ME 575
Homework #4: Computing Derivatives
Due Feb. 24 at 11:50 p.m. Rev 1.0

<u>Note:</u> The description of the assignment is subject to change slightly. However, I don't anticipate changes to the requested tasks. Indeed, I may add something on Automatic Differentiation. I am posting this now to provide it for this who wish to get started early.

<u>Overview</u>
For this assignment you will optimize a 10-bar truss problem where you will provide derivatives to the optimizer. Our goals are to learn several approaches for numerically computing derivatives, understand their advantages and disadvantages, and apply these methods to a problem to better understand the need for accurate gradients. You will use software already developed for truss analysis.

<u>Truss Specification</u>
Everyone will optimize the same 10-bar truss. This is a problem taken from the literature that is well known. The description of the problem (along with the solution) is given in the Appendix. The truss analysis software (Truss.m) reads in a data file (Data.m) that describes the particular problem. These files will be provided to you.

<u>Truss Derivatives</u>
Before we can optimize the truss we need to provide the following derivatives:
1) Derivatives of weight (objective) with respect to cross-sectional areas (design variables), i.e.

$$\frac{\partial w}{\partial a_i} \quad i = 1, 2, \ldots n$$

2) Derivatives of stress (constraints) with respect to cross-sectional areas, i.e.,

$$\frac{\partial \sigma_j}{\partial a_i} \quad i = 1, 2, \ldots n; j = 1, 2, \ldots m$$

There are m constraints (m=10), and n variables (n=10) so this will be a 10 by 10 matrix.

The truss analysis function is already provided that returns weight and stress for given cross-sectional areas (Truss.m). There is also a driver program to call this routine as a stand-alone program (StandAloneTruss.m), a data file which describes the 10-bar truss (Data.m), and a file (OptimizeTruss.m) that is already setup to optimize the truss. All files are available under Content > MATLAB Examples on Learning Suite. Executing OptimizeTruss.m will optimize the truss using fmincon and built-in derivatives. You will modify this file to add your own derivatives to the code. For the direct method you will change Truss.m directly since that method requires modification of the source code.

You should compute the derivatives of the objective (weight) and the constraints (stress) using the following methods:

(a) A finite-difference formula of your choice.
(b) A different finite-difference approach (either a different formula, or a different step size).
(c) The complex-step derivative method.
(d) The analytic direct method.

Truss Optimization

After you have developed the gradients, optimize the truss using fmincon. Previously we have used the default solver, which is the Interior Point method. The fmincon optimizer has three other solvers, and you might want to experiment to find which works best on this problem. Solve this optimization problem, from the same starting point, using all the methods.

Report the following:
a) Provide a listing giving your MATLAB code for each of the four methods with comments. Provide a brief explanation of how you integrated derivatives into the code and discuss any difficulties encountered.
b) Evaluate the relative errors of the derivatives of the various methods at the starting point. Discuss your findings. What are the relative merits of the approaches?
c) Provide a table giving 1) the number of function calls required by each derivative method, 2) the time of execution (use the "tic/toc" function of MATLAB), and 3) the stopping criterion given by MATLAB (one starting point is fine). Discuss differences you observe.

Notes

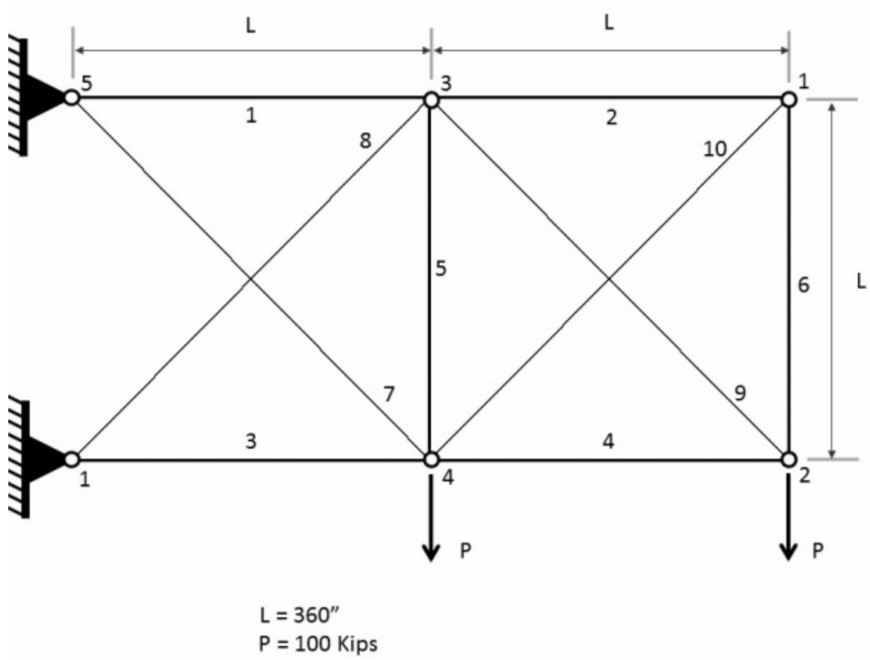
There is an option in fmincon to run a check on your gradients (see CheckGradients). This allows you to not only check the accuracy of your derivatives but to make sure they are in the correct order

Ten Bar Truss Sizing Optimization

The ten bar truss model was developed by Venkayya for:
Venkayya, V. B. (1971). "Design of Optimum Structures," Computers & Structures, 1(1), 265-309.

The design problems for this page are described in:
Haftka, Raphael T. Elements of Structural Optimization. Springer, 1992. Pages 238, 244.

Example CoFE input files for this page are provided here:
NASTRAN_CoFE/CoFE_examples/o1_tenbarOpt/



Details

- Young's modulus: 10^7 psi
- Specific mass: 0.1 lbm/in^3
- Minimum area: 0.1 in^2
- Allowable stress: +/- 25000 psi
- The ten design variables are the areas of the ten members.

Design Problem 1

Design for minimum weight with stress constraints.

Design Variable	Design from CoFE + SQP	Design in Haftka
x_1	7.94	7.94
x_2	0.1	0.1
x_3	8.06	8.06
x_4	3.94	3.94
x_5	0.1	0.1
x_6	0.1	0.1
x_7	5.74	5.74
x_8	5.57	5.57
x_9	5.57	5.57
x_{10}	0.1	0.1

Objective Function	Design from CoFE + SQP	Design in Haftka
Weight	1593.2 lb	1593.2 lb