Assignment 1
AI1110: Probability and Random Variables
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Question 2(b)

Question: Given that:

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

$$and\ \mathbf{B} = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$$

, find $\mathbf{AB}$.

Using this result, solve the following system of equation:

$$x - y = 3,\ 2x + 3y + 4z = 17\ and\ y + 2z = 7$$

in matrix form

$$x - y = 3,\ 2x + 3y + 4z = 17\ and\ y + 2z = 7$$

$$\implies \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix}$$

Solution:

$$\mathbf{AB} = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{pmatrix}$$

$$\implies \mathbf{AB} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \tag{1}$$

$$\implies \mathbf{AB} = 6\mathbf{I} \tag{2}$$

Where $\mathbf{I}$ is the Identity Matrix of order 3

Now, let us write the following system of equations

Now $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ is basically $\mathbf{A}$.

Let $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be $\mathbf{C}$ and $\begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix}$ be $\mathbf{D}$ $\therefore$ $\mathbf{AC} = \mathbf{D}$

To solve this system of equations we need to find $\mathbf{C}$

$$\implies \mathbf{C} = \mathbf{A}^{-1}\mathbf{D} \tag{3}$$

$$and, \mathbf{AB} = 6\mathbf{I} \tag{4}$$

$$\implies \mathbf{A}^{-1} = \frac{\mathbf{B}}{6} \tag{5}$$

$$\implies \mathbf{C} = \frac{\mathbf{BD}}{6} \tag{6}$$

$$\therefore \mathbf{C} = \frac{1}{6}\begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix} \tag{7}$$

$$\implies \mathbf{C} = \frac{1}{6}\begin{pmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{pmatrix} \tag{8}$$

$$\implies \mathbf{C} = \frac{1}{6}\begin{pmatrix} 12 \\ -6 \\ 24 \end{pmatrix} \tag{9}$$

$$\implies \mathbf{C} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \tag{10}$$

$$\therefore x = 2, y = -1, z = 4 \tag{11}$$

is the solution for this system of equations.