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Actividad 1.3

0.1.1. Ejercicios en clase

$$\begin{aligned} Z/2, \bar{0} &= \{2k \in \mathbb{Z} \mid k \in \mathbb{Z}\} \\ &= 2(1) = 2 \\ &\quad 2(0) = 0 \\ &\quad 2(-1) = -2 \end{aligned}$$

$$\begin{aligned} Z/2, \bar{1} &= \{2k + 1 \in \mathbb{Z} \mid k \in \mathbb{Z}\} \\ &= 2(1) + 1 = 3 \\ &\quad 2(0) + 1 = 1 \\ &\quad 2(-1) + 1 = -1 \end{aligned}$$

$$\begin{aligned} Z/5, \bar{1} &= \{5k + 1 \in \mathbb{Z} \mid k \in \mathbb{Z}\} \\ &= 5(-4) + 1 = -19 \\ &\quad 5(-3) + 1 = -14 \\ &\quad 5(-2) + 1 = -9 \\ &\quad 5(-1) + 1 = -4 \\ &\quad 5(0) + 1 = 1 \\ &\quad 5(1) + 1 = 6 \\ &\quad 5(2) + 1 = 11 \\ &\quad 5(3) + 1 = 16 \\ &\quad 5(4) + 1 = 21 \end{aligned}$$

$$\begin{aligned} Z/5, \bar{2} &= \{5k + 2 \in \mathbb{Z} \mid k \in \mathbb{Z}\} \\ &= 5(-4) + 2 = -18 \\ &\quad 5(-3) + 2 = -13 \\ &\quad 5(-2) + 2 = -8 \\ &\quad 5(-1) + 2 = -3 \\ &\quad 5(0) + 2 = 2 \\ &\quad 5(1) + 2 = 7 \\ &\quad 5(2) + 2 = 12 \\ &\quad 5(3) + 2 = 17 \\ &\quad 5(4) + 2 = 22 \end{aligned}$$

$$
\begin{aligned}
Z/5, \bar{3} &= \{5k + 3 \in \mathbb{Z} \mid k \in \mathbb{Z}\} \\
&= 5(-4) + 3 = -17 \\
&\quad 5(-3) + 3 = -12 \\
&\quad 5(-2) + 3 = -7 \\
&\quad 5(-1) + 3 = -2 \\
&\quad 5(0) + 3 = 3 \\
&\quad 5(1) + 3 = 8 \\
&\quad 5(2) + 3 = 13 \\
&\quad 5(3) + 3 = 18 \\
&\quad 5(4) + 3 = 23
\end{aligned}
$$

$$
\begin{aligned}
Z/5, \bar{4} &= \{5k + 5 \in \mathbb{Z} \mid k \in \mathbb{Z}\} \\
&= 5(-4) + 4 = -16 \\
&\quad 5(-3) + 4 = -11 \\
&\quad 5(-2) + 4 = -6 \\
&\quad 5(-1) + 4 = -1 \\
&\quad 5(0) + 4 = 4 \\
&\quad 5(1) + 4 = 9 \\
&\quad 5(2) + 4 = 14 \\
&\quad 5(3) + 4 = 19 \\
&\quad 5(4) + 4 = 24
\end{aligned}
$$

0.2. Máximo Común Divisor

0.2.1. Ejercicios en clase

1. $\mathbf{mcd(56, 42)}$

$$
\begin{aligned}
56 &= 1 \times 42 + 14 \\
42 &= 3 \times 14 + 0 \\
14
\end{aligned}
$$

   Luego de dos iteraciones obtenemos un residuo r_2 de 0, por lo que el maximo comun divisor de 56 y 42 es $r_{2-1} \equiv r_1$, es decir 14.

2. $\mathbf{mcd(106, 46)}$

$$
\begin{aligned}
106 &= 2 \times 46 + 14 \\
46 &= 3 \times 14 + 4 \\
14 &= 3 \times 4 + 2 \\
4 &= 2 \times 2 + 0 \\
2
\end{aligned}
$$

   Luego de cuatro iteraciones obtenemos un residuo r_4 de 0, por lo que el maximo comun divisor de 106 y 46 es r_3, es decir 2.

0.2.2. Código

Puede correr usted mismo el codigo en https://play.rust-lang.org/?version=stable&mode=debug&edition=2021&gist=38d711475a45183961a50f39e2c85da6

```rust
/// Usando la función de división de clases pasadas
fn divide(n: i64, d: i64) -> (i64, i64) {
    assert!(d != 0);
```

```
    match (n, d) {
        (_, d) if d < 0 ⇒ {
            let (q, r) = divide(n, -d);
            (-q, r)
        }
        (n, _) if n < 0 ⇒ {
            let (q, r) = divide(-n, d);
            if r == 0 {
                (-q, 0)
            } else {
                (-q - 1, d - r)
            }
        }
        (_, _) ⇒ {
            let (mut q, mut r) = (0, n);
            while r >= d {
                q += 1;
                r -= d;
            }
            (q, r)
        }
    }
}

fn mcd(a: i64, b: i64) → i64 {
    assert!(b != 0);

    let (mut d, mut c) = (b.abs(), a.abs());

    while d != 0 {
        let (t, r) = divide(c, d);
        println!("{c: >3} = ({t}) {d: >2} + {r}");
        c = d;
        d = r;
    }

    println!("mcd({a: >3}, {b: >3}): {c}\n");
    c
}

fn main() {
    mcd(-56, -42);
    mcd(56, 42);
    mcd(106, 46);
    mcd(78, 32);
}
```

Con la salida:

```
    Finished release [optimized] target(s) in 6.70s
     Running `target/release/divi`
 56 = (1) 42 + 14
 42 = (3) 14 + 0
mcd(-56, -42): 14

 56 = (1) 42 + 14
 42 = (3) 14 + 0
mcd( 56,  42): 14

106 = (2) 46 + 14
 46 = (3) 14 + 4
 14 = (3)  4 + 2
  4 = (2)  2 + 0
mcd(106,  46): 2

 78 = (2) 32 + 14
 32 = (2) 14 + 4
 14 = (3)  4 + 2
  4 = (2)  2 + 0
mcd( 78,  32): 2
```