



$$\mathbf{A}_{BI} = \mathbf{A}_{BC} \mathbf{A}_{CD} \mathbf{A}_{DI} \Rightarrow \mathbf{B} \mathbf{r} = \mathbf{A}_{BI} \mathbf{I} \mathbf{r} \Leftrightarrow \mathbf{B} \mathbf{r} = \mathbf{A}_{I I}^B \mathbf{r}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos \psi \cos \theta & \cos \theta \sin \psi & -\sin \theta \\ \cos \psi \sin \phi \sin \theta - \cos \phi \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta & \cos \theta \sin \phi \\ \sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta & \cos \phi \sin \psi \sin \theta - \cos \psi \sin \phi & \cos \phi \cos \theta \end{pmatrix}$$

$$\mathbf{A}_{IB} = \mathbf{A}_{BI}^T$$

$$= \begin{pmatrix} \cos \psi \cos \theta & \cos \psi \sin \phi \sin \theta - \cos \phi \sin \psi & \sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta \\ \cos \theta \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta & \cos \phi \sin \psi \sin \theta - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \phi \cos \theta \end{pmatrix}$$