THE PROOF *EX MOTU* FOR THE EXISTENCE OF GOD. LOGICAL ANALYSIS OF ST. THOMAS AQUINAS' ARGUMENTS

Dowód *ex motu* na istnienie Boga.
Analiza logiczna argumentacji św. Tomasza z Akwinu
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The main reason that has induced me to work on this subject is shortly speaking quite paradoxical: I have worked this subject up because I could not do it in some special way — within the traditional logic.

For some time now I have accepted the point of view of those who say that the so-called traditional logic is not sufficient enough for a precise study of scientific problems, unless it is limited to relatively simple subjects. Mathematical logic, although historically still so new, provides us with many new subtle tools for precise thinking. Rejection of these tools would be similar to a kind of attitude whereby somebody would stubbornly want to use a mail-coach only, when he could be using a train, a car or an airplane.[1]

Although I express such opinions, of course it does not mean at all that I am an enthusiast of all mathematical logicians' products.

Extremely conservative opinion on logic is especially dangerous — even more dangerous because of many reasons that I will not go into, for philosophical work, than for any other domains of scientific work. Philosophical production then easily drops down to the level that even the producers themselves are not able to realize. At this point, philosophy stops being a science, it turns into *eine Dichtung*, and on the top of

[1] At the present I am not the only one representing such an opinion in a scholastical group. At the last international Thomistical meeting in Prague, Fr. Bocheński, OP, expressed the similar opinion: "*Il me parait clair aujourd'hui, bien que je l'ai nié autrefois, que* ***la logistique est de nos jours la seule logique formelle scientifique de la déduction.*** *Pour se convaincre qu'il en est ainsi, il suffit de comparer les traités de logistiques avec les oeuvres des logiciens de l'ancienne école; ils traitent beaucoup plus de problèmes et d'une manière de beaucoup supérieure à celle des anciens. Surtout au point de vue de la rigueur dans la démonstration la chose est plus qu'évidente*" (1933, p. 154).

it, even as such, it appears so poorly that even literary critics do not wish to include it in their research work range. Modern philosophy provides even ample evidence of this fall.

In comparison with many confused works of quite a few modern philosophers, pithy symptoms of the main medieval philosophy movement are a really refreshing bath for somebody who inclines to precise thinking.

The great philosophers of the past in their scientific work did not limit themselves to these weak logical tools that they were *explicite* given to use. Both problems themselves and scientific genius were pushing them into creating intellectual constructions that go far beyond their contemporary schemes. Many times, while working on correctness of Aristotle's or St. Thomas Aquinas' arguments, I was not able to force them into syllogistic forms.

Neither could I force either into syllogistic schemas or into any other schemas of reasoning known to the traditional logic St. Thomas' classical argument for the existence of God, known as the proof *ex motu*.

Because of its subtle and complicated structure, this proof is an extremely interesting subject for any logician. If we also remember how primitive logical tools St. Thomas had in use, we should recognize this proof as a beautiful gem in the Angelic Doctor's scientific work.

I would also like the analysis of this proof to be a modest tone in the chorus of praise for this great pioneer of Christian philosophical and theological thought by so many and so meritorious scientific workers.

1. Treatment of the subject

St. Thomas Aquinas formulated five proofs for the existence of God, which are known as: *quinque viae*. These proofs are named respectively: *ex motu, ex causalitate, ex contingentia mundi, ex finalitate, ex gradibus perfectionis*. St. Thomas presented these proofs in *Theological Summa* (I q. 2 a. 3) and in *Summa contra Gentiles* (1. 1 c.13).

All five proofs are recited in *Theological Summa*, in *Summa contra Gentiles* he presents: two proofs *ex motu*, one — *ex causalitate*, one — *ex gradibus perfectionis* and one — *ex finalitate*; the proof *ex contingentia* is omitted. In the second proof *ex motu* given in *Summa contra Gentiles* there are some conceptions connected *cum contingentia mundi*. St. Thomas is not satisfied with this proof and he finishes it with the following remark: *Praedictos autem processus duo videntur infirmare...* — and then some explanations follow, showing the weak sides of this proof. It is possible that a little later the second proof *ex motu* was reformulated, stated more precisely and presented as the proof *ex contingentia* in *Theological Summa*.[2]

The first proof *ex motu* mentioned in *Summa contra Gentiles* is repeated in a shorter form in *Theological Summa*, and this reduction is done in such a way that the proof *ex motu* from *Theological Summa* is a proper part of the proof given in *Summa contra Gentiles*.

As a material to my logical analysis, I take the first proof *ex motu* presented in *Summa contra Gentiles*.

In *Theological Summa* St. Thomas does not give any historical sources of his proofs; in *Summa contra Gentiles* with both proofs *ex motu*, with proofs *ex causalitate* and *ex gradibus perfectionis* he refers to Aristotle, with proof *ex finalitate* he refers to St. John of Damask and to Averroes.[3]

It seems that apart from the above, while formulating these proofs St. Thomas was also dependent on St. Augustin, Avicenna and Moses Maymonides (cf. Ueberweg 1928, p. 437).

In my work I completely omit the matter of historical sources and I am interested only in an analysis of the argument supplied by St. Thomas.

So far, *quinque viae* by St. Thomas have been the main proofs of the existence of God in Christian philosophy.

Next to them, also a modern variant of the proof *ex motu* is being given in a form of the so-called entropological proof, which is based on the principles of thermodynamics; some authors tried to present different variants of the ontological proof, but St. Thomas had already given a good retort to such attempts. The so-called moral proofs, although they can have an important psychological meaning, they do not have a proper proving power.

Because of all this, my work has not only a historical meaning; the more so because St. Thomas himself considers proof *ex motu* as a much stronger one than others; to him, it is the *prima et manifestior via* (I q. 2 a. 3).

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2 *Theological Summa* was chronologically written later than *Summa contra Gentiles*.

3 "Commentator" with no additions is the name given to Averroes by St. Thomas.

A thorough study on logical structure of the proof shows all the assumptions that the proof is based on and helps to understand its proving power much better.

2. Treatment of logical tools used in this work

I have already mentioned in quasi-preface that I cannot do a logical analysis of St. Thomas' argument within the limits of traditional logic. For this to be done, I need certain notions from the theory of deduction, theory of relations and set theory, and the skill of using quantifiers. Because I would like anybody who is used to abstract thinking even without knowing those theories, an elementary knowledge of which I assume, to be able to read my work, I am going to explain shortly the notions which I will use in my work.

In reconstruction of the different parts of St. Thomas' argument, I will use idiography because then: (1) the reasonings are shorter and more clear than when they are expressed in common language; (2) the reasonings are cleared of various confusing associations, linked with common language; finally (3) this way one protects oneself from different stylistic violations that one would have to perform on the common language, which is not adjusted to precise reasonings.

I use almost without any changes an idiography used in the well-known work *Principia Mathematica* by Russell and Whitehead, because to me this is the most clear idiography among others that are known.[4] All idiographical signs I will be explaining progressively.

From theory of deduction I borrow the following notions, which I will write down using symbols, that from now will be used all the time till the end of my work:

(1) the notion of logical sum, also called a disjunction:

$$p \vee q$$

[4] Although this way I am adopting much from mathematical logicians, it does not mean at all that I sympathize with their nominalistical point of view in logic and materialistic or positivistic tendencies in philosophy. I think that the same way as within traditional logic grounds different philosophical systems could occur equally in agreement or disagreement, it happens similarly within mathematical logic grounds, only in the second case more responsibility is required.

read: [at least] p or q;

(2) the notion of logical intersection:

$$p \cdot q \quad \boxed{p \wedge q}$$

read: p and q;

(3) the notion of negation:

$$\sim p$$

read: it is not true, that p;

(4) the notion of implication:

$$p \supset q \quad \boxed{p \to q}$$

read: if p, then q; q follows from p;

(5) the notion of logical equivalence:

$$p \equiv q$$

read: p is equivalent to q.

All these newly introduced notions are functors with arguments which are sentences or sentential variables.

I emphasize that (1) logical sum is used in the meaning that it is true if at least one of its components is true, but also both of them can be true; (2) logical intersection is true if both components are true; (3) negation refers to the whole sentence, not just to a part of it; (4) implication means that "$p \supset q$" $\boxed{\text{“}p \to q\text{”}}$ is equivalent to "$\sim p \vee q$"; (5) equivalence equals bilateral implication, that is "$p \equiv q$" means the same as $p \supset q$ $\boxed{p \to q}$ and $q \supset p$ $\boxed{q \to p}$.

I use two quantifiers: universal and particular.

(1) universal quantifier:

$$[x] \cdot \chi(x) \quad \boxed{\bigwedge x\, \chi(x)}$$

read: For every x: $\chi(x)$;

(2) particular quantifier:

$$[\exists x] \cdot \chi(x) \quad \boxed{\bigvee x\, \chi(x)}$$

read: For some x: $\chi(x)$; there can be found such an x, that $\chi(x)$; or: there exists such x, that $\chi(x)$.

Here, a little digression is needed on the notion of existence, the more so as this notion occurs in my further considerations.

Scholastic philosophy divides all objects (*entia*) into two main groups: real objects (*entia realia*) and unreal objects (*entia rationis*). A real object is anything independent from our cognition and mind, and also from what our cognition and mind depend on. An unreal object is anything dependent on our cognition and mind. In this terminology expression "x exists" means the same as: x is a real object.

Such principle divisions are being avoided in modern philosophy; at the very most, some groups of objects are being distinguished: physical objects, psychological objects, logical-mathematical objects etc. Therefore, also the notion of existence glitters with a lot of various meanings.

The weakest intension has mathematical-logical existence. A positive condition of this existence is the introduction of the object by a proper definition, negative condition is the consistency; in this sense "x exists" means the same as: x is introduced by a proper definition and x is consistent.

Something exists physically when it has all properties that characterize each physical object; something exists psychologically when it has all properties of psychological object etc.

Among mathematical logicians, the conviction comes and goes that the particular quantifier has the existential meaning. Of course, the existential meaning of the particular quantifier glitters intentionally the same way that the existential meaning itself glitters intentionally too; only the context precisely defines what kind of the existence we are talking about in each particular case.

I have already pointed out that I am going to use a dot as a sign for logical intersection. Besides, like Russell and Whitehead, I will also use dots instead of brackets to divide the sentence into proper parts.

The following description will help a reader to understand the functions of dots in symbolic expressions.

Main rules:

(1) Dots appearing directly next to the sign of implication, equivalence, disjunction, definitial equality or directly after a quantifier, are used instead of brackets. Dots appearing in other places are the signs of logical intersection.

(2) Greater amount of dots has a larger range.

But to avoid writing too many dots, all the cases in which they are being used are divided into three groups.

I. Dots by the sign of implication, equivalence, disjunction, definitial equality.

II. Dots put after quantifiers.

III. Dots as signs of logical intersection.

Group I is stronger than group II and III, group II is stronger than group III.

Dots occurring for any reason are reaching beyond all the sets of dots consisting of the lower number of dots or consisting of the same number of dots, but belonging to the weaker group; and their range ends with the higher number of dots, or the same number of dots, but belonging to a stronger group, or with the end of the expression.

Now I am going to provide some necessary information on the theory of relations and the set theory.

Not getting into any complicated discussions about the notion of relation, I am going to refer to an everyday example.

If x is a father of y, we say that x is in some relation with y, there is a fatherhood relation between x and y. If x is a husband of y, then again there is some relation between x and y — a marriage relation.

The fact that there is some relation between x and y, we will note in the following way:

$$xRy.$$

If we have any relation given, for example relation R_1, then the set of all objects between which this relation exists, we are going to call the field of this relation; the field of relation R_1 I will mark with the following sign:

$$C'R_1.$$

In previous examples, the set of all fathers and sons (that is, in this case, the set of all male human beings, because every male, even though he may not be anybody's father, is at least somebody's son) is the field of the first relation; the set of all husbands and wives, that is the set of all married men and all married women, is the field of the second relation.

The fact, that x belongs to the field of relation R_1, is an element of this field, I will note as follows:

$$x \varepsilon C'R_1 \quad \boxed{x \in C'R_1}$$

The precise definition of this shortening is as follows:

Df. 1. $[x,R] : x\varepsilon C'R \,.\, =. \,[\exists t] \,.\, tRx \vee xRt$[5].

$\boxed{\text{Df. 1.} \quad \bigwedge x \bigwedge R[x \in C'R \equiv \bigvee t(tRx \vee xRt)]}$

The field of relation may be a finite or an infinite set.

In the old logic and mathematics quite confusing notions of infinity were used. Together with the set theory development, the notion of infinity has become much more precise.

Today, following Dedekind, an infinite set we call such a set, which is equipollent to one of its proper parts;[6] if for example we take the following two sets: the set of all positive integers:

1, 2, 3, 4, ...

and the set of all positive even numbers:

2, 4, 6, 8, ...,

then we can see that the first set is equipollent to the second one (for each element x from the first set, we can assign element y from the second set in one-to-one way in accordance with the rule: $y=2x$), although the second set is the proper part of the first set; therefore the set of plus integers is an infinite set.

There are some relations of such a kind that they order their own field, that is they arrange the elements of their field in such way that each element has its fixed place among others; i.e. the relation of being greater than within the field of real numbers puts in order the set of

[5] I will be writing definitions in this way, connecting *definiens* and *definindum* by using the equality sign and putting *definiendum* on the left side of the equality sign, and *definiens* on the right side. I point out that — describing the notion of the relation field — I am deliberately omitting the related matter of the theory of types or semantical categories, because in this work we will be talking only about the relations, in which elements of domain and counterdomain are of the same type.

[6] I use an expression "proper part" following set theorists, that use an expression "part" in such a meaning that each object is a part of itself, but it is not a proper part of itself; the notion of proper part is connected with an existence of the some rest; x is a proper part of y if x is included in y, but y is not included in x.

real numbers, because if we take any two different real numbers, one of them is always greater than the other and the relation of being greater than determines which one of them comes before the second one; however, the fatherhood relation does not arrange its field, because for example it does not determine if the father of x comes before the father of y or conversely; any kind of the priority title in this case may occur only because of some additional and accidental reasons.

The relations that order their fields we call the ordering relations. If the relation R_1 is the ordering relation, I will note it as follows:

$$K(R_1).$$

The ordering relation arranges its field in such way that it makes it an order set.

Which relations are the ordering relations?

Relation R_1 is an ordering relation, always and only if it is: (a) irreflexive (*irreflexiv*), (b) transitive (*transitiv*) and (c) connected (*zusamenhanged*).

Relation R_1 is irreflexive, always and only if the following condition is fulfilled:

$$[x, y] : xR_1y \supset . \, x \neq y^{7} \quad \boxed{\bigwedge x \bigwedge y(xR_1y \rightarrow x \neq y)}$$

For example, the relation of equality in any aspect is reflexive, because each object equals itself; but the fatherhood relation is irreflexive relation, because nobody is his own father. I would like to point out that the fact that a relation is not reflexive does not imply that it is irreflexive; for example, if we assume that not every human being loves himself, then the relation: x loves y is neither reflexive nor irreflexive.

Relation R_1 is transitive, always and only if the following condition is fulfilled:

$$[x, y, z] : xR_1y \,.\, yR_1z \,.\, \supset xR_1z \quad \boxed{\bigwedge x \bigwedge y \bigwedge z[(xR_1y \wedge yR_1z) \rightarrow xR_1z]}$$

For example, linear kinship relation is a transitive relation, because if x is lineary related to y and y is lineary related to z, then x is linearly related to z, whereas the fatherhood relation is non-transitive.

I would also like to point out here that the fact that a relation is not transitive does not imply that it is non-transitive; for example, taking quite a possible assumption that the proverb "Friends of our friends

[7] The expression "$x \neq y$" means the same as: x is not identical to y.

are our friends" is not true, relation: x is a friend of y is neither transitive nor non-transitive.

Relation R_1 is connected, always and only if the following condition is fulfilled:

$$[x,y] : x \varepsilon C'R_1 \,.\, y \varepsilon C'R_1 \,.\, x \neq y \,.\, \supset .\, xR_1 y \vee yR_1 x$$

$$\boxed{\bigwedge x \bigwedge y[(x \in C'R_1 \wedge y \in C'R_1 \wedge x \neq y) \rightarrow (xR_1 y \vee yR_1 x)]}$$

For example, the relation of being greater than in the field of real numbers is a connected relation, because for any two different real numbers x and y the following connection occurs: $x>y$ or $y>x$, whereas the marriage connection is not a connected relation.

The precise definition of the ordering relation will look as follows:

Df. 2. $[R] :: K(R) \equiv \therefore [x,y] : xRy \supset .\, x \neq y \therefore [x,y,z] : xRy \,.\, xRz \,.\, \supset xRz \therefore [x,y] : x \varepsilon C'R \,.\, y \varepsilon C'R \,.\, x \neq y \,.\, \supset .\, xRy \vee yRx$ [8]

$$\boxed{\text{Df. 2.}\quad \bigwedge R[K(R) \equiv [\![\bigwedge x \bigwedge y(xRy \rightarrow x \neq y) \wedge \bigwedge x \bigwedge y \bigwedge z[(xRy \wedge yRz) \rightarrow xRz] \wedge \bigwedge x \bigwedge y\{[(x \in C'R \wedge y \in C'R \wedge x \neq y) \rightarrow (xRy \vee yRx)\}]\!]}$$

It follows from the fact that the given relation is an ordering relation that it is an asymmetric relation.

The relation R_1 is asymmetrical always and only if the following condition is fulfilled:

$$[x,y] \,.\, xR_1 y \supset \sim(yR_1 x) \quad \boxed{\bigwedge x \bigwedge y(xR_1 y \rightarrow \sim yR_1 x)}$$

For example, the fatherhood relation is asymmetrical, because nobody is a father of his own father; whereas the relation: x is a brother of y — is not an asymmetric relation, because sometimes in such situations also y is a brother of x, even though y may be a sister of x.

Thus, the negation of asymmetricalness together with transivity gives in result negation of irreflexivity; because if

$$[\exists x,y] \,.\, xRy \,.\, yRx \quad \boxed{\bigvee x \bigvee y(xRy \wedge yRx)}$$

[8] I describe the ordering relation in such a way that — according to this definition — it is possible to put in order only the sets which consist of at least two elements; I completely omit mathematical specifications, introduced to make it possible to order also one-element sets, and I am doing this for the following reasons: (1) because of St. Thomas' pluralistic conception of the world in this work, we will deal only with the sets that consist of at least two elements; (2) I think that the need to order a one-element set may be a mathematical need only.

and if at the same time the transivity occurs, then we have: xRx; which means that from the fact that the relation is an ordering relation, follows that it is an asymmetric relation.

In the ordered set we can look for the first or the last element; in a case of non-ordered set, the notions of the first and last elements do not have any sense at all.

With a lot of different confusing notions of infinity, in the past there also existed an opinion that an ordered infinite set could not have the first and the last element; it had to be unlimited on at least one side; therefore, it used to be argued, for example, in the following way: if an ordered set has the first and the last element, then it is a finite set.

With contemporary mathematical researches on infinity, this opinion turns out to be wrong; for example, the set of all real numbers included between: $1 \leqslant x \leqslant 2$, ordered in such way that each next element is greater than previous elements, is an infinite set, although it has the first and the last element.

With this, I finish the description of logical tools and also my introductory explanations. This preface turned out to be a very long one, but — thanks to this preface — it will be possible to present futher considerations shorter and more clearly. Without this introduction it would be necessary to continually supply many of explanations on the side during the logical analysis itself, and that would spoil the content and clearness of considerations.

3. Reconstruction of the main proof

St. Thomas presents his proof *ex motu* not in the form of inference but demonstration, *modo geometrico*. First he gives the main proof, and then he proves the rightness of the assumptions occurring in the main proof.

The main proof is included in St. Thomas' text between the sentences: "*Omne quod movetur ab alio movetur*" and: "*...ergo necesse est ponere aliquod primum movens immobile*".

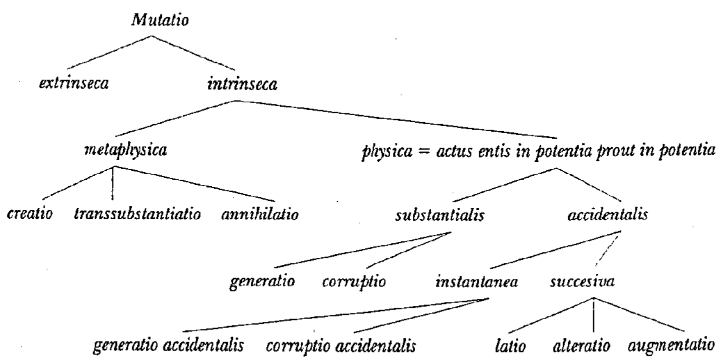
I introduce the following shortenings, which from now will be used throughout:

(1) constant functor "*f*" which will mean the same as: is in motion; so for example "*fx*" means the same as: x is in motion;[9]

[9] "Movement" in St. Thomas' understanding is an ambiguous term. Even though

the logical construction of the formal proof is independent from the meanings of the terms used, this meaning decides about the intension of theses present in the proof, and therefore gnosiologically is very important for the proof: (1) the intension of assumptions — and consequently their acceptance, recognition — depends on the meaning of each term; (2) the intension of the conclusion — and consequently its cognitive value — depends on the meaning of each term. Therefore, in my opinion, the notion of movement mentioned in the proof, should be — at least in the footnote — more carefully described.

Following St. Thomas, scholastics use the term "movement" (*motus*) as a synonymous of the term "change" (*mutatio*) and they build some classification of changes, that graphically can be presented as follows:



Apart from that, they use term "change" in such a broad sense that it includes — apart from all types of the changes mentioned above — also purely psychological acts (*intelligere et velle*).

What movement is St. Thomas talking about in his proof?

He gives an example of the local movement: ... *Patet autem sensu aliquid moveri, utputa solem...*

In the second digression included in the course of argument (starting with the words: *Sciendum autem quod Plato...*), he clearly indicates that right here he excludes purely psychological acts (*intelligere et velle*) from the extension of the notion of movement.

Reconstructing the proof *ex motu*, scholastics treat the notion of movement in such sense that it includes all types of changes shown in the above given graph, except only for the external changes (cf. i.a. Grendt 1926, p. 790).

However, I can see important reasons why St. Thomas uses in the proof the notion of movement only in the sense of the physical movement: (1) he introduces an assumption that if something is in motion, it is a body (compare with the following texts: "*Oportet etiam ipsum [scilicet quod movetur] divisibile esse et habere partes, quum omne quod movetur sit divisibile... Si in motoribus et motis proceditur in infinitum, oportet omnia huiusmodi infinita corpora esse, quia omne quod movetur est divisibile et corpus...*)"; (2) he refers to Aristotelian definition of movement: "*acuts existenis in potentia secundum quod huiusmodi...*"

(2) constant "R" which will mean the same as: moves, so for example "xRy" will mean the same as: x moves y.

I am doing the reconstruction of St. Thomas' main proof in the following way, marking it with letter T:

T. $[x] : fx \supset . [\exists t] . tRx \therefore K(R) :: [\exists y] \therefore y\varepsilon C'\mathcal{R} \therefore$
$[u] : u\varepsilon C'\mathcal{R} . u{\neq}y . \supset yRu :: \supset \therefore [\exists v] \therefore {\sim}(fv) \therefore [u] : u\varepsilon C'\mathcal{R} .$
$u{\neq}v . \supset vRu$

$$\boxed{T. \quad [\![\bigwedge x[fx\to\bigvee t(tRx)]\wedge K(R)\wedge\bigvee y\{y\in C'\mathcal{R}\wedge \bigwedge u[(u\in C'\mathcal{R}\wedge u\neq y)\to yRu]\}]\!]\to\bigvee v\{{\sim}fv\wedge\bigwedge u[(u\in C'\mathcal{R}\wedge u\neq v)\to vRu]\}}$$

4. Explanation of thesis T

Thesis T is a conditional; the antecedent is composed of three factors, the successor is composed of two factors put under one small quantifier.

The factors of the predecessor:

1. $[x] : fx \supset . [\exists t] . tRx$ $\boxed{\bigwedge x[fx\to\bigvee t(tRX)]}$ — means the same as: for any x there is a connection that if x is in motion, then there will be such t, that t moves x; this factor I mark for convenience with the sign "$c1$".

2. $K(R)$ — means the same as: movement relation is an ordering relation; this factor I mark for convenience with the sign "$c2$".

3. $[\exists y] \therefore y\varepsilon C'\mathcal{R} \therefore [u] : u\varepsilon C'\mathcal{R} . u{\neq}y . \supset yRu$ $\boxed{\bigvee y\{y\in C'\mathcal{R}\wedge\bigwedge u[(u\in C'\mathcal{R}\wedge u\neq y)\to yRu)]\}}$ — with the assumption that the relation R is an ordering relation, this factor means the same as: in the ordered field of the relation R, there exists the first element; this factor I mark with the sign "$c3$".

The successor means the same as: there exists such an object that is not in motion, but it moves anything that is in motion.

If all factors of the antecedent are accepted as true, then the successor should also be considered true, if thesis T is a true thesis.

5. Thesis T is a true thesis

I am constructing the proof in such a way that main steps of the proof I number with Arabic numerals, I put all numerals in round brackets before the given thesis, after the given thesis in square brackets I put numerals and letters which show how I obtained the given thesis.

To avoid the repetition of the same antecedents, I write the antecedent once at the top, and below I introduce one after the other the successors that I need; I numerate each successor with Greek letters put in round brackets before the given successor; the thesis marked with the given number is the thesis consisting of the antecedent written above, directly after the numeral digit, and of the last successor.

I will use this way of writing symbolic reasonings in all my work.

Dem.:

(1) $[x]:fx\supset.[\exists t].tRx:\supset:$
$[x]:[t].\sim(tRx).\supset\sim(fx)$ [counterposition][10]

(2) $K(R)\supset:$
$[y,u]:$
(α) $yRu\supset\sim(uRy)$ [asymmetric of relation R]
(β) $u\varepsilon C'R.u\neq y.yRu.\supset\sim(uRy)$ [α]

(3) $K(R)::[\exists y]\therefore y\varepsilon C'R\therefore[u]:u\varepsilon C'R.u\neq y.\supset yRu::\supset\therefore$
$[\exists v]\therefore$
$[u]:u\varepsilon C'R.u\neq v.\supset.$
(α) vRu [v/y][11]
(β) $\sim(uRv)$ [α, 2]
(γ) $\sim(uRv).vRu$ [β, α]

(4) $K(R)::[\exists y]\therefore y\varepsilon C'R\therefore[u]:u\varepsilon C'R.u\neq y.\supset yRu::\supset\therefore$
$[\exists v]\therefore[u]:u\varepsilon C'R.u\neq v.\supset\sim(uRv)\therefore[u]:u\varepsilon C'R.u\neq v.\supset vRu$[3][12]

(5) $[u,v].\sim(u\varepsilon C'R)\supset\sim(uRv)$ [Df. 1]

(6) $K(R)\supset:[u,v]:u=v.\supset\sim(uRv)$ [Df. 2]

(7) $K(R)::[\exists y]\therefore y\varepsilon C'R\therefore[u]:u\varepsilon C'R.u\neq y.\supset yRu::\supset\therefore$
$[\exists v]\therefore[u]:\sim(uRv)\therefore[u]:u\varepsilon C'R.u\neq v.\supset vRu$ [5,6,4][13]

[10] If q follows from p, then non-p follows from non-q.

[11] The successor is the repetition of the second factor of the antecedent, weakened by throwing out one of the factors and changed by substitution "v" for "y".

[12] If q and r follows from p, then q follows from p and r follows from p.

[13] This step is made according to the thesis of the deduction theory: $\sim p\supset q:r\supset.s\supset q:$

(8) $[x]:fx\supset.[\exists t].tRx\therefore K(R)::[\exists y]\therefore y\varepsilon C'R\therefore[u]:u\varepsilon C'R.u\neq y.\supset$
$yRu::\supset\therefore[\exists v]\therefore\sim(fv)\therefore[u]:u\varepsilon C'R.u\neq v.\supset vRu$ [7, 1]

Dem.:		
1.	$\bigwedge x[fx\rightarrow\bigvee t(tRx)]\rightarrow\bigwedge x[\bigwedge t(\sim tRx)\rightarrow\sim fx]$	
2a.	$K(R)\rightarrow\bigwedge y\bigwedge u(yRu\rightarrow\sim uRy)$	
2b.	$K(R)\rightarrow\bigwedge y\bigwedge u[(u\in C'R\wedge u\neq y\wedge yRu)\rightarrow\sim uRy]$	[2a]
3a.	$\llbracket K(R)\wedge\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rrbracket\rightarrow$ $\bigvee v\{\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow vRu]\}$	[v/y]
3b.	$\llbracket K(R)\wedge\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rrbracket\rightarrow$ $\bigvee v\{\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow\sim uRv]\}$	[2b, 3a]
3c.	$\llbracket K(R)\wedge\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rrbracket\rightarrow$ $\bigvee v\{\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow(\sim uRv\wedge vRu]\}$	[3b, 3a]
4.	$\llbracket K(R)\wedge\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rrbracket\rightarrow$ $\bigvee v\{\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow\sim uRv]\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow vRu]\}$	[3c]
5.	$\bigwedge u\bigwedge v(\sim u\in C'R\rightarrow\sim uRv)$	[Df. 1.]
6.	$K(R)\rightarrow\bigwedge u\bigwedge v(u=v\rightarrow\sim uRv)$	[Df. 2.]
7.	$\llbracket K(R)\wedge\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rrbracket\rightarrow$ $\bigvee v\{\bigwedge u(\sim uRv]\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow vRu]\}$	[5, 6, 4]
8.	$\llbracket\bigwedge x[fx\rightarrow\bigvee t(tRx)]\wedge K(R)\wedge\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rrbracket\rightarrow$ $\bigvee v\{\sim fv\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow vRu]\}$	[7, 1]

(8) it has the identical shape as thesis T; this way the demonstration is being done.

6. Comparison of thesis T with text of St. Thomas

St. Thomas does not formulate thesis T as precisely as I have done it; in his work we also cannot find anything similar to my proof of rightness of thesis T; his analogon of thesis T St. Thomas accepts as the thesis which is obvious without any proof.

What are the differences between thesis T and the text of St. Thomas? When it comes to a successor, there are only linguistical differences; St. Thomas expresses the conclusion this way: "*...ergo necessse est ponere aliquod movens immobile*".

$p.\sim s.\supset q:\supset.r\supset q$ $\boxed{\llbracket(\sim p\rightarrow q)\wedge\{[r\rightarrow(s\rightarrow q)]\wedge[(p\wedge\sim s)\rightarrow q)]\}\rrbracket\rightarrow(r\rightarrow q)}$.

There are quite significant differences in the formulating of antecedent.

The first factor is clearly given by St. Thomas, but — in a stronger form; translated into symbolical language this factor in St. Thomas' formulation looks as follows:

$$T_1.\ [x] : fx \supset .\ [\exists t]\ .\ tRx\ .\ t{\neq}x \quad \boxed{T_1.\ \wedge x[fx \rightarrow \vee t(tRx \wedge t{\neq}x)]}$$

In my reconstruction I have made the first factor weaker for two reasons: (1) in the weaker form together with other factors of the antecedent of thesis T, it is a sufficient condition of the successor; (2) from the first and the second factor of antecedent of thesis T follows thesis T_1.

Dem.:

(1) $K(R) \supset : [x,y] : xRy \supset .\ x{\neq}y$ [Df. 2[14].]

(2) $K(R) \therefore [x] : fx \supset .\ [\exists t]tRx \therefore \supset : [x] : fx \supset .\ [\exists t]\ .\ tRx\ .\ t{\neq}x$ [1]

Dem.:		
1.	$K(R) \rightarrow \wedge x \wedge y(xRy \rightarrow x{\neq}y)$	[Df. 2.]
2.	$\{K(R) \wedge \wedge x[fx \rightarrow \vee t(tRx)]\} \rightarrow \wedge x[fx \rightarrow \vee t(tRx \wedge t{\neq}x)]$	[1]

The second factor in the formulation of the main proof is not mentioned *explicite* — it can be only assumed; St. Thomas *explicite* mentions it only at the end of his argument, proving the rightness of the third factor. Of course, St. Thomas does not supply the definition of an ordering relation. The third factor of the antecedent of thesis T is formulated by St. Thomas in the main proof in a completely different way. This formulation is connected with some additional factor given by St. Thomas which is not included in thesis T at all.

St. Thomas also introduces *explicite* into the main proof some experiential factor: "*Patet autem sensu aliquid moveri, utputa solem...*"

Translated into symbolic language, in a precise formulation this factor can be presented in the following way:

$$[\exists z]\ .\ fz \quad \boxed{\vee z\, fz}$$

— with an addition that the rightness of this thesis has been experientially stated.

[14] Irreflexivity of relation R.

Because of the first factor of the antecedent of thesis T, this z is an element of the field of relation R.

St. Thomas formulates the third factor in the main proof in this way: the set of elements preceding this element z in the ordered field of relation R, cannot be an ordered infinite set, so it is an ordered finite set, and as such — it has the first element.

Further on in his argument St. Thomas gives three justifications for the rightness of his third factor.

The second and third justification are logically very simple, formulated in such way that they quite clearly talk directly about the existence of the first element in the field of relation R, and the matter of finity or infinity of this field is not mentioned at all.

The first justification is quite a complicated proof *per reductionem ad absurdum*. In short, this proof can be presented in the following way: if within the field of relation R the first element does not exist, then some absurdity occurs; therefore, in the field of relation R there exists the first element.

To connect these justifications with argument in the main proof, there comes the following completing idea: ... and if the first element exists in the ordered field of relation R, then the section of the field of relation R included between factual element z and the first element of the set, is not an infinite set. In this way, the truth of the third factor presented in the form of negation of infinity, would have been proved.

In *Theological Summa* (I q. 2 a. 3) in the first proof *ex motu* the following passage is being directly used: "*Hic autem non est procedere in infinitum, quia sic non esset aliquod primum movens...*"

In all this, the opinion reveals that if an ordered set is infinite, it is unlimited at least on one side, that is it does not have the first element or does not have the last element.

This opinion is perfectly understood in connection with long ago accepted in philosophy notion of infinity: *infinitum est id, quod limitem non habet*. But according to the present mathematical researches on infinity this opinion is wrong, because — as I have already mentioned — set theory already knows the ordered infinite sets possessing both the first and the last element.

Mathematical research on infinity is still struggling to overcome many of various difficulties and these difficulties lie in the basis; therefore, in my opinion, the results of this research cannot be moved without reserve into the domain of considerations on reality. But then again they should not be passed over in silence, as though they did not exist at all. A huge building of science is being built by a collective effort and the workers of different domains should be helping and respecting one another.

In the present stage of science I cannot accept as true the opinions that if an ordered set is infinite, then it is unlimited at least on one side.

In that case what kind of an attitude should one have towards St. Thomas reasoning? Two possibilities come to one's mind.

(1) To accept that the third factor of predecessor of thesis T is the thesis on the finity of ordered set of antecedent of element z. This factor would be then stronger than a factor concerning the existence of the first element, for if an ordered set is finite, then it has the first and the last element, so it also would be a sufficient factor; but in this case St. Thomas' long considerations on the truth of the third factor should be considered as not connected with this factor and needless.

(2) To relate St. Thomas' long considerations on the truth of the third factor with the main proof; but then this third factor should be expressed in the form of the first element's existence.[15]

I have chosen the second interpretation, because it deforms the run of St. Thomas' thought less significantly. This second interpretation seems to me better also because of some other reasons.

St. Thomas used to say that it cannot be philosophically proved that the time of the world existence is a finite time. The possibility of an infinite lasting of the world somehow goes together with, but — of course — does not follow from, the possibility of infinite series of motorial dependencies. Therefore, the second interpretation harmonizes better with the thesis of the possibility of everlasting world.

Using the second interpretation, the proof is stronger anyway, because *a fortiori* is then right for a finite series.

Because of the above reasons in my reconstruction, the third factor has taken the form that it has in thesis T.

[15] There is also the third possibility: to add additional, quite suggestive assumptions on the order type of relation R; i.e. that each section of the field of relation R is determined by a leap, then a sector of the field, included between any two fixed elements, is a finite set; but it seems to me that it would an unnecessary increase of the number of assumptions and therefore I omit this possibility.

I have completely omitted an experiential factor: $[\exists z] \,.\, fz$ $\boxed{\vee z\ fz}$ in thesis T, for the way I do the formulation it is logically needless; the other factors are a sufficient condition of the successor.

But even with the way I do the formulation, this factors plays an important part, although logically side part. This experiential factor indicates that the field of relation R is something real, not only a logical construction. Consequently, element v, which existence is being stated in the successor, exists not only in a logical-mathematical sense, but is a real object.

Shortly speaking, the way I formulate thesis T, St. Thomas' experimental factor equals the requirement of the reality of the field of relation R.

7. Are all factors of the antecedent of thesis *T* necessary?

The proof of the truth of thesis T also shows that the antecedent of thesis T is a sufficient justification for the successor, that is the set of the factors included in the antecedent of thesis T is sufficient to state the truth of the successor.

Now the second logical problem comes to one's mind. Are all these factors necessary, so that the successor could implicationally follow from the antecedent? Could any of these factors be omitted or could they be expressed less stronger?

Well, careful revision of the proof of the truth of thesis T is enough to say that the second factor could be expressed less stronger.

The proof of the truth of thesis T ***explicite*** includes the first and the third factor as a premise of reasoning and it is easy to prove that these two factors are necessary.

Factors two and three, with the omittance of the first factor, are not sufficient justification of the successor, because the first factor is the only one where occurs functor "f", which also occurs in the successor.

The necessity of the third factor, next to two first ones, can be shown with the following example.

Let's take a set of plus integers, put in order accordingly to relation of being greater than:

… 4, 3, 2, 1.

If "fx" means the same as: x is a plus integer, and "xRy" — x is greater than y, then both the first and the second factors of antecedent of thesis T will be satisfied, because the third factor of the antecedent is not satisfied.

However, the second factor is included in the truth proof only because an asymmetry and irreflexivity of relation R occurs there as a premise.

If a relation is an ordering relation, then it is also asymmetric, but not conversely. The assumption that relation R is an ordering relation is stronger than the assumption that relation R is an asymmetric relation.[16]

It can be also shown *via* an example that replacement of the second factor of the antecedent of thesis T with the postulate of asymmetry of relation R is sufficient to state the rightness of the successor.

Let's assume that ten copies of Raphael's *Madonna di Foligno* have been made.

Let's say that "fx" means the same as: x is a copy of *Madonna di Foligno*, and "xRy" — x is an original with respect to y.

Then the first and the third factor of the antecedent of thesis T will be fulfilled, relation R is asymmetric and — even though relation R is not an ordering relation, because it is neither connected nor transitive — the successor of thesis T will be true too.

If one took minimalistic position of many contemporary mathematical logicians, who give assumptions various acrobatic demands, almost not paying any attention to their intuitions, then in thesis T only the asymmetry of relation R should be assumed, instead of its being ordered. But then the third factor would lose the intuitional character of the first element and the whole antecedent of thesis T would become less obvious.

To obtain better obviousness, St. Thomas resigns postulates of logical minimalism.[17]

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[16] Irreflexivity follows from asymmetry.

[17] By the way, I point out that if in the proof we not only omitted transivity and connection of relation R, but also clearly denied them, which — of course — would be stronger than just omittance, then such point of view would be connected with some occasionatistical conception of the world, similar to the one represented by Moses Maymonides or Malebranche, or of some *harmoniae praestabilitae* style by Leibniz.

8. Proofs of irreflexivity of relation R

After the main proof had been done, St. Thomas picked up the proof of the truth of thesis T_1:

$$T_1.\ [x] : fx \supset .\ [\exists t]\ .\ tRx\ .\ t{\neq}x \quad \boxed{T_1.\ \bigwedge x[fx \rightarrow \bigvee t(tRx \wedge t{\neq}x)]}$$

This is the strengthened first factor of thesis T.

He gives three proofs for it.

T h e f i r s t p r o o f. This proof is included in the text between the words: "*Primo sic: Si aliquid movet seipsum...*" and the words: "*...necesse est ergo omne quod movetur ab alio moveri*".

In this proof, St. Thomas introduces some new assumptions that include new notion of the proper part. He does not supply any definition of the proper part; this notion could be quite easily defined, but it would require longer controversial considerations, and therefore — following St. Thomas — I will use this notion without a definition, referring to a common understanding.[18]

To shorten the run of the reasoning I introduce only one new shortening, a constant functor with one argument — M, with a proper index at the bottom; let "$M_x(a)$" $\boxed{\text{“}aMx\text{”}}$ means the same as: a is the proper part of x.

Assumption 1.1. $[x] : fx \supset .\ [\exists a, b]\ .\ M_x(a)\ .\ M_x(b)$

Assumption 1.2. $[x] \therefore [\exists a, b] : M_x(a)\ .\ M_x(b) : \sim(fa)\ .\ fb$
$.\vee.\ \sim(fa) \supset \sim(fb) : \supset \sim(xRx)$

Assumption 1.3. $[x] : fx \supset .\ [\exists t]\ .\ tRx$[19]

$$\boxed{\begin{array}{ll}\text{Assumption 1.1.} & \bigwedge x[fx \rightarrow \bigvee a \bigvee b(aMx \wedge bMx)] \\ \text{Assumption 1.2.} & \bigwedge x[\![\bigvee a \bigvee b\{(aMx \wedge bMx) \wedge [(\sim fa \wedge fb) \\ & \vee(\sim fa \rightarrow \sim fb)]\} \rightarrow \sim xRx]\!] \\ \text{Assumption 1.3.} & \bigwedge x[fx \rightarrow \bigvee t(tRx)]\end{array}}$$

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[18] In connection with the notion of the proper part, I emphasize that the logical theory about connections between things and their proper parts can be found in mereology of Prof. S. Leśniewski; this mereology is thoroughly discussed in (1928—1931).

[19] This assumption, identical with the first factor of thesis T, is not clearly formulated in this proof by St. Thomas, but it appears as premise in the proof; in this proof all the time the following conviction appears: if something is in motion, it either moves itself, or something else moves it.

Dem.:

(1) $[x] \therefore f(x) \,.\, xRx \,.\supset :$
$[\exists a, b]$
(α) $M_x(a) \,.\, M_x(b)$ [1.1]
(β) $M_x(a) \,.\, M_x(b) : \sim(fa) \,.\, fb$
$.\vee .\, fa \vee \sim(fb)$ [α[20]]
(γ) $M_x(a) \,.\, M_x(b) : \sim(fa) \,.\, fb \,.\vee .$
$\sim(fa) \supset \sim(fb)$ [β]

(2) $[x] : f(x) \,.\, xRx \,.\supset .\, \sim[\exists a, b] : M_x(a) \,.\, M_x(b) :$
$\sim(fa) \,.\, fb \,.\vee .\, \sim(fa) \supset \sim(fb))$ [1.2]

(3) $[x] : fx \supset .$
(α) $\sim(xRx)$ [1, 2[21]]
(β) $[\exists t] \,.\, tRx \,.\, \sim(xRx)$ [1.3, α]
(γ) $[\exists t] \,.\, tRx \,.\, t \neq x$ [β[22]]

Dem.:		
1a.	$\bigwedge x[(fx \wedge xRx) \rightarrow \bigvee a \bigvee b(aMx \wedge bMx)]$	[1.1]
1b.	$\bigwedge x[\![(fx \wedge xRx) \rightarrow \bigvee a \bigvee b\{(aMx \wedge bMx) \wedge [(\sim fa \wedge fb) \vee (fa \vee \sim fb)]\}]\!]$	[1a.]
1c.	$\bigwedge x[\![(fx \wedge xRx) \rightarrow \bigvee a \bigvee b\{(aMx \wedge bMx) \wedge [(\sim fa \wedge fb) \vee (\sim fa \rightarrow \sim fb)]\}]\!]$	[1b.]
2.	$\bigwedge x[\![(fx \wedge xRx) \rightarrow \sim \bigvee a \bigvee b\{(aMx \wedge bMx) \wedge [(\sim fa \wedge fb) \vee (\sim fa \rightarrow \sim fb)]\}]\!]$	[1.2]
3a.	$\bigwedge x[(fx \rightarrow \sim xRx)$	[1c., 2]
3b.	$\bigwedge x[(fx \rightarrow \bigvee t(tRx \wedge \sim xRx)]$	[1.3, 3a.]
3c.	$\bigwedge x[(fx \rightarrow \bigvee t(tRx \wedge t \neq x)]$	[3b.]

This way the thesis T_1 is proved.

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[20] β is different from α, because the successor is strengthened with some substitution of the principle of excluded middle; this step happens in accordance with the following thesis from deduction theory:

$$p \supset \therefore q \supset v \,.\supset : q \supset v \,.\, p \quad \boxed{p \rightarrow \{(q \rightarrow v) \rightarrow [q \rightarrow (v \wedge p)]\}} \,.$$

[21] If two contradictory sentences follow from a sentence, then this sentence is false; this step is done in accordance with the following thesis from deduction theory:

$$p \,.\, q \supset r : p \,.\, q \,.\supset \sim r : \supset .\, p \supset \sim q \quad \boxed{\{[(p \wedge q) \rightarrow r] \wedge [(p \wedge q) \rightarrow \sim r]\} \rightarrow (p \rightarrow \sim q)} \,.$$

[22] This step is correct not only with Leibnizian conception of identity (*identitas indiscernibilium*):

$$[x,y] : x=y \,.\equiv .\, [F].\, Fx \equiv Fy \quad \boxed{\bigwedge x \bigwedge y[x=y \equiv \bigwedge F(Fx \equiv Fy)]}$$

but also with the other, known to me conceptions of identity.

But here come the following remarks.

(1) Thesis T_1 together with the third factor of the antecedent of thesis T, with omission of the second factor, is not a sufficient condition of the successor of thesis T.

The following example shows it.

Let's say that "fx" means the same as: x is a member of the Brown family, "xRy" — x has the same surname as y. With the assumption that the Brown family consists of at least two members, and that one and only one family uses this surname, and that in the given period of time the Brown family consists only of men autochthonously belonging to this family, and also — at the most — of their wives, sons and unmarried daughters (I make these additional assumptions about the females of Brown family, considering surname complications which appear because of our custom connected with woman's marriage), in this example thesis T_1 will be satisfied, the third factor will be also satisfied, but the successor of thesis T will be not satisfied.

(2) Thesis T_1 follows from the first and second factor of thesis T (cf. p. 109).

Thus, the conversion of the first factor of the antecedent of thesis T into thesis T_1, while keeping other factors without any changes, is a needless reinforcement of the antecedent; but the replacement of the first factor in antecedent of thesis T with thesis T_1, together with getting rid of the second factor will cause the antecedent not to be a sufficient condition for the successor.

In St. Thomas' reasoning which I have just reconstructed in symbols, the first factor of thesis T occurs as assumption 1.3, so the whole reasoning as the only result gives the reinforcement of the first factor into thesis T_1. Therefore the reasoning in the presented form is not connected at all with thesis T and in fact it is not needed at all.

However, there are some right intuitions in the whole argument; inaccuracies occur only because of using too primitive logical tools. Namely, if we add to assumption 1.1 and 1.2 one more very intuitive assumption, then it is easy to transform all this argument into the proof of irrefexivity of relation R.

Assumption 1.4. $[x,y] \,.\, xRy \supset fy$

$\boxed{\text{Assumption 1.4. } \bigwedge x \bigwedge y(xRy \rightarrow fy)}$

Let's keep all the steps of the previous demonstration till the step 3α inclusive. Then:

(4)	$[x,y] : xRy \supset .$	
(α)	$\sim(yRy)$	[1.4, 3α]
(β)	$xRy \,.\, \sim(yRy)$	[α]
(γ)	$x \neq y$	[β]

4a.	$\bigwedge x \bigwedge y(xRy \rightarrow \sim yRy)$	[1.4, 3a]
4b.	$\bigwedge x \bigwedge y[xRy \rightarrow (xRy \wedge \sim yRy)]$	[4a.]
4c.	$\bigwedge x \bigwedge y(xRy \rightarrow x \neq y)$	[4b.]

In this way, we obtain the proof of irreflexivity of relation R.

Finally with the help of assumptions: 1.1, 1.2 and 1.4 it is possible to weaken the second factor of the predecessor of thesis T; if these three assumptions are accepted, then instead of the second factor of predecessor of thesis T, it is enough to introduce the assumption that relation R is connected and transitive.

9. Remarks on the newly introduced assumptions

As much as assumptions: 1.1 and 1.4 are quite intuitive, especially if we remember that the movement discussed in them is a physical movement, assumption 1.2 is not very intuitive, although it is quite suggestive.[23]

More intuitive would be here the weaker assumption:

[23] This assumption can be expressed much more simple, but then it becomes less suggestive. The antecedent of assumption 1.2 consists of three factors, put under one particular quantifier. The third factor is a substitution of the principle of excluded middle and according to the following thesis from deduction theory:

$$p \supset \therefore q \,.\, p \,.\supset r : \equiv .\, q \supset r \quad \boxed{p \rightarrow \{[(q \wedge p) \rightarrow r)] \equiv (q \rightarrow r)\}}$$

in the system in which this principle is valid, it can be completely omitted. The first and the second factors assume the existence of two proper parts in the same object x, but from the existence of one proper part infers the existence of at least one more different proper part in the same object. Convert consequence is obvious. Therefore assumption 1.2 is inferentially equivalent to the following assumption:

$$[x] : [\exists a] \,.\, M_x(a) \,.\supset \sim xRx \quad \boxed{\bigwedge x[\bigvee a(aMx) \rightarrow \sim xRx]}$$

at the same time it is also possible to put a under the universal quantifier and as the result we receive as follows:

$$[x, a] \,.\, M_x(a) \supset \sim(xRx) \quad \boxed{\bigwedge x \bigwedge x(aMx \rightarrow \sim xRx)}.$$

$$[x] \therefore [\exists a, b] : M_x(a) \,.\, M_x(b) : \sim(fa) \,.\, fb \,.\vee.\, fa \,.\, \sim(fb) : \supset \sim(xRx)$$

$$\boxed{\bigwedge x[\![\bigvee a \bigvee b\{(aMx \wedge bMx) \wedge [(\sim fa \wedge fb) \vee (fa \wedge \sim fb)]\} \rightarrow \sim xRx]\!]}$$

Shorter:

$$[x] \therefore [\exists a, b] \,.\, M_x(a) \,.\, M_x(b) \,.\, \sim(fa \equiv fb) \,.\supset \sim(xRx)$$

$$\boxed{\bigwedge x\{\bigvee a \bigvee b[(aMx \wedge bMx) \wedge \sim(fa \leftrightarrow fb)] \rightarrow \sim xRx\}}$$

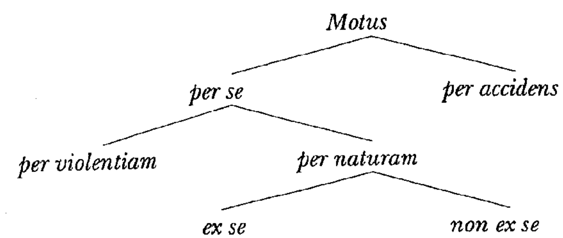
But if I add to assumptions: 1.1 and 1.4 this new weakened assumption, then I cannot prove irreflexivity of relation R on the basis of this group of assumptions.

10. The second proof

This proof is included in the text of St. Thomas between words: *Secundo probat per inductionem sic...* and words: *Ergo omne quod movetur ab alio movetur.*

This proof is logically very simple, its character is extremely referred to physics; I will call it shortly a physical proof.

Following Aristotle, St. Thomas gives the following classification of movement:



And shortly, analytically or referring to an experience, he shows that in each one of these types if something is in motion, then it is moved by something else.

The conclusion is not quite clear, but it seems that it also is thesis T_1, rather than the thesis about irreflexivity of relation R.

The argumentation, logically the weakest, has the nature of reduction; its value depends on the fact if the classification is adequate and experiential interpretations are right.

11. The third proof

This proof is included in St. Thomas' text between the words: "*Tertio probat sic...*" and the words: "*et sic nihil movet seipsum*".

I introduce new shortenings: let "xA_Sy" mean the same as: x is in aspect S *in actu* to y ; "xP_Sy" — x is in aspect S *in potentia* to y".

Assumption 2.1. $[x,y,S] . xA_Sy \supset \sim(xP_Sy)$
Assumption 2.2. $[x,y] : fx . yRx . \supset xP_Ry$
Assumption 2.3. $[x,y] : fx . yRx . \supset yA_Rx$
Assumption 2.4. $[x] : fx \supset . [\exists t] . tRx$[24]

Assumption 2.1.	$\bigwedge x \bigwedge y \bigwedge S(xA_Sy \rightarrow \sim xP_Sy)$
Assumption 2.2.	$\bigwedge x \bigwedge y[(fx \wedge yRx) \rightarrow xP_Ry]$
Assumption 2.3.	$\bigwedge x \bigwedge y[(fx \wedge yRx) \rightarrow yA_Rx]$
Assumption 2.4.	$\bigwedge x[fx \rightarrow \bigvee t(tRx)]$

Dem.:

(1)	$[x,y] . xA_Ry \supset \sim(xP_Ry)$	[2.1]
(2)	$[x,y] : fx . yRx . \supset . xP_Ry . yA_Rx$	[2.2, 2.3]
(3)	$[x] : fx . xRx . \supset .$	
	(α) $xP_Rx . xA_Rx$	[2]
	(β) $\sim(xA_Rx) \supset \sim(xP_Rx)$	[α]
(4)	$[x] . xA_Rx \supset \sim(xP_Rx)$	[1]
(5)	$[x] : fx . xRx . \supset . xA_Rx \supset \sim(xP_Rx)$	[4[25]]
(6)	$[x] . fx \supset \sim(xRx)$	[5, 3[26]]
(7)	$[x] : fx \supset . [\exists t] . tRx . \sim(xRx)$	[2.4, 6]
(8)	$[x] : fx \supset . [\exists t] . tRx . t \neq x$	[7]

Dem.:

1.	$\bigwedge x \bigwedge y(xA_Ry \rightarrow \sim xP_Ry)$	[2.1]
2.	$\bigwedge x \bigwedge y[(fx \wedge yRx) \rightarrow (xP_Ry \wedge yA_Rx)]$	[2.2, 2.3]
3a.	$\bigwedge x[(fx \wedge xRx) \rightarrow (xP_Rx \wedge xA_Rx)]$	[2]
3b.	$\bigwedge x[(fx \wedge xRx) \rightarrow \sim(xA_Rx \rightarrow \sim xP_Rx)]$	[3a]
4.	$\bigwedge x(xA_Rx \rightarrow \sim xP_Rx)$	[1]

[24] This assumption has not been directly formulated by St. Thomas, but it is used in the proof as a premise, like in the first proof.

[25] The true sentence follows from any sentence.

[26] If two contradictory sentences follow from a sentence, then this sentence is false.

5.	$\bigwedge x[(fx \wedge xRx) \rightarrow (xA_Rx \rightarrow \sim xP_Rx)]$	[4]
6.	$\bigwedge x(fx \rightarrow \sim xRx)$	[5, 3b]
7.	$\bigwedge x(fx \rightarrow [\bigvee t(tRx) \wedge \sim xRx]\}$	[2.4, 6]
8.	$\bigwedge x(fx \rightarrow \bigvee t(tRx \wedge t \neq x]\}$	[7]

All the remarks that I made about the first proof, should be repeated about the third proof, except for remarks concerning the assumptions of the first proof only.

I emphasize, that out of all the three proofs, only this proof is repeated in argument *ex motu* in *Theological Summa*.

If we leave a physical proof on the side, the result of the two remaining proofs, seemingly concerning thesis T_1, can be shortly expressed as follows: set of assumptions: 1.1, 1.2, 1.4 or set of assumptions: 2.1, 2.2, 2.3, 1.4[27] eliminate the postulate of irreflexivity of relation R from the second factor of the antecedent of thesis T.

12. Proofs for the existence of the first element in the ordered field of relation *R*

St. Thomas next proves, that the first element exists in the ordered field of relation R. He gives three proofs for this.

a) The first proof

This proof is included in St. Thomas' text between the words: "*Quarum prima talis est: Si in motoribus et motis proceditur in infinitum...*" and the words: "*Et sic unum infinitum movebitur tempore finito; quod est impossibile, ut probatur sexto Physicorum*".

Argument is being done *per reductionem ad absurdum*.

I introduce new shortenings: "Cx" — x is a body,[28] "t_iFx" — t_i is the duration of movement of x, "t" with the subscript is the time variable, "$F(t_i)$" $\boxed{\text{"}Ht_i\text{"}}$ — t_i is the finite period of time.

St. Thomas introduces the following new assumptions:

Assumption 3.1. $[x] . fx \supset Cx$

[27] Set of assumptions; 2.1, 2.2, 2.3, 1.4 can be easily simplified. In accordance with assumption 1.4, fx follows from yRx, therefore in assumptions: 2.2 and 2.3 the first factor of the predecessor can be omitted.

[28] St. Thomas characterizes a body as something divisible.

Assumption 3.2. $[x] : Cx \,.\, fx \,.\supset .\, [\exists t_1] \,.\, t_1 Fx$
Assumption 3.3. $[x,t_2] : Cx \supset .\, t_2 Fx \supset F(t_2)$
Assumption 3.4. $[x,y,t_1,t_2] : xRy \,.\, t_1Fx \,.\, t_2Fy \,.\supset .\, t_1=t_2$

Assumption 3.1.	$\bigwedge x(fx \rightarrow Cx)$
Assumption 3.2.	$\bigwedge x[(Cx \wedge fx) \rightarrow \bigvee t_1(t_1Fx)]$
Assumption 3.3.	$\bigwedge x \bigwedge t_2[Cx \rightarrow (t_2Fx \rightarrow Ht_2)]$
Assumption 3.4.	$\bigwedge x \bigwedge y \bigwedge t_1 \bigwedge t_2\{[xRy \wedge (t_1Fx \wedge t_2Fy)] \rightarrow t_1=t_2\}$

The thesis provisionally accepted in the argument, which is later being rejected because of the nonsensical consequences, is the following thesis:

$$T_p.\ [x,y] : xRy \supset .\, fx \,.\, fy \quad \boxed{T_p.\ \bigwedge x \bigwedge y[xRy \rightarrow (fx \wedge fy)]}$$

A few remarks are needed about thesis T_p.

(1) To connect the argumentation *per reductionem ad absurdum* with the problem of existence of the first element in the field of relation R, it is necessary that existence of the first element in the field of relation R follows from the negation of thesis T_p at the most added to the other accepted theses.

(2) Thesis T does not have an equivalent meaning to the negation of existence of the first element in the field of relation R; it is quite obvious even from the fact that in thesis T_p appears functor "f", which does not appear in thesis stating existence of the first element.

(3) Assumptions: 3.1, 3.2, 3.3 and 3.4 cannot interfere in connecting thesis T_p with the problem of existence of the first element in the field of relation R, because in each of them some reciprocally irreducible variables and functors appear, that cannot be found either in thesis T_p or in the thesis stating existence of the first element in the field of relation R.

(4) In connecting of thesis T_p with the problem of existence of the first element in the field of relation R, may at the most interfere the first and the second factors of the antecedent of thesis T, as theses already accepted.

Now, the negation of existence of the first element in the field of relation R follows from thesis T_p, the first and the second factors of thesis T.

Dem.:

(1)		$[x,y] : yRx \supset .\, fy \,.\, fx$	$[T_p, y/x, x/y]$
(2)		$[x,y] : xRy \vee yRx \,.\supset .\, fx \,.\, fy$	$[T_p, 1]$
(3)		$[x] : x \varepsilon C'R \supset .$	
		$[\exists t]\,.$	
	(α)	$tRx \vee xRt$	[Df. 1]
	(β)	$fx \,.\, ft$	[α, 2]
	(γ)	fx	(β)
		$[\exists u].$	
	(δ)	uRx	[γ, $c1$]
	(ε)	$u \varepsilon C'R \,.\, u \neq x \,.\, \sim(xRu)$	[δ, Df. 1, $c2$[29]]

Dem.:

1.	$\bigwedge x \bigwedge y[yRx \rightarrow (fy \wedge fx)]$	$[T_p, y/x, x/y]$
2.	$\bigwedge x \bigwedge y[(xRy \vee yRx) \rightarrow (fx \wedge fy)]$	$[T_p, 1]$
3a.	$\bigwedge x[x \in C'R \rightarrow \bigvee t(tRx \vee xRt)]$	[Df. 1]
3b.	$\bigwedge x[x \in C'R \rightarrow \bigvee t(fx \wedge ft)]$	[3a, 2]
3c.	$\bigwedge x(x \in C'R \rightarrow fx)$	[3b)
3d.	$\bigwedge x[x \in C'R \rightarrow \bigvee u(uRx)]$	[3c, $c1$]
3e.	$\bigwedge x\{x \in C'R \rightarrow \bigvee u[(u \in C'R \wedge u \neq x) \wedge \sim xRu]\}$	[3d, Df. 1, $c2$]

Thesis (3) $\boxed{3e}$ is equivalent to the negation of the first element in the field of relation R. But from the fact that the negation $c3$ follows from T_p, $c1$ and $c2$, does not follow at all that $c3$ follows from $c1$, $c2$ and the negation of T_p.

So if we want $c3$ to follow from $c1$, $c2$ and the negation of T_p, we at least need thesis T_p to follow from $c1$, $c2$ and the negation of $c3$.

Now, thesis T_p does not follow from $c1$, $c2$ and the negation of $c3$.

I will make a proof with the help of an example.

Let us take the set of plus and minus integers including zero, ordered by relation of being greater than: $x>y$:

..., 4, 3, 2, [1,] 0, –1, –2, –3,...

If "fx" means the same as: x is a plus integer, "xRy" — x is greater than y, then $c1$ and $c2$ are satisfied and there is no first element in this set, but thesis T_p is still not true, because for example $-1>-2$, and it is not true that –1 and –2 are the plus integers.

[29] I mean the irreflexivity and asymmetry of relation R.

I emphasize by the way, that — even more — thesis T_p does not follow from $c1$, $c2$ and the assumption of infinity of the field of relation R; the assumption of the ordered set infinity is weaker than the negation of the first element in the ordered set, because from the negation of existence of the first element in the ordered set follows, that this set is infinite, but not conversely. To link discussed argumentation *per reductionem ad absurdum* with the problem of existence of the first element in the field of relation R, it is necessary to add a new assumption. To do this, it is enough to strengthen the first factor of the antecedent of thesis T by changing it into an equivalence, which means adding as a new assumption the conversion of the first factor of the predecessor of thesis T.

Making St. Thomas' argument complete, I add the above assumption as the one that fits the context the best[30].

Assumption 3.5. $[x] : [\exists t] . tRx . \supset fx$

Assumption 3.5. $\bigwedge x[\bigvee t(tRx)\rightarrow fx]$

Thesis T_p follows from $c2$, the negation of $c3$ and the assumption 3.5.

Dem.:

(1) $[x,y] : xRy \supset . x\varepsilon C'R . y\varepsilon C'R$ [Df. 1]

(2) $\sim([\exists y] \therefore y\varepsilon C'R \therefore [u] : u\varepsilon C'R . u\neq y . \supset yRu) . \supset :$

$[y] : y\varepsilon C'R \supset .$

$[\exists u] .$

(α) $u\varepsilon C'R . u\neq y . \sim(yRu)$[31]

(β) uRy [α, $c2$[32]]

(γ) fy [β, 3.5]

(δ) $[x,y] : xRy \supset . fx . fy$ [1, γ]

Dem.:

1. $\bigwedge x\bigwedge R[xRy\rightarrow(x\in C'R\wedge y\in C'R)]$ [Df. 1]

2a. $\sim\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rightarrow$ $\bigwedge y\{y\in C'R\rightarrow\bigvee u[(u\in C'R\wedge u\neq y)\wedge\sim yRu]\}$

[30] In reasonings made with the use of common language, such assumptions can be easily introduced completely unconsciously. This assumption is inferentially equivalent to the already introduced assumption 1.4.

[31] The successor is equivalent to the antecedent.

[32] I mean the connection of relation R.

2b. $\sim\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rightarrow$ $\bigwedge y[y\in C'R\rightarrow\bigvee u(uRy)]$ [2a, $c2$]

2c. $\sim\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rightarrow$ $\bigwedge y(y\in C'R\rightarrow fy)$ [2b, 3.5]

2d. $\sim\bigvee y\{y\in C'R\wedge\bigwedge u[(u\in C'R\wedge u\neq y)\rightarrow yRu]\}\rightarrow$ $\bigwedge x\bigwedge y[xRy\rightarrow(fx\wedge fy)]$ [1, 2c]

From assumptions: 3.1, 3.2, 3.3, 3.4, thesis T_p and the second factor of the antecedent of thesis T follow some consequences, that according to St. Thomas, should not be accepted.

Dem.:

(1) $[x,y] : xRy\wedge yRx . \supset . fx . fy$ [T_p]

(2) $[x] : x\varepsilon C'R \supset .$

$[\exists z] .$

(α) $zRx\vee xRz$ [Df. 1]

(β) $fz . fx .$ [α, 1]

(γ) fx [β]

(3) $[x] . x\varepsilon C'R \supset Cx$ [2, 3, 1]

(4) $[x] : x\varepsilon C'R \supset . Cx . fx$ [3, 2]

(5) $[x] : x\varepsilon C'R \supset . [\exists t_1] . t_1Fx$ [4, 3.2]

(6) $[x] : x\varepsilon C'R \supset . [t_2] . t_2Fx\supset Ft$ [3, 3.3]

(7) $[x] \therefore x\varepsilon C'R \supset : [\exists t_1] . t_1Fx . Ft_1$ [5, 6]

(8) $[x,y,t_1,t_2] : xRy\vee yRx . t_1Fx . t_2Fx . \supset . t_1=t_2$ [3.4]

(9) $[x,y,t_1\ t_2] : x\varepsilon C'R . y\varepsilon C'R . x\neq y . t_1Fx . t_2Fx . \supset . t_1=t_2$ [$c2$,[33] 8]

Dem.:

1. $\bigwedge x\bigwedge y[(xRy\vee yRx)\rightarrow(fx\wedge fy)]$ [T_p]

2a. $\bigwedge x[x\in C'R\rightarrow\bigvee z(zRx\vee xRz)]$ [Df. 1]

2b. $\bigwedge x[x\in C'R\rightarrow\bigvee z(fz\wedge fx)]$ [2a, 1]

2c. $\bigwedge x(x\in C'R\rightarrow fx)$ [2b)

3. $\bigwedge x(x\in C'R\rightarrow Cx)$ [2c, 3.1]

4. $\bigwedge x[x\in C'R\rightarrow(Cx\wedge fx)]$ [3, 2c]

[33] I mean the connection of relation R. I emphasize that thesis (9) would look logically much better, if we removed factor: $x\neq y$ from its antecedent; in some conceptions of identity, for example Leibnizian *identitas indiscernibilium*, this factor could be easily removed; I do not do this, because of the following reasons: (1) thesis (9) is absolutely sufficient for the next considerations; (2) I do not want to get unnecessarily into quite complicated problems about the identity, especially because we deal here with the time indicator and this immediately brings to one's mind different relativistic complications.

5.	$\bigwedge x[x \in C'R \rightarrow \bigvee t_1(t_1 Fx)]$	[4, 3.2]
6.	$\bigwedge x[x \in C'R \rightarrow \bigwedge t_2(t_2 Fx \rightarrow Ht_2)]$	[3, 3.3]
7.	$\bigwedge x[x \in C'R \rightarrow \bigvee t_1(t_1 Fx \wedge Ht_2)]$	[5, 6]
8.	$\bigwedge x \bigwedge y \bigwedge t_1 \bigwedge t_2\{[xRy \vee yRx) \wedge (t_1 Fx \wedge t_2 Fy)] \rightarrow t_1 = t_2\}$	[3.4]
9.	$\bigwedge x \bigwedge y \bigwedge t_1 \bigwedge t_2\{[(x \in C'R \wedge y \in C'R) \wedge x \neq y \wedge (t_1 Fx \wedge t_2 Fy)] \rightarrow t_1 = t_2\}$	[$c2$, 8]

In step (7) it is said that each element of the field of relation R is in motion for some time and the duration of this movement is the limited period of time. In step (9) the exact synchronousness of the movement of all set elements is being stated. So it is sufficient to check the time of movement of any element of the set (according to line (7) it is a limited time); it will also be the time of movement of all elements.

In step (3) it is said that each element is a body. So if an ordered set of moving bodies and the bodies that move is an infinite set, then the infinite and ordered set of bodies is in motion for the limited time.

This is not possible because of the following assumptions.

Assumption 3.6. The infinite body — and even infinite set of bodies which make *per continuationem* or *per contiguationem* somehow one body — cannot be in motion for the limited period of time.

Assumption 3.7. The bodies cannot act from the distance.[34]

It follows form the assumption 3.7 that the set of moving bodies and bodies that move, makes at least *per contiguationem* one body.

From assumptions: 3.7 and 3.6 follows the thesis:

(A) An infinite and ordered set of moving bodies and bodies that move is not in motion for the limited period of time.

It follows from $c1$, $c2$ and T_p that the ordered field of relation R has no first element, and so it is an infinite set (cf. p. 122—123).

It follows from assumptions: 3.1, 3.2, 3.3, 3.4, from $c2$ and thesis T_p that if an ordered set of moving bodies and bodies that move is an infinite set then an infinite and ordered set of moving bodies and bodies that move is in motion for the limited time; besides that, each element of the field of relation R is a body.

Thus, from assumptions: 3.1, 3.2, 3.3, 3.4, from $c1$, $c2$ and thesis T_p follows the thesis:

(B) An infinite and ordered set of moving bodies and bodies and that move is in motion for the limited period of time.

[34] These physical assumptions St. Thomas explicitly formulates referring to Aristotle's *Physics*.

Thesis (B) is contradictory to thesis (A).

Accepting assumptions: 3.7 and 3.6 St. Thomas accepts thesis (A) and rejects thesis (B).

Rejecting thesis (B) and accepting assumptions: 3.1, 3.2, 3.3, 3.4 and also $c1$ and $c2$, we have to reject thesis T_p.

From assumption 3.5, from $c2$ and the negation of $c3$ follows thesis T_p (cf. p. 124).

Rejecting thesis T_p and accepting assumption 3.5 and $c2$, we have to deny the negation of $c3$, which means that we have to accept, that there exists the first element in the ordered field of relation R. The first proof for the existence of the first element in the field of relation R has such subtly complicated structure.

b) The second and the third proof

Apart from all this, St. Thomas gives two more reasons for existence of the first element in the field of relation R.

They are included in St. Thomas' text: the first — between the words: "*Secunda ratio ad idem probandum talis est...*" and the words: "*...et sic nihil movebitur in mundo*"; and the second one — between the words: "*...Tertia probatio in idem redit...*" and the words: "*...ergo nihil movebitur*".

These are the theorems taken from Aristotle's *Physics*. Wanted conclusion follows directly from them, with the help of little logical transformations.

Reason I. In the set of moving bodies and bodies that move, ordered in the way, that each element is in relation R to next elements, no body can be in motion if the first element does not exist.

Reason II. Dependent motors (*moventes instrumentaliter*) can act only when at least one independent motor exists (*movens principaliter*).

While talking about the first reason St. Thomas indicates quite strongly, that relation R is an ordering relation.

Of all three proofs for the existence of the first element in the field of relation R, only the second reason is mentioned in *Theological Summa*.

At this point ends St. Thomas' argumentation for the existence of God based on the phenomenon and notion of movement[35].

[35] There are two digressions in St. Thomas' argument *ex motu*.

The first digression, made together with the first proof for irreflexivity of relation R,

13. Analysis of the conclusion

In the successor of thesis T is stated the existence of object which: (1) does not move by itself; (2) moves each object that is in motion.

Because of the postulate of reality of the field of relation R in the conclusion the real existence of this object is stated.

However the uniqueness of this object is not stated, it is only being discussed in the conclusion that at least one such object exists.

But from the antecedent of the thesis T it also can be derived that only one such object exists.

Because it goes beyond the frame work of St. Thomas argument, I will not amplify the proof of uniqueness, I will just outline it.

Let us assume that there are two different first objects in the ordered field of relation R, some objects A and B.

If object A is the first object in the ordered field of relation R, then:

(1) $[x] : x\varepsilon C'\mathcal{R} \,.\, x{\neq}A \,.\supset ARx$ $\boxed{(1)\quad \bigwedge x[(x{\in}C'\mathcal{R}{\wedge}x{\neq}A){\rightarrow}Arx]}$.

If object B is the first object in the ordered field of relation R, then:

(2) $[x] : x\varepsilon C'\mathcal{R} \,.\, x{\neq}B \,.\supset BRx$ $\boxed{(2)\quad \bigwedge x[(x{\in}C'\mathcal{R}{\wedge}x{\neq}B){\rightarrow}BRx]}$.

Because objects A and B by assumption belong to the field of relation R and are different from each other, so — because of connection of relation R — occurs as follows:

(3) $ARB{\vee}BRA$

If ARB occurs, then — because of asymmetry of relation R — BRA does not occur, so (2) is false.

If BRA occurs, then — because of asymmetry of relation R — ARB does not occur, so (1) is false.

is included in the text between the words: "...*Nec obviat huic rationi*..." and the words: "...*Si homo est asinus, est irrationalis*". This digression is logically interesting. St. Thomas emphasizes in it that conditional sentence can be true even when the antecedent is false. It is still very distant from the notion of material implication; he probably means such substitution of formal implication, with which the antecedent changes into a false sentence.

The second digression, made together with the third proof for irreflexivity of relation R, is included in the text between the words: "*Sciendum autem quod Plato*..." and the words: "... *quod omnino sit immobile secundum Aristotelem*".

The difference between Plato's and Aristotle's notion of movement is indicated in this digression.

The assumption that there are at least two different first objects in the ordered field of relation R, leads to a contradiction.

So there exists one and only one such object.

In the argument *ex motu* the finity of the field of relation R is not assumed, so the argument is a proper one also with an assumption, that this field is an infinite field.

Against the background of this argument it may seem that the world is an ordered set of moving objects and objects that move, and the first element of this only sequence is God.

But such a conception of the world is not very probable.

More suggestive would be an apprehension of the world as a bunch of sequences consisting of moving objects and objects that move. If one continues to amplify this topological example, we would have to say that these sequences run on the straight and curved lines, they cross one another in various points, and this whole bunch originates from one point, which is *Motor Immobilis*, God.[36]

The basis of the argument *ex motu* for the existence of God, optionally can then be any of these sequences, fixed — more or less explicitly because of the sequences crossing — by an experiential factor, introduced in the argument in one form or another.

14. Specification of assumptions

At the end, I will specify all assumptions included in the Thomistic proof *ex motu* for the existence of God, except for logic assumptions; in the specification of assumptions I also omit physical proof for the irreflexivity of relation R.

1. $[x] : fx{\supset} \,.\, [\exists t] \,.\, tRx$
2. $[x,y,z] : xRy \,.\, yRz \,.\supset xRz$
3. $[x,y] : x\varepsilon C'\mathcal{R} \,.\, y\varepsilon C'\mathcal{R} \,.\, x{\neq}y \,.\supset .\, xRy{\vee}yRx$

[36] With such conception of the world the argument *ex motu* does not provide material sufficient to state the uniqueness of the First Motor; proof for uniqueness outlined previously only indicates that in each ordered sequence of moving objects and objects that move there is one and only one first element; from this of course does not follow at all that all first elements of different sequences are identical. With this conception it would be necessary to try to solve in a different way the problem, how many First Motors there are, but then again it brings us back to the same way that St. Thomas treats this matter; cf. *C. G.* l. 1 c. 42 and I q. 11 a. 3.

1.	$\bigwedge x[fx \rightarrow \bigvee t(tRx)]$
2.	$\bigwedge x \bigwedge y \bigwedge z[(xRy \land yRz) \rightarrow xRz]$
3.	$\bigwedge x \bigwedge y\{[(x \in C'R \land y \in C'R \land x \neq y)] \rightarrow (xRy \lor yRx)\}$

A or *B*[37]

A:	1.1.	$[x] : fx \supset . [\exists a,b] . M_x(a) . M_x(b)$
	1.2.	$[x] \therefore [\exists a,b] : M_x(a) . M_x(b) : \sim(fa) . fb . \lor . \sim(fa) \supset \sim(fb) : \supset \sim(xRx)$
	1.4.	$[x,y] . xRy \supset fy$
B:	2.1.	$[x,y,S] . xA_Sy \supset \sim(xP_Sy)$
	2.2.	$[x,y] : fx . yRx . \supset xP_Ry$
	2.3.	$[x,y] : fx . yRx . \supset yA_Rx$
	1.4.	$[x,y] . xRy \supset fy$

A:	1.1.	$\bigwedge x[fx \rightarrow \bigvee a \bigvee b(aMx \land bMx)]$
	1.2.	$\bigwedge x \llbracket \bigvee a \bigvee b\{(aMx \land bMx) \land [(\sim fa \land fb) \lor (\sim fa \rightarrow \sim fb)]\} \rightarrow \sim xRx \rrbracket$
	1.4.	$\bigwedge x \bigwedge y(xRy \rightarrow fy)$
B:	2.1.	$\bigwedge x \bigwedge y \bigwedge S(xA_Sy \rightarrow \sim xP_Sy)$
	2.2.	$\bigwedge x \bigwedge y[(fx \land yRx) \rightarrow xP_Ry]$
	2.3.	$\bigwedge x \bigwedge y[(fx \land yRx) \rightarrow yA_Rx]$
	1.4.	$\bigwedge x \bigwedge y(xRy \rightarrow fy)$

C or *D* or *E*

C:	3.1.	$[x] . fx \supset Cx$
	3.2.	$[x] : Cx . fx . \supset . [\exists t_1] . t_1Fx$
	3.3.	$[x] : Cx \supset . [t_2] . t_2Fx \supset F(t_2)$
	3.4.	$[x,y,t_1,t_2] : xRy . t_1Fx . t_2Fy . \supset . t_1=t_2$
	3.5.	$[x]:[\exists t] . tRx . \supset fx$

C:	3.1.	$\bigwedge x(fx \rightarrow Cx)$
	3.2.	$\bigwedge x[(Cx \land fx) \rightarrow \bigvee t_1(t_1Fx)]$
	3.3.	$\bigwedge x[Cx \rightarrow \bigwedge t_2(t_2Fx \rightarrow H\mathrm{t}_2)]$
	3.4.	$\bigwedge x \bigwedge y \bigwedge t_1 \bigwedge t_2\{[xRy \land (t_1Fx \land t_2Fy)] \rightarrow t_1=t_2\}$
	3.5.	$\bigwedge x[\bigvee t(tRx) \rightarrow fx]$

[37] The sets of assumptions treated alternatively I mark with the capital Latin letters.

3.6 An infinite body — and even an infinite set of bodies which makes *per continuationem* or *per contiguationem* kind of the one body — cannot move in a limited segment of time.

3.7 Bodies cannot act at a distance.[38]

D: In a set of bodies which are in motion and make others move, ordered in such way that each element is in relation *R* to the next ones, no body can be in motion if the first element does not exist.

E: Dependent motors (*moventes instrumentaliter*) can act only when there exists at least one independent motor (*movens principaliter*).

For the readers less accustomed to the symbolic language, I will formulate symbolic assumptions also in a common language.

1. If something is in motion, then there exists a motor which moves that is in motion.

2. Relation *R* (of movement) is a transitive relation.

3. Relation *R* (of movement) is connected relation.

1.1. If something is in motion, then it consists of proper parts.

1.2. If in any object *x* two such parts are included: *a* and *b*, that (1) *a* is not in motion, and *b* is in motion, or (2) if *a* is not in motion, then also *b* is not in motion, then it is not true that object *x* makes itself move.

1.4. If an object *y* is moved by an object *x*, then this object *y* is in motion.

2.1. If an object *x* is, under a certain aspect, *in actu* in respect to an object *y*, then it is not true that the same object *x* is under the same aspect *in potentia* in respect to the same object *y*.

2.2. If an object *x* is in motion and if this object *x* is moved by an object *y*, then, under the aspect of this movement, *x* is *in potentia* in respect to the object *y*.

2.3. If an object *x* is in motion and if this object *x* is moved by an object *y*, then, under the aspect of this movement, *y* is *in actu* in respect to the object *x*.

3.1. If an object *x* is in motion, then this object *x* is a body.

3.2. If a material object is in motion, then some segment of time is the period of the duration of this movement.

[38] *Actio in distans* is excluded.

3.3. If a material object is in motion, then the period of the duration of the movement of this object is a limited segment of time.

3.4. The duration of the movement of the motor in motion is the same as the duration of movement of the object moved.

3.5. If for an object x such object t can be found, that t moves x, then x is in motion.

In the specified assumptions, some logical simplifications can be done without any difficulties.

I did not make them in my work, because I wanted to reconstruct St. Thomas' proof, not to specify it; after all, it happens very often that logical simplifications argue with intuitivity and I would be afraid that by introduction of these simplifications I would deform St. Thomas' proof in intuitive respect.

However, to fulfill logical duty, I am hereby noting that the following simplifications simply intrude:

(1) In assumption 1.1 in the successor, one of the factors put under the particular quantificator can be omitted, because the second factor follows from the definition of the proper part.

(2) Assumption 1.2 is as I already mentioned in footnote 23 on p. 118, is inferentially equivalent to the following assumption:

$$[x, a] \,.\, M_x(a) \supset \sim(xRx) \quad \boxed{\wedge x \wedge a(aMx \rightarrow \sim xRx)}\,.$$

(3) Assumption 3.5 is inferentially equivalent to assumption 1.4.

(4) Because of assumption 1.4, the first factor of the antecedent can be omitted in assumptions: 2.2 and 2.3, in accordance with the following thesis from the deduction theory:

$$p \supset q \,.\, \supset \therefore q \,.\, p \,.\, \supset r : \equiv .\, p \supset r \quad \boxed{(p \rightarrow q) \rightarrow \{[(q \wedge p) \rightarrow r] \equiv (p \rightarrow r)\}}\,.$$

(5) Because of assumption 3.1 the first factor of the antecedent can be omitted in assumption 3.2; this simplification is being made because of the same reasons as in point (4).

I hope that the following words of St. Thomas sufficiently justify my new treatment of the traditional subject: "... *Quamvis scientia divina sit prima omnium scientiarum, naturaliter tamen quoad nos aliae scientiae sunt priores. Unde dicit Avic. in princ. suae Meta.: "Ordo illius scientiae est ut addiscatur post scientas naturales, in quibus sunt multa determinata, quibus ista scientia utitur...". Similiter etiam post mathematicam. Indiget enim haec scientia ad cognitionem substantiarum separatrum cognoscere numerum et ordinem*

orbium coelestium, quod non est possibile sine astrologia, ad quam tota mathematica praeexigitur" (*In Boëtii de Trin.* q. 5 a. 1). In the contemporary stage of the evolution of sciences, one would only need to change both terminology and reasons a little.

Translated from Polish by Kordula Świętorzecka