Assignment 1

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Area of Triangle

***Abstract*—This document contains the solution to find the area of a triangle, from the given coordinates of the vertices.**

Download all python codes from

https://github.com/AP1920/Assignment1/blob/main/assignment1.py

Download latex-tikz codes from

https://github.com/AP1920/Assignment1/blob/main/main.tex

1 Problem

1.1 Vector 2, Example-2,6

Find the area of the triangle the coordinates of whose angular points are respectively: $\mathbf{A}\left(-1,2\right)$, $\mathbf{B}\left(2,3\right)$ and $\mathbf{C}\left(4,-3\right)$.

2 Solution

We will be using vectors for calculating the area of the triangle formed by above three points.

$$(\mathbf{B}-\mathbf{A}) = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} \tag{2.0.1}$$

$$(\mathbf{B}-\mathbf{A}) = \begin{pmatrix}3\\1\end{pmatrix} \tag{2.0.2}$$

$$(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}4\\-3\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} \tag{2.0.3}$$

$$(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}5\\-5\end{pmatrix} \tag{2.0.4}$$

$$\because \text{Area of the Triangle} = \frac{1}{2}\left\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\right\| \tag{2.0.5}$$

As the vector cross product of two vectors can also be expressed as the product of a skew-symmetric matrix and a vector.

$$\mathbf{P}\times\mathbf{Q} = \begin{pmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{pmatrix} \times \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \tag{2.0.6}$$

Substituting values from equation 2.0.2 and 2.0.4 in above equation 2.0.6, we'll get:

$$(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}0 & 0 & 1\\ 0 & 0 & -3\\ -1 & 3 & 0\end{pmatrix} \times \begin{pmatrix}5\\-5\\0\end{pmatrix} \tag{2.0.7}$$

$$(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}0\\0\\-20\end{pmatrix} \tag{2.0.8}$$

$$\therefore \left\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\right\| = \sqrt{0^2+0^2+(-20)^2} = 20 \tag{2.0.9}$$

Substituting value from equation 2.0.9 in equation 2.0.5, we'll get area of triangle:

$\implies \frac{1}{2}(20) = 10units^2$

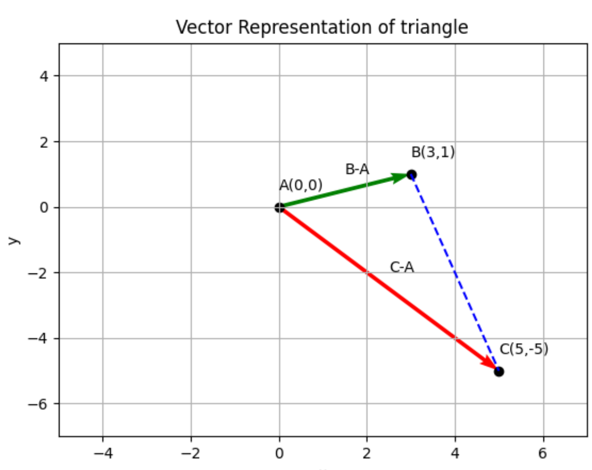

Fig. 1: Plot obtained from Python code