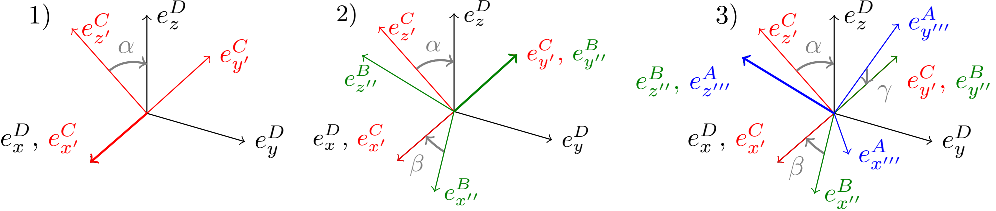

$$
\begin{aligned}
\mathbf{C}_{AD} &= \mathbf{C}_{AB}\,\mathbf{C}_{BC}\,\mathbf{C}_{CD} \qquad \Rightarrow \qquad {}_A\mathbf{r} = \mathbf{C}_{AD}\,{}_D\mathbf{r} \\
&= \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} \\
&= \begin{pmatrix} \cos\beta\cos\gamma & \cos\gamma\sin\alpha\sin\beta - \cos\alpha\sin\gamma & \sin\alpha\sin\gamma + \cos\alpha\cos\gamma\sin\beta \\ \cos\beta\sin\gamma & \cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma & \cos\alpha\sin\beta\sin\gamma - \cos\gamma\sin\alpha \\ -\sin\beta & \cos\beta\sin\alpha & \cos\alpha\cos\beta \end{pmatrix}
\end{aligned}
$$