Problem 7. Skip List

Experimental Set1up

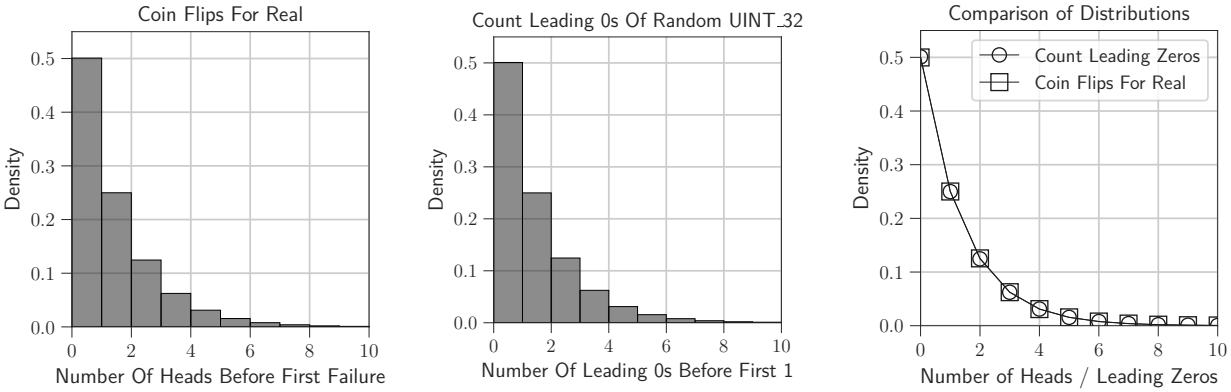
Conducted our benchmarks using Google Benchmark [1] on a system with the following hardware specifications:

- **Processor**: 11th Gen Intel(R) Core(TM) i7-11370H 4C/8T CPU @ 3.3 GHz
- **Cache Hierarchy**:
  - L1 Data: 48 KiB per core (×4)
  - L1 Instruction: 32 KiB per core (×4)
  - L2 Unified: 1280 KiB per core (×4)
  - L3 Unified: 12,288 KiB (shared)

All benchmarks were compiled using `-Ox` optimization level with `MSVC` (version 19.42.34436) and executed in a single-threaded environment to minimize external interference. Memory usage was measured using the Windows API `GetProcessMemory()` [3].

Questions And Experiments

a.1) Can we perform count coin tosses differently and is the alternative better?



Benchmark	Time (ns)	CPU (ns)	Iterations
Coin Flip For Real	29.8	29.3	22,400,000
Coin Flip Count Leading 0s	5.11	4.87	144,516,129

Table 1: Benchmark results comparing different coin flip implementations.

a.2) How to define vertical nodes withoud the downsides of introducing linked-list like overhead?

This is an awfully convenient question, because it is, I chose the question as an outlet to describe how I got to my current implementation of skip list. My first implementation of skip list was almost 2 weeks ago was horrendously slow. It took seconds to insert 1 million keys, never mind trying to insert 10s of millions of keys. It was so frustrating, I couldn't pinpoint exactly what was causing the bottleneck. The code was text-book, simple node struct, search for the insertion point, and stitch the new node/nodes into place.

After dilly dallying with debuggers and profilers, I thought that traversal was what was slowing everything down. Instead of using a linked-list to extend higher keys I thought of using an array instead as it is more cache friendly and might help solve the problem. I didn't have to store keys at every-level and vertical references. Since the data is more 'packed' I thought maybe it would solve the long traversal times.

At last I felt that inserting keys were faster however it still took almost a second to insert a million keys. After sulking in the group chat, Jorn asks whether I got my implementation ideas from a particular stack-overflow thread [4] as I had that vertical array going on. I sure didn't but I was surprised to see in many ways the shape of their function was kinda the same. We came to similar conclusions on implementation. However one thing that stood out in this implementation the author took special care in allocating nodes. They used a custom allocator, and allocated arrays manually. Their results were blazingly fast!

For my array data-structure I used `std::vector`, it was a big mistake, vectors have all kinds of fancy stuff to make them safe and easy to use. All the added indirection vector brought killed the performance of my skip-list. After swapping out std::vector to directly allocating the memory like the author in that stack-overflow thread, the speed was finally bearable to insert 2^{22} keys.

a.3) How does it perform against a reputable ordered map?

Skip List Insertion		
n	CPU Time (ms)	Memory Usage (KB)
8	1.14E-04	0
16	1.76E-04	0
32	5.78E-04	0
64	1.20E-03	0
128	3.25E-03	0
256	6.63E-03	0
512	1.60E-02	0
1024	6.98E-02	32
2048	2.00E-01	72
4096	4.39E-01	208
8192	1.20E+00	444
16384	2.54E+00	884
32768	6.56E+00	1724
65536	1.75E+01	3436
131072	4.97E+01	6988
262144	1.88E+02	15048
524288	6.72E+02	30228
524288	6.88E+02	30172
1048576	1.69E+03	60384
2097152	4.42E+03	120756
4194304	1.14E+04	241416
8388608	2.81E+04	483244

Ordered Map Insertion		
n	CPU Time (ms)	Memory Usage (KB)
8	0.00E+00	0
16	0.00E+00	0
32	0.00E+00	0
64	1.32E+01	0
128	4.73E+00	0
256	1.25E+01	0
512	0.00E+00	0
1024	5.78E+00	0
2048	8.72E+00	0
4096	1.45E+01	0
8192	2.78E+01	0
16384	4.94E+01	0
32768	4.88E+01	0
65536	5.31E+01	68
131072	1.20E+02	136
262144	3.33E+02	11864
524288	5.26E+02	204
524288	5.23E+02	336
1048576	8.28E+02	24396
2097152	2.08E+03	98332
4194304	5.08E+03	197080
8388608	1.25E+04	394916

Skip List Search		
n	CPU Time (ms)	Memory Usage (KB)
8	2.90E-05	0
16	8.50E-05	0
32	3.49E-04	0
64	6.10E-04	0
128	1.22E-03	0
256	2.70E-03	0
512	7.12E-03	0
1024	6.00E-02	0
2048	1.57E-01	32
4096	3.99E-01	28
8192	9.83E-01	156
16384	2.68E+00	520
32768	6.28E+00	1476
65536	1.67E+01	3320
131072	4.69E+01	6972
262144	2.15E+02	14604
524288	5.31E+02	29776
524288	5.63E+02	29708
1048576	1.66E+03	60032
2097152	3.95E+03	120340
4194304	9.83E+03	241212
8388608	2.36E+04	482876

Ordered Map Search		
n	CPU Time (ms)	Memory Usage (KB)
8	3.20E-05	0
16	9.40E-05	0
32	2.00E-04	0
64	4.30E-04	0
128	9.84E-04	0
256	2.25E-03	0
512	6.70E-03	0
1024	3.84E-02	0
2048	1.09E-01	0
4096	2.51E-01	0
8192	6.14E-01	0
16384	1.38E+00	64
32768	4.62E+00	0
65536	1.00E+01	32
131072	2.40E+01	0
262144	8.48E+01	204
524288	2.73E+02	204
524288	2.73E+02	68
1048576	8.91E+02	23996
2097152	2.19E+03	49152
4194304	5.45E+03	98832
8388608	1.31E+04	290012

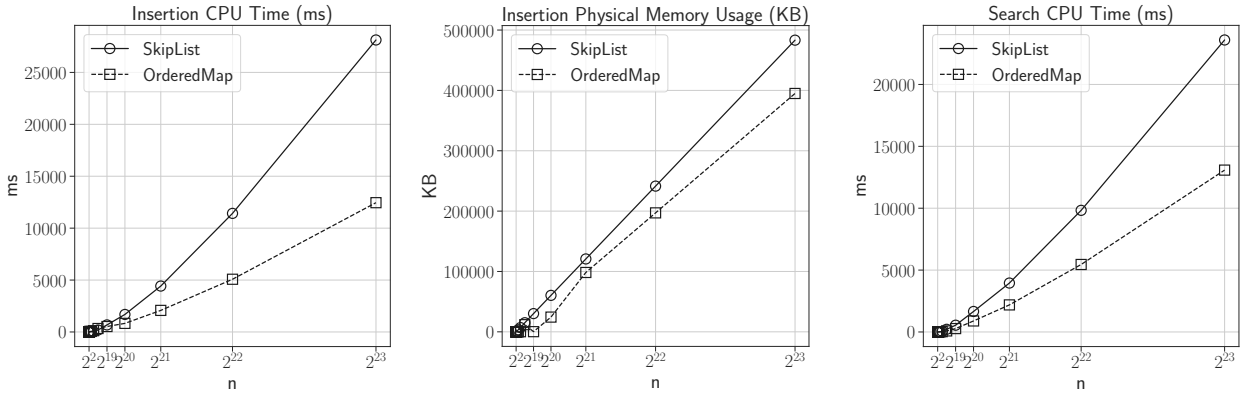


Figure 1: CPU Time and Memory Usage of Skip List and Ordered Map.

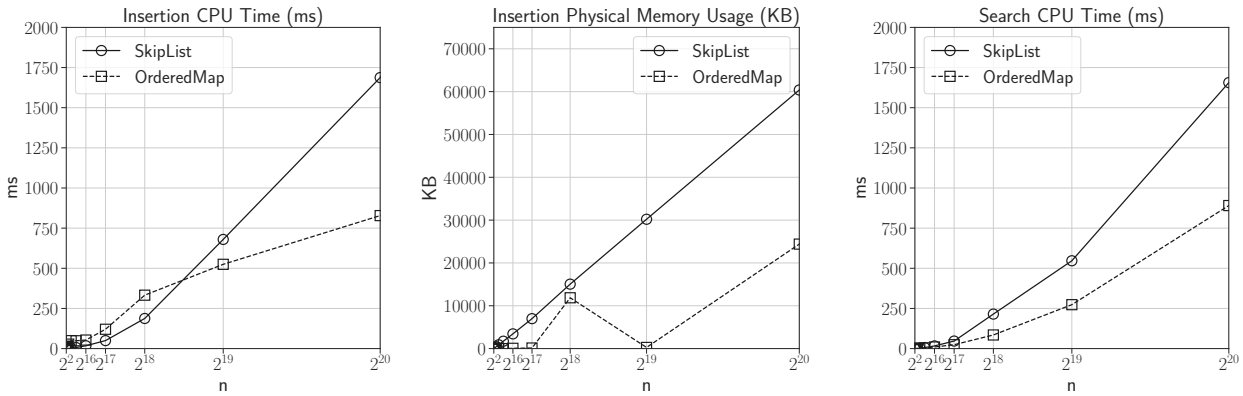


Figure 2: Crop of Figure 1.

Q4.) How might varying max height change performance of Skip List?

Skip List Insertion			Skip List Map Search		
Max Level	CPU Time (ms)	Memory Usage (KB)	Max Level	CPU Time (ms)	Memory Usage (KB)
2	3.03E+04	6888	2	3.03E+04	6888
4	2.91E+03	7232	4	2.91E+03	7232
6	7.66E+02	7424	6	7.66E+02	7424
8	2.97E+02	7480	8	2.97E+02	7480
10	1.32E+02	7456	10	1.32E+02	7456
12	5.86E+01	7424	12	5.86E+01	7424
14	4.69E+01	7416	14	4.69E+01	7416
16	6.53E+01	7404	16	6.53E+01	7404
18	5.40E+01	7440	18	5.40E+01	7440
20	5.00E+01	7484	20	5.00E+01	7484
22	4.69E+01	7520	22	4.69E+01	7520
24	5.16E+01	7420	24	5.16E+01	7420
26	4.98E+01	7404	26	4.98E+01	7404
28	5.16E+01	7388	28	5.16E+01	7388
30	4.55E+01	7420	30	4.55E+01	7420
32	4.83E+01	7500	32	4.83E+01	7500

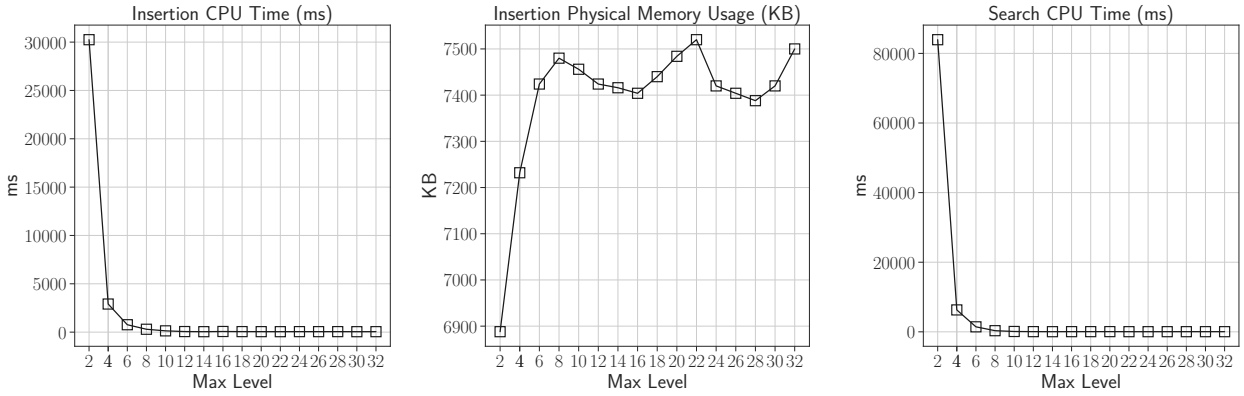


Figure 3: Crop of

b.) Search algorithm of Skip List when start is at the bottom left corner in $O(\log(d))$ where d is the number of elements smaller than the key?

Problem 8. (a, b) tree. $(2, 3)$ tree.

a.)

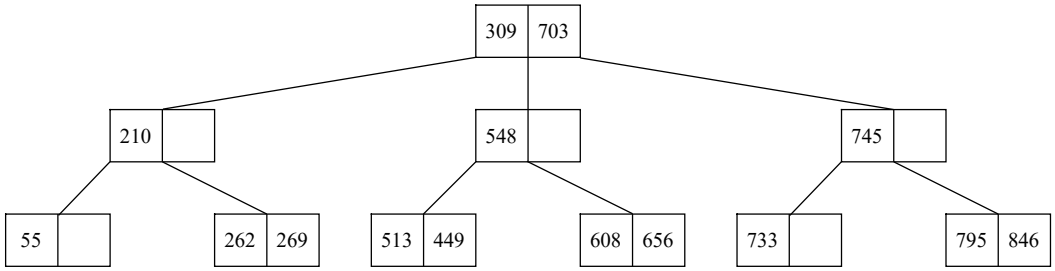


Figure 4: Keys $733, 703, 608, 846, 309, 269, 55, 745, 548, 449, 513, 210, 795, 656, 262$ inserted into a $(2, 3)$ tree.

b.)

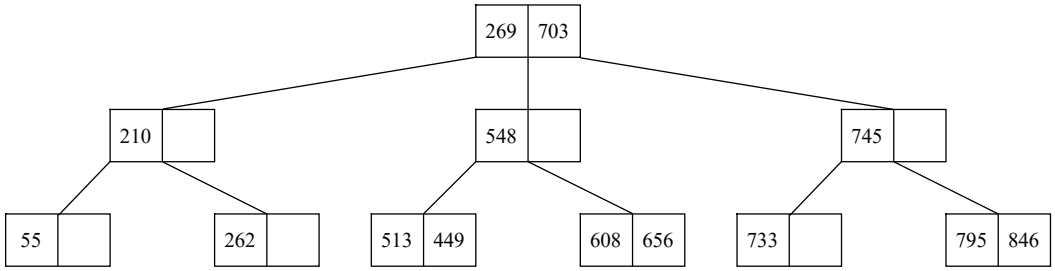


Figure 5: Key 309 removed from Figure 4 tree.

Problem 9. B-Tree Speed

Experimental Setup

Conducted our benchmarks using Google Benchmark [1] on a system with the following hardware specifications:

- **Processor**: 11th Gen Intel(R) Core(TM) i7-11370H 4C/8T CPU @ 3.3 GHz
- **Cache Hierarchy**:
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  - L3 Unified: 12,288 KiB (shared)

All benchmarks were compiled using `-Ox` optimization level with `MSVC` (version 19.42.34436) and executed in a single-threaded environment to minimize external interference. Memory usage was measured using the Windows API `GetProcessMemory()` [3].

The B-Tree implementation utilized in this experiment is sourced from the repository by frozenca on GitHub [2].

Finding Optimal Parameter b Of B-Tree

First I carried out benchmarks to measure performance, 20×10^6 random unique keys being inserted into the B-Tree while varying b parameter. Measured aggregate data, It is generally more stable than measure per operation. The operation per millisecond (ops/ms) is computed CPU-Time/20×10^6.

b	ops/ms	CPU Time (ms)	Memory Usage (KB)
2	336.93	59359.38	2544788
4	229.97	86968.75	1063908
6	187.82	106484.38	740108
8	162.25	123265.63	596900
16	147.98	135156.25	400248
32	138.27	144640.63	309792
64	130.08	153750.00	268680
128	122.46	163312.50	256996
256	114.90	174062.50	267676
512	107.48	186078.13	280852
1024	99.60	200796.88	285412
2048	90.49	221015.63	246636

Table 2: Benchmark results for B-Tree insertion with 20×10^6 inserts and varying b from 2 to 2048.

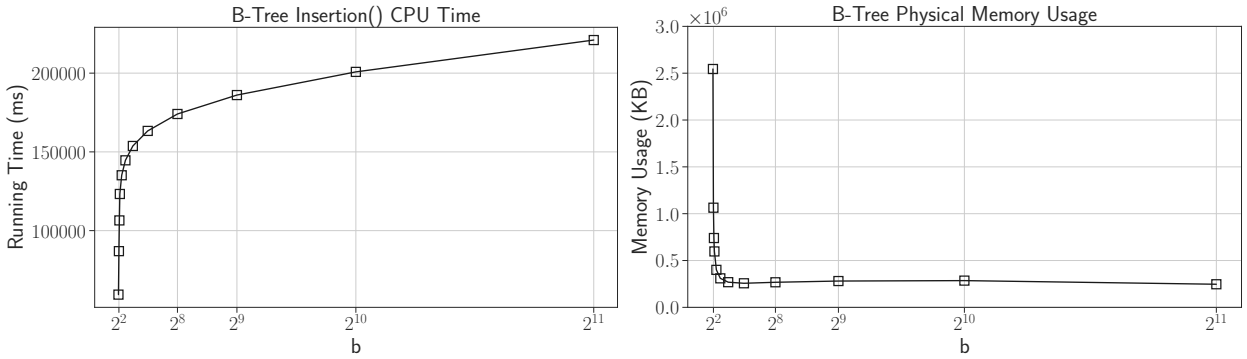

Figure 6: CPU Time and Memory Usage for B-Tree insertion with varying b values.

Notice that as the parameter of b the B-Tree increases, the physical memory usage decreases. My theory is that increasing b makes the tree shallower and more compact. Since a shallower tree reduces the number of pointers and improves spatial locality, nodes are more likely to fit within a cache line, leading to better memory efficiency?

Anywhow... we want the b such that the it minimizes CPU time and memory used. They way I did it is to simply normalize CPU time and memory used and get closest b with to the smallest differnce.

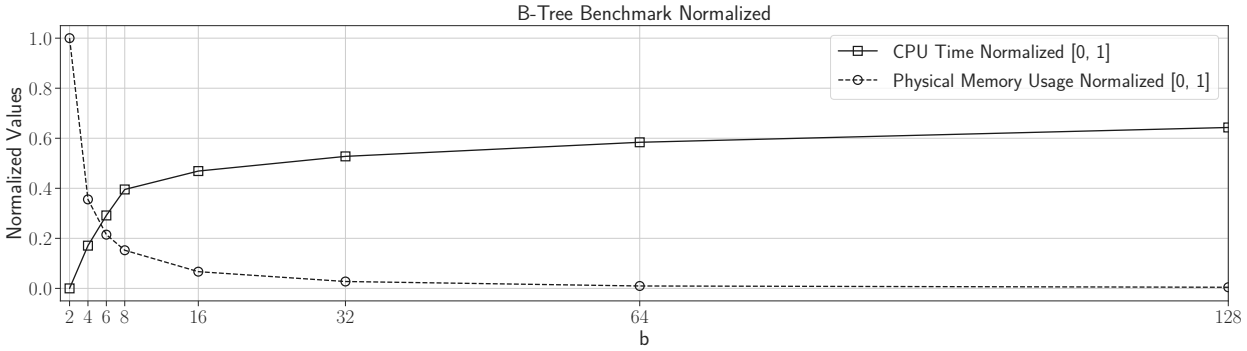

Figure 7: Normalized CPU Time and Memory Usage comparison for B-Tree insertion with varying b values.

Setting $b = 6$ minimizes both performance overhead and memory usage the most. However, for applications that prioritize memory efficiency over execution time, a larger value, such as $b = 16$, may be more optimal, as the performance trade-off becomes less significant. This version clarifies that $b = 6$ minimizes both performance overhead and memory usage while improving the flow of the second sentence.

Comparing B-Tree and Builtin Ordered-Map Performance

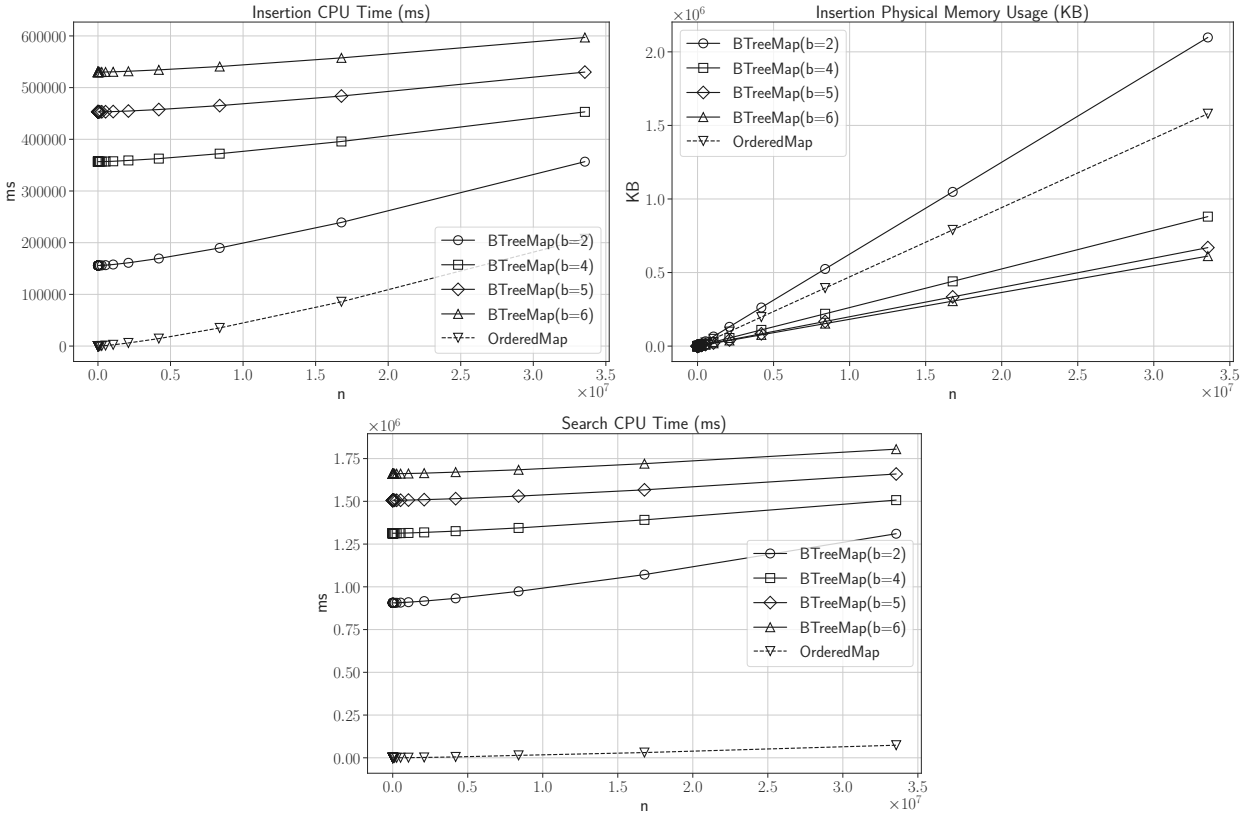

Figure 8: CPU Time and Memory Usage of B-Tree and C++ Builtin Ordered Map. Inserted 2^{24} unique and random keys.

Figure 8 illustrates that the running time for the C++ built-in Ordered Map outperforms the B-Tree in both insertion and search operations. However, it is important to note that for B-Trees with $b > 2$, memory efficiency improves, making them more memory-efficient than the built-in Ordered Map.

Ordered Map Insertion		
n	CPU Time (ms)	Memory Usage (KB)
8	0.00E+00	4
16	0.00E+00	0
32	0.00E+00	0
64	0.00E+00	0
128	0.00E+00	0
256	0.00E+00	4
512	0.00E+00	12
1024	0.00E+00	24
2048	0.00E+00	64
4096	0.00E+00	176
8192	0.00E+00	368
16384	3.13E+01	684
32768	1.56E+01	1584
65536	4.69E+01	3080
131072	1.41E+02	5936
262144	3.28E+02	11884
524288	8.91E+02	24628
1048576	2.42E+03	49260
2097152	6.02E+03	98572
4194304	1.44E+04	197348
8388608	3.51E+04	394748
16777216	8.57E+04	789744
33554432	2.08E+05	1579572

B-Tree b=6 Insertion		
n	CPU Time (ms)	Memory Usage (KB)
8	5.30E+05	4
16	5.30E+05	0
32	5.30E+05	0
64	5.30E+05	0
128	5.30E+05	4
256	5.30E+05	4
512	5.30E+05	8
1024	5.30E+05	16
2048	5.30E+05	32
4096	5.30E+05	68
8192	5.30E+05	124
16384	5.30E+05	308
32768	5.30E+05	576
65536	5.30E+05	1148
131072	5.30E+05	2216
262144	5.30E+05	4460
524288	5.30E+05	9356
1048576	5.31E+05	19092
2097152	5.32E+05	38316
4194304	5.34E+05	76508
8388608	5.41E+05	152888
16777216	5.58E+05	305684
33554432	5.97E+05	611656

Ordered Map Search		
n	CPU Time (ms)	Memory Usage (KB)
8	3.07E-05	4
16	7.67E-05	0
32	1.71E-04	0
64	4.53E-04	0
128	9.63E-04	0
256	2.22E-03	4
512	4.46E-03	12
1024	2.93E-02	24
2048	9.42E-02	64
4096	2.68E-01	176
8192	6.28E-01	368
16384	1.38E+00	684
32768	3.29E+00	1584
65536	8.54E+00	3080
131072	2.34E+01	5936
262144	1.02E+02	11884
524288	2.66E+02	24628
1048576	8.13E+02	49260
2097152	2.06E+03	98572
4194304	5.06E+03	197348
8388608	1.43E+04	394748
16777216	3.06E+04	789744
33554432	7.36E+04	1579572

B-Tree b=6 Search		
n	CPU Time (ms)	Memory Usage (KB)
8	1.66E+06	4
16	1.66E+06	0
32	1.66E+06	0
64	1.66E+06	0
128	1.66E+06	4
256	1.66E+06	4
512	1.66E+06	8
1024	1.66E+06	16
2048	1.66E+06	32
4096	1.66E+06	68
8192	1.66E+06	124
16384	1.66E+06	308
32768	1.66E+06	576
65536	1.66E+06	1148
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1048576	1.66E+06	19092
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4194304	1.67E+06	76508
8388608	1.68E+06	152888
16777216	1.72E+06	305684
33554432	1.81E+06	611656

References

[1] Google Benchmark. *A microbenchmark support library.* https://github.com/google/benchmark

[2] B-Tree Implementation on GitHub. *GitHub Repository.* https://github.com/frozenca/BTree

[3] GetProcessMemoryInfo function. *Microsoft Learn.* https://learn.microsoft.com/en-us/windows/win32/api/psapi/nf-psapi-getprocessmemoryinfo

[4] Skip Lists Stack Overflow. https://stackoverflow.com/questions/31580869/skip-lists-are-they-really-performing-as-good-as-pugh-paper-claim/34003558#34003558