Experiments in Computational Metaphysics:

Gödel’s Proof of God’s Existence

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**ABSTRACT**

“Computer scientists prove the existence of God” --- variants of this headline appeared in the international press in autumn 2013. Unfortunately, many media reports had only moderate success in communicating to the wider public what actually had been achieved and what not. This article outlines the main findings of the authors’ joint work in computational metaphysics. More precisely, the article focuses on their computer-supported analysis of variants and recent emendations of Kurt Gödel’s modern ontological argument for the existence of God. In the conducted experiments automated theorem provers discovered some interesting and relevant facts.

**1. Introduction**

In autumn 2013 headlines such as “Computer Scientist ‘Prove’ God Exists”, “God’s Existence Theorem Is Correct”, “God Is Alive”, etc., appeared in the media, first in Germany and Austria, and subsequently in the international press. Unfortunately, many of these media reports had only moderate success in communicating to the wider public what actually had been achieved and what not.

This paper provides some more detailed background information (in chronological order) on the actual research that triggered these media reports.

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| ScottsProof |
| **Fig. 1.**Gödel’s ontological argument for the existence of God [23.]; here the variant by Scott [30.] is presented. |

The results presented here were achieved in a close collaboration between the two authors. Both were introduced to each other after a presentation Benzmüller delivered in November 2012 to the Kurt Gödel Society in Vienna. In this presentation he demonstrated how quantified modal logics (QML) [28.][20.], and other non-classical logics, can be elegantly embedded [11.] in classical higher-order logic (HOL, Church’s type theory [2.]). Moreover, by employing the embedding approach, reasoning tools for HOL become readily and effectively applicable for reasoning within QML [6.][12.]. At the end of his talk, Benzmüller pointed out that the outlined approach should be applicable to formalize and verify Gödel’s modern version of the ontological argument [23.][30.], which is formulated in a higher-order QML (cf. Fig.1), with theorem provers for HOL. Benzmüller’s proposal was partly inspired by Fitting’s textbook [19.], and he had previously attempted some formalization along Fitting’s work, but at the time still without final success. This was due not only to insufficient insistence, but also to the comparably ambitious and demanding logic settings employed by Fitting.

Woltzenlogel Paleo, a Brazilian logician working in Vienna since 2006, travelled on holidays to Brazil in December 2012 and was diagnosed with a life-threatening illness soon after his arrival. He needed a lengthy and incapacitating treatment, during which he and his family were assisted not only by an excellent medical team led by Dr. Ana Maria Lobo but also by Priest Edvaldo and his church in Piracicaba. In order to thank him for his support, Woltzenlogel Paleo decided to present him Gödel’s proof. As he couldn’t find any sufficiently rigorous, complete and convincing formalization of Gödel’s proof, he started working on producing one on a natural deduction calculus for higher-order modal logics, which he created for this purpose. He stored his draft work in an open Github repository accessible to anyone interested in contributing and, remembering Benzmüller’s talk and interest in this proof, informed him about the repository.

Initially Benzmüller and Woltzenlogel Paleo worked largely independent, each one following his own approach. However, there was frequent and fruitful exchange of information (mainly by email).

**2. Initial Experiments: Scott’s variant of Gödel’s proof**

In late Spring 2013, two events happened independently and almost simultaneously: the hand-made natural deduction proof was completed, after two corrections proposed by Annika Siders, and Benzmüller reported success in some preliminary experiments on proving the lemmas and theorems of the Gödel’s proof automatically using the embedding approach. From that moment on both authors tightly joined forces and continued the studies together. A major motivation for joining forces was the complementarity of both approaches: the hand-made natural deduction proof was human-readable, but tedious to check and hence less reliable; the automatic proofs were machine-generated and more reliable, but not human-readable. The initial focus of the joint efforts was on the embedding approach. Further work on the hand-made natural deduction proof was delayed, partly because the embedding approach proved very fruitful and partly due to a career break of Siders. Nevertheless, the two quasi orthogonal approaches were integrated later, when Woltzenlogel Paleo implemented a natural calculus for higher-order QML in Coq on top of the embedding approach.

The initial series of experiments [7.] aimed at thoroughly checking the correctness of Gödel’s proof and precisely identifying the weakest possible assumptions under which it holds. The HOL automated theorem provers LEO-II [5.] and Satallax [18.] and the HOL model finder Nitpick [17.] were employed. The TPTP THF language [35.] served as a concrete syntax format for encoding higher-order QML in HOL. The THF language is supported as common input syntax by the above reasoning tools (and several others). The initial series of experiments concentrated on Scott’s version (cf. Fig.1) of Gödel’s ontological argument. An essential difference between Gödel’s and Scott’s versions is that the latter adds a conjunct in the definition D2 of essential properties (this will be discussed further below).

The findings from these experiments on Scott’s variant were manifold (they were obtained on a standard MacBook):

1. The axioms (and definitions) are consistent. This was confirmed by Nitpick, which presented a simple model within a few seconds.
2. Theorem T1 follows from Axioms A1 and A2 in modal logic K (which is entailed in stronger modal logics such as KB, S4 and S5, so that result also holds for these logics). This was proved by LEO-II and Satallax in a few milliseconds. In fact, the left to right direction of the equivalence A1 is sufficient to prove T1.
3. Corollary C follows from T1, D1 and A3, again already in modal logic K. This was proved by LEO-II and Satallax in a few milliseconds.
4. Theorem T2 follows from A1, D1, A4 and D2 in modal logic K. Again, the provers got this result quickly, Satallax within milliseconds and LEO-II within 20s.
5. Theorem T3, necessary existence of a God-like entity, follows from D1, C, T2, D3 and A5. Again, this was proved by LEO-II and Satallax in a few milliseconds. However, this time modal logic KB was required to obtain the result. KB strengthens modal logic K by postulating the B axiom, which enforces symmetry of the accessibility relation between possible worlds. In modal logic K theorem T2 does not follow from the axioms and definitions. This was confirmed by Nitpick, which reported a countermodel.

These results were announced in a short abstract uploaded to arXiv in August 2013.

The sufficiency of modal logic KB is philosophically profound. Our motivation to investigate the weakest modal logic sufficient for the ontological argument was our own perception that the modal logic S5 (which is usually assumed for ontological proofs) might be too strong, because it entails (perhaps counter-intuitively) that anything that is possibly necessary is necessary. Later we found out that the sufficiency of KB had already been conjectured by Anderson [1.] and acknowledged by Sobel [33.], but no formal proof had ever been presented. Soon after the automatic proof using only KB was found, Annika Siders submitted an improvement of the hand-made natural deduction proof which relied only on KB.

A few weeks later, the following two additional results were obtained and presented on the 1st of November 2013 at Freie Universität Berlin, in the first joint talk of Benzmüller and Woltzenlogel Paleo.

1. The God-like entity, whose existence was proved in Step (5), is flawless in the sense that it may only exemplify positive properties. The provers got this quickly (in modal logic KB) from A1 and D1; Satallax within milliseconds and LEO-II within 20s.
2. Moreover, this God-like entity is unique, i.e. monotheism is a consequence of Gödel’s theory. Satallax proved this in milliseconds from D1 and flawlessness of God.

In pen and paper proofs in philosophy, assumptions about the logical foundations are often not made explicit. This has also been the case for Gödel’s proof script. In computer formalizations, however, the detailed settings of the employed logic have to be explicitly provided and concrete choices cannot be avoided. That is, the very logic settings become fully transparent. The results reported above were achieved in a setting with full comprehension (which is inherited in the embedding approach from the HOL meta-logic), rigid terms and ‘possibilist’ (constant domain) quantification over individuals.

In a second series of experiments, the encoding of the quantifiers for individuals has been varied to capture also an ‘actualist’ version (varying domain). All our previous results remained valid with this modification as well. Together, the embedding approach and all experimental results mentioned up to this point were presented in detail at the European Conference on Artificial Intelligence [9.]

Automated theorem provers sometimes find interesting alternative proofs. This apparently was also the case in our experiments. By analyzing the proofs, one can see, for example, that the property of being self-identical, which is mentioned in Gödel’s manuscript and in Scott’s notes, can be avoided. In this particular case, however, the finding was not entirely new. The natural deduction proof of Theorem 1, constructed independently by Woltzenlogel Paleo, also did not use self-identity. Moreover, the possibility of a proof not relying on self-identity had already been pointed out by an anonymous referee to Anderson [1.].

**3. Possible and necessary truths and the modal collapse**

Anselm’s ontological argument does not properly differentiate between contingent, possible and necessary truths. Gödel in contrast formalized his proof in a modal (higher-order) logic, which supports such discrimination. For example, Gödel’s corollary C (cf. Fig.1) proves from preceding assumptions that it is *possible* for God to exists. Corollary C is then used further to prove T3, that *necessarily* God exists. This discrimination of possible and necessary truths via modal operators enabled Gödel to address a relevant critique, studied by Leibniz, about St. Anselm’s original work on the ontological argument: Anselm’s argument had presupposed that it is possible for a God-like being to exist, and without this presupposition the argument breaks. At first sight, it thus appears that Gödel’s argument very convincingly addresses these (and other) issues. However, as Sobel [32.][33.] showed, Gödel’s axioms and definitions are so powerful that they imply what is known as the *modal collapse*: contingent truth implies necessary truth, in formal notation P 🡪 []P. From this we also get that possible truth implies necessary truth and vice versa. In other words, there are no contingent truths. One may even see modal collapse as a result against free will.

The theorem provers were in fact able to confirm the modal collapse within a few seconds. Moreover, the provers also showed that the result is independent of using possibilist or actualist quantifiers (for individuals).

What does this now mean for Gödel’s ontological argument? Is it thus doomed to fail? Well, not necessarily. On the one hand note that modal collapse, being derivable from the assumptions of the ontological argument, may actually serve those philosophical views well which support forms of determinism. Kovác [27.] goes as far to argue that modal collapse may eventually be conform with Gödel‘s own philosophical viewpoints. On the other hand, modal collapse has recently incited several philosophers to develop emendations of Gödel’s argument in order to remedy the situation.

The authors, in collaboration with Leon Weber, have meanwhile extended their computer-supported analysis to several of these emendations [13.]. The findings of these more recent studied will be addressed in Sec. 5 below.

**4. A subtle difference in the notion of essence**

The above results apply to Scott’s variant [30.] of the Gödel’s proof which slightly differs from the version [23.] that was found in Gödel’s ‘Nachlass’. One difference is to be found in the notion of essence. In Scott’s version, “*an essence of an individual is a property possessed by it and necessarily implying any of its properties*”. For Gödel, in contrast, “*an essence of an individual is a property that is necessarily implying any of its properties*”, that is, he is omitting the conjunct “*possessed by it*”. So what happens if this conjunct in the definition of D2 (cf. Fig.1) is left out?

To study the consequences, we have replayed the experiments as reported above, but this time for the varied definition D2. Interestingly, in step (1), the model finder Nitpick failed to report a model. To assess the situation, we subsequently tried to use the HOL theorem provers to prove the inconsistency of the modified set of axioms and definitions. To our surprise, the prover LEO-II indeed succeeded (in about 30 seconds) in doing so. Both authors have previously not been aware of this inconsistency. In fact, related comments in philosophy papers often classify Scott’s modification only as a ‘cosmetic’ change to what is often addressed as a minor oversight by Gödel. So what causes this inconsistency and how is the argument working? Unfortunately, the technical, machine-oriented proof object that was returned by LEO-II has been so inaccessible that even Benzmüller, the developer of the tool, failed for a long time to extract a persuasive human-level argument from it.

**5. Repeating the experiments in Isabelle/HOL and Coq**

The automated theorem provers LEO-II and Satallax do not offer small and trusted kernels to represent their proofs in. Moreover, their low-level, machine-oriented proof calculi are making it particularly hard for humans to follow the very technical chain of proof steps they report. Though their background theory is sound, their actual reasoning could be flawed due to potential, yet undetected bugs in their implementations.

To address this issue and to add another layer of trust to their results, the authors therefore decided to repeat and verify all previous experiments by using the prominent proof assistants Isabelle/HOL [29.] and Coq [14.].

Isabelle/HOL in particular provides strong internal proof automation facilities, and it integrates (or links to) external automated theorem provers, including LEO-II and Satallax. Moreover, means to reconstruct external proofs within Isabelle/HOL’s highly trusted kernel have been developed in recent years [16.][34.]. By using these facilities we quickly succeeded in replaying their experiments within Isabelle/HOL, which reassured our previous findings [8.]. Unfortunately, however, proof reconstruction in Isabelle/HOL failed for one of the reported results, namely LEO-II’s inconsistency result as discussed above.

In addition to Isabelle/HOL, we also replayed the experiments in the Coq proof assistant. However, the main motivation now was to demonstrate that the embedding approach not only serves proof automation well but also enables user interaction. Woltzenlogel Paleo quickly succeeded in reconstructing the findings interactively within Coq. As part of this work we also demonstrated how a direct natural deduction calculus [31.] for higher-modal logic can be implemented within the embedding approach as tactics in Coq. This offers interesting perspectives for future work to integrate proof search in the direct and the embedding approach.

**6. Further Experiments: Emendations of Gödel’s proof**

The success of the experiments, particularly the observed nearly perfect match between the argumentation granularity in Gödel’s ontological argument and the proof automation capabilities of the HOL provers in the embedding approach was by no means expected. A very relevant question thus came up, namely whether the approach would scale also for the verification of other research papers in this area. The authors therefore decided to look at more recent emendations on the ontological argument attempting to remedy the modal collapse. The correctness of the emendations and of several meta-remarks about them is much more controversial, and this served as an additional motivation for the use of a computer-assisted approach.

Hájek [24.][25.] proposed the use of cautious instead of full comprehension principles, and Fitting [19.] took greater care of the semantics of higher-order quantifiers in the presence of modalities. Others, such as Anderson [1.], Hájek [26.] and Bjørdal [15.], proposed emendations of Gödel's axioms and definitions. They require neither comprehension restrictions nor more complex semantics. Therefore, they are technically simpler to analyze within the embedding approach.

The authors have formalized and studied those using Isabelle/HOL in combination with the automated HOL reasoners. The approach again performed very well. Like in the previous experiments, the HOL reasoners quickly responded to the formalized argumentation steps, either by automatically confirming them as valid, or by refuting them, in which case countermodels were presented.

Interestingly, the HOL provers not only confirmed many claimed results, but also exposed a few mistakes and produced novel insights. In particular, the provers were able to settle a long standing debate on the redundancy of axioms A4 and A5 in different settings that was going on between Magari, Anderson and Hájek [13.].

These additional experiments strikingly demonstrate the potential benefits of a computational metaphysics as exemplified here in which humans and computer programs join forces in order to settle philosophical disputes. As the authors believe, this is pretty much matches Leibniz vision known as “Calculemus!”.

**7. Leo-II’s inconsistency proof**

Inspired by a discussion of the inconsistency issue with Chad Brown, Benzmüller recently succeeded in reconstructing and verifying LEO-II’s argument by hand within Isabelle/HOL. Once revealed and understood, the argument is in fact surprisingly comprehensible:

1. The argument starts from axiom A1 “*Either a property or its negation is positive, but not both*” and axiom A2 “*A property necessarily implied by a positive property is positive*”. These two assumptions imply theorem T1 “*Positive properties are possibly exemplified*”, as we already know.
2. Now take the modified definition of D2 “*An essence of an individual is a property that is necessarily implying any of its properties*” and consider the empty property (or alternatively, the property of being self-different) as a candidate. Then Lemma 1 is derivable: “*The empty property is an essence of every entity*”.
3. From the definition of necessary existence “*Necessary existence of an individual is the necessary exemplification of all its essences*”, modal axiom B and Lemma 1 follows Lemma 2: “*Exemplification of necessary existence is not possible*”.
4. Axiom A5 “*Necessary existence is a positive property*”, theorem T1 and Lemma 2 now imply falsehood.

LEO-II’s proof object actually contains the crucial property instantiation performed in Step 2, however, this key step gets lost in the technical noise of the proof. It is important to remark, that this instantiation has not been synthesized by LEO-II, e.g. by employing higher-order (pre-)unification. Instead it has been guessed during proof search using the blindly guessing primitive substitution rule. As experts know, this rule can unfortunately not be abandoned in HOL theorem proving (without loosing Henkin-completeness) [4.]. This points to an interesting aspect: attempts to repeat this successful inconsistency proof of LEO-II with first-order theorem provers will likely be doomed to fail, since the only source to come up with this crucial instantiation of the empty property appears to be the comprehension schemes of set theory.

**8. Discussion**

Ontological arguments in the tradition of Anselm’s, since their first revelation, have fascinated generations of philosophers. In fact, they usually trigger strong reactions, against or in favor of them. In philosophical circles the debate is not yet settled and the allurement of ontological arguments seems far from fading. Readers of the public media reports that were triggered by the authors recent work, however, overwhelmingly seem to reject them. This becomes apparent from hundreds of blog entries linked at the respective media websites. Such blog entries have recently been further analysed by Fuhrmann [21.]. Clearly, without a certain level of education in metapyhics, modern logic and the axiomatic method, Gödel’s modal ontological argument, appears largely inaccessible. This may provide some level of explanation. Generally, a certain amount of philosophical education seems required for not being irritated by the ontological argument in the first place.

However, the media writers are also to be blamed, because of their apparent interest in creating ‘headline stories’, and in copying, nitpicking and obfuscating text passages from each other instead of presenting unbiased, properly investigated and individually prepared information. For example, in an early interview on the topic to the German edition of Spiegel Online, Benzmüller mentioned that ‘the initial experiments were conducted on his MacBook’. While this has been true, he should have better used the neutral term ‘notebook’, since the experiments can (and have been) repeated also with other computer technology. The original Spiegel Online article mentioned ‘MacBook’, however, it did not prominently overemphasize this point (and a good technology writer could have further clarified the independence of the exepriment results anyway). However, when the news subsequently made its way to the US, some intentionally (and very naively) obfuscated headlines appeared such as “Researchers say they used MacBook to prove Gödel’s God theorem” or “God exists, say Apple fanboy scientists”. Such headline stories were contributed even from ‘award winning creative directors’ such as Chris Matyszczyk, who in fact never talked to any of the authors directly. One may argue that these articles therefore say a lot more about media quality and media standards in the US than about the quality of the actual research content they address. And it is little surprise that such intentionally obfuscated and jaundiced media reports trigger negative attitudes of bloggers towards the ontological argument.

Finally, the authors think that philosophical tradition has to be blamed to a certain extent, since comparably few texts seem to be available which are trying to appropriately communicate the ontological argument to a wider audience. The authors themselves, who have began their studies on the topic only recently, have experienced that a large proportion of the existing texts are targeting a rather exclusive circle of readers sharing a significant background expertise, in particular, on the detailed historical development in the area. This in turn excludes, and presumably negatively affects, those readers not willing to clear their backlog before commenting on the topic. It seems that other academic disciplines have in fact achieved a higher level of historical and contextual independence in their modern literature. (But maybe this is not really a worthwile option for philosophy.)

But what is now the authors position on the ontological argument? Well, both share the opinion with proponents of Gödel’s work that prominent objections to his proof, including Gaunilo-like repectively Oppy-like parodies, are currently not on a par with Gödel’s work and its recent emendations with respect to technical precision and persuasive power. Investigating and eventually either confirming their correctness or unveiling flaws in them, with the assistance our technology, remains an exciting direction for future work.

On the other hand, there clearly are theologically and metaphysically relevant objections, including the modal collapse, which are not yet fully settled. Since several of these objections are beyond the authors’ expertise, it seems innapropriate for them, at least at the present moment, to comment on those. However, as a conclusion one may say that the ontological argument succeeds at least in the following sense: it shows that *belief in a (God-like) supreme being is not trivially irrational*. There are consistent axiomatizations that non-trivially entail the necessary existence of a God-like being. As for any axiomatization, and not only those with a religious theme, it often remains a ’matter of faith’ to believe in the truth of the proposed axioms in the actual universe.

The authors actual contribution is a technological approach and machinery that, as has been well demonstrated here, can fruitfully support further logical investigations in this area. This machinery may eventually even be helpful for settling some of the open questions. In particular, our technology seems ready to be used with the aim of minimizing logic related causes of defect in this area. As an expedient this machinery should (at least in the long run) be able to significantly ease the technically involved practical work in metaphysics.

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**References**

1. C.A. Anderson. Some emendations of Gödel’s ontological proof. Faith and Philosophy, 7(3), 1990.
2. C.A. Anderson and M. Gettings. Gödel ontological proof revisited. In Gödel’96: Logical Foundations of Mathematics, Computer Science, and Physics. Lecture Notes in Logic 6, pages 167-172. Springer, 1996.
3. P.B. Andrews. Church’s type theory. In E.N. Zalta (ed.), The Stanford Encyclopedia of Philosophy. Spring 2014 edition, 2014.
4. C. Benzmüller and D. Miller. Automation of Higher-Order Logic. Chapter in Handbook of the History of Logic, Volume 9 --- Computational Logic. North Holland, Elsevier, pp. 215-254, 2014.
5. C. Benzmüller, N. Sultana, L. Paulson, and F. Theiss. The Higher-Order Prover LEO-II. Journal of Automated Reasoning, 2015. In print. http://dx.doi.org/10.1007/s10817-015-9348-y
6. C. Benzmüller, J. Otten, and T. Raths. Implementing and evaluating provers for first-order modal logics. In Proc. of ECAI 2012, volume 242 of Frontiers in Artificial Intelligence and Applications, pages 163–168, 2012. IOS Press.
7. C. Benzmüller and B. Woltzenlogel Paleo. Formalization, Mechanization and Automation of Gödel’s Proof of God’s Existence. ArXiv e-prints, 2013.
8. C. Benzmüller and B. Woltzenlogel Paleo. Gödel’s God in Isabelle/HOL. Archive of Formal Proofs, 2013.
9. C. Benzmüller and B. Woltzenlogel Paleo. Automating Gödel’s ontological proof of God’s existence with higher-order automated theorem provers. In Proc. of ECAI 2014, volume 263 of Frontiers in Artificial Intelligence and Applications, pages 93 – 98. IOS Press, 2014.
10. C. Benzmüller and B. Woltzenlogel Paleo. Interacting with Modal Logics in the Coq Proof Assistant. In Computer Science - Theory and Applications - 10th International Computer Science Symposium in Russia, CSR 2015. Springer, LNCS, volume 9139, pp. 398--411, 2015.
11. C. Benzmüller and L. Paulson. Quantified multimodal logics in simple type theory. Logica Universalis, 7(1):7-20, 2013.
12. C. Benzmüller and T. Raths. HOL based first-order modal logic provers. In Proc. of LPAR 2013, volume 8312 of LNCS, pages 127-136. Springer, 2013
13. C. Benzmüller, L. Weber, and B. Woltzenlogel Paleo. Computer-assisted analysis of the Anderson- Hájek ontological controversy. Handbook of the 1st World Congress on Logic and Religion, Joao Pessoa, Brasil, 2015.
14. Y. Bertot and P. Casteran. Interactive Theorem Proving and Program Development. Springer, 2004.
15. F. Bjørdal. Understanding Gödel's Ontological Argument". In The Logica Yearbook 1998. Ed. by T. Childers. Filosofia, 1999.
16. J.C. Blanchette, S. Böhme, and L.C. Paulson. Extending Sledgehammer with SMT solvers. Journal of Automated Reasoning, 51(1):109-128, 2013.
17. J.C. Blanchette and T. Nipkow. Nitpick: A counterexample generator for higher-order logic based on a relational model finder. In Proc. of ITP 2010, number 6172 in LNCS, pages 131-146. Springer, 2010.
18. C.E. Brown. Satallax: An automated higher-order prover. In Proc. of IJCAR 2012, number 7364 in LNAI, pages 111-117. Springer, 2012.
19. M. Fitting. Types, Tableaux and Gödel’s God. Kluwer, 2002.
20. M.Fitting and R.L. Mendelsohn. First-Order Modal Logic, volume 277 of Synthese Library. Kluwer, 1998.
21. A. Fuhrmann. Blogging Gödel: His ontological argument in the public eye. To appear.
22. K. Gödel. Ontological proof. In Kurt Gödel: Collected Works Vol. 3: Unpublished Essays and Letters. Oxford University Press, 1970.
23. K. Gödel. Appx.A: Notes in Kurt Gödel’s Hand, pages 144-145. In [33.], 2004.
24. P. Hájek. Magari and others on Gödel's ontological proof. In Logic and algebra. A. Ursini and P. Agliano (eds.), Dekker, New York etc.,1996, pp. 125-135.
25. P. Hájek. Der Mathematiker und die Frage der Existenz Gottes. In Kurt Gödel. Wahrheit und Beweisbarkeit. B. Buldt, E. Köhler, M. Stöltzner, P. Weibel, C. Klein, and W. Depauli-Schimanowich-Göttig (eds.), ISBN 3-209-03835-X. öbv & hpt, Wien, 2001, pp. 325-336.
26. P. Hájek. A New Small Emendation of Gödel's Ontological Proof. Studia Logica 71(2):149-164, 2002.
27. S. Kovác. Modal collapse in Gödel’s ontological proof. In Ontological Proofs Today,M. Szatkowski (ed.), Ontos Verlag, 2012.
28. R. Muskens. Higher Order Modal Logic. In P. Blackburn et al. (eds.), Handbook of Modal Logic, pages 621–653. Elsevier, Dordrecht, 2006.
29. T. Nipkow, L.C. Paulson, and M.Wenzel. Isabelle/HOL: A Proof Assistant for Higher-Order Logic. Number 2283 in LNCS. Springer, 2002.
30. D. Scott. Appx.B: Notes in Dana Scott’s Hand, pages 145–146. In [33.], 2004.
31. A. Siders and B. Woltzenlogel Paleo. A variant of Gödel’s ontological proof in a natural deduction calculus. (github.com/FormalTheology/GoedelGod/blob/

master/Papers/InProgress/NaturalDeduction/GodProof-ND.pdf?raw=true).

1. J.H. Sobel. Gödel’s ontological proof. In On Being and Saying. Essays for Richard Cartwright, pages 241-261. MIT Press, 1987.
2. J.H. Sobel. Logic and Theism: Arguments for and Against Beliefs in God. Cambridge U. Press, 2004.
3. N. Sultana. Higher-order proof translation. PhD thesis, Computer Laboratory,

University of Cambridge, 2015. Available as Tech Report UCAM-CL-TR-867.

1. G. Sutcliffe and C. Benzmüller. Automated reasoning in higher-order logic using the TPTP THF infrastructure. Journal of Formalized Reasoning, 3(1):1-27, 2010.

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