Assignment 7 Papoulis-Pillai Chapter 6 Example 6.76

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Outline

Question

Question: Show that variance of a random variable x with the Student t distribution $t(n)$ is $\frac{n(n-2)}{2}$

Solution

Solution: We know that if x and y are two independent random variables, x is $N(0,1)$ and y is $\chi^2(n)$:

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \quad f_y(y) = \frac{1}{2^{n/2}\Gamma(n/2)} y^{n/2-1} e^{-y/2} U(y) \tag{2.1}$$

Then the random variable

$$z = \frac{x}{\sqrt{y/n}} \tag{2.2}$$

has a Student t distribution.

Hence $E(x^2) = 1$ and

$$E\left\{\frac{1}{y}\right\} = \frac{1}{2^{n/2}\Gamma(n/2)} \int_0^\infty y^{n/2-1} e^{-y/2}\, dy = \frac{2^{n/2-1}\Gamma(n/2-1)}{2^{n/2}\Gamma(n/2)} \tag{2.3}$$

$$\therefore E\left\{\frac{1}{y}\right\} = \frac{1}{n-2} \tag{2.4}$$

Also as x and y are independent we get that:

$$E(z^2) = n\, E(x^2) E\left\{\frac{1}{y}\right\} = \frac{n}{n-2} \tag{2.5}$$

$\therefore$ Variance of a random variable with the Student t distribution is $\frac{n}{n-2}$