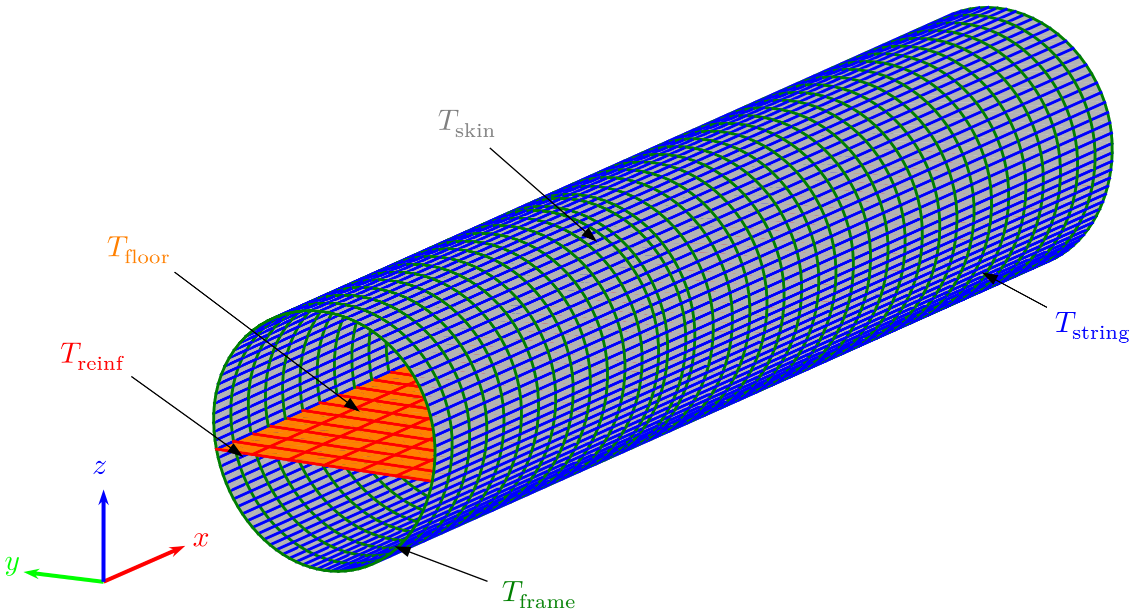
Task 2

In this task, a structural analysis of the fuselage of an Airbus A320 will be performed. To do so, the geometry will be split into different parts:



Modelled as **plate** elements:

- The outer **skin** elements: T_{skin} (plates of thickness $h_s = 4$ mm).
- The cabin **floor** elements: T_{floor} (plates of thickness $h_s = 8$ mm).

Modelled as **beams**:

- The **frames** elements: T_{frame} (beams with section A).
- The **stringers** elements: T_{string} (beams with section B).
- The cabin floor **reinforcements** elements: T_{reinf} (beams with section C).

Section A Section B Section C

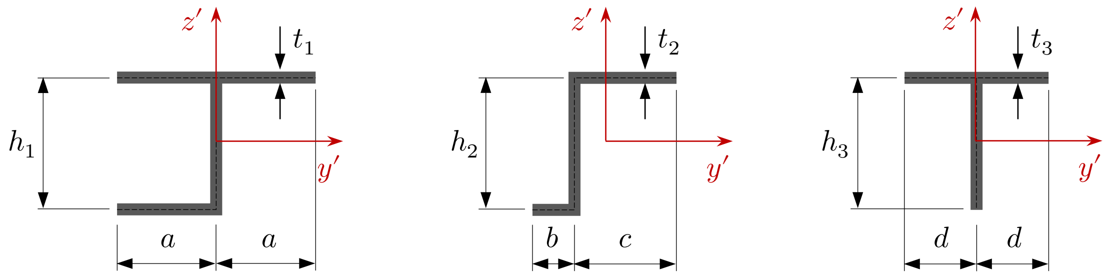


Table 1. Beams' section parameters

Parameter	Dim. (mm)	Parameter	Dim. (mm)	Parameter	Dim. (mm)
h_1	74	h_2	24	h_3	48
t_1	2	t_2	2.5	t_3	2.2
a	20	b	18	d	22
		c	20		

Table 2. Material properties

Component	Density (kg/m^3)	Young's Modulus (GPa)	Poisson's ratio
Frames Stringers	2650	70.1	0.30
Reinforcements Skin Floor	2910	76.3	0.35

Since the structure and the loading conditions that will be considered are <u>symmetric with respect to the XZ-plane</u>, the analysis will be performed only on half of the structure. For this reason, the defined sets of elements will contain <u>only</u> the elements on the right side of the symmetry plane.

The structural response of the fuselage to different loading conditions will be studied:

(A) Structural weight:

Weight will be considered as a distributed load:

$$\mathbf{w}_{\text{beams},e} = \rho_{\text{eff},e} A_e \mathbf{g}, \quad \forall e \in T_{\text{beams}}$$

$$\mathbf{w}_{\text{plates},e} = \rho_{\text{eff},e} h_e \mathbf{g}, \quad \forall e \in T_{\text{plates}}$$

where the effective density for each element, $\rho_{\text{eff},e}$, will be considered as the corresponding element's material density, ρ_e, plus an additional term $\hat{\rho}$ accounting for the remaining structural mass of the fuselage:

$$\rho_{\text{eff},e} = \rho_e + \hat{\rho},$$

$$\hat{\rho} = \frac{M_s - M_{\text{beams}} - M_{\text{plates}}}{V_{\text{beams}} + V_{\text{plates}}},$$

where $M_s = 22900$ kg and

$$V_{\text{beams}} = 2 \sum_{e \in T_{\text{beams}}} A_e \ell_e \, ; \quad V_{\text{plates}} = 2 \sum_{e \in T_{\text{plates}}} 4 a_e b_e h_e \, ;$$

$$M_{\text{beams}} = 2 \sum_{e \in T_{\text{beams}}} \rho_e A_e \ell_e \, ; \quad M_{\text{plates}} = 2 \sum_{e \in T_{\text{plates}}} 4 \rho_e a_e b_e h_e \, .$$

<u>Note</u>: Since the mesh contains only half the elements of the complete fuselage (due to symmetry with respect to the XZ-plane), to obtain the total mass and volume, a factor of 2 is considered.

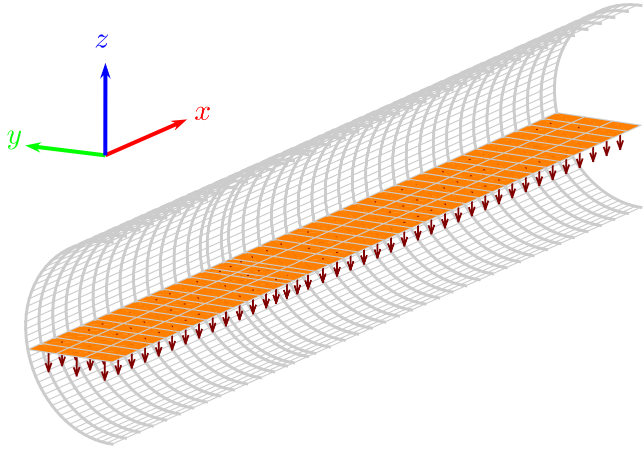
(B) Weight of the cabin passengers:

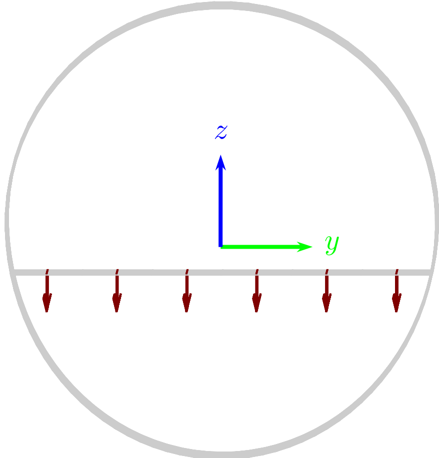
The cabin passengers' weight will be considered as a distributed load on the floor plates:

$$\mathbf{w}_{p,e} = \frac{M_p}{S_{\text{floor}}} \mathbf{g}, \quad \forall e \in T_{\text{floor}},$$

where $M_p = 13500$ kg and

$$S_{\text{floor}} = 2 \sum_{e \in T_{\text{floor}}} 4 a_e b_e .$$





(C) Loads transmitted by the wing:

The front and rear spars of the A320 aircraft are attached to the selected frames' elements, T_{front} and T_{rear}, respectively. Assuming the wing's system reaction forces and moments at the joint are given by:

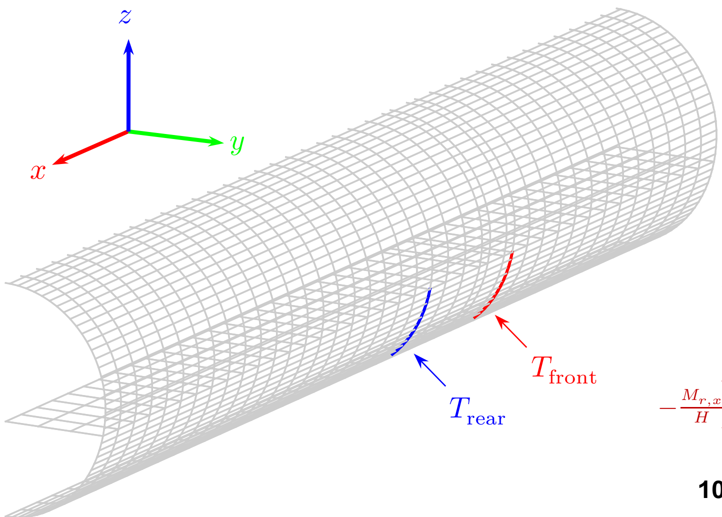
$$\mathbf{F}_f = \begin{bmatrix} 78.94 \\ 72.22 \\ -71.07 \end{bmatrix} \text{kN}; \quad \mathbf{F}_r = \begin{bmatrix} 17.18 \\ -72.22 \\ -25.47 \end{bmatrix} \text{kN},$$

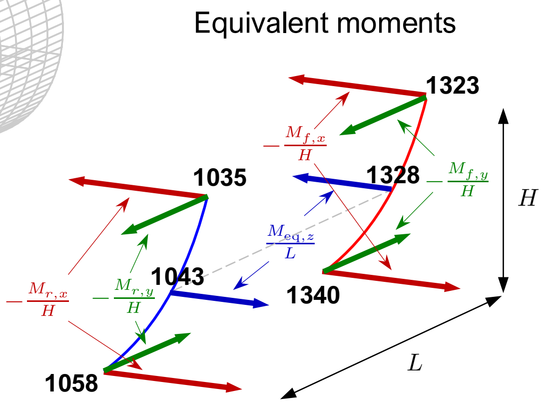
$$\mathbf{M}_f = \begin{bmatrix} -349.08 \\ 194.87 \\ -86.29 \end{bmatrix} \text{kN·m}; \quad \mathbf{M}_r = \begin{bmatrix} -203.02 \\ 83.85 \\ -91.93 \end{bmatrix} \text{kN·m}.$$

The force applied by the wing to the fuselage will be considered as an equivalent uniformly distributed load over each frame segment:

$$\mathbf{q}_{\text{front},e} = -\frac{\mathbf{F}_f}{\ell_{\text{front}}}, \quad \forall e \in T_{\text{front}}; \quad \ell_{\text{front}} = \sum_{e \in T_{\text{front}}} \ell_e,$$

$$\mathbf{q}_{\text{rear},e} = -\frac{\mathbf{F}_r}{\ell_{\text{rear}}}, \quad \forall e \in T_{\text{rear}}; \quad \ell_{\text{rear}} = \sum_{e \in T_{\text{rear}}} \ell_e,$$

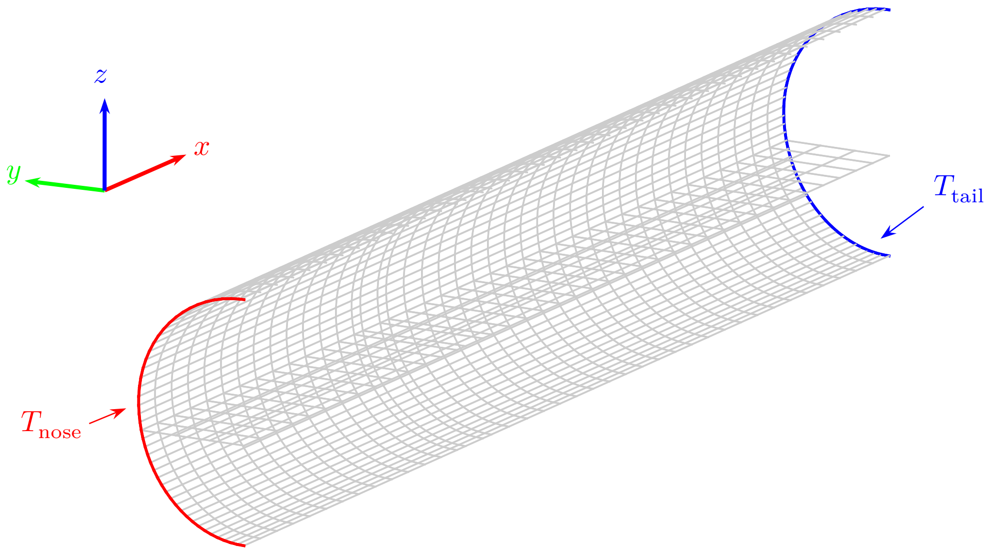




The moment applied by the wing to the fuselage will be accounted for by an equivalent system of point loads, as depicted in the figure above, where $H = z_{1323} - z_{1340}$, and $L = x_{1043} - x_{1328}$. The equivalent moment around z-axis is given by:

$$M_{eq,z} = -(M_{f,z} + M_{r,z}) - (M_{f,y} + M_{r,y})\frac{W}{H}; \quad W = y_{1323} - y_{1340}$$

(D) Loads transmitted by the nose and the tail cone:



The drag of the aircraft's nose (fuselage cross section) will be applied on the **first frame** elements, T_{nose}, as a uniformly distributed load:

$$\mathbf{q}_{\text{nose},e} = \frac{1}{\ell_{\text{nose}}}\begin{bmatrix} D_{\text{nose}} \\ 0 \\ 0 \end{bmatrix}, \qquad \forall e \in T_{\text{nose}}$$

where

$$\ell_{\text{nose}} = 2\sum_{e \in T_{\text{nose}}} \ell_e,$$

$$D_{\text{nose}} = \frac{1}{2}\rho V^2 S C_D,$$

with ρ = 1.225 kg/m^3, V = 210 m/s, C_D = 0.39 and S = 12.84 m^2.

The lift and drag caused by the aircraft's tail will be applied on the **last frame** elements, T_{tail}, as a uniformly distributed load:

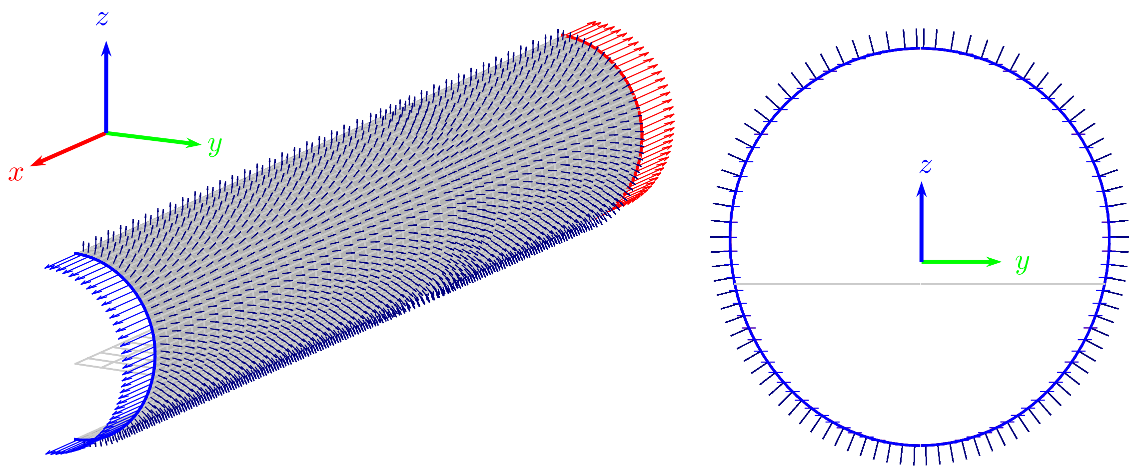
$$\mathbf{q}_{\text{tail},e} = \frac{1}{\ell_{\text{tail}}}\begin{bmatrix} D_{\text{tail}} \\ 0 \\ L_{\text{tail}} \end{bmatrix}, \qquad \forall e \in T_{\text{tail}}$$

where L_{tail} = 164.01 kN and D_{tail} = 17.58 kN and

$$\ell_{\text{tail}} = 2\sum_{e \in T_{\text{tail}}} \ell_e.$$

Note: As a first approximation, it is assumed that the resulting loads of the nose and tail sections are directly applied on the first and last frames of the fuselage section being studied, thus neglecting their equivalent moment around y-axis contribution.

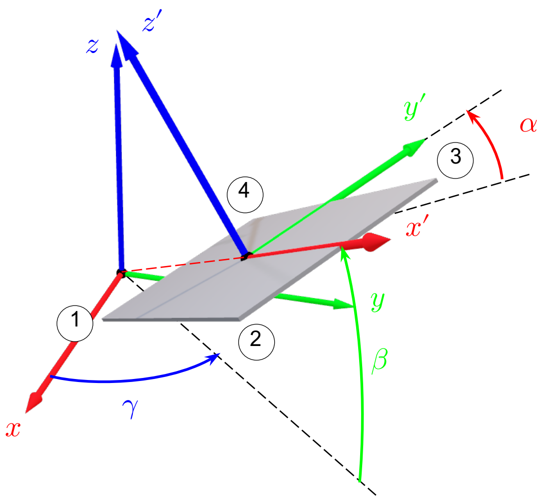
(E) Cabin pressure:



The cabin pressurization is considered as a differential pressure load applied normal to the skin elements:

$$\mathbf{p}_{\text{skin},e} = (p_{in} - p_{out})\mathbf{n}_e, \quad \forall e \in T_{\text{skin}}$$

where $p_{in} = 78191.21$ Pa, $p_{out} = 22632.06$ Pa and $\mathbf{n}_e$ is the unit normal to the element surface:



$$\mathbf{n}_e = \begin{bmatrix} -\cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ -\cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma \\ \cos\alpha\cos\beta \end{bmatrix}$$

Additionally, the equivalent axial stress produced by the same differential pressure on the x-direction (due to the nose and tail cabin sections) will be considered as a distributed load on the first and last frames elements:

$$\mathbf{p}_{\text{nose},e} = \frac{1}{\ell_{\text{nose}}} \begin{bmatrix} -(p_{in} - p_{out})S \\ 0 \\ 0 \end{bmatrix}, \quad \forall e \in T_{\text{nose}}$$

$$\mathbf{p}_{\text{tail},e} = \frac{1}{\ell_{\text{tail}}} \begin{bmatrix} (p_{in} - p_{out})S \\ 0 \\ 0 \end{bmatrix}, \quad \forall e \in T_{\text{tail}}$$

where $S = 12.84$ m^2.

Instructions

1. The input file **`input_fuselage.m`** contains all the mesh information, including:
   - Nodal coordinates: **`xnodes`** (in meters)
   - Beam element connectivities: **`Tbeams`**
   - Plate element connectivities: **`Tplates`**
   - Material connectivities for each beam and plates element: **`Tmat_beams`** ((1) frames, (2) stringers and (3) reinforcements), **`Tmat_plates`** ((1) skin and (2) floor).
   - Rotation angles for each beam and plate element: **`dat_beams`**, **`dat_plates`** (first column is α, second column is β and third column is γ, in radians).
   - List of frame elements: **`Tframe`**
   - List of stringer elements: **`Tstring`**
   - List of floor reinforcement elements: **`Treinf`**
   - List of skin elements: **`Tskin`**
   - List of floor elements: **`Tfloor`**
   - List of front spar attachment elements: **`Tfront`**
   - List of rear spar attachment elements: **`Trear`**
   - List of first frame elements: **`Tnose`**
   - List of last frame elements: **`Ttail`**
   - List of symmetry nodes (nodes on XZ-plane): **`Tsym`**

2. Develop a function to compute the geometric parameters of the beam elements: (a) the associated length ℓ_e, (b) the associated section area A_e, (c) the associated inertia $I_{y,e}$ and $I_{z,e}$, (d) the associated torsional constant J_e. Hint: Use the guide **ALG_BEAM_3D** for assistance in obtaining these parameters.

3. Compute the stiffness matrix $\mathbf{K}_e$ for each element and assembly those into the global stiffness matrix. Hint: You can use the provided local stiffness matrices expressions already implemented in **`local_K_matrices.m`** file and the guides **ALG_BEAM_3D** and **ALG_PLATES_3D** for assistance. Note that each node will have 6 degrees of freedom corresponding to:
   a. Displacement in the x-direction: u_x
   b. Displacement in the y-direction: u_y
   c. Displacement in the z-direction: u_z
   d. Rotation angle around the x-axis: θ_x
   e. Rotation angle around the y-axis: θ_y
   f. Rotation angle around the z-axis: θ_z

   Note: Since this is a problem with a considerable number of degrees of freedom, it is **highly recommended** to follow the steps in the guide **ALG_ASSEMBLY** to perform the stiffness matrix assembly process much more efficient.

4. Obtain the prescribed degrees of freedom vector by applying the symmetry condition on all nodes lying on the XZ-plane (list of nodes provided in **`Tsym`**). Hint: To apply symmetry conditions with respect to XZ-plane, one must prescribe the displacement in y-direction and the rotations around x and z-axes. Additionally, the displacement in x and z-directions of some node will need to be prescribed in order to avoid rigid body modes.

5. For each loading condition (separately):
    a. Compute the element force vector accounting for all the distributed loads on beam elements (if any).
    b. Compute the element force vector accounting for all the distributed loads on plate elements (if any).
    c. Assembly the element force vectors into the global force vector.
    d. Add additional point loads in the corresponding degrees of freedom of the global vector (if any).
    e. Solve the system for the free degrees of freedom and obtain the global displacement and rotations vector, $\mathbf{u}^{(\mathrm{X})}$, for the corresponding (X) loading conditions case.
    f. Obtain the local displacements and rotations for each element (beams and plates), $\mathbf{u}'^{(\mathrm{X})}_{int,e}$.
    g. Obtain the internal forces distribution for each beam element: (a) axial force for each element $N^{(\mathrm{X})}$, (b) shear forces for each element $Q_y^{(\mathrm{X})}, Q_z^{(\mathrm{X})}$, (c) bending moments for each element $M_y^{(\mathrm{X})}, M_z^{(\mathrm{X})}$ and (d) torsion moment for each element $T^{(\mathrm{X})}$. Hint: Use the guide **ALG_BEAM_3D** for assistance.

6. Use the **`plotFuselage`** function to postprocess the results. Hint: Look inside the script for the **`plotFuselage`** function to see the structure of the matrices expected as inputs.

A report with an analysis of this structure must be submitted including:

a. Plot of the deformed fuselage accounting for the sum of all loading systems combined at a relevant scale, showing the Von Mises stress distribution.
b. Plot of the deformed structure accounting only for the cabin pressure loads at a relevant scale, showing the hydrostatic pressure distribution.
c. Plots of the deformed structure accounting for each other loading contribution separately at a relevant scale, showing the Von Mises stress distribution.
d. Color plot of the internal distribution of the following parameters for the frames and the stringers:
    i. Axial force
    ii. Shear force in the local y' direction
    iii. Shear force in the local z' direction
    iv. Torsional moment
    v. Bending moment in the local y' direction
    vi. Bending moment in the local z' direction
e. Assessment of the effect of adding the frames, stringers and floor reinforcement beams in the structure (to do so, run the simulation considering only plate element stiffness matrices, and compare the results obtained with the previous simulation considering also the beam elements).
f. Any additional discussion/plot of the obtained results.