Assignment 1 — Applied Algorithms, T. II/2024–25

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Problem 1. Las Vegas and Monte Carlo

a.i) We want to show the probablity of running time of Monte Carlo is atleast the worst running time which is $4f(n)$. We can use markov inqualities to bound it..

$$\mathbf{P}(X \leq \lambda) \leq \frac{E[X]}{\lambda}$$

$$\begin{aligned}\mathbf{P}(T(n) \leq 4f(n)) &\leq \frac{f(n)}{4f(n)} \\ &\leq \frac{1}{4}\end{aligned}$$

a.ii) The worse-case running time happens at most 1/4 which produces incorrect answers. We can get the complement of the last answer...

$$1 - \mathbf{P}(T(n) \leq 4f(n)) \leq 1 - \frac{1}{4} = \frac{3}{4}$$

b.i) The LV algorithm running time is described as the follwing. Each iteration requires running A to produce an answer then run C to check the answer. So the running time for each trial is...

$$\text{1 iteration running time of LV} = f(n) + g(n)$$

So the question is what is the expected iterations needed to run LV to get a correct answer. If p is the probablity of success then the expected $1/p$.

$$\text{Running time of LV} = \frac{1}{p}(f(n) + g(n))$$

Problem 2. Chernoff-Hoeffding With Bounds

(Statement of problem goes here.)

$$\Pr[X > (1+\varepsilon)\mu H] \leq \exp\left(-\frac{\varepsilon^2}{2+\varepsilon}\mu H\right)$$

Proof. (Type your proof here.) □

Problem 3. Rescaling Trick

(Statement of problem goes here.)

Proof. (Type your proof here.) □

Problem 4. x^2 With π Degrees of Freedom

(Statement of problem goes here.)

Proof. (Type your proof here.) □

Problem 5. Simple Samplers.

(Statement of problem goes here.)

Proof. (Type your proof here.) □

Problem 6. Median of Means

(Statement of problem goes here.)

Proof. (Type your proof here.) □

Problem 7. Skip List

a.) Questions to experiment:

Q1.) Can we compute count coin tosses differently and is the alternative better?
Q2.) How does varying max height change performance?
Q3.) Linked lists are known to be cache unfriendly, is there a way we can modify
Q3.) How does it perform against a reputable ordered map?

b.) Search algorithm of Skip List when start is at the bottom left corner in $O(\log(d))$ where d is the number of elements smaller than the key?

Problem 8. (a, b) tree. $(2, 3)$ tree.

a.)

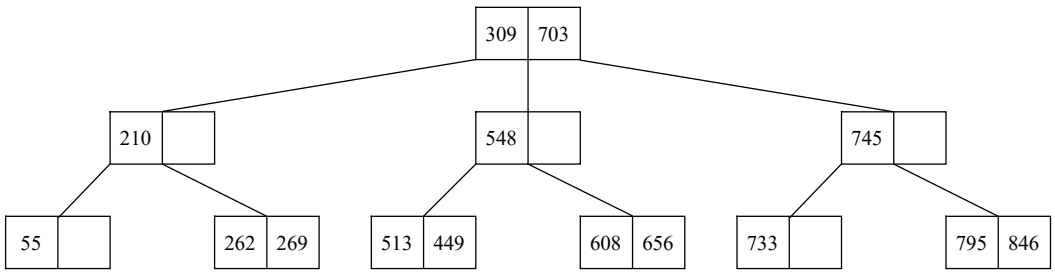

Figure 1: Keys $733, 703, 608, 846, 309, 269, 55, 745, 548, 449, 513, 210, 795, 656, 262$ inserted into a $(2, 3)$ tree.

b.)

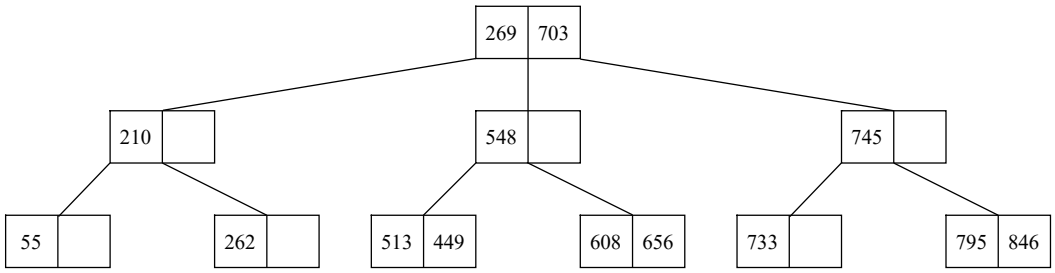

Figure 2: Key 309 removed from Figure 1 tree.

Problem 9. B-Tree Speed

(Statement of problem goes here.)

Proof. (Type your proof here.) □