Exercises 1.1

Terms and Concepts

1. In your own words, what does it mean to "find the limit of $f(x)$ as x approaches 3"?

2. An expression of the form $\frac{0}{0}$ is called ____.

3. T/F: The limit of $f(x)$ as x approaches 5 is $f(5)$.

4. Describe three situations where $\lim\limits_{x\to c} f(x)$ does not exist.

5. In your own words, what is a difference quotient?

6. When x is near 0, $\dfrac{\sin x}{x}$ is near what value?

Problems

In Exercises 7 – 16, approximate the given limits both numerically and graphically.

7. $\lim\limits_{x\to 1} x^2 + 3x - 5$

8. $\lim\limits_{x\to 0} x^3 - 3x^2 + x - 5$

9. $\lim\limits_{x\to 0} \dfrac{x+1}{x^2+3x}$

10. $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x^2-4x+3}$

11. $\lim\limits_{x\to -1} \dfrac{x^2+8x+7}{x^2+6x+5}$

12. $\lim\limits_{x\to 2} \dfrac{x^2+7x+10}{x^2-4x+4}$

13. $\lim\limits_{x\to 2} f(x)$, where
$$f(x) = \begin{cases} x+2 & x \le 2 \\ 3x-5 & x > 2 \end{cases}.$$

14. $\lim\limits_{x\to 3} f(x)$, where
$$f(x) = \begin{cases} x^2-x+1 & x \le 3 \\ 2x+1 & x > 3 \end{cases}.$$

15. $\lim\limits_{x\to 0} f(x)$, where
$$f(x) = \begin{cases} \cos x & x \le 0 \\ x^2+3x+1 & x > 0 \end{cases}.$$

16. $\lim\limits_{x\to \pi/2} f(x)$, where
$$f(x) = \begin{cases} \sin x & x \le \pi/2 \\ \cos x & x > \pi/2 \end{cases}.$$

In Exercises 17 – 24, a function f and a value a are given. Approximate the limit of the difference quotient, $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$, using $h = \pm 0.1, \pm 0.01$.

17. $f(x) = -7x + 2, \quad a = 3$

18. $f(x) = 9x + 0.06, \quad a = -1$

19. $f(x) = x^2 + 3x - 7, \quad a = 1$

20. $f(x) = \dfrac{1}{x+1}, \quad a = 2$

21. $f(x) = -4x^2 + 5x - 1, \quad a = -3$

22. $f(x) = \ln x, \quad a = 5$

23. $f(x) = \sin x, \quad a = \pi$

24. $f(x) = \cos x, \quad a = \pi$

Exercises 1.2

Terms and Concepts

1. What is wrong with the following "definition" of a limit?

   > "The limit of $f(x)$, as x approaches a, is K" means that given any $\delta > 0$ there exists $\varepsilon > 0$ such that whenever $|f(x) - K| < \varepsilon$, we have $|x - a| < \delta$.

2. Which is given first in establishing a limit, the x–tolerance or the y–tolerance?

3. T/F: ε must always be positive.

4. T/F: δ must always be positive.

Problems

In Exercises 5 – 14, prove the given limit using an $\varepsilon - \delta$ proof.

5. $\lim\limits_{x\to 4}(2x+5) = 13$

6. $\lim\limits_{x\to 5}(3-x) = -2$

7. $\lim\limits_{x\to 3}\left(x^2-3\right) = 6$

8. $\lim\limits_{x\to 4}\left(x^2+x-5\right) = 15$

9. $\lim\limits_{x\to 1}\left(2x^2+3x+1\right) = 6$

10. $\lim\limits_{x\to 2}\left(x^3-1\right) = 7$

11. $\lim\limits_{x\to 2} 5 = 5$

12. $\lim\limits_{x\to 0}\left(e^{2x}-1\right) = 0$

13. $\lim\limits_{x\to 1}\dfrac{1}{x} = 1$

14. $\lim\limits_{x\to 0}\sin x = 0$ (Hint: use the fact that $|\sin x| \leq |x|$, with equality only when $x = 0$.)

Exercises 1.3

Terms and Concepts

1. Explain in your own words, without using ε-δ formality, why $\lim\limits_{x\to c} b = b$.

2. Explain in your own words, without using ε-δ formality, why $\lim\limits_{x\to c} x = c$.

3. What does the text mean when it says that certain functions' "behavior is 'nice' in terms of limits"? What, in particular, is "nice"?

4. Sketch a graph that visually demonstrates the Squeeze Theorem.

5. You are given the following information:

   (a) $\lim\limits_{x\to 1} f(x) = 0$

   (b) $\lim\limits_{x\to 1} g(x) = 0$

   (c) $\lim\limits_{x\to 1} f(x)/g(x) = 2$

   What can be said about the relative sizes of $f(x)$ and $g(x)$ as x approaches 1?

6. T/F: $\lim\limits_{x\to 1} \ln x = 0$. Use a theorem to defend your answer.

Problems

In Exercises 7 – 14, use the following information to evaluate the given limit, when possible. If it is not possible to determine the limit, state why not.

- $\lim\limits_{x\to 9} f(x) = 6$, $\lim\limits_{x\to 6} f(x) = 9$, $f(9) = 6$
- $\lim\limits_{x\to 9} g(x) = 3$, $\lim\limits_{x\to 6} g(x) = 3$, $g(6) = 9$

7. $\lim\limits_{x\to 9} (f(x) + g(x))$

8. $\lim\limits_{x\to 9} (3f(x)/g(x))$

9. $\lim\limits_{x\to 9} \left(\dfrac{f(x) - 2g(x)}{g(x)}\right)$

10. $\lim\limits_{x\to 6} \left(\dfrac{f(x)}{3 - g(x)}\right)$

11. $\lim\limits_{x\to 9} g\big(f(x)\big)$

12. $\lim\limits_{x\to 6} f\big(g(x)\big)$

13. $\lim\limits_{x\to 6} g\big(f(f(x))\big)$

14. $\lim\limits_{x\to 6} f(x)g(x) - f^2(x) + g^2(x)$

In Exercises 15 – 18, use the following information to evaluate the given limit, when possible. If it is not possible to determine the limit, state why not.

- $\lim\limits_{x\to 1} f(x) = 2$, $\lim\limits_{x\to 10} f(x) = 1$, $f(1) = 1/5$
- $\lim\limits_{x\to 1} g(x) = 0$, $\lim\limits_{x\to 10} g(x) = \pi$, $g(10) = \pi$

15. $\lim\limits_{x\to 1} f(x)^{g(x)}$

16. $\lim\limits_{x\to 10} \cos\big(g(x)\big)$

17. $\lim\limits_{x\to 1} f(x)g(x)$

18. $\lim\limits_{x\to 1} g\big(5f(x)\big)$

In Exercises 19 – 34, evaluate the given limit.

19. $\lim\limits_{x\to 3} x^2 - 3x + 7$

20. $\lim\limits_{x\to \pi} \left(\dfrac{x-3}{x-5}\right)^7$

21. $\lim\limits_{x\to \pi/4} \cos x \sin x$

22. $\lim\limits_{x\to 1} \dfrac{2x-2}{x+4}$

23. $\lim\limits_{x\to 0} \ln x$

24. $\lim\limits_{x\to 3} 4^{x^3 - 8x}$

25. $\lim\limits_{x\to \pi/6} \csc x$

26. $\lim\limits_{x\to 0} \ln(1 + x)$

27. $\lim\limits_{x\to \pi} \dfrac{x^2 + 3x + 5}{5x^2 - 2x - 3}$

28. $\lim\limits_{x\to \pi} \dfrac{3x+1}{1-x}$

29. $\lim\limits_{x\to 6} \dfrac{x^2 - 4x - 12}{x^2 - 13x + 42}$

30. $\lim\limits_{x\to 0} \dfrac{x^2 + 2x}{x^2 - 2x}$

31. $\lim\limits_{x\to 2} \dfrac{x^2 + 6x - 16}{x^2 - 3x + 2}$

32. $\lim\limits_{x\to 2} \dfrac{x^2 - 10x + 16}{x^2 - x - 2}$

33. $\lim\limits_{x\to -2} \dfrac{x^2 - 5x - 14}{x^2 + 10x + 16}$

34. $\lim_{x\to -1} \frac{x^2+9x+8}{x^2-6x-7}$

Use the Squeeze Theorem in Exercises 35 – 38, where appropriate, to evaluate the given limit.

35. $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$

36. $\lim_{x\to 0} \sin x \cos\left(\frac{1}{x^2}\right)$

37. $\lim_{x\to 1} f(x)$, where $3x-2 \leq f(x) \leq x^3$.

38. $\lim_{x\to 3} f(x)$, where $6x-9 \leq f(x) \leq x^2$.

Exercises 39 – 43 challenge your understanding of limits but can be evaluated using the knowledge gained in this section.

39. $\lim_{x\to 0} \frac{\sin 3x}{x}$

40. $\lim_{x\to 0} \frac{\sin 5x}{8x}$

41. $\lim_{x\to 0} \frac{\ln(1+x)}{x}$

42. $\lim_{x\to 0} \frac{\sin x}{x}$, where x is measured in degrees, not radians.

43. Let $f(x) = 0$ and $g(x) = \frac{x}{x}$.

    (a) Show why $\lim_{x\to 2} f(x) = 0$.

    (b) Show why $\lim_{x\to 0} g(x) = 1$.

    (c) Show why $\lim_{x\to 2} g\big(f(x)\big)$ does not exist.

    (d) Show why the answer to part (c) does not violate the Composition Rule of Theorem 1.3.1.

Exercises 1.4

Terms and Concepts

1. What are the three ways in which a limit may fail to exist?

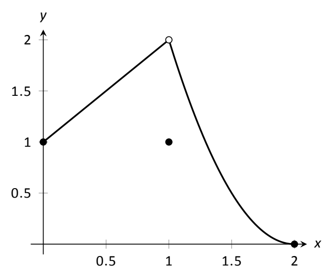
2. T/F: If $\lim\limits_{x\to 1^-} f(x) = 5$, then $\lim\limits_{x\to 1} f(x) = 5$

3. T/F: If $\lim\limits_{x\to 1^-} f(x) = 5$, then $\lim\limits_{x\to 1^+} f(x) = 5$

4. T/F: If $\lim\limits_{x\to 1} f(x) = 5$, then $\lim\limits_{x\to 1^-} f(x) = 5$

Problems

In Exercises 5 – 12, evaluate each expression using the given graph of $f(x)$.

5. 

(a) $\lim\limits_{x\to 1^-} f(x)$

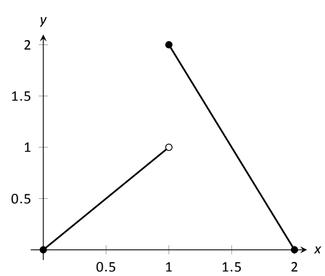
(b) $\lim\limits_{x\to 1^+} f(x)$

(c) $\lim\limits_{x\to 1} f(x)$

(d) $f(1)$

(e) $\lim\limits_{x\to 0^-} f(x)$

(f) $\lim\limits_{x\to 0^+} f(x)$

6. 

(a) $\lim\limits_{x\to 1^-} f(x)$

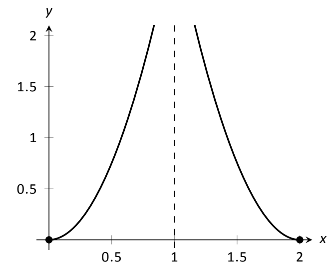
(b) $\lim\limits_{x\to 1^+} f(x)$

(c) $\lim\limits_{x\to 1} f(x)$

(d) $f(1)$

(e) $\lim\limits_{x\to 2^-} f(x)$

(f) $\lim\limits_{x\to 2^+} f(x)$

7. 

(a) $\lim\limits_{x\to 1^-} f(x)$

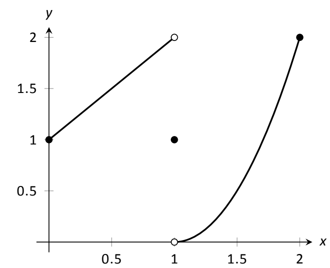
(b) $\lim\limits_{x\to 1^+} f(x)$

(c) $\lim\limits_{x\to 1} f(x)$

(d) $f(1)$

(e) $\lim\limits_{x\to 2^-} f(x)$

(f) $\lim\limits_{x\to 0^+} f(x)$

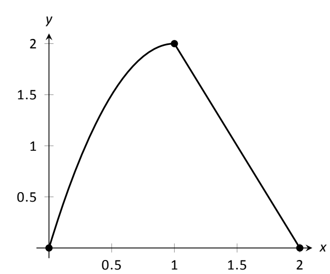
8. 

(a) $\lim\limits_{x\to 1^-} f(x)$

(b) $\lim\limits_{x\to 1^+} f(x)$

(c) $\lim\limits_{x\to 1} f(x)$

(d) $f(1)$

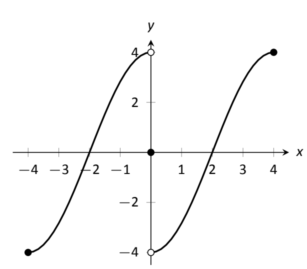
9. 

(a) $\lim\limits_{x\to 1^-} f(x)$

(b) $\lim\limits_{x\to 1^+} f(x)$

(c) $\lim\limits_{x\to 1} f(x)$

(d) $f(1)$

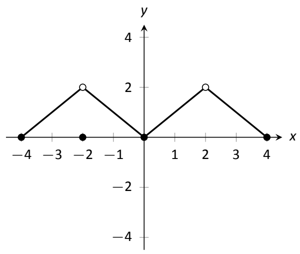
10. 

(a) $\lim\limits_{x\to 0^-} f(x)$

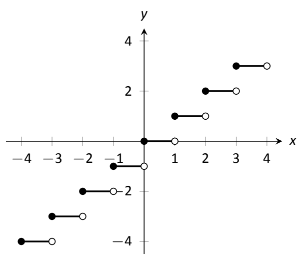
(b) $\lim\limits_{x\to 0^+} f(x)$

(c) $\lim\limits_{x\to 0} f(x)$

(d) $f(0)$

11. 

(a) $\lim\limits_{x\to -2^-} f(x)$
(b) $\lim\limits_{x\to -2^+} f(x)$
(c) $\lim\limits_{x\to -2} f(x)$
(d) $f(-2)$
(e) $\lim\limits_{x\to 2^-} f(x)$
(f) $\lim\limits_{x\to 2^+} f(x)$
(g) $\lim\limits_{x\to 2} f(x)$
(h) $f(2)$

12. 

Let $-3 \leq a \leq 3$ be an integer.

(a) $\lim\limits_{x\to a^-} f(x)$
(b) $\lim\limits_{x\to a^+} f(x)$
(c) $\lim\limits_{x\to a} f(x)$
(d) $f(a)$

In Exercises 13 – 21, evaluate the given limits of the piecewise defined functions f.

13. $f(x) = \begin{cases} x+1 & x \leq 1 \\ x^2-5 & x > 1 \end{cases}$

(a) $\lim\limits_{x\to 1^-} f(x)$
(b) $\lim\limits_{x\to 1^+} f(x)$
(c) $\lim\limits_{x\to 1} f(x)$
(d) $f(1)$

14. $f(x) = \begin{cases} 2x^2+5x-1 & x < 0 \\ \sin x & x \geq 0 \end{cases}$

(a) $\lim\limits_{x\to 0^-} f(x)$
(b) $\lim\limits_{x\to 0^+} f(x)$
(c) $\lim\limits_{x\to 0} f(x)$
(d) $f(0)$

15. $f(x) = \begin{cases} x^2-1 & x < -1 \\ x^3+1 & -1 \leq x \leq 1 \\ x^2+1 & x > 1 \end{cases}$

(a) $\lim\limits_{x\to -1^-} f(x)$
(b) $\lim\limits_{x\to -1^+} f(x)$
(c) $\lim\limits_{x\to -1} f(x)$
(d) $f(-1)$
(e) $\lim\limits_{x\to 1^-} f(x)$
(f) $\lim\limits_{x\to 1^+} f(x)$
(g) $\lim\limits_{x\to 1} f(x)$
(h) $f(1)$

16. $f(x) = \begin{cases} \cos x & x < \pi \\ \sin x & x \geq \pi \end{cases}$

(a) $\lim\limits_{x\to \pi^-} f(x)$
(b) $\lim\limits_{x\to \pi^+} f(x)$
(c) $\lim\limits_{x\to \pi} f(x)$
(d) $f(\pi)$

17. $f(x) = \begin{cases} 1-\cos^2 x & x < a \\ \sin^2 x & x \geq a \end{cases},$
where a is a real number.

(a) $\lim\limits_{x\to a^-} f(x)$
(b) $\lim\limits_{x\to a^+} f(x)$
(c) $\lim\limits_{x\to a} f(x)$
(d) $f(a)$

18. $f(x) = \begin{cases} x+1 & x < 1 \\ 1 & x = 1 \\ x-1 & x > 1 \end{cases}$

(a) $\lim\limits_{x\to 1^-} f(x)$
(b) $\lim\limits_{x\to 1^+} f(x)$
(c) $\lim\limits_{x\to 1} f(x)$
(d) $f(1)$

19. $f(x) = \begin{cases} x^2 & x < 2 \\ x+1 & x = 2 \\ -x^2+2x+4 & x > 2 \end{cases}$

(a) $\lim\limits_{x\to 2^-} f(x)$
(b) $\lim\limits_{x\to 2^+} f(x)$
(c) $\lim\limits_{x\to 2} f(x)$
(d) $f(2)$

20. $f(x) = \begin{cases} a(x-b)^2+c & x < b \\ a(x-b)+c & x \geq b \end{cases},$
where a, b and c are real numbers.

(a) $\lim\limits_{x\to b^-} f(x)$
(b) $\lim\limits_{x\to b^+} f(x)$
(c) $\lim\limits_{x\to b} f(x)$
(d) $f(b)$

21. $f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

(a) $\lim\limits_{x\to 0^-} f(x)$
(b) $\lim\limits_{x\to 0^+} f(x)$
(c) $\lim\limits_{x\to 0} f(x)$
(d) $f(0)$

Review

22. Evaluate the limit: $\lim\limits_{x\to -1} \dfrac{x^2+5x+4}{x^2-3x-4}$.

23. Evaluate the limit: $\lim\limits_{x\to -4} \dfrac{x^2-16}{x^2-4x-32}$.

24. Evaluate the limit: $\lim\limits_{x\to -6} \dfrac{x^2-15x+54}{x^2-6x}$.

25. Approximate the limit numerically: $\lim\limits_{x\to 0.4} \dfrac{x^2-4.4x+1.6}{x^2-0.4x}$.

26. Approximate the limit numerically: $\lim\limits_{x\to 0.2} \dfrac{x^2+5.8x-1.2}{x^2-4.2x+0.8}$.

Exercises 1.5

Terms and Concepts

1. In your own words, describe what it means for a function to be continuous.

2. In your own words, describe what the Intermediate Value Theorem states.

3. What is a "root" of a function?

4. Given functions f and g on an interval I, how can the Bisection Method be used to find a value c where $f(c) = g(c)$?

5. T/F: If f is defined on an open interval containing c, and $\lim\limits_{x\to c} f(x)$ exists, then f is continuous at c.

6. T/F: If f is continuous at c, then $\lim\limits_{x\to c} f(x)$ exists.

7. T/F: If f is continuous at c, then $\lim\limits_{x\to c^+} f(x) = f(c)$.

8. T/F: If f is continuous on $[a,b]$, then $\lim\limits_{x\to a^-} f(x) = f(a)$.

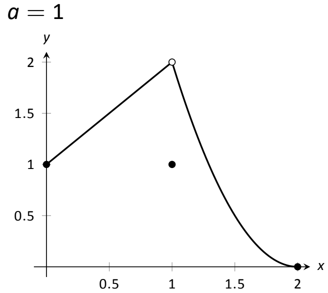
9. T/F: If f is continuous on $[0,1)$ and $[1,2)$, then f is continuous on $[0,2)$.

10. T/F: The sum of continuous functions is also continuous.

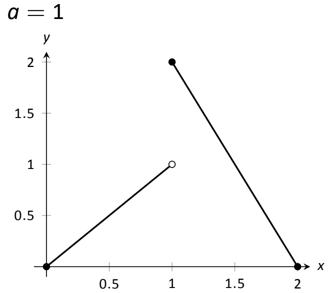
Problems

In Exercises 11 – 18, a graph of a function f is given along with a value a. Determine if f is continuous at a; if it is not, state why it is not.

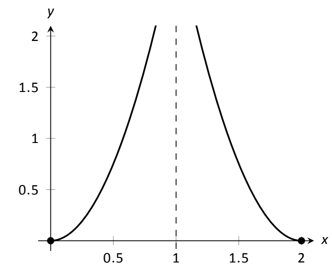
11. $a = 1$



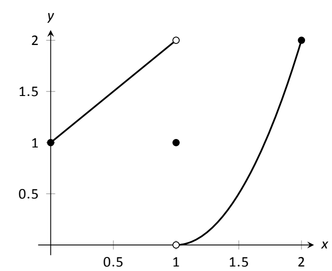
12. $a = 1$



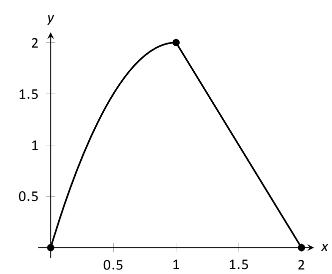
13. $a = 1$



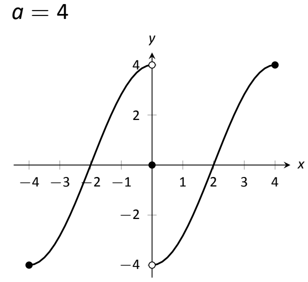
14. $a = 0$



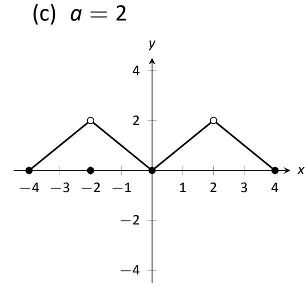
15. $a = 1$



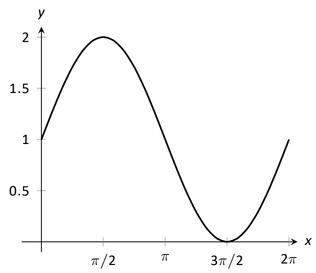
16. $a = 4$



17. (a) $a = -2$
    (b) $a = 0$
    (c) $a = 2$



18. $a = 3\pi/2$



In Exercises 19 – 22, determine if f is continuous at the indicated values. If not, explain why.

19. $f(x) = \begin{cases} 1 & x = 0 \\ \frac{\sin x}{x} & x > 0 \end{cases}$
    (a) $x = 0$
    (b) $x = \pi$

20. $f(x) = \begin{cases} x^3 - x & x < 1 \\ x - 2 & x \geq 1 \end{cases}$
    (a) $x = 0$
    (b) $x = 1$

21. $f(x) = \begin{cases} \frac{x^2+5x+4}{x^2+3x+2} & x \neq -1 \\ 3 & x = -1 \end{cases}$
    (a) $x = -1$
    (b) $x = 10$

22. $f(x) = \begin{cases} \frac{x^2-64}{x^2-11x+24} & x \neq 8 \\ 5 & x = 8 \end{cases}$
    (a) $x = 0$
    (b) $x = 8$

In Exercises 23 – 34, give the intervals on which the given function is continuous.

23. $f(x) = x^2 - 3x + 9$

24. $g(x) = \sqrt{x^2 - 4}$

25. $g(x) = \sqrt{4 - x^2}$

26. $h(k) = \sqrt{1 - k} + \sqrt{k + 1}$

27. $f(t) = \sqrt{5t^2 - 30}$

28. $g(t) = \dfrac{1}{\sqrt{1 - t^2}}$

29. $g(x) = \dfrac{1}{1 + x^2}$

30. $f(x) = e^x$

31. $g(s) = \ln s$

32. $h(t) = \cos t$

33. $f(k) = \sqrt{1 - e^k}$

34. $f(x) = \sin(e^x + x^2)$

Exercises 35 – 38 test your understanding of the Intermediate Value Theorem.

35. Let f be continuous on $[1, 5]$ where $f(1) = -2$ and $f(5) = -10$. Does a value $1 < c < 5$ exist such that $f(c) = -9$? Why/why not?

36. Let g be continuous on $[-3, 7]$ where $g(0) = 0$ and $g(2) = 25$. Does a value $-3 < c < 7$ exist such that $g(c) = 15$? Why/why not?

37. Let f be continuous on $[-1, 1]$ where $f(-1) = -10$ and $f(1) = 10$. Does a value $-1 < c < 1$ exist such that $f(c) = 11$? Why/why not?

38. Let h be a function on $[-1, 1]$ where $h(-1) = -10$ and $h(1) = 10$. Does a value $-1 < c < 1$ exist such that $h(c) = 0$? Why/why not?

In Exercises 39 – 42, use the Bisection Method to approximate, accurate to two decimal places, the value of the root of the given function in the given interval.

39. $f(x) = x^2 + 2x - 4$ on $[1, 1.5]$.

40. $f(x) = \sin x - 1/2$ on $[0.5, 0.55]$

41. $f(x) = e^x - 2$ on $[0.65, 0.7]$.

42. $f(x) = \cos x - \sin x$ on $[0.7, 0.8]$.

Review

43. Let $f(x) = \begin{cases} x^2 - 5 & x < 5 \\ 5x & x \geq 5 \end{cases}$.
    (a) $\lim\limits_{x\to 5^-} f(x)$
    (b) $\lim\limits_{x\to 5^+} f(x)$
    (c) $\lim\limits_{x\to 5} f(x)$
    (d) $f(5)$

44. Numerically approximate the following limits:
    (a) $\lim\limits_{x\to -4/5^+} \dfrac{x^2 - 8.2x - 7.2}{x^2 + 5.8x + 4}$
    (b) $\lim\limits_{x\to -4/5^-} \dfrac{x^2 - 8.2x - 7.2}{x^2 + 5.8x + 4}$

45. Give an example of function $f(x)$ for which $\lim\limits_{x\to 0} f(x)$ does not exist.

Exercises 1.6

Terms and Concepts

1. T/F: If $\lim\limits_{x\to 5} f(x) = \infty$, then we are implicitly stating that the limit exists.

2. T/F: If $\lim\limits_{x\to \infty} f(x) = 5$, then we are implicitly stating that the limit exists.

3. T/F: If $\lim\limits_{x\to 1^-} f(x) = -\infty$, then $\lim\limits_{x\to 1^+} f(x) = \infty$

4. T/F: If $\lim\limits_{x\to 5} f(x) = \infty$, then f has a vertical asymptote at $x = 5$.

5. T/F: $\infty/0$ is not an indeterminate form.

6. List 5 indeterminate forms.

7. Construct a function with a vertical asymptote at $x = 5$ and a horizontal asymptote at $y = 5$.

8. Let $\lim\limits_{x\to 7} f(x) = \infty$. Explain how we know that f is/is not continuous at $x = 7$.

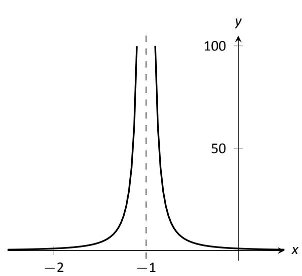
Problems

In Exercises 9 – 14, evaluate the given limits using the graph of the function.

9. $f(x) = \dfrac{1}{(x+1)^2}$

   (a) $\lim\limits_{x\to -1^-} f(x)$

   (b) $\lim\limits_{x\to -1^+} f(x)$



10. $f(x) = \dfrac{1}{(x-3)(x-5)^2}$.

    (a) $\lim\limits_{x\to 3^-} f(x)$

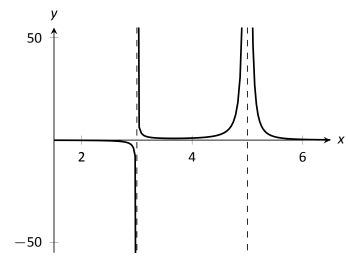
    (b) $\lim\limits_{x\to 3^+} f(x)$

    (c) $\lim\limits_{x\to 3} f(x)$

    (d) $\lim\limits_{x\to 5^-} f(x)$

    (e) $\lim\limits_{x\to 5^+} f(x)$

    (f) $\lim\limits_{x\to 5} f(x)$



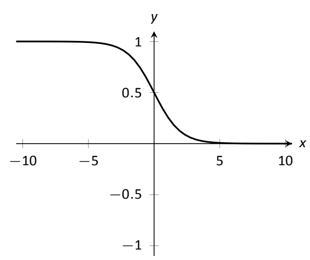
11. $f(x) = \dfrac{1}{e^x + 1}$

    (a) $\lim\limits_{x\to -\infty} f(x)$

    (b) $\lim\limits_{x\to \infty} f(x)$

    (c) $\lim\limits_{x\to 0^-} f(x)$

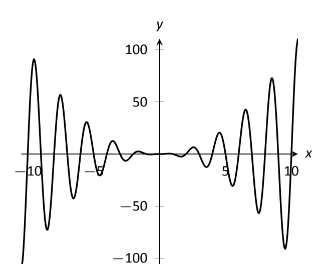
    (d) $\lim\limits_{x\to 0^+} f(x)$



12. $f(x) = x^2 \sin(\pi x)$

    (a) $\lim\limits_{x\to -\infty} f(x)$

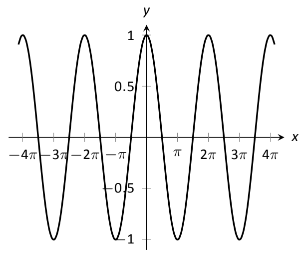
    (b) $\lim\limits_{x\to \infty} f(x)$



13. $f(x) = \cos(x)$

(a) $\lim\limits_{x\to-\infty} f(x)$

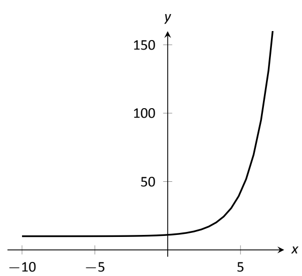
(b) $\lim\limits_{x\to\infty} f(x)$



14. $f(x) = 2^x + 10$

(a) $\lim\limits_{x\to-\infty} f(x)$

(b) $\lim\limits_{x\to\infty} f(x)$



In Exercises 15 – 18, numerically approximate the following limits:

(a) $\lim\limits_{x\to3^-} f(x)$

(b) $\lim\limits_{x\to3^+} f(x)$

(c) $\lim\limits_{x\to3} f(x)$

15. $f(x) = \dfrac{x^2-1}{x^2-x-6}$

16. $f(x) = \dfrac{x^2+5x-36}{x^3-5x^2+3x+9}$

17. $f(x) = \dfrac{x^2-11x+30}{x^3-4x^2-3x+18}$

18. $f(x) = \dfrac{x^2-9x+18}{x^2-x-6}$

In Exercises 19 – 24, identify the horizontal and vertical asymptotes, if any, of the given function.

19. $f(x) = \dfrac{2x^2-2x-4}{x^2+x-20}$

20. $f(x) = \dfrac{-3x^2-9x-6}{5x^2-10x-15}$

21. $f(x) = \dfrac{x^2+x-12}{7x^3-14x^2-21x}$

22. $f(x) = \dfrac{x^2-9}{9x-9}$

23. $f(x) = \dfrac{x^2-9}{9x+27}$

24. $f(x) = \dfrac{x^2-1}{-x^2-1}$

In Exercises 25 – 28, evaluate the given limit.

25. $\lim\limits_{x\to\infty} \dfrac{x^3+2x^2+1}{x-5}$

26. $\lim\limits_{x\to\infty} \dfrac{x^3+2x^2+1}{5-x}$

27. $\lim\limits_{x\to-\infty} \dfrac{x^3+2x^2+1}{x^2-5}$

28. $\lim\limits_{x\to-\infty} \dfrac{x^3+2x^2+1}{5-x^2}$

Review

29. Use an $\varepsilon - \delta$ proof to show that
$\lim\limits_{x\to1} 5x-2 = 3$.

30. Let $\lim\limits_{x\to2} f(x) = 3$ and $\lim\limits_{x\to2} g(x) = -1$. Evaluate the following limits.

(a) $\lim\limits_{x\to2} (f+g)(x)$

(b) $\lim\limits_{x\to2} (fg)(x)$

(c) $\lim\limits_{x\to2} (f/g)(x)$

(d) $\lim\limits_{x\to2} f(x)^{g(x)}$

31. Let $f(x) = \begin{cases} x^2-1 & x<3 \\ x+5 & x\geq 3 \end{cases}$.
Is f continuous everywhere?

32. Evaluate the limit: $\lim\limits_{x\to e} \ln x$.

Exercises 2.1

Terms and Concepts

1. T/F: Let f be a position function. The average rate of change on $[a,b]$ is the slope of the line through the points $(a,f(a))$ and $(b,f(b))$.

2. T/F: The definition of the derivative of a function at a point involves taking a limit.

3. In your own words, explain the difference between the average rate of change and instantaneous rate of change.

4. In your own words, explain the difference between Definitions 2.1.1 and 2.1.4.

5. Let $y = f(x)$. Give three different notations equivalent to "$f'(x)$."

6. If two lines are perpendicular, what is true of their slopes?

Problems

In Exercises 7 – 14, use the definition of the derivative to compute the derivative of the given function.

7. $f(x) = 6$

8. $f(x) = 2x$

9. $f(t) = 4 - 3t$

10. $g(x) = x^2$

11. $h(x) = x^3$

12. $f(x) = 3x^2 - x + 4$

13. $r(x) = \dfrac{1}{x}$

14. $r(s) = \dfrac{1}{s-2}$

In Exercises 15 – 22, a function and an x–value c are given. (Note: these functions are the same as those given in Exercises 7 through 14.)

(a) Give the equation of the tangent line at $x = c$.

(b) Give the equation of the normal line at $x = c$.

15. $f(x) = 6$, at $x = -2$.

16. $f(x) = 2x$, at $x = 3$.

17. $f(x) = 4 - 3x$, at $x = 7$.

18. $g(x) = x^2$, at $x = 2$.

19. $h(x) = x^3$, at $x = 4$.

20. $f(x) = 3x^2 - x + 4$, at $x = -1$.

21. $r(x) = \dfrac{1}{x}$, at $x = -2$.

22. $r(x) = \dfrac{1}{x-2}$, at $x = 3$.

In Exercises 23 – 26, a function f and an x–value a are given. Approximate the equation of the tangent line to the graph of f at $x = a$ by numerically approximating $f'(a)$, using $h = 0.1$.

23. $f(x) = x^2 + 2x + 1$, $x = 3$

24. $f(x) = \dfrac{10}{x+1}$, $x = 9$

25. $f(x) = e^x$, $x = 2$

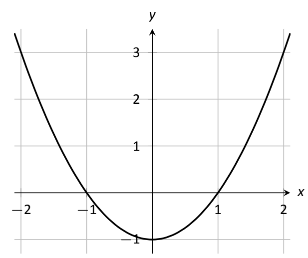
26. $f(x) = \cos x$, $x = 0$

27. The graph of $f(x) = x^2 - 1$ is shown.

    (a) Use the graph to approximate the slope of the tangent line to f at the following points: $(-1,0)$, $(0,-1)$ and $(2,3)$.

    (b) Using the definition, find $f'(x)$.

    (c) Find the slope of the tangent line at the points $(-1,0)$, $(0,-1)$ and $(2,3)$.

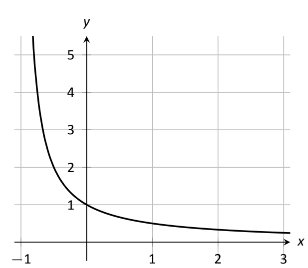


28. The graph of $f(x) = \dfrac{1}{x+1}$ is shown.

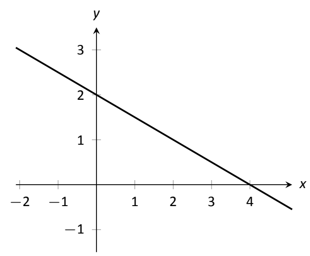
    (a) Use the graph to approximate the slope of the tangent line to f at the following points: $(0,1)$ and $(1,0.5)$.

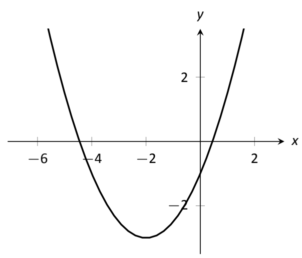
    (b) Using the definition, find $f'(x)$.

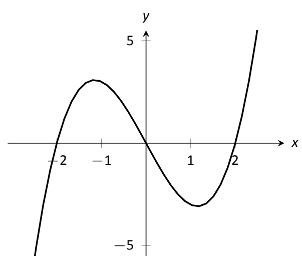
    (c) Find the slope of the tangent line at the points $(0,1)$ and $(1,0.5)$.

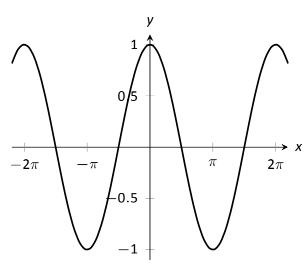


In Exercises 29 – 32, a graph of a function $f(x)$ is given. Using the graph, sketch $f'(x)$.

29. 

30. 

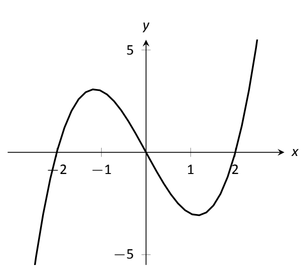
31. 

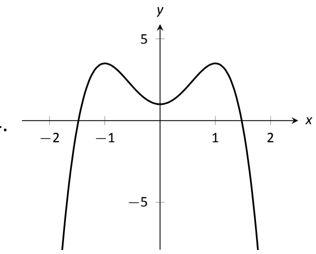
32. 

In Exercises 33 – 34, a graph of a function $g(x)$ is given. Using the graph, answer the following questions.

1. **Where is $g(x) > 0$?**
2. **Where is $g(x) < 0$?**
3. **Where is $g(x) = 0$?**

1. **Where is $g'(x) < 0$?**
2. **Where is $g'(x) > 0$?**
3. **Where is $g'(x) = 0$?**

33. 

34. 

In Exercises 35 – 36, a function $f(x)$ is given, along with its domain and derivative. Determine if $f(x)$ is differentiable on its domain.

35. $f(x) = \sqrt{x^5(1-x)}$, domain = $[0,1]$, $f'(x) = \dfrac{(5-6x)x^{3/2}}{2\sqrt{1-x}}$

36. $f(x) = \cos\left(\sqrt{x}\right)$, domain = $[0,\infty)$, $f'(x) = -\dfrac{\sin\left(\sqrt{x}\right)}{2\sqrt{x}}$

Review

37. Approximate $\displaystyle\lim_{x\to 5} \frac{x^2+2x-35}{x^2-10.5x+27.5}$.

38. Use the Bisection Method to approximate, accurate to two decimal places, the root of $g(x) = x^3+x^2+x-1$ on $[0.5, 0.6]$.

39. Give intervals on which each of the following functions are continuous.

(a) $\dfrac{1}{e^x+1}$

(b) $\dfrac{1}{x^2-1}$

(c) $\sqrt{5-x}$

(d) $\sqrt{5-x^2}$

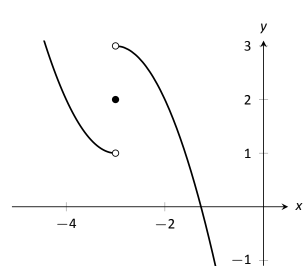
40. Use the graph of $f(x)$ provided to answer the following.

(a) $\displaystyle\lim_{x\to -3^-} f(x) = ?$

(b) $\displaystyle\lim_{x\to -3^+} f(x) = ?$

(c) $\displaystyle\lim_{x\to -3} f(x) = ?$

(d) Where is f continuous?



Exercises 2.2

Terms and Concepts

1. What is the instantaneous rate of change of position called?

2. Given a function $y = f(x)$, in your own words describe how to find the units of $f'(x)$.

3. What functions have a constant rate of change?

Problems

4. Given $f(5) = 10$ and $f'(5) = 2$, approximate $f(6)$.

5. Given $P(100) = -67$ and $P'(100) = 5$, approximate $P(110)$.

6. Given $z(25) = 187$ and $z'(25) = 17$, approximate $z(20)$.

7. Knowing $f(10) = 25$ and $f'(10) = 5$ and the methods described in this section, which approximation is likely to be most accurate: $f(10.1)$, $f(11)$, or $f(20)$? Explain your reasoning.

8. Given $f(7) = 26$ and $f(8) = 22$, approximate $f'(7)$.

9. Given $H(0) = 17$ and $H(2) = 29$, approximate $H'(2)$.

10. Let $V(x)$ measure the volume, in decibels, measured inside a restaurant with x customers. What are the units of $V'(x)$?

11. Let $v(t)$ measure the velocity, in ft/s, of a car moving in a straight line t seconds after starting. What are the units of $v'(t)$?

12. The height H, in feet, of a river is recorded t hours after midnight, April 1. What are the units of $H'(t)$?

13. P is the profit, in thousands of dollars, of producing and selling c cars.

    (a) What are the units of $P'(c)$?

    (b) What is likely true of $P(0)$?

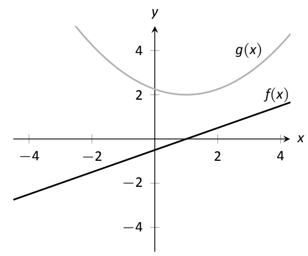
14. T is the temperature in degrees Fahrenheit, h hours after midnight on July 4 in Sidney, NE.

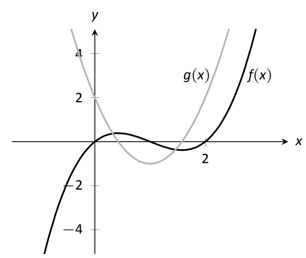
    (a) What are the units of $T'(h)$?

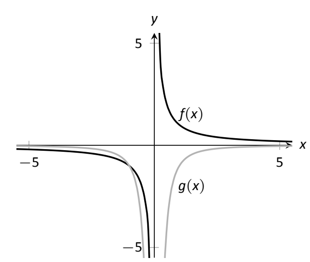
    (b) Is $T'(8)$ likely greater than or less than 0? Why?

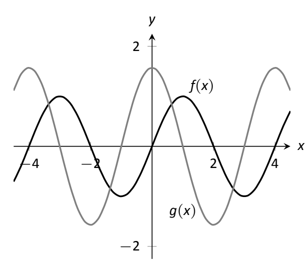
    (c) Is $T(8)$ likely greater than or less than 0? Why?

In Exercises 15 – 18, graphs of functions $f(x)$ and $g(x)$ are given. Identify which function is the derivative of the other.

15. 

16. 

17. 

18. 

Review

In Exercises 19 – 20, use the definition to compute the derivatives of the following functions.

19. $f(x) = 5x^2$

20. $f(x) = (x-2)^3$

In Exercises 21 – 22, numerically approximate the value of $f'(x)$ at the indicated x value.

21. $f(x) = \cos x$ at $x = \pi$.

22. $f(x) = \sqrt{x}$ at $x = 9$.

Exercises 2.3

Terms and Concepts

1. What is the name of the rule which states that $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$, where $n > 0$ is an integer?

2. What is $\frac{d}{dx}\left(\ln x\right)$?

3. Give an example of a function $f(x)$ where $f'(x) = f(x)$.

4. Give an example of a function $f(x)$ where $f'(x) = 0$.

5. The derivative rules introduced in this section explain how to compute the derivative of which of the following functions?
   - $f(x) = \frac{3}{x^2}$
   - $g(x) = 3x^2 - x + 17$
   - $h(x) = 5\ln x$
   - $j(x) = \sin x \cos x$
   - $k(x) = e^{x^2}$
   - $m(x) = \sqrt{x}$

6. Explain in your own words how to find the third derivative of a function $f(x)$.

7. Give an example of a function where $f'(x) \neq 0$ and $f''(x) = 0$.

8. Explain in your own words what the second derivative "means."

9. If $f(x)$ describes a position function, then $f'(x)$ describes what kind of function? What kind of function is $f''(x)$?

10. Let $f(x)$ be a function measured in pounds, where x is measured in feet. What are the units of $f''(x)$?

Problems

In Exercises 11 – 26, compute the derivative of the given function.

11. $f(x) = 7x^2 - 5x + 7$

12. $g(x) = 14x^3 + 7x^2 + 11x - 29$

13. $m(t) = 9t^5 - \frac{1}{8}t^3 + 3t - 8$

14. $f(\theta) = 9\sin\theta + 10\cos\theta$

15. $f(r) = 6e^r$

16. $g(t) = 10t^4 - \cos t + 7\sin t$

17. $f(x) = 2\ln x - x$

18. $p(s) = \frac{1}{4}s^4 + \frac{1}{3}s^3 + \frac{1}{2}s^2 + s + 1$

19. $h(t) = e^t - \sin t - \cos t$

20. $f(x) = \ln(5x^2)$

21. $f(t) = \ln(17) + e^2 + \sin\pi/2$

22. $g(t) = (1 + 3t)^2$

23. $g(x) = (2x - 5)^3$

24. $f(x) = (1 - x)^3$

25. $f(x) = (2 - 3x)^2$

26. A property of logarithms is that $\log_a x = \frac{\log_b x}{\log_b a}$, for all bases $a, b > 0, \neq 1$.
    (a) Rewrite this identity when $b = e$, i.e., using $\log_e x = \ln x$, with $a = 10$.
    (b) Use part (a) to find the derivative of $y = \log_{10} x$.
    (c) Use part (b) to find the derivative of $y = \log_a x$, for any $a > 0, \neq 1$.

In Exercises 27 – 32, compute the first four derivatives of the given function.

27. $f(x) = x^6$

28. $g(x) = 2\cos x$

29. $h(t) = t^2 - e^t$

30. $p(\theta) = \theta^4 - \theta^3$

31. $f(\theta) = \sin\theta - \cos\theta$

32. $f(x) = 1,100$

In Exercises 33 – 38, find the equations of the tangent and normal lines to the graph of the function at the given point.

33. $f(x) = x^3 - x$ at $x = 1$

34. $f(t) = e^t + 3$ at $t = 0$

35. $g(x) = \ln x$ at $x = 1$

36. $f(x) = 4\sin x$ at $x = \pi/2$

37. $f(x) = -2\cos x$ at $x = \pi/4$

38. $f(x) = 2x + 3$ at $x = 5$

Review

39. Given that $e^0 = 1$, approximate the value of $e^{0.1}$ using the tangent line to $f(x) = e^x$ at $x = 0$.

Exercises 2.4

Terms and Concepts

1. T/F: The Product Rule states that $\frac{d}{dx}\big(x^2\sin x\big) = 2x\cos x$.

2. T/F: The Quotient Rule states that $\frac{d}{dx}\left(\frac{x^2}{\sin x}\right) = \frac{\cos x}{2x}$.

3. T/F: The derivatives of the trigonometric functions that start with “c” have minus signs in them.

4. What derivative rule is used to extend the Power Rule to include negative integer exponents?

5. T/F: Regardless of the function, there is always exactly one right way of computing its derivative.

6. In your own words, explain what it means to make your answers “clear.”

Problems

In Exercises 7 – 10:

(a) Use the Product Rule to differentiate the function.

(b) Manipulate the function algebraically and differentiate without the Product Rule.

(c) Show that the answers from (a) and (b) are equivalent.

7. $f(x) = x(x^2+3x)$

8. $g(x) = 2x^2(5x^3)$

9. $h(s) = (2s-1)(s+4)$

10. $f(x) = (x^2+5)(3-x^3)$

In Exercises 11 – 14:

(a) Use the Quotient Rule to differentiate the function.

(b) Manipulate the function algebraically and differentiate without the Quotient Rule.

(c) Show that the answers from (a) and (b) are equivalent.

11. $f(x) = \frac{x^2+3}{x}$

12. $g(x) = \frac{x^3-2x^2}{2x^2}$

13. $h(s) = \frac{3}{4s^3}$

14. $f(t) = \frac{t^2-1}{t+1}$

In Exercises 15 – 36, compute the derivative of the given function.

15. $f(x) = x\sin x$

16. $f(x) = x^2\cos x$

17. $f(x) = e^x\ln x$

18. $f(t) = \frac{1}{t^2}(\csc t - 4)$

19. $g(x) = \frac{x+7}{x-5}$

20. $g(t) = \frac{t^5}{\cos t - 2t^2}$

21. $h(x) = \cot x - e^x$

22. $f(x) = \big(\tan x\big)\ln x$

23. $h(t) = 7t^2 + 6t - 2$

24. $f(x) = \frac{x^4+2x^3}{x+2}$

25. $f(x) = \big(3x^2+8x+7\big)e^x$

26. $g(t) = \frac{t^5-t^3}{e^t}$

27. $f(x) = (16x^3+24x^2+3x)\frac{7x-1}{16x^3+24x^2+3x}$

28. $f(t) = t^5(\sec t + e^t)$

29. $f(x) = \frac{\sin x}{\cos x + 3}$

30. $f(\theta) = \theta^3\sin\theta + \frac{\sin\theta}{\theta^3}$

31. $f(x) = \frac{\cos x}{x} + \frac{x}{\tan x}$

32. $g(x) = e^2\big(\sin(\pi/4) - 1\big)$

33. $g(t) = 4t^3e^t - \sin t\cos t$

34. $h(t) = \frac{t^2\sin t + 3}{t^2\cos t + 2}$

35. $f(x) = x^2e^x\tan x$

36. $g(x) = 2x\sin x\sec x$

In Exercises 37 – 40, find the equations of the tangent and normal lines to the graph of g at the indicated point.

37. $g(s) = e^s(s^2 + 2)$ at $(0, 2)$.

38. $g(t) = t \sin t$ at $\left(\frac{3\pi}{2}, -\frac{3\pi}{2}\right)$

39. $g(x) = \dfrac{x^2}{x - 1}$ at $(2, 4)$

40. $g(\theta) = \dfrac{\cos\theta - 8\theta}{\theta + 1}$ at $(0, 1)$

In Exercises 41 – 44, find the x–values where the graph of the function has a horizontal tangent line.

41. $f(x) = 6x^2 - 18x - 24$

42. $f(x) = x \sin x$ on $[-1, 1]$

43. $f(x) = \dfrac{x}{x + 1}$

44. $f(x) = \dfrac{x^2}{x + 1}$

In Exercises 45 – 48, find the requested derivative.

45. $f(x) = x \sin x$; find $f''(x)$.

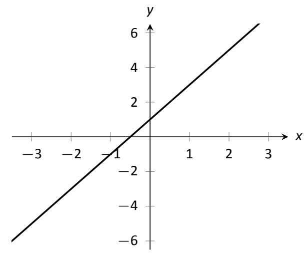
46. $f(x) = x \sin x$; find $f^{(4)}(x)$.

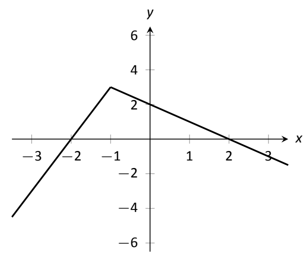
47. $f(x) = \csc x$; find $f''(x)$.

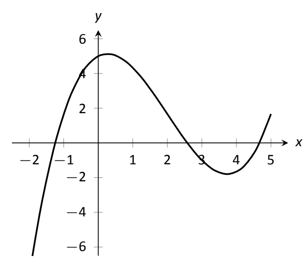
48. $f(x) = (x^3 - 5x + 2)(x^2 + x - 7)$; find $f^{(8)}(x)$.

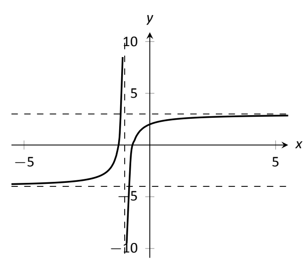
Review

In Exercises 49 – 52, use the graph of $f(x)$ to sketch $f'(x)$.

49. 

50. 

51. 

52. 

Exercises 2.5

Terms and Concepts

1. T/F: The Chain Rule describes how to evaluate the derivative of a composition of functions.

2. T/F: The Generalized Power Rule states that $\frac{d}{dx}\Big(g(x)^n\Big) = n\big(g(x)\big)^{n-1}$.

3. T/F: $\frac{d}{dx}\big(\ln(x^2)\big) = \frac{1}{x^2}$.

4. T/F: $\frac{d}{dx}\big(3^x\big) \approx 1.1 \cdot 3^x$.

5. T/F: $\frac{dx}{dy} = \frac{dx}{dt} \cdot \frac{dt}{dy}$

6. $f(x) = \big(\ln x + x^2\big)^3$

Problems

In Exercises 7 – 36, compute the derivative of the given function.

7. $f(x) = (4x^3 - x)^{10}$

8. $f(t) = (3t - 2)^5$

9. $g(\theta) = (\sin\theta + \cos\theta)^3$

10. $h(t) = e^{3t^2+t-1}$

11. $f(x) = \big(\ln x + x^2\big)^3$

12. $f(x) = 2^{x^3+3x}$

13. $f(x) = \big(x + \frac{1}{x}\big)^4$

14. $f(x) = \cos(3x)$

15. $g(x) = \tan(5x)$

16. $h(\theta) = \tan\big(\theta^2 + 4\theta\big)$

17. $g(t) = \sin\big(t^5 + \frac{1}{t}\big)$

18. $h(t) = \sin^4(2t)$

19. $p(t) = \cos^3(t^2 + 3t + 1)$

20. $f(x) = \ln(\cos x)$

21. $f(x) = \ln(x^2)$

22. $f(x) = 2\ln(x)$

23. $g(r) = 4^r$

24. $g(t) = 5^{\cos t}$

25. $g(t) = 15^2$

26. $m(w) = \frac{3^w}{2^w}$

27. $h(t) = \frac{2^t + 3}{3^t + 2}$

28. $m(w) = \frac{3^w + 1}{2^w}$

29. $f(x) = \frac{3^{x^2} + x}{2^{x^2}}$

30. $f(x) = x^2\sin(5x)$

31. $f(x) = (x^2 + x)^5(3x^4 + 2x)^3$

32. $g(t) = \cos(t^2 + 3t)\sin(5t - 7)$

33. $f(x) = \sin(3x + 4)\cos(5 - 2x)$

34. $g(t) = \cos(\frac{1}{t})e^{5t^2}$

35. $f(x) = \frac{\sin\big(4x + 1\big)}{(5x - 9)^3}$

36. $f(x) = \frac{(4x + 1)^2}{\tan(5x)}$

In Exercises 37 – 40, find the equations of tangent and normal lines to the graph of the function at the given point. Note: the functions here are the same as in Exercises 7 through 10.

37. $f(x) = (4x^3 - x)^{10}$ at $x = 0$

38. $f(t) = (3t - 2)^5$ at $t = 1$

39. $g(\theta) = (\sin\theta + \cos\theta)^3$ at $\theta = \pi/2$

40. $h(t) = e^{3t^2+t-1}$ at $t = -1$

41. Compute $\frac{d}{dx}\big(\ln(kx)\big)$ two ways:

   (a) Using the Chain Rule, and

   (b) by first using the logarithm rule $\ln(ab) = \ln a + \ln b$, then taking the derivative.

42. Compute $\frac{d}{dx}\big(\ln(x^k)\big)$ two ways:

   (a) Using the Chain Rule, and

   (b) by first using the logarithm rule $\ln(a^p) = p\ln a$, then taking the derivative.

Review

43. The "wind chill factor" is a measurement of how cold it "feels" during cold, windy weather. Let $W(w)$ be the wind chill factor, in degrees Fahrenheit, when it is 25°F outside with a wind of w mph.

   (a) What are the units of $W'(w)$?

   (b) What would you expect the sign of $W'(10)$ to be?

44. Find the derivatives of the following functions.

   (a) $f(x) = x^2e^x\cot x$

   (b) $g(x) = 2^x3^x4^x$

Exercises 2.6

Terms and Concepts

1. In your own words, explain the difference between implicit functions and explicit functions.

2. Implicit differentiation is based on what other differentiation rule?

3. T/F: Implicit differentiation can be used to find the derivative of $y = \sqrt{x}$.

4. T/F: Implicit differentiation can be used to find the derivative of $y = x^{3/4}$.

Problems

In Exercises 5 – 12, compute the derivative of the given function.

5. $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

6. $f(x) = \sqrt[3]{x} + x^{2/3}$

7. $f(t) = \sqrt{1-t^2}$

8. $g(t) = \sqrt{t}\sin t$

9. $h(x) = x^{1.5}$

10. $f(x) = x^{\pi} + x^{1.9} + \pi^{1.9}$

11. $g(x) = \dfrac{x+7}{\sqrt{x}}$

12. $f(t) = \sqrt[5]{t}(\sec t + e^t)$

In Exercises 13 – 25, find $\dfrac{dy}{dx}$ using implicit differentiation.

13. $x^4 + y^2 + y = 7$

14. $x^{2/5} + y^{2/5} = 1$

15. $\cos(x) + \sin(y) = 1$

16. $\dfrac{x}{y} = 10$

17. $\dfrac{y}{x} = 10$

18. $x^2e^2 + 2^y = 5$

19. $x^2 \tan y = 50$

20. $(3x^2 + 2y^3)^4 = 2$

21. $(y^2 + 2y - x)^2 = 200$

22. $\dfrac{x^2+y}{x+y^2} = 17$

23. $\dfrac{\sin(x)+y}{\cos(y)+x} = 1$

24. $\ln(x^2+y^2) = e$

25. $\ln(x^2+xy+y^2) = 1$

26. Show that $\dfrac{dy}{dx}$ is the same for each of the following implicitly defined functions.

    (a) $xy = 1$

    (b) $x^2y^2 = 1$

    (c) $\sin(xy) = 1$

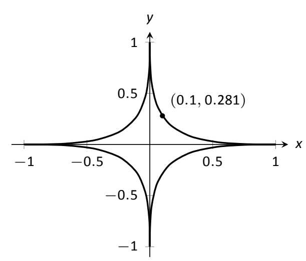
    (d) $\ln(xy) = 1$

In Exercises 27 – 32, find the equation of the tangent line to the graph of the implicitly defined function at the indicated points. As a visual aid, each function is graphed.

27. $x^{2/5} + y^{2/5} = 1$

    (a) At $(1, 0)$.

    (b) At $(0.1, 0.281)$ (which does not *exactly* lie on the curve, but is very close).

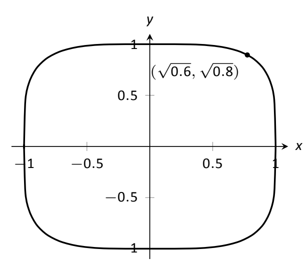


28. $x^4 + y^4 = 1$

    (a) At $(1, 0)$.

    (b) At $(\sqrt{0.6}, \sqrt{0.8})$.

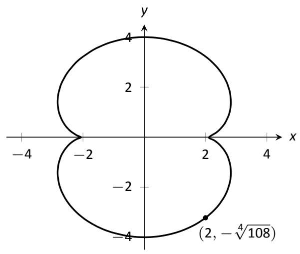
    (c) At $(0, 1)$.



29. $(x^2 + y^2 - 4)^3 = 108y^2$

   (a) At $(0, 4)$.

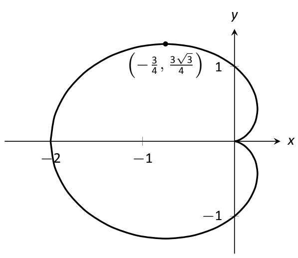
   (b) At $(2, -\sqrt[4]{108})$.



30. $(x^2 + y^2 + x)^2 = x^2 + y^2$

   (a) At $(0, 1)$.

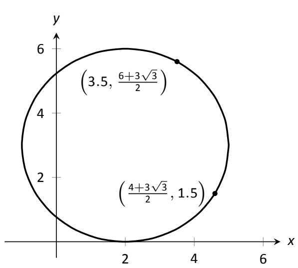
   (b) At $\left(-\dfrac{3}{4}, \dfrac{3\sqrt{3}}{4}\right)$.



31. $(x - 2)^2 + (y - 3)^2 = 9$

   (a) At $\left(\dfrac{7}{2}, \dfrac{6 + 3\sqrt{3}}{2}\right)$.

   (b) At $\left(\dfrac{4 + 3\sqrt{3}}{2}, \dfrac{3}{2}\right)$.

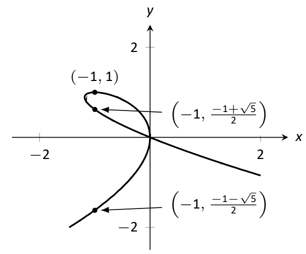


32. $x^2 + y^3 + 2xy = 0$

   (a) At $(-1, 1)$.

   (b) At $\left(-1, \dfrac{1}{2}(-1 + \sqrt{5})\right)$.

   (c) At $\left(-1, \dfrac{1}{2}(-1 - \sqrt{5})\right)$.



In Exercises 33 – 36, an implicitly defined function is given. Find $\dfrac{d^2y}{dx^2}$. Note: these are the same problems used in Exercises 13 through 16.

33. $x^4 + y^2 + y = 7$

34. $x^{2/5} + y^{2/5} = 1$

35. $\cos x + \sin y = 1$

36. $\dfrac{x}{y} = 10$

In Exercises 37 – 42, use logarithmic differentiation to find $\dfrac{dy}{dx}$, then find the equation of the tangent line at the indicated x–value.

37. $y = (1 + x)^{1/x}, \quad x = 1$

38. $y = (2x)^{x^2}, \quad x = 1$

39. $y = \dfrac{x^x}{x + 1}, \quad x = 1$

40. $y = x^{\sin(x)+2}, \quad x = \pi/2$

41. $y = \dfrac{x + 1}{x + 2}, \quad x = 1$

42. $y = \dfrac{(x + 1)(x + 2)}{(x + 3)(x + 4)}, \quad x = 0$

Exercises 2.7

Terms and Concepts

1. T/F: Every function has an inverse.

2. In your own words explain what it means for a function to be "one to one."

3. If $(1, 10)$ lies on the graph of $y = f(x)$, what can be said about the graph of $y = f^{-1}(x)$?

4. If $(1, 10)$ lies on the graph of $y = f(x)$ and $f'(1) = 5$, what can be said about $y = f^{-1}(x)$?

Problems

In Exercises 5 – 8, verify that the given functions are inverses.

5. $f(x) = 2x + 6$ and $g(x) = \frac{1}{2}x - 3$

6. $f(x) = x^2 + 6x + 11$, $x \geq 3$ and $g(x) = \sqrt{x-2} - 3$, $x \geq 2$

7. $f(x) = \dfrac{3}{x-5}$, $x \neq 5$ and $g(x) = \dfrac{3+5x}{x}$, $x \neq 0$

8. $f(x) = \dfrac{x+1}{x-1}$, $x \neq 1$ and $g(x) = f(x)$

In Exercises 9 – 14, an invertible function $f(x)$ is given along with a point that lies on its graph. Using Theorem 2.7.1, evaluate $\left(f^{-1}\right)'(x)$ at the indicated value.

9. $f(x) = 5x + 10$
   Point$=(2, 20)$
   Evaluate $\left(f^{-1}\right)'(20)$

10. $f(x) = x^2 - 2x + 4$, $x \geq 1$
    Point$=(3, 7)$
    Evaluate $\left(f^{-1}\right)'(7)$

11. $f(x) = \sin 2x$, $-\pi/4 \leq x \leq \pi/4$
    Point$=(\pi/6, \sqrt{3}/2)$
    Evaluate $\left(f^{-1}\right)'(\sqrt{3}/2)$

12. $f(x) = x^3 - 6x^2 + 15x - 2$
    Point$=(1, 8)$
    Evaluate $\left(f^{-1}\right)'(8)$

13. $f(x) = \dfrac{1}{1+x^2}$, $x \geq 0$
    Point$=(1, 1/2)$
    Evaluate $\left(f^{-1}\right)'(1/2)$

14. $f(x) = 6e^{3x}$
    Point$=(0, 6)$
    Evaluate $\left(f^{-1}\right)'(6)$

In Exercises 15 – 24, compute the derivative of the given function.

15. $h(t) = \sin^{-1}(2t)$

16. $f(t) = \sec^{-1}(2t)$

17. $g(x) = \tan^{-1}(2x)$

18. $f(x) = x \sin^{-1} x$

19. $g(t) = \sin t \cos^{-1} t$

20. $f(t) = \ln te^t$

21. $h(x) = \dfrac{\sin^{-1} x}{\cos^{-1} x}$

22. $g(x) = \tan^{-1}(\sqrt{x})$

23. $f(x) = \sec^{-1}(1/x)$

24. $f(x) = \sin(\sin^{-1} x)$

In Exercises 25 – 26, compute the derivative of the given function in two ways:

(a) By simplifying first, then taking the derivative, and

(b) by using the Chain Rule first then simplifying.

Verify that the two answers are the same.

25. $f(x) = \sin(\sin^{-1} x)$

26. $f(x) = \tan^{-1}(\tan x)$

In Exercises 27 – 28, find the equation of the line tangent to the graph of f at the indicated x value.

27. $f(x) = \sin^{-1} x$ at $x = \frac{\sqrt{2}}{2}$

28. $f(x) = \cos^{-1}(2x)$ at $x = \frac{\sqrt{3}}{4}$

Review

29. Find $\frac{dy}{dx}$, where $x^2y - y^2x = 1$.

30. Find the equation of the line tangent to the graph of $x^2 + y^2 + xy = 7$ at the point $(1, 2)$.

31. Let $f(x) = x^3 + x$.
    Evaluate $\displaystyle\lim_{s \to 0} \frac{f(x+s) - f(x)}{s}$.

Exercises 3.1

Terms and Concepts

1. Describe what an "extreme value" of a function is in your own words.

2. Sketch the graph of a function f on $(-1, 1)$ that has both a maximum and minimum value.

3. Describe the difference between absolute and relative maxima in your own words.

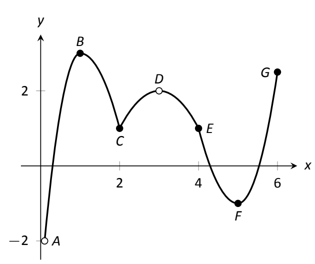
4. Sketch the graph of a function f where f has a relative maximum at $x = 1$ and $f'(1)$ is undefined.

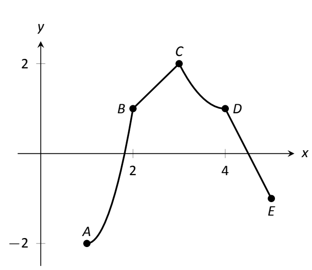
5. T/F: If c is a critical value of a function f, then f has either a relative maximum or relative minimum at $x = c$.

6. Fill in the blanks: The critical points of a function f are found where $f'(x)$ is equal to ________ or where $f'(x)$ is ________.

Problems

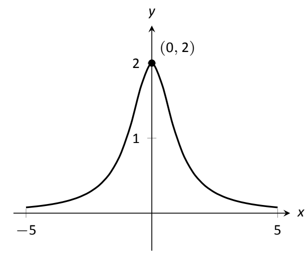
In Exercises 7 – 8, identify each of the marked points as being an absolute maximum or minimum, a relative maximum or minimum, or none of the above.

7. 

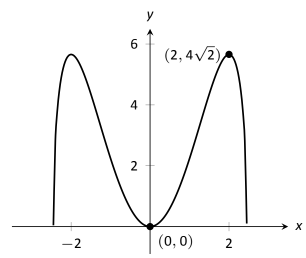
8. 

In Exercises 9 – 16, evaluate $f'(x)$ at the points indicated in the graph.

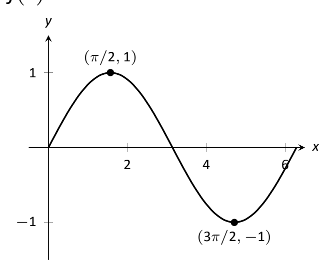
9. $f(x) = \dfrac{2}{x^2+1}$



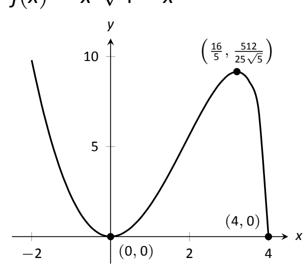
10. $f(x) = x^2\sqrt{6 - x^2}$



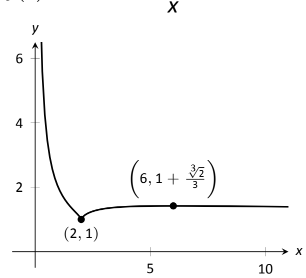
11. $f(x) = \sin x$



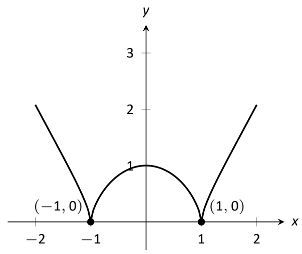
12. $f(x) = x^2\sqrt{4 - x}$



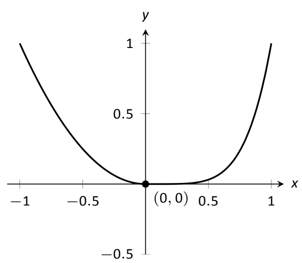
13. $f(x) = 1 + \dfrac{(x-2)^{2/3}}{x}$



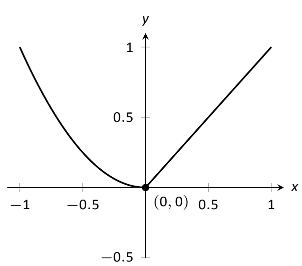
14. $f(x) = \sqrt[3]{x^4 - 2x + 1}$



15. $f(x) = \begin{cases} x^2 & x \leq 0 \\ x^5 & x > 0 \end{cases}$



16. $f(x) = \begin{cases} x^2 & x \leq 0 \\ x & x > 0 \end{cases}$



In Exercises 17 – 26, find the extreme values of the function on the given interval.

17. $f(x) = x^2 + x + 4 \quad \text{on} \quad [-1, 2]$.

18. $f(x) = x^3 - \dfrac{9}{2}x^2 - 30x + 3 \quad \text{on} \quad [0, 6]$.

19. $f(x) = 3\sin x \quad \text{on} \quad [\pi/4, 2\pi/3]$.

20. $f(x) = x^2\sqrt{4 - x^2} \quad \text{on} \quad [-2, 2]$.

21. $f(x) = x + \dfrac{3}{x} \quad \text{on} \quad [1, 5]$.

22. $f(x) = \dfrac{x^2}{x^2 + 5} \quad \text{on} \quad [-3, 5]$.

23. $f(x) = e^x \cos x \quad \text{on} \quad [0, \pi]$.

24. $f(x) = e^x \sin x \quad \text{on} \quad [0, \pi]$.

25. $f(x) = \dfrac{\ln x}{x} \quad \text{on} \quad [1, 4]$.

26. $f(x) = x^{2/3} - x \quad \text{on} \quad [0, 2]$.

Review

27. Find $\frac{dy}{dx}$, where $x^2y - y^2x = 1$.

28. Find the equation of the line tangent to the graph of $x^2 + y^2 + xy = 7$ at the point $(1, 2)$.

29. Let $f(x) = x^3 + x$.
Evaluate $\displaystyle\lim_{s \to 0} \frac{f(x + s) - f(x)}{s}$.

Exercises 3.2

Terms and Concepts

1. Explain in your own words what the Mean Value Theorem states.

2. Explain in your own words what Rolle's Theorem states.

Problems

In Exercises 3 – 10, a function $f(x)$ and interval $[a, b]$ are given. Check if Rolle's Theorem can be applied to f on $[a, b]$; if so, find c in $[a, b]$ such that $f'(c) = 0$.

3. $f(x) = 6$ on $[-1, 1]$.

4. $f(x) = 6x$ on $[-1, 1]$.

5. $f(x) = x^2 + x - 6$ on $[-3, 2]$.

6. $f(x) = x^2 + x - 2$ on $[-3, 2]$.

7. $f(x) = x^2 + x$ on $[-2, 2]$.

8. $f(x) = \sin x$ on $[\pi/6, 5\pi/6]$.

9. $f(x) = \cos x$ on $[0, \pi]$.

10. $f(x) = \dfrac{1}{x^2 - 2x + 1}$ on $[0, 2]$.

In Exercises 11 – 20, a function $f(x)$ and interval $[a, b]$ are given. Check if the Mean Value Theorem can be applied to f on $[a, b]$; if so, find a value c in $[a, b]$ guaranteed by the Mean Value Theorem.

11. $f(x) = x^2 + 3x - 1$ on $[-2, 2]$.

12. $f(x) = 5x^2 - 6x + 8$ on $[0, 5]$.

13. $f(x) = \sqrt{9 - x^2}$ on $[0, 3]$.

14. $f(x) = \sqrt{25 - x}$ on $[0, 9]$.

15. $f(x) = \dfrac{x^2 - 9}{x^2 - 1}$ on $[0, 2]$.

16. $f(x) = \ln x$ on $[1, 5]$.

17. $f(x) = \tan x$ on $[-\pi/4, \pi/4]$.

18. $f(x) = x^3 - 2x^2 + x + 1$ on $[-2, 2]$.

19. $f(x) = 2x^3 - 5x^2 + 6x + 1$ on $[-5, 2]$.

20. $f(x) = \sin^{-1} x$ on $[-1, 1]$.

Review

21. Find the extreme values of $f(x) = x^2 - 3x + 9$ on $[-2, 5]$.

22. Describe the critical points of $f(x) = \cos x$.

23. Describe the critical points of $f(x) = \tan x$.

Exercises 3.3

Terms and Concepts

1. In your own words describe what it means for a function to be increasing.

2. What does a decreasing function "look like"?

3. Sketch a graph of a function on $[0, 2]$ that is increasing, where it is increasing "quickly" near $x = 0$ and increasing "slowly" near $x = 2$.

4. Give an example of a function describing a situation where it is "bad" to be increasing and "good" to be decreasing.

5. T/F: Functions always switch from increasing to decreasing, or decreasing to increasing, at critical points.

6. A function f has derivative $f'(x) = (\sin x + 2)e^{x^2+1}$, where $f'(x) > 1$ for all x. Is f increasing, decreasing, or can we not tell from the given information?

Problems

In Exercises 7 – 14, a function $f(x)$ is given.

(a) Compute $f'(x)$.

(b) Graph f and f' on the same axes (using technology is permitted) and verify Theorem 3.3.1.

7. $f(x) = 2x + 3$

8. $f(x) = x^2 - 3x + 5$

9. $f(x) = \cos x$

10. $f(x) = \tan x$

11. $f(x) = x^3 - 5x^2 + 7x - 1$

12. $f(x) = 2x^3 - x^2 + x - 1$

13. $f(x) = x^4 - 5x^2 + 4$

14. $f(x) = \dfrac{1}{x^2 + 1}$

In Exercises 15 – 24, a function $f(x)$ is given.

(a) Give the domain of f.

(b) Find the critical numbers of f.

(c) Create a number line to determine the intervals on which f is increasing and decreasing.

(d) Use the First Derivative Test to determine whether each critical point is a relative maximum, minimum, or neither.

15. $f(x) = x^2 + 2x - 3$

16. $f(x) = x^3 + 3x^2 + 3$

17. $f(x) = 2x^3 + x^2 - x + 3$

18. $f(x) = x^3 - 3x^2 + 3x - 1$

19. $f(x) = \dfrac{1}{x^2 - 2x + 2}$

20. $f(x) = \dfrac{x^2 - 4}{x^2 - 1}$

21. $f(x) = \dfrac{x}{x^2 - 2x - 8}$

22. $f(x) = \dfrac{(x-2)^{2/3}}{x}$

23. $f(x) = \sin x \cos x$ on $(-\pi, \pi)$.

24. $f(x) = x^5 - 5x$

Review

25. Consider $f(x) = x^2 - 3x + 5$ on $[-1, 2]$; find c guaranteed by the Mean Value Theorem.

26. Consider $f(x) = \sin x$ on $[-\pi/2, \pi/2]$; find c guaranteed by the Mean Value Theorem.

Exercises 3.4

Terms and Concepts

1. Sketch a graph of a function $f(x)$ that is concave up on $(0,1)$ and is concave down on $(1,2)$.

2. Sketch a graph of a function $f(x)$ that is:

   (a) Increasing, concave up on $(0,1)$,

   (b) increasing, concave down on $(1,2)$,

   (c) decreasing, concave down on $(2,3)$ and

   (d) increasing, concave down on $(3,4)$.

3. Is is possible for a function to be increasing and concave down on $(0,\infty)$ with a horizontal asymptote of $y=1$? If so, give a sketch of such a function.

4. Is is possible for a function to be increasing and concave up on $(0,\infty)$ with a horizontal asymptote of $y=1$? If so, give a sketch of such a function.

Problems

In Exercises 5 – 14, a function $f(x)$ is given.

(a) Compute $f''(x)$.

(b) Graph f and f'' on the same axes (using technology is permitted) and verify Theorem 3.4.1.

5. $f(x)=-7x+3$

6. $f(x)=-4x^2+3x-8$

7. $f(x)=4x^2+3x-8$

8. $f(x)=x^3-3x^2+x-1$

9. $f(x)=-x^3+x^2-2x+5$

10. $f(x)=\sin x$

11. $f(x)=\tan x$

12. $f(x)=\dfrac{1}{x^2+1}$

13. $f(x)=\dfrac{1}{x}$

14. $f(x)=\dfrac{1}{x^2}$

In Exercises 15 – 28, a function $f(x)$ is given.

(a) Find the possible points of inflection of f.

(b) Create a number line to determine the intervals on which f is concave up or concave down.

15. $f(x)=x^2-2x+1$

16. $f(x)=-x^2-5x+7$

17. $f(x)=x^3-x+1$

18. $f(x)=2x^3-3x^2+9x+5$

19. $f(x)=\dfrac{x^4}{4}+\dfrac{x^3}{3}-2x+3$

20. $f(x)=-3x^4+8x^3+6x^2-24x+2$

21. $f(x)=x^4-4x^3+6x^2-4x+1$

22. $f(x)=\sec x$ on $(-3\pi/2, 3\pi/2)$

23. $f(x)=\dfrac{1}{x^2+1}$

24. $f(x)=\dfrac{x}{x^2-1}$

25. $f(x)=\sin x+\cos x$ on $(-\pi,\pi)$

26. $f(x)=x^2e^x$

27. $f(x)=x^2\ln x$

28. $f(x)=e^{-x^2}$

In Exercises 29 – 42, a function $f(x)$ is given. Find the critical points of f and use the Second Derivative Test, when possible, to determine the relative extrema. (Note: these are the same functions as in Exercises 15 – 28.)

29. $f(x)=x^2-2x+1$

30. $f(x)=-x^2-5x+7$

31. $f(x)=x^3-x+1$

32. $f(x)=2x^3-3x^2+9x+5$

33. $f(x)=\dfrac{x^4}{4}+\dfrac{x^3}{3}-2x+3$

34. $f(x)=-3x^4+8x^3+6x^2-24x+2$

35. $f(x)=x^4-4x^3+6x^2-4x+1$

36. $f(x)=\sec x$ on $(-3\pi/2, 3\pi/2)$

37. $f(x) = \dfrac{1}{x^2+1}$

38. $f(x) = \dfrac{x}{x^2-1}$

39. $f(x) = \sin x + \cos x$ on $(-\pi, \pi)$

40. $f(x) = x^2 e^x$

41. $f(x) = x^2 \ln x$

42. $f(x) = e^{-x^2}$

In Exercises 43 – 56, a function $f(x)$ is given. Find the x values where $f'(x)$ has a relative maximum or minimum. (Note: these are the same functions as in Exercises 15 – 28.)

43. $f(x) = x^2 - 2x + 1$

44. $f(x) = -x^2 - 5x + 7$

45. $f(x) = x^3 - x + 1$

46. $f(x) = 2x^3 - 3x^2 + 9x + 5$

47. $f(x) = \dfrac{x^4}{4} + \dfrac{x^3}{3} - 2x + 3$

48. $f(x) = -3x^4 + 8x^3 + 6x^2 - 24x + 2$

49. $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$

50. $f(x) = \sec x$ on $(-3\pi/2, 3\pi/2)$

51. $f(x) = \dfrac{1}{x^2+1}$

52. $f(x) = \dfrac{x}{x^2-1}$

53. $f(x) = \sin x + \cos x$ on $(-\pi, \pi)$

54. $f(x) = x^2 e^x$

55. $f(x) = x^2 \ln x$

56. $f(x) = e^{-x^2}$

Exercises 3.5

Terms and Concepts

1. Why is sketching curves by hand beneficial even though technology is ubiquitous?

2. What does "ubiquitous" mean?

3. T/F: When sketching graphs of functions, it is useful to find the critical points.

4. T/F: When sketching graphs of functions, it is useful to find the possible points of inflection.

5. T/F: When sketching graphs of functions, it is useful to find the horizontal and vertical asymptotes.

6. T/F: When sketching graphs of functions, one need not plot any points at all.

Problems

In Exercises 7 – 12, practice using Key Idea 3.5.1 by applying the principles to the given functions with familiar graphs.

7. $f(x) = 2x + 4$

8. $f(x) = -x^2 + 1$

9. $f(x) = \sin x$

10. $f(x) = e^x$

11. $f(x) = \dfrac{1}{x}$

12. $f(x) = \dfrac{1}{x^2}$

In Exercises 13 – 26, sketch a graph of the given function using Key Idea 3.5.1. Show all work; check your answer with technology.

13. $f(x) = x^3 - 2x^2 + 4x + 1$

14. $f(x) = -x^3 + 5x^2 - 3x + 2$

15. $f(x) = x^3 + 3x^2 + 3x + 1$

16. $f(x) = x^3 - x^2 - x + 1$

17. $f(x) = (x - 2)\ln(x - 2)$

18. $f(x) = (x - 2)^2 \ln(x - 2)$

19. $f(x) = \dfrac{x^2 - 4}{x^2}$

20. $f(x) = \dfrac{x^2 - 4x + 3}{x^2 - 6x + 8}$

21. $f(x) = \dfrac{x^2 - 2x + 1}{x^2 - 6x + 8}$

22. $f(x) = x\sqrt{x + 1}$

23. $f(x) = x^2 e^x$

24. $f(x) = \sin x \cos x$ on $[-\pi, \pi]$

25. $f(x) = (x - 3)^{2/3} + 2$

26. $f(x) = \dfrac{(x - 1)^{2/3}}{x}$

In Exercises 27 – 30, a function with the parameters a and b are given. Describe the critical points and possible points of inflection of f in terms of a and b.

27. $f(x) = \dfrac{a}{x^2 + b^2}$

28. $f(x) = ax^2 + bx + 1$

29. $f(x) = \sin(ax + b)$

30. $f(x) = (x - a)(x - b)$

31. Given $x^2 + y^2 = 1$, use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Use this information to justify the sketch of the unit circle.

Exercises 4.1

Terms and Concepts

1. T/F: Given a function $f(x)$, Newton's Method produces an exact solution to $f(x) = 0$.

2. T/F: In order to get a solution to $f(x) = 0$ accurate to d places after the decimal, at least $d + 1$ iterations of Newtons' Method must be used.

Problems

In Exercises 3 – 8, the roots of $f(x)$ are known or are easily found. Use 5 iterations of Newton's Method with the given initial approximation to approximate the root. Compare it to the known value of the root.

3. $f(x) = \cos x$, $x_0 = 1.5$

4. $f(x) = \sin x$, $x_0 = 1$

5. $f(x) = x^2 + x - 2$, $x_0 = 0$

6. $f(x) = x^2 - 2$, $x_0 = 1.5$

7. $f(x) = \ln x$, $x_0 = 2$

8. $f(x) = x^3 - x^2 + x - 1$, $x_0 = 1$

In Exercises 9 – 12, use Newton's Method to approximate all roots of the given functions accurate to 3 places after the decimal. If an interval is given, find only the roots that lie in that interval. Use technology to obtain good initial approximations.

9. $f(x) = x^3 + 5x^2 - x - 1$

10. $f(x) = x^4 + 2x^3 - 7x^2 - x + 5$

11. $f(x) = x^{17} - 2x^{13} - 10x^8 + 10$ on $(-2, 2)$

12. $f(x) = x^2 \cos x + (x - 1) \sin x$ on $(-3, 3)$

In Exercises 13 – 16, use Newton's Method to approximate when the given functions are equal, accurate to 3 places after the decimal. Use technology to obtain good initial approximations.

13. $f(x) = x^2$, $g(x) = \cos x$

14. $f(x) = x^2 - 1$, $g(x) = \sin x$

15. $f(x) = e^{x^2}$, $g(x) = \cos x$

16. $f(x) = x$, $g(x) = \tan x$ on $[-6, 6]$

17. Why does Newton's Method fail in finding a root of $f(x) = x^3 - 3x^2 + x + 3$ when $x_0 = 1$?

18. Why does Newton's Method fail in finding a root of $f(x) = -17x^4 + 130x^3 - 301x^2 + 156x + 156$ when $x_0 = 1$?

Exercises 4.2

Terms and Concepts

1. T/F: Implicit differentiation is often used when solving "related rates" type problems.

2. T/F: A study of related rates is part of the standard police officer training.

Problems

3. Water flows onto a flat surface at a rate of 5cm^3/s forming a circular puddle 10mm deep. How fast is the radius growing when the radius is:

   (a) 1 cm?

   (b) 10 cm?

   (c) 100 cm?

4. A circular balloon is inflated with air flowing at a rate of 10cm^3/s. How fast is the radius of the balloon increasing when the radius is:

   (a) 1 cm?

   (b) 10 cm?

   (c) 100 cm?

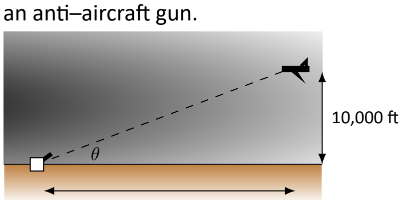
5. Consider the traffic situation introduced in Example 4.2.3. How fast is the "other car" traveling if the officer and the other car are each 1/2 mile from the intersection, the other car is traveling *due west*, the officer is traveling north at 50mph, and the radar reading is -80mph?

6. Consider the traffic situation introduced in Example 4.2.3. Calculate how fast the "other car" is traveling in each of the following situations.

   (a) The officer is traveling due north at 50mph and is 1/2 mile from the intersection, while the other car is 1 mile from the intersection traveling west and the radar reading is -80mph?

   (b) The officer is traveling due north at 50mph and is 1 mile from the intersection, while the other car is 1/2 mile from the intersection traveling west and the radar reading is -80mph?

7. An F-22 aircraft is flying at 500mph with an elevation of 10,000ft on a straight–line path that will take it directly over an anti–aircraft gun.



   How fast must the gun be able to turn to accurately track the aircraft when the plane is:

   (a) 1 mile away?

   (b) 1/5 mile away?

   (c) Directly overhead?

8. An F-22 aircraft is flying at 500mph with an elevation of 100ft on a straight–line path that will take it directly over an anti–aircraft gun as in Exercise 7 (note the lower elevation here).

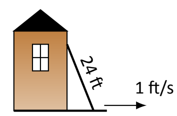
   How fast must the gun be able to turn to accurately track the aircraft when the plane is:

   (a) 1000 feet away?

   (b) 100 feet away?

   (c) Directly overhead?

9. A 24ft. ladder is leaning against a house while the base is pulled away at a constant rate of 1ft/s.



   At what rate is the top of the ladder sliding down the side of the house when the base is:

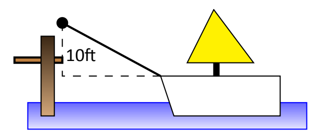
   (a) 1 foot from the house?

   (b) 10 feet from the house?

   (c) 23 feet from the house?

   (d) 24 feet from the house?

10. A boat is being pulled into a dock at a constant rate of 30ft/min by a winch located 10ft above the deck of the boat.



   At what rate is the boat approaching the dock when the boat is:

   (a) 50 feet out?

   (b) 15 feet out?

   (c) 1 foot from the dock?

   (d) What happens when the length of rope pulling in the boat is less than 10 feet long?

11. An inverted cylindrical cone, 20ft deep and 10ft across at the top, is being filled with water at a rate of 10ft^3/min. At what rate is the water rising in the tank when the depth of the water is:

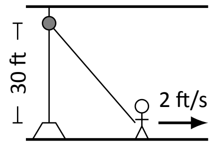
   (a) 1 foot?

   (b) 10 feet?

   (c) 19 feet?

   How long will the tank take to fill when starting at empty?

12. A rope, attached to a weight, goes up through a pulley at the ceiling and back down to a worker. The man holds the rope at the same height as the connection point between rope and weight.



Suppose the man stands directly next to the weight (i.e., a total rope length of 60 ft) and begins to walk away at a rate of 2ft/s. How fast is the weight rising when the man has walked:

(a) 10 feet?

(b) 40 feet?

How far must the man walk to raise the weight all the way to the pulley?

13. Consider the situation described in Exercise 12. Suppose the man starts 40ft from the weight and begins to walk away at a rate of 2ft/s.

(a) How long is the rope?

(b) How fast is the weight rising after the man has walked 10 feet?

(c) How fast is the weight rising after the man has walked 30 feet?

(d) How far must the man walk to raise the weight all the way to the pulley?

14. A hot air balloon lifts off from ground rising vertically. From 100 feet away, a 5' woman tracks the path of the balloon. When her sightline with the balloon makes a 45° angle with the horizontal, she notes the angle is increasing at about 5°/min.

(a) What is the elevation of the balloon?

(b) How fast is it rising?

15. A company that produces landscaping materials is dumping sand into a conical pile. The sand is being poured at a rate of 5ft^3/sec; the physical properties of the sand, in conjunction with gravity, ensure that the cone's height is roughly 2/3 the length of the diameter of the circular base.

How fast is the cone rising when it has a height of 30 feet?

Exercises 4.3

Terms and Concepts

1. T/F: An "optimization problem" is essentially an "extreme values" problem in a "story problem" setting.

2. T/F: This section teaches one to find the extreme values of a function that has more than one variable.

Problems

3. Find the maximum product of two numbers (not necessarily integers) that have a sum of 100.

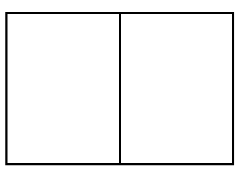
4. Find the minimum sum of two positive numbers whose product is 500.

5. Find the maximum sum of two positive numbers whose product is 500.

6. Find the maximum sum of two numbers, each of which is in $[0, 300]$ whose product is 500.

7. Find the maximal area of a right triangle with hypotenuse of length 1.

8. A rancher has 1000 feet of fencing in which to construct adjacent, equally sized rectangular pens. What dimensions should these pens have to maximize the enclosed area?



9. A standard soda can is roughly cylindrical and holds 355cm^3 of liquid. What dimensions should the cylinder be to minimize the material needed to produce the can? Based on your dimensions, determine whether or not the standard can is produced to minimize the material costs.

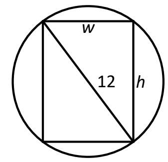
10. Find the dimensions of a cylindrical can with a volume of 206in^3 that minimizes the surface area.

    The "#10 can"is a standard sized can used by the restaurant industry that holds about 206in^3 with a diameter of 6 2/16in and height of 7in. Does it seem these dimensions were chosen with minimization in mind?

11. The United States Postal Service charges more for boxes whose combined length and girth exceeds 108" (the "length" of a package is the length of its longest side; the girth is the perimeter of the cross section, i.e., $2w + 2h$).

    What is the maximum volume of a package with a square cross section ($w = h$) that does not exceed the 108" standard?

12. The strength S of a wooden beam is directly proportional to its cross sectional width w and the square of its height h; that is, $S = kwh^2$ for some constant k.



    Given a circular log with diameter of 12 inches, what sized beam can be cut from the log with maximum strength?

13. A power line is to be run to an offshore facility in the manner described in Example 4.3.3. The offshore facility is 2 miles at sea and 5 miles along the shoreline from the power plant. It costs \$50,000 per mile to lay a power line underground and \$80,000 to run the line underwater.

    How much of the power line should be run underground to minimize the overall costs?

14. A power line is to be run to an offshore facility in the manner described in Example 4.3.3. The offshore facility is 5 miles at sea and 2 miles along the shoreline from the power plant. It costs \$50,000 per mile to lay a power line underground and \$80,000 to run the line underwater.

    How much of the power line should be run underground to minimize the overall costs?

15. A woman throws a stick into a lake for her dog to fetch; the stick is 20 feet down the shore line and 15 feet into the water from there. The dog may jump directly into the water and swim, or run along the shore line to get closer to the stick before swimming. The dog runs about 22ft/s and swims about 1.5ft/s.

    How far along the shore should the dog run to minimize the time it takes to get to the stick? (Hint: the figure from Example 4.3.3 can be useful.)

16. A woman throws a stick into a lake for her dog to fetch; the stick is 15 feet down the shore line and 30 feet into the water from there. The dog may jump directly into the water and swim, or run along the shore line to get closer to the stick before swimming. The dog runs about 22ft/s and swims about 1.5ft/s.

    How far along the shore should the dog run to minimize the time it takes to get to the stick? *(Google "calculus dog" to learn more about a dog's ability to minimize times.)*

17. What are the dimensions of the rectangle with largest area that can be drawn inside the unit circle?

Exercises 4.4

Terms and Concepts

1. T/F: Given a differentiable function $y = f(x)$, we are generally free to choose a value for dx, which then determines the value of dy.

2. T/F: The symbols "dx" and "Δx" represent the same concept.

3. T/F: The symbols "dy" and "Δy" represent the same concept.

4. T/F: Differentials are important in the study of integration.

5. How are differentials and tangent lines related?

6. T/F: In real life, differentials are used to approximate function values when the function itself is not known.

Problems

In Exercises 7 – 16, use differentials to approximate the given value by hand.

7. 2.05^2

8. 5.93^2

9. 5.1^3

10. 6.8^3

11. $\sqrt{16.5}$

12. $\sqrt{24}$

13. $\sqrt[3]{63}$

14. $\sqrt[3]{8.5}$

15. $\sin 3$

16. $e^{0.1}$

In Exercises 17 – 30, compute the differential *dy*.

17. $y = x^2 + 3x - 5$

18. $y = x^7 - x^5$

19. $y = \dfrac{1}{4x^2}$

20. $y = (2x + \sin x)^2$

21. $y = x^2 e^{3x}$

22. $y = \dfrac{4}{x^4}$

23. $y = \dfrac{2x}{\tan x + 1}$

24. $y = \ln(5x)$

25. $y = e^x \sin x$

26. $y = \cos(\sin x)$

27. $y = \dfrac{x+1}{x+2}$

28. $y = 3^x \ln x$

29. $y = x \ln x - x$

30. $f(x) = \ln\big(\sec x\big)$

Exercises 31 – 34 use differentials to approximate propagated error.

31. A set of plastic spheres are to be made with a diameter of 1cm. If the manufacturing process is accurate to 1mm, what is the propagated error in volume of the spheres?

32. The distance, in feet, a stone drops in t seconds is given by $d(t) = 16t^2$. The depth of a hole is to be approximated by dropping a rock and listening for it to hit the bottom. What is the propagated error if the time measurement is accurate to $2/10^{\text{ths}}$ of a second and the measured time is:

    (a) 2 seconds?

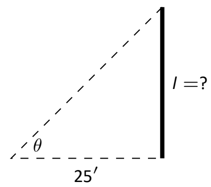
    (b) 5 seconds?

33. What is the propagated error in the measurement of the cross sectional area of a circular log if the diameter is measured at $15''$, accurate to $1/4''$?

34. A wall is to be painted that is $8'$ high and is measured to be $10',\ 7''$ long. Find the propagated error in the measurement of the wall's surface area if the measurement is accurate to $1/2''$.

Exercises 35 – 39 explore some issues related to surveying in which distances are approximated using other measured distances and measured angles. *(Hint: Convert all angles to radians before computing.)*

35. The length l of a long wall is to be approximated. The angle θ, as shown in the diagram (not to scale), is measured to be 85.2°, accurate to 1°. Assume that the triangle formed is a right triangle.



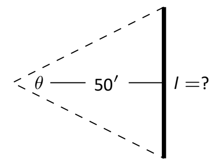
(a) What is the measured length l of the wall?

(b) What is the propagated error?

(c) What is the percent error?

36. Answer the questions of Exercise 35, but with a measured angle of 71.5°, accurate to 1°, measured from a point $100'$ from the wall.

37. The length l of a long wall is to be calculated by measuring the angle θ shown in the diagram (not to scale). Assume the formed triangle is an isosceles triangle. The measured angle is 143°, accurate to 1°.



(a) What is the measured length of the wall?

(b) What is the propagated error?

(c) What is the percent error?

38. The length of the walls in Exercises 35 – 37 are essentially the same. Which setup gives the most accurate result?

39. Consider the setup in Exercise 37. This time, assume the angle measurement of 143° is exact but the measured $50'$ from the wall is accurate to $6''$. What is the approximate percent error?

Exercises 5.1

Terms and Concepts

1. Define the term “antiderivative” in your own words.

2. Is it more accurate to refer to “the” antiderivative of $f(x)$ or “an” antiderivative of $f(x)$?

3. Use your own words to define the indefinite integral of $f(x)$.

4. Fill in the blanks: “Inverse operations do the ________ things in the ________ order.”

5. What is an “initial value problem”?

6. The derivative of a position function is a ________ function.

7. The antiderivative of an acceleration function is a ________ function.

8. If $F(x)$ is an antiderivative of $f(x)$, and $G(x)$ is an antiderivative of $g(x)$, give an antiderivative of $f(x)+g(x)$.

Problems

In Exercises 9 – 27, evaluate the given indefinite integral.

9. $\int 3x^3\,dx$

10. $\int x^8\,dx$

11. $\int (10x^2-2)\,dx$

12. $\int dt$

13. $\int 1\,ds$

14. $\int \frac{1}{3t^2}\,dt$

15. $\int \frac{3}{t^2}\,dt$

16. $\int \frac{1}{\sqrt{x}}\,dx$

17. $\int \sec^2\theta\,d\theta$

18. $\int \sin\theta\,d\theta$

19. $\int (\sec x\tan x+\csc x\cot x)\,dx$

20. $\int 5e^\theta\,d\theta$

21. $\int 3^t\,dt$

22. $\int \frac{5^t}{2}\,dt$

23. $\int (2t+3)^2\,dt$

24. $\int (t^2+3)(t^3-2t)\,dt$

25. $\int x^2x^3\,dx$

26. $\int e^\pi\,dx$

27. $\int a\,dx$

28. This problem investigates why Theorem 5.1.2 states that $\int \frac{1}{x}\,dx=\ln|x|+C$.
    (a) What is the domain of $y=\ln x$?
    (b) Find $\frac{d}{dx}\big(\ln x\big)$.
    (c) What is the domain of $y=\ln(-x)$?
    (d) Find $\frac{d}{dx}\big(\ln(-x)\big)$.
    (e) You should find that $1/x$ has two types of antiderivatives, depending on whether $x>0$ or $x<0$. In one expression, give a formula for $\int \frac{1}{x}\,dx$ that takes these different domains into account, and explain your answer.

In Exercises 29 – 39, find $f(x)$ described by the given initial value problem.

29. $f'(x)=\sin x$ and $f(0)=2$

30. $f'(x)=5e^x$ and $f(0)=10$

31. $f'(x)=4x^3-3x^2$ and $f(-1)=9$

32. $f'(x)=\sec^2 x$ and $f(\pi/4)=5$

33. $f'(x)=7^x$ and $f(2)=1$

34. $f''(x)=5$ and $f'(0)=7$, $f(0)=3$

35. $f''(x)=7x$ and $f'(1)=-1$, $f(1)=10$

36. $f''(x) = 5e^x$ and $f'(0) = 3, f(0) = 5$

37. $f''(\theta) = \sin\theta$ and $f'(\pi) = 2, f(\pi) = 4$

38. $f''(x) = 24x^2 + 2^x - \cos x$ and $f'(0) = 5, f(0) = 0$

39. $f''(x) = 0$ and $f'(1) = 3, f(1) = 1$

Review

40. Use information gained from the first and second derivatives to sketch $f(x) = \dfrac{1}{e^x + 1}$.

41. Given $y = x^2 e^x \cos x$, find dy.

Exercises 5.2

Terms and Concepts

1. What is “total signed area”?

2. What is “displacement”?

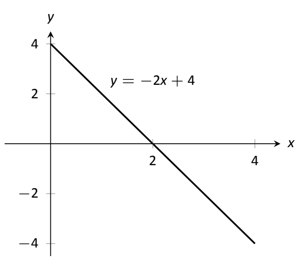
3. What is $\int_3^3 \sin x\, dx$?

4. Give a single definite integral that has the same value as

$$\int_0^1 (2x+3)\, dx + \int_1^2 (2x+3)\, dx.$$

Problems

In Exercises 5 – 10, a graph of a function $f(x)$ is given. Using the geometry of the graph, evaluate the definite integrals.

5. 

(a) $\int_0^1 (-2x+4)\, dx$

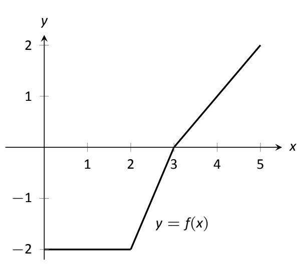
(b) $\int_0^2 (-2x+4)\, dx$

(c) $\int_0^3 (-2x+4)\, dx$

(d) $\int_1^3 (-2x+4)\, dx$

(e) $\int_2^4 (-2x+4)\, dx$

(f) $\int_0^1 (-6x+12)\, dx$

6. 

(a) $\int_0^2 f(x)\, dx$

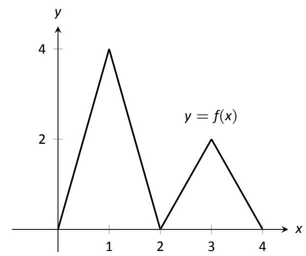
(b) $\int_0^3 f(x)\, dx$

(c) $\int_0^5 f(x)\, dx$

(d) $\int_2^5 f(x)\, dx$

(e) $\int_5^3 f(x)\, dx$

(f) $\int_0^3 -2f(x)\, dx$

7. 

(a) $\int_0^2 f(x)\, dx$

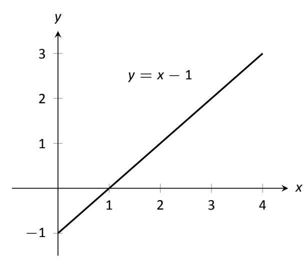
(b) $\int_2^4 f(x)\, dx$

(c) $\int_2^4 2f(x)\, dx$

(d) $\int_0^1 4x\, dx$

(e) $\int_2^3 (2x-4)\, dx$

(f) $\int_2^3 (4x-8)\, dx$

8. 

(a) $\int_0^1 (x-1)\, dx$

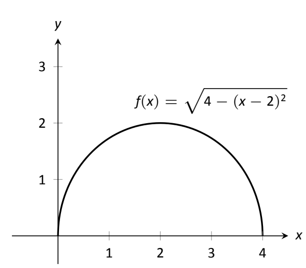
(b) $\int_0^2 (x-1)\, dx$

(c) $\int_0^3 (x-1)\, dx$

(d) $\int_2^3 (x-1)\, dx$

(e) $\int_1^4 (x-1)\, dx$

(f) $\int_1^4 \big((x-1)+1\big)\, dx$

9. 

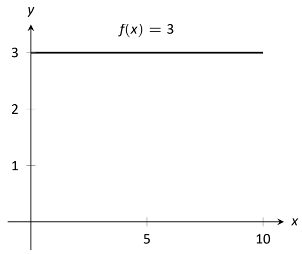
(a) $\int_0^2 f(x)\, dx$

(b) $\int_2^4 f(x)\, dx$

(c) $\int_0^4 f(x)\, dx$

(d) $\int_0^4 5f(x)\, dx$

10.



(a) $\int_0^5 f(x)\,dx$

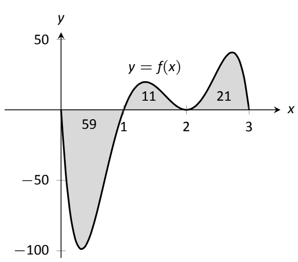
(b) $\int_3^7 f(x)\,dx$

(c) $\int_0^0 f(x)\,dx$

(d) $\int_a^b f(x)\,dx$, where $0 \leq a \leq b \leq 10$

In Exercises 11 – 14, a graph of a function $f(x)$ is given; the numbers inside the shaded regions give the area of that region. Evaluate the definite integrals using this area information.

11.



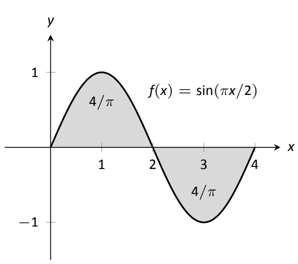
(a) $\int_0^1 f(x)\,dx$

(b) $\int_0^2 f(x)\,dx$

(c) $\int_0^3 f(x)\,dx$

(d) $\int_1^2 -3f(x)\,dx$

12.



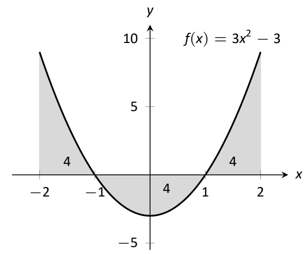
(a) $\int_0^2 f(x)\,dx$

(b) $\int_2^4 f(x)\,dx$

(c) $\int_0^4 f(x)\,dx$

(d) $\int_0^1 f(x)\,dx$

13.



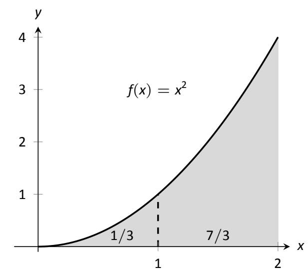
(a) $\int_{-2}^{-1} f(x)\,dx$

(b) $\int_1^2 f(x)\,dx$

(c) $\int_{-1}^1 f(x)\,dx$

(d) $\int_0^1 f(x)\,dx$

14.



(a) $\int_0^2 5x^2\,dx$

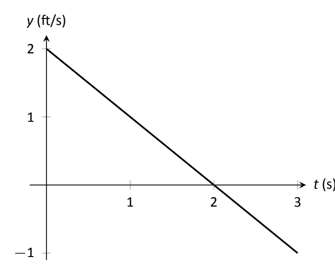
(b) $\int_0^2 (x^2 + 3)\,dx$

(c) $\int_1^3 (x-1)^2\,dx$

(d) $\int_2^4 \left((x-2)^2 + 5\right)\,dx$

In Exercises 15 – 16, a graph of the velocity function of an object moving in a straight line is given. Answer the questions based on that graph.

15.

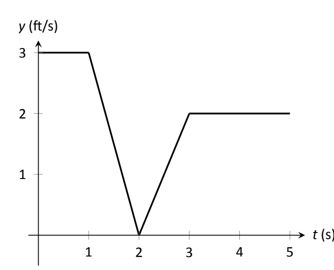


(a) What is the object's maximum velocity?

(b) What is the object's maximum displacement?

(c) What is the object's total displacement on $[0, 3]$?

16.



(a) What is the object's maximum velocity?

(b) What is the object's maximum displacement?

(c) What is the object's total displacement on $[0, 5]$?

17. An object is thrown straight up with a velocity, in ft/s, given by $v(t) = -32t + 64$, where t is in seconds, from a height of 48 feet.
    (a) What is the object's maximum velocity?
    (b) What is the object's maximum displacement?
    (c) When does the maximum displacement occur?
    (d) When will the object reach a height of 0? (Hint: find when the displacement is -48ft.)

18. An object is thrown straight up with a velocity, in ft/s, given by $v(t) = -32t + 96$, where t is in seconds, from a height of 64 feet.
    (a) What is the object's initial velocity?
    (b) When is the object's displacement 0?
    (c) How long does it take for the object to return to its initial height?
    (d) When will the object reach a height of 210 feet?

In Exercises 19 – 22, let

- $\displaystyle\int_0^2 f(x)\,dx = 5,$
- $\displaystyle\int_0^3 f(x)\,dx = 7,$
- $\displaystyle\int_0^2 g(x)\,dx = -3,$ **and**
- $\displaystyle\int_2^3 g(x)\,dx = 5.$

Use these values to evaluate the given definite integrals.

19. $\displaystyle\int_0^2 \big(f(x) + g(x)\big)\,dx$

20. $\displaystyle\int_0^3 \big(f(x) - g(x)\big)\,dx$

21. $\displaystyle\int_2^3 \big(3f(x) + 2g(x)\big)\,dx$

22. Find nonzero values for a and b such that $\displaystyle\int_0^3 \big(af(x) + bg(x)\big)\,dx = 0$

In Exercises 23 – 26, let

- $\displaystyle\int_0^3 s(t)\,dt = 10,$
- $\displaystyle\int_3^5 s(t)\,dt = 8,$
- $\displaystyle\int_3^5 r(t)\,dt = -1,$ **and**
- $\displaystyle\int_0^5 r(t)\,dt = 11.$

Use these values to evaluate the given definite integrals.

23. $\displaystyle\int_0^3 \big(s(t) + r(t)\big)\,dt$

24. $\displaystyle\int_5^0 \big(s(t) - r(t)\big)\,dt$

25. $\displaystyle\int_3^3 \big(\pi s(t) - 7r(t)\big)\,dt$

26. Find nonzero values for a and b such that $\displaystyle\int_0^5 \big(ar(t) + bs(t)\big)\,dt = 0$

Review

In Exercises 27 – 30, evaluate the given indefinite integral.

27. $\displaystyle\int \big(x^3 - 2x^2 + 7x - 9\big)\,dx$

28. $\displaystyle\int \big(\sin x - \cos x + \sec^2 x\big)\,dx$

29. $\displaystyle\int \big(\sqrt[3]{t} + \frac{1}{t^2} + 2^t\big)\,dt$

30. $\displaystyle\int \left(\frac{1}{x} - \csc x \cot x\right)\,dx$

Exercises 5.3

Terms and Concepts

1. A fundamental calculus technique is to use _________ to refine approximations to get an exact answer.

2. What is the upper bound in the summation $\displaystyle\sum_{i=7}^{14}(48i-201)$?

3. This section approximates definite integrals using what geometric shape?

4. T/F: A sum using the Right Hand Rule is an example of a Riemann Sum.

Problems

In Exercises 5 – 12, write out each term of the summation and compute the sum.

5. $\displaystyle\sum_{i=2}^{4} i^2$

6. $\displaystyle\sum_{i=-1}^{3}(4i-2)$

7. $\displaystyle\sum_{i=-2}^{2}\sin(\pi i/2)$

8. $\displaystyle\sum_{i=1}^{10} 5$

9. $\displaystyle\sum_{i=1}^{5}\frac{1}{i}$

10. $\displaystyle\sum_{i=1}^{6}(-1)^i i$

11. $\displaystyle\sum_{i=1}^{4}\left(\frac{1}{i}-\frac{1}{i+1}\right)$

12. $\displaystyle\sum_{i=0}^{5}(-1)^i\cos(\pi i)$

In Exercises 13 – 16, write each sum in summation notation.

13. $3+6+9+12+15$

14. $-1+0+3+8+15+24+35+48+63$

15. $\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}+\dfrac{4}{5}$

16. $1-e+e^2-e^3+e^4$

In Exercises 17 – 24, evaluate the summation using Theorem 5.3.1.

17. $\displaystyle\sum_{i=1}^{10} 5$

18. $\displaystyle\sum_{i=1}^{25} i$

19. $\displaystyle\sum_{i=1}^{10}(3i^2-2i)$

20. $\displaystyle\sum_{i=1}^{15}(2i^3-10)$

21. $\displaystyle\sum_{i=1}^{10}(-4i^3+10i^2-7i+11)$

22. $\displaystyle\sum_{i=1}^{10}(i^3-3i^2+2i+7)$

23. $1+2+3+\ldots+99+100$

24. $1+4+9+\ldots+361+400$

Theorem 5.3.1 states

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{k} a_i + \sum_{i=k+1}^{n} a_i \text{, so}$$

$$\sum_{i=k+1}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{k} a_i \text{.}$$

Use this fact, along with other parts of Theorem 5.3.1, to evaluate the summations given in Exercises 25 – 28.

25. $\displaystyle\sum_{i=11}^{20} i$

26. $\displaystyle\sum_{i=16}^{25} i^3$

27. $\displaystyle\sum_{i=7}^{12} 4$

28. $\displaystyle\sum_{i=5}^{10} 4i^3$

In Exercises 29 – 34, a definite integral $\int_a^b f(x)\,dx$ is given.

(a) Graph $f(x)$ on $[a, b]$.

(b) Add to the sketch rectangles using the provided rule.

(c) Approximate $\int_a^b f(x)\,dx$ by summing the areas of the rectangles.

29. $\int_{-3}^{3} x^2\,dx$, with 6 rectangles using the Left Hand Rule.

30. $\int_0^2 (5 - x^2)\,dx$, with 4 rectangles using the Midpoint Rule.

31. $\int_0^{\pi} \sin x\,dx$, with 6 rectangles using the Right Hand Rule.

32. $\int_0^3 2^x\,dx$, with 5 rectangles using the Left Hand Rule.

33. $\int_1^2 \ln x\,dx$, with 3 rectangles using the Midpoint Rule.

34. $\int_1^9 \frac{1}{x}\,dx$, with 4 rectangles using the Right Hand Rule.

In Exercises 35 – 40, a definite integral $\int_a^b f(x)\,dx$ is given. As demonstrated in Examples 5.3.6 and 5.3.7, do the following.

(a) Find a formula to approximate $\int_a^b f(x)\,dx$ using n subintervals and the provided rule.

(b) Evaluate the formula using $n = 10$, 100 and $1,000$.

(c) Find the limit of the formula, as $n \to \infty$, to find the exact value of $\int_a^b f(x)\,dx$.

35. $\int_0^1 x^3\,dx$, using the Right Hand Rule.

36. $\int_{-1}^{1} 3x^2\,dx$, using the Left Hand Rule.

37. $\int_{-1}^{3} (3x - 1)\,dx$, using the Midpoint Rule.

38. $\int_1^4 (2x^2 - 3)\,dx$, using the Left Hand Rule.

39. $\int_{-10}^{10} (5 - x)\,dx$, using the Right Hand Rule.

40. $\int_0^1 (x^3 - x^2)\,dx$, using the Right Hand Rule.

Review

In Exercises 41 – 46, find an antiderivative of the given function.

41. $f(x) = 5\sec^2 x$

42. $f(x) = \frac{7}{x}$

43. $g(t) = 4t^5 - 5t^3 + 8$

44. $g(t) = 5 \cdot 8^t$

45. $g(t) = \cos t + \sin t$

46. $f(x) = \frac{1}{\sqrt{x}}$

Exercises 5.4

Terms and Concepts

1. How are definite and indefinite integrals related?

2. What constant of integration is most commonly used when evaluating definite integrals?

3. T/F: If f is a continuous function, then $F(x) = \int_a^x f(t)\, dt$ is also a continuous function.

4. The definite integral can be used to find "the area under a curve." Give two other uses for definite integrals.

Problems

In Exercises 5 – 28, evaluate the definite integral.

5. $\int_1^3 (3x^2 - 2x + 1)\, dx$

6. $\int_0^4 (x-1)^2\, dx$

7. $\int_{-1}^1 (x^3 - x^5)\, dx$

8. $\int_{\pi/2}^{\pi} \cos x\, dx$

9. $\int_0^{\pi/4} \sec^2 x\, dx$

10. $\int_1^e \frac{1}{x}\, dx$

11. $\int_{-1}^1 5^x\, dx$

12. $\int_{-2}^{-1} (4 - 2x^3)\, dx$

13. $\int_0^{\pi} (2\cos x - 2\sin x)\, dx$

14. $\int_1^3 e^x\, dx$

15. $\int_0^4 \sqrt{t}\, dt$

16. $\int_9^{25} \frac{1}{\sqrt{t}}\, dt$

17. $\int_1^8 \sqrt[3]{x}\, dx$

18. $\int_1^2 \frac{1}{x}\, dx$

19. $\int_1^2 \frac{1}{x^2}\, dx$

20. $\int_1^2 \frac{1}{x^3}\, dx$

21. $\int_0^1 x\, dx$

22. $\int_0^1 x^2\, dx$

23. $\int_0^1 x^3\, dx$

24. $\int_0^1 x^{100}\, dx$

25. $\int_{-4}^4 dx$

26. $\int_{-10}^{-5} 3\, dx$

27. $\int_{-2}^2 0\, dx$

28. $\int_{\pi/6}^{\pi/3} \csc x \cot x\, dx$

29. Explain why:

    (a) $\int_{-1}^1 x^n\, dx = 0$, when n is a positive, odd integer, and

    (b) $\int_{-1}^1 x^n\, dx = 2\int_0^1 x^n\, dx$ when n is a positive, even integer.

30. Explain why $\int_a^{a+2\pi} \sin t\, dt = 0$ for all values of a.

In Exercises 31 – 34, find a value c guaranteed by the Mean Value Theorem.

31. $\displaystyle\int_0^2 x^2\,dx$

32. $\displaystyle\int_{-2}^2 x^2\,dx$

33. $\displaystyle\int_0^1 e^x\,dx$

34. $\displaystyle\int_0^{16} \sqrt{x}\,dx$

In Exercises 35 – 40, find the average value of the function on the given interval.

35. $f(x) = \sin x$ on $[0, \pi/2]$

36. $y = \sin x$ on $[0, \pi]$

37. $y = x$ on $[0, 4]$

38. $y = x^2$ on $[0, 4]$

39. $y = x^3$ on $[0, 4]$

40. $g(t) = 1/t$ on $[1, e]$

In Exercises 41 – 46, a velocity function of an object moving along a straight line is given. Find the displacement of the object over the given time interval.

41. $v(t) = -32t + 20$ft/s on $[0, 5]$

42. $v(t) = -32t + 200$ft/s on $[0, 10]$

43. $v(t) = 10$ft/s on $[0, 3]$.

44. $v(t) = 2^t$mph on $[-1, 1]$

45. $v(t) = \cos t$ ft/s on $[0, 3\pi/2]$

46. $v(t) = \sqrt[4]{t}$ ft/s on $[0, 16]$

In Exercises 47 – 50, an acceleration function of an object moving along a straight line is given. Find the change of the object's velocity over the given time interval.

47. $a(t) = -32$ft/s^2 on $[0, 2]$

48. $a(t) = 10$ft/s^2 on $[0, 5]$

49. $a(t) = t$ ft/s^2 on $[0, 2]$

50. $a(t) = \cos t$ ft/s^2 on $[0, \pi]$

In Exercises 51 – 54, sketch the given functions and find the area of the enclosed region.

51. $y = 2x$, $y = 5x$, and $x = 3$.

52. $y = -x + 1$, $y = 3x + 6$, $x = 2$ and $x = -1$.

53. $y = x^2 - 2x + 5$, $y = 5x - 5$.

54. $y = 2x^2 + 2x - 5$, $y = x^2 + 3x + 7$.

In Exercises 55 – 58, find $F'(x)$.

55. $\displaystyle F(x) = \int_2^{x^3+x} \frac{1}{t}\,dt$

56. $\displaystyle F(x) = \int_{x^3}^0 t^3\,dt$

57. $\displaystyle F(x) = \int_x^{x^2} (t+2)\,dt$

58. $\displaystyle F(x) = \int_{\ln x}^{e^x} \sin t\,dt$

Exercises 5.5

Terms and Concepts

1. T/F: Simpson's Rule is a method of approximating antiderivatives.

2. What are the two basic situations where approximating the value of a definite integral is necessary?

3. Why are the Left and Right Hand Rules rarely used?

4. Simpson's Rule is based on approximating portions of a function with what type of function?

Problems

In Exercises 5 – 12, a definite integral is given.

(a) Approximate the definite integral with the Trapezoidal Rule and $n = 4$.

(b) Approximate the definite integral with Simpson's Rule and $n = 4$.

(c) Find the exact value of the integral.

5. $\displaystyle\int_{-1}^{1} x^2\, dx$

6. $\displaystyle\int_{0}^{10} 5x\, dx$

7. $\displaystyle\int_{0}^{\pi} \sin x\, dx$

8. $\displaystyle\int_{0}^{4} \sqrt{x}\, dx$

9. $\displaystyle\int_{0}^{3} (x^3 + 2x^2 - 5x + 7)\, dx$

10. $\displaystyle\int_{0}^{1} x^4\, dx$

11. $\displaystyle\int_{0}^{2\pi} \cos x\, dx$

12. $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

In Exercises 13 – 20, approximate the definite integral with the Trapezoidal Rule and Simpson's Rule, with $n = 6$.

13. $\displaystyle\int_{0}^{1} \cos\left(x^2\right)\, dx$

14. $\displaystyle\int_{-1}^{1} e^{x^2}\, dx$

15. $\displaystyle\int_{0}^{5} \sqrt{x^2 + 1}\, dx$

16. $\displaystyle\int_{0}^{\pi} x \sin x\, dx$

17. $\displaystyle\int_{0}^{\pi/2} \sqrt{\cos x}\, dx$

18. $\displaystyle\int_{1}^{4} \ln x\, dx$

19. $\displaystyle\int_{-1}^{1} \frac{1}{\sin x + 2}\, dx$

20. $\displaystyle\int_{0}^{6} \frac{1}{\sin x + 2}\, dx$

In Exercises 21 – 24, find n such that the error in approximating the given definite integral is less than 0.0001 when using:

(a) the Trapezoidal Rule

(b) Simpson's Rule

21. $\displaystyle\int_{0}^{\pi} \sin x\, dx$

22. $\displaystyle\int_{1}^{4} \frac{1}{\sqrt{x}}\, dx$

23. $\displaystyle\int_{0}^{\pi} \cos\left(x^2\right)\, dx$

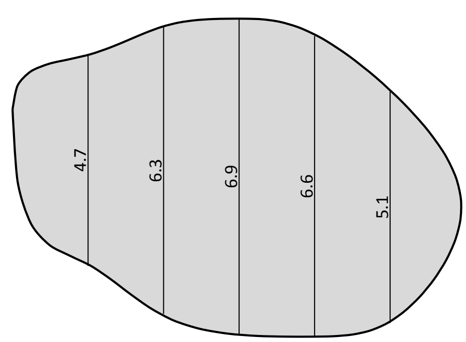
24. $\displaystyle\int_{0}^{5} x^4\, dx$

In Exercises 25 – 26, a region is given. Find the area of the region using Simpson's Rule:

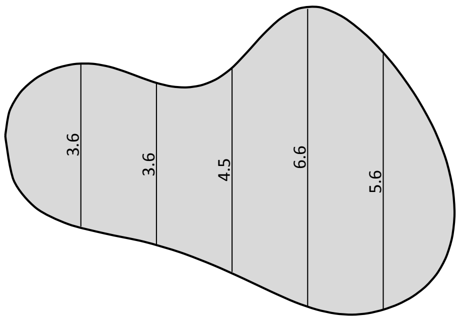
(a) where the measurements are in centimeters, taken in 1 cm increments, and

(b) where the measurements are in hundreds of yards, taken in 100 yd increments.

25.



26.



Exercises 6.1

Terms and Concepts

1. Substitution "undoes" what derivative rule?

2. T/F: One can use algebra to rewrite the integrand of an integral to make it easier to evaluate.

Problems

In Exercises 3 – 14, evaluate the indefinite integral to develop an understanding of Substitution.

3. $\displaystyle\int 3x^2\left(x^3-5\right)^7\,dx$

4. $\displaystyle\int (2x-5)\left(x^2-5x+7\right)^3\,dx$

5. $\displaystyle\int x\left(x^2+1\right)^8\,dx$

6. $\displaystyle\int (12x+14)\left(3x^2+7x-1\right)^5\,dx$

7. $\displaystyle\int \frac{1}{2x+7}dx$

8. $\displaystyle\int \frac{1}{\sqrt{2x+3}}dx$

9. $\displaystyle\int \frac{x}{\sqrt{x+3}}dx$

10. $\displaystyle\int \frac{x^3-x}{\sqrt{x}}dx$

11. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

12. $\displaystyle\int \frac{x^4}{\sqrt{x^5+1}}dx$

13. $\displaystyle\int \frac{\frac{1}{x}+1}{x^2}dx$

14. $\displaystyle\int \frac{\ln(x)}{x}dx$

In Exercises 15 – 24, use Substitution to evaluate the indefinite integral involving trigonometric functions.

15. $\displaystyle\int \sin^2(x)\cos(x)dx$

16. $\displaystyle\int \cos^3(x)\sin(x)dx$

17. $\displaystyle\int \cos(3-6x)dx$

18. $\displaystyle\int \sec^2(4-x)dx$

19. $\displaystyle\int \sec(2x)dx$

20. $\displaystyle\int \tan^2(x)\sec^2(x)dx$

21. $\displaystyle\int x\cos\left(x^2\right)\,dx$

22. $\displaystyle\int \tan^2(x)dx$

23. $\displaystyle\int \cot x\,dx$. Do not just refer to Theorem 6.1.2 for the answer; justify it through Substitution.

24. $\displaystyle\int \csc x\,dx$. Do not just refer to Theorem 6.1.2 for the answer; justify it through Substitution.

In Exercises 25 – 32, use Substitution to evaluate the indefinite integral involving exponential functions.

25. $\displaystyle\int e^{3x-1}dx$

26. $\displaystyle\int e^{x^3}x^2dx$

27. $\displaystyle\int e^{x^2-2x+1}(x-1)dx$

28. $\displaystyle\int \frac{e^x+1}{e^x}dx$

29. $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

30. $\displaystyle\int \frac{e^x-e^{-x}}{e^{2x}}dx$

31. $\displaystyle\int 3^{3x}dx$

32. $\displaystyle\int 4^{2x}dx$

In Exercises 33 – 36, use Substitution to evaluate the indefinite integral involving logarithmic functions.

33. $\displaystyle\int \frac{\ln x}{x}dx$

34. $\displaystyle\int \frac{\left(\ln x\right)^2}{x}dx$

35. $\displaystyle\int \frac{\ln\left(x^3\right)}{x}\,dx$

36. $\displaystyle\int \frac{1}{x\ln\left(x^2\right)}\,dx$

In Exercises 37 – 42, use Substitution to evaluate the indefinite integral involving rational functions.

37. $\displaystyle\int \frac{x^2+3x+1}{x}\,dx$

38. $\displaystyle\int \frac{x^3+x^2+x+1}{x}\,dx$

39. $\displaystyle\int \frac{x^3-1}{x+1}\,dx$

40. $\displaystyle\int \frac{x^2+2x-5}{x-3}\,dx$

41. $\displaystyle\int \frac{3x^2-5x+7}{x+1}\,dx$

42. $\displaystyle\int \frac{x^2+2x+1}{x^3+3x^2+3x}\,dx$

In Exercises 43 – 52, use Substitution to evaluate the indefinite integral involving inverse trigonometric functions.

43. $\displaystyle\int \frac{7}{x^2+7}\,dx$

44. $\displaystyle\int \frac{3}{\sqrt{9-x^2}}\,dx$

45. $\displaystyle\int \frac{14}{\sqrt{5-x^2}}\,dx$

46. $\displaystyle\int \frac{2}{x\sqrt{x^2-9}}\,dx$

47. $\displaystyle\int \frac{5}{\sqrt{x^4-16x^2}}\,dx$

48. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

49. $\displaystyle\int \frac{1}{x^2-2x+8}\,dx$

50. $\displaystyle\int \frac{2}{\sqrt{-x^2+6x+7}}\,dx$

51. $\displaystyle\int \frac{3}{\sqrt{-x^2+8x+9}}\,dx$

52. $\displaystyle\int \frac{5}{x^2+6x+34}\,dx$

In Exercises 53 – 78, evaluate the indefinite integral.

53. $\displaystyle\int \frac{x^2}{\left(x^3+3\right)^2}\,dx$

54. $\displaystyle\int \left(3x^2+2x\right)\left(5x^3+5x^2+2\right)^8\,dx$

55. $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$

56. $\displaystyle\int x^2\csc^2\left(x^3+1\right)\,dx$

57. $\displaystyle\int \sin(x)\sqrt{\cos(x)}\,dx$

58. $\displaystyle\int \sin\left(5x+1\right)\,dx$

59. $\displaystyle\int \frac{1}{x-5}\,dx$

60. $\displaystyle\int \frac{7}{3x+2}\,dx$

61. $\displaystyle\int \frac{3x^3+4x^2+2x-22}{x^2+3x+5}\,dx$

62. $\displaystyle\int \frac{2x+7}{x^2+7x+3}\,dx$

63. $\displaystyle\int \frac{9(2x+3)}{3x^2+9x+7}\,dx$

64. $\displaystyle\int \frac{-x^3+14x^2-46x-7}{x^2-7x+1}\,dx$

65. $\displaystyle\int \frac{x}{x^4+81}\,dx$

66. $\displaystyle\int \frac{2}{4x^2+1}\,dx$

67. $\displaystyle\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

68. $\displaystyle\int \frac{1}{\sqrt{16-9x^2}}\,dx$

69. $\displaystyle\int \frac{3x-2}{x^2-2x+10}\,dx$

70. $\displaystyle\int \frac{7-2x}{x^2+12x+61}\,dx$

71. $\displaystyle\int \frac{x^2+5x-2}{x^2-10x+32}\,dx$

72. $\displaystyle\int \frac{x^3}{x^2+9}\,dx$

73. $\displaystyle\int \frac{x^3 - x}{x^2 + 4x + 9} dx$

74. $\displaystyle\int \frac{\sin(x)}{\cos^2(x) + 1} dx$

75. $\displaystyle\int \frac{\cos(x)}{\sin^2(x) + 1} dx$

76. $\displaystyle\int \frac{\cos(x)}{1 - \sin^2(x)} dx$

77. $\displaystyle\int \frac{3x - 3}{\sqrt{x^2 - 2x - 6}} dx$

78. $\displaystyle\int \frac{x - 3}{\sqrt{x^2 - 6x + 8}} dx$

In Exercises 79 – 86, evaluate the definite integral.

79. $\displaystyle\int_1^3 \frac{1}{x - 5} dx$

80. $\displaystyle\int_2^6 x\sqrt{x - 2} dx$

81. $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$

82. $\displaystyle\int_0^1 2x(1 - x^2)^4 \, dx$

83. $\displaystyle\int_{-2}^{-1} (x + 1)e^{x^2+2x+1} \, dx$

84. $\displaystyle\int_{-1}^1 \frac{1}{1 + x^2} \, dx$

85. $\displaystyle\int_2^4 \frac{1}{x^2 - 6x + 10} \, dx$

86. $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}} \, dx$

Exercises 6.2

Terms and Concepts

1. T/F: Integration by Parts is useful in evaluating integrands that contain products of functions.

2. T/F: Integration by Parts can be thought of as the "opposite of the Chain Rule."

3. For what is "LIATE" useful?

4. T/F: If the integral that results from Integration by Parts appears to also need Integration by Parts, then a mistake was made in the orginal choice of "u".

Problems

In Exercises 5 – 34, evaluate the given indefinite integral.

5. $\displaystyle\int x\sin x\,dx$

6. $\displaystyle\int xe^{-x}\,dx$

7. $\displaystyle\int x^2\sin x\,dx$

8. $\displaystyle\int x^3\sin x\,dx$

9. $\displaystyle\int xe^{x^2}\,dx$

10. $\displaystyle\int x^3e^x\,dx$

11. $\displaystyle\int xe^{-2x}\,dx$

12. $\displaystyle\int e^x\sin x\,dx$

13. $\displaystyle\int e^{2x}\cos x\,dx$

14. $\displaystyle\int e^{2x}\sin(3x)\,dx$

15. $\displaystyle\int e^{5x}\cos(5x)\,dx$

16. $\displaystyle\int \sin x\cos x\,dx$

17. $\displaystyle\int \sin^{-1}x\,dx$

18. $\displaystyle\int \tan^{-1}(2x)\,dx$

19. $\displaystyle\int x\tan^{-1}x\,dx$

20. $\displaystyle\int \sin^{-1}x\,dx$

21. $\displaystyle\int x\ln x\,dx$

22. $\displaystyle\int (x-2)\ln x\,dx$

23. $\displaystyle\int x\ln(x-1)\,dx$

24. $\displaystyle\int x\ln(x^2)\,dx$

25. $\displaystyle\int x^2\ln x\,dx$

26. $\displaystyle\int (\ln x)^2\,dx$

27. $\displaystyle\int (\ln(x+1))^2\,dx$

28. $\displaystyle\int x\sec^2x\,dx$

29. $\displaystyle\int x\csc^2x\,dx$

30. $\displaystyle\int x\sqrt{x-2}\,dx$

31. $\displaystyle\int x\sqrt{x^2-2}\,dx$

32. $\displaystyle\int \sec x\tan x\,dx$

33. $\displaystyle\int x\sec x\tan x\,dx$

34. $\displaystyle\int x\csc x\cot x\,dx$

In Exercises 35 – 40, evaluate the indefinite integral after first making a substitution.

35. $\displaystyle\int \sin(\ln x)\,dx$

36. $\displaystyle\int e^{2x}\cos\left(e^x\right)\,dx$

37. $\displaystyle\int \sin(\sqrt{x})\,dx$

38. $\displaystyle\int \ln(\sqrt{x})\,dx$

39. $\displaystyle\int e^{\sqrt{x}}\,dx$

40. $\displaystyle\int e^{\ln x}\,dx$

In Exercises 41 – 49, evaluate the definite integral. Note: the corresponding indefinite integrals appear in Exercises 5 – 13.

41. $\displaystyle\int_0^{\pi} x\sin x\,dx$

42. $\displaystyle\int_{-1}^{1} xe^{-x}\,dx$

43. $\displaystyle\int_{-\pi/4}^{\pi/4} x^2\sin x\,dx$

44. $\displaystyle\int_{-\pi/2}^{\pi/2} x^3\sin x\,dx$

45. $\displaystyle\int_0^{\sqrt{\ln 2}} xe^{x^2}\,dx$

46. $\displaystyle\int_0^{1} x^3e^x\,dx$

47. $\displaystyle\int_1^{2} xe^{-2x}\,dx$

48. $\displaystyle\int_0^{\pi} e^x\sin x\,dx$

49. $\displaystyle\int_{-\pi/2}^{\pi/2} e^{2x}\cos x\,dx$

Exercises 6.3

Terms and Concepts

1. T/F: $\int \sin^2 x \cos^2 x\, dx$ cannot be evaluated using the techniques described in this section since both powers of $\sin x$ and $\cos x$ are even.

2. T/F: $\int \sin^3 x \cos^3 x\, dx$ cannot be evaluated using the techniques described in this section since both powers of $\sin x$ and $\cos x$ are odd.

3. T/F: This section addresses how to evaluate indefinite integrals such as $\int \sin^5 x \tan^3 x\, dx$.

4. T/F: Sometimes computer programs evaluate integrals involving trigonometric functions differently than one would using the techniques of this section. When this is the case, the techniques of this section have failed and one should only trust the answer given by the computer.

Problems

In Exercises 5 – 28, evaluate the indefinite integral.

5. $\int \sin x \cos^4 x\, dx$

6. $\int \sin^3 x \cos x\, dx$

7. $\int \sin^3 x \cos^2 x\, dx$

8. $\int \sin^3 x \cos^3 x\, dx$

9. $\int \sin^6 x \cos^5 x\, dx$

10. $\int \sin^2 x \cos^7 x\, dx$

11. $\int \sin^2 x \cos^2 x\, dx$

12. $\int \sin x \cos x\, dx$

13. $\int \sin(5x)\cos(3x)\, dx$

14. $\int \sin(x)\cos(2x)\, dx$

15. $\int \sin(3x)\sin(7x)\, dx$

16. $\int \sin(\pi x)\sin(2\pi x)\, dx$

17. $\int \cos(x)\cos(2x)\, dx$

18. $\int \cos\left(\frac{\pi}{2}x\right)\cos(\pi x)\, dx$

19. $\int \tan^4 x \sec^2 x\, dx$

20. $\int \tan^2 x \sec^4 x\, dx$

21. $\int \tan^3 x \sec^4 x\, dx$

22. $\int \tan^3 x \sec^2 x\, dx$

23. $\int \tan^3 x \sec^3 x\, dx$

24. $\int \tan^5 x \sec^5 x\, dx$

25. $\int \tan^4 x\, dx$

26. $\int \sec^5 x\, dx$

27. $\int \tan^2 x \sec x\, dx$

28. $\int \tan^2 x \sec^3 x\, dx$

In Exercises 29 – 35, evaluate the definite integral. Note: the corresponding indefinite integrals appear in the previous set.

29. $\int_0^{\pi} \sin x \cos^4 x\, dx$

30. $\int_{-\pi}^{\pi} \sin^3 x \cos x\, dx$

31. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^7 x\, dx$

32. $\int_0^{\pi/2} \sin(5x)\cos(3x)\, dx$

33. $\int_{-\pi/2}^{\pi/2} \cos(x)\cos(2x)\, dx$

34. $\int_0^{\pi/4} \tan^4 x \sec^2 x\, dx$

35. $\int_{-\pi/4}^{\pi/4} \tan^2 x \sec^4 x\, dx$

Exercises 6.4

Terms and Concepts

1. Fill in the blank: Partial Fraction Decomposition is a method of rewriting ________ functions.

2. T/F: It is sometimes necessary to use polynomial division before using Partial Fraction Decomposition.

3. Decompose $\dfrac{1}{x^2-3x}$ without solving for the coefficients, as done in Example 6.5.1.

4. Decompose $\dfrac{7-x}{x^2-9}$ without solving for the coefficients, as done in Example 6.5.1.

5. Decompose $\dfrac{x-3}{x^2-7}$ without solving for the coefficients, as done in Example 6.5.1.

6. Decompose $\dfrac{2x+5}{x^3+7x}$ without solving for the coefficients, as done in Example 6.5.1.

Problems

In Exercises 7 – 26, evaluate the indefinite integral.

7. $\displaystyle\int \frac{7x+7}{x^2+3x-10}\,dx$

8. $\displaystyle\int \frac{7x-2}{x^2+x}\,dx$

9. $\displaystyle\int \frac{-4}{3x^2-12}\,dx$

10. $\displaystyle\int \frac{6x+4}{3x^2+4x+1}\,dx$

11. $\displaystyle\int \frac{x+7}{(x+5)^2}\,dx$

12. $\displaystyle\int \frac{-3x-20}{(x+8)^2}\,dx$

13. $\displaystyle\int \frac{9x^2+11x+7}{x(x+1)^2}\,dx$

14. $\displaystyle\int \frac{-12x^2-x+33}{(x-1)(x+3)(3-2x)}\,dx$

15. $\displaystyle\int \frac{94x^2-10x}{(7x+3)(5x-1)(3x-1)}\,dx$

16. $\displaystyle\int \frac{x^2+x+1}{x^2+x-2}\,dx$

17. $\displaystyle\int \frac{x^3}{x^2-x-20}\,dx$

18. $\displaystyle\int \frac{2x^2-4x+6}{x^2-2x+3}\,dx$

19. $\displaystyle\int \frac{1}{x^3+2x^2+3x}\,dx$

20. $\displaystyle\int \frac{x^2+x+5}{x^2+4x+10}\,dx$

21. $\displaystyle\int \frac{12x^2+21x+3}{(x+1)(3x^2+5x-1)}\,dx$

22. $\displaystyle\int \frac{6x^2+8x-4}{(x-3)(x^2+6x+10)}\,dx$

23. $\displaystyle\int \frac{2x^2+x+1}{(x+1)(x^2+9)}\,dx$

24. $\displaystyle\int \frac{x^2-20x-69}{(x-7)(x^2+2x+17)}\,dx$

25. $\displaystyle\int \frac{9x^2-60x+33}{(x-9)(x^2-2x+11)}\,dx$

26. $\displaystyle\int \frac{6x^2+45x+121}{(x+2)(x^2+10x+27)}\,dx$

In Exercises 27 – 30, evaluate the definite integral.

27. $\displaystyle\int_1^2 \frac{8x+21}{(x+2)(x+3)}\,dx$

28. $\displaystyle\int_0^5 \frac{14x+6}{(3x+2)(x+4)}\,dx$

29. $\displaystyle\int_{-1}^1 \frac{x^2+5x-5}{(x-10)(x^2+4x+5)}\,dx$

30. $\displaystyle\int_0^1 \frac{x}{(x+1)(x^2+2x+1)}\,dx$

Exercises 6.5

Terms and Concepts

1. In Key Idea 6.6.1, the equation $\int \tanh x\, dx = \ln(\cosh x) + C$ is given. Why is "$\ln|\cosh x|$" not used – i.e., why are absolute values not necessary?

2. The hyperbolic functions are used to define points on the right hand portion of the hyperbola $x^2 - y^2 = 1$, as shown in Figure 6.6.1. How can we use the hyperbolic functions to define points on the left hand portion of the hyperbola?

Problems

In Exercises 3 – 10, verify the given identity using Definition 6.6.1, as done in Example 6.6.1.

3. $\coth^2 x - \operatorname{csch}^2 x = 1$

4. $\cosh 2x = \cosh^2 x + \sinh^2 x$

5. $\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

6. $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

7. $\dfrac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$

8. $\dfrac{d}{dx}[\coth x] = -\operatorname{csch}^2 x$

9. $\int \tanh x\, dx = \ln(\cosh x) + C$

10. $\int \coth x\, dx = \ln|\sinh x| + C$

In Exercises 11 – 22, find the derivative of the given function.

11. $f(x) = \sinh 2x$

12. $f(x) = \cosh^2 x$

13. $f(x) = \tanh(x^2)$

14. $f(x) = \ln(\sinh x)$

15. $f(x) = \sinh x \cosh x$

16. $f(x) = x \sinh x - \cosh x$

17. $f(x) = \operatorname{sech}^{-1}(x^2)$

18. $f(x) = \sinh^{-1}(3x)$

19. $f(x) = \cosh^{-1}(2x^2)$

20. $f(x) = \tanh^{-1}(x + 5)$

21. $f(x) = \tanh^{-1}(\cos x)$

22. $f(x) = \cosh^{-1}(\sec x)$

In Exercises 23 – 28, find the equation of the line tangent to the function at the given x-value.

23. $f(x) = \sinh x$ at $x = 0$

24. $f(x) = \cosh x$ at $x = \ln 2$

25. $f(x) = \tanh x$ at $x = -\ln 3$

26. $f(x) = \operatorname{sech}^2 x$ at $x = \ln 3$

27. $f(x) = \sinh^{-1} x$ at $x = 0$

28. $f(x) = \cosh^{-1} x$ at $x = \sqrt{2}$

In Exercises 29 – 44, evaluate the given indefinite integral.

29. $\int \tanh(2x)\, dx$

30. $\int \cosh(3x - 7)\, dx$

31. $\int \sinh x \cosh x\, dx$

32. $\int x \cosh x\, dx$

33. $\int x \sinh x\, dx$

34. $\int \dfrac{1}{\sqrt{x^2 + 1}}\, dx$

35. $\int \dfrac{1}{\sqrt{x^2 - 9}}\, dx$

36. $\int \dfrac{1}{9 - x^2}\, dx$

37. $\int \dfrac{2x}{\sqrt{x^4 - 4}}\, dx$

38. $\int \dfrac{\sqrt{x}}{\sqrt{1 + x^3}}\, dx$

39. $\int \dfrac{1}{x^4 - 16}\, dx$

40. $\int \dfrac{1}{x^2 + x}\, dx$

41. $\int \frac{e^x}{e^{2x}+1}\,dx$

42. $\int \sinh^{-1} x\,dx$

43. $\int \tanh^{-1} x\,dx$

44. $\int \operatorname{sech} x\,dx$ (Hint: multiply by $\frac{\cosh x}{\cosh x}$; set $u = \sinh x$.)

In Exercises 45 – 48, evaluate the given definite integral.

45. $\int_{-1}^{1} \sinh x\,dx$

46. $\int_{-\ln 2}^{\ln 2} \cosh x\,dx$

47. $\int_{0}^{1} \tanh^{-1} x\,dx$

48. $\int_{0}^{2} \frac{1}{\sqrt{x^2+1}}\,dx$

Exercises 6.6

Terms and Concepts

1. List the different indeterminate forms described in this section.

2. T/F: l'Hôpital's Rule provides a faster method of computing derivatives.

3. T/F: l'Hôpital's Rule states that $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)}{g'(x)}$.

4. Explain what the indeterminate form "1^∞" means.

5. Fill in the blanks:
   The Quotient Rule is applied to $\frac{f(x)}{g(x)}$ when taking ________;
   l'Hôpital's Rule is applied to $\frac{f(x)}{g(x)}$ when taking certain ________.

6. Create (but do not evaluate!) a limit that returns "∞^0".

7. Create a function $f(x)$ such that $\lim\limits_{x\to 1} f(x)$ returns "0^0".

8. Create a function $f(x)$ such that $\lim\limits_{x\to\infty} f(x)$ returns "$0 \cdot \infty$".

Problems

In Exercises 9 – 54, evaluate the given limit.

9. $\lim\limits_{x\to 1} \frac{x^2+x-2}{x-1}$

10. $\lim\limits_{x\to 2} \frac{x^2+x-6}{x^2-7x+10}$

11. $\lim\limits_{x\to \pi} \frac{\sin x}{x-\pi}$

12. $\lim\limits_{x\to \pi/4} \frac{\sin x - \cos x}{\cos(2x)}$

13. $\lim\limits_{x\to 0} \frac{\sin(5x)}{x}$

14. $\lim\limits_{x\to 0} \frac{\sin(2x)}{x+2}$

15. $\lim\limits_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$

16. $\lim\limits_{x\to 0} \frac{\sin(ax)}{\sin(bx)}$

17. $\lim\limits_{x\to 0^+} \frac{e^x-1}{x^2}$

18. $\lim\limits_{x\to 0^+} \frac{e^x-x-1}{x^2}$

19. $\lim\limits_{x\to 0^+} \frac{x-\sin x}{x^3-x^2}$

20. $\lim\limits_{x\to \infty} \frac{x^4}{e^x}$

21. $\lim\limits_{x\to \infty} \frac{\sqrt{x}}{e^x}$

22. $\lim\limits_{x\to \infty} \frac{1}{x^2}e^x$

23. $\lim\limits_{x\to \infty} \frac{e^x}{\sqrt{x}}$

24. $\lim\limits_{x\to \infty} \frac{e^x}{2^x}$

25. $\lim\limits_{x\to \infty} \frac{e^x}{3^x}$

26. $\lim\limits_{x\to 3} \frac{x^3-5x^2+3x+9}{x^3-7x^2+15x-9}$

27. $\lim\limits_{x\to -2} \frac{x^3+4x^2+4x}{x^3+7x^2+16x+12}$

28. $\lim\limits_{x\to \infty} \frac{\ln x}{x}$

29. $\lim\limits_{x\to \infty} \frac{\ln(x^2)}{x}$

30. $\lim\limits_{x\to \infty} \frac{\left(\ln x\right)^2}{x}$

31. $\lim\limits_{x\to 0^+} x \cdot \ln x$

32. $\lim\limits_{x\to 0^+} \sqrt{x} \cdot \ln x$

33. $\lim\limits_{x\to 0^+} xe^{1/x}$

34. $\lim\limits_{x\to \infty} x^3 - x^2$

35. $\lim\limits_{x\to \infty} \sqrt{x} - \ln x$

36. $\lim\limits_{x\to -\infty} xe^x$

37. $\lim\limits_{x\to 0^+} \frac{1}{x^2}e^{-1/x}$

38. $\lim\limits_{x\to 0^+} (1+x)^{1/x}$

39. $\lim\limits_{x\to 0^+} (2x)^x$

40. $\lim\limits_{x\to 0^+} (2/x)^x$

41. $\lim\limits_{x\to 0^+} (\sin x)^x$ Hint: use the Squeeze Theorem.

42. $\lim\limits_{x\to 1^+} (1-x)^{1-x}$

43. $\lim\limits_{x\to \infty} (x)^{1/x}$

44. $\lim\limits_{x\to \infty} (1/x)^x$

45. $\lim\limits_{x\to 1^+} (\ln x)^{1-x}$

46. $\lim\limits_{x\to \infty} (1+x)^{1/x}$

47. $\lim\limits_{x\to \infty} (1+x^2)^{1/x}$

48. $\lim\limits_{x\to \pi/2} \tan x \cos x$

49. $\lim\limits_{x\to \pi/2} \tan x \sin(2x)$

50. $\lim\limits_{x\to 1^+} \dfrac{1}{\ln x} - \dfrac{1}{x-1}$

51. $\lim\limits_{x\to 3^+} \dfrac{5}{x^2-9} - \dfrac{x}{x-3}$

52. $\lim\limits_{x\to \infty} x\tan(1/x)$

53. $\lim\limits_{x\to \infty} \dfrac{(\ln x)^3}{x}$

54. $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{\ln x}$

Exercises 6.7

Terms and Concepts

1. The definite integral was defined with what two stipulations?

2. If $\lim_{b\to\infty} \int_0^b f(x)\ dx$ exists, then the integral $\int_0^\infty f(x)\ dx$ is said to ________________.

3. If $\int_1^\infty f(x)\ dx = 10$, and $0 \leq g(x) \leq f(x)$ for all x, then we know that $\int_1^\infty g(x)\ dx$ ________________.

4. For what values of p will $\int_1^\infty \frac{1}{x^p}\ dx$ converge?

5. For what values of p will $\int_{10}^\infty \frac{1}{x^p}\ dx$ converge?

6. For what values of p will $\int_0^1 \frac{1}{x^p}\ dx$ converge?

Problems

In Exercises 7 – 34, evaluate the given improper integral.

7. $\int_0^\infty e^{5-2x}\ dx$

8. $\int_1^\infty \frac{1}{x^3}\ dx$

9. $\int_1^\infty x^{-4}\ dx$

10. $\int_{-\infty}^\infty \frac{1}{x^2+9}\ dx$

11. $\int_{-\infty}^0 2^x\ dx$

12. $\int_{-\infty}^0 \left(\frac{1}{2}\right)^x\ dx$

13. $\int_{-\infty}^\infty \frac{x}{x^2+1}\ dx$

14. $\int_3^\infty \frac{1}{x^2-4}\ dx$

15. $\int_2^\infty \frac{1}{(x-1)^2}\ dx$

16. $\int_1^2 \frac{1}{(x-1)^2}\ dx$

17. $\int_2^\infty \frac{1}{x-1}\ dx$

18. $\int_1^2 \frac{1}{x-1}\ dx$

19. $\int_{-1}^1 \frac{1}{x}\ dx$

20. $\int_1^3 \frac{1}{x-2}\ dx$

21. $\int_0^\pi \sec^2 x\ dx$

22. $\int_{-2}^1 \frac{1}{\sqrt{|x|}}\ dx$

23. $\int_0^\infty xe^{-x}\ dx$

24. $\int_0^\infty xe^{-x^2}\ dx$

25. $\int_{-\infty}^\infty xe^{-x^2}\ dx$

26. $\int_{-\infty}^\infty \frac{1}{e^x+e^{-x}}\ dx$

27. $\int_0^1 x \ln x\ dx$

28. $\int_0^1 x^2 \ln x\ dx$

29. $\int_1^\infty \frac{\ln x}{x}\ dx$

30. $\int_0^1 \ln x\ dx$

31. $\int_1^\infty \frac{\ln x}{x^2}\ dx$

32. $\int_1^\infty \frac{\ln x}{\sqrt{x}}\ dx$

33. $\int_0^\infty e^{-x} \sin x\ dx$

34. $\int_0^\infty e^{-x} \cos x\ dx$

In Exercises 35 – 44, use the Direct Comparison Test or the Limit Comparison Test to determine whether the given definite integral converges or diverges. Clearly state what test is being used and what function the integrand is being compared to.

35. $\displaystyle\int_{10}^{\infty} \frac{3}{\sqrt{3x^2+2x-5}}\,dx$

36. $\displaystyle\int_{2}^{\infty} \frac{4}{\sqrt{7x^3-x}}\,dx$

37. $\displaystyle\int_{0}^{\infty} \frac{\sqrt{x+3}}{\sqrt{x^3-x^2+x+1}}\,dx$

38. $\displaystyle\int_{1}^{\infty} e^{-x}\ln x\,dx$

39. $\displaystyle\int_{5}^{\infty} e^{-x^2+3x+1}\,dx$

40. $\displaystyle\int_{0}^{\infty} \frac{\sqrt{x}}{e^x}\,dx$

41. $\displaystyle\int_{2}^{\infty} \frac{1}{x^2+\sin x}\,dx$

42. $\displaystyle\int_{0}^{\infty} \frac{x}{x^2+\cos x}\,dx$

43. $\displaystyle\int_{0}^{\infty} \frac{1}{x+e^x}\,dx$

44. $\displaystyle\int_{0}^{\infty} \frac{1}{e^x-x}\,dx$

Exercises 6.8

Terms and Concepts

1. Trigonometric Substitution works on the same principles as Integration by Substitution, though it can feel "________".

2. If one uses Trigonometric Substitution on an integrand containing $\sqrt{25-x^2}$, then one should set $x =$________.

3. Consider the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$.

   (a) What identity is obtained when both sides are divided by $\cos^2\theta$?

   (b) Use the new identity to simplify $9\tan^2\theta + 9$.

4. Why does Key Idea 6.4.1(a) state that $\sqrt{a^2-x^2} = a\cos\theta$, and not $|a\cos\theta|$?

Problems

In Exercises 5 – 16, apply Trigonometric Substitution to evaluate the indefinite integrals.

5. $\displaystyle\int \sqrt{x^2+1}\,dx$

6. $\displaystyle\int \sqrt{x^2+4}\,dx$

7. $\displaystyle\int \sqrt{1-x^2}\,dx$

8. $\displaystyle\int \sqrt{9-x^2}\,dx$

9. $\displaystyle\int \sqrt{x^2-1}\,dx$

10. $\displaystyle\int \sqrt{x^2-16}\,dx$

11. $\displaystyle\int \sqrt{4x^2+1}\,dx$

12. $\displaystyle\int \sqrt{1-9x^2}\,dx$

13. $\displaystyle\int \sqrt{16x^2-1}\,dx$

14. $\displaystyle\int \frac{8}{\sqrt{x^2+2}}\,dx$

15. $\displaystyle\int \frac{3}{\sqrt{7-x^2}}\,dx$

16. $\displaystyle\int \frac{5}{\sqrt{x^2-8}}\,dx$

In Exercises 17 – 26, evaluate the indefinite integrals. Some may be evaluated without Trigonometric Substitution.

17. $\displaystyle\int \frac{\sqrt{x^2-11}}{x}\,dx$

18. $\displaystyle\int \frac{1}{(x^2+1)^2}\,dx$

19. $\displaystyle\int \frac{x}{\sqrt{x^2-3}}\,dx$

20. $\displaystyle\int x^2\sqrt{1-x^2}\,dx$

21. $\displaystyle\int \frac{x}{(x^2+9)^{3/2}}\,dx$

22. $\displaystyle\int \frac{5x^2}{\sqrt{x^2-10}}\,dx$

23. $\displaystyle\int \frac{1}{(x^2+4x+13)^2}\,dx$

24. $\displaystyle\int x^2(1-x^2)^{-3/2}\,dx$

25. $\displaystyle\int \frac{\sqrt{5-x^2}}{7x^2}\,dx$

26. $\displaystyle\int \frac{x^2}{\sqrt{x^2+3}}\,dx$

In Exercises 27 – 32, evaluate the definite integrals by making the proper trigonometric substitution *and* changing the bounds of integration. (Note: each of the corresponding indefinite integrals has appeared previously in this Exercise set.)

27. $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\,dx$

28. $\displaystyle\int_{4}^{8} \sqrt{x^2-16}\,dx$

29. $\displaystyle\int_{0}^{2} \sqrt{x^2+4}\,dx$

30. $\displaystyle\int_{-1}^{1} \frac{1}{(x^2+1)^2}\,dx$

31. $\displaystyle\int_{-1}^{1} \sqrt{9-x^2}\,dx$

32. $\displaystyle\int_{-1}^{1} x^2\sqrt{1-x^2}\,dx$

Exercises 7.1

Terms and Concepts

1. T/F: The area between curves is always positive.

2. T/F: Calculus can be used to find the area of basic geometric shapes.

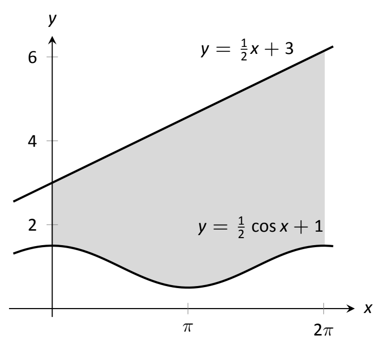
3. In your own words, describe how to find the total area enclosed by $y = f(x)$ and $y = g(x)$.

4. Describe a situation where it is advantageous to find an area enclosed by curves through integration with respect to y instead of x.

Problems

In Exercises 5 – 12, find the area of the shaded region in the given graph.

5.



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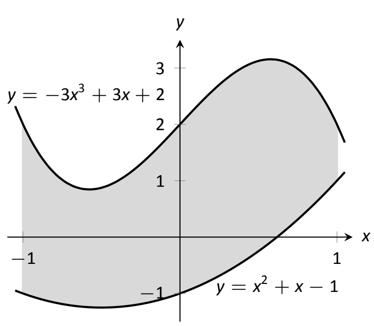

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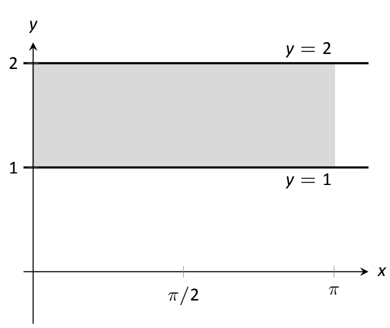

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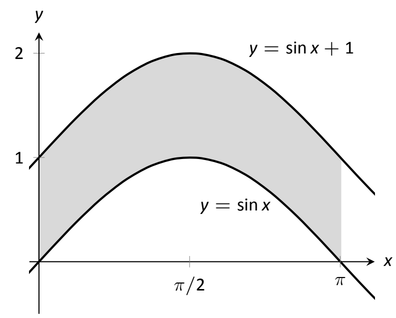

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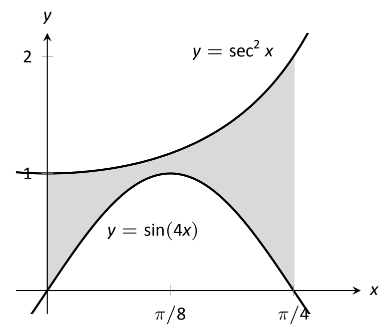

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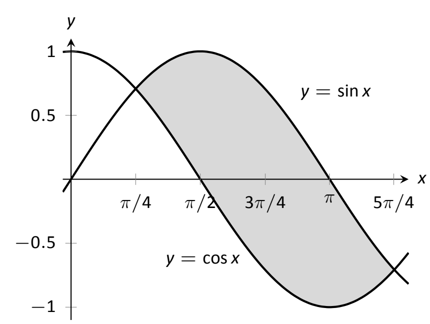

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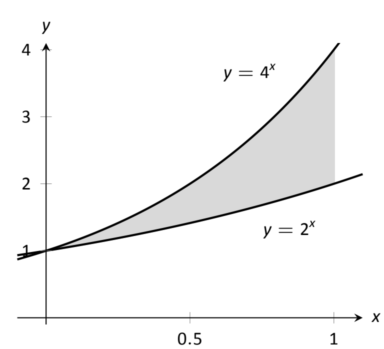
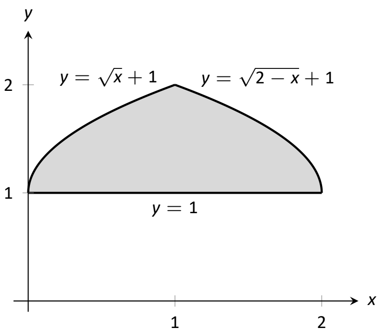


12. 

In Exercises 13 – 20, find the total area enclosed by the functions f and g.

13. $f(x) = 2x^2 + 5x - 3$, $g(x) = x^2 + 4x - 1$

14. $f(x) = x^2 - 3x + 2$, $g(x) = -3x + 3$

15. $f(x) = \sin x$, $g(x) = 2x/\pi$

16. $f(x) = x^3 - 4x^2 + x - 1$, $g(x) = -x^2 + 2x - 4$

17. $f(x) = x$, $g(x) = \sqrt{x}$

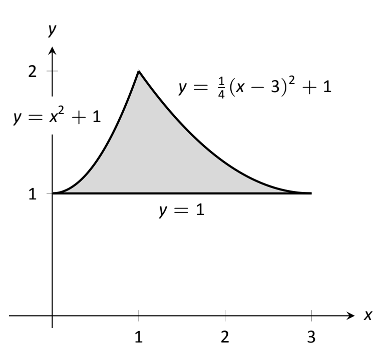
18. $f(x) = -x^3 + 5x^2 + 2x + 1$, $g(x) = 3x^2 + x + 3$

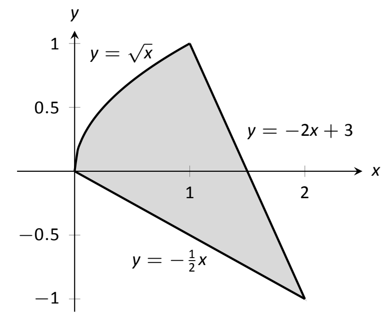
19. The functions $f(x) = \cos(x)$ and $g(x) = \sin x$ intersect infinitely many times, forming an infinite number of repeated, enclosed regions. Find the areas of these regions.

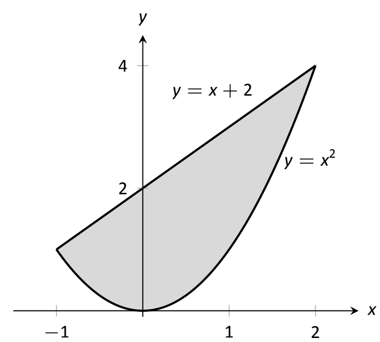
20. The functions $f(x) = \cos(2x)$ and $g(x) = \sin x$ intersect infinitely many times, forming an infinite number of repeated, enclosed regions. Find the areas of these regions.

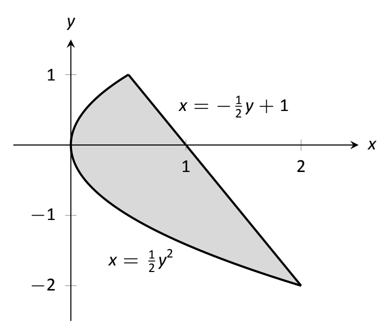
In Exercises 21 – 26, find the area of the enclosed region in two ways:

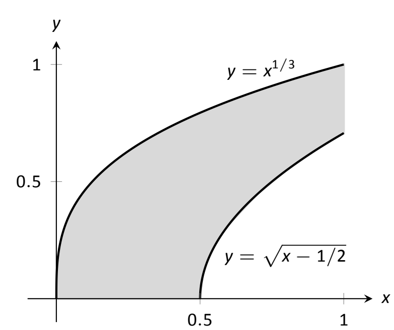
1. **by treating the boundaries as functions of x, and**
2. **by treating the boundaries as functions of y.**

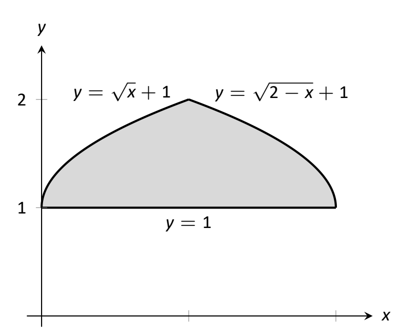
21. 

22. 

23. 

24. 

25. 

26. 

In Exercises 27 – 30, find the area triangle formed by the given three points.

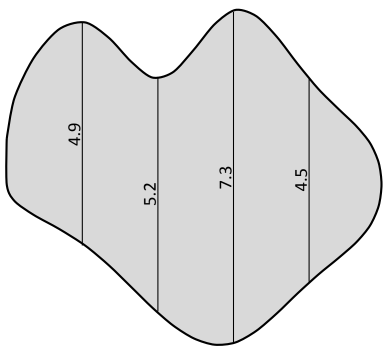
27. $(1,1)$, $(2,3)$, and $(3,3)$

28. $(-1,1)$, $(1,3)$, and $(2,-1)$

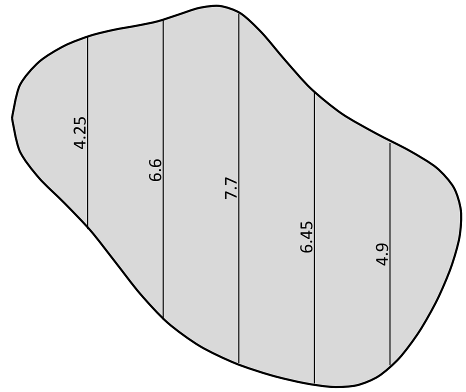
29. $(1,1)$, $(3,3)$, and $(3,3)$

30. $(0,0)$, $(2,5)$, and $(5,2)$

31. Use the Trapezoidal Rule to approximate the area of the pictured lake whose lengths, in hundreds of feet, are measured in 100-foot increments.



32. Use Simpson's Rule to approximate the area of the pictured lake whose lengths, in hundreds of feet, are measured in 200-foot increments.



Exercises 7.2

Terms and Concepts

1. T/F: A solid of revolution is formed by revolving a shape around an axis.

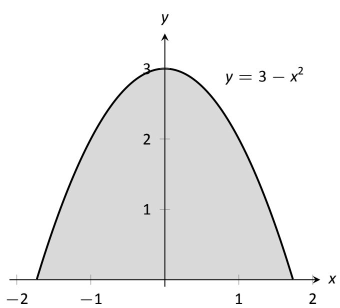
2. In your own words, explain how the Disk and Washer Methods are related.

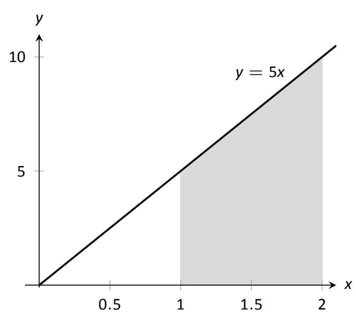
3. Explain the how the units of volume are found in the integral of Theorem 7.2.1: if $A(x)$ has units of in^2, how does $\int A(x)\,dx$ have units of in^3?

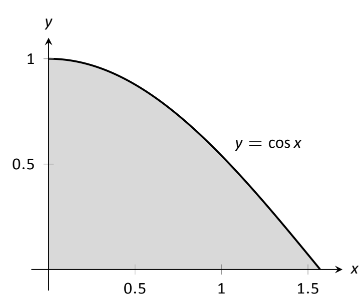
4. A fundamental principle of this section is "_________ can be found by integrating an area function."

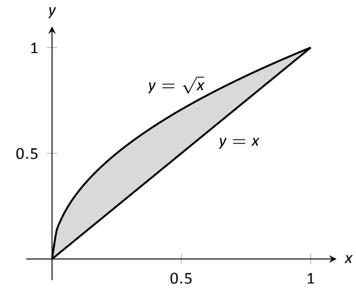
Problems

In Exercises 5 – 8, a region of the Cartesian plane is shaded. Use the Disk/Washer Method to find the volume of the solid of revolution formed by revolving the region about the x-axis.

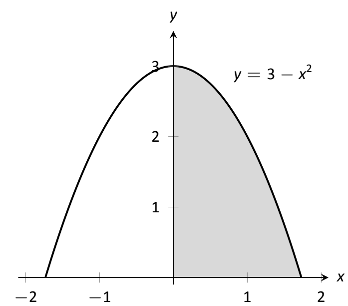
5. 

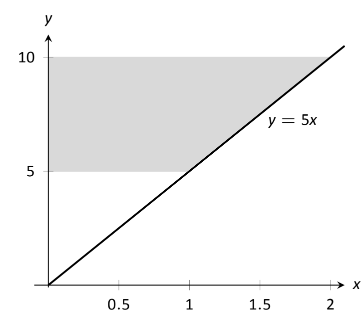
6. 

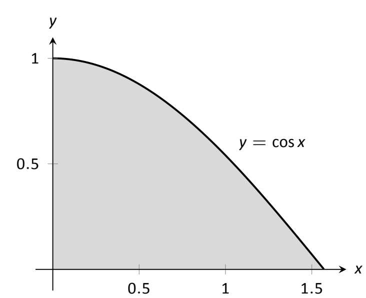
7. 

8. 

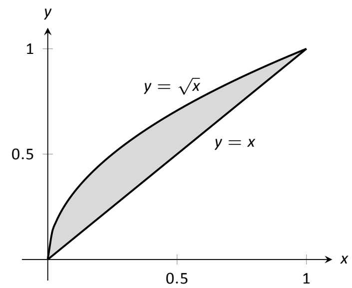
In Exercises 9 – 12, a region of the Cartesian plane is shaded. Use the Disk/Washer Method to find the volume of the solid of revolution formed by revolving the region about the y-axis.

9. 

10. 

11. 

(Hint: Integration By Parts will be necessary, twice. First let $u = \arccos^2 x$, then let $u = \arccos x$.)

12. 

In Exercises 13 – 18, a region of the Cartesian plane is described. Use the Disk/Washer Method to find the volume of the solid of revolution formed by rotating the region about each of the given axes.

13. Region bounded by: $y = \sqrt{x}$, $y = 0$ and $x = 1$.
    Rotate about:
    (a) the x-axis
    (b) $y = 1$
    (c) the y-axis
    (d) $x = 1$

14. Region bounded by: $y = 4 - x^2$ and $y = 0$.
    Rotate about:
    (a) the x-axis
    (b) $y = 4$
    (c) $y = -1$
    (d) $x = 2$

15. The triangle with vertices $(1, 1)$, $(1, 2)$ and $(2, 1)$.
    Rotate about:
    (a) the x-axis
    (b) $y = 2$
    (c) the y-axis
    (d) $x = 1$

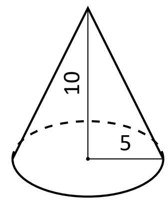
16. Region bounded by $y = x^2 - 2x + 2$ and $y = 2x - 1$.
    Rotate about:
    (a) the x-axis
    (b) $y = 1$
    (c) $y = 5$

17. Region bounded by $y = 1/\sqrt{x^2 + 1}$, $x = -1$, $x = 1$ and the x-axis.
    Rotate about:
    (a) the x-axis
    (b) $y = 1$
    (c) $y = -1$

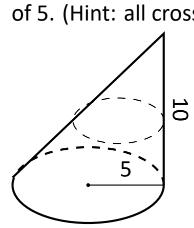
18. Region bounded by $y = 2x$, $y = x$ and $x = 2$.
    Rotate about:
    (a) the x-axis
    (b) $y = 4$
    (c) the y-axis
    (d) $x = 2$

In Exercises 19 – 22, a solid is described. Orient the solid along the x-axis such that a cross-sectional area function $A(x)$ can be obtained, then apply Theorem 7.2.1 to find the volume of the solid.

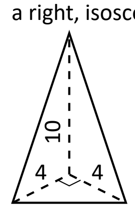
19. A right circular cone with height of 10 and base radius of 5.



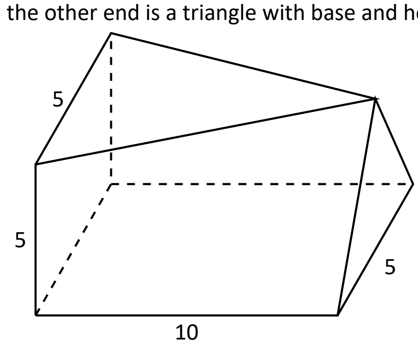
20. A skew right circular cone with height of 10 and base radius of 5. (Hint: all cross-sections are circles.)



21. A right triangular cone with height of 10 and whose base is a right, isosceles triangle with side length 4.



22. A solid with length 10 with a rectangular base and triangular top, wherein one end is a square with side length 5 and the other end is a triangle with base and height of 5.



Exercises 7.3

Terms and Concepts

1. T/F: A solid of revolution is formed by revolving a shape around an axis.

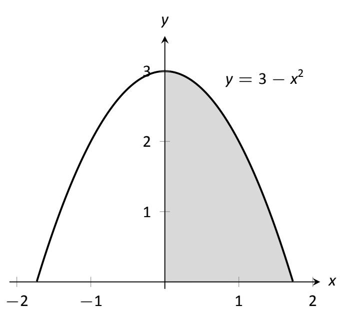
2. T/F: The Shell Method can only be used when the Washer Method fails.

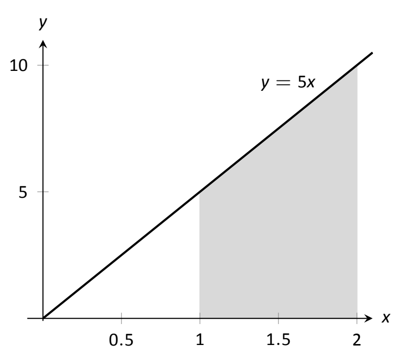
3. T/F: The Shell Method works by integrating cross–sectional areas of a solid.

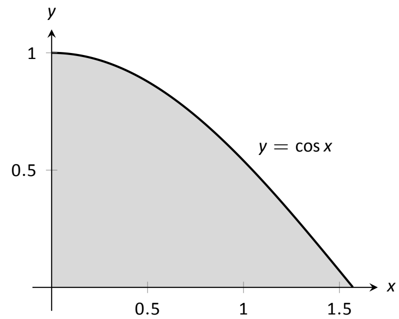
4. T/F: When finding the volume of a solid of revolution that was revolved around a vertical axis, the Shell Method integrates with respect to *x*.

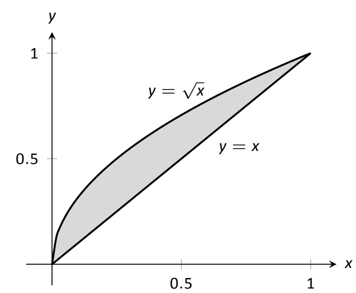
Problems

In Exercises 5 – 8, a region of the Cartesian plane is shaded. Use the Shell Method to find the volume of the solid of revolution formed by revolving the region about the *y*-axis.

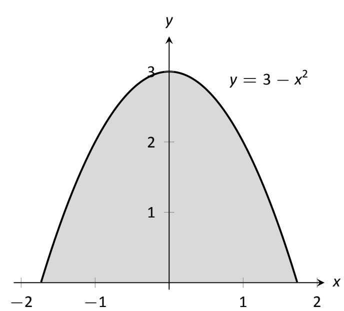
5. 

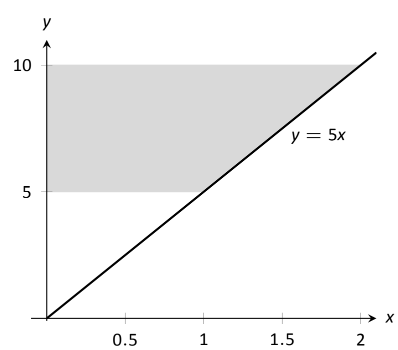
6. 

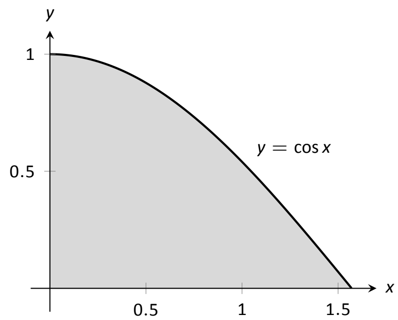
7. 

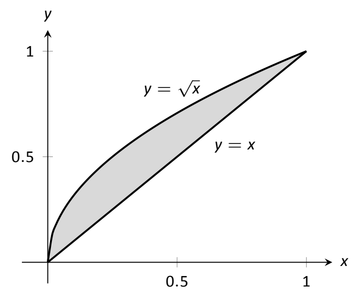
8. 

In Exercises 9 – 12, a region of the Cartesian plane is shaded. Use the Shell Method to find the volume of the solid of revolution formed by revolving the region about the *x*-axis.

9. 

10. 

11. 

12. 

In Exercises 13 – 18, a region of the Cartesian plane is described. Use the Shell Method to find the volume of the solid of revolution formed by rotating the region about each of the given axes.

13. Region bounded by: $y = \sqrt{x}$, $y = 0$ and $x = 1$.
Rotate about:
(a) the y-axis
(b) $x = 1$
(c) the x-axis
(d) $y = 1$

14. Region bounded by: $y = 4 - x^2$ and $y = 0$.
Rotate about:
(a) $x = 2$
(b) $x = -2$
(c) the x-axis
(d) $y = 4$

15. The triangle with vertices $(1, 1)$, $(1, 2)$ and $(2, 1)$.
Rotate about:
(a) the y-axis
(b) $x = 1$
(c) the x-axis
(d) $y = 2$

16. Region bounded by $y = x^2 - 2x + 2$ and $y = 2x - 1$.
Rotate about:
(a) the y-axis
(b) $x = 1$
(c) $x = -1$

17. Region bounded by $y = 1/\sqrt{x^2 + 1}$, $x = 1$ and the x and y-axes.
Rotate about:
(a) the y-axis
(b) $x = 1$

18. Region bounded by $y = 2x$, $y = x$ and $x = 2$.
Rotate about:
(a) the y-axis
(b) $x = 2$
(c) the x-axis
(d) $y = 4$

Exercises 7.4

Terms and Concepts

1. T/F: The integral formula for computing Arc Length was found by first approximating arc length with straight line segments.

2. T/F: The integral formula for computing Arc Length includes a square–root, meaning the integration is probably easy.

Problems

In Exercises 3 – 12, find the arc length of the function on the given interval.

3. $f(x) = x$ on $[0, 1]$.

4. $f(x) = \sqrt{8}x$ on $[-1, 1]$.

5. $f(x) = \dfrac{1}{3}x^{3/2} - x^{1/2}$ on $[0, 1]$.

6. $f(x) = \dfrac{1}{12}x^3 + \dfrac{1}{x}$ on $[1, 4]$.

7. $f(x) = 2x^{3/2} - \dfrac{1}{6}\sqrt{x}$ on $[0, 9]$.

8. $f(x) = \cosh x$ on $[-\ln 2, \ln 2]$.

9. $f(x) = \dfrac{1}{2}\left(e^x + e^{-x}\right)$ on $[0, \ln 5]$.

10. $f(x) = \dfrac{1}{12}x^5 + \dfrac{1}{5x^3}$ on $[.1, 1]$.

11. $f(x) = \ln\big(\sin x\big)$ on $[\pi/6, \pi/2]$.

12. $f(x) = \ln\big(\cos x\big)$ on $[0, \pi/4]$.

In Exercises 13 – 20, set up the integral to compute the arc length of the function on the given interval. Do not evaluate the integral.

13. $f(x) = x^2$ on $[0, 1]$.

14. $f(x) = x^{10}$ on $[0, 1]$.

15. $f(x) = \sqrt{x}$ on $[0, 1]$.

16. $f(x) = \ln x$ on $[1, e]$.

17. $f(x) = \sqrt{1 - x^2}$ on $[-1, 1]$. (Note: this describes the top half of a circle with radius 1.)

18. $f(x) = \sqrt{1 - x^2/9}$ on $[-3, 3]$. (Note: this describes the top half of an ellipse with a major axis of length 6 and a minor axis of length 2.)

19. $f(x) = \dfrac{1}{x}$ on $[1, 2]$.

20. $f(x) = \sec x$ on $[-\pi/4, \pi/4]$.

In Exercises 21 – 28, use Simpson's Rule, with $n = 4$, to approximate the arc length of the function on the given interval. Note: these are the same problems as in Exercises 13–20.

21. $f(x) = x^2$ on $[0, 1]$.

22. $f(x) = x^{10}$ on $[0, 1]$.

23. $f(x) = \sqrt{x}$ on $[0, 1]$. (Note: $f'(x)$ is not defined at $x = 0$.)

24. $f(x) = \ln x$ on $[1, e]$.

25. $f(x) = \sqrt{1 - x^2}$ on $[-1, 1]$. (Note: $f'(x)$ is not defined at the endpoints.)

26. $f(x) = \sqrt{1 - x^2/9}$ on $[-3, 3]$. (Note: $f'(x)$ is not defined at the endpoints.)

27. $f(x) = \dfrac{1}{x}$ on $[1, 2]$.

28. $f(x) = \sec x$ on $[-\pi/4, \pi/4]$.

In Exercises 29 – 33, find the surface area of the described solid of revolution.

29. The solid formed by revolving $y = 2x$ on $[0, 1]$ about the x-axis.

30. The solid formed by revolving $y = x^2$ on $[0, 1]$ about the y-axis.

31. The solid formed by revolving $y = x^3$ on $[0, 1]$ about the x-axis.

32. The solid formed by revolving $y = \sqrt{x}$ on $[0, 1]$ about the x-axis.

33. The sphere formed by revolving $y = \sqrt{1 - x^2}$ on $[-1, 1]$ about the x-axis.

Exercises 7.5

Terms and Concepts

1. What are the typical units of work?

2. If a man has a mass of 80 kg on Earth, will his mass on the moon be bigger, smaller, or the same?

3. If a woman weighs 130 lb on Earth, will her weight on the moon be bigger, smaller, or the same?

4. Fill in the blanks:

   Some integrals in this section are set up by multiplying a variable ________ by a constant distance; others are set up by multiplying a constant force by a variable ________.

Problems

5. A 100 ft rope, weighing 0.1 lb/ft, hangs over the edge of a tall building.

   (a) How much work is done pulling the entire rope to the top of the building?

   (b) How much rope is pulled in when half of the total work is done?

6. A 50 m rope, with a mass density of 0.2 kg/m, hangs over the edge of a tall building.

   (a) How much work is done pulling the entire rope to the top of the building?

   (b) How much work is done pulling in the first 20 m?

7. A rope of length ℓ ft hangs over the edge of tall cliff. (Assume the cliff is taller than the length of the rope.) The rope has a weight density of d lb/ft.

   (a) How much work is done pulling the entire rope to the top of the cliff?

   (b) What percentage of the total work is done pulling in the first half of the rope?

   (c) How much rope is pulled in when half of the total work is done?

8. A 20 m rope with mass density of 0.5 kg/m hangs over the edge of a 10 m building. How much work is done pulling the rope to the top?

9. A crane lifts a 2,000 lb load vertically 30 ft with a 1” cable weighing 1.68 lb/ft.

   (a) How much work is done lifting the cable alone?

   (b) How much work is done lifting the load alone?

   (c) Could one conclude that the work done lifting the cable is negligible compared to the work done lifting the load?

10. A 100 lb bag of sand is lifted uniformly 120 ft in one minute. Sand leaks from the bag at a rate of 1/4 lb/s. What is the total work done in lifting the bag?

11. A box weighing 2 lb lifts 10 lb of sand vertically 50 ft. A crack in the box allows the sand to leak out such that 9 lb of sand is in the box at the end of the trip. Assume the sand leaked out at a uniform rate. What is the total work done in lifting the box and sand?

12. A force of 1000 lb compresses a spring 3 in. How much work is performed in compressing the spring?

13. A force of 2 N stretches a spring 5 cm. How much work is performed in stretching the spring?

14. A force of 50 lb compresses a spring from a natural length of 18 in to 12 in. How much work is performed in compressing the spring?

15. A force of 20 lb stretches a spring from a natural length of 6 in to 8 in. How much work is performed in stretching the spring?

16. A force of 7 N stretches a spring from a natural length of 11 cm to 21 cm. How much work is performed in stretching the spring from a length of 16 cm to 21 cm?

17. A force of f N stretches a spring d m from its natural length. How much work is performed in stretching the spring?

18. A 20 lb weight is attached to a spring. The weight rests on the spring, compressing the spring from a natural length of 1 ft to 6 in.

    How much work is done in lifting the box 1.5 ft (i.e, the spring will be stretched 1 ft beyond its natural length)?

19. A 20 lb weight is attached to a spring. The weight rests on the spring, compressing the spring from a natural length of 1 ft to 6 in.

    How much work is done in lifting the box 6 in (i.e, bringing the spring back to its natural length)?

20. A 5 m tall cylindrical tank with radius of 2 m is filled with 3 m of gasoline, with a mass density of 737.22 kg/m^3. Compute the total work performed in pumping all the gasoline to the top of the tank.

21. A 6 ft cylindrical tank with a radius of 3 ft is filled with water, which has a weight density of 62.4 lb/ft^3. The water is to be pumped to a point 2 ft above the top of the tank.

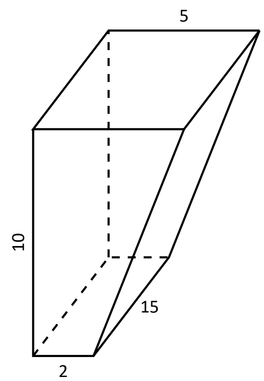
    (a) How much work is performed in pumping all the water from the tank?

    (b) How much work is performed in pumping 3 ft of water from the tank?

    (c) At what point is 1/2 of the total work done?

22. A gasoline tanker is filled with gasoline with a weight density of 45.93 lb/ft^3. The dispensing valve at the base is jammed shut, forcing the operator to empty the tank via pumping the gas to a point 1 ft above the top of the tank. Assume the tank is a perfect cylinder, 20 ft long with a diameter of 7.5 ft. How much work is performed in pumping all the gasoline from the tank?

23. A fuel oil storage tank is 10 ft deep with trapezoidal sides, 5 ft at the top and 2 ft at the bottom, and is 15 ft wide (see diagram below). Given that fuel oil weighs 55.46 lb/ft^3, find the work performed in pumping all the oil from the tank to a point 3 ft above the top of the tank.



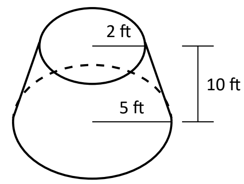
24. A conical water tank is 5 m deep with a top radius of 3 m. (This is similar to Example 7.5.6.) The tank is filled with pure water, with a mass density of 1000 kg/m^3.

    (a) Find the work performed in pumping all the water to the top of the tank.

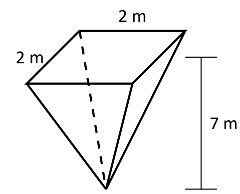
    (b) Find the work performed in pumping the top 2.5 m of water to the top of the tank.

    (c) Find the work performed in pumping the top half of the water, by volume, to the top of the tank.

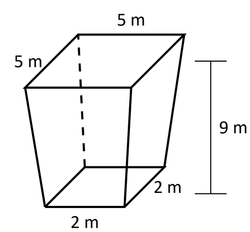
25. A water tank has the shape of a truncated cone, with dimensions given below, and is filled with water with a weight density of 62.4 lb/ft^3. Find the work performed in pumping all water to a point 1 ft above the top of the tank.



26. A water tank has the shape of an inverted pyramid, with dimensions given below, and is filled with water with a mass density of 1000 kg/m^3. Find the work performed in pumping all water to a point 5 m above the top of the tank.



27. A water tank has the shape of an truncated, inverted pyramid, with dimensions given below, and is filled with water with a mass density of 1000 kg/m^3. Find the work performed in pumping all water to a point 1 m above the top of the tank.



Exercises 7.6

Terms and Concepts

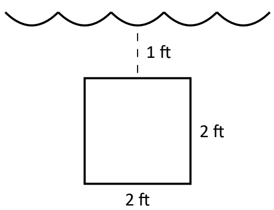
1. State in your own words Pascal's Principle.

2. State in your own words how pressure is different from force.

Problems

In Exercises 3 – 12, find the fluid force exerted on the given plate, submerged in water with a weight density of 62.4 lb/ft^3.

3.



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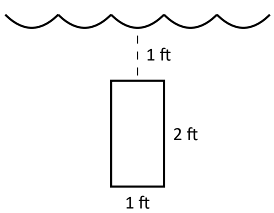

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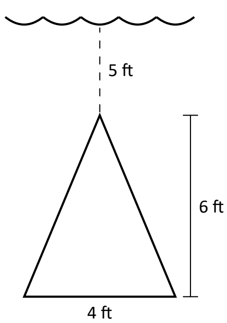

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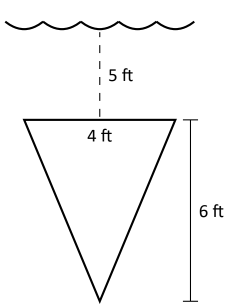

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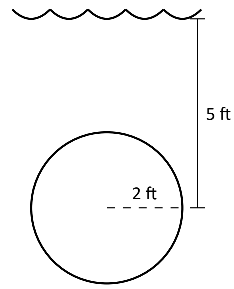

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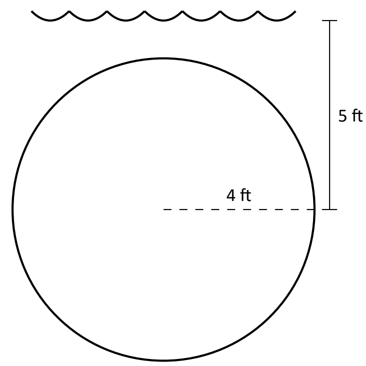

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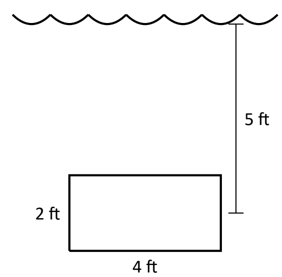

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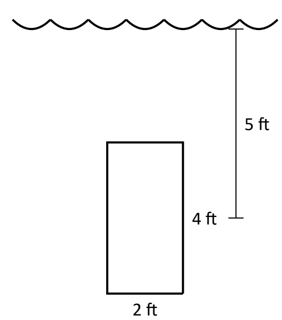

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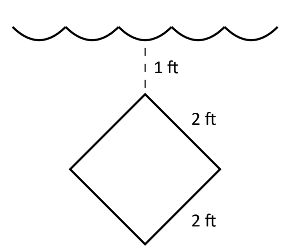

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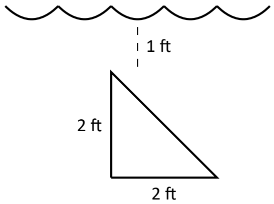
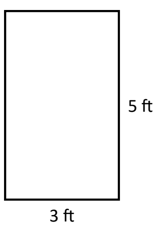


In Exercises 13 – 18, the side of a container is pictured. Find the fluid force exerted on this plate when the container is full of:

1. **water, with a weight density of 62.4 lb/ft^3, and**
2. **concrete, with a weight density of 150 lb/ft^3.**

13.



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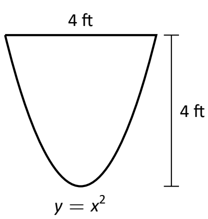

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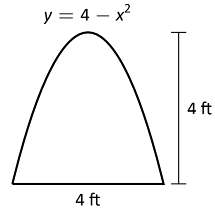

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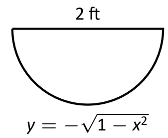

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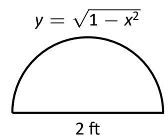

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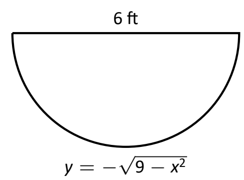


19. How deep must the center of a vertically oriented circular plate with a radius of 1 ft be submerged in water, with a weight density of 62.4 lb/ft^3, for the fluid force on the plate to reach 1,000 lb?

20. How deep must the center of a vertically oriented square plate with a side length of 2 ft be submerged in water, with a weight density of 62.4 lb/ft^3, for the fluid force on the plate to reach 1,000 lb?

Exercises 8.1

Terms and Concepts

1. Use your own words to define a *sequence.*

2. The domain of a sequence is the ________ numbers.

3. Use your own words to describe the *range* of a sequence.

4. Describe what it means for a sequence to be *bounded*.

Problems

In Exercises 5 – 8, give the first five terms of the given sequence.

5. $\{a_n\} = \left\{ \dfrac{4^n}{(n+1)!} \right\}$

6. $\{b_n\} = \left\{ \left(-\dfrac{3}{2}\right)^n \right\}$

7. $\{c_n\} = \left\{ -\dfrac{n^{n+1}}{n+2} \right\}$

8. $\{d_n\} = \left\{ \dfrac{1}{\sqrt{5}} \left(\left(\dfrac{1+\sqrt{5}}{2}\right)^n - \left(\dfrac{1-\sqrt{5}}{2}\right)^n \right) \right\}$

In Exercises 9 – 12, determine the n^{th} term of the given sequence.

9. 4, 7, 10, 13, 16, . . .

10. $3, \ -\dfrac{3}{2}, \ \dfrac{3}{4}, \ -\dfrac{3}{8}, \ \ldots$

11. 10, 20, 40, 80, 160, . . .

12. $1, 1, \ \dfrac{1}{2}, \ \dfrac{1}{6}, \ \dfrac{1}{24}, \ \dfrac{1}{120}, \ \ldots$

In Exercises 13 – 16, use the following information to determine the limit of the given sequences.

- $\{a_n\} = \left\{ \dfrac{2^n - 20}{2^n} \right\}$; $\quad \lim\limits_{n\to\infty} a_n = 1$
- $\{b_n\} = \left\{ \left(1 + \dfrac{2}{n}\right)^n \right\}$; $\quad \lim\limits_{n\to\infty} b_n = e^2$
- $\{c_n\} = \{\sin(3/n)\}$; $\quad \lim\limits_{n\to\infty} c_n = 0$

13. $\{a_n\} = \left\{ \dfrac{2^n - 20}{7 \cdot 2^n} \right\}$

14. $\{a_n\} = \{3b_n - a_n\}$

15. $\{a_n\} = \left\{ \sin(3/n) \left(1 + \dfrac{2}{n}\right)^n \right\}$

16. $\{a_n\} = \left\{ \left(1 + \dfrac{2}{n}\right)^{2n} \right\}$

In Exercises 17 – 28, determine whether the sequence converges or diverges. If convergent, give the limit of the sequence.

17. $\{a_n\} = \left\{ (-1)^n \dfrac{n}{n+1} \right\}$

18. $\{a_n\} = \left\{ \dfrac{4n^2 - n + 5}{3n^2 + 1} \right\}$

19. $\{a_n\} = \left\{ \dfrac{4^n}{5^n} \right\}$

20. $\{a_n\} = \left\{ \dfrac{n-1}{n} - \dfrac{n}{n-1} \right\}, n \geq 2$

21. $\{a_n\} = \{\ln(n)\}$

22. $\{a_n\} = \left\{ \dfrac{3n}{\sqrt{n^2+1}} \right\}$

23. $\{a_n\} = \left\{ \left(1 + \dfrac{1}{n}\right)^n \right\}$

24. $\{a_n\} = \left\{ 5 - \dfrac{1}{n} \right\}$

25. $\{a_n\} = \left\{ \dfrac{(-1)^{n+1}}{n} \right\}$

26. $\{a_n\} = \left\{ \dfrac{1.1^n}{n} \right\}$

27. $\{a_n\} = \left\{ \dfrac{2n}{n+1} \right\}$

28. $\{a_n\} = \left\{ (-1)^n \dfrac{n^2}{2^n - 1} \right\}$

In Exercises 29 – 34, determine whether the sequence is bounded, bounded above, bounded below, or none of the above.

29. $\{a_n\} = \{\sin n\}$

30. $\{a_n\} = \{\tan n\}$

31. $\{a_n\} = \left\{ (-1)^n \dfrac{3n-1}{n} \right\}$

32. $\{a_n\} = \left\{ \dfrac{3n^2 - 1}{n} \right\}$

33. $\{a_n\} = \{n \cos n\}$

34. $\{a_n\} = \{2^n - n!\}$

In Exercises 35 – 38, determine whether the sequence is monotonically increasing or decreasing. If it is not, determine if there is an m such that it is monotonic for all $n \geq m$.

35. $\{a_n\} = \left\{\dfrac{n}{n+2}\right\}$

36. $\{a_n\} = \left\{\dfrac{n^2 - 6n + 9}{n}\right\}$

37. $\{a_n\} = \left\{(-1)^n \dfrac{1}{n^3}\right\}$

38. $\{a_n\} = \left\{\dfrac{n^2}{2^n}\right\}$

Exercises 39 – 42 explore further the theory of sequences.

39. Prove Theorem 8.1.2; that is, use the definition of the limit of a sequence to show that if $\lim\limits_{n\to\infty} |a_n| = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.

40. Let $\{a_n\}$ and $\{b_n\}$ be sequences such that $\lim\limits_{n\to\infty} a_n = L$ and $\lim\limits_{n\to\infty} b_n = K$.

    (a) Show that if $a_n < b_n$ for all n, then $L \leq K$.

    (b) Give an example where $L = K$.

41. Prove the Squeeze Theorem for sequences: Let $\{a_n\}$ and $\{b_n\}$ be such that $\lim\limits_{n\to\infty} a_n = L$ and $\lim\limits_{n\to\infty} b_n = L$, and let $\{c_n\}$ be such that $a_n \leq c_n \leq b_n$ for all n. Then $\lim\limits_{n\to\infty} c_n = L$

42. Prove the statement "Let $\{a_n\}$ be a bounded, monotonic sequence. Then $\{a_n\}$ converges; i.e., $\lim\limits_{n\to\infty} a_n$ exists." is equivalent to Theorem 8.1.5. That is,

    (a) Show that if Theorem 8.1.5 is true, then above statement is true, and

    (b) Show that if the above statement is true, then Theorem 8.1.5 is true.

Exercises 8.2

Terms and Concepts

1. Use your own words to describe how sequences and series are related.

2. Use your own words to define a *partial sum*.

3. Given a series $\sum_{n=1}^{\infty} a_n$, describe the two sequences related to the series that are important.

4. Use your own words to explain what a geometric series is.

5. T/F: If $\{a_n\}$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is also convergent.

6. T/F: If $\{a_n\}$ converges to 0, then $\sum_{n=0}^{\infty} a_n$ converges.

Problems

In Exercises 7 – 14, a series $\sum_{n=1}^{\infty} a_n$ is given.

(a) Give the first 5 partial sums of the series.

(b) Give a graph of the first 5 terms of a_n and S_n on the same axes.

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

9. $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

10. $\displaystyle\sum_{n=1}^{\infty} n$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$

13. $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{9}{10}\right)^n$

14. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$

In Exercises 15 – 20, use Theorem 8.2.4 to show the given series diverges.

15. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{n(n+2)}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n - n^5}{5^n + n^5}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}}$

20. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

In Exercises 21 – 30, state whether the given series converges or diverges.

21. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

22. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{5^n}$

23. $\displaystyle\sum_{n=0}^{\infty} \frac{6^n}{5^n}$

24. $\displaystyle\sum_{n=1}^{\infty} n^{-4}$

25. $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{10}{n!}$

27. T/F: If $\{a_n\}$ converges to 0, then $\sum_{n=0}^{\infty} a_n$ converges.

28. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(2n+8)^2}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

In Exercises 31 – 46, a series is given.

(a) Find a formula for S_n, the n^{th} partial sum of the series.

(b) Determine whether the series converges or diverges. If it converges, state what it converges to.

31. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^n}$

32. $\displaystyle\sum_{n=1}^{\infty} 2$

33. $1^3 + 2^3 + 3^3 + 4^3 + \cdots$

34. $\displaystyle\sum_{n=1}^{\infty} (-1)^n n$

35. $\displaystyle\sum_{n=0}^{\infty} \frac{5}{2^n}$

36. $\displaystyle\sum_{n=1}^{\infty} e^{-n}$

37. $1 - \dfrac{1}{3} + \dfrac{1}{9} - \dfrac{1}{27} + \dfrac{1}{81} + \cdots$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$

40. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

41. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

43. $\dfrac{1}{1\cdot 4} + \dfrac{1}{2\cdot 5} + \dfrac{1}{3\cdot 6} + \dfrac{1}{4\cdot 7} + \cdots$

44. $2 + \left(\dfrac{1}{2} + \dfrac{1}{3}\right) + \left(\dfrac{1}{4} + \dfrac{1}{9}\right) + \left(\dfrac{1}{8} + \dfrac{1}{27}\right) + \cdots$

45. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

46. $\displaystyle\sum_{n=0}^{\infty} \left(\sin 1\right)^n$

47. Break the Harmonic Series into the sum of the odd and even terms:

$$\sum_{n=1}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{2n-1} + \sum_{n=1}^{\infty} \frac{1}{2n}.$$

The goal is to show that each of the series on the right diverge.

(a) Show why $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1} > \sum_{n=1}^{\infty} \frac{1}{2n}$.

(Compare each n^{th} partial sum.)

(b) Show why $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1} < 1 + \sum_{n=1}^{\infty} \frac{1}{2n}$

(c) Explain why (a) and (b) demonstrate that the series of odd terms is convergent, if, and only if, the series of even terms is also convergent. (That is, show both converge or both diverge.)

(d) Explain why knowing the Harmonic Series is divergent determines that the even and odd series are also divergent.

48. Show the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)}$ diverges.

Exercises 8.3

Terms and Concepts

1. In order to apply the Integral Test to a sequence $\{a_n\}$, the function $a(n) = a_n$ must be _____, _____ and _____.

2. T/F: The Integral Test can be used to determine the sum of a convergent series.

3. What test(s) in this section do not work well with factorials?

4. Suppose $\sum_{n=0}^{\infty} a_n$ is convergent, and there are sequences $\{b_n\}$ and $\{c_n\}$ such that $0 \leq b_n \leq a_n \leq c_n$ for all n. What can be said about the series $\sum_{n=0}^{\infty} b_n$ and $\sum_{n=0}^{\infty} c_n$?

Problems

In Exercises 5 – 12, use the Integral Test to determine the convergence of the given series.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

10. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

In Exercises 13 – 22, use the Direct Comparison Test to determine the convergence of the given series; state what series is used for comparison.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+3n-5}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n+n^2-n}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!+n}$

17. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$

18. $\displaystyle\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}-2}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^3-5}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^n+10}$

21. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2-1}$

22. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$

In Exercises 23 – 32, use the Limit Comparison Test to determine the convergence of the given series; state what series is used for comparison.

23. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2-3n+5}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n-n^2}$

25. $\displaystyle\sum_{n=4}^{\infty} \frac{\ln n}{n-3}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{n-10}{n^2+10n+10}$

29. $\displaystyle\sum_{n=1}^{\infty} \sin\left(1/n\right)$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{n^3-5}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{n^2+17}$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+100}$

In Exercises 33 – 40, determine the convergence of the given series. State the test used; more than one test may be appropriate.

33. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+5)^3}$

35. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n!}$

37. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n+n}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{n-2}{10n+5}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

40. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(1/n)}{\sqrt{n}}$

41. Given that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, state which of the following series converges, may converge, or does not converge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} a_n a_{n+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (a_n)^2$

   (d) $\displaystyle\sum_{n=1}^{\infty} n a_n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n}$

Exercises 8.4

Terms and Concepts

1. The Ratio Test is not effective when the terms of a sequence only contain ______ functions.

2. The Ratio Test is most effective when the terms of a sequence contains ______ and/or ______ functions.

3. What three convergence tests do not work well with terms containing factorials?

4. The Root Test works particularly well on series where each term is ______ to a ______.

Problems

In Exercises 5 – 14, determine the convergence of the given series using the Ratio Test. If the Ratio Test is inconclusive, state so and determine convergence with another test.

5. $\displaystyle\sum_{n=0}^{\infty} \frac{2n}{n!}$

6. $\displaystyle\sum_{n=0}^{\infty} \frac{5^n - 3n}{4^n}$

7. $\displaystyle\sum_{n=0}^{\infty} \frac{n!10^n}{(2n)!}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n + n^4}{7^n + n^2}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^3 + 7}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{10 \cdot 5^n}{7^n - 3}$

12. $\displaystyle\sum_{n=1}^{\infty} n \cdot \left(\frac{3}{5}\right)^n$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots 2n}{3 \cdot 6 \cdot 9 \cdot 12 \cdots 3n}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{5 \cdot 10 \cdot 15 \cdots (5n)}$

In Exercises 15 – 24, determine the convergence of the given series using the Root Test. If the Root Test is inconclusive, state so and determine convergence with another test.

15. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n + 5}{3n + 11}\right)^n$

16. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{.9n^2 - n - 3}{n^2 + n + 3}\right)^n$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^2}{3^n}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2 2^{n+1}}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+7}}{7^n}$

21. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2 - n}{n^2 + n}\right)^n$

22. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\ln n)^n}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(\ln n)^n}$

In Exercises 25 – 34, determine the convergence of the given series. State the test used; more than one test may be appropriate.

25. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 4n - 2}{n^3 + 4n^2 - 3n + 7}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{n^4 4^n}{n!}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n + n}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^n}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 4n + 1}}$

30. $\sum_{n=1}^{\infty} \frac{n!n!n!}{(3n)!}$

31. $\sum_{n=1}^{\infty} \frac{1}{\ln n}$

32. $\sum_{n=1}^{\infty} \left(\frac{n+2}{n+1}\right)^n$

33. $\sum_{n=1}^{\infty} \frac{n^3}{(\ln n)^n}$

34. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

Exercises 8.5

Terms and Concepts

1. Why is $\sum_{n=1}^{\infty} \sin n$ not an alternating series?

2. A series $\sum_{n=1}^{\infty} (-1)^n a_n$ converges when $\{a_n\}$ is ______, ______ and $\lim_{n\to\infty} a_n =$______.

3. Give an example of a series where $\sum_{n=0}^{\infty} a_n$ converges but $\sum_{n=0}^{\infty} |a_n|$ does not.

4. The sum of a ______ convergent series can be changed by rearranging the order of its terms.

Problems

In Exercises 5 – 20, an alternating series $\sum_{n=i}^{\infty} a_n$ is given.

(a) Determine if the series converges or diverges.

(b) Determine if $\sum_{n=0}^{\infty} |a_n|$ converges or diverges.

(c) If $\sum_{n=0}^{\infty} a_n$ converges, determine if the convergence is conditional or absolute.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n!}}$

7. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n+5}{3n-5}$

8. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^2}$

9. $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{3n+5}{n^2-3n+1}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n+1}$

11. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+3+5+\cdots+(2n-1)}$

13. $\displaystyle\sum_{n=1}^{\infty} \cos\left(\pi n\right)$

14. $\displaystyle\sum_{n=2}^{\infty} \frac{\sin\left((n+1/2)\pi\right)}{n\ln n}$

15. $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$

16. $\displaystyle\sum_{n=0}^{\infty} (-e)^{-n}$

17. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n!}$

18. $\displaystyle\sum_{n=0}^{\infty} (-1)^n 2^{-n^2}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1000)^n}{n!}$

Let S_n be the n^{th} partial sum of a series. In Exercises 21 – 24, a convergent alternating series is given and a value of n. Compute S_n and S_{n+1} and use these values to find bounds on the sum of the series.

21. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}, \quad n=5$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}, \quad n=4$

23. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}, \quad n=6$

24. $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n, \quad n=9$

In Exercises 25 – 28, a convergent alternating series is given along with its sum and a value of ε. Use Theorem 8.5.2 to find n such that the n^{th} partial sum of the series is within ε of the sum of the series.

25. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}, \quad \varepsilon = 0.001$

26. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}, \quad \varepsilon = 0.0001$

27. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}, \quad \varepsilon = 0.001$

28. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = \cos 1, \quad \varepsilon = 10^{-8}$

Exercises 8.6

Terms and Concepts

1. We adopt the convention that $x^0 =$______, regardless of the value of x.

2. What is the difference between the radius of convergence and the interval of convergence?

3. If the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} a_nx^n$ is 5, what is the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} n\cdot a_nx^{n-1}$?

4. If the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} a_nx^n$ is 5, what is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty}(-1)^na_nx^n$?

Problems

In Exercises 5 – 8, write out the sum of the first 5 terms of the given power series.

5. $\displaystyle\sum_{n=0}^{\infty} 2^nx^n$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}x^n$

7. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}x^n$

8. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$

In Exercises 9 – 24, a power series is given.

(a) Find the radius of convergence.

(b) Find the interval of convergence.

9. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!}x^n$

10. $\displaystyle\sum_{n=0}^{\infty} nx^n$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n}$

12. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+4)^n}{n!}$

13. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n}$

14. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-5)^n}{10^n}$

15. $\displaystyle\sum_{n=0}^{\infty} 5^n(x-1)^n$

16. $\displaystyle\sum_{n=0}^{\infty} (-2)^nx^n$

17. $\displaystyle\sum_{n=0}^{\infty} \sqrt{n}x^n$

18. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{3^n}x^n$

19. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}(x-5)^n$

20. $\displaystyle\sum_{n=0}^{\infty} (-1)^nn!(x-10)^n$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^3}$

23. $\displaystyle\sum_{n=0}^{\infty} n!\left(\frac{x}{10}\right)^n$

24. $\displaystyle\sum_{n=0}^{\infty} n^2\left(\frac{x+4}{4}\right)^n$

In Exercises 25 – 30, a function $f(x) = \displaystyle\sum_{n=0}^{\infty} a_nx^n$ is given.

(a) Give a power series for $f'(x)$ and its interval of convergence.

(b) Give a power series for $\int f(x)\,dx$ and its interval of convergence.

25. $\displaystyle\sum_{n=0}^{\infty} nx^n$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

27. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

28. $\displaystyle\sum_{n=0}^{\infty}(-3x)^n$

29. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

30. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$

In Exercises 31 – 36, give the first 5 terms of the series that is a solution to the given differential equation.

31. $y' = 3y, \quad y(0) = 1$

32. $y' = 5y, \quad y(0) = 5$

33. $y' = y^2, \quad y(0) = 1$

34. $y' = y + 1, \quad y(0) = 1$

35. $y'' = -y, \quad y(0) = 0, y'(0) = 1$

36. $y'' = 2y, \quad y(0) = 1, y'(0) = 1$

Exercises 8.7

Terms and Concepts

1. What is the difference between a Taylor polynomial and a Maclaurin polynomial?

2. T/F: In general, $p_n(x)$ approximates $f(x)$ better and better as n gets larger.

3. For some function $f(x)$, the Maclaurin polynomial of degree 4 is $p_4(x) = 6 + 3x - 4x^2 + 5x^3 - 7x^4$. What is $p_2(x)$?

4. For some function $f(x)$, the Maclaurin polynomial of degree 4 is $p_4(x) = 6 + 3x - 4x^2 + 5x^3 - 7x^4$. What is $f'''(0)$?

Problems

In Exercises 5 – 12, find the Maclaurin polynomial of degree n for the given function.

5. $f(x) = e^{-x}, \quad n = 3$

6. $f(x) = \sin x, \quad n = 8$

7. $f(x) = x \cdot e^x, \quad n = 5$

8. $f(x) = \tan x, \quad n = 6$

9. $f(x) = e^{2x}, \quad n = 4$

10. $f(x) = \dfrac{1}{1-x}, \quad n = 4$

11. $f(x) = \dfrac{1}{1+x}, \quad n = 4$

12. $f(x) = \dfrac{1}{1+x}, \quad n = 7$

In Exercises 13 – 20, find the Taylor polynomial of degree n, at $x = c$, for the given function.

13. $f(x) = \sqrt{x}, \quad n = 4, \quad c = 1$

14. $f(x) = \ln(x+1), \quad n = 4, \quad c = 1$

15. $f(x) = \cos x, \quad n = 6, \quad c = \pi/4$

16. $f(x) = \sin x, \quad n = 5, \quad c = \pi/6$

17. $f(x) = \dfrac{1}{x}, \quad n = 5, \quad c = 2$

18. $f(x) = \dfrac{1}{x^2}, \quad n = 8, \quad c = 1$

19. $f(x) = \dfrac{1}{x^2+1}, \quad n = 3, \quad c = -1$

20. $f(x) = x^2 \cos x, \quad n = 2, \quad c = \pi$

In Exercises 21 – 24, approximate the function value with the indicated Taylor polynomial and give approximate bounds on the error.

21. Approximate $\sin 0.1$ with the Maclaurin polynomial of degree 3.

22. Approximate $\cos 1$ with the Maclaurin polynomial of degree 4.

23. Approximate $\sqrt{10}$ with the Taylor polynomial of degree 2 centered at $x = 9$.

24. Approximate $\ln 1.5$ with the Taylor polynomial of degree 3 centered at $x = 1$.

Exercises 25 – 28 ask for an n to be found such that $p_n(x)$ approximates $f(x)$ within a certain bound of accuracy.

25. Find n such that the Maclaurin polynomial of degree n of $f(x) = e^x$ approximates e within 0.0001 of the actual value.

26. Find n such that the Taylor polynomial of degree n of $f(x) = \sqrt{x}$, centered at $x = 4$, approximates $\sqrt{3}$ within 0.0001 of the actual value.

27. Find n such that the Maclaurin polynomial of degree n of $f(x) = \cos x$ approximates $\cos \pi/3$ within 0.0001 of the actual value.

28. Find n such that the Maclaurin polynomial of degree n of $f(x) = \sin x$ approximates $\cos \pi$ within 0.0001 of the actual value.

In Exercises 29 – 34, find the n^{th} term of the indicated Taylor polynomial.

29. Find a formula for the n^{th} term of the Maclaurin polynomial for $f(x) = e^x$.

30. Find a formula for the n^{th} term of the Maclaurin polynomial for $f(x) = \cos x$.

31. Find a formula for the n^{th} term of the Maclaurin polynomial for $f(x) = \sin x$.

32. Find a formula for the n^{th} term of the Maclaurin polynomial for $f(x) = \dfrac{1}{1-x}$.

33. Find a formula for the n^{th} term of the Maclaurin polynomial for $f(x) = \dfrac{1}{1+x}$.

34. Find a formula for the n^{th} term of the Taylor polynomial for $f(x) = \ln x$ centered at $x = 1$.

In Exercises 35 – 37, approximate the solution to the given differential equation with a degree 4 Maclaurin polynomial.

35. $y' = y, \qquad y(0) = 1$

36. $y' = 5y, \qquad y(0) = 3$

37. $y' = \dfrac{2}{y}, \qquad y(0) = 1$

Exercises 8.8

Terms and Concepts

1. What is the difference between a Taylor polynomial and a Taylor series?

2. What theorem must we use to show that a function is equal to its Taylor series?

Problems

Key Idea 8.8.1 gives the n^{th} term of the Taylor series of common functions. In Exercises 3 – 6, verify the formula given in the Key Idea by finding the first few terms of the Taylor series of the given function and identifying a pattern.

3. $f(x) = e^x; \quad c = 0$

4. $f(x) = \sin x; \quad c = 0$

5. $f(x) = 1/(1-x); \quad c = 0$

6. $f(x) = \tan^{-1} x; \quad c = 0$

In Exercises 7 – 12, find a formula for the n^{th} term of the Taylor series of $f(x)$, centered at c, by finding the coefficients of the first few powers of x and looking for a pattern. (The formulas for several of these are found in Key Idea 8.8.1; show work verifying these formula.)

7. $f(x) = \cos x; \quad c = \pi/2$

8. $f(x) = 1/x; \quad c = 1$

9. $f(x) = e^{-x}; \quad c = 0$

10. $f(x) = \ln(1+x); \quad c = 0$

11. $f(x) = x/(x+1); \quad c = 1$

12. $f(x) = \sin x; \quad c = \pi/4$

In Exercises 13 – 16, show that the Taylor series for $f(x)$, as given in Key Idea 8.8.1, is equal to $f(x)$ by applying Theorem 8.8.1; that is, show $\lim\limits_{n\to\infty} R_n(x) = 0$.

13. $f(x) = e^x$

14. $f(x) = \sin x$

15. $f(x) = \ln x$ (show equality only on $(1, 2)$)

16. $f(x) = 1/(1-x)$ (show equality only on $(-1, 0)$)

In Exercises 17 – 20, use the Taylor series given in Key Idea 8.8.1 to verify the given identity.

17. $\cos(-x) = \cos x$

18. $\sin(-x) = -\sin x$

19. $\frac{d}{dx}\big(\sin x\big) = \cos x$

20. $\frac{d}{dx}\big(\cos x\big) = -\sin x$

In Exercises 21 – 24, write out the first 5 terms of the Binomial series with the given k-value.

21. $k = 1/2$

22. $k = -1/2$

23. $k = 1/3$

24. $k = 4$

In Exercises 25 – 30, use the Taylor series given in Key Idea 8.8.1 to create the Taylor series of the given functions.

25. $f(x) = \cos\left(x^2\right)$

26. $f(x) = e^{-x}$

27. $f(x) = \sin\left(2x+3\right)$

28. $f(x) = \tan^{-1}\left(x/2\right)$

29. $f(x) = e^x \sin x$ (only find the first 4 terms)

30. $f(x) = (1+x)^{1/2}\cos x$ (only find the first 4 terms)

In Exercises 31 – 32, approximate the value of the given definite integral by using the first 4 nonzero terms of the integrand's Taylor series.

31. $\displaystyle\int_0^{\sqrt{\pi}} \sin\left(x^2\right)\,dx$

32. $\displaystyle\int_0^{\pi^2/4} \cos\left(\sqrt{x}\right)\,dx$

Exercises 9.1

Terms and Concepts

1. What is the difference between degenerate and nondegenerate conics?

2. Use your own words to explain what the eccentricity of an ellipse measures.

3. What has the largest eccentricity: an ellipse or a hyperbola?

4. Explain why the following is true: "If the coefficient of the x^2 term in the equation of an ellipse in standard form is smaller than the coefficient of the y^2 term, then the ellipse has a horizontal major axis."

5. Explain how one can quickly look at the equation of a hyperbola in standard form and determine whether the transverse axis is horizontal or vertical.

6. Fill in the blank: It can be said that ellipses and hyperbolas share the *same* reflective property: "A ray emanating from one focus will reflect off the conic along a ________ that contains the other focus."

Problems

In Exercises 7 – 14, find the equation of the parabola defined by the given information. Sketch the parabola.

7. Focus: $(3, 2)$; directrix: $y = 1$

8. Focus: $(-1, -4)$; directrix: $y = 2$

9. Focus: $(1, 5)$; directrix: $x = 3$

10. Focus: $(1/4, 0)$; directrix: $x = -1/4$

11. Focus: $(1, 1)$; vertex: $(1, 2)$

12. Focus: $(-3, 0)$; vertex: $(0, 0)$

13. Vertex: $(0, 0)$; directrix: $y = -1/16$

14. Vertex: $(2, 3)$; directrix: $x = 4$

In Exercises 15 – 16, the equation of a parabola and a point on its graph are given. Find the focus and directrix of the parabola, and verify that the given point is equidistant from the focus and directrix.

15. $y = \frac{1}{4}x^2$, $P = (2, 1)$

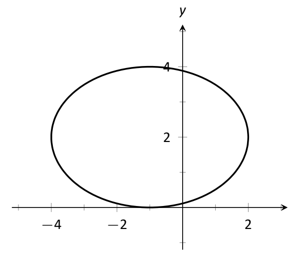
16. $x = \frac{1}{8}(y-2)^2 + 3$, $P = (11, 10)$

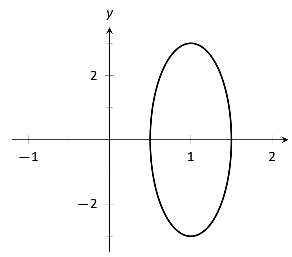
In Exercises 17 – 18, sketch the ellipse defined by the given equation. Label the center, foci and vertices.

17. $\dfrac{(x-1)^2}{3} + \dfrac{(y-2)^2}{5} = 1$

18. $\dfrac{1}{25}x^2 + \dfrac{1}{9}(y+3)^2 = 1$

In Exercises 19 – 20, find the equation of the ellipse shown in the graph. Give the location of the foci and the eccentricity of the ellipse.

19. 

20. 

In Exercises 21 – 24, find the equation of the ellipse defined by the given information. Sketch the elllipse.

21. Foci: $(\pm 2, 0)$; vertices: $(\pm 3, 0)$

22. Foci: $(-1, 3)$ and $(5, 3)$; vertices: $(-3, 3)$ and $(7, 3)$

23. Foci: $(2, \pm 2)$; vertices: $(2, \pm 7)$

24. Focus: $(-1, 5)$; vertex: $(-1, -4)$; center: $(-1, 1)$

In Exercises 25 – 28, write the equation of the given ellipse in standard form.

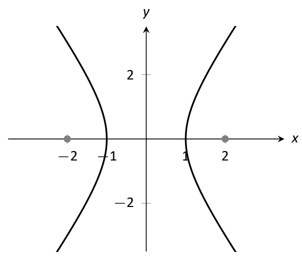
25. $x^2 - 2x + 2y^2 - 8y = -7$

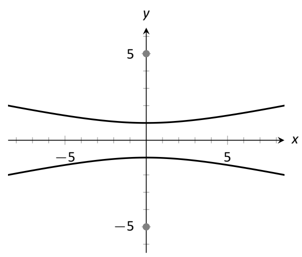
26. $5x^2 + 3y^2 = 15$

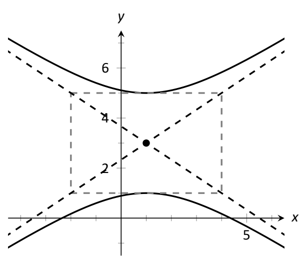
27. $3x^2 + 2y^2 - 12y + 6 = 0$

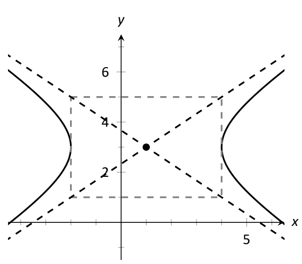
28. $x^2 + y^2 - 4x - 4y + 4 = 0$

In Exercises 29 – 32, find the equation of the hyperbola shown in the graph.

29. 

30. 

31. 

32. 

In Exercises 33 – 34, sketch the hyperbola defined by the given equation. Label the center and foci.

33. $\dfrac{(x-1)^2}{16} - \dfrac{(y+2)^2}{9} = 1$

34. $(y-4)^2 - \dfrac{(x+1)^2}{25} = 1$

In Exercises 35 – 38, find the equation of the hyperbola defined by the given information. Sketch the hyperbola.

35. Foci: $(\pm 3, 0)$; vertices: $(\pm 2, 0)$

36. Foci: $(0, \pm 3)$; vertices: $(0, \pm 2)$

37. Foci: $(-2, 3)$ and $(8, 3)$; vertices: $(-1, 3)$ and $(7, 3)$

38. Foci: $(3, -2)$ and $(3, 8)$; vertices: $(3, 0)$ and $(3, 6)$

In Exercises 39 – 42, write the equation of the hyperbola in standard form.

39. $3x^2 - 4y^2 = 12$

40. $3x^2 - y^2 + 2y = 10$

41. $x^2 - 10y^2 + 40y = 30$

42. $(4y - x)(4y + x) = 4$

43. Consider the ellipse given by $\dfrac{(x-1)^2}{4} + \dfrac{(y-3)^2}{12} = 1$.
    (a) Verify that the foci are located at $(1, 3 \pm 2\sqrt{2})$.
    (b) The points $P_1 = (2, 6)$ and $P_2 = (1+\sqrt{2}, 3+\sqrt{6}) \approx (2.414, 5.449)$ lie on the ellipse. Verify that the sum of distances from each point to the foci is the same.

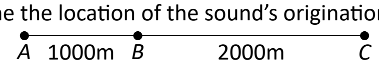
44. Johannes Kepler discovered that the planets of our solar system have elliptical orbits with the Sun at one focus. The Earth's elliptical orbit is used as a standard unit of distance; the distance from the center of Earth's elliptical orbit to one vertex is 1 Astronomical Unit, or A.U.

    The following table gives information about the orbits of three planets.

	Distance from center to vertex	eccentricity
Mercury	0.387 A.U.	0.2056
Earth	1 A.U.	0.0167
Mars	1.524 A.U.	0.0934

    (a) In an ellipse, knowing $c^2 = a^2 - b^2$ and $e = c/a$ allows us to find b in terms of a and e. Show $b = a\sqrt{1-e^2}$.
    (b) For each planet, find equations of their elliptical orbit of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (This places the center at $(0, 0)$, but the Sun is in a different location for each planet.)
    (c) Shift the equations so that the Sun lies at the origin. Plot the three elliptical orbits.

45. A loud sound is recorded at three stations that lie on a line as shown in the figure below. Station A recorded the sound 1 second after Station B, and Station C recorded the sound 3 seconds after B. Using the speed of sound as 340m/s, determine the location of the sound's origination.



Exercises 9.2

Terms and Concepts

1. T/F: When sketching the graph of parametric equations, the x and y values are found separately, then plotted together.

2. The direction in which a graph is "moving" is called the ____ of the graph.

3. An equation written as $y = f(x)$ is written in ____ form.

4. Create parametric equations $x = f(t)$, $y = g(t)$ and sketch their graph. Explain any interesting features of your graph based on the functions f and g.

Problems

In Exercises 5 – 8, sketch the graph of the given parametric equations by hand, making a table of points to plot. Be sure to indicate the orientation of the graph.

5. $x = t^2 + t, \quad y = 1 - t^2, \quad -3 \leq t \leq 3$

6. $x = 1, \quad y = 5\sin t, \quad -\pi/2 \leq t \leq \pi/2$

7. $x = t^2, \quad y = 2, \quad -2 \leq t \leq 2$

8. $x = t^3 - t + 3, \quad y = t^2 + 1, \quad -2 \leq t \leq 2$

In Exercises 9 – 18, sketch the graph of the given parametric equations; using a graphing utility is advisable. Be sure to indicate the orientation of the graph.

9. $x = t^3 - 2t^2, \quad y = t^2, \quad -2 \leq t \leq 3$

10. $x = 1/t, \quad y = \sin t, \quad 0 < t \leq 10$

11. $x = 3\cos t, \quad y = 5\sin t, \quad 0 \leq t \leq 2\pi$

12. $x = 3\cos t + 2, \quad y = 5\sin t + 3, \quad 0 \leq t \leq 2\pi$

13. $x = \cos t, \quad y = \cos(2t), \quad 0 \leq t \leq \pi$

14. $x = \cos t, \quad y = \sin(2t), \quad 0 \leq t \leq 2\pi$

15. $x = 2\sec t, \quad y = 3\tan t, \quad -\pi/2 < t < \pi/2$

16. $x = \cosh t, \quad y = \sinh t, \quad -2 \leq t \leq 2$

17. $x = \cos t + \frac{1}{4}\cos(8t), \quad y = \sin t + \frac{1}{4}\sin(8t), \quad 0 \leq t \leq 2\pi$

18. $x = \cos t + \frac{1}{4}\sin(8t), \quad y = \sin t + \frac{1}{4}\cos(8t), \quad 0 \leq t \leq 2\pi$

In Exercises 19 – 20, four sets of parametric equations are given. Describe how their graphs are similar and different. Be sure to discuss orientation and ranges.

19. (a) $x = t \quad y = t^2, \quad -\infty < t < \infty$

    (b) $x = \sin t \quad y = \sin^2 t, \quad -\infty < t < \infty$

    (c) $x = e^t \quad y = e^{2t}, \quad -\infty < t < \infty$

    (d) $x = -t \quad y = t^2, \quad -\infty < t < \infty$

20. (a) $x = \cos t \quad y = \sin t, \quad 0 \leq t \leq 2\pi$

    (b) $x = \cos(t^2) \quad y = \sin(t^2), \quad 0 \leq t \leq 2\pi$

    (c) $x = \cos(1/t) \quad y = \sin(1/t), \quad 0 < t < 1$

    (d) $x = \cos(\cos t) \quad y = \sin(\cos t), \quad 0 \leq t \leq 2\pi$

In Exercises 21 – 30, eliminate the parameter in the given parametric equations.

21. $x = 2t + 5, \quad y = -3t + 1$

22. $x = \sec t, \quad y = \tan t$

23. $x = 4\sin t + 1, \quad y = 3\cos t - 2$

24. $x = t^2, \quad y = t^3$

25. $x = \dfrac{1}{t+1}, \quad y = \dfrac{3t+5}{t+1}$

26. $x = e^t, \quad y = e^{3t} - 3$

27. $x = \ln t, \quad y = t^2 - 1$

28. $x = \cot t, \quad y = \csc t$

29. $x = \cosh t, \quad y = \sinh t$

30. $x = \cos(2t), \quad y = \sin t$

In Exercises 31 – 34, eliminate the parameter in the given parametric equations. Describe the curve defined by the parametric equations based on its rectangular form.

31. $x = at + x_0, \quad y = bt + y_0$

32. $x = r\cos t, \quad y = r\sin t$

33. $x = a\cos t + h, \quad y = b\sin t + k$

34. $x = a\sec t + h, \quad y = b\tan t + k$

In Exercises 35 – 38, find parametric equations for the given rectangular equation using the parameter $t = \dfrac{dy}{dx}$. Verify that at $t = 1$, the point on the graph has a tangent line with slope of 1.

35. $y = 3x^2 - 11x + 2$

36. $y = e^x$

37. $y = \sin x$ on $[0, \pi]$

38. $y = \sqrt{x}$ on $[0, \infty)$

In Exercises 39 – 42, find the values of t where the graph of the parametric equations crosses itself.

39. $x = t^3 - t + 3, \quad y = t^2 - 3$

40. $x = t^3 - 4t^2 + t + 7, \quad y = t^2 - t$

41. $x = \cos t, \quad y = \sin(2t)$ on $[0, 2\pi]$

42. $x = \cos t \cos(3t), \quad y = \sin t \cos(3t)$ on $[0, \pi]$

In Exercises 43 – 46, find the value(s) of t where the curve defined by the parametric equations is not smooth.

43. $x = t^3 + t^2 - t, \quad y = t^2 + 2t + 3$

44. $x = t^2 - 4t, \quad y = t^3 - 2t^2 - 4t$

45. $x = \cos t, \quad y = 2\cos t$

46. $x = 2\cos t - \cos(2t), \quad y = 2\sin t - \sin(2t)$

In Exercises 47 – 55, find parametric equations that describe the given situation.

47. A projectile is fired from a height of 0ft, landing 16ft away in 4s.

48. A projectile is fired from a height of 0ft, landing 200ft away in 4s.

49. A projectile is fired from a height of 0ft, landing 200ft away in 20s.

50. A circle of radius 2, centered at the origin, that is traced clockwise once on $[0, 2\pi]$.

51. A circle of radius 3, centered at $(1, 1)$, that is traced once counter–clockwise on $[0, 1]$.

52. An ellipse centered at $(1, 3)$ with vertical major axis of length 6 and minor axis of length 2.

53. An ellipse with foci at $(\pm 1, 0)$ and vertices at $(\pm 5, 0)$.

54. A hyperbola with foci at $(5, -3)$ and $(-1, -3)$, and with vertices at $(1, -3)$ and $(3, -3)$.

55. A hyperbola with vertices at $(0, \pm 6)$ and asymptotes $y = \pm 3x$.

Exercises 9.3

Terms and Concepts

1. T/F: Given parametric equations $x = f(t)$ and $y = g(t)$, $\frac{dy}{dx} = f'(t)/g'(t)$, as long as $g'(t) \neq 0$.

2. Given parametric equations $x = f(t)$ and $y = g(t)$, the derivative $\frac{dy}{dx}$ as given in Key Idea 9.3.1 is a function of ________?

3. T/F: Given parametric equations $x = f(t)$ and $y = g(t)$, to find $\frac{d^2y}{dx^2}$, one simply computes $\frac{d}{dt}\left(\frac{dy}{dx}\right)$.

4. T/F: If $\frac{dy}{dx} = 0$ at $t = t_0$, then the normal line to the curve at $t = t_0$ is a vertical line.

Problems

In Exercises 5 – 12, parametric equations for a curve are given.

(a) Find $\dfrac{dy}{dx}$.

(b) Find the equations of the tangent and normal line(s) at the point(s) given.

(c) Sketch the graph of the parametric functions along with the found tangent and normal lines.

5. $x = t, y = t^2; \quad t = 1$

6. $x = \sqrt{t}, y = 5t + 2; \quad t = 4$

7. $x = t^2 - t, y = t^2 + t; \quad t = 1$

8. $x = t^2 - 1, y = t^3 - t; \quad t = 0$ and $t = 1$

9. $x = \sec t, y = \tan t$ on $(-\pi/2, \pi/2); \quad t = \pi/4$

10. $x = \cos t, y = \sin(2t)$ on $[0, 2\pi]; \quad t = \pi/4$

11. $x = \cos t \sin(2t), y = \sin t \sin(2t)$ on $[0, 2\pi]; \quad t = 3\pi/4$

12. $x = e^{t/10} \cos t, y = e^{t/10} \sin t; \quad t = \pi/2$

In Exercises 13 – 20, find t-values where the curve defined by the given parametric equations has a horizontal tangent line. Note: these are the same equations as in Exercises 5 – 12.

13. $x = t, y = t^2$

14. $x = \sqrt{t}, y = 5t + 2$

15. $x = t^2 - t, y = t^2 + t$

16. $x = t^2 - 1, y = t^3 - t$

17. $x = \sec t, y = \tan t$ on $(-\pi/2, \pi/2)$

18. $x = \cos t, y = \sin(2t)$ on $[0, 2\pi]$

19. $x = \cos t \sin(2t), y = \sin t \sin(2t)$ on $[0, 2\pi]$

20. $x = e^{t/10} \cos t, y = e^{t/10} \sin t$

In Exercises 21 – 24, find $t = t_0$ where the graph of the given parametric equations is not smooth, then find $\displaystyle\lim_{t\to t_0} \frac{dy}{dx}$.

21. $x = \dfrac{1}{t^2 + 1}, \quad y = t^3$

22. $x = -t^3 + 7t^2 - 16t + 13, \quad y = t^3 - 5t^2 + 8t - 2$

23. $x = t^3 - 3t^2 + 3t - 1, \quad y = t^2 - 2t + 1$

24. $x = \cos^2 t, \quad y = 1 - \sin^2 t$

In Exercises 25 – 32, parametric equations for a curve are given. Find $\frac{d^2y}{dx^2}$, then determine the intervals on which the graph of the curve is concave up/down. Note: these are the same equations as in Exercises 5 – 12.

25. $x = t, \quad y = t^2$

26. $x = \sqrt{t}, \quad y = 5t + 2$

27. $x = t^2 - t, \quad y = t^2 + t$

28. $x = t^2 - 1, \quad y = t^3 - t$

29. $x = \sec t, \quad y = \tan t$ on $(-\pi/2, \pi/2)$

30. $x = \cos t, \quad y = \sin(2t)$ on $[0, 2\pi]$

31. $x = \cos t \sin(2t), \quad y = \sin t \sin(2t)$ on $[-\pi/2, \pi/2]$

32. $x = e^{t/10} \cos t, \quad y = e^{t/10} \sin t$

In Exercises 33 – 36, find the arc length of the graph of the parametric equations on the given interval(s).

33. $x = -3\sin(2t), \quad y = 3\cos(2t)$ on $[0, \pi]$

34. $x = e^{t/10} \cos t, \quad y = e^{t/10} \sin t$ on $[0, 2\pi]$ and $[2\pi, 4\pi]$

35. $x = 5t + 2, \quad y = 1 - 3t$ on $[-1, 1]$

36. $x = 2t^{3/2}, \quad y = 3t$ on $[0, 1]$

In Exercises 37 – 40, numerically approximate the given arc length.

37. Approximate the arc length of one petal of the rose curve $x = \cos t \cos(2t), \quad y = \sin t \cos(2t)$ using Simpson's Rule and $n = 4$.

38. Approximate the arc length of the "bow tie curve" $x = \cos t, \quad y = \sin(2t)$ using Simpson's Rule and $n = 6$.

39. Approximate the arc length of the parabola $x = t^2 - t$, $y = t^2 + t$ on $[-1, 1]$ using Simpson's Rule and $n = 4$.

40. A common approximate of the circumference of an ellipse given by $x = a\cos t, \quad y = b\sin t$ is $C \approx 2\pi\sqrt{\dfrac{a^2+b^2}{2}}$. Use this formula to approximate the circumference of $x = 5\cos t, \quad y = 3\sin t$ and compare this to the approximation given by Simpson's Rule and $n = 6$.

In Exercises 41 – 44, a solid of revolution is described. Find or approximate its surface area as specified.

41. Find the surface area of the sphere formed by rotating the circle $x = 2\cos t, \quad y = 2\sin t$ about:

    (a) the x-axis and

    (b) the y-axis.

42. Find the surface area of the torus (or "donut") formed by rotating the circle $x = \cos t + 2, \quad y = \sin t$ about the y-axis.

43. Approximate the surface area of the solid formed by rotating the "upper right half" of the bow tie curve $x = \cos t$, $y = \sin(2t)$ on $[0, \pi/2]$ about the x-axis, using Simpson's Rule and $n = 4$.

44. Approximate the surface area of the solid formed by rotating the one petal of the rose curve $x = \cos t\cos(2t)$, $y = \sin t\cos(2t)$ on $[0, \pi/4]$ about the x-axis, using Simpson's Rule and $n = 4$.

Exercises 9.4

Terms and Concepts

1. In your own words, describe how to plot the polar point $P(r, \theta)$.

2. T/F: When plotting a point with polar coordinate $P(r, \theta)$, r must be positive.

3. T/F: Every point in the Cartesian plane can be represented by a polar coordinate.

4. T/F: Every point in the Cartesian plane can be represented uniquely by a polar coordinate.

Problems

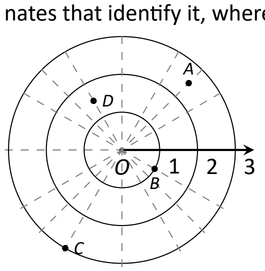
5. Plot the points with the given polar coordinates.

   (a) $A = P(2, 0)$
   (b) $B = P(1, \pi)$
   (c) $C = P(-2, \pi/2)$
   (d) $D = P(1, \pi/4)$

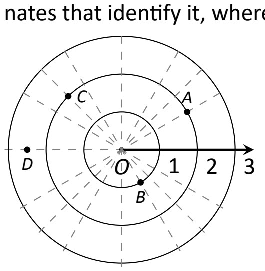
6. Plot the points with the given polar coordinates.

   (a) $A = P(2, 3\pi)$
   (b) $B = P(1, -\pi)$
   (c) $C = P(1, 2)$
   (d) $D = P(1/2, 5\pi/6)$

7. For each of the given points give two sets of polar coordinates that identify it, where $0 \leq \theta \leq 2\pi$.



8. For each of the given points give two sets of polar coordinates that identify it, where $-\pi \leq \theta \leq \pi$.



9. Convert each of the following polar coordinates to rectangular, and each of the following rectangular coordinates to polar.

   (a) $A = P(2, \pi/4)$
   (b) $B = P(2, -\pi/4)$
   (c) $C = (2, -1)$
   (d) $D = (-2, 1)$

10. Convert each of the following polar coordinates to rectangular, and each of the following rectangular coordinates to polar.

    (a) $A = P(3, \pi)$
    (b) $B = P(1, 2\pi/3)$
    (c) $C = (0, 4)$
    (d) $D = (1, -\sqrt{3})$

In Exercises 11 – 30, graph the polar function on the given interval.

11. $r = 2, \quad 0 \leq \theta \leq \pi/2$

12. $\theta = \pi/6, \quad -1 \leq r \leq 2$

13. $r = 1 - \cos\theta, \quad [0, 2\pi]$

14. $r = 2 + \sin\theta, \quad [0, 2\pi]$

15. $r = 2 - \sin\theta, \quad [0, 2\pi]$

16. $r = 1 - 2\sin\theta, \quad [0, 2\pi]$

17. $r = 1 + 2\sin\theta, \quad [0, 2\pi]$

18. $r = \cos(2\theta), \quad [0, 2\pi]$

19. $r = \sin(3\theta), \quad [0, \pi]$

20. $r = \cos(\theta/3), \quad [0, 3\pi]$

21. $r = \cos(2\theta/3), \quad [0, 6\pi]$

22. $r = \theta/2, \quad [0, 4\pi]$

23. $r = 3\sin(\theta), \quad [0, \pi]$

24. $r = 2\cos(\theta), \quad [0, \pi/2]$

25. $r = \cos\theta\sin\theta, \quad [0, 2\pi]$

26. $r = \theta^2 - (\pi/2)^2, \quad [-\pi, \pi]$

27. $r = \dfrac{3}{5\sin\theta - \cos\theta}, \quad [0, 2\pi]$

28. $r = \dfrac{-2}{3\cos\theta - 2\sin\theta}, \quad [0, 2\pi]$

29. $r = 3\sec\theta, \quad (-\pi/2, \pi/2)$

30. $r = 3\csc\theta, \quad (0, \pi)$

In Exercises 31 – 40, convert the polar equation to a rectangular equation.

31. $r = 6\cos\theta$

32. $r = -4\sin\theta$

33. $r = \cos\theta + \sin\theta$

34. $r = \dfrac{7}{5\sin\theta - 2\cos\theta}$

35. $r = \dfrac{3}{\cos\theta}$

36. $r = \dfrac{4}{\sin\theta}$

37. $r = \tan\theta$

38. $r = \cot\theta$

39. $r = 2$

40. $\theta = \pi/6$

In Exercises 41 – 48, convert the rectangular equation to a polar equation.

41. $y = x$

42. $y = 4x + 7$

43. $x = 5$

44. $y = 5$

45. $x = y^2$

46. $x^2y = 1$

47. $x^2 + y^2 = 7$

48. $(x + 1)^2 + y^2 = 1$

In Exercises 49 – 56, find the points of intersection of the polar graphs.

49. $r = \sin(2\theta)$ and $r = \cos\theta$ on $[0, \pi]$

50. $r = \cos(2\theta)$ and $r = \cos\theta$ on $[0, \pi]$

51. $r = 2\cos\theta$ and $r = 2\sin\theta$ on $[0, \pi]$

52. $r = \sin\theta$ and $r = \sqrt{3} + 3\sin\theta$ on $[0, 2\pi]$

53. $r = \sin(3\theta)$ and $r = \cos(3\theta)$ on $[0, \pi]$

54. $r = 3\cos\theta$ and $r = 1 + \cos\theta$ on $[-\pi, \pi]$

55. $r = 1$ and $r = 2\sin(2\theta)$ on $[0, 2\pi]$

56. $r = 1 - \cos\theta$ and $r = 1 + \sin\theta$ on $[0, 2\pi]$

57. Pick a integer value for n, where $n \neq 2, 3$, and use technology to plot $r = \sin\left(\frac{m}{n}\theta\right)$ for three different integer values of m. Sketch these and determine a minimal interval on which the entire graph is shown.

58. Create your own polar function, $r = f(\theta)$ and sketch it. Describe why the graph looks as it does.

Exercises 9.5

Terms and Concepts

1. Given polar equation $r = f(\theta)$, how can one create parametric equations of the same curve?

2. With rectangular coordinates, it is natural to approximate area with ________; with polar coordinates, it is natural to approximate area with ________.

Problems

In Exercises 3 – 10, find:

(a) $\dfrac{dy}{dx}$

(b) the equation of the tangent and normal lines to the curve at the indicated θ–value.

3. $r = 1; \quad \theta = \pi/4$

4. $r = \cos\theta; \quad \theta = \pi/4$

5. $r = 1 + \sin\theta; \quad \theta = \pi/6$

6. $r = 1 - 3\cos\theta; \quad \theta = 3\pi/4$

7. $r = \theta; \quad \theta = \pi/2$

8. $r = \cos(3\theta); \quad \theta = \pi/6$

9. $r = \sin(4\theta); \quad \theta = \pi/3$

10. $r = \dfrac{1}{\sin\theta - \cos\theta}; \quad \theta = \pi$

In Exercises 11 – 14, find the values of θ in the given interval where the graph of the polar function has horizontal and vertical tangent lines.

11. $r = 3; \quad [0, 2\pi]$

12. $r = 2\sin\theta; \quad [0, \pi]$

13. $r = \cos(2\theta); \quad [0, 2\pi]$

14. $r = 1 + \cos\theta; \quad [0, 2\pi]$

In Exercises 15 – 16, find the equation of the lines tangent to the graph at the pole.

15. $r = \sin\theta; \quad [0, \pi]$

16. $r = \sin(3\theta); \quad [0, \pi]$

In Exercises 17 – 28, find the area of the described region.

17. Enclosed by the circle: $r = 4\sin\theta$

18. Enclosed by the circle $r = 5$

19. Enclosed by one petal of $r = \sin(3\theta)$

20. Enclosed by one petal of the rose curve $r = \cos(n\,\theta)$, where n is a positive integer.

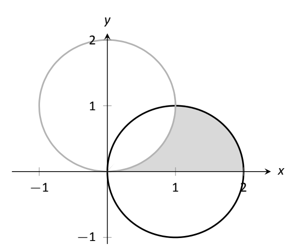
21. Enclosed by the cardioid $r = 1 - \sin\theta$

22. Enclosed by the inner loop of the limaçon $r = 1 + 2\cos\theta$

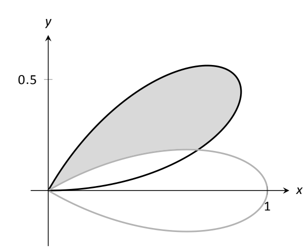
23. Enclosed by the outer loop of the limaçon $r = 1 + 2\cos\theta$ (including area enclosed by the inner loop)

24. Enclosed between the inner and outer loop of the limaçon $r = 1 + 2\cos\theta$

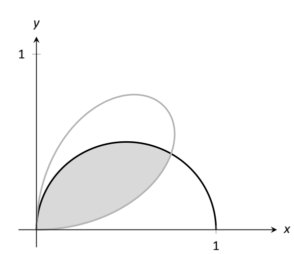
25. Enclosed by $r = 2\cos\theta$ and $r = 2\sin\theta$, as shown:



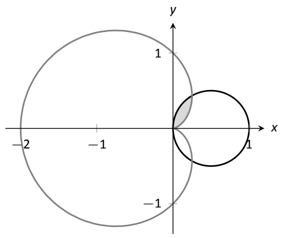
26. Enclosed by $r = \cos(3\theta)$ and $r = \sin(3\theta)$, as shown:



27. Enclosed by $r = \cos\theta$ and $r = \sin(2\theta)$, as shown:



28. Enclosed by $r = \cos\theta$ and $r = 1 - \cos\theta$, as shown:



In Exercises 29 – 34, answer the questions involving arc length.

29. Use the arc length formula to compute the arc length of the circle $r = 2$.

30. Use the arc length formula to compute the arc length of the circle $r = 4\sin\theta$.

31. Use the arc length formula to compute the arc length of $r = \cos\theta + \sin\theta$.

32. Use the arc length formula to compute the arc length of the cardioid $r = 1 + \cos\theta$. (Hint: apply the formula, simplify, then use a Power–Reducing Formula to convert $1 + \cos\theta$ into a square.)

33. Approximate the arc length of one petal of the rose curve $r = \sin(3\theta)$ with Simpson's Rule and $n = 4$.

34. Let $x(\theta) = f(\theta)\cos\theta$ and $y(\theta) = f(\theta)\sin\theta$. Show, as suggested by the text, that

$$x'(\theta)^2 + y'(\theta)^2 = f'(\theta)^2 + f(\theta)^2.$$

In Exercises 35 – 40, answer the questions involving surface area.

35. Find the surface area of the sphere formed by revolving the circle $r = 2$ about the initial ray.

36. Find the surface area of the sphere formed by revolving the circle $r = 2\cos\theta$ about the initial ray.

37. Find the surface area of the solid formed by revolving the cardioid $r = 1 + \cos\theta$ about the initial ray.

38. Find the surface area of the solid formed by revolving the circle $r = 2\cos\theta$ about the line $\theta = \pi/2$.

39. Find the surface area of the solid formed by revolving the line $r = 3\sec\theta$, $-\pi/4 \leq \theta \leq \pi/4$, about the line $\theta = \pi/2$.

40. Find the surface area of the solid formed by revolving the line $r = 3\sec\theta$, $0 \leq \theta \leq \pi/4$, about the initial ray.

Exercises 10.1

Terms and Concepts

1. Axes drawn in space must conform to the ________ ________ rule.

2. In the plane, the equation $x = 2$ defines a ________; in space, $x = 2$ defines a ________.

3. In the plane, the equation $y = x^2$ defines a ________; in space, $y = x^2$ defines a ________.

4. Which quadric surface looks like a Pringles® chip?

5. Consider the hyperbola $x^2 - y^2 = 1$ in the plane. If this hyperbola is rotated about the x-axis, what quadric surface is formed?

6. Consider the hyperbola $x^2 - y^2 = 1$ in the plane. If this hyperbola is rotated about the y-axis, what quadric surface is formed?

Problems

7. The points $A = (1, 4, 2)$, $B = (2, 6, 3)$ and $C = (4, 3, 1)$ form a triangle in space. Find the distances between each pair of points and determine if the triangle is a right triangle.

8. The points $A = (1, 1, 3)$, $B = (3, 2, 7)$, $C = (2, 0, 8)$ and $D = (0, -1, 4)$ form a quadrilateral $ABCD$ in space. Is this a parallelogram?

9. Find the center and radius of the sphere defined by
$x^2 - 8x + y^2 + 2y + z^2 + 8 = 0$.

10. Find the center and radius of the sphere defined by
$x^2 + y^2 + z^2 + 4x - 2y - 4z + 4 = 0$.

In Exercises 11 – 14, describe the region in space defined by the inequalities.

11. $x^2 + y^2 + z^2 < 1$

12. $0 \leq x \leq 3$

13. $x \geq 0,\ y \geq 0,\ z \geq 0$

14. $y \geq 3$

In Exercises 15 – 18, sketch the cylinder in space.

15. $z = x^3$

16. $y = \cos z$

17. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

18. $y = \dfrac{1}{x}$

In Exercises 19 – 22, give the equation of the surface of revolution described.

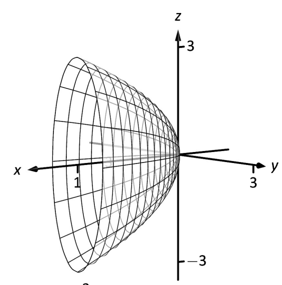
19. Revolve $z = \dfrac{1}{1 + y^2}$ about the y-axis.

20. Revolve $y = x^2$ about the x-axis.

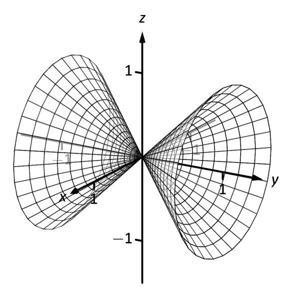
21. Revolve $z = x^2$ about the z-axis.

22. Revolve $z = 1/x$ about the z-axis.

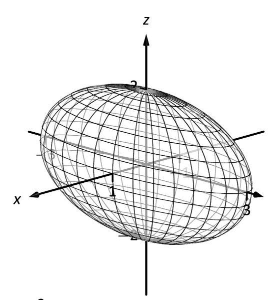
In Exercises 23 – 26, a quadric surface is sketched. Determine which of the given equations best fits the graph.

23. 

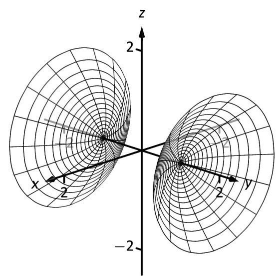
(a) $x = y^2 + \dfrac{z^2}{9}$ (b) $x = y^2 + \dfrac{z^2}{3}$

24. 

(a) $x^2 - y^2 - z^2 = 0$ (b) $x^2 - y^2 + z^2 = 0$

25. 

(a) $x^2 + \dfrac{y^2}{3} + \dfrac{z^2}{2} = 1$ (b) $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

26. 

(a) $y^2 - x^2 - z^2 = 1$ (b) $y^2 + x^2 - z^2 = 1$

In Exercises 27 – 32, sketch the quadric surface.

27. $z - y^2 + x^2 = 0$

28. $z^2 = x^2 + \dfrac{y^2}{4}$

29. $x = -y^2 - z^2$

30. $16x^2 - 16y^2 - 16z^2 = 1$

31. $\dfrac{x^2}{9} - y^2 + \dfrac{z^2}{25} = 1$

32. $4x^2 + 2y^2 + z^2 = 4$

Exercises 10.2

Terms and Concepts

1. Name two different things that cannot be described with just one number, but rather need 2 or more numbers to fully describe them.

2. What is the difference between $(1, 2)$ and $\langle 1, 2\rangle$?

3. What is a unit vector?

4. Unit vectors can be thought of as conveying what type of information?

5. What does it mean for two vectors to be parallel?

6. What effect does multiplying a vector by -2 have?

Problems

In Exercises 7 – 10, points P and Q are given. Write the vector $\overrightarrow{PQ}$ in component form and using the standard unit vectors.

7. $P = (2, -1), \quad Q = (3, 5)$

8. $P = (3, 2), \quad Q = (7, -2)$

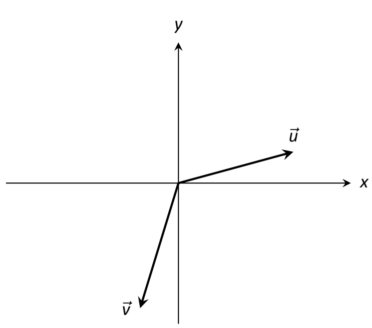
9. $P = (0, 3, -1), \quad Q = (6, 2, 5)$

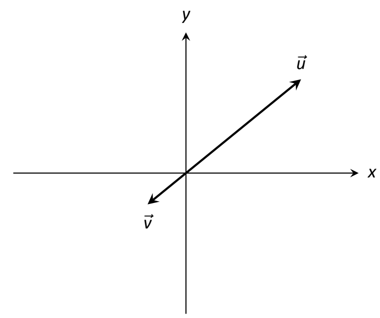
10. $P = (2, 1, 2), \quad Q = (4, 3, 2)$

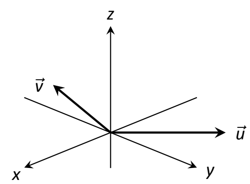
11. Let $\vec{u} = \langle 1, -2\rangle$ and $\vec{v} = \langle 1, 1\rangle$.
    (a) Find $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $2\vec{u} - 3\vec{v}$.
    (b) Sketch the above vectors on the same axes, along with $\vec{u}$ and $\vec{v}$.
    (c) Find $\vec{x}$ where $\vec{u} + \vec{x} = 2\vec{v} - \vec{x}$.

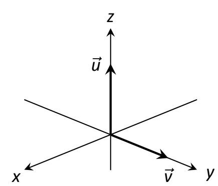
12. Let $\vec{u} = \langle 1, 1, -1\rangle$ and $\vec{v} = \langle 2, 1, 2\rangle$.
    (a) Find $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $\pi\vec{u} - \sqrt{2}\vec{v}$.
    (b) Sketch the above vectors on the same axes, along with $\vec{u}$ and $\vec{v}$.
    (c) Find $\vec{x}$ where $\vec{u} + \vec{x} = \vec{v} + 2\vec{x}$.

In Exercises 13 – 16, sketch $\vec{u}$, $\vec{v}$, $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ on the same axes.

13. 

14. 

15. 

16. 

In Exercises 17 – 20, find $||\,\vec{u}\,||$, $||\,\vec{v}\,||$, $||\,\vec{u}+\vec{v}\,||$ and $||\,\vec{u}-\vec{v}\,||$.

17. $\vec{u} = \langle 2, 1\rangle, \quad \vec{v} = \langle 3, -2\rangle$

18. $\vec{u} = \langle -3, 2, 2\rangle, \quad \vec{v} = \langle 1, -1, 1\rangle$

19. $\vec{u} = \langle 1, 2\rangle, \quad \vec{v} = \langle -3, -6\rangle$

20. $\vec{u} = \langle 2, -3, 6\rangle, \quad \vec{v} = \langle 10, -15, 30\rangle$

21. Under what conditions is $||\,\vec{u}\,|| + ||\,\vec{v}\,|| = ||\,\vec{u} + \vec{v}\,||$?

In Exercises 22 – 25, find the unit vector $\vec{u}$ in the direction of $\vec{v}$.

22. $\vec{v} = \langle 3, 7\rangle$

23. $\vec{v} = \langle 6, 8\rangle$

24. $\vec{v} = \langle 1, -2, 2\rangle$

25. $\vec{v} = \langle 2, -2, 2\rangle$

26. Find the unit vector in the first quadrant of $\mathbb{R}^2$ that makes a 50° angle with the x-axis.

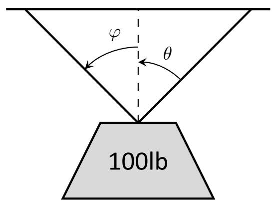
27. Find the unit vector in the second quadrant of $\mathbb{R}^2$ that makes a 30° angle with the y-axis.

28. Verify, from Key Idea 10.2.1, that

$$\vec{u} = \langle \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\rangle$$

is a unit vector for all angles θ and φ.

A weight of 100lb is suspended from two chains, making angles with the vertical of θ and φ as shown in the figure below.



In Exercises 29 – 32, angles θ and φ are given. Find the magnitude of the force applied to each chain.

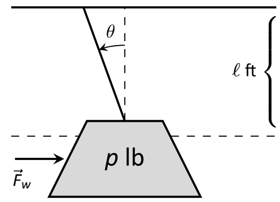
29. $\theta = 30^\circ$, $\varphi = 30^\circ$

30. $\theta = 60^\circ$, $\varphi = 60^\circ$

31. $\theta = 20^\circ$, $\varphi = 15^\circ$

32. $\theta = 0^\circ$, $\varphi = 0^\circ$

A weight of $p$lb is suspended from a chain of length ℓ while a constant force of $\vec{F}_w$ pushes the weight to the right, making an angle of θ with the vertical, as shown in the figure below.



In Exercises 33 – 36, a force $\vec{F}_w$ and length ℓ are given. Find the angle θ and the height the weight is lifted as it moves to the right.

33. $\vec{F}_w = 1\text{lb}$, $\ell = 1\text{ft}$, $p = 1\text{lb}$

34. $\vec{F}_w = 1\text{lb}$, $\ell = 1\text{ft}$, $p = 10\text{lb}$

35. $\vec{F}_w = 1\text{lb}$, $\ell = 10\text{ft}$, $p = 1\text{lb}$

36. $\vec{F}_w = 10\text{lb}$, $\ell = 10\text{ft}$, $p = 1\text{lb}$

Exercises 10.3

Terms and Concepts

1. The dot product of two vectors is a ________, not a vector.

2. How are the concepts of the dot product and vector magnitude related?

3. How can one quickly tell if the angle between two vectors is acute or obtuse?

4. Give a synonym for "orthogonal."

Problems

In Exercises 5 – 10, find the dot product of the given vectors.

5. $\vec{u} = \langle 2, -4 \rangle$, $\vec{v} = \langle 3, 7 \rangle$

6. $\vec{u} = \langle 5, 3 \rangle$, $\vec{v} = \langle 6, 1 \rangle$

7. $\vec{u} = \langle 1, -1, 2 \rangle$, $\vec{v} = \langle 2, 5, 3 \rangle$

8. $\vec{u} = \langle 3, 5, -1 \rangle$, $\vec{v} = \langle 4, -1, 7 \rangle$

9. $\vec{u} = \langle 1, 1 \rangle$, $\vec{v} = \langle 1, 2, 3 \rangle$

10. $\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle 0, 0, 0 \rangle$

11. Create your own vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$ and show that $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$.

12. Create your own vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ and scalar c and show that $c(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (c\vec{v})$.

In Exercises 13 – 16, find the measure of the angle between the two vectors in both radians and degrees.

13. $\vec{u} = \langle 1, 1 \rangle$, $\vec{v} = \langle 1, 2 \rangle$

14. $\vec{u} = \langle -2, 1 \rangle$, $\vec{v} = \langle 3, 5 \rangle$

15. $\vec{u} = \langle 8, 1, -4 \rangle$, $\vec{v} = \langle 2, 2, 0 \rangle$

16. $\vec{u} = \langle 1, 7, 2 \rangle$, $\vec{v} = \langle 4, -2, 5 \rangle$

In Exercises 17 – 20, a vector $\vec{v}$ is given. Give two vectors that are orthogonal to $\vec{v}$.

17. $\vec{v} = \langle 4, 7 \rangle$

18. $\vec{v} = \langle -3, 5 \rangle$

19. $\vec{v} = \langle 1, 1, 1 \rangle$

20. $\vec{v} = \langle 1, -2, 3 \rangle$

In Exercises 21 – 26, vectors $\vec{u}$ and $\vec{v}$ are given. Find $\text{proj}_{\vec{v}}\,\vec{u}$, the orthogonal projection of $\vec{u}$ onto $\vec{v}$, and sketch all three vectors with the same initial point.

21. $\vec{u} = \langle 1, 2 \rangle$, $\vec{v} = \langle -1, 3 \rangle$

22. $\vec{u} = \langle 5, 5 \rangle$, $\vec{v} = \langle 1, 3 \rangle$

23. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 1, 1 \rangle$

24. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 2, 3 \rangle$

25. $\vec{u} = \langle 1, 5, 1 \rangle$, $\vec{v} = \langle 1, 2, 3 \rangle$

26. $\vec{u} = \langle 3, -1, 2 \rangle$, $\vec{v} = \langle 2, 2, 1 \rangle$

In Exercises 27 – 32, vectors $\vec{u}$ and $\vec{v}$ are given. Write $\vec{u}$ as the sum of two vectors, one of which is parallel to $\vec{v}$ and one of which is perpendicular to $\vec{v}$. Note: these are the same pairs of vectors as found in Exercises 21 – 26.

27. $\vec{u} = \langle 1, 2 \rangle$, $\vec{v} = \langle -1, 3 \rangle$

28. $\vec{u} = \langle 5, 5 \rangle$, $\vec{v} = \langle 1, 3 \rangle$

29. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 1, 1 \rangle$

30. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 2, 3 \rangle$

31. $\vec{u} = \langle 1, 5, 1 \rangle$, $\vec{v} = \langle 1, 2, 3 \rangle$

32. $\vec{u} = \langle 3, -1, 2 \rangle$, $\vec{v} = \langle 2, 2, 1 \rangle$

33. A 10lb box sits on a ramp that rises 4ft over a distance of 20ft. How much force is required to keep the box from sliding down the ramp?

34. A 10lb box sits on a 15ft ramp that makes a 30° angle with the horizontal. How much force is required to keep the box from sliding down the ramp?

35. How much work is performed in moving a box horizontally 10ft with a force of 20lb applied at an angle of 45° to the horizontal?

36. How much work is performed in moving a box horizontally 10ft with a force of 20lb applied at an angle of 10° to the horizontal?

37. How much work is performed in moving a box up the length of a ramp that rises 2ft over a distance of 10ft, with a force of 50lb applied horizontally?

38. How much work is performed in moving a box up the length of a ramp that rises 2ft over a distance of 10ft, with a force of 50lb applied at an angle of 45° to the horizontal?

39. How much work is performed in moving a box up the length of a 10ft ramp that makes a 5° angle with the horizontal, with 50lb of force applied in the direction of the ramp?

Exercises 10.4

Terms and Concepts

1. The cross product of two vectors is a ________, not a scalar.

2. One can visualize the direction of $\vec{u} \times \vec{v}$ using the ________ ________ ________.

3. Give a synonym for "orthogonal."

4. T/F: A fundamental principle of the cross product is that $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$.

5. ________ is a measure of the turning force applied to an object.

6. T/F: If $\vec{u}$ and $\vec{v}$ are parallel, then $\vec{u} \times \vec{v} = \vec{0}$.

Problems

In Exercises 7 – 16, vectors $\vec{u}$ and $\vec{v}$ are given. Compute $\vec{u} \times \vec{v}$ and show this is orthogonal to both $\vec{u}$ and $\vec{v}$.

7. $\vec{u} = \langle 3, 2, -2 \rangle, \quad \vec{v} = \langle 0, 1, 5 \rangle$

8. $\vec{u} = \langle 5, -4, 3 \rangle, \quad \vec{v} = \langle 2, -5, 1 \rangle$

9. $\vec{u} = \langle 4, -5, -5 \rangle, \quad \vec{v} = \langle 3, 3, 4 \rangle$

10. $\vec{u} = \langle -4, 7, -10 \rangle, \quad \vec{v} = \langle 4, 4, 1 \rangle$

11. $\vec{u} = \langle 1, 0, 1 \rangle, \quad \vec{v} = \langle 5, 0, 7 \rangle$

12. $\vec{u} = \langle 1, 5, -4 \rangle, \quad \vec{v} = \langle -2, -10, 8 \rangle$

13. $\vec{u} = \langle a, b, 0 \rangle, \quad \vec{v} = \langle c, d, 0 \rangle$

14. $\vec{u} = \vec{i}, \quad \vec{v} = \vec{j}$

15. $\vec{u} = \vec{i}, \quad \vec{v} = \vec{k}$

16. $\vec{u} = \vec{j}, \quad \vec{v} = \vec{k}$

17. Pick any vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ and show that $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$.

18. Pick any vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ and show that $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$.

In Exercises 19 – 22, the magnitudes of vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ are given, along with the angle θ between them. Use this information to find the magnitude of $\vec{u} \times \vec{v}$.

19. $||\,\vec{u}\,|| = 2, \quad ||\,\vec{v}\,|| = 5, \quad \theta = 30^\circ$

20. $||\,\vec{u}\,|| = 3, \quad ||\,\vec{v}\,|| = 7, \quad \theta = \pi/2$

21. $||\,\vec{u}\,|| = 3, \quad ||\,\vec{v}\,|| = 4, \quad \theta = \pi$

22. $||\,\vec{u}\,|| = 2, \quad ||\,\vec{v}\,|| = 5, \quad \theta = 5\pi/6$

In Exercises 23 – 26, find the area of the parallelogram defined by the given vectors.

23. $\vec{u} = \langle 1, 1, 2 \rangle, \quad \vec{v} = \langle 2, 0, 3 \rangle$

24. $\vec{u} = \langle -2, 1, 5 \rangle, \quad \vec{v} = \langle -1, 3, 1 \rangle$

25. $\vec{u} = \langle 1, 2 \rangle, \quad \vec{v} = \langle 2, 1 \rangle$

26. $\vec{u} = \langle 2, 0 \rangle, \quad \vec{v} = \langle 0, 3 \rangle$

In Exercises 27 – 30, find the area of the triangle with the given vertices.

27. Vertices: $(0, 0, 0)$, $(1, 3, -1)$ and $(2, 1, 1)$.

28. Vertices: $(5, 2, -1)$, $(3, 6, 2)$ and $(1, 0, 4)$.

29. Vertices: $(1, 1)$, $(1, 3)$ and $(2, 2)$.

30. Vertices: $(3, 1)$, $(1, 2)$ and $(4, 3)$.

In Exercises 31 – 32, find the area of the quadrilateral with the given vertices. (Hint: break the quadrilateral into 2 triangles.)

31. Vertices: $(0, 0)$, $(1, 2)$, $(3, 0)$ and $(4, 3)$.

32. Vertices: $(0, 0, 0)$, $(2, 1, 1)$, $(-1, 2, -8)$ and $(1, -1, 5)$.

In Exercises 33 – 34, find the volume of the parallelepiped defined by the given vectors.

33. $\vec{u} = \langle 1, 1, 1 \rangle, \quad \vec{v} = \langle 1, 2, 3 \rangle, \quad \vec{w} = \langle 1, 0, 1 \rangle$

34. $\vec{u} = \langle -1, 2, 1 \rangle, \quad \vec{v} = \langle 2, 2, 1 \rangle, \quad \vec{w} = \langle 3, 1, 3 \rangle$

In Exercises 35 – 38, find a unit vector orthogonal to both $\vec{u}$ and $\vec{v}$.

35. $\vec{u} = \langle 1, 1, 1 \rangle, \quad \vec{v} = \langle 2, 0, 1 \rangle$

36. $\vec{u} = \langle 1, -2, 1 \rangle, \quad \vec{v} = \langle 3, 2, 1 \rangle$

37. $\vec{u} = \langle 5, 0, 2 \rangle, \quad \vec{v} = \langle -3, 0, 7 \rangle$

38. $\vec{u} = \langle 1, -2, 1 \rangle, \quad \vec{v} = \langle -2, 4, -2 \rangle$

39. A bicycle rider applies 150lb of force, straight down, onto a pedal that extends 7in horizontally from the crankshaft. Find the magnitude of the torque applied to the crankshaft.

40. A bicycle rider applies 150lb of force, straight down, onto a pedal that extends 7in from the crankshaft, making a 30° angle with the horizontal. Find the magnitude of the torque applied to the crankshaft.

41. To turn a stubborn bolt, 80lb of force is applied to a 10in wrench. What is the maximum amount of torque that can be applied to the bolt?

42. To turn a stubborn bolt, 80lb of force is applied to a 10in wrench in a confined space, where the direction of applied force makes a 10° angle with the wrench. How much torque is subsequently applied to the wrench?

43. Show, using the definition of the Cross Product, that $\vec{u}\cdot(\vec{u}\times\vec{v}) = 0$; that is, that $\vec{u}$ is orthogonal to the cross product of $\vec{u}$ and $\vec{v}$.

44. Show, using the definition of the Cross Product, that $\vec{u}\times\vec{u} = \vec{0}$.

Exercises 10.5

Terms and Concepts

1. To find an equation of a line, what two pieces of information are needed?

2. Two distinct lines in the plane can intersect or be ________.

3. Two distinct lines in space can intersect, be ________ or be ________.

4. Use your own words to describe what it means for two lines in space to be skew.

Problems

In Exercises 5 – 14, write the vector, parametric and symmetric equations of the lines described.

5. Passes through $P = (2, -4, 1)$, parallel to $\vec{d} = \langle 9, 2, 5\rangle$.

6. Passes through $P = (6, 1, 7)$, parallel to $\vec{d} = \langle -3, 2, 5\rangle$.

7. Passes through $P = (2, 1, 5)$ and $Q = (7, -2, 4)$.

8. Passes through $P = (1, -2, 3)$ and $Q = (5, 5, 5)$.

9. Passes through $P = (0, 1, 2)$ and orthogonal to both $\vec{d}_1 = \langle 2, -1, 7\rangle$ and $\vec{d}_2 = \langle 7, 1, 3\rangle$.

10. Passes through $P = (5, 1, 9)$ and orthogonal to both $\vec{d}_1 = \langle 1, 0, 1\rangle$ and $\vec{d}_2 = \langle 2, 0, 3\rangle$.

11. Passes through the point of intersection of $\vec{\ell}_1(t)$ and $\vec{\ell}_2(t)$ and orthogonal to both lines, where $\vec{\ell}_1(t) = \langle 2, 1, 1\rangle + t\,\langle 5, 1, -2\rangle$ and $\vec{\ell}_2(t) = \langle -2, -1, 2\rangle + t\,\langle 3, 1, -1\rangle$.

12. Passes through the point of intersection of $\ell_1(t)$ and $\ell_2(t)$ and orthogonal to both lines, where
$$\ell_1 = \begin{cases} x = t \\ y = -2 + 2t \\ z = 1 + t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 2 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}.$$

13. Passes through $P = (1, 1)$, parallel to $\vec{d} = \langle 2, 3\rangle$.

14. Passes through $P = (-2, 5)$, parallel to $\vec{d} = \langle 0, 1\rangle$.

In Exercises 15 – 22, determine if the described lines are the same line, parallel lines, intersecting or skew lines. If intersecting, give the point of intersection.

15. $\vec{\ell}_1(t) = \langle 1, 2, 1\rangle + t\,\langle 2, -1, 1\rangle$,
$\vec{\ell}_2(t) = \langle 3, 3, 3\rangle + t\,\langle -4, 2, -2\rangle$.

16. $\vec{\ell}_1(t) = \langle 2, 1, 1\rangle + t\,\langle 5, 1, 3\rangle$,
$\vec{\ell}_2(t) = \langle 14, 5, 9\rangle + t\,\langle 1, 1, 1\rangle$.

17. $\vec{\ell}_1(t) = \langle 3, 4, 1\rangle + t\,\langle 2, -3, 4\rangle$,
$\vec{\ell}_2(t) = \langle -3, 3, -3\rangle + t\,\langle 3, -2, 4\rangle$.

18. $\vec{\ell}_1(t) = \langle 1, 1, 1\rangle + t\,\langle 3, 1, 3\rangle$,
$\vec{\ell}_2(t) = \langle 7, 3, 7\rangle + t\,\langle 6, 2, 6\rangle$.

19. $$\ell_1 = \begin{cases} x = 1 + 2t \\ y = 3 - 2t \\ z = t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 3 - t \\ y = 3 + 5t \\ z = 2 + 7t \end{cases}$$

20. $$\ell_1 = \begin{cases} x = 1.1 + 0.6t \\ y = 3.77 + 0.9t \\ z = -2.3 + 1.5t \end{cases} \text{ and } \ell_2 = \begin{cases} x = 3.11 + 3.4t \\ y = 2 + 5.1t \\ z = 2.5 + 8.5t \end{cases}$$

21. $$\ell_1 = \begin{cases} x = 0.2 + 0.6t \\ y = 1.33 - 0.45t \\ z = -4.2 + 1.05t \end{cases} \text{ and } \ell_2 = \begin{cases} x = 0.86 + 9.2t \\ y = 0.835 - 6.9t \\ z = -3.045 + 16.1t \end{cases}$$

22. $$\ell_1 = \begin{cases} x = 0.1 + 1.1t \\ y = 2.9 - 1.5t \\ z = 3.2 + 1.6t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 4 - 2.1t \\ y = 1.8 + 7.2t \\ z = 3.1 + 1.1t \end{cases}$$

In Exercises 23 – 26, find the distance from the point to the line.

23. $Q = (1, 1, 1)$, $\quad \vec{\ell}(t) = \langle 2, 1, 3\rangle + t\,\langle 2, 1, -2\rangle$

24. $Q = (2, 5, 6)$, $\quad \vec{\ell}(t) = \langle -1, 1, 1\rangle + t\,\langle 1, 0, 1\rangle$

25. $Q = (0, 3)$, $\quad \vec{\ell}(t) = \langle 2, 0\rangle + t\,\langle 1, 1\rangle$

26. $Q = (1, 1)$, $\quad \vec{\ell}(t) = \langle 4, 5\rangle + t\,\langle -4, 3\rangle$

In Exercises 27 – 28, find the distance between the two lines.

27. $\vec{\ell}_1(t) = \langle 1, 2, 1\rangle + t\,\langle 2, -1, 1\rangle$,
$\vec{\ell}_2(t) = \langle 3, 3, 3\rangle + t\,\langle 4, 2, -2\rangle$.

28. $\vec{\ell}_1(t) = \langle 0, 0, 1\rangle + t\,\langle 1, 0, 0\rangle$,
$\vec{\ell}_2(t) = \langle 0, 0, 3\rangle + t\,\langle 0, 1, 0\rangle$.

Exercises 29 – 31 explore special cases of the distance formulas found in Key Idea 10.5.1.

29. Let Q be a point on the line $\vec{\ell}(t)$. Show why the distance formula correctly gives the distance from the point to the line as 0.

30. Let lines $\vec{\ell}_1(t)$ and $\vec{\ell}_2(t)$ be intersecting lines. Show why the distance formula correctly gives the distance between these lines as 0.

31. Let lines $\vec{\ell}_1(t)$ and $\vec{\ell}_2(t)$ be parallel.

   (a) Show why the distance formula for distance between lines cannot be used as stated to find the distance between the lines.

   (b) Show why letting $\vec{c} = (\overrightarrow{P_1P_2} \times \vec{d}_2) \times \vec{d}_2$ allows one to use the formula.

   (c) Show how one can use the formula for the distance between a point and a line to find the distance between parallel lines.

Exercises 10.6

Terms and Concepts

1. In order to find the equation of a plane, what two pieces of information must one have?

2. What is the relationship between a plane and one of its normal vectors?

Problems

In Exercises 3 – 6, give any two points in the given plane.

3. $2x - 4y + 7z = 2$

4. $3(x + 2) + 5(y - 9) - 4z = 0$

5. $x = 2$

6. $4(y + 2) - (z - 6) = 0$

In Exercises 7 – 20, give the equation of the described plane in standard and general forms.

7. Passes through $(2, 3, 4)$ and has normal vector $\vec{n} = \langle 3, -1, 7\rangle$.

8. Passes through $(1, 3, 5)$ and has normal vector $\vec{n} = \langle 0, 2, 4\rangle$.

9. Passes through the points $(1, 2, 3)$, $(3, -1, 4)$ and $(1, 0, 1)$.

10. Passes through the points $(5, 3, 8)$, $(6, 4, 9)$ and $(3, 3, 3)$.

11. Contains the intersecting lines $\vec{\ell_1}(t) = \langle 2, 1, 2\rangle + t\,\langle 1, 2, 3\rangle$ and $\vec{\ell_2}(t) = \langle 2, 1, 2\rangle + t\,\langle 2, 5, 4\rangle$.

12. Contains the intersecting lines $\vec{\ell_1}(t) = \langle 5, 0, 3\rangle + t\,\langle -1, 1, 1\rangle$ and $\vec{\ell_2}(t) = \langle 1, 4, 7\rangle + t\,\langle 3, 0, -3\rangle$.

13. Contains the parallel lines $\vec{\ell_1}(t) = \langle 1, 1, 1\rangle + t\,\langle 1, 2, 3\rangle$ and $\vec{\ell_2}(t) = \langle 1, 1, 2\rangle + t\,\langle 1, 2, 3\rangle$.

14. Contains the parallel lines $\vec{\ell_1}(t) = \langle 1, 1, 1\rangle + t\,\langle 4, 1, 3\rangle$ and $\vec{\ell_2}(t) = \langle 4, 4, 4\rangle + t\,\langle 4, 1, 3\rangle$.

15. Contains the point $(2, -6, 1)$ and the line
$$\ell(t) = \begin{cases} x = 2 + 5t \\ y = 2 + 2t \\ z = -1 + 2t \end{cases}$$

16. Contains the point $(5, 7, 3)$ and the line
$$\ell(t) = \begin{cases} x = t \\ y = t \\ z = t \end{cases}$$

17. Contains the point $(5, 7, 3)$ and is orthogonal to the line $\vec{\ell}(t) = \langle 4, 5, 6\rangle + t\,\langle 1, 1, 1\rangle$.

18. Contains the point $(4, 1, 1)$ and is orthogonal to the line
$$\ell(t) = \begin{cases} x = 4 + 4t \\ y = 1 + 1t \\ z = 1 + 1t \end{cases}$$

19. Contains the point $(-4, 7, 2)$ and is parallel to the plane $3(x - 2) + 8(y + 1) - 10z = 0$.

20. Contains the point $(1, 2, 3)$ and is parallel to the plane $x = 5$.

In Exercises 21 – 22, give the equation of the line that is the intersection of the given planes.

21. $p1:\ 3(x - 2) + (y - 1) + 4z = 0$, and
$p2:\ 2(x - 1) - 2(y + 3) + 6(z - 1) = 0$.

22. $p1:\ 5(x - 5) + 2(y + 2) + 4(z - 1) = 0$, and
$p2:\ 3x - 4(y - 1) + 2(z - 1) = 0$.

In Exercises 23 – 26, find the point of intersection between the line and the plane.

23. line: $\langle 5, 1, -1\rangle + t\,\langle 2, 2, 1\rangle$,
plane: $5x - y - z = -3$

24. line: $\langle 4, 1, 0\rangle + t\,\langle 1, 0, -1\rangle$,
plane: $3x + y - 2z = 8$

25. line: $\langle 1, 2, 3\rangle + t\,\langle 3, 5, -1\rangle$,
plane: $3x - 2y - z = 4$

26. line: $\langle 1, 2, 3\rangle + t\,\langle 3, 5, -1\rangle$,
plane: $3x - 2y - z = -4$

In Exercises 27 – 30, find the given distances.

27. The distance from the point $(1, 2, 3)$ to the plane $3(x - 1) + (y - 2) + 5(z - 2) = 0$.

28. The distance from the point $(2, 6, 2)$ to the plane $2(x - 1) - y + 4(z + 1) = 0$.

29. The distance between the parallel planes
$x + y + z = 0$ and
$(x - 2) + (y - 3) + (z + 4) = 0$

30. The distance between the parallel planes
$2(x-1)+2(y+1)+(z-2)=0$ and
$2(x-3)+2(y-1)+(z-3)=0$

31. Show why if the point Q lies in a plane, then the distance formula correctly gives the distance from the point to the plane as 0.

32. How is Exercise 30 in Section 10.5 easier to answer once we have an understanding of planes?

Exercises 11.1

Terms and Concepts

1. Vector–valued functions are closely related to ________ ________ of graphs.

2. When sketching vector–valued functions, technically one isn't graphing points, but rather ________.

3. It can be useful to think of ________ as a vector that points from a starting position to an ending position.

4. In the context of vector–valued functions, average rate of change is ________ divided by time.

Problems

In Exercises 5 – 12, sketch the vector–valued function on the given interval.

5. $\vec{r}(t) = \left\langle t^2, t^2 - 1\right\rangle$, for $-2 \leq t \leq 2$.

6. $\vec{r}(t) = \left\langle t^2, t^3\right\rangle$, for $-2 \leq t \leq 2$.

7. $\vec{r}(t) = \left\langle 1/t, 1/t^2\right\rangle$, for $-2 \leq t \leq 2$.

8. $\vec{r}(t) = \left\langle \frac{1}{10}t^2, \sin t\right\rangle$, for $-2\pi \leq t \leq 2\pi$.

9. $\vec{r}(t) = \left\langle \frac{1}{10}t^2, \sin t\right\rangle$, for $-2\pi \leq t \leq 2\pi$.

10. $\vec{r}(t) = \langle 3\sin(\pi t), 2\cos(\pi t)\rangle$, on $[0, 2]$.

11. $\vec{r}(t) = \langle 3\cos t, 2\sin(2t)\rangle$, on $[0, 2\pi]$.

12. $\vec{r}(t) = \langle 2\sec t, \tan t\rangle$, on $[-\pi, \pi]$.

In Exercises 13 – 16, sketch the vector–valued function on the given interval in $\mathbb{R}^3$. Technology may be useful in creating the sketch.

13. $\vec{r}(t) = \langle 2\cos t, t, 2\sin t\rangle$, on $[0, 2\pi]$.

14. $\vec{r}(t) = \langle 3\cos t, \sin t, t/\pi\rangle$ on $[0, 2\pi]$.

15. $\vec{r}(t) = \langle \cos t, \sin t, \sin t\rangle$ on $[0, 2\pi]$.

16. $\vec{r}(t) = \langle \cos t, \sin t, \sin(2t)\rangle$ on $[0, 2\pi]$.

In Exercises 17 – 20, find $||\,\vec{r}(t)\,||$.

17. $\vec{r}(t) = \left\langle t, t^2\right\rangle$.

18. $\vec{r}(t) = \langle 5\cos t, 3\sin t\rangle$.

19. $\vec{r}(t) = \langle 2\cos t, 2\sin t, t\rangle$.

20. $\vec{r}(t) = \left\langle \cos t, t, t^2\right\rangle$.

In Exercises 21 – 30, create a vector–valued function whose graph matches the given description.

21. A circle of radius 2, centered at $(1, 2)$, traced counter–clockwise once on $[0, 2\pi]$.

22. A circle of radius 3, centered at $(5, 5)$, traced clockwise once on $[0, 2\pi]$.

23. An ellipse, centered at $(0, 0)$ with vertical major axis of length 10 and minor axis of length 3, traced once counter–clockwise on $[0, 2\pi]$.

24. An ellipse, centered at $(3, -2)$ with horizontal major axis of length 6 and minor axis of length 4, traced once clockwise on $[0, 2\pi]$.

25. A line through $(2, 3)$ with a slope of 5.

26. A line through $(1, 5)$ with a slope of $-1/2$.

27. The line through points $(1, 2, 3)$ and $(4, 5, 6)$, where $\vec{r}(0) = \langle 1, 2, 3\rangle$ and $\vec{r}(1) = \langle 4, 5, 6\rangle$.

28. The line through points $(1, 2)$ and $(4, 4)$, where $\vec{r}(0) = \langle 1, 2\rangle$ and $\vec{r}(1) = \langle 4, 4\rangle$.

29. A vertically oriented helix with radius of 2 that starts at $(2, 0, 0)$ and ends at $(2, 0, 4\pi)$ after 1 revolution on $[0, 2\pi]$.

30. A vertically oriented helix with radius of 3 that starts at $(3, 0, 0)$ and ends at $(3, 0, 3)$ after 2 revolutions on $[0, 1]$.

In Exercises 31 – 34, find the average rate of change of $\vec{r}(t)$ on the given interval.

31. $\vec{r}(t) = \left\langle t, t^2\right\rangle$ on $[-2, 2]$.

32. $\vec{r}(t) = \langle t, t + \sin t\rangle$ on $[0, 2\pi]$.

33. $\vec{r}(t) = \langle 3\cos t, 2\sin t, t\rangle$ on $[0, 2\pi]$.

34. $\vec{r}(t) = \left\langle t, t^2, t^3\right\rangle$ on $[-1, 3]$.

Exercises 11.2

Terms and Concepts

1. Limits, derivatives and integrals of vector–valued functions are all evaluated ________–wise.

2. The definite integral of a rate of change function gives ________.

3. Why is it generally not useful to graph both $\vec{r}(t)$ and $\vec{r}\,'(t)$ on the same axes?

4. Theorem 11.2.4 contains three product rules. What are the three different types of products used in these rules?

Problems

In Exercises 5 – 8, evaluate the given limit.

5. $\lim\limits_{t\to 5} \langle 2t+1, 3t^2-1, \sin t\rangle$

6. $\lim\limits_{t\to 3} \left\langle e^t, \dfrac{t^2-9}{t+3}\right\rangle$

7. $\lim\limits_{t\to 0} \left\langle \dfrac{t}{\sin t}, (1+t)^{\frac{1}{t}}\right\rangle$

8. $\lim\limits_{h\to 0} \dfrac{\vec{r}(t+h)-\vec{r}(t)}{h}$, where $\vec{r}(t) = \langle t^2, t, 1\rangle$.

In Exercises 9 – 10, identify the interval(s) on which $\vec{r}(t)$ is continuous.

9. $\vec{r}(t) = \langle t^2, 1/t\rangle$

10. $\vec{r}(t) = \langle \cos t, e^t, \ln t\rangle$

In Exercises 11 – 16, find the derivative of the given function.

11. $\vec{r}(t) = \langle \cos t, e^t, \ln t\rangle$

12. $\vec{r}(t) = \left\langle \dfrac{1}{t}, \dfrac{2t-1}{3t+1}, \tan t\right\rangle$

13. $\vec{r}(t) = (t^2)\,\langle \sin t, 2t+5\rangle$

14. $r(t) = \langle t^2+1, t-1\rangle \cdot \langle \sin t, 2t+5\rangle$

15. $\vec{r}(t) = \langle t^2+1, t-1, 1\rangle \times \langle \sin t, 2t+5, 1\rangle$

16. $\vec{r}(t) = \langle \cosh t, \sinh t\rangle$

In Exercises 17 – 20, find $\vec{r}\,'(t)$. Sketch $\vec{r}(t)$ and $\vec{r}\,'(1)$, with the initial point of $\vec{r}\,'(1)$ at $\vec{r}(1)$.

17. $\vec{r}(t) = \langle t^2+t, t^2-t\rangle$

18. $\vec{r}(t) = \langle t^2-2t+2, t^3-3t^2+2t\rangle$

19. $\vec{r}(t) = \langle t^2+1, t^3-t\rangle$

20. $\vec{r}(t) = \langle t^2-4t+5, t^3-6t^2+11t-6\rangle$

In Exercises 21 – 24, give the equation of the line tangent to the graph of $\vec{r}(t)$ at the given t value.

21. $\vec{r}(t) = \langle t^2+t, t^2-t\rangle$ at $t=1$.

22. $\vec{r}(t) = \langle 3\cos t, \sin t\rangle$ at $t=\pi/4$.

23. $\vec{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$ at $t=\pi$.

24. $\vec{r}(t) = \langle e^t, \tan t, t\rangle$ at $t=0$.

In Exercises 25 – 28, find the value(s) of t for which $\vec{r}(t)$ is not smooth.

25. $\vec{r}(t) = \langle \cos t, \sin t - t\rangle$

26. $\vec{r}(t) = \langle t^2-2t+1, t^3+t^2-5t+3\rangle$

27. $\vec{r}(t) = \langle \cos t - \sin t, \sin t - \cos t, \cos(4t)\rangle$

28. $\vec{r}(t) = \langle t^3-3t+2, -\cos(\pi t), \sin^2(\pi t)\rangle$

Exercises 29 – 32 ask you to verify parts of Theorem 11.2.4. In each let $f(t) = t^3$, $\vec{r}(t) = \langle t^2, t-1, 1\rangle$ and $\vec{s}(t) = \langle \sin t, e^t, t\rangle$. Compute the various derivatives as indicated.

29. Simplify $f(t)\vec{r}(t)$, then find its derivative; show this is the same as $f\,'(t)\vec{r}(t)+f(t)\vec{r}\,'(t)$.

30. Simplify $\vec{r}(t)\cdot\vec{s}(t)$, then find its derivative; show this is the same as $\vec{r}\,'(t)\cdot\vec{s}(t)+\vec{r}(t)\cdot\vec{s}\,'(t)$.

31. Simplify $\vec{r}(t)\times\vec{s}(t)$, then find its derivative; show this is the same as $\vec{r}\,'(t)\times\vec{s}(t)+\vec{r}(t)\times\vec{s}\,'(t)$.

32. Simplify $\vec{r}\big(f(t)\big)$, then find its derivative; show this is the same as $\vec{r}\,'\big(f(t)\big)f\,'(t)$.

In Exercises 33 – 36, evaluate the given definite or indefinite integral.

33. $\displaystyle\int \langle t^3, \cos t, te^t\rangle\ dt$

34. $\displaystyle\int \left\langle \frac{1}{1+t^2}, \sec^2 t\right\rangle\ dt$

35. $\displaystyle\int_0^{\pi} \langle -\sin t, \cos t\rangle\ dt$

36. $\displaystyle\int_{-2}^{2} \langle 2t+1, 2t-1\rangle\ dt$

In Exercises 37 – 40, solve the given initial value problems.

37. Find $\vec{r}(t)$, given that $\vec{r}\,'(t) = \langle t, \sin t\rangle$ and $\vec{r}(0) = \langle 2, 2\rangle$.

38. Find $\vec{r}(t)$, given that $\vec{r}\,'(t) = \langle 1/(t+1), \tan t\rangle$ and $\vec{r}(0) = \langle 1, 2\rangle$.

39. Find $\vec{r}(t)$, given that $\vec{r}\,''(t) = \left\langle t^2, t, 1\right\rangle$, $\vec{r}\,'(0) = \langle 1, 2, 3\rangle$ and $\vec{r}(0) = \langle 4, 5, 6\rangle$.

40. Find $\vec{r}(t)$, given that $\vec{r}\,''(t) = \left\langle \cos t, \sin t, e^t\right\rangle$, $\vec{r}\,'(0) = \langle 0, 0, 0\rangle$ and $\vec{r}(0) = \langle 0, 0, 0\rangle$.

In Exercises 41 – 44 , find the arc length of $\vec{r}(t)$ on the indicated interval.

41. $\vec{r}(t) = \langle 2\cos t, 2\sin t, 3t\rangle$ on $[0, 2\pi]$.

42. $\vec{r}(t) = \langle 5\cos t, 3\sin t, 4\sin t\rangle$ on $[0, 2\pi]$.

43. $\vec{r}(t) = \left\langle t^3, t^2, t^3\right\rangle$ on $[0, 1]$.

44. $\vec{r}(t) = \left\langle e^{-t}\cos t, e^{-t}\sin t\right\rangle$ on $[0, 1]$.

45. Prove Theorem 11.2.5; that is, show if $\vec{r}(t)$ has constant length and is differentiable, then $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$. (Hint: use the Product Rule to compute $\frac{d}{dt}\big(\vec{r}(t)\cdot\vec{r}(t)\big)$.)

Exercises 11.3

Terms and Concepts

1. How is *velocity* different from *speed*?

2. What is the difference between *displacement* and *distance traveled*?

3. What is the difference between *average velocity* and *average speed*?

4. *Distance traveled* is the same as ________ ________, just viewed in a different context.

5. Describe a scenario where an object's average speed is a large number, but the magnitude of the average velocity is not a large number.

6. Explain why it is not possible to have an average velocity with a large magnitude but a small average speed.

Problems

In Exercises 7 – 10 , a position function $\vec{r}(t)$ is given. Find $\vec{v}(t)$ and $\vec{a}(t)$.

7. $\vec{r}(t) = \langle 2t+1, 5t-2, 7\rangle$

8. $\vec{r}(t) = \left\langle 3t^2-2t+1, -t^2+t+14\right\rangle$

9. $\vec{r}(t) = \langle \cos t, \sin t\rangle$

10. $\vec{r}(t) = \langle t/10, -\cos t, \sin t\rangle$

In Exercises 11 – 14 , a position function $\vec{r}(t)$ is given. Sketch $\vec{r}(t)$ on the indicated interval. Find $\vec{v}(t)$ and $\vec{a}(t)$, then add $\vec{v}(t_0)$ and $\vec{a}(t_0)$ to your sketch, with their initial points at $\vec{r}(t_0)$, for the given value of t_0.

11. $\vec{r}(t) = \langle t, \sin t\rangle$ on $[0, \pi/2]$; $t_0 = \pi/4$

12. $\vec{r}(t) = \left\langle t^2, \sin t^2\right\rangle$ on $[0, \pi/2]$; $t_0 = \sqrt{\pi/4}$

13. $\vec{r}(t) = \left\langle t^2+t, -t^2+2t\right\rangle$ on $[-2, 2]$; $t_0 = 1$

14. $\vec{r}(t) = \left\langle \dfrac{2t+3}{t^2+1}, t^2\right\rangle$ on $[-1, 1]$; $t_0 = 0$

In Exercises 15 – 24 , a position function $\vec{r}(t)$ of an object is given. Find the speed of the object in terms of t, and find where the speed is minimized/maximized on the indicated interval.

15. $\vec{r}(t) = \left\langle t^2, t\right\rangle$ on $[-1, 1]$

16. $\vec{r}(t) = \left\langle t^2, t^2-t^3\right\rangle$ on $[-1, 1]$

17. $\vec{r}(t) = \langle 5\cos t, 5\sin t\rangle$ on $[0, 2\pi]$

18. $\vec{r}(t) = \langle 2\cos t, 5\sin t\rangle$ on $[0, 2\pi]$

19. $\vec{r}(t) = \langle \sec t, \tan t\rangle$ on $[0, \pi/4]$

20. $\vec{r}(t) = \langle t+\cos t, 1-\sin t\rangle$ on $[0, 2\pi]$

21. $\vec{r}(t) = \langle 12t, 5\cos t, 5\sin t\rangle$ on $[0, 4\pi]$

22. $\vec{r}(t) = \left\langle t^2-t, t^2+t, t\right\rangle$ on $[0, 1]$

23. $\vec{r}(t) = \left\langle t, t^2, \sqrt{1-t^2}\right\rangle$ on $[-1, 1]$

24. **Projectile Motion:** $\vec{r}(t) = \left\langle (v_0\cos\theta)t, -\dfrac{1}{2}gt^2+(v_0\sin\theta)t\right\rangle$ on $\left[0, \dfrac{2v_0\sin\theta}{g}\right]$

In Exercises 25 – 28 , position functions $\vec{r}_1(t)$ and $\vec{r}_2(s)$ for two objects are given that follow the same path on the respective intervals.

(a) Show that the positions are the same at the indicated t_0 and s_0 values; i.e., show $\vec{r}_1(t_0) = \vec{r}_2(s_0)$.

(b) Find the velocity, speed and acceleration of the two objects at t_0 and s_0, respectively.

25. $\vec{r}_1(t) = \left\langle t, t^2\right\rangle$ on $[0, 1]$; $t_0 = 1$
$\vec{r}_2(s) = \left\langle s^2, s^4\right\rangle$ on $[0, 1]$; $s_0 = 1$

26. $\vec{r}_1(t) = \langle 3\cos t, 3\sin t\rangle$ on $[0, 2\pi]$; $t_0 = \pi/2$
$\vec{r}_2(s) = \langle 3\cos(4s), 3\sin(4s)\rangle$ on $[0, \pi/2]$; $s_0 = \pi/8$

27. $\vec{r}_1(t) = \langle 3t, 2t\rangle$ on $[0, 2]$; $t_0 = 2$
$\vec{r}_2(s) = \langle 6s-6, 4s-4\rangle$ on $[1, 2]$; $s_0 = 2$

28. $\vec{r}_1(t) = \left\langle t, \sqrt{t}\right\rangle$ on $[0, 1]$; $t_0 = 1$
$\vec{r}_2(s) = \left\langle \sin t, \sqrt{\sin t}\right\rangle$ on $[0, \pi/2]$; $s_0 = \pi/2$

In Exercises 29 – 32 , find the position function of an object given its acceleration and initial velocity and position.

29. $\vec{a}(t) = \langle 2, 3\rangle$; $\vec{v}(0) = \langle 1, 2\rangle$, $\vec{r}(0) = \langle 5, -2\rangle$

30. $\vec{a}(t) = \langle 2, 3\rangle$; $\vec{v}(1) = \langle 1, 2\rangle$, $\vec{r}(1) = \langle 5, -2\rangle$

31. $\vec{a}(t) = \langle \cos t, -\sin t\rangle$; $\vec{v}(0) = \langle 0, 1\rangle$, $\vec{r}(0) = \langle 0, 0\rangle$

32. $\vec{a}(t) = \langle 0, -32\rangle$; $\vec{v}(0) = \langle 10, 50\rangle$, $\vec{r}(0) = \langle 0, 0\rangle$

In Exercises 33 – 36 , find the displacement, distance traveled, average velocity and average speed of the described object on the given interval.

33. An object with position function $\vec{r}(t) = \langle 2\cos t, 2\sin t, 3t\rangle$, where distances are measured in feet and time is in seconds, on $[0, 2\pi]$.

34. An object with position function $\vec{r}(t) = \langle 5\cos t, -5\sin t\rangle$, where distances are measured in feet and time is in seconds, on $[0, \pi]$.

35. An object with velocity function $\vec{v}(t) = \langle \cos t, \sin t\rangle$, where distances are measured in feet and time is in seconds, on $[0, 2\pi]$.

36. An object with velocity function $\vec{v}(t) = \langle 1, 2, -1\rangle$, where distances are measured in feet and time is in seconds, on $[0, 10]$.

Exercises 37 – 42 ask you to solve a variety of problems based on the principles of projectile motion.

37. A boy whirls a ball, attached to a 3ft string, above his head in a counter–clockwise circle. The ball makes 2 revolutions per second.

    At what *t*-values should the boy release the string so that the ball heads directly for a tree standing 10ft in front of him?

38. David faces Goliath with only a stone in a 3ft sling, which he whirls above his head at 4 revolutions per second. They stand 20ft apart.

    (a) At what *t*-values must David release the stone in his sling in order to hit Goliath?

    (b) What is the speed at which the stone is traveling when released?

    (c) Assume David releases the stone from a height of 6ft and Goliath's forehead is 9ft above the ground. What angle of elevation must David apply to the stone to hit Goliath's head?

39. A hunter aims at a deer which is 40 yards away. Her crossbow is at a height of 5ft, and she aims for a spot on the deer 4ft above the ground. The crossbow fires her arrows at 300ft/s.

    (a) At what angle of elevation should she hold the crossbow to hit her target?

    (b) If the deer is moving perpendicularly to her line of sight at a rate of 20mph, by approximately how much should she lead the deer in order to hit it in the desired location?

40. A baseball player hits a ball at 100mph, with an initial height of 3ft and an angle of elevation of 20°, at Boston's Fenway Park. The ball flies towards the famed "Green Monster," a wall 37ft high located 310ft from home plate.

    (a) Show that as hit, the ball hits the wall.

    (b) Show that if the angle of elevation is 21°, the ball clears the Green Monster.

41. A Cessna flies at 1000ft at 150mph and drops a box of supplies to the professor (and his wife) on an island. Ignoring wind resistance, how far horizontally will the supplies travel before they land?

42. A football quarterback throws a pass from a height of 6ft, intending to hit his receiver 20yds away at a height of 5ft.

    (a) If the ball is thrown at a rate of 50mph, what angle of elevation is needed to hit his intended target?

    (b) If the ball is thrown at with an angle of elevation of 8°, what initial ball speed is needed to hit his target?

Exercises 11.4

Terms and Concepts

1. If $\vec{T}(t)$ is a unit tangent vector, what is $||\ \vec{T}(t)\ ||$?

2. If $\vec{N}(t)$ is a unit normal vector, what is $\vec{N}(t) \cdot \vec{r}\,'(t)$?

3. The acceleration vector $\vec{a}(t)$ lies in the plane defined by what two vectors?

4. a_{T} measures how much the acceleration is affecting the ________ of an object.

Problems

In Exercises 5 – 8 , given $\vec{r}(t)$, find $\vec{T}(t)$ and evaluate it at the indicated value of t.

5. $\vec{r}(t) = \langle 2t^2, t^2 - t \rangle, \quad t = 1$

6. $\vec{r}(t) = \langle t, \cos t \rangle, \quad t = \pi/4$

7. $\vec{r}(t) = \langle \cos^3 t, \sin^3 t \rangle, \quad t = \pi/4$

8. $\vec{r}(t) = \langle \cos t, \sin t \rangle, \quad t = \pi$

In Exercises 9 – 12 , find the equation of the line tangent to the curve at the indicated t-value using the unit tangent vector. Note: these are the same problems as in Exercises 5 – 8.

9. $\vec{r}(t) = \langle 2t^2, t^2 - t \rangle, \quad t = 1$

10. $\vec{r}(t) = \langle t, \cos t \rangle, \quad t = \pi/4$

11. $\vec{r}(t) = \langle \cos^3 t, \sin^3 t \rangle, \quad t = \pi/4$

12. $\vec{r}(t) = \langle \cos t, \sin t \rangle, \quad t = \pi$

In Exercises 13 – 16 , find $\vec{N}(t)$ using Definition 11.4.2. Confirm the result using Theorem 11.4.1.

13. $\vec{r}(t) = \langle 3\cos t, 3\sin t \rangle$

14. $\vec{r}(t) = \langle t, t^2 \rangle$

15. $\vec{r}(t) = \langle \cos t, 2\sin t \rangle$

16. $\vec{r}(t) = \langle e^t, e^{-t} \rangle$

In Exercises 17 – 20 , a position function $\vec{r}(t)$ is given along with its unit tangent vector $\vec{T}(t)$ evaluated at $t = a$, for some value of a.

(a) Confirm that $\vec{T}(a)$ is as stated.

(b) Using a graph of $\vec{r}(t)$ and Theorem 11.4.1, find $\vec{N}(a)$.

17. $\vec{r}(t) = \langle 3\cos t, 5\sin t \rangle; \quad \vec{T}(\pi/4) = \left\langle -\frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle.$

18. $\vec{r}(t) = \left\langle t, \frac{1}{t^2+1} \right\rangle; \quad \vec{T}(1) = \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle.$

19. $\vec{r}(t) = (1 + 2\sin t)\langle \cos t, \sin t \rangle; \quad \vec{T}(0) = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle.$

20. $\vec{r}(t) = \langle \cos^3 t, \sin^3 t \rangle; \quad \vec{T}(\pi/4) = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle.$

In Exercises 21 – 24 , find $\vec{N}(t)$.

21. $\vec{r}(t) = \langle 4t, 2\sin t, 2\cos t \rangle$

22. $\vec{r}(t) = \langle 5\cos t, 3\sin t, 4\sin t \rangle$

23. $\vec{r}(t) = \langle a\cos t, a\sin t, bt \rangle; \quad a > 0$

24. $\vec{r}(t) = \langle \cos(at), \sin(at), t \rangle$

In Exercises 25 – 30 , find a_{T} and a_{N} given $\vec{r}(t)$. Sketch $\vec{r}(t)$ on the indicated interval, and comment on the relative sizes of a_{T} and a_{N} at the indicated t values.

25. $\vec{r}(t) = \langle t, t^2 \rangle$ on $[-1, 1]$; consider $t = 0$ and $t = 1$.

26. $\vec{r}(t) = \langle t, 1/t \rangle$ on $(0, 4]$; consider $t = 1$ and $t = 2$.

27. $\vec{r}(t) = \langle 2\cos t, 2\sin t \rangle$ on $[0, 2\pi]$; consider $t = 0$ and $t = \pi/2$.

28. $\vec{r}(t) = \langle \cos(t^2), \sin(t^2) \rangle$ on $(0, 2\pi]$; consider $t = \sqrt{\pi/2}$ and $t = \sqrt{\pi}$.

29. $\vec{r}(t) = \langle a\cos t, a\sin t, bt \rangle$ on $[0, 2\pi]$, where $a, b > 0$; consider $t = 0$ and $t = \pi/2$.

30. $\vec{r}(t) = \langle 5\cos t, 4\sin t, 3\sin t \rangle$ on $[0, 2\pi]$; consider $t = 0$ and $t = \pi/2$.

Exercises 11.5

Terms and Concepts

1. It is common to describe position in terms of both ________ and/or ________.

2. A measure of the "curviness" of a curve is ________.

3. Give two shapes with constant curvature.

4. Describe in your own words what an "osculating circle" is.

5. Complete the identity: $\vec{T}\,'(s) =$ ________$\vec{N}(s)$.

6. Given a position function $\vec{r}(t)$, how are a_T and a_N affected by the curvature?

Problems

In Exercises 7 – 10 , a position function $\vec{r}(t)$ is given, where $t = 0$ corresponds to the initial position. Find the arc length parameter s, and rewrite $\vec{r}(t)$ in terms of s; that is, find $\vec{r}(s)$.

7. $\vec{r}(t) = \langle 2t, t, -2t\rangle$

8. $\vec{r}(t) = \langle 7\cos t, 7\sin t\rangle$

9. $\vec{r}(t) = \langle 3\cos t, 3\sin t, 2t\rangle$

10. $\vec{r}(t) = \langle 5\cos t, 13\sin t, 12\cos t\rangle$

In Exercises 11 – 22 , a curve C is described along with 2 points on C.

(a) Using a sketch, determine at which of these points the curvature is greater.

(b) Find the curvature κ of C, and evaluate κ at each of the 2 given points.

11. C is defined by $y = x^3 - x$; points given at $x = 0$ and $x = 1/2$.

12. C is defined by $y = \dfrac{1}{x^2+1}$; points given at $x = 0$ and $x = 2$.

13. C is defined by $y = \cos x$; points given at $x = 0$ and $x = \pi/2$.

14. C is defined by $y = \sqrt{1-x^2}$ on $(-1, 1)$; points given at $x = 0$ and $x = 1/2$.

15. C is defined by $\vec{r}(t) = \langle \cos t, \sin(2t)\rangle$; points given at $t = 0$ and $t = \pi/4$.

16. C is defined by $\vec{r}(t) = \left\langle \cos^2 t, \sin t\cos t\right\rangle$; points given at $t = 0$ and $t = \pi/3$.

17. C is defined by $\vec{r}(t) = \left\langle t^2 - 1, t^3 - t\right\rangle$; points given at $t = 0$ and $t = 5$.

18. C is defined by $\vec{r}(t) = \langle \tan t, \sec t\rangle$; points given at $t = 0$ and $t = \pi/6$.

19. C is defined by $\vec{r}(t) = \langle 4t+2, 3t-1, 2t+5\rangle$; points given at $t = 0$ and $t = 1$.

20. C is defined by $\vec{r}(t) = \left\langle t^3 - t, t^3 - 4, t^2 - 1\right\rangle$; points given at $t = 0$ and $t = 1$.

21. C is defined by $\vec{r}(t) = \langle 3\cos t, 3\sin t, 2t\rangle$; points given at $t = 0$ and $t = \pi/2$.

22. C is defined by $\vec{r}(t) = \langle 5\cos t, 13\sin t, 12\cos t\rangle$; points given at $t = 0$ and $t = \pi/2$.

In Exercises 23 – 26 , find the value of x or t where curvature is maximized.

23. $y = \dfrac{1}{6}x^3$

24. $y = \sin x$

25. $\vec{r}(t) = \left\langle t^2 + 2t, 3t - t^2\right\rangle$

26. $\vec{r}(t) = \langle t, 4/t, 3/t\rangle$

In Exercises 27 – 30 , find the radius of curvature at the indicated value.

27. $y = \tan x$, at $x = \pi/4$

28. $y = x^2 + x - 3$, at $x = \pi/4$

29. $\vec{r}(t) = \langle \cos t, \sin(3t)\rangle$, at $t = 0$

30. $\vec{r}(t) = \langle 5\cos(3t), t\rangle$, at $t = 0$

In Exercises 31 – 34 , find the equation of the osculating circle to the curve at the indicated t-value.

31. $\vec{r}(t) = \left\langle t, t^2\right\rangle$, at $t = 0$

32. $\vec{r}(t) = \langle 3\cos t, \sin t\rangle$, at $t = 0$

33. $\vec{r}(t) = \langle 3\cos t, \sin t\rangle$, at $t = \pi/2$

34. $\vec{r}(t) = \left\langle t^2 - t, t^2 + t\right\rangle$, at $t = 0$

Exercises 12.1

Terms and Concepts

1. Give two examples (other than those given in the text) of "real world" functions that require more than one input.

2. The graph of a function of two variables is a ________.

3. Most people are familiar with the concept of level curves in the context of ________ maps.

4. T/F: Along a level curve, the output of a function does not change.

5. The analogue of a level curve for functions of three variables is a level ________.

6. What does it mean when level curves are close together? Far apart?

Problems

In Exercises 7 – 14, give the domain and range of the multivariable function.

7. $f(x,y) = x^2 + y^2 + 2$

8. $f(x,y) = x + 2y$

9. $f(x,y) = x - 2y$

10. $f(x,y) = \dfrac{1}{x+2y}$

11. $f(x,y) = \dfrac{1}{x^2+y^2+1}$

12. $f(x,y) = \sin x \cos y$

13. $f(x,y) = \sqrt{9 - x^2 - y^2}$

14. $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-9}}$

In Exercises 15 – 22, describe in words and sketch the level curves for the function and given c values.

15. $f(x,y) = 3x - 2y$; $c = -2, 0, 2$

16. $f(x,y) = x^2 - y^2$; $c = -1, 0, 1$

17. $f(x,y) = x - y^2$; $c = -2, 0, 2$

18. $f(x,y) = \dfrac{1 - x^2 - y^2}{2y - 2x}$; $c = -2, 0, 2$

19. $f(x,y) = \dfrac{2x - 2y}{x^2 + y^2 + 1}$; $c = -1, 0, 1$

20. $f(x,y) = \dfrac{y - x^3 - 1}{x}$; $c = -3, -1, 0, 1, 3$

21. $f(x,y) = \sqrt{x^2 + 4y^2}$; $c = 1, 2, 3, 4$

22. $f(x,y) = x^2 + 4y^2$; $c = 1, 2, 3, 4$

In Exercises 23 – 26, give the domain and range of the functions of three variables.

23. $f(x,y,z) = \dfrac{x}{x + 2y - 4z}$

24. $f(x,y,z) = \dfrac{1}{1 - x^2 - y^2 - z^2}$

25. $f(x,y,z) = \sqrt{z - x^2 + y^2}$

26. $f(x,y,z) = z^2 \sin x \cos y$

In Exercises 27 – 30, describe the level surfaces of the given functions of three variables.

27. $f(x,y,z) = x^2 + y^2 + z^2$

28. $f(x,y,z) = z - x^2 + y^2$

29. $f(x,y,z) = \dfrac{x^2 + y^2}{z}$

30. $f(x,y,z) = \dfrac{z}{x - y}$

31. Compare the level curves of Exercises 21 and 22. How are they similar, and how are they different? Each surface is a quadric surface; describe how the level curves are consistent with what we know about each surface.

Exercises 12.2

Terms and Concepts

1. Describe in your own words the difference between boundary and interior points of a set.

2. Use your own words to describe (informally) what $\lim\limits_{(x,y)\to(1,2)} f(x,y) = 17$ means.

3. Give an example of a closed, bounded set.

4. Give an example of a closed, unbounded set.

5. Give an example of a open, bounded set.

6. Give an example of a open, unbounded set.

Problems

In Exercises 7 – 10, a set S is given.

(a) Give one boundary point and one interior point, when possible, of S.

(b) State whether S is open, closed, or neither.

(c) State whether S is bounded or unbounded.

7. $S = \left\{(x,y) \;\middle|\; \dfrac{(x-1)^2}{4} + \dfrac{(y-3)^2}{9} \leq 1\right\}$

8. $S = \left\{(x,y) \,|\, y \neq x^2\right\}$

9. $S = \left\{(x,y) \,|\, x^2 + y^2 = 1\right\}$

10. $S = \{(x,y) | y > \sin x\}$

In Exercises 11 – 14:

(a) Find the domain D of the given function.

(b) State whether D is an open or closed set.

(c) State whether D is bounded or unbounded.

11. $f(x,y) = \sqrt{9 - x^2 - y^2}$

12. $f(x,y) = \sqrt{y - x^2}$

13. $f(x,y) = \dfrac{1}{\sqrt{y - x^2}}$

14. $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$

In Exercises 15 – 20, a limit is given. Evaluate the limit along the paths given, then state why these results show the given limit does not exist.

15. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$

    (a) Along the path $y = 0$.

    (b) Along the path $x = 0$.

16. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x + y}{x - y}$

    (a) Along the path $y = mx$.

17. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy - y^2}{y^2 + x}$

    (a) Along the path $y = mx$.

    (b) Along the path $x = 0$.

18. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2)}{y}$

    (a) Along the path $y = mx$.

    (b) Along the path $y = x^2$.

19. $\lim\limits_{(x,y)\to(1,2)} \dfrac{x + y - 3}{x^2 - 1}$

    (a) Along the path $y = 2$.

    (b) Along the path $y = x + 1$.

20. $\lim\limits_{(x,y)\to(\pi,\pi/2)} \dfrac{\sin x}{\cos y}$

    (a) Along the path $x = \pi$.

    (b) Along the path $y = x - \pi/2$.

Exercises 12.3

Terms and Concepts

1. What is the difference between a constant and a coefficient?

2. Given a function $z = f(x, y)$, explain in your own words how to compute f_x.

3. In the mixed partial fraction f_{xy}, which is computed first, f_x or f_y?

4. In the mixed partial fraction $\dfrac{\partial^2 f}{\partial x \partial y}$, which is computed first, f_x or f_y?

Problems

In Exercises 5 – 8, evaluate $f_x(x, y)$ and $f_y(x, y)$ at the indicated point.

5. $f(x, y) = x^2y - x + 2y + 3$ at $(1, 2)$

6. $f(x, y) = x^3 - 3x + y^2 - 6y$ at $(-1, 3)$

7. $f(x, y) = \sin y \cos x$ at $(\pi/3, \pi/3)$

8. $f(x, y) = \ln(xy)$ at $(-2, -3)$

In Exercises 9 – 26, find f_x, f_y, f_{xx}, f_{yy}, f_{xy} and f_{yx}.

9. $f(x, y) = x^2y + 3x^2 + 4y - 5$

10. $f(x, y) = y^3 + 3xy^2 + 3x^2y + x^3$

11. $f(x, y) = \dfrac{x}{y}$

12. $f(x, y) = \dfrac{4}{xy}$

13. $f(x, y) = e^{x^2+y^2}$

14. $f(x, y) = e^{x+2y}$

15. $f(x, y) = \sin x \cos y$

16. $f(x, y) = (x + y)^3$

17. $f(x, y) = \cos(5xy^3)$

18. $f(x, y) = \sin(5x^2 + 2y^3)$

19. $f(x, y) = \sqrt{4xy^2 + 1}$

20. $f(x, y) = (2x + 5y)\sqrt{y}$

21. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$

22. $f(x, y) = 5x - 17y$

23. $f(x, y) = 3x^2 + 1$

24. $f(x, y) = \ln(x^2 + y)$

25. $f(x, y) = \dfrac{\ln x}{4y}$

26. $f(x, y) = 5e^x \sin y + 9$

In Exercises 27 – 30, form a function $z = f(x, y)$ such that f_x and f_y match those given.

27. $f_x = \sin y + 1, \quad f_y = x \cos y$

28. $f_x = x + y, \quad f_y = x + y$

29. $f_x = 6xy - 4y^2, \quad f_y = 3x^2 - 8xy + 2$

30. $f_x = \dfrac{2x}{x^2 + y^2}, \quad f_y = \dfrac{2y}{x^2 + y^2}$

In Exercises 31 – 34, find f_x, f_y, f_z, f_{yz} and f_{zy}.

31. $f(x, y, z) = x^2e^{2y-3z}$

32. $f(x, y, z) = x^3y^2 + x^3z + y^2z$

33. $f(x, y, z) = \dfrac{3x}{7y^2z}$

34. $f(x, y, z) = \ln(xyz)$

Exercises 12.4

Terms and Concepts

1. T/F: If $f(x, y)$ is differentiable on S, the f is continuous on S.

2. T/F: If f_x and f_y are continuous on S, then f is differentiable on S.

3. T/F: If $z = f(x, y)$ is differentiable, then the change in z over small changes dx and dy in x and y is approximately dz.

4. Finish the sentence: "The new z-value is approximately the old z-value plus the approximate ________."

Problems

In Exercises 5 – 8, find the total differential dz.

5. $z = x\sin y + x^2$

6. $z = (2x^2 + 3y)^2$

7. $z = 5x - 7y$

8. $z = xe^{x+y}$

In Exercises 9 – 12, a function $z = f(x, y)$ is given. Give the indicated approximation using the total differential.

9. $f(x, y) = \sqrt{x^2 + y}$. Approximate $f(2.95, 7.1)$ knowing $f(3, 7) = 4$.

10. $f(x, y) = \sin x \cos y$. Approximate $f(0.1, -0.1)$ knowing $f(0, 0) = 0$.

11. $f(x, y) = x^2y - xy^2$. Approximate $f(2.04, 3.06)$ knowing $f(2, 3) = -6$.

12. $f(x, y) = \ln(x - y)$. Approximate $f(5.1, 3.98)$ knowing $f(5, 4) = 0$.

Exercises 13 – 16 ask a variety of questions dealing with approximating error and sensitivity analysis.

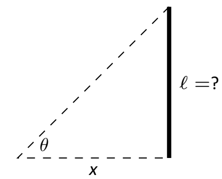
13. A cylindrical storage tank is to be 2ft tall with a radius of 1ft. Is the volume of the tank more sensitive to changes in the radius or the height?

14. **Projectile Motion:** The x-value of an object moving under the principles of projectile motion is $x(\theta, v_0, t) = (v_0\cos\theta)t$. A particular projectile is fired with an initial velocity of $v_0 = 250$ft/s and an angle of elevation of $\theta = 60^\circ$. It travels a distance of 375ft in 3 seconds.

    Is the projectile more sensitive to errors in initial speed or angle of elevation?

15. The length ℓ of a long wall is to be approximated. The angle θ, as shown in the diagram (not to scale), is measured to be 85°, and the distance x is measured to be 30'. Assume that the triangle formed is a right triangle.

    Is the measurement of the length of ℓ more sensitive to errors in the measurement of x or in θ?



16. It is "common sense" that it is far better to measure a long distance with a long measuring tape rather than a short one. A measured distance D can be viewed as the product of the length ℓ of a measuring tape times the number n of times it was used. For instance, using a 3' tape 10 times gives a length of 30'. To measure the same distance with a 12' tape, we would use the tape 2.5 times. (I.e., $30 = 12 \times 2.5$.) Thus $D = n\ell$.

    Suppose each time a measurement is taken with the tape, the recorded distance is within 1/16" of the actual distance. (I.e., $d\ell = 1/16'' \approx 0.005$ft). Using differentials, show why common sense proves correct in that it is better to use a long tape to measure long distances.

In Exercises 17 – 18, find the total differential dw.

17. $w = x^2yz^3$

18. $w = e^x \sin y \ln z$

In Exercises 19 – 22, use the information provided and the total differential to make the given approximation.

19. $f(3, 1) = 7$, $f_x(3, 1) = 9$, $f_y(3, 1) = -2$. Approximate $f(3.05, 0.9)$.

20. $f(-4, 2) = 13$, $f_x(-4, 2) = 2.6$, $f_y(-4, 2) = 5.1$. Approximate $f(-4.12, 2.07)$.

21. $f(2, 4, 5) = -1$, $f_x(2, 4, 5) = 2$, $f_y(2, 4, 5) = -3$, $f_z(2, 4, 5) = 3.7$. Approximate $f(2.5, 4.1, 4.8)$.

22. $f(3, 3, 3) = 5$, $f_x(3, 3, 3) = 2$, $f_y(3, 3, 3) = 0$, $f_z(3, 3, 3) = -2$. Approximate $f(3.1, 3.1, 3.1)$.

Exercises 12.5

Terms and Concepts

1. What is the difference between a directional derivative and a partial derivative?

2. For what $\vec{u}$ is $D_{\vec{u}}f = f_x$?

3. For what $\vec{u}$ is $D_{\vec{u}}f = f_y$?

4. The gradient is ________ to level curves.

5. The gradient points in the direction of ________ increase.

6. It is generally more informative to view the directional derivative not as the result of a limit, but rather as the result of a ________ product.

Problems

In Exercises 7 – 12, a function $z = f(x,y)$ is given. Find ∇f.

7. $f(x,y) = -x^2y + xy^2 + xy$

8. $f(x,y) = \sin x \cos y$

9. $f(x,y) = \dfrac{1}{x^2+y^2+1}$

10. $f(x,y) = -4x + 3y$

11. $f(x,y) = x^2 + 2y^2 - xy - 7x$

12. $f(x,y) = x^2y^3 - 2x$

In Exercises 13 – 18, a function $z = f(x,y)$ and a point P are given. Find the directional derivative of f in the indicated directions. Note: these are the same functions as in Exercises 7 through 12.

13. $f(x,y) = -x^2y + xy^2 + xy$, $P = (2,1)$
   (a) In the direction of $\vec{v} = \langle 3,4\rangle$
   (b) In the direction toward the point $Q = (1,-1)$.

14. $f(x,y) = \sin x \cos y$, $P = \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$
   (a) In the direction of $\vec{v} = \langle 1,1\rangle$.
   (b) In the direction toward the point $Q = (0,0)$.

15. $f(x,y) = \dfrac{1}{x^2+y^2+1}$, $P = (1,1)$.
   (a) In the direction of $\vec{v} = \langle 1,-1\rangle$.
   (b) In the direction toward the point $Q = (-2,-2)$.

16. $f(x,y) = -4x + 3y$, $P = (5,2)$
   (a) In the direction of $\vec{v} = \langle 3,1\rangle$.
   (b) In the direction toward the point $Q = (2,7)$.

17. $f(x,y) = x^2 + 2y^2 - xy - 7x$, $P = (4,1)$
   (a) In the direction of $\vec{v} = \langle -2,5\rangle$
   (b) In the direction toward the point $Q = (4,0)$.

18. $f(x,y) = x^2y^3 - 2x$, $P = (1,1)$
   (a) In the direction of $\vec{v} = \langle 3,3\rangle$
   (b) In the direction toward the point $Q = (1,2)$.

In Exercises 19 – 24, a function $z = f(x,y)$ and a point P are given.

(a) Find the direction of maximal increase of f at P.

(b) What is the maximal value of $D_{\vec{u}}f$ at P?

(c) Find the direction of minimal increase of f at P.

(d) Give a direction $\vec{u}$ such that $D_{\vec{u}}f = 0$ at P.

Note: these are the same functions and points as in Exercises 13 through 18.

19. $f(x,y) = -x^2y + xy^2 + xy$, $P = (2,1)$

20. $f(x,y) = \sin x \cos y$, $P = \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

21. $f(x,y) = \dfrac{1}{x^2+y^2+1}$, $P = (1,1)$.

22. $f(x,y) = -4x + 3y$, $P = (5,4)$.

23. $f(x,y) = x^2 + 2y^2 - xy - 7x$, $P = (4,1)$

24. $f(x,y) = x^2y^3 - 2x$, $P = (1,1)$

In Exercises 25 – 28, a function $w = F(x,y,z)$, a vector $\vec{v}$ and a point P are given.

(a) Find $\nabla F(x,y,z)$.

(b) Find $D_{\vec{u}}F$ at P, where $\vec{u}$ is the unit vector in the direction of $\vec{v}$.

25. $F(x,y,z) = 3x^2z^3 + 4xy - 3z^2$, $\vec{v} = \langle 1,1,1\rangle$, $P = (3,2,1)$

26. $F(x,y,z) = \sin(x)\cos(y)e^z$, $\vec{v} = \langle 2,2,1\rangle$, $P = (0,0,0)$

27. $F(x,y,z) = x^2y^2 - y^2z^2$, $\vec{v} = \langle -1,7,3\rangle$, $P = (1,0,-1)$

28. $F(x,y,z) = \dfrac{2}{x^2+y^2+z^2}$, $\vec{v} = \langle 1,1,-2\rangle$, $P = (1,1,1)$

Exercises 12.6

Terms and Concepts

1. Explain how the vector $\vec{v} = \langle 1, 0, 3\rangle$ can be thought of as having a "slope" of 3.

2. Explain how the vector $\vec{v} = \langle 0.6, 0.8, -2\rangle$ can be thought of as having a "slope" of -2.

3. T/F: Let $z = f(x, y)$ be differentiable at P. If $\vec{n}$ is a normal vector to the tangent plane of f at P, then $\vec{n}$ is orthogonal to ℓ_x and ℓ_y at P.

4. Explain in your own words why we do not refer to *the* tangent line to a surface at a point, but rather to *directional* tangent line*s* to a surface at a point.

Problems

In Exercises 5 – 8, a function $z = f(x, y)$, a vector $\vec{v}$ and a point P are given. Give the parametric equations of the following directional tangent lines to f at P:

(a) $\ell_x(t)$

(b) $\ell_y(t)$

(c) $\ell_{\vec{u}}(t)$**, where $\vec{u}$ is the unit vector in the direction of $\vec{v}$.**

5. $f(x, y) = 2x^2y - 4xy^2$, $\vec{v} = \langle 1, 3\rangle$, $P = (2, 3)$.

6. $f(x, y) = 3\cos x \sin y$, $\vec{v} = \langle 1, 2\rangle$, $P = (\pi/3, \pi/6)$.

7. $f(x, y) = 3x - 5y$, $\vec{v} = \langle 1, 1\rangle$, $P = (4, 2)$.

8. $f(x, y) = x^2 - 2x - y^2 + 4y$, $\vec{v} = \langle 1, 1\rangle$, $P = (1, 2)$.

In Exercises 9 – 12, a function $z = f(x, y)$ and a point P are given. Find the equation of the normal line to f at P. Note: these are the same functions as in Exercises 5 – 8.

9. $f(x, y) = 2x^2y - 4xy^2$, $P = (2, 3)$.

10. $f(x, y) = 3\cos x \sin y$, $P = (\pi/3, \pi/6)$.

11. $f(x, y) = 3x - 5y$, $P = (4, 2)$.

12. $f(x, y) = x^2 - 2x - y^2 + 4y$, $P = (1, 2)$.

In Exercises 13 – 16, a function $z = f(x, y)$ and a point P are given. Find the two points that are 2 units from the surface f at P. Note: these are the same functions as in Exercises 5 – 8.

13. $f(x, y) = 2x^2y - 4xy^2$, $P = (2, 3)$.

14. $f(x, y) = 3\cos x \sin y$, $P = (\pi/3, \pi/6)$.

15. $f(x, y) = 3x - 5y$, $P = (4, 2)$.

16. $f(x, y) = x^2 - 2x - y^2 + 4y$, $P = (1, 2)$.

In Exercises 17 – 20, a function $z = f(x, y)$ and a point P are given. Find the equation of the tangent plane to f at P. Note: these are the same functions as in Exercises 5 – 8.

17. $f(x, y) = 2x^2y - 4xy^2$, $P = (2, 3)$.

18. $f(x, y) = 3\cos x \sin y$, $P = (\pi/3, \pi/6)$.

19. $f(x, y) = 3x - 5y$, $P = (4, 2)$.

20. $f(x, y) = x^2 - 2x - y^2 + 4y$, $P = (1, 2)$.

In Exercises 21 – 24, an implicitly defined function of x, y and z is given along with a point P that lies on the surface. Use the gradient ∇F to:

(a) find the equation of the normal line to the surface at P, and

(b) find the equation of the plane tangent to the surface at P.

21. $\dfrac{x^2}{8} + \dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$, at $P = (1, \sqrt{2}, \sqrt{6})$

22. $z^2 - \dfrac{x^2}{4} - \dfrac{y^2}{9} = 0$, at $P = (4, -3, \sqrt{5})$

23. $xy^2 - xz^2 = 0$, at $P = (2, 1, -1)$

24. $\sin(xy) + \cos(yz) = 0$, at $P = (2, \pi/12, 4)$

Exercises 12.7

Terms and Concepts

1. T/F: Theorem 12.8.1 states that if f has a critical point at P, then f has a relative extrema at P.

2. T/F: A point P is a critical point of f if f_x and f_y are both 0 at P.

3. T/F: A point P is a critical point of f if f_x or f_y are undefined at P.

4. Explain what it means to "solve a constrained optimization" problem.

Problems

In Exercises 5 – 14, find the critical points of the given function. Use the Second Derivative Test to determine if each critical point corresponds to a relative maximum, minimum, or saddle point.

5. $f(x,y) = \frac{1}{2}x^2 + 2y^2 - 8y + 4x$

6. $f(x,y) = x^2 + 4x + y^2 - 9y + 3xy$

7. $f(x,y) = x^2 + 3y^2 - 6y + 4xy$

8. $f(x,y) = \dfrac{1}{x^2 + y^2 + 1}$

9. $f(x,y) = x^2 + y^3 - 3y + 1$

10. $f(x,y) = \dfrac{1}{3}x^3 - x + \dfrac{1}{3}y^3 - 4y$

11. $f(x,y) = x^2y^2$

12. $f(x,y) = x^4 - 2x^2 + y^3 - 27y - 15$

13. $f(x,y) = \sqrt{16 - (x-3)^2 - y^2}$

14. $f(x,y) = \sqrt{x^2 + y^2}$

In Exercises 15 – 18, find the absolute maximum and minimum of the function subject to the given constraint.

15. $f(x,y) = x^2 + y^2 + y + 1$, constrained to the triangle with vertices $(0,1)$, $(-1,-1)$ and $(1,-1)$.

16. $f(x,y) = 5x - 7y$, constrained to the region bounded by $y = x^2$ and $y = 1$.

17. $f(x,y) = x^2 + 2x + y^2 + 2y$, constrained to the region bounded by the circle $x^2 + y^2 = 4$.

18. $f(x,y) = 3y - 2x^2$, constrained to the region bounded by the parabola $y = x^2 + x - 1$ and the line $y = x$.

Exercises 12.8

Terms and Concepts

1. Let a level curve of $z = f(x, y)$ be described by $x = g(t)$, $y = h(t)$. Explain why $\frac{dz}{dt} = 0$.

2. Fill in the blank: The single variable Chain Rule states $\dfrac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)\cdot$ ________.

3. Fill in the blank: The Multivariable Chain Rule states
$\dfrac{df}{dt} = \dfrac{\partial f}{\partial x} \cdot$ ________ $+$ ________ $\cdot \dfrac{dy}{dt}$.

4. If $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$, we can substitute and write z as an explicit function of t.
T/F: Using the Multivariable Chain Rule to find $\frac{dz}{dt}$ is sometimes easier than first substituting and then taking the derivative.

5. T/F: The Multivariable Chain Rule is only useful when all the related functions are known explicitly.

6. The Multivariable Chain Rule allows us to compute implicit derivatives easily by just computing two ________ derivatives.

Problems

In Exercises 7 – 12, functions $z = f(x, y)$, $x = g(t)$ and $y = h(t)$ are given.

(a) Use the Multivariable Chain Rule to compute $\dfrac{dz}{dt}$.

(b) Evaluate $\dfrac{dz}{dt}$ at the indicated t-value.

7. $z = 3x + 4y, \quad x = t^2, \quad y = 2t; \quad t = 1$

8. $z = x^2 - y^2, \quad x = t, \quad y = t^2 - 1; \quad t = 1$

9. $z = 5x + 2y, \quad x = 2\cos t + 1, \quad y = \sin t - 3; \quad t = \pi/4$

10. $z = \dfrac{x}{y^2 + 1}, \quad x = \cos t, \quad y = \sin t; \quad t = \pi/2$

11. $z = x^2 + 2y^2, \quad x = \sin t, \quad y = 3\sin t; \quad t = \pi/4$

12. $z = \cos x \sin y, \quad x = \pi t, \quad y = 2\pi t + \pi/2; \quad t = 3$

In Exercises 13 – 18, functions $z = f(x, y)$, $x = g(t)$ and $y = h(t)$ are given. Find the values of t where $\frac{dz}{dt} = 0$. Note: these are the same surfaces/curves as found in Exercises 7 – 12.

13. $z = 3x + 4y, \quad x = t^2, \quad y = 2t$

14. $z = x^2 - y^2, \quad x = t, \quad y = t^2 - 1$

15. $z = 5x + 2y, \quad x = 2\cos t + 1, \quad y = \sin t - 3$

16. $z = \dfrac{x}{y^2 + 1}, \quad x = \cos t, \quad y = \sin t$

17. $z = x^2 + 2y^2, \quad x = \sin t, \quad y = 3\sin t$

18. $z = \cos x \sin y, \quad x = \pi t, \quad y = 2\pi t + \pi/2$

In Exercises 19 – 22, functions $z = f(x, y)$, $x = g(s, t)$ and $y = h(s, t)$ are given.

(a) Use the Multivariable Chain Rule to compute $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

(b) Evaluate $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ at the indicated s and t values.

19. $z = x^2 y, \quad x = s - t, \quad y = 2s + 4t; \quad s = 1, t = 0$

20. $z = \cos\left(\pi x + \frac{\pi}{2} y\right), \quad x = st^2, \quad y = s^2 t; \quad s = 1, t = 1$

21. $z = x^2 + y^2, \quad x = s\cos t, \quad y = s\sin t; \quad s = 2, t = \pi/4$

22. $z = e^{-(x^2+y^2)}, \quad x = t, \quad y = st^2; \quad s = 1, t = 1$

In Exercises 23 – 26, find $\dfrac{dy}{dx}$ using Implicit Differentiation and Theorem 12.5.3.

23. $x^2 \tan y = 50$

24. $(3x^2 + 2y^3)^4 = 2$

25. $\dfrac{x^2 + y}{x + y^2} = 17$

26. $\ln(x^2 + xy + y^2) = 1$

In Exercises 27 – 30, find $\dfrac{dz}{dt}$, or $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$, using the supplied information.

27. $\dfrac{\partial z}{\partial x} = 2, \quad \dfrac{\partial z}{\partial y} = 1, \quad \dfrac{dx}{dt} = 4, \quad \dfrac{dy}{dt} = -5$

28. $\dfrac{\partial z}{\partial x} = 1, \quad \dfrac{\partial z}{\partial y} = -3, \quad \dfrac{dx}{dt} = 6, \quad \dfrac{dy}{dt} = 2$

29. $\dfrac{\partial z}{\partial x} = -4, \quad \dfrac{\partial z}{\partial y} = 9,$
$\dfrac{\partial x}{\partial s} = 5, \quad \dfrac{\partial x}{\partial t} = 7, \quad \dfrac{\partial y}{\partial s} = -2, \quad \dfrac{\partial y}{\partial t} = 6$

30. $\dfrac{\partial z}{\partial x} = 2, \quad \dfrac{\partial z}{\partial y} = 1,$
$\dfrac{\partial x}{\partial s} = -2, \quad \dfrac{\partial x}{\partial t} = 3, \quad \dfrac{\partial y}{\partial s} = 2, \quad \dfrac{\partial y}{\partial t} = -1$

Exercises 13.1

Terms and Concepts

1. When integrating $f_x(x,y)$ with respect to x, the constant of integration C is really which: $C(x)$ or $C(y)$? What does this mean?

2. Integrating an integral is called ________ ________.

3. When evaluating an iterated integral, we integrate from ________ to ________, then from ________ to ________.

4. One understanding of an iterated integral is that $\int_a^b \int_{g_1(x)}^{g_2(x)} dy\,dx$ gives the ________ of a plane region.

Problems

In Exercises 5 – 10, evaluate the integral and subsequent iterated integral.

5. (a) $\displaystyle\int_2^5 \left(6x^2 + 4xy - 3y^2\right) dy$

   (b) $\displaystyle\int_{-3}^{-2}\int_2^5 \left(6x^2 + 4xy - 3y^2\right) dy\,dx$

6. (a) $\displaystyle\int_0^{\pi} \left(2x\cos y + \sin x\right) dx$

   (b) $\displaystyle\int_0^{\pi/2}\int_0^{\pi} \left(2x\cos y + \sin x\right) dx\,dy$

7. (a) $\displaystyle\int_1^{x} \left(x^2y - y + 2\right) dy$

   (b) $\displaystyle\int_0^{2}\int_1^{x} \left(x^2y - y + 2\right) dy\,dx$

8. (a) $\displaystyle\int_y^{y^2} \left(x - y\right) dx$

   (b) $\displaystyle\int_{-1}^{1}\int_y^{y^2} \left(x - y\right) dx\,dy$

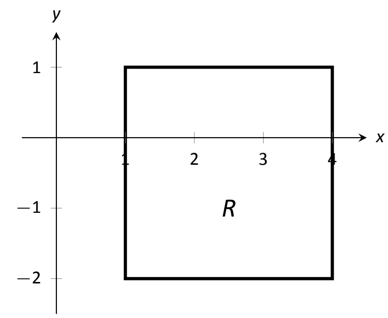
9. (a) $\displaystyle\int_0^{y} \left(\cos x \sin y\right) dx$

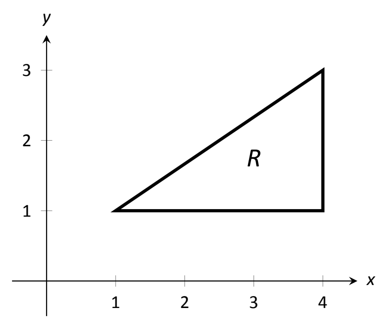
   (b) $\displaystyle\int_0^{\pi}\int_0^{y} \left(\cos x \sin y\right) dx\,dy$

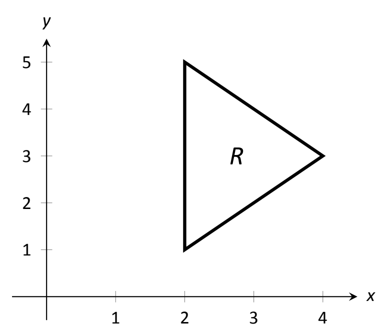
10. (a) $\displaystyle\int_0^{x} \left(\frac{1}{1+x^2}\right) dy$

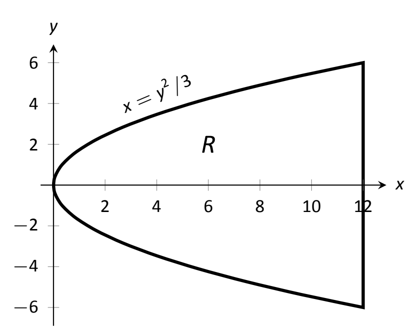
    (b) $\displaystyle\int_1^{2}\int_0^{x} \left(\frac{1}{1+x^2}\right) dy\,dx$

In Exercises 11 – 16, a graph of a planar region R is given. Give the iterated integrals, with both orders of integration $dy\,dx$ and $dx\,dy$, that give the area of R. Evaluate one of the iterated integrals to find the area.

11. 

12. 

13. 

14. 

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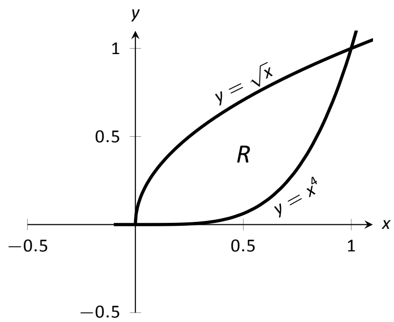

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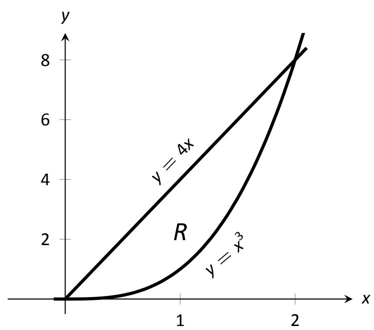


In Exercises 17 – 22, iterated integrals are given that compute the area of a region *R* in the *x*-*y* plane. Sketch the region *R*, and give the iterated integral(s) that give the area of *R* with the opposite order of integration.

17. $\displaystyle\int_{-2}^{2}\int_{0}^{4-x^2} dy\,dx$

18. $\displaystyle\int_{0}^{1}\int_{5-5x}^{5-5x^2} dy\,dx$

19. $\displaystyle\int_{-2}^{2}\int_{0}^{2\sqrt{4-y^2}} dx\,dy$

20. $\displaystyle\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dy\,dx$

21. $\displaystyle\int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy + \int_{1}^{4}\int_{y-2}^{\sqrt{y}} dx\,dy$

22. $\displaystyle\int_{-1}^{1}\int_{(x-1)/2}^{(1-x)/2} dy\,dx$

Exercises 13.2

Terms and Concepts

1. An integral can be interpreted as giving the signed area over an interval; a double integral can be interpreted as giving the signed ________ over a region.

2. Explain why the following statement is false: "Fubini's Theorem states that $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\ dy\ dx = \int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y)\ dx\ dy$."

3. Explain why if $f(x,y) > 0$ over a region R, then $\iint_R f(x,y)\ dA > 0$.

4. If $\iint_R f(x,y)\ dA = \iint_R g(x,y)\ dA$, does this imply $f(x,y) = g(x,y)$?

Problems

In Exercises 5 – 10,

(a) Evaluate the given iterated integral, and

(b) rewrite the integral using the other order of integration.

5. $\displaystyle\int_1^2 \int_{-1}^1 \left(\frac{x}{y}+3\right)\ dx\ dy$

6. $\displaystyle\int_{-\pi/2}^{\pi/2} \int_0^{\pi} (\sin x \cos y)\ dx\ dy$

7. $\displaystyle\int_0^4 \int_0^{-x/2+2} \left(3x^2 - y + 2\right)\ dy\ dx$

8. $\displaystyle\int_1^3 \int_y^3 \left(x^2y - xy^2\right)\ dx\ dy$

9. $\displaystyle\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} (x+y+2)\ dx\ dy$

10. $\displaystyle\int_0^9 \int_{y/3}^{\sqrt{y}} \left(xy^2\right)\ dx\ dy$

In Exercises 11 – 18:

(a) Sketch the region R given by the problem.

(b) Set up the iterated integrals, in both orders, that evaluate the given double integral for the described region R.

(c) Evaluate one of the iterated integrals to find the signed volume under the surface $z = f(x,y)$ over the region R.

11. $\displaystyle\iint_R x^2y\ dA$, where R is bounded by $y = \sqrt{x}$ and $y = x^2$.

12. $\displaystyle\iint_R x^2y\ dA$, where R is bounded by $y = \sqrt[3]{x}$ and $y = x^3$.

13. $\displaystyle\iint_R x^2 - y^2\ dA$, where R is the rectangle with corners $(-1,-1)$, $(1,-1)$, $(1,1)$ and $(-1,1)$.

14. $\displaystyle\iint_R ye^x\ dA$, where R is bounded by $x = 0$, $x = y^2$ and $y = 1$.

15. $\displaystyle\iint_R \left(6-3x-2y\right)\ dA$, where R is bounded by $x = 0$, $y = 0$ and $3x+2y = 6$.

16. $\displaystyle\iint_R e^y\ dA$, where R is bounded by $y = \ln x$ and $y = \frac{1}{e-1}(x-1)$.

17. $\displaystyle\iint_R \left(x^3y - x\right)\ dA$, where R is the half of the circle $x^2+y^2 = 9$ in the first and second quadrants.

18. $\displaystyle\iint_R \left(4-3y\right)\ dA$, where R is bounded by $y = 0$, $y = x/e$ and $y = \ln x$.

In Exercises 19 – 22, state why it is difficult/impossible to integrate the iterated integral in the given order of integration. Change the order of integration and evaluate the new iterated integral.

19. $\displaystyle\int_0^4 \int_{y/2}^2 e^{x^2}\ dx\ dy$

20. $\displaystyle\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \cos\left(y^2\right)\ dy\ dx$

21. $\displaystyle\int_0^1 \int_y^1 \frac{2y}{x^2+y^2}\ dx\ dy$

22. $\displaystyle\int_{-1}^1 \int_1^2 \frac{x\tan^2 y}{1+\ln y}\ dy\ dx$

In Exercises 23 – 26, find the average value of f over the region R. Notice how these functions and regions are related to the iterated integrals given in Exercises 5 – 8.

23. $f(x,y) = \frac{x}{y} + 3$; R is the rectangle with opposite corners $(-1,1)$ and $(1,2)$.

24. $f(x,y) = \sin x \cos y$; R is bounded by $x = 0$, $x = \pi$, $y = -\pi/2$ and $y = \pi/2$.

25. $f(x,y) = 3x^2 - y + 2$; R is bounded by the lines $y = 0$, $y = 2 - x/2$ and $x = 0$.

26. $f(x,y) = x^2y - xy^2$; R is bounded by $y = x$, $y = 1$ and $x = 3$.

Exercises 13.3

Terms and Concepts

1. When evaluating $\iint_R f(x,y)\ dA$ using polar coordinates, $f(x,y)$ is replaced with ________ and dA is replaced with ________.

2. Why would one be interested in evaluating a double integral with polar coordinates?

Problems

In Exercises 3 – 10, a function $f(x,y)$ is given and a region R of the x-y plane is described. Set up and evaluate $\iint_R f(x,y)\ dA$ using polar coordinates.

3. $f(x,y) = 3x - y + 4$; R is the region enclosed by the circle $x^2 + y^2 = 1$.

4. $f(x,y) = 4x + 4y$; R is the region enclosed by the circle $x^2 + y^2 = 4$.

5. $f(x,y) = 8 - y$; R is the region enclosed by the circles with polar equations $r = \cos\theta$ and $r = 3\cos\theta$.

6. $f(x,y) = 4$; R is the region enclosed by the petal of the rose curve $r = \sin(2\theta)$ in the first quadrant.

7. $f(x,y) = \ln\left(x^2 + y^2\right)$; R is the annulus enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

8. $f(x,y) = 1 - x^2 - y^2$; R is the region enclosed by the circle $x^2 + y^2 = 1$.

9. $f(x,y) = x^2 - y^2$; R is the region enclosed by the circle $x^2 + y^2 = 36$ in the first and fourth quadrants.

10. $f(x,y) = (x-y)/(x+y)$; R is the region enclosed by the lines $y = x$, $y = 0$ and the circle $x^2 + y^2 = 1$ in the first quadrant.

In Exercises 11 – 14, an iterated integral in rectangular coordinates is given. Rewrite the integral using polar coordinates and evaluate the new double integral.

11. $\displaystyle\int_0^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{x^2+y^2}\ dy\ dx$

12. $\displaystyle\int_{-4}^4 \int_{-\sqrt{16-y^2}}^{0} \left(2y - x\right)\ dx\ dy$

13. $\displaystyle\int_0^2 \int_y^{\sqrt{8-y^2}} \left(x + y\right)\ dx\ dy$

14. $\displaystyle\int_{-2}^{-1} \int_0^{\sqrt{4-x^2}} \left(x+5\right)\ dy\ dx + \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} \left(x+5\right)\ dy\ dx + \int_1^2 \int_0^{\sqrt{4-x^2}} \left(x+5\right)\ dy\ dx$

    Hint: draw the region of each integral carefully and see how they all connect.

In Exercises 15 – 16, special double integrals are presented that are especially well suited for evaluation in polar coordinates.

15. Consider $\displaystyle\iint_R e^{-(x^2+y^2)}\ dA$.

    (a) Why is this integral difficult to evaluate in rectangular coordinates, regardless of the region R?

    (b) Let R be the region bounded by the circle of radius a centered at the origin. Evaluate the double integral using polar coordinates.

    (c) Take the limit of your answer from (b), as $a \to \infty$. What does this imply about the volume under the surface of $e^{-(x^2+y^2)}$ over the entire x-y plane?

16. The surface of a right circular cone with height h and base radius a can be described by the equation $f(x,y) = h - h\sqrt{\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2}}$, where the tip of the cone lies at $(0,0,h)$ and the circular base lies in the x-y plane, centered at the origin.

    Confirm that the volume of a right circular cone with height h and base radius a is $V = \dfrac{1}{3}\pi a^2 h$ by evaluating $\displaystyle\iint_R f(x,y)\ dA$ in polar coordinates.

Exercises 13.4

Terms and Concepts

1. Why is it easy to use "mass" and "weight" interchangeably, even though they are different measures?

2. Given a point (x, y), the value of x is a measure of distance from the ________-axis.

3. We can think of $\iint_R dm$ as meaning "sum up lots of ________"

4. What is a "discrete planar system?"

5. Why does M_x use $\iint_R y\delta(x, y)\,dA$ instead of $\iint_R x\delta(x, y)\,dA$; that is, why do we use "y" and not "x"?

6. Describe a situation where the center of mass of a lamina does not lie within the region of the lamina itself.

Problems

In Exercises 7 – 10, point masses are given along a line or in the plane. Find the center of mass $\bar{x}$ or $(\bar{x}, \bar{y})$, as appropriate. (All masses are in grams and distances are in cm.)

7. $m_1 = 4$ at $x = 1$; $m_2 = 3$ at $x = 3$; $m_3 = 5$ at $x = 10$

8. $m_1 = 2$ at $x = -3$; $m_2 = 2$ at $x = -1$;
$m_3 = 3$ at $x = 0$; $m_4 = 3$ at $x = 7$

9. $m_1 = 2$ at $(-2, -2)$; $m_2 = 2$ at $(2, -2)$;
$m_3 = 20$ at $(0, 4)$

10. $m_1 = 1$ at $(-1, -1)$; $m_2 = 2$ at $(-1, 1)$;
$m_3 = 2$ at $(1, 1)$; $m_4 = 1$ at $(1, -1)$

In Exercises 11 – 18, find the mass/weight of the lamina described by the region R in the plane and its density function $\delta(x, y)$.

11. R is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = 5\text{gm/cm}^2$

12. R is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = (x + y^2)\text{gm/cm}^2$

13. R is the triangle with corners $(-1, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = 2\text{lb/in}^2$

14. R is the triangle with corners $(0, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = (x^2 + y^2 + 1)\text{lb/in}^2$

15. R is the disk centered at the origin with radius 2; $\delta(x, y) = (x + y + 4)\text{kg/m}^2$

16. R is the circle sector bounded by $x^2 + y^2 = 25$ in the first quadrant; $\delta(x, y) = (\sqrt{x^2 + y^2} + 1)\text{kg/m}^2$

17. R is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = 4\text{lb/ft}^2$

18. R is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = \sqrt{x^2 + y^2}\text{lb/ft}^2$

In Exercises 19 – 26, find the center of mass of the lamina described by the region R in the plane and its density function $\delta(x, y)$.
Note: these are the same lamina as in Exercises 11 – 18.

19. R is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = 5\text{gm/cm}^2$

20. R is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = (x + y^2)\text{gm/cm}^2$

21. R is the triangle with corners $(-1, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = 2\text{lb/in}^2$

22. R is the triangle with corners $(0, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = (x^2 + y^2 + 1)\text{lb/in}^2$

23. R is the disk centered at the origin with radius 2; $\delta(x, y) = (x + y + 4)\text{kg/m}^2$

24. R is the circle sector bounded by $x^2 + y^2 = 25$ in the first quadrant; $\delta(x, y) = (\sqrt{x^2 + y^2} + 1)\text{kg/m}^2$

25. R is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = 4\text{lb/ft}^2$

26. R is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = \sqrt{x^2 + y^2}\text{lb/ft}^2$

The *moment of inertia* I is a measure of the tendency of a lamina to resist rotating about an axis or continue to rotate about an axis. I_x is the moment of inertia about the x-axis, I_y is the moment of inertia about the x-axis, and I_O is the moment of inertia about the origin. These are computed as follows:

- $I_x = \displaystyle\iint_R y^2\,dm$
- $I_y = \displaystyle\iint_R x^2\,dm$
- $I_O = \displaystyle\iint_R \left(x^2 + y^2\right)\,dm$

In Exercises 27 – 30, a lamina corresponding to a planar region R is given with a mass of 16 units. For each, compute I_x, I_y and I_O.

27. R is the 4×4 square with corners at $(-2, -2)$ and $(2, 2)$ with density $\delta(x, y) = 1$.

28. R is the 8×2 rectangle with corners at $(-4, -1)$ and $(4, 1)$ with density $\delta(x, y) = 1$.

29. R is the 4×2 rectangle with corners at $(-2, -1)$ and $(2, 1)$ with density $\delta(x, y) = 2$.

30. R is the disk with radius 2 centered at the origin with density $\delta(x, y) = 4/\pi$.

Exercises 13.5

Terms and Concepts

1. "Surface area" is analogous to what previously studied concept?

2. To approximate the area of a small portion of a surface, we computed the area of its ________ plane.

3. We interpret $\iint_R \, dS$ as "sum up lots of little ________ ________."

4. Why is it important to know how to set up a double integral to compute surface area, even if the resulting integral is hard to evaluate?

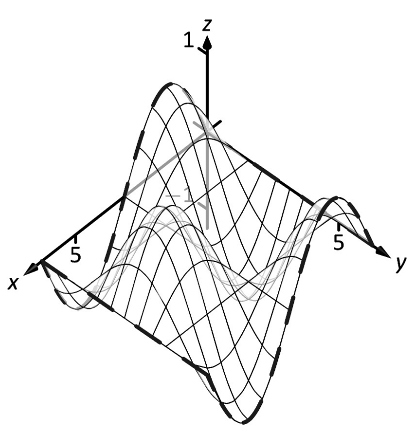
5. Why do $z = f(x, y)$ and $z = g(x, y) = f(x, y) + h$, for some real number h, have the same surface area over a region R?

6. Let $z = f(x, y)$ and $z = g(x, y) = 2f(x, y)$. Why is the surface area of g over a region R not twice the surface area of f over R?

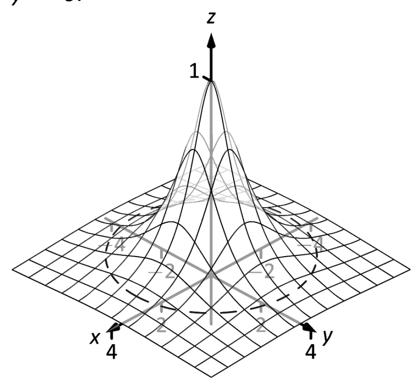
Problems

In Exercises 7 – 10, *set up* the iterated integral that computes the surface area of the given surface over the region R.

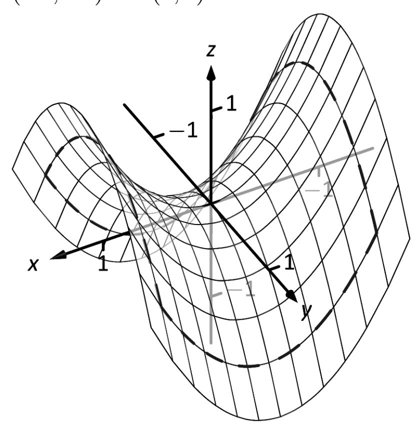
7. $f(x, y) = \sin x \cos y$; R is the rectangle with bounds $0 \le x \le 2\pi$, $0 \le y \le 2\pi$.



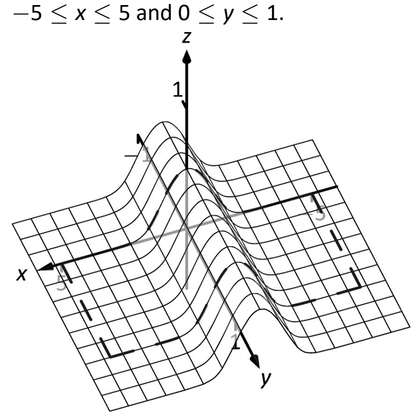
8. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$; R is bounded by the circle $x^2 + y^2 = 9$.



9. $f(x, y) = x^2 - y^2$; R is the rectangle with opposite corners $(-1, -1)$ and $(1, 1)$.



10. $f(x, y) = \dfrac{1}{e^{x^2} + 1}$; R is the rectangle bounded by $-5 \le x \le 5$ and $0 \le y \le 1$.



In Exercises 11 – 19, find the area of the given surface over the region R.

11. $f(x, y) = 3x - 7y + 2$; R is the rectangle with opposite corners $(-1, 0)$ and $(1, 3)$.

12. $f(x, y) = 2x + 2y + 2$; R is the triangle with corners $(0, 0)$,

$(1, 0)$ and $(0, 1)$.

13. $f(x, y) = x^2 + y^2 + 10$; R is bounded by the circle $x^2 + y^2 = 16$.

14. $f(x, y) = -2x + 4y^2 + 7$ over R, the triangle bounded by $y = -x$, $y = x$, $0 \leq y \leq 1$.

15. $f(x, y) = x^2 + y$ over R, the triangle bounded by $y = 2x$, $y = 0$ and $x = 2$.

16. $f(x, y) = \frac{2}{3}x^{3/2} + 2y^{3/2}$ over R, the rectangle with opposite corners $(0, 0)$ and $(1, 1)$.

17. $f(x, y) = 10 - 2\sqrt{x^2 + y^2}$ over R, bounded by the circle $x^2 + y^2 = 25$. (This is the cone with height 10 and base radius 5; be sure to compare your result with the known formula.)

18. Find the surface area of the sphere with radius 5 by doubling the surface area of $f(x, y) = \sqrt{25 - x^2 - y^2}$ over R, bounded by the circle $x^2 + y^2 = 25$. (Be sure to compare your result with the known formula.)

19. Find the surface area of the ellipse formed by restricting the plane $f(x, y) = cx + dy + h$ to the region R, bounded by the circle $x^2 + y^2 = 1$, where c, d and h are some constants. Your answer should be given in terms of c and d; why does the value of h not matter?

Exercises 13.6

Terms and Concepts

1. The strategy for establishing bounds for triple integrals is "________ to ________, ________ to ________ and ________ to ________."

2. Give an informal interpretation of what "$\iiint_D dV$" means.

3. Give two uses of triple integration.

4. If an object has a constant density δ and a volume V, what is its mass?

Problems

In Exercises 5 – 8, two surfaces $f_1(x, y)$ and $f_2(x, y)$ and a region R in the x, y plane are given. Set up and evaluate the double integral that finds the volume between these surfaces over R.

5. $f_1(x, y) = 8 - x^2 - y^2$, $f_2(x, y) = 2x + y$;
   R is the square with corners $(-1, -1)$ and $(1, 1)$.

6. $f_1(x, y) = x^2 + y^2$, $f_2(x, y) = -x^2 - y^2$;
   R is the square with corners $(0, 0)$ and $(2, 3)$.

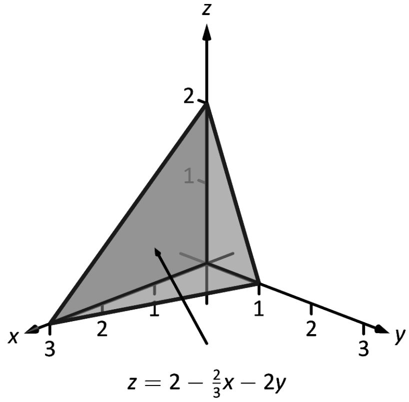
7. $f_1(x, y) = \sin x \cos y$, $f_2(x, y) = \cos x \sin y + 2$;
   R is the triangle with corners $(0, 0)$, $(\pi, 0)$ and (π, π).

8. $f_1(x, y) = 2x^2 + 2y^2 + 3$, $f_2(x, y) = 6 - x^2 - y^2$;
   R is the disk bounded by $x^2 + y^2 = 1$.

In Exercises 9 – 16, a domain D is described by its bounding surfaces, along with a graph. Set up the triple integrals that give the volume of D in all 6 orders of integration, and find the volume of D by evaluating the indicated triple integral.

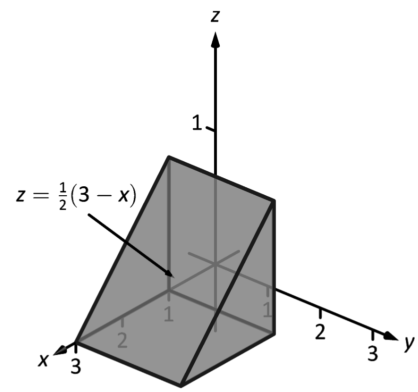
9. D is bounded by the coordinate planes and $z = 2 - 2x/3 - 2y$.

   Evaluate the triple integral with order $dz\,dy\,dx$.



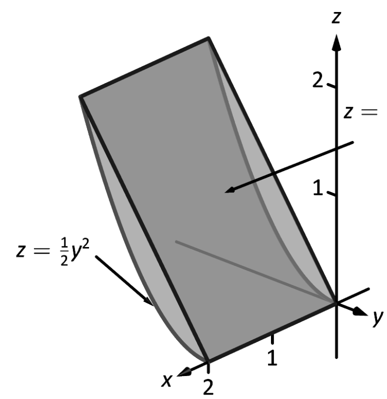
10. D is bounded by the planes $y = 0$, $y = 2$, $x = 1$, $z = 0$ and $z = (3 - x)/2$.

    Evaluate the triple integral with order $dx\,dy\,dz$.



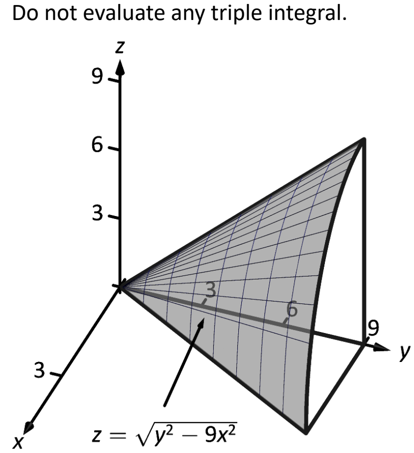
11. D is bounded by the planes $x = 0$, $x = 2$, $z = -y$ and by $z = y^2/2$.

    Evaluate the triple integral with the order $dy\,dz\,dx$.



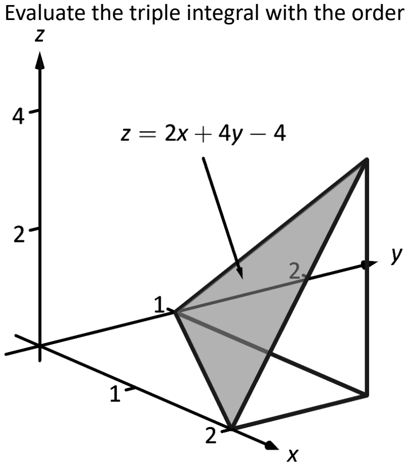
12. D is bounded by the planes $z = 0$, $y = 9$, $x = 0$ and by $z = \sqrt{y^2 - 9x^2}$.

    Do not evaluate any triple integral.



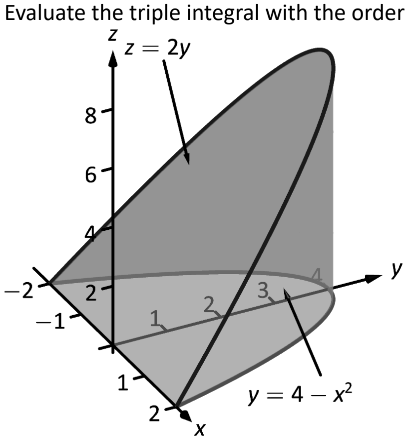
13. D is bounded by the planes $x = 2$, $y = 1$, $z = 0$ and $z = 2x + 4y - 4$.

Evaluate the triple integral with the order $dx\,dy\,dz$.

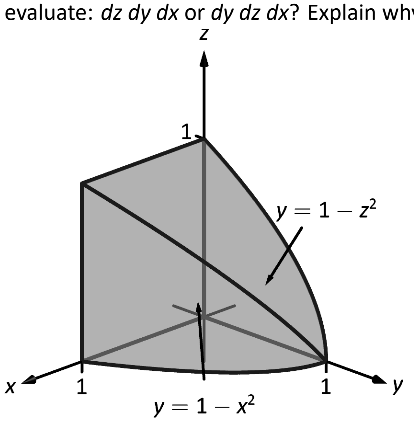


14. D is bounded by the plane $z = 2y$ and by $y = 4 - x^2$.

Evaluate the triple integral with the order $dz\,dy\,dx$.

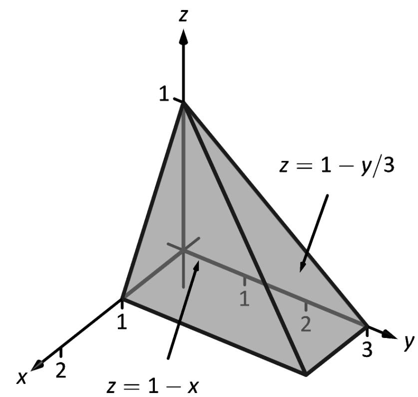


15. D is bounded by the coordinate planes and by $y = 1 - x^2$ and $y = 1 - z^2$.
Do not evaluate any triple integral. Which order is easier to evaluate: $dz\,dy\,dx$ or $dy\,dz\,dx$? Explain why.



16. D is bounded by the coordinate planes and by $z = 1 - y/3$ and $z = 1 - x$.

Evaluate the triple integral with order $dx\,dy\,dz$.



In Exercises 17 – 20, evaluate the triple integral.

17. $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^{\pi}\int_0^{\pi}\big(\cos x\sin y\sin z\big)\,dz\,dy\,dx$

18. $\displaystyle\int_0^1\int_0^x\int_0^{x+y}\big(x+y+z\big)\,dz\,dy\,dx$

19. $\displaystyle\int_0^{\pi}\int_0^1\int_0^z\big(\sin(yz)\big)\,dx\,dy\,dz$

20. $\displaystyle\int_{\pi}^{\pi^2}\int_x^{x^3}\int_{-y^2}^{y^2}\left(z\frac{x^2y+y^2x}{e^{x^2+y^2}}\right)\,dz\,dy\,dx$

In Exercises 21 – 24, find the center of mass of the solid represented by the indicated space region D with density function $\delta(x,y,z)$.

21. D is bounded by the coordinate planes and $z = 2 - 2x/3 - 2y$; $\quad\delta(x,y,z) = 10\text{gm/cm}^3$.
(Note: this is the same region as used in Exercise 9.)

22. D is bounded by the planes $y = 0$, $y = 2$, $x = 1$, $z = 0$ and $z = (3 - x)/2$; $\quad\delta(x,y,z) = 2\text{gm/cm}^3$.
(Note: this is the same region as used in Exercise 10.)

23. D is bounded by the planes $x = 2$, $y = 1$, $z = 0$ and $z = 2x + 4y - 4$; $\quad\delta(x,y,z) = x^2\text{lb/in}^3$.
(Note: this is the same region as used in Exercise 13.)

24. D is bounded by the plane $z = 2y$ and by $y = 4 - x^2$. $\delta(x,y,z) = y^2\text{lb/in}^3$.
(Note: this is the same region as used in Exercise 14.)

Exercises 13.7

Terms and Concepts

1. Explain the difference between the roles r, in cylindrical coordinates, and ρ, in spherical coordinates, play in determining the location of a point.

2. Why are points on the z-axis not determined uniquely when using cylindrical and spherical coordinates?

3. What surfaces are naturally defined using cylindrical coordinates?

4. What surfaces are naturally defined using spherical coordinates?

Problems

In Exercises 5 – 6, points are given in either the rectangular, cylindrical or spherical coordinate systems. Find the coordinates of the points in the other systems.

5. (a) Points in rectangular coordinates: $(2, 2, 1)$ and $(-\sqrt{3}, 1, 0)$
   (b) Points in cylindrical coordinates: $(2, \pi/4, 2)$ and $(3, 3\pi/2, -4)$
   (c) Points in spherical coordinates: $(2, \pi/4, \pi/4)$ and $(1, 0, 0)$

6. (a) Points in rectangular coordinates: $(0, 1, 1)$ and $(-1, 0, 1)$
   (b) Points in cylindrical coordinates: $(0, \pi, 1)$ and $(2, 4\pi/3, 0)$
   (c) Points in spherical coordinates: $(2, \pi/6, \pi/2)$ and $(3, \pi, \pi)$

In Exercises 7 – 8, describe the curve, surface or region in space determined by the given bounds.

7. Bounds in cylindrical coordinates:

   (a) $r = 1, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq 1$
   (b) $1 \leq r \leq 2, \quad 0 \leq \theta \leq \pi, \quad 0 \leq z \leq 1$

   Bounds in spherical coordinates:

   (c) $\rho = 3, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \varphi \leq \pi/2$
   (d) $2 \leq \rho \leq 3, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \varphi \leq \pi$

8. Bounds in cylindrical coordinates:

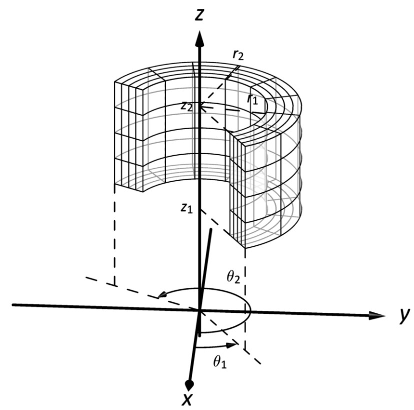
   (a) $1 \leq r \leq 2, \quad \theta = \pi/2, \quad 0 \leq z \leq 1$
   (b) $r = 2, \quad 0 \leq \theta \leq 2\pi, \quad z = 5$

   Bounds in spherical coordinates:

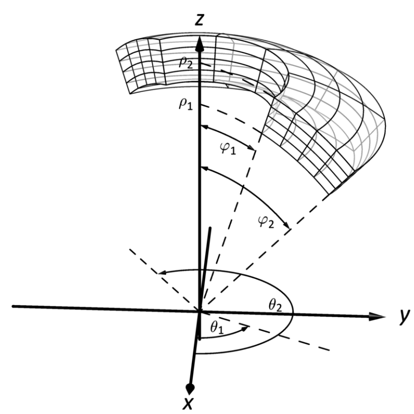
   (c) $0 \leq \rho \leq 2, \quad 0 \leq \theta \leq \pi, \quad \varphi = \pi/4$
   (d) $\rho = 2, \quad 0 \leq \theta \leq 2\pi, \quad \varphi = \pi/6$

In Exercises 9 – 10, standard regions in space, as defined by cylindrical and spherical coordinates, are shown. Set up the triple integral that integrates the given function over the graphed region.

9. Cylindrical coordinates, integrating $h(r, \theta, z)$:



10. Cylindrical coordinates, integrating $h(\rho, \theta, \varphi)$:



In Exercises 11 – 16, a triple integral in cylindrical coordinates is given. Describe the region in space defined by the bounds of the integral.

11. $\displaystyle\int_0^{\pi/2}\int_0^2\int_0^2 r\,dz\,dr\,d\theta$

12. $\displaystyle\int_0^{2\pi}\int_3^4\int_0^5 r\,dz\,dr\,d\theta$

13. $\displaystyle\int_0^{2\pi}\int_0^1\int_0^{1-r} r\,dz\,dr\,d\theta$

14. $\displaystyle\int_0^{\pi}\int_0^1\int_0^{2-r} r\,dz\,dr\,d\theta$

15. $\displaystyle\int_0^{\pi}\int_0^3\int_0^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta$

16. $\displaystyle\int_0^{2\pi}\int_0^{a}\int_0^{\sqrt{a^2-r^2}+b} r\, dz\, dr\, d\theta$

In Exercises 17 – 22, a triple integral in spherical coordinates is given. Describe the region in space defined by the bounds of the integral.

17. $\displaystyle\int_0^{\pi/2}\int_0^{\pi}\int_0^{1} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

18. $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_1^{1.1} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

19. $\displaystyle\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

20. $\displaystyle\int_0^{2\pi}\int_{\pi/6}^{\pi/4}\int_0^{2} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

21. $\displaystyle\int_0^{2\pi}\int_0^{\pi/6}\int_0^{\sec\varphi} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

22. $\displaystyle\int_0^{2\pi}\int_0^{\pi/6}\int_0^{a\sec\varphi} \rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

In Exercises 23 – 26, a solid is described along with its density function. Find the mass of the solid using cylindrical coordinates.

23. Bounded by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $z=4$ with density function $\delta(x,y,z)=\sqrt{x^2+y^2}+1$.

24. Bounded by the cylinders $x^2+y^2=4$ and $x^2+y^2=9$, between the planes $z=0$ and $z=10$ with density function $\delta(x,y,z)=z$.

25. Bounded by $y\geq 0$, the cylinder $x^2+y^2=1$, and between the planes $z=0$ and $z=4-y$ with density function $\delta(x,y,z)=1$.

26. The upper half of the unit ball, bounded between $z=0$ and $z=\sqrt{1-x^2-y^2}$, with density function $\delta(x,y,z)=1$.

In Exercises 27 – 30, a solid is described along with its density function. Find the center of mass of the solid using cylindrical coordinates. (Note: these are the same solids and density functions as found in Exercises 23 through 26.)

27. Bounded by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $z=4$ with density function $\delta(x,y,z)=\sqrt{x^2+y^2}+1$.

28. Bounded by the cylinders $x^2+y^2=4$ and $x^2+y^2=9$, between the planes $z=0$ and $z=10$ with density function $\delta(x,y,z)=z$.

29. Bounded by $y\geq 0$, the cylinder $x^2+y^2=1$, and between the planes $z=0$ and $z=4-y$ with density function $\delta(x,y,z)=1$.

30. The upper half of the unit ball, bounded between $z=0$ and $z=\sqrt{1-x^2-y^2}$, with density function $\delta(x,y,z)=1$.

In Exercises 31 – 34, a solid is described along with its density function. Find the mass of the solid using spherical coordinates.

31. The upper half of the unit ball, bounded between $z=0$ and $z=\sqrt{1-x^2-y^2}$, with density function $\delta(x,y,z)=1$.

32. The spherical shell bounded between $x^2+y^2+z^2=16$ and $x^2+y^2+z^2=25$ with density function $\delta(x,y,z)=\sqrt{x^2+y^2+z^2}$.

33. The conical region bounded above $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$ with density function $\delta(x,y,z)=z$.

34. The cone bounded above $z=\sqrt{x^2+y^2}$ and below the plane $z=1$ with density function $\delta(x,y,z)=z$.

In Exercises 35 – 38, a solid is described along with its density function. Find the center of mass of the solid using spherical coordinates. (Note: these are the same solids and density functions as found in Exercises 31 through 34.)

35. The upper half of the unit ball, bounded between $z=0$ and $z=\sqrt{1-x^2-y^2}$, with density function $\delta(x,y,z)=1$.

36. The spherical shell bounded between $x^2+y^2+z^2=16$ and $x^2+y^2+z^2=25$ with density function $\delta(x,y,z)=\sqrt{x^2+y^2+z^2}$.

37. The conical region bounded above $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$ with density function $\delta(x,y,z)=z$.

38. The cone bounded above $z=\sqrt{x^2+y^2}$ and below the plane $z=1$ with density function $\delta(x,y,z)=z$.

In Exercises 39 – 42, a region is space is described. Set up the triple integrals that find the volume of this region using rectangular, cylindrical and spherical coordinates, then comment on which of the three appears easiest to evaluate.

39. The region enclosed by the unit sphere, $x^2+y^2+z^2=1$.

40. The region enclosed by the cylinder $x^2+y^2=1$ and planes $z=0$ and $z=1$.

41. The region enclosed by the cone $z=\sqrt{x^2+y^2}$ and plane $z=1$.

42. The cube enclosed by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ and $z=1$. (Hint: in spherical, use order of integration $d\rho\, d\varphi\, d\theta$.)

Exercises 14.1

Terms and Concepts

1. Explain how a line integral can be used to find the area under a curve.

2. How does the evaluation of a line integral given as $\int_C f(s)\ ds$ differ from a line integral given as $\oint_C f(s)\ ds$?

3. Why are most line integrals evaluated using Key Idea 14.1.1 instead of "directly" as $\int_C f(s)\ ds$?

4. Sketch a closed, piecewise smooth curve composed of three subcurves.

Problems

In Exercises 5 – 10, a planar curve C is given along with a surface f that is defined over C. Evaluate the line integral $\displaystyle\int_C f(s)\ ds$.

5. C is the line segment joining the points $(-2,-1)$ and $(1,2)$; the surface is $f(x,y)=x^2+y^2+2$.

6. C is the segment of $y=3x+2$ on $[1,2]$; the surface is $f(x,y)=5x+2y$.

7. C is the circle with radius 2 centered at the point $(4,2)$; the surface is $f(x,y)=3x-y$.

8. C is the curve given by $\vec{r}(t)=\langle \cos t+t\sin t, \sin t-t\cos t\rangle$ on $[0,2\pi]$; the surface is $f(x,y)=5$.

9. C is the piecewise curve composed of the line segments that connect $(0,1)$ to $(1,1)$, then connect $(1,1)$ to $(1,0)$; the surface is $f(x,y)=x+y^2$.

10. C is the piecewise curve composed of the line segment joining the points $(0,0)$ and $(1,1)$, along with the quarter–circle parametrized by $\langle \cos t, -\sin t+1\rangle$ on $[0,\pi/2]$(which starts at the point $(1,1)$ and ends at $(0,0)$; the surface is $f(x,y)=x^2+y^2$.

In Exercises 11 – 14, a planar curve C is given along with a surface f that is defined over C. Set up the line integral $\displaystyle\int_C f(s)\ ds$, then approximate its value using technology.

11. C is the portion of the parabola $y=2x^2+x+1$ on $[0,1]$; the surface is $f(x,y)=x^2+2y$.

12. C is the portion of the curve $y=\sin x$ on $[0,\pi]$; the surface is $f(x,y)=x$.

13. C is the ellipse given by $\vec{r}(t)=\langle 2\cos t, \sin t\rangle$ on $[0,2\pi]$; the surface is $f(x,y)=10-x^2-y^2$.

14. C is the portion of $y=x^3$ on $[-1,1]$; the surface is $f(x,y)=2x+3y+5$.

In Exercises 15 – 18, a parametrized curve C in space is given. Find the area above the x-y plane that is under C.

15. C: $\vec{r}(t)=\langle 5t, t, t^2\rangle$ for $1\leq t\leq 2$.

16. C: $\vec{r}(t)=\langle \cos t, \sin t, \sin(2t)+1\rangle$ for $0\leq t\leq 2\pi$.

17. C: $\vec{r}(t)=\langle 3\cos t, 3\sin t, t^2\rangle$ for $0\leq t\leq 2\pi$.

18. C: $\vec{r}(t)=\langle 3t, 4t, t\rangle$ for $0\leq t\leq 1$.

In Exercises 19 – 20, a parametrized curve C is given that represents a thin wire with density δ. Find the mass and center of mass of the thin wire.

19. C: $\vec{r}(t)=\langle \cos t, \sin t, t\rangle$ for $0\leq t\leq 4\pi$; $\delta(x,y,z)=z$.

20. C: $\vec{r}(t)=\langle t-t^2, t^2-t^3, t^3-t^4\rangle$ for $0\leq t\leq 1$; $\delta(x,y,z)=x+2y+2z$. Use technology to approximate the value of each integral.

Exercises 14.2

Terms and Concepts

1. Give two quantities that can be represented by a vector field in the plane or in space.

2. In your own words, describe what it means for a vector field to have a negative divergence at a point.

3. In your own words, describe what it means for a vector field to have a negative curl at a point.

4. The divergence of a vector field $\vec{F}$ at a particular point is 0. Does this mean that $\vec{F}$ is incompressible? Why/why not?

Problems

In Exercises 5 – 8, sketch the given vector field over the rectangle with opposite corners $(-2,-2)$ and $(2,2)$, sketching one vector for every point with integer coordinates (i.e., at $(0,0)$, $(1,2)$, etc.).

5. $\vec{F} = \langle x, 0\rangle$

6. $\vec{F} = \langle 0, x\rangle$

7. $\vec{F} = \langle 1, -1\rangle$

8. $\vec{F} = \langle y^2, 1\rangle$

In Exercises 9 – 18, find the divergence and curl of the given vector field.

9. $\vec{F} = \langle x, y^2\rangle$

10. $\vec{F} = \langle -y^2, x\rangle$

11. $\vec{F} = \langle \cos(xy), \sin(xy)\rangle$

12. $\vec{F} = \left\langle \dfrac{-2x}{(x^2+y^2)^2}, \dfrac{-2y}{(x^2+y^2)^2}\right\rangle$

13. $\vec{F} = \langle x+y, y+z, x+z\rangle$

14. $\vec{F} = \langle x^2+z^2, x^2+y^2, y^2+z^2\rangle$

15. $\vec{F} = \nabla f$, where $f(x,y) = \frac{1}{2}x^2 + \frac{1}{3}y^3$.

16. $\vec{F} = \nabla f$, where $f(x,y) = x^2y$.

17. $\vec{F} = \nabla f$, where $f(x,y,z) = x^2y + \sin z$.

18. $\vec{F} = \nabla f$, where $f(x,y,z) = \dfrac{1}{x^2+y^2+z^2}$.

Exercises 14.3

Terms and Concepts

1. T/F: In practice, the evaluation of line integrals over vector fields involves computing the magnitude of a vector–valued function.

2. Let $\vec{F}(x, y)$ be a vector field in the plane and let $\vec{r}(t)$ be a two–dimensional vector–valued function. Why is "$\vec{F}(\vec{r}(t))$" an "abuse of notation"?

3. T/F: The orientation of a curve C matters when computing a line integral over a vector field.

4. T/F: The orientation of a curve C matters when computing a line integral over a scalar field.

5. Under "reasonable conditions," if curl $\vec{F} = \vec{0}$, what can we conclude about the vector field $\vec{F}$?

6. Let $\vec{F}$ be a conservative field and let C be a closed curve. Why are we able to conclude that $\oint_C \vec{F} \cdot d\vec{r} = 0$?

Problems

In Exercises 7 – 12, a vector field $\vec{F}$ and a curve C are given. Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.

7. $\vec{F} = \langle y, y^2 \rangle$; C is the line segment from $(0, 0)$ to $(3, 1)$.

8. $\vec{F} = \langle x, x + y \rangle$; C is the portion of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

9. $\vec{F} = \langle y, x \rangle$; C is the top half of the unit circle, beginning at $(1, 0)$ and ending at $(-1, 0)$.

10. $\vec{F} = \langle xy, x \rangle$; C is the portion of the curve $y = x^3$ on $-1 \leq x \leq 1$.

11. $\vec{F} = \langle z, x^2, y \rangle$; C is the line segment from $(1, 2, 3)$ to $(4, 3, 2)$.

12. $\vec{F} = \langle y + z, x + z, x + y \rangle$; C is the helix $\vec{r}(t) = \langle \cos t, \sin t, t/(2\pi) \rangle$ on $0 \leq t \leq 2\pi$.

In Exercises 13 – 16, find the work performed by the force field $\vec{F}$ moving a particle along the path C.

13. $\vec{F} = \langle y, x^2 \rangle$ N; C is the segment of the line $y = x$ from $(0, 0)$ to $(1, 1)$, where distances are measured in meters.

14. $\vec{F} = \langle y, x^2 \rangle$ N; C is the portion of $y = \sqrt{x}$ from $(0, 0)$ to $(1, 1)$, where distances are measured in meters.

15. $\vec{F} = \langle 2xy, x^2, 1 \rangle$ lbs; C is the path from $(0, 0, 0)$ to $(2, 4, 8)$ via $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ on $0 \leq t \leq 2$, where distance are measured in feet.

16. $\vec{F} = \langle 2xy, x^2, 1 \rangle$ lbs; C is the path from $(0, 0, 0)$ to $(2, 4, 8)$ via $\vec{r}(t) = \langle t, 2t, 4t \rangle$ on $0 \leq t \leq 2$, where distance are measured in feet.

In Exercises 17 – 20, a conservative vector field $\vec{F}$ and a curve C are given.

1. **Find a potential function f for $\vec{F}$.**
2. **Compute** curl $\vec{F}$.
3. **Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ directly, i.e., using Key Idea 14.3.1.**
4. **Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ using the Fundamental Theorem of Line Integrals.**

17. $\vec{F} = \langle y + 1, x \rangle$, C is the line segment from $(0, 1)$ to $(1, 0)$.

18. $\vec{F} = \langle 2x + y, 2y + x \rangle$, C is curve parametrized by $\vec{r}(t) = \langle t^2 - t, t^3 - t \rangle$ on $0 \leq t \leq 1$.

19. $\vec{F} = \langle 2xyz, x^2z, x^2y \rangle$, C is curve parametrized by $\vec{r}(t) = \langle 2t + 1, 3t - 1, t \rangle$ on $0 \leq t \leq 2$.

20. $\vec{F} = \langle 2x, 2y, 2z \rangle$, C is curve parametrized by $\vec{r}(t) = \langle \cos t, \sin t, \sin(2t) \rangle$ on $0 \leq t \leq 2\pi$.

21. Prove part of Theorem 14.3.3: let $\vec{F} = \langle M, N, P \rangle$ be a conservative vector field. Show that curl $\vec{F} = 0$.

Exercises 14.4

Terms and Concepts

1. Let $\vec{F}$ be a vector field and let C be a curve. *Flow* is a measure of the amount of $\vec{F}$ going ________________ C; *flux* is a measure of the amount of $\vec{F}$ going ________________ C.

2. What is circulation?

3. Green's Theorem states, informally, that the circulation around a closed curve that bounds a region R is equal to the sum of ________________ across R.

4. The Divergence Theorem states, informally, that the outward flux across a closed curve that bounds a region R is equal to the sum of ________________ across R.

5. Let $\vec{F}$ be a vector field and let C_1 and C_2 be any nonintersecting paths except that each starts at point A and ends at point B. If ________ $= 0$, then $\int_{C_1} \vec{F} \cdot \vec{T}\, ds = \int_{C_2} \vec{F} \cdot \vec{T}\, ds$.

6. Let $\vec{F}$ be a vector field and let C_1 and C_2 be any nonintersecting paths except that each starts at point A and ends at point B. If ________ $= 0$, then $\int_{C_1} \vec{F} \cdot \vec{n}\, ds = \int_{C_2} \vec{F} \cdot \vec{n}\, ds$.

Problems

In Exercises 7 – 12, a vector field $\vec{F}$ and a curve C are given. Evaluate $\int_C \vec{F} \cdot \vec{n}\, ds$, the flux of $\vec{F}$ over C.

7. $\vec{F} = \langle x+y, x-y \rangle$; C is the curve with initial and terminal points $(3,-2)$ and $(3,2)$, respectively, parametrized by $\vec{r}(t) = \langle 3t^2, 2t \rangle$ on $-1 \leq t \leq 1$.

8. $\vec{F} = \langle x+y, x-y \rangle$; C is the curve with initial and terminal points $(3,-2)$ and $(3,2)$, respectively, parametrized by $\vec{r}(t) = \langle 3, t \rangle$ on $-2 \leq t \leq 2$.

9. $\vec{F} = \langle x^2, y+1 \rangle$; C is line segment from $(0,0)$ to $(2,4)$.

10. $\vec{F} = \langle x^2, y+1 \rangle$; C is the portion of the parabola $y = x^2$ from $(0,0)$ to $(2,4)$.

11. $\vec{F} = \langle y, 0 \rangle$; C is the line segment from $(0,0)$ to $(0,1)$.

12. $\vec{F} = \langle y, 0 \rangle$; C is the line segment from $(0,0)$ to $(1,1)$.

In Exercises 13 – 16, a vector field $\vec{F}$ and a closed curve C, enclosing a region R, are given. Verify Green's Theorem by evaluating $\oint_C \vec{F} \cdot d\vec{r}$ and $\iint_R \text{curl}\, \vec{F}\, dA$, showing they are equal.

13. $\vec{F} = \langle x-y, x+y \rangle$; C is the closed curve composed of the parabola $y = x^2$ on $0 \leq x \leq 2$ followed by the line segment from $(2,4)$ to $(0,0)$.

14. $\vec{F} = \langle -y, x \rangle$; C is the unit circle.

15. $\vec{F} = \langle 0, x^2 \rangle$; C the triangle with corners at $(0,0)$, $(2,0)$ and $(1,1)$.

16. $\vec{F} = \langle x+y, 2x \rangle$; C the curve that starts at $(0,1)$, follows the parabola $y = (x-1)^2$ to $(3,4)$, then follows a line back to $(0,1)$.

In Exercises 17 – 20, a closed curve C enclosing a region R is given. Find the area of R by computing $\oint_C \vec{F} \cdot d\vec{r}$ for an appropriate choice of vector field $\vec{F}$.

17. C is the ellipse parametrized by $\vec{r}(t) = \langle 4\cos t, 3\sin t \rangle$ on $0 \leq t \leq 2\pi$.

18. C is the curve parametrized by $\vec{r}(t) = \langle \cos t, \sin(2t) \rangle$ on $-\pi/2 \leq t \leq \pi/2$.

19. C is the curve parametrized by $\vec{r}(t) = \langle \cos t, \sin(2t) \rangle$ on $0 \leq t \leq 2$.

20. C is the curve parametrized by $\vec{r}(t) = \langle 2\cos t + \frac{1}{10}\cos(10t), 2\sin t + \frac{1}{10}\sin(10t) \rangle$ on $0 \leq t \leq 2\pi$.

In Exercises 21 – 24, a vector field $\vec{F}$ and a closed curve C, enclosing a region R, are given. Verify the Divergence Theorem by evaluating $\oint_C \vec{F} \cdot \vec{n}\, ds$ and $\iint_R \text{div}\, \vec{F}\, dA$, showing they are equal.

21. $\vec{F} = \langle x-y, x+y \rangle$; C is the closed curve composed of the parabola $y = x^2$ on $0 \leq x \leq 2$ followed by the line segment from $(2,4)$ to $(0,0)$.

22. $\vec{F} = \langle -y, x \rangle$; C is the unit circle.

23. $\vec{F} = \langle 0, y^2 \rangle$; C the triangle with corners at $(0,0)$, $(2,0)$ and $(1,1)$.

24. $\vec{F} = \langle x^2/2, y^2/2 \rangle$; C the curve that starts at $(0,1)$, follows the parabola $y = (x-1)^2$ to $(3,4)$, then follows a line back to $(0,1)$.

Exercises 14.5

Terms and Concepts

1. In your own words, describe what an orientable surface is.

2. Give an example of a non-orientable surface.

Problems

In Exercises 3 – 4, parametrize the surface defined by the function $z = f(x,y)$ over each of the given regions R of the x-y plane.

3. $z = 3x^2y$;

   (a) R is the rectangle bounded by $-1 \leq x \leq 1$ and $0 \leq y \leq 2$.

   (b) R is the circle of radius 3, centered at $(1,2)$.

   (c) R is the triangle with vertices $(0,0)$, $(1,0)$ and $(0,2)$.

   (d) R is the region bounded by the x-axis and the graph of $y = 1 - x^2$.

4. $z = 4x + 2y^2$;

   (a) R is the rectangle bounded by $1 \leq x \leq 4$ and $5 \leq y \leq 7$.

   (b) R is the ellipse with major axis of length 8 parallel to the x-axis, and minor axis of length 6 parallel to the y-axis, centered at the origin.

   (c) R is the triangle with vertices $(0,0)$, $(2,2)$ and $(0,4)$.

   (d) R is the annulus bounded between the circles, centered at the origin, with radius 2 and radius 5.

In Exercises 5 – 8, a surface $\mathcal{S}$ in space is described that cannot be defined in terms of a function $z = f(x,y)$. Give a parametrization of $\mathcal{S}$.

5. $\mathcal{S}$ is the rectangle in space with corners at $(0,0,0)$, $(0,2,0)$, $(0,2,1)$ and $(0,0,1)$.

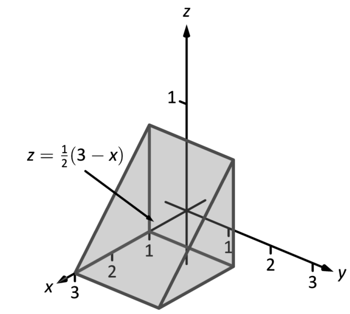
6. $\mathcal{S}$ is the triangle in space with corners at $(1,0,0)$, $(1,0,1)$ and $(0,0,1)$.

7. $\mathcal{S}$ is the ellipsoid $\dfrac{x^2}{9} + \dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$.

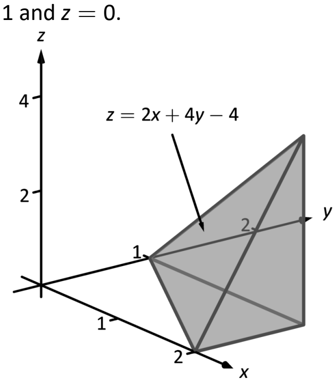
8. $\mathcal{S}$ is the elliptic cone $y^2 = x^2 + \dfrac{z^2}{16}$, for $-1 \leq y \leq 5$.

In Exercises 9 – 16, a domain D in space is given. Parametrize each of the bounding surfaces of D.

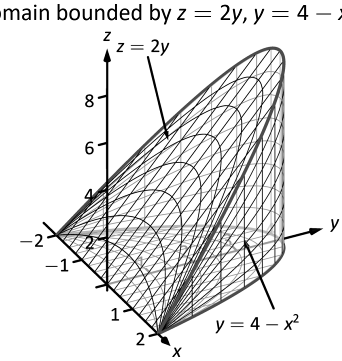
9. D is the domain bounded by the planes $z = \frac{1}{2}(3-x)$, $x = 1$, $y = 0$, $y = 2$ and $z = 0$.



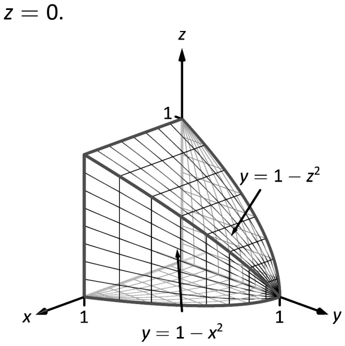
10. D is the domain bounded by the planes $z = 2x + 4y - 4$, $x = 2$, $y = 1$ and $z = 0$.



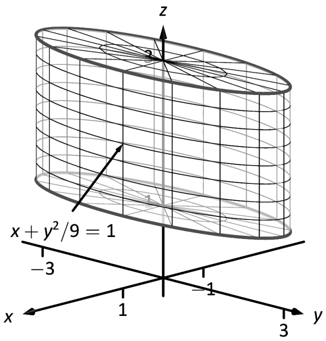
11. D is the domain bounded by $z = 2y$, $y = 4 - x^2$ and $z = 0$.



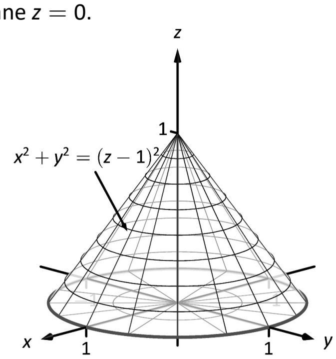
12. D is the domain bounded by $y = 1 - z^2$, $y = 1 - x^2$, $x = 0$, $y = 0$ and $z = 0$.



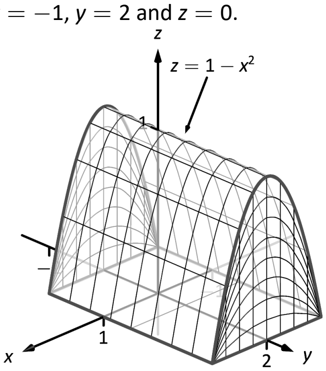
13. D is the domain bounded by the cylinder $x + y^2/9 = 1$ and the planes $z = 1$ and $z = 3$.



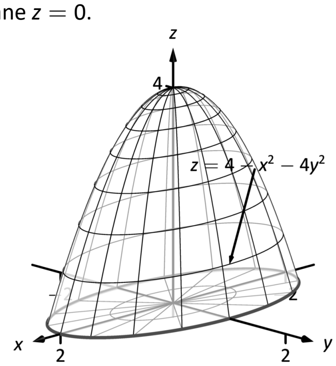
14. D is the domain bounded by the cone $x^2 + y^2 = (z - 1)^2$ and the plane $z = 0$.



15. D is the domain bounded by the cylinder $z = 1 - x^2$ and the planes $y = -1$, $y = 2$ and $z = 0$.



16. D is the domain bounded by the paraboloid $z = 4 - x^2 - 4y^2$ and the plane $z = 0$.



In Exercises 17 – 20, find the surface area S of the given surface $\mathcal{S}$. (The associated integrals are computable without the assistance of technology.)

17. $\mathcal{S}$ is the plane $z = 2x + 3y$ over the rectangle $-1 \le x \le 1$, $2 \le v \le 3$.

18. $\mathcal{S}$ is the plane $z = x + 2y$ over the triangle with vertices at $(0, 0)$, $(1, 0)$ and $(0, 1)$.

19. $\mathcal{S}$ is the plane $z = x + y$ over the circular disk, centered at the origin, with radius 2.

20. $\mathcal{S}$ is the plane $z = x + y$ over the annulus bounded by the circles, centered at the origin, with radius 1 and radius 2.

In Exercises 21 – 24, set up the double integral that finds the surface area S of the given surface $\mathcal{S}$, then use technology to approximate its value.

21. $\mathcal{S}$ is the paraboloid $z = x^2 + y^2$ over the circular disk of radius 3 centered at the origin.

22. $\mathcal{S}$ is the paraboloid $z = x^2 + y^2$ over the triangle with vertices at $(0, 0)$, $(0, 1)$ and $(1, 1)$.

23. $\mathcal{S}$ is the plane $z = 5x - y$ over the region enclosed by the parabola $y = 1 - x^2$ and the x-axis.

24. $\mathcal{S}$ is the hyperbolic paraboloid $z = x^2 - y^2$ over the circular disk of radius 1 centered at the origin.

Exercises 14.6

Terms and Concepts

1. In the plane, flux is a measurement of how much of the vector field passes across a ________; in space, flux is a measurement of how much of the vector field passes across a ________.

2. When computing flux, what does it mean when the result is a negative number?

3. When $\mathcal{S}$ is a closed surface, we choose the normal vector so that it points to the ________ of the surface.

4. If $\mathcal{S}$ is a plane, and $\vec{F}$ is always parallel to $\mathcal{S}$, then the flux of $\vec{F}$ across $\mathcal{S}$ will be ________.

Problems

In Exercises 5 – 6, a surface $\mathcal{S}$ that represents a thin sheet of material with density δ is given. Find the mass of each thin sheet.

5. $\mathcal{S}$ is the plane $f(x,y) = x+y$ on $-2 \leq x \leq 2$, $-3 \leq y \leq 3$, with $\delta(x,y,z) = z$.

6. $\mathcal{S}$ is the unit sphere, with $\delta(x,y,z) = x+y+z+10$.

In Exercises 7 – 14, a surface $\mathcal{S}$ and a vector field $\vec{F}$ are given. Compute the flux of $\vec{F}$ across $\mathcal{S}$. (If $\mathcal{S}$ is not a closed surface, choose $\vec{n}$ so that it has a positive z-component, unless otherwise indicated.)

7. $\mathcal{S}$ is the plane $f(x,y) = 3x+y$ on $0 \leq x \leq 1$, $1 \leq y \leq 4$; $\vec{F} = \langle x^2, -z, 2y\rangle$.

8. $\mathcal{S}$ is the plane $f(x,y) = 8-x-y$ over the triangle with vertices at $(0,0)$, $(1,0)$ and $(1,5)$; $\vec{F} = \langle 3,1,2\rangle$.

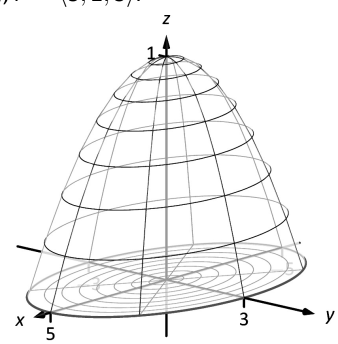
9. $\mathcal{S}$ is the paraboloid $f(x,y) = x^2+y^2$ over the unit disk; $\vec{F} = \langle 1,0,0\rangle$.

10. $\mathcal{S}$ is the unit sphere; $\vec{F} = \langle y-z, z-x, x-y\rangle$.

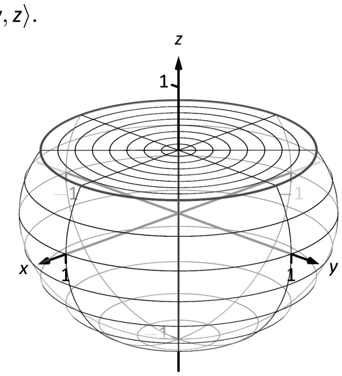
11. $\mathcal{S}$ is the square in space with corners at $(0,0,0)$, $(1,0,0)$, $(1,0,1)$ and $(0,0,1)$ (choose $\vec{n}$ such that it has a positive y-component); $\vec{F} = \langle 0, -z, y\rangle$.

12. $\mathcal{S}$ is the disk in the y-z plane with radius 1, centered at $(0,1,1)$ (choose $\vec{n}$ such that it has a positive x-component); $\vec{F} = \langle y,z,x\rangle$.

13. $\mathcal{S}$ is the closed surface composed of $\mathcal{S}_1$, whose boundary is the ellipse in the x-y plane described by $\frac{x^2}{25}+\frac{y^2}{9} = 1$ and $\mathcal{S}_2$, part of the elliptical paraboloid $f(x,y) = 1-\frac{x^2}{25}-\frac{y^2}{9}$ (see graph); $\vec{F} = \langle 5,2,3\rangle$.



14. $\mathcal{S}$ is the closed surface composed of $\mathcal{S}_1$, part of the unit sphere and $\mathcal{S}_2$, part of the plane $z = 1/2$ (see graph); $\vec{F} = \langle x, -y, z\rangle$.



Exercises 14.7

Terms and Concepts

1. What are the differences between the Divergence Theorems of Section 14.4 and this section?

2. What property of a vector field does the Divergence Theorem relate to flux?

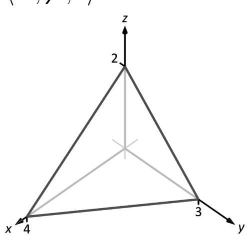
3. What property of a vector field does Stokes' Theorem relate to circulation?

4. Stokes' Theorem is the spatial version of what other theorem?

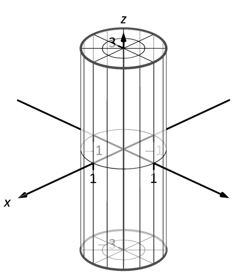
Problems

In Exercises 5 – 8, a closed surface $\mathcal{S}$ enclosing a domain D and a vector field $\vec{F}$ are given. Verify the Divergence Theorem on $\mathcal{S}$; that is, show $\iint_{\mathcal{S}} \vec{F} \cdot \vec{n}\, dS = \iiint_D \operatorname{div} \vec{F}\, dV$.

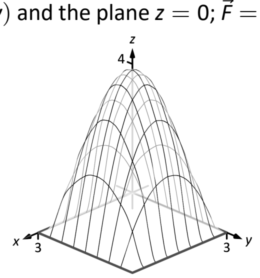
5. $\mathcal{S}$ is the surface bounding the domain D enclosed by the plane $z = 2 - x/2 - 2y/3$ and the coordinate planes in the first octant; $\vec{F} = \langle x^2, y^2, x \rangle$.



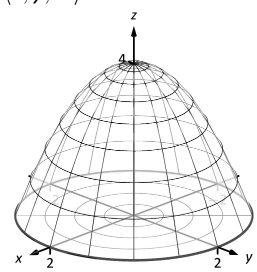
6. $\mathcal{S}$ is the surface bounding the domain D enclosed by the cylinder $x^2 + y^2 = 1$ and the planes $z = -3$ and $z = 3$; $\vec{F} = \langle -x, y, z \rangle$.



7. $\mathcal{S}$ is the surface bounding the domain D enclosed by $z = xy(3-x)(3-y)$ and the plane $z = 0$; $\vec{F} = \langle 3x, 4y, 5z + 1 \rangle$.

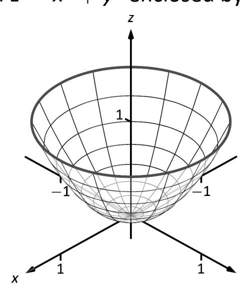


8. $\mathcal{S}$ is the surface composed of $\mathcal{S}_1$, the paraboloid $z = 4 - x^2 - y^2$ for $z \geq 0$, and $\mathcal{S}_2$, the disk of radius 2 centered at the origin; $\vec{F} = \langle x, y, z^2 \rangle$.

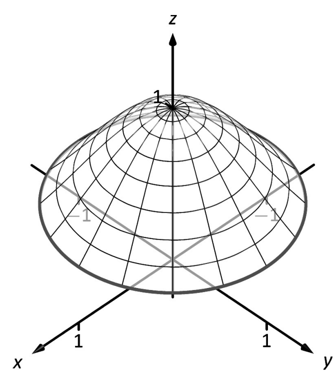


In Exercises 9 – 12, a closed curve C that is the boundary of a surface $\mathcal{S}$ is given along with a vector field $\vec{F}$. Verify Stokes' Theorem on C; that is, show $\oint_C \vec{F} \cdot d\vec{r} = \iint_{\mathcal{S}} \left(\operatorname{curl} \vec{F} \right) \cdot \vec{n}\, dS$.

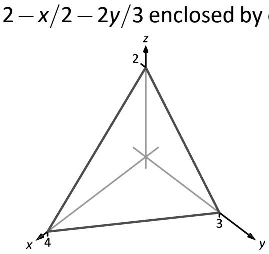
9. C is the curve parametrized by $\vec{r}(t) = \langle \cos t, \sin t, 1 \rangle$ and $\mathcal{S}$ is the portion of $z = x^2 + y^2$ enclosed by C; $\vec{F} = \langle z, -x, y \rangle$.



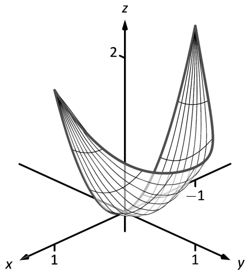
10. C is the curve parametrized by $\vec{r}(t) = \langle \cos t, \sin t, e^{-1} \rangle$ and $\mathcal{S}$ is the portion of $z = e^{-x^2-y^2}$ enclosed by C; $\vec{F} = \langle -y, x, 1 \rangle$.



11. C is the curve that follows the triangle with vertices at $(0,0,2)$, $(4,0,0)$ and $(0,3,0)$, traversing the the vertices in that order and returning to $(0,0,2)$, and $\mathcal{S}$ is the portion of the plane $z = 2 - x/2 - 2y/3$ enclosed by C; $\vec{F} = \langle y, -z, y \rangle$.

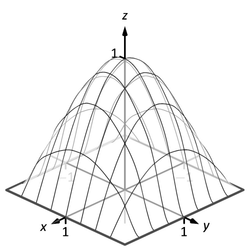


12. C is the curve whose x and y coordinates follow the parabola $y = 1 - x^2$ from $x = 1$ to $x = -1$, then follow the line from $(-1, 0)$ back to $(1, 0)$, where the z coordinates of C are determined by $f(x, y) = 2x^2 + y^2$, and $\mathcal{S}$ is the portion of $z = 2x^2 + y^2$ enclosed by C; $\vec{F} = \langle y^2 + z, x, x^2 - y\rangle$.

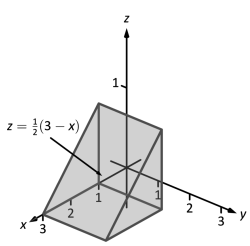


In Exercises 13 – 16, a closed surface $\mathcal{S}$ and a vector field $\vec{F}$ are given. Find the outward flux of $\vec{F}$ over $\mathcal{S}$ either through direct computation or through the Divergence Theorem.

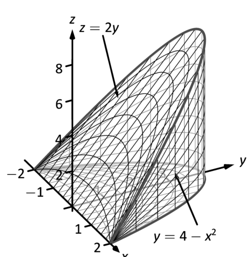
13. $\mathcal{S}$ is the surface formed by the intersections of $z = 0$ and $z = (x^2 - 1)(y^2 - 1)$; $\vec{F} = \langle x^2 + 1, yz, xz^2\rangle$.



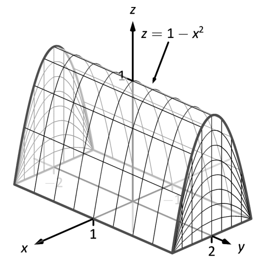
14. $\mathcal{S}$ is the surface formed by the intersections of the planes $z = \frac{1}{2}(3 - x)$, $x = 1$, $y = 0$, $y = 2$ and $z = 0$; $\vec{F} = \langle x, y^2, z\rangle$.



15. $\mathcal{S}$ is the surface formed by the intersections of the planes $z = 2y$, $y = 4 - x^2$ and $z = 0$; $\vec{F} = \langle xz, 0, xz\rangle$.

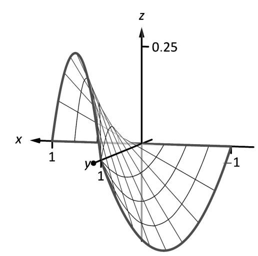


16. $\mathcal{S}$ is the surface formed by the intersections of the cylinder $z = 1 - x^2$ and the planes $y = -2$, $y = 2$ and $z = 0$; $\vec{F} = \langle 0, y^3, 0\rangle$.

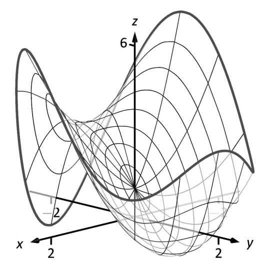


In Exercises 17 – 20, a closed curve C that is the boundary of a surface $\mathcal{S}$ is given along with a vector field $\vec{F}$. Find the circulation of $\vec{F}$ around C either through direct computation or through Stokes' Theorem.

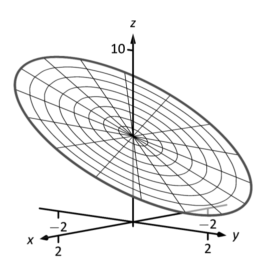
17. C is the curve whose x- and y-values are determined by the three sides of a triangle with vertices at $(-1, 0)$, $(1, 0)$ and $(0, 1)$, traversed in that order, and the z-values are determined by the function $z = xy$; $\vec{F} = \langle z - y^2, x, z\rangle$.



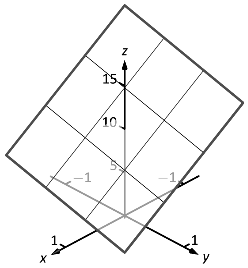
18. C is the curve whose x- and y-values are given by $\vec{r}(t) = \langle 2\cos t, 2\sin t\rangle$ and the z-values are determined by the function $z = x^2 + y^3 - 3y + 1$; $\vec{F} = \langle -y, x, z\rangle$.



19. C is the curve whose x- and y-values are given by $\vec{r}(t) = \langle \cos t, 3\sin t\rangle$ and the z-values are determined by the function $z = 5 - 2x - y$; $\vec{F} = \langle -\frac{1}{3}y, 3x, \frac{2}{3}y - 3x\rangle$.



20. C is the curve whose x- and y-values are sides of the square with vertices at $(1,1)$, $(-1,1)$, $(-1,-1)$ and $(1,-1)$, traversed in that order, and the z-values are determined by the function $z = 10 - 5x - 2y$; $\vec{F} = \langle 5y^2, 2y^2, y^2\rangle$.



Exercises 21 – 24 are designed to challenge your understanding and require no computation.

21. Let $\mathcal{S}$ be any closed surface enclosing a domain D. Consider $\vec{F}_1 = \langle x, 0, 0\rangle$ and $\vec{F}_2 = \langle y, y^2, z - 2yz\rangle$.
These fields are clearly very different. Why is it that the total outward flux of each field across $\mathcal{S}$ is the same?

22. (a) Green's Theorem can be used to find the area of a region enclosed by a curve by evaluating a line integral with the appropriate choice of vector field $\vec{F}$. What condition on $\vec{F}$ makes this possible?

    (b) Likewise, Stokes' Theorem can be used to find the surface area of a region enclosed by a curve in space by evaluating a line integral with the appropriate choice of vector field $\vec{F}$. What condition on $\vec{F}$ makes this possible?

23. The Divergence Theorem establishes equality between a particular double integral and a particular triple integral. What types of circumstances would lead one to choose to evaluate the triple integral over the double integral?

24. Stokes' Theorem establishes equality between a particular line integral and a particular double integral. What types of circumstances would lead one to choose to evaluate the double integral over the line integral?

A: Solutions To Selected Problems

1. Answers will vary.
2. An indeterminate form.
3. F
4. The function may approach different values from the left and right, the function may grow without bound, or the function might oscillate.
5. Answers will vary.
6. 1
7. -1
8. -5
9. Limit does not exist
10. 2
11. 1.5
12. Limit does not exist.
13. Limit does not exist.
14. 7
15. 1
16. Limit does not exist.
17. The limit seems to be exactly 7.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-7
-0.01	-7
0.01	-7
0.1	-7

18. The limit seems to be exactly 9.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	9
-0.01	9
0.01	9
0.1	9

19. The limit is approx. 5.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	4.9
-0.01	4.99
0.01	5.01
0.1	5.1

20. The limit is approx. -0.11.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.114943
-0.01	-0.111483
0.01	-0.110742
0.1	-0.107527

21. The limit is approx. 29.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	29.4
-0.01	29.04
0.01	28.96
0.1	28.6

22. The limit is approx. 0.2.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	0.202027
-0.01	0.2002
0.01	0.1998
0.1	0.198026

23. The limit is approx. -1.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.998334
-0.01	-0.999983
0.01	-0.999983
0.1	-0.998334

24. The limit is approx. 0.005.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.0499583
-0.01	-0.00499996
0.01	0.00499996
0.1	0.0499583

Section 1.2

1. ε should be given first, and the restriction $|x-a|<\delta$ implies $|f(x)-K|<\varepsilon$, not the other way around.
2. The y–tolerance.
3. T
4. T
5. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-4|<\delta$, $|f(x)-13|<\varepsilon$.
   Consider $|f(x)-13|<\varepsilon$:
   $$\begin{aligned} |f(x)-13| &< \varepsilon \\ |(2x+5)-13| &< \varepsilon \\ |2x-8| &< \varepsilon \\ 2|x-4| &< \varepsilon \\ -\varepsilon/2 < x-4 &< \varepsilon/2. \end{aligned}$$
   This implies we can let $\delta=\varepsilon/2$. Then:
   $$\begin{aligned} |x-4| &< \delta \\ -\delta < x-4 &< \delta \\ -\varepsilon/2 < x-4 &< \varepsilon/2 \\ -\varepsilon < 2x-8 &< \varepsilon \\ -\varepsilon < (2x+5)-13 &< \varepsilon \\ |(2x+5)-13| &< \varepsilon, \end{aligned}$$
   which is what we wanted to prove.
6. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-5|<\delta$, $|f(x)-(-2)|<\varepsilon$.
   Consider $|f(x)-(-2)|<\varepsilon$:
   $$\begin{aligned} |f(x)+2| &< \varepsilon \\ |(3-x)+2| &< \varepsilon \\ |5-x| &< \varepsilon \\ -\varepsilon < 5-x &< \varepsilon \\ -\varepsilon < x-5 &< \varepsilon. \end{aligned}$$
   This implies we can let $\delta=\varepsilon$. Then:
   $$\begin{aligned} |x-5| &< \delta \\ -\delta < x-5 &< \delta \\ -\varepsilon < x-5 &< \varepsilon \\ -\varepsilon < (x-3)-2 &< \varepsilon \\ -\varepsilon < (-x+3)-(-2) &< \varepsilon \\ |3-x-(-2)| &< \varepsilon, \end{aligned}$$
   which is what we wanted to prove.

7. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-3| < \delta$, $|f(x) - 6| < \varepsilon$.
   Consider $|f(x) - 6| < \varepsilon$, keeping in mind we want to make a statement about $|x-3|$:

$$\begin{aligned}|f(x) - 6| &< \varepsilon\\ |x^2 - 3 - 6| &< \varepsilon\\ |x^2 - 9| &< \varepsilon\\ |x-3| \cdot |x+3| &< \varepsilon\\ |x-3| &< \varepsilon/|x+3|\end{aligned}$$

   Since x is near 3, we can safely assume that, for instance, $2 < x < 4$. Thus

$$\begin{aligned}2+3 < x+3 &< 4+3\\ 5 < x+3 &< 7\\ \frac{1}{7} < \frac{1}{x+3} &< \frac{1}{5}\\ \frac{\varepsilon}{7} < \frac{\varepsilon}{x+3} &< \frac{\varepsilon}{5}\end{aligned}$$

   Let $\delta = \frac{\varepsilon}{7}$. Then:

$$\begin{aligned}|x-3| &< \delta\\ |x-3| &< \frac{\varepsilon}{7}\\ |x-3| &< \frac{\varepsilon}{x+3}\\ |x-3| \cdot |x+3| &< \frac{\varepsilon}{x+3} \cdot |x+3|\end{aligned}$$

   Assuming x is near 3, $x+3$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned}|x-3| \cdot |x+3| &< \frac{\varepsilon}{x+3} \cdot (x+3)\\ |x^2 - 9| &< \varepsilon\\ |(x^2 - 3) - 6| &< \varepsilon,\end{aligned}$$

   which is what we wanted to prove.

8. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-4| < \delta$, $|f(x) - 15| < \varepsilon$.
   Consider $|f(x) - 15| < \varepsilon$, keeping in mind we want to make a statement about $|x-4|$:

$$\begin{aligned}|f(x) - 15| &< \varepsilon\\ |x^2 + x - 5 - 15| &< \varepsilon\\ |x^2 + x - 20| &< \varepsilon\\ |x-4| \cdot |x+5| &< \varepsilon\\ |x-4| &< \varepsilon/|x+5|\end{aligned}$$

   Since x is near 4, we can safely assume that, for instance, $3 < x < 5$. Thus

$$\begin{aligned}3+5 < x+5 &< 5+5\\ 8 < x+5 &< 10\\ \frac{1}{10} < \frac{1}{x+5} &< \frac{1}{8}\\ \frac{\varepsilon}{10} < \frac{\varepsilon}{x+5} &< \frac{\varepsilon}{8}\end{aligned}$$

   Let $\delta = \frac{\varepsilon}{10}$. Then:

$$\begin{aligned}|x-4| &< \delta\\ |x-4| &< \frac{\varepsilon}{10}\\ |x-4| &< \frac{\varepsilon}{x+5}\\ |x-4| \cdot |x+5| &< \frac{\varepsilon}{x+5} \cdot |x+5|\end{aligned}$$

   Assuming x is near 4, $x+5$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned}|x-4| \cdot |x+5| &< \frac{\varepsilon}{x+5} \cdot (x+5)\\ |x^2 + x - 20| &< \varepsilon\\ |(x^2 + x - 5) - 15| &< \varepsilon,\end{aligned}$$

   which is what we wanted to prove.

9. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-1| < \delta$, $|f(x) - 6| < \varepsilon$.
   Consider $|f(x) - 6| < \varepsilon$, keeping in mind we want to make a statement about $|x-1|$:

$$\begin{aligned}|f(x) - 6| &< \varepsilon\\ |(2x^2 + 3x + 1) - 6| &< \varepsilon\\ |2x^2 + 3x - 5| &< \varepsilon\\ |2x+5| \cdot |x-1| &< \varepsilon\\ |x-1| &< \varepsilon/|2x+5|\end{aligned}$$

   Since x is near 1, we can safely assume that, for instance, $0 < x < 2$. Thus

$$\begin{aligned}0+5 < 2x+5 &< 4+5\\ 5 < 2x+5 &< 9\\ \frac{1}{9} < \frac{1}{2x+5} &< \frac{1}{5}\\ \frac{\varepsilon}{9} < \frac{\varepsilon}{2x+5} &< \frac{\varepsilon}{5}\end{aligned}$$

   Let $\delta = \frac{\varepsilon}{9}$. Then:

$$\begin{aligned}|x-1| &< \delta\\ |x-1| &< \frac{\varepsilon}{9}\\ |x-1| &< \frac{\varepsilon}{2x+5}\\ |x-1| \cdot |2x+5| &< \frac{\varepsilon}{2x+5} \cdot |2x+5|\end{aligned}$$

   Assuming x is near 1, $2x+5$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned}|x-1| \cdot |2x+5| &< \frac{\varepsilon}{2x+5} \cdot (2x+5)\\ |2x^2 + 3x - 5| &< \varepsilon\\ |(2x^2 + 3x + 1) - 6| &< \varepsilon,\end{aligned}$$

   which is what we wanted to prove.

10. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x) - 7| < \varepsilon$.

Consider $|f(x) - 7| < \varepsilon$, keeping in mind we want to make a statement about $|x - 2|$:

$$|f(x) - 7| < \varepsilon$$
$$|x^3 - 1 - 7| < \varepsilon$$
$$|x^3 - 8| < \varepsilon$$
$$|x - 2| \cdot |x^2 + 2x + 4| < \varepsilon$$
$$|x - 3| < \varepsilon/|x^2 + 2x + 4|$$

Since x is near 2, we can safely assume that, for instance, $1 < x < 3$. Thus

$$1^2 + 2 \cdot 1 + 4 < x^2 + 2x + 4 < 3^2 + 2 \cdot 3 + 4$$
$$7 < x^2 + 2x + 4 < 19$$
$$\frac{1}{19} < \frac{1}{x^2 + 2x + 4} < \frac{1}{7}$$
$$\frac{\varepsilon}{19} < \frac{\varepsilon}{x^2 + 2x + 4} < \frac{\varepsilon}{7}$$

Let $\delta = \frac{\varepsilon}{19}$. Then:

$$|x - 2| < \delta$$
$$|x - 2| < \frac{\varepsilon}{19}$$
$$|x - 2| < \frac{\varepsilon}{x^2 + 2x + 4}$$
$$|x - 2| \cdot |x^2 + 2x + 4| < \frac{\varepsilon}{x^2 + 2x + 4} \cdot |x^2 + 2x + 4|$$

Assuming x is near 2, $x^2 + 2x + 4$ is positive and we can drop the absolute value signs on the right.

$$|x - 2| \cdot |x^2 + 2x + 4| < \frac{\varepsilon}{x^2 + 2x + 4} \cdot (x^2 + 2x + 4)$$
$$|x^3 - 8| < \varepsilon$$
$$|(x^3 - 1) - 7| < \varepsilon,$$

which is what we wanted to prove.

11. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x - 2| < \delta$, $|f(x) - 5| < \varepsilon$. However, since $f(x) = 5$, a constant function, the latter inequality is simply $|5 - 5| < \varepsilon$, which is always true. Thus we can choose any δ we like; we arbitrarily choose $\delta = \varepsilon$.

12. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x - 0| < \delta$, $|f(x) - 0| < \varepsilon$.

    Consider $|f(x) - 0| < \varepsilon$, keeping in mind we want to make a statement about $|x - 0|$ (i.e., $|x|$):

$$|f(x) - 0| < \varepsilon$$
$$|e^{2x} - 1| < \varepsilon$$
$$-\varepsilon < e^{2x} - 1 < \varepsilon$$
$$1 - \varepsilon < e^{2x} < 1 + \varepsilon$$
$$\ln(1 - \varepsilon) < 2x < \ln(1 + \varepsilon)$$
$$\frac{\ln(1 - \varepsilon)}{2} < x < \frac{\ln(1 + \varepsilon)}{2}$$

Let $\delta = \min\left\{\left|\frac{\ln(1-\varepsilon)}{2}\right|, \frac{\ln(1+\varepsilon)}{2}\right\} = \frac{\ln(1+\varepsilon)}{2}$.

Thus:

$$|x| < \delta$$
$$|x| < \frac{\ln(1 + \varepsilon)}{2} < \left|\frac{\ln(1 - \varepsilon)}{2}\right|$$
$$\frac{\ln(1 - \varepsilon)}{2} < x < \frac{\ln(1 + \varepsilon)}{2}$$
$$\ln(1 - \varepsilon) < 2x < \ln(1 + \varepsilon)$$
$$1 - \varepsilon < e^{2x} < 1 + \varepsilon$$
$$-\varepsilon < e^{2x} - 1 < \varepsilon$$
$$|e^{2x} - 1 - (0)| < \varepsilon,$$

which is what we wanted to prove.

13. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x - 1| < \delta$, $|f(x) - 1| < \varepsilon$.

    Consider $|f(x) - 1| < \varepsilon$, keeping in mind we want to make a statement about $|x - 1|$:

$$|f(x) - 1| < \varepsilon$$
$$|1/x - 1| < \varepsilon$$
$$|(1 - x)/x| < \varepsilon$$
$$|x - 1|/|x| < \varepsilon$$
$$|x - 1| < \varepsilon \cdot |x|$$

Since x is near 1, we can safely assume that, for instance, $1/2 < x < 3/2$. Thus $\varepsilon/2 < \varepsilon \cdot x$.

Let $\delta = \frac{\varepsilon}{2}$. Then:

$$|x - 1| < \delta$$
$$|x - 1| < \frac{\varepsilon}{2}$$
$$|x - 1| < \varepsilon \cdot x$$
$$|x - 1|/x < \varepsilon$$

Assuming x is near 1, x is positive and we can bring it into the absolute value signs on the left.

$$|(x - 1)/x| < \varepsilon$$
$$|1 - 1/x| < \varepsilon$$
$$|(1/x) - 1| < \varepsilon,$$

which is what we wanted to prove.

14. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x - 0| < \delta$, $|f(x) - 0| < \varepsilon$. In simpler terms, we want to show that when $|x| < \delta$, $|\sin x| < \varepsilon$.

    Set $\delta = \varepsilon$. We start with assuming that $|x| < \delta$. Using the hint, we have that $|\sin x| < |x| < \delta = \varepsilon$. Hence if $|x| < \delta$, we know immediately that $|\sin x| < \varepsilon$.

Section 1.3

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. As x is near 1, both f and g are near 0, but f is approximately twice the size of g. (I.e., $f(x) \approx 2g(x)$.)
6. T; by Theorem 1.3.3, $\lim_{x\to 1} \ln x = \ln 1 = 0$.
7. 9
8. 6

9. 0
10. Limit does not exist.
11. 3
12. Not possible to know; as x approaches 6, $g(x)$ approaches 3, but we know nothing of the behavior of $f(x)$ when x is near 3.
13. 3
14. -45
15. 1
16. -1
17. 0
18. π
19. 7
20. $-0.000000015 \approx 0$
21. $1/2$
22. 0
23. Limit does not exist
24. 64
25. 2
26. 0
27. $\frac{\pi^2+3\pi+5}{5\pi^2-2\pi-3} \approx 0.6064$
28. $\frac{3\pi+1}{1-\pi}$
29. -8
30. -1
31. 10
32. -2
33. $-3/2$
34. $-7/8$
35. 0
36. 0
37. 1
38. 9
39. 3
40. $5/8$
41. 1
42. $\pi/180$
43. (a) Apply Part 1 of Theorem 1.3.1.
    (b) Apply Theorem 1.3.6; $g(x) = \frac{x}{x}$ is the same as $g(x) = 1$ everywhere except at $x = 0$. Thus $\lim\limits_{x\to 0} g(x) = \lim\limits_{x\to 0} 1 = 1$.
    (c) The function $f(x)$ is always 0, so $g\big(f(x)\big)$ is never defined as $g(x)$ is not defined at $x = 0$. Therefore the limit does not exist.
    (d) The Composition Rule requires that $\lim\limits_{x\to 0} g(x)$ be equal to $g(0)$. They are not equal, so the conditions of the Composition Rule are not satisfied, and hence the rule is not violated.

Section 1.4

1. The function approaches different values from the left and right; the function grows without bound; the function oscillates.
2. F
3. F
4. T
5. (a) 2
   (b) 2
   (c) 2
   (d) 1
   (e) As f is not defined for $x < 0$, this limit is not defined.
   (f) 1
6. (a) 1
   (b) 2
   (c) Does not exist.
   (d) 2
   (e) 0
   (f) As f is not defined for $x > 2$, this limit is not defined.
7. (a) Does not exist.
   (b) Does not exist.
   (c) Does not exist.
   (d) Not defined.
   (e) 0
   (f) 0
8. (a) 2
   (b) 0
   (c) Does not exist.
   (d) 1
9. (a) 2
   (b) 2
   (c) 2
   (d) 2
10. (a) 4
    (b) -4
    (c) Does not exist.
    (d) 0
11. (a) 2
    (b) 2
    (c) 2
    (d) 0
    (e) 2
    (f) 2
    (g) 2
    (h) Not defined
12. (a) $a - 1$
    (b) a
    (c) Does not exist.
    (d) a
13. (a) 2
    (b) -4
    (c) Does not exist.
    (d) 2
14. (a) -1
    (b) 0
    (c) Does not exist.

(d) 0

15. (a) 0
    (b) 0
    (c) 0
    (d) 0
    (e) 2
    (f) 2
    (g) 2
    (h) 2

16. (a) -1
    (b) 0
    (c) Does not exist.
    (d) 0

17. (a) $1 - \cos^2 a = \sin^2 a$
    (b) $\sin^2 a$
    (c) $\sin^2 a$
    (d) $\sin^2 a$

18. (a) 2
    (b) 0
    (c) Does not exist
    (d) 1

19. (a) 4
    (b) 4
    (c) 4
    (d) 3

20. (a) c
    (b) c
    (c) c
    (d) c

21. (a) -1
    (b) 1
    (c) Does not exist
    (d) 0

22. $-3/5$
23. $2/3$
24. 2.5
25. -9
26. -1.63

Section 1.5

1. Answers will vary.
2. Answers will vary.
3. A root of a function f is a value c such that $f(c) = 0$.
4. Consider the function $h(x) = g(x) - f(x)$, and use the Bisection Method to find a root of h.
5. F
6. T
7. T
8. F
9. F
10. T
11. No; $\lim\limits_{x\to 1} f(x) = 2$, while $f(1) = 1$.
12. No; $\lim\limits_{x\to 1} f(x)$ does not exist.
13. No; $f(1)$ does not exist.
14. Yes
15. Yes
16. Yes
17. (a) No; $\lim\limits_{x\to -2} f(x) \neq f(-2)$
    (b) Yes
    (c) No; $f(2)$ is not defined.
18. Yes; $\lim\limits_{x\to 3\pi/2} \sin x + 1 = 0$, and $\sin(3\pi/2) + 1 = 0$.
19. (a) Yes
    (b) Yes
20. (a) Yes
    (b) No; the left and right hand limits at 1 are not equal.
21. (a) Yes
    (b) Yes
22. (a) Yes
    (b) No. $\lim_{x\to 8} f(x) = 16/5 \neq f(8) = 5$.
23. $(-\infty, \infty)$
24. $(-\infty, -2]$ and $[2, \infty)$
25. $[-2, 2]$
26. $[-1, 1]$
27. $(-\infty, -\sqrt{6}]$ and $[\sqrt{6}, \infty)$
28. $(-1, 1)$
29. $(-\infty, \infty)$
30. $(-\infty, \infty)$
31. $(0, \infty)$
32. $(-\infty, \infty)$
33. $(-\infty, 0]$
34. $(-\infty, \infty)$
35. Yes, by the Intermediate Value Theorem.
36. Yes, by the Intermediate Value Theorem. In fact, we can be more specific and state such a value c exists in $(0, 2)$, not just in $(-3, 7)$.
37. We cannot say; the Intermediate Value Theorem only applies to function values between -10 and 10; as 11 is outside this range, we do not know.
38. We cannot say; the Intermediate Value Theorem only applies to continuous functions. As we do know know if h is continuous, we cannot say.
39. Approximate root is $x = 1.23$. The intervals used are:
    $[1, 1.5]$ $[1, 1.25]$ $[1.125, 1.25]$
    $[1.1875, 1.25]$ $[1.21875, 1.25]$ $[1.234375, 1.25]$
    $[1.234375, 1.2421875]$ $[1.234375, 1.2382813]$
40. Approximate root is $x = 0.52$. The intervals used are:
    $[0.5, 0.55]$ $[0.5, 0.525]$ $[0.5125, 0.525]$
    $[0.51875, 0.525]$ $[0.521875, 0.525]$
41. Approximate root is $x = 0.69$. The intervals used are:
    $[0.65, 0.7]$ $[0.675, 0.7]$ $[0.6875, 0.7]$
    $[0.6875, 0.69375]$ $[0.690625, 0.69375]$

42. Approximate root is $x = 0.78$. The intervals used are:
$[0.7, 0.8]$ $[0.75, 0.8]$ $[0.775, 0.8]$
$[0.775, 0.7875]$ $[0.78125, 0.7875]$
(A few more steps show that 0.79 is better as the root is $\pi/4 \approx 0.78539$.)

43. (a) 20
(b) 25
(c) Limit does not exist
(d) 25

44.

x	$f(x)$
-0.81	-2.34129
-0.801	-2.33413
-0.79	-2.32542
-0.799	-2.33254

The top two lines give an approximation of the limit from the left: -2.33. The bottom two lines give an approximation from the right: -2.33 as well.

45. Answers will vary.

Section 1.6

1. F
2. T
3. F
4. T
5. T
6. Answers will vary.
7. Answers will vary.
8. The limit of f as x approaches 7 does not exist, hence f is not continuous. (Note: f could be defined at 7!)
9. (a) ∞
(b) ∞
10. (a) $-\infty$
(b) ∞
(c) Limit does not exist
(d) ∞
(e) ∞
(f) ∞
11. (a) 1
(b) 0
(c) $1/2$
(d) $1/2$
12. (a) Limit does not exist
(b) Limit does not exist
13. (a) Limit does not exist
(b) Limit does not exist
14. (a) 10
(b) ∞
15. Tables will vary.

(a)

x	$f(x)$
2.9	-15.1224
2.99	-159.12
2.999	-1599.12

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

x	$f(x)$
3.1	16.8824
3.01	160.88
3.001	1600.88

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x)$ does not exist.

16. Tables will vary.

(a)

x	$f(x)$
2.9	-335.64
2.99	-30350.6

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

x	$f(x)$
3.1	-265.61
3.01	-29650.6

It seems $\lim_{x\to 3^+} f(x) = -\infty$.

(c) It seems $\lim_{x\to 3} f(x) = -\infty$.

17. Tables will vary.

(a)

x	$f(x)$
2.9	132.857
2.99	12124.4

It seems $\lim_{x\to 3^-} f(x) = \infty$.

(b)

x	$f(x)$
3.1	108.039
3.01	11876.4

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x) = \infty$.

18. Tables will vary.

(a)

x	$f(x)$
2.9	-0.632
2.99	-0.6032
2.999	-0.60032

It seems $\lim_{x\to 3^-} f(x) = -0.6$.

(b)

x	$f(x)$
3.1	-0.5686
3.01	-0.5968
3.001	-0.59968

It seems $\lim_{x\to 3^+} f(x) = -0.6$.

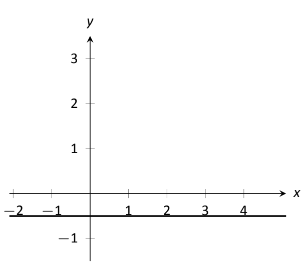
(c) It seems $\lim_{x\to 3} f(x) = -0.6$.

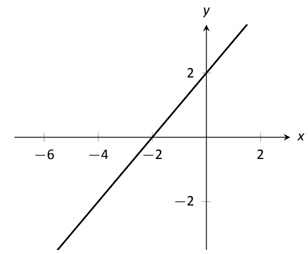
19. Horizontal asymptote at $y = 2$; vertical asymptotes at $x = -5, 4$.
20. Horizontal asymptote at $y = -3/5$; vertical asymptote at $x = 3$.
21. Horizontal asymptote at $y = 0$; vertical asymptotes at $x = -1, 0$.
22. No horizontal asymptote; vertical asymptote at $x = 1$.
23. No horizontal or vertical asymptotes.
24. Horizontal asymptote at $y = -1$; no vertical asymptotes
25. ∞
26. $-\infty$
27. $-\infty$
28. ∞
29. Solution omitted.
30. (a) 2
(b) -3
(c) -3
(d) $1/3$
31. Yes. The only "questionable" place is at $x = 3$, but the left and right limits agree.
32. 1

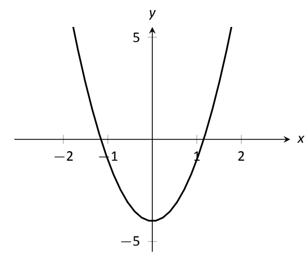
Chapter 2

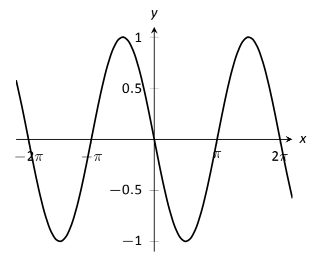
Section 2.1

1. T
2. T
3. Answers will vary.

4. Answers will vary.
5. Answers will vary.
6. The two lines have opposite–reciprocal slopes.
7. $f'(x) = 0$
8. $f'(x) = 2$
9. $f'(t) = -3$
10. $g'(x) = 2x$
11. $h'(x) = 3x^2$
12. $f''(x) = 6x - 1$
13. $r'(x) = \frac{-1}{x^2}$
14. $r'(s) = \frac{-1}{(s-2)^2}$
15. (a) $y = 6$
    (b) $x = -2$
16. (a) $y = 2x$
    (b) $y = -1/2x$
17. (a) $y = -3x + 4$
    (b) $y = 1/3(x - 7) - 17$
18. (a) $y = 4(x - 2) + 4$
    (b) $y = -1/4(x - 2) + 4$
19. (a) $y = 48(x - 4) + 64$
    (b) $y = -\frac{1}{48}(x - 4) + 64$
20. (a) $y = -7(x + 1) + 8$
    (b) $y = 1/7(x + 1) + 8$
21. (a) $y = -1/4(x + 2) - 1/2$
    (b) $y = 4(x + 2) - 1/2$
22. (a) $y = -1(x - 3) + 1$
    (b) $y = 1(x - 3) + 1$
23. $y = 8.1(x - 3) + 16$
24. $y = -0.099(x - 9) + 1$
25. $y = 7.77(x - 2) + e^2$, or $y = 7.77(x - 2) + 7.39$.
26. $y = -0.05x + 1$
27. (a) Approximations will vary; they should match (c) closely.
    (b) $f'(x) = 2x$
    (c) At $(-1, 0)$, slope is -2. At $(0, -1)$, slope is 0. At $(2, 3)$, slope is 4.
28. (a) Approximations will vary; they should match (c) closely.
    (b) $f'(x) = -1/(x + 1)^2$
    (c) At $(0, 1)$, slope is -1. At $(1, 0.5)$, slope is $-1/4$.
29. 

30. 

31. 

32. 

33. (a) Approximately on $(-2, 0)$ and $(2, \infty)$.
    (b) Approximately on $(-\infty, -2)$ and $(0, 2)$.
    (c) Approximately at $x = 0, \pm 2$.
    (d) Approximately on $(-\infty, -1)$ and $(1, \infty)$.
    (e) Approximately on $(-1, 1)$.
    (f) Approximately at $x = \pm 1$.
34. (a) Approximately on $(-1.5, 1.5)$.
    (b) Approximately on $(-\infty, -1.5)$ and $(1.5, \infty)$.
    (c) Approximately at $x = \pm 1.5$.
    (d) On $(-\infty, -1)$ and $(0, 1)$.
    (e) On $(-1, 0)$ and $(1, \infty)$.
    (f) At $x = \pm 1$.
35. $\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = 0$; note also that $\lim_{x\to 0^+} f'(x) = 0$. So f is differentiable at $x = 0$.
    $\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = -\infty$; note also that $\lim_{x\to 1^-} f'(x) = -\infty$. So f is not differentiable at $x = 1$.
    f is differentiable on $[0, 1)$, not its entire domain.
36. The limit of the difference quotient is difficult to evaluate. Using Theorem 1.3.5, we can determine $\lim_{x\to 0^+} f'(x) = -1/2$.
    Since f' is defined on $(0, \infty)$, we conclude f is differentiable on $[0, \infty)$.
37. Approximately 24.
38. Approximately 0.54.
39. (a) $(-\infty, \infty)$
    (b) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
    (c) $(-\infty, 5]$
    (d) $[-\sqrt{5}, \sqrt{5}]$
40. (a) 1
    (b) 3
    (c) Does not exist
    (d) $(-\infty, -3) \cup (3, \infty)$

Section 2.2

1. Velocity
2. Answers will vary.
3. Linear functions.
4. 12
5. -17
6. 102
7. $f(10.1)$ is likely most accurate, as accuracy is lost the farther from $x = 10$ we go.
8. -4
9. 6
10. decibels per person
11. ft/s^2
12. ft/h
13. (a) thousands of dollars per car
    (b) It is likely that $P(0) < 0$. That is, negative profit for not producing any cars.
14. (a) degrees Fahrenheit per hour
    (b) It is likely that $T'(8) > 0$ since at 8 in the morning, the temperature is likely rising.
    (c) It is very likely that $T(8) > 0$, as at 8 in the morning on July 4, we would expect the temperature to be well above 0.
15. $f(x) = g'(x)$
16. $g(x) = f'(x)$
17. Either $g(x) = f'(x)$ or $f(x) = g'(x)$ is acceptable. The actual answer is $g(x) = f'(x)$, but is very hard to show that $f(x) \neq g'(x)$ given the level of detail given in the graph.
18. $g(x) = f'(x)$
19. $f'(x) = 10x$
20. $f'(x) = 3x^2 - 12x + 12$
21. $f'(\pi) \approx 0.$
22. $f'(9) \approx 0.1667.$

Section 2.3

1. Power Rule.
2. $1/x$
3. One answer is $f(x) = 10e^x$.
4. One answer is $f(x) = 10$.
5. $g(x)$ and $h(x)$
6. Answers will vary.
7. One possible answer is $f(x) = 17x - 205$.
8. Answers will vary.
9. $f'(x)$ is a velocity function, and $f''(x)$ is acceleration.
10. lbs/ft^2.
11. $f'(x) = 14x - 5$
12. $g'(x) = 42x^2 + 14x + 11$
13. $m'(t) = 45t^4 - \frac{3}{8}t^2 + 3$
14. $f'(\theta) = 9\cos\theta - 10\sin\theta$
15. $f'(r) = 6e^r$
16. $g'(t) = 40t^3 + \sin t + 7\cos t$
17. $f'(x) = \frac{2}{x} - 1$
18. $p'(s) = s^3 + s^2 + s + 1$
19. $h'(t) = e^t - \cos t + \sin t$
20. $f'(x) = \frac{2}{x}$
21. $f'(t) = 0$
22. $g'(t) = 18t + 6$
23. $g'(x) = 24x^2 - 120x + 150$
24. $f'(x) = -3x^2 + 6x - 3$
25. $f'(x) = 18x - 12$
26. (a) $\log_{10} x = \frac{\ln x}{\ln 10}$.
    (b) $\frac{d}{dx}\left(\log_{10} x\right) = \frac{d}{dx}\left(\frac{1}{\ln 10}\ln x\right) = \frac{1}{\ln 10}\frac{1}{x}$.
    (c) $\frac{d}{dx}\left(\log_a x\right) = \frac{1}{\ln a}\frac{1}{x}$
27. $f'(x) = 6x^5$ $f''(x) = 30x^4$ $f'''(x) = 120x^3$ $f^{(4)}(x) = 360x^2$
28. $g'(x) = -2\sin x$ $g''(x) = -2\cos x$ $g'''(x) = 2\sin x$ $g^{(4)}(x) = 2\cos x$
29. $h'(t) = 2t - e^t$ $h''(t) = 2 - e^t$ $h'''(t) = -e^t$ $h^{(4)}(t) = -e^t$
30. $p'(\theta) = 4\theta^3 - 3\theta^2$ $p''(\theta) = 12\theta^2 - 6\theta$ $p'''(\theta) = 24\theta - 6$ $p^{(4)}(\theta) = 24$
31. $f'(\theta) = \cos\theta + \sin\theta$ $f''(\theta) = -\sin\theta + \cos\theta$ $f'''(\theta) = -\cos\theta - \sin\theta$ $f^{(4)}(\theta) = \sin\theta - \cos\theta$
32. $f'(x) = f''(x) = f'''(x) = f^{(4)}(x) = 0$
33. Tangent line: $y = 2(x-1)$
    Normal line: $y = -1/2(x-1)$
34. Tangent line: $y = t + 4$
    Normal line: $y = -t + 4$
35. Tangent line: $y = x - 1$
    Normal line: $y = -x + 1$
36. Tangent line: $y = 4$
    Normal line: $x = \pi/2$
37. Tangent line: $y = \sqrt{2}(x - \frac{\pi}{4}) - \sqrt{2}$
    Normal line: $y = \frac{-1}{\sqrt{2}}(x - \frac{\pi}{4}) - \sqrt{2}$
38. Tangent line: $y = 2x + 3$
    Normal line: $y = -1/2(x-5) + 13$
39. The tangent line to $f(x) = e^x$ at $x = 0$ is $y = x + 1$; thus $e^{0.1} \approx y(0.1) = 1.1$.

Section 2.4

1. F
2. F
3. T
4. Quotient Rule
5. F
6. Answers will vary.
7. (a) $f'(x) = (x^2 + 3x) + x(2x + 3)$
   (b) $f'(x) = 3x^2 + 6x$
   (c) They are equal.
8. (a) $g'(x) = 4x(5x^3) + 2x^2(15x^2)$
   (b) $g'(x) = 50x^4$
   (c) They are equal.
9. (a) $h'(s) = 2(s + 4) + (2s - 1)(1)$
   (b) $h'(s) = 4s + 7$
   (c) They are equal.

10. (a) $f'(x) = 2x(3 - x^3) + (x^2 + 5)(-3x^2)$
(b) $f'(x) = -5x^4 - 15x^2 + 6x$
(c) They are equal.

11. (a) $f'(x) = \frac{x(2x)-(x^2+3)1}{x^2}$
(b) $f'(x) = 1 - \frac{3}{x^2}$
(c) They are equal.

12. (a) $g'(x) = \frac{2x^2(3x^2-4x)-(x^3-2x^2)(4x)}{4x^4}$
(b) $g'(x) = 1/2$
(c) They are equal.

13. (a) $h'(s) = \frac{4s^3(0)-3(12s^2)}{16s^6}$
(b) $h'(s) = -9/4s^{-4}$
(c) They are equal.

14. (a) $f'(t) = \frac{(t+1)(2t)-(t^2-1)(1)}{(t+1)^2}$
(b) $f(t) = t - 1$ when $t \neq -1$, so $f'(t) = 1$.
(c) They are equal.

15. $f'(x) = \sin x + x \cos x$

16. $f'(x) = 2x \cos x - x^2 \sin x$

17. $f'(x) = e^x \ln x + e^x \frac{1}{x}$

18. $f'(t) = \frac{-2}{t^3}(\csc t - 4) + \frac{1}{t^2}(-\csc t \cot t)$

19. $g'(x) = \frac{-12}{(x-5)^2}$

20. $g'(t) = \frac{(\cos t-2t^2)(5t^4)-(t^5)(-\sin t-4t)}{(\cos t-2t^2)^2}$

21. $h'(x) = -\csc^2 x - e^x$

22. $f'(x) = \left(\sec^2 x\right) \ln x + \left(\tan x\right) \frac{1}{x}$

23. $h'(t) = 14t + 6$

24. (a) $f'(x) = \frac{(x+2)(4x^3+6x^2)-(x^4+2x^3)(1)}{(x+2)^2}$
(b) $f(x) = x^3$ when $x \neq -2$, so $f'(x) = 3x^2$.
(c) They are equal.

25. $f'(x) = \left(6x + 8\right)e^x + \left(3x^2 + 8x + 7\right)e^x$

26. $g'(t) = \frac{e^t(5t^4-3t^2)-(t^5-t^3)e^t}{(e^t)^2}$

27. $f'(x) = 7$

28. $f'(t) = 5t^4(\sec t + e^t) + t^5(\sec t \tan t + e^t)$

29. $f'(x) = \frac{\sin^2(x)+\cos^2(x)+3\cos(x)}{(\cos(x)+3)^2}$

30. $f'(\theta) = 3\theta^2 \sin\theta + \theta^3 \cos\theta + \frac{\theta^3 \cos\theta-(\sin\theta)(3\theta^2)}{\theta^6}$

31. $f'(x) = \frac{-x\sin x-\cos x}{x^2} + \frac{\tan x-x\sec^2 x}{\tan^2 x}$

32. $g'(x) = 0$

33. $g'(t) = 12t^2e^t + 4t^3e^t - \cos^2 t + \sin^2 t$

34. $f'(x) = \frac{(t^2\cos t+2)(2t\sin t+t^2\cos t)-(t^2\sin t+3)(2t\cos t-t^2\sin t)}{(t^2\cos t+2)^2}$

35. $f'(x) = 2xe^x \tan x = x^2e^x \tan x + x^2e^x \sec^2 x$

36. $g'(x) = 2\sin x \sec x + 2x \cos x \sec x + 2x \sin x \sec x \tan x = 2\tan x + 2x + 2x\tan^2 x = 2\tan x + 2x\sec^2 x$

37. Tangent line: $y = 2x + 2$
Normal line: $y = -1/2x + 2$

38. Tangent line: $y = -(x - \frac{3\pi}{2}) - \frac{3\pi}{2} = -x$
Normal line: $y = (x - \frac{3\pi}{2}) - \frac{3\pi}{2} = x - 3\pi$

39. Tangent line: $y = 4$
Normal line: $x = 2$

40. Tangent line: $y = -9x + 1$
Normal line: $y = 1/9x + 1$

41. $x = 3/2$

42. $x = 0$

43. $f'(x)$ is never 0.

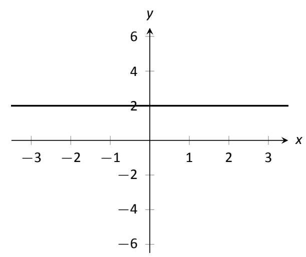
44. $x = -2, 0$

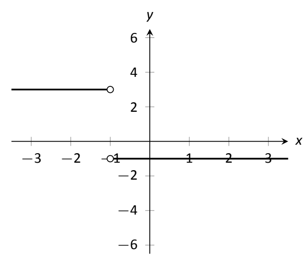
45. $f''(x) = 2\cos x - x\sin x$

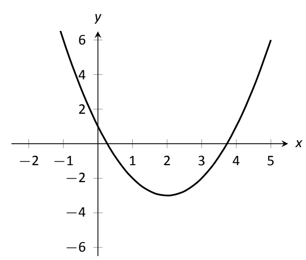
46. $f^{(4)}(x) = -4\cos x + x\sin x$

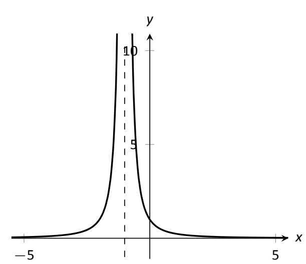
47. $f''(x) = \cot^2 x \csc x + \csc^3 x$

48. $f^{(8)} = 0$

49. 

50. 

51. 

52. 

Section 2.5

1. T

2. F

3. F

4. T

5. T

6. $f'(x) = 3\left(\ln x + x^2\right)2\left(\frac{1}{x} + 2x\right)$

7. $f'(x) = 10(4x^3 - x)^9 \cdot (12x^2 - 1) = (120x^2 - 10)(4x^3 - x)^9$
8. $f'(t) = 15(3t - 2)^4$
9. $g'(\theta) = 3(\sin\theta + \cos\theta)^2(\cos\theta - \sin\theta)$
10. $h'(t) = (6t + 1)e^{3t^2+t-1}$
11. $f'(x) = 3\big(\ln x + x^2\big)2(\frac{1}{x} + 2x)$
12. $f'(x) = (\ln 2)(2^{x^3+3x})(3x^2 + 3)$
13. $f'(x) = 4\big(x + \frac{1}{x}\big)^3\big(1 - \frac{1}{x^2}\big)$
14. $f'(x) = -3\sin(3x)$
15. $g'(x) = 5\sec^2(5x)$
16. $h'(\theta) = \sec^2\big(\theta^2 + 4\theta\big)(2\theta + 4)$
17. $g'(t) = \cos\big(t^5 + \frac{1}{t}\big)\left(5t^4 - \frac{1}{t^3}\right)$
18. $h'(t) = 8\sin^3(2t)\cos(2t)$
19. $p'(t) = -3\cos^2(t^2 + 3t + 1)\sin(t^2 + 3t + 1)(2t + 3)$
20. $f'(x) = -\tan x$
21. $f'(x) = 2/x$
22. $f'(x) = 2/x$
23. $g'(r) = \ln 4 \cdot 4^r$
24. $g'(t) = -\ln 5 \cdot 5^{\cos t}\sin t$
25. $g'(t) = 0$
26. $m'(w) = \ln(3/2)(3/2)^w$
27. $f'(x) = \frac{(3^t+2)\left((\ln 2)2^t\right) - (2^t+3)\left((\ln 3)3^t\right)}{(3^t+2)^2}$
28. $m'(w) = \frac{2^w\left(\ln 3 \cdot 3^w - \ln 2 \cdot (3^w+1)\right)}{2^{2w}}$
29. $f'(x) = \frac{2^{x^2}(\ln 3 \cdot 3^{x^2} 2x+1) - (3^{x^2}+x)(\ln 2 \cdot 2^{x^2} 2x)}{2^{2x^2}}$
30. $f'(x) = 5x^2\cos(5x) + 2x\sin(5x)$
31. $f'(x) =$ $5(x^2+x)^4(2x+1)(3x^4+2x)^3 + 3(x^2+x)^5(3x^4+2x)^2(12x^3+2)$
32. $g'(t) = 5\cos(t^2+3t)\cos(5t-7) - (2t+3)\sin(t^2+3t)\sin(5t-7)$
33. $f'(x) = 3\cos(3x+4)\cos(5-2x) + 2\sin(3x+4)\sin(5-2x)$
34. $g'(t) = 10t\cos(\frac{1}{t})e^{5t^2} + \frac{1}{t^2}\sin(\frac{1}{t})e^{5t^2}$
35. $f'(x) = \frac{4(5x-9)^3\cos(4x+1) - 15\sin(4x+1)(5x-9)^2}{(5x-9)^6}$
36. $f'(x) = \frac{8\tan(5x)(4x+1) - 5(4x+1)^2\sec^2(5x)}{\tan^2(5x)}$
37. Tangent line: $y = 0$
    Normal line: $x = 0$
38. Tangent line: $y = 15(t-1) + 1$
    Normal line: $y = -1/15(t-1) + 1$
39. Tangent line: $y = -3(\theta - \pi/2) + 1$
    Normal line: $y = 1/3(\theta - \pi/2) + 1$
40. Tangent line: $y = -5e(t+1) + e$
    Normal line: $y = 1/(5e)(t+1) + e$
41. In both cases the derivative is the same: $1/x$.
42. In both cases the derivative is the same: k/x.
43. (a) $^\circ$ F/mph
    (b) The sign would be negative; when the wind is blowing at 10 mph, any increase in wind speed will make it feel colder, i.e., a lower number on the Fahrenheit scale.
44. (a) $2xe^x\cot x + x^2e^x\cot x - x^2e^x\csc^2 x$
    (b) $\ln(48)48^x$

Section 2.6

1. Answers will vary.
2. The Chain Rule.
3. T
4. T
5. $f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$
6. $f'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-1/3} = \frac{1}{3\sqrt[3]{x^2}} + \frac{2}{3\sqrt[3]{x}}$
7. $f'(t) = \frac{-t}{\sqrt{1-t^2}}$
8. $g'(t) = \sqrt{t}\cos t + \frac{\sin t}{2\sqrt{t}}$
9. $h'(x) = 1.5x^{0.5} = 1.5\sqrt{x}$
10. $f'(x) = \pi x^{\pi-1} + 1.9x^{0.9}$
11. $g'(x) = \frac{\sqrt{x}(1)-(x+7)(1/2x^{-1/2})}{x} = \frac{1}{2\sqrt{x}} - \frac{7}{2\sqrt{x^3}}$
12. $f'(t) = \frac{1}{5}x^{-4/5}(\sec t + e^t) + \sqrt[5]{t}(\sec t\tan t + e^t)$
13. $\frac{dy}{dx} = \frac{-4x^3}{2y+1}$
14. $\frac{dy}{dx} = -\frac{y^{3/5}}{x^{3/5}}$
15. $\frac{dy}{dx} = \sin(x)\sec(y)$
16. $\frac{dy}{dx} = \frac{y}{x}$
17. $\frac{dy}{dx} = \frac{y}{x}$
18. $\frac{dy}{dx} = -\frac{e^x x(x+2)2^{-y}}{\ln|2|}$
19. $\frac{dy}{dx} = -\frac{2\sin(y)\cos(y)}{x}$
20. $\frac{dy}{dx} = -\frac{x}{y^2}$
21. $\frac{dy}{dx} = \frac{1}{2y+2}$
22. If one takes the derivative of the equation, as shown, using the Quotient Rule, one finds $\frac{dy}{dx} = \frac{x^2+2xy^2-y}{2x^2y-x+y^2}$.
    If one first clears the denominator and writes $x^2 + y = 17(x + y^2)$ then takes the derivative of both sides, one finds $\frac{dy}{dx} = \frac{2x-17}{34y-1}$.
    These expressions, by themselves, are not equal. However, for values of x and y that satisfy the original equation (i.e, for x and y such that $\frac{x^2+y}{x+y^2} = 17$), these expressions are equal.
23. If one takes the derivative of the equation, as shown, using the Quotient Rule, one finds $\frac{dy}{dx} = \frac{-\cos(x)(x+\cos(y))+\sin(x)+y}{\sin(y)(\sin(x)+y)+x+\cos(y)}$.
    If one first clears the denominator and writes $\sin(x) + y = \cos(y) + x$ then takes the derivative of both sides, one finds $\frac{dy}{dx} = \frac{1-\cos(x)}{1+\sin(y)}$.
    These expressions, by themselves, are not equal. However, for values of x and y that satisfy the original equation (i.e, for x and y such that $\frac{\sin(x)+y)}{\cos(y)+x)} = 1$), these expressions are equal.
24. $\frac{dy}{dx} = -\frac{x}{y}$
25. $\frac{dy}{dx} = -\frac{2x+y}{2y+x}$
26. In each, $\frac{dy}{dx} = -\frac{y}{x}$.
27. (a) $y = 0$
    (b) $y = -1.859(x - 0.1) + 0.281$
28. (a) $x = 1$

(b) $y = -\frac{3\sqrt{3}}{8}(x - \sqrt{.6}) + \sqrt{.8} \approx -0.65(x - 0.775) + 0.894$

(c) $y = 1$

29. (a) $y = 4$

(b) $y = 0.93(x - 2) + \sqrt[4]{108}$

30. (a) $y = -1/3x + 1$

(b) $y = 3\sqrt{3}/4$

31. (a) $y = -\frac{1}{\sqrt{3}}(x - \frac{7}{2}) + \frac{6+3\sqrt{3}}{2}$

(b) $y = \sqrt{3}(x - \frac{4+3\sqrt{3}}{2}) + \frac{3}{2}$

32. (a) $y = 1$

(b) $y = -\frac{2}{\sqrt{5}}(x + 1) + \frac{1}{2}(-1 + \sqrt{5})$

(c) $y = \frac{2}{\sqrt{5}}(x + 1) + \frac{1}{2}(-1 - \sqrt{5})$

33. $\frac{d^2y}{dx^2} = \frac{(2y+1)(-12x^2)+4x^3\left(2\frac{-4x^3}{2y+1}\right)}{(2y+1)^2}$

34. $\frac{d^2y}{dx^2} = \frac{3}{5}\frac{y^{3/5}}{x^{8/5}} + \frac{3}{5}\frac{1}{yx^{6/5}}$

35. $\frac{d^2y}{dx^2} = \frac{\cos x \cos y + \sin^2 x \tan y}{\cos^2 y}$

36. $\frac{d^2y}{dx^2} = 0$

37. $y' = (1 + x)^{1/x}\left(\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2}\right)$
Tangent line: $y = (1 - 2\ln 2)(x - 1) + 2$

38. $y' = (2x)^{x^2}\left(2x\ln(2x) + x\right)$
Tangent line: $y = (2 + 4\ln 2)(x - 1) + 2$

39. $y' = \frac{x^x}{x+1}\left(\ln x + 1 - \frac{1}{x+1}\right)$
Tangent line: $y = (1/4)(x - 1) + 1/2$

40. $y' = x^{\sin(x)+2}\left(\cos x \ln x + \frac{\sin x + 2}{x}\right)$
Tangent line: $y = (3\pi^2/4)(x - \pi/2) + (\pi/2)^3$

41. $y' = \frac{x+1}{x+2}\left(\frac{1}{x+1} - \frac{1}{x+2}\right)$
Tangent line: $y = 1/9(x - 1) + 2/3$

42. $y' = \frac{(x+1)(x+2)}{(x+3)(x+4)}\left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3} - \frac{1}{x+4}\right)$
Tangent line: $y = 11/72x + 1/6$

Section 2.7

1. F
2. Answers will vary.
3. The point $(10, 1)$ lies on the graph of $y = f^{-1}(x)$ (assuming f is invertible).
4. The point $(10, 1)$ lies on the graph of $y = f^{-1}(x)$ (assuming f is invertible) and $(f^{-1})'(10) = 1/5$.
5. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
6. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
7. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
8. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
9. $\left(f^{-1}\right)'(20) = \frac{1}{f'(2)} = 1/5$
10. $\left(f^{-1}\right)'(7) = \frac{1}{f'(3)} = 1/4$
11. $\left(f^{-1}\right)'(\sqrt{3}/2) = \frac{1}{f'(\pi/6)} = 1$
12. $\left(f^{-1}\right)'(8) = \frac{1}{f'(1)} = 1/6$
13. $\left(f^{-1}\right)'(1/2) = \frac{1}{f'(1)} = -2$
14. $\left(f^{-1}\right)'(6) = \frac{1}{f'(0)} = 1/6$
15. $h'(t) = \frac{2}{\sqrt{1-4t^2}}$
16. $f'(t) = \frac{1}{|t|\sqrt{4t^2+1}}$
17. $g'(x) = \frac{2}{1+4x^2}$
18. $f'(x) = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$
19. $g'(t) = \cos^{-1}(t)\cos(t) - \frac{\sin(t)}{\sqrt{1-t^2}}$
20. $f'(t) = \frac{e^t}{t} + \ln t e^t$
21. $h'(x) = \frac{\sin^{-1}(x)+\cos^{-1}(x)}{\sqrt{1-x^2}\cos^{-1}(x)^2}$
22. $g'(x) = \frac{1}{\sqrt{x}(2x+2)}$
23. $f'(x) = -\frac{1}{\sqrt{1-x^2}}$
24. (a) $f(x) = x$, so $f'(x) = 1$
    (b) $f'(x) = \cos(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}} = 1.$
25. (a) $f(x) = x$, so $f'(x) = 1$
    (b) $f'(x) = \cos(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}} = 1.$
26. (a) $f(x) = x$, so $f'(x) = 1$
    (b) $f'(x) = \frac{1}{1+\tan^2 x}\sec^2 x = 1$
27. $y = \sqrt{2}(x - \sqrt{2}/2) + \pi/4$
28. $y = -4(x - \sqrt{3}/4) + \pi/6$
29. $\frac{dy}{dx} = \frac{y(y-2x)}{x(x-2y)}$
30. $y = -4/5(x - 1) + 2$
31. $3x^2 + 1$

Chapter 3

Section 3.1

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. F
6. Where $f'(x)$ is equal to $\underline{0}$ or where $f'(x)$ is $\underline{\text{undefined}}$.
7. A: none; the function isn't defined here. B: abs. max & rel. max C: rel. min D: none; the function isn't defined here. E: none F: rel. min G: rel. max
8. A: abs. min & rel. min B: none C: abs. max & rel. max D: none E: rel. min
9. $f'(0) = 0$
10. $f'(0) = 0\, f'(2) = 0$
11. $f'(\pi/2) = 0\, f'(3\pi/2) = 0$
12. $f'(0) = 0\, f'(3.2) = 0\, f'(4)$ is undefined
13. $f'(2)$ is not defined $f'(6) = 0$
14. Both $f'(-1)$ and $f'(1)$ are undefined.
15. $f'(0) = 0$
16. $f'(0)$ is not defined
17. min: $(-0.5, 3.75)$
    max: $(2, 10)$

18. min: $(5, -134.5)$
max: $(0, 3)$
19. min: $(\pi/4, 3\sqrt{2}/2)$
max: $(\pi/2, 3)$
20. min: $(0, 0)$ and $(\pm 2, 0)$
max: $(\pm 2\sqrt{2/3}, 16\sqrt{3}/9)$
21. min: $(\sqrt{3}, 2\sqrt{3})$
max: $(5, 28/5)$
22. min: $(0, 0)$
max: $(5, 5/6)$
23. min: $(\pi, -e^{\pi})$
max: $(\pi/4, \frac{\sqrt{2}e^{\pi/4}}{2})$
24. min: $(0, 0)$ and $(\pi, 0)$
max: $(3\pi/4, \frac{\sqrt{2}e^{3\pi/4}}{2})$
25. min: $(1, 0)$
max: $(e, 1/e)$
26. min: $(2, 2^{2/3} - 2)$
max: $(8/27, 4/27)$
27. $\frac{dy}{dx} = \frac{y(y-2x)}{x(x-2y)}$
28. $y = -4/5(x-1) + 2$
29. $3x^2 + 1$

Section 3.2

1. Answers will vary.
2. Answers will vary.
3. Any c in $[-1, 1]$ is valid.
4. Rolle's Thm. does not apply.
5. $c = -1/2$
6. $c = -1/2$
7. Rolle's Thm. does not apply.
8. $c = \pi/2$
9. Rolle's Thm. does not apply.
10. Rolle's Theorem does not apply.
11. $c = 0$
12. $c = 5/2$
13. $c = 3/\sqrt{2}$
14. $c = 19/4$
15. The Mean Value Theorem does not apply.
16. $c = 4/\ln 5$
17. $c = \pm\sec^{-1}(2/\sqrt{\pi})$
18. $c = -2/3$
19. $c = \frac{5-7\sqrt{7}}{6}$
20. $c = \frac{\pm\sqrt{\pi^2-4}}{\pi}$
21. Max value of 19 at $x = -2$ and $x = 5$; min value of 6.75 at $x = 1.5$.
22. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)
23. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)

Section 3.3

1. Answers will vary.
2. Answers will vary.
3. Answers will vary; graphs should be steeper near $x = 0$ than near $x = 2$.
4. Answers will vary.
5. False; for instance, $y = x^3$ is always increasing though it has a critical point at $x = 0$.
6. Increasing
7. Graph and verify.
8. Graph and verify.
9. Graph and verify.
10. Graph and verify.
11. Graph and verify.
12. Graph and verify.
13. Graph and verify.
14. Graph and verify.
15. domain: $(-\infty, \infty)$
c.p. at $c = -1$;
decreasing on $(-\infty, -1)$;
increasing on $(-1, \infty)$;
rel. min at $x = -1$.
16. domain=$(-\infty, \infty)$
c.p. at $c = -2, 0$;
increasing on $(-\infty, -2)$ and $(0, \infty)$;
decreasing on $(-2, 0)$;
rel. min at $x = 0$;
rel. max at $x = -2$.
17. domain=$(-\infty, \infty)$
c.p. at $c = \frac{1}{6}(-1 \pm \sqrt{7})$;
decreasing on $(\frac{1}{6}(-1-\sqrt{7}), \frac{1}{6}(-1+\sqrt{7})))$;
increasing on $(-\infty, \frac{1}{6}(-1-\sqrt{7}))$ and $(\frac{1}{6}(-1+\sqrt{7}), \infty)$;
rel. min at $x = \frac{1}{6}(-1+\sqrt{7})$;
rel. max at $x = \frac{1}{6}(-1-\sqrt{7})$.
18. domain=$(-\infty, \infty)$
c.p. at $c = 1$;
increasing on $(-\infty, \infty)$;
19. domain=$(-\infty, \infty)$
c.p. at $c = 1$;
decreasing on $(1, \infty)$
increasing on $(-\infty, 1)$;
rel. max at $x = 1$.
20. domain=$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
c.p. at $c = 0$;
decreasing on $(-\infty, -1)$ and $(-1, 0)$;
increasing on $(0, 1)$ and $(1, \infty)$;
rel. min at $x = 0$;
21. domain=$(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$
no c.p.;
decreasing on entire domain, $(-\infty, -2)$, $(-2, 4)$ and $(4, \infty)$
22. domain=$(-\infty, 0) \cup (0, \infty)$;
c.p. at $c = 2, 6$;
decreasing on $(-\infty, 0)$, $(0, 2)$ and $(6, \infty)$;
increasing on $(2, 6)$;
rel. min at $x = 2$; rel. max at $x = 6$.

23. domain=$(-\infty, \infty)$
c.p. at $c = -3\pi/4, -\pi/4, \pi/4, 3\pi/4$;
decreasing on $(-3\pi/4, -\pi/4)$ and $(\pi/4, 3\pi/4)$;
increasing on $(-\pi, -3\pi/4)$, $(-\pi/4, \pi/4)$ and $(3\pi/4, \pi)$;
rel. min at $x = -\pi/4, 3\pi/4$;
rel. max at $x = -3\pi/4, \pi/4$.
24. domain = $(-\infty, \infty)$;
c.p. at $c = -1, 1$;
decreasing on $(-1, 1)$;
increasing on $(-\infty, -1)$ and $(1, \infty)$;
rel. min at $x = 1$;
rel. max at $x = -1$
25. $c = 1/2$
26. $c = \pm \cos^{-1}(2/\pi)$

Section 3.4

1. Answers will vary.
2. Answers will vary.
3. Yes; Answers will vary.
4. No.
5. Graph and verify.
6. Graph and verify.
7. Graph and verify.
8. Graph and verify.
9. Graph and verify.
10. Graph and verify.
11. Graph and verify.
12. Graph and verify.
13. Graph and verify.
14. Graph and verify.
15. Possible points of inflection: none; concave up on $(-\infty, \infty)$
16. Possible points of inflection: none; concave down on $(-\infty, \infty)$
17. Possible points of inflection: $x = 0$; concave down on $(-\infty, 0)$; concave up on $(0, \infty)$
18. Possible points of inflection: $x = 1/2$; concave down on $(-\infty, 1/2)$; concave up on $(1/2, \infty)$
19. Possible points of inflection: $x = -2/3, 0$; concave down on $(-2/3, 0)$; concave up on $(-\infty, -2/3)$ and $(0, \infty)$
20. Possible points of inflection: $x = (1/3)(2 \pm \sqrt{7})$; concave up on $((1/3)(2 - \sqrt{7}), (1/3)(2 + \sqrt{7}))$; concave down on $(-\infty, (1/3)(2 - \sqrt{7}))$ and $((1/3)(2 + \sqrt{7}), \infty)$
21. Possible points of inflection: $x = 1$; concave up on $(-\infty, \infty)$
22. Possible points of inflection: $f''(x)$ is not defined (nor is f) at $x = -\pi/2, \pi/2$; concave down on $(-3\pi/2, -\pi/2)$ and $(\pi/2, 3\pi/2)$ concave up on $(-\pi/2, \pi/2)$
23. Possible points of inflection: $x = \pm 1/\sqrt{3}$; concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$; concave up on $(-\infty, -1/\sqrt{3})$ and $(1/\sqrt{3}, \infty)$
24. Possible points of inflection: $x = 0, \pm 1$; concave down on $(-\infty, -1)$ and $(0, 1)$ concave up on $(-1, 0)$ and $(1, \infty)$
25. Possible points of inflection: $x = -\pi/4, 3\pi/4$; concave down on $(-\pi/4, 3\pi/4)$ concave up on $(-\pi, -\pi/4)$ and $(3\pi/4, \pi)$
26. Possible points of inflection: $x = -2 \pm \sqrt{2}$; concave down on $(-2 - \sqrt{2}, -2 + \sqrt{2})$ concave up on $(-\infty, -2 - \sqrt{2})$ and $(-2 + \sqrt{2}, \infty)$
27. Possible points of inflection: $x = 1/e^{3/2}$; concave down on $(0, 1/e^{3/2})$ concave up on $(1/e^{3/2}, \infty)$
28. Possible points of inflection: $x = \pm 1/\sqrt{2}$; concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$ concave up on $(-\infty, -1/\sqrt{2})$ and $(1/\sqrt{2}, \infty)$
29. min: $x = 1$
30. max: $x = -5/2$
31. max: $x = -1/\sqrt{3}$ min: $x = 1/\sqrt{3}$
32.
33. min: $x = 1$
34. max: $x = -1, 2$; min: $x = 1$
35. min: $x = 1$
36. max: at $x = \pm\pi$ min: at $x = 0$
37. max: $x = 0$
38. critical values: $x = -1, 1$; no max/min
39. max: $x = \pi/4$; min: $x = -3\pi/4$
40. max: $x = -2$; min: $x = 0$
41. min: $x = 1/\sqrt{e}$
42. max: $x = 0$
43. f' has no maximal or minimal value.
44. f' has no maximal or minimal value
45. f' has a minimal value at $x = 0$
46. f' has a minimal value at $x = 1/2$
47. Possible points of inflection: $x = -2/3, 0$; f' has a relative min at: $x = 0$; relative max at: $x = -2/3$
48. f' has a relative max at: $x = (1/3)(2 + \sqrt{7})$ relative min at: $x = (1/3)(2 - \sqrt{7})$
49. f' has no relative extrema
50. $f'(x)$ has no relative extrema
51. f' has a relative max at $x = -1/\sqrt{3}$; relative min at $x = 1/\sqrt{3}$
52. f' has a relative max at $x = 0$
53. f' has a relative min at $x = 3\pi/4$; relative max at $x = -\pi/4$
54. f' has a relative max at $x = -2 - \sqrt{2}$; relative min at $x = -2 + \sqrt{2}$
55. f' has a relative min at $x = 1/\sqrt{e^3} = e^{-3/2}$
56. f' has a relative max at $x = -1/\sqrt{2}$; a relative min at $x = 1/\sqrt{2}$

Section 3.5

1. Answers will vary.
2. Found everywhere.
3. T
4. T
5. T
6. F
7. A good sketch will include the x and y intercepts and draw the appropriate line.
8. A good sketch will include the x and y intercepts..
9. Use technology to verify sketch.
10. Use technology to verify sketch.
11. Use technology to verify sketch.
12. Use technology to verify sketch.

13. Use technology to verify sketch.
14. Use technology to verify sketch.
15. Use technology to verify sketch.
16. Use technology to verify sketch.
17. Use technology to verify sketch.
18. Use technology to verify sketch.
19. Use technology to verify sketch.
20. Use technology to verify sketch.
21. Use technology to verify sketch.
22. Use technology to verify sketch.
23. Use technology to verify sketch.
24. Use technology to verify sketch.
25. Use technology to verify sketch.
26. Use technology to verify sketch.
27. Critical point: $x = 0$ Points of inflection: $\pm b/\sqrt{3}$
28. Critical point: $x = -b/(2a)$ No points of inflection
29. Critical points: $x = \frac{n\pi/2-b}{a}$, where n is an odd integer Points of inflection: $(n\pi - b)/a$, where n is an integer.
30. Critical point: $x = (a+b)/2$ Points of inflection: none
31. $\frac{dy}{dx} = -x/y$, so the function is increasing in second and fourth quadrants, decreasing in the first and third quadrants.
    $\frac{d^2y}{dx^2} = -1/y - x^2/y^3$, which is positive when $y < 0$ and is negative when $y > 0$. Hence the function is concave down in the first and second quadrants and concave up in the third and fourth quadrants.

Chapter 4

Section 4.1

1. F
2. F
3. $x_0 = 1.5$, $x_1 = 1.5709148$, $x_2 = 1.5707963$, $x_3 = 1.5707963$, $x_4 = 1.5707963$, $x_5 = 1.5707963$
4. $x_0 = 1$, $x_1 = -0.55740772$, $x_2 = 0.065936452$, $x_3 = -0.000095721919$, $x_4 = 2.9235662 * 10^{-13}$, $x_5 = 0$
5. $x_0 = 0$, $x_1 = 2$, $x_2 = 1.2$, $x_3 = 1.0117647$, $x_4 = 1.0000458$, $x_5 = 1$
6. $x_0 = 1.5$, $x_1 = 1.4166667$, $x_2 = 1.4142157$, $x_3 = 1.4142136$, $x_4 = 1.4142136$, $x_5 = 1.4142136$
7. $x_0 = 2$, $x_1 = 0.6137056389$, $x_2 = 0.9133412072$, $x_3 = 0.9961317034$, $x_4 = 0.9999925085$, $x_5 = 1$
8. $x_0 = 1$, $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $x_4 = 1$, $x_5 = 1$
9. roots are: $x = -5.156$, $x = -0.369$ and $x = 0.525$
10. roots are: $x = -3.714$, $x = -0.857$, $x = 1$ and $x = 1.571$
11. roots are: $x = -1.013$, $x = 0.988$, and $x = 1.393$
12. roots are: $x = -2.165$, $x = 0$, $x = 0.525$ and $x = 1.813$
13. $x = \pm 0.824$,
14. $x = -0.637$, $x = 1.410$
15. $x = \pm 0.743$
16. $x = \pm 4.493$, $x = 0$
17. The approximations alternate between $x = 1$ and $x = 2$.
18. The approximations alternate between $x = 1$, $x = 2$ and $x = 3$.

Section 4.2

1. T
2. F
3. (a) $5/(2\pi) \approx 0.796$cm/s
   (b) $1/(4\pi) \approx 0.0796$ cm/s
   (c) $1/(40\pi) \approx 0.00796$ cm/s
4. (a) $5/(2\pi) \approx 0.796$cm/s
   (b) $1/(40\pi) \approx 0.00796$ cm/s
   (c) $1/(4000\pi) \approx 0.0000796$ cm/s
5. 63.14mph
6. (a) 64.44 mph
   (b) 78.89 mph
7. Due to the height of the plane, the gun does not have to rotate very fast.
   (a) 0.0573 rad/s
   (b) 0.0725 rad/s
   (c) In the limit, rate goes to 0.0733 rad/s
8. Due to the height of the plane, the gun does not have to rotate very fast.
   (a) 0.073 rad/s
   (b) 3.66 rad/s (about 1/2 revolution/sec)
   (c) In the limit, rate goes to 7.33 rad/s (more than 1 revolution/sec)
9. (a) 0.04 ft/s
   (b) 0.458 ft/s
   (c) 3.35 ft/s
   (d) Not defined; as the distance approaches 24, the rates approaches ∞.
10. (a) 30.59 ft/min
    (b) 36.1 ft/min
    (c) 301 ft/min
    (d) The boat no longer floats as usual, but is being pulled up by the winch (assuming it has the power to do so).
11. (a) 50.92 ft/min
    (b) 0.509 ft/min
    (c) 0.141 ft/min
    As the tank holds about 523.6ft^3, it will take about 52.36 minutes.
12. (a) 0.63 ft/sec
    (b) 1.6 ft/sec
    About 52 ft.
13. (a) The rope is 80ft long.
    (b) 1.71 ft/sec
    (c) 1.84 ft/sec
    (d) About 34 feet.
14. (a) The balloon is 105ft in the air.
    (b) The balloon is rising at a rate of 17.45ft/min. (Hint: convert all angles to radians.)
15. The cone is rising at a rate of 0.003ft/s.

Section 4.3

1. T

2. F
3. 2500; the two numbers are each 50.
4. The minimum sum is $2\sqrt{500}$; the two numbers are each $\sqrt{500}$.
5. There is no maximum sum; the fundamental equation has only 1 critical value that corresponds to a minimum.
6. The only critical point of the fundamental equation corresponds to a minimum; to find maximum, we check the endpoints.

   If one number is 300, the other number y satisfies $300y = 500$; $y = 5/3$. Thus the sum is $300 + 5/3$.

   The other endpoint, 0, is not feasible as we cannot solve $0 \cdot y = 500$ for y. In fact, if $0 < x < 5/3$, then $x \cdot y = 500$ forces $y > 300$, which is not a feasible solution.

   Hence the maximum sum is $301.\overline{6}$.
7. Area = 1/4, with sides of length $1/\sqrt{2}$.
8. Each pen should be $500/3 \approx 166.67$ feet by 125 feet.
9. The radius should be about 3.84cm and the height should be $2r = 7.67$cm. No, this is not the size of the standard can.
10. The radius should be about 3.2in and the height should be $2r = 6.4$in. As the #10 is not a perfect cylinder (with extra material to aid in stacking, etc.), the dimensions are close enough to assume that minimizing surface area was a consideration.
11. The height and width should be 18 and the length should be 36, giving a volume of $11,664\text{in}^3$.
12. $w = 4\sqrt{3}$, $h = 4\sqrt{6}$
13. $5 - 10/\sqrt{39} \approx 3.4$ miles should be run underground, giving a minimum cost of \$374,899.96.
14. The power line should be run directly to the off shore facility, skipping any underground, giving a cost of about \$430,813.
15. The dog should run about 19 feet along the shore before starting to swim.
16. The dog should run about 13 feet along the shore before starting to swim.
17. The largest area is 2 formed by a square with sides of length $\sqrt{2}$.

Section 4.4

1. T
2. T
3. F
4. T
5. Answers will vary.
6. T
7. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 2$ and $dx = 0.05$. Thus $dy = .2$; knowing $2^2 = 4$, we have $2.05^2 \approx 4.2$.
8. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 6$ and $dx = -0.07$. Thus $dy = -0.84$; knowing $6^2 = 36$, we have $5.93^2 \approx 35.16$.
9. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 5$ and $dx = 0.1$. Thus $dy = 7.5$; knowing $5^3 = 125$, we have $5.1^3 \approx 132.5$.
10. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 7$ and $dx = -0.2$. Thus $dy = -29.4$; knowing $7^3 = 343$, we have $6.8^3 \approx 313.6$.
11. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 16$ and $dx = 0.5$. Thus $dy = .0625$; knowing $\sqrt{16} = 4$, we have $\sqrt{16.5} \approx 4.0625$.
12. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 25$ and $dx = -1$. Thus $dy = -0.1$; knowing $\sqrt{25} = 5$, we have $\sqrt{24} \approx 4.9$.
13. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 64$ and $dx = -1$. Thus $dy = -1/48 \approx 0.0208$; we could use $-1/48 \approx -1/50 = -0.02$; knowing $\sqrt[3]{64} = 4$, we have $\sqrt[3]{63} \approx 3.98$.
14. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 8$ and $dx = 0.5$. Thus $dy = 1/24 \approx 1/25 = 0.04$; knowing $\sqrt[3]{8} = 2$, we have $\sqrt[3]{8.5} \approx 2.04$.
15. Use $y = \sin x$; $dy = \cos x \cdot dx$ with $x = \pi$ and $dx \approx -0.14$. Thus $dy = 0.14$; knowing $\sin\pi = 0$, we have $\sin 3 \approx 0.14$.
16. Use $y = e^x$; $dy = e^x \cdot dx$ with $x = 0$ and $dx = 0.1$. Thus $dy = 0.1$; knowing $e^0 = 1$, we have $e^{0.1} \approx 1.1$.
17. $dy = (2x + 3)dx$
18. $dy = (7x^6 - 5x^4)dx$
19. $dy = \frac{-2}{4x^3}dx$
20. $dy = 2(2x + \sin x)(2 + \cos x)dx$
21. $dy = \left(2xe^{3x} + 3x^2e^{3x}\right)dx$
22. $dy = \frac{-16}{x^5}dx$
23. $dy = \frac{2(\tan x+1)-2x\sec^2 x}{(\tan x+1)^2}dx$
24. $dy = \frac{1}{x}dx$
25. $dy = (e^x \sin x + e^x \cos x)dx$
26. $dy = (-\sin(\sin x)\cos x)dx$
27. $dy = \frac{1}{(x+2)^2}dx$
28. $dy = ((\ln 3)3^x \ln x + \frac{3^x}{x})dx$
29. $dy = (\ln x)dx$
30. $dy = \left(\tan x\right)dx$
31. $dV = \pm 0.157$
32. (a) ± 12.8 feet
    (b) ± 32 feet
33. $\pm 15\pi/8 \approx \pm 5.89\text{in}^2$
34. $\pm 48\text{in}^2$, or $1/3\text{ft}^2$
35. (a) 297.8 feet
    (b) ± 62.3 ft
    (c) $\pm 20.9\%$
36. (a) 298.8 feet
    (b) ± 17.3 ft
    (c) $\pm 5.8\%$
37. (a) 298.9 feet
    (b) ± 8.67 ft
    (c) $\pm 2.9\%$
38. The isosceles triangle setup works the best with the smallest percent error.
39. 1%

Chapter 5

Section 5.1

1. Answers will vary.
2. "an"
3. Answers will vary.
4. opposite; opposite
5. Answers will vary.
6. velocity
7. velocity

8. $F(x)+G(x)$
9. $3/4x^4+C$
10. $1/9x^9+C$
11. $10/3x^3-2x+C$
12. $t+C$
13. $s+C$
14. $-1/(3t)+C$
15. $-3/(t)+C$
16. $2\sqrt{x}+C$
17. $\tan\theta+C$
18. $-\cos\theta+C$
19. $\sec x-\csc x+C$
20. $5e^{\theta}+C$
21. $3^t/\ln 3+C$
22. $\frac{5^t}{2\ln 5}+C$
23. $4/3t^3+6t^2+9t+C$
24. $t^6/6+t^4/4-3t^2+C$
25. $x^6/6+C$
26. $e^{\pi}x+C$
27. $ax+C$
28. (a) $x>0$
    (b) $1/x$
    (c) $x<0$
    (d) $1/x$
    (e) $\ln|x|+C$. Explanations will vary.
29. $-\cos x+3$
30. $5e^x+5$
31. x^4-x^3+7
32. $\tan x+4$
33. $7^x/\ln 7+1-49/\ln 7$
34. $5/2x^2+7x+3$
35. $\frac{7x^3}{6}-\frac{9x}{2}+\frac{40}{3}$
36. $5e^x-2x$
37. $\theta-\sin(\theta)-\pi+4$
38. $2x^4+\cos x+\frac{2^x}{(\ln 2)^2}+(5-\frac{1}{\ln 2})x-1-\frac{1}{(\ln 2)^2}$
39. $3x-2$
40. No answer provided.
41. $dy=(2xe^x\cos x+x^2e^x\cos x-x^2e^x\sin x)dx$

Section 5.2

1. Answers will vary.
2. Answers will vary.
3. 0
4. $\int 0^2(2x+3)\,dx$
5. (a) 3
   (b) 4
   (c) 3
   (d) 0
   (e) -4
   (f) 9
6. (a) -4
   (b) -5
   (c) -3
   (d) 1
   (e) -2
   (f) 10
7. (a) 4
   (b) 2
   (c) 4
   (d) 2
   (e) 1
   (f) 2
8. (a) $-1/2$
   (b) 0
   (c) $3/2$
   (d) $3/2$
   (e) $9/2$
   (f) $15/2$
9. (a) π
   (b) π
   (c) 2π
   (d) 10π
10. (a) 15
    (b) 12
    (c) 0
    (d) $3(b-a)$
11. (a) -59
    (b) -48
    (c) -27
    (d) -33
12. (a) $4/\pi$
    (b) $-4/\pi$
    (c) 0
    (d) $2/\pi$
13. (a) 4
    (b) 4
    (c) -4
    (d) -2
14. (a) $40/3$
    (b) $26/3$
    (c) $8/3$
    (d) $38/3$
15. (a) 2ft/s
    (b) 2ft
    (c) 1.5ft
16. (a) 3ft/s
    (b) 9.5ft
    (c) 9.5ft
17. (a) 64ft/s

(b) 64ft
(c) $t = 2$
(d) $t = 2 + \sqrt{7} \approx 4.65$ seconds

18. (a) 96ft/s
(b) 6 seconds
(c) 6 seconds
(d) Never; the maximum height is 208ft.

19. 2
20. 5
21. 16
22. Answers can vary; one solution is $a = -2$, $b = 7$
23. 24
24. -7
25. -7
26. Answers can vary; one solution is $a = -18$, $b = 11$
27. $1/4x^4 - 2/3x^3 + 7/2x^2 - 9x + C$
28. $-\cos x - \sin x + \tan x + C$
29. $3/4t^{4/3} - 1/t + 2^t/\ln 2 + C$
30. $\ln|x| + \csc x + C$

Section 5.3

1. limits
2. 14
3. Rectangles.
4. T
5. $2^2 + 3^2 + 4^2 = 29$
6. $-6 - 2 + 2 + 6 + 10 = 10$
7. $0 - 1 + 0 + 1 + 0 = 0$
8. $5 + 5 + \ldots + 5 = 50$
9. $1 + 1/2 + 1/3 + 1/4 + 1/5 = 137/60$
10. $-1 + 2 - 3 + 4 - 5 + 6 = 3$
11. $1/2 + 1/6 + 1/12 + 1/20 = 4/5$
12. $1 + 1 + 1 + 1 + 1 + 1 = 6$
13. Answers may vary; $\sum_{i=1}^{5} 3i$
14. Answers may vary; $\sum_{i=0}^{8}(i^2 - 1)$
15. Answers may vary; $\sum_{i=1}^{4} \frac{i}{i+1}$
16. Answers may vary; $\sum_{i=0}^{4}(-1)^i e^i$
17. $5 \cdot 10 = 50$
18. 325
19. 1045
20. 28,650
21. -8525
22. 2050
23. 5050
24. 2870
25. 155
26. $91,225$
27. 24
28. $11,700$
29. 19
30. $59/8$
31. $\pi/3 + \pi/(2\sqrt{3}) \approx 1.954$
32. 8.144
33. 0.388584
34. $496/315 \approx 1.5746$
35. (a) Exact expressions will vary; $\frac{(1+n)^2}{4n^2}$.
(b) $121/400$, $10201/40000$, $1002001/4000000$
(c) $1/4$
36. (a) Exact expressions will vary; $2 + 4/n^2$.
(b) $51/25$, $5001/2500$, $500001/250000$
(c) 2
37. (a) 8.
(b) 8, 8, 8
(c) 8
38. (a) Exact expressions will vary; $20/3 - 96/(3n) + 64/(3n^2)$.
(b) $92/25$, $3968/625$, $103667/15625$
(c) $20/3$
39. (a) Exact expressions will vary; $100 - 200/n$.
(b) 80, 98, $499/5$
(c) 100
40. (a) Exact expressions will vary; $-1/12(1 - 1/n^2)$.
(b) $-33/400$, $-3333/40000$, $-333333/4000000$
(c) $-1/12$
41. $F(x) = 5\tan x + 4$
42. $F(x) = 7\ln|x| + 14$
43. $G(t) = 4/6t^6 - 5/4t^4 + 8t + 9$
44. $G(t) = 5 \cdot 8^t/\ln 8 + 900$
45. $G(t) = \sin t - \cos t - 78$
46. $F(x) = 2\sqrt{x} - \pi$

Section 5.4

1. Answers will vary.
2. 0
3. T
4. Answers will vary.
5. 20
6. $28/3$
7. 0
8. 1
9. 1
10. 1
11. $(5 - 1/5)/\ln 5$
12. $23/2$
13. -4
14. $e^3 - e$
15. $16/3$
16. 4

17. $45/4$
18. $\ln 2$
19. $1/2$
20. $3/8$
21. $1/2$
22. $1/3$
23. $1/4$
24. $1/101$
25. 8
26. 15
27. 0
28. $2 - 2/\sqrt{3}$
29. Explanations will vary. A sketch will help.
30. $\int_a^{a+2\pi} \sin t\,dt = \cos(a+2\pi) - \cos(a)$. Since cosine is periodic with period 2π, $\cos(a+2\pi) = \cos(a)$, and hence the integral is 0.
31. $c = 2/\sqrt{3}$
32. $c = \pm 2/\sqrt{3}$
33. $c = \ln(e-1) \approx 0.54$
34. $c = 64/9 \approx 7.1$
35. $2/\pi$
36. $2/pi$
37. 2
38. $16/3$
39. 16
40. $1/(e-1)$
41. -300ft
42. 400ft
43. 30ft
44. $1.5/\ln(2) \approx 2.164$miles
45. -1ft
46. $128/5$ft
47. -64ft/s
48. 50ft/s
49. 2ft/s
50. 0ft/s
51. $27/2$
52. 21
53. $9/2$
54. $343/6$
55. $F'(x) = (3x^2+1)\frac{1}{x^3+x}$
56. $F'(x) = -3x^{11}$
57. $F'(x) = 2x(x^2+2) - (x+2)$
58. $F'(x) = e^x \sin(e^x) - 1/x \sin(\ln x)$

Section 5.5

1. F
2. When the antiderivative cannot be computed and when the integrand is unknown.
3. They are superseded by the Trapezoidal Rule; it takes an equal amount of work and is generally more accurate.
4. A quadratic function (i.e., parabola)
5. (a) $3/4$
   (b) $2/3$
   (c) $2/3$
6. (a) 250
   (b) 250
   (c) 250
7. (a) $\frac{1}{4}(1+\sqrt{2})\pi \approx 1.896$
   (b) $\frac{1}{6}(1+2\sqrt{2})\pi \approx 2.005$
   (c) 2
8. (a) $2+\sqrt{2}+\sqrt{3} \approx 5.15$
   (b) $2/3(3+\sqrt{2}+2\sqrt{3}) \approx 5.25$
   (c) $16/3 \approx 5.33$
9. (a) 38.5781
   (b) $147/4 \approx 36.75$
   (c) $147/4 \approx 36.75$
10. (a) 0.2207
    (b) 0.2005
    (c) $1/5$
11. (a) 0
    (b) 0
    (c) 0
12. (a) $9/2(1+\sqrt{3}) \approx 12.294$
    (b) $3+6\sqrt{3} \approx 13.392$
    (c) $9\pi/2 \approx 14.137$
13. Trapezoidal Rule: 0.9006
    Simpson's Rule: 0.90452
14. Trapezoidal Rule: 3.0241
    Simpson's Rule: 2.9315
15. Trapezoidal Rule: 13.9604
    Simpson's Rule: 13.9066
16. Trapezoidal Rule: 3.0695
    Simpson's Rule: 3.14295
17. Trapezoidal Rule: 1.1703
    Simpson's Rule: 1.1873
18. Trapezoidal Rule: 2.52971
    Simpson's Rule: 2.5447
19. Trapezoidal Rule: 1.0803
    Simpson's Rule: 1.077
20. Trapezoidal Rule: 3.5472
    Simpson's Rule: 3.6133
21. (a) $n = 161$ (using max $\big(f''(x)\big) = 1$)
    (b) $n = 12$ (using max $\big(f^{(4)}(x)\big) = 1$)
22. (a) $n = 150$ (using max $\big(f''(x)\big) = 1$)
    (b) $n = 18$ (using max $\big(f^{(4)}(x)\big) = 7$)
23. (a) $n = 1004$ (using max $\big(f''(x)\big) = 39$)
    (b) $n = 62$ (using max $\big(f^{(4)}(x)\big) = 800$)
24. (a) $n = 5591$ (using max $\big(f''(x)\big) = 300$)

(b) $n = 46$ (using max $\left(f^{(4)}(x)\right) = 24$)

25. (a) Area is 30.8667 cm^2.

    (b) Area is 308, 667 yd^2.

26. (a) Area is 25.0667 cm^2

    (b) Area is 250,667 yd^2

Chapter 6

Section 6.1

1. Chain Rule.
2. T
3. $\frac{1}{8}(x^3-5)^8+C$
4. $\frac{1}{4}(x^2-5x+7)^4+C$
5. $\frac{1}{18}\left(x^2+1\right)^9+C$
6. $\frac{1}{3}(3x^2+7x-1)^6+C$
7. $\frac{1}{2}\ln|2x+7|+C$
8. $\sqrt{2x+3}+C$
9. $\frac{2}{3}(x+3)^{3/2}-6(x+3)^{1/2}+C=\frac{2}{3}(x-6)\sqrt{x+3}+C$
10. $\frac{2}{21}x^{3/2}\left(3x^2-7\right)+C$
11. $2e^{\sqrt{x}}+C$
12. $\frac{2\sqrt{x^5+1}}{5}+C$
13. $-\frac{1}{2x^2}-\frac{1}{x}+C$
14. $\frac{\ln^2(x)}{2}+C$
15. $\frac{\sin^3(x)}{3}+C$
16. $-\frac{\cos^4(x)}{4}+C$
17. $-\frac{1}{6}\sin(3-6x)+C$
18. $-\tan(4-x)+C$
19. $\frac{1}{2}\ln|\sec(2x)+\tan(2x)|+C$
20. $\frac{\tan^3(x)}{3}+C$
21. $\frac{\sin\left(x^2\right)}{2}+C$
22. $\tan(x)-x+C$
23. The key is to rewrite $\cot x$ as $\cos x/\sin x$, and let $u=\sin x$.
24. The key is to multiply $\csc x$ by 1 in the form $(\csc x+\cot x)/(\csc x+\cot x)$.
25. $\frac{1}{3}e^{3x-1}+C$
26. $\frac{e^{x^3}}{3}+C$
27. $\frac{1}{2}e^{(x-1)^2}+C$
28. $x-e^{-x}+C$
29. $\ln\left(e^x+1\right)+C$
30. $\frac{e^{-3x}}{3}-e^{-x}+C$
31. $\frac{27^x}{\ln 27}+C$
32. $\frac{16^x}{\ln(16)}+C$
33. $\frac{1}{2}\ln^2(x)+C$
34. $\frac{\left(\ln x\right)^3}{3}+C$
35. $\frac{3}{2}(\ln x)^2+C$
36. $\frac{1}{2}\ln\left(\ln\left(x^2\right)\right)+C$
37. $\frac{x^2}{2}+3x+\ln|x|+C$
38. $\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x|+C$
39. $\frac{x^3}{3}-\frac{x^2}{2}+x-2\ln|x+1|+C$
40. $\frac{1}{2}\left(x^2+10x+20\ln|x-3|\right)+C$
41. $\frac{3}{2}x^2-8x+15\ln|x+1|+C$
42. $\frac{1}{3}\ln\left|x^2+3x+3\right|+\frac{\ln|x|}{3}+C$
43. $\sqrt{7}\tan^{-1}\left(\frac{x}{\sqrt{7}}\right)+C$
44. $3\sin^{-1}\left(\frac{x}{3}\right)+C$
45. $14\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$
46. $\frac{2}{3}\sec^{-1}(|x|/3)+C$
47. $\frac{5}{4}\sec^{-1}(|x|/4)+C$
48. $\frac{1}{2}\sin^{-1}\left(x^2\right)+C$
49. $\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{7}}\right)}{\sqrt{7}}+C$
50. $2\sin^{-1}\left(\frac{x-3}{4}\right)+C$
51. $3\sin^{-1}\left(\frac{x-4}{5}\right)+C$
52. $\tan^{-1}\left(\frac{x+3}{5}\right)+C$
53. $-\frac{1}{3\left(x^3+3\right)}+C$
54. $\frac{1}{45}(5x^3+5x^2+2)^9+C$
55. $-\sqrt{1-x^2}+C$
56. $-\frac{1}{3}\cot\left(x^3+1\right)+C$
57. $-\frac{2}{3}\cos^{\frac{3}{2}}(x)+C$
58. $-\cos\left(5x+1\right)/5+C$
59. $\ln|x-5|+C$
60. $\frac{7}{3}\ln|3x+2|+C$
61. $\frac{3x^2}{2}+\ln\left|x^2+3x+5\right|-5x+C$
62. $\ln\left|x^2+7x+3\right|+C$
63. $3\ln\left|3x^2+9x+7\right|+C$
64. $-\frac{x^2}{2}+2\ln\left|x^2-7x+1\right|+7x+C$
65. $\frac{1}{18}\tan^{-1}\left(\frac{x^2}{9}\right)+C$
66. $\tan^{-1}(2x)+C$
67. $\sec^{-1}(|2x|)+C$
68. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$
69. $\frac{3}{2}\ln\left|x^2-2x+10\right|+\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right)+C$
70. $\frac{19}{5}\tan^{-1}\left(\frac{x+6}{5}\right)-\ln\left|x^2+12x+61\right|+C$
71. $\frac{15}{2}\ln\left|x^2-10x+32\right|+x+\frac{41\tan^{-1}\left(\frac{x-5}{\sqrt{7}}\right)}{\sqrt{7}}+C$
72. $\frac{x^2}{2}-\frac{9}{2}\ln\left|x^2+9\right|+C$
73. $\frac{x^2}{2}+3\ln\left|x^2+4x+9\right|-4x+\frac{24\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)}{\sqrt{5}}+C$

74. $-\tan^{-1}(\cos(x)) + C$

75. $\tan^{-1}(\sin(x)) + C$

76. $\ln|\sec x + \tan x| + C$ (integrand simplifies to $\sec x$)

77. $3\sqrt{x^2-2x-6} + C$

78. $\sqrt{x^2-6x+8} + C$

79. $-\ln 2$

80. $352/15$

81. $2/3$

82. $1/5$

83. $(1-e)/2$

84. $\pi/2$

85. $\pi/2$

86. $\pi/6$

Section 6.2

1. T

2. F

3. Determining which functions in the integrand to set equal to "u" and which to set equal to "dv".

4. F; it is not uncommon to need to use Integration by Parts several times to fully evaluate an integral.

5. $\sin x - x\cos x + C$

6. $-e^{-x} - xe^{-x} + C$

7. $-x^2\cos x + 2x\sin x + 2\cos x + C$

8. $-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

9. $1/2e^{x^2} + C$

10. $x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

11. $-\frac{1}{2}xe^{-2x} - \frac{e^{-2x}}{4} + C$

12. $1/2e^x(\sin x - \cos x) + C$

13. $1/5e^{2x}(\sin x + 2\cos x) + C$

14. $1/13e^{2x}(2\sin(3x) - 3\cos(3x)) + C$

15. $1/10e^{5x}(\sin(5x) + \cos(5x)) + C$

16. $-1/2\cos^2 x + C$

17. $\sqrt{1-x^2} + x\sin^{-1}(x) + C$

18. $x\tan^{-1}(2x) - \frac{1}{4}\ln\left|4x^2+1\right| + C$

19. $\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + C$

20. $\sqrt{1-x^2} + x\sin^{-1}x + C$

21. $\frac{1}{2}x^2\ln|x| - \frac{x^2}{4} + C$

22. $-\frac{x^2}{4} + \frac{1}{2}x^2\ln|x| + 2x - 2x\ln|x| + C$

23. $-\frac{x^2}{4} + \frac{1}{2}x^2\ln|x-1| - \frac{x}{2} - \frac{1}{2}\ln|x-1| + C$

24. $\frac{1}{2}x^2\ln\left(x^2\right) - \frac{x^2}{2} + C$

25. $\frac{1}{3}x^3\ln|x| - \frac{x^3}{9} + C$

26. $2x + x(\ln x)^2 - 2x\ln x + C$

27. $2(x+1) + (x+1)(\ln(x+1))^2 - 2(x+1)\ln(x+1) + C$

28. $x\tan(x) + \ln|\cos(x)| + C$

29. $\ln|\sin(x)| - x\cot(x) + C$

30. $\frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + C$

31. $\frac{1}{3}(x^2-2)^{3/2} + C$

32. $\sec x + C$

33. $x\sec x - \ln|\sec x + \tan x| + C$

34. $-x\csc x - \ln|\csc x + \cot x| + C$

35. $1/2x\big(\sin(\ln x) - \cos(\ln x)\big) + C$

36. $\cos\left(e^x\right) + e^x\sin\left(e^x\right) + C$

37. $2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right) + C$

38. $\frac{1}{2}x\ln|x| - \frac{x}{2} + C$

39. $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

40. $1/2x^2 + C$

41. π

42. $-2/e$

43. 0

44. $\frac{3\pi^2}{2} - 12$

45. $1/2$

46. $6 - 2e$

47. $\frac{3}{4e^2} - \frac{5}{4e^4}$

48. $\frac{1}{2} + \frac{e^\pi}{2}$

49. $1/5\left(e^\pi + e^{-\pi}\right)$

Section 6.3

1. F

2. F

3. F

4. F

5. $-\frac{1}{5}\cos^5(x) + C$

6. $\frac{1}{4}\sin^4(x) + C$

7. $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

8. $\frac{1}{6}\cos^6 x - \frac{1}{4}\cos^4 x + C$

9. $\frac{1}{11}\sin^{11} x - \frac{2}{9}\sin^9 x + \frac{1}{7}\sin^7 x + C$

10. $-\frac{1}{9}\sin^9(x) + \frac{3\sin^7(x)}{7} - \frac{3\sin^5(x)}{5} + \frac{\sin^3(x)}{3} + C$

11. $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$

12. $\frac{1}{2}\sin^2 x + C$ or $-\frac{1}{2}\cos^2 x + C$, depending on the choice of substitution

13. $\frac{1}{2}\left(-\frac{1}{8}\cos(8x) - \frac{1}{2}\cos(2x)\right) + C$

14. $\frac{1}{2}\left(-\frac{1}{3}\cos(3x) + \cos(-x)\right) + C$

15. $\frac{1}{2}\left(\frac{1}{4}\sin(4x) - \frac{1}{10}\sin(10x)\right) + C$

16. $\frac{1}{2}\left(\frac{1}{\pi}\sin(\pi x) - \frac{1}{3\pi}\sin(3\pi x)\right) + C$

17. $\frac{1}{2}\left(\sin(x) + \frac{1}{3}\sin(3x)\right) + C$

18. $\frac{1}{\pi}\sin(\frac{\pi}{2}x) + \frac{1}{3\pi}\sin(\pi x) + C$

19. $\frac{\tan^5(x)}{5} + C$

20. $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$

21. $\frac{\tan^6(x)}{6} + \frac{\tan^4(x)}{4} + C$

22. $\frac{\tan^4(x)}{4} + C$

23. $\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3} + C$

24. $\frac{\sec^9(x)}{9} - \frac{2\sec^7(x)}{7} + \frac{\sec^5(x)}{5} + C$

25. $\frac{1}{3}\tan^3 x - \tan x + x + C$

26. $\frac{1}{4}\tan x\sec^3 x + \frac{3}{8}\left(\sec x\tan x + \ln|\sec x + \tan x|\right) + C$

27. $\frac{1}{2}\left(\sec x\tan x - \ln|\sec x + \tan x|\right) + C$

28. $\frac{1}{4}\tan x\sec^3 x - \frac{1}{8}\left(\sec x\tan x + \ln|\sec x + \tan x|\right) + C$

29. $\frac{2}{5}$

30. 0

31. $32/315$

32. $1/2$

33. $2/3$

34. $1/5$

35. $16/15$

Section 6.4

1. rational
2. T
3. $\frac{A}{x} + \frac{B}{x-3}$
4. $\frac{A}{x-3} + \frac{B}{x+3}$
5. $\frac{A}{x-\sqrt{7}} + \frac{B}{x+\sqrt{7}}$
6. $\frac{A}{x} + \frac{Bx+C}{x^2+7}$
7. $3\ln|x-2| + 4\ln|x+5| + C$
8. $9\ln|x+1| - 2\ln|x| + C$
9. $\frac{1}{3}(\ln|x+2| - \ln|x-2|) + C$
10. $\ln|x+1| + \ln|3x+1| + C$
11. $\ln|x+5| - \frac{2}{x+5} + C$
12. $-\frac{4}{x+8} - 3\ln|x+8| + C$
13. $\frac{5}{x+1} + 7\ln|x| + 2\ln|x+1| + C$
14. $-\ln|2x-3| + 5\ln|x-1| + 2\ln|x+3| + C$
15. $-\frac{1}{5}\ln|5x-1| + \frac{2}{3}\ln|3x-1| + \frac{3}{7}\ln|7x+3| + C$
16. $x + \ln|x-1| - \ln|x+2| + C$
17. $\frac{x^2}{2} + x + \frac{125}{9}\ln|x-5| + \frac{64}{9}\ln|x+4| - \frac{35}{2} + C$
18. $2x + C$
19. $\frac{1}{6}\left(-\ln\left|x^2+2x+3\right| + 2\ln|x| - \sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)\right) + C$
20. $-\frac{3}{2}\ln\left|x^2+4x+10\right| + x + \frac{\tan^{-1}\left(\frac{x+2}{\sqrt{6}}\right)}{\sqrt{6}} + C$
21. $\ln\left|3x^2+5x-1\right| + 2\ln|x+1| + C$
22. $2\ln|x-3| + 2\ln|x^2+6x+10| - 4\tan^{-1}(x+3) + C$
23. $\frac{9}{10}\ln\left|x^2+9\right| + \frac{1}{5}\ln|x+1| - \frac{4}{15}\tan^{-1}\left(\frac{x}{3}\right) + C$
24. $\frac{1}{2}\left(3\ln\left|x^2+2x+17\right| - 4\ln|x-7| + \tan^{-1}\left(\frac{x+1}{4}\right)\right) + C$
25. $3\left(\ln\left|x^2-2x+11\right| + \ln|x-9|\right) + 3\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{x-1}{\sqrt{10}}\right) + C$
26. $\frac{1}{2}\ln\left|x^2+10x+27\right| + 5\ln|x+2| - 6\sqrt{2}\tan^{-1}\left(\frac{x+5}{\sqrt{2}}\right) + C$
27. $\ln(2000/243) \approx 2.108$
28. $5\ln(9/4) - \frac{1}{3}\ln(17/2) \approx 3.3413$
29. $-\pi/4 + \tan^{-1}3 - \ln(11/9) \approx 0.263$
30. $1/8$

Section 6.5

1. Because $\cosh x$ is always positive.

2. The points on the left hand side can be defined as $(-\cosh x, \sinh x)$.

3.
$$\begin{aligned}\coth^2 x - \operatorname{csch}^2 x &= \left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)^2 - \left(\frac{2}{e^x-e^{-x}}\right)^2\\ &= \frac{(e^{2x}+2+e^{-2x})-(4)}{e^{2x}-2+e^{-2x}}\\ &= \frac{e^{2x}-2+e^{-2x}}{e^{2x}-2+e^{-2x}}\\ &= 1\end{aligned}$$

4.
$$\begin{aligned}\cosh^2 x + \sinh^2 x &= \left(\frac{e^x+e^{-x}}{2}\right)^2 + \left(\frac{e^x-e^{-x}}{2}\right)^2\\ &= \frac{e^{2x}+2+e^{-2x}}{4} + \frac{e^{2x}-2+e^{-2x}}{4}\\ &= \frac{2e^{2x}+2e^{-2x}}{4}\\ &= \frac{e^{2x}+e^{-2x}}{2}\\ &= \cosh 2x.\end{aligned}$$

5.
$$\begin{aligned}\cosh^2 x &= \left(\frac{e^x+e^{-x}}{2}\right)^2\\ &= \frac{e^{2x}+2+e^{-2x}}{4}\\ &= \frac{1}{2}\,\frac{(e^{2x}+e^{-2x})+2}{2}\\ &= \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}+1\right)\\ &= \frac{\cosh 2x+1}{2}.\end{aligned}$$

6.
$$\begin{aligned}\sinh^2 x &= \left(\frac{e^x-e^{-x}}{2}\right)^2\\ &= \frac{e^{2x}-2+e^{-2x}}{4}\\ &= \frac{1}{2}\,\frac{(e^{2x}+e^{-2x})-2}{2}\\ &= \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}-1\right)\\ &= \frac{\cosh 2x-1}{2}.\end{aligned}$$

7.
$$\begin{aligned}\frac{d}{dx}\left[\operatorname{sech} x\right] &= \frac{d}{dx}\left[\frac{2}{e^x+e^{-x}}\right]\\ &= \frac{-2(e^x-e^{-x})}{(e^x+e^{-x})^2}\\ &= -\frac{2(e^x-e^{-x})}{(e^x+e^{-x})(e^x+e^{-x})}\\ &= -\frac{2}{e^x+e^{-x}}\cdot\frac{e^x-e^{-x}}{e^x+e^{-x}}\\ &= -\operatorname{sech} x\tanh x\end{aligned}$$

8. $$\frac{d}{dx}[\coth x] = \frac{d}{dx}\left[\frac{e^x+e^{-x}}{e^x-e^{-x}}\right]$$
$$= \frac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$$
$$= \frac{e^{2x}+e^{-2x}-2-(e^{2x}+e^{-2x}+2)}{(e^x-e^{-x})^2}$$
$$= -\frac{4}{(e^x-e^{-x})^2}$$
$$= -\operatorname{csch}^2 x$$

9. $\int \tanh x\,dx = \int \frac{\sinh x}{\cosh x}\,dx$
Let $u = \cosh x$; $du = (\sinh x)dx$
$$= \int \frac{1}{u}\,du$$
$$= \ln|u| + C$$
$$= \ln(\cosh x) + C.$$

10. $$\int \coth x\,dx = \int \frac{\cosh x}{\sinh x}\,dx$$
Let $u = \sinh x$; $du = (\cosh x)dx$
$$= \int \frac{1}{u}\,du$$
$$= \ln|u| + C$$
$$= \ln|\sinh x| + C.$$

11. $2\cosh 2x$
12. Taking the derivative of $(\cosh x)^2$ directly, one gets $2\cosh x \sinh x$; using the identity $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$ first, one gets $\sinh 2x$; by Key Idea 6.6.1, these are equal.
13. $2x\sec^2(x^2)$
14. $\coth x$
15. $\sinh^2 x + \cosh^2 x$
16. $x\cosh x$
17. $\frac{-2x}{(x^2)\sqrt{1-x^4}}$
18. $\frac{3}{\sqrt{9x^2+1}}$
19. $\frac{4x}{\sqrt{4x^4-1}}$
20. $\frac{1}{1-(x+5)^2}$
21. $-\csc x$
22. $\sec x$
23. $y = x$
24. $y = \frac{3}{4}(x - \ln 2) + \frac{5}{4}$
25. $y = \frac{9}{25}(x + \ln 3) - \frac{4}{5}$
26. $y = -\frac{72}{125}(x - \ln 3) + \frac{9}{25}$
27. $y = x$
28. $y = (x - \sqrt{2}) + \cosh^{-1}(\sqrt{2}) \approx (x - 1.414) + 0.881$
29. $1/2\ln(\cosh(2x)) + C$
30. $1/3\sinh(3x-7) + C$
31. $1/2\sinh^2 x + C$ or $1/2\cosh^2 x + C$
32. $x\sinh(x) - \cosh(x) + C$
33. $x\cosh(x) - \sinh(x) + C$
34. $\sinh^{-1} x + C = \ln\left(x + \sqrt{x^2+1}\right) + C$
35. $\cosh^{-1} x/3 + C = \ln\left(x + \sqrt{x^2-9}\right) + C$
36. $\begin{cases} \frac{1}{3}\tanh^{-1}\left(\frac{x}{3}\right) + C & x^2 < 9 \\ \frac{1}{3}\coth^{-1}\left(\frac{x}{3}\right) + C & 9 < x^2 \end{cases} =$
$\frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + C$
37. $\cosh^{-1}(x^2/2) + C = \ln(x^2 + \sqrt{x^4-4}) + C$
38. $2/3\sinh^{-1} x^{3/2} + C = 2/3\ln(x^{3/2} + \sqrt{x^3+1}) + C$
39. $\frac{1}{16}\tan^{-1}(x/2) + \frac{1}{32}\ln|x-2| + \frac{1}{32}\ln|x+2| + C$
40. $\ln x - \ln|x+1| + C$
41. $\tan^{-1}(e^x) + C$
42. $x\sinh^{-1} x - \sqrt{x^2+1} + C$
43. $x\tanh^{-1} x + 1/2\ln|x^2-1| + C$
44. $\tan^{-1}(\sinh x) + C$
45. 0
46. $3/2$
47. 2
48. $\sinh^{-1} 2 = \ln\left(2 + \sqrt{5}\right) \approx 1.444$

Section 6.6

1. $0/0, \infty/\infty, 0\cdot\infty, \infty-\infty, 0^0, 1^\infty, \infty^0$
2. F
3. F
4. The base of an expression is approaching 1 while its power is growing without bound.
5. derivatives; limits
6. Answers will vary.
7. Answers will vary.
8. Answers will vary.
9. 3
10. $-5/3$
11. -1
12. $-\sqrt{2}/2$
13. 5
14. 0
15. $2/3$
16. a/b
17. ∞
18. $1/2$
19. 0
20. 0
21. 0
22. ∞
23. ∞
24. ∞
25. 0
26. 2
27. -2
28. 0
29. 0
30. 0
31. 0

32. 0
33. ∞
34. ∞
35. ∞
36. 0
37. 0
38. e
39. 1
40. 1
41. 1
42. 1
43. 1
44. 0
45. 1
46. 1
47. 1
48. 1
49. 2
50. $1/2$
51. $-\infty$
52. 1
53. 0
54. 3

Section 6.7

1. The interval of integration is finite, and the integrand is continuous on that interval.
2. converge
3. converges; could also state < 10.
4. $p > 1$
5. $p > 1$
6. $p < 1$
7. $e^5/2$
8. $1/2$
9. $1/3$
10. $\pi/3$
11. $1/\ln 2$
12. diverges
13. diverges
14. $-1/4\ln(1/5) = 1/2\tanh^{-1}(2/3) \approx 0.4024$
15. 1
16. diverges
17. diverges
18. diverges
19. diverges
20. diverges
21. diverges
22. $2 + 2\sqrt{2}$
23. 1
24. $1/2$
25. 0
26. $\pi/2$
27. $-1/4$
28. $-1/9$
29. diverges
30. -1
31. 1
32. diverges
33. $1/2$
34. $1/2$
35. diverges; Limit Comparison Test with $1/x$.
36. converges; Limit Comparison Test with $1/x^{3/2}$.
37. diverges; Limit Comparison Test with $1/x$.
38. converges; Direct Comparison Test with xe^{-x}.
39. converges; Direct Comparison Test with e^{-x}.
40. converges; Direct Comparison Test with xe^{-x}.
41. converges; Direct Comparison Test with $1/(x^2 - 1)$.
42. diverges; Direct Comparison Test with $x/(x^2 + 1)$.
43. converges; Direct Comparison Test with $1/e^x$.
44. converges; Limit Comparison Test with $1/e^x$.

Section 6.8

1. backwards
2. $5\sin\theta$
3. (a) $\tan^2\theta + 1 = \sec^2\theta$
   (b) $9\sec^2\theta$.
4. Because we are considering $a > 0$ and $x = a\sin\theta$, which means $\theta = \sin^{-1}(x/a)$. The arcsine function has a domain of $-\pi/2 \leq \theta \leq \pi/2$; on this domain, $\cos\theta \geq 0$, so $a\cos\theta$ is always non-negative, allowing us to drop the absolute value signs.
5. $\frac{1}{2}\left(x\sqrt{x^2+1} + \ln|\sqrt{x^2+1} + x|\right) + C$
6. $2\left(\frac{x}{4}\sqrt{x^2+4} + \ln|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}|\right) + C$
7. $\frac{1}{2}\left(\sin^{-1}x + x\sqrt{1-x^2}\right) + C$
8. $\frac{1}{2}\left(9\sin^{-1}(x/3) + x\sqrt{9-x^2}\right) + C$
9. $\frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln|x + \sqrt{x^2-1}| + C$
10. $\frac{1}{2}x\sqrt{x^2-16} - 8\ln\left|\frac{x}{4} + \frac{\sqrt{x^2-16}}{4}\right| + C$
11. $x\sqrt{x^2+1/4} + \frac{1}{4}\ln|2\sqrt{x^2+1/4} + 2x| + C =$ $\frac{1}{2}x\sqrt{4x^2+1} + \frac{1}{4}\ln|\sqrt{4x^2+1} + 2x| + C$
12. $\frac{1}{6}\sin^{-1}(3x) + \frac{3}{2}\sqrt{1/9 - x^2} + C = \frac{1}{6}\sin^{-1}(3x) + \frac{1}{2}\sqrt{1-9x^2} + C$
13. $4\left(\frac{1}{2}x\sqrt{x^2-1/16} - \frac{1}{32}\ln|4x + 4\sqrt{x^2-1/16}|\right) + C =$ $\frac{1}{2}x\sqrt{16x^2-1} - \frac{1}{8}\ln|4x + \sqrt{16x^2-1}| + C$
14. $8\ln\left|\frac{\sqrt{x^2+2}}{\sqrt{2}} + \frac{x}{\sqrt{2}}\right| + C$; with Section 6.6, we can state the answer as $8\sinh^{-1}(x/\sqrt{2}) + C$.

15. $3\sin^{-1}\left(\frac{x}{\sqrt{7}}\right)+C$ (Trig. Subst. is not needed)

16. $5\ln\left|\frac{x}{\sqrt{8}}+\frac{\sqrt{x^2-8}}{\sqrt{8}}\right|+C$

17. $\sqrt{x^2-11}-\sqrt{11}\sec^{-1}(x/\sqrt{11})+C$

18. $\frac{1}{2}\left(\tan^{-1}x+\frac{x}{x^2+1}\right)+C$

19. $\sqrt{x^2-3}+C$ (Trig. Subst. is not needed)

20. $\frac{1}{8}\sin^{-1}x-\frac{1}{8}x\sqrt{1-x^2}(1-2x^2)+C$

21. $-\frac{1}{\sqrt{x^2+9}}+C$ (Trig. Subst. is not needed)

22. $\frac{5}{2}x\sqrt{x^2-10}+25\ln\left|\frac{x}{\sqrt{10}}+\frac{\sqrt{x^2-10}}{\sqrt{10}}\right|+C$

23. $\frac{1}{18}\frac{x+2}{x^2+4x+13}+\frac{1}{54}\tan^{-1}\left(\frac{x+2}{2}\right)+C$

24. $\frac{x}{\sqrt{1-x^2}}-\sin^{-1}x+C$

25. $\frac{1}{7}\left(-\frac{\sqrt{5-x^2}}{x}-\sin^{-1}(x/\sqrt{5})\right)+C$

26. $\frac{1}{2}x\sqrt{x^2+3}-\frac{3}{2}\ln\left|\frac{\sqrt{x^2+3}}{\sqrt{3}}+\frac{x}{\sqrt{3}}\right|+C$

27. $\pi/2$

28. $16\sqrt{3}-8\ln(2+\sqrt{3})$

29. $2\sqrt{2}+2\ln(1+\sqrt{2})$

30. $\pi/4+1/2)$

31. $9\sin^{-1}(1/3)+\sqrt{8}$ Note: the new lower bound is $\theta=\sin^{-1}(-1/3)$ and the new upper bound is $\theta=\sin^{-1}(1/3)$. The final answer comes with recognizing that $\sin^{-1}(-1/3)=-\sin^{-1}(1/3)$ and that $\cos\left(\sin^{-1}(1/3)\right)=\cos\left(\sin^{-1}(-1/3)\right)=\sqrt{8}/3$.

32. $\pi/8$

Chapter 7

Section 7.1

1. T
2. T
3. Answers will vary.
4. Answers may vary; one common answer is when the region has two or more "top" or "bottom" functions when viewing the region with respect to *x*, but has only 1 "top" function and 1 "bottom" function when viewed with respect to *y*. The former area requires multiple integrals to compute, whereas the latter area requires one.
5. $4\pi+\pi^2\approx 22.436$
6. $16/3$
7. π
8. π
9. $1/2$
10. $2\sqrt{2}$
11. $1/\ln 4$
12. $4/3$
13. 4.5
14. $4/3$
15. $2-\pi/2$
16. 8
17. $1/6$
18. $37/12$
19. All enclosed regions have the same area, with regions being the reflection of adjacent regions. One region is formed on $[\pi/4, 5\pi/4]$, with area $2\sqrt{2}$.
20. On regions such as $[\pi/6, 5\pi/6]$, the area is $3\sqrt{3}/2$. On regions such as $[-\pi/2, \pi/6]$, the area is $3\sqrt{3}/4$.
21. 1
22. $5/3$
23. $9/2$
24. $9/4$
25. $1/12(9-2\sqrt{2})\approx 0.514$
26. $4/3$
27. 1
28. 5
29. 4
30. $133/20$
31. 219,000 ft^2
32. 623,333 ft^2

Section 7.2

1. T
2. Answers will vary.
3. Recall that "dx" does not just "sit there;" it is multiplied by $A(x)$ and represents the thickness of a small slice of the solid. Therefore dx has units of in, giving $A(x)\,dx$ the units of in^3.
4. volume
5. $48\pi\sqrt{3}/5$ units^3
6. $175\pi/3$ units^3
7. $\pi^2/4$ units^3
8. $\pi/6$ units^3
9. $9\pi/2$ units^3
10. $35\pi/3$ units^3
11. $\pi^2-2\pi$ units^3
12. $2\pi/15$ units^3
13. (a) $\pi/2$
    (b) $5\pi/6$
    (c) $4\pi/5$
    (d) $8\pi/15$
14. (a) $512\pi/15$
    (b) $256\pi/5$
    (c) $832\pi/15$
    (d) $128\pi/3$
15. (a) $4\pi/3$
    (b) $2\pi/3$
    (c) $4\pi/3$
    (d) $\pi/3$
16. (a) $104\pi/15$
    (b) $64\pi/15$
    (c) $32\pi/5$

17. (a) $\pi^2/2$
    (b) $\pi^2/2 - 4\pi\sinh^{-1}(1)$
    (c) $\pi^2/2 + 4\pi\sinh^{-1}(1)$
18. (a) 8π
    (b) 8π
    (c) $16\pi/3$
    (d) $8\pi/3$
19. Placing the tip of the cone at the origin such that the x-axis runs through the center of the circular base, we have $A(x) = \pi x^2/4$. Thus the volume is $250\pi/3$ units^3.
20. The cross–sections of this cone are the same as the cone in Exercise 19. Thus they have the same volume of $250\pi/3$ units^3.
21. Orient the cone such that the tip is at the origin and the x-axis is perpendicular to the base. The cross–sections of this cone are right, isosceles triangles with side length $2x/5$; thus the cross–sectional areas are $A(x) = 2x^2/25$, giving a volume of $80/3$ units^3.
22. Orient the solid so that the x-axis is parallel to long side of the base. All cross–sections are trapezoids (at the far left, the trapezoid is a square; at the far right, the trapezoid has a top length of 0, making it a triangle). The area of the trapezoid at x is $A(x) = 1/2(-1/2x + 5 + 5)(5) = -5/4x + 25$. The volume is 187.5 units^3.

Section 7.3

1. T
2. F
3. F
4. T
5. $9\pi/2$ units^3
6. $70\pi/3$ units^3
7. $\pi^2 - 2\pi$ units^3
8. $2\pi/15$ units^3
9. $48\pi\sqrt{3}/5$ units^3
10. $350\pi/3$ units^3
11. $\pi^2/4$ units^3
12. $\pi/6$ units^3
13. (a) $4\pi/5$
    (b) $8\pi/15$
    (c) $\pi/2$
    (d) $5\pi/6$
14. (a) $128\pi/3$
    (b) $128\pi/3$
    (c) $512\pi/15$
    (d) $256\pi/5$
15. (a) $4\pi/3$
    (b) $\pi/3$
    (c) $4\pi/3$
    (d) $2\pi/3$
16. (a) $16\pi/3$
    (b) $8\pi/3$
    (c) 8π
17. (a) $2\pi(\sqrt{2} - 1)$
    (b) $2\pi(1 - \sqrt{2} + \sinh^{-1}(1))$
18. (a) $16\pi/3$
    (b) $8\pi/3$
    (c) 8π
    (d) 8π

Section 7.4

1. T
2. F
3. $\sqrt{2}$
4. 6
5. $4/3$
6. 6
7. $109/2$
8. $3/2$
9. $12/5$
10. $79953333/400000 \approx 199.883$
11. $-\ln(2 - \sqrt{3}) \approx 1.31696$
12. $\sinh^{-1} 1$
13. $\int_0^1 \sqrt{1 + 4x^2}\, dx$
14. $\int_0^1 \sqrt{1 + 100x^{18}}\, dx$
15. $\int_0^1 \sqrt{1 + \frac{1}{4x}}\, dx$
16. $\int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$
17. $\int_{-1}^1 \sqrt{1 + \frac{x^2}{1-x^2}}\, dx$
18. $\int_{-3}^3 \sqrt{1 + \frac{x^2}{81-9x^2}}\, dx$
19. $\int_1^2 \sqrt{1 + \frac{1}{x^4}}\, dx$
20. $\int_{-\pi/4}^{\pi/4} \sqrt{1 + \sec^2 x \tan^2 x}\, dx$
21. 1.4790
22. 1.8377
23. Simpson's Rule fails, as it requires one to divide by 0. However, recognize the answer should be the same as for $y = x^2$; why?
24. 2.1300
25. Simpson's Rule fails.
26. Simpson's Rule fails.
27. 1.4058
28. 1.7625
29. $2\pi\int_0^1 2x\sqrt{5}\, dx = 2\pi\sqrt{5}$
30. $2\pi\int_0^1 x\sqrt{1 + 4x^2}\, dx = \pi/6(5\sqrt{5} - 1)$
31. $2\pi\int_0^1 x^3\sqrt{1 + 9x^4}\, dx = \pi/27(10\sqrt{10} - 1)$
32. $2\pi\int_0^1 \sqrt{x}\sqrt{1 + 1/(4x)}\, dx = \pi/6(5\sqrt{5} - 1)$
33. $2\pi\int_0^1 \sqrt{1 - x^2}\sqrt{1 + x/(1 - x^2)}\, dx = 4\pi$

Section 7.5

1. In SI units, it is one joule, i.e., one Newton–meter, or kg·m/s^2·m. In Imperial Units, it is ft–lb.
2. The same.
3. Smaller.

4. force; distance
5. (a) 500 ft–lb
   (b) $100 - 50\sqrt{2} \approx 29.29$ ft–lb
6. (a) 2450 J
   (b) 1568 J
7. (a) $\frac{1}{2} \cdot d \cdot l^2$ ft–lb
   (b) 75 %
   (c) $\ell(1 - \sqrt{2}/2) \approx 0.2929\ell$
8. 735 J
9. (a) 756 ft–lb
   (b) 60,000 ft–lb
   (c) Yes, for the cable accounts for about 1% of the total work.
10. 11,100 ft–lb
11. 575 ft–lb
12. 125 ft–lb
13. 0.05 J
14. 12.5 ft–lb
15. 5/3 ft–lb
16. 0.2625 = 21/80 J
17. $f \cdot d/2$ J
18. 45 ft–lb
19. 5 ft–lb
20. $953,284$ J
21. (a) 52,929.6 ft–lb
    (b) 18,525.3 ft–lb
    (c) When 3.83 ft of water have been pumped from the tank, leaving about 2.17 ft in the tank.
22. 192,767 ft–lb. Note that the tank is oriented horizontally. Let the origin be the center of one of the circular ends of the tank. Since the radius is 3.75 ft, the fluid is being pumped to $y = 4.75$; thus the distance the gas travels is $h(y) = 4.75 - y$. A differential element of water is a rectangle, with length 20 and width $2\sqrt{3.75^2 - y^2}$. Thus the force required to move that slab of gas is $F(y) = 40 \cdot 45.93 \cdot \sqrt{3.75^2 - y^2}dy$. Total work is $\int_{-3.75}^{3.75} 40 \cdot 45.93 \cdot (4.75 - y)\sqrt{3.75^2 - y^2}\, dy$. This can be evaluated without actual integration; split the integral into $\int_{-3.75}^{3.75} 40 \cdot 45.93 \cdot (4.75)\sqrt{3.75^2 - y^2}\, dy + \int_{-3.75}^{3.75} 40 \cdot 45.93 \cdot (-y)\sqrt{3.75^2 - y^2}\, dy$. The first integral can be evaluated as measuring half the area of a circle; the latter integral can be shown to be 0 without much difficulty. (Use substitution and realize the bounds are both 0.)
23. 212,135 ft–lb
24. (a) approx. 577,000 J
    (b) approx. 399,000 J
    (c) approx 110,000 J (By volume, half of the water is between the base of the cone and a height of 3.9685 m. If one rounds this to 4 m, the work is approx 104,000 J.)
25. 187,214 ft–lb
26. 617,400 J
27. 4,917,150 J

Section 7.6

1. Answers will vary.
2. Answers will vary.
3. 499.2 lb
4. 249.6 lb
5. 6739.2 lb
6. 5241.6 lb
7. 3920.7 lb
8. 15682.8 lb
9. 2496 lb
10. 2496 lb
11. 602.59 lb
12. 291.2 lb
13. (a) 2340 lb
    (b) 5625 lb
14. (a) 1064.96 lb
    (b) 2560 lb
15. (a) 1597.44 lb
    (b) 3840 lb
16. (a) 41.6 lb
    (b) 100 lb
17. (a) 56.42 lb
    (b) 135.62 lb
18. (a) 1123.2 lb
    (b) 2700 lb
19. 5.1 ft
20. 4.1 ft

Chapter 8

Section 8.1

1. Answers will vary.
2. natural
3. Answers will vary.
4. Answers will vary.
5. $2, \frac{8}{3}, \frac{8}{3}, \frac{32}{15}, \frac{64}{45}$
6. $-\frac{3}{2}, \frac{9}{4}, -\frac{27}{8}, \frac{81}{16}, -\frac{243}{32}$
7. $-\frac{1}{3}, -2, -\frac{81}{5}, -\frac{512}{3}, -\frac{15625}{7}$
8. $1, 1, 2, 3, 5$
9. $a_n = 3n + 1$
10. $a_n = (-1)^{n+1}\frac{3}{2^{n-1}}$
11. $a_n = 10 \cdot 2^{n-1}$
12. $a_n = 1/(n-1)!$
13. 1/7
14. $3e^2 - 1$
15. 0
16. e^4
17. diverges
18. converges to $4/3$
19. converges to 0
20. converges to 0

21. diverges
22. converges to 3
23. converges to e
24. converges to 5
25. converges to 0
26. diverges
27. converges to 2
28. converges to 0
29. bounded
30. neither bounded above or below
31. bounded
32. bounded below
33. neither bounded above or below
34. bounded above
35. monotonically increasing
36. monotonically increasing for $n \geq 3$
37. never monotonic
38. monotonically decreasing for $n \geq 3$
39. Let $\{a_n\}$ be given such that $\lim\limits_{n\to\infty} |a_n| = 0$. By the definition of the limit of a sequence, given any $\varepsilon > 0$, there is a m such that for all $n > m$, $\big| |a_n| - 0 \big| < \varepsilon$. Since $\big| |a_n| - 0 \big| = |a_n - 0|$, this directly implies that for all $n > m$, $|a_n - 0| < \varepsilon$, meaning that $\lim\limits_{n\to\infty} a_n = 0$.
40. (a) A sketch of one proof method:
    By Theorem 8.1.3 we know that the limit of the sequence $\{b_n - a_n\} = K - L$. One can show that since $b_n - a_n \geq 0$ for all n, then $K - L \geq 0$, concluding that $L \leq K$.
    (b) $a_n = 1/3^n$ and $b_n = 1/2^n$
41. A sketch of one proof method:
    Let any $\varepsilon > 0$ be given. Since $\{a_n\}$ and $\{b_n\}$ converge, there exists an $N > 0$ such that for all $n \geq N$, both a_n and b_n are within $\varepsilon/2$ of L; we can conclude that they are at most ε apart from each other. Since $a_n \leq c_n \leq b_n$, one can show that c_n is within ε of L, showing that $\{c_n\}$ also converges to L.
42. A sketch of one proof method:
    (a) Assume that Theorem 8.1.5 is true, and let $\{a_n\}$ be bounded and monotonic. Since $\{a_n\}$ is bounded, it is bounded both above and below. If it is increasing, it is bounded above and we apply Theorem 8.1.5; if it is decreasing, it is bounded below and we apply the theorem. Either way, $\{a_n\}$ converges and the statement is true.
    (b) Assume the statement is true, and let $\{a_n\}$ be a monotonically increasing sequence that is bounded above. Since $\{a_n\}$ is monotonically increasing, $a_1 \leq a_2 \leq \ldots$; that is, a_1 bounds $\{a_n\}$ from below. Therefore $\{a_n\}$ is bounded and monotonic; by the statement, $\{a_n\}$ converges. A similar statement can be made for when $\{a_n\}$ is monotonically decreasing and bounded below. Therefore Theorem 8.1.5 is true.

Section 8.2

1. Answers will vary.
2. Answers will vary.
3. One sequence is the sequence of terms $\{a_n\}$. The other is the sequence of n^{th} partial sums, $\{S_n\} = \{\sum_{i=1}^{n} a_i\}$.
4. Answers will vary.
5. F
6. F
7. (a) $-1, -\frac{1}{2}, -\frac{5}{6}, -\frac{7}{12}, -\frac{47}{60}$
   (b) Plot omitted
8. (a) $1, \frac{5}{4}, \frac{49}{36}, \frac{205}{144}, \frac{5269}{3600}$
   (b) Plot omitted
9. (a) $-1, 0, -1, 0, -1$
   (b) Plot omitted
10. (a) $1, 3, 6, 10, 15$
    (b) Plot omitted
11. (a) $1, \frac{3}{2}, \frac{5}{3}, \frac{41}{24}, \frac{103}{60}$
    (b) Plot omitted
12. (a) $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \frac{121}{243}$
    (b) Plot omitted
13. (a) $-0.9, -0.09, -0.819, -0.1629, -0.75339$
    (b) Plot omitted
14. (a) $0.1, 0.11, 0.111, 0.1111, 0.11111$
    (b) Plot omitted
15. $\lim\limits_{n\to\infty} a_n = 3$; by Theorem 8.2.4 the series diverges.
16. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 8.2.4 the series diverges.
17. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 8.2.4 the series diverges.
18. $\lim\limits_{n\to\infty} a_n = 1$; by Theorem 8.2.4 the series diverges.
19. $\lim\limits_{n\to\infty} a_n = 1/2$; by Theorem 8.2.4 the series diverges.
20. $\lim\limits_{n\to\infty} a_n = e$; by Theorem 8.2.4 the series diverges.
21. Converges; p-series with $p = 5$.
22. Converges; geometric series with $r = 1/5$.
23. Diverges; geometric series with $r = 6/5$.
24. Converges; p-series with $p = 4$.
25. Diverges; fails n^{th} term test
26. Converges; by Key Idea 8.2.1 and Theorem 8.2.3, series converges to $10e$.
27. F
28. Converges; general p-series with $p = 2$.
29. Diverges; by Theorem 8.2.3 this is half the Harmonic Series, which diverges by growing without bound. "Half of growing without bound" is still growing without bound.
30. Diverges; general p-series with $p = 1$.
31. (a) $S_n = \frac{1-(1/4)^n}{3/4}$
    (b) Converges to $4/3$.
32. (a) $S_n = 2n$
    (b) Diverges.
33. (a) $S_n = \left(\frac{n(n+1)}{2}\right)^2$
    (b) Diverges
34. (a) $S_n = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ -\frac{n}{2} & \text{n is even} \end{cases}$
    (b) Diverges
35. (a) $S_n = 5\frac{1-1/2^n}{1/2}$

(b) Converges to 10.

36. (a) $S_n = \frac{1-(1/e)^{n+1}}{1-1/e}$.

(b) Converges to $1/(1-1/e) = e/(e-1)$.

37. (a) $S_n = \frac{1-(-1/3)^n}{4/3}$

(b) Converges to $3/4$.

38. (a) With partial fractions, $a_n = \frac{1}{n} - \frac{1}{n+1}$. Thus $S_n = 1 - \frac{1}{n+1}$.

(b) Converges to 1.

39. (a) With partial fractions, $a_n = \frac{3}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$. Thus $S_n = \frac{3}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$.

(b) Converges to $9/4$

40. (a) Use partial fraction decomposition to recognize the telescoping series: $a_n = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$. Then $S_n = \frac{n}{2n+1}$.

(b) Converges to $1/2$.

41. (a) $S_n = \ln\big(1/(n+1)\big)$

(b) Diverges (to $-\infty$).

42. (a) $S_n = 1 - \frac{1}{(n+1)^2}$

(b) Converges to 1.

43. (a) $a_n = \frac{1}{n(n+3)}$; using partial fractions, the resulting telescoping sum reduces to $S_n = \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right)$

(b) Converges to $11/18$.

44. (a) $a_n = 1/2^n + 1/3^n$ for $n \geq 0$. Thus $S_n = \frac{1-1/2^2}{1/2} + \frac{1-1/3^n}{2/3}$.

(b) Converges to $2 + 3/2 = 7/2$.

45. (a) With partial fractions, $a_n = \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$. Thus $S_n = \frac{1}{2}\left(3/2 - \frac{1}{n} - \frac{1}{n+1}\right)$.

(b) Converges to $3/4$.

46. (a) $S_n = \frac{1-(\sin 1)^{n+1}}{1-\sin 1}$

(b) Converges to $\frac{1}{1-\sin 1}$..

47. (a) The n^{th} partial sum of the odd series is $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. The n^{th} partial sum of the even series is $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n}$. Each term of the even series is less than the corresponding term of the odd series, giving us our result.

(b) The n^{th} partial sum of the odd series is $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. The n^{th} partial sum of 1 plus the even series is $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2(n-1)}$. Each term of the even series is now greater than or equal to the corresponding term of the odd series, with equality only on the first term. This gives us the result.

(c) If the odd series converges, the work done in (a) shows the even series converges also. (The sequence of the n^{th} partial sum of the even series is bounded and monotonically increasing.) Likewise, (b) shows that if the even series converges, the odd series will, too. Thus if either series converges, the other does.

Similarly, (a) and (b) can be used to show that if either series diverges, the other does, too.

(d) If both the even and odd series converge, then their sum would be a convergent series. This would imply that the Harmonic Series, their sum, is convergent. It is not. Hence each series diverges.

48. Using partial fractions, we can show that $a_n = \frac{1}{4}\left(\frac{1}{2n-1} + \frac{1}{2n+1}\right)$. The series is effectively twice the sum of the odd terms of the Harmonic Series which was shown to diverge in Exercise 47. Thus this series diverges.

Section 8.3

1. continuous, positive and decreasing
2. F
3. The Integral Test (we do not have a continuous definition of $n!$ yet) and the Limit Comparison Test (same as above, hence we cannot take its derivative).
4. $\displaystyle\sum_{n=0}^{\infty} b_n$ converges; we cannot conclude anything about $\displaystyle\sum_{n=0}^{\infty} c_n$
5. Converges
6. Converges
7. Diverges
8. Diverges
9. Converges
10. Converges
11. Converges
12. Converges
13. Converges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$, as $1/(n^2+3n-5) \leq 1/n^2$ for all $n > 1$.
14. Converges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$, as $1/(4^n + n^2 - n) \leq 1/4^n$ for all $n \geq 1$.
15. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, as $1/n \leq \ln n/n$ for all $n \geq 3$.
16. Converges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$, as $1/(n!+n) \leq 1/n!$ for all $n \geq 1$.
17. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Since $n = \sqrt{n^2} > \sqrt{n^2-1}$, $1/n \leq 1/\sqrt{n^2-1}$ for all $n \geq 2$.
18. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, as $1/\sqrt{n} \leq 1/(\sqrt{n}-2)$ for all $n \geq 5$.
19. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$:
$$\frac{1}{n} = \frac{n^2}{n^3} < \frac{n^2+n+1}{n^3} < \frac{n^2+n+1}{n^3-5},$$
for all $n \geq 1$.
20. Converges; compare to $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$, as $2^n/(5^n+10) < 2^n/5^n$ for all $n \geq 1$.
21. Diverges; compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Note that
$$\frac{n}{n^2-1} = \frac{n^2}{n^2-1} \cdot \frac{1}{n} > \frac{1}{n},$$
as $\frac{n^2}{n^2-1} > 1$, for all $n \geq 2$.

22. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, as $1/(n^2 \ln n) \leq 1/n^2$ for all $n \geq 2$.

23. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

24. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{4^n}$.

25. Diverges; compare to $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

26. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

27. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

28. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

29. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$. Just as $\lim_{n\to 0} \frac{\sin n}{n} = 1$, $\lim_{n\to\infty} \frac{\sin(1/n)}{1/n} = 1$.

30. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

31. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

32. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$.

33. Converges; Integral Test

34. Converges; Integral Test, p-Series Test, Direct & Limit Comparison Tests can all be used.

35. Diverges; the n^{th} Term Test and Direct Comparison Test can be used.

36. Converges; the Direct Comparison Test can be used with sequence $1/(n-1)!$.

37. Converges; the Direct Comparison Test can be used with sequence $1/3^n$.

38. Diverges; the n^{th} Term Test can be used, along with the Limit Comparison Test (compare with 1/10).

39. Diverges; the n^{th} Term Test can be used, along with the Integral Test.

40. Diverges; the Limit Comparison Test can be used with sequence $1/\sqrt{n}$.

41. (a) Converges; use Direct Comparison Test as $\frac{a_n}{n} < n$.
    (b) Converges; since original series converges, we know $\lim_{n\to\infty} a_n = 0$. Thus for large n, $a_n a_{n+1} < a_n$.
    (c) Converges; similar logic to part (b) so $(a_n)^2 < a_n$.
    (d) May converge; certainly $na_n > a_n$ but that does not mean it does not converge.
    (e) Does not converge, using logic from (b) and n^{th} Term Test.

Section 8.4

1. algebraic, or polynomial.
2. factorial and/or exponential
3. Integral Test, Limit Comparison Test, and Root Test
4. raised to a power
5. Converges
6. Diverges
7. Converges
8. Converges
9. The Ratio Test is inconclusive; the p-Series Test states it diverges.
10. The Ratio Test is inconclusive; the Direct Comparison Test with $1/n^3$ shows it converges.
11. Converges
12. Converges
13. Converges; note the summation can be rewritten as $\sum_{n=1}^{\infty} \frac{2^n n!}{3^n n!}$, to which the Ratio Test or Geometric Series Test can be applied.
14. Converges; rewrite the summation as $\sum_{n=1}^{\infty} \frac{n!}{5^n n!}$ then apply the Ratio Test or the Geometric Series Test.
15. Converges
16. Converges
17. Converges
18. Converges
19. Diverges
20. Converges
21. Diverges. The Root Test is inconclusive, but the n^{th}-Term Test shows divergence. (The terms of the sequence approach e^2, not 0, as $n \to \infty$.)
22. Converges
23. Converges
24. Converges
25. Diverges; Limit Comparison Test with $1/n$.
26. Converges; Ratio Test
27. Converges; Ratio Test or Limit Comparison Test with $1/3^n$.
28. Converges; Root Test
29. Diverges; n^{th}-Term Test or Limit Comparison Test with 1.
30. Converges; Ratio Test
31. Diverges; Direct Comparison Test with $1/n$
32. Diverges; n^{th}-Term Test (n^{th} term approaches e.)
33. Converges; Root Test
34. Converges; Limit Comparison Test with $1/n^2$ (get common denominator first). It is also a Telescoping Series.

Section 8.5

1. The signs of the terms do not alternate; in the given series, some terms are negative and the others positive, but they do not necessarily alternate.
2. postive, decreasing, 0
3. Many examples exist; one common example is $a_n = (-1)^n/n$.
4. conditionally
5. (a) converges
   (b) converges (p-Series)
   (c) absolute
6. (a) converges

(b) converges (use Ratio Test)

(c) absolute

7. (a) diverges (limit of terms is not 0)

(b) diverges

(c) n/a; diverges

8. (a) diverges (limit of terms is not 0)

(b) diverges

(c) n/a; diverges

9. (a) converges

(b) diverges (Limit Comparison Test with $1/n$)

(c) conditional

10. (a) converges

(b) diverges (Direct Comparison Test with $1/n$)

(c) conditional

11. (a) diverges (limit of terms is not 0)

(b) diverges

(c) n/a; diverges

12. (a) converges

(b) converges (the sum in the denominator is n^2)

(c) absolute

13. (a) diverges (terms oscillate between ± 1)

(b) diverges

(c) n/a; diverges

14. (a) converges

(b) diverges (Integral Test)

(c) conditional

15. (a) converges

(b) converges (Geometric Series with $r = 2/3$)

(c) absolute

16. (a) converges

(b) converges (Geometric Series with $r = 1/e$)

(c) absolute

17. (a) converges

(b) converges (Ratio Test)

(c) absolute

18. (a) converges

(b) converges (Ratio Test)

(c) absolute

19. (a) converges

(b) diverges (p-Series Test with $p = 1/2$)

(c) conditional

20. (a) converges

(b) converges (Ratio Test)

(c) absolute

21. $S_5 = -1.1906$; $S_6 = -0.6767$;
$$-1.1906 \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \leq -0.6767$$

22. $S_4 = 0.9459$; $S_5 = 0.9475$;
$$0.9459 \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \leq 0.9475$$

23. $S_6 = 0.3681$; $S_7 = 0.3679$;
$$0.3681 \leq \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \leq 0.3679$$

24. $S_9 = 0.666016$; $S_{10} = 0.666992$;
$$0.666016 \leq \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \leq 0.666992$$

25. $n = 5$

26. $n = 7$

27. Using the theorem, we find $n = 499$ guarantees the sum is within 0.001 of $\pi/4$. (Convergence is actually faster, as the sum is within ε of $\pi/24$ when $n \geq 249$.)

28. $n = 5$ ($(2n)! > 10^8$ when $n \geq 6$)

Section 8.6

1. 1

2. The radius of convergence is a **value** R such that a power series, centered at $x = c$, converges for all values of x in $(c - R, c + R)$. The interval of convergence is an **interval** on which the power series converges; it may differ from $(c - R, c + R)$ only at the endpoints.

3. 5

4. 5

5. $1 + 2x + 4x^2 + 8x^3 + 16x^4$

6. $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16} + \frac{x^5}{25}$

7. $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

8. $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$

9. (a) $R = \infty$

(b) $(-\infty, \infty)$

10. (a) $R = 1$

(b) $(-1, 1)$

11. (a) $R = 1$

(b) $(2, 4]$

12. (a) $R = \infty$

(b) $(-\infty, \infty)$

13. (a) $R = 2$

(b) $(-2, 2)$

14. (a) $R = 10$

(b) $(-5, 15)$

15. (a) $R = 1/5$

(b) $(4/5, 6/5)$

16. (a) $R = 1/2$

(b) $(-1/2, 1/2)$

17. (a) $R = 1$

(b) $(-1, 1)$

18. (a) $R = 3$

(b) $(-3, 3)$

19. (a) $R = \infty$

(b) $(-\infty, \infty)$

20. (a) $R = 0$

(b) $x = 10$

21. (a) $R = 1$

(b) $[-1, 1]$

22. (a) $R = 1$

(b) $[-3, -1]$

23. (a) $R = 0$

(b) $x = 0$

24. (a) $R = 4$

(b) $x = (-8, 0)$

25. (a) $f'(x) = \sum_{n=1}^{\infty} n^2 x^{n-1}; \quad (-1, 1)$

(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{n}{n+1} x^{n+1}; \quad (-1, 1)$

26. (a) $f'(x) = \sum_{n=1}^{\infty} x^{n-1}; \quad (-1, 1)$

(b) $\int f(x)\,dx = C + \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^{n+1}; \quad [-1, 1]$

27. (a) $f'(x) = \sum_{n=1}^{\infty} \frac{n}{2^n} x^{n-1}; \quad (-2, 2)$

(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{1}{(n+1)2^n} x^{n+1}; \quad [-2, 2)$

28. (a) $f'(x) = \sum_{n=1}^{\infty} n(-3)^n x^{n-1}; \quad (-1/3, 1/3)$

(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-3)^n}{n+1} x^{n+1}; \quad (-1/3, 1/3]$

29. (a) $f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!};$ $(-\infty, \infty)$

(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}; \quad (-\infty, \infty)$

30. (a) $f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n!}; \quad (-\infty, \infty)$

(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}; \quad (-\infty, \infty)$

31. $1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4$

32. $5 + 25x + \frac{125}{2}x^2 + \frac{625}{6}x^3 + \frac{3125}{24}x^4$

33. $1 + x + x^2 + x^3 + x^4$

34. $1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4$

35. $0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4$

36. $1 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4$

Section 8.7

1. The Maclaurin polynomial is a special case of Taylor polynomials. Taylor polynomials are centered at a specific x-value; when that x-value is 0, it is a Maclauring polynomial.
2. T
3. $p_2(x) = 6 + 3x - 4x^2$.
4. $f'''(0) = 30$
5. $p_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$
6. $p_8(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7$
7. $p_5(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5$
8. $p_6(x) = \frac{2x^5}{15} + \frac{x^3}{3} + x$
9. $p_4(x) = \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1$
10. $p_4(x) = x^4 + x^3 + x^2 + x + 1$
11. $p_4(x) = x^4 - x^3 + x^2 - x + 1$
12. $p_7(x) = -\frac{x^7}{7} + \frac{x^5}{5} - \frac{x^3}{3} + x$
13. $p_4(x) = 1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4$
14. $p_4(x) =$ $\ln(2) + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{24}(-1+x)^3 - \frac{1}{64}(-1+x)^4$
15. $p_6(x) = \frac{1}{\sqrt{2}} - \frac{-\frac{\pi}{4}+x}{\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^2}{2\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^3}{6\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^4}{24\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^5}{120\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^6}{720\sqrt{2}}$
16. $p_5(x) = \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(-\frac{\pi}{6}+x\right) - \frac{1}{4}\left(-\frac{\pi}{6}+x\right)^2 - \frac{\left(-\frac{\pi}{6}+x\right)^3}{4\sqrt{3}} + \frac{1}{48}\left(-\frac{\pi}{6}+x\right)^4 + \frac{\left(-\frac{\pi}{6}+x\right)^5}{80\sqrt{3}}$
17. $p_5(x) = \frac{1}{2} - \frac{x-2}{4} + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \frac{1}{32}(x-2)^4 - \frac{1}{64}(x-2)^5$
18. $p_8(x) = 1 - 2(-1+x) + 3(-1+x)^2 - 4(-1+x)^3 + 5(-1+x)^4 - 6(-1+x)^5 + 7(-1+x)^6 - 8(-1+x)^7 + 9(-1+x)^8$
19. $p_3(x) = \frac{1}{2} + \frac{1+x}{2} + \frac{1}{4}(1+x)^2$
20. $p_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{1}{2}\left(\pi^2 - 2\right)(x-\pi)^2$
21. $p_3(x) = x - \frac{x^3}{6}$; $p_3(0.1) = 0.09983$. Error is bounded by $\pm\frac{1}{4!}\cdot 0.1^4 \approx \pm 0.000004167$.
22. $p_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$; $p_4(1) = 13/24 \approx 0.54167$. Error is bounded by $\pm\frac{1}{5!}\cdot 1^5 \approx \pm 0.00833$
23. $p_2(x) = 3 + \frac{1}{6}(-9+x) - \frac{1}{216}(-9+x)^2$; $p_2(10) = 3.16204$. The third derivative of $f(x) = \sqrt{x}$ is bounded on $(8, 11)$ by 0.003. Error is bounded by $\pm\frac{0.003}{3!}\cdot 1^3 = \pm 0.0005$.
24. $p_3(x) = -1 + x - \frac{1}{2}(-1+x)^2 + \frac{1}{3}(-1+x)^3$; $p_3(1.5) = 0.41667$. The fourth derivative of $f(x) = \ln x$ is bounded on $(.9, 2)$ by 10. Error is bounded by $\pm\frac{10}{4!}\cdot .5^4 = \pm 0.026$.
25. The n^{th} derivative of $f(x) = e^x$ is bounded by 3 on intervals containing 0 and 1. Thus $|R_n(1)| \leq \frac{3}{(n+1)!}1^{(n+1)}$. When $n = 7$, this is less than 0.0001.
26. The n^{th} derivative of $f(x) = \sqrt{x}$ is bounded by 0.1 on intervals containing 3 and 4. Thus $|R_n(\pi)| \leq \frac{0.1}{(n+1)!}(1)^{(n+1)}$. When $n = 4$, this is less than 0.0001.
27. The n^{th} derivative of $f(x) = \cos x$ is bounded by 1 on intervals containing 0 and $\pi/3$. Thus $|R_n(\pi/3)| \leq \frac{1}{(n+1)!}(\pi/3)^{(n+1)}$. When $n = 7$, this is less than 0.0001. Since the Maclaurin polynomial of $\cos x$ only uses even powers, we can actually just use $n = 6$.
28. The n^{th} derivative of $f(x) = \sin x$ is bounded by 1 on intervals containing 0 and π. Thus $|R_n(\pi)| \leq \frac{1}{(n+1)!}(\pi)^{(n+1)}$. When $n = 12$, this is less than 0.0001. Since the Maclaurin polynomial of $\sin x$ only uses odd powers, we can actually just use $n = 11$.
29. The n^{th} term is $\frac{1}{n!}x^n$.
30. The n^{th} term is: when n is even, $\frac{(-1)^{n/2}}{n!}x^n$; when n is odd, 0.
31. The n^{th} term is: when n even, 0; when n is odd, $\frac{(-1)^{(n-1)/2}}{n!}x^n$.
32. The n^{th} term is x^n.

33. The n^{th} term is $(-1)^n x^n$.

34. The n^{th} term is $(-1)^n \frac{(x-1)^n}{n}$.

35. $1 + x + \dfrac{1}{2}x^2 + \dfrac{1}{6}x^3 + \dfrac{1}{24}x^4$

36. $3 + 15x + \dfrac{75}{2}x^2 + \dfrac{375}{6}x^3 + \dfrac{1875}{24}x^4$

37. $1 + 2x - 2x^2 + 4x^3 - 10x^4$

Section 8.8

1. A Taylor polynomial is a **polynomial**, containing a finite number of terms. A Taylor series is a **series**, the summation of an infinite number of terms.

2. Theorem 8.8.1, entitled "Function and Taylor Series Equality"

3. All derivatives of e^x are e^x which evaluate to 1 at $x = 0$.
The Taylor series starts $1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

4. All derivatives of $\sin x$ are either $\pm \cos x$ or $\pm \sin x$, which evaluate to ± 1 or 0 at $x = 0$. The Taylor series starts
$0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4 + \frac{1}{120}x^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

5. The n^{th} derivative of $1/(1-x)$ is $f^{(n)}(x) = (n)!/(1-x)^{n+1}$, which evaluates to $n!$ at $x = 0$.
The Taylor series starts $1 + x + x^2 + x^3 + \cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} x^n$

6. The derivative of $\tan^{-1} x$ is $1/(1+x^2)$. Taking successive derivatives using the Quotient Rule, the derivatives of $\tan^{-1} x$ fall into two categories in terms of their evaluation at $x = 0$.
When n is even, $f^{(n)}(x) = (-1)^{(n-1)/2} \dfrac{p(x)}{(1+x^2)^n}$, where $p(x)$ is a polynomial such that $p(0) = 0$. Hence $f^{(n)}(0) = 0$ when n is even.
When n is odd, $f^{(n)}(x) = (-1)^{(n-1)/2} \dfrac{p(x)}{(1+x^2)^n}$, where $p(x)$ is a polynomial such that $p(0) = (n-1)!$. Hence
$f^{(n)}(0) = (-1)^{(n-1)/2}(n-1)!$ when n is odd. (The unusual power of (-1) is such that every other odd term is negative.)
The Taylor series starts $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots$; by reindexing to only obtain odd powers of x, we get
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$.

7. The Taylor series starts
$0 - (x - \pi/2) + 0x^2 + \frac{1}{6}(x-\pi/2)^3 + 0x^4 - \frac{1}{120}(x-\pi/2)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi/2)^{2n+1}}{(2n+1)!}$

8. The Taylor series starts
$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - (x-1)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n (x-1)^n$

9. $f^{(n)}(x) = (-1)^n e^{-x}$; at $x = 0$, $f^{(n)}(0) = -1$ when n is odd and $f^{(n)}(0) = 1$ when n is even.
The Taylor series starts $1 - x + \frac{1}{2}x^2 - \frac{1}{3!}x^3 + \cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$.

10. $f^{(n)}(x) = (-1)^{n+1} \frac{(n-1)!}{(1+x)^n}$; at $x = 0$, $f^{(n)}(0) = (-1)^{n+1}(n-1)!$
The Taylor series starts $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$;
the Taylor series is $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$.

11. $f^{(n)}(x) = (-1)^{n+1} \frac{n!}{(x+1)^{n+1}}$; at $x = 1$, $f^{(n)}(1) = (-1)^{n+1} \frac{n!}{2^{n+1}}$
The Taylor series starts
$\frac{1}{2} + \frac{1}{4}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 \cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{2^{n+1}}$.

12. The derivatives of $\sin x$ are $\pm \cos x$ and $\pm \sin x$; at $x = \pi/4$, these derivatives evaluate to $\pm\sqrt{2}/2$.
The Taylor series starts $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x-\pi/4) - \frac{\sqrt{2}}{2}\frac{(x-\pi/4)^2}{2} - \frac{\sqrt{2}}{2}\frac{(x-\pi/4)^3}{3!} + \frac{\sqrt{2}}{2}\frac{(x-\pi/4)^4}{4!} + \frac{\sqrt{2}}{2}\frac{(x-\pi/4)^5}{5!} \cdots$. Note how the signs are "even, even, odd, odd, even, even, odd, odd,. . . We saw signs like these in Example 8.1.1 of Section 8.1; one way of producing such signs is to raise (-1) to a special quadratic power. While many possibilities exist, one such quadratic is $(n+3)(n+4)/2$.
Thus the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^{\frac{(n+3)(n+4)}{2}} \frac{\sqrt{2}}{2} \frac{(x-\pi/4)^n}{n!}$.

13. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$
where z is between 0 and x.
If $x > 0$, then $z < x$ and $f^{(n+1)}(z) = e^z < e^x$. If $x < 0$, then $x < z < 0$ and $f^{(n+1)}(z) = e^z < 1$. So given a fixed x value, let $M = \max\{e^x, 1\}$; $f^{(n)}(z) < M$. This allows us to state
$$\left|R_n(x)\right| \leq \frac{M}{(n+1)!}\left|x^{(n+1)}\right|.$$
For any x, $\displaystyle\lim_{n\to\infty} \frac{M}{(n+1)!}\left|x^{(n+1)}\right| = 0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty} R_n(x) = 0$ for all x, and hence
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for all } x.$$

14. The following argument is essentially the same as that given for $f(x) = \cos x$ in Example 8.8.3.
Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|.$$
Since all derivatives of $\sin x$ are $\pm \cos x$ or $\pm \sin x$, whose magnitudes are bounded by 1, we can state
$$\left|R_n(x)\right| \leq \frac{1}{(n+1)!}\left|x^{(n+1)}\right|.$$
For any x, $\displaystyle\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)!} = 0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty} R_n(x) = 0$ for all x, and hence
$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \text{for all } x.$$

15. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|(x-1)^{(n+1)}\right|,$$
where z is between 1 and x.
Note that $\left|f^{(n+1)}(x)\right| = \frac{n!}{x^{n+1}}$.
Per the statement of the problem, we only consider the case $1 < x < 2$.
If $1 < x < 2$, then $1 < z < x$ and $f^{(n+1)}(z) = \frac{n!}{z^{n+1}} < n!$. Thus
$$\left|R_n(x)\right| \leq \frac{n!}{(n+1)!}\left|(x-1)^{(n+1)}\right| = \frac{(x-1)^{n+1}}{n+1} < \frac{1}{n+1}.$$
Thus
$$\lim_{n\to\infty}\left|R_n(x)\right| < \lim_{n\to\infty}\frac{1}{n+1} = 0,$$
hence
$$\ln x = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n} \text{ on } (1,2).$$

16. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$
where z is between 0 and x.
Note that $\left|f^{(n+1)}(x)\right| = \frac{(n+1)!}{(1-x)^{n+2}}$.
If $-1 < x < 0$, then $x < z < 0$ and $f^{(n+1)}(z) = \frac{(n+1)!}{(1-z)^{n+2}} < (n+1)!$. Thus
$$\left|R_n(x)\right| \leq \frac{(n+1)!}{(n+1)!}\left|x^{n+1}\right| = \left|x^{n+1}\right|.$$
For a fixed x,
$$\lim_{n\to\infty}\left|x^{n+1}\right| = 0 \text{ as } |x| < 1,$$
hence
$$\frac{1}{1-x} = \sum_{n=0}^{\infty}x^n \text{ on } (-1,0).$$

17. Given $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,
$\cos(-x) = \sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos x$, as all powers in the series are even.

18. Given $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,
$\sin(-x) = \sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{-x^{2n+1}}{(2n+1)!} = -\sin x$, as all powers in the series are odd.

19. Given $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,
$\frac{d}{dx}\left(\sin x\right) = \frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\right) = \sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos x$. (The summation still starts at $n=0$ as there was no constant term in the expansion of $\sin x$).

20. Given $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,
$\frac{d}{dx}\left(\cos x\right) = \frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\right) = \sum_{n=1}^{\infty}(-1)^n\frac{(2n)x^{2n-1}}{(2n)!} = \sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!}$. We can re-index this summation to start at $n=0$ by replacing n with $n+1$ in the summation:
$$\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}.$$
Note that this series has the opposite sign of the Taylor series for $\sin x$; thus $\frac{d}{dx}(\cos x) = -\sin x$.

21. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128}$

22. $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128}$

23. $1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} - \frac{10x^4}{243}$

24. $1 + 4x + 6x^2 + 4x^3 + x^4$ (note the series is finite, and the formula still applies)

25. $\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$.

26. $\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}$.

27. $\sum_{n=0}^{\infty}(-1)^n\frac{(2x+3)^{2n+1}}{(2n+1)!}$.

28. $\sum_{n=0}^{\infty}(-1)^n\frac{(x/2)^{2n+1}}{(2n+1)}$.

29. $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30}$

30. $1 + \frac{x}{2} - \frac{5x^2}{8} - \frac{3x^3}{16}$

31. $\int_0^{\sqrt{\pi}}\sin\left(x^2\right)\,dx \approx \int_0^{\sqrt{\pi}}\left(x^2 - \frac{x^6}{6} + \frac{x^{10}}{120} - \frac{x^{14}}{5040}\right)dx = 0.8877$

32. $\int_0^{\pi^2/4}\cos\left(\sqrt{x}\right)\,dx \approx \int_0^{\pi^2/4}\left(1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720}\right)\,dx = 1.1412$. (Actual answer: $\pi - 2$)

Chapter 9

Section 9.1

1. When defining the conics as the intersections of a plane and a double napped cone, degenerate conics are created when the plane intersects the tips of the cones (usually taken as the origin). Nondegenerate conics are formed when this plane does not contain the origin.

2. Answers will vary.

3. Hyperbola

4. With the equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, the ellipse has a horizontal major axis if $a > b$. But the coefficient of the x^2 term is $1/a^2$ (not a^2), so if $1/a^2 < 1/b^2$, then $a > b$ and the major axis is horizontal.

5. With a horizontal transverse axis, the x^2 term has a positive coefficient; with a vertical transverse axis, the y^2 term has a positive coefficient.

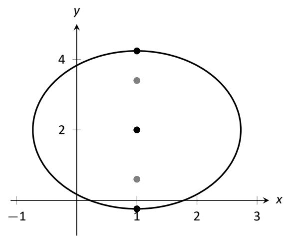
6. line

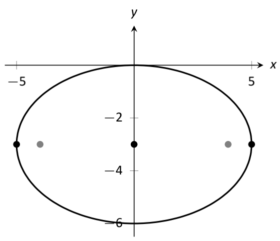
7. $y = \frac{1}{2}(x-3)^2 + \frac{3}{2}$

8. $y = \frac{-1}{12}(x+1)^2 - 1$

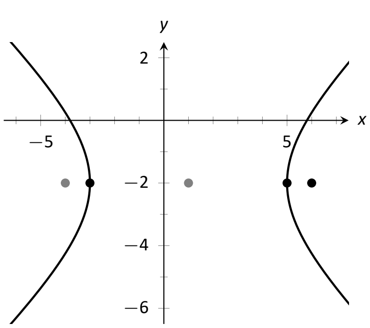
9. $x = -\frac{1}{4}(y-5)^2 + 2$

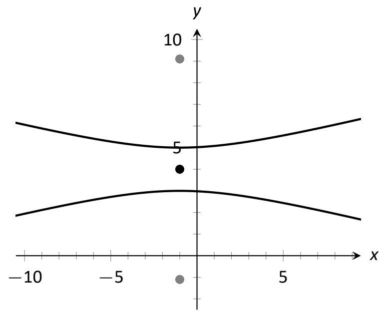
10. $x = y^2$
11. $y = -\frac{1}{4}(x-1)^2 + 2$
12. $x = -\frac{1}{12}y^2$
13. $y = 4x^2$
14. $x = -\frac{1}{8}(y-3)^2 + 2$
15. focus: $(0, 1)$; directrix: $y = -1$. The point P is 2 units from each.
16. focus: $(5, 2)$; directrix: $x = 1$. The point P is 10 units from each.

17. 

18. 

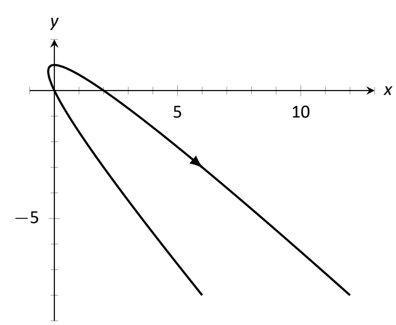
19. $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$; foci at $(-1 \pm \sqrt{5}, 2)$; $e = \sqrt{5}/3$
20. $\frac{(x-1)^2}{1/4} + \frac{y^2}{9} = 1$; foci at $(1, \pm\sqrt{8.75})$; $e = \sqrt{8.75}/3 \approx 0.99$
21. $\frac{x^2}{9} + \frac{y^2}{5} = 1$
22. $\frac{(x-2)^2}{25} + \frac{(y-3)^2}{16} = 1$
23. $\frac{(x-2)^2}{45} + \frac{y^2}{49} = 1$
24. $\frac{(x+1)^2}{9} + \frac{(y-1)^2}{25} = 1$
25. $\frac{(x-1)^2}{2} + (y-2)^2 = 1$
26. $\frac{x^2}{3} + \frac{y^2}{5} = 1$
27. $\frac{x^2}{4} + \frac{(y-3)^2}{6} = 1$
28. $\frac{(x-2)^2}{4} + \frac{(y-2)^2}{4} = 1$
29. $x^2 - \frac{y^2}{3} = 1$
30. $y^2 - \frac{x^2}{24} = 1$
31. $\frac{(y-3)^2}{4} - \frac{(x-1)^2}{9} = 1$
32. $\frac{(x-1)^2}{9} - \frac{(y-3)^2}{4} = 1$

33. 

34. 

35. $\frac{x^2}{4} - \frac{y^2}{5} = 1$
36. $\frac{y^2}{4} - \frac{x^2}{5} = 1$
37. $\frac{(x-3)^2}{16} - \frac{(y-3)^2}{9} = 1$
38. $\frac{(y-3)^2}{9} - \frac{(x-3)^2}{16} = 1$
39. $\frac{x^2}{4} - \frac{y^2}{3} = 1$
40. $\frac{x^2}{3} - \frac{(y-1)^2}{9} = 1$
41. $(y-2)^2 - \frac{x^2}{10} = 1$
42. $4y^2 - \frac{x^2}{4} = 1$
43. (a) $c = \sqrt{12-4} = 2\sqrt{2}$.
    (b) The sum of distances for each point is $2\sqrt{12} \approx 6.9282$.
44. (a) Solve for c in $e = c/a$: $c = ae$. Thus $a^2e^2 = a^2 - b^2$, and $b^2 = a^2 - a^2e^2$. The result follows.
    (b) Mercury: $x^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
    Earth: $x^2 + y^2/(0.99986)^2 = 1$
    Mars: $x^2/(1.524)^2 + y^2/(1.517)^2 = 1$
    (c) Mercury: $(x-0.08)^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
    Earth: $(x-0.0167)^2 + y^2/(0.99986)^2 = 1$
    Mars: $(x-0.1423)^2/(1.524)^2 + y^2/(1.517)^2 = 1$
45. The sound originated from a point approximately 31m to the left of B and 1340m above it.

Section 9.2

1. T
2. orientation
3. rectangular
4. Answers will vary.
5. 

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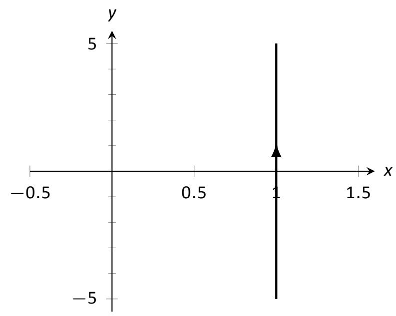

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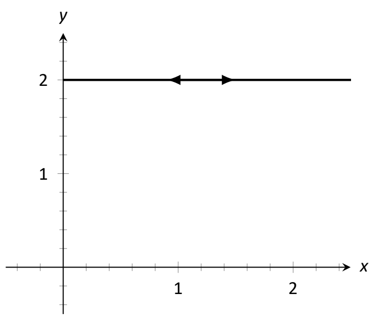

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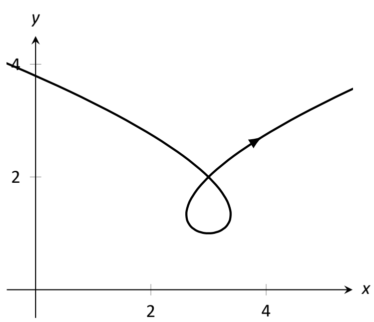

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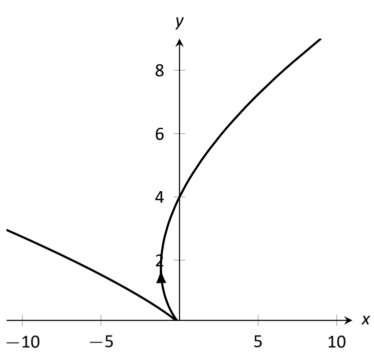

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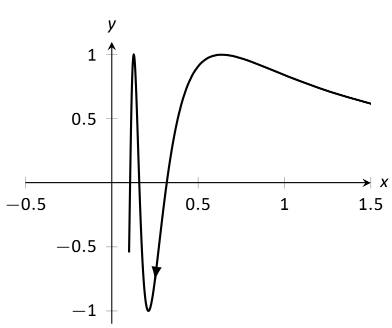

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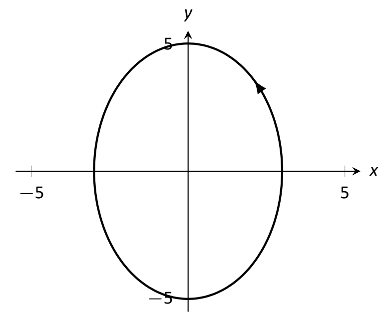

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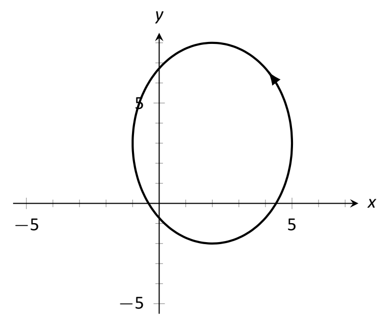

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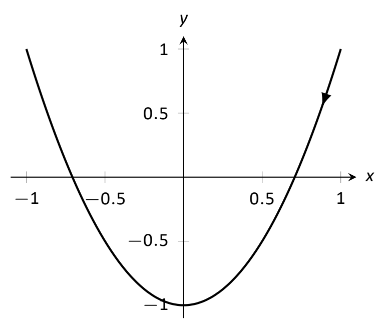

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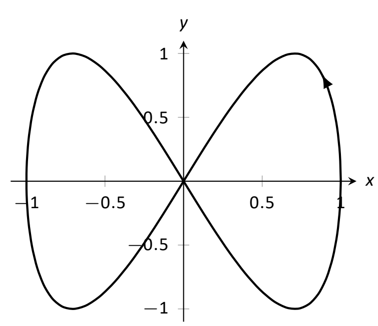

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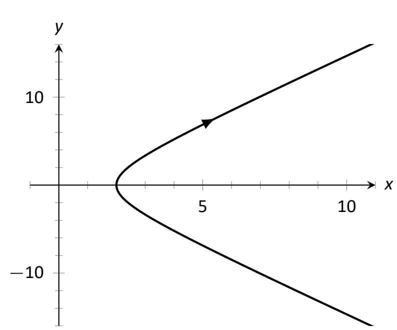
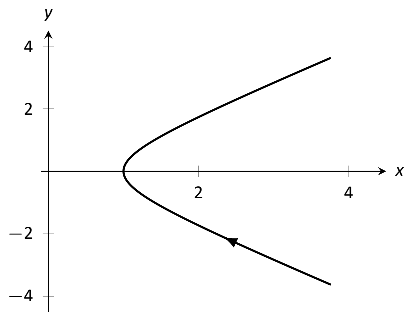
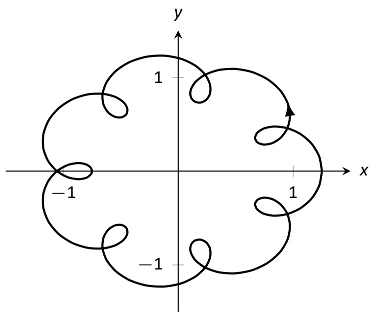
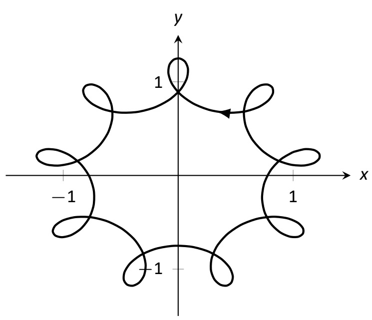


16. 

17. 

18. 

19. (a) Traces the parabola $y = x^2$, moves from left to right.
    (b) Traces the parabola $y = x^2$, but only from $-1 \leq x \leq 1$; traces this portion back and forth infinitely.
    (c) Traces the parabola $y = x^2$, but only for $0 < x$. Moves left to right.
    (d) Traces the parabola $y = x^2$, moves from right to left.
20. (a) Traces a circle of radius 1 counterclockwise once.
    (b) Traces a circle of radius 1 counterclockwise over 6 times.
    (c) Traces a circle of radius 1 clockwise infinite times.
    (d) Traces an arc of a circle of radius 1, from an angle of -1 radians to 1 radian, twice.
21. $y = -1.5x + 8.5$
22. $x^2 - y^2 = 1$
23. $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$
24. $y = x^{3/2}$
25. $y = 2x + 3$
26. $y = x^3 - 3$
27. $y = e^{2x} - 1$
28. $y^2 - x^2 = 1$
29. $x^2 - y^2 = 1$
30. $x = 1 - 2y^2$
31. $y = \frac{b}{a}(x - x_0) + y_0$; line through (x_0, y_0) with slope b/a.
32. $x^2 + y^2 = r^2$; circle centered at $(0, 0)$ with radius r.
33. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$; ellipse centered at (h, k) with horizontal axis of length $2a$ and vertical axis of length $2b$.
34. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$; hyperbola centered at (h, k) with horizontal transverse axis and asymptotes with slope b/a. The parametric equations only give half of the hyperbola. When $a > 0$, the right half; when $a < 0$, the left half.
35. $x = (t + 11)/6$, $y = (t^2 - 97)/12$. At $t = 1$, $x = 2$, $y = -8$.
    $y' = 6x - 11$; when $x = 2$, $y' = 1$.
36. $x = \ln t$, $y = t$. At $t = 1$, $x = 0$, $y = 1$.
    $y' = e^x$; when $x = 0$, $y' = 1$.
37. $x = \cos^{-1} t$, $y = \sqrt{1 - t^2}$. At $t = 1$, $x = 0$, $y = 0$.
    $y' = \cos x$; when $x = 0$, $y' = 1$.
38. $x = 1/(4t^2)$, $y = 1/(2t)$. At $t = 1$, $x = 1/4$, $y = 1/2$.
    $y' = 1/(2\sqrt{x})$; when $x = 1/4$, $y' = 1$.
39. $t = \pm 1$
40. $t = -1,\ 2$
41. $t = \pi/2, 3\pi/2$
42. $t = \pi/6, \pi/2, 5\pi/6$
43. $t = -1$
44. $t = 2$
45. $t = \ldots \pi/2,\ 3\pi/2,\ 5\pi/2,\ \ldots$
46. $t = \ldots 0,\ 2\pi,\ 4\pi,\ \ldots$
47. $x = 4t$, $y = -16t^2 + 64t$
48. $x = 50t$, $y = -16t^2 + 64t$
49. $x = 10t$, $y = -16t^2 + 320t$
50. $x = 2\cos t$, $y = -2\sin t$; other answers possible
51. $x = 3\cos(2\pi t) + 1$, $y = 3\sin(2\pi t) + 1$; other answers possible
52. $x = \cos t + 1$, $y = 3\sin t + 3$; other answers possible
53. $x = 5\cos t$, $y = \sqrt{24}\sin t$; other answers possible
54. $x = \pm\sec t + 2$, $y = \sqrt{8}\tan t - 3$; other answers possible
55. $x = 2\tan t$, $y = \pm 6\sec t$; other answers possible

Section 9.3

1. F
2. t
3. F
4. T
5. (a) $\frac{dy}{dx} = 2t$
   (b) Tangent line: $y = 2(x - 1) + 1$; normal line: $y = -1/2(x - 1) + 1$
6. (a) $\frac{dy}{dx} = 10\sqrt{t}$
   (b) Tangent line: $y = 20(x - 2) + 22$; normal line: $y = -1/20(x - 2) + 22$
7. (a) $\frac{dy}{dx} = \frac{2t+1}{2t-1}$
   (b) Tangent line: $y = 3x + 2$; normal line: $y = -1/3x + 2$
8. (a) $\frac{dy}{dx} = \frac{3t^2-1}{2t}$
   (b) $t = 0$: Tangent line: $x = -1$; normal line: $y = 0$
   $t = 1$: Tangent line: $y = x$; normal line: $y = -x$
9. (a) $\frac{dy}{dx} = \csc t$

(b) $t = \pi/4$: Tangent line: $y = \sqrt{2}(x - \sqrt{2}) + 1$; normal line: $y = -1/\sqrt{2}(x - \sqrt{2}) + 1$

10. (a) $\frac{dy}{dx} = -2\cos(2t)\csc t$

(b) $t = \pi/4$: Tangent line: $y = 1$; normal line: $x = \sqrt{2}/2$

11. (a) $\frac{dy}{dx} = \frac{\cos t \sin(2t) + \sin t \cos(2t)}{-\sin t \sin(2t) + 2\cos t \cos(2t)}$

(b) Tangent line: $y = x - \sqrt{2}$; normal line: $y = -x - \sqrt{2}$

12. (a) $\frac{dy}{dx} = \frac{\sin(t) + 10\cos(t)}{\cos(t) - 10\sin(t)}$

(b) Tangent line: $y = -x/10 + e^{\pi/20}$; normal line: $y = 10x + e^{\pi/20}$

13. $t = 0$

14. $t = 0$ (though this uses a one–sided limit, as $x(t)$ is not defined for $t < 0$.

15. $t = -1/2$

16. $t = \pm 1/\sqrt{3}$

17. The graph does not have a horizontal tangent line.

18. $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

19. The solution is non-trivial; use identities $\sin(2t) = 2\sin t\cos t$ and $\cos(2t) = \cos^2 t - \sin^2 t$ to rewrite $g'(t) = 2\sin t(2\cos^2 t - \sin^2 t)$. On $[0, 2\pi]$, $\sin t = 0$ when $t = 0, \pi, 2\pi$, and $2\cos^2 t - \sin^2 t = 0$ when $t = \tan^{-1}(\sqrt{2}),\ \pi \pm \tan^{-1}(\sqrt{2}),\ 2\pi - \tan^{-1}(\sqrt{2})$.

20. $t = \tan^{-1}(-10),\ \tan^{-1}(-10) + \pi$

21. $t_0 = 0$; $\lim_{t\to 0} \frac{dy}{dx} = 0$.

22. $t_0 = 2$; $\lim_{t\to 2} \frac{dy}{dx} = 1$.

23. $t_0 = 1$; $\lim_{t\to 1} \frac{dy}{dx} = \infty$.

24. $t_0 = \ldots, -\pi/2, 0, \pi/2, \pi, \ldots$; $\lim_{t\to 0} \frac{dy}{dx} = 1$.

25. $\frac{d^2y}{dx^2} = 2$; always concave up

26. $\frac{d^2y}{dx^2} = 10$; always concave up

27. $\frac{d^2y}{dx^2} = -\frac{4}{(2t-1)^3}$; concave up on $(-\infty, 1/2)$; concave down on $(1/2, \infty)$.

28. $\frac{d^2y}{dx^2} = \frac{3t^2+1}{4t^3}$; concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.

29. $\frac{d^2y}{dx^2} = -\cot^3 t$; concave up on $(-\infty, 0)$; concave down on $(0, \infty)$.

30. $\frac{d^2y}{dx^2} = \frac{\cos t \sin(2t) + 2\sin t \cos(2t)}{(-\sin t \sin(2t) + 2\cos t \cos(2t))^2}$; concavity switches at $t = \tan^{-1}(1/\sqrt{2}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{2}),\ \pi + \tan^{-1}(1/\sqrt{2}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{2})$

31. $\frac{d^2y}{dx^2} = \frac{4(13+3\cos(4t))}{(\cos t + 3\cos(3t))^3}$, obtained with a computer algebra system; concave up on $\big(-\tan^{-1}(\sqrt{2}/2), \tan^{-1}(\sqrt{2}/2)\big)$, concave down on $\big(-\pi/2, -\tan^{-1}(\sqrt{2}/2)\big) \cup \big(\tan^{-1}(\sqrt{2}/2), \pi/2\big)$

32. $\frac{d^2y}{dx^2} = \frac{1010}{e^{t/10}\big(\cos t - 10\sin t\big)^3}$; concavity switches at $t = \tan^{-1}(1/10) + n\pi$, where n is an integer.

33. $L = 6\pi$

34. On $[0, 2\pi]$, arc length is $L = \sqrt{101}(e^{\pi/5} - 1)$; on $[2\pi, 4\pi]$, $L = \sqrt{101}(e^{2\pi/5} - 1)$.

35. $L = 2\sqrt{34}$

36. $L = 4\sqrt{2} - 2$

37. $L \approx 2.4416$ (actual value: $L = 2.42211$)

38. $L \approx 9.73004$ (actual value: $L = 9.42943$)

39. $L \approx 4.19216$ (actual value: $L = 4.18308$)

40. Formula: $C \approx 25.9062$; Simpson's Rule: $C \approx 25.4786$ (actual value: $C = 25.527$)

41. The answer is 16π for both (of course), but the integrals are different.

42. $8\pi^2$.

43. $SA \approx 8.50101$ (actual value $SA = 8.02851$)

44. $SA \approx 1.36751$ (actual value $SA = 1.36707$)

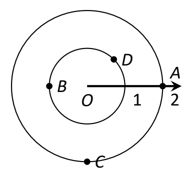
Section 9.4

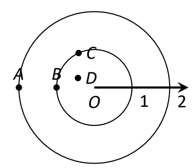
1. Answers will vary.

2. F

3. T

4. F

5. 

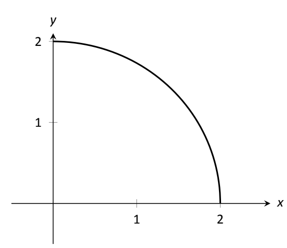
6. 

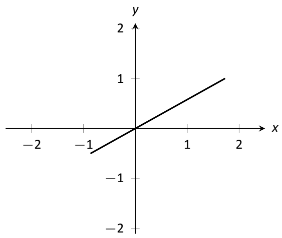
7. $A = P(2.5, \pi/4)$ and $P(-2.5, 5\pi/4)$;
$B = P(-1, 5\pi/6)$ and $P(1, 11\pi/6)$;
$C = P(3, 4\pi/3)$ and $P(-3, \pi/3)$;
$D = P(1.5, 2\pi/3)$ and $P(-1.5, 5\pi/3)$;

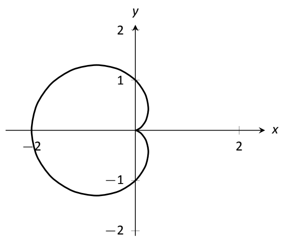
8. $A = P(2, \pi/6)$ and $P(-2, -5\pi/6)$;
$B = P(1, -\pi/3)$ and $P(-1, 2\pi/3)$;
$C = P(2, 3\pi/4)$ and $P(-2, -\pi/4)$;
$D = P(2.5, \pi)$ and $P(2.5, -\pi)$;

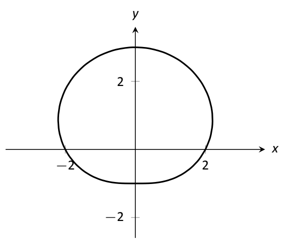
9. $A = (\sqrt{2}, \sqrt{2})$
$B = (\sqrt{2}, -\sqrt{2})$
$C = P(\sqrt{5}, -0.46)$
$D = P(\sqrt{5}, 2.68)$

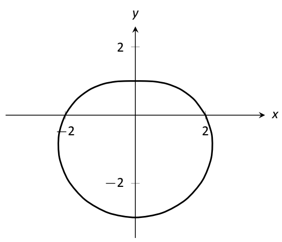
10. $A = (-3, 0)$
$B = (-1/2, \sqrt{3}/2)$
$C = P(4, \pi/2)$
$D = P(2, -\pi/3)$

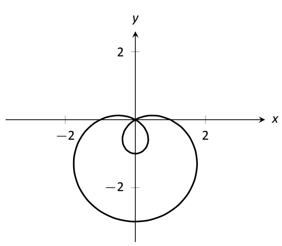
11. 

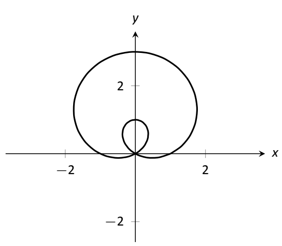
12. 

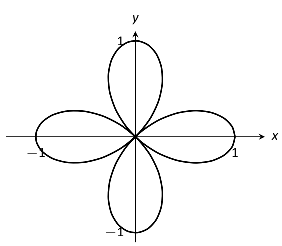
13. 

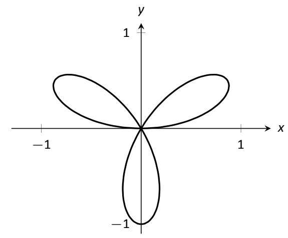
14. 

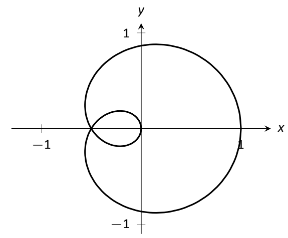
15. 

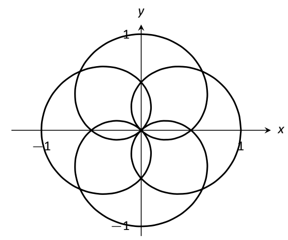
16. 

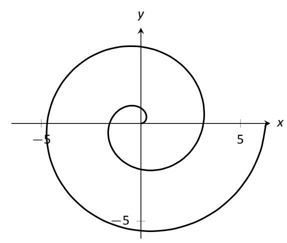
17. 

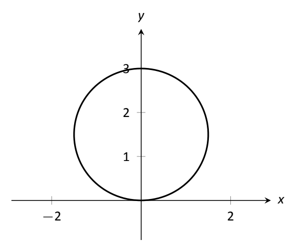
18. 

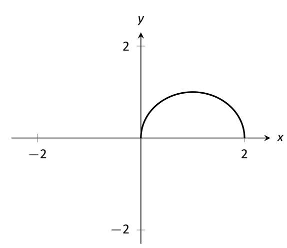
19. 

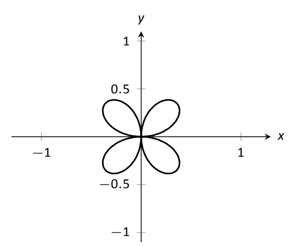
20. 

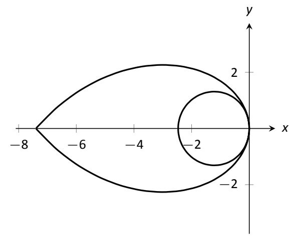
21. 

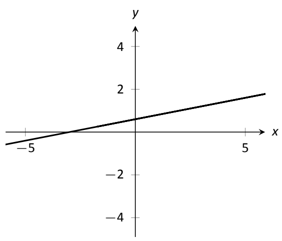
22. 

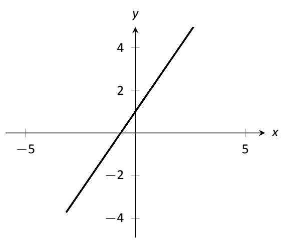
23. 

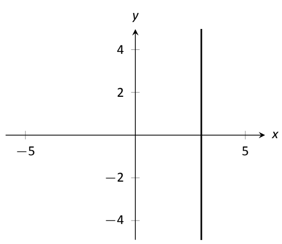
24. 

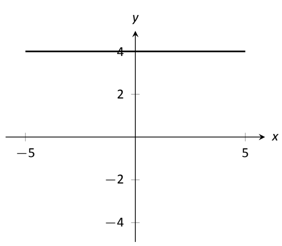
25. 

26. 

27. 

28. 

29. 

30. 

31. $(x-3)^2+y^2=3$
32. $x^2+(y+2)^2=4$
33. $(x-1/2)^2+(y-1/2)^2=1/2$
34. $y=2/5x+7/5$
35. $x=3$
36. $y=4$
37. $x^4+x^2y^2-y^2=0$
38. $y^4+x^2y^2-x^2=0$
39. $x^2+y^2=4$
40. $y=x/\sqrt{3}$
41. $\theta=\pi/4$
42. $r=7/(\sin\theta-4\cos\theta)$
43. $r=5\sec\theta$
44. $r=5\csc\theta$
45. $r=\cos\theta/\sin^2\theta$
46. $r=1/\sqrt[3]{\cos^2\theta\sin\theta}$
47. $r=\sqrt{7}$
48. $r=-2\cos\theta$
49. $P(\sqrt{3}/2,\pi/6)$, $P(0,\pi/2)$, $P(-\sqrt{3}/2,5\pi/6)$
50. $P(1,0)$, $P(0,\pi/2)=P(0,\pi/4)$, $P(-1/2,\pi/3)$
51. $P(0,0)=P(0,\pi/2)$, $P(\sqrt{2},\pi/4)$
52. $P(\sqrt{3}/2,\pi/3)=P(-\sqrt{3}/2,4\pi/3)$, $P(\sqrt{3}/2,2\pi/3)=P(-\sqrt{3}/2,5\pi/3)$, $P(0,\pi/2)$
53. $P(\sqrt{2}/2,\pi/12)$, $P(-\sqrt{2}/2,5\pi/12)$, $P(\sqrt{2}/2,3\pi/4)$
54. $P(3/2,\pi/3)$, $P(3/2,-\pi/3)$
55. For all points, $r=1$; $\theta=\pi/12,\ 5\pi/12,\ 7\pi/12,\ 11\pi/12,\ 13\pi/12,\ 17\pi/12,\ 19\pi/12,\ 23\pi/12$.
56. $P(0,0)=P(0,3\pi/2)$, $P(1+\sqrt{2}/2,3\pi/4)$, $P(1-\sqrt{2}/2,7\pi/4)$
57. Answers will vary. If m and n do not have any common factors, then an interval of $2n\pi$ is needed to sketch the entire graph.
58. Answers will vary.

Section 9.5

1. Using $x=r\cos\theta$ and $y=r\sin\theta$, we can write $x=f(\theta)\cos\theta$, $y=f(\theta)\sin\theta$.
2. rectangles; sectors of circles
3. (a) $\frac{dy}{dx}=-\cot\theta$
   (b) tangent line: $y=-(x-\sqrt{2}/2)+\sqrt{2}/2$; normal line: $y=x$
4. (a) $\frac{dy}{dx}=1/2(\tan\theta-\cot\theta)$
   (b) tangent line: $y=1/2$; normal line: $x=1/2$
5. (a) $\frac{dy}{dx}=\frac{\cos\theta(1+2\sin\theta)}{\cos^2\theta-\sin\theta(1+\sin\theta)}$
   (b) tangent line: $x=3\sqrt{3}/4$; normal line: $y=3/4$
6. (a) $\frac{dy}{dx}=\frac{3\sin^2(t)+(1-3\cos(t))\cos(t)}{3\sin(t)\cos(t)-\sin(t)(1-3\cos(t))}$
   (b) tangent line: $y=\frac{1}{1+3\sqrt{2}}(x+(1/\sqrt{2}+3/2))+1/\sqrt{2}+3/2\approx y=0.19(x+2.21)+2.21$; normal line: $y=-(1+3\sqrt{2})(x+(1/\sqrt{2}+3/2))+1/\sqrt{2}+3/2$
7. (a) $\frac{dy}{dx}=\frac{\theta\cos\theta+\sin\theta}{\cos\theta-\theta\sin\theta}$
   (b) tangent line: $y=-2/\pi x+\pi/2$; normal line: $y=\pi/2x+\pi/2$
8. (a) $\frac{dy}{dx}=\frac{\cos\theta\cos(3\theta)-3\sin\theta\sin(3\theta)}{-\cos(3\theta)\sin\theta-3\cos\theta\sin(3\theta)}$
   (b) tangent line: $y=x/\sqrt{3}$; normal line: $y=-\sqrt{3}x$
9. (a) $\frac{dy}{dx}=\frac{4\sin(\theta)\cos(4\theta)+\sin(4\theta)\cos(\theta)}{4\cos(\theta)\cos(4\theta)-\sin(\theta)\sin(4\theta)}$
   (b) tangent line: $y=5\sqrt{3}(x+\sqrt{3}/4)-3/4$; normal line: $y=-1/5\sqrt{3}(x+\sqrt{3}/4)-3/4$
10. (a) $\frac{dy}{dx}=1$
    (b) tangent line: $y=x+1$; normal line: $y=-x-1$
11. horizontal: $\theta=\pi/2,3\pi/2$;
    vertical: $\theta=0,\pi,2\pi$
12. horizontal: $\theta=0,\pi/2,\pi$;
    vertical: $\theta=\pi/4,3\pi/4$

13. horizontal: $\theta = \tan^{-1}(1/\sqrt{5}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{5}),\ \pi + \tan^{-1}(1/\sqrt{5}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{5})$;
vertical: $\theta = 0,\ \tan^{-1}(\sqrt{5}),\ \pi - \tan^{-1}(\sqrt{5}),\ \pi,\ \pi + \tan^{-1}(\sqrt{5}),\ 2\pi - \tan^{-1}(\sqrt{5})$

14. horizontal: $\theta = \pi/3,\ 5\pi/3$;
vertical: $\theta = 0,\ 2\pi/3,\ 4\pi/3,\ 2\pi$
At $\theta = \pi$, $\frac{dy}{dx} = 0/0$; apply L'Hopital's Rule to find that $\frac{dy}{dx} \to 0$ as $\theta \to \pi$.

15. In polar: $\theta = 0 \cong \theta = \pi$
In rectangular: $y = 0$

16. In polar: $\theta = 0,\ \theta = \pi/3,\ \theta = 2\pi/3$.
In rectangular: $y = 0$, $y = \sqrt{3}x$, and $y = -\sqrt{3}x$.

17. area = 4π

18. area = 25π

19. area = $\pi/12$

20. area = $\pi/(4n)$

21. area = $3\pi/2$

22. area = $\pi - 3\sqrt{3}/2$

23. area = $2\pi + 3\sqrt{3}/2$

24. area = $\pi + 3\sqrt{3}$

25. area = 1

26. area = $\displaystyle\int_{\pi/12}^{\pi/3} \frac{1}{2}\sin^2(3\theta)\,d\theta - \int_{\pi/12}^{\pi/6} \frac{1}{2}\cos^2(3\theta)\,d\theta = \frac{1}{12} + \frac{\pi}{24}$

27. area = $\frac{1}{32}(4\pi - 3\sqrt{3})$

28. area = $\displaystyle\int_0^{\pi/3} \frac{1}{2}(1-\cos\theta)^2\,d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\cos\theta)^2\,d\theta = \frac{7\pi}{24} - \frac{\sqrt{3}}{2} \approx 0.0503$

29. 4π

30. 4π

31. area = $\sqrt{2}\pi$

32. 8

33. $L \approx 2.2592$; (actual value $L = 2.22748$)

34. $x'(\theta) = f'(\theta)\cos\theta - f(\theta)\sin\theta$, $y'(\theta) = f'(\theta)\sin\theta + f(\theta)\cos\theta$. Square each and add; applying the Pythagoran Theorem twice achieves the result.

35. $SA = 16\pi$

36. $SA = 4\pi$

37. $SA = 32\pi/5$

38. $SA = 4\pi^2$

39. $SA = 36\pi$

40. $SA = 9\pi$

Chapter 10

Section 10.1

1. right hand

2. line; plane

3. curve (a parabola); surface (a cylinder)

4. a hyperbolic paraboloid

5. a hyperboloid of two sheets

6. a hyperboloid of one sheet

7. $||\,\overline{AB}\,|| = \sqrt{6}$; $||\,\overline{BC}\,|| = \sqrt{17}$; $||\,\overline{AC}\,|| = \sqrt{11}$. Yes, it is a right triangle as $||\,\overline{AB}\,||^2 + ||\,\overline{AC}\,||^2 = ||\,\overline{BC}\,||^2$.

8. Yes, as opposite sides have equal length.
$||\,\overline{AB}\,|| = \sqrt{21} = ||\,\overline{CD}\,||$; $||\,\overline{BC}\,|| = \sqrt{6} = ||\,\overline{AD}\,||$.

9. Center at $(4, -1, 0)$; radius = 3

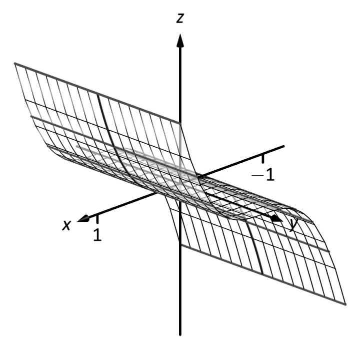
10. Center at $(-2, 1, 2)$; radius = $\sqrt{5}$

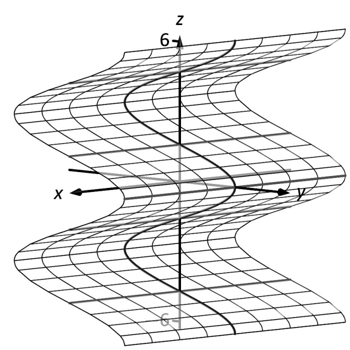
11. Interior of a sphere with radius 1 centered at the origin.

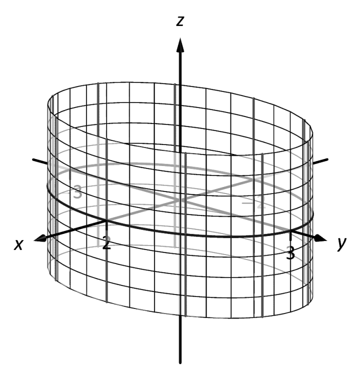
12. Region bounded between the planes $x = 0$ (the $y - z$ coordinate plane) and $x = 3$.

13. The first octant of space; all points (x, y, z) where each of x, y and z are non-negative. (Analogous to the first quadrant in the plane.)

14. All points in space where the y value is greater than 3; viewing space as often depicted in this text, this is the region "to the right" of the plane $y = 3$ (which is parallel to the $x - z$ coordinate plane.)

15. 

16. 

17. 

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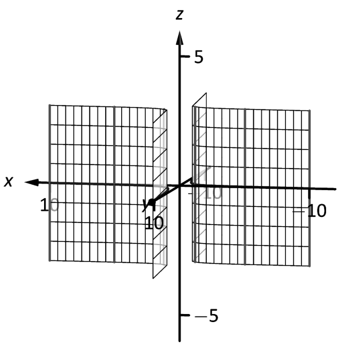


19. $x^2 + z^2 = \frac{1}{(1+y^2)^2}$

20. $y^2 + z^2 = x^4$

21. $z = (\sqrt{x^2+y^2})^2 = x^2 + y^2$

22. $z = \frac{1}{\sqrt{x^2+y^2}}$

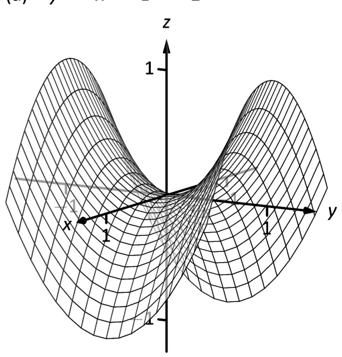
23. (a) $x = y^2 + \dfrac{z^2}{9}$

24. (b) $x^2 - y^2 + z^2 = 0$

25. (b) $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

26. (a) $y^2 - x^2 - z^2 = 1$

27.



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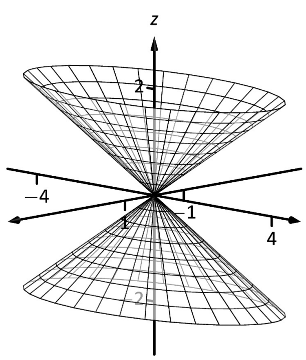

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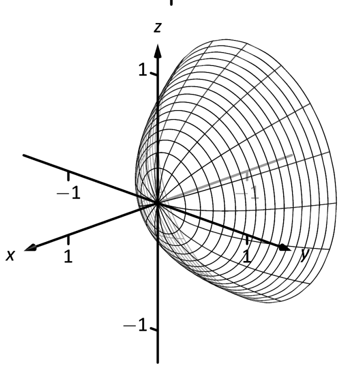

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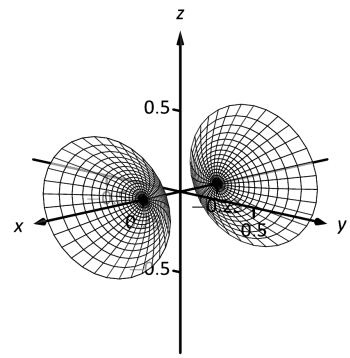

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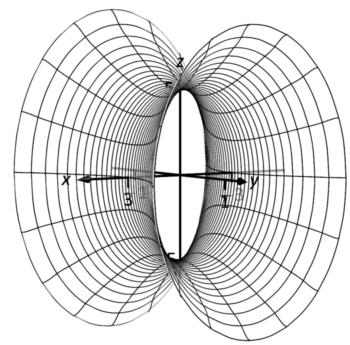

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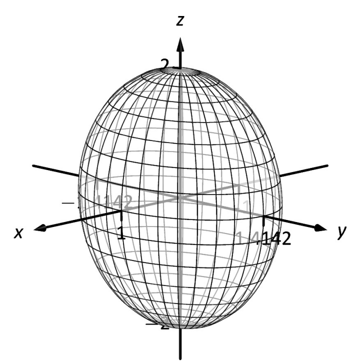


Section 10.2

1. Answers will vary.

2. $(1, 2)$ is a point; $\langle 1, 2\rangle$ is a vector that describes a displacement of 1 unit in the x-direction and 2 units in the y-direction.

3. A vector with magnitude 1.

4. Direction

5. Their respective unit vectors are parallel; unit vectors $\vec{u}_1$ and $\vec{u}_2$ are parallel if $\vec{u}_1 = \pm\vec{u}_2$.

6. It stretches the vector by a factor of 2, and points it in the opposite direction.

7. $\overrightarrow{PQ} = \langle 1, 6\rangle = 1\vec{i} + 6\vec{j}$

8. $\overrightarrow{PQ} = \langle 4, -4\rangle = 4\vec{i} - 4\vec{j}$

9. $\overrightarrow{PQ} = \langle 6, -1, 6\rangle = 6\vec{i} - \vec{j} + 6\vec{k}$

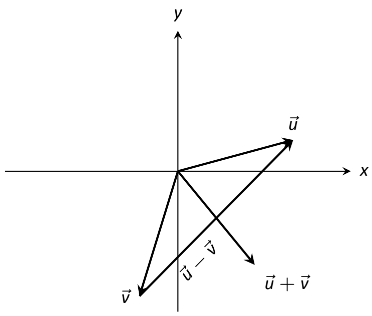
10. $\overrightarrow{PQ} = \langle 2, 2, 0\rangle = 2\vec{i} + 2\vec{j}$

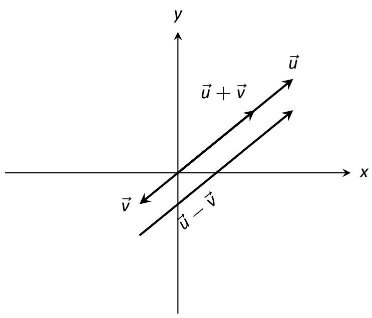
11. (a) $\vec{u} + \vec{v} = \langle 2, -1\rangle$; $\vec{u} - \vec{v} = \langle 0, -3\rangle$; $2\vec{u} - 3\vec{v} = \langle -1, -7\rangle$.

    (c) $\vec{x} = \langle 1/2, 2\rangle$.

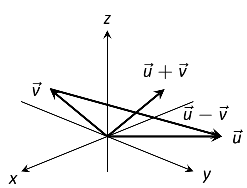
12. (a) $\vec{u} + \vec{v} = \langle 3, 2, 1\rangle$; $\vec{u} - \vec{v} = \langle -1, 0, -3\rangle$; $\pi\vec{u} - \sqrt{2}\vec{v} = \langle \pi - 2\sqrt{2}, \pi - \sqrt{2}, -\pi - 2\sqrt{2}\rangle$.

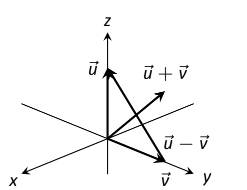
    (c) $\vec{x} = \langle -1, 0, -3\rangle$.

13. 

14. 

Sketch of $\vec u - \vec v$ shifted for clarity.

15. 

16. 

17. $||\,\vec u\,|| = \sqrt{5}$, $||\,\vec v\,|| = \sqrt{13}$, $||\,\vec u + \vec v\,|| = \sqrt{26}$, $||\,\vec u - \vec v\,|| = \sqrt{10}$
18. $||\,\vec u\,|| = \sqrt{17}$, $||\,\vec v\,|| = \sqrt{3}$, $||\,\vec u + \vec v\,|| = \sqrt{14}$, $||\,\vec u - \vec v\,|| = \sqrt{26}$
19. $||\,\vec u\,|| = \sqrt{5}$, $||\,\vec v\,|| = 3\sqrt{5}$, $||\,\vec u + \vec v\,|| = 2\sqrt{5}$, $||\,\vec u - \vec v\,|| = 4\sqrt{5}$
20. $||\,\vec u\,|| = 7$, $||\,\vec v\,|| = 35$, $||\,\vec u + \vec v\,|| = 42$, $||\,\vec u - \vec v\,|| = 28$
21. When $\vec u$ and $\vec v$ have the same direction. (Note: parallel is not enough.)
22. $\vec u = \left\langle 3/\sqrt{58}, 7/\sqrt{58}\right\rangle$
23. $\vec u = \langle 0.6, 0.8\rangle$
24. $\vec u = \langle 1/3, -2/3, 2/3\rangle$
25. $\vec u = \left\langle 1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3}\right\rangle$
26. $\vec u = \langle \cos 50^\circ, \sin 50^\circ\rangle \approx \langle 0.643, 0.766\rangle$.
27. $\vec u = \langle \cos 120^\circ, \sin 120^\circ\rangle = \left\langle -1/2, \sqrt{3}/2\right\rangle$.
28.

$$\begin{aligned} ||\,\vec u\,|| &= \sqrt{\sin^2\theta\cos^2\varphi + \sin^2\theta\sin^2\varphi + \cos^2\theta} \\ &= \sqrt{\sin^2\theta(\cos^2\varphi + \sin^2\varphi) + \cos^2\theta} \\ &= \sqrt{\sin^2\theta + \cos^2\theta} \\ &= 1. \end{aligned}$$

29. The force on each chain is $100/\sqrt{3} \approx 57.735$lb.
30. The force on each chain is 100lb.
31. The force on the chain with angle θ is approx. 45.124lb; the force on the chain with angle φ is approx. 59.629lb.
32. The force on each chain is 50lb.
33. $\theta = 45^\circ$; the weight is lifted 0.29 ft (about 3.5in).
34. $\theta = 5.71^\circ$; the weight is lifted 0.005 ft (about 1/16th of an inch).
35. $\theta = 45^\circ$; the weight is lifted 2.93 ft.
36. $\theta = 84.29^\circ$; the weight is lifted 9 ft.

Section 10.3

1. Scalar
2. The magnitude of a vectors is the square root of the dot product of a vector with itself; that is, $||\,\vec v\,|| = \sqrt{\vec v \cdot \vec v}$.
3. By considering the sign of the dot product of the two vectors. If the dot product is positive, the angle is acute; if the dot product is negative, the angle is obtuse.
4. "Perpendicular" is one answer.
5. -22
6. 33
7. 3
8. 0
9. not defined
10. 0
11. Answers will vary.
12. Answers will vary.
13. $\theta = 0.3218 \approx 18.43^\circ$
14. $\theta = 1.6476 \approx 94.4^\circ$
15. $\theta = \pi/4 = 45^\circ$
16. $\theta = \pi/2 = 90^\circ$
17. Answers will vary; two possible answers are $\langle -7, 4\rangle$ and $\langle 14, -8\rangle$.
18. Answers will vary; two possible answers are $\langle 5, 3\rangle$ and $\langle -15, -9\rangle$.
19. Answers will vary; two possible answers are $\langle 1, 0, -1\rangle$ and $\langle 4, 5, -9\rangle$.
20. Answers will vary; two possible answers are $\langle 2, 1, 0\rangle$ and $\langle 1, 1, 1/3\rangle$.
21. $\text{proj}_{\vec v}\,\vec u = \langle -1/2, 3/2\rangle$.
22. $\text{proj}_{\vec v}\,\vec u = \langle 2, 6\rangle$.
23. $\text{proj}_{\vec v}\,\vec u = \langle -1/2, -1/2\rangle$.
24. $\text{proj}_{\vec v}\,\vec u = \langle 0, 0\rangle$.
25. $\text{proj}_{\vec v}\,\vec u = \langle 1, 2, 3\rangle$.
26. $\text{proj}_{\vec v}\,\vec u = \langle 4/3, 4/3, 2/3\rangle$.
27. $\vec u = \langle -1/2, 3/2\rangle + \langle 3/2, 1/2\rangle$.
28. $\vec u = \langle 2, 6\rangle + \langle 3, -1\rangle$.

29. $\vec{u} = \langle -1/2, -1/2\rangle + \langle -5/2, 5/2\rangle$.

30. $\vec{u} = \langle 0, 0\rangle + \langle -3, 2\rangle$.

31. $\vec{u} = \langle 1, 2, 3\rangle + \langle 0, 3, -2\rangle$.

32. $\vec{u} = \langle 4/3, 4/3, 2/3\rangle + \langle 5/3, -7/3, 4/3\rangle$.

33. 1.96lb

34. 5lb

35. 141.42ft–lb

36. 196.96ft–lb

37. 500ft–lb

38. 424.26ft–lb

39. 500ft–lb

Section 10.4

1. vector

2. right hand rule

3. "Perpendicular" is one answer.

4. T

5. Torque

6. T

7. $\vec{u} \times \vec{v} = \langle 12, -15, 3\rangle$

8. $\vec{u} \times \vec{v} = \langle 11, 1, -17\rangle$

9. $\vec{u} \times \vec{v} = \langle -5, -31, 27\rangle$

10. $\vec{u} \times \vec{v} = \langle 47, -36, -44\rangle$

11. $\vec{u} \times \vec{v} = \langle 0, -2, 0\rangle$

12. $\vec{u} \times \vec{v} = \langle 0, 0, 0\rangle$

13. $\vec{u} \times \vec{v} = \langle 0, 0, ad - bc\rangle$

14. $\vec{i} \times \vec{j} = \vec{k}$

15. $\vec{i} \times \vec{k} = -\vec{j}$

16. $\vec{j} \times \vec{k} = \vec{i}$

17. Answers will vary.

18. Answers will vary.

19. 5

20. 21

21. 0

22. 5

23. $\sqrt{14}$

24. $\sqrt{230}$

25. 3

26. 6

27. $5\sqrt{2}/2$

28. $3\sqrt{30}$

29. 1

30. $5/2$

31. 7

32. $8\sqrt{7/2}$

33. 2

34. 15

35. $\pm\frac{1}{\sqrt{6}}\langle 1, 1, -2\rangle$

36. $\pm\frac{1}{\sqrt{21}}\langle -2, 1, 4\rangle$

37. $\langle 0, \pm 1, 0\rangle$

38. Since $\vec{u}$ and $\vec{v}$ are parallel, any unit vector orthogonal to $\vec{u}$ works (such as $\frac{1}{\sqrt{2}}\langle 1, 0, -1\rangle$).

39. 87.5ft–lb

40. $43.75\sqrt{3} \approx 75.78$ft–lb

41. $200/3 \approx 66.67$ft–lb

42. 11.58ft–lb

43. With $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$, we have

$$\begin{aligned}\vec{u}\cdot(\vec{u}\times\vec{v}) &= \langle u_1, u_2, u_3\rangle \cdot (\langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1\rangle)\\ &= u_1(u_2v_3 - u_3v_2) - u2(u_1v_3 - u_3v_1) + u3(u_1v_2 - u_2v_1)\\ &= 0.\end{aligned}$$

44. With $\vec{u} = \langle u_1, u_2, u_3\rangle$, we have

$$\begin{aligned}\vec{u}\times\vec{u} &= \langle u_2u_3 - u_3u_2, -(u_1u_3 - u_3u_1), u_1u_2 - u_2u_1\rangle)\\ &= \langle 0, 0, 0\rangle\\ &= \vec{0}.\end{aligned}$$

Section 10.5

1. A point on the line and the direction of the line.

2. parallel

3. parallel, skew

4. Answers will vary

5. vector: $\ell(t) = \langle 2, -4, 1\rangle + t\langle 9, 2, 5\rangle$
parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$
symmetric: $(x-2)/9 = (y+4)/2 = (z-1)/5$

6. vector: $\ell(t) = \langle 6, 1, 7\rangle + t\langle -3, 2, 5\rangle$
parametric: $x = 6 - 3t$, $y = 1 + 2t$, $z = 7 + 5t$
symmetric: $-(x-6)/3 = (y-1)/2 = (z-7)/5$

7. Answers can vary: vector: $\ell(t) = \langle 2, 1, 5\rangle + t\langle 5, -3, -1\rangle$
parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$
symmetric: $(x-2)/5 = -(y-1)/3 = -(z-5)$

8. Answers can vary: vector: $\ell(t) = \langle 1, -2, 3\rangle + t\langle 4, 7, 2\rangle$
parametric: $x = 1 + 4t$, $y = -2 + 7t$, $z = 3 + 2t$
symmetric: $(x-1)/4 = (y+2)/7 = (z-3)/2$

9. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 0, 1, 2\rangle + t\langle -10, 43, 9\rangle$
parametric: $x = -10t$, $y = 1 + 43t$, $z = 2 + 9t$
symmetric: $-x/10 = (y-1)/43 = (z-2)/9$

10. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 5, 1, 9\rangle + t\langle 0, -1, 0\rangle$
parametric: $x = 5$, $y = 1 - t$, $z = 9$
symmetric: not defined, as some components of the direction are 0.

11. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 7, 2, -1\rangle + t\langle 1, -1, 2\rangle$
parametric: $x = 7 + t$, $y = 2 - t$, $z = -1 + 2t$
symmetric: $x - 7 = 2 - y = (z+1)/2$

12. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 2, 2, 3\rangle + t\langle 5, -1, -3\rangle$
parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
symmetric: $(x-2)/5 = -(y-2) = -(z-3)/3$

13. vector: $\ell(t) = \langle 1, 1\rangle + t\langle 2, 3\rangle$
parametric: $x = 1 + 2t$, $y = 1 + 3t$
symmetric: $(x-1)/2 = (y-1)/3$

14. vector: $\ell(t) = \langle -2, 5\rangle + t\,\langle 0, 1\rangle$
parametric: $x = -2$, $y = 5 + t$
symmetric: not defined

15. parallel

16. intersecting; $\ell_1(2) = \ell_2(-2) = \langle 12, 3, 7\rangle$

17. intersecting; $\vec{\ell}_1(3) = \vec{\ell}_2(4) = \langle 9, -5, 13\rangle$

18. same

19. skew

20. parallel

21. same

22. skew

23. $\sqrt{41}/3$

24. $3\sqrt{2}$

25. $5\sqrt{2}/2$

26. 5

27. $3/\sqrt{2}$

28. 2

29. Since both P and Q are on the line, $\overrightarrow{PQ}$ is parallel to $\vec{d}$. Thus $\overrightarrow{PQ} \times \vec{d} = \vec{0}$, giving a distance of 0.

30. (Note: this solution is easier once one has studied Section 10.6.) Since the two lines intersect, we can state $P_2 = P_1 + a\vec{d}_1 + b\vec{d}_2$ for some scalars a and b. (Here we abuse notation slightly and add points to vectors.) Thus $\overrightarrow{P_1P_2} = a\vec{d}_1 + b\vec{d}_2$. Vector $\vec{c}$ is the cross product of $\vec{d}_1$ and $\vec{d}_2$, hence is orthogonal to both, and hence is orthogonal to $\overrightarrow{P_1P_2}$. Thus $\overrightarrow{P_1P_2} \cdot \vec{c} = 0$, and the distance between lines is 0.

31. (a) The distance formula cannot be used because since $\vec{d}_1$ and $\vec{d}_2$ are parallel, $\vec{c}$ is $\vec{0}$ and we cannot divide by $||\,\vec{0}\,||$.
    (b) Since $\vec{d}_1$ and $\vec{d}_2$ are parallel, $\overrightarrow{P_1P_2}$ lies in the plane formed by the two lines. Thus $\overrightarrow{P_1P_2} \times \vec{d}_2$ is orthogonal to this plane, and $\vec{c} = (\overrightarrow{P_1P_2} \times \vec{d}_2) \times \vec{d}_2$ is parallel to the plane, but still orthogonal to both $\vec{d}_1$ and $\vec{d}_2$. We desire the length of the projection of $\overrightarrow{P_1P_2}$ onto $\vec{c}$, which is what the formula provides.
    (c) Since the lines are parallel, one can measure the distance between the lines at any location on either line (just as to find the distance between straight railroad tracks, one can use a measuring tape anywhere along the track, not just at one specific place.) Let $P = P_1$ and $Q = P_2$ as given by the equations of the lines, and apply the formula for distance between a point and a line.

Section 10.6

1. A point in the plane and a normal vector (i.e., a direction orthogonal to the plane).

2. A normal vector is orthogonal to the plane.

3. Answers will vary.

4. Answers will vary.

5. Answers will vary.

6. Answers will vary.

7. Standard form: $3(x-2) - (y-3) + 7(z-4) = 0$
general form: $3x - y + 7z = 31$

8. Standard form: $2(y-3) + 4(z-5) = 0$
general form: $2y + 4z = 26$

9. Answers may vary;
Standard form: $8(x-1) + 4(y-2) - 4(z-3) = 0$
general form: $8x + 4y - 4z = 4$

10. Answers may vary;
Standard form: $-5(x-5) + 3(y-3) + 2(z-8) = 0$
general form: $-5x + 3y + 2z = 0$

11. Answers may vary;
Standard form: $-7(x-2) + 2(y-1) + (z-2) = 0$
general form: $-7x + 2y + z = -10$

12. Answers may vary;
Standard form: $3(x-5) + 3(z-3) = 0$
general form: $3x + 3z = 24$

13. Answers may vary;
Standard form: $2(x-1) - (y-1) = 0$
general form: $2x - y = 1$

14. Answer forms may vary; normal vectors should be multiples of $\langle 2, 1, -3\rangle$.
Standard form: $6(x-1) + 3(y-1) - 9(z-1) = 0$
general form: $6x + 3y - 9z = 0$

15. Answers may vary;
Standard form: $2(x-2) - (y+6) - 4(z-1) = 0$
general form: $2x - y - 4z = 6$

16. Answers may vary;
Standard form: $4(x-5) - 2(y-7) - 2(z-3) = 0$
general form: $4x - 2y - 2z = 0$

17. Answers may vary;
Standard form: $(x-5) + (y-7) + (z-3) = 0$
general form: $x + y + z = 15$

18. Answers may vary;
Standard form: $4(x-4) + (y-1) + (z-1) = 0$
general form: $4x + y + z = 18$

19. Answers may vary;
Standard form: $3(x+4) + 8(y-7) - 10(z-2) = 0$
general form: $3x + 8y - 10z = 24$

20. Standard form: $x - 1 = 0$
general form: $x = 1$

21. Answers may vary:
$$\ell = \begin{cases} x = 14t \\ y = -1 - 10t \\ z = 2 - 8t \end{cases}$$

22. Answers may vary:
$$\ell = \begin{cases} x = 1 + 20t \\ y = 3 + 2t \\ z = 3.5 - 26t \end{cases}$$

23. $(-3, -7, -5)$

24. $(3, 1, 1)$

25. No point of intersection; the plane and line are parallel.

26. The plane contains the line, so every point on the line is a "point of intersection."

27. $\sqrt{5/7}$

28. $8/\sqrt{21}$

29. $1/\sqrt{3}$

30. 3

31. If P is any point in the plane, and Q is also in the plane, then $\overrightarrow{PQ}$ lies parallel to the plane and is orthogonal to $\vec{n}$, the normal vector. Thus $\vec{n} \cdot \overrightarrow{PQ} = 0$, giving the distance as 0.

32. The intersecting lines define a plane with normal vector $\vec{n} = \vec{c} = \vec{d}_1 \times \vec{d}_2$. Since points P_1 and P_2 lie in the plane, $\vec{c}$ is orthogonal to $\overrightarrow{P_1P_2}$, hence $\overrightarrow{P_1P_2} \cdot \vec{c} = 0$, giving a distance of 0. Knowing the principles of planes, especially their normal vectors, makes this simpler.

Chapter 11

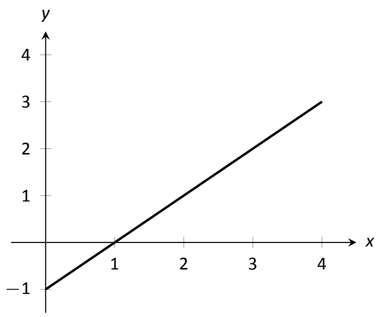
Section 11.1

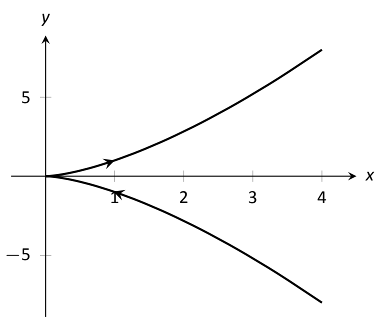
1. parametric equations

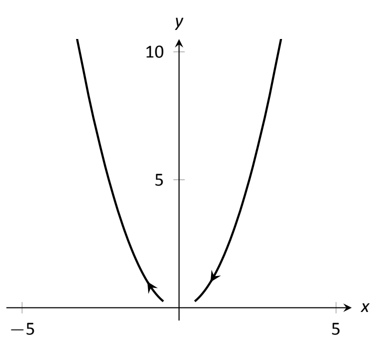
2. vectors

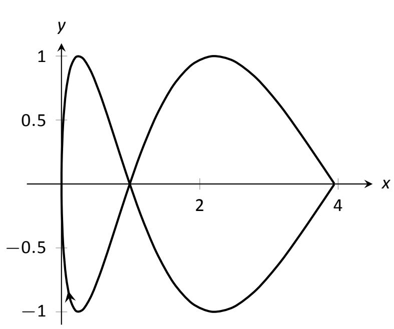
3. displacement

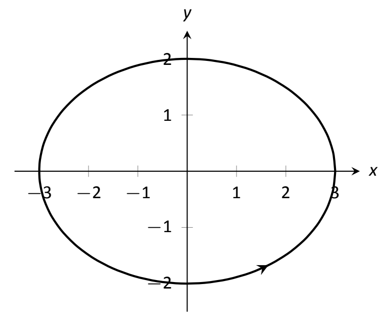
4. displacement

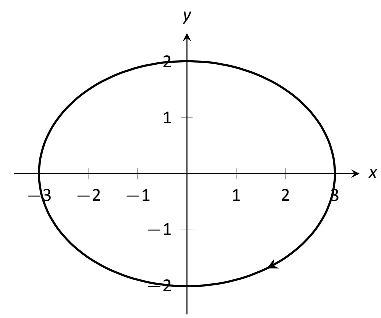
5. 

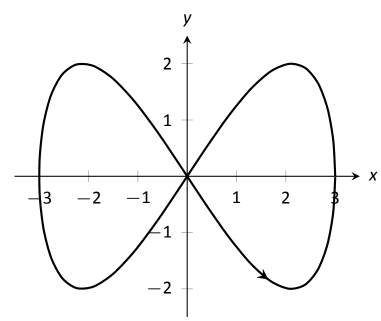
6. 

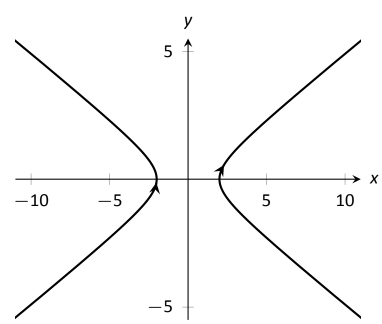
7. 

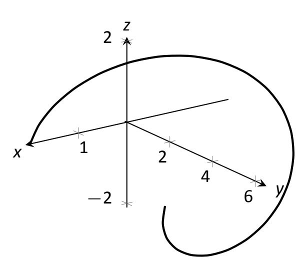
8. 

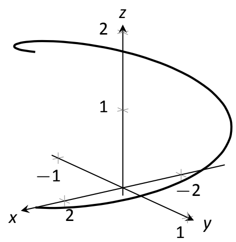
9. 

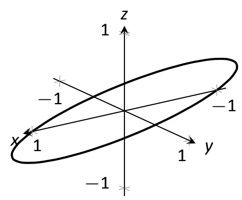
10. 

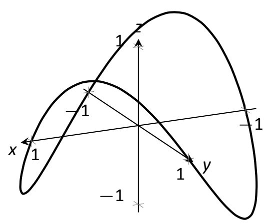
11. 

12. 

13. 

14. 

15. 

16. 

17. $\|\vec{r}(t)\| = \sqrt{t^2 + t^4} = |t|\sqrt{t^2 + 1}$.

18. $\|\vec{r}(t)\| = \sqrt{25\cos^2 t + 9\sin^2 t}$.

19. $\|\vec{r}(t)\| = \sqrt{4\cos^2 t + 4\sin^2 t + t^2} = \sqrt{t^2 + 4}$.

20. $\|\vec{r}(t)\| = \sqrt{\cos^2 t + t^2 + t^4}$.

21. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 2\cos t + 1, 2\sin t + 2\rangle$.

22. Answers may vary; three solutions are $\vec{r}(t) = \langle 3\sin t + 5, 3\cos t + 5\rangle$, $\vec{r}(t) = \langle -3\cos t + 5, 3\sin t + 5\rangle$ and $\vec{r}(t) = \langle 3\cos t + 5, -3\sin t + 5\rangle$.

23. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 1.5\cos t, 5\sin t\rangle$.

24. Answers may vary, though most direct solutions are $\vec{r}(t) = \langle -3\cos t + 3, 2\sin t - 2\rangle$, $\vec{r}(t) = \langle 3\cos t + 3, -2\sin t - 2\rangle$ and $\vec{r}(t) = \langle 3\sin t + 3, 2\cos t - 2\rangle$.

25. Answers may vary, though most direct solutions are $\vec{r}(t) = \langle t, 5(t-2) + 3\rangle$ and $\vec{r}(t) = \langle t + 2, 5t + 3\rangle$.

26. Answers may vary, though most direct solutions are $\vec{r}(t) = \langle t, -1/2(t-1) + 5\rangle$, $\vec{r}(t) = \langle t + 1, -1/2t + 5\rangle$, $\vec{r}(t) = \langle -2t + 1, t + 5\rangle$ and $\vec{r}(t) = \langle 2t + 1, -t + 5\rangle$.

27. Specific forms may vary, though most direct solutions are $\vec{r}(t) = \langle 1, 2, 3\rangle + t\langle 3, 3, 3\rangle$ and $\vec{r}(t) = \langle 3t + 1, 3t + 2, 3t + 3\rangle$.

28. Specific forms may vary, though most direct solutions are $\vec{r}(t) = \langle 1, 2\rangle + t\langle 3, 2\rangle$ and $\vec{r}(t) = \langle 3t + 1, 2t + 2\rangle$.

29. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 2\cos t, 2\sin t, 2t\rangle$.

30. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 3\cos(4\pi t), 3\sin(4\pi t), 3t\rangle$.

31. $\langle 1, 0\rangle$

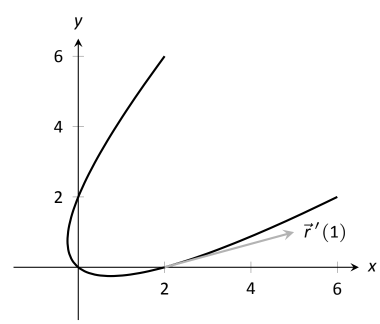
32. $\langle 1, 1\rangle$

33. $\langle 0, 0, 1\rangle$

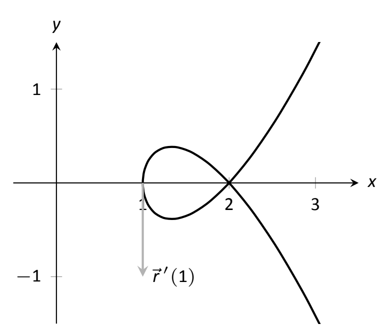
34. $\langle 1, 2, 7\rangle$

Section 11.2

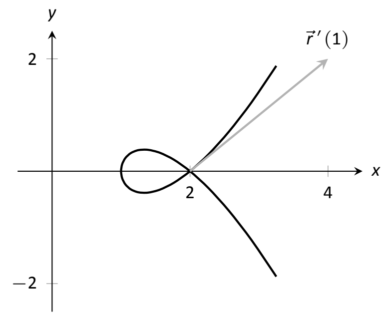
1. component
2. displacement
3. It is difficult to identify the points on the graphs of $\vec{r}(t)$ and $\vec{r}\,'(t)$ that correspond to each other.
4. A scalar-vector product, a dot product and a cross product.
5. $\langle 11, 74, \sin 5\rangle$
6. $\langle e^3, 0\rangle$
7. $\langle 1, e\rangle$
8. $\langle 2t, 1, 0\rangle$
9. $(-\infty, 0)\bigcup(0, \infty)$
10. $(0, \infty)$
11. $\vec{r}\,'(t) = \langle -\sin t, e^t, 1/t\rangle$
12. $\vec{r}\,'(t) = \langle -1/t^2, 5/(3t+1)^2, \sec^2 t\rangle$
13. $\vec{r}\,'(t) = (2t)\langle \sin t, 2t + 5\rangle + (t^2)\langle \cos t, 2\rangle = \langle 2t\sin t + t^2\cos t, 6t^2 + 10t\rangle$
14. $\vec{r}\,'(t) = \langle 2t, 1\rangle \cdot \langle \sin t, 2t + 5\rangle + \langle t^2 + 1, t - 1\rangle \cdot \langle \cos t, 2\rangle = (t^2 + 1)\cos t + 2t\sin t + 4t + 3$
15. $\vec{r}\,'(t) = \langle 2t, 1, 0\rangle \times \langle \sin t, 2t + 5, 1\rangle + \langle t^2 + 1, t - 1, 1\rangle \times \langle \cos t, 2, 0\rangle = \langle -1, \cos t - 2t, 6t^2 + 10t + 2 + \cos t - \sin t - t\cos t\rangle$
16. $\vec{r}\,'(t) = \langle \sinh t, \cosh t\rangle$

17. 

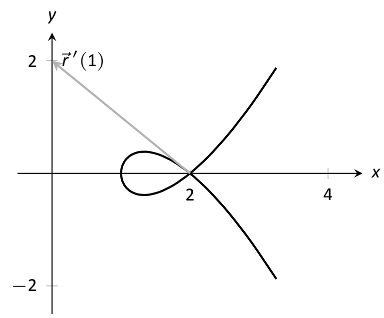
$\vec{r}\,'(t) = \langle 2t + 1, 2t - 1\rangle$

18. 

$\vec{r}\,'(t) = \langle 2t - 2, 3t^2 - 6t + 2\rangle$

19. 

$\vec{r}\,'(t) = \langle 2t, 3t^2 - 1 \rangle$

20. 

$\vec{r}\,'(t) = \langle 2t - 4, 3t^2 - 12t + 11 \rangle$

21. $\ell(t) = \langle 2, 0 \rangle + t\,\langle 3, 1 \rangle$

22. $\ell(t) = \langle 3\sqrt{2}/2, \sqrt{2}/2 \rangle + t\,\langle -3\sqrt{2}/2, \sqrt{2}/2 \rangle$

23. $\ell(t) = \langle -3, 0, \pi \rangle + t\,\langle 0, -3, 1 \rangle$

24. $\ell(t) = \langle 1, 0, 0 \rangle + t\,\langle 1, 1, 1 \rangle$

25. $t = 2n\pi$, where n is an integer; so $t = \ldots - 4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots$

26. $t = 1$

27. $\vec{r}(t)$ is not smooth at $t = 3\pi/4 + n\pi$, where n is an integer

28. $t = \pm 1$

29. Both derivatives return $\langle 5t^4, 4t^3 - 3t^2, 3t^2 \rangle$.

30. Both derivatives return $2\sin t + t^2 \cos t + te^t + 1$.

31. Both derivatives return $\langle 2t - e^t - 1, \cos t - 3t^2, (t^2 + 2t)e^t - (t-1)\cos t - \sin t \rangle$.

32. Both derivatives return $\langle 6t^5, 3t^2, 0 \rangle$

33. $\langle \frac{1}{4}t^4, \sin t, te^t - e^t \rangle + \vec{C}$

34. $\langle \tan^{-1} t, \tan t \rangle + \vec{C}$

35. $\langle -2, 0 \rangle$

36. $\langle 4, -4 \rangle$

37. $\vec{r}(t) = \langle \frac{1}{2}t^2 + 2, -\cos t + 3 \rangle$

38. $\vec{r}(t) = \langle \ln|t+1| + 1, -\ln|\cos t| + 2 \rangle$

39. $\vec{r}(t) = \langle t^4/12 + t + 4,\ t^3/6 + 2t + 5,\ t^2/2 + 3t + 6 \rangle$

40. $\vec{r}(t) = \langle -\cos t + 1, t - \sin t, e^t - t - 1 \rangle$

41. $2\sqrt{13}\pi$

42. 10π

43. $\frac{1}{54}\left((22)^{3/2} - 8\right)$

44. $\sqrt{2}(1 - e^{-1})$

45. As $\vec{r}(t)$ has constant length, $\vec{r}(t) \cdot \vec{r}(t) = c^2$ for some constant c. Thus

$$\begin{aligned} \vec{r}(t) \cdot \vec{r}(t) &= c^2 \\ \frac{d}{dt}\left(\vec{r}(t) \cdot \vec{r}(t)\right) &= \frac{d}{dt}\left(c^2\right) \\ \vec{r}\,'(t) \cdot \vec{r}(t) + \vec{r}(t) \cdot \vec{r}\,'(t) &= 0 \\ 2\vec{r}(t) \cdot \vec{r}\,'(t) &= 0 \\ \vec{r}(t) \cdot \vec{r}\,'(t) &= 0. \end{aligned}$$

Section 11.3

1. Velocity is a vector, indicating an objects direction of travel and its rate of distance change (i.e., its speed). Speed is a scalar.

2. Displacement is a vector, indicating the difference between the starting and ending positions of an object. Distance traveled is a scalar, indicating the arc length of the path followed.

3. The average velocity is found by dividing the displacement by the time traveled – it is a vector. The average speed is found by dividing the distance traveled by the time traveled – it is a scalar.

4. arc length

5. One example is traveling at a constant speed s in a circle, ending at the starting position. Since the displacement is $\vec{0}$, the average velocity is $\vec{0}$, hence $||\,\vec{0}\,|| = 0$. But traveling at constant speed s means the average speed is also $s > 0$.

6. Distance traveled is always greater than or equal to the magnitude of displacement, therefore average speed will always be at least as large as the magnitude of the average velocity.

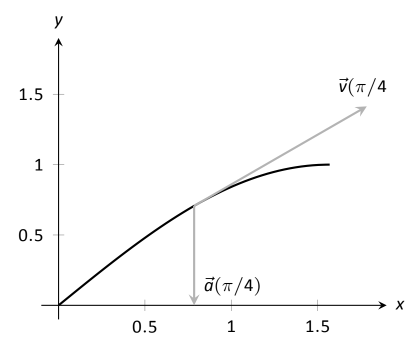
7. $\vec{v}(t) = \langle 2, 5, 0 \rangle$, $\vec{a}(t) = \langle 0, 0, 0 \rangle$

8. $\vec{v}(t) = \langle 6t - 2, -2t + 1 \rangle$, $\vec{a}(t) = \langle 6, -2 \rangle$

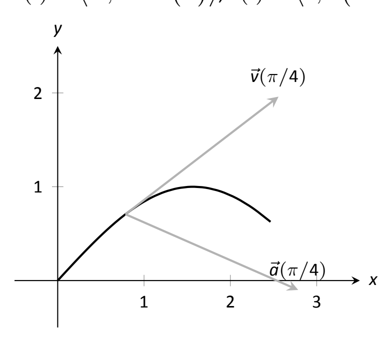
9. $\vec{v}(t) = \langle -\sin t, \cos t \rangle$, $\vec{a}(t) = \langle -\cos t, -\sin t \rangle$

10. $\vec{v}(t) = \langle 1/10, \sin t, \cos t \rangle$, $\vec{a}(t) = \langle 0, \cos t, -\sin t \rangle$

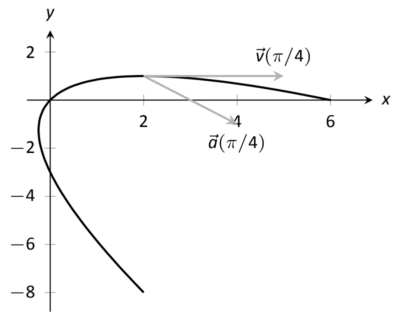
11. $\vec{v}(t) = \langle 1, \cos t \rangle$, $\vec{a}(t) = \langle 0, -\sin t \rangle$



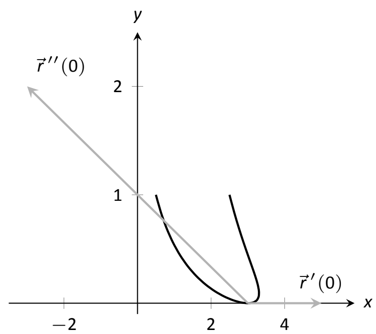
12. $\vec{v}(t) = \langle 2t, 2t\cos(t^2) \rangle$, $\vec{a}(t) = \langle 2, 2\left(\cos(t^2) - 2t^2\sin(t^2)\right) \rangle$



13. $\vec{v}(t) = \langle 2t+1, -2t+2\rangle$, $\vec{a}(t) = \langle 2, -2\rangle$



14. $\vec{v}(t) = \left\langle -\frac{2(t^2+3t-1)}{(t^2+1)^2}, 2t\right\rangle$, $\vec{a}(t) = \left\langle \frac{2(2t^3+9t^2-6t-3)}{(t^2+1)^3}, 2\right\rangle$



15. $||\,\vec{v}(t)\,|| = \sqrt{4t^2+1}$.
Min at $t=0$; Max at $t=\pm 1$.

16. $||\,\vec{v}(t)\,|| = |t|\sqrt{9t^2-12t+8}$.
min: $t=0$; max: $t=-1$

17. $||\,\vec{v}(t)\,|| = 5$.
Speed is constant, so there is no difference between min/max

18. $||\,\vec{v}(t)\,|| = \sqrt{4\sin^2 t + 25\cos^2 t}$.
min: $t=\pi/2,\ 3\pi/2$; max: $t=0,\ 2\pi$

19. $||\,\vec{v}(t)\,|| = |\sec t|\sqrt{\tan^2 t + \sec^2 t}$.
min: $t=0$; max: $t=\pi/4$

20. $||\,\vec{v}(t)\,|| = \sqrt{2-2\sin t}$.
min: $t=\pi/2$; max: $t=3\pi/2$

21. $||\,\vec{v}(t)\,|| = 13$.
speed is constant, so there is no difference between min/max

22. $||\,\vec{v}(t)\,|| = \sqrt{8t^2+3}$.
min: $t=0$; max: $t=1$

23. $||\,\vec{v}(t)\,|| = \sqrt{4t^2+1+t^2/(1-t^2)}$.
min: $t=0$; max: there is no max; speed approaches ∞ as $t\to\pm 1$

24. $||\,\vec{v}(t)\,|| = \sqrt{g^2t^2 - (2gv_0\sin\theta)t + v_0^2}$.
min: $t=(v_0\sin\theta)/g$; max: $t=0, t=(2v_0\sin\theta)/g$

25. (a) $\vec{r}_1(1) = \langle 1,1\rangle$; $\vec{r}_2(1) = \langle 1,1\rangle$
(b) $\vec{v}_1(1) = \langle 1,2\rangle$; $||\,\vec{v}_1(1)\,|| = \sqrt{5}$; $\vec{a}_1(1) = \langle 0,2\rangle$
$\vec{v}_2(1) = \langle 2,4\rangle$; $||\,\vec{v}_2(1)\,|| = 2\sqrt{5}$; $\vec{a}_2(1) = \langle 2,12\rangle$

26. (a) $\vec{r}_1(\pi/2) = \langle 0,3\rangle$; $\vec{r}_2(\pi/8) = \langle 0,3\rangle$
(b) $\vec{v}_1(\pi/2) = \langle -3,0\rangle$; $||\,\vec{v}_1(\pi/2)\,|| = 3$; $\vec{a}_1(\pi/2) = \langle 0,-3\rangle$
$\vec{v}_2(\pi/8) = \langle -12,0\rangle$; $||\,\vec{v}_2(\pi/8)\,|| = 12$;
$\vec{a}_2(\pi/8) = \langle 0,-48\rangle$

27. (a) $\vec{r}_1(2) = \langle 6,4\rangle$; $\vec{r}_2(2) = \langle 6,4\rangle$
(b) $\vec{v}_1(2) = \langle 3,2\rangle$; $||\,\vec{v}_1(2)\,|| = \sqrt{13}$; $\vec{a}_1(2) = \langle 0,0\rangle$
$\vec{v}_2(2) = \langle 6,4\rangle$; $||\,\vec{v}_2(2)\,|| = 2\sqrt{13}$; $\vec{a}_2(2) = \langle 0,0\rangle$

28. (a) $\vec{r}_1(1) = \langle 1,1\rangle$; $\vec{r}_2(\pi/2) = \langle 1,1\rangle$
(b) $\vec{v}_1(1) = \langle 1,1/2\rangle$; $||\,\vec{v}_1(1)\,|| = \sqrt{5}/2$; $\vec{a}_1(1) = \langle 0,-1/4\rangle$
$\vec{v}_2(\pi/2) = \langle 0,0\rangle$; $||\,\vec{v}_2(\pi/2)\,|| = 0$;
$\vec{a}_2(\pi/2) = \langle -1,-1/2\rangle$

29. $\vec{v}(t) = \langle 2t+1, 3t+2\rangle$, $\vec{r}(t) = \left\langle t^2+t+5, 3t^2/2+2t-2\right\rangle$

30. $\vec{v}(t) = \langle 2t-1, 3t-1\rangle$, $\vec{r}(t) = \left\langle t^2-t+5, 3t^2/2-t-5/2\right\rangle$

31. $\vec{v}(t) = \langle \sin t, \cos t\rangle$, $\vec{r}(t) = \langle 1-\cos t, \sin t\rangle$

32. $\vec{v}(t) = \langle 10, -32t+50\rangle$, $\vec{r}(t) = \left\langle 10t, -16t^2+50t\right\rangle$

33. Displacement: $\langle 0,0,6\pi\rangle$; distance traveled: $2\sqrt{13}\pi \approx 22.65$ft; average velocity: $\langle 0,0,3\rangle$; average speed: $\sqrt{13}\approx 3.61$ft/s

34. Displacement: $\langle -10,0\rangle$; distance traveled: $5\pi\approx 15.71$ft; average velocity: $\langle -10/\pi, 0\rangle \approx \langle -3.18, 0\rangle$; average speed: 5ft/s

35. Displacement: $\langle 0,0\rangle$; distance traveled: $2\pi\approx 6.28$ft; average velocity: $\langle 0,0\rangle$; average speed: 1ft/s

36. Displacement: $\langle 10,20,-20\rangle$; distance traveled: 30ft; average velocity: $\langle 1,2,-2\rangle$; average speed: 3ft/s

37. At t-values of $\sin^{-1}(9/30)/(4\pi)+n/2 \approx 0.024+n/2$ seconds, where n is an integer.

38. The stone, while whirling, can be modeled by $\vec{r}(t) = \langle 3\cos(8\pi t), 3\sin(8\pi t)\rangle$.
(a) For t-values $t=\sin^{-1}(3/20)/(8\pi)+n/4 \approx 0.006+n/4$, where n is an integer.
(b) $||\,\vec{r}\,'(t)\,|| = 24\pi \approx 51.4$ft/s
(c) At $t=0.006$, the stone is approximately 19.77ft from Goliath. Using the formula for projectile motion, we want the angle of elevation that lets a projectile starting at $\langle 0,6\rangle$ with a initial velocity of 51.4ft/s arrive at $\langle 19.77, 9\rangle$. The desired angle is 0.27 radians, or 15.69°.

39. (a) Holding the crossbow at an angle of 0.013 radians, $\approx 0.745°$ will hit the target 0.4s later. (Another solution exists, with an angle of 89°, landing 18.75s later, but this is impractical.)
(b) In the .4 seconds the arrow travels, a deer, traveling at 20mph or 29.33ft/s, can travel 11.7ft. So she needs to lead the deer by 11.7ft.

40. The position function of the ball is $\vec{r}(t) = \left\langle (146.67\cos\theta)t, -16t^2+(146.67\sin\theta)t+3\right\rangle$, where θ is the angle of elevation.
(a) With $\theta=20°$, the ball reaches 310ft from home plate in 2.25 seconds; at this time, the height of the ball is 34.9ft, not enough to clear the Green Monster.
(b) With $\theta=21°$, the ball reaches 310ft from home plate in 2.26s, with a height of 40ft, clearing the wall.

41. The position function is $\vec{r}(t) = \left\langle 220t, -16t^2+1000\right\rangle$. The y-component is 0 when $t=7.9$; $\vec{r}(7.9) = \langle 1739.25, 0\rangle$, meaning the box will travel about 1740ft horizontally before it lands.

42. The position function of the ball is $\vec{r}(t) = \left\langle (v_0\cos\theta)t, -16t^2+(v_0\sin\theta)t+6\right\rangle$, where θ is the angle of elevation and v_0 is the initial ball speed.
(a) With $v_0 = 73.33$ft/s, there are two angles of elevation possible. An angle of $\theta=9.47°$ delivers the ball in 0.83s, while an angle of 79.57° delivers the ball in 4.5s.
(b) With $\theta=8°$, the initial speed must be 53.8mph$\approx$ 78.9ft/s.

Section 11.4

1. 1
2. 0
3. $\vec{T}(t)$ and $\vec{N}(t)$.

4. the speed

5. $\vec{T}(t) = \left\langle \frac{4t}{\sqrt{20t^2-4t+1}}, \frac{2t-1}{\sqrt{20t^2-4t+1}} \right\rangle$; $\vec{T}(1) = \langle 4/\sqrt{17}, 1/\sqrt{17} \rangle$

6. $\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+\sin^2 t}}, -\frac{\sin t}{\sqrt{1+\sin^2 t}} \right\rangle$; $\vec{T}(\pi/4) = \left\langle \sqrt{2/3}, -1/\sqrt{3} \right\rangle$

7. $\vec{T}(t) = \frac{\cos t \sin t}{\sqrt{\cos^2 t \sin^2 t}} \langle -\cos t, \sin t \rangle$. (Be careful; this cannot be simplified as just $\langle -\cos t, \sin t \rangle$ as $\sqrt{\cos^2 t \sin^2 t} \neq \cos t \sin t$, but rather $|\cos t \sin t|$.) $\vec{T}(\pi/4) = \langle -\sqrt{2}/2, \sqrt{2}/2 \rangle$

8. $\vec{T}(t) = \langle -\sin t, \cos t \rangle$; $\vec{T}(\pi) = \langle 0, -1 \rangle$

9. $\ell(t) = \langle 2, 0 \rangle + t \langle 4/\sqrt{17}, 1/\sqrt{17} \rangle$; in parametric form, $\ell(t) = \begin{cases} x = 2 + 4t/\sqrt{17} \\ y = t/\sqrt{17} \end{cases}$

10. $\ell(t) = \langle \pi/4, \sqrt{2}/2 \rangle + t \left\langle \sqrt{2/3}, -1/\sqrt{3} \right\rangle$; in parametric form, $\ell(t) = \begin{cases} x = \pi/4 + \sqrt{2/3}t \\ y = \sqrt{2}/2 - t/\sqrt{3} \end{cases}$

11. $\ell(t) = \langle \sqrt{2}/4, \sqrt{2}/4 \rangle + t \langle -\sqrt{2}/2, \sqrt{2}/2 \rangle$; in parametric form, $\ell(t) = \begin{cases} x = \sqrt{2}/4 - \sqrt{2}t/2 \\ y = \sqrt{2}/4 + \sqrt{2}t/2 \end{cases}$

12. $\ell(t) = \langle -1, 0 \rangle + t \langle 0, -1 \rangle$; in parametric form, $\ell(t) = \begin{cases} x = -1 \\ y = -t \end{cases}$

13. $\vec{T}(t) = \langle -\sin t, \cos t \rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t \rangle$

14. $\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}} \right\rangle$; $\vec{N}(t) = \left\langle -\frac{2t}{\sqrt{1+4t^2}}, \frac{1}{\sqrt{1+4t^2}} \right\rangle$

15. $\vec{T}(t) = \left\langle -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}}, \frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}} \right\rangle$;
$\vec{N}(t) = \left\langle -\frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}}, -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}} \right\rangle$

16. $\vec{T}(t) = \left\langle \frac{e^t}{\sqrt{e^{2t}+e^{-2t}}}, -\frac{e^{-t}}{\sqrt{e^{2t}+e^{-2t}}} \right\rangle$;
$\vec{N}(t) = \left\langle \frac{e^{-t}}{\sqrt{e^{2t}+e^{-2t}}}, \frac{e^t}{\sqrt{e^{2t}+e^{-2t}}} \right\rangle$

17. (a) Be sure to show work
 (b) $\vec{N}(\pi/4) = \langle -5/\sqrt{34}, -3/\sqrt{34} \rangle$

18. (a) Be sure to show work
 (b) $\vec{N}(1) = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

19. (a) Be sure to show work
 (b) $\vec{N}(0) = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

20. (a) Be sure to show work
 (b) $\vec{N}(\pi/4) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

21. $\vec{T}(t) = \frac{1}{\sqrt{5}} \langle 2, \cos t, -\sin t \rangle$; $\vec{N}(t) = \langle 0, -\sin t, -\cos t \rangle$

22. $\vec{T}(t) = \langle -\sin t, 3/5 \cos t, 4/5 \cos t \rangle$;
$\vec{N}(t) = \langle -\cos t, -3/5 \sin t, -4/5 \sin t \rangle$

23. $\vec{T}(t) = \frac{1}{\sqrt{a^2+b^2}} \langle -a \sin t, a \cos t, b \rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t, 0 \rangle$

24. $\vec{T}(t) = \frac{1}{\sqrt{a^2+1}} \langle -a \sin(at), a \cos(at), 1 \rangle$;
$\vec{N}(t) = \langle -\cos t, -\sin t, 0 \rangle$

25. $a_T = \frac{4t}{\sqrt{1+4t^2}}$ and $a_N = \sqrt{4 - \frac{16t^2}{1+4t^2}}$
At $t = 0$, $a_T = 0$ and $a_N = 2$;
At $t = 1$, $a_T = 4/\sqrt{5}$ and $a_N = 2/\sqrt{5}$.
At $t = 0$, all acceleration comes in the form of changing the direction of velocity and not the speed; at $t = 1$, more acceleration comes in changing the speed than in changing direction.

26. $a_T = \frac{-2/t^5}{\sqrt{1+1/t^4}}$ and $a_N = \sqrt{\frac{4}{t^6} - \frac{4/t^{10}}{1+1/t^4}}$
At $t = 1$, $a_T = \sqrt{2}$ and $a_N = -\sqrt{2}$;
At $t = 2$, $a_T = -\frac{1}{4\sqrt{17}}$ and $a_N = \frac{1}{\sqrt{17}}$.
At $t = 1$, acceleration comes from changing speed and changing direction in "equal measure;" at $t = 2$, acceleration is nearly $\vec{0}$ as it is; the low value of a_T shows that the speed is nearly constant and the low value of a_N shows the direction is not changing quickly.

27. $a_T = 0$ and $a_N = 2$
At $t = 0$, $a_T = 0$ and $a_N = 2$;
At $t = \pi/2$, $a_T = 0$ and $a_N = 2$.
The object moves at constant speed, so all acceleration comes from changing direction, hence $a_T = 0$. $\vec{a}(t)$ is always parallel to $\vec{N}(t)$, but twice as long, hence $a_N = 2$.

28. $a_T = 2$ and $a_N = 4t^2$
At $t = \sqrt{\pi/2}$, $a_T = 2$ and $a_N = 2\pi$;
At $t = \sqrt{\pi}$, $a_T = 2$ and $a_N = 4\pi$.
The object moves at increasing speed (increasing at a constant rate of acceleration), hence $a_T = 2$. Since the object is increasing speed yet always traveling in a circle of radius 1, the direction must change more quickly; the amount of acceleration that changes direction increases over time.

29. $a_T = 0$ and $a_N = a$
At $t = 0$, $a_T = 0$ and $a_N = a$;
At $t = \pi/2$, $a_T = 0$ and $a_N = a$.
The object moves at constant speed, meaning that a_T is always 0. The object "rises" along the z-axis at a constant rate, so all acceleration comes in the form of changing direction circling the z-axis. The greater the radius of this circle the greater the acceleration, hence $a_N = a$.

30. $a_T = 0$ and $a_N = 5$
At $t = 0$, $a_T = 0$ and $a_N = 5$;
At $t = \pi/2$, $a_T = 0$ and $a_N = 5$.
The object moves at constant speed, meaning that a_T is always 0. Acceleration is thus always perpendicular to the direction of travel; in this particular case, it is always 5 times the unit vector pointing orthogonal to the direction of travel.

Section 11.5

1. time and/or distance

2. curvature

3. Answers may include lines, circles, helixes

4. Answers will vary; they should mention the circle is tangent to the curve and has the same curvature as the curve at that point.

5. κ

6. a_T is not affected by curvature; the greater the curvature, the larger a_N becomes.

7. $s = 3t$, so $\vec{r}(s) = \langle 2s/3, s/3, -2s/3 \rangle$

8. $s = 7t$, so $\vec{r}(s) = \langle 7\cos(s/7), 7\sin(s/7) \rangle$

9. $s = \sqrt{13}t$, so $\vec{r}(s) = \langle 3\cos(s/\sqrt{13}), 3\sin(s/\sqrt{13}), 2s/\sqrt{13} \rangle$

10. $s = 13t$, so $\vec{r}(s) = \langle 5\cos(s/13), 13\sin(s/13), 12\cos(s/13) \rangle$

11. $\kappa = \frac{|6x|}{\left(1+(3x^2-1)^2\right)^{3/2}}$;
$\kappa(0) = 0$, $\kappa(1/2) = \frac{192}{17\sqrt{17}} \approx 2.74$.

12. $\kappa = \frac{\left|\frac{6x^2-2}{(x^2+1)^3}\right|}{\left(1+\frac{4x^2}{(x^2+1)^4}\right)^{3/2}}$;
$\kappa(0) = 2$, $\kappa(2) = \frac{2750}{641\sqrt{641}} \approx 0.169$.

13. $\kappa = \frac{|\cos x|}{\left(1+\sin^2 x\right)^{3/2}}$;
$\kappa(0) = 1$, $\kappa(\pi/2) = 0$

14. $\kappa = 1$;
$\kappa(0) = 1$, $\kappa(1/2) = 1$

15. $\kappa = \frac{|2\cos t\cos(2t)+4\sin t\sin(2t)|}{\left(4\cos^2(2t)+\sin^2 t\right)^{3/2}}$;
$\kappa(0) = 1/4$, $\kappa(\pi/4) = 8$

16. $\kappa = 2$;
$\kappa(0) = 2$, $\kappa(\pi/3) = 2$

17. $\kappa = \frac{|6t^2+2|}{\left(4t^2+(3t^2-1)^2\right)^{3/2}}$;
$\kappa(0) = 2$, $\kappa(5) = \frac{19}{1394\sqrt{1394}} \approx 0.0004$

18. $\kappa = \frac{|\sec^3 t|}{\left(\sec^4 t+\sec^2 t\tan^2 t\right)^{3/2}}$;
$\kappa(0) = 1$, $\kappa(\pi/6) = \frac{3\sqrt{3}}{5\sqrt{5}} \approx 0.465$

19. $\kappa = 0$;
$\kappa(0) = 0$, $\kappa(1) = 0$

20. $\kappa = \frac{2\sqrt{18t^4+15t^2+1}}{\left(18t^4-2t^2+1\right)^{3/2}}$;
$\kappa(0) = 2$, $\kappa(1) = 2\sqrt{2}/17 \approx 0.166378$

21. $\kappa = \frac{3}{13}$;
$\kappa(0) = 3/13$, $\kappa(\pi/2) = 3/13$

22. $\kappa = \frac{1}{13}$;
$\kappa(0) = 1/13$, $\kappa(\pi/2) = 1/13$

23. maximized at $x = \pm\frac{\sqrt{2}}{\sqrt[4]{5}}$

24. maximized at $x = \ldots - 3\pi/2, -\pi/2, \pi/2, \ldots$

25. maximized at $t = 1/4$

26. maximized at $t = \pm\sqrt{5}$

27. radius of curvature is $5\sqrt{5}/4$.

28. radius of curvature is $5\sqrt{10}$.

29. radius of curvature is 9.

30. radius of curvature is $1/45$.

31. $x^2 + (y - 1/2)^2 = 1/4$, or $\vec{c}(t) = \langle 1/2\cos t, 1/2\sin t + 1/2\rangle$

32. $(x - 8/3)^2 + y^2 = 1/9$, or $\vec{c}(t) = \left\langle \frac{1}{3}\cos t + \frac{8}{3}, \frac{1}{3}\sin t\right\rangle$

33. $x^2 + (y + 8)^2 = 81$, or $\vec{c}(t) = \langle 9\cos t, 9\sin t - 8\rangle$

34. $(x - 1/2)^2 + (y - 1/2)^2 = 1/2$, or
$\vec{c}(t) = \left\langle \frac{\sqrt{2}}{2}\cos t + \frac{1}{2}, \frac{\sqrt{2}}{2}\sin t + \frac{1}{2}\right\rangle$

Chapter 12

Section 12.1

1. Answers will vary.

2. surface

3. topographical

4. T

5. surface

6. When level curves are close together, it means the function is changing z-values rapidly. When far apart, it changes z-values slowly.

7. domain: $\mathbb{R}^2$
range: $z \geq 2$

8. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

9. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

10. domain: $x \neq 2y$; in set notation, $\{(x, y) \mid x \neq 2y\}$
range: $z \neq 0$

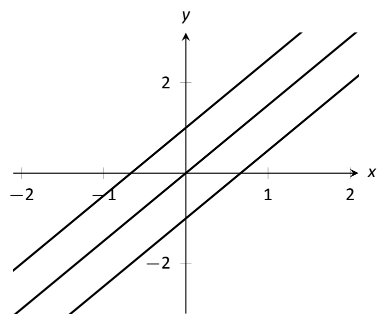
11. domain: $\mathbb{R}^2$
range: $0 < z \leq 1$

12. domain: $\mathbb{R}^2$
range: $-1 \leq z \leq 1$

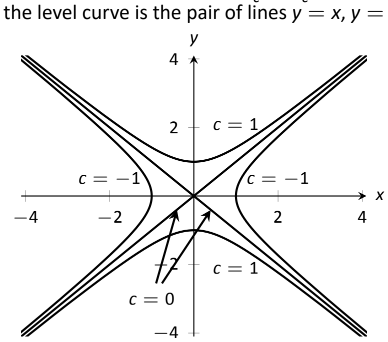
13. domain: $\{(x, y) \mid x^2 + y^2 \leq 9\}$, i.e., the domain is the circle and interior of a circle centered at the origin with radius 3.
range: $0 \leq z \leq 3$

14. domain: $\{(x, y) \mid x^2 + y^2 \geq 9\}$, i.e., the domain is the exterior of the circle (not including the circle itself) centered at the origin with radius 3.
range: $0 < z < \infty$, or $(0, \infty)$

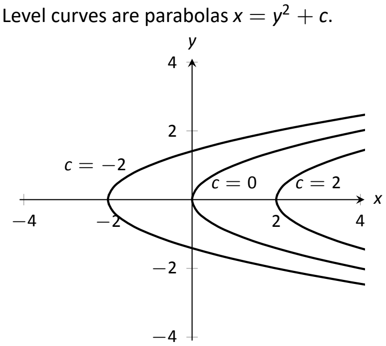
15. Level curves are lines $y = (3/2)x - c/2$.



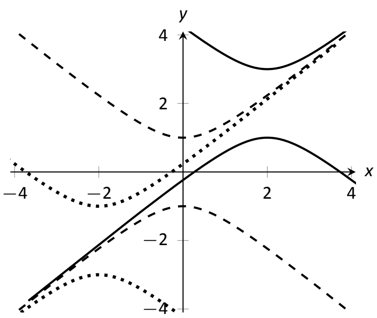
16. Level curves are hyperbolas $\frac{x^2}{c} - \frac{y^2}{c} = 1$, except for $c = 0$, where the level curve is the pair of lines $y = x$, $y = -x$.



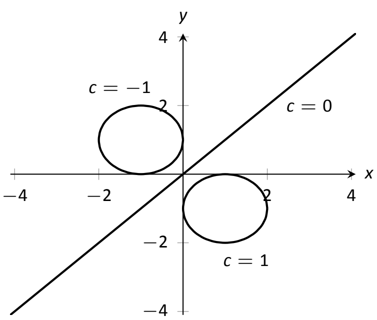
17. Level curves are parabolas $x = y^2 + c$.



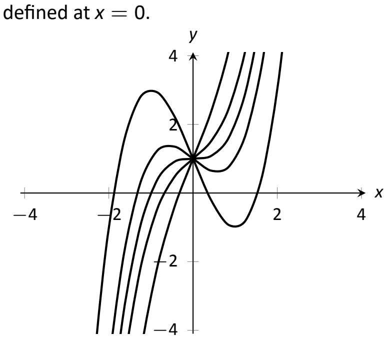
18. Level curves are hyperbolas $(x-c)^2-(y-c)^2=1$, drawn in graph in different styles to differentiate the curves.



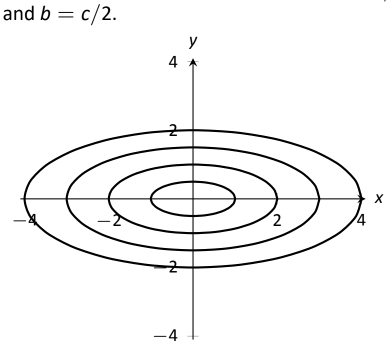
19. When $c \neq 0$, the level curves are circles, centered at $(1/c, -1/c)$ with radius $\sqrt{2/c^2-1}$. When $c=0$, the level curve is the line $y=x$.



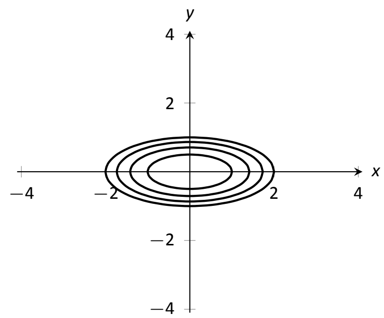
20. Level curves are cubics of the form $y=x^3+cx+1$. Note how each curve passes through $(0,1)$ and that the function is not defined at $x=0$.



21. Level curves are ellipses of the form $\frac{x^2}{c^2}+\frac{y^2}{c^2/4}=1$, i.e., $a=c$ and $b=c/2$.



22. Level curves are ellipses of the form $\frac{x^2}{c}+\frac{y^2}{c/4}=1$, i.e., $a=\sqrt{c}$ and $b=\sqrt{c}/2$.



23. domain: $x+2y-4z \neq 0$; the set of points in $\mathbb{R}^3$ NOT in the domain form a plane through the origin.
range: $\mathbb{R}$

24. domain: $x^2+y^2+z^2 \neq 1$; the set of points in $\mathbb{R}^3$ NOT in the domain form a sphere of radius 1.
range: $(-\infty, 0) \cup [1, \infty)$

25. domain: $z \geq x^2-y^2$; the set of points in $\mathbb{R}^3$ above (and including) the hyperbolic paraboloid $z=x^2-y^2$.
range: $[0, \infty)$

26. domain: $\mathbb{R}^3$
range: $\mathbb{R}$

27. The level surfaces are spheres, centered at the origin, with radius $\sqrt{c}$.

28. The level surfaces are hyperbolic paraboloids of the form $z=x^2-y^2+c$; each is shifted up/down by c.

29. The level surfaces are paraboloids of the form $z=\frac{x^2}{c}+\frac{y^2}{c}$; the larger c, the "wider" the paraboloid.

30. The level surfaces are planes through the origin of the form $cx-cy-z=0$, that is, planes through the origin with normal vector $\langle c, -c, -1\rangle$.

31. The level curves for each surface are similar; for $z=\sqrt{x^2+4y^2}$ the level curves are ellipses of the form $\frac{x^2}{c^2}+\frac{y^2}{c^2/4}=1$, i.e., $a=c$ and $b=c/2$; whereas for $z=x^2+4y^2$ the level curves are ellipses of the form $\frac{x^2}{c}+\frac{y^2}{c/4}=1$, i.e., $a=\sqrt{c}$ and $b=\sqrt{c}/2$.
The first set of ellipses are spaced evenly apart, meaning the function grows at a constant rate; the second set of ellipses are more closely spaced together as c grows, meaning the function grows faster and faster as c increases.
The function $z=\sqrt{x^2+4y^2}$ can be rewritten as $z^2=x^2+4y^2$, an elliptic cone; the function $z=x^2+4y^2$ is a paraboloid, each matching the description above.

Section 12.2

1. Answers will vary.

2. Answers will vary. One answer is "As (x,y) gets close to $(1,2)$, $f(x,y)$ gets close to 17."

3. Answers will vary.
One possible answer: $\{(x,y) | x^2+y^2 \leq 1\}$

4. Answers will vary.
One possible answer: $\{(x,y) | y \geq x^2\}$

5. Answers will vary.
One possible answer: $\{(x,y) | x^2+y^2 < 1\}$

6. Answers will vary.
One possible answer: $\{(x,y) | y > x^2\}$

7. (a) Answers will vary.
interior point: $(1,3)$
boundary point: $(3,3)$

(b) S is a closed set

(c) S is bounded

8. (a) Answers will vary.
interior point: $(-5, 28)$
boundary point: $(3, 9)$

(b) S is an open set

(c) S is unbounded

9. (a) Answers will vary.
interior point: none
boundary point: $(0, -1)$

(b) S is a closed set, consisting only of boundary points

(c) S is bounded

10. (a) Answers will vary.
Interior point: $(0, 1)$
Boundary point: $(0, 0)$

(b) S is a closed set, containing all of its boundary points.

(c) S is unbounded.

11. (a) $D = \{(x, y) \mid 9 - x^2 - y^2 \geq 0\}$.

(b) D is a closed set.

(c) D is bounded.

12. (a) $D = \{(x, y) \mid y \geq x^2\}$.

(b) D is a closed set.

(c) D is unbounded.

13. (a) $D = \{(x, y) \mid y > x^2\}$.

(b) D is an open set.

(c) D is unbounded.

14. (a) $D = \{(x, y) \mid (x, y) \neq (0, 0)\}$.

(b) D is an open set.

(c) D is unbounded.

15. (a) Along $y = 0$, the limit is 1.

(b) Along $x = 0$, the limit is -1.

Since the above limits are not equal, the limit does not exist.

16. (a) Along $y = mx$, the limit is $\frac{m+1}{m-1}$.

Since the above limit varies according to what m is used, each limit is different, meaning the overall limit does not exist.

17. (a) Along $y = mx$, the limit is $\dfrac{mx(1-m)}{m^2x+1} = 0$ for all m.

(b) Along $x = 0$, the limit is -1.

Since the above limits are not equal, the limit does not exist.

18. (a) Along $y = mx$, the limit is:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y} = \lim_{x\to 0} \frac{\sin(x^2)}{mx}$$

apply L'Hôpital's Rule

$$= \lim_{x\to 0} \frac{2x\cos(x^2)}{m} = 0.$$

(b) Along $x = 0$, the limit is:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y} = \lim_{x\to 0} \frac{\sin(x^2)}{x^2}.$$

This can be evaluated with L'Hôpital's Rule or from known limits; it is 1.

Since the limits along the lines $y = mx$ are not the same as the limit along $y = x^2$, the overall limit does not exist.

19. (a) Along $y = 2$, the limit is:

$$\lim_{(x,y)\to(1,2)} \frac{x+y-3}{x^2-1} = \lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{1}{x+1} = 1/2.$$

(b) Along $y = x + 1$, the limit is:

$$\lim_{(x,y)\to(1,2)} \frac{x+y-3}{x^2-1} = \lim_{x\to 1} \frac{2(x-1)}{x^2-1} = \lim_{x\to 1} \frac{2}{x+1} = 1.$$

Since the limits along the lines $y = 2$ and $y = x + 1$ differ, the overall limit does not exist.

20. (a) Along $x = \pi$, the limit is:

$$\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y} = \lim_{y\to\pi/2} \frac{0}{\cos y} = 0.$$

(b) Along $y = x - \pi/2$, the limit is:

$$\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y} = \lim_{x\to\pi} \frac{\sin x}{\cos(x-\pi/2)}$$

Apply L'Hôpital's Rule:

$$= \lim_{x\to\pi} \frac{\cos x}{\sin(x-\pi/2)} = -1.$$

Since the limits along the lines $x = \pi$ and $y = x - \pi$ differ, the overall limit does not exist.

Section 12.3

1. A constant is a number that is added or subtracted in an expression; a coefficient is a number that is being multiplied by a nonconstant function.

2. Answers will vary; each should include something about treating y as a constant or a coefficient.

3. f_x

4. f_y

5. $f_x = 2xy - 1, f_y = x^2 + 2$
$f_x(1, 2) = 3, f_y(1, 2) = 3$

6. $f_x = 3x^2 - 3, f_y = 2y - 6$
$f_x(-1, 3) = 0, f_y(-1, 3) = 0$

7. $f_x = -\sin x \sin y, f_y = \cos x \cos y$
$f_x(\pi/3, \pi/3) = -3/4, f_y(\pi/3, \pi/3) = 1/4$

8. $f_x = 1/x, f_y = 1/y$
$f_x(-2, -3) = -1/2, f_y(-2, -3) = -1/3$

9. $f_x = 2xy + 6x, f_y = x^2 + 4$
$f_{xx} = 2y + 6, f_{yy} = 0$
$f_{xy} = 2x, f_{yx} = 2x$

10. $f_x = 3x^2 + 6xy + 3y^2, f_y = 3x^2 + 6xy + 3y^2$
$f_{xx} = 6x + 6y, f_{yy} = 6x + 6y$
$f_{xy} = 6x + 6y, f_{yx} = 6x + 6y$

11. $f_x = 1/y, f_y = -x/y^2$
$f_{xx} = 0, f_{yy} = 2x/y^3$
$f_{xy} = -1/y^2, f_{yx} = -1/y^2$

12. $f_x = -4/(x^2y), f_y = -4/(xy^2)$
$f_{xx} = 8/(x^3y), f_{yy} = 8/(xy^3)$
$f_{xy} = 4/(x^2y^2), f_{yx} = 4/(x^2y^2)$

13. $f_x = 2xe^{x^2+y^2}, f_y = 2ye^{x^2+y^2}$
$f_{xx} = 2e^{x^2+y^2} + 4x^2e^{x^2+y^2}, f_{yy} = 2e^{x^2+y^2} + 4y^2e^{x^2+y^2}$
$f_{xy} = 4xye^{x^2+y^2}, f_{yx} = 4xye^{x^2+y^2}$

14. $f_x = e^{x+2y}, f_y = 2e^{x+2y}$
$f_{xx} = e^{x+2y}, f_{yy} = 4e^{x+2y}$
$f_{xy} = 2e^{x+2y}, f_{yx} = 2e^{x+2y}$

15. $f_x = \cos x \cos y, f_y = -\sin x \sin y$
$f_{xx} = -\sin x \cos y, f_{yy} = -\sin x \cos y$
$f_{xy} = -\sin y \cos x, f_{yx} = -\sin y \cos x$

16. $f_x = 3(x+y)^2, f_y = 3(x+y)^2$
$f_{xx} = 6(x+y), f_{yy} = 6(x+y)$
$f_{xy} = 6(x+y), f_{yx} = 6(x+y)$

17. $f_x = -5y^3\sin(5xy^3), f_y = -15xy^2\sin(5xy^3)$
$f_{xx} = -25y^6\cos(5xy^3),$
$f_{yy} = -225x^2y^4\cos(5xy^3) - 30xy\sin(5xy^3)$
$f_{xy} = -75xy^5\cos(5xy^3) - 15y^2\sin(5xy^3),$
$f_{yx} = -75xy^5\cos(5xy^3) - 15y^2\sin(5xy^3)$

18. $f_x = 10x\cos(5x^2+2y^3), f_y = 6y^2\cos(5x^2+2y^3)$
$f_{xx} = 10\cos(5x^2+2y^3) - 100x^2\sin(5x^2+2y^3),$
$f_{yy} = 12y\cos(5x^2+2y^3) - 36y^4\sin(5x^2+2y^3)$
$f_{xy} = -60xy^2\sin(5x^2+2y^3), f_{yx} = -60xy^2\sin(5x^2+2y^3)$

19. $f_x = \frac{2y^2}{\sqrt{4xy^2+1}}, f_y = \frac{4xy}{\sqrt{4xy^2+1}}$
$f_{xx} = -\frac{4y^4}{\sqrt{4xy^2+1}^3}, f_{yy} = -\frac{16x^2y^2}{\sqrt{4xy^2+1}^3} + \frac{4x}{\sqrt{4xy^2+1}}$
$f_{xy} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}, f_{yx} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}$

20. $f_x = 2\sqrt{y}, f_y = 5\sqrt{y} + \frac{2x+5y}{2\sqrt{y}}$
$f_{xx} = 0, f_{yy} = \frac{5}{\sqrt{y}} - \frac{2x+5y}{4y^{3/2}}$
$f_{xy} = \frac{1}{\sqrt{y}}, f_{yx} = \frac{1}{\sqrt{y}}$

21. $f_x = -\frac{2x}{(x^2+y^2+1)^2}, f_y = -\frac{2y}{(x^2+y^2+1)^2}$
$f_{xx} = \frac{8x^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}, f_{yy} = \frac{8y^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}$
$f_{xy} = \frac{8xy}{(x^2+y^2+1)^3}, f_{yx} = \frac{8xy}{(x^2+y^2+1)^3}$

22. $f_x = 5, f_y = -17$
$f_{xx} = 0, f_{yy} = 0$
$f_{xy} = 0, f_{yx} = 0$

23. $f_x = 6x, f_y = 0$
$f_{xx} = 6, f_{yy} = 0$
$f_{xy} = 0, f_{yx} = 0$

24. $f_x = \frac{2x}{(x^2+y)}, f_y = \frac{1}{(x^2+y)}$
$f_{xx} = -\frac{4x^2}{(x^2+y)^2} + \frac{2}{(x^2+y)}, f_{yy} = -\frac{1}{(x^2+y)^2}$
$f_{xy} = -\frac{2x}{(x^2+y)^2}, f_{yx} = -\frac{2x}{(x^2+y)^2}$

25. $f_x = \frac{1}{4xy}, f_y = -\frac{\ln x}{4y^2}$
$f_{xx} = -\frac{1}{4x^2y}, f_{yy} = \frac{\ln x}{2y^3}$
$f_{xy} = -\frac{1}{4xy^2}, f_{yx} = -\frac{1}{4xy^2}$

26. $f_x = 5e^x\sin y, f_y = 5e^x\cos y$
$f_{xx} = 5e^x\sin y, f_{yy} = -5e^x\sin y$
$f_{xy} = 5e^x\cos y, f_{yx} = 5e^x\cos y$

27. $f(x,y) = x\sin y + x + C$, where C is any constant.

28. $f(x,y) = \frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + C$, where C is any constant.

29. $f(x,y) = 3x^2y - 4xy^2 + 2y + C$, where C is any constant.

30. $f(x,y) = \ln(x^2+y^2) + C$, where C is any constant.

31. $f_x = 2xe^{2y-3z}, f_y = 2x^2e^{2y-3z}, f_z = -3x^2e^{2y-3z}$
$f_{yz} = -6x^2e^{2y-3z}, f_{zy} = -6x^2e^{2y-3z}$

32. $f_x = 3x^2y^2 + 3x^2z, f_y = 2x^3y + 2yz, f_z = x^3 + y^2$
$f_{yz} = 2y, f_{zy} = 2y$

33. $f_x = \frac{3}{7y^2z}, f_y = -\frac{6x}{7y^3z}, f_z = -\frac{3x}{7y^2z^2}$
$f_{yz} = \frac{6x}{7y^3z^2}, f_{zy} = \frac{6x}{7y^3z^2}$

34. $f_x = \frac{1}{x}, f_y = \frac{1}{y}, f_z = \frac{1}{z}$
$f_{yz} = 0, f_{zy} = 0$

Section 12.4

1. T
2. T
3. T
4. amount of change
5. $dz = (\sin y + 2x)dx + (x\cos y)dy$
6. $dz = 8x(2x^2+3y)dx + 6(2x^2+3y)dy$
7. $dz = 5dx - 7dy$
8. $dz = (e^{x+y} + xe^{x+y})dx + xe^{x+y}dy$
9. $dz = \frac{x}{\sqrt{x^2+y}}dx + \frac{1}{2\sqrt{x^2+y}}dy$, with $dx = -0.05$ and $dy = .1$. At $(3,7)$, $dz = 3/4(-0.05) + 1/8(.1) = -0.025$, so $f(2.95, 7.1) \approx -0.025 + 4 = 3.975$.
10. $dz = (\cos x\cos y)dx - (\sin x\sin y)dy$, with $dx = 0.1$ and $dy = -0.1$. At $(0,0)$, $dz = 1(.1) - (0)(-0.1) = 0.1$, so $f(0.1, -0.1) \approx 0.1 + 0 = 0.1$.
11. $dz = (2xy - y^2)dx + (x^2 - 2xy)dy$, with $dx = 0.04$ and $dy = 0.06$. At $(2,3)$, $dz = 3(0.04) + (-8)(0.06) = -0.36$, so $f(2.04, 3.06) \approx -0.36 - 6 = -6.36$.
12. $dz = \frac{1}{x-y}dx - \frac{1}{x-y}dy$, with $dx = 0.1$ and $dy = -0.02$. At $(5,4)$, $dz = 1(0.1) + (-1)(-0.02) = 0.12$, so $f(5.1, 3.98) \approx 0.12 + 0 = 0.12$.
13. The total differential of volume is $dV = 4\pi dr + \pi dh$. The coefficient of dr is greater than the coefficient of dh, so the volume is more sensitive to changes in the radius.
14. Distance of the projectile is a function of two variables (leaving $t = 3$): $D(v_0, \theta) = 3v_0\cos\theta$. The total differential of D is $dD = 3\cos\theta dv_0 - 3v_0\sin\theta d\theta$. The coefficient of $d\theta$ has a much greater magnitude than the coefficient of dv_0, so a small change in the angle of elevation has a much greater effect on distance traveled than a small change in initial velocity.
15. Using trigonometry, $\ell = x\tan\theta$, so $d\ell = \tan\theta dx + x\sec^2\theta d\theta$. With $\theta = 85^\circ$ and $x = 30$, we have $d\ell = 11.43dx + 3949.38d\theta$. The measured length of the wall is much more sensitive to errors in θ than in x. While it can be difficult to compare sensitivities between measuring feet and measuring degrees (it is somewhat like "comparing apples to oranges"), here the coefficients are so different that the result is clear: a small error in degree has a much greater impact than a small error in distance.
16. With $D = n\ell$, the total differential is $dD = \ell\, dn + n\, d\ell$. If one measures with a short tape, n must be large and hence $n\, d\ell$ is going to be greater than when a large tape is used (wherein n will be small).
17. $dw = 2xyz^3\, dx + x^2z^3\, dy + 3x^2yz^2\, dz$
18. $dw = e^x\sin y\ln z\, dx + e^x\cos y\ln z\, dy + e^x\sin y\frac{1}{z}\, dz$
19. $dx = 0.05, dy = -0.1$. $dz = 9(.05) + (-2)(-0.1) = 0.65$. So $f(3.05, 0.9) \approx 7 + 0.65 = 7.65$.
20. $dx = -0.12, dy = 0.07$. $dz = 2.6(-.12) + (5.1)(0.07) = 0.045$. So $f(-4.12, 2.07) \approx 13 + 0.045 = 13.045$.
21. $dx = 0.5, dy = 0.1, dz = -0.2$.
$dw = 2(0.5) + (-3)(0.1) + 3.7(-0.2) = -0.04$, so $f(2.5, 4.1, 4.8) \approx -1 - 0.04 = -1.04$.
22. $dx = 0.1, dy = 0.1, dz = 0.1$.
$dw = 2(0.1) + (0)(0.1) + (-2)(.1) = 0$, so $f(3.1, 3.1, 3.1) \approx 5 + 0 = 5$.

Section 12.5

1. A partial derivative is essentially a special case of a directional derivative; it is the directional derivative in the direction of x or y, i.e., $\langle 1,0\rangle$ or $\langle 0,1\rangle$.
2. $\vec{u}=\langle 1,0\rangle$
3. $\vec{u}=\langle 0,1\rangle$
4. orthogonal
5. maximal, or greatest
6. dot
7. $\nabla f=\left\langle -2xy+y^2+y, -x^2+2xy+x\right\rangle$
8. $\nabla f=\langle \cos x\cos y, -\sin x\sin y\rangle$
9. $\nabla f=\left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$
10. $\nabla f=\langle -4,3\rangle$
11. $\nabla f=\langle 2x-y-7, 4y-x\rangle$
12. $\nabla f=\left\langle 2xy^3-2, 3x^2y^2\right\rangle$
13. $\nabla f=\left\langle -2xy+y^2+y, -x^2+2xy+x\right\rangle$; $\nabla f(2,1)=\langle -2,2\rangle$. Be sure to change all directions to unit vectors.
    (a) $2/5$ ($\vec{u}=\langle 3/5,4/5\rangle$)
    (b) $-2/\sqrt{5}$ ($\vec{u}=\left\langle -1/\sqrt{5}, -2/\sqrt{5}\right\rangle$)
14. $\nabla f=\langle \cos x\cos y, -\sin x\sin y\rangle$; $\nabla f(\frac{\pi}{4},\frac{\pi}{3})=\left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle$. Be sure to change all directions to unit vectors.
    (a) $\frac{1}{4}(1-\sqrt{3})$ ($\vec{u}=\left\langle 1/\sqrt{2},1/\sqrt{2}\right\rangle$)
    (b) $\frac{4\sqrt{3}-1}{10\sqrt{2}}$ ($\vec{u}=\langle -3/5,-4/5\rangle$)
15. $\nabla f=\left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$; $\nabla f(1,1)=\langle -2/9,-2/9\rangle$. Be sure to change all directions to unit vectors.
    (a) 0 ($\vec{u}=\left\langle 1/\sqrt{2},-1/\sqrt{2}\right\rangle$)
    (b) $2\sqrt{2}/9$ ($\vec{u}=\left\langle -1/\sqrt{2},-1/\sqrt{2}\right\rangle$)
16. $\nabla f=\langle -4,3\rangle$; $\nabla f(5,2)=\langle -4,3\rangle$. Be sure to change all directions into unit vectors.
    (a) $-9/\sqrt{10}$ ($\vec{u}=\left\langle 3/\sqrt{10},1/\sqrt{10}\right\rangle$)
    (b) $27/\sqrt{34}$ ($\vec{u}=\left\langle -3/\sqrt{34},5/\sqrt{34}\right\rangle$)
17. $\nabla f=\langle 2x-y-7, 4y-x\rangle$; $\nabla f(4,1)=\langle 0,0\rangle$.
    (a) 0
    (b) 0
18. $\nabla f=\left\langle 2xy^3-2, 3x^2y^2\right\rangle$; $\nabla f(1,1)=\langle 0,3\rangle$ Be sure to change all directions to unit vectors.
    (a) $3/\sqrt{2}$; ($\vec{u}=\left\langle 1/\sqrt{2},1/\sqrt{2}\right\rangle$)
    (b) 3
19. $\nabla f=\left\langle -2xy+y^2+y, -x^2+2xy+x\right\rangle$
    (a) $\nabla f(2,1)=\langle -2,2\rangle$
    (b) $||\,\nabla f(2,1)\,||=||\,\langle -2,2\rangle\,||=\sqrt{8}$
    (c) $\langle 2,-2\rangle$
    (d) $\left\langle 1/\sqrt{2},1/\sqrt{2}\right\rangle$
20. $\nabla f=\langle \cos x\cos y, -\sin x\sin y\rangle$
    (a) $\nabla f(\frac{\pi}{4},\frac{\pi}{3})=\left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle$
    (b) $||\,\nabla f(\frac{\pi}{4},\frac{\pi}{3})\,||=||\,\left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle\,||=1/\sqrt{2}$
    (c) $\left\langle -\frac{1}{2\sqrt{2}}, \frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle$
    (d) $\left\langle \frac{1}{2}\sqrt{\frac{3}{2}}, \frac{1}{2\sqrt{2}}\right\rangle$
21. $\nabla f=\left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$
    (a) $\nabla f(1,1)=\langle -2/9,-2/9\rangle$.
    (b) $||\,\nabla f(1,1)\,||=||\,\langle -2/9,-2/9\rangle\,||=2\sqrt{2}/9$
    (c) $\langle 2/9,2/9\rangle$
    (d) $\left\langle 1/\sqrt{2},-1/\sqrt{2}\right\rangle$
22. $\nabla f=\langle -4,3\rangle$
    (a) $\nabla f(5,4)=\langle -4,3\rangle$.
    (b) $||\,\nabla f(5,4)\,||=||\,\langle -4,3\rangle\,||=5$
    (c) $\langle 4,-3\rangle$
    (d) $\langle 3/5,4/5\rangle$
23. $\nabla f=\langle 2x-y-7, 4y-x\rangle$
    (a) $\nabla f(4,1)=\langle 0,0\rangle$
    (b) 0
    (c) $\langle 0,0\rangle$
    (d) All directions give a directional derivative of 0.
24. $\nabla f=\left\langle 2xy^3-2, 3x^2y^2\right\rangle$
    (a) $\nabla f(1,1)=\langle 0,3\rangle$
    (b) 3
    (c) $\langle 0,-3\rangle$
    (d) $\vec{u}=\langle 1,0\rangle$
25. (a) $\nabla F(x,y,z)=\left\langle 6xz^3+4y, 4x, 9x^2z^2-6z\right\rangle$
    (b) $113/\sqrt{3}$
26. (a) $\nabla F(x,y,z)=\langle \cos x\cos ye^z, -\sin x\sin ye^z, \sin x\cos ye^z\rangle$
    (b) $2/3$
27. (a) $\nabla F(x,y,z)=\left\langle 2xy^2, 2y(x^2-z^2), -2y^2z\right\rangle$
    (b) 0
28. (a) $\nabla F(x,y,z)=$ $\left\langle -\frac{4x}{(x^2+y^2+z^2)^2}, -\frac{4y}{(x^2+y^2+z^2)^2}, -\frac{4z}{(x^2+y^2+z^2)^2}\right\rangle$
    (b) 0

Section 12.6

1. Answers will vary. The displacement of the vector is one unit in the x-direction and 3 units in the z-direction, with no change in y. Thus along a line parallel to $\vec{v}$, the change in z is 3 times the change in x – i.e., a "slope" of 3. Specifically, the line in the x-z plane parallel to z has a slope of 3.
2. Answers will vary. Let $\vec{u}=\langle 0.6,0.8\rangle$; this is a unit vector. The displacement of the vector is one unit in the $\vec{u}$-direction and -2 units in the z-direction. In the plane containing the z-axis and the vector $\vec{u}$, the line parallel to $\vec{v}$ has slope -2.
3. T
4. On a surface through a point, there are many different smooth curves, each with a tangent line at the point. Each of these tangent lines is also "tangent" to the surface. There is not just one tangent line, but many, each in a different direction. Therefore we refer to directional tangent lines, not just *the* tangent line.
5. (a) $\ell_x(t)=\begin{cases} x=2+t \\ y=3 \\ z=-48-12t \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = 2 \\ y = 3 + t \\ z = -48 - 40t \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = 2 + t/\sqrt{10} \\ y = 3 + 3t/\sqrt{10} \\ z = -48 - 66\sqrt{2/5}t \end{cases}$

6. (a) $\ell_x(t) = \begin{cases} x = \pi/3 + t \\ y = \pi/6 \\ z = 3/4 - \frac{3\sqrt{3}}{4}t \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = \pi/3 \\ y = \pi/6 + t \\ z = 3/4 + \frac{3\sqrt{3}}{4}t \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = \pi/3 + t/\sqrt{5} \\ y = \pi/6 + 2t/\sqrt{5} \\ z = 3/4 + \frac{3\sqrt{3/5}}{4}t \end{cases}$

7. (a) $\ell_x(t) = \begin{cases} x = 4 + t \\ y = 2 \\ z = 2 + 3t \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = 4 \\ y = 2 + t \\ z = 2 - 5t \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = 4 + t/\sqrt{2} \\ y = 2 + t/\sqrt{2} \\ z = 2 - \sqrt{2}t \end{cases}$

8. (a) $\ell_x(t) = \begin{cases} x = 1 + t \\ y = 2 \\ z = 3 \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = 1 \\ y = 2 + t \\ z = 3 \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = 1 + t/\sqrt{2} \\ y = 2 + t/\sqrt{2} \\ z = 3 \end{cases}$

9. $\ell_{\vec{n}}(t) = \begin{cases} x = 2 - 12t \\ y = 3 - 40t \\ z = -48 - t \end{cases}$

10. $\ell_{\vec{n}}(t) = \begin{cases} x = \pi/3 - \frac{3\sqrt{3}}{4}t \\ y = \pi/6 + \frac{3\sqrt{3}}{4}t \\ z = 3/4 - t \end{cases}$

11. $\ell_{\vec{n}}(t) = \begin{cases} x = 4 + 3t \\ y = 2 - 5t \\ z = 2 - t \end{cases}$

12. $\ell_{\vec{n}}(t) = \begin{cases} x = 1 \\ y = 2 \\ z = 3 - t \end{cases}$

13. $(1.425, 1.085, -48.078)$, $(2.575, 4.915, -47.952)$

14. $(-0.195, 1.766, -0.206)$ and $(2.289, -0.719, 1.706)$

15. $(5.014, 0.31, 1.662)$ and $(2.986, 3.690, 2.338)$

16. $(1, 2, 1)$ and $(1, 2, 5)$

17. $-12(x-2) - 40(y-3) - (z+48) = 0$

18. $-\frac{3\sqrt{3}}{4}(x - \pi/3) + \frac{3\sqrt{3}}{4}(y - \pi/6) - (z - 3/4) = 0$

19. $3(x-4) - 5(y-2) - (z-2) = 0$ (Note that this tangent plane is the same as the original function, a plane.)

20. $-(z-3) = 0$, or $z = 3$

21. $\nabla F = \langle x/4, y/2, z/8 \rangle$; at P, $\nabla F = \langle 1/4, \sqrt{2}/2, \sqrt{6}/8 \rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 1 + t/4 \\ y = \sqrt{2} + \sqrt{2}t/2 \\ z = \sqrt{6} + \sqrt{6}t/8 \end{cases}$

(b) $\frac{1}{4}(x-1) + \frac{\sqrt{2}}{2}(y - \sqrt{2}) + \frac{\sqrt{6}}{8}(z - \sqrt{6}) = 0.$

22. $\nabla F = \left\langle -\frac{x}{2}, -\frac{2y}{9}, 2z \right\rangle$; at P, $\nabla F = \langle -2, 2/3, 2\sqrt{5} \rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 4 - 2t \\ y = -3 + 2t/3 \\ z = \sqrt{5} + 2\sqrt{5}t \end{cases}$

(b) $-2(x-4) + \frac{2}{3}(y+3) + 2\sqrt{5}(z - \sqrt{5}) = 0.$

23. $\nabla F = \langle y^2 - z^2, 2xy, -2xz \rangle$; at P, $\nabla F = \langle 0, 4, 4 \rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2 \\ y = 1 + 4t \\ z = -1 + 4t \end{cases}$

(b) $4(y-1) + 4(z+1) = 0.$

24. $\nabla F = \langle y\cos(xy), x\cos(xy) - z\sin(yz), -y\sin(yz) \rangle$; at P, $\nabla F = \left\langle \frac{\pi}{8\sqrt{3}}, -\sqrt{3}, -\frac{\pi}{8\sqrt{3}} \right\rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2 + \frac{\pi}{8\sqrt{3}}t \\ y = \frac{\pi}{12} - \sqrt{3}t \\ z = 4 - \frac{\pi}{8\sqrt{3}}t \end{cases}$

(b) $\frac{\pi}{8\sqrt{3}}(x-2) - \sqrt{3}(y - \frac{\pi}{12}) - \frac{\pi}{8\sqrt{3}}(z-4) = 0.$

Section 12.7

1. F; it is the “other way around.”
2. T
3. T
4. Answers will vary. A good answer will state that we are optimizing a function subject to a constraint, or limit, on the domain of the function. We are looking to maximize/minimize the function while “looking” at only a certain part of the domain.
5. One critical point at $(-4, 2)$; $f_{xx} = 1$ and $D = 4$, so this point corresponds to a relative minimum.
6. One critical point at $(7, -6)$; $D = -5$, so this point corresponds to a saddle point.
7. One critical point at $(6, -3)$; $D = -4$, so this point corresponds to a saddle point.
8. One critical point at $(0, 0)$; $f_{xx} = -2$ and $D = 4$, so this point corresponds to a relative maximum.
9. Two critical points: at $(0, -1)$; $f_{xx} = 2$ and $D = -12$, so this point corresponds to a saddle point;
at $(0, 1)$, $f_{xx} = 2$ and $D = 12$, so this corresponds to a relative minimum.
10. There are 4 critical points:
$(-1, -2)$, rel. max; $(1, -2)$, saddle point;
$(-1, 2)$, saddle point; $(1, 2)$, rel. min.,
where $f_{xx} = 2x$ and $D = 4xy$.
11. There are infinite critical points, whenever $x = 0$ or $y = 0$. With $D = -12x^2y^2$, at each critical point $D = 0$ and the test is inconclusive. (Some elementary thought shows that each is an absolute minimum.)
12. Six critical points: $f_x = 0$ when $x = -1, 0$ and 1; $f_y = 0$ when $y = -3, 3$. Together, we get the points:
$(-1, -3)$ saddle point; $(-1, 3)$ rel. min
$(0, -3)$ rel. max; $(0, 3)$ saddle point
$(1, -3)$ saddle point; $(1, 3)$ relative min
where $f_{xx} = 12x^2 - 4$ and $D = 24y(3x^2 - 1)$.

13. One critical point: $f_x = 0$ when $x = 3$; $f_y = 0$ when $y = 0$, so one critical point at $(3, 0)$, which is a relative maximum, where $f_{xx} = \frac{y^2-16}{(16-(x-3)^2-y^2)^{3/2}}$ and $D = \frac{16}{(16-(x-3)^2-y^2)^2}$.
Both f_x and f_y are undefined along the circle $(x-3)^2 + y^2 = 16$; at any point along this curve, $f(x, y) = 0$, the absolute minimum of the function.

14. One critical point: $f_x = 0$ when $x = 0$; $f_y = 0$ when $y = 0$, so one critical point at $(0, 0)$ (although it should be noted that at $(0, 0)$, both f_x and f_y are undefined.) The Second Derivative Test fails at $(0, 0)$, with $D = 0$. A graph, or simple calculation, shows that $(0, 0)$ is the absolute minimum of f.

15. The triangle is bound by the lines $y = -1$, $y = 2x + 1$ and $y = -2x + 1$.
Along $y = -1$, there is a critical point at $(0, -1)$.
Along $y = 2x + 1$, there is a critical point at $(-3/5, -1/5)$.
Along $y = -2x + 1$, there is a critical point at $(3/5, -1/5)$.
The function f has one critical point, irrespective of the constraint, at $(0, -1/2)$.
Checking the value of f at these four points, along with the three vertices of the triangle, we find the absolute maximum is at $(0, 1, 3)$ and the absolute minimum is at $(0, -1/2, 3/4)$.

16. The region has two "corners" at $(1, 1)$ and $(-1, 1)$.
Along $y = 1$, there is no critical point.
Along $y = x^2$, there is a critical point at $(5/14, 25/196) \approx (0.357, 0.128)$.
The function f itself has no critical points. Checking the value of f at the corners $(1, 1)$, $(-1, 1)$ and the critical point $(5/14, 25/196)$, we find the absolute maximum is at $(5/14, 25/196, 25/28) \approx (0.357, 0.128, 0.893)$ and the absolute minimum is at $(-1, 1, -12)$.

17. The region has no "corners" or "vertices," just a smooth edge. To find critical points along the circle $x^2 + y^2 = 4$, we solve for y^2: $y^2 = 4 - x^2$. We can go further and state $y = \pm\sqrt{4 - x^2}$.
We can rewrite f as
$f(x) = x^2 + 2x + (4 - x^2) + 2\sqrt{4 - x^2} = 2x + 4 + 2\sqrt{4 - x^2}$.
(We will return and use $-\sqrt{4 - x^2}$ later.) Solving $f'(x) = 0$, we get $x = \sqrt{2} \Rightarrow y = \sqrt{2}$. $f'(x)$ is also undefined at $x = \pm 2$, where $y = 0$.
Using $y = -\sqrt{4 - x^2}$, we rewrite $f(x, y)$ as
$f(x) = 2x + 4 - 2\sqrt{4 - x^2}$. Solving $f'(x) = 0$, we get $x = -\sqrt{2}$, $y = -\sqrt{2}$. Again, $f'(x)$ is undefined at $x = \pm 2$.
The function $z = f(x, y)$ itself has a critical point at $(-1, -1)$.
Checking the value of f at $(-1, -1)$, $(\sqrt{2}, \sqrt{2})$, $(-\sqrt{2}, -\sqrt{2})$, $(2, 0)$ and $(-2, 0)$, we find the absolute maximum is at $(\sqrt{2}, \sqrt{2}, 4 + 4\sqrt{2})$ and the absolute minimum is at $(-1, -1, -2)$.

18. The region has two "corners" at $(-1, -1)$ and $(1, 1)$.
Along the line $y = x$, $f(x, y)$ becomes $f(x) = 3x - 2x^2$. Along this line, we have a critical point at $(3/4, 3/4)$.
Along the curve $y = x^2 + x - 1$, $f(x, y)$ becomes $f(x) = x^2 + 3x - 3$. There is a critical point along this curve at $(-3/2, -1/4)$. Since $x = -3/2$ lies outside our bounded region, we ignore this critical point.
The function f itself has no critical points.
Checking the value of f at $(-1, -1)$, $(1, 1)$, $(3/4, 3/4)$, we find the absolute maximum is at $(3/4, 3/4, 9/8)$ and the absolute minimum is at $(-1, -1, -5)$.

Section 12.8

1. Because the parametric equations describe a level curve, z is constant for all t. Therefore $\frac{dz}{dt} = 0$.
2. $g'(x)$
3. $\frac{dx}{dt}$, and $\frac{\partial f}{\partial y}$
4. T
5. F
6. partial
7. (a) $\frac{dz}{dt} = 3(2t) + 4(2) = 6t + 8$.
   (b) At $t = 1$, $\frac{dz}{dt} = 14$.
8. (a) $\frac{dz}{dt} = 2x(1) - 2y(2t) = 2x - 4yt$
   (b) At $t = 1$, $x = 1$, $y = 0$ and $\frac{dz}{dt} = 2$.
9. (a) $\frac{dz}{dt} = 5(-2\sin t) + 2(\cos t) = -10\sin t + 2\cos t$
   (b) At $t = \pi/4$, $\frac{dz}{dt} = -4\sqrt{2}$.
10. (a) $\dfrac{dz}{dt} = \dfrac{1}{1 + y^2}(-\sin t) - \dfrac{2xy}{(y^2 + 1)^2}(\cos t)$.
   (b) At $t = \pi/2$, $x = 0$, $y = 1$, and $\frac{dz}{dt} = -1/2$.
11. (a) $\dfrac{dz}{dt} = 2x(\cos t) + 4y(3\cos t)$.
   (b) At $t = \pi/4$, $x = \sqrt{2}/2$, $y = 3\sqrt{2}/2$, and $\frac{dz}{dt} = 19$.
12. (a) $\frac{dz}{dt} = -\sin x \sin y(\pi) + \cos x \cos y(2\pi)$.
   (b) At $t = 3$, $x = 3\pi$, $y = 13\pi/2$, and $\frac{dz}{dt} = 0$.
13. $t = -4/3$; this corresponds to a minimum
14. $t = 0, \pm\sqrt{3/2}$
15. $t = \tan^{-1}(1/5) + n\pi$, where n is an integer
16. We find that
$$\frac{dz}{dt} = -\frac{2\cos^2 t \sin t}{(1 + \sin^2 t)^2} - \frac{\sin t}{1 + \sin^2 t}.$$
Setting this equal to 0, finding a common denominator and factoring out $\sin t$, we get
$$\sin t\left(\frac{2\cos^2 t + \sin^2 t + 1}{(1 + \sin^2 t)^2}\right) = 0.$$
We have $\sin t = 0$ when $t = \pi n$, where n is an integer. The expression in the parenthesis above is always positive, and hence never equal 0. So all solutions are $t = \pi n$, n is an integer.
17. We find that
$$\frac{dz}{dt} = 38\cos t \sin t.$$
Thus $\frac{dz}{dt} = 0$ when $t = \pi n$ or $\pi n + \pi/2$, where n is any integer.
18. We find that
$$\frac{dz}{dt} = -\pi\sin(\pi t)\sin(2\pi t + \pi/2) + 2\pi\cos(\pi t)\cos(2\pi t + \pi/2).$$
One can "easily" see that when t is an integer, $\sin(\pi t) = 0$ and $\cos(2\pi t + \pi/2) = 0$, hence $\frac{dz}{dt} = 0$ when t is an integer. There are other places where z has a relative max/min that require more work. First, verify that $\sin(2\pi t + \pi/2) = \cos(2\pi t)$, and $\cos(2\pi t + \pi/2) = -\sin(2\pi t)$. This lets us rewrite $\frac{dz}{dt} = 0$ as
$$-\sin(\pi t)\cos(2\pi t) - 2\cos(\pi t)\sin(2\pi t) = 0.$$
By bringing one term to the other side of the equality then dividing, we can get
$$2\tan(2\pi t) = -\tan(\pi t).$$
Using the angle sum/difference formulas found in the back of the book, we know
$$\tan(2\pi t) = \tan(\pi t) + \tan(\pi t) = \frac{\tan(\pi t) + \tan(\pi t)}{1 - \tan^2(\pi t)}.$$
Thus we write
$$2\frac{\tan(\pi t) + \tan(\pi t)}{1 - \tan^2(\pi t)} = -\tan(\pi t).$$

Solving for $\tan^2(\pi t)$, we find

$$\tan^2(\pi t) = 5 \quad \Rightarrow \quad \tan(\pi t) = \pm\sqrt{5},$$

and so

$$\pi t = \tan^{-1}(\pm\sqrt{5}) = \pm\tan^{-1}(\sqrt{5}).$$

Since the period of tangent is π, we can adjust our answer to be

$$\pi t = \pm\tan^{-1}(\sqrt{5}) + n\pi, \text{ where } n \text{ is an integer.}$$

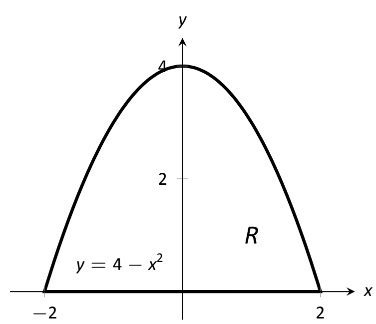
Dividing by π, we find

$$t = \pm\frac{1}{\pi}\tan^{-1}(\sqrt{5}) + n, \text{ where } n \text{ is an integer.}$$

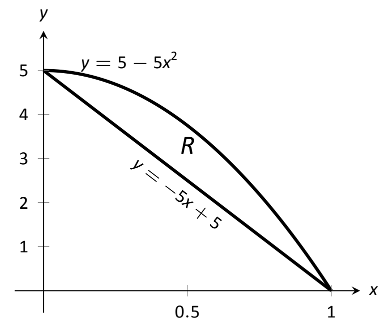
19. (a) $\frac{\partial z}{\partial s} = 2xy(1) + x^2(2) = 2xy + 2x^2$;
$\frac{\partial z}{\partial t} = 2xy(-1) + x^2(4) = -2xy + 4x^2$
    (b) With $s = 1$, $t = 0$, $x = 1$ and $y = 2$. Thus $\frac{\partial z}{\partial s} = 6$ and $\frac{\partial z}{\partial t} = 0$
20. (a) $\frac{\partial z}{\partial s} = -\pi\sin(\pi x + \pi y/2)(t^2) - \frac{1}{2}\pi\sin(\pi x + \pi y/2)(2st) = -\pi\Big(t^2\sin(\pi x + \pi y/2) + st\sin(\pi x + \pi y/2)\Big)$;
$\frac{\partial z}{\partial t} = -\pi\sin(\pi x + \pi y/2)(2st) - \frac{1}{2}\pi\sin(\pi x + \pi y/2)(s^2) = -\pi\Big(2st\sin(\pi x + \pi y/2) + \frac{1}{2}s^2\sin(\pi x + \pi y/2)\Big)$
    (b) With $s = 1$, $t = 1$, $x = 1$ and $y = 1$. Thus $\frac{\partial z}{\partial s} = 2\pi$ and $\frac{\partial z}{\partial t} = 5\pi/2$
21. (a) $\frac{\partial z}{\partial s} = 2x(\cos t) + 2y(\sin t) = 2x\cos t + 2y\sin t$;
$\frac{\partial z}{\partial t} = 2x(-s\sin t) + 2y(s\cos t) = -2xs\sin t + 2ys\cos t$
    (b) With $s = 2$, $t = \pi/4$, $x = \sqrt{2}$ and $y = \sqrt{2}$. Thus $\frac{\partial z}{\partial s} = 4$ and $\frac{\partial z}{\partial t} = 0$
22. (a) $\frac{\partial z}{\partial s} = -2xe^{-(x^2+y^2)}(0) - 2ye^{-(x^2+y^2)}(t^2) = -2yt^2e^{-(x^2+y^2)}$;
$\frac{\partial z}{\partial t} = -2xe^{-(x^2+y^2)}(1) - 2ye^{-(x^2+y^2)}(2st) = -2xe^{-(x^2+y^2)} - 4stye^{-(x^2+y^2)}$
    (b) With $s = 1$, $t = 1$, $x = 1$ and $y = 1$. Thus $\frac{\partial z}{\partial s} = -2/e^2$ and $\frac{\partial z}{\partial t} = -6/e^2$
23. $f_x = 2x\tan y$, $f_y = x^2\sec^2 y$;
$\frac{dy}{dx} = -\frac{2\tan y}{x\sec^2 y}$
24. $f_x = 4(3x^2 + 2y^3)^3(6x)$, $f_y = 4(3x^2 + 2y^3)^3(6y^2)$;
$\frac{dy}{dx} = -\frac{x}{y^2}$
25. $f_x = \frac{(x + y^2)(2x) - (x^2 + y)(1)}{(x + y^2)^2}$,
$f_y = \frac{(x + y^2)(1) - (x^2 + y)(2y)}{(x + y^2)^2}$;
$\frac{dy}{dx} = -\frac{2x(x + y^2) - (x^2 + y)}{x + y^2 - 2y(x^2 + y)}$
26. $f_x = \frac{2x + y}{x^2 + xy + y^2}$, $f_y = \frac{x + 2y}{x^2 + xy + y^2}$;
$\frac{dy}{dx} = -\frac{2x + y}{2y + x}$
27. $\frac{dz}{dt} = 2(4) + 1(-5) = 3$.
28. $\frac{dz}{dt} = 1(6) + (-3)(2) = 0$.
29. $\frac{\partial z}{\partial s} = -4(5) + 9(-2) = -38$,
$\frac{\partial z}{\partial t} = -4(7) + 9(6) = 26$.
30. $\frac{\partial z}{\partial s} = 2(-2) + 1(2) = -2$,
$\frac{\partial z}{\partial t} = 2(3) + 1(-1) = 5$.

Chapter 13

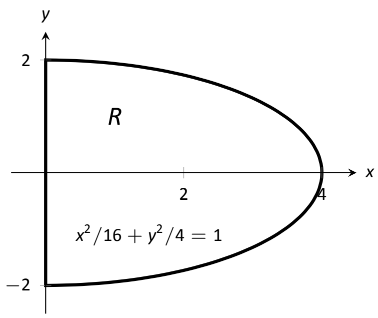
Section 13.1

1. $C(y)$, meaning that instead of being just a constant, like the number 5, it is a function of y, which acts like a constant when taking derivatives with respect to x.
2. iterated integration
3. curve to curve, then from point to point
4. area
5. (a) $18x^2 + 42x - 117$
    (b) -108
6. (a) $2 + \pi^2\cos y$
    (b) $\pi^2 + \pi$
7. (a) $x^4/2 - x^2 + 2x - 3/2$
    (b) $23/15$
8. (a) $y^4/2 - y^3 + y^2/2$
    (b) $8/15$
9. (a) $\sin^2 y$
    (b) $\pi/2$
10. (a) $x/(1 + x^2)$
    (b) $\frac{1}{2}\ln\left(\frac{5}{2}\right)$
11. $\int_1^4\int_{-2}^1 dy\,dx$ and $\int_{-2}^1\int_1^4 dx\,dy$.
area of $R = 9u^2$
12. $\int_1^4\int_1^{\frac{2}{3}x+\frac{1}{3}} dy\,dx$ and $\int_1^3\int_{\frac{3}{2}y-\frac{1}{2}}^4 dx\,dy$.
area of $R = 3u^2$
13. $\int_2^4\int_{x-1}^{7-x} dy\,dx$. The order $dx\,dy$ needs two iterated integrals as x is bounded above by two different functions. This gives:
$$\int_1^3\int_2^{y+1} dx\,dy + \int_3^5\int_2^{7-y} dx\,dy.$$
area of $R = 4u^2$
14. $\int_0^{12}\int_{-\sqrt{3x}}^{\sqrt{3x}} dy\,dx$ and $\int_{-6}^6\int_{y^2/3}^{12} dx\,dy$
area of $R = 96u^2$
15. $\int_0^1\int_{x^4}^{\sqrt{x}} dy\,dx$ and $\int_0^1\int_{y^2}^{\sqrt[4]{y}} dx\,dy$
area of $R = 7/15u^2$
16. $\int_0^2\int_{x^3}^{4x} dy\,dx$ and $\int_0^8\int_{y/4}^{\sqrt[3]{y}} dx\,dy$
area of $R = 4u^2$
17. 

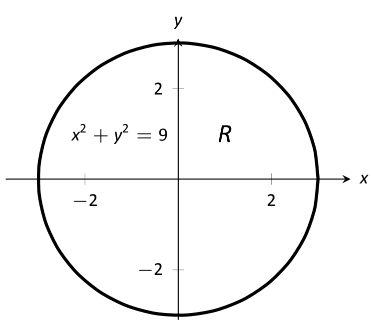
area of $R = \int_0^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dy$

18. 

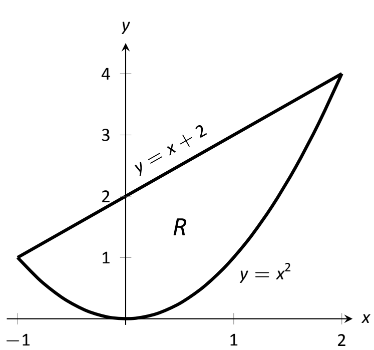
area of $R = \int_0^5 \int_{1-y/5}^{\sqrt{1-y/5}} \, dx \, dy$

19. 

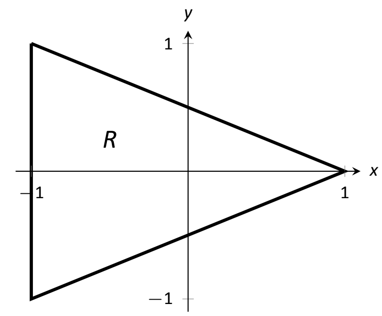
area of $R = \int_0^4 \int_{-\sqrt{4-x^2/4}}^{\sqrt{4-x^2/4}} \, dy \, dx$

20. 

area of $R = \int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \, dx \, dy$

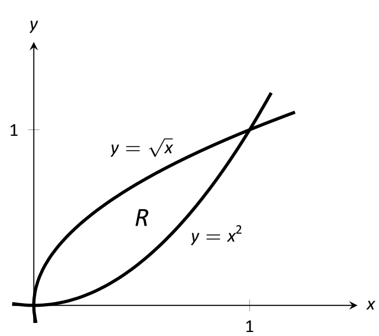
21. 

area of $R = \int_{-1}^{2} \int_{x^2}^{x+2} \, dy \, dx$

22. 

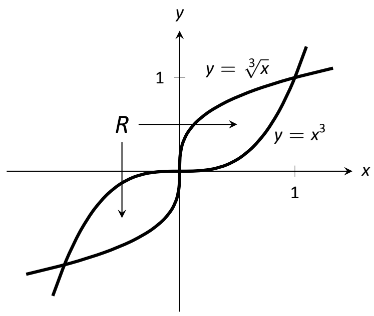
area of $R = \int_{-1}^{0} \int_{0}^{2y+1} \, dx \, dy + \int_0^1 \int_0^{1-2y} \, dx \, dy$

Section 13.2

1. volume
2. When switching the order of integration, the bounds integrals must change to reflect the bounds of the region of integration. You cannot merely change the letters x and y in a few places.
3. The double integral gives the signed volume under the surface. Since the surface is always positive, it is always above the x-y plane and hence produces only "positive" volume.
4. No. It means that there is the same amount of signed volume under f and g over R, but the functions could be very different.
5. 6; $\int_{-1}^{1} \int_{1}^{2} \left(\frac{x}{y} + 3\right) \, dy \, dx$
6. 4; $\int_{0}^{\pi} \int_{-\pi/2}^{\pi/2} (\sin x \cos y) \, dy \, dx$
7. 112/3; $\int_0^2 \int_0^{4-2y} \left(3x^2 - y + 2\right) \, dx \, dy$
8. 76/15; $\int_1^3 \int_1^x \left(x^2 y - xy^2\right) \, dy \, dx$
9. 16/5; $\int_{-1}^{1} \int_0^{1-x^2} (x + y + 2) \, dy \, dx$
10. 6561/40; $\int_0^3 \int_{x^2}^{3x} \left(xy^2\right) \, dy \, dx$
11. (a) 

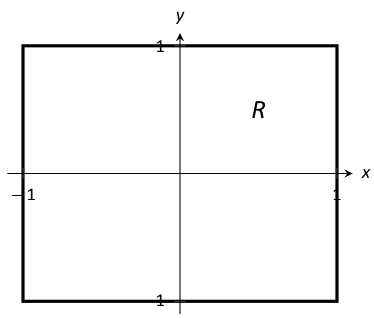
(b) $\int_0^1 \int_{x^2}^{\sqrt{x}} x^2 y \, dy \, dx = \int_0^1 \int_{y^2}^{\sqrt{y}} x^2 y \, dx \, dy.$

(c) $\frac{3}{56}$

12. (a) 

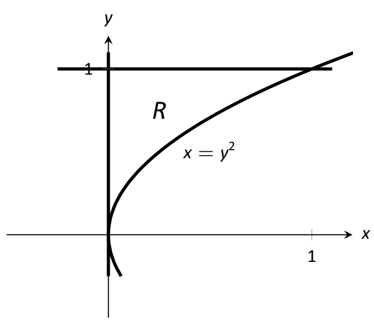
(b) $\int_0^1 \int_{x^3}^{\sqrt[3]{x}} x^2 y \, dy \, dx + \int_{-1}^0 \int_{\sqrt[3]{x}}^{x^3} x^2 y \, dy \, dx$
$= \int_0^1 \int_{y^3}^{\sqrt[3]{y}} x^2 y \, dx \, dy + \int_{-1}^0 \int_{\sqrt[3]{y}}^{y^3} x^2 y \, dx \, dy.$

(c) 0

13. (a) 

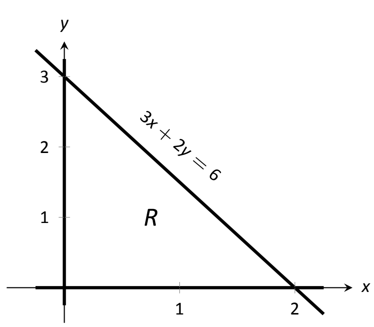
(b) $\int_{-1}^1 \int_{-1}^1 x^2 - y^2 \, dy \, dx = \int_{-1}^1 \int_{-1}^1 x^2 - y^2 \, dx \, dy.$

(c) 0

14. (a) 

(b) $\int_0^1 \int_0^{y^2} ye^x \, dx \, dy = \int_0^1 \int_{\sqrt{x}}^1 ye^x \, dy \, dx.$

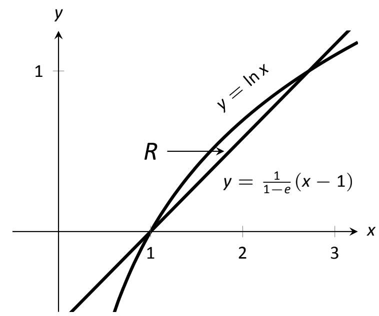
(c) $e/2 - 1$

15. (a) 

(b)

(c) $\int_0^2 \int_0^{3-3/2x} \big(6 - 3x - 2y\big) \, dy \, dx =$
$\int_0^3 \int_0^{2-2/3y} \big(6 - 3x - 2y\big) \, dx \, dy.$

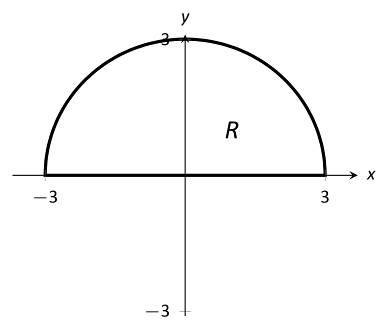
(d) 6

16. (a) 

(b)

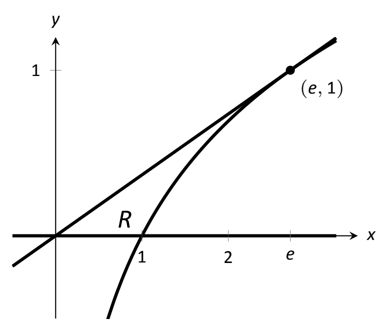
(c) $\int_1^e \int_{\frac{x-1}{e-1}}^{\ln x} e^y \, dy \, dx = \int_0^1 \int_{e^y}^{y(e-1)+1} e^y \, dx \, dy.$

(d) $-\frac{1}{2}e^2 + 2e - \frac{3}{2}$

17. (a) 

(b) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \big(x^3 y - x\big) \, dy \, dx =$
$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \big(x^3 y - x\big) \, dx \, dy.$

(c) 0

18. (a) 

(b) $\int_0^1 \int_{ey}^{e^y} \big(4 - 3y\big) \, dx \, dy =$
$\int_0^1 \int_0^{x/e} \big(4 - 3y\big) \, dy \, dx + \int_1^e \int_{\ln x}^{x/e} \big(4 - 3y\big) \, dy \, dx.$

(c) $3e - 7$

19. Integrating e^{x^2} with respect to x is not possible in terms of elementary functions. $\int_0^2 \int_0^{2x} e^{x^2} \, dy \, dx = e^4 - 1.$

20. Integrating $\cos(y^2)$ with respect to y is not possible in terms of elementary functions. $\int_0^{\sqrt{\pi/2}} \int_0^y \cos(y^2) \, dx \, dy = \frac{1}{2}.$

21. Integrating $\int_y^1 \frac{2y}{x^2+y^2}\,dx$ gives $\tan^{-1}(1/y)-\pi/4$; integrating $\tan^{-1}(1/y)$ is hard.
$\int_0^1\int_0^x \frac{2y}{x^2+y^2}\,dy\,dx = \ln 2.$

22. Integrating in the order shown is hard/impossible. By changing the order of integration, we have $\int_1^2\int_{-1}^1 \frac{x\tan^2 y}{1+\ln y}\,dx\,dy = 0$, since the integrand is an odd function with respect to x. Thus the iterated integral evaluates to 0.

23. average value of $f = 6/2 = 3$

24. average value of $f = 4/\pi^2$

25. average value of $f = \frac{112/3}{4} = 28/3$

26. average value of $f = \frac{76/15}{2} = \frac{38}{15} \approx 2.53$

Section 13.3

1. $f\big(r\cos\theta, r\sin\theta\big)$, $r\,dr\,d\theta$

2. Some regions in the x-y plane are easier to describe using polar coordinates than using rectangular coordinates. Also, some integrals are easier to evaluate one the polar substitutions have been made.

3. $\int_0^{2\pi}\int_0^1 \big(3r\cos\theta - r\sin\theta + 4\big)r\,dr\,d\theta = 4\pi$

4. $\int_0^{2\pi}\int_0^2 \big(4r\cos\theta + 4r\sin\theta\big)r\,dr\,d\theta = 0$

5. $\int_0^{\pi}\int_{\cos\theta}^{3\cos\theta} \big(8 - r\sin\theta\big)r\,dr\,d\theta = 16\pi$

6. $\int_0^{\pi/2}\int_0^{\sin(2\theta)} \big(4\big)r\,dr\,d\theta = \pi/2$

7. $\int_0^{2\pi}\int_1^2 \big(\ln(r^2)\big)r\,dr\,d\theta = 2\pi\big(\ln 16 - 3/2\big)$

8. $\int_0^{2\pi}\int_0^1 \big(1-r^2\big)r\,dr\,d\theta = \pi/2$

9. $\int_{-\pi/2}^{\pi/2}\int_0^6 \big(r^2\cos^2\theta - r^2\sin^2\theta\big)r\,dr\,d\theta =$
$\int_{-\pi/2}^{\pi/2}\int_0^6 \big(r^2\cos(2\theta)\big)r\,dr\,d\theta = 0$

10. $\int_0^{\pi/4}\int_0^1 \left(\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right) r\,dr\,d\theta = \ln 2$

11. $\int_{-\pi/2}^{\pi/2}\int_0^5 \big(r^2\big)\,dr\,d\theta = 125\pi/3$

12. $\int_{\pi/2}^{3\pi/2}\int_0^4 \big(2r\sin\theta - r\cos\theta\big)r\,dr\,d\theta = 128/3$

13. $\int_0^{\pi/4}\int_0^{\sqrt{8}} \big(r\cos\theta + r\sin\theta\big)r\,dr\,d\theta = 16\sqrt{2}/3$

14. $\int_0^{\pi}\int_1^2 \big(r\cos\theta + 5\big)r\,dr\,d\theta = 15\pi/2$

15. (a) This is impossible to integrate with rectangular coordinates as $e^{-(x^2+y^2)}$ does not have an antiderivative in terms of elementary functions.
(b) $\int_0^{2\pi}\int_0^a re^{r^2}\,dr\,d\theta = \pi(1-e^{-a^2})$.
(c) $\lim_{a\to\infty} \pi(1-e^{-a^2}) = \pi$. This implies that there is a finite volume under the surface $e^{-(x^2+y^2)}$ over the entire x-y plane.

16.
$$\begin{aligned}\iint_R f(x,y)\,dA &= \int_0^{2\pi}\int_0^a \left(h - h\sqrt{\frac{r^2\cos^2\theta}{a^2} + \frac{r^2\sin^2\theta}{a^2}}\right) r\,dr\,d\theta \\ &= \int_0^{2\pi}\int_0^a \left(hr - h\frac{r^2}{a}\right)\,dr\,d\theta \\ &= \int_0^{2\pi}\left(\frac{1}{2}hr^2 - \frac{h}{3a}r^3\right)\Big|_0^a\,d\theta \\ &= \int_0^{2\pi}\left(\frac{1}{6}a^2h\right)\,d\theta \\ &= \frac{1}{3}\pi a^2 h.\end{aligned}$$

Section 13.4

1. Because they are scalar multiples of each other.
2. y
3. "little masses"
4. A collection of individual masses in the plane. Each mass is a point mass, i.e., mass located at a point, not across a region.
5. M_x measures the moment about the x-axis, meaning we need to measure distance from the x-axis. Such measurements are measures in the y-direction.
6. If the lamina is an annulus, the center of mass will likely be in the middle, outside of the region. (See Example 13.4.9.)
7. $\bar{x} = 5.25$
8. $\bar{x} = 1.3$
9. $(\bar{x},\bar{y}) = (0,3)$
10. $(\bar{x},\bar{y}) = (0,1/3)$
11. $M = 150$gm;
12. $M = 190$gm
13. $M = 2$lb
14. $M = 2/3$lb
15. $M = 16\pi \approx 50.27$kg
16. $M = 325\pi/12 \approx 85$kg
17. $M = 54\pi \approx 169.65$lb
18. $M = 63\pi \approx 197.92$lb
19. $M = 150$gm; $M_y = 600$; $M_x = -75$; $(\bar{x},\bar{y}) = (4,-0.5)$
20. $M = 190$gm; $M_y = 850$; $M_x = -315/2$; $(\bar{x},\bar{y}) = (4.47,-0.83)$
21. $M = 2$lb; $M_y = 0$; $M_x = 2/3$; $(\bar{x},\bar{y}) = (0,1/3)$
22. $M = 2/3$lb; $M_y = 7/30$; $M_x = 7/30$; $(\bar{x},\bar{y}) = (0.35,0.35)$
23. $M = 16\pi \approx 50.27$kg; $M_y = 4\pi$; $M_x = 4\pi$; $(\bar{x},\bar{y}) = (1/4,1/4)$
24. $M = 325\pi/12 \approx 85$kg; $M_y = 2375/12$; $M_x = 2375/12$; $(\bar{x},\bar{y}) = (2.33,2.33)$
25. $M = 54\pi \approx 169.65$lb; $M_y = 0$; $M_x = 504$; $(\bar{x},\bar{y}) = (0,2.97)$
26. $M = 63\pi \approx 197.92$lb; $M_y = 0$; $M_x = 1215/2$; $(\bar{x},\bar{y}) = (0,3.07)$
27. $I_x = 64/3$; $I_y = 64/3$; $I_O = 128/3$
28. $I_x = 16/3$; $I_y = 256/3$; $I_O = 272/3$
29. $I_x = 16/3$; $I_y = 64/3$; $I_O = 80/3$
30. $I_x = 16$; $I_y = 16$; $I_O = 32$

Section 13.5

1. arc length
2. tangent

3. surface areas
4. Technology makes good approximations accessible, if not exact answers.
5. Intuitively, adding h to f only shifts f up (i.e., parallel to the z-axis) and does not change its shape. Therefore it will not change the surface area over R.

   Analytically, $f_x = g_x$ and $f_y = g_y$; therefore, the surface area of each is computed with identical double integrals.
6. Analytically, $g_x = 2f_x$ and $g_y = 2f_y$. The double integral to compute the surface area of f over R is $\iint_R \sqrt{1+f_x^2+f_y^2}\,dA$; the double integral to compute the surface area of g over R is $\iint_R \sqrt{1+4f_x^2+4f_y^2}\,dA$, which is *not* twice the double integral used to calculate the surface area of f.
7. $SA = \displaystyle\int_0^{2\pi}\int_0^{2\pi} \sqrt{1+\cos^2 x\cos^2 y+\sin^2 x\sin^2 y}\,dx\,dy$
8. $SA = \displaystyle\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \sqrt{1+\frac{4x^2+4y^2}{(1+x^2+y^2)^4}}\,dx\,dy$

   Polar offers simpler bounds:

   $SA = \displaystyle\int_0^{2\pi}\int_0^{3} r\sqrt{1+\frac{4r^2}{(1+r^2)^4}}\,dr\,d\theta$
9. $SA = \displaystyle\int_{-1}^{1}\int_{-1}^{1} \sqrt{1+4x^2+4y^2}\,dx\,dy$
10. $SA = \displaystyle\int_{-5}^{5}\int_0^{1} \sqrt{1+\frac{4x^2e^{2x^2}}{\left(1+e^{x^2}\right)^4}}\,dy\,dx$
11. $SA = \displaystyle\int_0^{3}\int_{-1}^{1} \sqrt{1+9+49}\,dx\,dy = 6\sqrt{59} \approx 46.09$
12. $SA = \displaystyle\int_0^{1}\int_0^{1-x} \sqrt{1+4+4}\,dy\,dx = 18$
13. This is easier in polar:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^{4} r\sqrt{1+4r^2\cos^2 t+4r^2\sin^2 t}\,dr\,d\theta \\ &= \int_0^{2\pi}\int_0^{4} r\sqrt{1+4r^2}\,dr\,d\theta \\ &= \frac{\pi}{6}\left(65\sqrt{65}-1\right) \approx 273.87 \end{aligned}$$

14.

$$\begin{aligned} SA &= \int_0^{1}\int_{-y}^{y} \sqrt{1+4+64y^2}\,dx\,dy \\ &= \int_0^{1} \left(2y\sqrt{5+64y^2}\right)dy \\ &= \frac{1}{96}\left(69\sqrt{69}-5\sqrt{5}\right) \approx 5.85 \end{aligned}$$

15.

$$\begin{aligned} SA &= \int_0^{2}\int_0^{2x} \sqrt{1+1+4x^2}\,dy\,dx \\ &= \int_0^{2} \left(2x\sqrt{2+4x^2}\right)dx \\ &= \frac{26}{3}\sqrt{2} \approx 12.26 \end{aligned}$$

16.

$$\begin{aligned} SA &= \int_0^{1}\int_0^{1} \sqrt{1+x+9y}\,dx\,dy \\ &= \int_0^{1} \frac{2}{3}\left((9y+2)^{3/2}-(9y+1)^{3/2}\right)dy \\ &= \frac{4}{135}\left(121\sqrt{11}-100\sqrt{10}-4\sqrt{2}+1\right) \approx 2.383 \end{aligned}$$

17. This is easier in polar:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^{5} r\sqrt{1+\frac{4r^2\cos^2\theta+4r^2\sin^2\theta}{r^2\sin^2\theta+r^2\cos^2\theta}}\,dr\,d\theta \\ &= \int_0^{2\pi}\int_0^{5} r\sqrt{5}\,dr\,d\theta \\ &= 25\pi\sqrt{5} \approx 175.62 \end{aligned}$$

18. This is easier in polar:

$$\begin{aligned} SA &= 2\int_0^{2\pi}\int_0^{5} r\sqrt{1+\frac{r^2\cos^2 t+r^2\sin^2 t}{25-r^2\sin^2 t-r^2\cos^2 t}}\,dr\,d\theta \\ &= 2\int_0^{2\pi}\int_0^{5} r\sqrt{\frac{1}{25-r^2}}\,dr\,d\theta \\ &= 100\pi \approx 314.16 \end{aligned}$$

19. Integrating in polar is easiest considering R:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^{1} r\sqrt{1+c^2+d^2}\,dr\,d\theta \\ &= \int_0^{2\pi} \frac{1}{2}\left(\sqrt{1+c^2+d^2}\right)d\theta \\ &= \pi\sqrt{1+c^2+d^2}. \end{aligned}$$

The value of h does not matter as it only shifts the plane vertically (i.e., parallel to the z-axis). Different values of h do not create different ellipses in the plane.

Section 13.6

1. surface to surface, curve to curve and point to point
2. One possible answer is "sum up lots of little volumes over D."
3. Answers can vary. From this section we used triple integration to find the volume of a solid region, the mass of a solid, and the center of mass of a solid.
4. δV.
5. $V = \int_{-1}^{1}\int_{-1}^{1}\left(8-x^2-y^2-(2x+y)\right)dx\,dy = 88/3$
6. $V = \int_0^{2}\int_0^{3}\left(x^2+y^2-(-x^2-y^2)\right)dy\,dx = 52$
7. $V = \int_0^{\pi}\int_0^{x}\left(\cos x\sin y+2-\sin x\cos y\right)dy\,dx = \pi^2-\pi \approx 6.728$
8. $V = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\left(6-x^2-y^2-(2x^2+2y^2+3)\right)dy\,dx$.

   Integrating in polar is easier, giving

   $V = \int_0^{2\pi}\int_0^{1}\left(3-3r^2\right)r\,dr\,d\theta = 3\pi/2$.
9. $dz\,dy\,dx$: $\displaystyle\int_0^{3}\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx$

   $dz\,dx\,dy$: $\displaystyle\int_0^{1}\int_0^{3-3y}\int_0^{2-2x/3-2y} dz\,dx\,dy$

   $dy\,dz\,dx$: $\displaystyle\int_0^{3}\int_0^{2-2x/3}\int_0^{1-x/3-z/2} dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^2\int_0^{3-3z/2}\int_0^{1-x/3-z/2} dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^1\int_0^{2-2y}\int_0^{3-3y-3z/2} dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^2\int_0^{1-z/2}\int_0^{3-3y-3z/2} dx\,dy\,dz$

$V=\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx = 1.$

10. $dz\,dy\,dx$: $\int_1^3\int_0^2\int_0^{(3-x)/2} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_0^2\int_1^3\int_0^{(3-x)/2} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_1^3\int_0^{(3-x)/2}\int_0^2 dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^1\int_1^{3-2z}\int_0^2 dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^2\int_0^1\int_1^{3-2z} dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^1\int_0^2\int_1^{3-2z} dx\,dy\,dz$

$V=\int_0^1\int_0^2\int_1^{3-2z} dx\,dy\,dz = 2.$

11. $dz\,dy\,dx$: $\int_0^2\int_{-2}^0\int_{y^2/2}^{-y} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_{-2}^0\int_0^2\int_{y^2/2}^{-y} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dx\,dz$

$dx\,dz\,dy$: $\int_{-2}^0\int_{y^2/2}^{-y}\int_0^2 dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^2\int_{-\sqrt{2z}}^{-z}\int_0^2 dx\,dy\,dz$

$V=\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx = 4/3.$

12. $dz\,dy\,dx$: $\int_0^3\int_{3x}^9\int_0^{\sqrt{y^2-9x^2}} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_0^9\int_0^{y/3}\int_0^{\sqrt{y^2-9x^2}} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_0^3\int_0^{\sqrt{81-9x^2}}\int_{\sqrt{z^2+9x^2}}^9 dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^9\int_0^{\sqrt{9-z^2/9}}\int_{\sqrt{z^2+9x^2}}^9 dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^9\int_0^y\int_0^{\frac{1}{3}\sqrt{y^2-z^2}} dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^9\int_z^9\int_0^{\frac{1}{3}\sqrt{y^2-z^2}} dx\,dy\,dz$

13. $dz\,dy\,dx$: $\int_0^2\int_{1-x/2}^1\int_0^{2x+4y-4} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_0^1\int_{2-2y}^2\int_0^{2x+4y-4} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_0^2\int_0^{2x}\int_{z/4-x/2+1}^1 dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^4\int_{z/2}^2\int_{z/4-x/2+1}^1 dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^1\int_0^{4y}\int_{z/2-2y+2}^2 dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^4\int_{z/4}^1\int_{z/2-2y+2}^2 dx\,dy\,dz$

$V=\int_0^4\int_{z/4}^1\int_{z/2-2y+2}^2 dx\,dy\,dz = 4/3.$

14. $dz\,dy\,dx$: $\int_{-2}^2\int_0^{4-x^2}\int_0^{2y} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_0^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_0^{2x+4y-4} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_{-2}^2\int_0^{8-2x^2}\int_{z/2}^{4-x^2} dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^8\int_{-\sqrt{4-z/2}}^{\sqrt{4-z/2}}\int_{z/2}^{4-x^2} dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^4\int_0^{2y}\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^8\int_{z/2}^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dy\,dz$

$V=\int_{-2}^2\int_0^{4-x^2}\int_0^{2y} dz\,dy\,dx = 512/15.$

15. $dz\,dy\,dx$: $\int_0^1\int_0^{1-x^2}\int_0^{\sqrt{1-y}} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_0^1\int_0^x\int_0^{1-x^2} dy\,dz\,dx + \int_0^1\int_x^1\int_0^{1-z^2} dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^1\int_0^z\int_0^{1-z^2} dy\,dx\,dz + \int_0^1\int_z^1\int_0^{1-x^2} dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^1\int_0^{1-z^2}\int_0^{\sqrt{1-y}} dx\,dy\,dz$

Answers will vary. Neither order is particularly "hard." The order *dz dy dx* requires integrating a square root, so powers can be messy; the order *dy dz dx* requires two triple integrals, but each uses only polynomials.

16. $dz\,dy\,dx$: $\int_0^1\int_0^{3x}\int_0^{1-x} dz\,dy\,dx + \int_0^1\int_{3x}^3\int_0^{1-y/3} dz\,dy\,dx$

$dz\,dx\,dy$: $\int_0^3\int_0^{y/3}\int_0^{1-y/3} dz\,dy\,dx + \int_0^3\int_{y/3}^1\int_0^{1-x} dz\,dx\,dy$

$dy\,dz\,dx$: $\int_0^1\int_0^{1-x}\int_0^{3-3z} dy\,dz\,dx$

$dy\,dx\,dz$: $\int_0^1\int_0^{1-z}\int_0^{3-3z} dy\,dx\,dz$

$dx\,dz\,dy$: $\int_0^3\int_0^{1-y/3}\int_0^{1-z} dx\,dz\,dy$

$dx\,dy\,dz$: $\int_0^1\int_0^{3-3z}\int_0^{1-z} dx\,dy\,dz$

$V=\int_0^1\int_0^{3-3z}\int_0^{1-z} dx\,dy\,dz = 1.$

17. 8

18. $7/8$

19. π

20. 0

21. $M=10$, $M_{yz}=15/2$, $M_{xz}=5/2$, $M_{xy}=5$; $(\bar{x},\bar{y},\bar{z})=(3/4,1/4,1/2)$

22. $M=4$, $M_{yz}=20/3$, $M_{xz}=4$, $M_{xy}=4/3$; $(\bar{x},\bar{y},\bar{z})=(5/3,1,1/3)$

23. $M=16/5$, $M_{yz}=16/3$, $M_{xz}=104/45$, $M_{xy}=32/9$; $(\bar{x},\bar{y},\bar{z})=(5/3,13/18,10/9)\approx(1.67,0.72,1.11)$

24. $M = \frac{65{,}536}{15} \approx 208.05$, $M_{yz} = 0$, $M_{xz} = \frac{2{,}097{,}152}{3465} \approx 605.24$, $M_{xy} = \frac{2{,}097{,}152}{3465} \approx 605.24$; $(\bar{x}, \bar{y}, \bar{z}) = (0, 32/11, 32/11) \approx (0, 2.91, 2.91)$

Section 13.7

1. In cylindrical, r determines how far from the origin one goes in the x-y plane before considering the z-component. Equivalently, if on projects a point in cylindrical coordinates onto the x-y plane, r will be the distance of this projection from the origin.

   In spherical, ρ is the distance from the origin to the point.

2. If $r = 0$ or $\rho = 0$, then the point in each coordinate system lies on the z-axis regardless of the value of θ.

3. Cylinders (tubes) centered at the origin, parallel to the z-axis; planes parallel to the z-axis that intersect the z-axis; planes parallel to the x-y plane.

4. Spheres centered at the origin; planes parallel to the z-axis that intersect the z-axis; cones centered on the z-axis with point at the origin.

5. (a) Cylindrical: $(2\sqrt{2}, \pi/4, 1)$ and $(2, 5\pi/6, 0)$
   Spherical: $(3, \pi/4, \cos^{-1}(1/3))$ and $(2, 5\pi/6, \pi/2)$

   (b) Rectangular: $(\sqrt{2}, \sqrt{2}, 2)$ and $(0, -3, -4)$
   Spherical: $(2\sqrt{2}, \pi/4, \pi/4)$ and $(5, 3\pi/2, \pi - \tan^{-1}(3/4))$

   (c) Rectangular: $(1, 1, \sqrt{2})$ and $(0, 0, 1)$
   Cylindrical: $(\sqrt{2}, \pi/4, \sqrt{2})$ and $(0, 0, 1)$

6. (a) Cylindrical: $(1, \pi/2, 1)$ and $(1, \pi, 1)$
   Spherical: $(\sqrt{2}, \pi/2, \pi/4)$ and $(\sqrt{2}, \pi, \pi/4)$

   (b) Rectangular: $(0, 0, 1)$ and $(-1, -\sqrt{3}, 0)$
   Spherical: $(1, \pi, 0)$ and $(2, 4\pi/3, \pi/2)$

   (c) Rectangular: $(\sqrt{3}, 1, 0)$ and $(0, 0, -3)$
   Cylindrical: $(2, \pi/6, 0)$ and $(0, \pi, -3)$

7. (a) A cylindrical surface or tube, centered along the z-axis of radius 1, extending from the x-y plane up to the plane $z = 1$ (i.e., the tube has a length of 1).

   (b) This is a region of space, being half of a tube with "thick" walls of inner radius 1 and outer radius 2, centered along the z-axis with a length of 1, where the half "below" the x-z plane is removed.

   (c) This is upper half of the sphere of radius 3 centered at the origin (i.e., the upper hemisphere).

   (d) This is a region of space, where the ball of radius 2, centered at the origin, is removed from the ball of radius 3, centered at the origin.

8. (a) A square portion of the y-z plane with corners at $(0, 1, 0)$, $(0, 1, 1)$, $(0, 2, 1)$ and $(0, 2, 0)$.

   (b) This is a curve, a circle of radius 2, centered at $(0, 0, 5)$, lying parallel to the x-y plane (i.e., in the plane $z = 5$).

   (c) This is a region of space, a half of a solid cone with rounded top, where the rounded top is a portion of the ball of radius 2 centered at the origin and the sides of the cone make an angle of $\pi/4$ with the positive z-axis. The bounds on θ mean only the portion "above" the x-z plane are retained.

   (d) This is a curve, a circle of radius 1 centered at $(0, 0, \sqrt{3})$, lying parallel to the x-y plane.

9. $\displaystyle\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} h(r, \theta, z) r\, dz\, dr\, d\theta$

10. $\displaystyle\int_{\varphi_1}^{\varphi_2}\int_{\theta_1}^{\theta_2}\int_{\rho_1}^{\rho_2} h(\rho, \theta, \varphi)\rho^2 \sin(\varphi)\, d\rho\, d\theta\, d\varphi$

11. The region in space is bounded between the planes $z = 0$ and $z = 2$, inside of the cylinder $x^2 + y^2 = 4$, and the planes $\theta = 0$ and $\theta = \pi/2$: describes a "wedge" of a cylinder of height 2 and radius 2; the angle of the wedge is $\pi/2$, or 90°.

12. Bounded between the planes $z = 0$ and $z = 5$, between the cylinders $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$: describes a "pipe" or "tube" of length 5, an inner radius of 3 and outer radius of 4.

13. Bounded between the plane $z = 1$ and the cone $z = 1 - \sqrt{x^2 + y^2}$: describes an inverted cone, with height of 1, point at $(0, 0, 1)$ and base radius of 1.

14. Bounded between $y \geq 0$, inside the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z = 2 - \sqrt{x^2 + y^2}$: describes cylindrical solid of height 1 and radius 2, topped with an inverted cone of height 1 and base radius 1 with point at $(0, 0, 2)$.

15. Describes a quarter of a ball of radius 3, centered at the origin; the quarter resides above the x-y plane and above the x-z plane.

16. Bounded between the plane $z = 0$, inside the cylinder $x^2 + y^2 = a^2$, and below the upper hemisphere $z = \sqrt{a^2 - x^2 - y^2} + b$, with radius a and centered at $(0, 0, b)$: describes a cylindrical solid of radius a and height b, topped with the upper hemisphere of radius a.

17. Describes the portion of the unit ball that resides in the first octant.

18. Describes half of a spherical shell (i.e., $y \geq 0$) with inner radius of 1 and outer radius of 1.1 centered at the origin.

19. Bounded above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$: describes a shape that is somewhat "diamond"-like; some think of it as looking like an ice cream cone (see Figure 13.7.8). It describes a cone, where the side makes an angle of $\pi/4$ with the positive z-axis, topped by the portion of the ball of radius 2, centered at the origin.

20. It is the region is space bounded below by $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 4$, with the portion above the cone $z = \sqrt{3}\sqrt{x^2 + y^2}$ removed: it describes a cone, where the side makes an angle of $\pi/4$ with the positive z-axis, topped by the portion of the ball of radius 2, centered at the origin, with the inner cone with angle $\pi/6$ removed, along with corresponding portion of the ball of radius 2.

21. The region in space is bounded below by the cone $z = \sqrt{3}\sqrt{x^2 + y^2}$ and above by the plane $z = 1$: it describes a cone, with point at the origin, centered along the positive z-axis, with height of 1 and base radius of $\tan(\pi/6) = 1/\sqrt{3}$.

22. The region in space is bounded below by the cone $z = \sqrt{3}\sqrt{x^2 + y^2}$ and above by the plane $z = a$: it describes a cone, with point at the origin, centered along the positive z-axis, with height of a and base radius of $a\tan(\pi/6)$.

23. In cylindrical coordinates, the density is $\delta(r, \theta, z) = r + 1$. Thus mass is
$$\int_0^{2\pi}\int_0^2\int_0^4 (r + 1) r\, dz\, dr\, d\theta = 112\pi/3.$$

24. In cylindrical coordinates, the density is $\delta(r, \theta, z) = z$. Thus mass is
$$\int_0^{2\pi}\int_2^3\int_0^{10} zr\, dz\, dr\, d\theta = 250\pi.$$

25. In cylindrical coordinates, the density is $\delta(r, \theta, z) = 1$. Thus mass is
$$\int_0^{\pi}\int_0^1\int_0^{4 - r\sin\theta} r\, dz\, dr\, d\theta = 2\pi - 2/3 \approx 5.617.$$

26. In cylindrical coordinates, the density is $\delta(r, \theta, z) = 1$. Thus mass is
$$\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\, dz\, dr\, d\theta = 4\pi/3.$$

27. In cylindrical coordinates, the density is $\delta(r,\theta,z) = r + 1$. Thus mass is

$$M = \int_0^{2\pi}\int_0^2\int_0^4 (r+1)r\,dz\,dr\,d\theta = 112\pi/3.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = 224\pi/3$, placing the center of mass at $(0,0,2)$.

28. In cylindrical coordinates, the density is $\delta(r,\theta,z) = z$. Thus mass is

$$M = \int_0^{2\pi}\int_2^3\int_0^{10} zr\,dz\,dr\,d\theta = 250\pi.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = 5000\pi/3$, placing the center of mass at $(0,0,20/3)$.

29. In cylindrical coordinates, the density is $\delta(r,\theta,z) = 1$. Thus mass is

$$\int_0^{\pi}\int_0^1\int_0^{4-r\sin\theta} r\,dz\,dr\,d\theta = 2\pi - 2/3 \approx 5.617.$$

We find $M_{yz} = 0$, $M_{xz} = 8/3 - \pi/8$, and $M_{xy} = 65\pi/16 - 8/3$, placing the center of mass at $\approx (0, 0.405, 1.80)$.

30. In cylindrical coordinates, the density is $\delta(r,\theta,z) = 1$. Thus mass is

$$\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta = 2\pi/3.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = \pi/4$, placing the center of mass at $(0,0,3/8)$.

31. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = 1$. Thus mass is

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^1 \rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = 2\pi/3.$$

32. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = \rho$. Thus mass is

$$\int_0^{\pi}\int_0^{2\pi}\int_4^5 (\rho)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = 369\pi.$$

33. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = \rho\cos\varphi$. Thus mass is

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^1 \big(\rho\cos(\varphi)\big)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = \pi/8.$$

34. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = \rho\cos\varphi$. Thus mass is

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec(\varphi)} \big(\rho\cos(\varphi)\big)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = \pi/4.$$

35. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = 1$. Thus mass is

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^1 \rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = 2\pi/3.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = \pi/4$, placing the center of mass at $(0,0,3/8)$.

36. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = \rho$. Thus mass is

$$\int_0^{\pi}\int_0^{2\pi}\int_4^5 (\rho)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = 369\pi.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = 0$, placing the center of mass at $(0,0,0)$.

37. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = \rho\cos\varphi$. Thus mass is

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^1 \big(\rho\cos(\varphi)\big)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = \pi/8.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = (4-\sqrt{2})\pi/30$, placing the center of mass at $(0,0,4(4-\sqrt{2})/15)$.

38. In spherical coordinates, the density is $\delta(\rho,\theta,\varphi) = \rho\cos\varphi$. Thus mass is

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec(\varphi)} \big(\rho\cos(\varphi)\big)\rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi = \pi/4.$$

We find $M_{yz} = 0$, $M_{xz} = 0$, and $M_{xy} = \pi/5$, placing the center of mass at $(0,0,4/5)$.

39. Rectangular: $\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx$

Cylindrical: $\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta$

Spherical: $\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi$

Spherical appears simplest, avoiding the integration of square-roots and using techniques such as Substitution; all bounds are constants.

40. Rectangular: $\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^1 dz\,dy\,dx$

Cylindrical: $\int_0^{2\pi}\int_0^1\int_0^1 r\,dz\,dr\,d\theta$

Spherical: $\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec\varphi} \rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi +$
$\int_{\pi/4}^{\pi/2}\int_0^{2\pi}\int_0^{\csc\varphi} \rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi$

Cylindrical appears simplest, avoiding the integration of square-roots and two triple integrals; all bounds are constants.

41. Rectangular: $\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^1 dz\,dy\,dx$

Cylindrical: $\int_0^{2\pi}\int_0^1\int_r^1 r\,dz\,dr\,d\theta$

Spherical: $\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec\varphi} \rho^2\sin(\varphi)\,d\rho\,d\theta\,d\varphi$

Cylindrical appears simplest, avoiding the integration of square-roots that rectangular uses. Spherical is not difficult, though it requires Substitution, an extra step.

42. Rectangular: $\int_0^1\int_0^1\int_0^1 dz\,dy\,dx$

Cylindrical: $\int_0^{\pi/4}\int_0^{\sec\theta}\int_0^1 r\,dz\,dr\,d\theta + \int_{\pi/4}^{\pi/2}\int_0^{\csc\theta}\int_0^1 r\,dz\,dr\,d\theta$

Spherical: $\int_0^{\pi/4}\int_0^{\tan^{-1}(\sec\theta)}\int_0^{\sec\varphi} \rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta +$
$\int_0^{\pi/4}\int_{\tan^{-1}(\sec\theta)}^{\pi/2}\int_0^{\sec\theta\csc\varphi} \rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta +$
$\int_{\pi/4}^{\pi/2}\int_0^{\tan^{-1}(\csc\theta)}\int_0^{\sec\varphi} \rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta +$
$\int_{\pi/4}^{\pi/2}\int_{\tan^{-1}(\csc\theta)}^{\pi/2}\int_0^{\csc\theta\csc\varphi} \rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta.$

Rectangular is clearly the simplest.

Chapter 14

Section 14.1

1. When C is a curve in the plane and f is a surface defined over C, then $\int_C f(s)\,ds$ describes the area under the spatial curve that lies on f, over C.

2. The evaluation is the same. The $\oint$ notation signifies that the curve C is a closed curve, though the evaluation is the same.

3. The variable s denotes the arc-length parameter, which is generally difficult to use. The Key Idea allows one to parametrize a curve using another, ideally easier-to-use, parameter.

4. Answers will vary.

5. $12\sqrt{2}$

6. $41\sqrt{10}/2$

7. 40π

8. $10\pi^2$

9. Over the first subcurve of C, the line integral has a value of $3/2$; over the second subcurve, the line integral has a value of $4/3$. The total value of the line integral is thus $17/6$.

10. Over the first subcurve of C, the line integral has a value of $2\sqrt{2}/3$; over the second subcurve, the line integral has a value of $\pi - 2$. The total value of the line integral is thus $\pi + 2\sqrt{2}/3 - 2$.

11. $\int_0^1 (5t^2 +_2 t + 2)\sqrt{(4t+1)^2+1}\,dt \approx 17.071$

12. $\int_0^\pi t\sqrt{1+\cos^2 t}\,dt \approx 6.001$

13. $\oint_0^{2\pi} \left(10 - 4\cos^2 t - \sin^2 t\right)\sqrt{\cos^2 t + 4\sin^2 t}\,dt \approx 74.986$

14. $\int_{-1}^1 \left(3t^3 + 2t + 5\right)\sqrt{9t^4+1}\,dt \approx 15.479$

15. $7\sqrt{26}/3$

16. 2π

17. $8\pi^3$

18. $5/2$

19. $M = 8\sqrt{2}\pi^2$; center of mass is $(0, -1/(2\pi), 8\pi/3)$.

20. $M \approx 0.237$; center of mass is approximately $(0.173, 0.099, 0.065)$.

Section 14.2

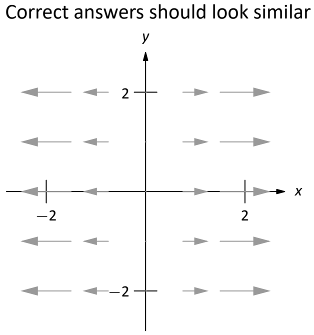
1. Answers will vary. Appropriate answers include velocities of moving particles (air, water, etc.); gravitational or electromagnetic forces.

2. Specific answers will vary, though should relate to the idea that "more of the vector field is moving into that point than out of that point."

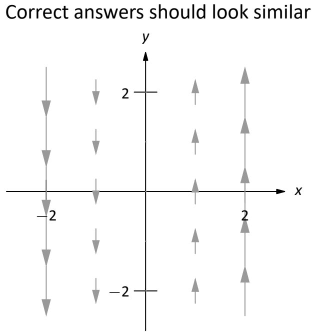
3. Specific answers will vary, though should relate to the idea that the vector field is spinning clockwise at that point.

4. No; to be incompressible, the divergence needs to be 0 everywhere, not just at one point.

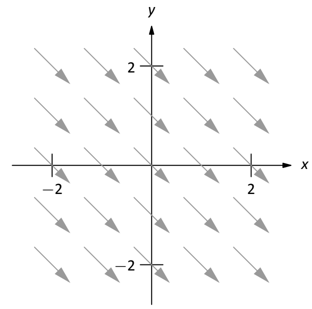
5. Correct answers should look similar to



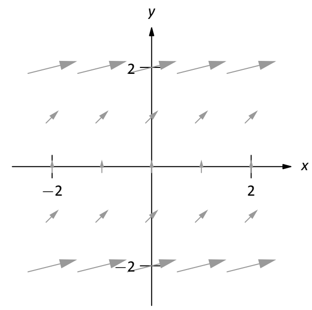
6. Correct answers should look similar to



7. Correct answers should look similar to



8. Correct answers should look similar to



9. $\text{div}\,\vec F = 1 + 2y$
   $\text{curl}\,\vec F = 0$

10. $\text{div}\,\vec F = 0$
    $\text{curl}\,\vec F = 1 + 2y$

11. $\text{div}\,\vec F = x\cos(xy) - y\sin(xy)$
    $\text{curl}\,\vec F = y\cos(xy) + x\sin(xy)$

12. $\text{div}\,\vec F = \frac{4}{(x^2+y^2)^2}$
    $\text{curl}\,\vec F = 0$

13. $\text{div}\,\vec F = 3$
    $\text{curl}\,\vec F = \langle -1, -1, -1\rangle$

14. $\text{div}\,\vec F = 2x + 2y + 2z$
    $\text{curl}\,\vec F = \langle 2y, 2z, 2x\rangle$

15. $\text{div}\,\vec F = 1 + 2y$
    $\text{curl}\,\vec F = 0$

16. $\text{div}\,\vec F = 2y$
    $\text{curl}\,\vec F = 0$

17. $\text{div}\,\vec F = 2y - \sin z$
    $\text{curl}\,\vec F = \vec 0$

18. $\text{div}\,\vec F = \frac{2}{(x^2+y^2+z^2)^2}$
    $\text{curl}\,\vec F = \vec 0$

Section 14.3

1. False. It is true for line integrals over scalar fields, though.

2. The input of $\vec F$ should be a point in the plane, not a two dimensional vector.

3. True.

4. False.

5. We can conclude that $\vec F$ is conservative.

6. By the Fundamental Theorem of Line Integrals, since $\vec F$ is conservative, $\oint_C \vec F \cdot d\vec r = f(B) - f(A)$, where f is a potential function for $\vec F$ and A and B are the initial and terminal points of C, respectively. Since C is a closed curve, $A = B$, and hence $f(B) - f(A) = 0$.

7. $11/6$. (One parametrization for C is $\vec{r}(t) = \langle 3t, t\rangle$ on $0 \le t \le 1$.)
8. $5/3$. (One parametrization for C is $\vec{r}(t) = \langle t, t^2\rangle$ on $0 \le t \le 1$.)
9. 0. (One parametrization for C is $\vec{r}(t) = \langle \cos t, \sin t\rangle$ on $0 \le t \le \pi$.)
10. $2/5$. (One parametrization for C is $\vec{r}(t) = \langle t, t^3\rangle$ on $-1 \le t \le 1$.)
11. 12. (One parametrization for C is $\vec{r}(t) = \langle 1,2,3\rangle + t\langle 3,1,-1\rangle$ on $0 \le t \le 1$.)
12. 1.
13. $5/6$ joules. (One parametrization for C is $\vec{r}(t) = \langle t, t\rangle$ on $0 \le t \le 1$.)
14. $13/15$ joules. (One parametrization for C is $\vec{r}(t) = \langle t, \sqrt{t}\rangle$ on $0 \le t \le 1$.)
15. 24 ft-lbs.
16. 24 ft-lbs.
17.
   (a) $f(x,y) = xy + x$
   (b) curl $\vec{F} = 0$.
   (c) 1. (One parametrization for C is $\vec{r}(t) = \langle t, -1t\rangle$ on $0 \le t \le 1$.)
   (d) 1 (with $A = (0,1)$ and $B = (1,0)$, $f(B) - f(A) = 1$.)
18.
   (a) $f(x,y) = x^2 + xy + y^2$
   (b) curl $\vec{F} = 0$.
   (c) 0.
   (d) 0 (with $A = (0,0)$ and $B = (0,0)$, $f(B) - f(A) = 0$.)
19.
   (a) $f(x,y) = x^2yz$
   (b) curl $\vec{F} = \vec{0}$.
   (c) 250.
   (d) 250 (with $A = (1,-1,0)$ and $B = (5,5,2)$, $f(B) - f(A) = 250$.)
20.
   (a) $f(x,y) = x^2 + y^2 + z^2$
   (b) curl $\vec{F} = \vec{0}$.
   (c) 0.
   (d) 0 (with $A = (1,0,0)$ and $B = (1,0,0)$, $f(B) - f(A) = 250$.)
21. Since $\vec{F}$ is conservative, it is the gradient of some potential function. That is, $\nabla f = \langle f_x, f_y, f_z\rangle = \vec{F} = \langle M, N, P\rangle$. In particular, $M = f_x$, $N = f_y$ and $P = f_z$.
   Note that curl $\vec{F} = \langle P_y - N_z, M_z - P_x, N_x - M_y\rangle = \langle f_{zy} - f_{yz}, f_{xz} - f_{zx}, f_{yx} - f_{xy}\rangle$, which, by Theorem 12.3.1, is $\langle 0,0,0\rangle$.

Section 14.4

1. along, across
2. It is the measure of flow around the entirety of a closed curve C.
3. the curl of $\vec{F}$, or curl $\vec{F}$
4. the divergence of $\vec{F}$, or div $\vec{F}$
5. curl $\vec{F}$
6. div $\vec{F}$
7. 12
8. 12
9. $-2/3$
10. $10/3$
11. $1/2$
12. $1/2$
13. The line integral $\oint_C \vec{F}\cdot d\vec{r}$, over the parabola, is $38/3$; over the line, it is -10. The total line integral is thus $38/3 - 10 = 8/3$. The double integral of curl $\vec{F} = 2$ over R also has value $8/3$.
14. Both the line integral and double integral have value of 2π.
15. Three line integrals need to be computed to compute $\oint_C \vec{F}\cdot d\vec{r}$. It does not matter which corner one starts from first, but be sure to proceed around the triangle in a counterclockwise fashion.
   From $(0,0)$ to $(2,0)$, the line integral has a value of 0. From $(2,0)$ to $(1,1)$ the integral has a value of $7/3$. From $(1,1)$ to $(0,0)$ the line integral has a value of $-1/3$. Total value is 2.
   The double integral of curl $\vec{F}$ over R also has value 2.
16. Two line integrals need to be computed to compute $\oint_C \vec{F}\cdot d\vec{r}$. Along the parabola, the line integral has value 25.5. Along the line, the line integral has value -21. Together, the total value is 4.5
   The double integral of curl $\vec{F}$ over R also has value 4.5.
17. Any choice of $\vec{F}$ is appropriate as long as curl $\vec{F} = 1$. When $\vec{F} = \langle -y/2, x/2\rangle$, the integrand of the line integral is simply 6. The area of R is 12π.
18. Any choice of $\vec{F}$ is appropriate as long as curl $\vec{F} = 1$. The choices of $\vec{F} = \langle -y, 0\rangle$ and $\langle 0, x\rangle$ each lead to reasonable integrands. The area of R is $4/3$.
19. Any choice of $\vec{F}$ is appropriate as long as curl $\vec{F} = 1$. The choices of $\vec{F} = \langle -y, 0\rangle$, $\langle 0, x\rangle$ and $\langle -y/2, x/2\rangle$ each lead to reasonable integrands. The area of R is $16/15$.
20. Any choice of $\vec{F}$ is appropriate as long as curl $\vec{F} = 1$. The choice of $\vec{F} = \langle -y/2, x/2\rangle$ leads to a reasonable integrand after simplification. The area of R is $41\pi/10$.
21. The line integral $\oint_C \vec{F}\cdot\vec{n}\,ds$, over the parabola, is $-22/3$; over the line, it is 10. The total line integral is thus $-22/3 + 10 = 8/3$. The double integral of div $\vec{F} = 2$ over R also has value $8/3$.
22. Both the line integral and double integral have value of 0.
23. Three line integrals need to be computed to compute $\oint_C \vec{F}\cdot\vec{n}\,ds$. It does not matter which corner one starts from first, but be sure to proceed around the triangle in a counterclockwise fashion.
   From $(0,0)$ to $(2,0)$, the line integral has a value of 0. From $(2,0)$ to $(1,1)$ the integral has a value of $1/3$. From $(1,1)$ to $(0,0)$ the line integral has a value of $1/3$. Total value is $2/3$.
   The double integral of div $\vec{F}$ over R also has value $2/3$.
24. Two line integrals need to be computed to compute $\oint_C \vec{F}\cdot\vec{n}\,ds$. Along the parabola, the line integral has value $159/20$. Along the line, the line integral has value 6. Together, the total value is $279/20$.
   The double integral of div $\vec{F}$ over R also has value $279/20$.

Section 14.5

1. Answers will vary, though generally should meaningfully include terms like "two sided".
2. Many possible answers exist; the one given by the book is the Möbius band.
3.
   (a) $\vec{r}(u,v) = \langle u, v, 3u^2v\rangle$ on $-1 \le u \le 1$, $0 \le v \le 2$.
   (b) $\vec{r}(u,v) = \langle 3v\cos u + 1, 3v\sin u + 2, 3(3v\cos u + 1)^2(3v\sin u + 2)\rangle$, on $0 \le u \le 2\pi$, $0 \le v \le 1$.
   (c) $\vec{r}(u,v) = \langle u, v(2-2u), 3u^2v(2-2u)\rangle$ on $0 \le u, v \le 1$.
   (d) $\vec{r}(u,v) = \langle u, v(1-u^2), 3u^2v(1-u^2)\rangle$ on $-1 \le u \le 1$, $0 \le v \le 1$.
4.
   (a) $\vec{r}(u,v) = \langle u, v, 4u + 2u^2\rangle$ on $1 \le u \le 4$, $5 \le v \le 7$.
   (b) $\vec{r}(u,v) = \langle 4v\cos u, 3v\sin u, 16v\cos u + 2(3v\sin u)^2\rangle$, on $0 \le u \le 2\pi$, $0 \le v \le 1$.

(c) $\vec{r}(u,v) = \langle u, u+v(4-2u), 4u+2\big(u+v(4-2u)\big)^2\rangle$ on $0\leq u\leq 2, 0\leq v\leq 1$.

(d) $\vec{r}(u,v) = \langle v\cos u, v\sin u, 4v\cos u + 2(v\sin u)^2\rangle$ on $0\leq u\leq 2\pi, 2\leq v\leq 5$.

5. $\vec{r}(u,v) = \langle 0,u,v\rangle$ with $0\leq u\leq 2, 0\leq v\leq 1$.
6. $\vec{r}(u,v) = \langle u, 0, 1-u+vu\rangle$ with $0\leq u\leq 1, 0\leq v\leq 1$.
7. $\vec{r}(u,v) = \langle 3\sin u\cos v, 2\sin u\sin v, 4\cos u\rangle$ with $0\leq u\leq\pi$, $0\leq v\leq 2\pi$.
8. Answers may vary; one solution is $\vec{r}(u,v) = \langle v\cos u, v, 4v\sin u\rangle$ with $0\leq u\leq 2\pi$, $-1\leq v\leq 5$.
9. Answers may vary.
For $z=\frac{1}{2}(3-x)$: $\vec{r}(u,v) = \langle u, v, \frac{1}{2}(3-u)\rangle$, with $1\leq u\leq 3$ and $0\leq v\leq 2$.
For $x=1$: $\vec{r}(u,v) = \langle 0,u,v\rangle$, with $0\leq u\leq 2, 0\leq v\leq 1$
For $y=0$: $\vec{r}(u,v) = \langle u,0,v/2(3-u)\rangle$, with $1\leq u\leq 3$, $0\leq v\leq 1$
For $y=2$: $\vec{r}(u,v) = \langle u,2,v/2(3-u)\rangle$, with $1\leq u\leq 3$, $0\leq v\leq 1$
For $z=0$: $\vec{r}(u,v) = \langle u,v,0\rangle$, with $1\leq u\leq 3, 0\leq v\leq 2$
10. Answers may vary.
For $z=2x+4y-4$:
$\vec{r}(u,v) = \langle u, 1-u/2+uv/2, 2u+4(1-u/2+uv/2)-4\rangle$, with $0\leq u\leq 2, 0\leq v\leq 1$.
For $x=2$: $\vec{r}(u,v) = \langle 2,u,4uv\rangle$, with $0\leq u\leq 1, 0\leq v\leq 1$
For $y=1$: $\vec{r}(u,v) = \langle u,1,2uv\rangle$, with $0\leq u\leq 2, 0\leq v\leq 1$
For $z=0$: $\vec{r}(u,v) = \langle u, 1-u/2+uv/2, 0\rangle$, with $0\leq u\leq 2$, $0\leq v\leq 1$
11. Answers may vary.
For $z=2y$: $\vec{r}(u,v) = \langle u, v(4-u^2), 2v(4-u^2)\rangle$ with $-2\leq u\leq 2$ and $0\leq v\leq 1$.
For $y=4-x^2$: $\vec{r}(u,v) = \langle u, 4-u^2, 2v(4-u^2)\rangle$ with $-2\leq u\leq 2$ and $0\leq v\leq 1$.
For $z=0$: $\vec{r}(u,v) = \langle u, v(4-u^2), 0\rangle$ with $-2\leq u\leq 2$ and $0\leq v\leq 1$.
12. Answers may vary.
For $y=1-z^2$: $\vec{r}(u,v) = \langle u, v(1-u^2), \sqrt{1-v(1-u^2)}\rangle$ with $0\leq u\leq 1$ and $0\leq v\leq 1$.
For $y=1-x^2$: $\vec{r}(u,v) = \langle u, 1-u^2, uv\rangle$ with $0\leq u\leq 1$ and $0\leq v\leq 1$.
For $x=0$: $\vec{r}(u,v) = \langle 0, v(1-u^2), u\rangle$ with $0\leq u\leq 1$ and $0\leq v\leq 1$.
For $y=0$: $\vec{r}(u,v) = \langle u,0,v\rangle$ with $0\leq u\leq 1$ and $0\leq v\leq 1$.
For $z=0$: $\vec{r}(u,v) = \langle u, v(1-u^2), 0 rangle$ with $0\leq u\leq 1$ and $0\leq v\leq 1$.
13. Answers may vary.
For $x+y^2/9=1$: $\vec{r}(u,v) = \langle\cos u, 3\sin u, v\rangle$ with $0\leq u\leq 2\pi$ and $1\leq v\leq 3$.
For $z=1$: $\vec{r}(u,v) = \langle v\cos u, 3v\sin u, 1\rangle$ with $0\leq u\leq 2\pi$ and $0\leq v\leq 1$.
For $z=3$: $\vec{r}(u,v) = \langle v\cos u, 3v\sin u, 3\rangle$ with $0\leq u\leq 2\pi$ and $0\leq v\leq 1$.
14. Answers may vary.
For $x^2+y^2=(z-1)^2$: $\vec{r}(u,v) = \langle v\cos u, v\sin u, 1-v\rangle$ with $0\leq u\leq 2\pi$ and $0\leq v\leq 1$.
For $z=0$: $\vec{r}(u,v) = \langle v\cos u, v\sin u, 0\rangle$ with $0\leq u\leq 2\pi$ and $0\leq v\leq 1$.
15. Answers may vary.
For $z=1-x^2$: $\vec{r}(u,v) = \langle u,v,1-u^2\rangle$ with $-1\leq u\leq 1$ and $-1\leq v\leq 2$.
For $y=-1$: $\vec{r}(u,v) = \langle u,-1,v(1-u^2)\rangle$ with $-1\leq u\leq 1$ and $0\leq v\leq 1$.
For $y=2$: $\vec{r}(u,v) = \langle u,2,v(1-u^2)\rangle$ with $-1\leq u\leq 1$ and $0\leq v\leq 1$.
For $z=0$: $\vec{r}(u,v) = \langle u,v,0\rangle$ with $-1\leq u\leq 1$ and $-1\leq v\leq 2$.
16. Answers may vary.
For $z=4-x^2-4y^2$:
$\vec{r}(u,v) = \langle 2v\cos u, v\sin u, 4-(2v\cos u)^2-4(v\sin u)^2\rangle$ with $0\leq u\leq 2\pi$ and $0\leq v\leq 1$.
For $z=0$: $\vec{r}(u,v) = \langle 2v\cos u, v\sin u, 0\rangle$ with $0\leq u\leq 2\pi$ and $0\leq v\leq 1$.
17. $S=2\sqrt{14}$.
18. $S=\sqrt{6}/2$.
19. $S=4\sqrt{3}\pi$.
20. $S=3\sqrt{3}\pi$.
21. $S=\int_0^3\int_0^{2\pi}\sqrt{v^2+4v^4}\,du\,dv = (37\sqrt{37}-1)\pi/6 \approx 117.319$.
22. $S=\int_0^1\int_0^1\sqrt{v^2+4u^2v^2+4v^4}\,du\,dv\approx 0.931$.
23. $S=\int_0^1\int_{-1}^1\sqrt{(5u^2-2uv-5)^2+u^4+(1-u^2)^2}\,du\,dv\approx 7.084$.
24. $S=\int_0^1\int_0^{2\pi}\sqrt{v^2+4v^4}\,du\,dv = (5\sqrt{5}-1)\pi/6\approx 5.330$.

Section 14.6

1. curve; surface
2. Answers will vary; in general, it means that more of the vector field passes through the surface opposite the direction of the normal vector than in the same direction of the normal vector.
3. outside
4. 0.
5. $240\sqrt{3}$
6. 40π
7. 24
8. 15
9. 0
10. 0
11. $-1/2$
12. π
13. 0; the flux over $\mathcal{S}_1$ is -45π and the flux over $\mathcal{S}_2$ is 45π.
14. $9\pi/8$; the flux over $\mathcal{S}_1$ is $3\pi/4$ (use $\vec{r}(u,v) = \langle\sin u\cos v, \sin u\sin v, \cos u\rangle$ on $\pi/3\leq u\leq\pi$, $0\leq v\leq 2\pi$) and the flux over $\mathcal{S}_2$ is $3\pi/8$ (use $\vec{r}(u,v) = \langle v\sqrt{3}\cos(u)/2, v\sqrt{3}\sin(u)/2, 1/2\rangle$ for $0\leq u\leq 2\pi$, $0\leq v\leq 1$.

Section 14.7

1. Answers will vary; in Section 14.4, the Divergence Theorem connects outward flux over a closed curve in the plane to the divergence of the vector field, whereas in this section the Divergence Theorem connects outward flux over a closed surface in space to the divergence of the vector field.
2. Divergence.
3. Curl.
4. Green's Theorem.
5. Outward flux across the plane $z=2-x/2-2y/3$ is 14; across the plane $z=0$ the outward flux is -8; across the planes $x=0$ and $y=0$ the outward flux is 0.
Total outward flux: 14.
$\iint_D \text{div}\,\vec{F}\,dV = \int_0^4\int_0^{3-3x/4}\int_0^{2-x/2-2y/3}(2x+2y)\,dz\,dy\,dx = 14$.

6. Outward flux across the cylinder $x^2 + y^2 = 1$ is 0; across the plane $z = 3$ the outward flux is 3π; across the plane $z = -3$ the outward flux is 3π.

   Total outward flux: 6π.

   $\iint_D \text{div}\,\vec{F}\,dV = \int_0^{2\pi}\int_0^1\int_{-3}^3 r\,dz\,dr\,d\theta = 6\pi$.

7. Outward flux across the surface $z = xy(3-x)(3-y)$ is 252; across the plane $z = 0$ the outward flux is -9.

   Total outward flux: 243.

   $\iint_D \text{div}\,\vec{F}\,dV = \int_0^3\int_0^3\int_0^{xy(3-x)(3-y)} 12\,dz\,dy\,dx = 243$.

8. Outward flux across the paraboloid is $112\pi/3$; across the disk the outward flux is 0.

   Total outward flux: $112\pi/3$.

   $\iint_D \text{div}\,\vec{F}\,dV = \int_0^{2\pi}\int_0^2\int_0^{4-r^2}(2z+2)r\,dz\,dr\,d\theta = 112\pi/3$.

9. Circulation on C: $\oint_C \vec{F}\cdot d\vec{r} = \pi$

   $\iint_{\mathcal{S}} \left(\text{curl}\,\vec{F}\right)\cdot\vec{n}\,dS = \pi$.

10. Circulation on C: $\oint_C \vec{F}\cdot d\vec{r} = \pi$

    $\iint_{\mathcal{S}} \left(\text{curl}\,\vec{F}\right)\cdot\vec{n}\,dS = \pi$.

11. Circulation on C: The flow along the line from $(0,0,2)$ to $(4,0,0)$ is 0; from $(4,0,0)$ to $(0,3,0)$ it is -6, and from $(0,3,0)$ to $(0,0,2)$ it is 6. The total circulation is $0 + (-6) + 6 = 0$.

    $\iint_{\mathcal{S}} \left(\text{curl}\,\vec{F}\right)\cdot\vec{n}\,dS = \iint_{\mathcal{S}} 0\,dS = 0$.

12. Circulation on C: The flow along the parabola is $-32/15$; the flow along the line is $4/3$. The total circulation is $4/3 - 32/15 = -4/5$.

    $\iint_{\mathcal{S}} \left(\text{curl}\,\vec{F}\right)\cdot\vec{n}\,dS = -4/5$.

13. $128/225$
14. 8
15. $8192/105 \approx 78.019$
16. $64/3$
17. $5/3$
18. 8π
19. 23π
20. 0
21. Each field has a divergence of 1; by the Divergence Theorem, the total outward flux across $\mathcal{S}$ is $\iint_D 1\,dS$ for each field.
22. (a) $\text{curl}\,\vec{F} = 1$.

    (b) $\text{curl}\,\vec{F}\cdot\vec{n} = 1$, where $\vec{n}$ is a unit vector normal to $\mathcal{S}$.
23. Answers will vary. Often the closed surface $\mathcal{S}$ is composed of several smooth surfaces. To measure total outward flux, this may require evaluating multiple double integrals. Each double integral requires the parametrization of a surface and the computation of the cross product of partial derivatives. One triple integral may require less work, especially as the divergence of a vector field is generally easy to compute.
24. Answers will vary. Often the closed curve C is composed of several smooth curves. To measure the total circulation, one may have to evaluate line integrals along each curve. Each line integral requires the parameterization of its curve. It may be less work to evaluate one single double (i.e., surface) integral.