Optimal Spring Sizing

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Summary

0.1 Design variable values

The optimal force was found to be 6.454 lbs. The determined optimum values for the design variables are as follows:

Table 1: Optimum values for design

Variable	**Value**
Wire Diameter	0.0724 in
Coil Diameter	0.6776 in
Number Coils	0.5928
Free Height	1.3691

0.2 Design function values

Table 2: Values of design functions at optimum

Design Function	**Value**
Maximize F_0	$6.4541 lbs$
$h_S + 0.05 \leq h_{def}$	$0.60 in \leq 0.60 in$
$\tau_a \leq \frac{s_e}{s_f}$	$18352 psi \leq 30000 psi$
$\tau_a + \tau_m \leq \frac{s_y}{s_f}$	$70576 psi \leq 70576 psi$
$4 \geq \frac{D}{d} \leq 16$	$4 in \leq 9.359 in \leq 16 in$
$D + d \leq 0.75$	$0.75 in \leq 0.75 in$
$\tau_{solid} \leq s_y$	$75165 psi \leq 105860 psi$

0.3 Binding Constraints

The binding constraints are:

- $h_S + 0.05 \leq h_{def}$
- $D + d \leq 0.75$
- $\tau_a + \tau_m \leq \frac{s_y}{s_f}$

1 Setup

1.1 Variable Mapping

Table 3: My caption

Analysis Variables	**Design Variables**
Wire Diameter	Wire Diameter
Coil Diameter	Coil Diameter
Number of Coils	Number of Coils
Free height	Free height
Preload Height	
Preload Deflection	
Shear Modulus	
Safety Factor	
Endurance Limit	

Analysis Functions	**Design Functions**
Force	Maximize
Spring Stiffness	Solid Height $\leq$ Deflected height
Whal Factor	Alternating Stress $\leq \frac{s_e}{s_f}$
Solid Height	Alternating and Mean Stress $\leq \frac{s_y}{s_f}$
Alternating Stress	$4 \leq$ Coil to Wire Ratio ≤ 16
Mean Stress	Solid Stress $\leq s_y$
Yield Strength	
Coil to Wire Ratio	
Spring Diameter	
Deflected Height	

2 Results

2.1 Optimum values of variables and functions

Table 4: Optimum Values of Variables and Functions (binding functions are highlighted)

Variable/Function	**Value**
Wire Diameter	0.0724 in
Coil Diameter	0.6776 in
Number Coils	0.5928
Free Height	1.3691
Preload Height	1.0 in
δ_0	0.4 in
h_{def}	0.6 in
h_s	0.55in
F_0	6.454 lbs
k	17.4853 lbs/in
K	1.1561
τ_{max}	70576 psi
τ_{min}	33871 psi
τ_a	18353 psi
τ_m	52224 psi
$\tau_a + \tau_m$	70576 psi
$\frac{S_e}{S_f}$	30000 psi
$\frac{S_y}{S_f}$	70576 psi
$\frac{D}{d}$	9.3539
$D + d$	0.75

2.2 Starting points and obtained values

Table 5: Optimized Values from Given Starting Point

	Initial Values				**Optimized Values**			
Trial	Wire Diameter	Coil Diameter	Number of Coils	Free Height	Wire Diameter	Coil Diameter	Number of Coils	Free Height
1	0.04735309758	0.2632045077	13.47275949	1.525959964	0.07243674785	0.6775631377	7.592829475	1.369115417
2	0.07681530634	0.6400386081	12.94948955	1.594751247	0.07243674785	0.6775631377	7.592829475	1.369115417
3	0.1842667961	0.2857953622	15.87240389	1.778356185	0.07243674785	0.6775631377	7.592829475	1.369115417
4	0.08228471093	0.4690840665	4.289522923	1.148555107	0.07243674785	0.6775631377	7.592829475	1.369115417
5	0.1108515351	0.6064586996	18.87818163	1.216915588	0.07243674785	0.6775631377	7.592829475	1.369115417
6	0.1180764956	0.4051039167	3.202335182	1.40341038	0.07243674785	0.6775631377	7.592829475	1.369115417
7	0.04081463856	0.6162849514	8.290655715	1.575679822	0.07243674785	0.6775631377	7.592829475	1.369115417
8	0.0414732586	0.4912882619	7.470511837	1.688671189	0.07243674785	0.6775631377	7.592829475	1.369115417
9	0.1409507556	0.5862985353	10.65920717	1.17543924	0.07243674785	0.6775631377	7.592829475	1.369115417
10	0.05350562406	0.693669285	5.590426322	1.84323528	0.07243674785	0.6775631377	7.592829475	1.369115417

2.3 Design space contour plot

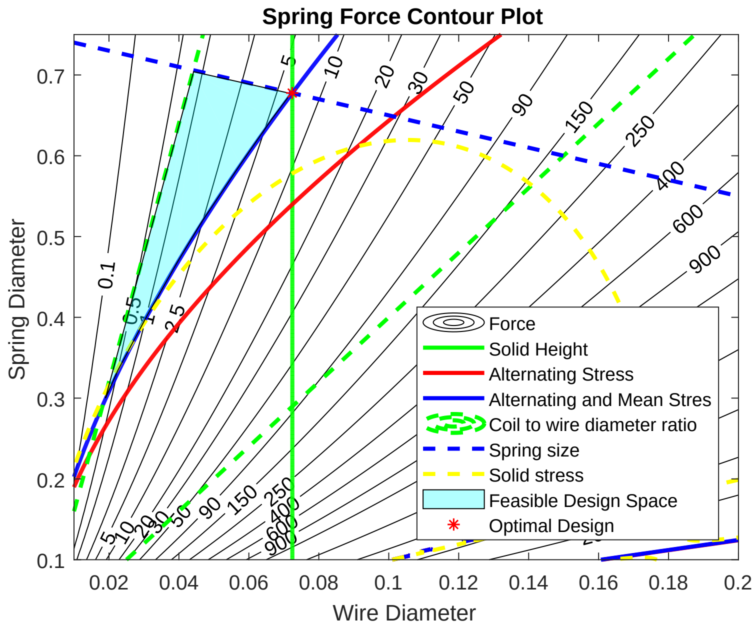

Figure 1: Contour plot showing the design space

2.4 Feasible design space contour plot

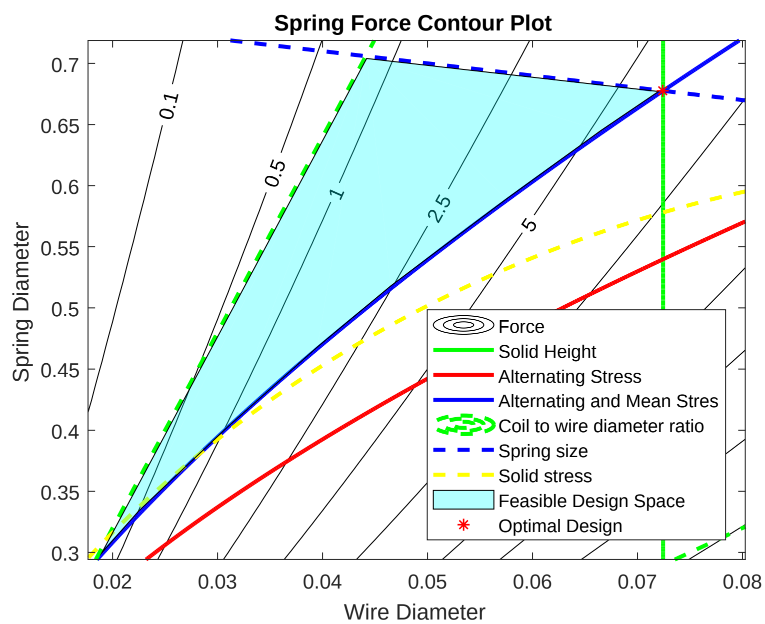

Figure 2: Contour plot showing the feasible design space

3 Discussion

As can be seen in figure 1 the design space for this problem is quite restricted. There are several contraints and the feasible design space is quite small compared to the total design space, at least when viewed on a contour plot of wire diameter and spring diameter. Due to the fact that the contours shown in Figure 1 are quite linear and that the various initial values tried all converged to the same value as shown in Table 5 I conclude that we have found a global optimum for the spring given the applied constraints. It is also of interest that the optimum appears to be bound by three constraints, meaning that if we wanted to increase the increase the rate of the spring by only changing the spring or wire diameter we would need to either relax the alternating and mean stress constraint(ie. find a stronger material), or we would need to relax both the solid height and spring size constraints. It is important to remember that Figures 1 and 2 are only showing a portion of the design space and other options should also be considered

4 Appendix

4.1 Matlab files

4.1.1 Optmization code

```
function [xopt, fopt, exitflag, output] = Opt()

    % ------------Starting point and bounds------------
    % design variables d, D, n, h_f
%     x0 = [0.03, 0.6, 4, 1.2];
    n_tries = 10;
    x0 = zeros(n_tries, 4);
    ub = [0.2, 0.75, 20, 2];
    lb = [0.01, 0.1, 3, 1.1];
    for i=1:n_tries
        for j = 1:4
            x0(i, j) = lb(1, j) + (ub(1, j) - lb(1, j)) * rand();
        end
    end

    % ------------Linear constraints------------
    A = [];
    b = [];
    Aeq = [];
    beq = [];

    % ------------Objective and Non-linear Constraints------------
    function [f, c, ceq] = objcon(x)
        % extract design variables
        d = x(1);      % wire diameter (in)
        D = x(2);      % spring diameter (in)
        n = x(3);      % number coils
        h_f = x(4);    % free height (in)

        % constants
        pi = 3.14159;
        del_0 = 0.4; %(in)
        h_0 = 1; %(in)
        h_def = h_0 - del_0; %(in)
        G = 12000000; %(psi)
```

```
    sf = 1.5;
    se = 45000; %(psi)
    Q = 150000; %(psi)
    w = 0.18;

    % analysis functions
    k = (G * d^4) / (8 * D^3 * n);
    h_s = n*d;
    del_free = h_f - h_0;
    F_0 = k * del_free;
    F_def = k * (del_free + del_0);
    F_solid = k * (h_f - h_s);
    K = (4 * D - d) / (4 * (D - d)) + 0.62 * d / D;
    tau_max = (8 * F_def * D) / (pi * d^3) * K;
    tau_min = (8 * F_0 * D) / (pi * d^3) * K;
    tau_solid = (8 * F_solid * D) / (pi * d^3) * K;
    tau_a = (tau_max - tau_min) / 2;
    tau_m = (tau_max + tau_min) / 2;
    sy = 0.44 * Q / d^w;

    % Objective function
    f = -F_0;

    % constraints
    c = zeros(6, 1);
    c(1) = h_s + 0.05 - h_def;
    c(2) = tau_a - se/sf;
    c(3) = tau_a + tau_m - sy/sf;
    c(4) = D/d - 16;
    c(5) = -D/d + 4;
    c(6) = D + d - 0.75;
    c(7) = tau_solid - sy;

    % equality constraints
    ceq = [];
end

% ------------Call fmincon------------
options = optimoptions(@fmincon, 'display', 'iter-detailed');
opts = zeros(n_tries, 4);
for i = 1:n_tries
    x1 = x0(1, :)
    [xopt, fopt, exitflag, output] = fmincon(@obj, x1, A, b, Aeq, beq, lb(1,:), ub(1,:), @con, optic
    opts(i, :) = xopt;
    fopt
end

% ------------Separate obj/con (do not change)------------
function [f] = obj(x)
        [f, ~, ~] = objcon(x);
end
function [c, ceq] = con(x)
        [~, c, ceq] = objcon(x);
end
```

```
end
```

4.1.2 Plotting Code

```
% constants
pi = 3.14159;
del_0 = 0.4; %(in)
h_0 = 1; %(in)
h_def = h_0 - del_0; %(in)
G = 12000000; %(psi)
sf = 1.5;
se = 45000; %(psi)
Q = 150000; %(psi)
w = 0.18;



%design variables
n = 7.5928;
h_f = 1.3691;
[d, D] = meshgrid(0.01:0.001:0.2, 0.1:0.001:0.75);


%equations
k = (G * d.^4) ./ (8 * D.^3. .* n);
h_s = n.*d;
del_free = h_f - h_0;
F_0 = k .* del_free;
F_def = k .* (del_free + del_0);
F_solid = k .* (-h_s + h_f);
K = (4 .* D - d) ./ (4 .* (D - d)) + 0.62 .* d ./ D;
tau_max = (8 .* F_def .* D) ./ (pi .* d.^3) .* K;
tau_min = (8 .* F_0 .* D) ./ (pi .* d.^3) .* K;
tau_solid = (8 .* F_solid .* D) ./ (pi .* d.^3) .* K;
tau_a = (tau_max - tau_min) ./ 2;
tau_m = (tau_max + tau_min) ./ 2;
sy = 0.44 .* Q ./ d.^w;

figure(1)
[C,h] = contour(d, D, F_0, [0.1 0.5 1 2.5 5 10 20 30 50 90 150 250 400 600 ...
    900 1300 2500 5000 10000 20000], 'k');
clabel(C,h,'Labelspacing',250);
title('Spring Force Contour Plot');
xlabel('Wire Diameter');
ylabel('Spring Diameter');
hold on;
contour(d, D, h_s, [h_def-0.05 h_def-0.05], 'g-', 'LineWidth', 2);
contour(d, D, tau_a, [se/sf se/sf], 'r-', 'LineWidth', 2);
contour(d, D, tau_m + tau_a - sy/sf, [0, 0], 'b-', 'LineWidth', 2); % stress
contour(d, D, D./d, [4 16], 'g--', 'LineWidth', 2); % diameter ratio
contour(d, D, d + D, [0.75, 0.75], 'b--', 'LineWidth', 2); %spring diam
contour(d, D, tau_solid - sy, [0,0], 'y--', 'LineWidth', 2);
x_patch = [0.072, 0.0665, 0.0575, 0.0465, 0.0395, 0.031, 0.0225, 0.019, 0.0442];
y_patch = [0.677, 0.645, 0.589, 0.518, 0.468, 0.403, 0.334, 0.3, 0.704];
```

```
patch('XData', x_patch,'YData',  y_patch, 'FaceColor', 'Cyan', 'FaceAlpha', 0.3);
plot(0.0724, 0.6776, 'r*')
legend('Force', 'Solid Height', 'Alternating Stress',...
    'Alternating and Mean Stress', 'Coil to wire diameter ratio',...
    'Spring size', 'Solid stress', 'Feasible Design Space',...
    'Optimal Design', 'Location', 'SouthEast');
```