

Binary Numbers

Learning to count!

What is...

1

Decimal System

- has 10 digits/symbols (0-9)
- has a *base* of 10

What does each digit in **123** represent?

We could rewrite **123** as

$$1 * 10^2 + 2 * 10^1 + 3 * 10^0$$

Binary System

- has 2 digits/symbols (0 and 1)
 - "off" and "on"
 - called **bits** (*binary digit*)
 - a **byte** is 8 bits
- has a *base* of 2

Binary to Decimal Conversion

Example

Convert **111** (binary) to a decimal number

Convert **111** (binary) to a decimal number:

$$1 * 2^2 + 1 * 2^1 + 1 * 2^0 =$$

$$1 * 4 + 1 * 2 + 1 * 1 =$$

$$4 + 2 + 1 =$$

$$7$$

Binary Flipper Tool

Make one 😊

Try it out!

- `0b1100`
- `0b1011`
- `0b10010`
- `0b10101`

Decimal to Binary Conversion

Example

Convert 255 to binary

Hint: **Be greedy!** 😈

2^8

2^7

2^6

2^5

2^4

2^3

2^2

2^1

2^0

?

?

?

?

?

?

?

?

?

2^8

2^7

2^6

2^5

2^4

2^3

2^2

2^1

2^0

Try it out!

- 9
- 17
- 109
- 200

Binary Addition

$$\begin{array}{r} 1 \\ 09 \\ + 03 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 1 \\ 01 \\ + 01 \\ \hline 10 \end{array}$$

Binary Subtraction

is really addition of the negative!

$$2 - 3 = 2 + (-3)$$

How might we represent negative numbers?

Binary Widths

What happens if our computer only has the hardware to deal with 2-bit numbers?

Overflow/Underflow Errors

- **overflow**: occurs when the number is *too big* to be represented (wraps to smallest number)
- **underflow**: occurs when the number is *too small* to be represented (wraps to the largest number)

What will happen if we add 1 and 255 on an 8-bit machine?

How about 1 and 256 ?

```
  0b11111111
+ 0b00000001
-----
  0b100000000
```

We started off with 2 8-bit numbers, and our result is
256 , a 9-bit number!

Since the computer cannot manage 9-bit numbers, the
extra bit gets chopped off.

So 255 + 1 = 0 !

Rounding Errors

- machines do not have the hardware to represent an infinite number of digits
 - some fractions (e.g. π , e , $\frac{1}{3}$) cannot be represented accurately
 - must approximate (round)

Why Learn Binary?