Task 1

Consider the internal wing structure of a typical commercial aircraft composed of a front and rear spars and ribs (see Figure 1). Both spars and ribs are modelled as beam elements with the properties given in Figure 2.

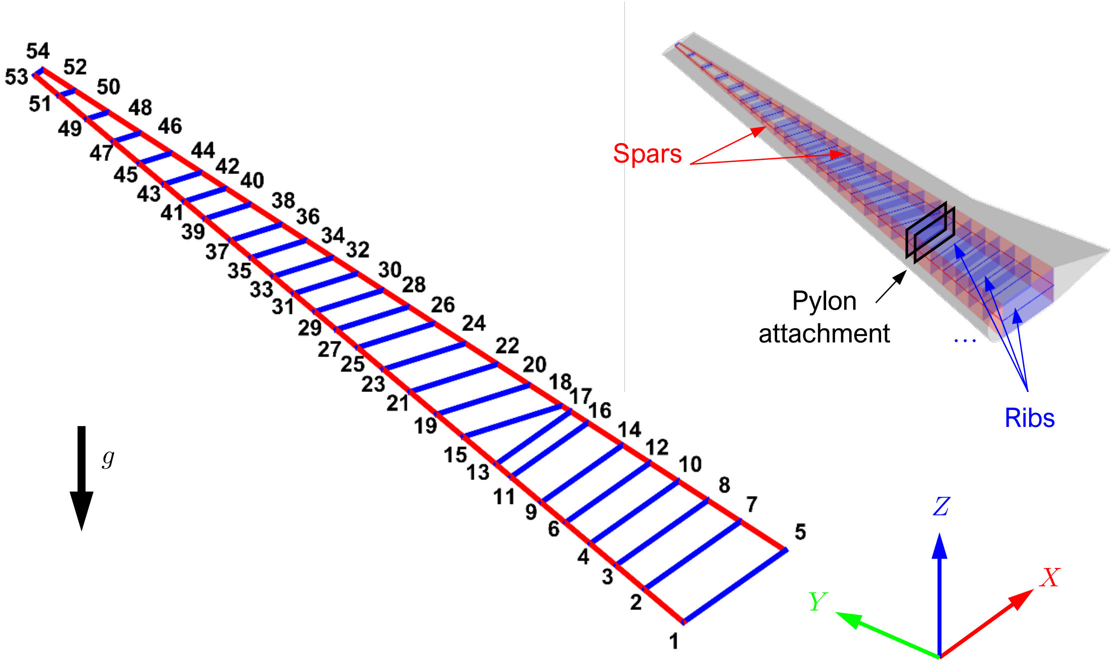
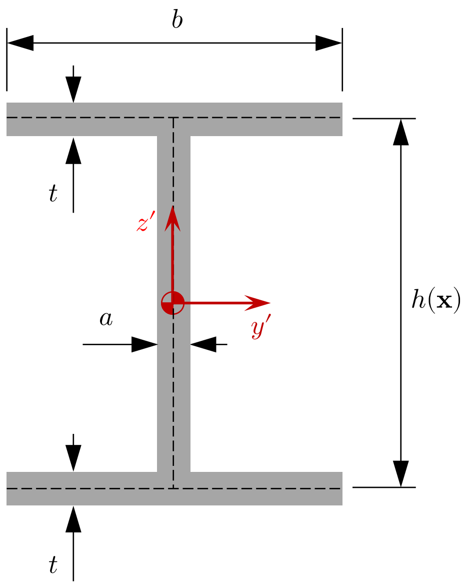


Figure 1. Structural representation of the wing.



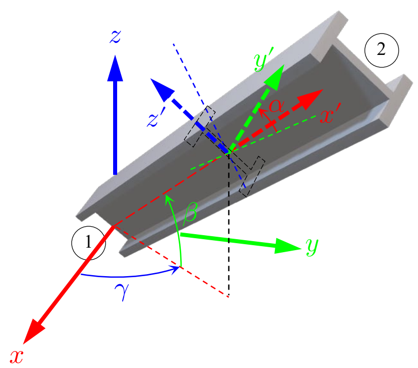
	SPARS	RIBS
E (GPa)	72.5	58.1
ν	0.3	0.27
ρ (kg/m^3)	2320	1850
a (mm)	11	5
b (mm)	87	12
t (mm)	5	3

* Note that the height h is variable

Figure 2. Cross section area of the ribs and spars.

Find the following data in the Matlab script file **`input_wing.m`**:

- Nodal coordinates **x**: the i-th row gives the coordinates $\{x, y, z\}$ of the i-th node in meters.
- Nodal connectivities **`Tnod`**: the i-th row gives the nodes conforming the i-th beam element.
    - Ribs elements: **1** to **27**.
    - Front spar elements: **28** to **53**.
    - Rear spar elements: **54** to **79**.
- Material connectivities **`Tmat`**: the i-th row gives the index of the material for the i-th element.
    - Material index 1 for spars.
    - Material index 2 for ribs.
- Geometrical parameters **`dat`**: the i-th row gives the following data for the i-th element.
    - Column 1: Height of the beam element h in meters.
    - Columns 2 to 4: Rotation angles $\{\alpha, \beta, \gamma\}$ for the beam element from local to global system in radians.



The following loading conditions will be considered:

- Assume that the wing is attached to the aircraft's fuselage at nodes **1** and **5**, where displacements and rotations will be prescribed.
- The weight of the wing structure will be estimated with an effective density accounting for the total mass of the wing:

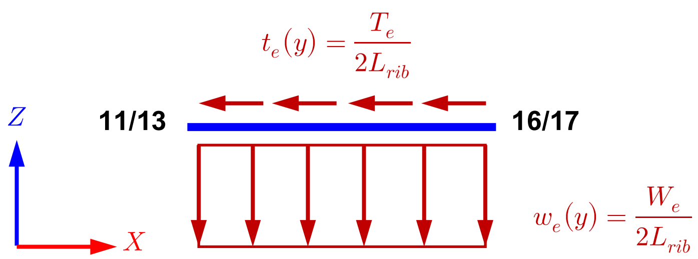
$$\rho_{\mathrm{eff},e} = \rho_e + \hat{\rho}$$

where ρ_e corresponds to the material density of the rib/spar, and $\hat{\rho}$ is a pseudo-density accounting for the mass of the rest of the wing elements, that can be estimated as

$$\hat{\rho} = \frac{M_w - M_s - M_r}{V_s + V_r},$$

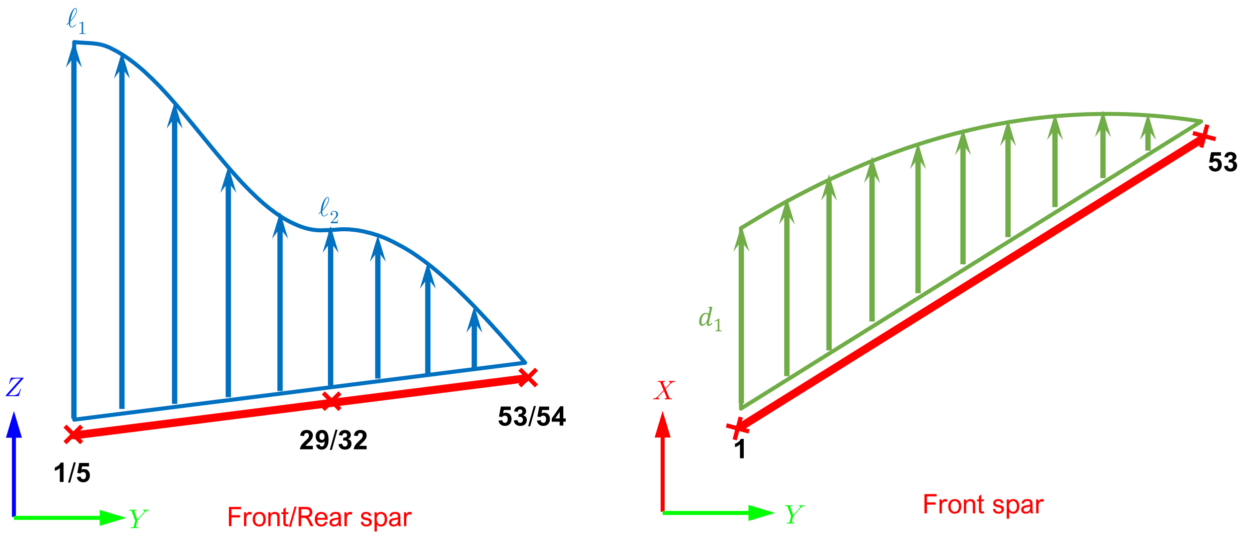
with $M_w = 1450$ kg being the mass of the whole wing, M_s and M_r the mass of the spars and ribs, respectively, while V_s and V_r refer to their corresponding total volume.

- The thrust T_e and weight W_e of the engine is transmitted through the pylon as a distributed load into ribs **11-16** (element **7**) and **13-17** (element **8**). Therefore, half the forces are then applied at each rib as distributed load (per unit length).



The value of T_e must be such to compensate the total drag on the wing. The mass of the engine is $M_e =$ 980 kg.

- The aerodynamic loads will be distributed in the front and rear spars as follows:



$$\ell(y) = \begin{cases} \frac{1}{2}\left(\ell_1 + \ell_2 + (\ell_1 - \ell_2)\cos\left[\pi\left(\frac{y-y_1}{y_2-y_1}\right)\right]\right) & y_1 < y < y_2 \\ \ell_2 \cos\left[\frac{\pi}{2}\left(\frac{y-y_2}{y_3-y_2}\right)\right] & y_2 < y < y_3 \end{cases}$$

$$d(y) = d_1\left[1 - \left(\frac{y-y_1}{y_3-y_1}\right)^2\right]$$

	FRONT SPAR	**REAR SPAR**
ℓ_1 (kN/m)	15	4.5
ℓ_2 (kN/m)	4.5	4.5
d_1 (kN/m)	1	-

where the lift is applied both on the front and rear spar in the z-direction, while the drag load is only applied on the front spar in the x-direction, as it is observed in the previous figure.

Instructions

1. Develop a function to compute the geometric parameters of the beam element: (a) the associated length L_e, (b) the associated section area A_e, (c) the associated inertia $I_{y,e}$ and $I_{z,e}$, (d) the associated torsional constant J_e. Hint: Use the guide **ALG_BEAM_3D** for assistance in obtaining these parameters. Note that the function can be generalized so it can be applied to any thin-walled open section that can be split into several rectangular segments if the section parameters in terms of each rectangular segment (i.e., the corresponding centroid coordinates y'_i, z'_i and its side lengths $\Delta y'_i, \Delta z'_i$) are given as inputs.
2. Compute each beam's volume and mass.
3. Obtain V_r, V_s, M_r and M_s as the sum of the corresponding elements volumes and masses and use them to compute the pseudo-density $\hat{\rho}$. Then, compute the effective density $\rho_{\text{eff},e}$ for each element.

4. Compute the total thrust T_e that the engines must produce to compensate the drag (as a force).

5. Compute the stiffness matrix $\mathbf{K}_e$ for each element and assembly those into the global stiffness matrix. Hint: Use the guide **ALG_BEAM_3D** for assistance. Note that each node will have 6 degrees of freedom corresponding to:
   (1) Displacement in the x-direction: u_x
   (2) Displacement in the y-direction: u_y
   (3) Displacement in the z-direction: u_z
   (4) Rotation angle around the x-axis: θ_x
   (5) Rotation angle around the y-axis: θ_y
   (6) Rotation angle around the z-axis: θ_z

6. For all the distributed loads (i.e. (a) the weight of each beam $\mathbf{w}_e = \rho_{\text{eff},e} A_e \mathbf{g}$, (b) the lift and drag in the spars' elements, ℓ_e and d_e, and (c) the distributed thrust t_e in the corresponding ribs) compute the element force vector and assembly them into the global force vector. Hint: Follow the guide **ALG_BEAM_3D** for assistance in computing the element force vector. The assembly process of this element force vector $\mathbf{f}_e$ into the global vector of external forces $\mathbf{f}$ is the same to that of the stiffness matrix, but only for the rows (since it is a vector instead of a matrix).

7. Solve the system $\mathbf{Ku} = \mathbf{f}$ after prescribing the corresponding degrees of freedom and obtain the solution displacements and rotations vector $\mathbf{u}$.

8. Obtain the local displacements, rotations and internal forces distribution for each element: (a) internal local displacement and rotations vector for each element $\mathbf{u}'_{int,e}$, (b) axial force for each element N, (c) shear forces for each element Q_y, Q_z, (d) bending moments for each element M_y, M_z and (e) torsion moment for each element T. Hint: Use the guide **ALG_BEAM_3D** for assistance.

9. Use the **`plotWing`** function to obtain the wing displacements along with the internal forces distribution as well as the local deformation, deflection and rotation angles of each beam element. Hint: Look inside the script for the **`plotWing`** function to see the structure of the matrices expected as inputs.

10. Use the **`plotBeams3D`** function to plot the deformed shape (displacement and rotations) of the most critical beams. Hint: the number of subdivisions is ruled by **`nsub`** parameter (a value of 20 subdivisions can be used). In addition, the element to plot (**`e_plot`**) must be given as input to this function (see inside the script for further information).

11. Use the **`plotBeams3D_def`** function to obtain the deformed wing using the polynomial displacements within each beam. Remember to adjust the number of subdivisions and the scale factor to amplify the displacements.

A report with an analysis of this structure must be submitted including:

a. Nominal parameters regarding the structure: (i) total mass as well as mass of the ribs and of the spars, (ii) total lift, (iii) total drag and (iv) total thrust produced.
b. Reactions at the wing joint with the fuselage.
c. Plot of the deformed wing using the **`plotBeams3D_def`** function and compare the results with the ones obtained using **`plotWing`** function.
d. Color plot of the beams' internal distribution of the following parameters (using **`plotWing`**):
   i. Axial force
   ii. Shear force in the local y' direction
   iii. Shear force in the local z' direction
   iv. Torsional moment
   v. Bending moment in the local y' direction

vi. Bending moment in the local z' direction

e. Plot of the displacements and rotations of the three most critical beams: the one with the highest axial force, the one with the highest shear force and the one with the highest bending moment (using **`plotBeams3D`**).
f. Short discussion of the obtained results.