$$Z_1 = D(1) = (1-0.2)^2 + (1-0.4)^2$$

$$= 1.17 \text{ (ans)}$$

$$Z_2 = D(2) = z_2 = 0.92 \text{ (ans)}$$

at the end of iteration 1

Updated value of V . $\begin{bmatrix} 0.2 & 0.51 \\ 0.4 & 0.65 \end{bmatrix}$

$$W = \begin{bmatrix} 0.1 & 0.21 \\ 0.6 & 0.79 \end{bmatrix}$$