Binary Numbers

Learning to count!

What is...

Decimal System

- has 10 digits/symbols (0-9)
- has a *base* of 10

What does each digit in `123` represent?

We could rewrite `123` as

$$1 * 10^2 + 2 * 10^1 + 3 * 10^0$$

Binary System

- has 2 digits/symbols (`0` and `1`)
  - "off" and "on"
  - called **bits** (*binary digit*)
  - a **byte** is 8 bits
- has a *base* of 2

Binary to Decimal Conversion Example

Convert `111` (binary) to a decimal number

Convert `111` (binary) to a decimal number:

$$1 * 2^2 + 1 * 2^1 + 1 * 2^0 =$$

$$1 * 4 + 1 * 2 + 1 * 1 =$$

$$4 + 2 + 1 =$$

$$7$$

Binary Flipper Tool

Make one 😀

Try it out!

- `0b1100`
- `0b1011`
- `0b10010`
- `0b10101`

Decimal to Binary Conversion Example

Convert `255` to binary

Hint: **Be greedy!** 😈

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
?	?	?	?	?	?	?	?	?

```
2^8    2^7     2^6     2^5     2^4     2^3     2^2     2^1     2^0
```

Try it out!

- 9
- 17
- 109
- 200

Binary Addition

```
  1                1
  09               01
+ 03             + 01
------           ------
  12               10
```

Binary Subtraction

is really addition of the negative!

$$2 - 3 = 2 + (-3)$$

How might we represent negative numbers?

Binary Widths

What happens if our computer only has the hardware to deal with 2-bit numbers?

Overflow/Underflow Errors

- **overflow**: occurs when the number is *too big* to be represented (wraps to smallest number)
- **underflow**: occurs when the number is *too small* to be represented (wraps to the largest number)

What will happen if we add `1` *and* `255` *on an 8-bit machine?*

How about `1` *and* `256` *?*

```
  0b11111111
+ 0b00000001
--------------
 0b100000000
```

We started off with 2 8-bit numbers, and our result is `256`, a 9-bit number!

Since the computer cannot manage 9-bit numbers, the extra bit gets chopped off.

So `255 + 1 = 0`!

Rounding Errors

- machines do not have the hardware to represent an infinite number of digits
  - some fractions (e.g. π, e, $\frac{1}{3}$) cannot be represented accurately
  - must approximate (round)

Why Learn Binary?