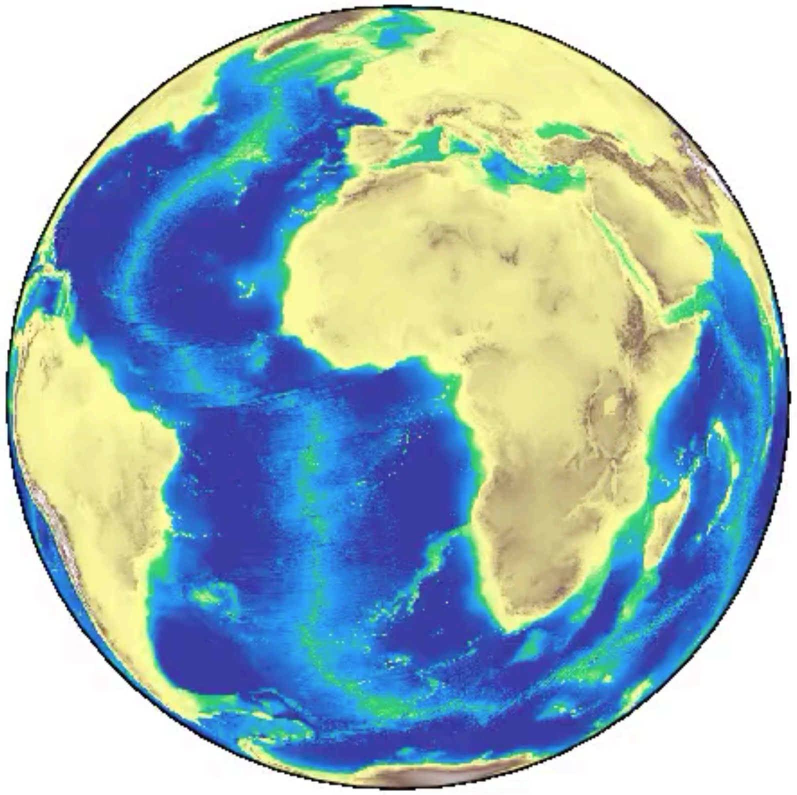
Australian National University

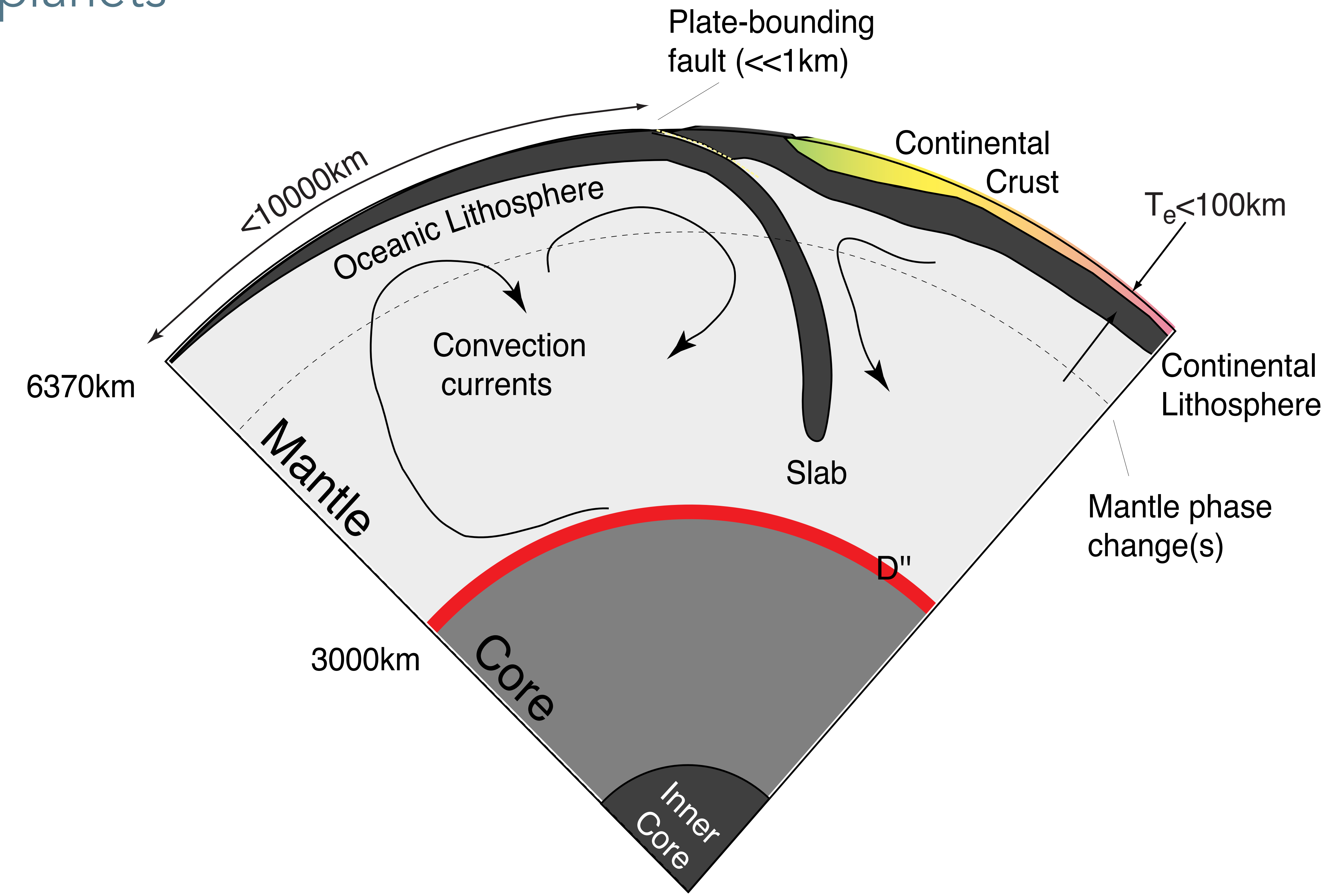
PHYS 3070 Section 2:

Plate Tectonics & Mantle Convection



In which we examine the connection between the Earth’s heat engine and surface plate motions.

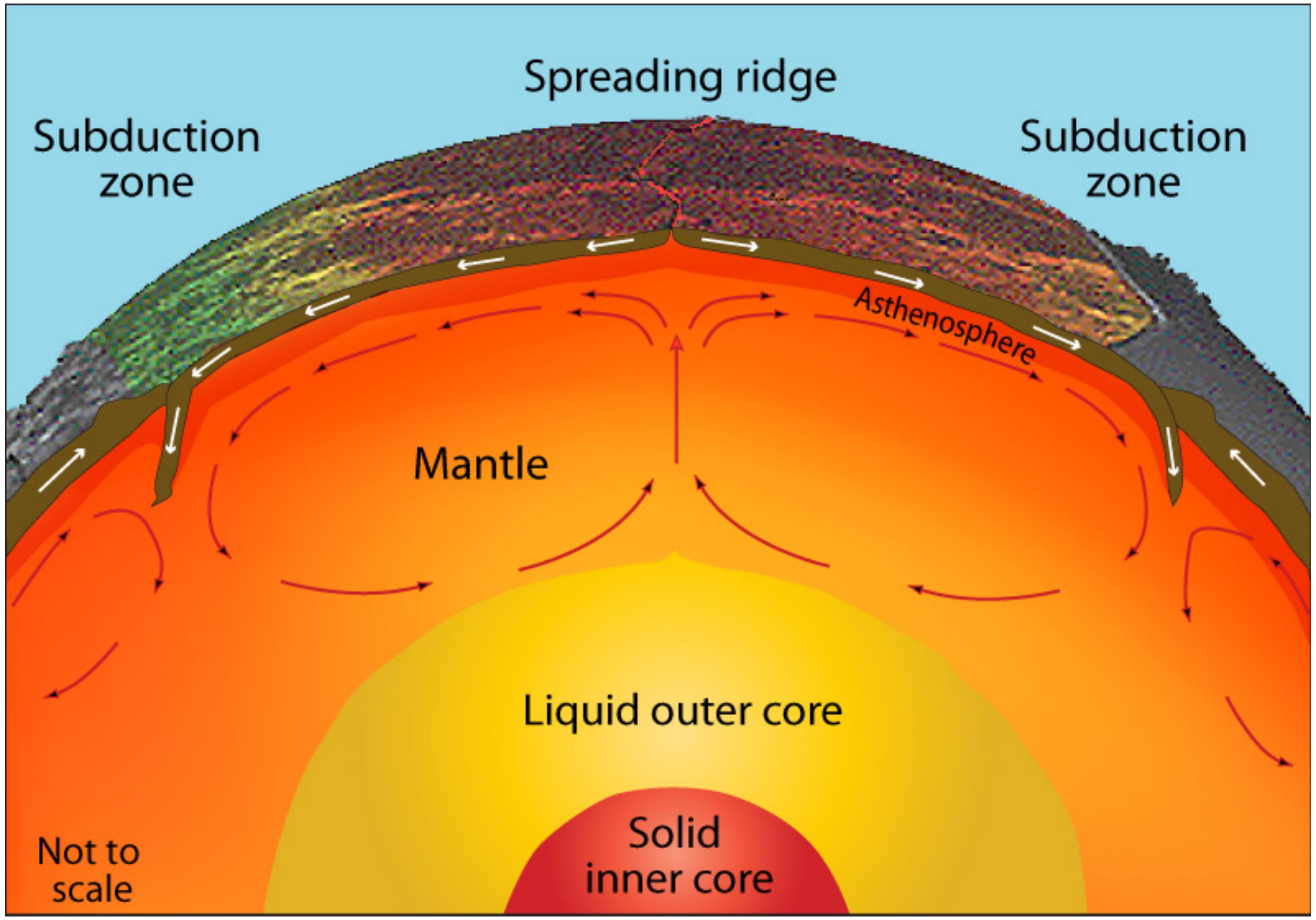
Dynamics of planets



We have a reasonable 3D picture of the Earth from the many methods of remote sensing + theoretical models and any textbook will show a picture like this to explain how plates move.

Mantle Convection

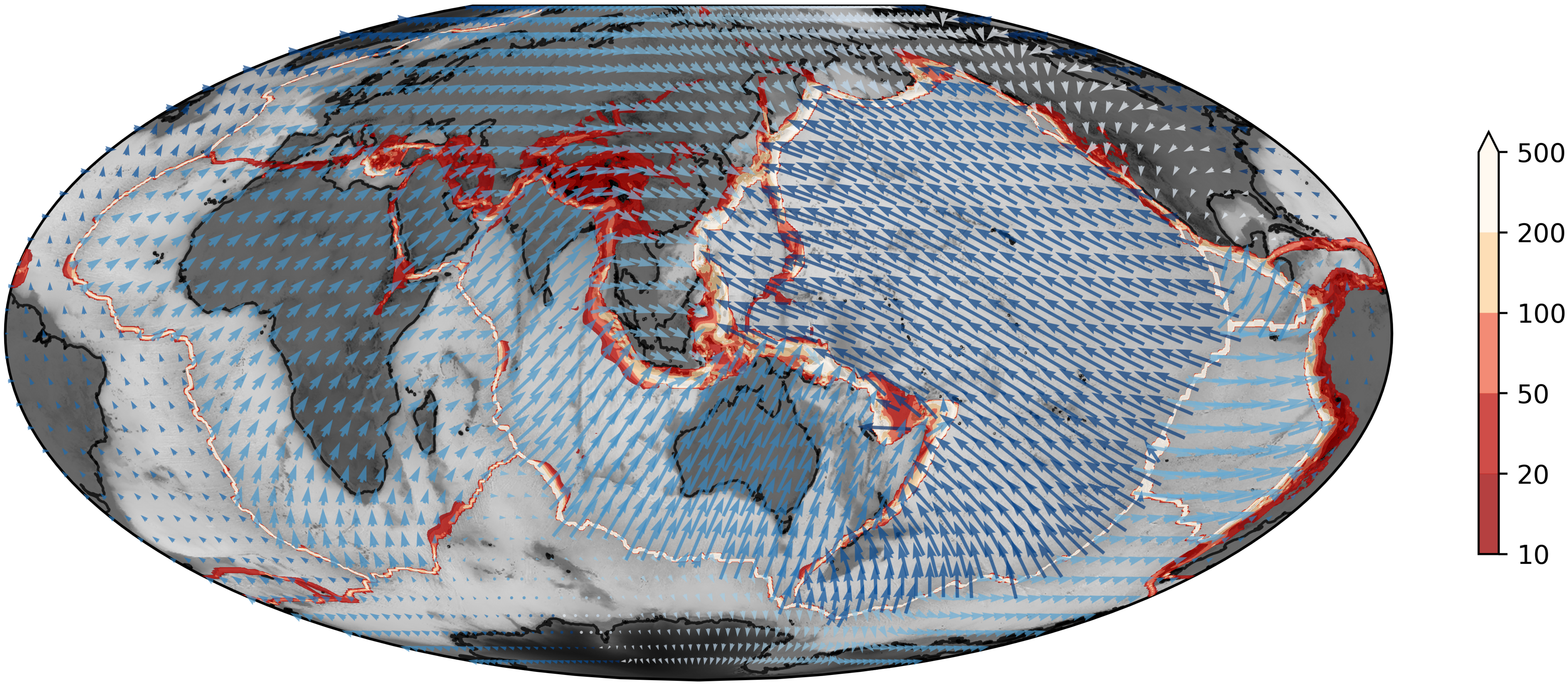
Convection in Earth's interior is (a little bit) like a boiling pot (see previous lecture)



Modified from USGS Graphics

The hot soup rises to the surface, spreads and begins to cool, and then sinks back to the bottom of the pot where it is reheated and rises again. **Why does hot soup rise and cold soup sink ?**

Clearly, the ocean basins and continents are behave differently

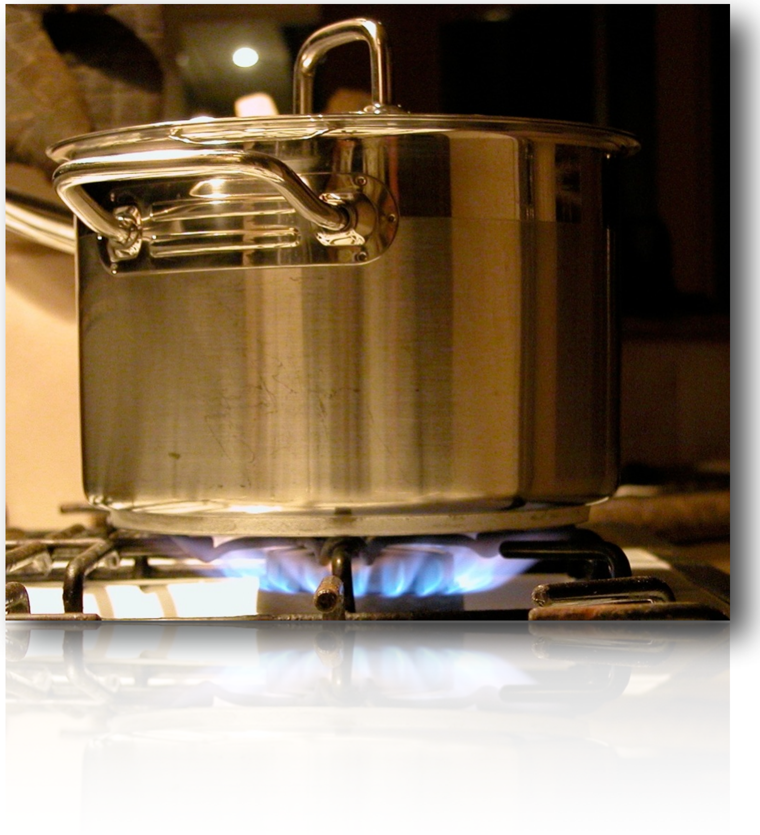


The ocean basins have a systematic variation in heat flow and thickness from mid ocean ridge to subduction zone that we can try to interpret in terms of the cooling boundary layer model. The continents do not fit that model at all because they are not part of the global 3D circulation.

Why does soup convect ?

What happens when we heat the pot of soup ?

- *Hot liquid is more buoyant than cold material and it* ***tends to rise***
- *Cooler liquid is less buoyant and therefore* ***tends to sink***
- *This can only happen if the two can* ***move past each other***
- *Convection produces a self-stirring*



Buoyancy forces are at work and viscous forces counteract these forces.

$$\text{buoyancy} \propto g\rho_0\alpha(1-\Delta T)$$

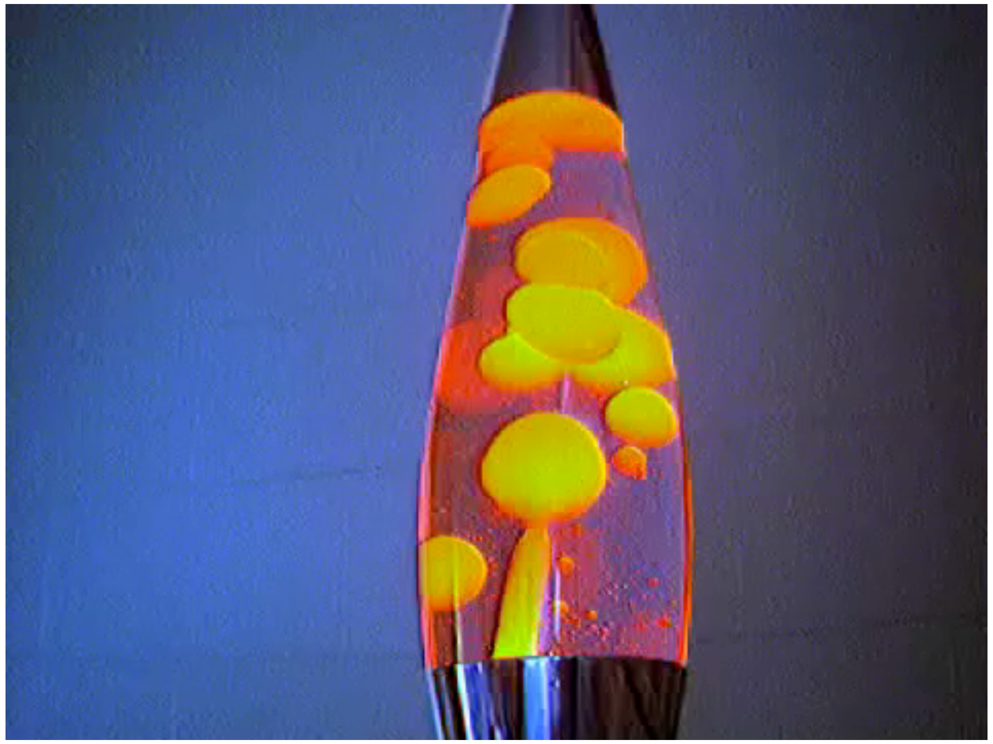
Convection like this will only work when the soup is heated from below or, in the case of the Earth, if it is heated from within by radioactivity.

(Why ?)

Why does soup convect ?

Definition of convection

Convection is the transfer of heat by the self-organised movement of a fluid. Free convection is when the fluid is stirred entirely by rising buoyant material and sinking negatively-buoyant material. (Forced convection is produced when the fluid is stirred mechanically).

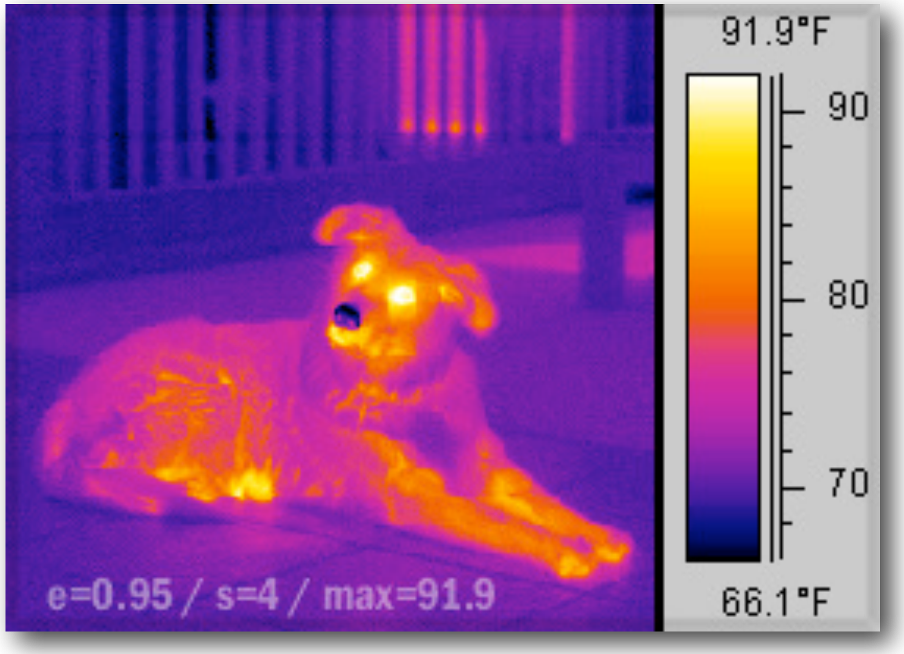


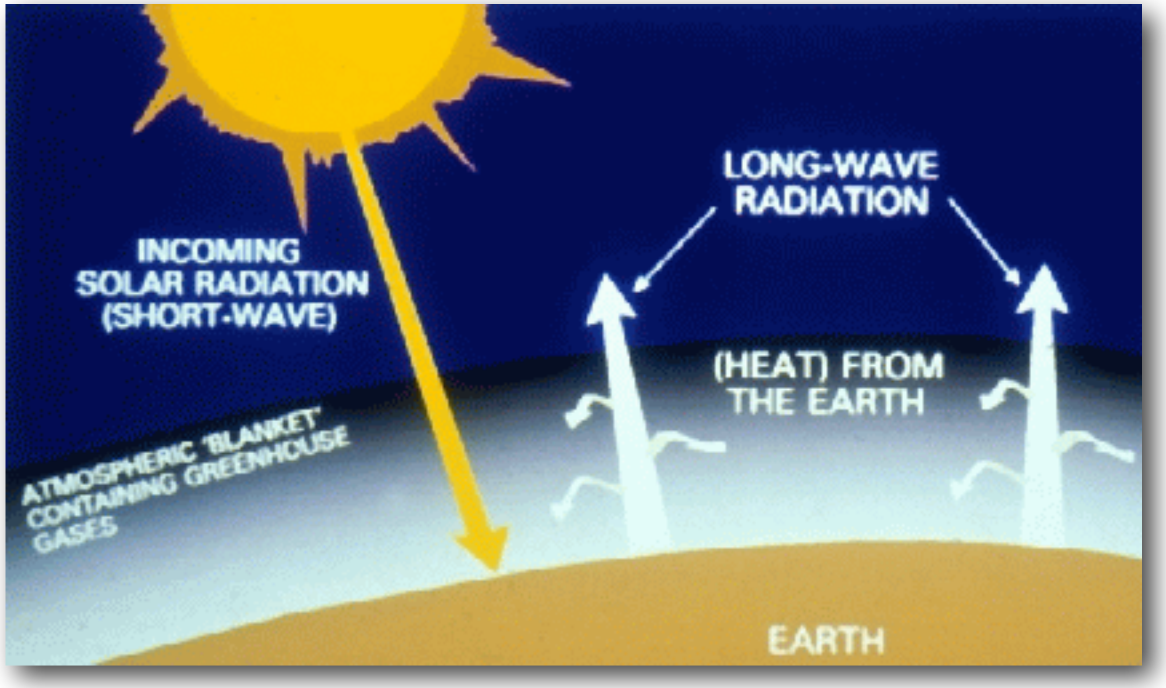
Convection is **one** of the ways we transfer heat from hot regions to cold regions. Other ways that heat can be transferred include radiation and conduction.

Radiation

Definition

- The transfer of (heat) energy by photons from one material to another.
- Does not require physical contact, or intermediary material (i.e. can transfer heat across a vacuum).
- Range of operation depends on how translucent the intermediary material is (to infra red)





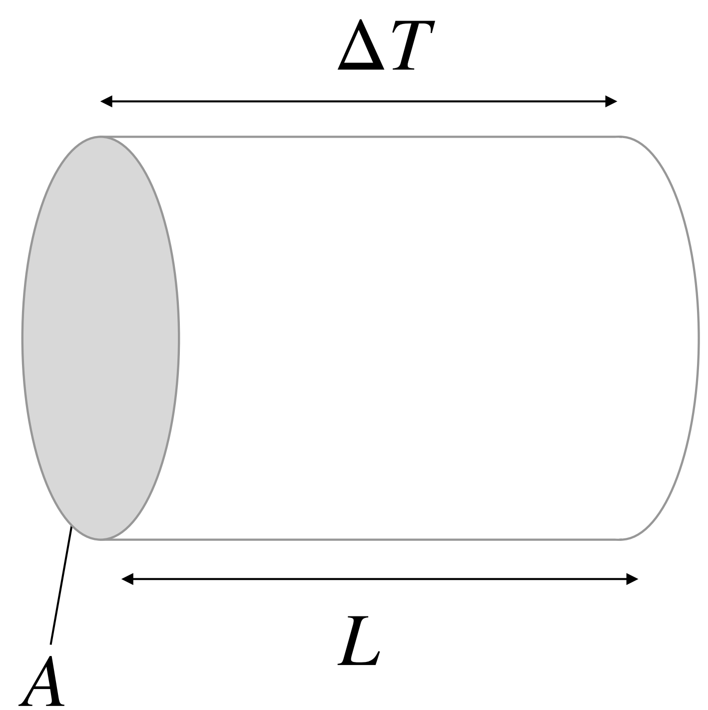
Conduction

Definition of heat energy transported across a bar in unit time:

- Conduction is the transfer of heat (or electric current) from one substance to another by direct contact (lattice vibrations / electrons)

- The transfer is always from a higher temperature to a lower temperature. Denser substances are usually better conductors; metals are excellent conductors.

$$Q = -\frac{kA\Delta T}{L}$$



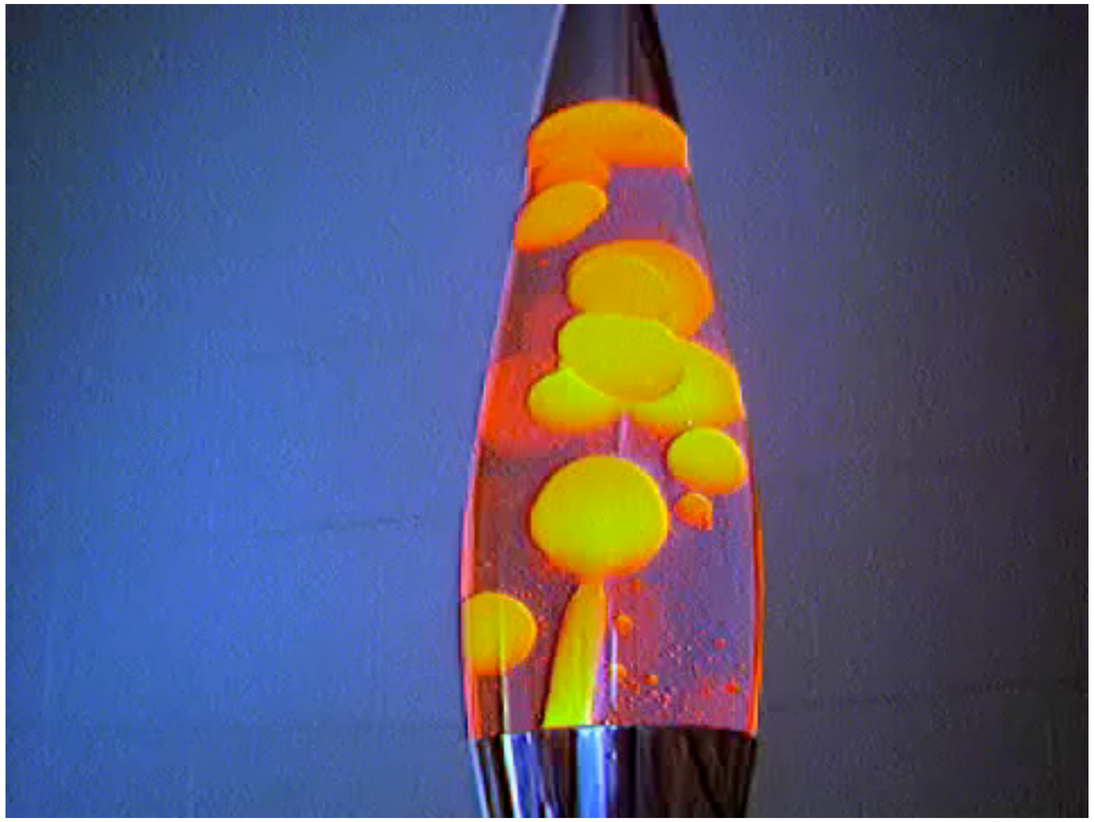
Fourier's Law:

Q is the heat transferred per unit time, A is the cross sectional area of the conducting sample, L is its width, k is a thermal conductivity ("constant") dependent on the nature of the material and often also on the temperature, and ΔT is the temperature difference across the sample (NOTE the minus sign ... **why is that there** ?)

Advection

Definition of Heat advection:

- Physical transport of heat by motion of the material from place to place which carries the heat along before it can be conducted away.

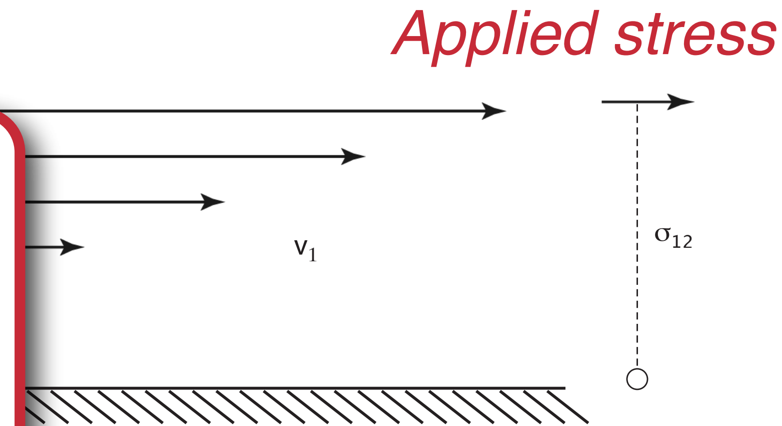


Convection is actually a balance between conduction of heat through a fluid, radiation (which is probably a minor effect in the mantle) and advection:

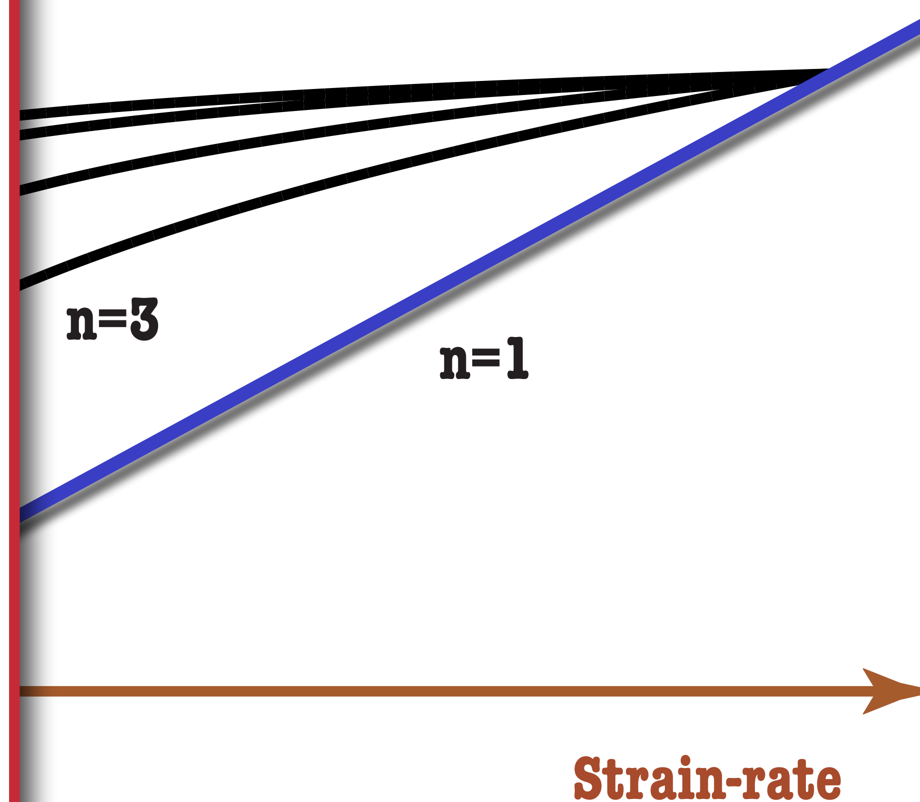
- Diffusion acts to suppress the temperature difference between the blobs and the background
- Advection carries heat from place to place as a result of the self-stirring of the fluid
- Heat can only enter or exit the convecting fluid by conduction (or radioactivity within a planet)

What is viscosity ?

Viscous deformation is irreversible **flow**



Viscosity is a measure of the resistance of a fluid to deform under shear stress. It is commonly perceived as "thickness", or resistance to flow. Viscosity describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction. Water is runny, having a lower viscosity, while honey is "thick" having a higher viscosity

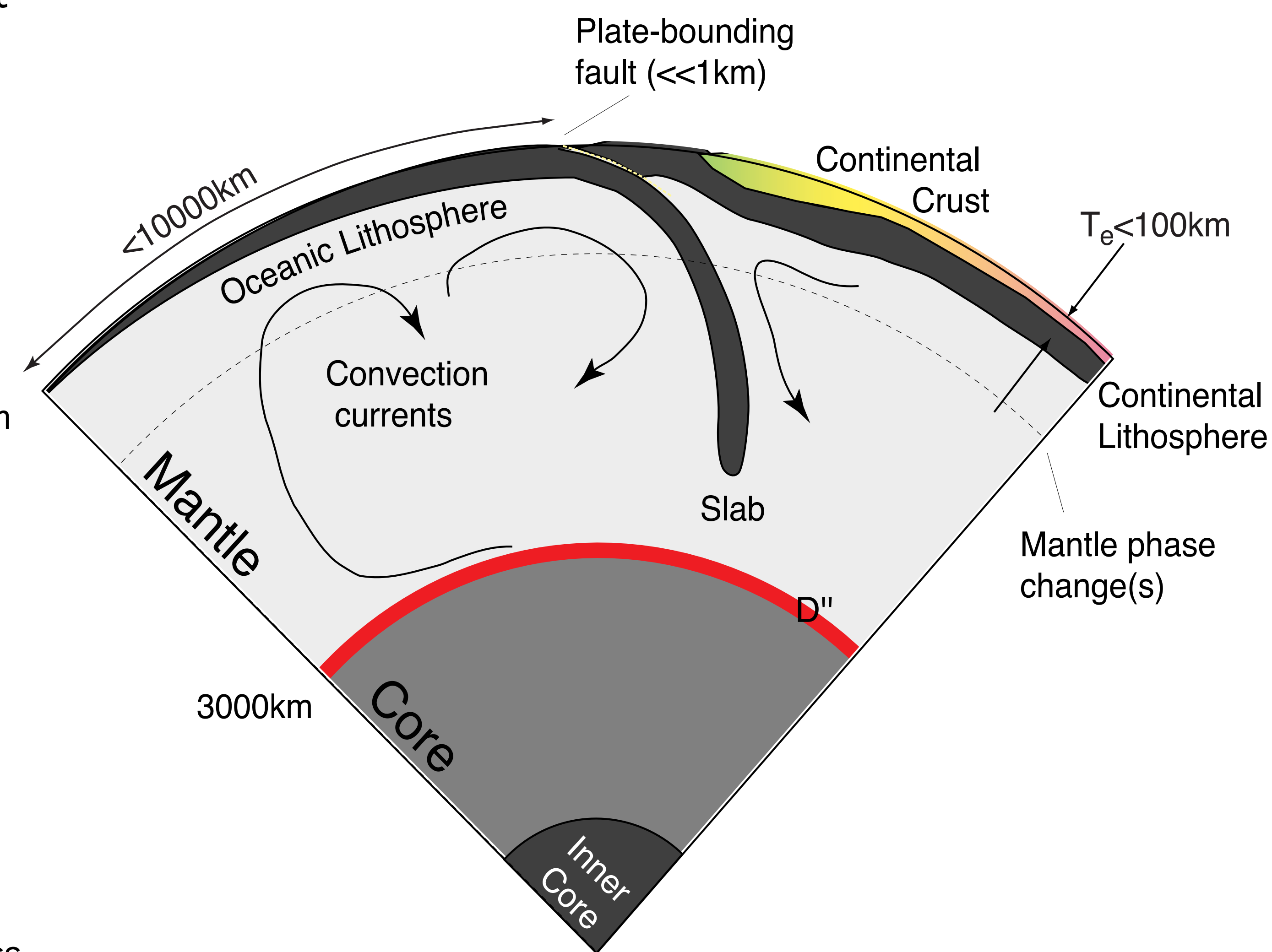


ses are familiar fluids, but so are

The Earth's mantle is not actually soup

The soup analogy is quite helpful because it demonstrates how heat escaping from the interior of the planet can do mechanical work (a.k.a. a *heat engine*).

This heat engine is the driving force behind tectonic motion and the constructive side of of the geological cycle.



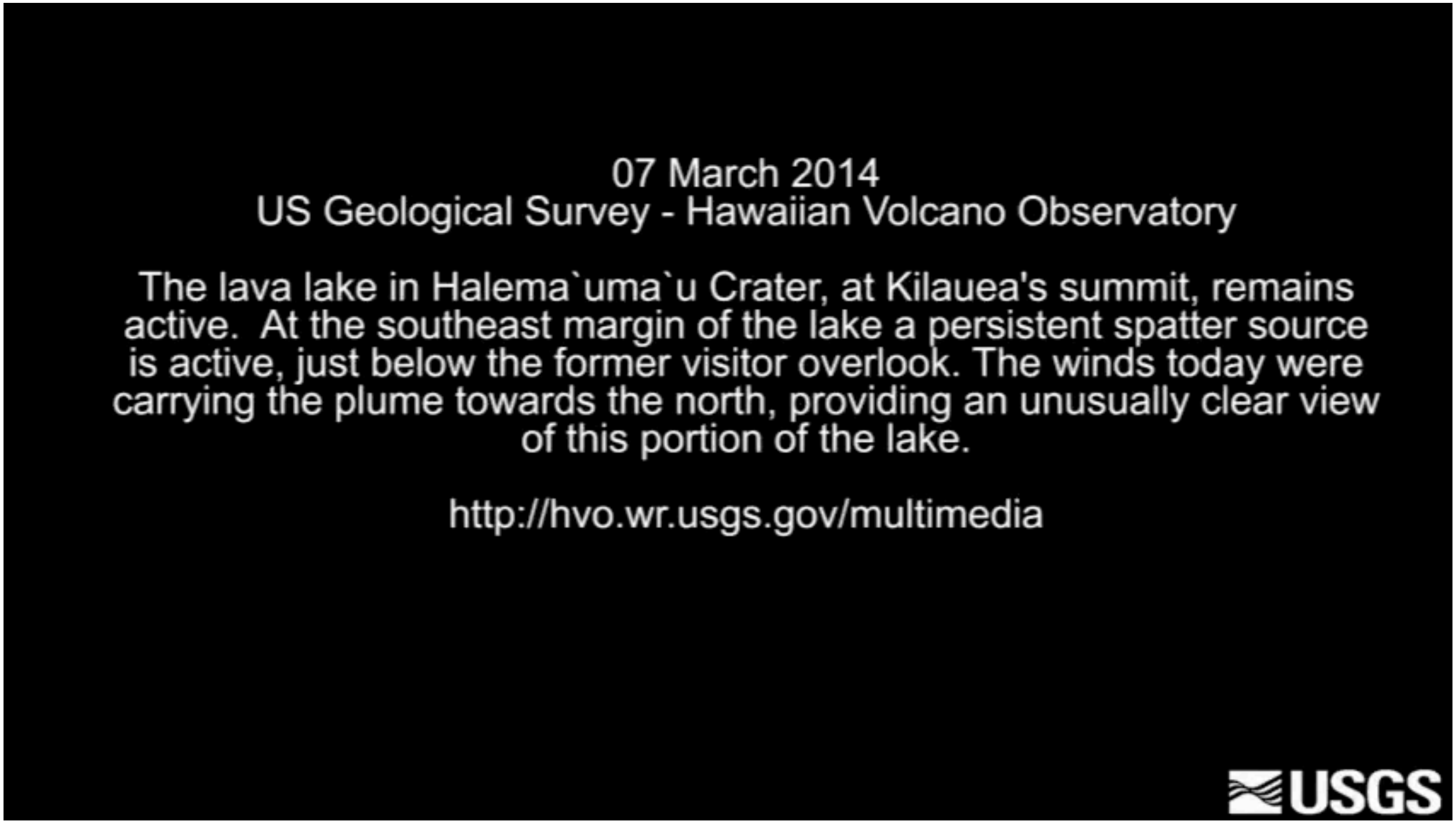
How does the Earth's mantle differ from the conceptual model (i.e. the pot of soup) ?

- The Earth's mantle is solid, the soup is liquid / a slurry
  (How do we know that the mantle is solid*?)
  (When does a solid act like a viscous fluid† ?)
- The soup does not show any evidence of plate tectonics
  (What would we see if it did ?)
- Hot and cold rocks have quite different physical properties (e.g. viscosity) even if they don't melt

* *Hint: how do we know that the outer core is not solid ?*

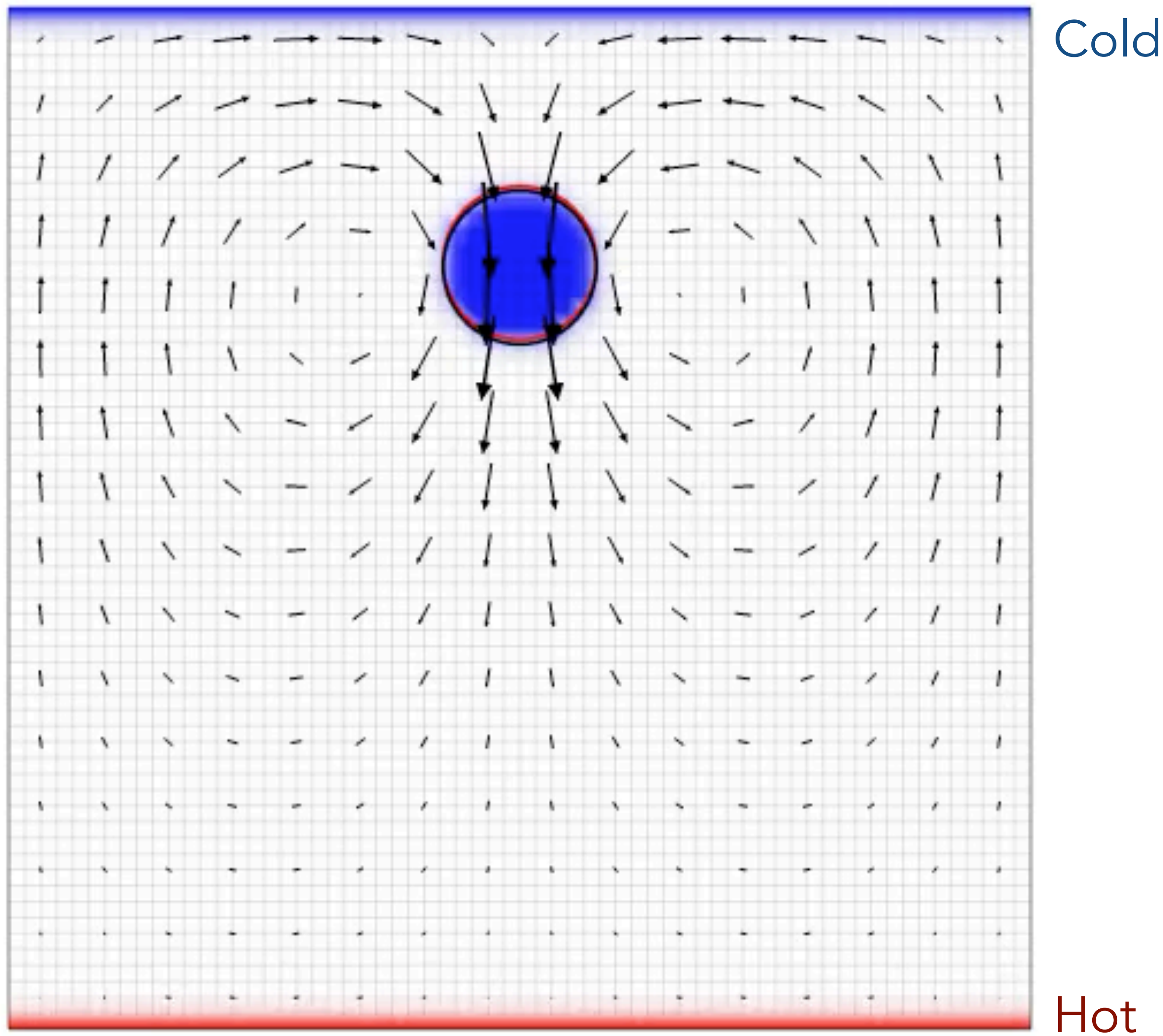
† *Later we will talk about crystal defects, deformation and viscosity.*

Lava Lake



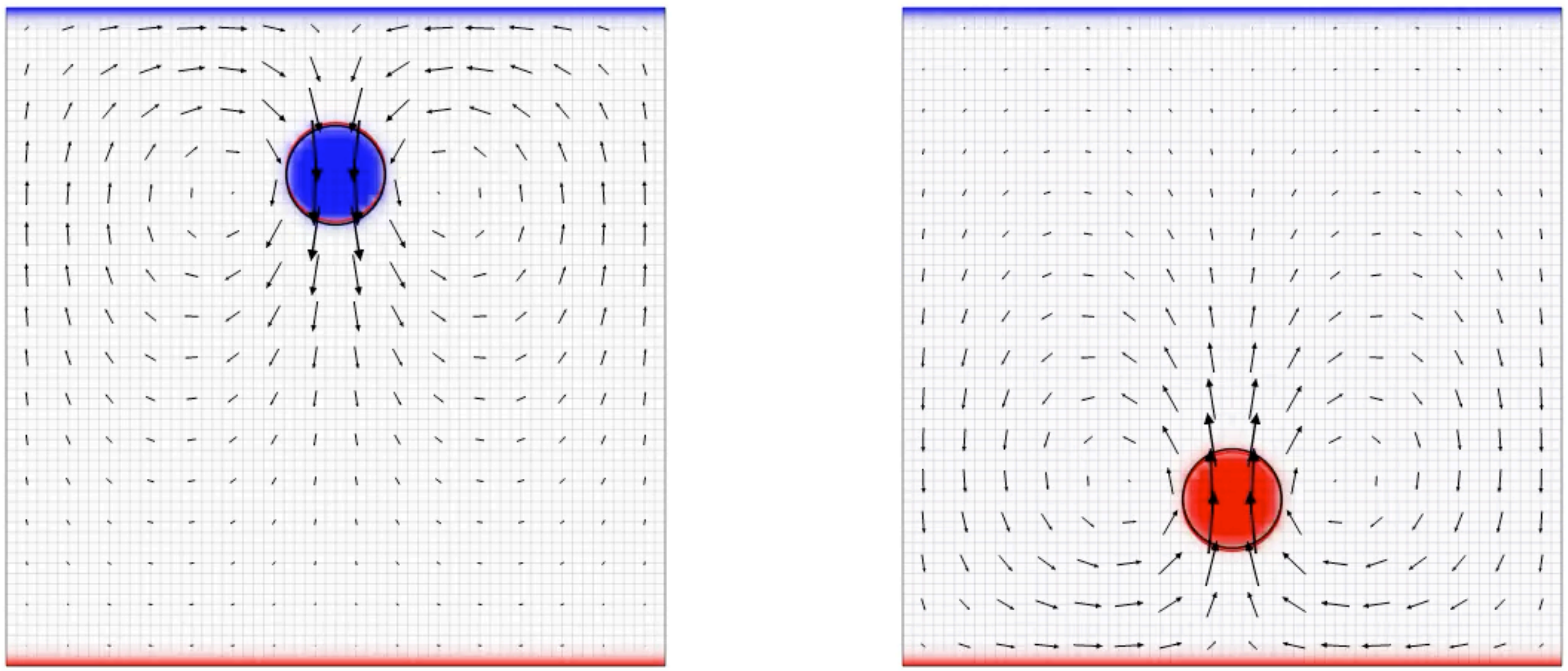
There are some hints at plate-like surface motions in caldera lava lakes like this one

A toy model



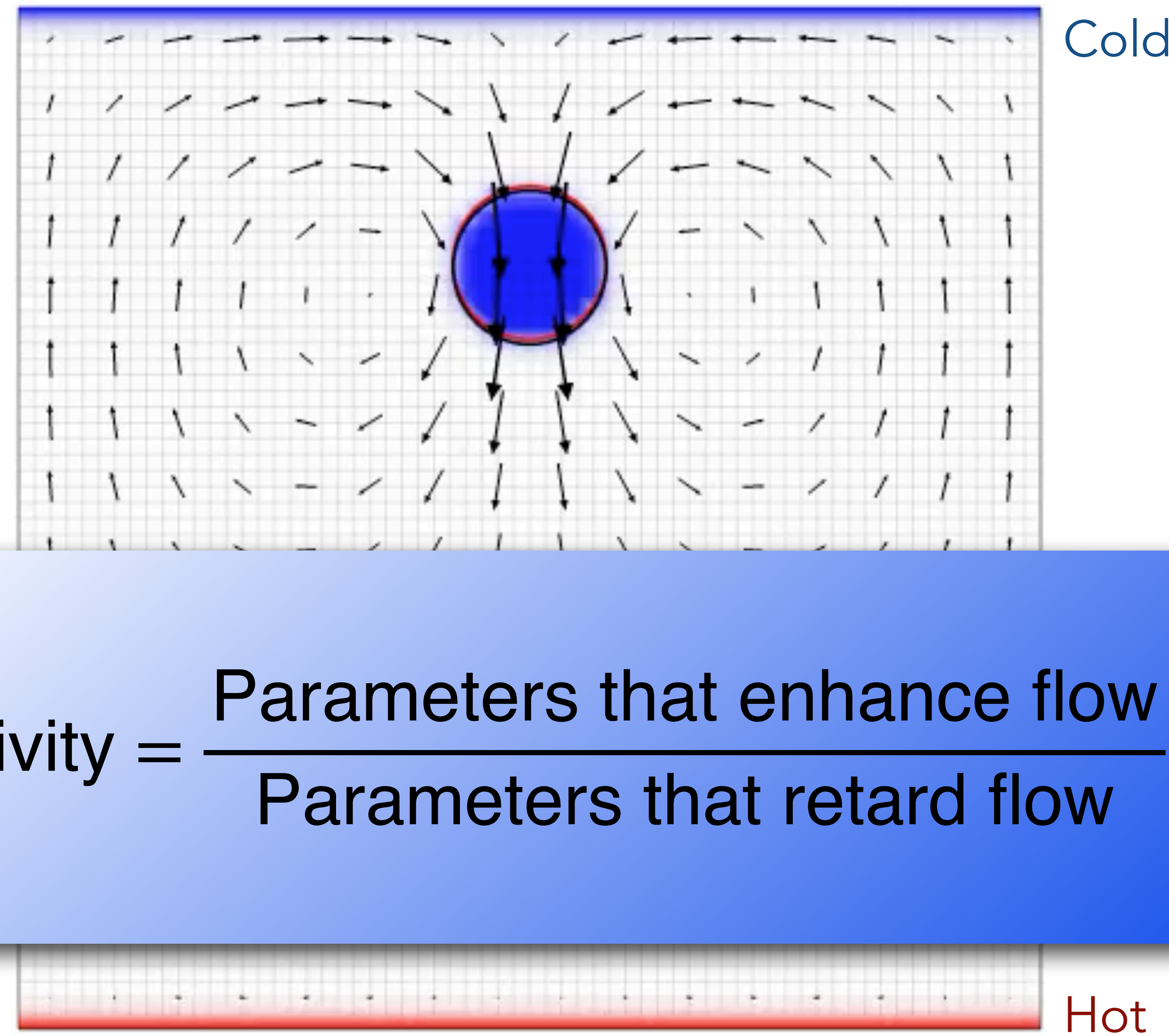
Imagine a tank of thick, viscous fluid (like syrup, for example). This turns out to be a reasonable model for the interior of a solid (rocky / icy) planet if we consider the evolution on a geological timescale. A cold / dense blob of material sinks and stirs a tank of fluid just like this computer model shows.

A toy model



In this toy model, do hot blobs and cold blobs behave in the same way (other than moving up v. down ?)

A toy model



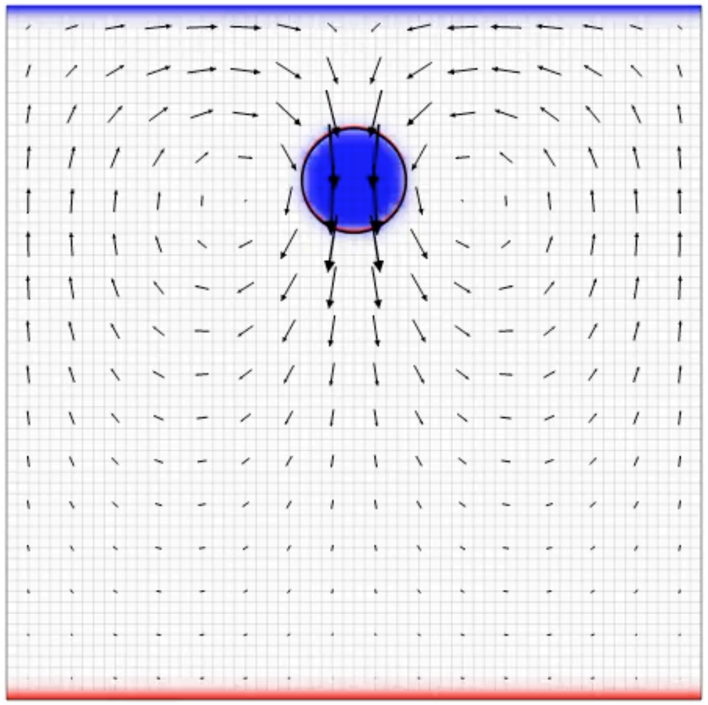
$$\text{Activity} = \frac{\text{Parameters that enhance flow}}{\text{Parameters that retard flow}}$$

Now let us think about how the physical properties of the fluid and the boundary conditions change how this blob behaves. We will consider how the behaviour changes with various parameters.

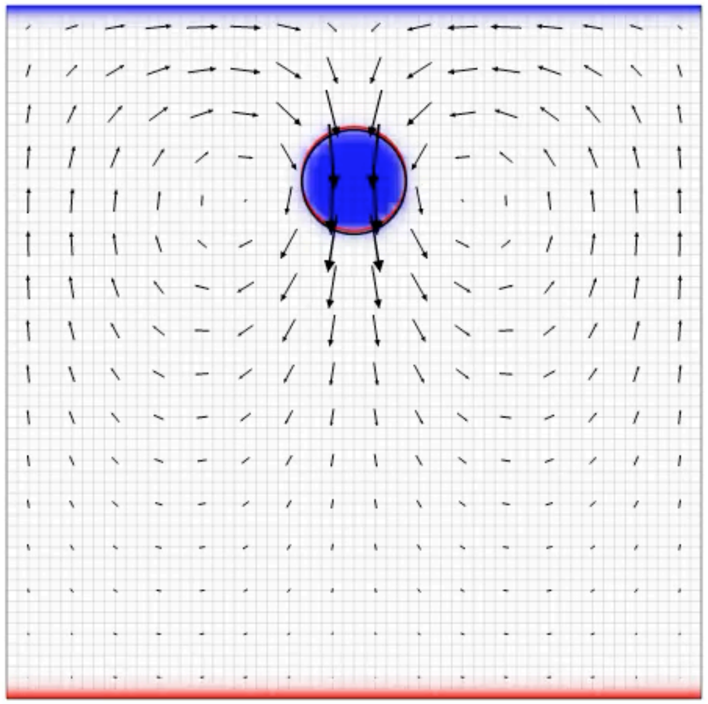
When we change a parameter does it tend to enhance flow or retard it ?

Buoyancy forces

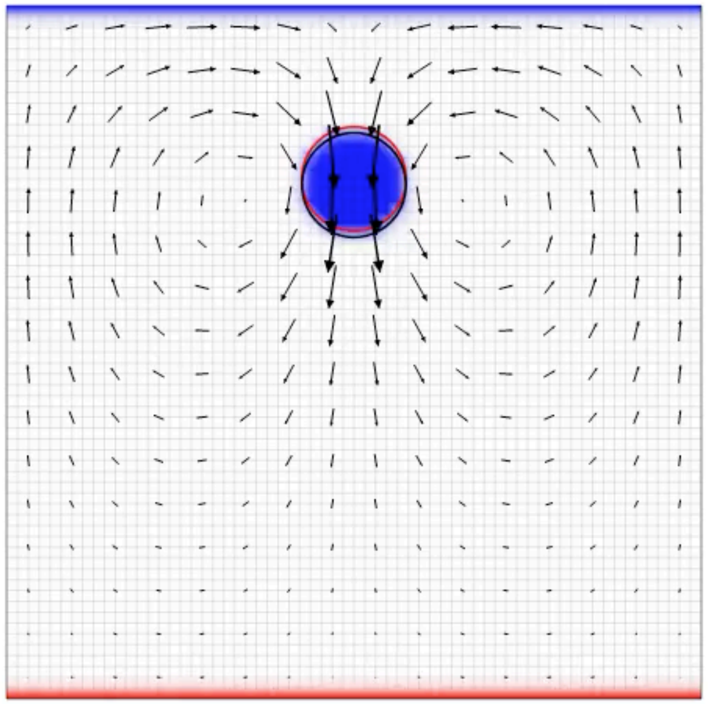
buoyancy $\propto g\rho_0\alpha(1-\Delta T)$



Lower buoyancy contrast



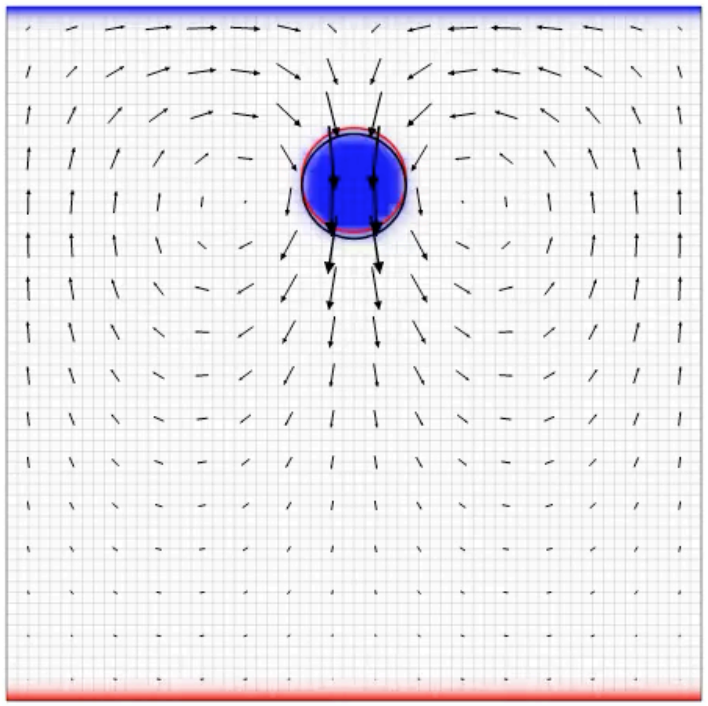
Reference case



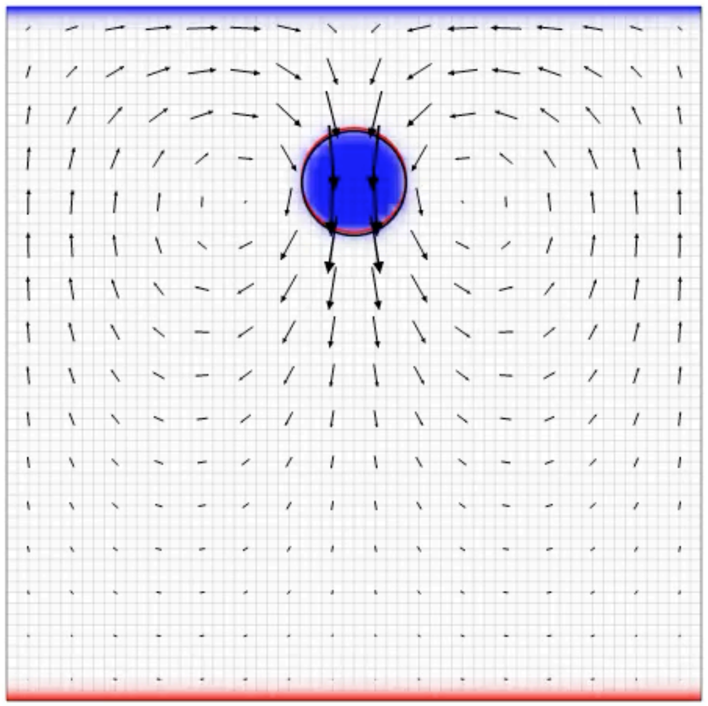
Higher buoyancy contrast

When we change this parameter does it tend to enhance flow or retard it ?

Viscous resistance



Lower background viscosity



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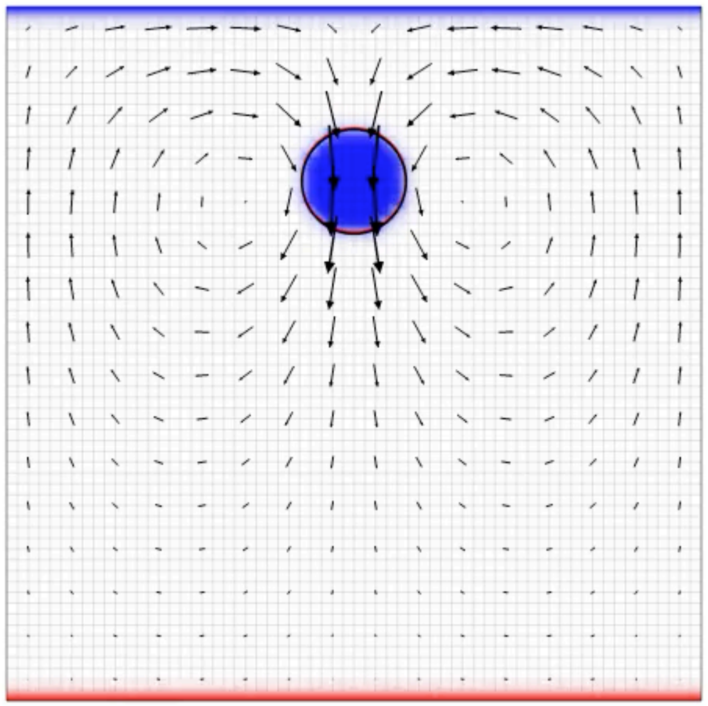
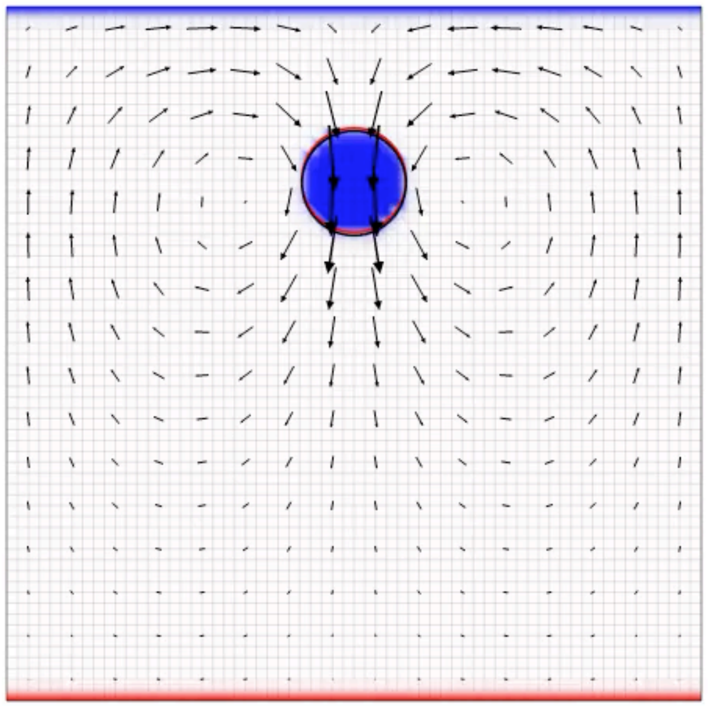


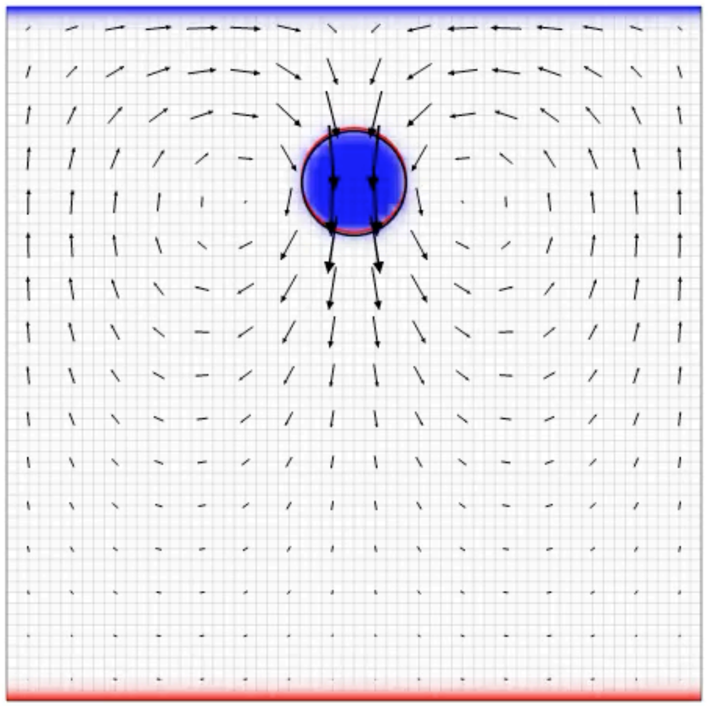
Higher background viscosity

When we change this parameter does it tend to enhance flow or retard it ?

Thermal conductivity (or diffusivity)



Lower thermal conductivity



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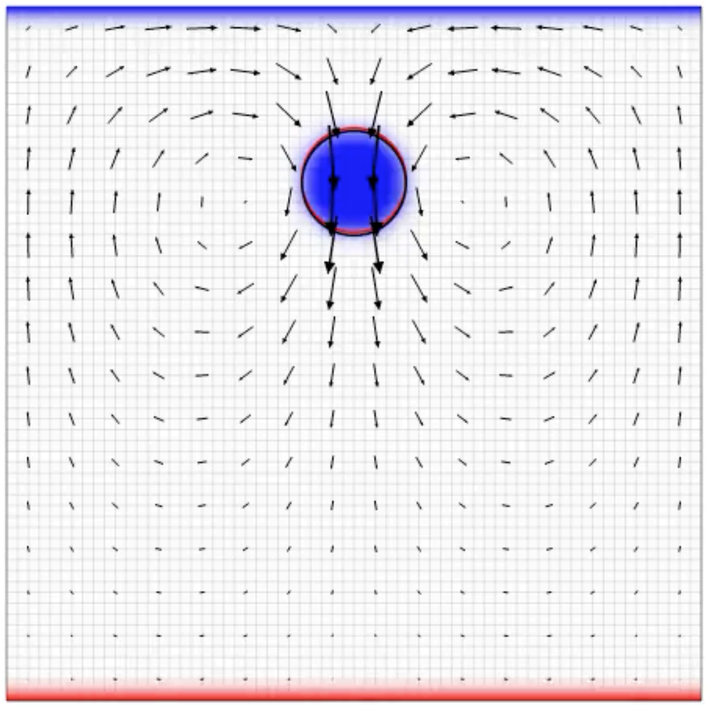
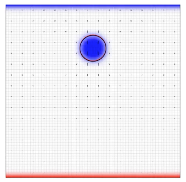


Higher thermal conductivity

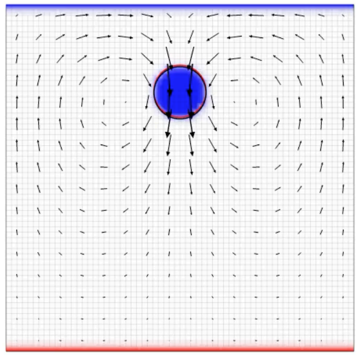
When we change this parameter does it tend to enhance flow or retard it ?

(Note - perhaps we need to look more closely at what is happening in this example)

Problem scale



Half-size box



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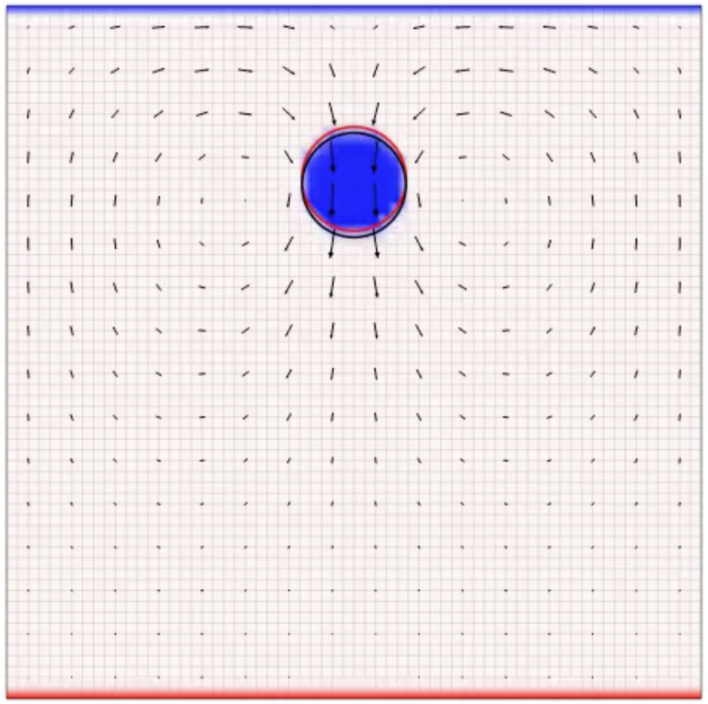
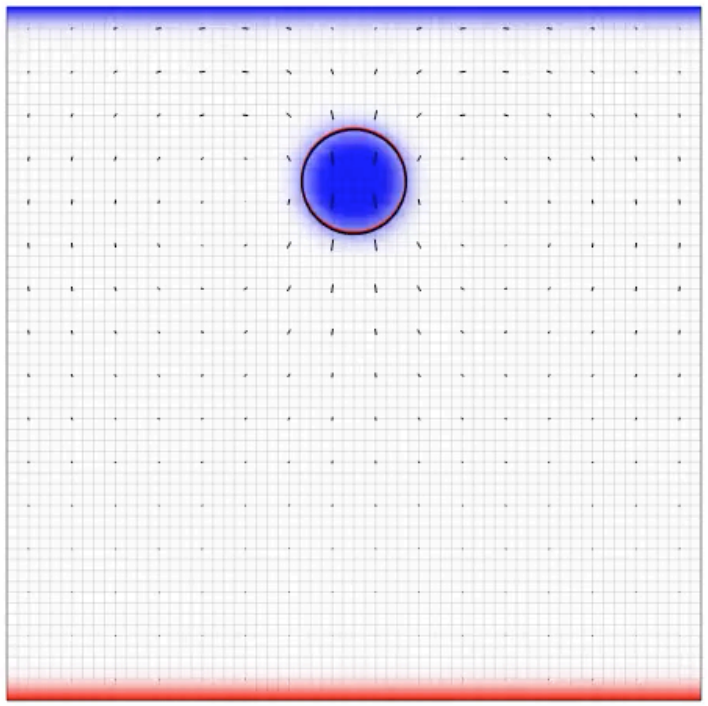


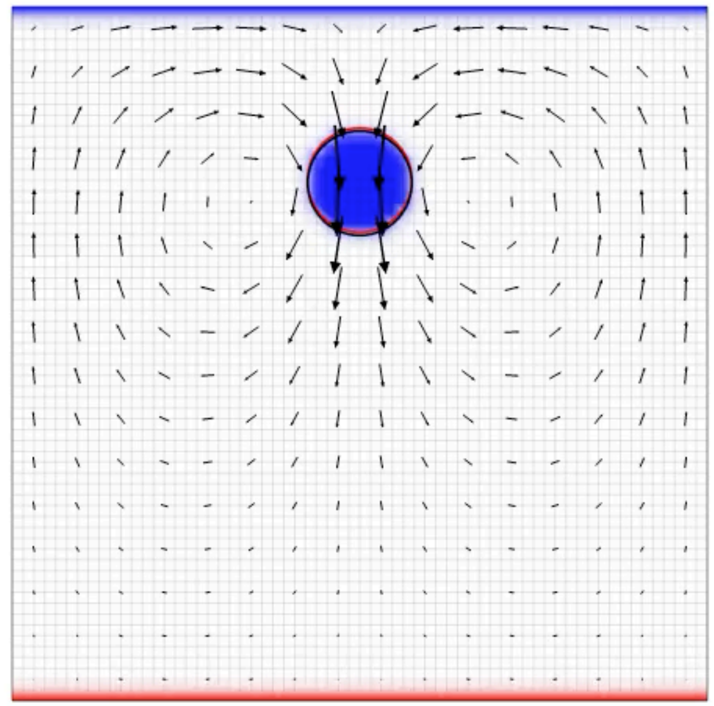
Double size box

When we change this parameter does it tend to enhance flow or retard it ?

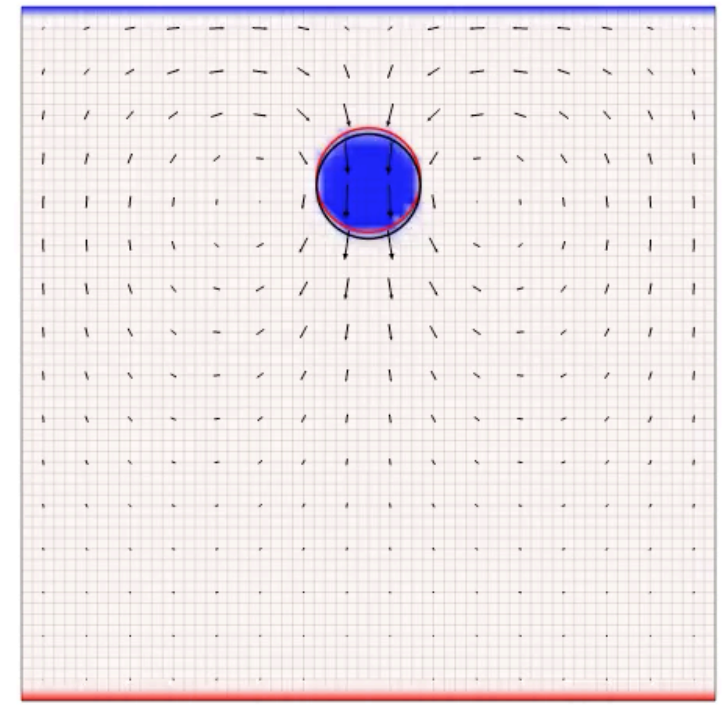
Problem scale (all viewed at the same size)



Half-size box



Reference case



Double size box

When we change this parameter does it tend to enhance flow or retard it ?

Convective activity

Intuition from these simple toy models suggests that the vigour of convection may depend upon the various system parameters according to the following equation:

$$\text{Activity} \propto \frac{\text{buoyancy} \times d}{\kappa\eta}$$

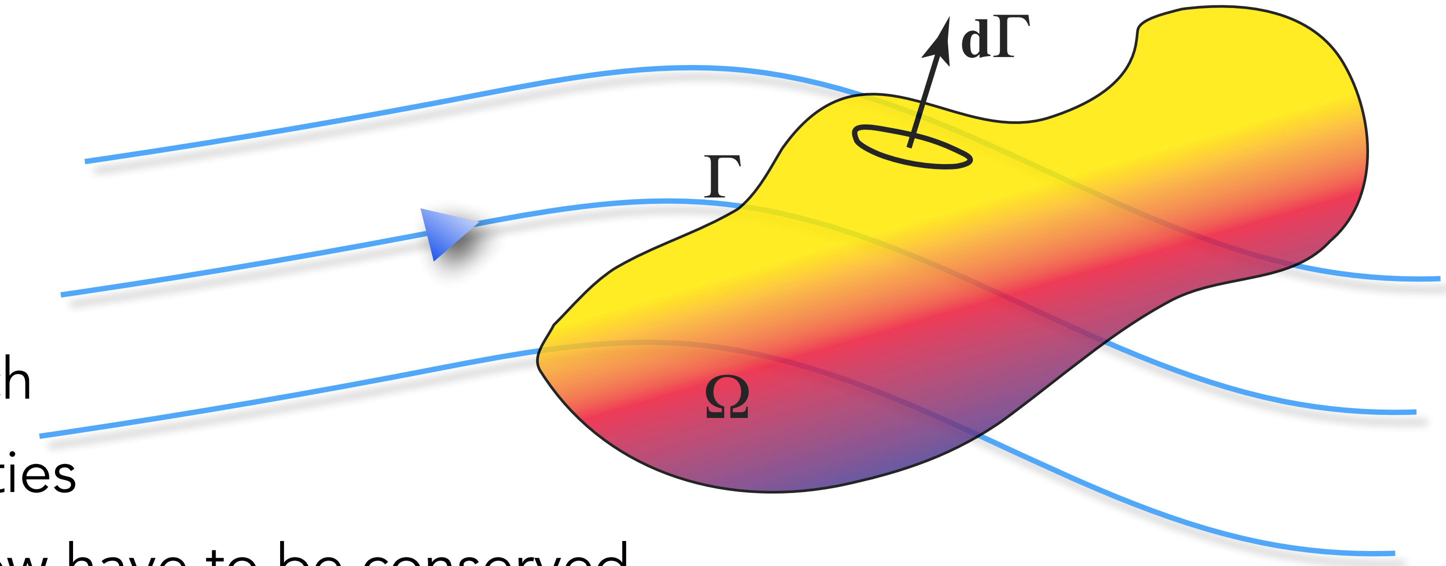
This turns out to be roughly correct, but there is an actual relationship that looks like this

$$\boxed{\text{Activity} \propto \frac{g\rho\alpha\Delta T d^3}{\kappa\eta}}$$

How do we know this is correct, and that we have considered all the relevant effects / parameters ?

Conservation Equations

(See lecture notes)



Consider an arbitrary volume in space in which there is fluid flowing. The fluid carries properties such as momentum and energy which we know have to be conserved.

A general expression for the evolution of some quantity, ϕ is this:

$$\frac{d}{dt}\int_\Omega \rho\phi d\Omega = \boxed{-\int_\Gamma \mathbf{F}\cdot\mathbf{d\Gamma}} + \boxed{\int_\Omega \rho H d\Omega} - \boxed{\int_\Gamma \rho\phi\mathbf{v}\cdot\mathbf{d\Gamma}}$$

Flux of ϕ through Γ absent any fluid transport

Sources of ϕ within Ω

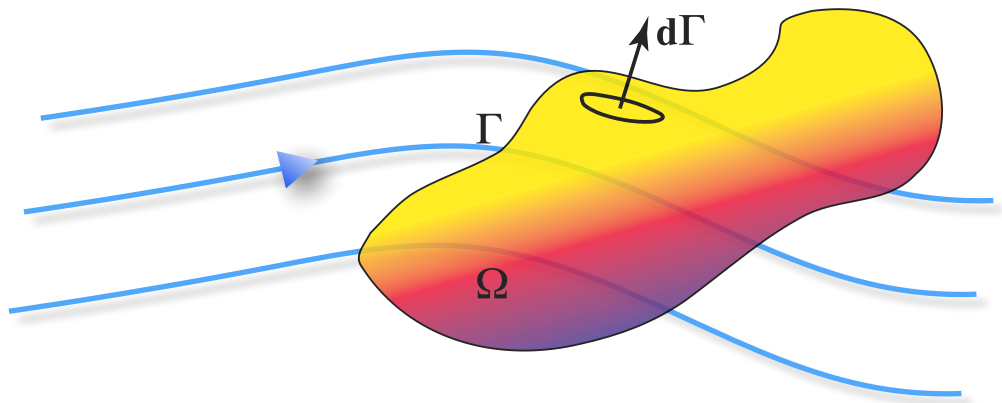
ϕ carried by fluid in/out of Ω

Gauss theorem, all integrals on Ω,
Independent of exact choice of Ω

$$\frac{d\rho\phi}{dt} + \nabla\cdot(\mathbf{F}+\rho\phi\mathbf{v}) - \rho H = 0$$

Conservation Equations — Mass

$$\frac{d}{dt}\int_\Omega \rho\phi d\Omega = \boxed{-\int_\Gamma \mathbf{F}\cdot\mathbf{d\Gamma}} + \boxed{\int_\Omega \rho H d\Omega} - \boxed{\int_\Gamma \rho\phi\mathbf{v}\cdot\mathbf{d\Gamma}}$$



Flux of ϕ through Γ absent any fluid transport

Sources of ϕ within Ω

ϕ carried by fluid in/out of Ω

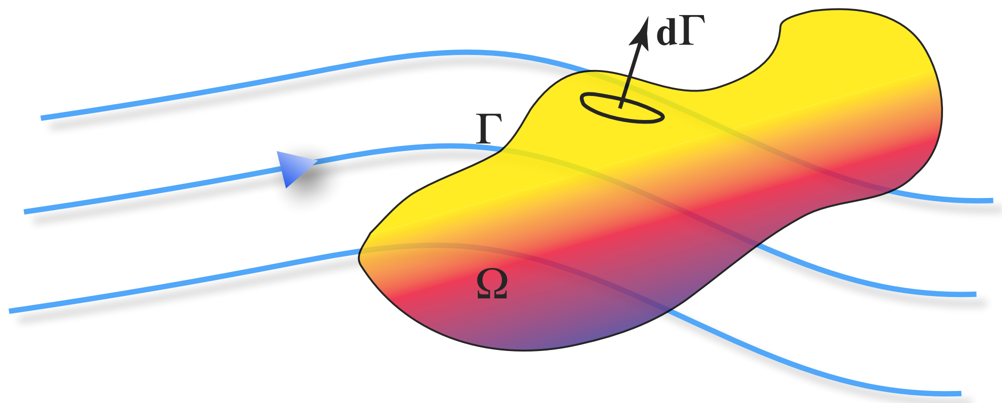
Mass: $\phi \leftarrow 1$

- no source term in classical mechanics
- no flux across the boundary if there is not any fluid flow

Hence: $\dfrac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{v} = 0$

Conservation Equations — Heat energy

$$\frac{d}{dt}\int_{\Omega} \rho\phi d\Omega = -\int_{\Gamma} \mathbf{F}\cdot\mathbf{d\Gamma} + \int_{\Omega} \rho H d\Omega - \int_{\Gamma} \rho\phi\mathbf{v}\cdot\mathbf{d\Gamma}$$



Flux of ϕ through Γ absent any fluid transport

Sources of ϕ within Ω

ϕ carried by fluid in/out of Ω

Heat Energy: $\phi \leftarrow C_p T$

- source terms possible, including "pure source" such as internal radioactive decay.
- flux across the boundary is heat-diffusion given by Fourier's Law

$\dfrac{\partial(\rho C_p T)}{\partial t} + \nabla\cdot\left(-k\nabla T + \rho C_p T\mathbf{v}\right) - \rho H = 0$ which we can rearrange, using mass conservation, to

$\left(\dfrac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = \kappa\nabla^2 T + \dfrac{H}{C_p}$ — note the directional derivative $\mathbf{v}\cdot\nabla$

Conservation Equations — Momentum

Momentum is a vector quantity so the equation has a vector form

$$\frac{d}{dt}\int_\Omega \rho \mathbf{v} d\Omega = -\int_\Omega \rho g \hat{\mathbf{z}} d\Omega + \int_\Gamma \boldsymbol{\sigma}\cdot \mathbf{d\Gamma} - \int_\Gamma \rho \mathbf{v}(\mathbf{v}\cdot \mathbf{d\Gamma})$$

- source terms are body force (vectors)
- flux across the boundary is tractions applies on the boundary by stresses in the fluid: $t_i = \sigma_{ij} n_j$ where $\hat{\mathbf{n}} = \left\{ n_j \right\}$ is the normal vector at a po
- Momentum is carried by fluid (an inertial term)

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \rho g \hat{\mathbf{z}} - \nabla\cdot\boldsymbol{\sigma} + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = 0$$ or $\frac{\partial \rho}{\partial}$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = \nabla\cdot\boldsymbol{\sigma} - g\rho\hat{\mathbf{z}}$$ where we h

$$(\mathbf{v}\cdot\nabla)\mathbf{u} = \hat{\imath}\left(v_1\frac{\partial u_1}{\partial x_1} + v_2\frac{\partial u_1}{\partial x_2} + v_3\frac{\partial u_1}{\partial x_3}\right)$$
$$\hat{\jmath}\left(v_1\frac{\partial u_2}{\partial x_1} + v_2\frac{\partial u_2}{\partial x_2} + v_3\frac{\partial u_2}{\partial x_3}\right)$$
$$\hat{k}\left(v_1\frac{\partial u_3}{\partial x_1} + v_2\frac{\partial u_3}{\partial x_2} + v_3\frac{\partial u_3}{\partial x_3}\right)$$

Also, the term $(\mathbf{v}\cdot\nabla)\mathbf{u}$ hides quite a bit in this

Stress and Viscosity

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = \nabla \cdot \boldsymbol{\sigma} - g\rho\hat{\mathbf{z}}$$

but to close this system of equations, we need to know how $\boldsymbol{\sigma}$ relates to the momentum or velocity. In a Newtonian, viscous fluid:

$$\tau_{ij} = \eta \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right]$$ (the deviatoric stress) and

$$\sigma_{ij} = \eta \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] - \frac{\partial p}{\partial x_i}$$ (the total stress)

Hence $\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = \eta \nabla^2 \mathbf{v} - \nabla p - g\rho\hat{\mathbf{z}}$ (assuming constant η)

Navier Stokes equation / Energy equation for fluid flow

$$\rho_0 \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = \eta \nabla^2 \mathbf{v} - \nabla p - g\rho_0 \alpha (T - T_0)\hat{\mathbf{z}}$$

$$\left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T \right) = \kappa \nabla^2 T$$

This is Navier-Stokes equation plus the advection-diffusion equation for thermal evolution. These together define the convection problem — boundary and initial conditions are needed too.

Navier-Stokes is a balance of accelerations (on the left) with changes in forces on the right (the fluid equivalent of $F = ma$). It is complicated because the balance occurs in 3D. Changes in one direction can balance those in another.

The equivalent for temperature is actually a generalisation of the Fourier expression to account for heat transport by advection. Again, temperature gradients in one direction might balance thermal advection in another direction.

That is what it means to write a partial differential equation. The best way to approach this system is to understand what it does physically BEFORE looking at the mathematics.

Navier Stokes equation

$$\rho_0 \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = \eta \nabla^2 \mathbf{v} - \nabla p - g\rho_0 \alpha (T - T_0)\hat{\mathbf{z}}$$

$$\left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T \right) = \kappa \nabla^2 T$$

Depending upon the following physical parameters which vary from experiment to experiment & planet to planet

viscosity	η	$Pa.s$
temperature scale	$\Delta T = T - T_0$	K
length scale	x	m
density	ρ_0	$kg.m^{-3}$
thermal expansivity	α	K^{-1}
gravity	g	$m.s^{-2}$
thermal diffusivity	κ	$m^2 s^{-1}$

Non-dimensionalization of Navier Stokes

It should not matter if we work in **SI** units **cgs** units or **Imperial** units — the physics does not change and the system does not care about our choice. That means there should be a way to express the equations without having to define specific units at all (or some sort of natural units for the problem that make everything close to 1).*

Consider a prototype planet which is completely flat — the mantle is a layer of fixed depth with fixed top and bottom temperatures and no-shear-stress boundaries.

We can rescale all our numbers by typical values of length, time, temperature etc. We have to be consistent (remember dimensional analysis) so these choices work, for example:

$$x = d.x' \qquad \partial/\partial x = (1/d)\partial/\partial x' \qquad \nabla = (1/d)\nabla'$$

$$t = (d^2/\kappa)t' \qquad \partial/\partial t = (\kappa/d^2)\partial/\partial t'$$

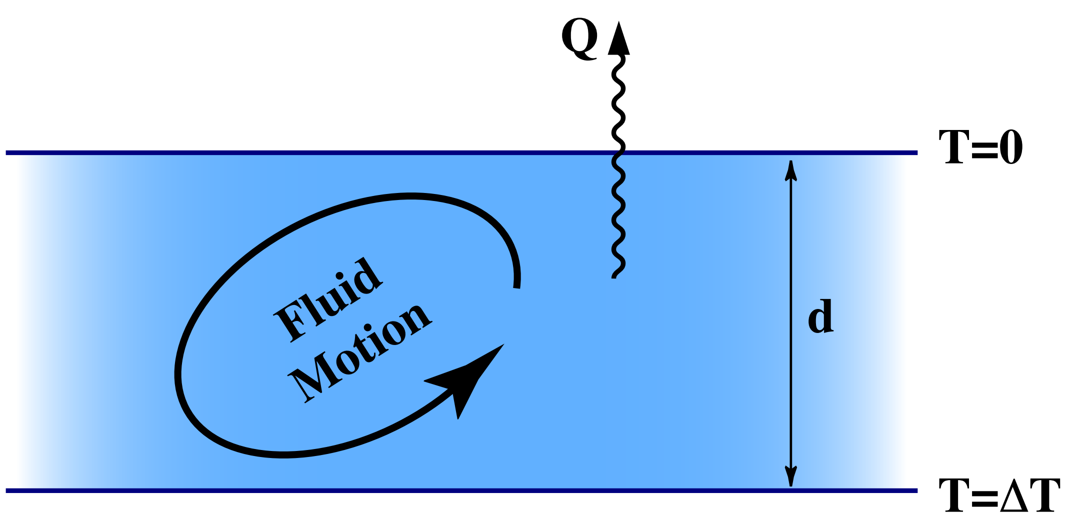
$$T = \Delta T T'$$

$$v = (\kappa/d)v'$$

$$p = p_0 + (\eta\kappa/d^2)p'$$

where

$$\nabla p_0 = -g\rho_0$$



(*) this is what people used to do — feet / inches for carpenters, furlongs for farmers / horses.

Non-dimensionalization

Substituting for all the existing terms in the Navier-Stokes equation

$$\frac{\rho_0 \kappa}{d^2} \frac{D}{Dt'} \left(\frac{\kappa}{d} \mathbf{v}' \right) = \frac{\eta}{d^2} \acute{\nabla}^2 \left(\frac{\kappa}{d} \mathbf{v}' \right) - \frac{\eta \kappa}{d^3} \acute{\nabla} p' + g \rho_0 \alpha \Delta T T' \hat{\mathbf{z}}$$

Collect up terms:

$$\frac{\rho \kappa}{\eta} \frac{D\mathbf{v}'}{Dt'} = \acute{\nabla}^2 \mathbf{v}' - \acute{\nabla} p' + \frac{g \rho_0 \alpha \Delta T d^3}{\kappa \eta} T' \hat{\mathbf{z}}$$

or

$$\boxed{\frac{1}{\mathrm{Pr}} \frac{D\mathbf{v}'}{Dt'} = \acute{\nabla}^2 \mathbf{v}' - \acute{\nabla} p' + \mathrm{Ra} T' \hat{\mathbf{z}}}$$

There are primes everywhere now because we converted all the numbers to ratios and put all the units into one place. These units in those numbers MUST cancel out or the equation would be sensitive to our choices about measurement.

There we have our dimensionless quantities such as Rayleigh number appearing automatically.

If we estimate the Prandtl number for the Earth it is $\gg 10^{20}$ so the left hand side is much smaller than the right and can be ignored completely (no accelerations / no inertia and thus no turbulence).

Dimensionless numbers

Rayleigh number - measures the intensity of the buoyancy forces

$$\mathrm{Ra} = \frac{g\rho\alpha\Delta T d^3}{\eta\kappa}$$

Reynolds number - measures the relative importance of inertial to viscous forces

$$\mathrm{Re} = \frac{\rho V_0 d}{\eta} = \frac{V_0 d}{\nu}$$

Prandtl number - measures the relative importance of stress diffusion and thermal diffusion

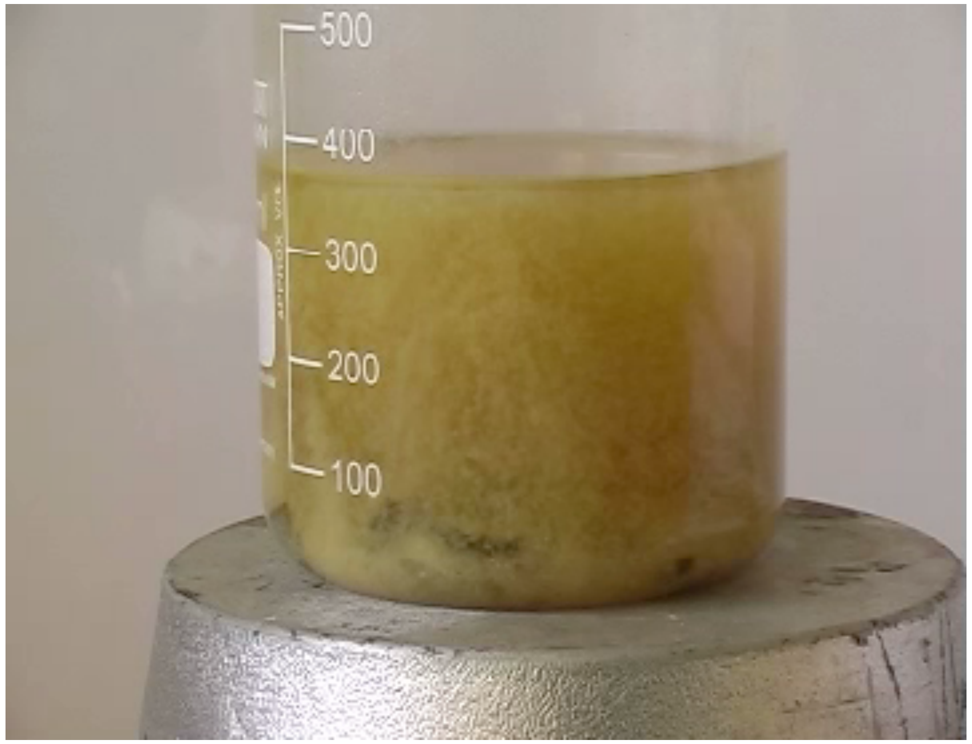
$$\mathrm{Pr} = \frac{\eta}{\rho\kappa} = \frac{\nu}{\kappa}$$

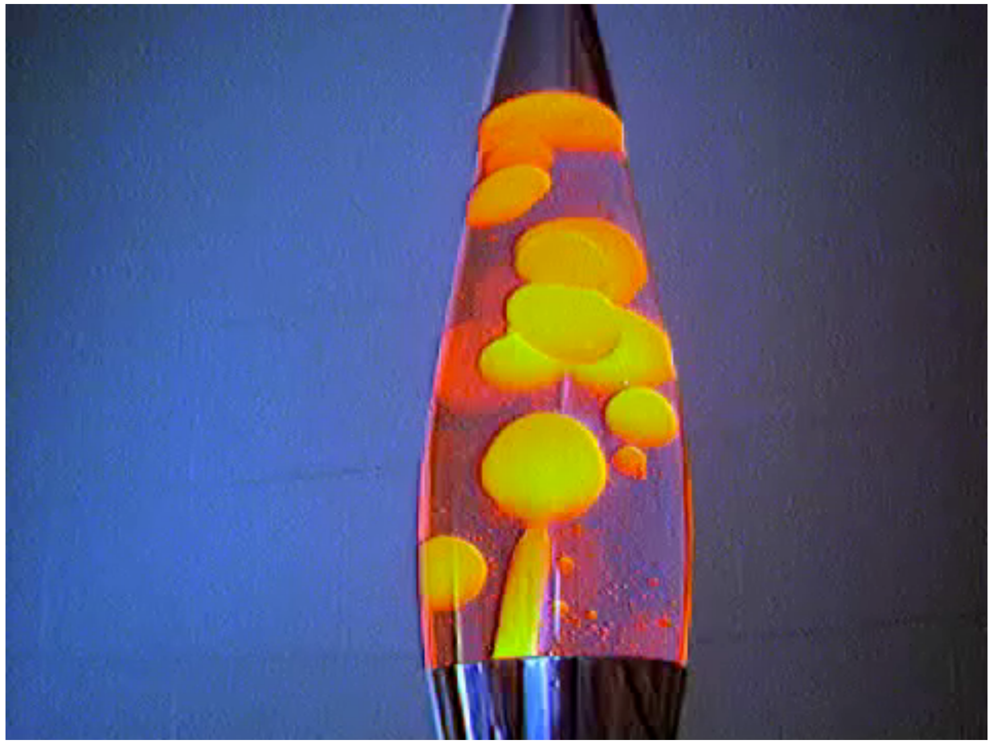
Convection

These dimensionless numbers are useful. For example, we can build an equivalent of Fourier's law (earlier slide) that tells us how much heat can be transported by a convecting layer.

$$Q = Q_c \left[\frac{g\rho\alpha\Delta T d^3}{\kappa\eta} \right]^{\frac{1}{3}} \quad \text{or} \quad Q = Q_c \mathrm{Ra}^{\frac{1}{3}}$$

Here Q_c is the heat transported by a non-convecting layer of the same thermal properties and this ratio (Q/Q_c) is also dimensionless and is known as the Nusselt number.





Incompressible flow

In all the derivations so far we assume two incompatible things:

- ρ is constant ($\nabla \cdot \rho \mathbf{v} \equiv \rho_0 \nabla \cdot \mathbf{v}$)
- flow is driven by buoyancy forces that result from changes in density

It is not unusual to consider incompressible flows (even for air if the Mach number is "small") and we do not lose a lot of detail if we do this. We assume that the density variations are small so they are only important when they appear in a term that does not depend on the actual density but only the density gradient (e.g. buoyancy)

This is called the *Boussinesq approximation*.

Pressure becomes a constraint parameter rather than a physical pressure (cf a Lagrange multiplier) and the constraint equation is:

$$\nabla \cdot \mathbf{v} = 0$$

One way to see how this works is to rewrite the equations to show that pressure can be eliminated if we assume incompressible flow.

The stream function

The stream-function is a scalar quantity which defines a 2D incompressible flow everywhere. It satisfies:

$$v_1 = -\frac{\partial \psi}{\partial x_2}; \quad v_2 = \frac{\partial \psi}{\partial x_1}$$ — compare this to $\mathbf{v} = \nabla \times (\psi \hat{\mathbf{k}})$ an out-of-plane vector quantity

and this automatically implies

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0$$ (i.e. incompressibility)

and

$$(\mathbf{v} \cdot \nabla)\psi = v_1 \frac{\partial \psi}{\partial x_1} + v_2 \frac{\partial \psi}{\partial x_2} = \frac{\partial \psi}{\partial x_2}\frac{\partial \psi}{\partial x_1} - \frac{\partial \psi}{\partial x_1}\frac{\partial \psi}{\partial x_2} = 0$$ i.e ψ does not change during advection

Note: a more general, 3D version of this derivation is the Helmholtz decomposition of a vector field into irrotational and a solenoidal (divergence free) parts:

$$\mathbf{u} = \nabla \phi + \nabla \times \mathbf{A}$$

The biharmonic equation

Vorticity is defined as $\boldsymbol{\omega} = \nabla \times \mathbf{v}$ but in 2D, there is just one component of this equation and we treat it as a scalar ($\omega \leftarrow \omega_3$)

$$\omega = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}$$

If we take the curl of the Navier-Stokes equation we find we can eliminate the pressure gradients as $\nabla \times \nabla P \equiv 0$. For infinite Prandtl number, the L.H.S vanishes.

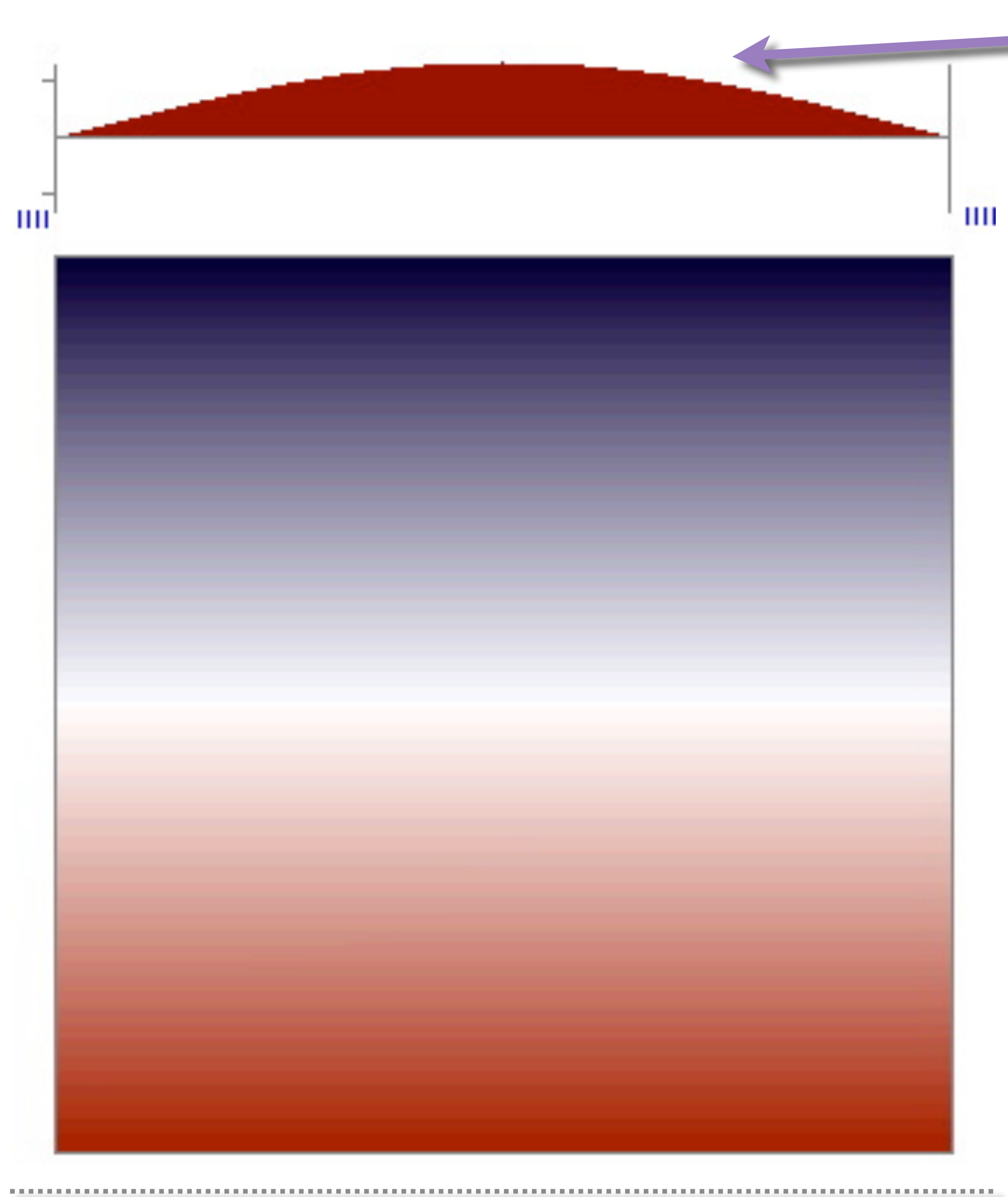
$$\frac{1}{\mathrm{Pr}} \left(\frac{D\boldsymbol{\omega}}{Dt} - (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} \right) = \eta \nabla^2 \boldsymbol{\omega} + \mathrm{Ra} \frac{\partial T}{\partial x_1} \quad (1)$$

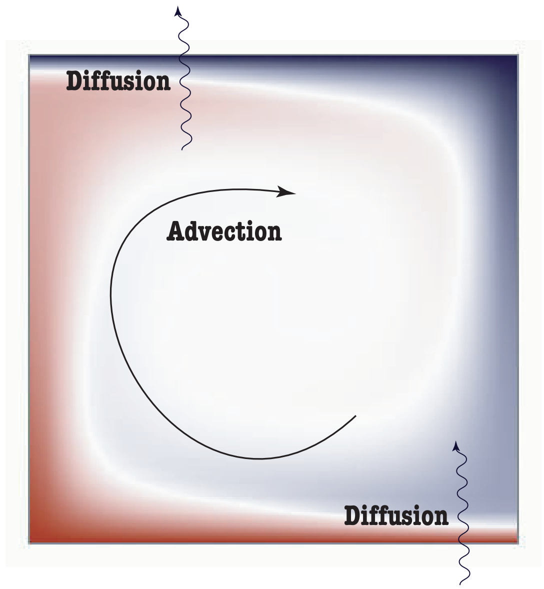
Now, $\mathbf{v} = (-\partial\psi/\partial x_2, \partial\psi/\partial x_1, 0)$ implies that $\nabla \times \mathbf{v} = \nabla^2 \psi = \omega$ and, if we substitute this into (1):

$$\nabla^4 \psi = -\,\mathrm{Ra} \frac{\partial T}{\partial x_1} \quad \text{(the biharmonic equation)}$$

where ∇^4 is known as the biharmonic operator and $\nabla^4 \equiv \nabla^2(\nabla^2) \equiv \left(\frac{\partial^4}{\partial x_1^4} + \frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2} + \frac{\partial^4}{\partial x_2^4} \right)$

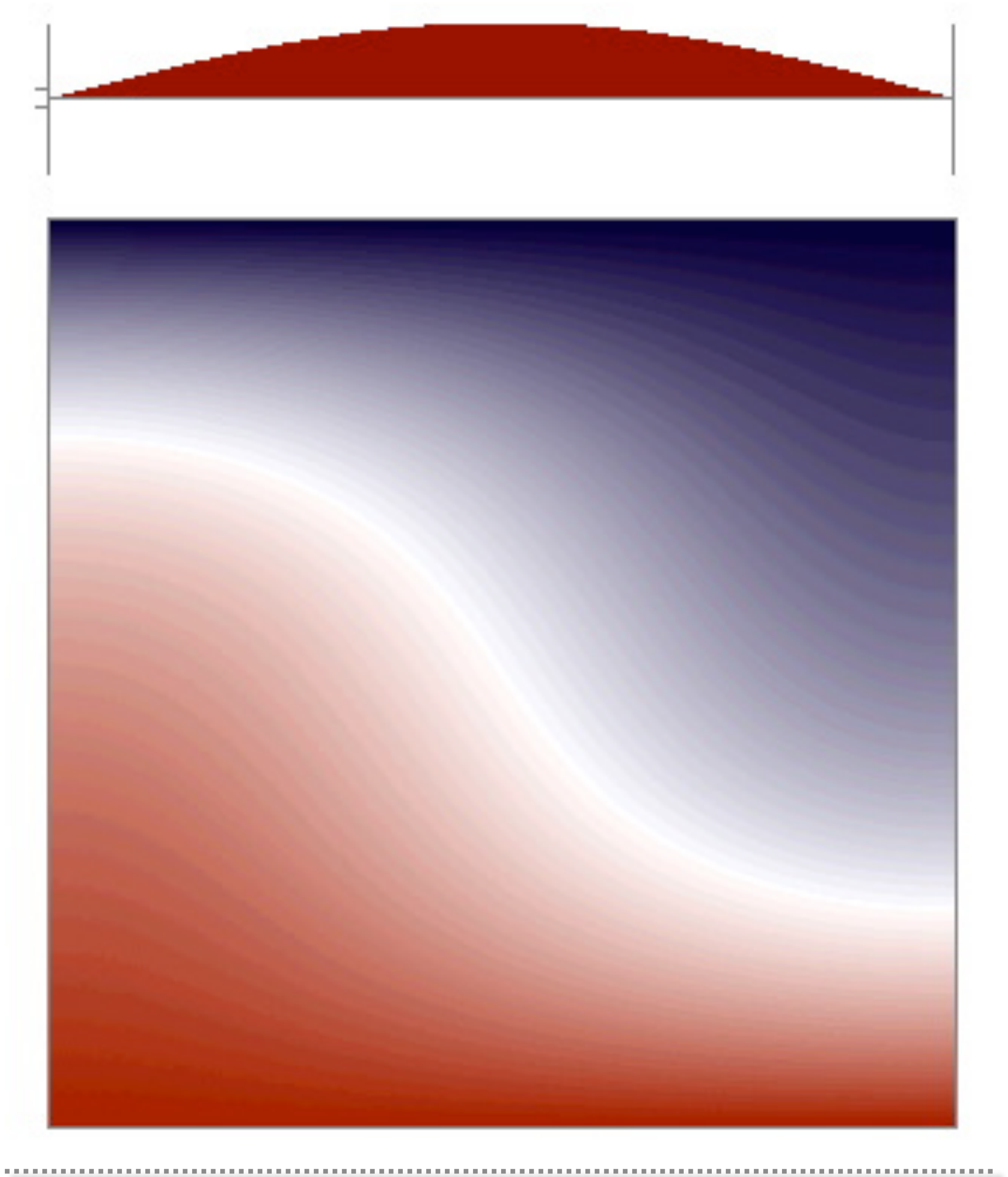
Thermal convection

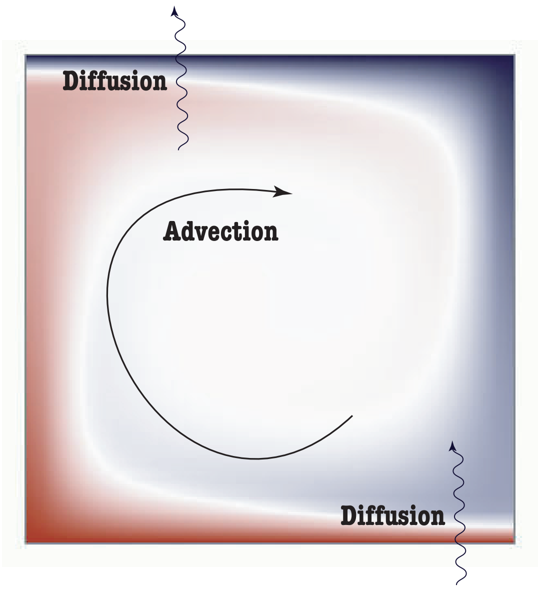




Rayleigh number = 100

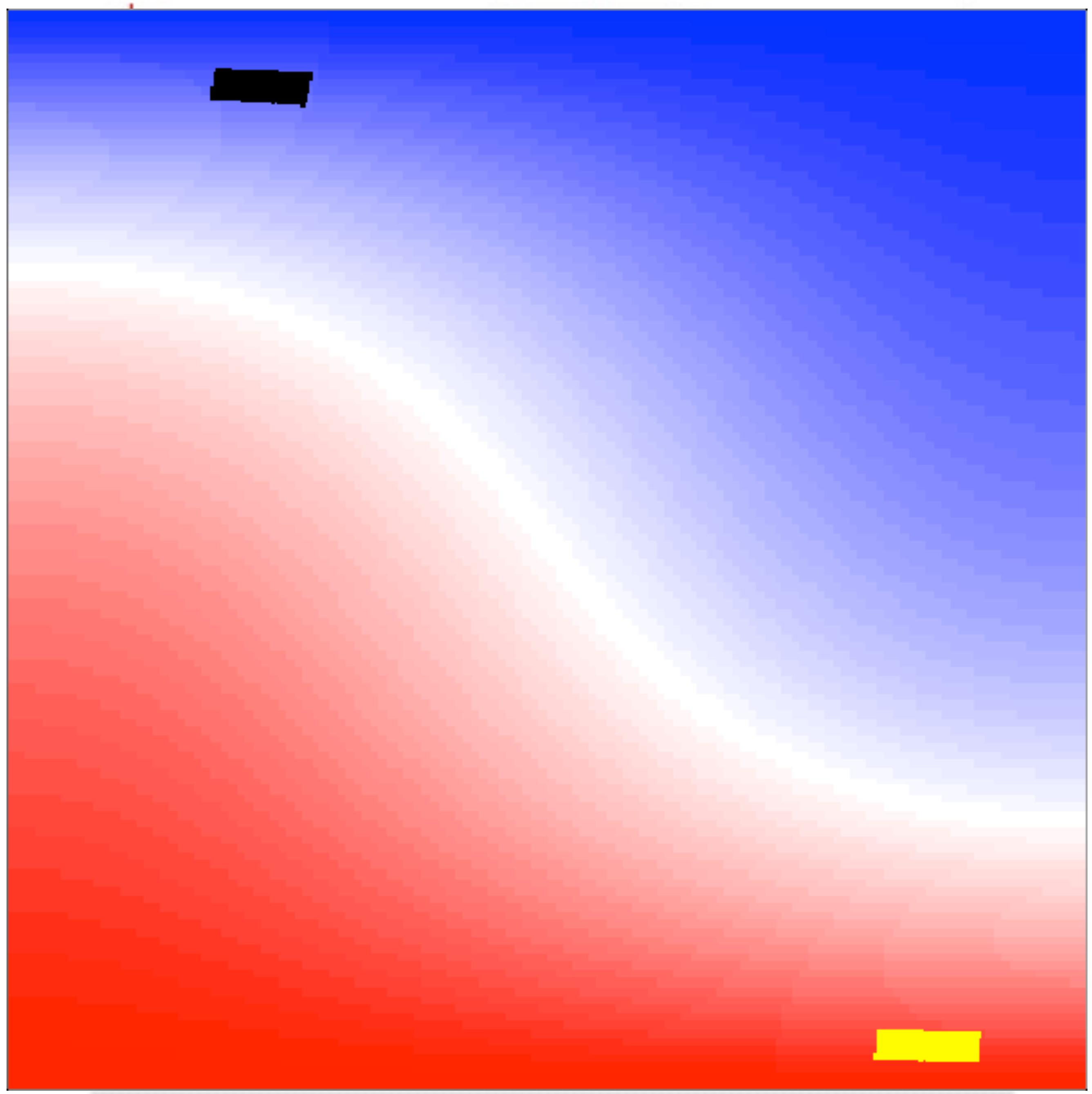
Thermal convection

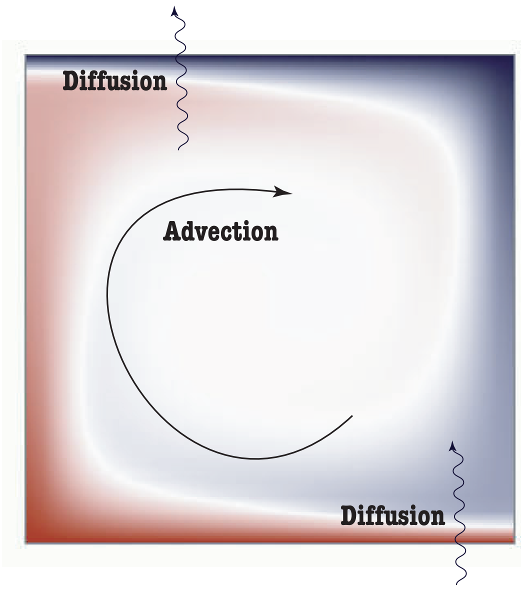




Rayleigh number = 1000

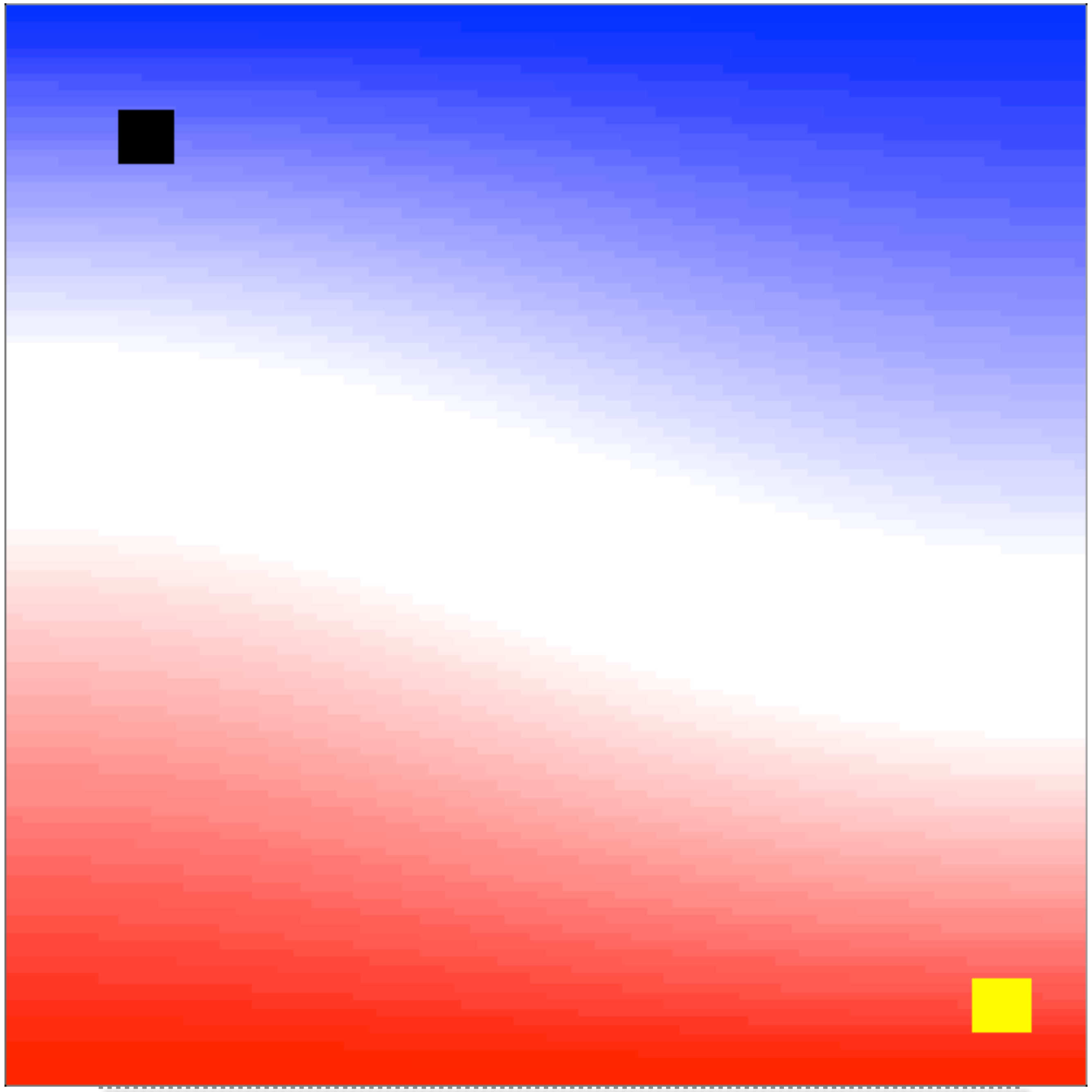
Thermal convection

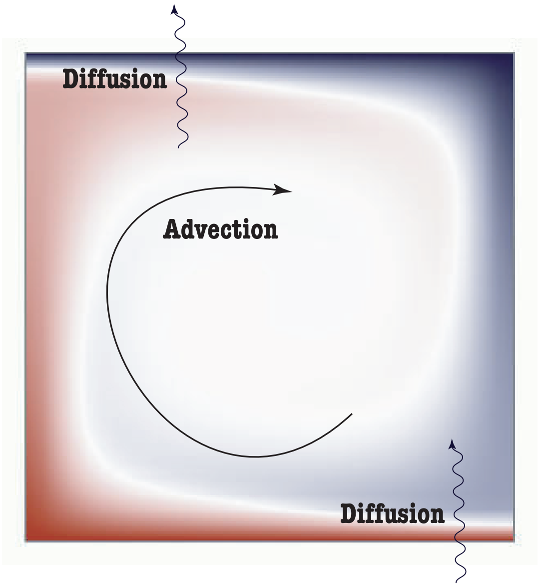




Rayleigh number = 10000

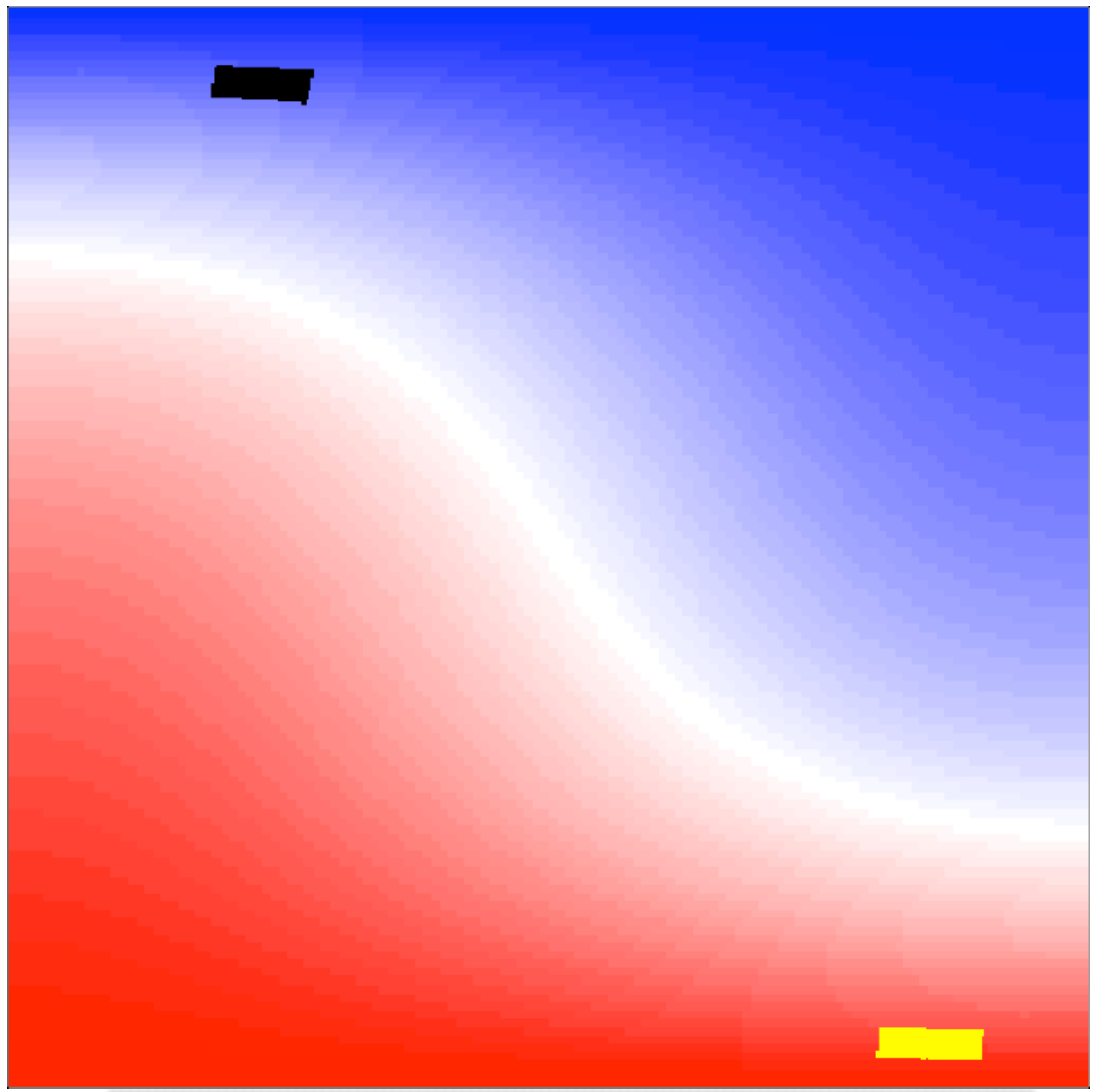
Thermal convection

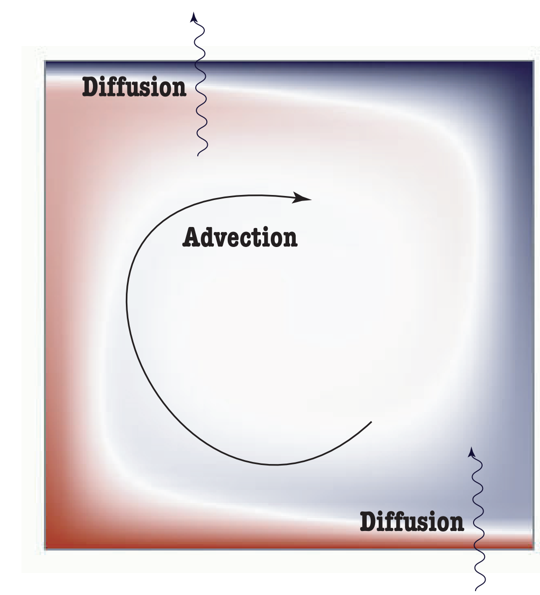




Rayleigh number = 100000

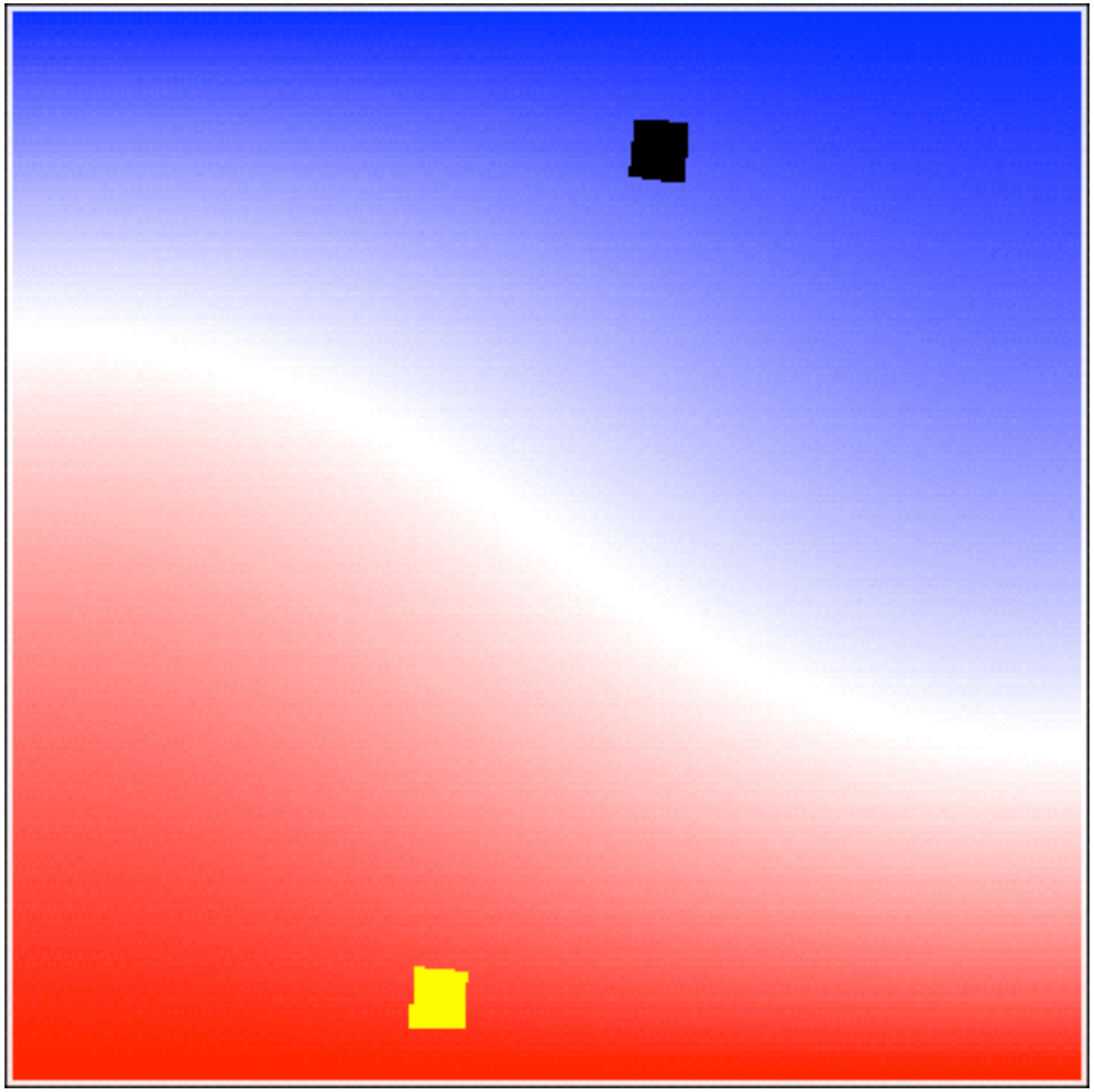
Thermal convection

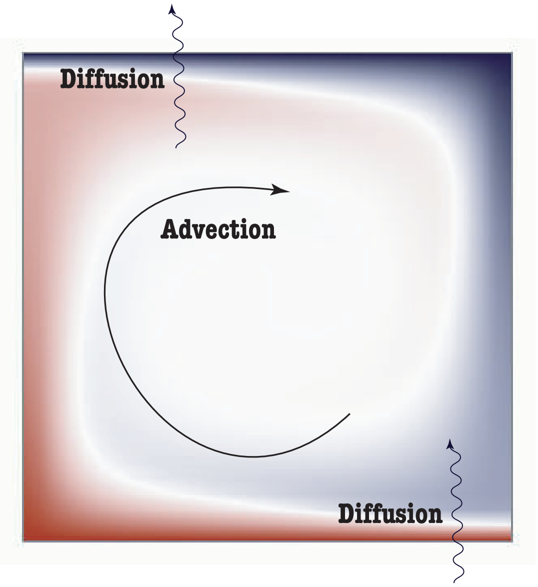




Rayleigh number = 1000000

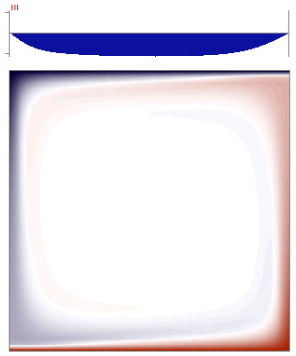
Thermal convection

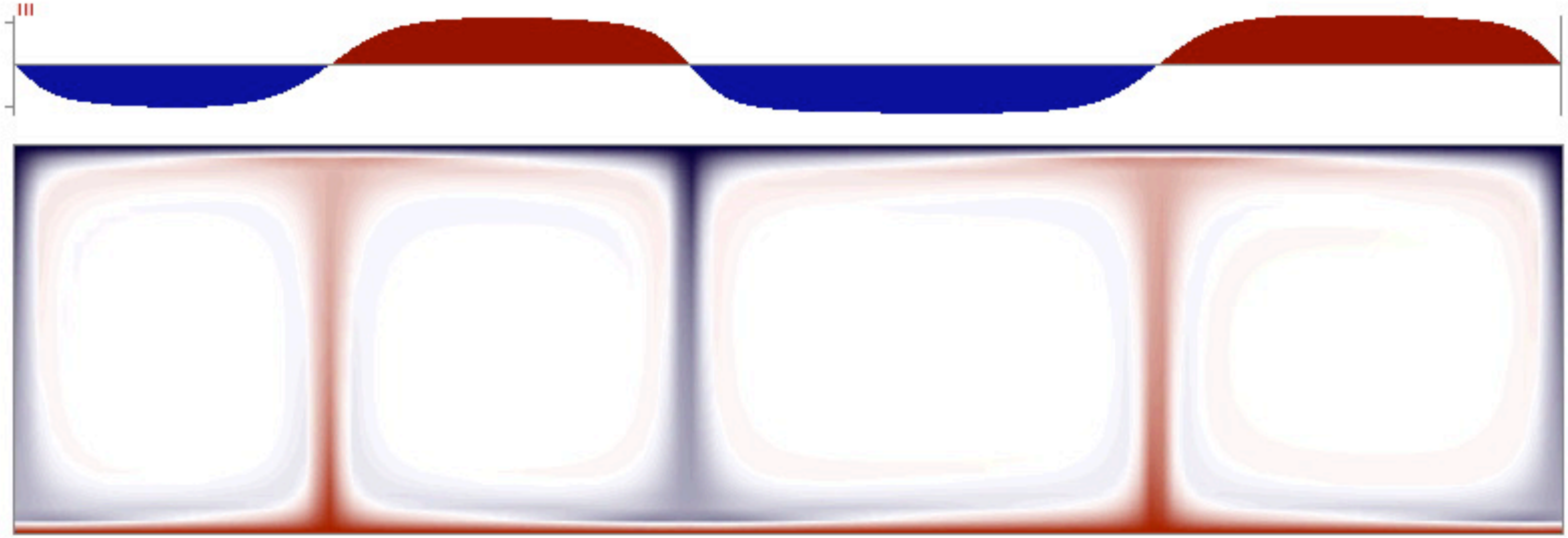




Rayleigh number = 10000000

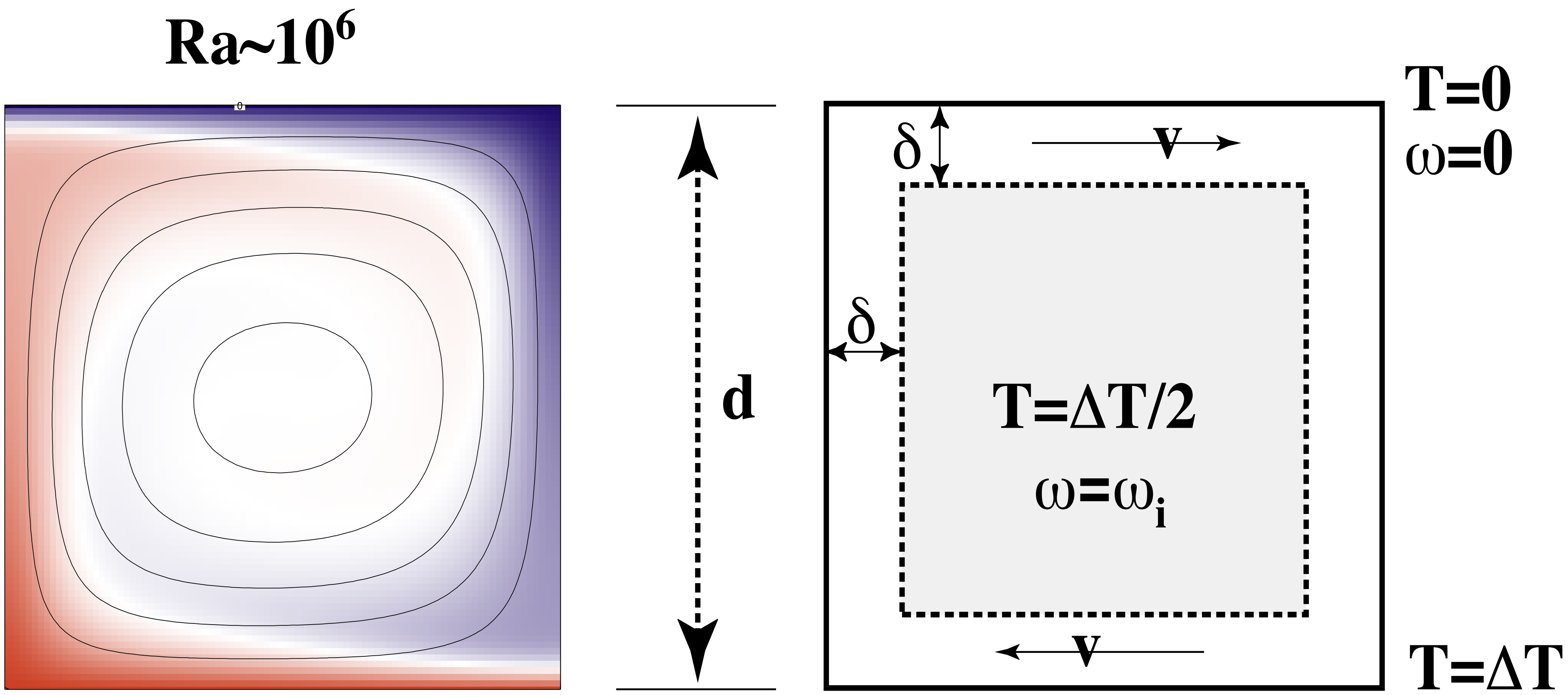
Thermal convection





Rayleigh number = 1000000, wide aspect ratio

Thermal convection

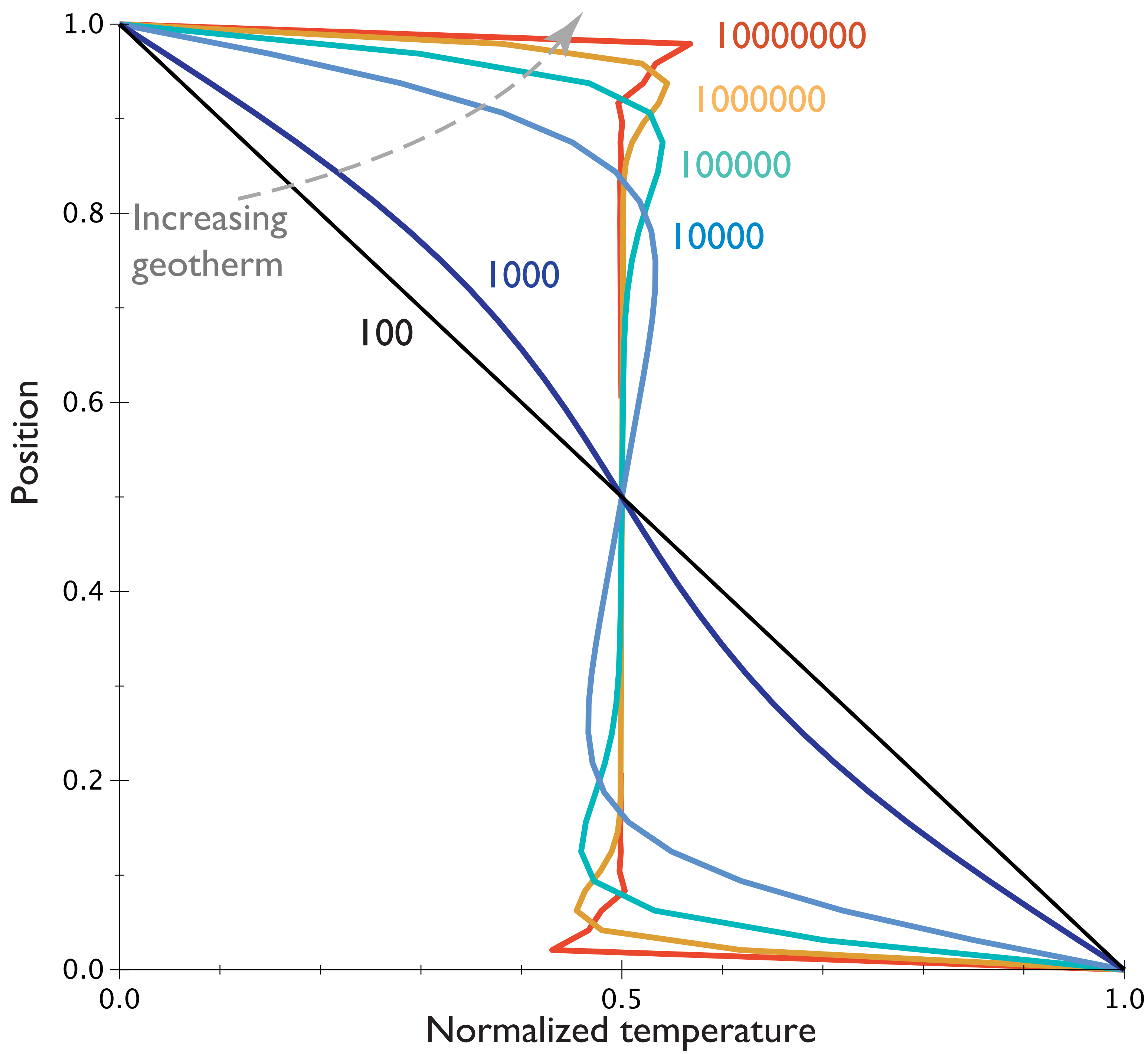


A relatively simple pattern emerges from the competition between advection of heat by fluid flow and diffusion

We will see this picture again when we consider the patterns of heat flow in the Earth's ocean floor.

Convection length and timescales are determined by the Rayleigh number.

Thermal convection



$$Ra = \frac{g\rho\alpha\Delta T d^3}{\kappa\eta}$$

The temperature (velocity, stress etc) profiles vary systematically with Rayleigh number.

Nusselt number

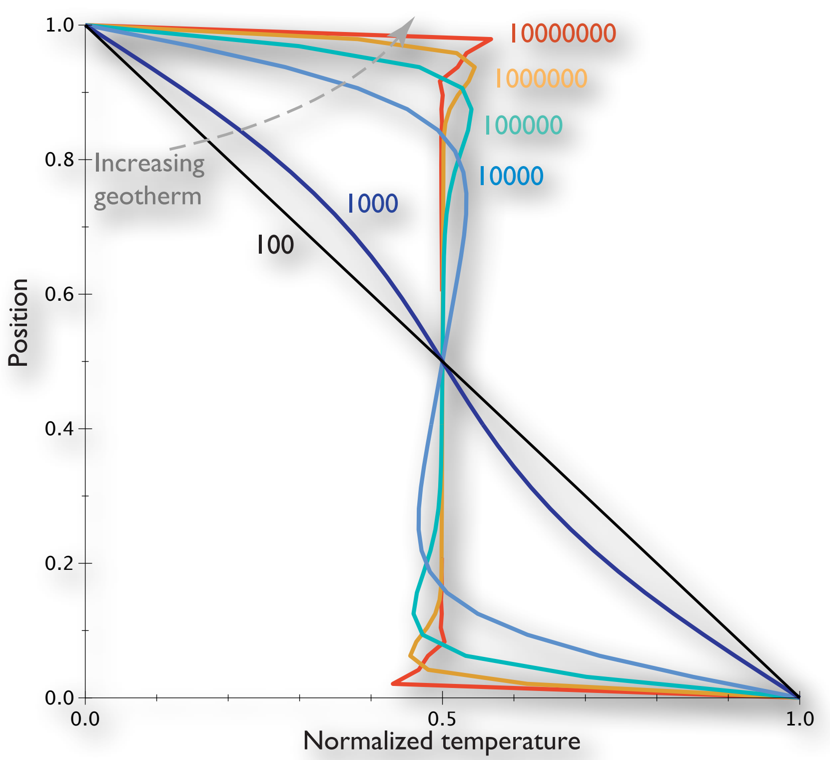
A third, independent, dimensionless number can be derived for the thermally driven flow equations. This is the Nusselt number

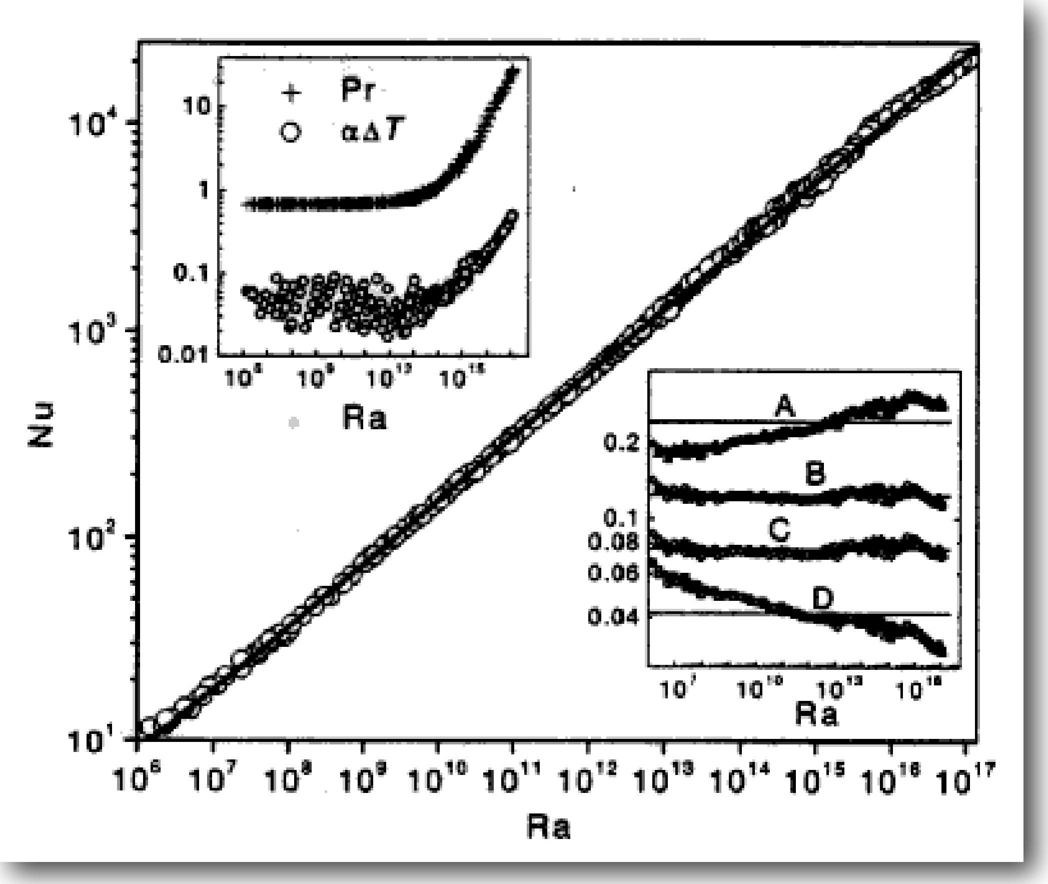
$$\text{Nu} = \frac{qd}{k\Delta T}$$

It is the ratio of heat transported by fluid motions in the layer (q) compared to that transported conductively in the absence of fluid motion.

It can be shown that

$$\text{Nu} \propto \text{Ra}^{1/3}$$



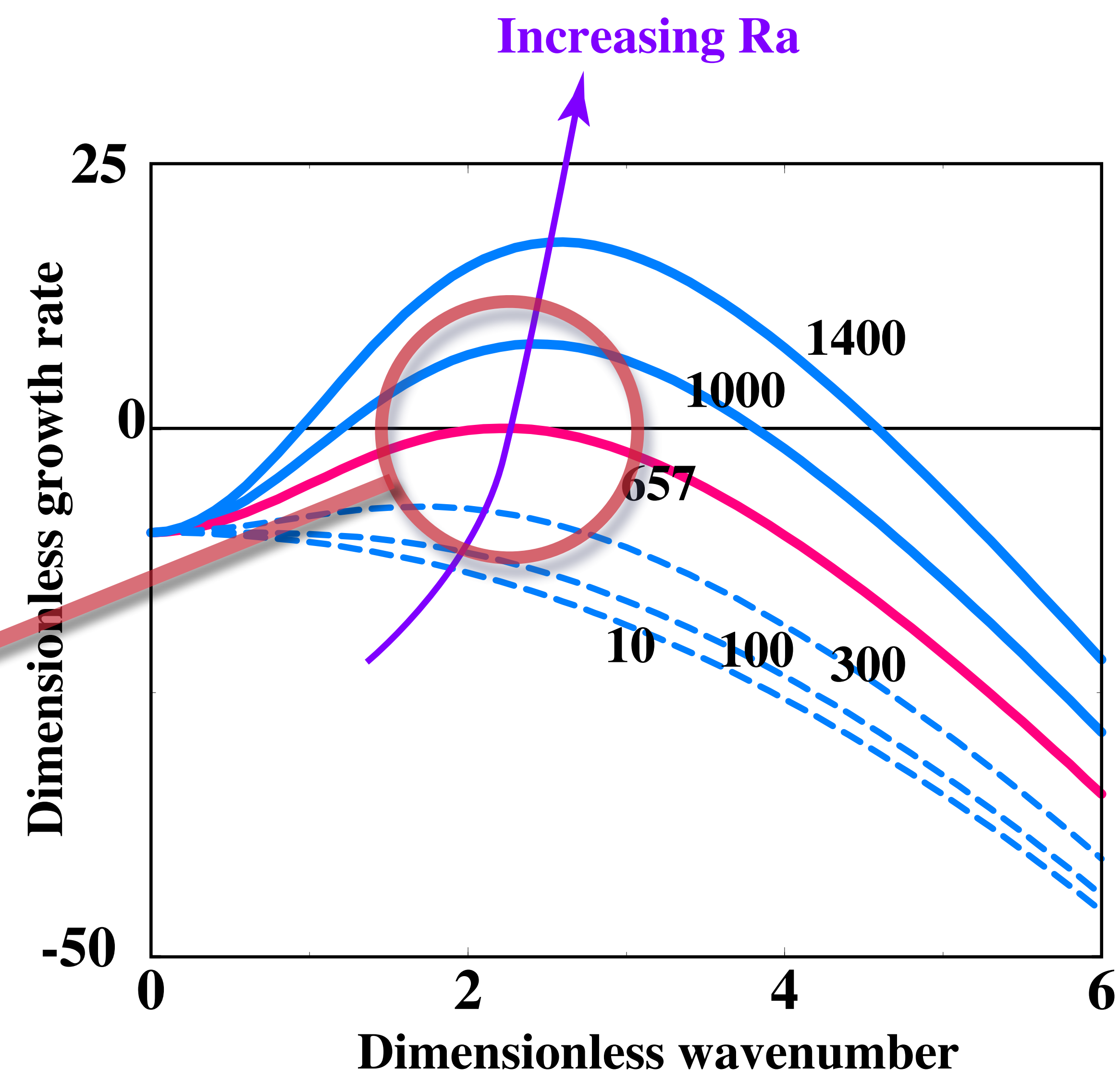


Critical Rayleigh number

At low Rayleigh number diffusion dominates advection: fluid motion and any lateral temperature variations die away. At high **Ra**, the reverse is true: lateral temperature variations are not damped out, but grow rapidly

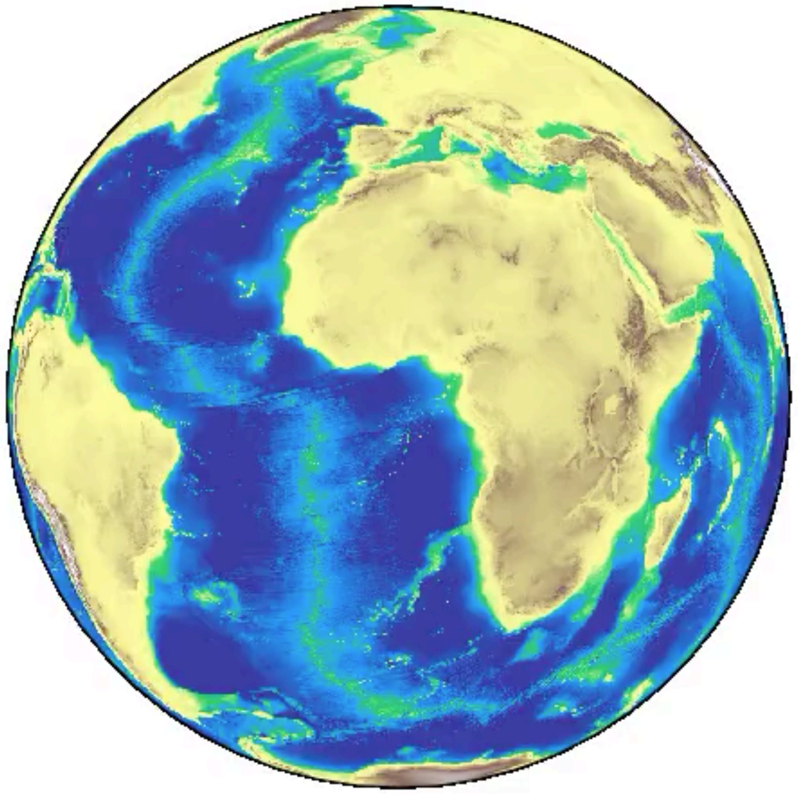
$$Ra_c = \frac{27}{4}\pi^4 = 657.51$$

$$k = \frac{\pi}{2^{1/2}} = 2.22$$



What is the Rayleigh number of the Earth's mantle ?

- Gravity is about $10 ms^{-2}$ throughout
- Density 3000 kg m^{-3}
- Thermal expansivity 10^{-5} K^{-1}
- Temperature drop 1300 K
- Thickness of the mantle is roughly 3000km
- Thermal diffusivity 10^{-7} m^2s^{-1}
- Viscosity 10^{23} Pa.s



Is it above the critical value ?

What does that tell us ?

Summary

Convection is a heat engine (i.e. it converts heat energy into mechanical work)

Convection is a balance between heat transported by fluid motion and diffusion. The fluid self-organises to create large scale patterns “out of nowhere”

In tanks of viscous fluids like syrup or honey, convection depends on just the one free parameter which is a combination of fluid properties, geometry and boundary conditions — this is called the Rayleigh number.

If we know the Rayleigh number, we can predict the heat flow and typical velocity of the system

If the Rayleigh number is below a critical value, convection dies away even if it is stirred up to begin with … but if the Rayleigh number is more than (about) ten times this value then convection cannot be suppressed.

The Earth’s mantle is super-critical and so it is guaranteed to be convecting.

None of these simple models actually produce plate tectonics but they do still tell us about the heat flow in the Earth.