

Monads and Mathematics: Gödel and Husserl

Richard Tieszen

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Abstract In 1928 Edmund Husserl wrote that “The ideal of the future is essentially that of phenomenologically based (“philosophical”) sciences, in unitary relation to an absolute theory of monads” (“Phenomenology”, *Encyclopedia Britannica* draft) There are references to phenomenological monadology in various writings of Husserl. Kurt Gödel began to study Husserl’s work in 1959. On the basis of his later discussions with Gödel, Hao Wang tells us that “Gödel’s own main aim in philosophy was to develop metaphysics—specifically, something like the monadology of Leibniz transformed into exact theory—with the help of phenomenology.” (*A Logical Journey: From Gödel to Philosophy*, p. 166) In the *Cartesian Meditations* and other works Husserl identifies ‘monads’ (in his sense) with ‘transcendental egos in their full concreteness’. In this paper I explore some prospects for a Gödelian monadology that result from this identification, with reference to texts of Gödel and to aspects of Leibniz’s original monadology.

Keywords Husserl · Gödel · Leibniz · Monadology · Logic

In a book that contains an extensive record of his discussions with Kurt Gödel, *A Logical Journey: From Gödel to Philosophy*, Hao Wang tells us that

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See also my forthcoming book *After Gödel: Platonism and Rationalism in Mathematics and Logic* (Oxford University Press).

R. Tieszen (✉)

Department of Philosophy, San José State University, San Jose, CA 95192-0096, USA
e-mail: RichardTieszen@aol.com

Gödel's own main aim in philosophy was to develop metaphysics—specifically, something like the monadology of Leibniz transformed into exact theory—with the help of phenomenology (Wang 1996, p. 166).

Now consider the following statement:

The ideal of the future is essentially that of phenomenologically based (“philosophical”) sciences, in unitary relation to an absolute theory of monads.

Is this latter statement a quote from Gödel? It seems that it very well could be, but in fact it is a passage from a draft of Edmund Husserl's *Encyclopedia Britannica* article “Phenomenology”, written in 1928 (Husserl 1927–1928, pp. 191–194). It is known that Gödel began to study Husserl's work in 1959 and that he continued this study through the nineteen seventies. He had studied Leibniz's philosophy already in the nineteen thirties and Wang tells us that he worked intensively on Leibniz from 1943 until 1946. There are notes on both Leibniz and Husserl in Gabelsberger shorthand in the Gödel *Nachlass* but we are only starting to learn what is in them. Gödel also read and evidently appreciated the essay *Eine neue Monadologie* (1917) by one of Husserl's students, Dietrich Mahnke (van Atten and Kennedy 2003, p. 457). Mahnke obtained his doctoral degree with Husserl in 1922, writing a thesis titled *Leibnizens Synthese von Universalmathematik und Individualmetaphysik*. This thesis was published in Husserl's *Jahrbuch für Philosophie und phänomenologische Forschung* in 1925.¹

In Sect. 1 of this paper I present some comments of Gödel on monads and mathematics and report some impressions of Hao Wang on this topic, while in Sect. 2 I present some of Husserl's comments about monads and mathematics. Section 3 indicates what some of the elements of a new Gödelian monadology might be were we to follow Husserl's comments. Finally, in Sect. 4 I indicate briefly how we might understand Gödel's critiques of Hilbert, Carnap, and mechanist views of mind if ‘monads’ are in the picture. In Sects. 1 and 2 I quote a lot of text, both to inform readers who are not aware of this material and also to provide the background for the comments in the later sections.

Since Gödel himself does not tell us much about how a new Gödelian monadology should be developed with the help of Husserl's phenomenology (unless there are notes in Gabelsberger shorthand on the matter in the *Nachlass*), it is difficult to determine which elements of Leibniz's original monadology he would want to keep and whether he would favor modifications of it that are suggested by Husserl's transcendental phenomenology. On a number of key points one could go either Leibniz's way or one could go Husserl's way. Leibniz's *Monadology* is suffused with theology, for example, but Husserl's phenomenology is not. Theological ideas are virtually non-existent in Husserl's philosophical writings, and it is not clear how substantive theological claims could be supported on the basis of Husserl's methodology of the *epochē* and his views on evidence and intuition. In §58 of *Ideas I*, for example, Husserl extends the phenomenological

¹ Mahnke's work predates Gödel's technical and philosophical work. Although his writings are certainly of interest, he does not consider what a new Gödelian monadology would look like.

reduction to the existence of God. There are not many Leibnizian monadologists around these days, and perhaps there never were very many. There are many contemporary philosophers, however, who would regard various claims in Leibniz's monadology as preposterous. If Gödel means to go Leibniz's way on a number of the main points then I think his view will be much more difficult to defend. I do not know which path Gödel would want to take on many of the issues but I would argue that with all of the philosophical and mathematical work that occurred between the time of Leibniz and Husserl the phenomenological monadology suggested by Husserl, on the whole, is more defensible. I will provide a few indications below about why I think this is so.

1 Some Comments of Gödel on Plato, Leibniz, Kant and Husserl

We know from the work of Wang and others that Gödel's favorite philosophers were Plato, Leibniz, and Husserl. Let me quote some passages from Wang that are, I think, important for indicating, if only very generally, how ideas in the work of Plato, Leibniz, and Husserl were related in Gödel's thinking:

In his philosophy Gödel tried to combine and go beyond the main contributions of his three heroes: Plato, Leibniz and Husserl. Leibniz had defined the ideal by giving a preliminary formulation of monadology. Husserl had supplied the method for attaining this ideal. Plato had proposed, in his rudimentary objectivism in mathematics, an approach that could serve as foundation for Husserl's method and, at the same time, make plausible for Gödel the crucial belief that we are indeed capable of perceiving the primitive concepts of metaphysics clearly enough to set up the axioms (Wang 1996, p. 289).

Note that the claim here is that Leibniz gave a 'preliminary' formulation of monadology. The method that Husserl supplied, as we will see below, is the method of the phenomenological reduction or *epoché*, which includes what Husserl calls the 'eidetic reduction'. Without the eidetic reduction, which is supposed to help us to make clear the *essence* of the phenomena to be studied, the entire project would be indistinguishable from the forms of naturalism, empiricism and positivism that Gödel rejects. Plato's 'objectivism' is described in this passage as 'rudimentary' but as providing a foundation for Husserl's method of the eidetic phenomenological reduction, especially insofar as it makes plausible for Gödel the idea that we are capable of perceiving the primitive concepts of metaphysics clearly enough to set up axioms of metaphysics. For the later Husserl, (abstract) concepts in any domain of thinking are said to be intuited or perceived and clarified in eidetic or 'categorical' intuition. I return to these themes below.

Kant is, of course, also an important figure in Gödel's thinking. We know that Gödel was interested in aspects of Kant's transcendental idealism, and that Kant figures into Gödel's views on time and relativity theory in a significant way. The following passage from Wang provides some indication of how Gödel thought that Kant fits into the mix with his favorite philosophers Plato, Leibniz and Husserl:

Before 1959 Gödel had studied Plato, Leibniz, and Kant with care: his sympathies were with Plato and Leibniz. Yet he felt he needed to take Kant's critique of Leibniz seriously and find a way to meet Kant's objections to rationalism. He was not satisfied with Kant's dualism or with his restriction of intuition to sense intuition, which ruled out the possibility of intellectual or categorial intuition. It seems likely that, in the process of working on his Carnap paper in the 1950s, Gödel had realized that his realism about the conceptual world called for a more solid foundation than he then possessed. At this juncture it was not surprising for him to turn to Husserl's phenomenology, which promises a general framework for justifying certain fundamental beliefs that Gödel shared: realism about the conceptual world, the analogy of concepts and mathematical objects to physical objects, the possibility and importance of categorial intuition or immediate conceptual knowledge, and the one-sidedness of what Husserl calls "the naive or natural standpoint" (Wang 1996, p. 164).

This passage suggests that the way to meet Kant's objections to rationalism, which certainly apply to Leibnizian rationalism, is to turn to Husserl's work.² Wang explicitly mentions Husserl's notion of categorial intuition (or 'intellectual intuition') here. In a draft letter of 1972 to Gian-Carlo Rota (van Atten and Kennedy, p. 446) Gödel says that

I believe that his [Husserl's] transcendental phenomenology, carried through, would be nothing less than Kant's critique of pure reason transformed into an exact science, except for the fact that [in footnote: Kant's subjectivism and negativism for the most part would be eliminated] the result (of the 'critique') would be far more favorable for human reason.

The Kantian critique of reason was clearly too restrictive by Gödel's sights. Husserl agrees that it is too restrictive, as we will see below in a number of quotations in which Husserl portrays the phenomenological method as a way to develop and defend a new kind of scientific rationalism that avoids the excesses of older forms of rationalism and yet skirts any kind of mysticism.

Coming now to Leibniz, we have seen that Gödel was interested in some updated version of a monadology that used the methodology of transcendental phenomenology (see also Wang 1996, pp. 55, 288, 309). There is, however, one point on which he gives fairly clear signs of diverging significantly from Leibniz. This concerns Leibniz's aspirations for the universal characteristic. In Leibniz's writings on logic, universal characteristic, and *calculus ratiocinator* one finds assertions such as the following from *On the Universal Science: Characteristic (XIV)*:

² All of the 'fundamental beliefs' mentioned at the end of this passage—realism about the conceptual world, the analogy of concepts and mathematical objects to physical objects, the possibility and importance of categorial intuition or immediate conceptual knowledge, and the one-sidedness of what Husserl calls "the naive or natural standpoint—were the subject of discussions I had with Wang in the nineteen eighties about Gödel and Husserl. Many of my comments below are shaped by the exchanges I had with Wang.

I think that controversies will never end nor silence be imposed on the *sects*, unless complicated reasonings can be reduced to simple *calculations*, and words of vague and uncertain meaning to determinate *characters*.

Once this is done, then when a controversy arises, disputation will be no more needed between two philosophers than between two computers. It will suffice that, pen in hand, they sit down to their abacus and...say to each other: *let us calculate*.

Gödel evidently thought his own work in the foundations of mathematics, the incompleteness theorems in particular, showed that a mechanist or computational view of reason of the kind found in Leibniz's work on logic would have to be qualified or perhaps even abandoned. In a note in his *Nachlass* (see van Atten and Kennedy 2003, p. 433) Gödel says: "The universal characteristic claimed by Leibniz (1677) does not exist. Any systematic procedure for solving problems of all kinds would have to be nonmechanical." Gödel amended the first sentence of this note to read: "The universal characteristic claimed by Leibniz (1677) if interpreted as a formal system does not exist."

For Gödel, however, this did not mean abandoning a rationalistic optimism about solving open problems in mathematics and logic. Leibniz is a rationalist who is interested in philosophy as a rational (not empirical) universal science. He is interested in deciding mathematical and other problems by human reason through the analysis of concepts, although in his writings on logic he tends to think of decidability in a mechanical way. For Leibniz, as for other classical rationalists, concepts of reason, including those of logic and mathematics, are exact and our grasp of such concepts either is or can be made clear and distinct, whereas empirical knowledge lacks, in various degrees, just these features. Leibniz holds that the science of possibilities and necessities precedes sciences of actualities. In Husserlian language, one might say that in science, as an undertaking of reason, essence precedes existence. I would argue that Gödel retained a rationalistic optimism about mathematical problem-solving on the basis of analyses of concepts but that by the time of his 1961 manuscript on Husserl he shifted the philosophical foundation for his optimism from Leibniz and Hilbert to Husserl (Tieszen 2006). The rationalistic optimism in his later writings is not based on a mechanist conception of reason but rather on a conception of the capacity of the monad for a form of reason that allows for the possibility of finding systematic and finite but non-mechanical methods for the decision of mathematical questions on the basis of clarification of the intuition of the abstract concepts (meanings of the terms) involved in the problems (Tieszen 1992, 1998, 2002). The appeals here to the grasp or intuition of meaning, and to the fact that this meaning is 'abstract' (connecting meaning with a kind of Platonism), are closely related to Gödel's view of the philosophical consequences of his incompleteness theorems (see Sect. 4 below). I think the ideas about Husserl and meaning clarification that Gödel presents in his 1961 manuscript are compatible with the rationalist's belief that reason can arrive at clarity, exactness, rigor. It is just that these latter features are not to be associated exclusively with mechanical calculation. One can contrast the confused, inexact ideas of sensory perception (intuition) with clear, exact ideas of reason, i.e., of eidetic or categorial intuition,

independently of the supposition that all mathematical ideas have to be amenable to mechanical calculation. There are plenty of examples of clarification of the meaning of mathematical concepts in the history and practice of mathematics that have not issued in mechanically decidable statements or definitions.

I now turn to some of Gödel's specific comments about Husserl, as recorded by Wang. We already saw above that Wang says that it was not surprising for Gödel to turn to Husserl's phenomenology in the late nineteen fifties after becoming dissatisfied with his Carnap paper, because it promised a general framework for justifying certain fundamental beliefs that Gödel shared: realism about the conceptual world, the analogy of concepts and mathematical objects to physical objects, the possibility and importance of categorial intuition or immediate conceptual knowledge, and the one-sidedness of what Husserl calls 'the naive or natural standpoint'.

Wang also recorded in his notes certain direct comments of Gödel on Husserl. I reproduce a few of these here, in order to refer to them in the analysis that follows:

Husserl's is a very important method as an entrance into philosophy, so as to finally arrive at some metaphysics. Transcendental phenomenology with *epochē* as its methodology is the investigation (without knowledge of scientific facts) of the cognitive process, so as to find out what really appears to be—to find the objective concepts (Wang 1996, p. 166).

Husserl used Kant's terminology to reach, for now, the foundations and, afterwards, used Leibniz to get the world picture. Husserl reached the end, arrived at the science of metaphysics [Wang notes that this last sentence is different from what Gödel said on other occasions.] (Wang 1996, p. 166).

Some reductionism is right: reduce to concepts and truths, but not to sense perceptions. Really it should be the other way around: Platonic ideas... are what things are to be reduced to. Phenomenology makes them [the ideas] clear (Wang 1996, p. 167).

Leibniz believed in the ideal of seeing the primitive concepts clearly and distinctly. When Husserl affirmed our ability to 'intuit essences' he had in mind something like what Leibniz believed (Wang 1996, p. 168).

Among other things, these comments of Gödel and Wang reinforce the point that it is Husserl's *transcendental* phenomenology, with its *epochē* (= phenomenological reduction), that is of most interest to Gödel. Gödel mentions the *epochē* explicitly in one of the comments cited above and he also makes other remarks about it that Wang recorded. For example, he says that

Epochē concerns how introspection should be used, for example, to detach oneself from influences of external stimuli (such as fashions of the day) (Wang 1996, p. 169).

For the purposes of this paper, we can briefly characterize the *epochē* as the turning of our regard from objects to the *consciousness of objects*. We typically take ourselves in the 'natural attitude' to be concerned straightforwardly with objects in

our experience in various domains. In reflection, however, we can attend to the nature of our consciousness of objects. The so-called ‘eidetic reduction’ is involved in this turning of our regard, so that we are not concerned with what is individual, private or subjective in consciousness of objects. Unlike one of the standard notions of introspection or ‘inner sense’, the essence analysis associated with the *epochē* is not about what is particular or about what makes one human subject different from another. Instead, it involves a kind of abstraction in which universal features of human consciousness are sought.

One such universal feature that is found in various modes of human consciousness is intentionality. Gödel takes note of this central concept of Husserlian phenomenology:

One fundamental discovery of introspection marks the true beginning of psychology. This discovery is that the basic form of consciousness distinguishes between an intentional object and our being pointed (directed) toward it in some way (willing, feeling, cognizing). There are various kinds of intentional object. There is nothing analogous in physics. This discovery marks the first division of phenomena between the psychological and the physical. Introspection [*epochē*] calls for learning how to direct attention in an unnatural way (Wang 1996, p. 169).

The concept of intentionality is a centerpiece of phenomenological psychology and, as we will see below, in transcendental phenomenology it is viewed as the basis of the constitution of the meaning of being of the world by the monad. When Gödel says that introspection, as used in the *epochē*, calls for learning how to direct attention in an ‘unnatural way’ he is directly mirroring comments of Husserl on how the practice of phenomenological *reflection* on consciousness of objects is not our ‘natural’ frame of mind. Indeed, we are usually engaged in the ‘natural attitude’.

2 Husserl on Plato, Leibniz, and Kant

Husserl was beginning to connect his transcendental phenomenology with ideas in Leibniz’s monadology already around 1917, and this continued in his writings throughout the nineteen twenties and early thirties. In thinking about what a Gödelian monadology must be like, it is very interesting to see how several themes concerning the work of Plato, Leibniz, and Kant mentioned above come together in Husserl’s own writings, especially in works such as the “London Lectures” (Husserl 1922), *Erste Philosophie* (Husserl 1923–1924), the drafts of the *Encyclopedia Britannica* article (Husserl 1927–1928), *Cartesian Meditations* (Husserl 1931), and others.

In the *Cartesian Meditations*, for example, Husserl says that

The [transcendental] ego, taken in full concreteness, we propose to call by the Leibnizian name: monad. Since the monadically concrete ego includes also the whole of actual and potential conscious life, it is clear that the problem of *explicating this monadic ego phenomenologically* (the problem of his

constitution for himself) must include *all constitutional problems without exception*. Consequently the phenomenology of *self-constitution* coincides with *phenomenology as a whole* (Husserl 1931, p. 67).

This is a crucial remark, because we now see for the first time how Husserl is thinking of ‘monads’. *Monads are transcendental egos in their full concreteness*. Transcendental egos in their full concreteness are not ‘mere poles of identity’, but are rather egos with all of the predicates that attach to these poles of identity, so that each monad is distinct from every other monad. We know that Leibniz has a range of different kinds of monads but Husserl’s focus is much narrower. It is on the kinds of ‘monads’ that we are. What Leibniz calls *reasonable* souls (or ‘spirits’ or ‘minds’) are monads that are not only capable of *perception*, which is the internal state of the monad that represents external things, but also of *apperception*, which is consciousness or reflective knowledge of this internal state. Souls with reason are capable of acts of reflection. Acts of reflection, Leibniz says, make possible our knowledge of a priori or necessary truths.

If we are to think of monads as transcendental egos in their full concreteness then monads are just you and me as regarded under the *epochē*. What does this mean? It is not a mystery. As a short gloss on this, we can explain it in the following manner. In reflecting on consciousness of objects I am reflecting on the way in which objects *appear* to me in consciousness. I must restrict myself to appearances, which is another way of saying that I must restrict myself to ‘phenomena’ (from which phenomenology derives its name). In so doing I suspend or withhold claims about any alleged ‘reality’ that is supposed to lie behind appearances of objects, including myself as an object of consciousness. ‘Monads’, in Husserl’s sense, have to be understood accordingly. All I have to go on are appearances of objects, appearances of myself included, and any legitimate claims about what exists or what is real will have to be erected on this basis, in accordance with evidence in our experience that we actually have or can have for objects. Husserl thus says that

Phenomenological transcendental idealism has presented itself as a *monadology*, which, despite all our deliberate suggestions of Leibniz’s metaphysics, draws its content purely from phenomenological explication of the transcendental experience laid open by transcendental reduction, accordingly from the most originary evidence, wherein all conceivable evidences must be grounded... Actually, therefore, phenomenological explication is nothing like “metaphysical construction”... (CM, Husserl 1931, p. 150).

In claiming in this passage that phenomenological explication is nothing like “metaphysical construction” Husserl is distinguishing his view in an important way from Leibniz’s monadology. He is saying, among other things, that phenomenology is not engaged in the ‘naive’ metaphysics of earlier philosophical projects. Monads, as transcendental egos in full concreteness, are not metaphysical objects in an objectionable sense. The *epochē* suspends or brackets claims about any alleged reality that is supposed to lie behind appearances of selves, including souls or spirits. It requires us to stick to evidence and intuition regarding what appears in consciousness. There is no such method of suspension in Leibniz’s philosophy.

The *epoché* is intended to put us on a more secure footing in philosophy. It does not issue in the radical rejection of metaphysics of the logical positivists. Instead, it is a view that emerged in the twentieth century, like logical positivism, but it seeks to go between the radical empiricist skepticism of logical positivism on the one hand and, on the other hand, the questionable aspects of traditional rationalism, in order to proceed beyond the ‘preliminary’ formulation of monadology by Leibniz. In language that Gödel uses in his 1961 text on Husserl (Gödel *1961/?), phenomenology seeks to avoid “the death-defying leap into a new metaphysics” that would only amount to another dubious metaphysical scheme.

Husserl speaks of intentional constitution in these passages, and the concept of intentionality can perhaps be seen as related to Leibniz’s view that monads are entelechies. Leibniz means by this that monads are living active forces that are governed by laws of final causes, not by laws of efficient causes. Leibniz’s view of monads as entelechies has sometimes been associated with vitalism, but I think that the concept of intentionality does not commit us to any kind of vitalism. Leibniz holds that souls act according to laws of final causes, through appetite, ends and means, while bodies act according to laws of efficient causes (see e.g., §78 *Monadology*), and he offers the bold metaphysical hypothesis that the union of soul and body is to be explained on the basis of the pre-established harmony prevailing among all substances. The two realms, that of efficient causes and that of final causes, are in mutual harmony. Once we engage the phenomenological *epoché*, however, and adhere to Husserl’s strictures on evidence and intuition as fulfillment of intention, I do not see how there could be a basis for a *metaphysical* explanation of the union or conformity of the soul and the body in a theory of pre-established harmony. Nonetheless, there are certain vestiges of the division between explanation by efficient causes and explanation by final causes if intentionality is regarded as a real feature of our mental life that is not reducible to purely naturalistic or physical notions. I cannot go into this here.

Continuing with Husserl, there is a long interesting passage from a draft of the *Encyclopedia Britannica* entry that contains language quite similar to some of Gödel’s remarks on Husserl. We are told that

Remarkable consequences arise when one weighs the significance of transcendental phenomenology. In its systematic development, it brings to realization the Leibnizian idea of a universal ontology as the systematic unity of all conceivable a priori sciences, but on a new foundation which overcomes “dogmatism” through the use of the transcendental phenomenological method. Phenomenology as the science of all conceivable transcendental phenomena and especially the synthetic total structures in which alone they are concretely possible—those of the transcendental single subjects [monads] bound to communities of subjects [monads] is *eo ipso* the apriori science of all conceivable beings [*Seienden*]. But [it is the science], then, not merely of the totality of objectively existing beings taken in an attitude of natural positivity, but rather of the being as such in full concretion, which produces its sense of being and its validity through the correlative intentional constitution. It also deals with the being of transcendental subjectivity itself, whose nature is to be

demonstrably constituted transcendently in and for itself. Accordingly, a phenomenology properly carried through is the truly universal ontology, as over against the only illusorily all-embracing ontology in positivity—and precisely for this reason it overcomes the dogmatic one-sidedness and hence the unintelligibility of the latter, while at the same time it comprises within itself the truly legitimate content [of an ontology of positivity] as grounded originally in intentional constitution (Husserl 1927–1928, p. 175).

In this passage Husserl speaks of realizing the Leibnizian idea of the systematic unity of all conceivable a priori sciences on the basis of a new foundation that overcomes dogmatism through the use of the transcendental phenomenological method. The language here is similar to a number of Gödel's pronouncements, as we have seen above. The science that should result, on the basis of transcendental eidetic phenomenology, is not just another 'positive' science. Instead, it is the science of the monads that internally produce the meaning of being through intentional constitution. Human monads, whose consciousness exhibits intentionality, internally constitute the meaning of being of their objects in science through founded acts of consciousness, such as acts of reflection, abstraction, formalization, and imaginative variation. To intend objects, in the sense of intentionality, is to *mean* objects in a certain way. Husserl says that on this basis we can overcome the dogmatic one-sidedness of the positive sciences and provide a correct foundation for such sciences by showing how they have their origins in intentional constitution.

In notes for the "London Lectures" some of these themes are repeated:

Transcendental phenomenological subjectivity or monadologism as [is a] necessary consequence of the transcendental phenomenological attitude. The knowledge that any objectivity is only what it is through intentional meaning or significance shows that there is only one possibility for an absolute and concrete being: the being of a concretely full transcendental subjectivity. It is the only genuine "Substance". The *ego* is what it is from its own fundamental meaning. The *ego* is in so far as it constitutes itself for itself as being. All other being is merely relative to the ego and is encompassed within the regulated intentionality of subjectivity (Husserl 1922, p. 72).

Here Husserl says that any objectivity, which would include mathematical and logical objectivity, is what it is only through intentional meaning conferral by the monad. The monad is the only genuine 'substance'. Note that Husserl puts the term 'substance' in scare quotes. This is to indicate how its sense is modified by the *epoché*: Leibniz says that a monad is a simple substance, but now we cannot speak of such a simple substance as though it were some reality behind all possible appearances. The idea that in science monads constitute the meaning of being of the objects toward which they are (intentionally) directed by their (founded) mental acts ('reflection' in Leibniz) plays a very important role in my view of how Gödel's ideas can be developed into a defensible position.

Finally, in a formulation that brings together ideas in Leibniz, Plato, and transcendental philosophy, Husserl says

Thus, as Phenomenology is developed, the Leibnizian foreshadowing of a Universal Ontology, the unification of all conceivable a priori sciences, is improved, and realized upon the new and non-dogmatic basis of phenomenological method. For Phenomenology as the science of all concrete Phenomena proper to Subjectivity and Intersubjectivity, is *eo ipso* an a priori science of all possible existence and existences. Phenomenology is universal in its scope, because there is no a priori which does not depend upon its intentional constitution, and derive from this its power of engendering habits in the consciousness that knows it, so that the establishment of any a priori must reveal the subjective process by which it is established.

... Once the a priori disciplines, such as the mathematical sciences, are incorporated within Phenomenology, they cannot thereafter be beset by “paradoxes” or disputes concerning principles: and those sciences which have become a priori independently of Phenomenology, can only hope to set their methods and premises beyond criticism by founding themselves upon it. For their very claim to be positive, dogmatic sciences, bears witness to their dependency, as branches merely, of that universal, eidetic ontology which is Phenomenology.

... The endless task, this exposition of the *Universum* of the a priori, by referring all objectives to their transcendental “origin”, may be considered as one function in the construction of a universal science of Fact, where every department, including the positive, will be settled on its a priori.

... Thus the ancient conception of Philosophy as the Universal Science, Philosophy in the Platonic, Philosophy in the Cartesian, sense, that shall embrace all knowledge, is once more justly restored (Husserl 1927–1928, pp. 191–194).

This passage also uses language that seems to agree with Gödel’s view that Leibniz’s preliminary formulation of monadology is to be improved and realized upon the new and non-dogmatic basis of phenomenological method. We are, on the basis of this method, to reveal the monadic (subjective) processes by which the a priori is established. Husserl even holds out the promise that, on these grounds, the mathematical sciences cannot be beset with paradoxes, although there is little indication here about how to cash out this promise.

3 Outlines of a Gödelian Monadology?

As I indicated at the beginning of this paper, it is not clear just how closely wedded to Leibniz’s original monadology Gödel wanted to be. We also do not know how close to Husserl’s transcendental phenomenology he wished to remain. If he were to go Husserl’s way, however, then I think that some of the following features would be part of the view: Philosophy, as transcendental phenomenology, can become a rigorous, universal, apriori science, on the basis of its use of the phenomenological eidetic method (*epoché*) to develop a new kind of monadology. This monadology

would be combined with recognition of the objectivity of ideal or abstract objects and concepts of mathematics, logic, and philosophy, and would acknowledge and seek to cultivate either the categorial or eidetic intuition of such objects in order to clarify the meaning of primitive concepts of logic, mathematics, and even metaphysics.³ It is clearly a form of rationalism. It would ideally be used, in connection with Gödel's technical results, in helping to decide open problems in the foundations of mathematics and logic, including higher set theory, and also in providing a foundation for the sciences and for philosophy itself.

It would be a viewpoint that is opposed to logical positivism, naturalism, conventionalism, nominalism, and empiricism about logic, mathematics, and philosophy. It would also be opposed to subjectivism, psychologism, and Aristotelian realism about the concepts and objects of logic, mathematics, and philosophy. It would be the kind of view that lies behind Gödel's critique of Carnap's view of mathematics as syntax of language, and his objections to what in his 1961 manuscript he calls the 'leftward' aspects of Hilbert's formalism about the foundations of mathematics. It would buttress the argument that the human mind (monad) is not a (Turing) machine. We will see below how Gödel connects this last point with his study of Husserl's phenomenology.

What are human monads (as 'simple substances') on this view? They are transcendental egos in their full concreteness. They internally constitute the meaning of being of the world through their intentionality. In the case of (classical) mathematics, logic, and the other a priori sciences, including phenomenology itself, they internally constitute the meaning of the being of their objects (essences, categorial objects) as ideal or abstract and non-mental in a rationally motivated way in founded acts of reflection. Evidence in these domains is acquired on the basis of categorial or eidetic intuition, not just Kantian sensory intuition. As in Kant, however, mere concepts without intuition are empty. Categorial intuition is required for knowledge and is the source of objectivity in mathematics, logic, and the eidetic sciences. In §§29–30 of the *Monadology* Leibniz already said that

Knowledge of necessary and eternal truths... distinguishes us from mere animals and grants us *reason* and the sciences, elevating us to knowledge of ourselves and of God. This possession is what is called our reasonable soul or *spirit*.

By this knowledge of necessary truths and by the abstractions made possible through them, we also are raised to *acts of reflection*... These acts of reflection furnish the principal objects of our reasoning.

Leibniz, as we saw, calls the souls of *rational* animals 'minds'. In *Principles of Nature and Grace, Based on Reason* he says that

³ In the *Logical Investigations* and other works from this period Husserl speaks of 'categorial intuition' in connection with the objects of logic and mathematics but in later works he speaks mostly of 'eidetic intuition', i.e., intuition of essences. Both can be viewed as types of rational intuition, with eidetic intuition focused on essences in particular. I do not have space to go into the differences here but I am interested in both as species of rational intuition.

These souls are capable of performing reflective acts, and capable of considering what is called “I”, substance, soul, mind—in brief, immaterial things and immaterial truths.

For Husserl, the constitution of mathematics and logic indeed takes place through founded acts of reflection and abstraction of different types. It is these kinds of acts that underlie categorial or eidetic intuition.

Gödel’s platonism, in this scheme, would reflect the extent to which the monad internally constitutes in a rational manner the meaning of being of the objects of logic and mathematics as abstract and mind-independent or transcendent. Given the *epochē*, however, this would have to be a form of platonism that I have elsewhere called *constituted platonism* (see Tieszen 2010). Constituted platonism is unlike traditional mathematical platonism since traditional platonists have not been transcendental (phenomenological) idealists. Plato certainly did not speak of the internal constitution of the meaning of being by “monads”, and he is engaged, by Husserl’s sights, in naive metaphysics. This is also true of other traditional mathematical platonists. Constituted platonism has to restrict itself to appearances or phenomena. It cannot access some alleged reality that is supposed to lie behind all of our possible experience and practice with categorial or eidetic intuition.

In this monadology the monad (as a transcendental ego in its full concreteness) can now be combined with a kind of (constituted) platonism about logic and mathematics (unlike in Leibniz and Kant), and with the idea of universal science (as in Leibniz and Plato) in a way that keeps a Kantian transcendental approach in some respects but extends it to mathematics, logic, and philosophy itself, avoiding Kant’s dualism (phenomena and noumena), his restrictions on intuition, his critique of rationalism and his skepticism about ideal or abstract objects (concepts). Elements in the work of Plato, Leibniz, Kant, and Husserl come together in a position in which the monad (as a concrete transcendental ego), in a community of monads, constitutes the meaning of being of its objects in mathematics and logic as ideal or abstract and non-mental and acquires evidence in these domains on the basis of categorial intuition or *Wesensanalyse*.

We have been saying, in accordance with a Leibnizian theme, that monads *internally* constitute the meaning of being of mathematical and logical objects. Leibniz is famous for saying that monads have no windows. Monads do not, for example, interact causally with one another or indeed with anything else. All of their acts are internal. They are not subject to laws of efficient causes. Leibniz says that they are subject instead to laws of final causes. Now if we skip ahead from Leibniz’s views on these matters in the late seventeenth and early eighteenth century to Husserl’s in the twentieth century we find an interesting shift. Husserl’s monads are transcendental egos in their full concreteness. Once we engage the *epochē* we are to refrain from analyzing transcendental egos in terms of causal relations. We do not take them to interact causally with one another or with anything else, in the sense that with the *epochē* we detach from or suspend causal explanation. Causal explanation is indeed to be sought in the natural sciences, including the natural science of psychology, but the *epochē* is supposed to open up a space for the purely phenomenological description and analysis of consciousness,

that is, for the analysis of how monads constitute the meaning of being of the world through their intentional activities. The object-directedness of human consciousness (i.e., intentionality), in particular, is not to be understood in terms of a causal relation to an object. Rather, it has the form ‘monad M *means* x’. Engaging the *epochē*, we speak in terms of appearances or phenomena. Appearances as such are not themselves items in the natural world to be studied by natural sciences. Instead, we appeal to appearances or phenomena in order to provide a *philosophical* account of how consciousness of the natural world is possible. The *epochē* allows us to focus on the internal activities of monads. We can thus understand Leibniz’s claim in this new way. Phenomenology brings ‘monads’ into the twentieth century in this manner, distilling what was a bold metaphysical hypothesis of Leibniz into a much more palatable claim about how to proceed with an analysis of the *phenomenology* of human consciousness.

Eidetic phenomenology is supposed to allow us to investigate how monads internally constitute the meaning of being of the world, and this will extend to causal relations themselves. Monads M will mean that x causes y for certain categories of objects x,y. They can be said to *know* that x causes y for particular x’s and y’s when they have acquired evidence for their causal conjectures. Causal relations are themselves constituted by the monad in accordance with various conditions on evidence. Monads are the source or origin of all constitution. Instead of being derived from sense experience, as empiricism would have it, the category of (efficient) causality, as a formal structure, would be regarded as an a priori condition for the possibility of sense experience and natural science.

In §57 of the *Monadology* Leibniz says that each monad has its own point of view or perspective (he adds that this is the means of obtaining the greatest possible variety, together with the greatest possible order; in other words, it is the means of obtaining as much perfection as possible). Husserl would certainly agree that human consciousness is perspectival. Moreover, Leibniz says that each monad mirrors or reflects all of the others. Husserl’s gloss on this is to say that the constitution of the meaning of being of one objective world (which would include one objective world of mathematics) requires the community of monads, a single universe of compossible monads. Intersubjectivity and the constitution of the meaning of the objective world is held to require a universe of compossible (“harmonious”) monads. Each monad (transcendental ego), to extend the analogy, would presumably be ‘windowless’ but would mirror all of the others if there is to be constitution of one objective world.

Issues about intersubjectivity and the layers of constitution involved in the meaning of being of the objective world have of course been analyzed in great detail by Husserl and others in the phenomenological movement. Husserl presents an analysis in a number of works of how human monads internally constitute awareness of other human monads. He argues that there is a structure internal to each monad that makes it possible to experience other monads. Without going into the many details, one of his ideas is that there is an associative ‘pairing function’ that makes possible the *analogical apperception* of other monads. We do not literally perceive the consciousness of another monad in the way that we perceive the body of the monad. Husserl holds that the body of the other is perceived in sense

experience, but in order to experience another mind I must not only experience the other as an object but also as a subject of experience. I must experience that I am experienced as a subject by another or else there would be no reflection or mirroring. This is supposed to be possible because in my own case my lived body is paired with my consciousness and, on the basis of the perception of the similar lived body of the other, we associate consciousness with that lived body. The consciousness of the other is not literally perceived but is thereby analogically apperceived. The monads have no windows but they are held to have the internal structure of the associative pairing function that makes the constitution of the consciousness of other monads possible. The monads ‘mirror’ one another in this manner, each from its own perspective.

Let us now consider in a little more detail a point about the constitution of mathematical objectivity by the monads. As we saw above, Leibniz says that he thinks controversies will never end nor silence be imposed upon the sects unless complicated reasoning can be reduced to calculations and words to characters in the universal characteristic. If we can affect such a reduction then to settle a dispute we only need to sit down and calculate. Combining Leibniz’s ideas on the universal characteristic and the *calculus ratiocinator* with his monadology we evidently arrive at the view that we will have complete harmony among the monads in mathematics when all mathematical problem-solving is a matter of mechanical decidability (calculation, computation). This is how we can eliminate disagreements among the ‘sects’ in mathematics and logic. Mathematical objectivity ideally requires, as it were, complete harmony among the monads, and complete harmony among the monads is only possible when mathematical problem-solving is reducible to mechanical decidability. In Husserlian language, all constitution by the monads in mathematics would have to be suitably algorithmic. On a Leibnizian view, this is presumably how mathematical objectivity would be constituted for the kinds of limited monads that we are.

As we saw above, Gödel says that the universal characteristic claimed by Leibniz (if interpreted as a formal system) does not exist. Any systematic procedure for solving problems of all kinds would have to be non-mechanical. Gödel thinks that the incompleteness theorems show this. There could not be a single formal system in which these calculations are carried out. So, for Gödel, what is the alternative? The alternative is evidently to be found in the method of the phenomenological *epochē*, with its eidetic reduction, which offers the possibility of finding systematic and finite but non-mechanical methods for the decision of mathematical questions on the basis of clarification of the intuition of the abstract meanings (concepts) associated with the terms involved in the problems. This is the reason why Gödel speaks of meaning clarification in the 1961 manuscript on Husserl. Mathematical objectivity will now have to depend on such meaning clarification, in addition to any assistance that can be derived from formal rigor. Decidability by human reason has to be distinguished from decidability on the basis of mechanical calculation. Harmony among the monads in mathematics might still be possible on such grounds. It would at least remain as an ideal of science that should not be abandoned.

It is evidently to this kind of meaning clarification that we will especially have to resort in unfolding our knowledge of the infinite. In §60 of the *Monadology* Leibniz says that

It is not in the objects of their knowledge, but in the modes of this knowledge that the monads are limited. All of them have a confused knowledge of the infinite; that is of the whole; but they are limited and distinguished by the degrees of distinct perception.

What will be needed is clarification of the meaning of the infinite, which is how Gödel describes the situation in his paper on Cantor's continuum hypothesis (Gödel 1964). There is an item in the *Nachlass* that shows how Gödel was thinking of Husserl's work when he prepared the 1964 version of the Cantor paper. Speaking of higher set theory in connection with his view of mathematical intuition, he says that "Perhaps a further development of phenomenology will, some day, make it possible to decide questions regarding the soundness of primitive terms and their axioms in a completely convincing manner" (*Nachlass* series 4, folder 101, item 040311).

It might be suggested that in Leibniz's work there is a kind of synthesis of Aristotelian and Platonic views in the following sense: human minds come to know about abstract concepts and objects by abstraction (Aristotle), which is limited, whereas all the abstract concepts or objects that exist *already exist* in the mind of God and are in this sense independent of human minds (a kind of Platonism).⁴ As they perfect their knowledge, limited monads come to know about some of these abstract concepts or objects that exist in the mind of God. Unlike Leibniz, Husserl does not employ a hypothesis about the mind of God in order to support a version of platonism. As I see it, he instead thinks of platonism about mathematics and logic as itself constituted by founded activities of human monads. Rational monads constitute the mind-independence and abstractness of mathematical objects on the basis of acts of reflection, abstraction, formalization, idealization, and imaginative variation. One could compare Leibniz and Husserl point by point on monads and mathematics to take note of the occasions on which Leibniz appeals to the existence of God and Husserl refrains from such a hypothesis.

It is possible to read the sketch of the monadology in this paper with or without the God hypothesis in the background but, as indicated above, I think that with the *epoché* in effect we are required to bracket the claim that God exists.

4 Monads and Philosophical Consequences of the Incompleteness Theorems

In his 1961 manuscript "The Modern Development of the Foundations of Mathematics in the Light of Philosophy" Gödel sets up a general schema of possible philosophical worldviews according to their degree and manner of affinity to metaphysics. We obtain a division into two groups, with skepticism, materialism and positivism on one side and spiritualism, idealism, and theology on the other. If one thinks of philosophical doctrines as arranged along a line from left to right in

⁴ Shinji Ikeda suggested such a view in conversation.

this manner then empiricism belongs on the left side and rationalism and a priorism belong on the right. Gödel says that the development of philosophy since the Renaissance has on the whole gone from right to left, and that one sees the impact of this development in mathematics itself. Gödel's philosophical viewpoint, as we indicated, is opposed to 'leftward' views about logic, mathematics, and philosophy such as logical positivism, naturalism, conventionalism, nominalism, and empiricism. It is also opposed to subjectivism, psychologism, and Aristotelian realism about the concepts and objects of logic, mathematics, and philosophy. Gödel brings the philosophical consequences of his incompleteness theorems to bear in his own writing in three main areas in which one sees the impact of the leftward development on mathematics and logic: Hilbert's program, Carnap's early view of mathematics as syntax of language, and the issue whether human minds are machines. I consider each of these briefly in relation to the sketch of phenomenological monadology above.

4.1 Monads and Hilbertian Proof Theory

On Gödel's analysis in the 1961 manuscript, the problem with Hilbert's original view of proof theory is that it tries to combine the 'rightward' and 'leftward' directions but it does so in an unworkable way. It seeks to do justice both to the empiricist *Zeitgeist* and to the 'rightward' nature of mathematics. In conformity with the ideas prevailing in recent empiricism, it acknowledges that the truth of the axioms from which mathematics starts cannot be justified or recognized in any way and therefore that the drawing of consequences from them has meaning only in a hypothetical sense. The drawing of consequences itself, to further satisfy the spirit of the time, is construed as a mere game with symbols according to rules, where this is likewise not thought of as supported by insight or intuition. In accordance with the earlier rationalist's philosophy of mathematics and the mathematician's instinct, however, it is held that a proof of a proposition must provide a secure grounding for the proposition and that every precisely formulated yes-or-no question in mathematics must have a clear cut answer. One aims to prove, that is, for the inherently unfounded rules of the game with symbols that of two sentences A and A , exactly one can always be derived. Such a system is consistent if not both can be derived, and if one can be derived then the mathematical question expressed by A can be unambiguously answered. In order to justify the assertions of consistency and completeness a certain part of mathematics must be acknowledged to be true in the sense of the old rightward philosophy. The part in question, however, is much less opposed to the spirit of the time than the high abstractions of set theory. It is the part that refers only to concrete and finite objects in space, namely the combinations of symbols. This is Hilbert's finitism.

In Hilbert's program we thus see an interesting mixture of rationalist and empiricist elements. The two rightward, rationalistic elements, namely, the belief in the decidability of clearly formulated mathematical problems and the demand that proofs provide a secure grounding for propositions, are translated into the context of empirically given formal systems. Empirically given formal systems are in accord with the spirit of the time. The next step in the development comes with Gödel's

incompleteness theorems: it turns out that it is impossible to rescue the older rightward aspects of mathematics in such a way as to be in accord with the spirit of the time.

Gödel's first incompleteness theorem says that for formal theories T containing enough mathematics to make Gödel numbering possible, if T is consistent then there is a sentence G_T , the Gödel sentence for T , such that neither G_T nor $\neg G_T$ is provable in T . Now for Hilbert, finitist mathematics is supposed to be the mathematics of the concrete, finite sign configurations (as tokens) given to us in space and time in immediate sensory intuition. These objects are in the causal nexus. (If it is sign types that are involved then the types might still be regarded as quasi-concrete, following Charles Parsons' work.) Hilbert invokes a Kantian conception of intuition. Furthermore, there is no need to consider the meanings of terms in the axiomatic formal systems for which we are to provide finitist consistency proofs. In fact, we are supposed to avoid any appeals to meaning and work only with syntax. Let T in the above statement of the first incompleteness theorem be finitist mathematics F . For the argument at hand we do not need to decide exactly what finitist mathematics is. Perhaps it is primitive recursive arithmetic (PRA), or even Peano arithmetic (PA). In any case, F will not be able to decide the Gödel sentence G_F for F if F is consistent. Nonetheless, G_F is true if F is consistent. Since F is the mathematics of the concrete finite spatiotemporal objects given to us in space and time in sensory intuition, our grasp of the truth of G_F must be a grasp of something abstract, infinitary, which could not be a matter of the presentation of signs in space and time in immediate sensory experience. How, therefore, is it possible to know that G_F is true if F is consistent? On the basis of our monadology the claim would be that a condition for the possibility of this knowledge is that there be monads that 'see' or intuit the truth of G_F . Monads decide G_F in a rational way on the basis of (founded) acts of reflection in which they do not merely intend but in fact categorically intuit the truth of G_F . If knowledge requires not mere conception but also intuition, so that intuition is required for objectivity, then a form of rational intuition must be involved. Kantian or Hilbertian intuition will not suffice. Evidence is acquired on the basis of rational intuition. The idea is that human monads decide such finitistically undecidable sentences on the basis of a capacity for reason that does not reduce to mechanical decidability in formal systems.

According to the first incompleteness theorem there is of course a condition on knowing G_F to be true. It is that F be consistent. Now let us consider the second incompleteness theorem. The second incompleteness theorem says that if a formal theory T is consistent then $\text{CON}(T)$ is not provable in T , where ' $\text{CON}(T)$ ' is a particular formalized statement that asserts the consistency of T . Again, let $T = F$. Finitist mathematics F will not be able to prove the consistency of F if F is consistent. There are, however, consistency proofs for F , supposing F to be PRA, PA or even a more powerful theory. How is this possible? Again, on the basis of our monadology we might argue that a condition for the possibility of such proofs is that human monads can prove the consistency of F on the basis of reflection on abstract, infinitary concepts or objects. Such concepts or objects would not be (fully) available in sensory or what Husserl calls "straightforward" intuition. If knowledge requires not only mere conception or meaning-intention but also intuition, and if we

can be said to know that F is consistent because we have a proof of it, then “categorical” intuition must be involved. Gödel thinks the incompleteness theorems show that we need a broader notion of intuition, albeit one that still bears certain important analogies to ordinary sensory intuition. Contrary to Hilbert’s view, proofs requiring such ideal or abstract objects would require insights not derived merely from reflection on combinatorial, space–time properties of symbols but would rather require reflection on the meanings or contents involved. If F is Peano arithmetic, for example, then a consistency proof in the style of Gentzen will employ transfinite induction on ordinals $< \epsilon_0$, while a consistency proof in the style of Gödel’s *Dialectica* interpretation will require primitive recursive functionals of finite type.

The conception of monads we outlined above allows for the possibility of finding systematic and finite but non-mechanical methods for the decision of mathematical questions on the basis of clarification of the intuition of the abstract concepts (meanings of the terms) involved in the problems. Finitist proofs cannot provide a secure grounding for mathematical propositions, as Hilbert intended, because the consistency of F cannot be proved using F . We will instead have to rely on the rational intuition of monads for such a secure grounding. The problem is that we cannot sustain Hilbertian optimism about deciding clearly posed mathematical problems nor can we hold that a proof should provide secure grounding for a proposition once these ideas are translated into the kind of purely formal setting that is supposed to be acceptable to empiricists. These two rationalistic elements can be preserved, however, on the basis of a phenomenological monadology.

One might argue that the incompleteness theorems in fact issue in a kind of meaning clarification. They show us, for example, that we should not mix two distinct concepts—the concept of purely formal provability and the concept of mathematical truth—together in one concept. Indeed, it can be argued that they show us that the purely formal concept of proof should not be confused with the concept of proof as ‘that which provides evidence’.⁵

4.1.1 Monads and Carnap’s Early Positivism

The new monadology is also quite different from Carnap’s early view of mathematics as syntax of language. Carnap’s view is clearly on the left in Gödel’s schema, and the arguments of Gödel *1953/9 are meant to refute it. Against Carnap’s early view of the foundations of mathematics, Gödel argues that mathematics cannot be syntax of language (Gödel *1953/1959). The nominalism and conventionalism of Carnap’s program is refutable. Carnap recognizes two kinds of statements: analytic (tautologies and logical falsehoods) and synthetic (empirical). Statements of mathematics and logic are supposed to be true on the basis of linguistic or syntactical conventions (rules) alone. They are without content or object. All other statements are about the world and are to be considered meaningful

⁵ In various writings going back to the nineteen thirties, Gödel in fact distinguishes the purely formal and relative concept of proof from the ‘abstract’ concept of proof as “that which provides evidence”. See, e.g., Gödel (1937, p. 164, *1951, p. 318, footnote 27, *1953/1959, p. 341, footnote 20, and 1972b, p. 273, footnote).

or not and accepted or rejected on the grounds of their empirical verifiability. Metaphysical statements are, famously, rejected on these grounds.

In order for the syntactical view of mathematics and logic to be correct it is required that there be consistency proofs for the sets of syntactical conventions (rules), for if the rules are inconsistent then all statements will follow from them, including all empirical statements. The consistency proof, by Carnap's own sights, would have to be either mathematical or empirical in nature. If a consistency proof for the syntactical rules is mathematical then by the second incompleteness theorem it will require resources going beyond the concrete, finitary, and sensory objects needed for the nominalism and conventionalism of the syntactical program. Hence, we would again be faced with content, meaning, the ideal or abstract, the infinitary, and categorial intuition. And, again, we *do* have such consistency proofs. The claim would be that this is possible on the grounds of the phenomenological monadology we have sketched.

On the other hand, suppose the consistency 'proof' is empirical in nature. In this case the claim to consistency is based on the fact that the syntactical conventions have thus far (in our use of them) not been found to lead to inconsistency. The evidence for consistency is based on past experience, i.e., it is inductive evidence. This reliance on empirical evidence or empirical facts to maintain syntactical conventionalism about mathematical truths again violates the claim that the latter truths should be based solely on syntactical (linguistic) conventions, come what may in the empirical world. Furthermore, the empirical assertions used to support the consistency claim in this case would have content, so that content will again be required, albeit empirical (as opposed to mathematical) content. Under this alternative mathematical statements completely lose their a priori character, their character as linguistic conventions, and their alleged lack of content. Thus, we can again not hold to strict linguistic conventionalism about mathematics.

In sum, it is not possible without a consistency proof to be a conventionalist/nominalist about mathematics in the manner of Carnap's early logical positivism, but what is needed for the consistency proof, whether it is mathematical or empirical in nature, undermines the conventionalism and nominalism of the logical positivists. For Gödel, the alternative to Carnap's view would be a phenomenological monadology. With its *epoché*, phenomenological monadology is supposed to prevent the dubious metaphysics of earlier forms of platonic rationalism, but without the radical anti-metaphysical stance of logical positivism. Logic is deeper than logical positivism. Phenomenological ontology is still a possibility.

4.1.2 *Monads and Machines*

One can argue that the identification of minds with machines is yet another manifestation of the leftward direction in philosophy that has been in development since the Renaissance. Gödel says, however, that the incompleteness theorems disrupt the purely materialist, empiricist *Zeitgeist*. Some of Gödel's basic claims about minds and machines can be found in the 1964 Postscriptum of Gödel (*1951, 1972a). In his later thinking the view would evidently be that the human mind ('monad') cannot be replaced by any Turing machine (TM), whereas such a view

would be more plausible if the incompleteness theorems and related undecidability results had not been proved. Consider the following assertion: The mind is a finite combinatorial mechanism and there are for it no absolutely undecidable number-theoretic questions. The incompleteness theorems refute this assertion if we take “finite combinatorial mechanism” to mean TM. Restating the negation of the assertion as a disjunction we obtain: “Either there exist infinitely many number-theoretic questions which the human mind is unable to answer or the human mind contains an element totally different from a finite combinatorial mechanism.” A disjunction such as this is stated in various places in Gödel’s writing (see especially Gödel *1951). In a note in the *Nachlass* (cited in van Atten 2006, p. 257) Gödel says “I conjecture that the second alternative is true and perhaps can be verified by a phenomenological investigation of the processes of reasoning.” Phenomenological investigation of the processes of reasoning shows that consciousness in human monads exhibits intentionality. Human monads can be directed in their thinking toward abstract concepts or objects. They are directed by virtue of meanings and can reflect on these meanings. A central idea is that the human monad must use systematic and finite but non-mechanical methods for the decision of open problems in number theory, based on a grasp of the abstract meanings of the terms involved. This is required for invention or discovery, if we are to obtain something new in our thinking.

Gödel wants to use phenomenological considerations to investigate the decidability of (mathematical) problems posed by human reason. Human reason, on this view, is not to be understood in a completely mechanical manner, as Leibniz and others might have it. Indeed, if human minds, as finite ‘monads’, can know about mind-independent ideal concepts or objects on the grounds of categorial intuition or *Wesensanschauung* then human minds are not (Turing) machines, even though they might use such forms of intuition systematically. In light of the philosophical uses to which Gödel puts the incompleteness theorems, we might view the incompleteness theorems themselves as examples of philosophy become rigorous science.

There are many other interesting points of comparison between Leibniz, Gödel and Husserl that I do not have space to discuss in this paper.

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