

Assignment 1

Aman Pratap Singh

Area of Triangle

Abstract—This document contains the solution to find the area of a triangle, from the given coordinates of the vertices.

Download all python codes from

<https://github.com/AP1920/Assignment1/blob/main/assignment1.py>

Download latex-tikz codes from

<https://github.com/AP1920/Assignment1/blob/main/main.tex>

1 PROBLEM

1.1 Vector 2, Example-2,6

Find the area of the triangle the coordinates of whose angular points are respectively:

$$\mathbf{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad (1.1.1)$$

2 SOLUTION

We will be using vectors for calculating the area of the triangle formed by above three points.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 5 \\ -5 \end{pmatrix} \quad (2.0.4)$$

$$\therefore \text{Area of the Triangle} = \frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| \quad (2.0.5)$$

As the vector cross product of two vectors can also be expressed as the product of a skew-symmetric matrix and a vector.

$$\mathbf{P} \times \mathbf{Q} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (2.0.6)$$

Substituting values from equation (2.0.2) and (2.0.4) in above equation (2.0.6), we'll get:

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -3 \\ -1 & 3 & 0 \end{pmatrix} \times \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix} \quad (2.0.7)$$

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 0 \\ 0 \\ -20 \end{pmatrix} \quad (2.0.8)$$

$$\therefore \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| = \sqrt{0^2 + 0^2 + (-20)^2} = 20 \quad (2.0.9)$$

Substituting value from equation (2.0.9) in equation (2.0.5), we'll get area of triangle:

$$\Rightarrow \frac{1}{2}(20) = 10 \text{units}^2$$

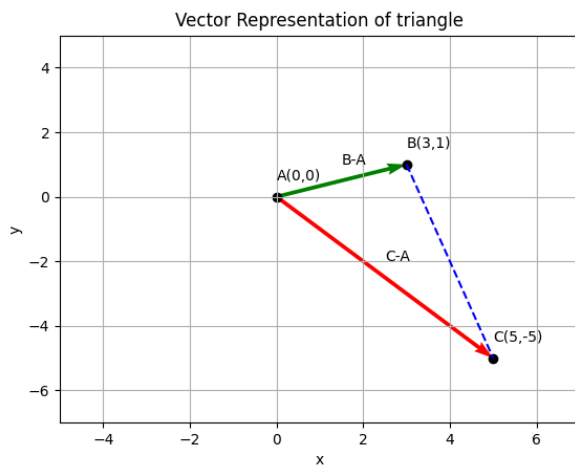


Fig. 1: Plot obtained from Python code