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Ontological Proofs of Existence and Non-Existence

Abstract. Caramuels' proof of non-existence of God is compared with Gödel's proof of existence.

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Gödel's ontological proof of necessary existence of a "godlike being" was published posthumously in Volume III of his Collected works [3], but was known and discussed before in several papers (see the references) and in two books, one by Fitting [2] and and one Sobel [7]. Recently Sousedík published his paper *Leibniz and Caramuel's Leptotatos* [9] where he discussed, among other things, the proof of non-existence of God contained in the book *Leptotatos Latine Subtilissimus* by Johann Caramuel von Lobkowicz (1606-1682). Various proofs of non-existence of God are discussed in [7], Chap. XI-XII, mainly in connection with the problem of evil. We shall not discuss them; Caramuel's proof is interesting since it is "ontological" and very simple. In this paper I compare Gödel's proof with Caramuel's explicitly formulating their assumptions (axioms) and showing how Gödel's proof can be changed to give Caramuel's proof. Both systems have consistent sets of axioms but of course their union is inconsistent. At the end I shortly comment, like in my previous papers, on possible relevance (if any) of "ontological proofs" for religious faith.

1. Recalling the logic used and Gödel's proof.

We shall use two-sorted first-order predicate modal logic S5 as in [5, 6]: there is a sort of individuals and a sort of properties. We have variables $x, y, u, \ldots$ for individuals, $X, Y, \ldots$ for properties, two constants G (property of being godlike) and E (property of actually existing), one binary predicate of application *Appl* (the property X applies to the individual x, i.e. x has the property X, notation $Appl(X, x)$ or just $X(x)$) and one unary predicate P on properties ($P(X)$ is read "X is a positive property"). Further

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there are logical connectives, quantifiers $\forall, \exists$ and modalities $\Box, \Diamond$ (necessarily, possibly). $\exists$ is defined from $\forall$ and $\Diamond$ is defined from $\Box$ in the obvious way. We also define $(\exists^E x)\varphi$ to be $(\exists x)(E(x)\&\varphi)$ and similarly $(\forall^E x)\varphi$ to be $(\forall x)(E(x) \to \varphi)$. (If the reader prefers to use second order logic with variables for properties as second order variables, it is possible but has no advantage.)

The semantics is that of Kripke models with varying sets of actually existing individuals (depending on possible worlds), all being subsets of a set of potentially existing individuals. Details of this semantics are postponed to Section 3. Axioms are those of propositional and predicate logic plus modal axioms of the logic S5; deduction rules are modus ponens, generalization and necessitation. (See [5, 6] for detail; note that there is no assumption on E, in particular it is not assumed that in each possible world there are some actually existing individuals.)

Our variant of Gödel's proof has three axioms; first we present the two of them:

(A1) $G(x) \equiv (\forall Y)(P(Y) \to \Box(Y(x)))$,(A2) $P(G)$ and $P(E)$

(thus: x is godlike iff it necessarily has all positive properties; and both godlikeness and actual existence is a positive property).

Lemma 1. *The following formulas are provable from (A1) and (A2):*

$G(u) \to \Box G(u)$, $G(u) \to \Box E(u)$, *thus*

$G(u) \to \Box(G(u) \,\&\, E(u))$.

This is immediate.

Lemma 2. *From (A1) and (A2) one proves*

$$\Diamond(\exists^E u)G(u) \to \Box(\exists^E u)G(u).$$

(In words: If possibly there actually exists a godlike being then necessarily there actually exists a godlike being.)

Proof. We have proved $G(u) \to \Box(G(u) \,\&\, E(u))$, thus

$(\exists u)G(u) \to (\exists u)\Box(G(u) \,\&\, E(u))$,

$(\exists u)G(u) \to \Box(\exists u)(G(u) \,\&\, E(u))$,

$(\exists u)G(u) \to \Box(\exists^E u)G(u)$,

$(\exists^E u)G(u) \to \Box(\exists^E u)G(u)$.

Let φ stand for $(\exists^E u)G(u)$. We have just proved $\varphi \to \Box\varphi$. This gives the provability of the following formulas:

$\Box(\varphi \to \Box\varphi)$, $\Diamond\varphi \to \Diamond\Box\varphi$, $\Diamond\varphi \to \Box\varphi$. ∎

Now we formulate a third axiom (A3), just saying

$$(\forall Y)(P(Y) \rightarrow \Diamond(\exists^E u)Y(u).$$

(Some variants of Gödel's proof have (A3) as a theorem, proved from some other axioms, but this is not important for or purpose.) The axiom is equivalent to the statement that a necessarily actually empty property is not positive. Now the following theorem is almost evident:

THEOREM 1. *(A1), (A2), (A3) prove necessary actual existence of a godlike being, i.e.* $\Box(\exists^E u)G(u)$.

To close this section remembering Gödel's ontological proof; let us mention that the theory with the axioms (A1), (A2), (A3) is consistent, see Section 3.

2. Caramuel's proof of non-existence.

Caramuel's proof goes as follows (freely translated from [9]): The statement "there is nothing" is false but not necessarily false, it is possible. But if it is possible that there is nothing then it is possible that there is no god. But a god which can not be is no real god. Thus there is no god.

Caramuel's argument for the possibility of the statement "there is nothing" is strange; he argues that from the assumption "there is nothing" one can deduce no affirmative statement (ex propositione pure negativa, qualis est 'nihil est' nulla inferi potest affirmativa), hence the the assumption is non-contradictory and possible. Note that this depends on the logic you have: e.g. in the classical predicate logic with equality the formula $(\exists x)(x = x)$ is provable and hence its negation, saying that there is nothing, is contradictory and implies everything – ex falso quodlibet. Also note that as Sousedík tells, there is no doubt that Caramuel did not have any doubts about the existence of God; he also offered some refutation of "there is nothing".

But let us formalize, again using modal logic. We take two axioms:

(A4) $\Diamond(\forall x)\neg E(x)$,

(*) $(\forall x)[G(x) \rightarrow \Box(G(x)\&E(x))]$.

Recall that (*) is a consequence of the axioms (A1) and (A2), see above. The axiom (A4) says that possibly there is no really existing individuum; and (*) says that a godlike individuum is necessarily godlike and necessarily actually existing. Now it is clear that $(\exists x)G(x)\&(*)\&(A4)$ implies a contradiction (since the godlike x would necessarily actually exist, i.e. actually exist in each possible world so that there would be no possible world in which no object actually exists. Thus we have the following

THEOREM 2. *The axioms (*) and (A4) prove necessary actual non-existence of a godlike being.*

PROOF. From what we just have said we see that the axioms (*) and (A4) prove $\neg(\exists x)G(x)$ by the deduction theorem; thus we get $\neg(\exists x)(G(x)\&E(x))$ and $\Box\neg(\exists^E x)G(x)$ by necessitation. ■

COROLLARY. *The axioms (A1), (A2), (A4) prove necessary actual non-existence of a godlike being.*

The last axioms present just a Caramuel's style variant of Gödel's axioms; we may suppose that Caramuel would accept (A1), (A2) as good basis for the claim $(*)$ that a god who possibly does not (actually) exist is not God.

3. Consistency and summary.

To show the consistency of the two theories of Theorem 1 and 2 we produce Kripke models of them (see [5, 6] for details). A Kripke model $\mathbf{K}$ (for our language) consists of a non-empty set W of possible worlds, non-empty sets M and $Prop$ of individuals and properties, a subset Pos of $Prop$ (set of positive properties) two elements $\hat{G}$ and $\hat{E}$ of Pos and a mapping $Eval$ assigning to each triple $\langle$property, individual, possible world$\rangle$ a truth value 1 or 0 (determining whether the individual has the property in the given possible world). Thus $\mathbf{K} = \langle W, M, Prop, Pos, \hat{G}, \hat{E}, Eval\rangle$. For each formula φ, each evaluation of its free individual variables by elements of M, each evaluation of its free property variables by elements of $Prop$ and each possible world this defines the truth value of φ in the world w (for the individuals and properties taken). In particular, the truth value of a formula beginning by a modality $\Diamond, \Box$ is independent of w; this is the truth value of φ in the Kripke model. Obviously, for each w the set of individuals actually existing in w is the set of individuals x for which the truth value of $E(x)$ in w is 1, i.e. the triple $\langle \hat{E}, x, w\rangle$ gets value 1 in $Eval$.

EXAMPLE 1. Take any non-empty $W, M, Prop$ with $Prop$ containing $\hat{G}, \hat{E}$ and an element $g \in M$. Define the truth evaluation by postulating that in each possible world w, $G(x)$ is true iff x is g, $E(x)$ is true (all individuals exist actually in each world), properties different from G and E are interpreted arbitrarily and each property Y is positive iff g necessarily has Y. Clearly this is a model of (A1), (A2), (A3), thus the system is consistent.

Example 2. Take $W, M, Prop$ as above and a world $w_0 \in W$. Let $E(x)$ be false in w_0 for all individuals x (nothing actually exists in w_0, $G(x)$ false in each w, otherwise arbitrary. This is a model of (A1), (A2), (A4), thus the system is consistent.

Summary. We have two ontological proofs, both starting with some axioms that are consistent. The reader wanting to rely on one of them has to decide which of these axiom systems is intuitively more acceptable (if any). In particular, the proof of Caramuel is not a proof from nothing but uses the assumption that possibly there exists nothing, which is a consistent but unprovable assumption as our Example 1 shows: in that model every possible world has a non-empty set of actually existing individuals. To conclude let us refer to the discussion of questionable relevance of ontological proofs for religious faith in my [6, 5]. Let us at least repeat that religious (Christian) belief is not first a matter of accepting some axioms but accepting a style of life (following an invitation); as Pascal said: Fire. God of Abraham, of Isaac, of Jacob, not of philosophers and scientists. Nevertheless, "ontological proofs" are interesting both from the point of modal logic as well as in relation to historical theological attempts to prove God.

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