Assignment 6 Papoulis-Pillai Chapter 4 Example 4.34

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Outline

Question

Question: We place at random 200 points in the interval $(0, 100)$. Find the probability that in the interval (0, 2) there will be one and only one point (a) exactly and (b) using the Poisson approximation.

Solution

Solution: Probability of a point landing in the interval (0,2) is 0.02. i.e $p = 0.02$
Probability of k points from 200 points landing in the interval (0,2) is

$$p_k = {}^nC_k p^k (1-p)^{n-k} \tag{2.1}$$

p=0.02, k=1, n=200.
(a) For exact probability

$$p_1 = {}^nC_1 p(1-p)^{n-1} \tag{2.2}$$

$$p_1 = n \times p \times (1-p)^{n-1} \tag{2.3}$$

$$p_1 = 200 \times 0.02 \times (0.98)^{199} \tag{2.4}$$

$$\therefore p_1 = 0.0718 \tag{2.5}$$

(b) For Poisson approximation, let $\lambda = n \times p$.
in Poisson approximation we take limit n tends to ∞ and p tends to 0. λ is a constant.
Substituting value of $p = \dfrac{\lambda}{n}$

$$p_k = {}^nC_k p^k (1-p)^{n-k} \tag{2.6}$$

$$\implies p_k = \frac{n!}{(n-k)!k!}\frac{\lambda^k}{n^k}(1-\frac{\lambda}{n})^n(1-\frac{\lambda}{n})^{-k} \tag{2.7}$$

$$p_k = \frac{n(n-1)(n-2)....(n-k+1)}{k!}\frac{\lambda^k}{n^k}(1-\frac{\lambda}{n})^n(1-\frac{\lambda}{n})^{-k} \tag{2.8}$$

$$p_k = \frac{n}{n}\frac{n-1}{n}\frac{n-2}{n}.....\frac{n-k+1}{n}\frac{\lambda^k}{k!}(1-\frac{\lambda}{n})^n(1-\frac{\lambda}{n})^{-k} \tag{2.9}$$

$$\lim_{n\to\infty} p_k = 1.1.1.....1.\frac{\lambda^k}{k!}(1-\frac{\lambda}{n})^n.1 \tag{2.10}$$

$$\tag{2.11}$$

$$\lim_{n\to\infty}(1-\frac{\lambda}{n})^n = e^{-\lambda} \tag{2.12}$$

$$\therefore \lim_{n\to\infty} p_k = \frac{\lambda^k}{k!}e^{-\lambda} \tag{2.13}$$

$$\therefore \lim_{n\to\infty} p_1 = \lambda e^{-\lambda} \tag{2.14}$$

$$\lambda = n \times p \implies \lambda = 0.02 \times 200 = 4 \tag{2.15}$$

$$\therefore \lim_{n\to\infty} p_1 = 4e^{-4} = 0.0732 \tag{2.16}$$

Value of p_1 exactly is 0.0718 and by Poisson approximation it is 0.0732