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MSc in Aerospace Engineering

—– Computational Engineering —–
Computational Structural Analysis

TASK 01: Analysis of an internal wing structure of a typical commercial aircraft composed of a front and rear spars and ribs.

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1 Introduction

The aim of this report is the structural study of a wing from a typical commercial aircraft. For that purpose, a Matlab code is computed in order to obtain the displacement, rotations and internal forces and moments in the structure by using the input data given.

First of all, a problem description is carried out in order to show the properties of the wing. Then, the solving methodology used in the Matlab code is explained. Finally, the results obtained are shown and therefore some conclusions are extracted.

2 Problem description

First of all, the statement of the problem is defined in order to introduce the important aspects that have been considered for the approach. The first step is defining the physical characteristics of the problem, then the numerical description is done and finally the considerations done for the approach are explained.

2.1 Physical description

The problem consists on the internal structure of a wing from a typical commercial aircraft. The wing is composed of front and rear spars and ribs. On Figure 2.1 the structural representation of the wing is shown.

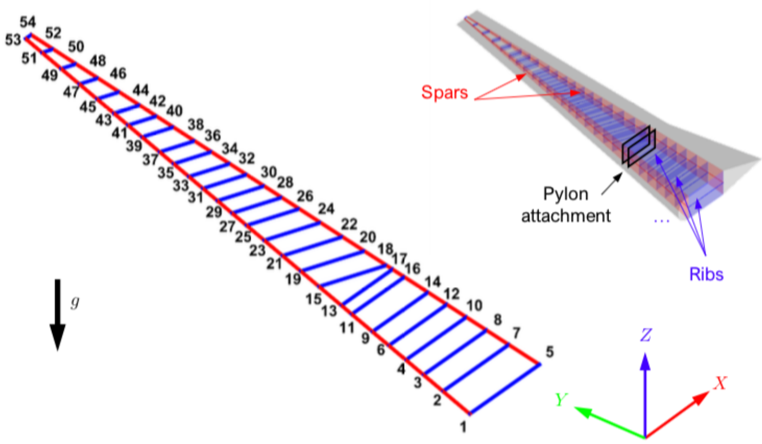

Figure 2.1: Wing structure

Regarding to the spars and ribs, they are modelled with "I" cross-section shaped beams, as it can be seen on the following figure.

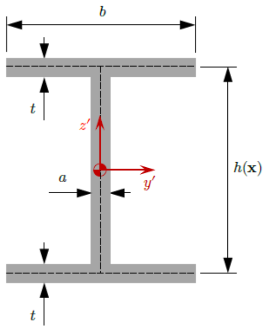

Figure 2.2: Cross-section for the spars and ribs.

The physical properties of the mentioned elements are defined on the following table:

	Spars	**Ribs**
E (GPa)	72.5	58.1
ν	0.3	0.27
$\rho (kg/m^3)$	2320	1850
a (mm)	11	5
b (mm)	87	12
t (mm)	5	3

Table 2.1: Physical properties for spars and ribs.

2.2 Numerical description

To define the wing, the following variables are defined:

- **Nodal coordinates (x).** The wing is defined in 3D with 54 nodes.
- **Nodal connectivities (Tnod).** This matrix (2x79) defines the two nodes which conform each element, being 79 the total amount of elements. Each element, belongs to a different part of the structure:
  - Ribs elements: 1 to 27.
  - Front spar elements: 28 to 53.
  - Rear spar elements: 54 to 79.
- **Material connectivities (Tmat).** Where each element is associated with a 1 if it is a spar or 2 in case it is a rib.
- **Geometrical parameters (dat).** Each column contains different physical information:
  - Column 1: height of the beam element h in meters.

– Column 2 to 4: Rotation angles (α, β, γ) for the beam element from local to global system in radians.

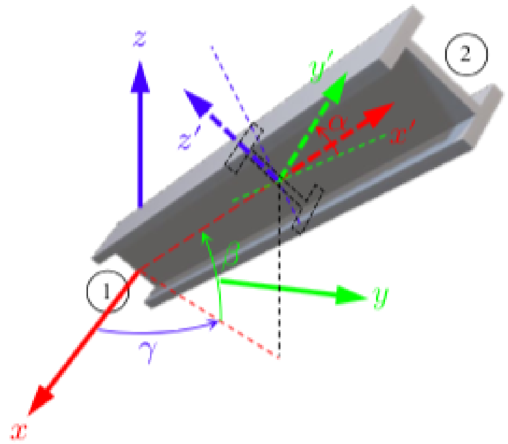

Figure 2.3: Local and global definition for angles.

2.3 Considerations

To define the problem, the following loading conditions are considered:

- The wing is attached to the aircraft's fuselage at nodes 1 and 5.
- The weight of the wing strcuture is estimated with an effective density accounting for the total mass of the wing:

$$\rho_{eff,e} = \rho_e + \hat{\rho} \tag{2.1}$$

where ρ_e correspond for the material density of the rib or spar, and $\hat{\rho}$ is a pseudo-density accounting for the mass of the rest of the wing elements:

$$\hat{\rho} = \frac{M_w - M_s - M_r}{V_s + V_r} \tag{2.2}$$

where $M_w = 1450kg$ is the mass of the whole wing, M_s the mass of the spars and M_r the ribs one. Being V_r and V_S their respective volumes.

- The thrust (T_e) and weight (W_e) of the engine is transmitted through the pylon as a distributed load into ribs 11-16 and 13-17.

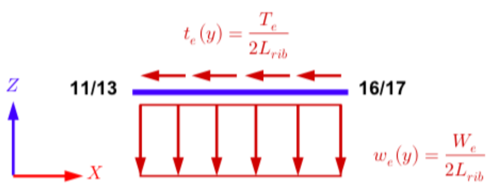

Figure 2.4: Thrust and weight distribution.

T_e must be such to compensate the total drag of the wing. The mass of the engine is $M_e = 980kg$.

- The aerodynamic loads are distributed in the front and rear spars as follows:

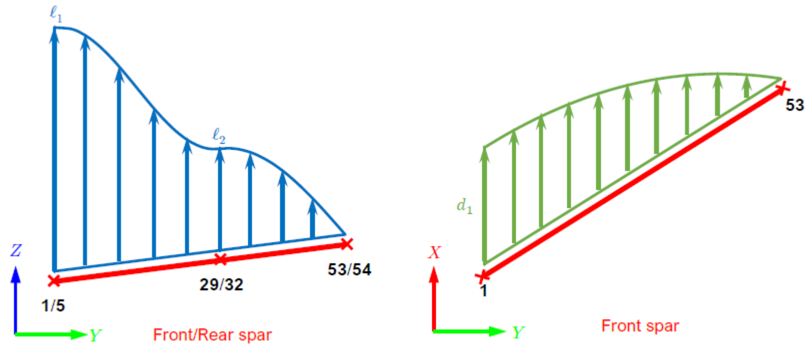

Figure 2.5: Aerodynamic loads distribution.

And are defined with the following equations:

$$l(y) = \begin{cases} \left(l_1 + l_2 + (l_1 - l_2)cost\left[\pi\left(\frac{y-y_1}{y_2-y_1}\right)\right]\right) & y_1 < yy_2 \\ l_2 cos\left[\frac{\pi}{2}\left(\frac{y-y_2}{y_3-y_2}\right)\right] & y_2 < y < y_3 \end{cases} \tag{2.3}$$

$$d(y) = d_1\left[1 - \left(\frac{y - y_1}{y_3 - y_1}\right)^2\right] \tag{2.4}$$

The values of the parameters are shown in the following table:

	Front spar	**Rear spar**
$l_1(kN/m)$	15	4.5
$l_2(kN/m)$	4.5	4.5
$d_1(kN/m)$	1	-

Table 2.2: Equation's parameters definition for front and rear spar.

3 Solving methodology

The code is developed in Matlab. It consists on a main script which can be divided in the following parts: data, pre-process, solution and plots. Each part calls different functions to calculate the necessary variables. In the following subsections, the different parts of the structure of the main script are explained.

3.1 Data

On data, all physical and numerical data mentioned in the problem description is defined. The function *input_wing* is run in order to define: x, Tnode, Tmat and dat, which are given values.

3.2 Pre-process

On the pre-process, the degrees of freedom connectivities matrix is computed. And some input parameters of the problem are previously entered. Also, the lift and drag components for each element is computed.

3.3 Solution

There is a function called *solution*, which calls all the following functions:

- *density_calculus.* This function calculates the different nominal parameters, such as, the total mass, total volume, lift and drag among others.
- *element.* This function computes the rotation matrix, the element's stiffness matrix, the forces matrix and the thrust.
- *assembly.* This function does the assembly of the total stifness and forces matrix.
- *systemEqs.* This function solves the equations in order to obtain the displacements.

The internal forces are computed in the main code.

3.4 Outputs

Once the results are obtained the given functions (*plotWing, plotBeams3D_def* and *plotBeams3D*) are used to do the different plots.

Also, the most critical beams in terms of axial force, shear force and bending moments are displayed.

In order to obtain the solution, the steps to follow and the definition and meaning of each matrix and term can be explained with more detail at [1].

4 Results

Once the code is computed, the results and plots obtained are presented and analysed in order to proof its veracity.

4.1 Nominal parameters

As it has been mentioned, with the function *density_ calculus* the nominal parameters are calculated. The results are shown in the following table.

Spar mass	535.90 kg
Ribs mass	250.66 kg
Total mass	786.56 kg
Lift	175467.63 N
Drag	11302.98 N
Thrust	-11302.98 N

Table 4.1: Nominal parameters

After calculating the spar and ribs mass, the total mass is computed by doing the sum of these two terms. Although the total mass of the wing given as an input data is 1550 kg, the obtained total mass can be considered correct. This is due to the fact that in the code only the mass related to spars and ribs have been computed. However, the whole wing is composed by some other elements which increase the total mass. For this reason, the total mass calculated is lower than the one given. Another fact worth mentioning is that the lift is higher than the total weight using the given mass.

On the other hand, the lift and drag have been computed using equations 2.3 and 2.4, respectively. As it is seen, the total thrust compensates the drag. For that purpose, equilibrium of forces in the longitudinal axis has been imposed. Regarding to the relationship among the lift and drag, it can be seen that the lift is one order of magnitude higher than the drag, as it could be expected.

4.2 Reactions at the wing joint with the fuselage

There are two nodes (1 and 5) at the wing joint with the fuselage, each one with six degrees of freedom. The reaction at each one of these DoF are available in the table below. As seen, the first 3 DoFs correspond to forces in x, y and z direction respectively, and DoFs 4, 5 and 6 to the corresponding moments in the same directions.

As expected, forces in x and y direction in both nodes have the same modulus as the structure is in equilibrium. However, for the other values these present different values especially for the moments.

The reason for this is because of the distribution of the lift that is placed along the front spars and rear spars with different values on each of them. One can observe that the forces in z-direction (direction where the lift acts) and moments in both nodes are different.

Node	Degree of Freedom	Value
1	1	$-3.86 \cdot 10^3$ N
1	2	$-7.93 \cdot 10^3$ N
1	3	$-1.0263 \cdot 10^5$ N
1	4	$-4.6137 \cdot 10^5$ Nm
1	5	$2.4723 \cdot 10^5$ Nm
1	6	$-0.0272 \cdot 10^5$ Nm
5	1	$3.86 \cdot 10^3$ N
5	2	$7.93 \cdot 10^3$ N
5	3	$-0.4900 \cdot 10^5$ N
5	4	$-3.4909 \cdot 10^5$ Nm
5	5	$1.2700 \cdot 10^5$ Nm
5	6	$-0.0299 \cdot 10^5$ Nm

4.3 Deformed structure

Using the function $plotBeams3D\_def$ the deformed wing (Figure 4.1) is obtained by using the polynomial displacements within each beam, the number of subdivisions was set up to 20.

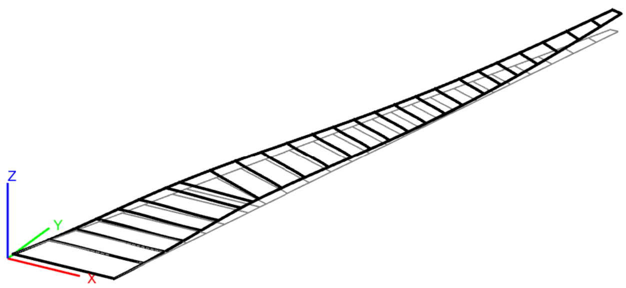

Figure 4.1: Deformed structure using $plotBeams3D\_def$

On the other hand, the solution obtained using plotWing function can be observed in Figure 4.2.

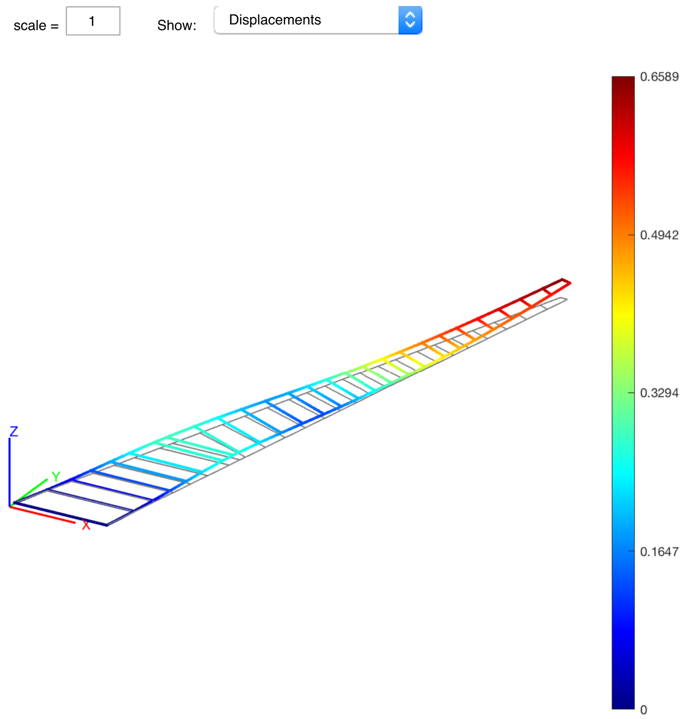

Figure 4.2: Deformed structure using plotWing function

As expected both functions give us the same shape in the figure, however plotWing gives us useful information by also providing the value of the displacement of each node with a colorbar on the right with value that corresponds to the colour.

The maximum displacement is obtained at the wingtip as we expect in a wing of an aircraft. It is also relevant to see that the elements where the engine is attached suffer from a horizontal displacement too.

4.4 Colour plot of the beams' internal distribution of different magnitudes

Using the plotWing function, the plot of the beams' internal distribution of the following parameters have been obtained: Axial force (Figure 4.3), Shear force in y' direction (Figure 4.4), Shear force in z' direction (Figure 4.5), Torsional moment (Figure 4.6), Bending moment in y' direction (Figure 4.7) and Bending moment in z' direction (Figure 4.8).

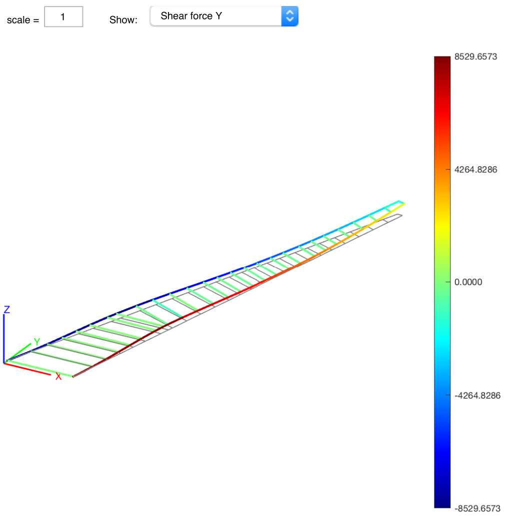

Figure 4.3: Axial forces in the wing structure

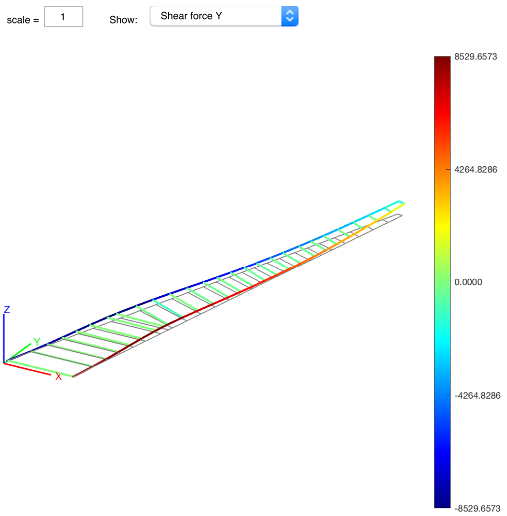

Figure 4.4: Shear force in y' direction in the wing structure

The highest values of forces are experienced in axial and shear in y' direction forces. Whereas for the moment, the values of bending moment in y' direction is the highest. The values obtained have a strong dependence on the cross-wise section used.

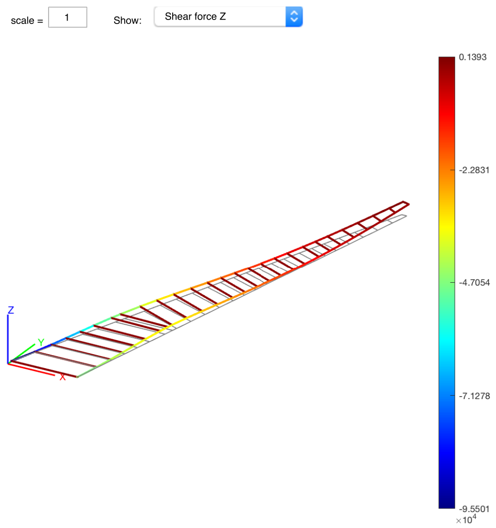

Figure 4.5: Shear force in z' direction in the wing structure

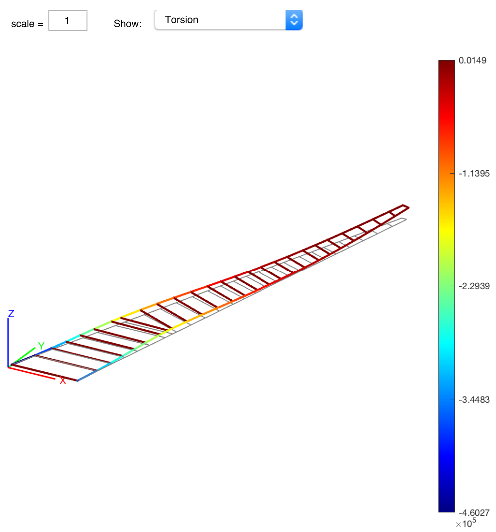

Figure 4.6: Torsional moment in the wing structure

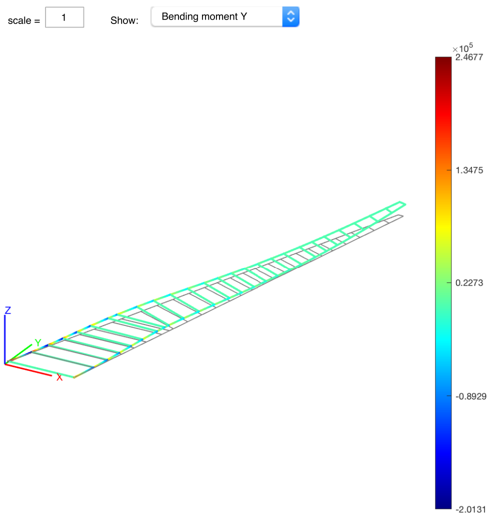

Figure 4.7: Bending moment in y' direction in the wing structure

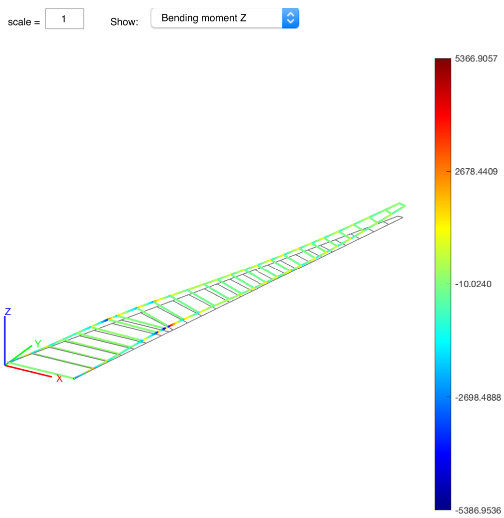

Figure 4.8: Bending moment in z' direction in the wing structure

4.5 Most critical beams

The highest values for forces (axial and shear) and moments (torsional and bending moment) of the solution for the wing structure were the following:

- Highest Axial Force (34) = 9669.8 N
- Highest Shear Force in y (35) = 8529.7 N
- Highest Shear Force in z (28) = 95501.5 N
- Highest Torsional Moment in x (28) = 375563.2 Nm
- Highest Bending Moment in y (51) = 831.2 Nm
- Highest Bending Moment in z (54) = 2107.3 Nm

For the axial force, the beam with the highest value is element 34 that corresponds to a front spar element (Figure 4.9). It is important to highlight that this element is not the element where the maximum displacement obtained.

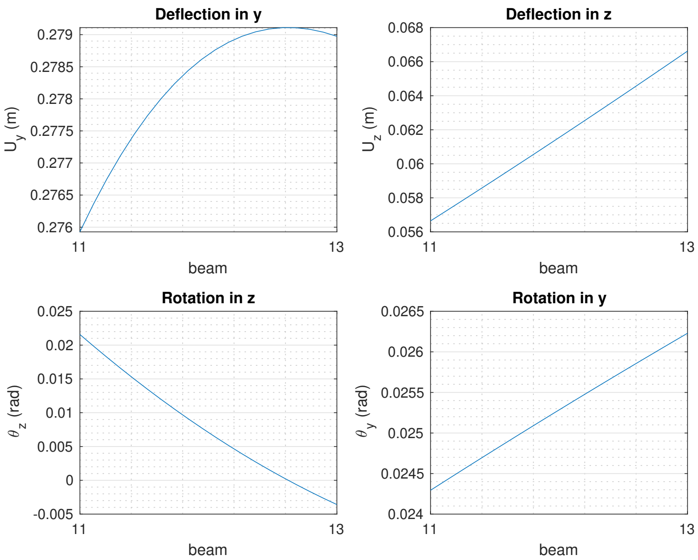

Figure 4.9: Beam with the highest axial force (element 34)

For the shear force, the value obtained for z-direction was 1 order of magnitude higher than the one obtained for the y-direction, the beam with the highest value is element 28 that corresponds to a front-spar element (Figure 4.10).

Additionally, the maximum value of the torsional moment in the x-direction is also achieved in element 28, which also experiences the highest Shear Force in z' direction.

As observed in the plots in this case, both the deflections in y and z directions can be considered lineal.

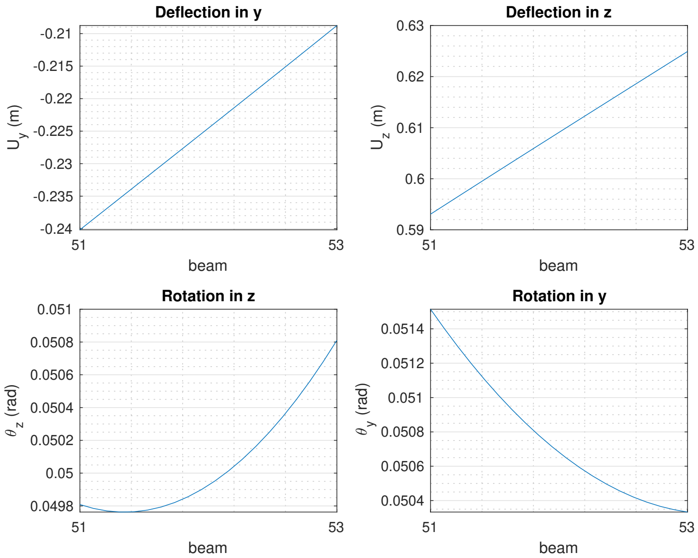

Figure 4.10: Displacements and rotations of the beam with the highest Shear Force (element 28)

Finally for the bending moment force, the value obtained for y-direction was smaller than the one obtained for the z-direction, the beam with the highest value is element 54 that corresponds to a rear spar element.

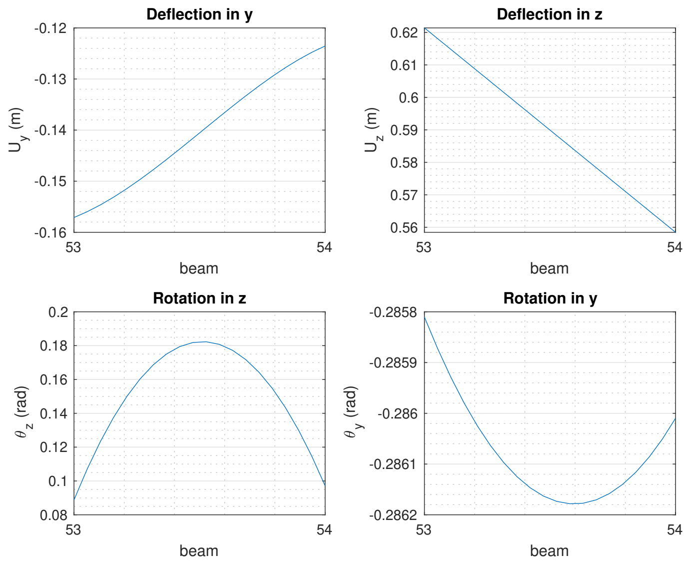

Figure 4.11: Displacements and rotations of the beam with the highest Bending Moment (element 54)

In this case, both the rotations seem to fit the second order perfectly. Whereas, deflection in z can be considered lineal.

For the 3 previous plots for the highest values for axial and shear forces and bending moment, the rotations have been calculated in [rad]. And in the case of the displacements, these have been computed in [m].

The plots for the deflections and rotations in y and z axes have been obtained with cubic polynomial deflection formulas in [1]. These formulas are already included in the function plotBeams3D.

5 Conclusions and discussion of the results

The resolution carried out in this problem has allowed to draw the following conclusions.

First of all, the main parameters of the structure (mass, density, section areas and inertias...) have been obtained. As stated previously, the total mass has been M = 786.56 kg which is lower than the total mass of the wing. Moreover, the lift is higher than the weight of the wing, and the total thrust produced is equal to the drag force.

Regarding the deformation of the structure, this has been plotted using two functions (*plotWing* and *plotBeams3D_def*), showing the maximum displacement at the wingtip.

Secondly, one has seen which are the most critical beams for the maximum values of forces (axial and shear) and moments (bending and torsional too). As observed the elements where the highest values of forces and moments are achieved do not correspond to the elements with the highest displacements.

It is important also to mention that the structure seems to be in equilibrium and that setting up nodes 1 and 5 as rigid were a good selection. Even though a further analysis regarding the moments should be done considering the other half of the wing.

A further study of the stress on each element could be done in order to see which are the most critical sections in the element taking into consideration the combination of axial and shear forces and bending and torsional moments.

6 Bibliography

References

[1] UPC Computational Engineering. “Element computations for 3D beams”. In: (). URL: https://atenea.upc.edu/mod/assign/view.php?id=2555093/ALG_BEAMS_3D.pdf.