Assignment 1

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Area of Triangle

***Abstract*—This document contains the solution to find the area of a triangle, from the given coordinates of the vertices.**

Download all python codes from

```
https://github.com/AP1920/Assignment1/blob/main/
    assignment1.py
```

Download latex-tikz codes from

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https://github.com/AP1920/Assignment1/blob/main/
    main.tex
```

1 Problem

1.1 Vector 2, Example-2,6

Find the area of the triangle the coordinates of whose angular points are respectively:

$$A = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \ B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ and } \ C = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

2 Solution

We will be using vectors for calculating the area of the triangle formed by above three points.

$$(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{2.0.1}$$

$$(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{2.0.2}$$

$$(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{2.0.3}$$

$$(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 5 \\ -5 \end{pmatrix} \tag{2.0.4}$$

$$\because \text{Area of the Triangle} = \frac{1}{2} \left\| (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) \right\| \tag{2.0.5}$$

As the vector cross product of two vectors can also be expressed as the product of a skew-symmetric matrix and a vector.

$$\mathbf{P} \times \mathbf{Q} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2.0.6}$$

Substituting values from equation (2.0.2) and (2.0.4) in above equation (2.0.6), we'll get:

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -3 \\ -1 & 3 & 0 \end{pmatrix} \times \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 0 \\ 0 \\ -20 \end{pmatrix} \tag{2.0.8}$$

$$\therefore \left\| (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) \right\| = \sqrt{0^2 + 0^2 + (-20)^2} = 20 \tag{2.0.9}$$

Substituting value from equation (2.0.9) in equation (2.0.5), we'll get area of triangle:
$\implies \frac{1}{2}(20) = 10 units^2$

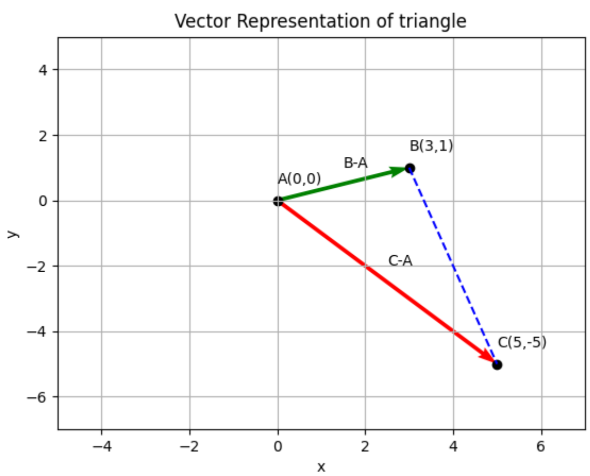

Fig. 1: Plot obtained from Python code