



$$\begin{aligned}
 \mathbf{C}_{AD} &= \mathbf{C}_{AB} \mathbf{C}_{BC} \mathbf{C}_{CD} \Rightarrow \mathbf{A} \mathbf{r} = \mathbf{C}_{AD} \mathbf{D} \mathbf{r} \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} \cos \theta \cos \psi & -\cos \theta \sin \psi & \sin \theta \\ \cos \phi \sin \psi + \cos \psi \sin \phi \sin \theta & \cos \phi \cos \psi - \sin \phi \sin \theta \sin \psi & -\cos \theta \sin \phi \\ \sin \phi \sin \psi - \cos \phi \cos \psi \sin \theta & \cos \psi \sin \phi + \cos \phi \sin \theta \sin \psi & \cos \phi \cos \theta \end{pmatrix}
 \end{aligned}$$