# 1 Local Redundancy

We first consider an example first posed by Postan:

$$
\dfrac{\dfrac{\dfrac{L_2\colon P(A) \qquad \eta_2\colon \neg P(x), \neg Q(x,B)}{\neg Q(x,B)} \qquad \eta_1\colon \neg P(z), Q(z,y)}{\neg P(z)} \qquad L_1\colon P(x)}{\bot}
$$

Which is locally redundant; see the compressed version in his document.

# 2 "Example 2"

We consider example 2, from the LU/RPI paper, modified for first order predicates in a trivial way:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\eta_1\colon \neg P(A) \qquad \eta_3\colon P(A), Q(B)}{\eta_4\colon Q(B)} \qquad \eta_2\colon P(A), R(C), \neg Q(B)}{\eta_5\colon P(A), R(C)} \qquad \eta_1\colon \neg P(A)}{\eta_6\colon R(C)} \qquad \dfrac{\dfrac{\eta_4\colon Q(B) \qquad \eta_7\colon P(A), \neg Q(B), \neg R(C)}{\eta_8\colon P(A), \neg R(C)} \qquad \eta_1\colon \neg P(A)}{\eta_9\colon \neg R(C)}}{\bot}
$$

## 2.1 Lower Units

Proceeds exactly the same as in the paper.
**TODO:** show exact steps?
**TODO:** cases where we can lower larger formulas, namely non-units, as long as they're unifiable (and subject to the other same conditions?)

## 2.2 RecyclePivots

Again, proceeds like in the paper.

# 3 A larger proof

**TODO:** We consider the following proof:

$$
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\dfrac{\vdash \text{(eq (f U) 0), (eq (f U) 1)} \qquad \dfrac{\vdash \text{(le U U)} \qquad \text{(eq (f V) 0), (eq (f U) 0), (le (s U) V)} \vdash}{\text{(eq (f (s U)) 0), (eq (f U) 0)} \vdash}}{\text{(eq (f U) 0)} \vdash \text{(eq (f (s U)) 1)}}
\qquad
\dfrac{
\dfrac{\dfrac{\dfrac{\vdash \text{(le U U)} \qquad \text{(le (max U V) W)} \vdash \text{(le V W)}}{\vdash \text{(le V (max U V))}} \qquad \text{(eq (f V) 0), (eq (f U) 0), (le (s U) V)} \vdash}{\text{(eq (f (max X (s U))) 0), (eq (f U) 0)} \vdash} \qquad \vdash \text{(eq (f U) 0), (eq (f U) 1)}}{\text{(eq (f U) 0)} \vdash \text{(eq (f (max X (s U))) 1)}}
\qquad
\dfrac{\dfrac{\text{(le (max U V) W)} \vdash \text{(le U W)} \qquad \vdash \text{(le U U)}}{\vdash \text{(le U (max U V))}} \qquad \text{(eq (f V) 1), (eq (f U) 1), (le (s U) V)} \vdash}{\text{(eq (f (max (s U) X)) 1), (eq (f U) 1)} \vdash}
}{\text{(eq (f W) 0), (eq (f U) 1)} \vdash}
}{\text{(eq (f X) 0), (eq (f W) 0)} \vdash}
}{\text{(eq (f X) 0)} \vdash}
\qquad \vdash \text{(eq (f U) 0), (eq (f U) 1)}
}{\vdash \text{(eq (f X) 1)}}
\qquad
\dfrac{\dfrac{\text{(eq (f V) 1), (eq (f U) 1), (le (s U) V)} \vdash \qquad \text{(eq (f V) 1), (eq (f U) 1), (le (s U) V)}}{\text{(eq (f (s U)) 1), (eq (f U) 1)} \vdash}}{\text{(eq (f (s U)) 1)} \vdash}
}{\vdash}
$$