

# Optimal Spring Sizing

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## Summary

### 0.1 Design variable values

The optimal cost was found to be \$400,140. The determined optimum values for the design variables are as follows:

Table 1: Optimum values for design

Variable	Value
Velocity	$7.8161 \frac{ft}{s}$
Pipe Diameter	$0.1816 ft$
Particle Size	$0.0005 ft$
Water Flow Rate	$0.1128 \frac{ft^3}{s}$
Volumetric Concentration	0.4
Slurry Density	$104.84 \frac{lbm}{ft^3}$

### 0.2 Design function values

Table 2: Values of design functions at optimum

Design Function	Value
Maximize $F_0$	$6.4541 lbs$
$h_S + 0.05 \leq h_{def}$	$0.60 in \leq 0.60 in$
$\tau_a \leq \frac{s_e}{s_f}$	$18352 psi \leq 30000 psi$
$\tau_a + \tau_m \leq \frac{s_y}{s_f}$	$70576 psi \leq 70576 psi$
$4 \geq \frac{D}{d} \leq 16$	$4 in \leq 9.359 in \leq 16 in$
$D + d \leq 0.75$	$0.75 in \leq 0.75 in$
$\tau_{solid} \leq s_y$	$75165 psi \leq 105860 psi$

### 0.3 Binding Constraints

The binding constraints are:

- $h_S + 0.05 \leq h_{def}$
- $D + d \leq 0.75$
- $\tau_a + \tau_m \leq \frac{s_y}{s_f}$

# 1 Setup

## 1.1 Variable Mapping

Table 3: My caption

### Analysis Variables

Wire Diameter  
Coil Diameter  
Number of Coils  
Free height  
Preload Height  
Preload Deflection  
Shear Modulus  
Safety Factor  
Endurance Limit

### Analysis Functions

Force  
Spring Stiffness  
Whal Factor  
Solid Height  
Alternating Stress  
Mean Stress  
Yield Strength  
Coil to Wire Ratio  
Spring Diameter  
Deflected Height

### Design Variables

Wire Diameter  
Coil Diameter  
Number of Coils  
Free height

### Design Functions

Maximize  
Solid Height  $\leq$  Deflected height  
Alternating Stress  $\leq \frac{s_e}{s_f}$   
Alternating and Mean Stress  $\leq \frac{s_y}{s_f}$   
 $4 \leq$  Coil to Wire Ratio  $\leq 16$   
Solid Stress  $\leq s_y$

## 2 Results

### 2.1 Optimum values of variables and functions

Table 4: Optimum Values of Variables and Functions (binding functions are highlighted)

Variable/Function	Value
Wire Diameter	0.0724 in
Coil Diameter	0.6776 in
Number Coils	0.5928
Free Height	1.3691
Preload Height	1.0 in
$\delta_0$	0.4 in
$h_{def}$	0.6 in
$h_s$	0.55in
$F_0$	6.454 lbs
k	17.4853 lbs/in
K	1.1561
$\tau_{max}$	70576 psi
$\tau_{min}$	33871 psi
$\tau_a$	18353 psi
$\tau_m$	52224 psi
$\tau_a + \tau_m$	70576 psi
$\frac{S_e}{S_f}$	30000 psi
$\frac{S_y}{S_f}$	70576 psi
$\frac{D}{d}$	9.3539
$D + d$	0.75

### 2.2 Starting points and obtained values

Table 5: Optimized Values from Given Starting Point

Trial	Initial Values				Optimized Values			
	Wire Diameter	Coil Diameter	Number of Coils	Free Height	Wire Diameter	Coil Diameter	Number of Coils	Free Height
1	0.04735309758	0.2632045077	13.47275949	1.525959964	0.07243674785	0.6775631377	7.592829475	1.369115417
2	0.07681530634	0.6400386081	12.94948955	1.594751247	0.07243674785	0.6775631377	7.592829475	1.369115417
3	0.1842667961	0.2857953622	15.87240389	1.778356185	0.07243674785	0.6775631377	7.592829475	1.369115417
4	0.08228471093	0.4690840665	4.289522923	1.148555107	0.07243674785	0.6775631377	7.592829475	1.369115417
5	0.1108515351	0.6064586996	18.87818163	1.216915588	0.07243674785	0.6775631377	7.592829475	1.369115417
6	0.1180764956	0.4051039167	3.202335182	1.40341038	0.07243674785	0.6775631377	7.592829475	1.369115417
7	0.04081463856	0.6162849514	8.290655715	1.575679822	0.07243674785	0.6775631377	7.592829475	1.369115417
8	0.0414732586	0.4912882619	7.470511837	1.688671189	0.07243674785	0.6775631377	7.592829475	1.369115417
9	0.1409507556	0.5862985353	10.65920717	1.17543924	0.07243674785	0.6775631377	7.592829475	1.369115417
10	0.05350562406	0.693669285	5.590426322	1.84323528	0.07243674785	0.6775631377	7.592829475	1.369115417

## 2.3 Design space contour plot

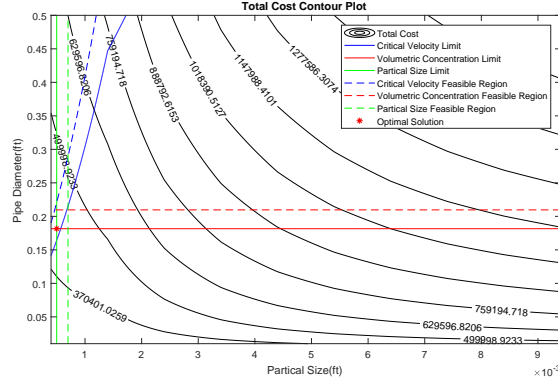


Figure 1: Contour plot showing the design space

## 2.4 Feasible design space contour plot

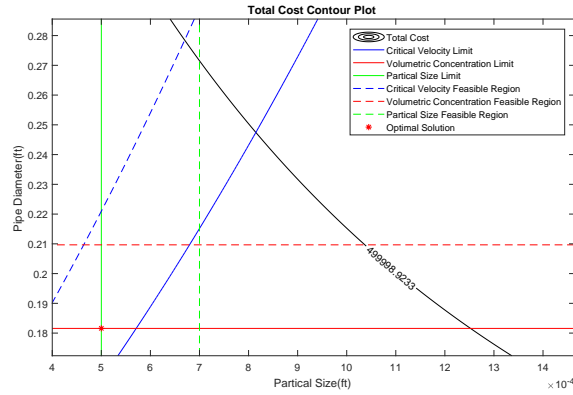


Figure 2: Contour plot showing the feasible design space

## 3 Discussion

As can be seen in figure 1 the design space for this problem is quite restricted. There are several constraints and the feasible design space is quite small compared to the total design space, at least when viewed on a contour plot of wire diameter and spring diameter. Due to the fact that the contours shown in Figure 1 are quite linear and that the various initial values tried all converged to the same value as shown in Table 5 I conclude that we have found a global optimum for the spring given the applied constraints. It is also of interest that the optimum appears to be bound by three constraints, meaning that if we wanted to increase the increase the rate of the spring by only changing the spring or wire diameter we would need to either relax the alternating and mean stress constraint (ie. find a stronger material), or we would need to relax both the solid height and spring size constraints. It is important to remember that Figures 1 and 2 are only showing a portion of the design space and other options should also be considered

## 4 Appendix

### 4.1 Matlab files

#### 4.1.1 Optmization code

#### 4.1.2 Plotting Code