8. Doubling time is given by $\ln 2/\ln(1 + r) \doteqdot 0.693/\ln(1 + r)$, if r is the rate of increase compounded annually. Show

(a) that $0.70/r$ is on the whole a better approximation than $0.693/r$ for values of r of 0.01, 0.02, 0.03, 0.04, and

(b) that for those r values the error in $0.70/r$ is never in excess of one percent.

9. A population of size 5,000,000 closed to migration has fewer births than deaths. If it decreases at an annual rate of 0.0075, how many years will it take to reach half its present size?

10. Suppose that world population numbered 2 persons 1,000,000 years ago and 4,000,000,000 now.

(a) How many doublings does this represent?

(b) What is the average time per doubling?

(c) What is the average rate of exponential increase?

11. Given that 6^{12} is about 2 billion and that ln 6 is 1.79176, work out the average rate of increase in Problem 10 with logarithms to base 6 rather than base 2.

12. A population growing exponentially stood at 1,000,000 in 1930 and at 3,000,000 in 1970.

(a) What is its rate of increase?

(b) What is its doubling time?

(c) How long would it take to multiply by 9?

13. Consider a logistic population model: $P(t) = [A + Be^{-ut}]^{-1}$, $t \geq 0$. Verify that $P(t)$ satisfies the differential equation

$$\frac{dP(t)}{dt} = uP(t) - uA[P(t)]^2.$$

14. Find the coordinates of the point of inflection for $P(t)$.

15. Fit a logistic curve to the populations of Michigan from 1810 to 1970 as found in Census Bureau publications. Which volume or volumes did you use?

16. You have decided to project the United States population by a logistic curve

$$P(t) = \frac{1}{A + Be^{-ut}}, \qquad t \geq 0.$$

You know the populations on July 1, 1979, and July 1, 1980, are 221.1 and 223.9 millions. Determine A, B, and u for low, medium, and high growth patterns leading to year 2076 populations of 350, 450, and 550 millions. As an approximation, you may replace $P(\infty)$ by $P(2076)$. How satisfactory are such projections?

17. Use the volume(s) from Problem 15 to do the following project. Set up a differential equation of the form $dy/dt = uy$ for a Standard Metropolitan Statistical Area (SMSA) which interests you. Let k = population in 1950.

Use the data for 1960 and 1970 to determine u. Determine the appropriate units for t. Solve for y.

Note: The following maps could be consulted in choosing a SMSA:

(i) Standard Metropolitan Statistical Areas, United States Maps, GE-50, No. 55;
(ii) Population Distribution, Urban and Rural, in the United States: 1970, United States Maps, GE-50, No. 45;
(iii) Population Distribution, Urban and Rural, in the United States: 1970, United States Maps, GE-70, No. 1.

Many university libraries have Map Rooms, and the study of the above maps and others can be both enjoyable and enlightening.

18. This problem is based on the pamphlets "Current Population Reports of the United States Bureau of the Census." Consider a differential equation of the form $dy/dt = uy$ for Hawaii. Use the data for July 1, 1977 and July 1, 1978 to determine the growth rate u. Here $t = 1$ corresponds to 1 year from July 1, 1977. Project the population to 1990.

19. Repeat Problem 18 for a state of your choice.

20. A certain country had a population on July 1, 1980 of 15,000,000. The births and deaths during 1980 were 750,000 and 300,000, respectively.

(a) What is the annual crude rate of growth?

(b) Use an exponential model to project the population to 2030 (July 1).

(c) It is thought that the "ultimate" population will be 145,000,000. Set up a logistic curve for the projections.

(d) What is the first year in which a population of 144,500,000 occurs?

21. Consult pages 254–256 of *World Population 1979*, U.S. Dept. of Commerce, Bureau of the Census, 1980. These pages provide population data for Thailand. Verify the annual rates of growth for 1975 and 1978.

With the 1978 rate of growth, and an exponential model, obtain low and high population projections to July 1, 2029.

22. As an alternate model for Problem 21, use a logistic

$$P(t) = \frac{1/A}{1 + (B/A)e^{-ut}}, \qquad t \geq 0,$$

where $P(0) = 46{,}687{,}000$, $P(\infty) = 1/A = 245{,}000{,}000$, and an assumption that $u = 0.023$. What value of B/A results?

Determine $P(10)$ (population on July 1, 1989), $P(15)$, $P(20)$, $P(25)$, $P(30)$, $P(35)$, $P(40)$, and $P(50)$ (population on July 1, 2029).

23. Recall that for the logistic of Problem 22,

$$\frac{1}{P(t)}\frac{dP(t)}{dt} = u - uAP(t).$$

Use the right-hand side of this equation to obtain the relative rates of change for $t = 15, 25, 35$ in Problem 22.