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von Bruno Woltzenlogel Paleo

Automated Verification and Reconstruction of Gödel's Proof of God's Existence

Attempts to prove the existence (or non-existence) of God by means of purely logical arguments are an old tradition in philosophy and theology, dating back at least to 1078 a.C., when St. Anselm of Canterbury proposed his famous ontological argument. The proof proposed by the Austrian logic genius Kurt Gödel (1906-1978) is a modern culmination of this tradition, following particularly the footsteps of Leibniz.

Gödel worked on the proof for decades and shared a two-page manuscript with Dana Scott in 1970. There Gödel defines God as a being who possesses all positive properties. "Positiveness" of a property is a primitive notion in Gödel's theory: it is the only concept left undefined; however, its meaning is not arbitrary, but constrained by five reasonable (but certainly debatable) axioms. They are:

A1) Either a property or its negation is positive, but not both
A2) A property necessarily implied by a positive property is positive
A3) The conjunction of any number of positive properties is positive
A4) Positive properties are necessarily positive
A5) Necessary exemplification of an individual's essences is a positive property (and a property is an essence of an individual if it necessarily implies any property possessed by the individual).

For the derivation of the main theorem (T3: necessarily, God exists), the following auxiliary lemmas are proposed:

T1) Positive properties are possibly exemplified
C1) Possibly, God exists
T2) Being god is an essence of any God
L2) If God's existence is possible, then it is necessary

The computer-assisted verification of Gödel's proof by Bruno Woltzenlogel Paleo, at the Technische Universität Wien, and Christoph Benzmüller, at the Freie Universität Berlin, established a new bridge between computer science and theoretical philosophy and theology, two fields that had so far remained mostly untouched by advances in artificial intelligence.

Up to now, the use of computers for metaphysical investigations had been hindered by the lack of expressiveness of the logical languages employed by the available theorem proving systems. Gödel's proof, for example, requires a complex higher-order modal logic to adequately formalize adverbs such as "possibly" and "necessarily", to allow quantification over properties and reason about their positiveness.

As Gödel's manuscript does not contain detailed proofs of the intermediary lemmas, it can be considered more a proof sketch than an actual proof. Therefore, its verification also required the automatic reconstruction of all the missing parts. To overcome this challenge,

the production of a draft overview of the proof, written in natural deduction and showing the dependencies of lemmas on axioms and previous lemmas was an important first step.

INFORMAL OVERVIEW OF THE PROOF

Gödel proves God's necessary existence by first proving that, if God's existence is at all possible, then it must be necessary (Lemma L2). This idea is already present in St. Anselm's and Descartes' arguments. Leibniz argued that these arguments were incomplete because they assumed the possibility of God's existence (Corollary C1) without any proof. For Leibniz, this should be derivable from the definition of God as a perfect being and from the notion of perfection. This idea can be clearly recognized in Gödel's proof: from axioms A1 and A2, one can derive that, for any positive property, the existence of a being having this property is possible (Theorem T1). As God is defined as a being who possesses all positive properties, the property of being God-like must be, by axiom A3, a positive property as well. C1 then follows trivially from T1. The hardest lemmas in Gödel's proof are T2 and L2. For that, Gödel had to formulate a non-trivial definition of "essence", in order not only to derive T2 from A1 and A4 but also to state an arguably acceptable axiom A5 allowing the derivation of L2 from T2 and other axioms from the modal logic S5

The next step was the embedding of modal logic into higher-order logic, according to techniques described in previous works by Christoph Benzmüller and Lawrence Paulson. This allowed the use of existing higher-order automated and interactive theorem proving systems such as LEO-II, Satallax, Coq, Isabelle and Nitpick. The result is: not only we now know for sure that God's necessary existence indeed follows from Gödel's axioms, but a flexible infrastructure for reasoning with very expressive modal logics has been developed. This infrastructure can now be used for more mundane problems as well.

The fully automatic reconstruction of the gaps in Gödel's manuscript attests the maturity of our current theorem proving technology. Yet, there is still plenty of room for improvement. Although the automated theorem provers are able to fill the gaps between the lemmas and the final theorem, they are not yet able to prove the final theorem directly from the axioms without the help of the auxiliary lemmas, even when executed in powerful hardware such as the Vienna Scientific Cluster. This is evidence that Gödel's proof is non-trivial and may be used as a benchmark to measure the progress in automated reasoning in the near future.

The automated reasoning tools were also capable of quickly obtaining various new results of philosophical interest about Gödel's proof. They determined, for example, that Gödel's original axioms were actually inconsistent, but Scott's correction of the definition of essence makes them consistent. They showed that the weak modal logic KB is sufficient for Gödel's proof, and contrary to popular belief, the stronger modal logic S5 is not necessary. And they even confirmed Sobel's controversial criticism known as modal collapse.

This work illustrates the potential for a new computer-assisted theoretical philosophy, where the critical discussion of the underlying concepts and the acceptance of the proposed definitions and axioms remains a human responsibility, but computers can help us to reach correct conclusions more quickly and avoid mistakes of a logical nature. In case of controversies, a computer can check the competing arguments and partially fulfill Leibniz' dictum: Calculemus - Let us calculate! <<

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