Formalization and Automation of Variants of Gödel's Ontological Proof of God's Existence

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Attempts to prove the existence (or non-existence) of God by means of abstract ontological arguments are an old tradition in philosophy and theology. Gödel's proof [**?**] is a modern culmination of this tradition, following particularly the footsteps of Leibniz. Gödel defines God as a being who possesses all *positive* properties. He does not extensively discuss what positive properties are, but instead he states a few reasonable but debatable axioms that they should satisfy. In our recent work we did employ the following axioms[3]: (1) Any property strictly implied by a positive property is positive, (2) A property is positive if and only if its negation is not positive, and (3) the property of being God-like is positive. Moreover, the following definitions are employed: (A) x is God-like if and only if x incorporates all positive properties, (B) a property P is essential for x (and essence of x) if and only if P is a property of x and every property Q that x has is strictly implied by P, (C) x necessarily exists if and only if every essence of x is necessarily exemplified.

From these axioms and definitions we then infer:

- Positive properties are eventually exemplified.
- Eventually God exists.
- If x is God-like, then the property of being God-like is an essence of x.
- Necessarily God exists.

In the literature [**?**] slightly different versions of the axioms and definitions exist and derivations of the above mentioned claims are presented with various degrees of detail and formality. The above variant of Gödel's proof has now been constructed for the first-time with the utmost degree of detail and formality; cf. [**?**]. The following has been done:

- A detailed natural deduction proof.
- A formalization of the axioms and theorems in the TPTP THF format [6].
- Automatic verification of the consistency of the axioms and definitions with Nitpick [3].
- Automatic demonstration of the theorems with the provers LEO-II [2] and Satallax [4].
- A step-by-step formalization using the Coq proof assistant [**?**].

[3] Say where they come from and how they relate to Gödel.

– A formalization using the Isabelle proof assistant [5] partially automated with Sledgehammer [**?**] and Metis [**?**].

Gödel's proof is challenging to formalize and verify because it requires very expressive logical languages with modal operators (*possible* and *necessary*) and higher-order quantifiers. Our computer-assisted formalizations rely on an embedding of the modal logic S5 into classical higher-order logic with Henkin semantics [1] and employed recently developed interactive and automated deduction tools designed for this logic.

This work attests the maturity of contemporary interactive and automated deduction tools and opens new perspectives for a computer-assisted theoretical philosophy. The critical discussion of the underlying concepts, definitions and axioms remains a human responsibility, but the computer can assist in building and checking rigorously correct logical arguments. In case of logico-philosophical disputes, the computer can check the disputing arguments and partially fulfill Leibniz' dictum: Calculemus — Let us calculate!

Future work includes an extensive study of other formalizations of ontological arguments with our machinery.

References

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