

The Inconsistency in Gödel's Ontological Argument — A Success Story for AI in Metaphysics —

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Motivation

Vision of Leibniz (1646–1716): *Calculus!*



Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo ... dicere: calculemus. (Leibniz, 1684)



Required:
characteristica universalis and **calculus ratiocinator**

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other ...: Let us calculate.

(Translation by Russell)

Application: Gödel's Ontological Argument

Gödel's Manuscript: Identifying the Inconsistent Axioms

On the Logical Basis of the Ontological Argument Feb. 10, 1970

At 1: $P(\phi) \cdot P(\psi) \supset P(\phi \cdot \psi)$
At 2: $P(\phi) \cdot P(\psi) \supset P(\psi \cdot \phi)$
P1: $G(x) \equiv (\psi) [P(\psi) \supset \psi(x)]$ (God)
P2: $\phi \text{ Ess. } x \equiv (\psi) [\psi(x) \supset N(\exists y) [P(y) \supset \psi(y)]]$ Essence of x
P3: $\neg \phi \cdot \neg \psi = N(\phi \cdot \psi)$ Necessity
At 2: $P(\phi) \supset N \neg P(\phi)$
 $\neg P(\phi) \supset N \neg P(\phi)$ because it follows from the nature of the property
Th. $G(x) \supset G \text{ Ess. } x$
Def. $E(x) \equiv (\phi) [\phi \text{ Ess. } x \supset N \neg \phi(x)]$ Necessary Existence
At 3: $P(E)$
Th. $G(x) \supset N(\exists y) G(y)$
hence $(\exists x) G(x) \supset N(\exists y) G(y)$
" $M(\exists x) G(x) \supset M N(\exists y) G(y)$
" $\supset N(\exists y) G(y)$
any two essences of x are nec. equivalent
exclusive or * and for any number of individuals

the system of $M(\exists x) G(x)$ means "all pos. props. are compatible". This is true because of:
At 4: $P(\phi) \cdot \phi \supset N \neg \psi \supset P(\psi)$ which implies
 $\begin{cases} x=x & \text{is positive} \\ x \neq x & \text{is negative} \end{cases}$
But if a system S of pos. props. were inconsistent it would mean that the same prop. A (which is positive) would be $x \neq x$
Positive means positive in the moral axiomatic sense (independently of the accidental structure of the world). (Only in the ax. sense, it is not purely a logical matter.)

Inconsistency	Scott
$\forall \phi [P(\neg \phi) \rightarrow \neg P(\phi)]$	A1 (\supset)
$\forall \phi \forall \psi [(P(\phi) \wedge \Box \forall x [\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$	A2
$\phi \text{ ess. } x \leftrightarrow \forall \psi (\psi(x) \rightarrow \Box \forall y (\phi(y) \rightarrow \psi(y)))$	D2*
$NE(x) \leftrightarrow \forall \phi [\phi \text{ ess. } x \rightarrow \Box \exists y \phi(y)]$	D3
$P(NE)$	A5

Scott's vs. Gödel's Version

Scott's Version of Gödel's Axioms, Definitions and Theorems

- Axiom A1** Either a property or its negation is positive, but not both: $\forall \phi [P(\neg \phi) \leftrightarrow \neg P(\phi)]$
- Axiom A2** A property necessarily implied by a positive property is positive: $\forall \phi \forall \psi [(P(\phi) \wedge \Box \forall x [\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$
- Thm. T1** Positive properties are possibly exemplified: $\forall \phi [P(\phi) \rightarrow \Diamond \exists x \phi(x)]$
- Def. D1** A God-like being possesses all positive properties: $G(x) \leftrightarrow \forall \phi [P(\phi) \rightarrow \phi(x)]$
- Axiom A3** The property of being God-like is positive: $P(G)$
- Cor. C** Possibly, God exists: $\Diamond \exists x G(x)$
- Axiom A4** Positive properties are necessarily positive: $\forall \phi [P(\phi) \rightarrow \Box P(\phi)]$
- Def. D2** An essence of an individual is a property possessed by it and necessarily implying any of its properties: $\phi \text{ ess. } x \leftrightarrow \phi(x) \wedge \forall \psi (\psi(x) \rightarrow \Box \forall y (\phi(y) \rightarrow \psi(y)))$
- Thm. T2** Being God-like is an essence of any God-like being: $\forall x [G(x) \rightarrow G \text{ ess. } x]$
- Def. D3** Necessary existence of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall \phi [\phi \text{ ess. } x \rightarrow \Box \exists y \phi(y)]$
- Axiom A5** Necessary existence is a positive property: $P(NE)$
- Thm. T3** Necessarily, God exists: $\Box \exists x G(x)$

Difference to Gödel (who omits this conjunct)

Bla

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