Management Science Individual Assignment

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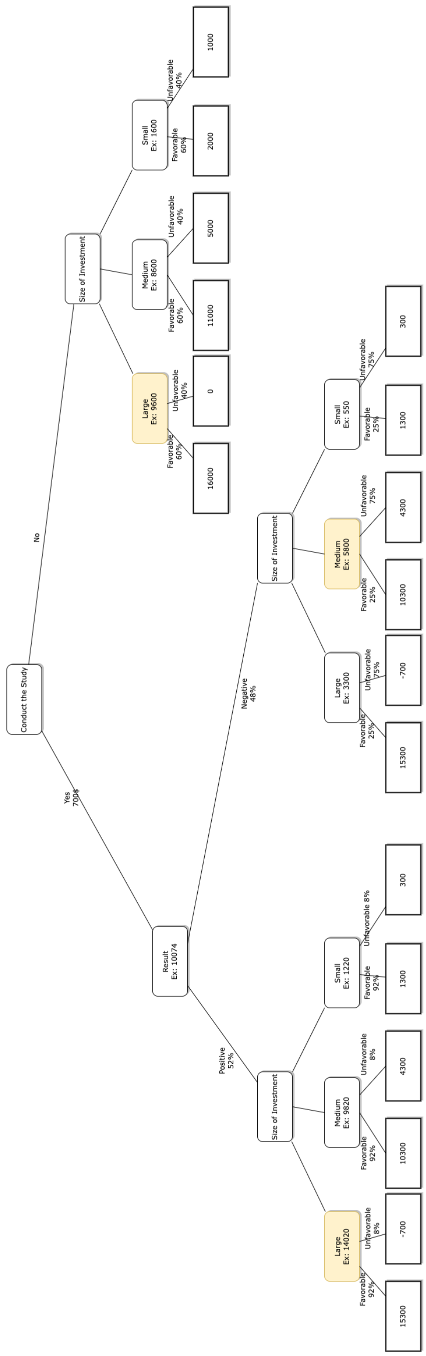
1 Question 1

Alternatives	Favorable	Unfavorable	Expected Profit
Small	2000	1000	1600
Medium	11000	5000	8600
Large	16000	0	9600
Probability	60%	40%	

Above, the payoff table is present, comparing the investment sizes for the different market conditions. As seen above, the favorable conditions, which is more likely than unfavorable conditions, lead to greater profit compared to unfavorable conditions. The expected profits reflect the increased likelihood of the conditions. In order to satisfy the goal of increasing expected profits, the large investment seems like the best option.

Below, a decision tree model is present for the second part of the question. The decision tree initially decides which size of investment should be done. Later, it decides whether should the study be conducted or not. The boxes highlight the expected profits after the study being conducted or not. The final profits according to the market conditions are given in the end as well.



The expected profits gained by doing the study seems to outweigh the expected profits of not doing the study. This is elaborated on the risk profiles below.

Profit	Probability
16000	60%
0	40%

The first table shows the risk analysis without the study. The expected profits are 9600.

Profit	Probability
14020	52%
5800	48%

The second table shows the risk analysis of the preferred conditions after the study is conducted. If the conditions are favorable (52%), the large investment is chosen and if the conditions are unfavorable (48%) the medium option is chosen. The expected profit is 10074, which is greater than 9600, hence conducting the study seem reasonable.

2 Question 2

2.1 Equation

In order to solve the logistics problem of the barrels, a linear equation was constructed.

Basic Rules

$$\begin{aligned}\text{Demand}_{\text{Pit}} &= 20000\\ \text{Demand}_{\text{At}} &= 25000\\ \text{Supply}_{\text{Tex}} &\leq 10000\\ \text{Supply}_{\text{Cal}} &\leq 50000\end{aligned}$$

Supplies

$$\begin{aligned}
\text{Tex}_{\text{NO}} + \text{Tex}_{\text{Char}} + \text{Tex}_{\text{Sea}} &= \text{Supply}_{\text{Tex}} \\
\text{Cal}_{\text{NO}} + \text{Cal}_{\text{Char}} + \text{Cal}_{\text{Sea}} &= \text{Supply}_{\text{Cal}} \\
\text{NO}_{\text{Pit}} + \text{NO}_{\text{At}} = \text{Supply}_{\text{NO}} &= \text{Tex}_{\text{NO}} + \text{Cal}_{\text{NO}} \\
\text{Char}_{\text{Pit}} + \text{Char}_{\text{At}} = \text{Supply}_{\text{Char}} &= \text{Tex}_{\text{Char}} + \text{Cal}_{\text{Char}} \\
\text{Sea}_{\text{Pit}} + \text{Sea}_{\text{At}} = \text{Supply}_{\text{Sea}} &= \text{Tex}_{\text{Sea}} + \text{Cal}_{\text{Sea}}
\end{aligned}$$

Demands

$$\begin{aligned}
\text{Demand}_{\text{Pit}} &= \text{NO}_{\text{Pit}} + \text{Char}_{\text{Pit}} + \text{Sea}_{\text{Pit}} \\
\text{Demand}_{\text{At}} &= \text{NO}_{\text{At}} + \text{Char}_{\text{At}} + \text{Sea}_{\text{At}}
\end{aligned}$$

The aim is to minimize the following costs

$$\begin{aligned}
\text{totalCost} = 11 * \text{Tex}_{\text{NO}} + 7 * \text{Tex}_{\text{Char}} + 2 * \text{Tex}_{\text{Sea}} \\
+7 * \text{Cal}_{\text{NO}} + 4 * \text{Cal}_{\text{Char}} + 8 * \text{Cal}_{\text{Sea}} \\
+11 * \text{NO}_{\text{Pit}} + 7 * \text{NO}_{\text{At}} \\
+7 * \text{Char}_{\text{Pit}} + 4 * \text{Char}_{\text{At}} \\
+5 * \text{Sea}_{\text{Pit}} + 3 * \text{Sea}_{\text{At}}
\end{aligned}$$

2.2 Gurobi

Gurobi was used for finding the optimal solution and conducting the sensitivity analysis. The code for both sections are present in the appendix.

The optimal solution has the cost of 38000 dollars. 10000 barrels will be delivered to Seattle from Texas. 35000 barrels will be sent to Charlton from California. From there, 10000 barrels are sent from Charlton to Pittsburgh and 25000 barrels are sent from Charlton to Atlanta. The 10000 barrels delivered to Seattle are sent to Pittsburgh as well.

2.3 Sensitivity

The first question looks at the supply of Texas. If the supply were to increase to 15000, the optimal solution would not change since the upper boundary of Texas supply is infinity.

The second question is quite similar to the first one. If the supply at California were to decrease to 40000, the solution would not change as well. Since the lower boundary is 35000, the new supply would be within the range of the optimal solution.

The third question looks at delivering extra 10000 barrels at either Atlanta or Pittsburgh. Looking at the upper boundaries of the demand, the optimal solution can have 15000 barrels delivered to either Atlanta or Pittsburgh and the optimal solution would not change. Hence, the Pi value is checked to find the cheaper option. Delivering the extra barrels to Atlanta seems like the cheaper option since it has a lower Pi value. The value of 8 is multiplied by 10000 to conclude that the extra cost would be 80000 dollars.

The final question looks at a new motorway between California and Seattle. The current cost of delivering between those cities is 8 dollars and the new road would decrease it to 7 dollars. The range has a lower boundary of 6 dollars, so the optimal solution would not change as well. However, at the moment, no barrels are delivered between those cities and since the optimal solution would not change, the new motorway would not be used. The optimal cost would not change as well.

3 Question 3

3.1 Equation

The equation concerning the sailing boats was quite tricky. In order to tackle the problem, equations were constructed.

Basic Rules

$$\begin{aligned}
\text{InventoryA}_{\text{Day1}} &= \text{PurchaseA}\\
\text{InventoryB}_{\text{Day1}} &= \text{PurchaseB}\\
\text{InventoryA}_{\text{DayX}} &\geq \text{DemandA}_{\text{DayX}}\\
\text{InventoryB}_{\text{DayX}} &\geq \text{DemandB}_{\text{DayX}}
\end{aligned}$$

Sails at A

$$\begin{aligned}
\text{InventoryA}_{\text{DayX}} = \text{InventoryA}_{\text{DayX}-1}\\
+\text{ExpensiveTransportB2A}_{\text{DayX}}\\
+\text{CheapTransportB2A}_{\text{DayX}-1}\\
-\text{CheapTransportA2B}_{\text{DayX}}\\
-\text{ExpensiveTransportA2B}_{\text{DayX}}
\end{aligned}$$

Sails at B

$$\begin{aligned}
\text{InventoryB}_{\text{DayX}} = \text{InventoryB}_{\text{DayX}-1}\\
+\text{ExpensiveTransportA2B}_{\text{DayX}}\\
+\text{CheapTransportA2B}_{\text{DayX}-1}\\
-\text{CheapTransportB2A}_{\text{DayX}}\\
-\text{ExpensiveTransportB2A}_{\text{DayX}}
\end{aligned}$$

The aim is to minimize the following cost

$$\sum_{x=1}^{7} 20 * (\text{ExpensiveTransportA2B}_{\text{DayX}} + \text{ExpensiveTransportB2A}_{\text{DayX}})$$
$$5 * (\text{CheapTransportA2B}_{\text{DayX}} + \text{CheapTransportB2A}_{\text{DayX}})$$
$$+\text{PurchaseA} + \text{PurchaseB}$$

3.2 Gurobi

Again, the Gurobi code is present in the appendix. The optimal solution costs 11265 dollars. 45 sails are purchased at A and 10 sails are purchased at 10.

Expensive transportation is used 8 times, on the first and on the 6^{th} days. Cheap transport is used 21 times on three different occasions. Most of the time, the transports are from A to B. Almost 98% of the cost is from purchasing the actual sails.

The final question deals with 365 days of operating shops A and B. The exact same code for the previous question was used, but the period was extended to 365 days and the demand was taken from the csv file provided.

The optimal solution costs 22850 dollars. It is very interesting that even though this version is running the shop for a complete year instead of a week, the cost is only the double of the first question. 107 sails were purchased in the beginning, 58 for A and 49 for B. Later, most of the transportation is done by the cheap method rather than the expensive way. The expensive method is only used in three different occasions.

4 Appendix

Question 2 Code

```python
model=gp.Model()

tNO=model.addVar(lb=0,ub=10000,name="t2no")
tChar=model.addVar(lb=0,ub=10000,name="t2char")
tSea=model.addVar(lb=0,ub=10000,name="t2sea")
model.addConstr(tNO+tChar+tSea<=10000,name="Texas Suppy")

cNO=model.addVar(lb=0,ub=50000,name="c2no")
cChar=model.addVar(lb=0,ub=50000,name="c2char")
cSea=model.addVar(lb=0,ub=50000,name="c2sea")
model.addConstr(cNO+cChar+cSea<=50000,name="California Suppy")

dPit=model.addVar(20000,name="demand pit")
dAt=model.addVar(25000,name="demand at")

costTex=(11*tNO+7*tChar+2*tSea)
costCal=(7*cNO+4*cChar+8*cSea)

supplyNO=(tNO+cNO)
supplyChar=(tChar+cChar)
supplySea=(tSea+cSea)

noPit=model.addVar(lb=0,ub=60000,name="no2pit")
noAt=model.addVar(lb=0,ub=60000,name="no2at")
model.addConstr(noPit+noAt<=supplyNO,name="New Orleeens Supply")

charPit=model.addVar(lb=0,ub=60000,name="char2pit")
charAt=model.addVar(lb=0,ub=60000,name="char2at")
model.addConstr(charPit+charAt<=supplyChar,name="Charlton Supply")

seaPit=model.addVar(lb=0,ub=60000,name="sea2pit")
seaAt=model.addVar(lb=0,ub=60000,name="sea2at")
model.addConstr(seaPit+seaAt<=supplySea,name="Seatle Supply")

costNO=(11*noPit+7*noAt)
costChar=(7*charPit+4*charAt)
costSea=(5*seaPit+3*seaAt)

model.addConstr((noPit+charPit+seaPit)==dPit,name="Pitssburgh Delivery")
model.addConstr((noAt+charAt+seaAt)==dAt,name="Atlanta Delivery")

model.setObjective(costTex+costCal+costNO+costChar+costSea,gp.GRB.MINIMIZE)
model.optimize()
if not model.status == gp.GRB.OPTIMAL:
    print("something went wrong")
print("Total cost is",model.objval)
model.printAttr("X")
```

•••

```python
model.printAttr(["X","Obj","SAObjLow","SAObjUP"])
model.printAttr(["RC","LB","SALBLow","SALBUp","UB","SAUBLow","SAUBUp"])
model.printAttr(["Sense","Slack","Pi","RHS","SARHSLow","SARHSUp"])
```

Question 3-2 Code

```
model = gp.Model()

#purchase
PA = model.addVar(lb = 0, ub = 100, name = 'PA',obj=200)
PB = model.addVar(lb = 0, ub = 100, name = 'PB',obj=200)

#basics
periods=7
demandA=[45,20,20,25,15,28,15]
demandB=[8,12,23,30,12,10,33]
expensive=20
cheap=5

#costs
EA2B=model.addVars(periods,name="ExpensiveA2B",obj=expensive)
CA2B=model.addVars(periods,name="cheapA2B",obj=cheap)

EB2A=model.addVars(periods,name="ExpensiveB2A",obj=expensive)
CB2A=model.addVars(periods,name="cheapB2A",obj=cheap)

#inventory
IA=model.addVars(periods,name="IA")
IB=model.addVars(periods,name="IB")

#constraints
model.addConstrs(IA[x]>=demandA[x] for x in range(periods))
model.addConstrs(IB[x]>=demandB[x] for x in range(periods))
model.addConstr(IA[0]==PA)
model.addConstr(IB[0]==PB)

model.addConstrs(IA[x]==IA[x-1]+EB2A[x]-EA2B[x]-CA2B[x] for x in range(1,2))
model.addConstrs(IB[x]==IB[x-1]+EA2B[x]-EB2A[x]-CB2A[x] for x in range(1,2))

model.addConstrs(IA[x]==IA[x-1]+EB2A[x]-EA2B[x]+CB2A[x-1]-CA2B[x] for x in range(2,periods)
model.addConstrs(IB[x]==IB[x-1]+EA2B[x]-EB2A[x]+CA2B[x-1]-CB2A[x] for x in range(2,periods)

#finish
model.optimize()
if not model.status == gp.GRB.OPTIMAL:
    print("something went wrong")
print("Optimal cost is", model.objval)
model.printAttr("X")
```

Question 3-3 Code

```python
import pandas as pd

df=pd.read_csv("sails.csv")

demandA = list(df["demandA"])
demandB = list(df["demandB"])

model = gp.Model()

#purchase
PA = model.addVar(lb = 0, ub = 1000, name = 'PA',obj=200)
PB = model.addVar(lb = 0, ub = 1000, name = 'PB',obj=200)

#basics
periods=365
expensive=20
cheap=5

#costs
EA2B=model.addVars(periods,name="ExpensiveA2B",obj=expensive)
CA2B=model.addVars(periods,name="cheapA2B",obj=cheap)

EB2A=model.addVars(periods,name="ExpensiveB2A",obj=expensive)
CB2A=model.addVars(periods,name="cheapB2A",obj=cheap)

#inventory
IA=model.addVars(periods,name="IA")
IB=model.addVars(periods,name="IB")

#constraints
model.addConstrs(IA[x]>=demandA[x] for x in range(periods))
model.addConstrs(IB[x]>=demandB[x] for x in range(periods))
model.addConstr(IA[0]==PA)
model.addConstr(IB[0]==PB)

model.addConstrs(IA[x]==IA[x-1]+EB2A[x]-EA2B[x]-CA2B[x] for x in range(1,2))
model.addConstrs(IB[x]==IB[x-1]+EA2B[x]-EB2A[x]-CB2A[x] for x in range(1,2))

model.addConstrs(IA[x]==IA[x-1]+EB2A[x]-EA2B[x]+CB2A[x-1]-CA2B[x] for x in range(2,periods))
model.addConstrs(IB[x]==IB[x-1]+EA2B[x]-EB2A[x]+CA2B[x-1]-CB2A[x] for x in range(2,periods))

#finish
model.optimize()

if not model.status == gp.GRB.OPTIMAL:
    print("something went wrong")
print("Optimal cost is", model.objval)
model.printAttr("X")
```