Optimal Spring Sizing

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Summary

0.1 Design variable values

The optimal force was found to be 6.454 lbs. The determined optimum values for the design variables are as follows:

Table 1: Optimum values for design

Variable	**Value**
Wire Diameter	0.0724 in
Coil Diameter	0.6776 in
Number Coils	0.5928
Free Height	1.3691

0.2 Design function values

Table 2: Values of design functions at optimum

Design Function	**Value**
Maximize F_0	$6.4541 lbs$
$h_S + 0.05 \leq h_{def}$	$0.60 in \leq 0.60 in$
$\tau_a \leq \frac{s_e}{s_f}$	$18352 psi \leq 30000 psi$
$\tau_a + \tau_m \leq \frac{s_y}{s_f}$	$70576 psi \leq 70576 psi$
$4 \geq \frac{D}{d} \leq 16$	$4 in \leq 9.359 in \leq 16 in$
$D + d \leq 0.75$	$0.75 in \leq 0.75 in$
$\tau_{solid} \leq s_y$	$75165 psi \leq 105860 psi$

0.3 Binding Constraints

The binding constraints are:

- $h_S + 0.05 \leq h_{def}$
- $D + d \leq 0.75$
- $\tau_a + \tau_m \leq \frac{s_y}{s_f}$

1 Setup

1.1 Variable Mapping

Table 3: My caption

Analysis Variables	**Design Variables**
Wire Diameter	Wire Diameter
Coil Diameter	Coil Diameter
Number of Coils	Number of Coils
Free height	Free height
Preload Height	
Preload Deflection	
Shear Modulus	
Safety Factor	
Endurance Limit	

Analysis Functions	**Design Functions**
Force	Maximize
Spring Stiffness	Solid Height $\leq$ Deflected height
Whal Factor	Alternating Stress $\leq \frac{s_e}{s_f}$
Solid Height	Alternating and Mean Stress $\leq \frac{s_y}{s_f}$
Alternating Stress	$4 \leq$ Coil to Wire Ratio ≤ 16
Mean Stress	Solid Stress $\leq s_y$
Yield Strength	
Coil to Wire Ratio	
Spring Diameter	
Deflected Height	

2 Results

2.1 Optimum values of variables and functions

Table 4: Optimum Values of Variables and Functions (binding functions are highlighted)

Variable/Function	**Value**
Wire Diameter	0.0724 in
Coil Diameter	0.6776 in
Number Coils	0.5928
Free Height	1.3691
Preload Height	1.0 in
δ_0	0.4 in
h_{def}	0.6 in
h_s	0.55in
F_0	6.454 lbs
k	17.4853 lbs/in
K	1.1561
τ_{max}	70576 psi
τ_{min}	33871 psi
τ_a	18353 psi
τ_m	52224 psi
$\tau_a + \tau_m$	70576 psi
$\frac{S_e}{S_f}$	30000 psi
$\frac{S_y}{S_f}$	70576 psi
$\frac{D}{d}$	9.3539
$D + d$	0.75

2.2 Starting points and obtained values

Table 5: Optimized Values from Given Starting Point

	Initial Values				**Optimized Values**			
Trial	Wire Diameter	Coil Diameter	Number of Coils	Free Height	Wire Diameter	Coil Diameter	Number of Coils	Free Height
1	0.04735309758	0.2632045077	13.47275949	1.525959964	0.07243674785	0.6775631377	7.592829475	1.369115417
2	0.07681530634	0.6400386081	12.94948955	1.594751247	0.07243674785	0.6775631377	7.592829475	1.369115417
3	0.1842667961	0.2857953622	15.87240389	1.778356185	0.07243674785	0.6775631377	7.592829475	1.369115417
4	0.08228471093	0.4690840665	4.289522923	1.148555107	0.07243674785	0.6775631377	7.592829475	1.369115417
5	0.1108515351	0.6064586996	18.87818163	1.216915588	0.07243674785	0.6775631377	7.592829475	1.369115417
6	0.1180764956	0.4051039167	3.202335182	1.40341038	0.07243674785	0.6775631377	7.592829475	1.369115417
7	0.04081463856	0.6162849514	8.290655715	1.575679822	0.07243674785	0.6775631377	7.592829475	1.369115417
8	0.0414732586	0.4912882619	7.470511837	1.688671189	0.07243674785	0.6775631377	7.592829475	1.369115417
9	0.1409507556	0.5862985353	10.65920717	1.17543924	0.07243674785	0.6775631377	7.592829475	1.369115417
10	0.05350562406	0.693669285	5.590426322	1.84323528	0.07243674785	0.6775631377	7.592829475	1.369115417

2.3 Design space contour plot

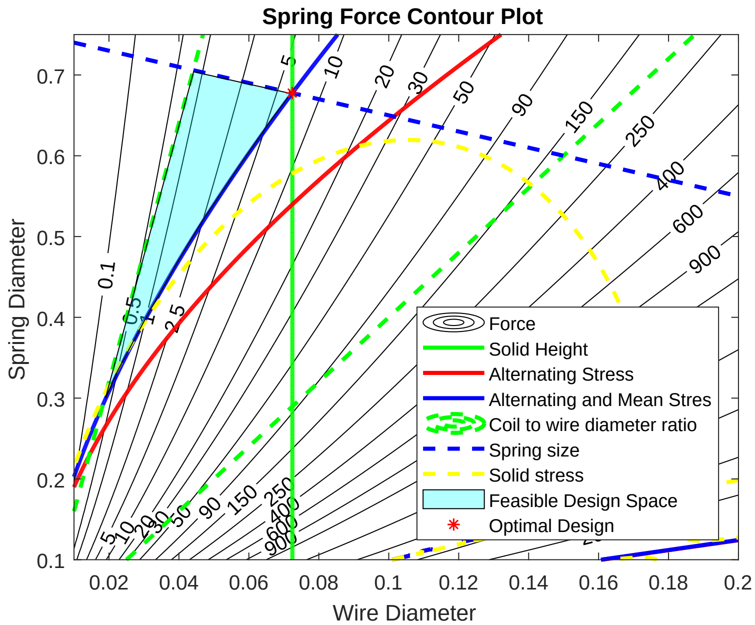


Figure 1: Contour plot showing the design space

2.4 Feasible design space contour plot

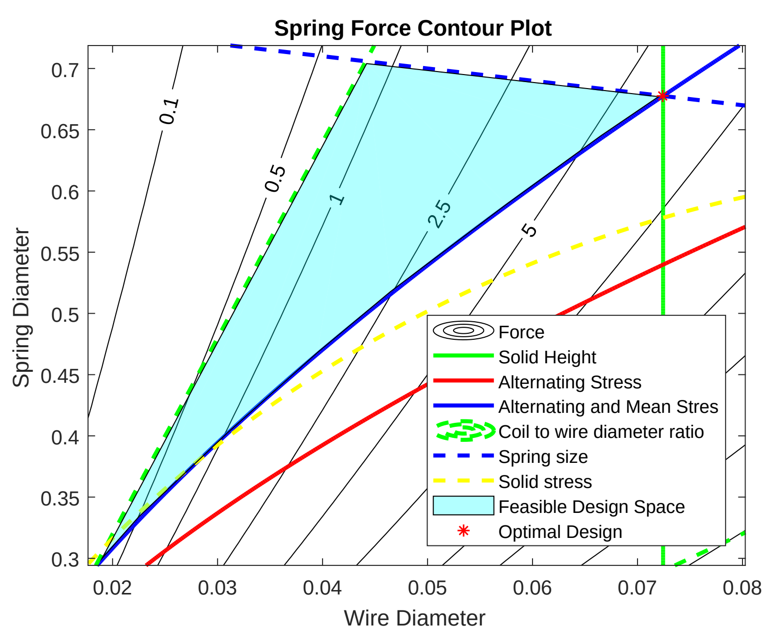


Figure 2: Contour plot showing the feasible design space

3 Discussion

As can be seen in figure 1 the design space for this problem is quite restricted. There are several contraints and the feasible design space is quite small compared to the total design space, at least when viewed on a contour plot of wire diameter and spring diameter. Due to the fact that the contours shown in Figure 1 are quite linear and that the various initial values tried all converged to the same value as shown in Table 5 I conclude that we have found a global optimum for the spring given the applied constraints. It is also of interest that the optimum appears to be bound by three constraints, meaning that if we wanted to increase the increase the rate of the spring by only changing the spring or wire diameter we would need to either relax the alternating and mean stress constraint(ie. find a stronger material), or we would need to relax both the solid height and spring size constraints. It is important to remember that Figures 1 and 2 are only showing a portion of the design space and other options should also be considered

4 Appendix

4.1 Matlab files

4.1.1 Optmization code

```
function [xopt, fopt, exitflag, output] = Opt()

    % ------------Starting point and bounds------------
    % design variables d, D, n, h_f
%     x0 = [0.03, 0.6, 4, 1.2];
    n_tries = 10;
    x0 = zeros(n_tries, 4);
    ub = [0.2, 0.75, 20, 2];
    lb = [0.01, 0.1, 3, 1.1];
    for i=1:n_tries
        for j = 1:4
            x0(i, j) = lb(1, j) + (ub(1, j) - lb(1, j)) * rand();
        end
    end

    % ------------Linear constraints------------
    A = [];
    b = [];
    Aeq = [];
    beq = [];

    % ------------Objective and Non-linear Constraints------------
    function [f, c, ceq] = objcon(x)
        % extract design variables
        d = x(1);      % wire diameter (in)
        D = x(2);      % spring diameter (in)
        n = x(3);      % number coils
        h_f = x(4);    % free height (in)

        % constants
        pi = 3.14159;
        del_0 = 0.4; %(in)
        h_0 = 1; %(in)
        h_def = h_0 - del_0; %(in)
        G = 12000000; %(psi)
```

```
    sf = 1.5;
    se = 45000; %(psi)
    Q = 150000; %(psi)
    w = 0.18;

    % analysis functions
    k = (G * d^4) / (8 * D^3 * n);
    h_s = n*d;
    del_free = h_f - h_0;
    F_0 = k * del_free;
    F_def = k * (del_free + del_0);
    F_solid = k * (h_f - h_s);
    K = (4 * D - d) / (4 * (D - d)) + 0.62 * d / D;
    tau_max = (8 * F_def * D) / (pi * d^3) * K;
    tau_min = (8 * F_0 * D) / (pi * d^3) * K;
    tau_solid = (8 * F_solid * D) / (pi * d^3) * K;
    tau_a = (tau_max - tau_min) / 2;
    tau_m = (tau_max + tau_min) / 2;
    sy = 0.44 * Q / d^w;

    % Objective function
    f = -F_0;

    % constraints
    c = zeros(6, 1);
    c(1) = h_s + 0.05 - h_def;
    c(2) = tau_a - se/sf;
    c(3) = tau_a + tau_m - sy/sf;
    c(4) = D/d - 16;
    c(5) = -D/d + 4;
    c(6) = D + d - 0.75;
    c(7) = tau_solid - sy;

    % equality constraints
    ceq = [];
end

% ------------Call fmincon------------
options = optimoptions(@fmincon, 'display', 'iter-detailed');
opts = zeros(n_tries, 4);
for i = 1:n_tries
    x1 = x0(1, :)
    [xopt, fopt, exitflag, output] = fmincon(@obj, x1, A, b, Aeq, beq, lb(1,:), ub(1,:), @con, optio
    opts(i, :) = xopt;
    fopt
end

% ------------Separate obj/con (do not change)------------
function [f] = obj(x)
        [f, ~, ~] = objcon(x);
end
function [c, ceq] = con(x)
        [~, c, ceq] = objcon(x);
end
```

```
end
```

4.1.2 Plotting Code

```
% constants
pi = 3.14159;
del_0 = 0.4; %(in)
h_0 = 1; %(in)
h_def = h_0 - del_0; %(in)
G = 12000000; %(psi)
sf = 1.5;
se = 45000; %(psi)
Q = 150000; %(psi)
w = 0.18;



%design variables
n = 7.5928;
h_f = 1.3691;
[d, D] = meshgrid(0.01:0.001:0.2, 0.1:0.001:0.75);


%equations
k = (G * d.^4) ./ (8 * D.^3. .* n);
h_s = n.*d;
del_free = h_f - h_0;
F_0 = k .* del_free;
F_def = k .* (del_free + del_0);
F_solid = k .* (-h_s + h_f);
K = (4 .* D - d) ./ (4 .* (D - d)) + 0.62 .* d ./ D;
tau_max = (8 .* F_def .* D) ./ (pi .* d.^3) .* K;
tau_min = (8 .* F_0 .* D) ./ (pi .* d.^3) .* K;
tau_solid = (8 .* F_solid .* D) ./ (pi .* d.^3) .* K;
tau_a = (tau_max - tau_min) ./ 2;
tau_m = (tau_max + tau_min) ./ 2;
sy = 0.44 .* Q ./ d.^w;

figure(1)
[C,h] = contour(d, D, F_0, [0.1 0.5 1 2.5 5 10 20 30 50 90 150 250 400 600 ...
    900 1300 2500 5000 10000 20000], 'k');
clabel(C,h,'Labelspacing',250);
title('Spring Force Contour Plot');
xlabel('Wire Diameter');
ylabel('Spring Diameter');
hold on;
contour(d, D, h_s, [h_def-0.05 h_def-0.05], 'g-', 'LineWidth', 2);
contour(d, D, tau_a, [se/sf se/sf], 'r-', 'LineWidth', 2);
contour(d, D, tau_m + tau_a - sy/sf, [0, 0], 'b-', 'LineWidth', 2); % stress
contour(d, D, D./d, [4 16], 'g--', 'LineWidth', 2); % diameter ratio
contour(d, D, d + D, [0.75, 0.75], 'b--', 'LineWidth', 2); %spring diam
contour(d, D, tau_solid - sy, [0,0], 'y--', 'LineWidth', 2);
x_patch = [0.072, 0.0665, 0.0575, 0.0465, 0.0395, 0.031, 0.0225, 0.019, 0.0442];
y_patch = [0.677, 0.645, 0.589, 0.518, 0.468, 0.403, 0.334, 0.3, 0.704];
```

```
patch('XData', x_patch,'YData',  y_patch, 'FaceColor', 'Cyan', 'FaceAlpha', 0.3);
plot(0.0724, 0.6776, 'r*')
legend('Force', 'Solid Height', 'Alternating Stress',...
    'Alternating and Mean Stress', 'Coil to wire diameter ratio',...
    'Spring size', 'Solid stress', 'Feasible Design Space',...
    'Optimal Design', 'Location', 'SouthEast');
```

ME 575
Homework #1 Spring Design
Due Friday, January 19, 2:50 p.m.

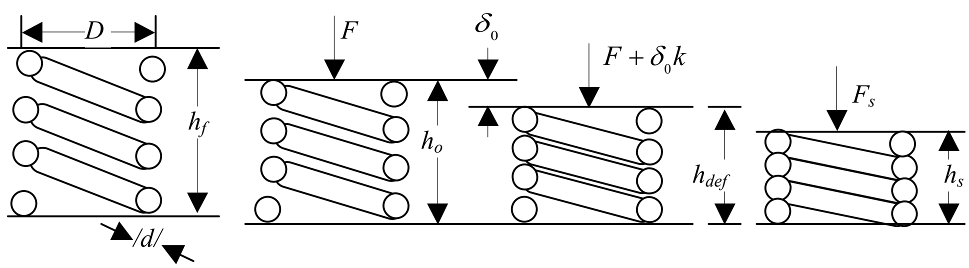
The specifications and modeling equations for compression spring design are given below. We wish to determine the spring design that maximizes the force of a spring at its preload height, h_o, of 1.0 inches. The spring is to operate an indefinite number of times through a deflection δ_o, of 0.4 inches, which is an additional deflection from h_o. The stress at the solid height, h_s, must be less than S_y to protect the spring from inadvertent damage.

The variables defining the design of a spring are d, D, n, h_o and h_f,

where	d	=	wire diameter
	D	=	coil diameter
	n	=	number of coils in the spring
	h_o	=	preload height
	h_f	=	free height (spring exerting no force)

and other variables/functions, as shown below, are,

δ_o	=	deflection from preload height
h_{def}	=	deflected height
h_s	=	solid height



The force in a linear spring is given by,

$$F = k\Delta x$$

where k is the spring stiffness and Δx is the deflection.

The spring stiffness is,

$$k = \frac{Gd^4}{8D^3n}$$

where G is the shear modulus of the material. The stress in a spring with an axial load of F is,

$$\tau = \frac{8FD}{\pi d^3} K$$

where K is the Wahl factor that accounts for stress concentration due to curvature of the spring as well as direct shear:

$$K = \frac{4D - d}{4(D - d)} + 0.62\frac{d}{D}$$

Solid height, h_S, is the height at which the coils of the compressed spring close up. It is simply,

$$h_s = nd$$

If the spring is to operate indefinitely through a deflection δ_0, it must be designed so that it does not fail in fatigue. A fatigue criterion for compression spring design is

$$\tau_a \le S_e / S_f$$

$$\tau_a + \tau_m \le S_y / S_f$$

where τ_m is the mean shear stress and τ_a is the alternating shear stress, defined to be,

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} \qquad \tau_a = \frac{\tau_{\max} - \tau_{\min}}{2}$$

and where S_f is a factor of safety, S_e is the endurance limit, and S_y is the yield strength in shear. S_f and S_e are constants, but S_y is a function of material properties Q and w, according to the relation,

$$S_y = 0.44\frac{Q}{d^w}$$

Also, to be reasonable, the ratio D/d should be $4 \le D/d \le 16$. The diameters of wire should be $0.01 \le d \le 0.2$ inches. The maximum allowable width for the spring, i.e., $(D + d)$, is 0.75 inches. To insure that the spring does not reach solid height in service, a clash allowance of 0.05 inches should be provided. This means the solid height should be at least 0.05 inches below the lowest point of deflection the spring reaches in service.

For this problem, assume

$G = 12 \times 10^6$ psi	$S_f = 1.5$
$S_e = 45{,}000$ psi	$Q = 150{,}000$ psi
$w = 0.18$	

Also assume the number of coils is continuous for optimization.

Sample Design:
(Note, no guarantee this is a feasible design)

```
wire diameter           0.050  (in)
coil diameter           0.500  (in)
number of coils         10.00
free height             1.500  (in)

Spring constant                 7.500 (lb/in)
Wahl Factor                     1.14533
Force at Preload height      3.75 (lb)
Alternating Stress       17499 (psi)
Mean Stress                     61248 (psi)
Yield Strength                  113170 (psi)
Clash Allowance                 0.1 (in)
Diameter Sum             0.55 (in)
```

Assignment: Using whatever programming language you wish, solve for the optimum for several starting points, as mentioned below. Also create the requested contour plots. Optional: Develop a table of derivatives.

Report:

1) Title page with main optimization results (include values for design variables and functions, and indicate which constraints are binding).
2) Setup:
    a. A table showing the mapping between analysis variables, design variables, analysis functions and design functions, similar to Figure 1.7 in the notes.
    b. Optional: Print out the gradients for the functions (derivatives of each design function with respect to the design variables). Comment on whether some scaling would be appropriate.
3) Results:
    a. A table showing the optimum values of all variables and functions (analysis and design). Indicate (with arrows, highlighter, etc.) binding constraints and/or variables at bounds. (Hint: use Courier font to keep values aligned.)
    b. A table giving several starting points which were tried along with the optimal objective value reached from each point.
    c. A "zoomed out" contour plot showing the design space (both feasible and infeasible space) for coil diameter vs. wire diameter, with all constraints shown and the feasible region shaded and optimum marked.
    d. A "zoomed in" contour plot of the design space (mostly feasible space) for coil diameter vs. wire diameter, with all constraints shown and the feasible region shaded and optimum marked.
4) Discussion:
    a. Include any observations or comments about the model, process of optimization or the design space. What did you learn about optimizing this problem? Do you feel this is a global optimum? Keep this a half page or less.
5) Appendix:
    a. Listing of `MATLAB or other files`
    b. Copy of the assignment

Turn in through Learning Suite as a pdf file.

Please note: any output from `MATLAB (or other language)` should be integrated into the report as given in the sections above. Tables and figures should all have explanatory captions. Please do **not** just staple pages of output to your assignment.