例題 4：

$E(X_i) = \mu$, $V(X_i) = \sigma^2 = E(X_i^2) - \mu^2$,

$E(\bar{X}) = \mu$, $V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

$\hat{\theta}_1 = \sum_{i=1}^{n}(X_i - \bar{X})^2/n$ ：不偏估計量

$\hat{\theta}_2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1)$ ：偏誤估計量

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