Member 
$$i \Rightarrow j$$
  $\frac{AE}{L}$   $0 < c \leq 5$ 

$$1 \quad 1 \Rightarrow 2 \quad \frac{12 \times 4(10)^{-4} 210(10)^{9}}{12} \quad 225 \quad \frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}}$$

$$2 \quad 1 \Rightarrow 3 \quad \frac{4(10)^{-4} 210(10)^{9}}{1} \quad -90 \quad 0 \quad -1$$

$$\mathcal{K}^{(e)} = \frac{AE}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ s^2 & -cs & -s^2 \end{bmatrix}$$

$$-sym - \begin{bmatrix} c^2 & cs \\ s^2 \end{bmatrix}$$

$$|C_{1} = 8.4(0)^{7} \begin{cases} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -sym & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{cases}$$

$$d_{S} = \begin{cases} d_{2x} \\ d_{2y} \\ d_{3x} \\ d_{3y} \end{cases} = \begin{cases} 0 \\ 0 \\ 0 \end{cases}$$

$$F_{S} = \begin{cases} F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{cases}$$

$$d_{F} = \begin{cases} d_{1x} \\ d_{1y} \end{cases}$$

$$F_{F} = \begin{cases} F_{1x} \\ F_{1y} \\ F_{1y} \end{cases} = \begin{cases} 50,000 \\ 0 \end{cases}$$

Assemble System Matrix

dix diy, dzx dzy dsx dzy k.FS

\[ \frac{1}{2} \frac{1} Fig 8.4(10) -Fzy Fzy Fzy Solve for Displacements KFF dF + KFS dS = FF

$$|C_{FF}| = d_{F} + |C_{FS}| = F_{F}$$

$$8.4(10)^{7} \left[\frac{1}{2} + \frac{1}{2}\right] \left(\frac{1}{2} + \frac{1}{2}\right) \left(\frac{1}{2} + \frac{$$

. .

$$\begin{cases}
-\frac{1}{2} - \frac{1}{2} \\
-\frac{1}{2} - \frac{1}{2}
\end{cases}
\begin{cases}
1.74(10)^{-3} \\
-5.45(10)^{-4}
\end{cases} = \begin{cases}
F_{2x} \\
F_{2y}
\end{cases}
= \begin{cases}
-5(10)^{4} \\
-5(10)^{4}
\end{cases}$$

$$\begin{cases}
F_{3x} \\
F_{3y}
\end{cases}$$

$$\begin{cases}
F_{3y}
\end{cases}$$

$$\begin{cases}
S(e) \\
S(x) \\
S(e)
\end{cases} = \frac{AE}{L} \begin{bmatrix} C & S & -C & -S \\
-C & -S & C \end{bmatrix} \begin{pmatrix} dix \\ diy \\ dix \\ diy
\end{cases}$$

$$8.4(10)^{7} \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{cases} 1.79(10)^{\frac{2}{3}} \\ -5.95(10)^{\frac{2}{3}} \\ 0 \end{cases} = \begin{cases} -7.07(10)^{\frac{2}{3}} \\ -7.07(10)^{\frac{2}{3}} \\ 0 \end{cases}$$
Tension
$$\Rightarrow \times$$

$$0$$

$$0$$

Element #2

$$8.4(10)^{7} \begin{cases} 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 \end{cases} \begin{cases} 1.79(10)^{-3} \\ -5.95(10)^{-4} \\ 0 & 0 \end{cases} = \begin{cases} 5(10)^{4} \\ -5(10)^{4} \\ 0 & 0 \end{cases}$$

$$\frac{5}{4(10)^{-4}} = 1.25(10)^{8} N/m^{2}$$

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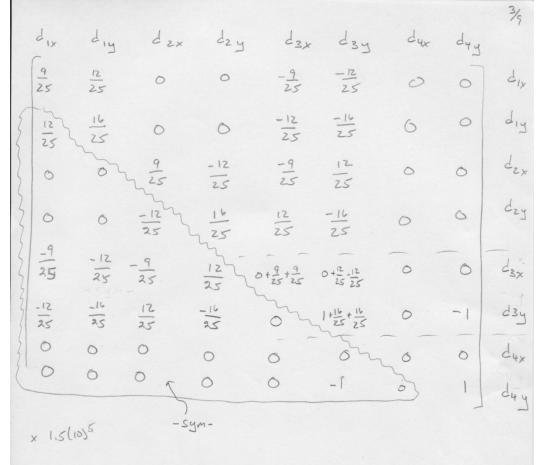
$$| \frac{1}{10} | \frac{1}{10$$

 $L_{2} = \begin{cases} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{9}{2} & \frac{1}{2} & \frac{9}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{9}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{9}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}$ 

Now Assemble System St. Places Matrix

Example: Show Row Column Swap

Example: Show Direct Assembly with proper order



Interested in Node 3

First Swap Rows 105
and Rows 206

de dy Esx 234 dyx -<u>9</u> 25 78 d 3x -16 034 0 0 -12 -9 25 25 0 dzx 0 0 0. 0 0 0 214 -16 0 0 0 0 dyx 0 0 0 0 0 0 × 1,5 × 10 5

> Wext Swap Columns 1 +5 and Colomns 2006

dzy  $\frac{12}{25} - \frac{9}{25} - \frac{12}{25}$   $\frac{-16}{25} - \frac{12}{25} - \frac{16}{25}$ -12 -symafter the last swap it is symetric again × 1.5(10)5 Note: this is extremly painfull much better to order degrees of Greedom Som the start is Assemble

Stiffness matrix correctly without row/column swap
See Sollowing page

$$\frac{d_{3x}}{2s} \quad d_{3y} \quad d_{2x} \quad d_{2y} \quad d_{1z} \quad d_{1y} \quad \frac{d_{1x}}{d_{2x}} \quad d_{4x} \quad d_{4y} \quad d_{5}$$

$$\frac{18}{2s} \quad 0 \quad \frac{7}{2s} \quad \frac{12}{2s} \quad \frac{7}{2s} \quad \frac{7}{2s} \quad 0 \quad 0 \quad 0$$

$$0 \quad \frac{57}{2s} \quad \frac{12}{2s} \quad \frac{12}{2s} \quad \frac{11}{2s} \quad 0 \quad 0 \quad 0$$

$$\frac{1}{2s} \quad 0 \quad 0 \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad \frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad \frac{1}{2s} \quad 0 \quad 0$$

$$\frac{1}{2s} \quad \frac{1}{2s} \quad \frac{1}{2s$$

A

$$1.5(10)^{4} \begin{bmatrix} 18 & 0 \\ 25 & 0 \\ 0 & \frac{57}{25} \end{bmatrix} \begin{pmatrix} 37 \\ 439 \end{pmatrix} + 0 = \begin{cases} 5(10)^{3} \\ 1(10)^{4} \end{cases}$$

$$\{d_{F}\}=\{d_{34}\}=\{0,046\}$$

Solve Sor Reactions

$$\begin{bmatrix}
-\frac{9}{25} & \frac{17}{25} \\
\frac{17}{25} & \frac{16}{25}
\end{bmatrix} = \begin{bmatrix}
-4.61(0)^{3} \\
6.14(10)^{3} \\
-3.95(10)^{2} \\
-5.26(10)^{2}
\end{bmatrix} = \begin{bmatrix}
R_{27} \\
R_{29} \\
R_{17} \\
R_{17} \\
R_{17} \\
R_{17} \\
R_{17}
\end{bmatrix}$$

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Soule for Element Sorces:

$$\begin{cases} S_{ix}^{(e)} \\ S_{ix}^{(e)} \end{cases} = \frac{AE}{L} \begin{bmatrix} C & S & -C & -S \\ -C & -S & C & S \end{bmatrix} \begin{pmatrix} d_{ix} \\ d_{iy} \\ d_{iy} \\ d_{iy} \end{pmatrix}$$

Element 1:

Flement 2:

Complession

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Element #3

7

a) Determine *Displacements*, *Reactions*, *element forces* and *stresses* for the truss shown.

F=1000 N 
$$\theta$$
 = 45 deg

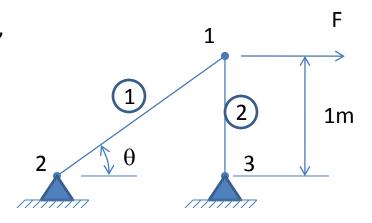
### For Elements:

$$A_1 = \sqrt{2} \times 4 \times 10^{-4} \text{ m}^2$$
  
 $A_2 = 4 \times 10^{-4} \text{ m}^2$   
 $E = 210 \text{ GPa}$ 

Problem a (Nastran Solution)

**Conclusions:** 

Manual computation matches Nastran solution

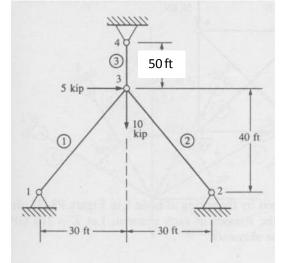


```
time 2
sol 101
cend
title=ME204 Homework 3
subtitle=Problem a
spc=100
disp=all
spcforces=all
subcase 1
   label=Load Set
   load=501
   forces=all
   stress=all
begin bulk
grid, 11,,1.,1.
grid, 12,,0.,0.
grid, 13,,1.,0.
crod, 21, 20, 11, 12
crod, 22, 21, 11, 13
prod, 20, 40, 5.657-4
prod, 21, 40, 4.-4
mat1,40,2.1+11,,0.3
spc1,100,12,12,13
force, 501, 11,, 50000., 1.
param, grdpnt, 0
enddat.a
```

### DISPLACEMENT VECTOR

POINT ID. 11 12 13	TYPE G G G	T1 1.785684E-03 0.0 0.0	T2 -5.952381E-04 0.0 0.0	T3 0.0 0.0 0.0	R1 0.0 0.0 0.0	R2 0.0 0.0 0.0	R3 0.0 0.0 0.0		
		FORC	ES OF SI	NGLE-PO	INT CON	ISTRAINT			
POINT ID.	TYPE	T1	Т2	Т3	R1	R2	R3		
12 13	G G	-5.000000E+04	5.000000E+04 5.000000E+04		Numbers r	natch manua	l computation		
			FORCES	IN ROD	ELEMEN		D )		
ELEMENT ID.		AXIAL FORCE	TORQUE	EL	EMENT	AXIAL FORCE	TORQUE		
21		71068E+04 0	IONGOE		+ 1	5.000000E+04 0.			
		S	TRESSES	IN ROD	ELEMENI	rs (cro	D )		
ELEMENT	AXI	AL SAFET	TORSIONAL	SAFETY	ELEMENT	AXIAL	SAFETY		
TORSIONAL SAFETY									
ID.	STRE	SS MARGIN	N STRESS	MARGIN	ID.	STRESS	MARGIN		
STRESS MAR		607.00	0.0		0.0	1 0500007.00	0.0		
21	1.2499	68E+08	0.0		22	-1.250000E+08	0.0		

b) Determine *Displacements*, *Reactions*, *element forces* and *stresses* for the truss shown.



Problem b (Nastran Solution)
Conclusions:
Manual computation matches Nastran solution

## For All Elements: A=3 in<sup>2</sup> E=3 x 10<sup>7</sup> psi

time 2

```
sol 101
cend
title=ME204 Homework 3
subtitle=Problem b
spc=100
disp=all
spcforces=all
subcase 1
   label=Load Set
   load=501
   forces=all
   stress=all
begin bulk
grid, 11,,-360.,-480.
grid, 12,, 360., -480.
grid, 13,,0.,0.
grid, 14,,0.,600.
crod, 21, 20, 11, 13
crod, 22, 20, 12, 13
crod, 23, 20, 14, 13
prod, 20, 40, 3.
mat1, 40, 3+7, 0.3
spc1,100,12,11,12,14
force, 501, 13,, 5000., 1.
force, 501, 13,, 10000., 0., -1.
param, grdpnt, 0
enddata
```

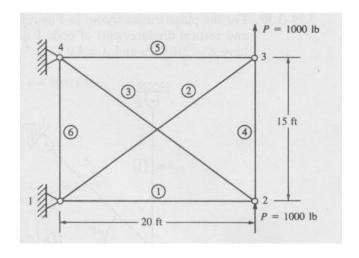
### DISPLACEMENT VECTOR

POINT	ID.	TYPE	Т1	T2	Т3	R1	R2	R3			
	11	G	0.0	0.0	0.0	0.0	0.0	0.0			
	12	G	0.0	0.0	0.0	0.0	0.0	0.0			
	13	G	4.629629E-02	-2.923977E-02	0.0	0.0	0.0	0.0			
	14	G	0.0	0.0	0.0	0.0	0.0	0.0			
FORCES OF SINGLE-POINT CONSTRAINT											
POIN'	T ID.	TYPE	Т1	Т2	Т3						
	11	G	-3.947368E+02	-5.263158E+02	< 0.0	Numh	ars match n	nanual computatio	n		
	12	G	-4.605263E+03	6.140351E+03		Numb	cis illattii il	nandar computatio			
	14	G	0.0	4.385965E+03		0.0	0.0	0.0			
ELE)	MENT		AXIAL	FORCES	IN ROI	D ELEMEN ELEMENT	T S C R O	O D )			
	D.		FORCE	TORQUE		ID.	FORCE	TORQUE			
	21		578947E+02 0.0					0.0			
	23	4.3	885965E+03 0.0	)							
			S 5	rress es	IN ROD	ELEMENT	S (CRO	D )			
ELE	MENT	AXIA		TORSIONAL	SAFETY	ELEMENT	AXIAL	SAFETY			
I,	D.	STRES	S MARGIN	STRESS	MARGIN	ID.	STRESS	MARGIN			
	21	2.19298	32E+02	0.0		22 -	-2.558479E+03	Y			
	23	1.46198	88E+03	0.0							

# Problem c (Nastran Solution) Use A=1in<sup>2</sup> E=10<sup>7</sup> psi

### **Conclusions:**

## Nastran Problem solves redundant system properly



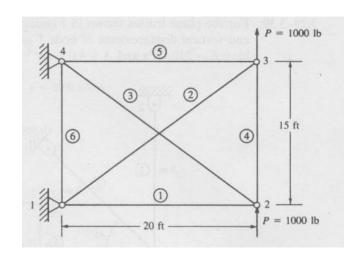
 $A=1in^2$ , and  $E=10^7$  psi

```
time 2
sol 101
cend
title=ME204 Homework 3
subtitle=Problem c
spc=100
disp=all
spcforces=all
subcase 1
   label=Load Set
   load=501
   forces=all
   stress=all
begin bulk
grid, 11,,0.,0.
grid, 12,,240.,0.
grid, 13,,240.,180.
grid, 14,,0.,180.
crod, 21, 20, 11, 12
crod, 22, 20, 11, 13
crod, 23, 20, 12, 14
crod, 24, 20, 12, 13
crod, 25, 20, 13, 14
crod, 26, 20, 11, 14
prod, 20, 40, 1.
mat1, 40, 1+7, 0.3, 2.6-4
,2000.,2000.
spc1,100,12,11,14
force, 501, 12, , 1000., 0., 1.
force, 501, 13,, 1000., 0., 1.
param, grdpnt, 0
enddata
```

# Key Results from .f06 file

DISPLACEMENT VECTOR													
1	ID. 11 12 13 14	G G 3 G -3	T1 0.0 8.2000000 8.2000000	E-02 E-02	T2 0.0 1.260000E-01 1.260000E-01 0.0		3	0.0 0.0 0.0 0.0	R1	0.0 0.0 0.0 0.0		0.0 0.0 0.0	R3
FORCES OF SINGLE-POINT CONSTRAINT													
_	ID. 11 14				T2 -1.000000E+03 -1.000000E+03		3	0.0	R1	0.0		0.0	R3
					FORCES	S IN F			ΜE		( C R	OD)	
2		AXI FOF 1.3333 -1.6666 -1.3333	RCE 333E+03 567E+03	0.0 0.0 0.0	1		]	ELEMENT ID. 22 24 26		AXIAL FORCE 1.666667E 0.0 0.0		TORQUE 0.0 0.0 0.0	
ELEMENT ID. 21 23	1 3	AXIAL STRESS 1.333333E+0 -1.666667E+0 -1.333333E+0	2.01	ETY	RESSES TORSIONAL STRESS 0.0 0.0 0.0	SAFET	Y	ELEME ID.	NT	S (     AXIAL     STRESS 1.666667F 0.0 0.0		D ) SAFETY MARGIN 2.0E-01	TORSIONAL STRESS 0.0 0.0

Problem d
Conclusions: When crossbars 2 and 3 are removed,
Nastran recognizes system as a singular
Matrix. Produces consistent error shown on following page



 $A=1in^2$ , and  $E=10^7$  psi

```
time 2
sol 101
cend
title=ME204 Homework 3
subtitle=Problem d
spc=100
disp=all
spcforces=all
subcase 1
   label=Load Set
   load=501
   forces=all
   stress=all
begin bulk
grid, 11,,0.,0.
grid, 12,,240.,0.
grid, 13,,240.,180.
grid, 14,,0.,180.
crod, 21, 20, 11, 12
crod, 24, 20, 12, 13
crod, 25, 20, 13, 14
crod, 26, 20, 11, 14
prod, 20, 40, 1.
mat1, 40, 1+7, 0.3, 2.6-4
,2000.,2000.
spc1,100,12,11,14
force, 501, 12,, 1000., 0., 1.
force, 501, 13,, 1000., 0., 1.
param, grdpnt, 0
enddata
```

Key Results from .f06 file: System is singular – is a mechanism (i.e. F=ma should be used)

```
*** USER INFORMATION MESSAGE 6137 (DFMN)
     INPUT MATRIX TO DECOMPOSITION HAS
                                              1 SINGULARITIES.
     SEE FOLLOWING MESSAGES FOR DETAILS.
 *** USER INFORMATION MESSAGE 4158 (DFMSA)
 ---- STATISTICS FOR SPARSE DECOMPOSITION OF DATA BLOCK KLL
                                                                  FOLLOW
        NUMBER OF NEGATIVE TERMS ON FACTOR DIAGONAL =
       MAXIMUM RATIO OF MATRIX DIAGONAL TO FACTOR DIAGONAL = 1.0E+15 AT ROW NUMBER
 *** SYSTEM INFORMATION MESSAGE 4159 (DFMSA)
     THE DECOMPOSITION OF KLL
                                   YIELDS A MAXIMUM MATRIX-TO-FACTOR-DIAGONAL RATIO OF
                                                                                        1.000000E+15
 *** USER WARNING MESSAGE 4698 (DCMPD)
     STATISTICS FOR DECOMPOSITION OF MATRIX KLL.
     THE FOLLOWING DEGREES OF FREEDOM HAVE FACTOR DIAGONAL RATIOS GREATER THAN
     1.000000E+07 OR HAVE NEGATIVE TERMS ON THE FACTOR DIAGONAL.
     USER INFORMATION:
     THIS MESSAGE MAY BE IGNORED IF NO GRID POINT IDS OR HIGH RATIO MESSAGES APPEAR IN THE TABLE ON THE NEXT PAGE.
   ME204 HOMEWORK 3
                                                                           SEPTEMBER 23, 2007
                                                                                                 MD NASTRAN
PAGE
        10
     PROBLEM 3.26
     LOAD SET
                                                                                                              SUBCASE
         GRID POINT ID
                             DEGREE OF FREEDOM
                                                 MATRIX/FACTOR DIAGONAL RATIO
                                                                                     MATRIX DIAGONAL
              12
                                    T2
                                                         1.00000E+15
                                                                                        5.55556E+04
        TERMINATED DUE TO EXCESSIVE PIVOT RATIOS IN MATRIX KLL.
                   CONSTRAIN MECHANISMS WITH SPCI OR SUPORTI ENTRIES OR SPECIFY PARAM, BAILOUT, -1 TO
     CONTINUE THE RUN WITH MECHANISMS.
```

This may help identifying problem if you think it **should** be properly posed, i.e. this is the FIRST grid point where a problem is identified