Assignment 7 Papoulis-Pillai Chapter 6 Example 6.76

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Outline

Question

Question: Show that if $\beta_x(t) = f_x(t|x > t)$, $\beta_y(t) = f_y(t|y > t)$ and $\beta_x(t) = k\beta_y(t)$ then $1 - F(x) = [1 - F(y)]^k$

Solution

Solution:

$$F(x|\mathbf{x}>t)=\frac{P(\mathbf{x}\leq x,\mathbf{x}>t)}{P(\mathbf{x}>t)}=\frac{F(x)-F(t)}{1-F(t)} \tag{2.1}$$

Diffrentiating w.r.t x

$$f(x|\mathbf{x}>t)=\frac{f(x)}{1-F(t)},\quad x>t \tag{2.2}$$

$$\because f(x)=F'(x)$$

$$\beta(t)=f(t|x>t)=\frac{f(t)}{1-F(t)} \tag{2.3}$$

$$Also\quad R(t)=1-F(t)$$

$$\implies \beta(t)=\frac{F'(t)}{1-F(t)}=\frac{-R'(t)}{R(t)} \tag{2.4}$$

Integrating this equation from 0 to x and using the fact that $\ln(R(0)) = 0$:

$$-\int_0^x \beta(t)dt = \ln(R(x)) \tag{2.5}$$

$$R(x) = 1 - F(x) = exp[-\int_0^x \beta(t)dt] \tag{2.6}$$

$$\therefore 1 - F_x(x) = exp[-\int_0^x \beta_x(t)dt] \; and \; 1 - F_y(y) = exp[-\int_0^x \beta_y(t)dt]$$

$$\beta_x(t) = k\beta_y(t) \tag{2.7}$$

$$\therefore 1 - F_x(x) = exp[-\int_0^x k\beta_y(t)dt] = exp[-k\int_0^x \beta_y(t)dt]$$

$$= exp[-\int_0^x \beta_y(t)dt]^k$$

$$\therefore 1 - F_x(x) = [1 - F_y(y)]^k \tag{2.8}$$