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Anselm's Ontological Proof: Consequences in System Theory

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To George Erik Lasker

1. Introduction

Philosophers have discussed the problem of the existence of a Supreme Being for thousands of years. Today the great majority of scientists think that such a question is not scientific as the actual prevalent episemological ideas affirm. So, the given complete philosphical proofs of God's existence are pratically ignored. But not to consider such proofs completely means losing not only their metaphysical aspects but also their logical, mathematical and systemic aspects that are often the most important. It is not by chance that the most recent of the important demonstrations of God's existence is a proof of Kurt Gödel, who is a mathematician,[1] and that such a proof has been discussed principally by mathematicians.[2] We think that such a proof does not add aspects of interest to the scope of this paper on Anselm's

[1] K. Gödel, *Ontologischer Beweis*, in J. H. Sobel, *Gödel's Ontological Argument*, On Being and Saying (Ed. by Thompson), MIT Press, Cambridge MA, 1987 pp. 256-257.

[2] C. A. Anderson, *Some Emendations to Gödel's Ontological Proof*, Faith and Phylosophy **7** (1990). R. Brecher, *Hartshorne's Modal Argument for Existence of God*, Ratio **XVII** (1975). J. Czermak, *Abriss des ontologisshen Argumentes*, Wahrheit und Beveisbarkeit - Leben und Werk K. Gödel's (Buld, Köhler and Schimanovitch, eds.) (1995). P. Hajeck, *Der Mathematiker und die frage der existenz Gottes (Betreffen Gödel's ontologishen Beweis)*, Wahrheit und Beweisbarkeit – Leben und Werk K. Gödel's (Buld, Köhler and Schimanovitch, eds.) (1995). P. Hajek, *Magari and others on Gödels ontological proof* – Logic and Algebra (A. Ursini and P. Aglianò eds. – Dekker, New York-Basel-Hong Kong) (1996). R. Magari, *Logica e teofilia*, Notizie di logica, VII **4** (1988). F. P. M. J. Vorbraak, *As for I know (Epistemic logic and uncertainity)*, Ph. D. thesis, Univ. Utrecht, 1993.

proof of God's existence. So, we shall consider directly this last proof to obtain useful results by them for the system theory.

2. Proof of God's Existence by Anselm of Canterbury

An English translation of Anselm of Canterbury's[3] proof of God's existence is the following:

" *Hence, Lord, who gives faith intellect, give me to understand, with all Your possible help, because You are as we*

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[3] Sancti Anselmi Cantuariensis Archiepiscopi, *Opera Omnia*, ad fidem codicum recensuit F. S. Schmitt, O.S.B., Edimburgi 1946–1961, 6 voll: vol I, *Proslogion*, cap. II: 1-5, cap. III: 1-2. The original latin text is the following:

" *Ergo, domine, qui das fidei intellectum, da mihi ut quantum scis expedire intelligam, quia es sicut credimus, et hoc quod credimus. Et quidem credimus te esse aliquid quo nihil maius cogitari possit.*

An ergo non est aliqua talis natura, quia 'dixit insipiens in corde suo: non est deus'? Sed certe ipse idem insipiens, cum audit hoc ipsum quod dico: 'aliquid quo maius nihil cogitari potest', intelligit in intellectu eius est, etiam si non intelligat illud esse.

Aliud enim est rem esse in intellectu, aliud intelligere rem esse. Nam cum pictor præcogitat quæ facturus est, habet quidem in intellectu, sed nondum intelligit esse quod nondum fecit. Cum vero iam pinxit, et habet in intellectu et intelligit esse quod iam fecit.

Convincitur ergo etiam insipiens esse vel in intellectu aliquid quo nihil maius cogitari potest, quia hoc cum audit intelligit, et quidquid intelligitur in intellectu est. Et certe id quo maius cogitari nequit, non potest esse in solo intellectu. Si enim vel in solo intellectu est, potest cogitari esse et in re, quod maius est.

Si ergo id quo maius cogitari non potest, est in solo intellectu: id ipsum quo maius cogitari non potest, est quo maius cogitari potest. Sed certe hoc esse non potest. Existit ergo procul dubio aliquid quo maius cogitari non valet et in intellectu et in re.

Quod utique sic vere est, ut nec cogitari possit non esse. Nam potest cogitari esse aliquid, quod non possit cogitari non esse; quod maius est quam quod non esse cogitari potest. Quare si id quo maius nequit cogitari, potest cogitari non esse: id ipsum quo maius cogitari nequit, non est id quo maius cogitari nequit; quod convenire non potest, ut nec cogitari possit non esse.

Et hoc est tu, domine deus noster. Sic ergo vere es, domine deus meus, ut nec cogitari possis non esse. Et merito. Si enim aliqua mens posset cogitare aliquid melius te, ascenderet creatura super creatorem, et iudicaret de creatore; quod valde est absurdum. Et quidem quidquid est aliud præter te solum, potest cogitari non esse. Solus igitur verissime omnium, et ideo maxime omnium habes esse: quia quidquid aliud est non sic vere, et idcirco minus habet esse."

believe and that You are that we believe. Now, we believe that You are whose nothing can be thought greater.

Or is not there perhaps anything of such a nature because 'the fool tells in his heart: God is not'? But of course this same fool, when he listens to that I tell 'something whose nothing can be thought greater', understands that he listens to. And that he understands is in his intellect, nevertheless he does not see it in reality.

In fact, to think something is not to see something in reality. Indeed a painter thinks that he will make, so he has something in his intellect, but he does not see such a thing in reality. When he painted it, either he thinks it or he sees it in reality.

Hence, the fool also must persuade himself that there is in his intellect something whose nothing can be thought greater because he understands it when he listens to it, and that it is understod in intellect. Indeed , of course,whose nothing can be thought greater cannot be in intellect only. In fact, if it is in intellect only, then it can be thought existing in reality also and therefore it should be greater.

Indeed , if whose nothing can be thought greater is in intellect only, then it is something can be thought greater. But, of course, this last conclusion is contradictory. Hence, whose nothing can be thought greater exists either in intellect or in reality without doubt.

So, such a thing is so true that it is not possible to think that it does not exist. In fact, it is possible to think that something, that cannot be thought not existing, exists, but such a thing is greater than everything can be thought not existing. Indeed, if whose nothing can be thought greater does not exist, then it is not whose nothing can be thought greater; this last conclusion is contradictory. Hence, whose nothing can be thought greater exists so really that it cannot be thought as not existing.

And you, Lord and our God, are such a thing. Indeed, Lord and my God, exist so truly that you cannot be thought as not existing. And with merit. If some mind could think something that is better than you, a creature should be better than its creator and should judge its creator, but such a conclusion is absurd too. In effect, everything that exists, except you only, can be thought as not existing. You only, hence, has the the most truth and great existence among all the things, as every other thing is not so true, and therefore it has a lesser existence."

Scholastic doctors like Anselm had achieved the highest level in formal logic, but they did not use a formalized language. A modern approach to Anselm's proof can be the following.

Consider the set of possible objects mo_i that can be thought by a human mind $U = \{mo_1, \ldots, mo_n, \ldots\}$. Let '$\leq$' be the order reflexive relation '… *isworse than or equal to* …' which gives a partial order on *U*, i. e. $U = (U, \leq)$ Read '*U is a partial ordered set or, briefly, a poset*'. A subset $X \subseteq U$ is *directed* if $X \neq \emptyset$ and $(\forall x)(\forall y)(\exists z)(x \in X \wedge y \in X \wedge z \in X \wedge x \leq z \wedge y \leq z)$, i. e. if it is not empty and, for every couple of its elements, there is an element of it that is better than or equal to every element of that couple. *U* is a *complete partial order* (cpo) if $(\exists x)(\forall y)(x \in D \wedge y \in D \wedge x \leq y)$ (call such an element *x* '*worst element*' of *D* and denote it with '$\perp$') and $(\forall X)(\exists x)(\forall y)(X \subseteq D \wedge x \in X \wedge y \in X \wedge y \leq x)$ (call such an element *x* '*best element*' of *X* and denote it with '$\bigcup X$'). A *complete lattice* is a poset *D* such that $(\forall X)(\exists x)(poset(D) \wedge X \subseteq D \wedge x = \bigcup X)$. A complete lattice is a cpo because $\perp = \bigcup \emptyset$ and it is a directed set because $D \subseteq D$. If $P(X)$ is the power set of the set *X*, then $(P(X), \subseteq)$ is a cpo (even a complete lattice).[4] So, every power set has a best element.

Consider now a first order theory as it is defined by standard mathematical logic.[5] We can put or add a set of individual

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[4] J. Barwise, D. Kaplan, H. J. Keisler, P. Suppes, A. S. Troelstra eds., *The Lambda Calculus – Its Syntax and Semantics*, North – Holland, Amsterdam – New York – Oxford, Revised edition 1984

[5] E. Mendelson, *Introduction to Mathematical Logic*, D. Van Nostrand Company, Princeton, New Jersey, 1964. Precisely, a first order theory is a logical mathematical structure with the following general axioms:

1) $\mathsf{A} \supset (\mathsf{B} \supset \mathsf{A})$,

2) $(\mathsf{A} \supset (\mathsf{B} \supset \mathsf{C})) \supset ((\mathsf{A} \supset \mathsf{B}) \supset (\mathsf{A} \supset \mathsf{C}))$,

3) $(\sim \mathsf{B} \supset \sim \mathsf{A}) \supset ((\sim \mathsf{B} \supset \mathsf{A}) \supset \mathsf{B})$,

4) $(\forall x_i)\mathsf{A}(x_i) \supset \mathsf{A}(t)$ if x_i is free for t in $\mathsf{A}(x_i)$,

5) $(\forall x_i)(\mathsf{A} \supset \mathsf{B}) \supset (\mathsf{A} \supset (\forall x_i)\mathsf{B})$ if x_i has no free occurrences in A;

with the following infererence schemes:

6) $\mathsf{A}, \mathsf{A} \supset \mathsf{B} \mapsto \mathsf{B}$,

7) $\mathsf{A} \mapsto (\forall x_i)\mathsf{A}$;

constants $\{a_1,\ldots,a_n,\ldots\}$ to such a theory always.[6] To denote such a set of p individual constants a_i we can use sequences of $q = \mathrm{int}(\log_2 p) + 1$ signs '1' and/or '0' between squared brackets because 2^q is greater than or equal to p by construction. E. g., given any $a_i \in \{a_1,\ldots,a_p\}$, put the integer i in q digit binary format ans call such a sequence of signs '1' and/or '0' $b_q(i)$. So, we can put $a_i = [b_q(i)]$. After, we can transform any sequence of '1' and/or '0' in a sentence function. E. g., consider the sequence '100111001'. It has nine signs '1' and/or '0'. The first sign is '1'. Replace this sign with the sentence function $I_1^1(x_k)$ in a way that can put '100111001' $\rightarrow$ $I_1^1(x_k) \wedge \wedge$ '00111001'. The second sign is '0'. Replace this sign with the sentence function $\sim I_2^1(x_k)$ in a way that can put '100111001' $\rightarrow$ $I_1^1(x_k) \wedge \sim I_2^1(x_k) \wedge$ '0111001'. The third sign is '0'. Replace this sign with the sentence function $\sim I_3^1(x_k)$ in a way that can put '100111001' $\rightarrow$ $I_1^1(x_k) \wedge \sim I_2^1(x_k) \wedge \sim I_3^1(x_k) \wedge$ '111001'. The fourth sign is '1'. Replace this sign with the sentence function $I_4^1(x_k)$ in a way that can put '100111001' $\rightarrow$ $\rightarrow I_1^1(x_k) \wedge \sim I_2^1(x_k) \wedge \sim I_3^1(x_k) \wedge I_4^1(x_k) \wedge$ '11001' and so on. Finally we have '100111001' $\rightarrow I_1^1(x_k) \wedge \ldots \wedge I_9^1(x_k)$. So, e. g., we have $\mathsf{A}(a_r) \equiv \mathsf{A}([100111001]) \equiv (\forall x_k)(\mathsf{A}(x_k) \wedge I_1^1(x_k) \wedge \ldots \wedge I_9^1(x_k))$. In this way, to introduce a set of individual constants in a first order predicative calculus without individual constants (remember that the introduction of new individual constants does not change a first order theory),[7] we can introduce a set of monadic predicates . It is easy to prove that if we introduce n new monadic predicates, then we introduce 2^n new individual constants. Every constant set

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and finally with its own peculiar axioms. A generic sentence of a first order theory contains n-adic predicates A_m^n for some n, and variables x_i for some i, also it can contain logic connectives, quantifiers $(\forall x_i)$ and/or $(\exists x_i)$ for some i, individual constants a_i for some i.

[6] See note 5.

[7] See note 5.

becomes a power set of an opportune monadic predicate set and every monadic predicate set generates a constant set that is its power set. Given a monadic predicate set $MPR = \{A_1^1, \ldots, A_n^1, \ldots\}$, call *good criterion* an order meta-relation '≤' such that, for every A_i^1 of *MPR*, $(x_k)('A_i^1(x_k)' \leq' \sim A_i^1(x_k)') \not\equiv (x_k)(' \sim A_i^1(x_k)' \leq' A_i^1(x_k)')$. Observe that if we have a good criterion for a monadic predicate set, then the power set of such a predicate set is directed, and, therefore, the individual constant set generated by *MPR* is directed, i. e. there is an individual constant which is the best element. It is very easy to prove that, for every monadic predicate set, we have many good criteria as generated individual constants, i. e. there is a mapping between the generated individual constants and the good criteria . Every individual constant becomes the best element when its corresponding criterion is applied.

From the previous logical and mathematical results we can reduce Anselm's ontologic proof to this brief argument:

1) There is in reality an objective good criterion … *is better than* … that transform our set of mind objects in a directed set,
2) For this good criterion and for the monadic predicate '… exists', is true that '…exists' *is better than* '… does not exist',
3) Given 1) and 2) the best object among our mind objects has to exist, and its existence is the consequence of the existence of a good criterion where '…exists' *is better than* '… does not exist' in the reality.

3. Use of Anselm's Proof in Modern System Theory

Consider a generic system.It is a relation set.[8] Also, we can affirm that every mind object can be represented by an opportune

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[8] Given an ordered subset *T* of the set *R* of the real numbers and two not empty sets at will *U* and *Y*, call *abstract directed system* a relation set:

$$\mathsf{S} \equiv \left\{ S_{t_0} \subset \left(U^{T(t_0)} \times Y^{T(t_0)} \right) \middle| t_0 \in T \right\}$$

such that:

$$(\forall t_0)(\forall t_1)\left(t_0 \in T \wedge t_1 \in T \wedge (u_0, y_0) \in S_{t_0} \Rightarrow \left(u_0 \middle| T(t_1), y_0 \middle| T(t_1)\right) \in S_{t_1}\right);$$

where *T* is the set of the time values, *U* the set of the input quantity and *Y* see: A. Ruberti and A. Isidori, *Teoria dei sistemi*, Boringhieri, Torino, 1979; W. J. Rugh, *Mathematical Description of Linear Systems*, Dekker, New York, system.[9] As every system is a mind object because it has been thought by its creator at less, we can affirm that the systems are the mind objects exactly. Now consider the set *MO* of all the possible systems (which is the set of all possible mind object too).

When is a mind object, i. e. a system, real? Epistemology would like to solve this problem. Given a cybernetic aproach, human knowledge of reality is possible iff reality is representable by mind objects, i. e. by systems; in other terms, iff the reality is structured in objects that are representable by systems. We can affirm that a mind object (a system) is real if is the *best descriptor* of a real object.This one appears to us with a space time extension of data that have to be summarizable by relations among them. Call H_{ro} the information quantity[10] which is necessary to describe the set of data that represent a given real object *ro*. If we suppose that a cybernetic can know the world, then one o more systems, i. e. one or more mind objects can describe the considered real object. Suppose that *ro* can be described by the totally or partially uncompatible mind objects $mo_1,\ldots,mo_n$. Call $H_{mo_1},\ldots H_{mo_n}$ the information quantities that describe $mo_1,\ldots,mo_n$ respectively. Let $mo_r \in \{mo_1,\ldots,mo_n\} | (\forall i \in \{1,\ldots,n\})(H_{ro} - H_{mo_r} \geq H_{ro} - H_{mo_i})$ be the best descriptor of *ro*. We can make such an assumption because in this way we choose as 'true' the most simple and compact system (i. e. mind object) which explains all the data of the real object. Observe that, given two systems (mind objects) mo_1 and mo_2 and a real object *ro* such that $H_{ro} - H_{mo_1} = H_{ro} - H_{mo_2}$, if a fact *f* of *ro* that falsify mo_1 but not mo_2 happens, then we have the following transformations: $H_{ro} \to H_{ro} + H_f$ because *f* adds itself to *ro* data,

1975. It is trivial that if an abstract direct system is a relation set, then a generic system has to be a relation set too.

[9] Because every mind object can be defined completely by an opportune relation set. E. g. consider the structure of the dictionary of any language where the meaning of the words is described by relations with other words.

[10] To evalue the information quantity that is necessary to describe data at will we use Shannon's measure $H = -\sum_{i=1}^{n} p_i \log_2 p_i$ where *n* is the number of possible distinct descriptions and p_i is the probability that the *i*-th description is true. See Shannon C. E., *A Mathematical Theory of Comunication*, Bell System Tech. J. **27**, 1948.

$H_{mo_1} \rightarrow H_{mo_1} + H_f$ because mo_1 does not explain *f* and therefore to explain all the *f* information has to be added to the falsified system,

$H_{mo_2} \rightarrow H_{mo_2}$ as mo_2 contained the *f* information before; so $H_{ro} - H_{mo_1} = H_{ro} - H_{mo_2} \rightarrow H_{ro} - H_{mo_1} \leq H_{ro} + H_f - H_{mo_2}$, hence Popper's epistemologic criterion is an easy consequence of our criterion. So, we can consider *real* any mind object (any system) when it is the best descriptor of a given real object.

Finally, consider the set *RS* of the real mind objects , i. e. of the real systems. In general, every system can be included in another opportune system. Given $S_1, S_2 \in RS$, If S_1 explains all the real data which S_2 explains, then we write $S_1 \rhd S_2$. So, $(RS, \rhd)$ is a poset.

After, $RS \not\equiv \emptyset$, otherwise no system represents real data and no science should be possible. If $S_1, S_2 \in RS$, the we can build such a $S_3 \in SR$ that explains either all the real data ro_1 explained by S_1 or all the real data ro_2 explained by S_2. In fact, $S_1, S_2 \in RS$ are relation sets and therefore we can put $S_3 \subseteq S_1 \cup S_2$. We can adjust $S_1 \cup S_2$ by unification of the symbols and by choosing among the uncompatible relations in $S_1 \cup S_2$ by the usual scientific methods.$S_1 \cup S_2$ becomes such a efficient system $S_1 \oplus S_2$ that $S_3 = S_1 \oplus S_2$.$H_{ro_1} - H_{S_1} + H_{ro_2} - H_{S_2} \leq H_{ro_1} + H_{ro_2} - H_{S_3}$ is trivial. Therefore $S_3 = S_1 \oplus S_2$ is preferible to $S_1 \cup S_2$ by our truth criterion . Hence, $(RS, \rhd)$ is an epistemological poset too, i. e. if $S_1 \rhd S_2$, then S_1 is 'more true' then S_2. Also, it is trivial that $(\forall S_1)(\forall S_2)(\exists S_3)(S_1 \in RS \wedge S_2 \in RS \wedge S_3 \in RS \wedge S_3 \rhd S_1 \wedge S_3 \rhd S_2)$, where $S_3 = S_1 \oplus S_2$, hence, $(RS, \rhd)$ is an epistemological directed set where the upper elements are 'more real' of the lower ones.

Consider the set *RD* of real data. Every subset has a best description system. So, *RS* is the power set of *RD*. Also, if $ro_1, ro_2, ro_3 \in RD$ are described by $S_1, S_2, S_3 \in RS$, then $ro_1 \supseteq ro_2 \cup ro_3$ iff $S_1 \rhd S_2 \oplus S_3$, i. e. $(RS, \rhd)$ is an epistemological cpo and complete lattice. So, all the reality is represented by the system $\cup RS$ which is more complex and scientifically true than any other system. Let the comprehension of its nature be left to philosophers.