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Intensional Quantified Multimodal Logic as Fragment of HOL

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QML

(predicate vars)

$\in IV$ (individual vars)

$$s,t ::= k^n(X^1,\ldots,X^n) \mid K^n(X^1,\ldots,X^n) \mid \neg s \mid s \vee t \mid \forall X.s \mid \forall K.s \mid \Box_r t$$

$\in Sym$ (constant symbols)

Model:

Domain (we here consider constant domain sem.)

$$M = (W, (R_r)_{r \in S}, D, (P_n)_{n \in \mathbb{N}}, (I_{(v,w)})_{v,w \in W})$$

possible worlds

non-empty subsets of $(\underbrace{W \times \ldots \times W}_{n \in \mathbb{N}})$

accessibility relations between worlds in W (S is an index set)

$I_{(v,w)}$ are interpretation functions mapping each n-place relation symbol k to some n-place relation on D in world w.

Important: The difference to the Logica Universalis paper is only wrt Interpretation I. I is here dependent on two worlds!

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variable assignment: $g = (g^{iv}, g^{pv})$ pair of maps

$g^{iv} : W \to W \to IV \to D$

$g^{pv} : W \to W \to PV \to D$

two worlds as first arguments

Validity: See next page

Validity: $\underset{\text{world we are currently in}}{M, g, \overset{\text{original actual world}}{v}, w \models s}$

$M, g, v, w \models k^n(X^1, \ldots, X^n)$ iff $\langle g^{iv}(X^1), \ldots, g^{iv}(X^n) \rangle \in I_{(v,w)}(k^n)$

$M, g, v, w \models K^n(X^1, \ldots, X^n)$ iff $\langle g^{iv}(X^1), \ldots, g^{iv}(X^n) \rangle \in g^{pr}(v,w)(K^n)$

$M, g, v, w \models \neg s$ iff $M, g, v, w \not\models s$

$M, g, v, w \models s \vee t$ iff $M, g, v, w \models s$ or $M, g, v, w \models t$

$M, g, v, w \models \forall^i X. s$ iff $M, ([d/X]g^{iv}(v,w), g^{pr}), v, w \models s$ for all $d \in D$

$M, g, v, w \models \forall^p \underset{\text{arity-}n}{K^n}. s$ iff $M, (g^{iv}, [p/K]g^{pv}(v,w)), v, w \models s$ for all $p \in P_n$

$M, g, v, w \models \Box_r t$ iff $M, g, v, u \models s$ for all $u \in W$ such that $\langle w, u \rangle \in R_r$

Embedding: $k \to \hat{k} : \underbrace{\overbrace{\mu \to \ldots \to \mu}^{n}}_{\text{individuals}} \to \underbrace{(\overbrace{i \to i}^{\text{worlds}} \to o)}_{\text{new type } \sigma}$

(ebenso: $K \to \hat{K}$)

$\neg_{\sigma \to \sigma} = \lambda \varsigma_\sigma \lambda V_i \lambda W_i . \neg(\varsigma V W)$

$\vee_{\sigma \to \sigma \to \sigma} = \lambda \varsigma_\sigma \lambda \varphi_\sigma \lambda V \lambda W . (\varsigma V W) \vee (\varphi V W)$

$\Box_r = \lambda \varsigma \lambda V \lambda W . \forall U . \neg(r W U) \vee \varsigma(V U)$

$\Pi^i_{(\mu \to \sigma) \to \sigma} = \lambda \Phi_{\mu \to \sigma} . \lambda V \lambda W \, \forall X_\mu . \Phi X V W$

$\Pi^p_{((\underbrace{\mu \to \ldots \to \mu}_{n} \to \sigma) \to \sigma) \to \sigma} = \lambda \Phi_{(\underbrace{\mu \to \ldots \to \mu}_{n} \to \sigma) \to \sigma} . \lambda V \lambda W \, \forall K^n_{(\mu \to \ldots \to \mu \to \sigma)} . \Phi K^n V W$