On Logic Embeddings and Gödel's God

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Logic embeddings provide an elegant means to formalize sophisticated non-classical logics in classical higher-order logic (HOL, Church's simple type theory [11]). In previous work (cf. [4] and the references therein) the embeddings approach has been successfully applied to automate object-level and meta-level reasoning for a range of logics and logic combinations with off-the-shelf HOL theorem provers. This also includes quantified modal logics (QML) [7] and quantified conditional logics (QCL) [3]. For many of the embedded logics few or none automated theorem provers did exist before. HOL is exploited in this approach to encode the semantics of the logics to be embedded, for example, Kripke semantics for QMLs [12] or selection function semantics for QCLs [23]. The embeddings approach is related to labelled deductive systems [15], which employ meta-level (world-)labeling techniques for the modeling and implementation of non-classical proof systems. In our embeddings approach such labels are instead encoded in the HOL logic.

In recent work [6, 5] we have applied the embeddings approach to verify and automate a philosophical argument that has fascinated philosophers and theologists for about 1000 years: the ontological argument for the existence of God [22]. We have thereby concentrated on Gödel's [16], respectively Scott's [21], modern version of this argument, which employs a second-order modal logic, for which, until now, no theorem provers were available. In our computer-assisted study of the argument, the HOL provers LEO-II [8] and Satallax [10] have made some interesting observations, some of which were unknown so far.

Ongoing and future work concentrates on the systematic study of Gödel's and Scott's proofs. We have also begun to study more recent variants of the argument [2, 1, 9, 14, 13, 17, 18], which claim to remedy some fundamental problem of Gödel's and Scott's proofs, known as the modal collapse. The long-term goal is to work out a landscape of the detailled logic parameters (e.g., constant vs. varying domains, rigid vs. non-rigid terms, logics KB vs. S4 vs. S5, etc.) under which the proposed variants of the modern ontological argument hold or fail.

There is little related work [19, 20], and this focuses solely on the comparably simpler, original ontological argument by Anselm of Canterbury.

Our work attests the maturity of contemporary interactive and automated deduction tools for HOL and demonstrates the elegance and practical relevance of the embeddings-based approach. Most importantly, our work opens new perspectives towards a computational metaphysics.

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