

Gödel’s God on the Computer*

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Attempts to prove the existence (or non-existence) of God by means of abstract ontological arguments are an old tradition in philosophy and theology. Gödel’s proof [12, 13] is a modern culmination of this tradition, following particularly the footsteps of Leibniz. Gödel defines God as a being who possesses all *positive* properties. He does not extensively discuss what positive properties are, but instead he states a few reasonable (but debatable) axioms that they should satisfy. Various slightly different versions of axioms and definitions have been considered by Gödel and by several philosophers who commented on his proof; cf. [17, 2, 11, 1, 10]. We have analyzed Scott’s version of Gödel’s proof [16] for the first-time with an unprecedented degree of detail and formality with the help of theorem provers; cf. <https://github.com/FormalTheology/GoedelGod>.

The following has been done (and in this order): (i) a detailed natural deduction proof; (ii) a formalization in TPTP THF syntax [18]; (iii) an automatic verification of the consistency of the axioms and definitions with Nitpick [8]; (iv) an automatic demonstration of the theorems with the provers LEO-II [5] and Satallax [9]; (v) a step-by-step formalization using the Coq proof assistant [6]; (vi) a formalization using the Isabelle proof assistant [15], where the theorems have been automated with the Isabelle tools Sledgehammer [7] and Metis [14].

Furthermore, the multi-step THF development of the proof has recently been encoded in Sutcliffe’s new TPI scripting language [19]. This scripting language offers important features that were missing in the TPTP world so far. Our collaboration with Sutcliffe has also led to some improvements of the TPI language.

Gödel’s proof is challenging to formalize and verify because it requires an expressive logical language with modal operators (*possibly* and *necessarily*) and with quantifiers for individuals and properties. Our computer-assisted formalizations rely on an embedding of the modal logic into classical higher-order logic with Henkin semantics. The formalization is thus essentially done in classical higher-order logic where quantified modal logic is emulated.

In our ongoing computer-assisted study of Gödel’s proof the proof tools have made some interesting observations, some of which were unknown so far.

This work attests the maturity of contemporary interactive and automated deduction tools for classical higher-order logic and demonstrates the elegance and practical relevance of the embeddings-based approach. Most importantly, our work opens new perspectives for a computer-assisted theoretical philosophy. The critical discussion of the underlying concepts, definitions and axioms remains a human responsibility, but the computer can assist in building and checking rigorously correct logical arguments. In case of logico-philosophical disputes, the computer can check the disputing arguments and partially fulfill Leibniz’ dictum: *Calculemus* — Let us calculate!

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