Group Theory

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Contents

1 Sets and Mappings

§1.1 Sets

Definition 1.1.1 (set)**.** A *set* is collection of elements.

Definition 1.1.2 (belongs to)**.** If x belongs to $S \Rightarrow x \in S$.

Definition 1.1.3 (does not belong to)**.** If x does not belong to S $x \Rightarrow x \notin S$.

Definition 1.1.4 (equal sets)**.** For sets A and B, $A = B \Leftrightarrow x \in A \Leftrightarrow x \in B$.

Definition 1.1.5 (null set)**.** Set with no elements: null/void set $\emptyset$.

Definition 1.1.6 (subset)**.** If $a \in A \Rightarrow a \in B$, $A \subset B$ or $A \subseteq B$.

Definition 1.1.7 (superset)**.** If $A \subset B$, $B \supset A$ or $B \supseteq A$.

Definition 1.1.8 (universal set)**.** Set U that contains all sets in a given context.

Definition 1.1.9 (union)**.** Union of A and B:

$$A \cup B = \{x \in U | x \in A \text{ or } x \in B\}$$

Definition 1.1.10 (intersection)**.** Intersection of A and B:

$$A \cap B = \{x \in U | x \in A \text{ and } x \in B\}$$

Definition 1.1.11 (difference)**.** Difference of A and B:

$$A - B = \{x \in U | x \in A \text{ and } x \notin B\}$$

Definition 1.1.12 (complement)**.** If $B \in A$, complement of B: $A - B$

Definition 1.1.13 (disjoint)**.** Disjoint: $A \cap B = \emptyset$

Theorem 1.1.14 (commutative, associative, distributive)

(i) $A \cup A = A = A \cap A$

(ii) $A \cup B = B \cup A; A \cap B = B \cap A$

(iii) $(A \cup B) \cup C = A \cup (B \cup C)$,
$(A \cap B) \cap C = A \cap (B \cap C)$

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(v) $A \cup (A \cap B) = A = A \cap (A \cup B)$

Theorem 1.1.15 (DeMorgan's rules)

(i) $X - (X - A) = X \cap A$

(ii) $X - (A \cup B) = (X - A) \cap (X - B)$

(iii) $X - (A \cap B) = (X - A) \cup (X - B)$

Definition 1.1.16 (union of all sets in a set)**.** $\bigcup_{x \in S} X = \{x | x \in X \text{ for some } X \text{ in } S\}$

Definition 1.1.17 (intersection of all sets in a set)**.** $\bigcap_{x \in S} X = \{x | x \in X \text{ for every } X \text{ in } S\}$

Definition 1.1.18 (power set)**.** $\mathcal{P}(X) = S | S \subset X$ *(including $\emptyset$)*