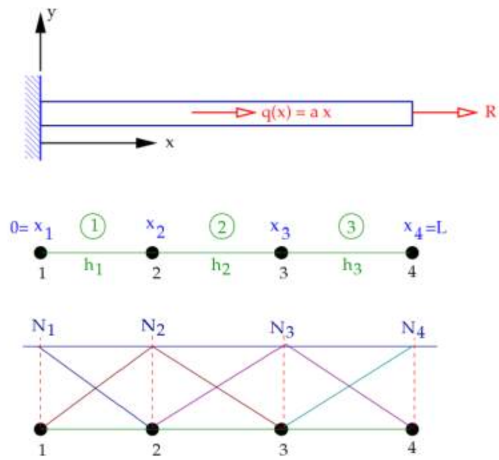
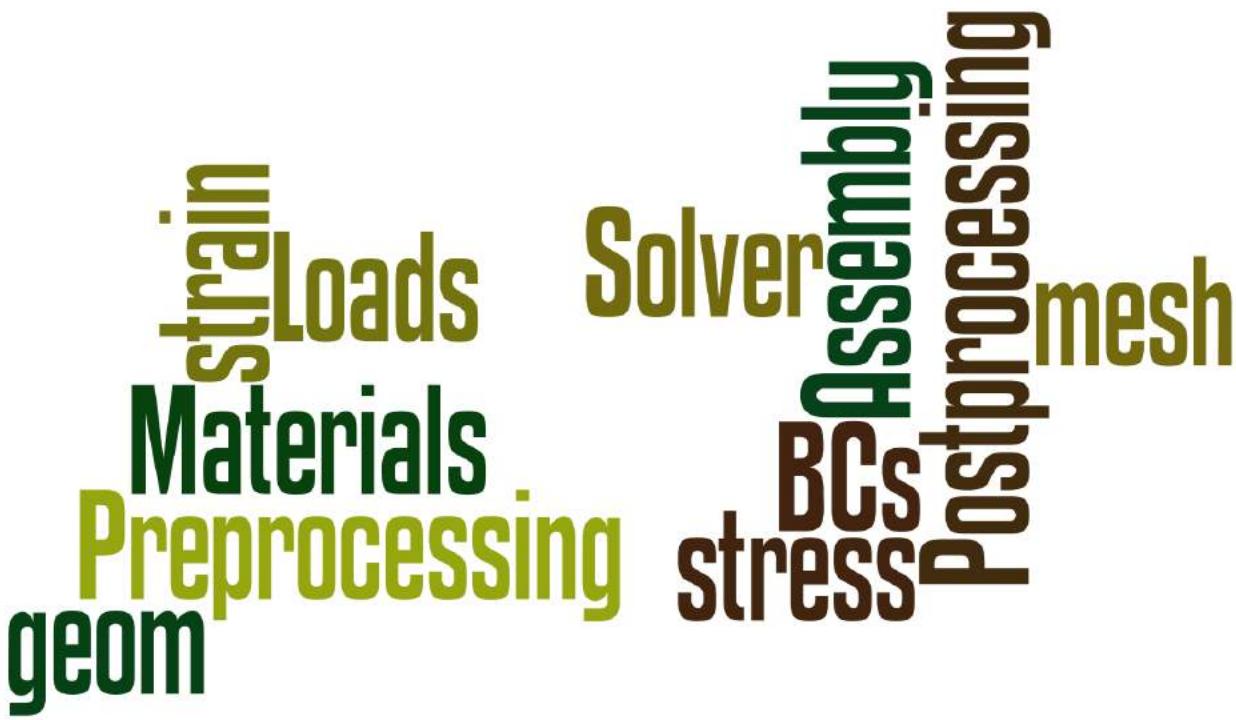
1MAE701 Exercices to be done/started in class

C1: Complete a simple FEA code





Fill the comments with above pictures wordles , specify the units

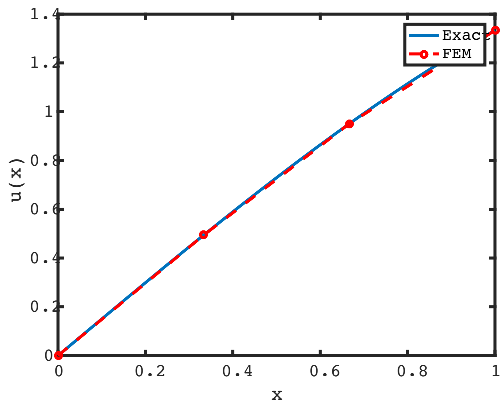
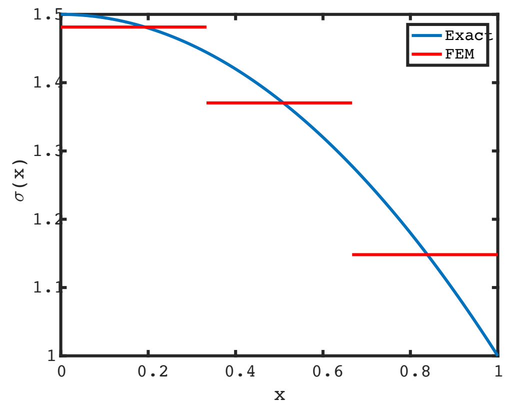
Copy paste part of the code in the ?? at the right place

B*u	[-1/h 1/h]	inv(Kred)*fred	(A*E/h)*[[1 -1];[-1 1]]	E*eps

```matlab
function AxialBarFEM
%
%Material and Length
%
       A = 1.0; %cross section mm2
       L = 1.0; %length mm2
       E = 1.0; %Young's modulus MPa (N/mm2)
%loads
       a = 1.0; %distributed force N.m
       R = 1.0; %Force (N)

%
%Prepostfem mesh
%
       e = 3;
       h = L/e;
       n = e+1;

       for i=1:n
         node(i) = (i-1)*h;
       end
       for i=1:e
         elem(i,:) = [i i+1];
       end
%
%Assembly
%
       K = zeros(n);
       f = zeros(n,1);
       for i=1:e
         node1 = elem(i,1);
         node2 = elem(i,2);
         Ke = elementStiffness(A, E, h);
         fe =
elementLoad(node(node1),node(node2), a, h);
         K(node1:node2,node1:node2) =
K(node1:node2,node1:node2) + Ke;
         f(node1:node2) = f(node1:node2) + fe;
       end

%
%BCs &Loads
%
       f(n) = f(n) + 1.0;
       Kred = K(2:n,2:n);
       fred = f(2:n);
%
%Solvers
%
       d = ??;
       dsol = [0 d']; %known solution at x=0
       fsol = K*dsol';
       sum(fsol)
%
%Postprocessing
%
       figure;
       p0 = plotDisp(E, A, L, R, a);
       p1 = plot(node, dsol, 'ro--',
'LineWidth', 3); hold on;
       legend([p0 p1],'Exact','FEM');

       for i=1:e
         node1 = elem(i,1);
         node2 = elem(i,2);
         u1 = dsol(node1);
         u2 = dsol(node2);
         [eps(i), sig(i)] =
elementStrainStress(u1, u2, E, h);
       end

       figure;
       p0 = plotStress(E, A, L, R, a);
       for i=1:e
         node1 = node(elem(i,1));
         node2 = node(elem(i,2));
         p1 = plot([node1 node2], [sig(i)
sig(i)], 'r-','LineWidth',3); hold on;
       end
       legend([p0 p1],'Exact','FEM');


    function [p] = plotDisp(E, A, L, R, a)
       dx = 0.01;
       nseg = L/dx;
       for i=1:nseg+1
         x(i) = (i-1)*dx;
         u(i) = (1/6*A*E)*(-a*x(i)^3 + (6*R +
3*a*L^2)*x(i));
       end
       p = plot(x, u, 'LineWidth', 3); hold
on;
       xlabel('x', 'FontName', 'palatino',
'FontSize', 18);
       ylabel('u(x)', 'FontName', 'palatino',
'FontSize', 18);
       set(gca, 'LineWidth', 3, 'FontName',
'palatino', 'FontSize', 18);

    function [p] = plotStress(E, A, L, R, a)
       dx = 0.01;
       nseg = L/dx;
       for i=1:nseg+1
         x(i) = (i-1)*dx;
         sig(i) = (1/2*A*E)*(-a*x(i)^2 + (2*R
+ a*L^2));
       end
       p = plot(x, sig, 'LineWidth', 3); hold
on;
       xlabel('x', 'FontName', 'palatino',
'FontSize', 18);
       ylabel('\sigma(x)', 'FontName',
'palatino', 'FontSize', 18);
       set(gca, 'LineWidth', 3, 'FontName',
'palatino', 'FontSize', 18);

    function [Ke] = elementStiffness(A, E, h)

       Ke = ??;

    function [fe] = elementLoad(node1, node2,
a, h)
       x1 = node1;
       x2 = node2;
       fe1 = a*x2/(2*h)*(x2^2-x1^2) -
a/(3*h)*(x2^3-x1^3);
       fe2 = -a*x1/(2*h)*(x2^2-x1^2) +
a/(3*h)*(x2^3-x1^3);
       fe = [fe1;fe2];

    function [eps, sig] =
elementStrainStress(u1, u2, E, h)

       B = ??
       u = [u1; u2];
       eps = ??
       sig = ??;
```

Please also comment some lines %? And the results under

Play with e. Conclude

	
?	?

C2: Fill up a more “complex” FEA code* publish needed on LMS by pair

A cantilever beam of length Lp = 3 m and define by a squared section of aera Ap = 30×30 cm2.
The BCs are C-F, and the vertical load is imposed at the free boundary conditions such as F_o = 75 KN.
The materials used is concrete of Young’s modulus Eb = 32000 MPa.

Matlab will help you A LOT to accelerate learning and deepen understanding. The prof wants that you zip your publish or livescript with m files and upload on LMS.

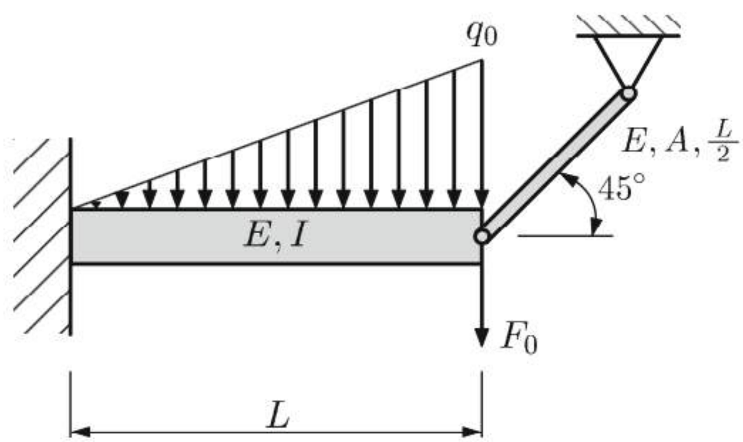


FIGURE 1: STRUCTURE TO BE DISCRETIZED USING 1 BEAM AND 1 BAR ELEMENT

Part 1 : Cantilever ONLY with Load case F_o

1) You can then compute the analytical beam deflection (to check)
2) Compute the rigidity matrix Kp and handwrite the system of equations respecting the nodes of figure above
3) Apply the BC Cantilver and solve the reduced system. Check using Matlab the solution and the resultant forces at the BC using the file “assignement2_student.m” on LMS

Part 2 : Cantilever beam+ inclined bar with the 2 load cases

The free end of the beam is connected to an inclined steel bar as shown in the figure1.
The section of the bar is Ab = 25 cm2 and the modulus of elasticity of the steel is Es = 200000 MPa.

0) Compute first the equivalent load for q_0 with max(q_0)=10kN
1) Calculate the stiffness matrix Kb of the inclined bar, taking into account the degrees of freedom of transverse displacement and rotation.
2) Introduce the axial displacement into the stiffness matrix Kp of the beam.
3) Assemble the two matrices Kb and Kp according to the numbering of the nodes given in the fiigure and write the global force vector (for all nodes of the structure).
4) Apply the boundary conditions and write the system of equations to solve.
5) Solve the system and give the displacements and rotations of the nodes. Check reactions
6) Calculate the axial force in the bar and the beam.

A SMALL HELP

CONCEPTS OF GAUSSIAN INTEGRATION

In Gaussian integration, also called Gaussian quadrature, the integral of a function in natural coordinates is substituted by an equivalent sum of this function evaluated at special points multiplied by a corresponding weight. Commonly, these special points are referred to as sampling points ξ_i.

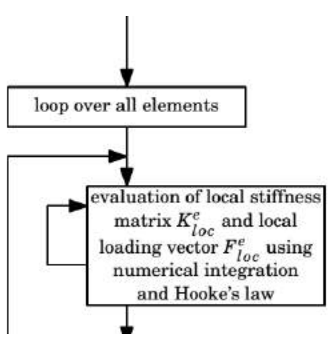
$$\int_{-1}^{1} F(\xi)d\xi = \sum_{i=1}^{NG} w_i F(\xi_i)$$

Although there is usually an error term associated with numerical integration, Gaussian quadrature has been shown to yield exact results for polynomials of degree 2m-1 or lower, where m is equal to the amount of weights / sampling points. This case directly applied to the common shape functions and their associated derivatives. Below is a table summarizing the position of the sampling points for Gaussian quadrature and their associated weights.

rule	point	coordinate	numerical value	weight	numerical value	order
1 point	1	0	0.0000000000	2	2.0000000000	1
2 points	1	$-\sqrt{\frac{1}{3}}$	-0.5773502692	1	1.0000000000	3
	2	$\sqrt{\frac{1}{3}}$	0.5773502692	1	1.0000000000	
3 points	1	$-\sqrt{\frac{3}{5}}$	-0.7745966692	$\frac{5}{9}$	0.5555555556	5
	2	0	0.0000000000	$\frac{8}{9}$	0.8888888889	
	3	$\sqrt{\frac{3}{5}}$	0.7745966692	$\frac{5}{9}$	0.5555555556	
4 points	1	$-\sqrt{\frac{3+2\sqrt{\frac{6}{5}}}{7}}$	-0.8611363116	$\frac{18-\sqrt{30}}{36}$	0.3478548452	7
	2	$-\sqrt{\frac{3-2\sqrt{\frac{6}{5}}}{7}}$	-0.3399810436	$\frac{18+\sqrt{30}}{36}$	0.6521451548	
	3	$\sqrt{\frac{3-2\sqrt{\frac{6}{5}}}{7}}$	0.3399810436	$\frac{18+\sqrt{30}}{36}$	0.6521451548	
	4	$\sqrt{\frac{3+2\sqrt{\frac{6}{5}}}{7}}$	0.8611363116	$\frac{18-\sqrt{30}}{36}$	0.3478548452	

SAMPLING POINTS AND CORRESPONDING WEIGHTS FOR GAUSSIAN QUADRATURE

PSEUDO CODE FOR IMPLEMENTATION IN A FINITE ELEMENT SOFTWARE



Processing:
for *each element* **do**
    recall element specific properties (length, cross-section, material)
    recall numerical integration points ξ_i and their weights w_i
    evaluate numerical integration of local stiffness matrix and load vector:
    for *each numerical integration point* **do**
        recall evaluated shape functions H_i and derivatives B_i
        evaluate Jacobian J and its determinant $|J|$
        evaluate local stiffness matrix $\mathbf{K^e_{loc}} = \mathbf{K^e_{loc}} + \mathbf{B^T D B |J|}\big|_{\xi_i} \cdot w_i$
        evaluate local loading vector $\mathbf{F^e_{loc}} = \mathbf{F^e_{loc}} + \mathbf{H^T f}\big|_{\xi_i} \cdot w_i$
    end

PSEUDO CODE FOR NUMERICAL INTEGRATION WITHIN A FINITE ELEMENT SOFTWARE

EXPECTED RESULTS

After first summation

K_local (4x4 double)

	1	2	3	4
1	27.6000	43.5349	-27.6000	11.6651
2	43.5349	68.6697	-43.5349	18.4000
3	-27.6000	-43.5349	27.6000	-11.6651
4	11.6651	18.4000	-11.6651	4.9303

F_local (4x1 doub

	1
1	8.8490
2	2.6289
3	1.1510
4	-0.7044

After second summation

K_local (4x4 double)

	1	2	3	4
1	55.2000	55.2000	-55.2000	55.2000
2	55.2000	73.6000	-55.2000	36.8000
3	-55.2000	-55.2000	55.2000	-55.2000
4	55.2000	36.8000	-55.2000	73.6000

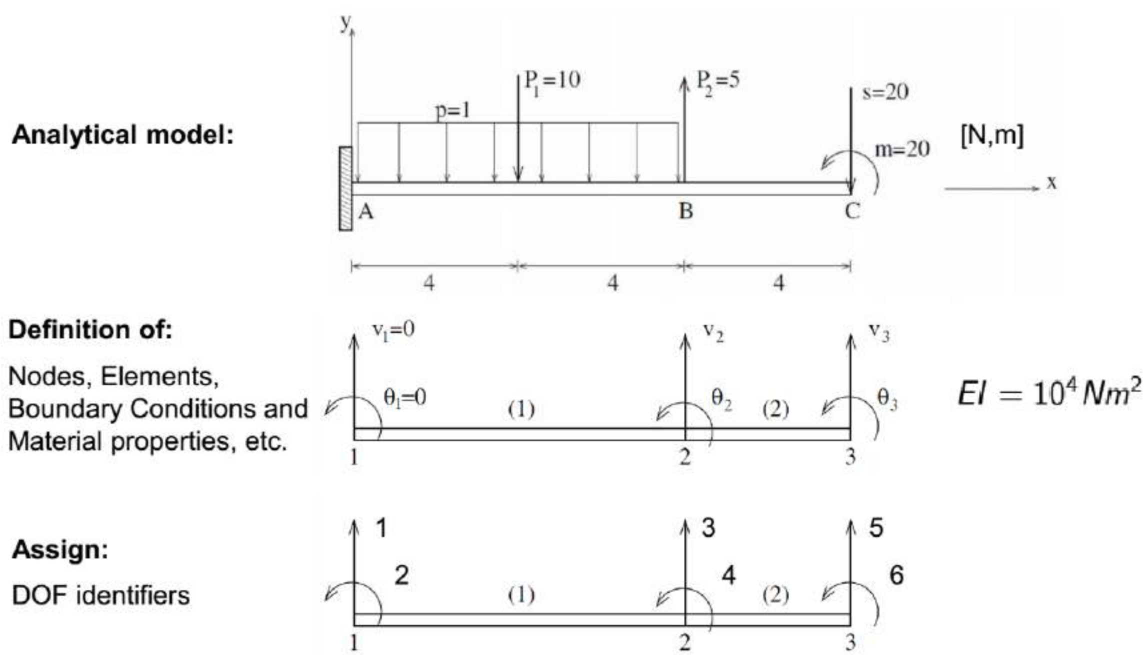
F_local (4x1 doubl

	1
1	10
2	3.3333
3	10
4	-3.3333

SOLVING A SAMPLE PROBLEM

Given the following sample beam, find the displacements, moments and shear forces across the beam. Modify your numerical integration method slightly to work within a skeletal finite element program provided, for both the integration of the stiffness matrix as well as the force vector.

Take a look at the skeletal finite element code provided and complete the sections marked with comments in the code. Some intermediate results are provided below.



Some intermediate results are the following:

K_local = (Beam 1)

K_local | K_system | F_local | F_system_el

4x4 double

	1	2	3	4
1	234.3750	937.5000	-234.3750	937.5000
2	937.5000	5.0000e+03	-937.5000	2.5000e+03
3	-234.3750	-937.5000	234.3750	-937.5000
4	937.5000	2.5000e+03	-937.5000	5.0000e+03

K_local = (Beam 2)

K_local | K_system | F_local | F_system_elemen

4x4 double

	1	2	3	4
1	1.8750e+03	3.7500e+03	-1.8750e+03	3.7500e+03
2	3.7500e+03	1.0000e+04	-3.7500e+03	5.0000e+03
3	-1.8750e+03	-3.7500e+03	1.8750e+03	-3.7500e+03
4	3.7500e+03	5.0000e+03	-3.7500e+03	1.0000e+04

K_system =

K_system | K_local | F_local | F_system_element | F_system_nodal

6x6 double

	1	2	3	4	5	6
1	234.3750	937.5000	-234.3750	937.5000	0	0
2	937.5000	5.0000e+03	-937.5000	2.5000e+03	0	0
3	-234.3750	-937.5000	2.1094e+03	2.8125e+03	-1.8750e+03	3.7500e+03
4	937.5000	2.5000e+03	2.8125e+03	1.5000e+04	-3.7500e+03	5.0000e+03
5	0	0	-1.8750e+03	-3.7500e+03	1.8750e+03	-3.7500e+03
6	0	0	3.7500e+03	5.0000e+03	-3.7500e+03	1.0000e+04

F_local = (distributed load)

F_local

4x1 doubl

	1
1	-4
2	-5.3333
3	-4
4	5.3333

F_local = (point load)

F_local

4x1 doub

	1
1	-5
2	-10
3	-5
4	10

F_system_element = (point and distributed load)

F_system_element

6x1 double

	1	2
1	-9	
2	-15.3333	
3	-9	
4	15.3333	
5	0	
6	0	

F_system_nodal = (all nodal loads)

F_system_nodal

6x1 double

	1	2
1	0	
2	0	
3	5	
4	0	
5	-20	
6	20	

F_system =

F_system

6x1 double

	1
1	-9
2	-15.3333
3	-4
4	15.3333
5	-20
6	20

d_system =

d_system

6x1 double

	1
1	0
2	0
3	-0.5525
4	-0.1125
5	-1.0293
6	-0.1205

C3: Approximation & Gauss Quadrature

Exercice 1

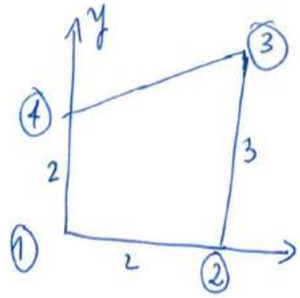
Integrate numerically $f(x) = e^{x}$ on $[0:6]$ using Gauss quadrature. compare with 'trapz', 'quad' Matlab's functions

$$\int_0^6 e^x \, dx = e^x \Big|_0^6 = e^6 - e^0 = e^6 - 1 \simeq 402{,}43$$

exact solution (see WOLFRAM α)

Compute the relative errors with exact solution

Exercice 2



Evaluate the integral $I = \iint_A (x^2 + y) \, dx \, dy$ over the quadrilateral shown.

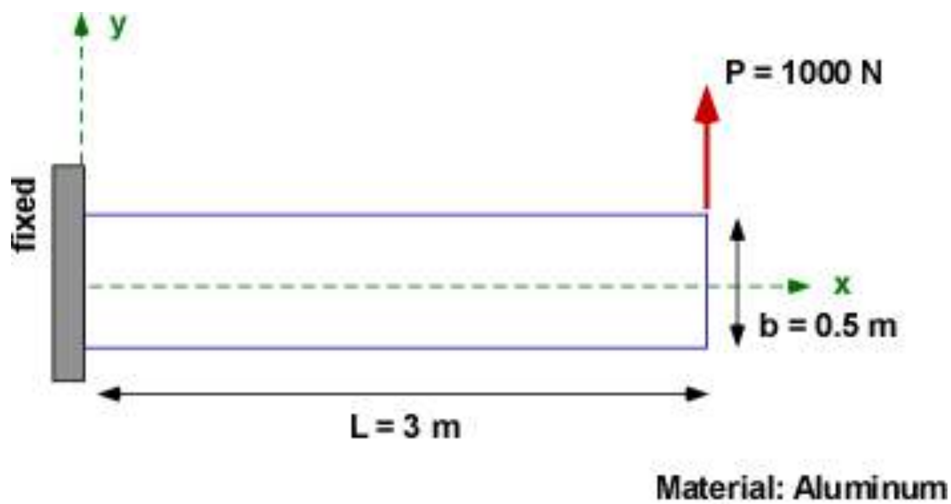
Corners: $x^T = [0 \ 2 \ 2 \ 0]$; $y^T = [0 \ 0 \ 3 \ 2]$

Using the function gaussQuad.m, find the optimal gauss points to reach tolerance 1E-4 for abs(I_exact-I_gauss)

C4: Play with a more “complex” FEA code* publish needed on LMS by pair

Using MATLAB PDE Toolbox

For **elliptical PDEs**, MATLAB provides you with a graphical tool that uses finite elements to solve the problem. However it is available only through the **Partial Differential Equation Toolbox**. The tool is started from the command line with the command ***pdetool***. This is graphical user interface (GUI) tool and requires you to follow a sequence of steps. Knowledge of finite elements is useful. We will use it as a tool to solve the 2D Elasticity problem shown below:



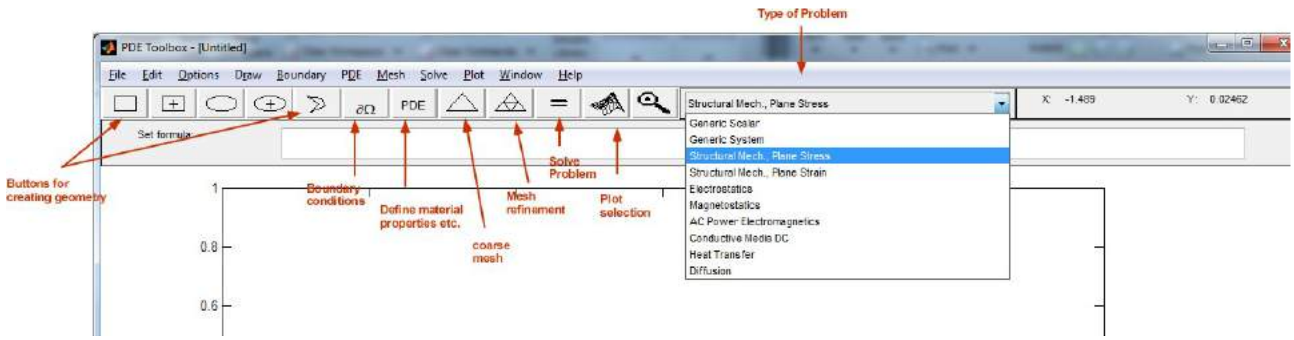
Thickness a is 1m.

The various steps in sequence of applying FEM is detailed below. It should be sufficient to get you started and solve similar problems as long as you remember to do it in sequence. I think the GUI makes it quite painless and interesting and that should be motivation enough.

We will be using the *pdetool* and therefore there is no code for solving this problem. Navigating the tool is through icons or the menu. Our problem is simple and we will use mainly the icons in the following. At the command prompt launch the tool

>> pdetool

Use the drop down menu and choose the Type of Problem : Plane Stress



Like many applications you should see a window with a menu on top, a list of icons below it, a line for formula entry, and a rectangular region for the problem.

Let us setup the problem. Remember you are solving for the displacements. You can get strain and stress from that solution.

Step 1 (Use menu to define problem region)

Options → Axes Limits (slightly more than our rectangle) Set X-axis range [-0.5 3.5]

Set Y-axis range [-0.5 1.0]

Options → Grid (Draw grid) Options → Snap

Click on Rectangle icon

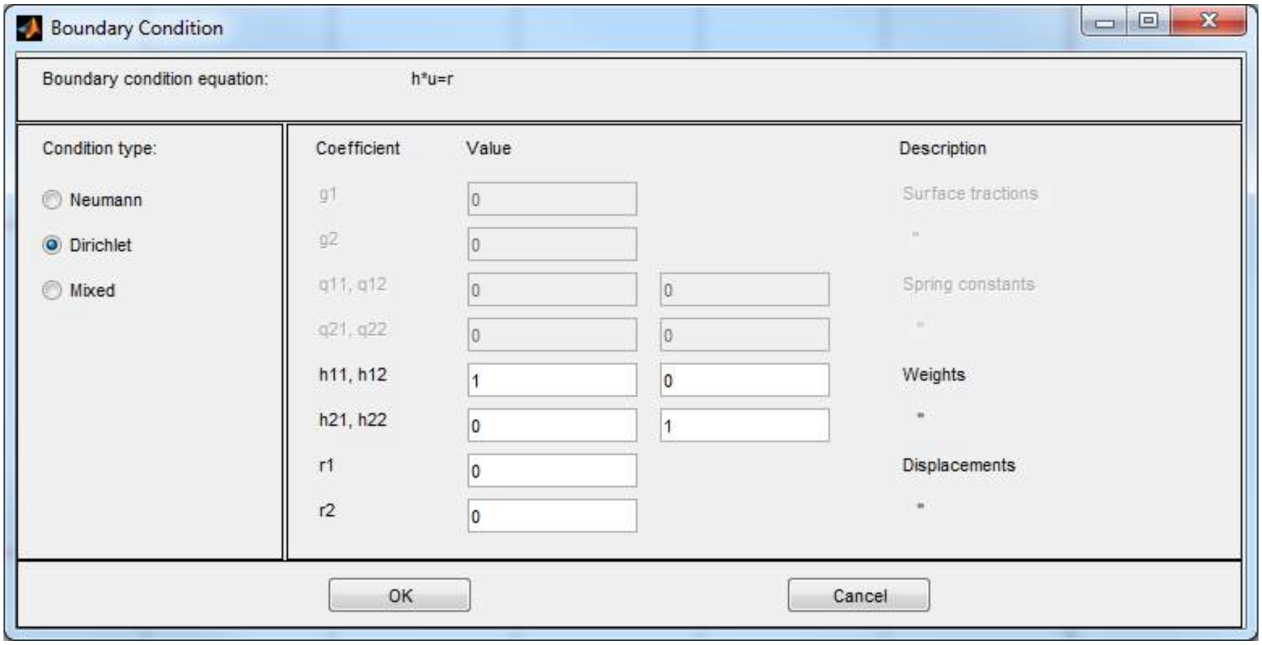
Use mouse to draw rectangle

Click at (0,0.5) and press and drag to (3,0)

Step 2 (Set Boundary condition)

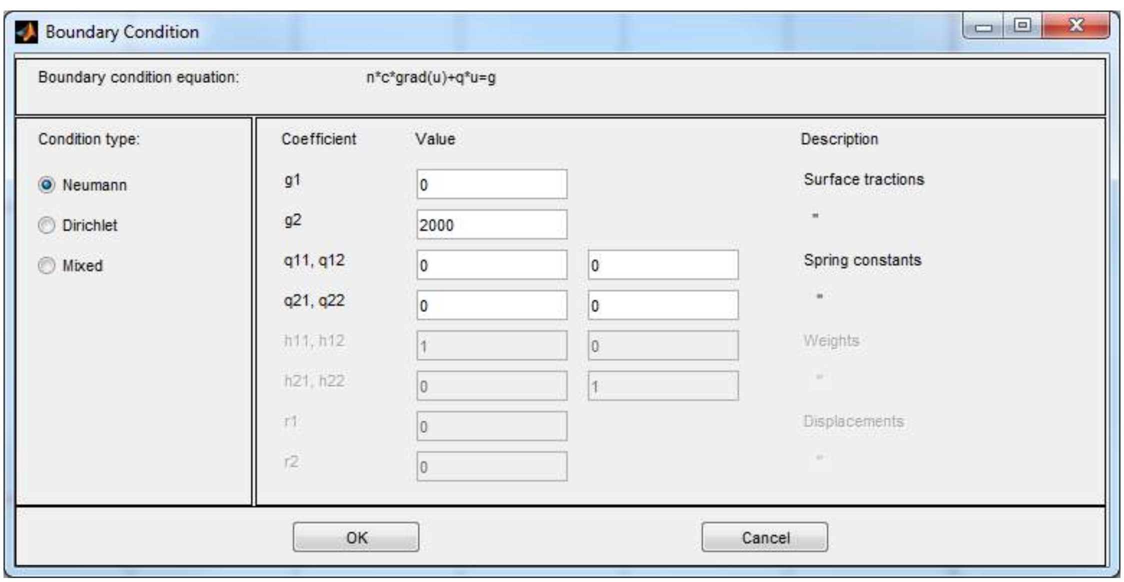
Boundary → remove all subdomain boundaries Click boundary conditions button

(i) Left Edge - Double click left edge (or x = 0 edge). There us no displacement (u = v = 0). Remember u in the figure below is the vector of u and v. We will use the Dirichlet Boundary condition.

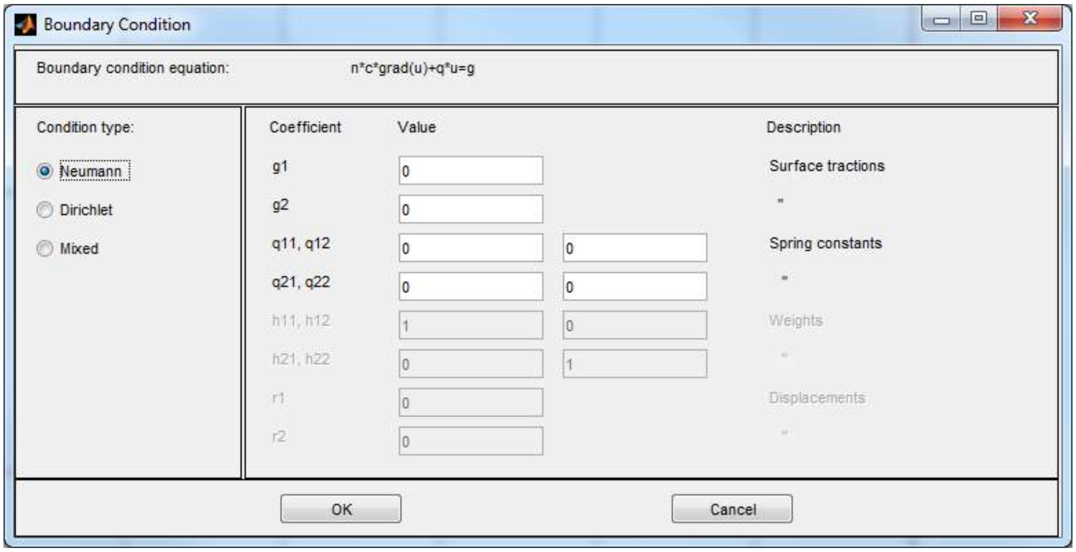


(ii) Right Boundary: This carries the load P. You cannot set up a point load. You have to specify it as a traction force - or Force/Area on the edge. That is P/b = 2000 N. We have to specify traction using the Neumann boundary condition

Double click right edge (or x = 3 edge)



(iii) Top Edge (y = 0.5) . All traction are zero. Use Neumann to accept default values



(iv) Same for Bottom edge

Step 3. Click on the PDE button to set the material properties for Aluminum

density = 2710 kg/m^3

Modulus of Elasticity (E) = 70 GPa Modulus of rigidity (G) = 26 GPa

Poisson ratio (nu) : calculate from G = E/(2(1 + nu))

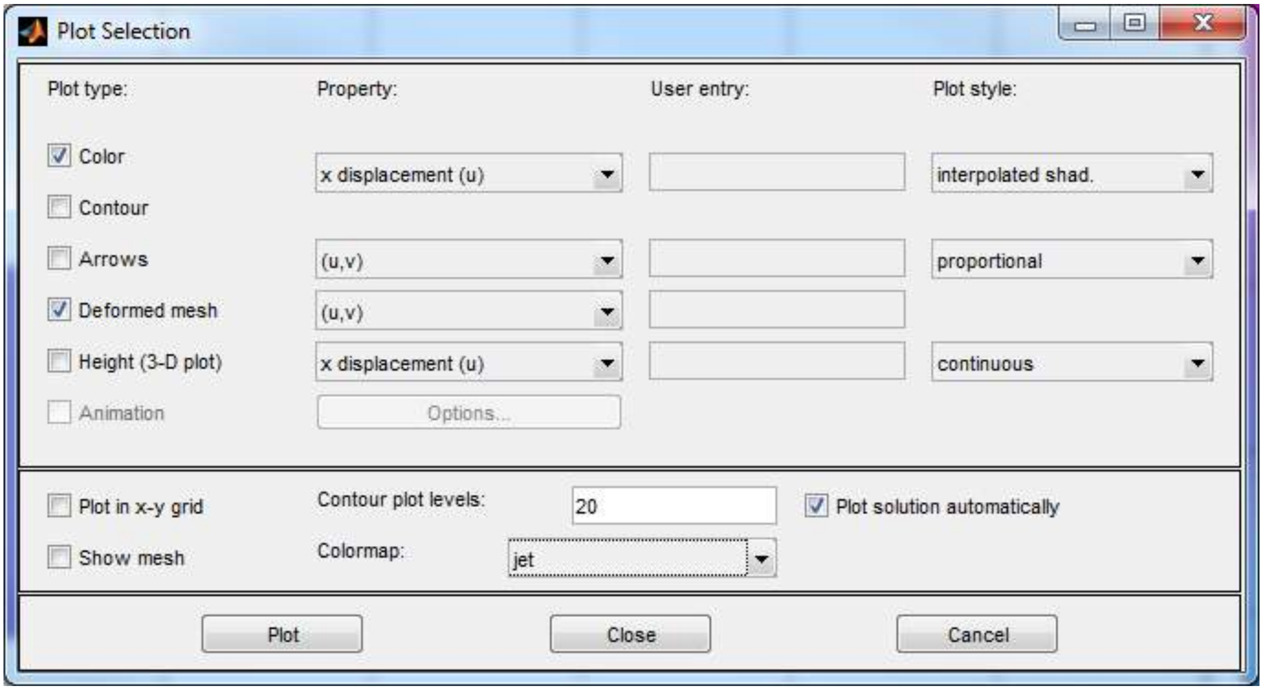
Step 4. Mesh the domain (default meshes) Click on coarse mesh

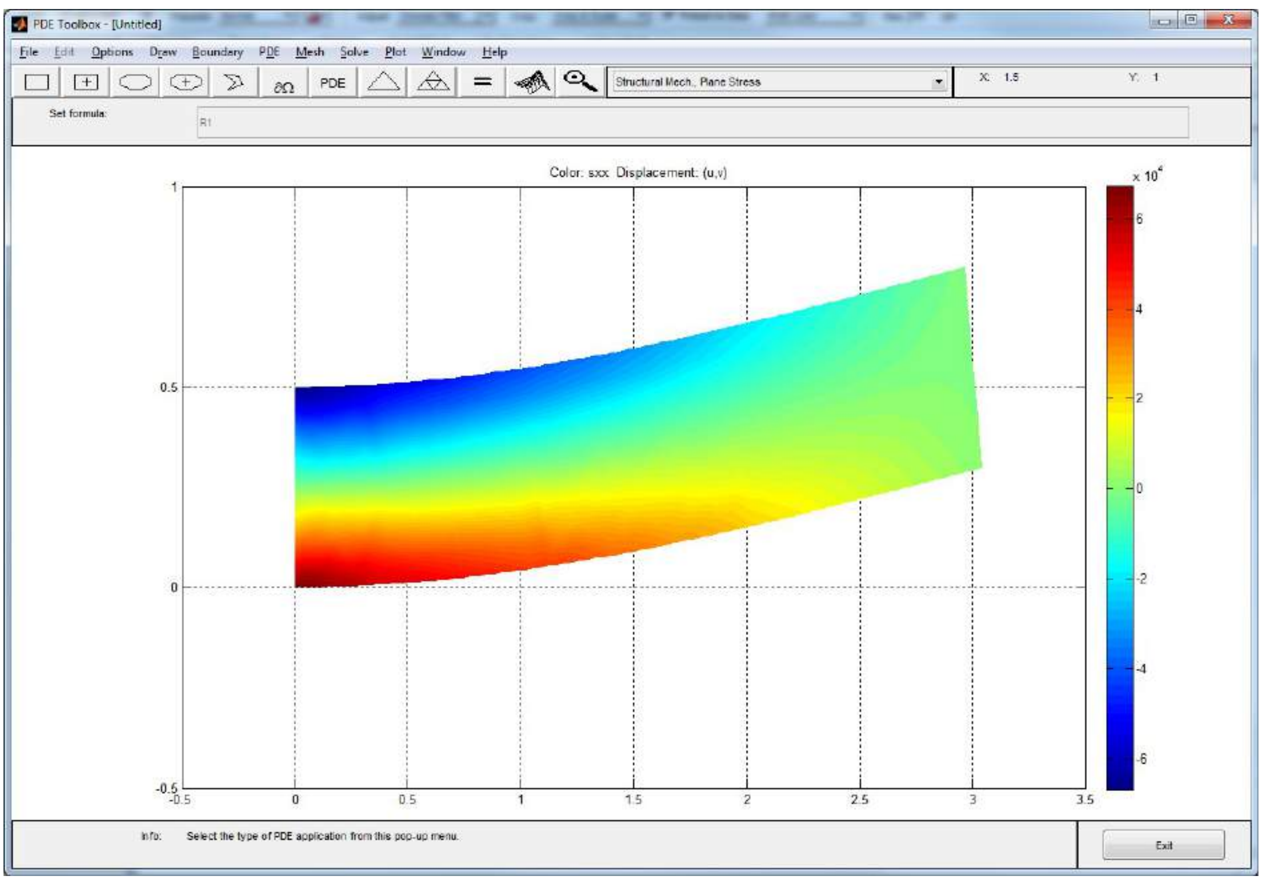
Refine the mesh twice

This program only uses triangular meshes or elements

Step 5 (Solve the problem) Click on = icon

You will see the solution. We will change the plot properties by clicking the plot button


Plot Selection
Plot type:
Property:
User entry:
Plot style:
Color
x displacement (u)
interpolated shad.
Contour
Arrows
(u,v)
proportional
Deformed mesh
(u,v)
Height (3-D plot)
x displacement (u)
continuous
Animation
Options...
Plot in x-y grid
Contour plot levels:
20
Plot solution automatically
Show mesh
Colormap:
jet
Plot
Close
Cancel


PDE Toolbox - [Untitled]
File Edit Options Draw Boundary PDE Mesh Solve Plot Window Help
PDE
Structural Mech., Plane Stress
X: 1.5
Y: 1
Set formula:
R1
Color: sxx Displacement: (u,v)
x 10^4
Info:
Select the type of PDE application from this pop-up menu.
Exit

You can now Export the required variables from pdetool and create a MATLAB script

Question 1: Export the stress fields and compute

VonMises using matlab (check with PDEtool interface)

$$\overline{\sigma} = \sqrt{\frac{3}{2}.\underline{dev}\left(\underline{\underline{\sigma}}\right):\underline{dev}\left(\underline{\underline{\sigma}}\right)} = \sqrt{\sigma_x^2 + 3.\tau_{xy}^2}$$

Question 2: Empirically fit with $\sigma_{xx} = K.(L - x).y$ Please. Comment !

Question 3: (BONUS) Add a hole center in L/2, h/2 of size defined from the relationship $(D/H = 0.6)$. Make a convergence study on sigma_max. Compare with standard stress concentration factor abaqus if possible.