

# Non-negative matrix factorization (NMF)

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**Benjamin Wilson**

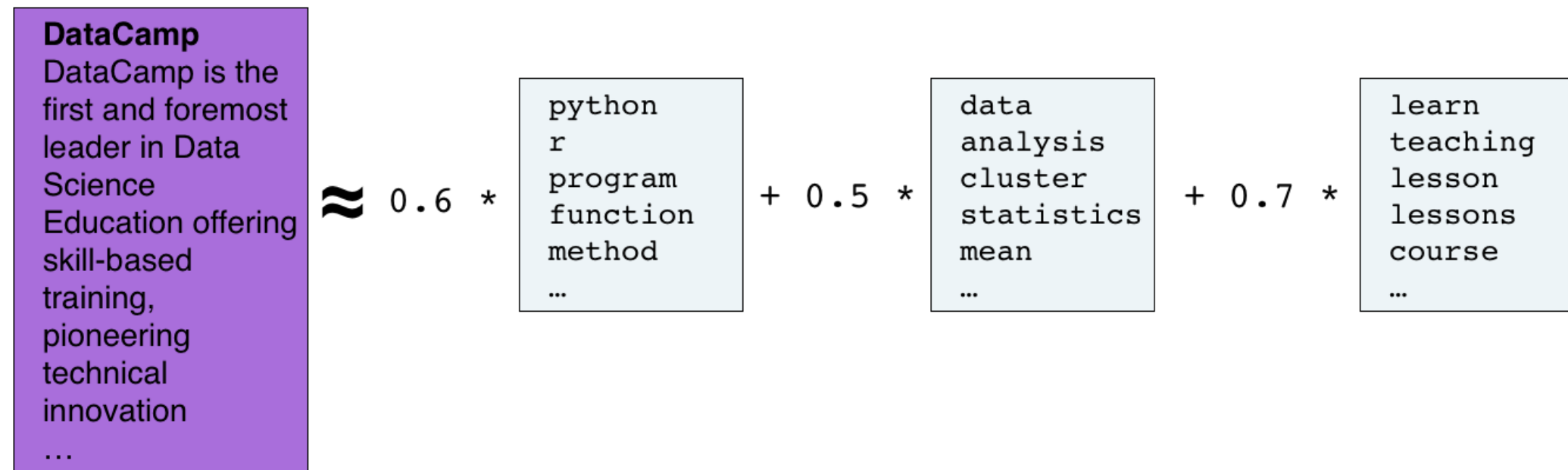
Director of Research at lateral.io

# Non-negative matrix factorization

- NMF = "non-negative matrix factorization"
- Dimension reduction technique
- NMF models are interpretable (unlike PCA)
- Easy to interpret means easy to explain!
- However, all sample features must be non-negative ( $\geq 0$ )

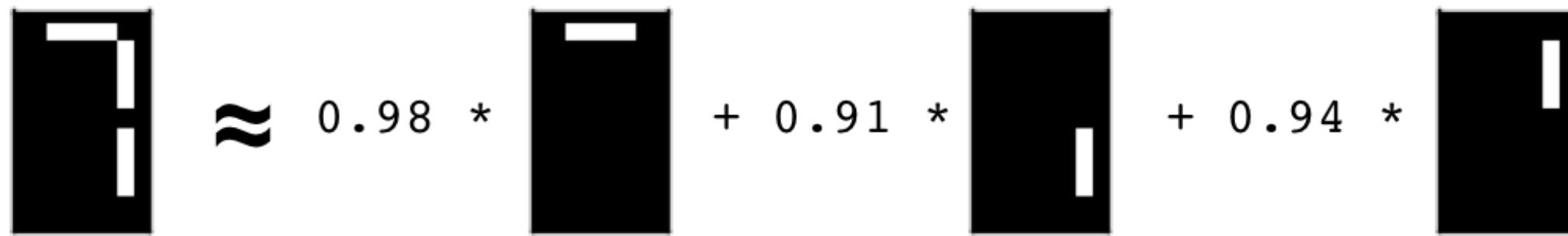
# Interpretable parts

- NMF expresses documents as combinations of topics (or "themes")



# Interpretable parts

- NMF expresses images as combinations of patterns


$$\text{Image} \approx 0.98 * \text{Pattern}_1 + 0.91 * \text{Pattern}_2 + 0.94 * \text{Pattern}_3$$

# Using scikit-learn NMF

- Follows `fit()` / `transform()` pattern
- Must specify number of components e.g.  
`NMF(n_components=2)`
- Works with NumPy arrays and with `csr_matrix`

# Example word-frequency array

- Word frequency array, 4 words, many documents
- Measure presence of words in each document using "tf-idf"
  - "tf" = frequency of word in document
  - "idf" reduces influence of frequent words

	course	datacamp	potato	the
document0	0.2,	0.3,	0.0,	0.1
document1	0.0,	0.0,	0.4,	0.1
...			...	

# Example usage of NMF

- `samples` is the word-frequency array

```
from sklearn.decomposition import NMF
model = NMF(n_components=2)
model.fit(samples)
```

```
NMF(n_components=2)
```

```
nmf_features = model.transform(samples)
```

# NMF components

- NMF has components
- ... just like PCA has principal components
- Dimension of components = dimension of samples
- Entries are non-negative

```
print(model.components_)
```

```
[[ 0.01  0.    2.13  0.54]  
 [ 0.99  1.47  0.    0.5 ]]
```



# NMF features

- NMF feature values are non-negative
- Can be used to reconstruct the samples
- ... combine feature values with components

```
print(nmf_features)
```

```
[[ 0.    0.2 ]  
 [ 0.19  0.  ]  
 ...  
 [ 0.15  0.12]]
```

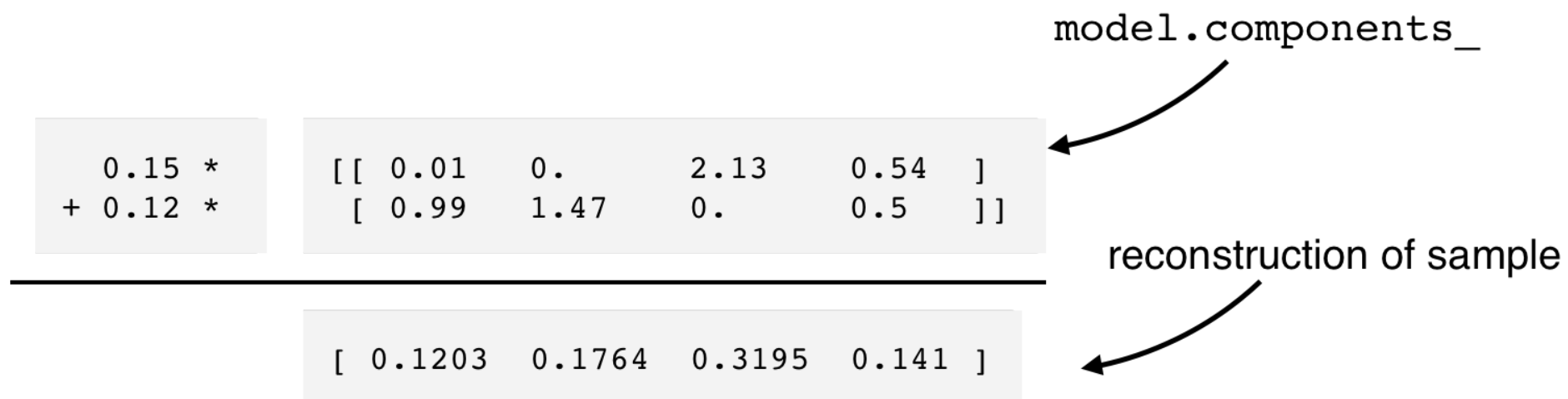
# Reconstruction of a sample

```
print(samples[i,:])
```

```
[ 0.12  0.18  0.32  0.14]
```

```
print(nmf_features[i,:])
```

```
[ 0.15  0.12]
```



# Sample reconstruction

- Multiply components by feature values, and add up
- Can also be expressed as a product of matrices
- This is the "**Matrix Factorization**" in "NMF"

# NMF fits to non-negative data only

- Word frequencies in each document
- Images encoded as arrays
- Audio spectrograms
- Purchase histories on e-commerce sites
- ... and many more!

# Let's practice!

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# NMF learns interpretable parts

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# Example: NMF learns interpretable parts

- Word-frequency array articles (tf-idf)
- 20,000 scientific articles (rows)
- 800 words (columns)



# Applying NMF to the articles

```
print(articles.shape)
```

```
(20000, 800)
```

```
from sklearn.decomposition import NMF  
nmf = NMF(n_components=10)  
nmf.fit(articles)
```

```
NMF(n_components=10)
```

```
print(nmf.components_.shape)
```

```
(10, 800)
```

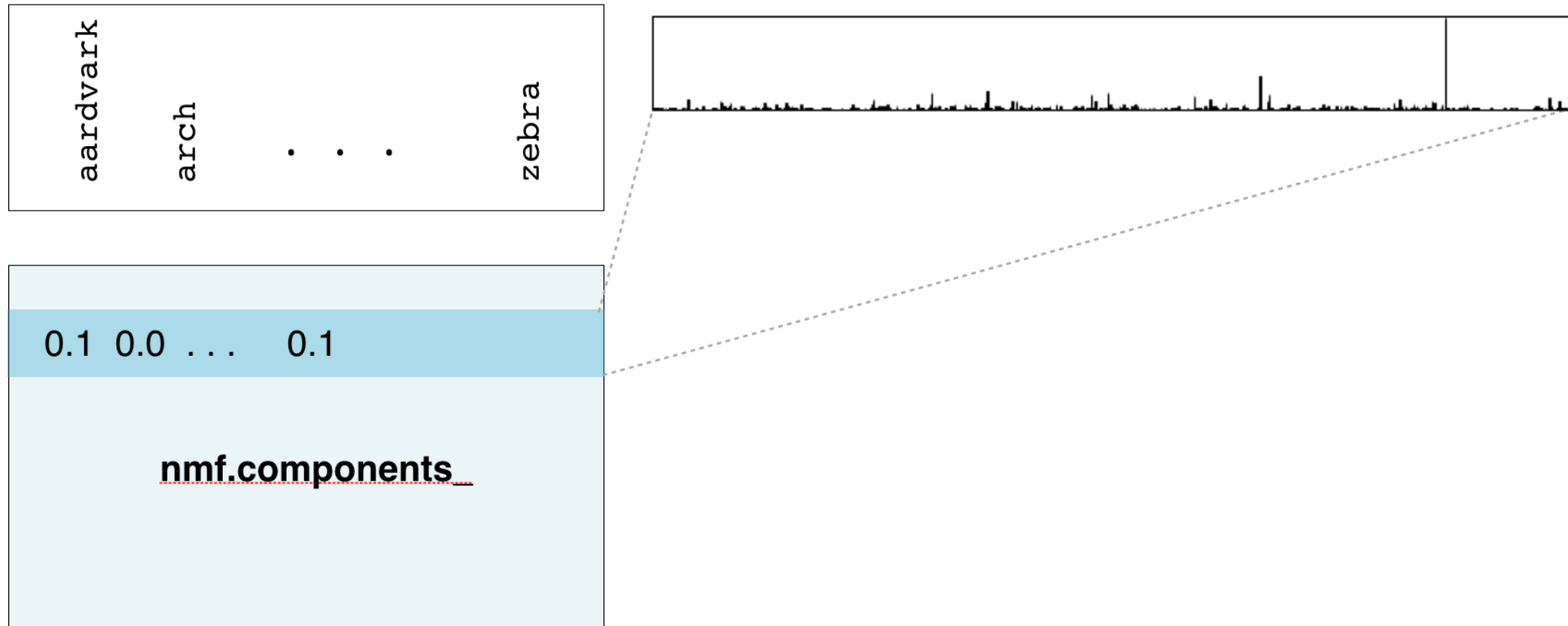


# NMF components are topics

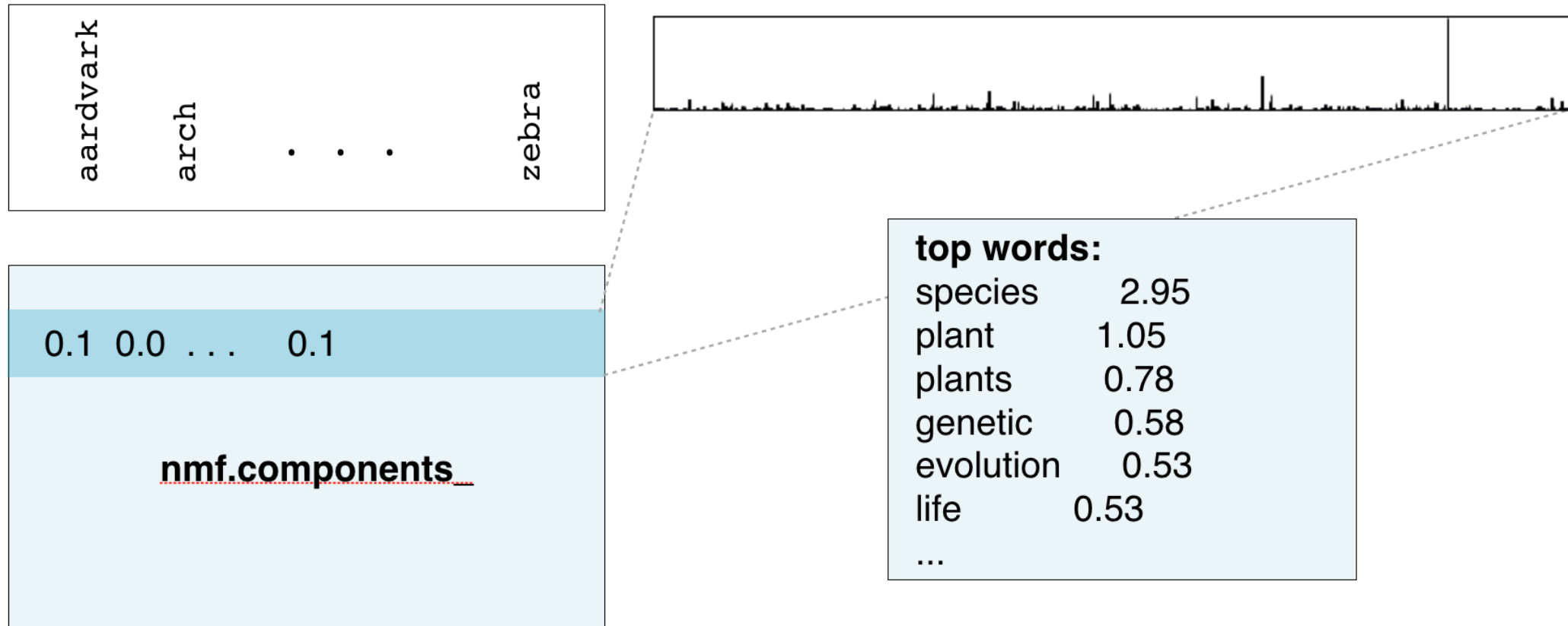
aardvark   arch   .   .   .   zebra

nmf.components\_

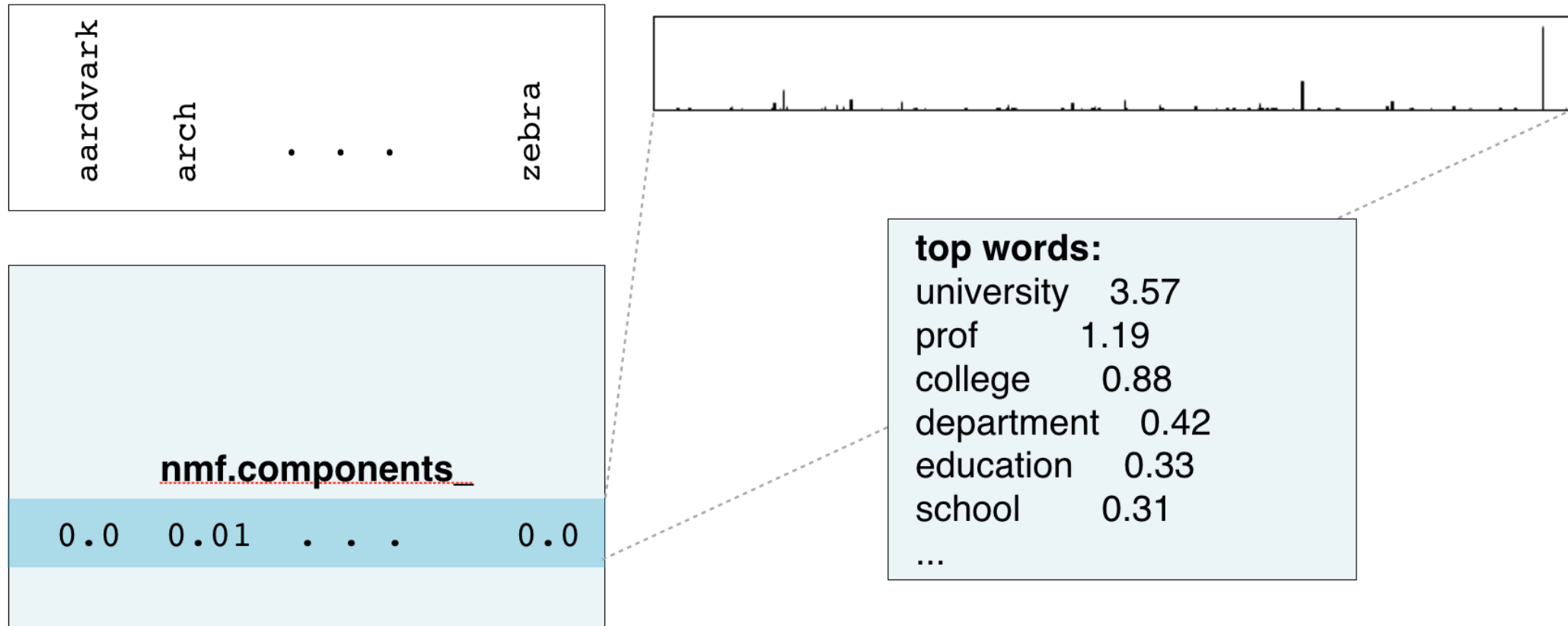
# NMF components are topics



# NMF components are topics

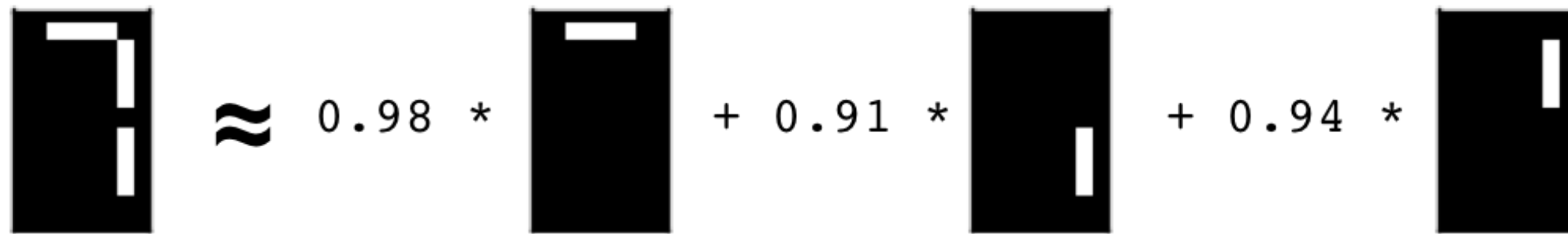


# NMF components are topics



# NMF components

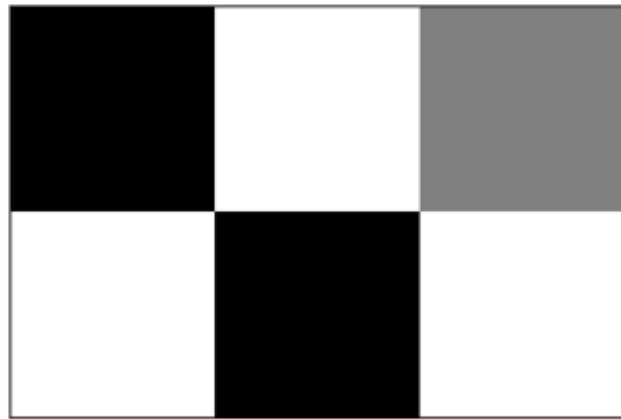
- For documents:
  - NMF components represent topics
  - NMF features combine topics into documents
- For images, NMF components are parts of images



The diagram shows a visual representation of Non-negative Matrix Factorization (NMF) for images. On the left is a target image of a black rectangle with a white 'H' shape. This is followed by an approximation symbol (≈). Then, three component images are shown, each multiplied by a weight and added together. The first component is a black rectangle with a white horizontal bar at the top, multiplied by 0.98. The second component is a black rectangle with a white vertical bar on the right side, multiplied by 0.91. The third component is a black rectangle with a white vertical bar on the left side, multiplied by 0.94. The equation is: 
$$\text{Target Image} \approx 0.98 * \text{Component 1} + 0.91 * \text{Component 2} + 0.94 * \text{Component 3}$$

# Grayscale images

- "Grayscale" image = no colors, only shades of gray
- Measure pixel brightness
- Represent with value between 0 and 1 (0 is black)
- Convert to 2D array



```
[[ 0.  1.  0.5]  
 [ 1.  0.  1. ]]
```

# Grayscale image example

- An 8×8 grayscale image of the moon, written as an array

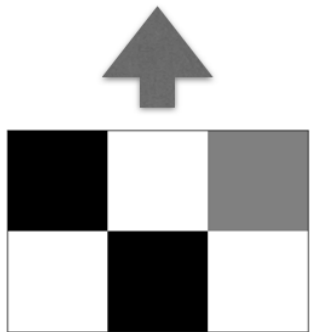


```
[ [ 0.  0.  0.  0.  0.  0.  0.  0. ]  
  [ 0.  0.  0.  0.7 0.8 0.  0.  0. ]  
  [ 0.  0.  0.8 0.8 0.9 1.  0.  0. ]  
  [ 0.  0.7 0.9 0.9 1.  1.  1.  0. ]  
  [ 0.  0.8 0.9 1.  1.  1.  1.  0. ]  
  [ 0.  0.  0.9 1.  1.  1.  0.  0. ]  
  [ 0.  0.  0.  0.9 1.  0.  0.  0. ]  
  [ 0.  0.  0.  0.  0.  0.  0.  0. ] ]
```

# Grayscale images as flat arrays

- Enumerate the entries
- Row-by-row
- From left to right, top to bottom

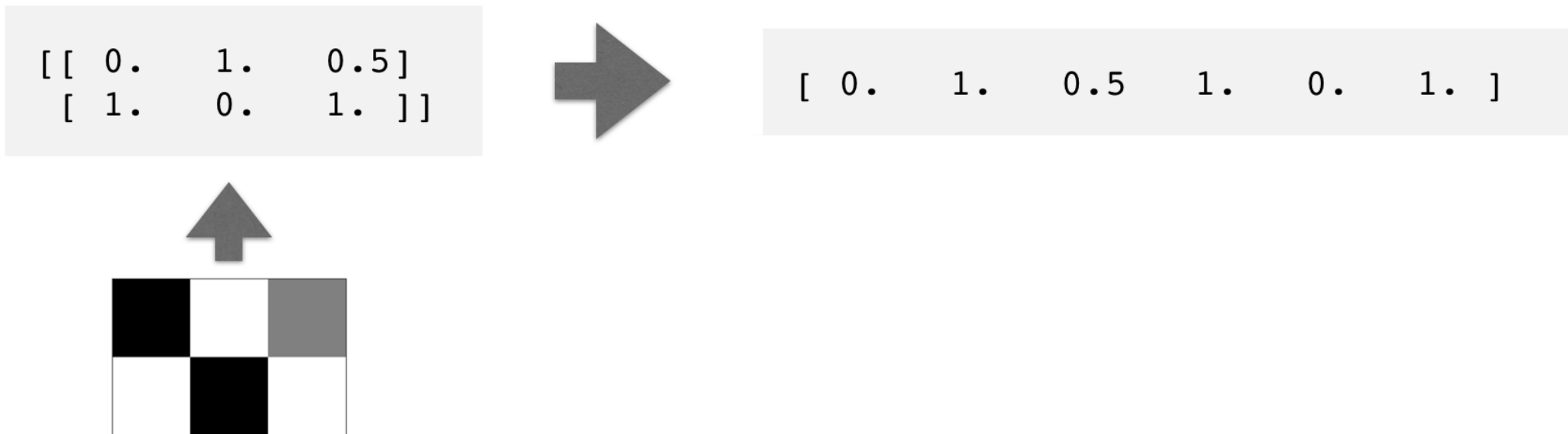
```
[[ 0.  1.  0.5]  
 [ 1.  0.  1. ]]
```





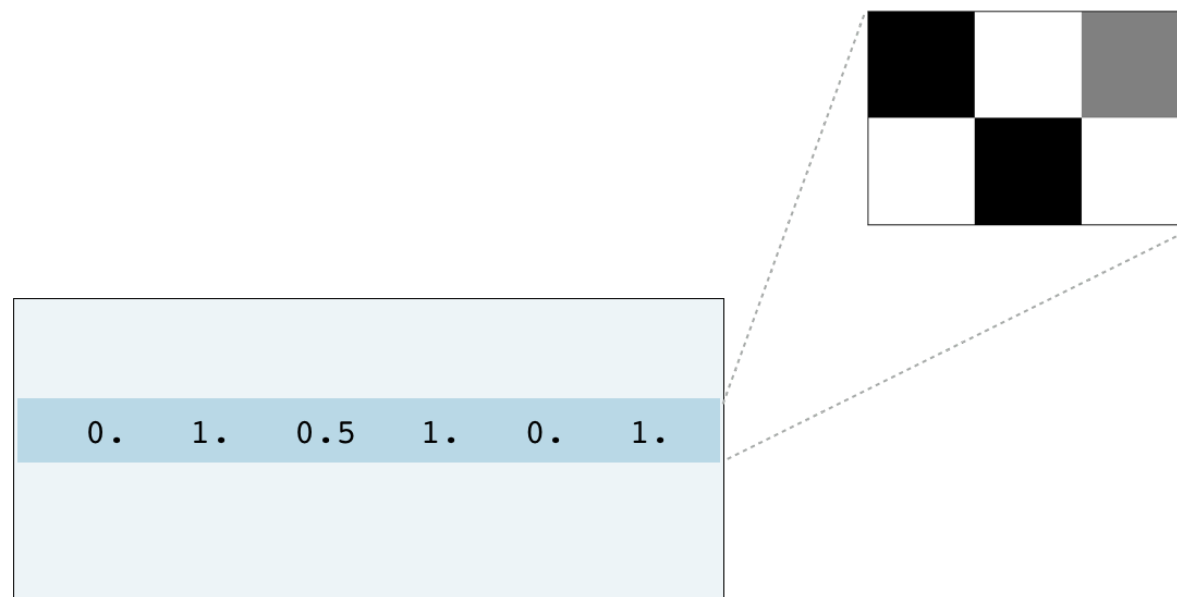
# Grayscale images as flat arrays

- Enumerate the entries
- Row-by-row
- From left to right, top to bottom



# Encoding a collection of images

- Collection of images of the same size
- Encode as 2D array
- Each row corresponds to an image
- Each column corresponds to a pixel
- ... can apply NMF!



# Visualizing samples

```
print(sample)
```

```
[ 0.  1.  0.5  1.  0.  1.]
```

```
bitmap = sample.reshape((2, 3))  
print(bitmap)
```

```
[[ 0.  1.  0.5]  
 [ 1.  0.  1. ]]
```

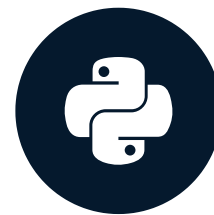
```
from matplotlib import pyplot as plt  
plt.imshow(bitmap, cmap='gray', interpolation='nearest')  
plt.show()
```

# Let's practice!

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# Building recommender systems using NMF

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# Finding similar articles

- Engineer at a large online newspaper
- Task: recommend articles similar to article being read by customer
- Similar articles should have similar topics

# Strategy

- Apply NMF to the word-frequency array
- NMF feature values describe the topics
- ... so similar documents have similar NMF feature values
- Compare NMF feature values?

# Apply NMF to the word-frequency array

- `articles` is a word frequency array

```
from sklearn.decomposition import NMF
nmf = NMF(n_components=6)
nmf_features = nmf.fit_transform(articles)
```



# Strategy

- Apply NMF to the word-frequency array
- NMF feature values describe the topics
- ... so similar documents have similar NMF feature values
- Compare NMF feature values?

# Versions of articles

- Different versions of the same document have same topic proportions
- ... exact feature values may be different!
- 
- 

## strong version

Dog bites man!  
Attack by terrible  
canine leaves man  
paralyzed...

# Versions of articles

- Different versions of the same document have same topic proportions
- ... exact feature values may be different!
- E.g. because one version uses many meaningless words
- 

## strong version

Dog bites man!  
Attack by terrible  
canine leaves man  
paralyzed...

## weak version

You may have heard,  
unfortunately it seems  
that a dog has perhaps  
bitten a man ...

# Versions of articles

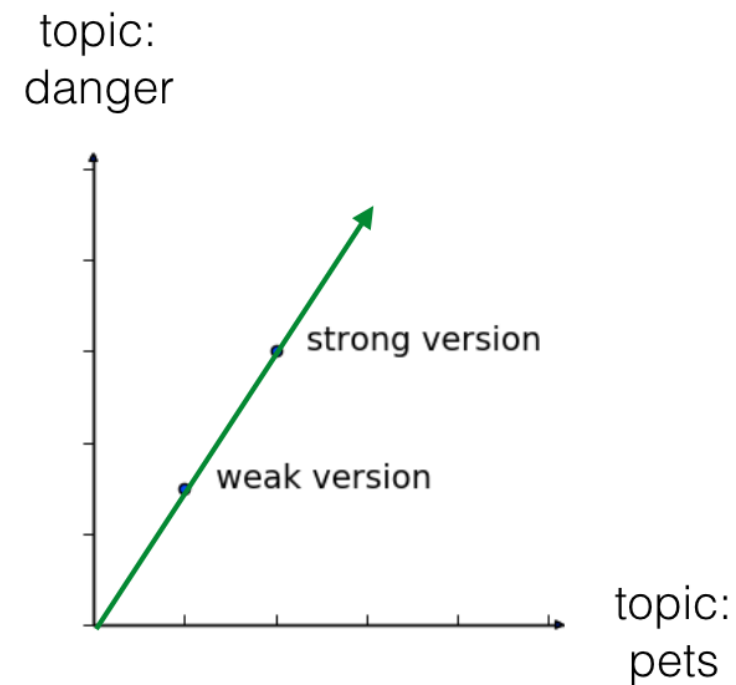
- Different versions of the same document have same topic proportions
- ... exact feature values may be different!
- E.g. because one version uses many meaningless words
- But all versions lie on the same line through the origin

## strong version

Dog bites man!  
Attack by terrible  
canine leaves man  
paralyzed...

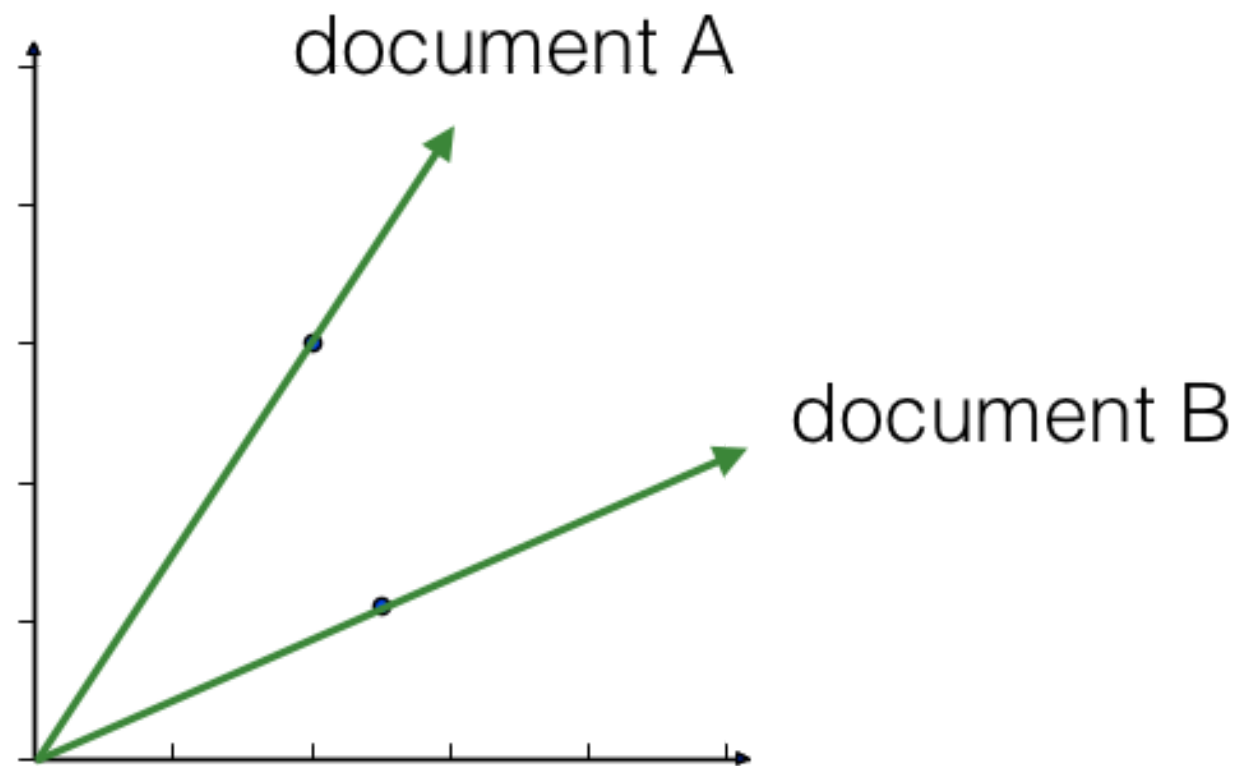
## weak version

You may have heard,  
unfortunately it seems  
that a dog has perhaps  
bitten a man ...



# Cosine similarity

- Uses the angle between the lines
- Higher values means more similar
- Maximum value is 1, when angle is 0 degrees



# Calculating the cosine similarities

```
from sklearn.preprocessing import normalize
norm_features = normalize(nmf_features)
# if has index 23
current_article = norm_features[23,:]
similarities = norm_features.dot(current_article)
print(similarities)
```

```
[ 0.7150569  0.26349967 ..., 0.20323616  0.05047817]
```

# DataFrames and labels

- Label similarities with the article titles, using a DataFrame
- Titles given as a list: `titles`

```
import pandas as pd
norm_features = normalize(nmf_features)
df = pd.DataFrame(norm_features, index=titles)
current_article = df.loc['Dog bites man']
similarities = df.dot(current_article)
```

# DataFrames and labels

```
print(similarities.nlargest())
```

```
Dog bites man                1.000000  
Hound mauls cat              0.979946  
Pets go wild!                0.979708  
Dachshunds are dangerous     0.949641  
Our streets are no longer safe 0.900474  
dtype: float64
```



# Let's practice!

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# Final thoughts

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# Congratulations!

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