

Assignment 1 — Applied Algorithms, T. II/2024–25

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Problem 1. Las Vegas and Monte Carlo

a.i) We want to show the probability of running time of Monte Carlo is at least the worst running time which is $4f(n)$. We can use Markov inequalities to bound it..

$$\mathbf{P}(X \leq \lambda) \leq \frac{E[X]}{\lambda}$$

$$\begin{aligned}\mathbf{P}(T(n) \leq 4f(n)) &\leq \frac{f(n)}{4f(n)} \\ &\leq \frac{1}{4}\end{aligned}$$

a.ii) The worst-case running time happens at most $1/4$ which produces incorrect answers. We can get the complement of the last answer...

$$1 - \mathbf{P}(T(n) \leq 4f(n)) \leq 1 - \frac{1}{4} = \frac{3}{4}$$

b.i) The LV algorithm running time is described as the following. Each iteration requires running A to produce an answer then run C to check the answer. So the running time for each trial is...

$$1 \text{ iteration running time of LV} = f(n) + g(n)$$

So the question is what is the expected iterations needed to run LV to get a correct answer. If p is the probability of success then the expected $1/p$.

$$\text{Running time of LV} = \frac{1}{p}(f(n) + g(n))$$

Problem 2. Chernoff-Hoeffding With Bounds

(Statement of problem goes here.)

Proof. (Type your proof here.)

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Problem 3. Rescaling Trick

(Statement of problem goes here.)

Proof. (Type your proof here.)

□

Problem 4. x^2 With π Degrees of Freedom

(Statement of problem goes here.)

Proof. (Type your proof here.)

□

Problem 5. Simple Samplers.

(Statement of problem goes here.)

Proof. (Type your proof here.)

□

Problem 6. Median of Means

(Statement of problem goes here.)

Proof. (Type your proof here.)

□

Problem 7. Skip List

a.) Questions to experiment:

- Q1.) Can we compute count coin tosses differently and is the alternative better?
- Q2.) How does varying max height change performance?
- Q3.) Linked lists are known to be cache unfriendly, is there a way we can modify
- Q3.) How does it perform against a reputable ordered map?

b.) Search algorithm of Skip List when start is at the bottom left corner in $O(\log(d))$ where d is the number of elements smaller than the key?

Problem 8. (a, b) tree. $(2, 3)$ tree.

a.)

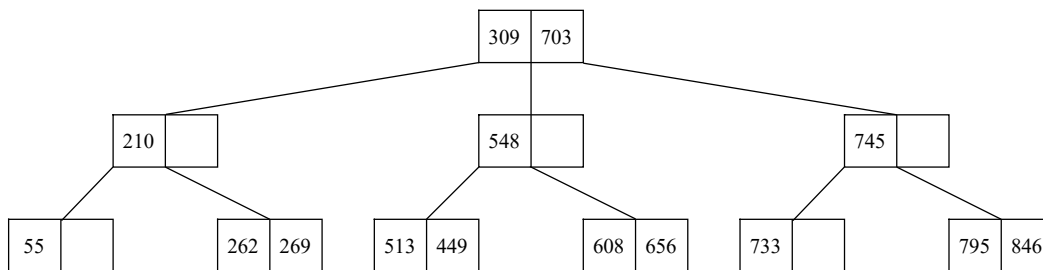


Figure 1: Keys 733, 703, 608, 846, 309, 269, 55, 745, 548, 449, 513, 210, 795, 656, 262 inserted into a $(2, 3)$ tree.

b.)

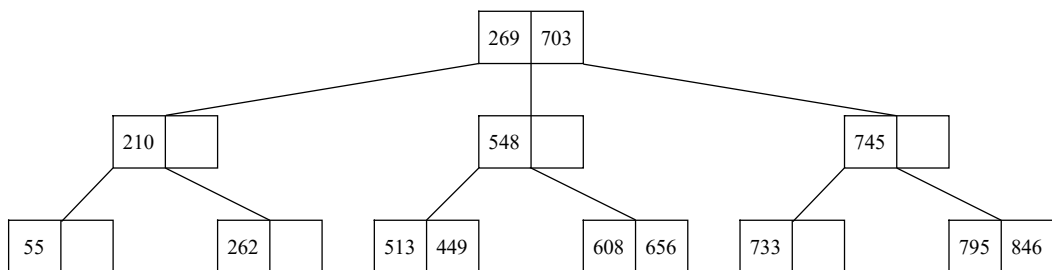


Figure 2: Key 309 removed from Figure 1 tree.

Problem 9. B-Tree Speed

(Statement of problem goes here.)

Proof. (Type your proof here.)

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