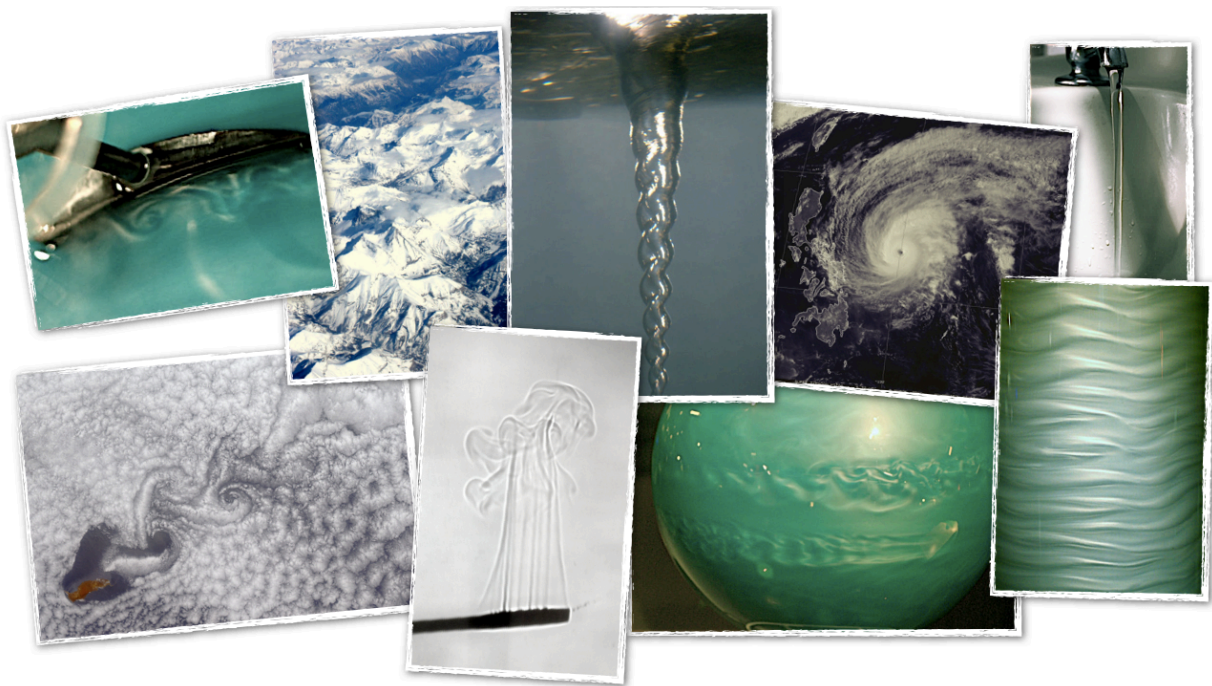




MTH3360 — Fluid Mechanics

Part 2: Incompressible Fluids

With solutions to selected examples



Bibliography

Acheson, D.J., 1990. Elementary Fluid Dynamics, Oxford University Press.

Batchelor, G.K., 1967. Introduction to Fluid Dynamics. Cambridge University Press.

Lighthill, J., 1986. An Informal Introduction to Theoretical Fluid Mechanics. Oxford University Press.

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Cover images



- a — Flow past a wing-cross-section visualised by suspended particles in a thin layer of slowly flowing water.
- b — The French alps seen from the air in the Spring of 2010. Viscous flow is an important component of mountain building (orogeny) on geological timescales.
- c — The free-surface of a rapidly rotating vortex in a cylindrical column of water seen from beneath.
- d — A Geostationary Meteorological Satellite (GMS) image of super typhoon Mitag, 5 March 2002. The image comes from the University of Madison, Wisconsin
- e — Water from a running tap showing how the cross sectional area of the flow reduces as the fluid gains speed
- f — Taylor-Couette flow at a rotation rate above the point where secondary vortices appear, but just below the onset of instability.
- g — Zonal flows which develop spontaneously in a rapidly-rotating, fluid-filled sphere
- h — Buoyant plume of warm water created by switching on a heating element in a tank of water and visualised by its shadow
- i — MISR images of a von Karman vortex street. The alternating double row of vortices form in the wake of an obstacle, in this instance the eastern Pacific island of Guadalupe. This volcanic Mexican island reaches a maximum elevation of 1.3 kilometers. The island is about 35 kilometers long and is located 260 kilometers west of Baja California. The vortex pattern is made visible by the marine stratocumulus clouds around Guadalupe Island.

Images in *(a,e,f,g,h)* taken at the Swiss Science Center Technorama Winterthur, Switzerland, 2010, by LM – (<http://www.technorama.ch>). Photograph of the alps *(b)* by LM of a view provided by Swiss International Airlines. Image *(d)* comes from the University of Madison, Wisconsin. *(i)* NASA visible earth – http://visibleearth.nasa.gov/view_rec.php?vev1id=6586

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