

$$\begin{aligned}
 w_{22}(\text{new}) &= 0.7 + 0.3(1 - 0.7) \\
 &= 0.7 + 0.09 \\
 &= \underline{0.79}
 \end{aligned}$$

$$V = \begin{bmatrix} 0.2 & 0.51 \\ 0.4 & 0.65 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.1 & 0.21 \\ 0.6 & 0.79 \end{bmatrix}$$

### Hamming NET

It is a classifier net that determines which of several exemplary vectors is most similar to an i/p vector.

The parameters used are

$n \rightarrow$  no of i/p nodes (i/p vectors)

$m \rightarrow$  no of o/p nodes (exemplary vectors)

$e_j = j^{\text{th}}$  exemplary vector

Step-1

Weights are to be initialized for storing 'm' exemplary vectors

$$w_{ij} = \frac{e_j(i)}{2} \quad \text{where } i=1, \dots, n, j=1, \dots, m$$

Initializing the bias

$$b_j = \frac{n}{2} \quad (j=1, \dots, m)$$

Step-2

For each vector  $x$ , perform steps 3 to 5

step-3

Compute the net i/p to each unit  $y_j$   

$$Y_{inj} = b_j + \sum_i x_i w_{ij} \text{ where } j=1, \dots, n$$

step-4

Initialize activations for network

$$AF = f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases} \quad y_j(0) = Y_{inj} \text{ where } j=1, \dots, n$$

Given the exemplar vectors  $e(1) = (-1 \ 1 \ -1 \ 1)$   
 $e(2) = (1 \ -1 \ -1 \ -1)$

Using hamming NET find which exemplar vector is close or similar to each of the i/p (bipolar) patterns

inputs  $(1 \ 1 \ -1 \ -1)$   
 $(-1 \ 1 \ 1 \ -1)$   
 $(-1 \ -1 \ -1 \ 1)$   
 $(-1 \ -1 \ 1 \ 1)$

i/p pattern  $x_1 = (1 \ 1 \ -1 \ -1)$   
 $x_2 = (-1 \ 1 \ 1 \ -1)$   
 $x_3 = (-1 \ -1 \ -1 \ 1)$   
 $x_4 = (-1 \ -1 \ 1 \ 1)$

step-1

Initialize the weights to extend the exemplar vectors

$$e(1) = (-1 \ 1 \ -1 \ 1) \\ = (-0.5 \ -0.5 \ -0.5 \ 0.5) \\ e(2) = (1 \ -1 \ -1 \ -1) \\ = (0.5 \ -0.5 \ -0.5 \ -0.5)$$



Date \_\_\_\_\_  
Page \_\_\_\_\_

$$W = \begin{matrix} & e(1) & e(2) \\ \begin{matrix} -0.5 & 0.5 \\ 0.5 & -0.5 \\ -0.5 & -0.5 \\ 0.5 & -0.5 \end{matrix} \end{matrix}$$

$$b_1 = b_2 = \frac{n}{2} = \frac{4}{2} = 2.$$

step-2

For the vector  $x_1 = (1 \ 1 \ -1 \ -1)$  perform steps 3 to 5.

$$Y_{in1} = 2 + (-0.5 + 0.5 + 0.5 - 0.5) = 2.$$

$$Y_{in2} = 2 + (0.5 + 0.5 + 0.5 - 0.5) = 3.$$

$$AF = Y_1(b) = Y_{in1} = 2.$$

$$Y_2(b) = Y_{in2} = 3.$$

$$e_1 = (1 \ 1 \ -1 \ -1) \} 2 \text{ matches}$$

$$x_1 = (1 \ 1 \ -1 \ -1)$$

$\Rightarrow$  close to 1<sup>st</sup>  $e$ .

$$e_2 = (1 \ -1 \ -1 \ -1) \} 3 \text{ matches}$$

$$x_1 = (1 \ 1 \ -1 \ -1)$$

For the vector  $x_2 = (-1 \ 1 \ 1 \ -1)$

$$Y_{in1} = 2 + (0.5 + 0.5 - 0.5 - 0.5) = 2.$$

$$Y_{in2} = 2 + (-0.5 - 0.5 - 0.5 + 0.5) = 1.$$

$$AF = Y_1(i) = Y_{in1} = 2.$$

$$Y_2(i) = Y_{in2} = 1.$$

$$e_1 \& x_2 \Rightarrow 2 \text{ matches}$$

$$e_2 \& x_2 \Rightarrow 1 \text{ match}$$

$\Rightarrow$  close to 2<sup>nd</sup>  $e$

For the vector  $x_3 = (-1 -1 -1 1)$   
 $y_{in1} = 2 + (-.5 - .5 + .5 + .5) = 2$   
 $y_{in2} = 2 + (-.5 + .5 + .5 - .5) = 2$

$y_1(2) = y_{in1} = 3$   
 $y_2(2) = y_{in2} = 2$   
 $e_1 \& x_3 \rightarrow 3 \text{ matches}$   
 $e_2 \& x_3 \rightarrow 2 \text{ matches}$

For the vector  $x_4 = (-1 -1 1 1)$   
 $y_{in1} = 2 + (-.5 - .5 - .5 + .5) = 2$   
 $y_{in2} = 2 + (-.5 + .5 - .5 - .5) = 1$

$e_1 \& x_4 \rightarrow 2 \text{ matches}$   
 $e_2 \& x_4 \rightarrow 1 \text{ match}$  }  $x_4 \text{ closer to } e_1$

②. o/p:  $e_1 = (1 1 1 1)$   
 $e_2 = (-1 -1 1 1)$

i/p:  $x_1 = (1 1 -1 -1)$   
 $x_2 = (1 -1 1 1)$

$W = \begin{bmatrix} .5 & .5 \\ .5 & .5 \\ .5 & .5 \\ .5 & .5 \\ .5 & .5 \end{bmatrix}$

$b_1 = b_2 = 5/2 = 2.5$

$y_{in1} = 2.5 + (-.5 + .5 - .5 - .5)$   
 $= 2.5 + (-.5) = 2$   
 $y_{in2} = 2.5 + (-.5 + .5 + .5 + .5)$   
 $= 4$



## Learning Vector Quantization

Construct and test L VQ with 4 vectors assigned to two classes assume learning rate  $\alpha = 0.1$  & perform 2 iteration

vector	class
1 0 1 0	1
0 0 1 1	2
<hr/>	
1 1 0 0	1
1 0 0 1	2

} weights (training)  
} Testing

Solution

Iter - 1

step - 1

Initialize the weights  $w_1 = (1 \ 0 \ 1 \ 0)$   
 $w_2 = (0 \ 0 \ 1 \ 1)$

$$\alpha = 0.1$$

step 2

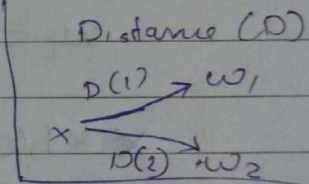
Beginning of training process

1<sup>st</sup> test vector  $x = (1 \ 1 \ 0 \ 0) \rightarrow c = 1$

step 3:

Calculating distance 'D'

$$D(i) = \sum_j (w_{ij} - x_j)^2$$



$$D(1) = (1-1)^2 + (0-1)^2 + (1-0)^2 + (0-0)^2$$

$$= 2$$

$$D(2) = (0-1)^2 + (0-1)^2 + (1-0)^2 + (1-0)^2$$

$$= 4$$

$w_2 = (0 \ 0 \ 1 \ 1)$   
 $x = (1 \ 1 \ 0 \ 0)$

$c = 1$ . check for  $c = 1$

$$c = 1 = 1$$

Formula for updating  $w$

$$w_{i, (new)} = w_{i, (old)} + \alpha (x - w_{i, (old)})$$

$$= (1 \ 0 \ 1 \ 0) + 0.1((1 \ 1 \ 0 \ 0) - (1 \ 0 \ 1 \ 0))$$

$$= [1 \ 1 \ 0.9 \ 0]$$

$$1/p \rightarrow x = [1 \ 0 \ 0] \rightarrow c \rightarrow 2$$

$$D(1) = 0^2 + 0^2 + 0^2 + 1^2 = 1$$

$$D(2) = 1^2 + 0^2 + 1^2 + 0^2 = 2$$

$$d=1$$

$$c=2$$

$$if \ 1 \neq c$$

$$w(new) = w(old) - \alpha (x - w(old))$$

$$1^{st} \text{ test vector}$$

$$w_1(new) = w_1(old) - \alpha (x - w_1(old))$$

$$= [1 \ 0 \ 0 \ 0] - 0.1(0 \ 0 \ 0 \ 1)$$

$$= [1 \ 0 \ 0 \ 0]$$

$$w_2 = [0 \ 0 \ 0 \ 1]$$

$$x_1 = [1 \ 0 \ 0 \ 0]$$

$$w_1 = [1 \ 0 \ 0 \ 0]$$

$$w_2 = [0 \ 0 \ 0 \ 1]$$

$$x_1 = [1 \ 0 \ 0 \ 0]$$

$$x_2 = [1 \ 0 \ 0 \ 1]$$

$$D(1) = (1-1)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 = 0$$

$$D(2) = (1-1)^2 + (0-0)^2 + (0-0)^2 + (1-1)^2 = 0$$

$$j=1; c=1$$

$$w_1(new) = w_1(old) + \alpha (x - w_1(old))$$



$$C(1 \cdot 11 \cdot 99 \cdot -1) + 1(0 \cdot 0 \cdot 89 \cdot -99 + 1)$$

$$= C(1 \cdot 19 \cdot 891 \cdot -1)$$

$X_2$

$$D(1) = 2.00$$

$$D(2) = 2$$

$$W_2 = W_{old} + \alpha (X_2 - W_{old})$$

$$= (0 \ 0 \ 1 \ 1) + 1(0 \ 1 \ 0 \ -1 \ 0)$$

$$= \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

Q-2.

V

1 1 0 0

0 0 0 1

0 0 1 1

1 0 0 0

0 1 1 0

C

1 } w (training)

2 }

2 }

1 }

2 }

testing

$$D(1) = 4$$

$$D(2) = 1$$

# Fuzzy Logic

## Crisp Set

ex

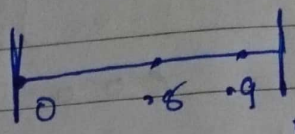
ON - 1 ; True  
OFF - 0 ; False

## Fuzzy set

Ex

Is RAM honest?

Fuzzy  $\rightarrow$  extremely honest (1)  
Very honest (0.9)  
occasionally honest (0.5)  
extremely dishonest (0)



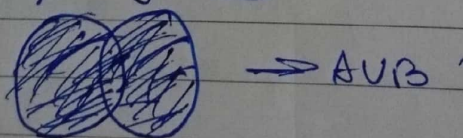
## Union ( $\cup$ )

$$A = \{a, b, c, 1, 2\}$$

$$B = \{1, 2, 3, a, c\}$$

$$A \cup B = \{a, b, c, 1, 2, 3\}$$

Venn diagram

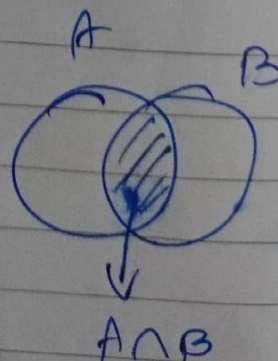


## Intersection ( $\cap$ )

$$A = \{a, b, c, 1, 2\}$$

$$B = \{1, 2, 3, a, c\}$$

$$A \cap B = \{a, c, 1, 2\}$$

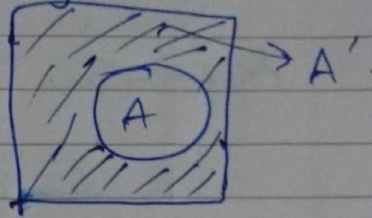




Complement of A ( $A'$ )

Universal set  $X = \{1, 2, 3, 4, 5, 6, 7\}$   
 $A = \{5, 4, 3\}$

$$A' = \{1, 2, 6, 7\}$$

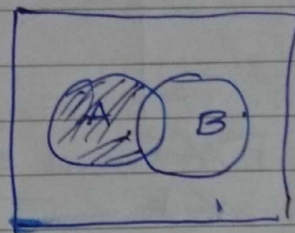


Difference

$$A = \{a, b, c, d, e\}$$

$$B = \{b, d\}$$

$$A - B = \{a, c, e\}$$

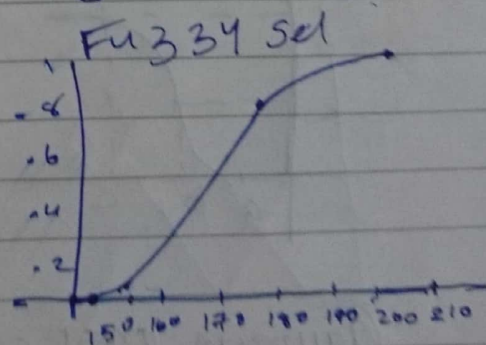
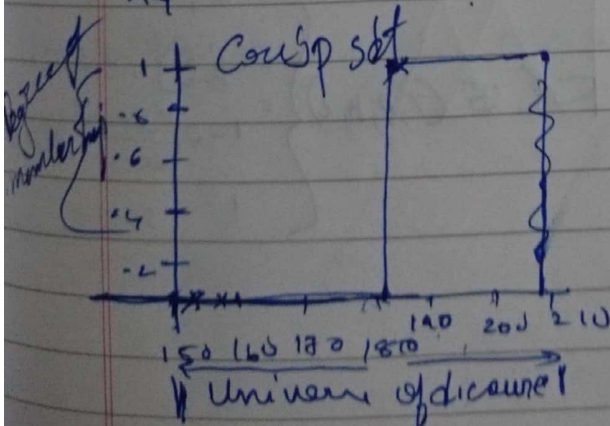


Plot membership function of Tall man

Degree of membership of tallness

Name      Height      Degree of membership  
    Crisp      Fuzzy

$A_1$	208	1	1	Very tall
$A_2$	181 nearest $\Rightarrow$	1	0.82	Tall
$A_3$	155	0	0.01	Short
$A_4$	152	0	0	Very short



$$\text{Fuzzy set } \left\{ \frac{1}{200}, \frac{0.82}{181}, \frac{0.01}{155}, \frac{0}{152} \right\} \rightarrow A$$

$$\text{Short man } \bar{A} = \left\{ \frac{0}{208}, \frac{0.18}{181}, \frac{0.99}{155}, \frac{1}{152} \right\}$$

## Properties of crisp set

Commutative,  $A \cup B = B \cup A$   
 $A \cap B = B \cap A$

Associative  $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

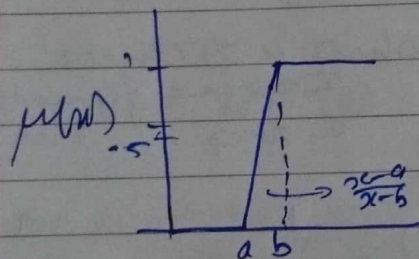
Distributive  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

## Different shapes of M.F

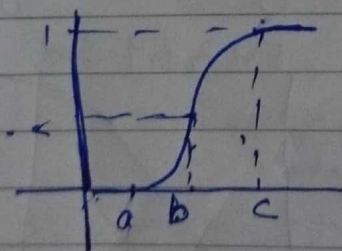
M.F

Linearly expressed gamma M.F



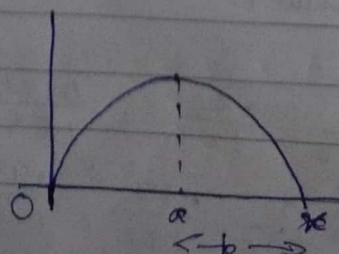
$$\Gamma(x, a, b) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x \geq b \end{cases}$$

## Sigmoidal M.F



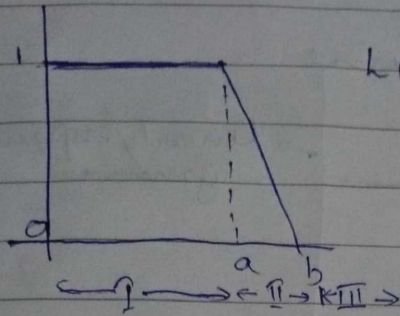
$$\Gamma_S(x, a, b, c) = \begin{cases} 0, & x < a \\ \frac{2(\frac{x-a}{c-a})^2}{1 + (\frac{x-a}{c-a})^2}, & a \leq x \leq b \\ 1 - \frac{2(\frac{x-b}{c-b})^2}{1 + (\frac{x-b}{c-b})^2}, & b \leq x \leq c \\ 1, & x \geq c \end{cases}$$

## Gaussian M.F

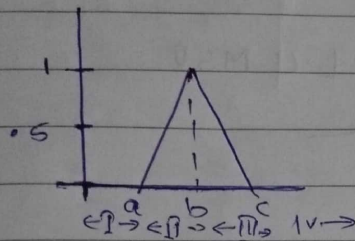


$$G(x, a, b) = e^{-b(a-x)^2}$$

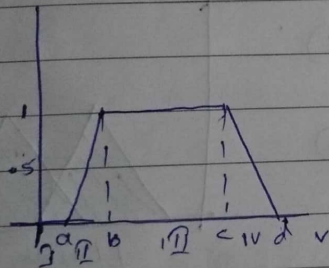


L-L MSF

$$L(x, a, b) = \begin{cases} 1, & x < a \\ \frac{b-x}{b-a}, & a \leq x \leq b \\ 0, & x \geq b \end{cases}$$

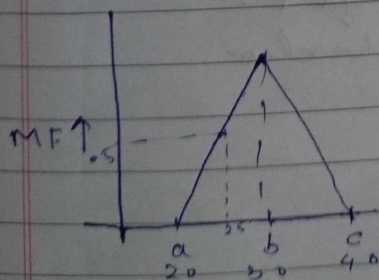
Triangle MSF ( $\lambda$ -MSF)

$$\lambda(x, a, b, c) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x \geq c \end{cases}$$

Trapezoidal MSF /  $\pi$ -MSF

$$\pi(x, a, b, c, d) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b < x < c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x \geq d \end{cases}$$

Plot triangular MSF;  $a=20$ ,  $b=30$ ,  $c=40$



Consider  $x=20$

$$\frac{x-a}{b-a} = \frac{20-20}{10} = 0$$

$$x=20$$

$$\frac{20-20}{10} = 0$$

$$x=25$$

$$\frac{25-20}{10} = 0.5$$

$$x=30$$

$$\frac{30-20}{10} = 1$$

$$x=35$$

$$\frac{c-x}{c-b} = \frac{40-35}{10} = 0.5$$

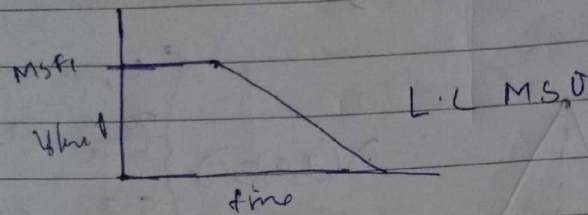
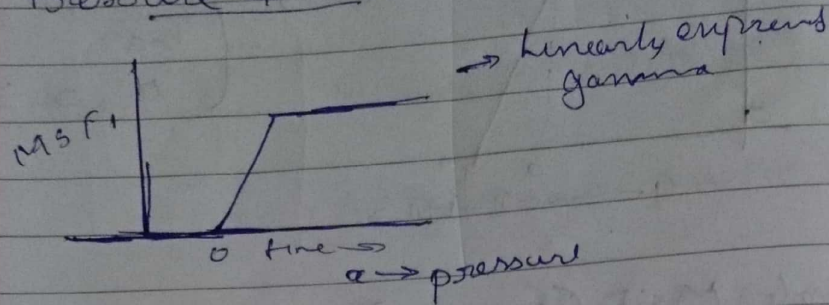
$$x=39$$

$$\frac{40-39}{10} = 0.1$$

$$x=40, \frac{40-40}{10} = 0$$

## Fuzzy Implications

### Pressure & Volume

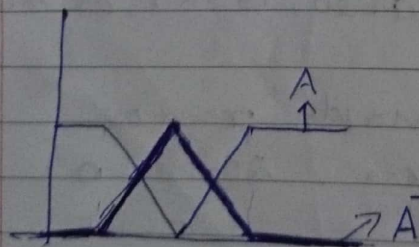
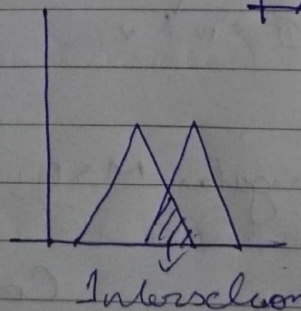
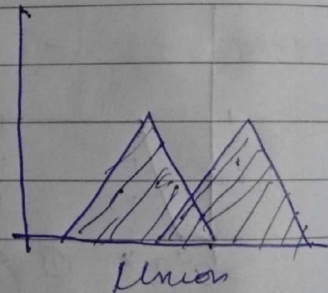


## Operation of Fuzzy Logic sets

$$\mu(A \cup B)(x) = \mu_A(x) \cup \mu_B(x)$$

$$\mu(A \cap B)(x) = \mu_A(x) \cap \mu_B(x)$$

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$



Consider 2 Fuzzy sets A and B  
Find complement, Union, Intersection  
Difference, Verification of De Morgan's Law



$$A = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.6}{4}, \frac{0.2}{5} + \frac{0.6}{6} \right\}$$

$$B = \left\{ \frac{0.5}{2} + \frac{0.8}{3} + \frac{0.4}{4} + \frac{0.7}{5} + \frac{0.3}{6} \right\}$$

$$\bar{A} = \left\{ \frac{0}{2} + \frac{.5}{3} + \frac{.4}{4} + \frac{.8}{5} + \frac{.4}{6} \right\}$$

$$\bar{B} = \left\{ \frac{.5}{2} + \frac{.2}{3} + \frac{.6}{4} + \frac{.3}{5} + \frac{.7}{6} \right\}$$

$$A \cup B = \left\{ \frac{1}{2} + \frac{.8}{3} + \frac{.6}{4} + \frac{.7}{5} + \frac{.6}{6} \right\}$$

$A \cup B$  is maximum value of corresponding elements

$$A \cap B = \left\{ \frac{.5}{2} + \frac{.5}{3} + \frac{.4}{4} + \frac{.2}{5} + \frac{.3}{6} \right\}$$

$$A/B = A \cap \bar{B}$$

$$= \left\{ \frac{.5}{2} + \frac{.2}{3} + \frac{.6}{4} + \frac{.2}{5} + \frac{.6}{6} \right\}$$

$$B/A = B \cap \bar{A}$$

$$= \left\{ \frac{.0}{2} + \frac{.5}{3} + \frac{.4}{4} + \frac{.7}{5} + \frac{.3}{6} \right\}$$

De Morgan's Law

$$\overline{A \cup B} = \bar{A} \cap \bar{B} = \left\{ \frac{.0}{2} + \frac{.2}{3} + \frac{.4}{4} + \frac{.3}{5} + \frac{.4}{6} \right\}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B} = \left\{ \frac{.5}{2} + \frac{.5}{3} + \frac{.6}{4} + \frac{.8}{5} + \frac{.7}{6} \right\}$$

Fuzzification

It is the conversion of crisp quantity into fuzzy quantity. Defuzzification

is the process of converting Fuzzy quantity into crisp quantity

Fuzzification

Height	Crisp quantity
$A_1$	50
$A_2$	100
$A_3$	25
$A_4$	10

↓

lowest = 10 → 0

Highest = 100 → 1

Fuzzy quantity Crisp

Normalized data

$$Z_i = \frac{x_i - \min(x)}{\max(x) - \min(x)}$$

$$A_1 = \frac{50 - 10}{100 - 10} = 0.44$$

$$A_2 = \frac{100 - 10}{100 - 10} = 1$$

$$A_3 = \frac{25 - 10}{90} = \frac{15}{90} = 0.17$$

$$A_4 = \frac{10 - 10}{90} = 0$$

Defuzzification

$$= x(\max(x) - \min(x)) + \min(x)$$

$$A_1 = 0.44(100 - 10) + 10$$

$$= 0.44(90) + 10 = 50 \rightarrow \text{Real}$$

$$A_2 = 1(100 - 10) + 10 = 100$$

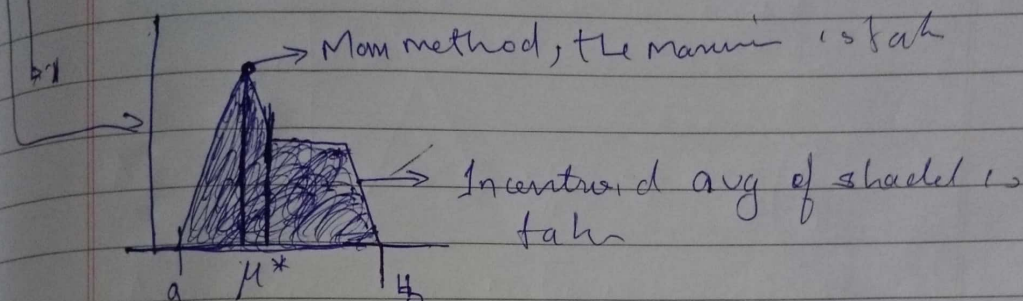
$$A_3 = 0.17(90) + 10 = 25$$

$$A_4 = 0(100 - 10) + 10 = 10$$



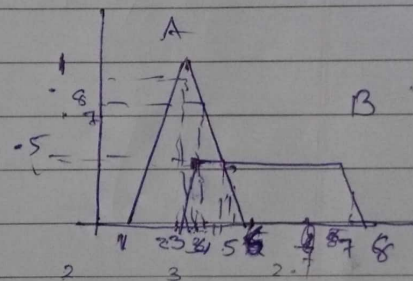
## Types of defuzzification

1. Max-membership method
2. Centroid method
3. Weighted average
4. Mean-Max membership method
5. Center of sums
6. Center of largest area
7. ~~Foot~~ Cause or loss of Maximizing



finding fuzzy value  
b/w a and b.

$$\mu^* = \frac{\sum_{i=1}^N u_i \mu_{out}(u_i)}{\sum_{i=1}^N \mu_{out}(u_i)}$$



$$\mu^* = \frac{(2 \times 0.6 + 3 \times 0.8 + 4 \times 0.7 + 5 \times 0.4 + 6 \times 0.5 + 7 \times 0.3 + 8 \times 0)}{0.6 + 0.8 + 0.7 + 0.4 + 0.5 + 0.3}$$

$$= \frac{11.3}{11.3} = 1.0$$