Optimal Spring Sizing

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1 Summary

1.1 Design variable values

The optimal cost was found to be **$400,140**. The determined optimum values for the design variables are as follows:

Table 1: Optimum values for design

Variable	**Value**
Velocity	$7.8161\frac{ft}{s}$
Pipe Diameter	$0.1816ft$
Particle Size	$0.0005ft$
Water Flow Rate	$0.1128\frac{ft^3}{s}$
Volumetric Concentration	0.4
Slurry Density	$104.84\frac{lbm}{ft^3}$

2 Procedure

2.1 Equation Sequence

Get values from optimizer (V, D, d)	Optimization code will set these variables
$Q = \pi * \frac{D^2}{4} * V$	Volumetric flow rate
$Q_l = \frac{W}{\rho_l}$	Lime volumetric flow rate
$Q_w = Q - Q_l$	Water volumetric flow rate
$c = \frac{Q_l}{Q}$	Volumetric concentration
$\rho = \rho_w + c * (\gamma - \rho_w)$	Slurry density
$C_d R_p^2 = 4g\rho_w d^3 \left(\frac{\gamma - \rho_w}{3\mu^2}\right)$	Eqn (3)
$C_d = e^{-0.001log(C_d R_p^2)^3 + 0.0583log(C_d R_p^2)^2 - 1.1497log(C_d R_p^2) + 6.4442}$	Fit of empirical data
$R_w = \frac{\rho_w V D}{\mu}$	Reynolds number
$\begin{cases} f_w = \frac{0.3164}{R_w^{0.25}} \quad \text{if } R_w \leq 10^5 \\ f_w = 0.0032 + 0.221 R_w^{-0.2377} \end{cases}$	Water friction factor
$f = f_w \left[\frac{\rho_w}{\rho} + 150c\frac{\rho_w}{\rho}\left(\frac{gD(S-1)}{V^2\sqrt{C_d}}\right)^{1.5}\right]$	Friction factor
$\delta p = \frac{f \rho L V^2}{D 2 g_c}$	Pressure loss due to friction
$P_p = \delta p * Q$	Pump power required
$P_g = 218W\left(\frac{1}{\sqrt{d}} - \frac{1}{\sqrt{a}}\right)$	Power grind required
$HP_p = \frac{P_g}{550}$	Horsepower to pump
$HP_g = \frac{P_g}{550}$	Horsepower to grind
$C_p = 300HP_g + 200HP_g$	Purchase cost of system
$C_o = (0.07HP_g + 0.05HP_p) * 8 * 300 * \left(\frac{1.07^7 - 1}{0.07 * 1.07^7}\right)$	Operational cost of system
$C = C_p + C_o$	Total system cost

2.2 Mapping Table

Variables	**Design Variables**
Average flow velocity (V)	Average flow velocity (V)
Internal pipe diameter (D)	Internal pipe diameter (D)
Average particle size after grinding (d)	Average particle size after grinding (d)
length of pipeline (w)	
Limestone flowrate (W)	
Average lump size before grinding (a)	
Gravity (g)	
Density of water (ρ_w)	
Viscosity of water (μ)	
Limestone density (γ)	
Analysis Functions	**Design Functions**
Volumetric slurry concentration (c)	Minimize cost
Water flow rate (Q_w)	s.t.
Density of slurry (ρ)	$V \geq 1.1 * V\_c$
Limestone flowrate (volumetric) (Q_l)	
Slurry flow rate (Q)	
Grinding Power (P_g)	
Friction factor (f)	
Water friction factor (f_w)	
Average drag coefficient (C_d)	
Limestone specific gravity (S)	
Reynolds number (R_w)	
$C_d R_p$	
Slurry density (ρ)	
Pipe pressure drop (δp)	
Pumping power (P_f)	
Critical velocity (V_c)	
Cost (C)	

2.3 Model Validation

The curve fit was performed in the statistical package JMP. It is a third order fit of the log of both C_d and $C_d R_p^2$ data that was provided in the problem statement. The R^2 value of the fit is exceptionally high at 0.9999 which suggests that the data is very well approximated by the fit.

3 Results and Discussion

3.1 Optimum Values

Variable/Function	**Value at optimum**
Average flow velocity (V)	7.2599
Internal pipe diameter (D)	0.1816
Average particle size after grinding (d)	0.0005
length of pipeline (w)	79200
Limestone flowrate (W)	12.67
Average lump size before grinding (a)	0.01
Gravity (g)	32.17
Density of water (ρ_w)	62.4
Viscosity of water (μ)	0.0007392
Limestone density (γ)	168.5
Volumetric slurry concentration (c)	0.4
Water flow rate (Q_w)	0.1128
Density of slurry (ρ)	104.84
Limestone flowrate (volumetric) (Q_l)	0.0752
Slurry flow rate (Q)	0.188
Grinding Power (P_g)	95902
Friction factor (f)	0.0175
Water friction factor (f_w)	0.0173
Average drag coefficient (C_d)	13.3093
Limestone specific gravity (S)	2.7003
Reynolds number (R_w)	111280
C_dR_p	64.9645
Pipe pressure drop (δp)	656410
Pumping power (P_f)	123390
Critical velocity (V_c)	6.5999
Pumping HP (HP_p)	224.35
Grinding HP (HP_g)	174.37
Cost (C)	400140

3.2 Optimum and Design Space

A slice of the design space is shown in figure 1. As can be seen in the figure the feasible design space is rather small in comparison to the total design space. It occupies a small triangular section of the space and shows that the optimum is heavily constrained. The optimum, marked with a red star in the figure is a constrained optimum and could be enhanced by relaxing the constraints on the volumetric concentration and the critical velocity, however it is unlikely that these constraints can be relaxed as they are governed by physical phenomena. The constraint is most likely a global optimum of the design space due to the somewhat simple nature of the contour lines.

3.3 Contour Plot

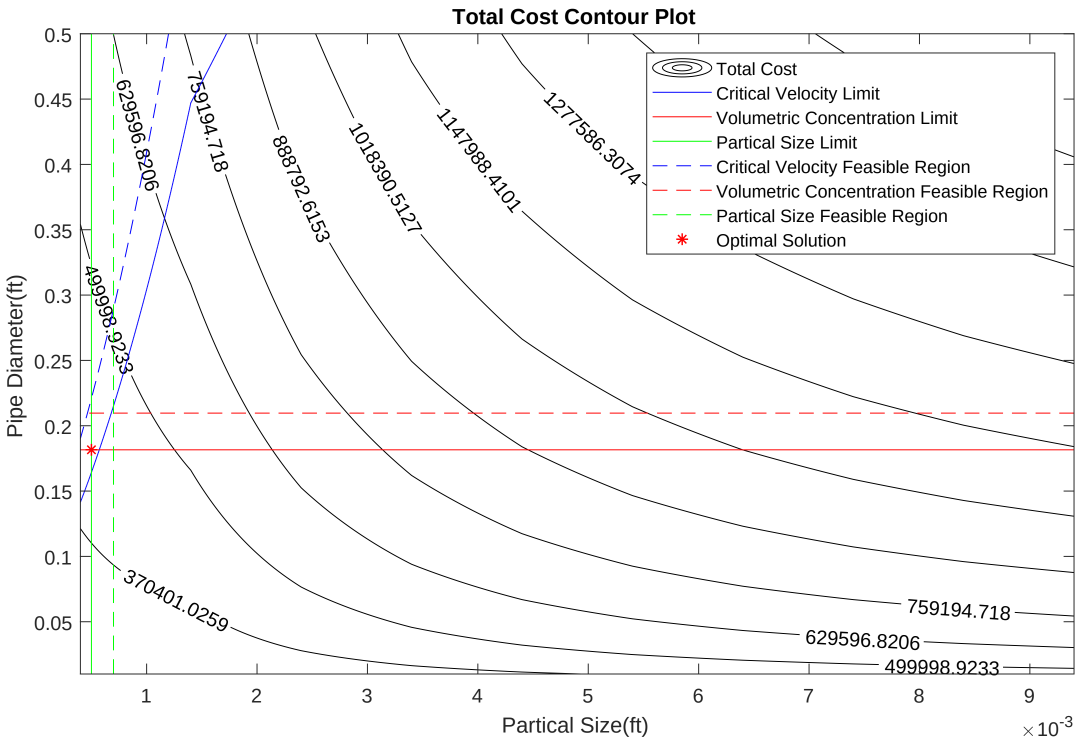


Figure 1: Plot of the design space

3.4 Other Observations

It is of interest that this model is constrained so tightly considering the relatively few number of constraints that exist in the problem, however all of the constraints shown in figure 1 contribute to constraining the optimum value of the problem. As mentioned above the optimum could be enhanced through the relaxation of the critical velocity and volumetric concentration limits, however this would likely lead to clogging issues in the system and is not recommendable. It is also important to not that this contour plot only shows three (two varied with one held constant) of a total 6 possible dimensions of the problem. To further understand the design space it would be necessary to create several similar plots of the other variables.

4 Appendix

4.1 Matlab files

4.1.1 Optmization code

```
function [xopt, fopt, exitflag, output] = Opt()

    % ------------Starting point and bounds------------
    % design variables vel, pipeDiam, particleSize
    n_params = 3;
    n_tries = 1;
    x0 = zeros(n_tries, n_params);
    ub = [100, 0.5, 0.01];
    lb = [0.01, 0.01, 0.0005];
```

```
    for i=1:n_tries
        for j = 1:n_params
            x0(i, j) = lb(1, j) + (ub(1, j) - lb(1, j)) * rand();
        end
    end
%     x0

    % ------------Linear constraints------------
    A = [];
    b = [];
    Aeq = [];
    beq = [];

    % ------------Objective and Non-linear Constraints------------


    % ------------Call fmincon------------
    options = optimoptions(@fmincon, 'display', 'iter-detailed');
    opts = zeros(n_tries, n_params);
    for i = 1:n_tries
        x1 = x0(1, :)
        [xopt, fopt, exitflag, output] = fmincon(@obj, x1, A, b, Aeq, beq, lb(1,:), ub(1,:), @con, o
        opts(i, :) = xopt;
        fopt
    end
    x0
    opts


    % ------------Separate obj/con (do not change)------------
    function [f] = obj(x)
            [f, ~, ~] = objcon(x);
    end
    function [c, ceq] = con(x)
            [~, c, ceq] = objcon(x);
    end
end
```

4.1.2 ObjCon Function

```
    function [f, c, ceq] = objcon(x)
        % extract design variables
        vel = x(1); % ft/s
        pipeDiam = x(2); % ft
        partSize = x(3); % ft

        % constants
        pipeLen = 15 .* 5280; % ft
        wLime = 12.67; % lbm/s
        partSizePre = 0.01; % ft
        grav = 32.17; % ft/s^2
        rhoWater = 62.4; % lbm/ft^3
        rhoLime = 168.5; % lbm/ft^3
        SGLime = rhoLime ./ rhoWater;
```

```
        muWater = 0.0007392; %lbm/(ft-sec)
        hrPerDay = 8;
        dayPerYear = 300;
        yrsLife = 7;

        % calculate other design variables
        volFlow = pi .* (pipeDiam.^2 ./ 4) .* vel;
        volFlowLime = wLime ./ rhoLime;
        volFlowWater = volFlow - volFlowLime;
        volCons = volFlowLime ./ volFlow;
        rhoSlurry = rhoWater + volCons .* (rhoLime - rhoWater);

        % analysis functions
        cdrp = 4 .* grav .* rhoWater .* partSize.^3 .* ((rhoLime - rhoWater) ...
            ./ (3 .* muWater.^2));
        % cd = 1.9116946 + 571.81334 .* (1/cdrp);
        cd = exp(-.001.*log(cdrp).^3+.0583.*log(cdrp).^2-1.1497.*log(cdrp)+6.4442);
        reynolds = (rhoWater .* vel .* pipeDiam) ./ muWater;
        critVel = ((40 .* grav .* volCons .* (SGLime - 1) .* pipeDiam) ...
            ./ sqrt(cd)).^0.5;

        if reynolds <= 100000
          fricWater = 0.3164 ./ reynolds.^0.25;
        else
          fricWater = 0.0032 + 0.221 .* reynolds.^-0.237;
        end

        fricFact = fricWater .* ((rhoWater ./ rhoSlurry) + 150 .* volCons .*...
            (rhoWater ./ rhoSlurry) .* ((grav .* pipeDiam .* (SGLime - 1)) ./...
            (vel.^2 .* sqrt(cd))).^1.5);
        deltaP = (fricFact .* rhoSlurry .* pipeLen .* vel.^2) ./...
            (pipeDiam .* 2 .* grav);
        powerPump = deltaP .* volFlow;
        powerGrind = 218 .* wLime .* ((1./sqrt(partSize)) - (1./sqrt(partSizePre)));
        hpPump = powerPump ./ 550;
        hpGrind = powerGrind ./ 550;

        purchaseCost = 300 .* hpGrind + 200 .* hpPump;
        yearlyPowerCost = (0.07 .* hpGrind + 0.05 .* hpPump) .* hrPerDay .* dayPerYear;
        totPowerCost = yearlyPowerCost .* ((1.07.^yrsLife - 1) ./ (0.07 .* 1.07.^7));
        totCost = purchaseCost + totPowerCost;
        totHP = hpPump + hpGrind;

        % Objective function
        f = totCost;

        % constraints
        c = zeros(2, 1);
        c(1) = -vel + critVel .* 1.1;
        c(2) = volCons - 0.4;

        % equality constraints
        ceq = [];

```

```
    end
```

4.1.3 Plotting Code

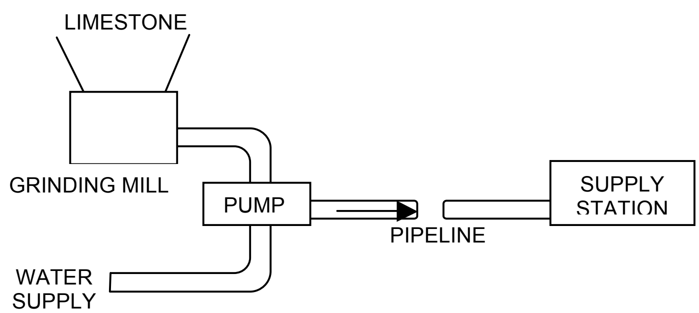
```
% extract design variables
vel = 7.2599; % ft/s
[partSize, pipeDiam] = meshgrid(0.0004:0.001:0.01, 0.01:0.001:0.5);

% constants
pipeLen = 15 .* 5280; % ft
wLime = 12.67; % lbm/s
partSizePre = 0.01; % ft
grav = 32.17; % ft/s^2
rhoWater = 62.4; % lbm/ft^3
rhoLime = 168.5; % lbm/ft^3
SGLime = rhoLime ./ rhoWater;
muWater = 0.0007392; %lbm/(ft-sec)
hrPerDay = 8;
dayPerYear = 300;
yrsLife = 7;

% calculate other design variables
volFlow = pi .* (pipeDiam.^2 ./ 4) .* vel;
volFlowLime = wLime ./ rhoLime;
volFlowWater = volFlow - volFlowLime;
volCons = volFlowLime ./ volFlow;
rhoSlurry = rhoWater + volCons .* (rhoLime - rhoWater);

% analysis functions
cdrp = 4 .* grav .* rhoWater .* partSize.^3 .* ((rhoLime - rhoWater) ...
    ./ (3 .* muWater.^2));
% cd = 1.9116946 + 571.81334 .* (1/cdrp);
cd = exp(-.001.*log(cdrp).^3+.0583.*log(cdrp).^2-1.1497.*log(cdrp)+6.4442);
reynolds = (rhoWater .* vel .* pipeDiam) ./ muWater;
critVel = ((40 .* grav .* volCons .* (SGLime - 1) .* pipeDiam) ...
    ./ sqrt(cd)).^0.5;

if reynolds <= 100000
  fricWater = 0.3164 ./ reynolds.^0.25;
else
  fricWater = 0.0032 + 0.221 .* reynolds.^-0.237;
end

fricFact = fricWater .* ((rhoWater ./ rhoSlurry) + 150 .* volCons .*...
    (rhoWater ./ rhoSlurry) .* ((grav .* pipeDiam .* (SGLime - 1)) ./...
    (vel.^2 .* sqrt(cd))).^1.5);
deltaP = (fricFact .* rhoSlurry .* pipeLen .* vel.^2) ./...
    (pipeDiam .* 2 .* grav);
powerPump = deltaP .* volFlow;
powerGrind = 218 .* wLime .* ((1./sqrt(partSize)) - (1./sqrt(partSizePre)));
hpPump = powerPump ./ 550;
hpGrind = powerGrind ./ 550;

```

```
purchaseCost = 300 .* hpGrind + 200 .* hpPump;
yearlyPowerCost = (0.07 .* hpGrind + 0.05 .* hpPump) .* hrPerDay .* dayPerYear;
totPowerCost = yearlyPowerCost .* ((1.07.^yrsLife - 1) ./ (0.07 .* 1.07.^7));
totCost = purchaseCost + totPowerCost;
totHP = hpPump + hpGrind;

% Objective function
f = totCost;

% constraints
c1 = -vel + critVel .* 1.1;
c2 = volCons - 0.4;


% constants
pi = 3.14159;
del_0 = 0.4; %(in)
h_0 = 1; %(in)
h_def = h_0 - del_0; %(in)
G = 12000000; %(psi)
sf = 1.5;
se = 45000; %(psi)
Q = 150000; %(psi)
w = 0.18;


fig = figure(1)
[C,h] = contour(partSize, pipeDiam, f, 10 , 'k');
clabel(C,h,'Labelspacing',500);
title('Total Cost Contour Plot');
xlabel('Partical Size(ft)');
ylabel('Pipe Diameter(ft)');
hold on;
y1=get(gca,'ylim');
contour(partSize, pipeDiam, c1, [0, 0], 'b')
contour(partSize, pipeDiam, c2, [0, 0], 'r')
plot([0.0005, 0.0005], y1, 'g')
contour(partSize, pipeDiam, c1, [-1, -1], 'b--')
contour(partSize, pipeDiam, c2, [-.1, -.1], 'r--')
plot([0.0007, 0.0007], y1, 'g--')

plot(0.0005, 0.1816, 'r*', 'linewidth', 6)

legend('Total Cost', 'Critical Velocity Limit', 'Volumetric Concentration Limit',...
      'Partical Size Limit', 'Critical Velocity Feasible Region',...
      'Volumetric Concentration Feasible Region', 'Partical Size Feasible Region',...
      'Optimal Solution', 'Location', 'NorthEast')

set(fig,'Units','Inches');
pos = get(fig,'Position');
set(fig,'PaperPositionMode','Auto','PaperUnits','Inches','PaperSize',[pos(3), pos(4)])
print(fig, '-dpdf', 'test.pdf');
```

ME 575
Homework #2 Pipeline Design
Due Jan 26 at 2:50 p.m.

For this assignment, you will gain additional experience in developing an engineering model and optimizing it. This problem includes experimental data, some conditional statements in the model, and an economic objective function. This problem is a modified version taken from James Siddall, *Optimal Engineering Design*, pp. 281-285, Dekker.

Minimize the total cost (capital and operating) for the source station (grinder and pump) for a pipeline which transports crushed limestone from a quarry to a terminal located some distance away, using water as a transporting medium. This will require that you determine the diameter of the pipeline, even though it will not be included in the cost.



SOURCE STATION

The limestone is crushed at the quarry, mixed with water to form a slurry, and pumped through the pipe.

The following specifications are given,

- L = length of pipeline = 15 miles
- W = flowrate of limestone = 12.67 lb_m/sec
- a = average lump size of limestone before grinding = 0.01 ft.

The designer wishes to determine:

- V = average flow velocity, ft/sec
- c = volumetric concentration of slurry
- D = internal diameter of pipe, ft.
- d = average limestone particle size after grinding, ft.
- Q_w = water flow rate, ft^3/sec
- ρ = density of slurry lb_m/ft^3

but only three of the above values can be changed; the others are then determined. For example, the volumetric concentration of slurry can be expressed as,

$$c = \frac{Q_l}{Q_w + Q_l} = \frac{Q_l}{Q}$$

where,

Q = slurry flow rate (ft^3/sec)
Q_l = limestone flow rate (ft^3/sec) (fixed in problem statement)
Q_w = water flow rate (ft^3/sec)

Recall also that mass flow rate is $\dot{m} = \rho A V$ and volumetric flow rate is $Q = AV$.

Other Considerations:
The velocity, V, must exceed that at which sedimentation and clogging would occur. The formula for grinding power is not valid for a particle sizes below 0.0005 ft (particle size after grinding). The concentration of limestone in the pipe must be less than that at which pipe blockage would occur. The pipe diameter should not exceed 6 inches, above which the initial cost for the pipeline would be excessive.

The following expressions will be used to build a model:

Power for Grinding
The power for grinding is given by,

$$P_g = 218\ W\left(\frac{1}{\sqrt{d}} - \frac{1}{\sqrt{a}}\right) \tag{1}$$

where P_g has units of ft-lb_f/sec; W is in lb_m/sec and d, a are in ft. The constant 218 is a conversion factor that also has units; we will assume the units are such as to give P_g in ft-lb_f/sec.

Power for Pumping
The friction factor for the slurry is estimated by

$$f = f_w\left[\frac{\rho_w}{\rho} + 150c\,\frac{\rho_w}{\rho}\left(\frac{gD(S-1)}{V^2\sqrt{C_d}}\right)^{1.5}\right] \tag{2}$$

where

f_w = friction factor of water
g = acceleration due to gravity = 32.17 ft/sec^2
ρ_w = density of water = 62.4 lb_m/ft^3
C_d = average drag coefficient of the particles
S = specific gravity of the limestone (density of limestone divided by the density of water)

The friction factor of water is given by,

$$f_w = \frac{0.3164}{R_w^{0.25}} \qquad \text{if} \qquad R_w \le 10^5$$

$$f_w = 0.0032 + 0.221 R_w^{-0.237} \qquad R_w \ge 10^5$$

where

$$R_w = \frac{\rho_w V D}{\mu}$$

where

ρ_w = density of water
V = Velocity
D = diameter of pipe
μ = viscosity of water = 7.392×10^{-4} $lb_m/(ft\text{-}sec)$

Equation (2) above contains C_d, the drag coefficient of the particles. The drag coefficient combines with Reynold's number for the particle as a dimensionless quantity which depends on particle diameter:

$$C_d R_p^2 = 4 g \rho_w d^3 \left(\frac{\gamma - \rho_w}{3\mu^2} \right) \tag{3}$$

where

γ = limestone density = 168.5 lb_m/ft^3
μ = viscosity of water ($lb_m/(ft\text{-}sec)$)
R_p = Reynolds number for the particle at terminal settling velocity (not calculated)

There is an empirical relationship between C_d and $C_dR_p^2$ defined by the following table:

C_d	240	120	80	49.5	36.5	26.5	14.6	10.4
$C_d R_p^2$	2.4	4.8	7.2	12.4	17.9	26.5	58.4	93.7

C_d	6.9	5.3	4.1	2.55	2.0	1.5	1.27	1.07
$C_d R_p^2$	173	260	410	1020	1800	3750	6230	10,700

C_d	0.77	0.65	0.55	0.5	0.46	0.42	0.40	0.385
$C_d R_p^2$	30,800	58,500	138,000	245,000	460,000	1,680,000	3,600,000	9,600,000

The slurry density can be expressed as,

$$\rho = \rho_w + c(\gamma - \rho_w) \qquad \text{(units are } lb_m/ft^3\text{)} \tag{4}$$

The pressure drop in the pipe due to friction is given by,

$$\Delta p = \frac{f \rho L V^2}{D 2 g_c} \qquad \text{(units are } lb_f/ft^2\text{)} \tag{5}$$

where

f = friction factor for the slurry, given by (2) above
ρ = density of slurry, given by (4) above
L = length of pipeline
V = velocity of slurry
D = diameter of pipe
g_c = conversion between lb_f and $lb_m = 32.17 \dfrac{lb_m\text{-ft}}{lb_f\text{-sec}^2}$

Finally, the friction power loss is given by

$$P_f = \Delta p \, Q \quad \text{(units in ft-lb}_f\text{/sec)} \tag{6}$$

where

Δp = is pressure drop from Eq. (5)
Q = slurry flow rate (ft^3/sec)

Sedimentation

Sedimentation and clogging may occur if the velocity V is less than a critical velocity V_c. This velocity is estimated by the equation,

$$V_c = \left(\frac{40 gc (S-1) D}{\sqrt{C_d}} \right)^{0.5}$$

As a factor of safety, we would like the slurry velocity to be 10% higher than V_c.

Pipe Blockage

Blockage can occur due to simply too high a fraction of solids in the slurry. If the particles were idealized into spheres of equal size and jammed together, the percent of unoccupied space, or voidage would be 26% or a concentration of 0.74. For irregular particles it is estimated that a safe concentration should be less than 0.4.

Cost

Cost should include the capital cost of the grinder and pump and the energy cost to operate the grinder and pump. As a first estimate, we will assume the cost of the grinder is \$300 per horsepower and the pump is \$200 per horsepower. Assume the plant will operate 8 hours per day, 300 days per year, with a plant life of seven years. The interest rate is 7%. For reasons we won't elaborate on, cost of energy is \$0.07 per hp-hr for the grinder and \$0.05 per hp-hour for the pump. Estimate the total cost using a net present value method.

Limits of Model

The model for grinding power is not valid for an average particle diameter below 0.0005 feet.

Comments

Note that this model requires a curve fit of some data to relate $C_d \,\mathrm{Re}^2$ to C_d. Make sure the curve fit is good enough that you do not introduce significant error in the problem (this implies the goodness-of-fit is high). There are several ways this might be approached, including curve fitting the log of the data.

As a "ballpark" value, the total power should be approximately 400 hp (±100).
Check your units!

Turn in a report with the following sections:

1) Title Page with Summary. The Summary should be short (less than 50 words), and give the values of the variables and objective at the optimum.

2) Procedure:
   a. Show the equation sequence in manner similar to that given for the Heat Pump example of Section 2.8.5 of the notes. This can be done by hand.
   b. Provide a table showing the mapping between analysis space and design space.
   c. Explain anything you did to validate the model. Discuss briefly how you handled the curve fit and how accurate your fit was.

3) Results and Discussion of Results:
   a. Provide a table showing the optimum values of variables and functions, with binding constraints and/or variables at bounds highlighted. Note, in addition to cost, include the power required by the pump and grinder in hp.
   b. Briefly discuss the optimum and the design space around the optimum. Do you feel this is a global optimum? Provide support for your conclusion.
   c. Include at least one contour plot with the feasible region and optimum marked. What did you learn from this plot?
   d. Include any other observations you feel are pertinent. These may relate to the model, the results, the optimization process, the nature of the optimum, etc. This section should not be longer than a paragraph.

4) Appendix:
   a. Listing of `MATLAB or other programs`
   b. Copy of the assignment

Please turn in as a pdf on Learning Suite. Note: Include requested items (such as graphs or tables) in their respective sections as given above, and not in the Appendix. Any output from MATLAB should be integrated into the report with captions, explanatory comments, etc.