If some Property is not Divine, then God exists.

In 1970 Kurt Gödel, in a two-page note called "Ontologischer Beweis" (now available in Feferman (1995)), put forward an original ontological argument for the existence of God, where he made use of second-order modal logical principles. In the talk I will remind the reader of the principles which govern Anderson's emendation of Gödel's argument in Anderson (1990), where he rectified Gödelian ontological arguments by developing a version closely analogous with Gödel's original one while at the same time avoiding the *modal collapse* which, as shown by Sobel (1987), characterises Gödel's original argument. I will then make recourse to Bjørdal (1999), where it is shown that one can obtain a version of Gödel's ontological argument which only makes use of two axioms instead of Anderson's five axioms. In Bjørdal's 1999-approach, one takes the property of being Godlike, i.e. the predicate G, as primitive, and defines the second order property of *being a divine* (or *"positive" in* Gödel's exposition) *propert*y as follows:

Definition D: $D(F)\equiv\Box(\forall x)(G(x)\supset F(x))$

Definition D (or more precisely, the bi-conditional supported by the understood definition, and written out in this bi-conditional form in the exposition here) turns out, as shown in Bjørdal (1996), to be equivalent under second order S4 (including certain instances of second order comprehension) with the conjunction of three of Anderson's axioms and Gödel's Def. 1. If we let N be Anderson's rectified property of necessary-existence, then Bjørdal (1999) established, in the terms used here, that one under second-order B only needs the axioms

Ax. 1: $D(F) \supset {\sim}D({\sim}F)$
Ax. 2: $D(N)$

in order to support the theistic conclusion.

Petr Hajek, in Hajek (1996), presupposing a comprehension principle for the predicate G, has shown that Anderson's AA4 and AA5 are superfluous. According to Hajek, then, the following assumptions, in our terms, suffice in the derivation of Gödel's main theorem:

DA	$Gx \equiv (\forall \mathbf{F})(\Box\mathbf{F}(x) \equiv D(\mathbf{F}))$
AA1	$D(\mathbf{F}) \supset {\sim}D({\sim}\mathbf{F})$
AA2	$[D(\mathbf{F})\ \&\ \Box(\forall x)(\mathbf{F}(x) \supset \mathbf{H}(x))] \supset D(\mathbf{H})$
AA3	$D(G)$

A presupposition in Hajek (1996) is the following second order comprehension principle

Hajek's Assumption $(\exists H)(H(x) \equiv (\forall \mathbf{F})(\Box\mathbf{F}(x) \equiv D(\mathbf{F})))$

It turns out that the results in Hajek (1996) and Bjørdal (1999) can be combined to obtain a stronger result. Let us in addition to Definition D presuppose one half of DA:

Definition G: $G(x)\equiv(\forall Y)(D(Y)\supset\Box Y(x))$

In combination, Definition D and Definition G express the following recursive definition of *divine property*:

Definition RD: $D(F)\equiv\Box(\forall x)((\forall Y)(D(Y)\supset\Box Y(x))\supset F(x))$

Of course, the seeming circularity in this recursive definition is only apparent. The occurrence of the definiendum in the definiens in Definition RD is positive, and is avoided if we presuppose third order machinery. Given this, Definition RD is perfectly acceptable as a definitional introduction of the second-order predicate *Divine*. Assuming Definition RD and Definition G therefore amounts to the same thing as assuming Definition D and Definition G.

A version of Hajek's Assumption will be operative in our argument insofar as we allow universally instantiating with the predicate G. We now give the argument. Assume that some property X is not divine, i.e. $\sim D(X)$. From Definition D it follows that $\Diamond\exists x(G(x)\ \&\ \sim X(x))$, and hence $\Diamond\exists xG(x)$. Now, Definition G supports, given our version of Hajek's Assumption, that $\Box(D(G)\supset(\forall x)(Gx\supset\Box Gx))$. Assuming the characteristic S4-principle $\Box F\supset\Box\Box F$, we have, given Definition D, that $D(G)\supset\Box D(G)$. But, given Definition D we clearly have that D(G), as $\Box(\forall x)(G(x)\supset G(x))$. It follows that $\Box(\forall x)(Gx\supset\Box Gx))$. By the necessity of predicate-logic we have that $\Box((\exists x)Gx\supset(\exists x)\Box Gx)$. As $\Box((\exists x)\Box Gx\supset\Box(\exists x)Gx)$, it follows that $\Box((\exists x)Gx\supset\Box(\exists x)Gx)$. It follows that $\Diamond\exists xG(x)\supset\Diamond\Box\exists xG(x)$. But we established that $\Diamond\exists xG(x)$ so we have that $\Diamond\Box\exists xG(x)$. By the characteristic B-principle $\Diamond\Box F\supset F$ it now follows that $\exists xG(x)$. As we did not use necessitation in our derivation, detachment supports the title of this abstract.

The argument given here presupposes second order S5, or rather, second order S5 minus the characteristic T-principle $\Box p\supset p$. Given Definition RD, or Theorem RD if Definition D and Definition G are assumed and the supported bi-conditionals made use of, our argument is amenable to a fixed-point analysis. From an atheist point of view, the appropriate fixed-point will be the maximal one, making all properties trivially divine. There is nothing which a priori prohibits smaller fixed-points for Definition RD, and under some arguably plausible conditions (e.g. that it is a divine property to be identical with someone who is God-like) even monotheistic results will follow. In all Kripke-style possible-worlds models of modal logic which do not allow empty possible worlds, there will be a wide variety of non-maximal fixed-points for bi-conditionals like the one exhibited in Definition RD. This fact does not have any untoward consequences, and thus also takes the sting out of the argument against Gödelian ontological arguments given in Oppy (1996).

References

Anderson, C. Anthony 1990: *Some Emendations of Gödel's Ontological Proof*, in *Faith and Philosophy*, Volume 7 (3), p. 291-303.

Bjørdal, Frode: *Understanding Gödel's Ontological Argument.* In *The LOGICA Yearbook 1998*, FILOSOFIA, Prague 1999.

Feferman, Solomon et.al. 1995: *Kurt Gödel - Collected Works*, Vol. III, Oxford University Press.

Oppy, Graham 1996: *Gödelian ontological arguments*, in *Analysis*, Vol. 56 No. 4, 1996, pp. 226-230.

Hajek, Petr 1996: *Magari and others on Gödel's ontological proof*, in Ursini and Agliano, editors: Logic and Algebra, Marcel Dekker, Inc., pp. 125-136.

Sobel, Jordan Howard 1987: *Gödel's Ontological Proof*, in J.J. Thomson, editor, *On Being and Saying*, Essays for Richard Cartwright, MIT Press.

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