

## Differentiation Rules

$$1. \frac{d}{dx}(cx) = c$$

$$2. \frac{d}{dx}(u \pm v) = u' \pm v'$$

$$3. \frac{d}{dx}(u \cdot v) = uv' + u'v$$

$$4. \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$5. \frac{d}{dx}(u(v)) = u'(v)v'$$

$$6. \frac{d}{dx}(c) = 0$$

$$7. \frac{d}{dx}(x) = 1$$

$$8. \frac{d}{dx}(x^n) = nx^{n-1}$$

$$9. \frac{d}{dx}(e^x) = e^x$$

$$10. \frac{d}{dx}(a^x) = \ln a \cdot a^x$$

$$11. \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$12. \frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$13. \frac{d}{dx}(\sin x) = \cos x$$

$$14. \frac{d}{dx}(\cos x) = -\sin x$$

$$15. \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$16. \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$17. \frac{d}{dx}(\tan x) = \sec^2 x$$

$$18. \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$19. \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$20. \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$21. \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$22. \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$23. \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$24. \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

## Integration Rules

$$1. \int c \cdot f(x) dx = c \int f(x) dx$$

$$2. \int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$3. \int 0 dx = C$$

$$4. \int 1 dx = x + C$$

$$5. \int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$$

$$6. \int e^x dx = e^x + C$$

$$7. \int a^x dx = \frac{1}{\ln a} \cdot a^x + C$$

$$8. \int \frac{1}{x} dx = \ln |x| + C$$

$$9. \int \cos x dx = \sin x + C$$

$$10. \int \sin x dx = -\cos x + C$$

$$11. \int \tan x dx = -\ln |\cos x| + C$$

$$12. \int \sec x dx = \ln |\sec x + \tan x| + C$$

$$13. \int \csc x dx = -\ln |\csc x + \cot x| + C$$

$$14. \int \cot x dx = \ln |\sin x| + C$$

$$15. \int \sec^2 x dx = \tan x + C$$

$$16. \int \csc^2 x dx = -\cot x + C$$

$$17. \int \sec x \tan x dx = \sec x + C$$

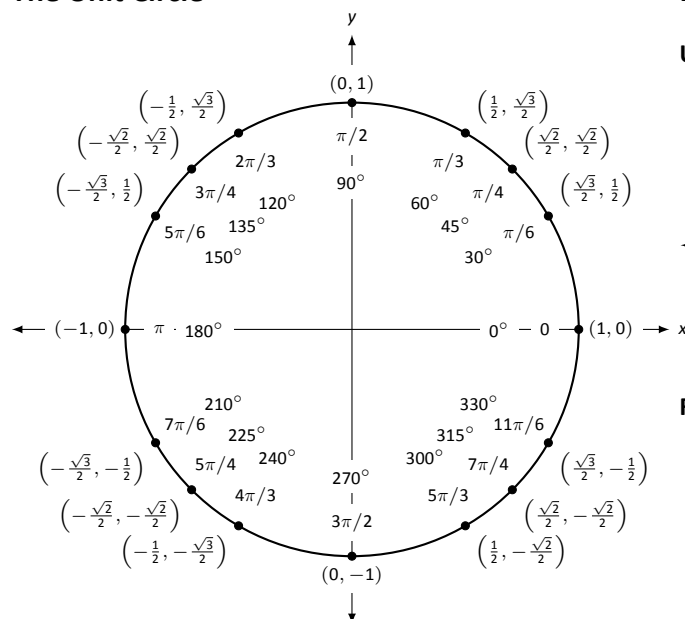
$$18. \int \csc x \cot x dx = -\csc x + C$$

$$19. \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$20. \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

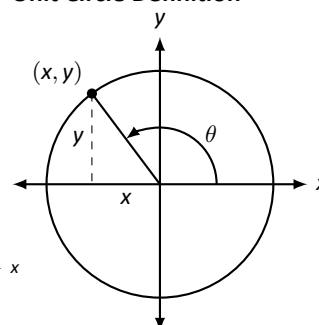
$$21. \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{|x|}{a}\right) + C$$

## The Unit Circle



## Definitions of the Trigonometric Functions

### Unit Circle Definition

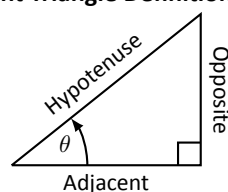


$$\sin \theta = y \quad \cos \theta = x$$

$$\csc \theta = \frac{1}{y} \quad \sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

### Right Triangle Definition



$$\sin \theta = \frac{O}{H} \quad \csc \theta = \frac{H}{O}$$

$$\cos \theta = \frac{A}{H} \quad \sec \theta = \frac{H}{A}$$

$$\tan \theta = \frac{O}{A} \quad \cot \theta = \frac{A}{O}$$

## Common Trigonometric Identities

### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

### Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

### Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

### Sum to Product Formulas

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

### Power-Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

### Even/Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

### Product to Sum Formulas

$$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x-y) + \cos(x+y))$$

$$\sin x \cos y = \frac{1}{2} (\sin(x+y) + \sin(x-y))$$

### Angle Sum/Difference Formulas

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

## Areas and Volumes

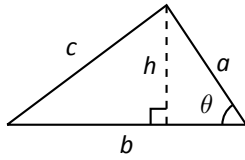
### Triangles

$$h = a \sin \theta$$

$$\text{Area} = \frac{1}{2}bh$$

Law of Cosines:

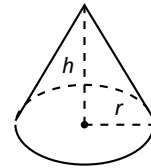
$$c^2 = a^2 + b^2 - 2ab \cos \theta$$



### Right Circular Cone

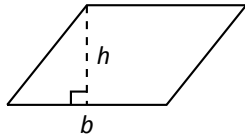
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Surface Area} = \pi r \sqrt{r^2 + h^2} + \pi r^2$$



### Parallelograms

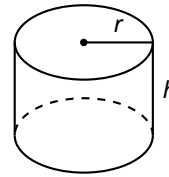
$$\text{Area} = bh$$



### Right Circular Cylinder

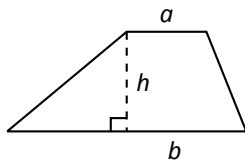
$$\text{Volume} = \pi r^2 h$$

$$\text{Surface Area} = 2\pi rh + 2\pi r^2$$



### Trapezoids

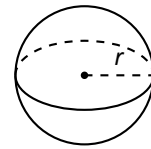
$$\text{Area} = \frac{1}{2}(a + b)h$$



### Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

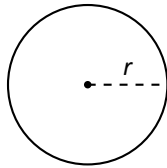
$$\text{Surface Area} = 4\pi r^2$$



### Circles

$$\text{Area} = \pi r^2$$

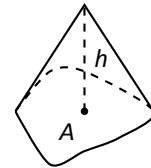
$$\text{Circumference} = 2\pi r$$



### General Cone

$$\text{Area of Base} = A$$

$$\text{Volume} = \frac{1}{3}Ah$$

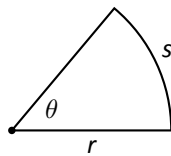


### Sectors of Circles

$\theta$  in radians

$$\text{Area} = \frac{1}{2}\theta r^2$$

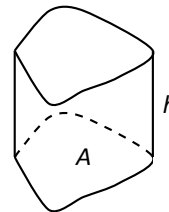
$$s = r\theta$$



### General Right Cylinder

$$\text{Area of Base} = A$$

$$\text{Volume} = Ah$$



# Algebra

## Factors and Zeros of Polynomials

Let  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$  be a polynomial. If  $p(a) = 0$ , then  $a$  is a *zero* of the polynomial and a solution of the equation  $p(x) = 0$ . Furthermore,  $(x - a)$  is a *factor* of the polynomial.

## Fundamental Theorem of Algebra

An  $n$ th degree polynomial has  $n$  (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

## Quadratic Formula

If  $p(x) = ax^2 + bx + c$ , and  $0 \leq b^2 - 4ac$ , then the real zeros of  $p$  are  $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

## Special Factors

$$\begin{aligned}x^2 - a^2 &= (x - a)(x + a) & x^3 - a^3 &= (x - a)(x^2 + ax + a^2) \\x^3 + a^3 &= (x + a)(x^2 - ax + a^2) & x^4 - a^4 &= (x^2 - a^2)(x^2 + a^2) \\(x + y)^n &= x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n \\(x - y)^n &= x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n\end{aligned}$$

## Binomial Theorem

$$\begin{aligned}(x + y)^2 &= x^2 + 2xy + y^2 & (x - y)^2 &= x^2 - 2xy + y^2 \\(x + y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 & (x - y)^3 &= x^3 - 3x^2y + 3xy^2 - y^3 \\(x + y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 & (x - y)^4 &= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4\end{aligned}$$

## Rational Zero Theorem

If  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$  has integer coefficients, then every *rational zero* of  $p$  is of the form  $x = r/s$ , where  $r$  is a factor of  $a_0$  and  $s$  is a factor of  $a_n$ .

## Factoring by Grouping

$$acx^3 + adx^2 + bcx + bd = ax^2(cs + d) + b(cx + d) = (ax^2 + b)(cx + d)$$

## Arithmetic Operations

$$\begin{aligned}ab + ac &= a(b + c) & \frac{a}{b} + \frac{c}{d} &= \frac{ad + bc}{bd} & \frac{a + b}{c} &= \frac{a}{c} + \frac{b}{c} \\ \left(\frac{a}{b}\right) \left(\frac{c}{d}\right) &= \left(\frac{a}{b}\right) \left(\frac{d}{c}\right) = \frac{ad}{bc} & \left(\frac{a}{b}\right) \frac{c}{c} &= \frac{a}{bc} & \frac{a}{\left(\frac{b}{c}\right)} &= \frac{ac}{b} \\ a \left(\frac{b}{c}\right) &= \frac{ab}{c} & \frac{a - b}{c - d} &= \frac{b - a}{d - c} & \frac{ab + ac}{a} &= b + c\end{aligned}$$

## Exponents and Radicals

$$\begin{aligned}a^0 &= 1, \quad a \neq 0 & (ab)^x &= a^x b^x & a^x a^y &= a^{x+y} & \sqrt{a} &= a^{1/2} & \frac{a^x}{a^y} &= a^{x-y} & \sqrt[n]{a} &= a^{1/n} \\ \left(\frac{a}{b}\right)^x &= \frac{a^x}{b^x} & \sqrt[n]{a^m} &= a^{m/n} & a^{-x} &= \frac{1}{a^x} & \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} & (a^x)^y &= a^{xy} & \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}}\end{aligned}$$

## Additional Formulas

### Summation Formulas:

$$\begin{aligned}\sum_{i=1}^n c &= cn & \sum_{i=1}^n i &= \frac{n(n+1)}{2} \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} & \sum_{i=1}^n i^3 &= \left(\frac{n(n+1)}{2}\right)^2\end{aligned}$$

### Trapezoidal Rule (Area):

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_1) + 2f(x_2) + 2f(x_3) + \dots + 2f(x_n) + f(x_{n+1})]$$

$$\text{with Error} \leq \frac{(b-a)^3}{12n^2} [\max |f''(x)|]$$

### Simpson's Rule (Area):

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \dots + 2f(x_{n-1}) + 4f(x_n) + f(x_{n+1})]$$

$$\text{with Error} \leq \frac{(b-a)^5}{180n^4} [\max |f^{(4)}(x)|]$$

### Arc Length:

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

### Surface of Revolution:

$$S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$$

(where  $f(x) \geq 0$ )

$$S = 2\pi \int_a^b x \sqrt{1 + f'(x)^2} dx$$

(where  $a, b \geq 0$ )

### Work Done by a Variable Force:

$$W = \int_a^b F(x) dx$$

### Force Exerted by a Fluid:

$$F = \int_a^b w d(y) \ell(y) dy$$

### Taylor Series Expansion for $f(x)$ :

$$p_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

### Maclaurin Series Expansion for $f(x)$ , where $c = 0$ :

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n$$

## Summary of Tests for Series:

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
$n$ th-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} r^n$	$ r  < 1$	$ r  \geq 1$	Sum = $\frac{1}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n \rightarrow \infty} b_n = L$		Sum = $b_1 - L$
$p$ -Series	$\sum_{n=1}^{\infty} \frac{1}{(an+b)^p}$	$p > 1$	$p \leq 1$	