# Exercises 1.1

## *Terms and Concepts*

01 01 ex 19 1. In your own words, what does it mean to "find the limit of $f(x)$ as $x$ approaches 3"?

01 01 ex 20 2. An expression of the form $\frac{0}{0}$ is called ____.

01 01 ex 21 3. T/F: The limit of $f(x)$ as $x$ approaches 5 is $f(5)$.

01 01 ex 22 4. Describe three situations where $\lim\limits_{x\to c} f(x)$ does not exist.

01 01 ex 23 5. In your own words, what is a difference quotient?

## *Problems*

01 01 exset 01 **In Exercises 6 – 16, approximate the given limits both numerically and graphically.**

01 01 ex 01 6. $\lim\limits_{x\to 1} x^2 + 3x - 5$

01 01 ex 02 7. $\lim\limits_{x\to 0} x^3 - 3x^2 + x - 5$

01 01 ex 03 8. $\lim\limits_{x\to 0} \dfrac{x+1}{x^2+3x}$

01 01 ex 04 9. $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x^2-4x+3}$

01 01 ex 05 10. $\lim\limits_{x\to -1} \dfrac{x^2+8x+7}{x^2+6x+5}$

01 01 ex 06 11. $\lim\limits_{x\to 2} \dfrac{x^2+7x+10}{x^2-4x+4}$

01 01 ex 07 12. $\lim\limits_{x\to 2} f(x)$, where

$$f(x) = \begin{cases} x+2 & x \le 2 \\ 3x-5 & x > 2 \end{cases}.$$

01 01 ex 08 13. $\lim\limits_{x\to 3} f(x)$, where

$$f(x) = \begin{cases} x^2-x+1 & x \le 3 \\ 2x+1 & x > 3 \end{cases}.$$

01 01 ex 09 14. $\lim\limits_{x\to 0} f(x)$, where

$$f(x) = \begin{cases} \cos x & x \le 0 \\ x^2+3x+1 & x > 0 \end{cases}.$$

01 01 ex 10 15. $\lim\limits_{x\to \pi/2} f(x)$, where

$$f(x) = \begin{cases} \sin x & x \le \pi/2 \\ \cos x & x > \pi/2 \end{cases}.$$

01 01 exset 02 **In Exercises 16 – 24, a function $f$ and a value $a$ are given. Approximate the limit of the difference quotient, $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$, using $h = \pm 0.1, \pm 0.01$.**

01 01 ex 11 16. $f(x) = -7x + 2, \quad a = 3$

01 01 ex 12 17. $f(x) = 9x + 0.06, \quad a = -1$

01 01 ex 13 18. $f(x) = x^2 + 3x - 7, \quad a = 1$

01 01 ex 14 19. $f(x) = \dfrac{1}{x+1}, \quad a = 2$

01 01 ex 15 20. $f(x) = -4x^2 + 5x - 1, \quad a = -3$

01 01 ex 16 21. $f(x) = \ln x, \quad a = 5$

01 01 ex 17 22. $f(x) = \sin x, \quad a = \pi$

01 01 ex 18 23. $f(x) = \cos x, \quad a = \pi$

# Exercises 1.2

## *Terms and Concepts*

01 02 ex 01 1. What is wrong with the following "definition" of a limit?

> "The limit of $f(x)$, as $x$ approaches $a$, is $K$" means that given any $\delta > 0$ there exists $\varepsilon > 0$ such that whenever $|f(x) - K| < \varepsilon$, we have $|x - a| < \delta$.

01 02 ex 02 2. Which is given first in establishing a limit, the $x$–tolerance or the $y$–tolerance?

01 02 ex 09 3. T/F: $\varepsilon$ must always be positive.

01 02 ex 10 4. T/F: $\delta$ must always be positive.

## *Problems*

01 02 exset 02 **In Exercises 5 – 11, prove the given limit using an $\varepsilon - \delta$ proof.**

01 02 ex 04 5. $\lim\limits_{x\to 5} 3 - x = -2$

01 02 ex 05 6. $\lim\limits_{x\to 3} x^2 - 3 = 6$

01 02 ex 11 7. $\lim\limits_{x\to 4} x^2 + x - 5 = 15$

01 02 ex 06 8. $\lim\limits_{x\to 2} x^3 - 1 = 7$

01 02 ex 03 9. $\lim\limits_{x\to 2} 5 = 5$

01 02 ex 07 10. $\lim\limits_{x\to 0} e^{2x} - 1 = 0$

01 02 ex 08 11. $\lim\limits_{x\to 0} \sin x = 0$ (Hint: use the fact that $|\sin x| \leq |x|$, with equality only when $x = 0$.)

# Exercises 1.3

## *Terms and Concepts*

01 03 ex 01 1. Explain in your own words, without using $\varepsilon$-$\delta$ formality, why $\lim_{x\to c} b = b$.

01 03 ex 02 2. Explain in your own words, without using $\varepsilon$-$\delta$ formality, why $\lim_{x\to c} x = c$.

01 03 ex 03 3. What does the text mean when it says that certain functions' "behavior is 'nice' in terms of limits"? What, in particular, is "nice"?

01 03 ex 04 4. Sketch a graph that visually demonstrates the Squeeze Theorem.

01 03 ex 05 5. You are given the following information:

   (a) $\lim_{x\to 1} f(x) = 0$

   (b) $\lim_{x\to 1} g(x) = 0$

   (c) $\lim_{x\to 1} f(x)/g(x) = 2$

   What can be said about the relative sizes of $f(x)$ and $g(x)$ as $x$ approaches 1?

## *Problems*

01 03 exset 02 **Using:**

$$\lim_{x\to 9} f(x) = 6 \qquad \lim_{x\to 6} f(x) = 9$$
$$\lim_{x\to 9} g(x) = 3 \qquad \lim_{x\to 6} g(x) = 3$$

**evaluate the limits given in Exercises 6 – 13, where possible. If it is not possible to know, state so.**

01 03 ex 06 6. $\lim_{x\to 9}\big(f(x)+g(x)\big)$

01 03 ex 07 7. $\lim_{x\to 9}\big(3f(x)/g(x)\big)$

01 03 ex 08 8. $\lim_{x\to 9}\left(\dfrac{f(x)-2g(x)}{g(x)}\right)$

01 03 ex 09 9. $\lim_{x\to 6}\left(\dfrac{f(x)}{3-g(x)}\right)$

01 03 ex 10 10. $\lim_{x\to 9} g\big(f(x)\big)$

01 03 ex 11 11. $\lim_{x\to 6} f\big(g(x)\big)$

01 03 ex 12 12. $\lim_{x\to 6} g\big(f(f(x))\big)$

01 03 ex 13 13. $\lim_{x\to 6} f(x)g(x) - f^2(x) + g^2(x)$

01 03 exset 03 **Using:**

$$\lim_{x\to 1} f(x) = 2 \qquad \lim_{x\to 10} f(x) = 1$$
$$\lim_{x\to 1} g(x) = 0 \qquad \lim_{x\to 10} g(x) = \pi$$

**evaluate the limits given in Exercises 14 – 17, where possible. If it is not possible to know, state so.**

01 03 ex 14 14. $\lim_{x\to 1} f(x)^{g(x)}$

01 03 ex 15 15. $\lim_{x\to 10} \cos\big(g(x)\big)$

01 03 ex 16 16. $\lim_{x\to 1} f(x)g(x)$

01 03 ex 17 17. $\lim_{x\to 1} g\big(5f(x)\big)$

01 03 exset 04 **In Exercises 18 – 32, evaluate the given limit.**

01 03 ex 18 18. $\lim_{x\to 3} x^2 - 3x + 7$

01 03 ex 21 19. $\lim_{x\to \pi}\left(\dfrac{x-3}{x-5}\right)^7$

01 03 ex 22 20. $\lim_{x\to \pi/4} \cos x \sin x$

01 03 ex 23 21. $\lim_{x\to 0} \ln x$

01 03 ex 24 22. $\lim_{x\to 3} 4^{x^3-8x}$

01 03 ex 25 23. $\lim_{x\to \pi/6} \csc x$

01 03 ex 26 24. $\lim_{x\to 0} \ln(1+x)$

01 03 ex 20 25. $\lim_{x\to \pi} \dfrac{x^2+3x+5}{5x^2-2x-3}$

01 03 ex 19 26. $\lim_{x\to \pi} \dfrac{3x+1}{1-x}$

01 03 ex 27 27. $\lim_{x\to 6} \dfrac{x^2-4x-12}{x^2-13x+42}$

01 03 ex 28 28. $\lim_{x\to 0} \dfrac{x^2+2x}{x^2-2x}$

01 03 ex 29 29. $\lim_{x\to 2} \dfrac{x^2+6x-16}{x^2-3x+2}$

01 03 ex 30 30. $\lim_{x\to 2} \dfrac{x^2-10x+16}{x^2-x-2}$

01 03 ex 31 31. $\lim_{x\to -2} \dfrac{x^2-5x-14}{x^2+10x+16}$

01 03 ex 32 32. $\lim_{x\to -1} \dfrac{x^2+9x+8}{x^2-6x-7}$

01 03 exset 06

**Use the Squeeze Theorem in Exercises 33 – 36, where appropriate, to evaluate the given limit.**

01 03 ex 38

33. $\lim\limits_{x\to 0} x \sin\left(\dfrac{1}{x}\right)$

01 03 ex 40

34. $\lim\limits_{x\to 0} \sin x \cos\left(\dfrac{1}{x^2}\right)$

01 03 ex 42

35. $\lim\limits_{x\to 1} f(x)$, where $3x - 2 \leq f(x) \leq x^3$.

01 03 ex 41

36. $\lim\limits_{x\to 3^+} f(x)$, where $6x - 9 \leq f(x) \leq x^2$ on $[0, 3]$.

01 03 exset 05

**Exercises 37 – 40 challenge your understanding of limits but can be evaluated using the knowledge gained in this section.**

01 03 ex 33

37. $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x}$

01 03 ex 34

38. $\lim\limits_{x\to 0} \dfrac{\sin 5x}{8x}$

01 03 ex 35

39. $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x}$

01 03 ex 36

40. $\lim\limits_{x\to 0} \dfrac{\sin x}{x}$, where $x$ is measured in degrees, not radians.

# Exercises 1.4

## *Terms and Concepts*

01 04 ex 01 1. What are the three ways in which a limit may fail to exist?

01 04 ex 02 2. T/F: If $\lim\limits_{x\to1^-} f(x) = 5$, then $\lim\limits_{x\to1} f(x) = 5$

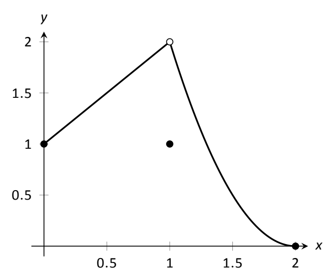
01 04 ex 03 3. T/F: If $\lim\limits_{x\to1^-} f(x) = 5$, then $\lim\limits_{x\to1^+} f(x) = 5$

01 04 ex 04 4. T/F: If $\lim\limits_{x\to1} f(x) = 5$, then $\lim\limits_{x\to1^-} f(x) = 5$

## *Problems*

01 04 exset 02 **In Exercises 5 – 12, evaluate each expression using the given graph of $f(x)$.**

01 04 ex 05 5.




(a) $\lim\limits_{x\to1^-} f(x)$

(b) $\lim\limits_{x\to1^+} f(x)$

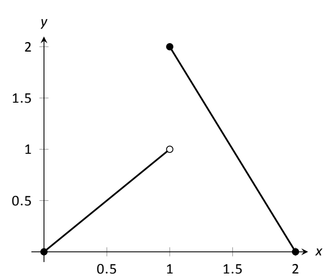
(c) $\lim\limits_{x\to1} f(x)$

(d) $f(1)$

(e) $\lim\limits_{x\to0^-} f(x)$

(f) $\lim\limits_{x\to0^+} f(x)$

01 04 ex 06 6.




(a) $\lim\limits_{x\to1^-} f(x)$

(b) $\lim\limits_{x\to1^+} f(x)$

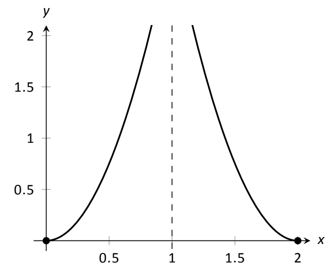
(c) $\lim\limits_{x\to1} f(x)$

(d) $f(1)$

(e) $\lim\limits_{x\to2^-} f(x)$

(f) $\lim\limits_{x\to2^+} f(x)$

01 04 ex 07 7.




(a) $\lim\limits_{x\to1^-} f(x)$

(b) $\lim\limits_{x\to1^+} f(x)$

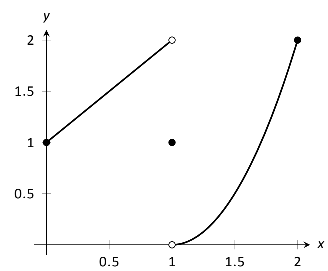
(c) $\lim\limits_{x\to1} f(x)$

(d) $f(1)$

(e) $\lim\limits_{x\to2^-} f(x)$

(f) $\lim\limits_{x\to0^+} f(x)$

01 04 ex 08 8.



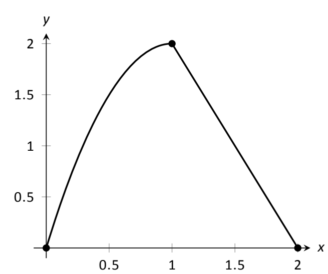

(a) $\lim\limits_{x\to1^-} f(x)$

(b) $\lim\limits_{x\to1^+} f(x)$

(c) $\lim\limits_{x\to1} f(x)$

(d) $f(1)$

01 04 ex 09 9.



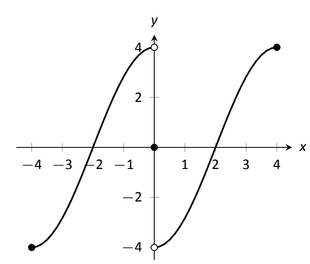

(a) $\lim\limits_{x\to1^-} f(x)$

(b) $\lim\limits_{x\to1^+} f(x)$

(c) $\lim\limits_{x\to1} f(x)$

(d) $f(1)$

01 04 ex 10 10.



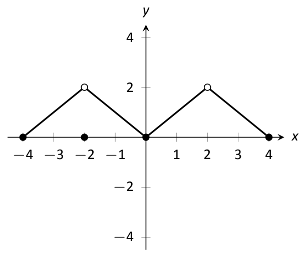

(a) $\lim\limits_{x\to0^-} f(x)$

(b) $\lim\limits_{x\to0^+} f(x)$

(c) $\lim\limits_{x\to0} f(x)$

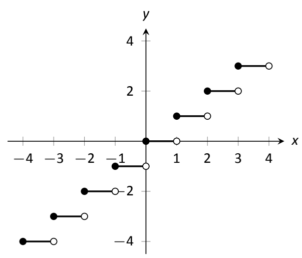
(d) $f(0)$

01 04 ex 11 11.




(a) $\lim\limits_{x\to -2^-} f(x)$ (e) $\lim\limits_{x\to 2^-} f(x)$
(b) $\lim\limits_{x\to -2^+} f(x)$ (f) $\lim\limits_{x\to 2^+} f(x)$
(c) $\lim\limits_{x\to -2} f(x)$ (g) $\lim\limits_{x\to 2} f(x)$
(d) $f(-2)$ (h) $f(2)$

01 04 ex 12 12.




Let $-3 \le a \le 3$ be an integer.

(a) $\lim\limits_{x\to a^-} f(x)$ (c) $\lim\limits_{x\to a} f(x)$
(b) $\lim\limits_{x\to a^+} f(x)$ (d) $f(a)$

01 04 exset 03 **In Exercises 13 – 21, evaluate the given limits of the piecewise defined functions $f$.**

01 04 ex 13 13. $f(x) = \begin{cases} x+1 & x \le 1 \\ x^2-5 & x > 1 \end{cases}$

(a) $\lim\limits_{x\to 1^-} f(x)$ (c) $\lim\limits_{x\to 1} f(x)$
(b) $\lim\limits_{x\to 1^+} f(x)$ (d) $f(1)$

01 04 ex 14 14. $f(x) = \begin{cases} 2x^2+5x-1 & x < 0 \\ \sin x & x \ge 0 \end{cases}$

(a) $\lim\limits_{x\to 0^-} f(x)$ (c) $\lim\limits_{x\to 0} f(x)$
(b) $\lim\limits_{x\to 0^+} f(x)$ (d) $f(0)$

01 04 ex 15 15. $f(x) = \begin{cases} x^2-1 & x < -1 \\ x^3+1 & -1 \le x \le 1 \\ x^2+1 & x > 1 \end{cases}$

(a) $\lim\limits_{x\to -1^-} f(x)$ (e) $\lim\limits_{x\to 1^-} f(x)$
(b) $\lim\limits_{x\to -1^+} f(x)$ (f) $\lim\limits_{x\to 1^+} f(x)$
(c) $\lim\limits_{x\to -1} f(x)$ (g) $\lim\limits_{x\to 1} f(x)$
(d) $f(-1)$ (h) $f(1)$

01 04 ex 16 16. $f(x) = \begin{cases} \cos x & x < \pi \\ \sin x & x \ge \pi \end{cases}$

(a) $\lim\limits_{x\to \pi^-} f(x)$ (c) $\lim\limits_{x\to \pi} f(x)$
(b) $\lim\limits_{x\to \pi^+} f(x)$ (d) $f(\pi)$

01 04 ex 17 17. $f(x) = \begin{cases} 1-\cos^2 x & x < a \\ \sin^2 x & x \ge a \end{cases}$,
where $a$ is a real number.

(a) $\lim\limits_{x\to a^-} f(x)$ (c) $\lim\limits_{x\to a} f(x)$
(b) $\lim\limits_{x\to a^+} f(x)$ (d) $f(a)$

01 04 ex 18 18. $f(x) = \begin{cases} x+1 & x < 1 \\ 1 & x = 1 \\ x-1 & x > 1 \end{cases}$

(a) $\lim\limits_{x\to 1^-} f(x)$ (c) $\lim\limits_{x\to 1} f(x)$
(b) $\lim\limits_{x\to 1^+} f(x)$ (d) $f(1)$

01 04 ex 19 19. $f(x) = \begin{cases} x^2 & x < 2 \\ x+1 & x = 2 \\ -x^2+2x+4 & x > 2 \end{cases}$

(a) $\lim\limits_{x\to 2^-} f(x)$ (c) $\lim\limits_{x\to 2} f(x)$
(b) $\lim\limits_{x\to 2^+} f(x)$ (d) $f(2)$

01 04 ex 20 20. $f(x) = \begin{cases} a(x-b)^2+c & x < b \\ a(x-b)+c & x \ge b \end{cases}$,
where $a$, $b$ and $c$ are real numbers.

(a) $\lim\limits_{x\to b^-} f(x)$ (c) $\lim\limits_{x\to b} f(x)$
(b) $\lim\limits_{x\to b^+} f(x)$ (d) $f(b)$

01 04 ex 21 21. $f(x) = \begin{cases} \frac{|x|}{x} & x \ne 0 \\ 0 & x = 0 \end{cases}$

(a) $\lim\limits_{x\to 0^-} f(x)$ (c) $\lim\limits_{x\to 0} f(x)$
(b) $\lim\limits_{x\to 0^+} f(x)$ (d) $f(0)$

## *Review*

01 04 ex 22 22. Evaluate the limit: $\lim\limits_{x\to -1} \dfrac{x^2+5x+4}{x^2-3x-4}$.

01 04 ex 23 23. Evaluate the limit: $\lim\limits_{x\to -4} \dfrac{x^2-16}{x^2-4x-32}$.

01 04 ex 24 24. Evaluate the limit: $\lim\limits_{x\to -6} \dfrac{x^2-15x+54}{x^2-6x}$.

01 04 ex 26 25. Approximate the limit numerically: $\lim\limits_{x\to 0.4} \dfrac{x^2-4.4x+1.6}{x^2-0.4x}$.

01 04 ex 27 26. Approximate the limit numerically: $\lim\limits_{x\to 0.2} \dfrac{x^2+5.8x-1.2}{x^2-4.2x+0.8}$.

# Exercises 1.5

## *Terms and Concepts*

01 05 ex 06 1. In your own words, describe what it means for a function to be continuous.

01 05 ex 08 2. In your own words, describe what the Intermediate Value Theorem states.

01 05 ex 09 3. What is a "root" of a function?

01 05 ex 10 4. Given functions $f$ and $g$ on an interval $I$, how can the Bisection Method be used to find a value $c$ where $f(c) = g(c)$?

01 05 ex 01 5. T/F: If $f$ is defined on an open interval containing $c$, and $\lim\limits_{x\to c} f(x)$ exists, then $f$ is continuous at $c$.

01 05 ex 02 6. T/F: If $f$ is continuous at $c$, then $\lim\limits_{x\to c} f(x)$ exists.

01 05 ex 03 7. T/F: If $f$ is continuous at $c$, then $\lim\limits_{x\to c^+} f(x) = f(c)$.

01 05 ex 04 8. T/F: If $f$ is continuous on $[a, b]$, then $\lim\limits_{x\to a^-} f(x) = f(a)$.

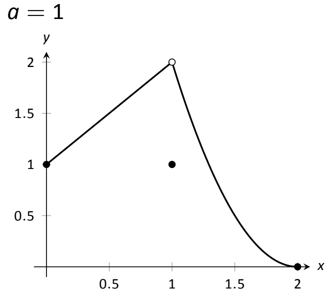
01 05 ex 05 9. T/F: If $f$ is continuous on $[0, 1)$ and $[1, 2)$, then $f$ is continuous on $[0, 2)$.

01 05 ex 07 10. T/F: The sum of continuous functions is also continuous.

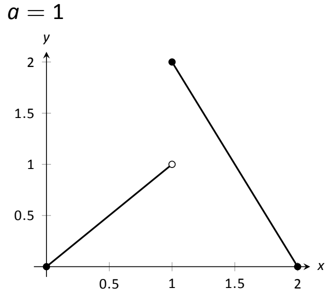
## *Problems*

01 05 exset 01 **In Exercises 11 – 17, a graph of a function $f$ is given along with a value $a$. Determine if $f$ is continuous at $a$; if it is not, state why it is not.**

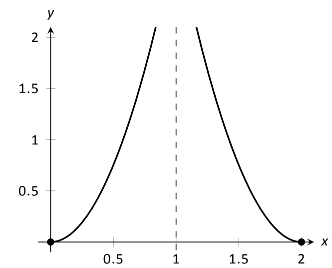
01 05 ex 11 11. $a = 1$



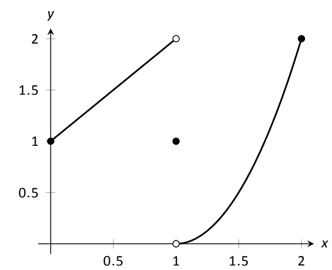
01 05 ex 12 12. $a = 1$



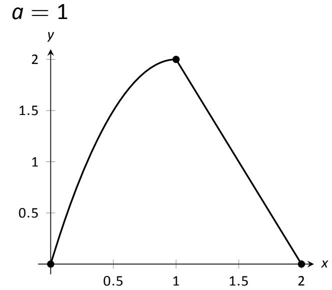
01 05 ex 13 13. $a = 1$



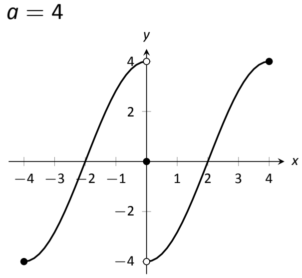
01 05 ex 14 14. $a = 0$



01 05 ex 15 15. $a = 1$



01 05 ex 16 16. $a = 4$

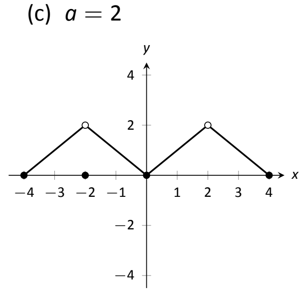


01 05 ex 17 17.

(a) $a = -2$

(b) $a = 0$

(c) $a = 2$



01 05 exset 02 **In Exercises 18 – 21, determine if $f$ is continuous at the indicated values. If not, explain why.**

01 05 ex 18 18. $f(x) = \begin{cases} 1 & x = 0 \\ \frac{\sin x}{x} & x > 0 \end{cases}$

(a) $x = 0$

(b) $x = \pi$

01 05 ex 19 19. $f(x) = \begin{cases} x^3 - x & x < 1 \\ x - 2 & x \geq 1 \end{cases}$

(a) $x = 0$

(b) $x = 1$

01 05 ex 20 20. $f(x) = \begin{cases} \frac{x^2+5x+4}{x^2+3x+2} & x \neq -1 \\ 3 & x = -1 \end{cases}$

(a) $x = -1$

(b) $x = 10$

01 05 ex 21 21. $f(x) = \begin{cases} \frac{x^2-64}{x^2-11x+24} & x \neq 8 \\ 5 & x = 8 \end{cases}$

(a) $x = 0$

(b) $x = 8$

01 05 exset 03 **In Exercises 22 – 32, give the intervals on which the given function is continuous.**

01 05 ex 22 22. $f(x) = x^2 - 3x + 9$

01 05 ex 23 23. $g(x) = \sqrt{x^2 - 4}$

01 05 ex 24 24. $h(k) = \sqrt{1 - k} + \sqrt{k + 1}$

01 05 ex 25 25. $f(t) = \sqrt{5t^2 - 30}$

01 05 ex 26 26. $g(t) = \dfrac{1}{\sqrt{1 - t^2}}$

01 05 ex 27 27. $g(x) = \dfrac{1}{1 + x^2}$

01 05 ex 28 28. $f(x) = e^x$

01 05 ex 29 29. $g(s) = \ln s$

01 05 ex 30 30. $h(t) = \cos t$

01 05 ex 31 31. $f(k) = \sqrt{1 - e^k}$

01 05 ex 32 32. $f(x) = \sin(e^x + x^2)$

01 05 ex 33 33. Let $f$ be continuous on $[1, 5]$ where $f(1) = -2$ and $f(5) = -10$. Does a value $1 < c < 5$ exist such that $f(c) = -9$? Why/why not?

01 05 ex 34 34. Let $g$ be continuous on $[-3, 7]$ where $g(0) = 0$ and $g(2) = 25$. Does a value $-3 < c < 7$ exist such that $g(c) = 15$? Why/why not?

01 05 ex 35 35. Let $f$ be continuous on $[-1, 1]$ where $f(-1) = -10$ and $f(1) = 10$. Does a value $-1 < c < 1$ exist such that $f(c) = 11$? Why/why not?

01 05 ex 36 36. Let $h$ be a function on $[-1, 1]$ where $h(-1) = -10$ and $h(1) = 10$. Does a value $-1 < c < 1$ exist such that $h(c) = 0$? Why/why not?

01 05 exset 04 **In Exercises 37 – 40, use the Bisection Method to approximate, accurate to two decimal places, the value of the root of the given function in the given interval.**

01 05 ex 37 37. $f(x) = x^2 + 2x - 4$ on $[1, 1.5]$.

01 05 ex 38 38. $f(x) = \sin x - 1/2$ on $[0.5, 0.55]$

01 05 ex 39 39. $f(x) = e^x - 2$ on $[0.65, 0.7]$.

01 05 ex 40 40. $f(x) = \cos x - \sin x$ on $[0.7, 0.8]$.

## *Review*

01 05 ex 41 41. Let $f(x) = \begin{cases} x^2 - 5 & x < 5 \\ 5x & x \geq 5 \end{cases}$.

(a) $\lim_{x \to 5^-} f(x)$

(b) $\lim_{x \to 5^+} f(x)$

(c) $\lim_{x \to 5} f(x)$

(d) $f(5)$

01 05 ex 42 42. Numerically approximate the following limits:

(a) $\lim_{x \to -4/5^+} \dfrac{x^2 - 8.2x - 7.2}{x^2 + 5.8x + 4}$

(b) $\lim_{x \to -4/5^-} \dfrac{x^2 - 8.2x - 7.2}{x^2 + 5.8x + 4}$

01 05 ex 43 43. Give an example of function $f(x)$ for which $\lim_{x \to 0} f(x)$ does not exist.

# Exercises 1.6

## *Terms and Concepts*

01 06 ex 01 1. T/F: If $\lim\limits_{x\to 5} f(x) = \infty$, then we are implicitly stating that the limit exists.

01 06 ex 02 2. T/F: If $\lim\limits_{x\to \infty} f(x) = 5$, then we are implicitly stating that the limit exists.

01 06 ex 03 3. T/F: If $\lim\limits_{x\to 1^-} f(x) = -\infty$, then $\lim\limits_{x\to 1^+} f(x) = \infty$

01 06 ex 04 4. T/F: If $\lim\limits_{x\to 5} f(x) = \infty$, then $f$ has a vertical asymptote at $x = 5$.

01 06 ex 05 5. T/F: $\infty/0$ is not an indeterminate form.

01 06 ex 06 6. List 5 indeterminate forms.

01 06 ex 07 7. Construct a function with a vertical asymptote at $x = 5$ and a horizontal asymptote at $y = 5$.

01 06 ex 08 8. Let $\lim\limits_{x\to 7} f(x) = \infty$. Explain how we know that $f$ is/is not continuous at $x = 7$.

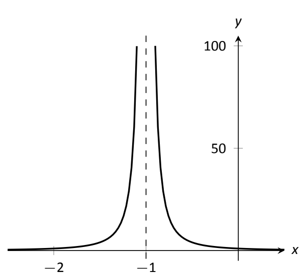
## *Problems*

01 06 exset 01 **In Exercises 9 – 14, evaluate the given limits using the graph of the function.**

01 06 ex 09 9. $f(x) = \dfrac{1}{(x+1)^2}$

(a) $\lim\limits_{x\to -1^-} f(x)$

(b) $\lim\limits_{x\to -1^+} f(x)$




01 06 ex 10 10. $f(x) = \dfrac{1}{(x-3)(x-5)^2}$.

(a) $\lim\limits_{x\to 3^-} f(x)$

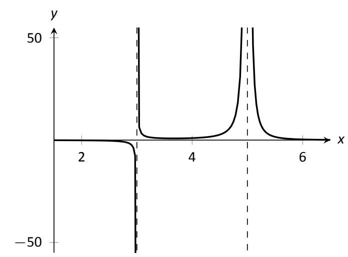
(b) $\lim\limits_{x\to 3^+} f(x)$

(c) $\lim\limits_{x\to 3} f(x)$

(d) $\lim\limits_{x\to 5^-} f(x)$

(e) $\lim\limits_{x\to 5^+} f(x)$

(f) $\lim\limits_{x\to 5} f(x)$



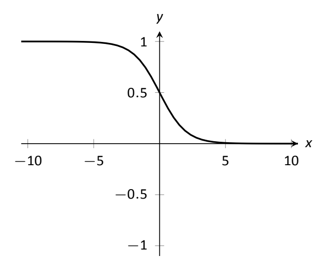

01 06 ex 11 11. $f(x) = \dfrac{1}{e^x + 1}$

(a) $\lim\limits_{x\to -\infty} f(x)$

(b) $\lim\limits_{x\to \infty} f(x)$

(c) $\lim\limits_{x\to 0^-} f(x)$

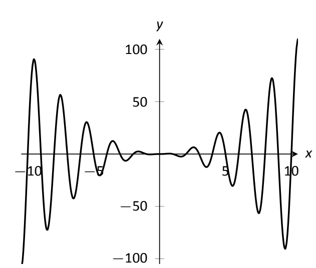
(d) $\lim\limits_{x\to 0^+} f(x)$




01 06 ex 12 12. $f(x) = x^2 \sin(\pi x)$

(a) $\lim\limits_{x\to -\infty} f(x)$

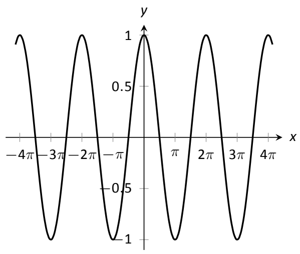
(b) $\lim\limits_{x\to \infty} f(x)$



01 06 ex 13 13. $f(x) = \cos(x)$

(a) $\lim\limits_{x\to -\infty} f(x)$

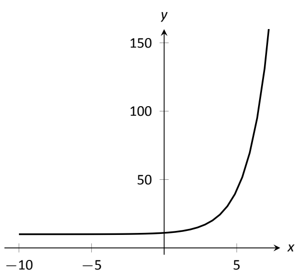
(b) $\lim\limits_{x\to \infty} f(x)$




01 06 ex 40 14. $f(x) = 2^x + 10$

(a) $\lim\limits_{x\to -\infty} f(x)$

(b) $\lim\limits_{x\to \infty} f(x)$




01 06 exset 02 **In Exercises 15 – 18, numerically approximate the following limits:**

**(a)** $\lim\limits_{x\to 3^-} f(x)$

**(b)** $\lim\limits_{x\to 3^+} f(x)$

**(c)** $\lim\limits_{x\to 3} f(x)$

01 06 ex 14 15. $f(x) = \dfrac{x^2-1}{x^2-x-6}$

01 06 ex 15 16. $f(x) = \dfrac{x^2+5x-36}{x^3-5x^2+3x+9}$

01 06 ex 16 17. $f(x) = \dfrac{x^2-11x+30}{x^3-4x^2-3x+18}$

01 06 ex 17 18. $f(x) = \dfrac{x^2-9x+18}{x^2-x-6}$

01 06 exset 03 **In Exercises 19 – 24, identify the horizontal and vertical asymptotes, if any, of the given function.**

01 06 ex 18 19. $f(x) = \dfrac{2x^2-2x-4}{x^2+x-20}$

01 06 ex 19 20. $f(x) = \dfrac{-3x^2-9x-6}{5x^2-10x-15}$

01 06 ex 20 21. $f(x) = \dfrac{x^2+x-12}{7x^3-14x^2-21x}$

01 06 ex 21 22. $f(x) = \dfrac{x^2-9}{9x-9}$

01 06 ex 22 23. $f(x) = \dfrac{x^2-9}{9x+27}$

01 06 ex 41 24. $f(x) = \dfrac{x^2-1}{-x^2-1}$

01 06 exset 04 **In Exercises 25 – 28, evaluate the given limit.**

01 06 ex 23 25. $\lim\limits_{x\to\infty} \dfrac{x^3+2x^2+1}{x-5}$

01 06 ex 24 26. $\lim\limits_{x\to\infty} \dfrac{x^3+2x^2+1}{5-x}$

01 06 ex 25 27. $\lim\limits_{x\to-\infty} \dfrac{x^3+2x^2+1}{x^2-5}$

01 06 ex 26 28. $\lim\limits_{x\to-\infty} \dfrac{x^3+2x^2+1}{5-x^2}$

## *Review*

01 06 ex 27 29. Use an $\varepsilon - \delta$ proof to show that $\lim\limits_{x\to 1} 5x-2 = 3$.

01 06 ex 28 30. Let $\lim\limits_{x\to 2} f(x) = 3$ and $\lim\limits_{x\to 2} g(x) = -1$. Evaluate the following limits.

(a) $\lim\limits_{x\to 2}(f+g)(x)$

(b) $\lim\limits_{x\to 2}(fg)(x)$

(c) $\lim\limits_{x\to 2}(f/g)(x)$

(d) $\lim\limits_{x\to 2} f(x)^{g(x)}$

01 06 ex 29 31. Let $f(x) = \begin{cases} x^2-1 & x<3 \\ x+5 & x\geq 3 \end{cases}$.
Is $f$ continuous everywhere?

01 06 ex 30 32. Evaluate the limit: $\lim\limits_{x\to e} \ln x$.

# Exercises 2.1

## Terms and Concepts

02 01 ex 01 1. T/F: Let $f$ be a position function. The average rate of change on $[a, b]$ is the slope of the line through the points $(a, f(a))$ and $(b, f(b))$.

02 01 ex 02 2. T/F: The definition of the derivative of a function at a point involves taking a limit.

02 01 ex 03 3. In your own words, explain the difference between the average rate of change and instantaneous rate of change.

02 01 ex 04 4. In your own words, explain the difference between Definitions 7 and 10.

02 01 ex 05 5. Let $y = f(x)$. Give three different notations equivalent to "$f'(x)$."

## Problems

02 01 exset 02 **In Exercises 6 – 12, use the definition of the derivative to compute the derivative of the given function.**

02 01 ex 10 6. $f(x) = 6$

02 01 ex 11 7. $f(x) = 2x$

02 01 ex 12 8. $f(t) = 4 - 3t$

02 01 ex 13 9. $g(x) = x^2$

02 01 ex 14 10. $f(x) = 3x^2 - x + 4$

02 01 ex 15 11. $r(x) = \dfrac{1}{x}$

02 01 ex 16 12. $r(s) = \dfrac{1}{s-2}$

02 01 exset 03 **In Exercises 13 – 19, a function and an $x$–value $c$ are given. (Note: these functions are the same as those given in Exercises 6 through 12.)**

**(a) Find the tangent line to the graph of the function at $c$.**

**(b) Find the normal line to the graph of the function at $c$.**

02 01 ex 17 13. $f(x) = 6$, at $x = -2$.

02 01 ex 18 14. $f(x) = 2x$, at $x = 3$.

02 01 ex 19 15. $f(x) = 4 - 3x$, at $x = 7$.

02 01 ex 20 16. $g(x) = x^2$, at $x = 2$.

02 01 ex 21 17. $f(x) = 3x^2 - x + 4$, at $x = -1$.

02 01 ex 22 18. $r(x) = \dfrac{1}{x}$, at $x = -2$.

02 01 ex 23 19. $r(x) = \dfrac{1}{x-2}$, at $x = 3$.

02 01 exset 01 **In Exercises 20 – 23, a function $f$ and an $x$–value $a$ are given. Approximate the equation of the tangent line to the graph of $f$ at $x = a$ by numerically approximating $f'(a)$, using $h = 0.1$.**

02 01 ex 06 20. $f(x) = x^2 + 2x + 1$, $x = 3$

02 01 ex 07 21. $f(x) = \dfrac{10}{x+1}$, $x = 9$

02 01 ex 08 22. $f(x) = e^x$, $x = 2$

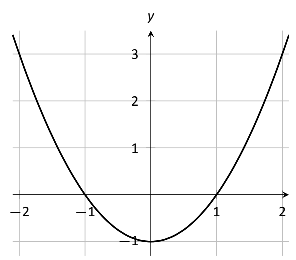
02 01 ex 09 23. $f(x) = \cos x$, $x = 0$

02 01 ex 24 24. The graph of $f(x) = x^2 - 1$ is shown.

(a) Use the graph to approximate the slope of the tangent line to $f$ at the following points: $(-1, 0)$, $(0, -1)$ and $(2, 3)$.

(b) Using the definition, find $f'(x)$.

(c) Find the slope of the tangent line at the points $(-1, 0)$, $(0, -1)$ and $(2, 3)$.

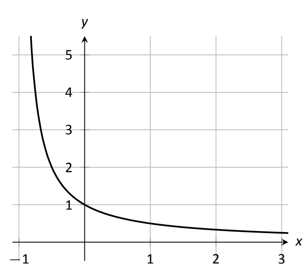



02 01 ex 25 25. The graph of $f(x) = \dfrac{1}{x+1}$ is shown.

(a) Use the graph to approximate the slope of the tangent line to $f$ at the following points: $(0, 1)$ and $(1, 0.5)$.

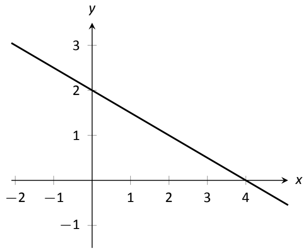
(b) Using the definition, find $f'(x)$.

(c) Find the slope of the tangent line at the points $(0, 1)$ and $(1, 0.5)$.

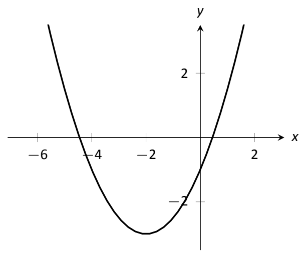


02 01 exset 04 **In Exercises 26 – 29, a graph of a function $f(x)$ is given. Using the graph, sketch $f'(x)$.**

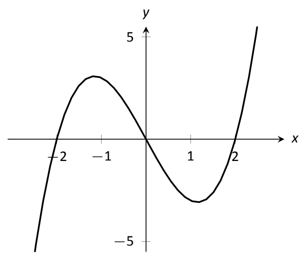
02 01 ex 26 26.



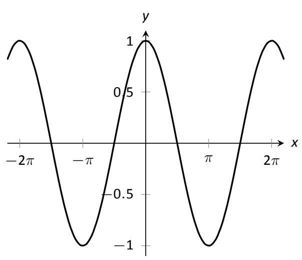

02 01 ex 27 27.




02 01 ex 28 28.

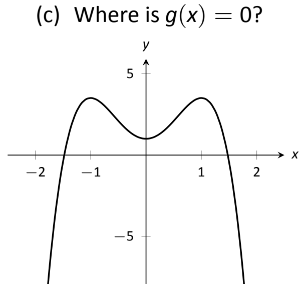



02 01 ex 29 29.




02 01 ex 30 30. Using the graph of $g(x)$ below, answer the following questions.

(a) Where is $g(x) > 0$?
(b) Where is $g(x) < 0$?
(c) Where is $g(x) = 0$?
(c) Where is $g'(x) < 0$?
(d) Where is $g'(x) > 0$?
(e) Where is $g'(x) = 0$?




## *Review*

02 01 ex 31 31. Approximate $\lim\limits_{x\to 5} \dfrac{x^2+2x-35}{x^2-10.5x+27.5}$.

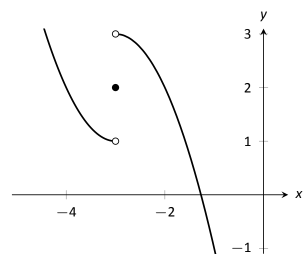
02 01 ex 32 32. Use the Bisection Method to approximate, accurate to two decimal places, the root of $g(x) = x^3 + x^2 + x - 1$ on $[0.5, 0.6]$.

02 01 ex 33 33. Give intervals on which each of the following functions are continuous.

(a) $\dfrac{1}{e^x+1}$
(b) $\dfrac{1}{x^2-1}$
(c) $\sqrt{5-x}$
(d) $\sqrt{5-x^2}$

02 01 ex 34 34. Use the graph of $f(x)$ provided to answer the following.

(a) $\lim\limits_{x\to -3^-} f(x) = ?$
(b) $\lim\limits_{x\to -3^+} f(x) = ?$
(c) $\lim\limits_{x\to -3} f(x) = ?$
(d) Where is $f$ continuous?



# Exercises 2.2

## Terms and Concepts

02 02 exset 01

02 02 ex 01 1. What is the instantaneous rate of change of position called?

02 02 ex 02 2. Given a function $y = f(x)$, in your own words describe how to find the units of $f'(x)$.

02 02 ex 03 3. What functions have a constant rate of change?

## Problems

02 02 ex 04 4. Given $f(5) = 10$ and $f'(5) = 2$, approximate $f(6)$.

02 02 ex 05 5. Given $P(100) = -67$ and $P'(100) = 5$, approximate $P(110)$.

02 02 ex 06 6. Given $z(25) = 187$ and $z'(25) = 17$, approximate $z(20)$.

02 02 ex 07 7. Knowing $f(10) = 25$ and $f'(10) = 5$ and the methods described in this section, which approximation is likely to be most accurate: $f(10.1)$, $f(11)$, or $f(20)$? Explain your reasoning.

02 02 ex 08 8. Given $f(7) = 26$ and $f(8) = 22$, approximate $f'(7)$.

02 02 ex 09 9. Given $H(0) = 17$ and $H(2) = 29$, approximate $H'(2)$.

02 02 ex 10 10. Let $V(x)$ measure the volume, in decibels, measured inside a restaurant with $x$ customers. What are the units of $V'(x)$?

02 02 ex 11 11. Let $v(t)$ measure the velocity, in ft/s, of a car moving in a straight line $t$ seconds after starting. What are the units of $v'(t)$?

02 02 ex 12 12. The height $H$, in feet, of a river is recorded $t$ hours after midnight, April 1. What are the units of $H'(t)$?

02 02 ex 13 13. $P$ is the profit, in thousands of dollars, of producing and selling $c$ cars.

(a) What are the units of $P'(c)$?

(b) What is likely true of $P(0)$?

02 02 ex 14 14. $T$ is the temperature in degrees Fahrenheit, $h$ hours after midnight on July 4 in Sidney, NE.

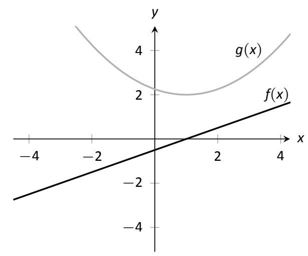
(a) What are the units of $T'(h)$?

(b) Is $T'(8)$ likely greater than or less than 0? Why?

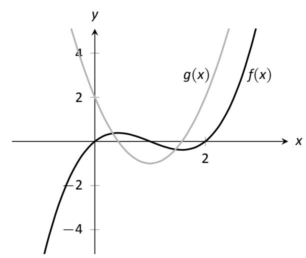
(c) Is $T(8)$ likely greater than or less than 0? Why?

**In Exercises 15 – 18, graphs of functions $f(x)$ and $g(x)$ are given. Identify which function is the derivative of the other.)**

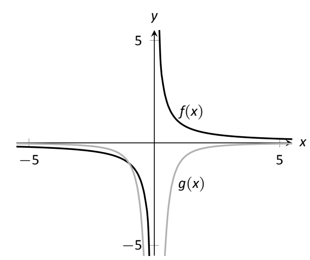
02 02 ex 15 15.



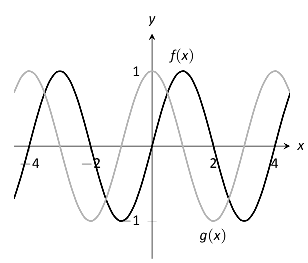

02 02 ex 16 16.




02 02 ex 17 17.




02 02 ex 18 18.




## Review

02 02 exset 02

**In Exercises 19 – 20, use the definition to compute the derivatives of the following functions.**

02 02 ex 19 19. $f(x) = 5x^2$

02 02 ex 20 20. $f(x) = (x-2)^3$

02 02 exset 03

**In Exercises 21 – 22, numerically approximate the value of $f'(x)$ at the indicated $x$ value.**

02 02 ex 21 21. $f(x) = \cos x$ at $x = \pi$.

02 02 ex 22 22. $f(x) = \sqrt{x}$ at $x = 9$.

# Exercises 2.3

## *Terms and Concepts*

02 03 ex 01 1. What is the name of the rule which states that $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$, where $n > 0$ is an integer?

02 03 ex 02 2. What is $\frac{d}{dx}\big(\ln x\big)$?

02 03 ex 03 3. Give an example of a function $f(x)$ where $f'(x) = f(x)$.

02 03 ex 04 4. Give an example of a function $f(x)$ where $f'(x) = 0$.

02 03 ex 05 5. The derivative rules introduced in this section explain how to compute the derivative of which of the following functions?

- $f(x) = \frac{3}{x^2}$
- $g(x) = 3x^2 - x + 17$
- $h(x) = 5\ln x$
- $j(x) = \sin x \cos x$
- $k(x) = e^{x^2}$
- $m(x) = \sqrt{x}$

02 03 ex 06 6. Explain in your own words how to find the third derivative of a function $f(x)$.

02 03 ex 07 7. Give an example of a function where $f'(x) \neq 0$ and $f''(x) = 0$.

02 03 ex 08 8. Explain in your own words what the second derivative "means."

02 03 ex 09 9. If $f(x)$ describes a position function, then $f'(x)$ describes what kind of function? What kind of function is $f''(x)$?

02 03 ex 10 10. Let $f(x)$ be a function measured in pounds, where $x$ is measured in feet. What are the units of $f''(x)$?

## *Problems*

02 03 exset 01 **In Exercises 11 – 25, compute the derivative of the given function.**

02 03 ex 11 11. $f(x) = 7x^2 - 5x + 7$

02 03 ex 12 12. $g(x) = 14x^3 + 7x^2 + 11x - 29$

02 03 ex 13 13. $m(t) = 9t^5 - \frac{1}{8}t^3 + 3t - 8$

02 03 ex 14 14. $f(\theta) = 9\sin\theta + 10\cos\theta$

02 03 ex 15 15. $f(r) = 6e^r$

02 03 ex 16 16. $g(t) = 10t^4 - \cos t + 7\sin t$

02 03 ex 17 17. $f(x) = 2\ln x - x$

02 03 ex 18 18. $p(s) = \frac{1}{4}s^4 + \frac{1}{3}s^3 + \frac{1}{2}s^2 + s + 1$

02 03 ex 19 19. $h(t) = e^t - \sin t - \cos t$

02 03 ex 20 20. $f(x) = \ln(5x^2)$

02 03 ex 21 21. $f(t) = \ln(17) + e^2 + \sin\pi/2$

02 03 ex 22 22. $g(t) = (1 + 3t)^2$

02 03 ex 23 23. $g(x) = (2x - 5)^3$

02 03 ex 24 24. $f(x) = (1 - x)^3$

02 03 ex 25 25. $f(x) = (2 - 3x)^2$

02 03 ex 26 26. A property of logarithms is that $\log_a x = \frac{\log_b x}{\log_b a}$, for all bases $a, b > 0, \neq 1$.

(a) Rewrite this identity when $b = e$, i.e., using $\log_e x = \ln x$.

(b) Use part (a) to find the derivative of $y = \log_a x$.

(c) Give the derivative of $y = \log_{10} x$.

02 03 exset 02 **In Exercises 27 – 32, compute the first four derivatives of the given function.**

02 03 ex 26 27. $f(x) = x^6$

02 03 ex 27 28. $g(x) = 2\cos x$

02 03 ex 28 29. $h(t) = t^2 - e^t$

02 03 ex 29 30. $p(\theta) = \theta^4 - \theta^3$

02 03 ex 30 31. $f(\theta) = \sin\theta - \cos\theta$

02 03 ex 31 32. $f(x) = 1,100$

02 03 exset 03 **In Exercises 33 – 38, find the equations of the tangent and normal lines to the graph of the function at the given point.**

02 03 ex 32 33. $f(x) = x^3 - x$ at $x = 1$

02 03 ex 33 34. $f(t) = e^t + 3$ at $t = 0$

02 03 ex 34 35. $g(x) = \ln x$ at $x = 1$

02 03 ex 35 36. $f(x) = 4\sin x$ at $x = \pi/2$

02 03 ex 36 37. $f(x) = -2\cos x$ at $x = \pi/4$

02 03 ex 37 38. $f(x) = 2x + 3$ at $x = 5$

## *Review*

02 03 ex 39 39. Given that $e^0 = 1$, approximate the value of $e^{0.1}$ using the tangent line to $f(x) = e^x$ at $x = 0$.

02 03 ex 40 40. Approximate the value of $(3.01)^4$ using the tangent line to $f(x) = x^4$ at $x = 3$.

# Exercises 2.4

## *Terms and Concepts*

02 04 ex 01

1. T/F: The Product Rule states that $\frac{d}{dx}\left(x^2\sin x\right) = 2x\cos x$.

02 04 ex 02

2. T/F: The Quotient Rule states that $\frac{d}{dx}\left(\frac{x^2}{\sin x}\right) = \frac{\cos x}{2x}$.

02 04 ex 03

3. T/F: The derivatives of the trigonometric functions that start with "c" have minus signs in them.

02 04 ex 04

4. What derivative rule is used to extend the Power Rule to include negative integer exponents?

02 04 ex 05

5. T/F: Regardless of the function, there is always exactly one right way of computing its derivative.

02 04 ex 06

6. In your own words, explain what it means to make your answers "clear."

## *Problems*

02 04 exset 01

**In Exercises 7 – 10:**

**(a) Use the Product Rule to differentiate the function.**

**(b) Manipulate the function algebraically and differentiate without the Product Rule.**

**(c) Show that the answers from (a) and (b) are equivalent.**

02 04 ex 07

7. $f(x) = x(x^2+3x)$

02 04 ex 08

8. $g(x) = 2x^2(5x^3)$

02 04 ex 09

9. $h(s) = (2s-1)(s+4)$

02 04 ex 10

10. $f(x) = (x^2+5)(3-x^3)$

02 04 exset 02

**In Exercises 11 – 14:**

**(a) Use the Quotient Rule to differentiate the function.**

**(b) Manipulate the function algebraically and differentiate without the Quotient Rule.**

**(c) Show that the answers from (a) and (b) are equivalent.**

02 04 ex 11

11. $f(x) = \frac{x^2+3}{x}$

02 04 ex 12

12. $g(x) = \frac{x^3-2x^2}{2x^2}$

02 04 ex 13

13. $h(s) = \frac{3}{4s^3}$

02 04 ex 14

14. $f(t) = \frac{t^2-1}{t+1}$

02 04 exset 03

**In Exercises 15 – 29, compute the derivative of the given function.**

02 04 ex 16

15. $f(x) = x\sin x$

02 04 ex 17

16. $f(t) = \frac{1}{t^2}(\csc t - 4)$

02 04 ex 18

17. $g(x) = \frac{x+7}{x-5}$

02 04 ex 19

18. $g(t) = \frac{t^5}{\cos t - 2t^2}$

02 04 ex 20

19. $h(x) = \cot x - e^x$

02 04 ex 21

20. $h(t) = 7t^2 + 6t - 2$

02 04 ex 15

21. $f(x) = \frac{x^4+2x^3}{x+2}$

02 04 ex 22

22. $f(x) = (16x^3+24x^2+3x)\frac{7x-1}{16x^3+24x^2+3x}$

02 04 ex 23

23. $f(t) = t^5(\sec t + e^t)$

02 04 ex 46

24. $f(x) = \frac{\sin x}{\cos x + 3}$

02 04 ex 24

25. $g(x) = e^2\big(\sin(\pi/4) - 1\big)$

02 04 ex 25

26. $g(t) = 4t^3e^t - \sin t\cos t$

02 04 ex 47

27. $h(t) = \frac{t^2\sin t + 3}{t^2\cos t + 2}$

02 04 ex 26

28. $f(x) = x^2e^x\tan x$

02 04 ex 27

29. $g(x) = 2x\sin x\sec x$

02 04 exset 04

**In Exercises 30 – 33, find the equations of the tangent and normal lines to the graph of $g$ at the indicated point.**

02 04 ex 28

30. $g(s) = e^s(s^2+2)$ at $(0, 2)$.

02 04 ex 29

31. $g(t) = t\sin t$ at $\left(\frac{3\pi}{2}, -\frac{3\pi}{2}\right)$

02 04 ex 30

32. $g(x) = \frac{x^2}{x-1}$ at $(2, 4)$

02 04 ex 31

33. $g(\theta) = \frac{\cos\theta - 8\theta}{\theta+1}$ at $(0, -5)$

02 04 exset 05

**In Exercises 34 – 37, find the $x$–values where the graph of the function has a horizontal tangent line.**

02 04 ex 32

34. $f(x) = 6x^2 - 18x - 24$

02 04 ex 33

35. $f(x) = x\sin x$ on $[-1, 1]$

02 04 ex 34 36. $f(x) = \dfrac{x}{x+1}$

02 04 ex 35 37. $f(x) = \dfrac{x^2}{x+1}$

02 04 exset 06 **In Exercises 38 – 41, find the requested derivative.**

02 04 ex 36 38. $f(x) = x\sin x$; find $f''(x)$.

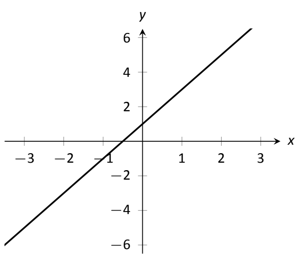
02 04 ex 37 39. $f(x) = x\sin x$; find $f^{(4)}(x)$.

02 04 ex 38 40. $f(x) = \csc x$; find $f''(x)$.

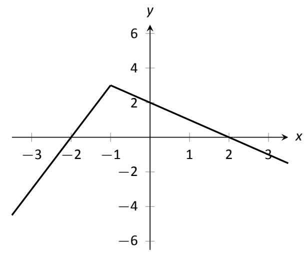
02 04 ex 39 41. $f(x) = (x^3 - 5x + 2)(x^2 + x - 7)$; find $f^{(8)}(x)$.

02 04 exset 07 **In Exercises 42 – 45, use the graph of $f(x)$ to sketch $f'(x)$.**

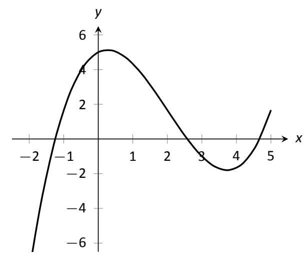
02 04 ex 42 42.



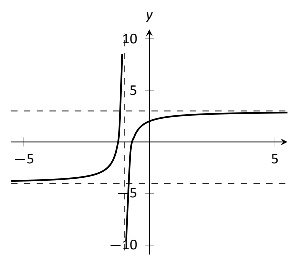

02 04 ex 43 43.




02 04 ex 44 44.




02 04 ex 45 45.



# Exercises 2.5

## *Terms and Concepts*

02 05 ex 05 1. T/F: The Chain Rule describes how to evaluate the derivative of a composition of functions.

02 05 ex 01 2. T/F: The Generalized Power Rule states that $\frac{d}{dx}\Big(g(x)^n\Big) = n\big(g(x)\big)^{n-1}$.

02 05 ex 02 3. T/F: $\frac{d}{dx}\big(\ln(x^2)\big) = \frac{1}{x^2}$.

02 05 ex 03 4. T/F: $\frac{d}{dx}\big(3^x\big) \approx 1.1 \cdot 3^x$.

02 05 ex 04 5. T/F: $\frac{dx}{dy} = \frac{dx}{dt} \cdot \frac{dt}{dy}$

02 05 ex 53 6. T/F: Taking the derivative of $f(x) = x^2 \sin(5x)$ requires the use of both the Product and Chain Rules.

## *Problems*

02 05 exset 01 **In Exercises 7 – 28, compute the derivative of the given function.**

02 05 ex 06 7. $f(x) = (4x^3 - x)^{10}$

02 05 ex 07 8. $f(t) = (3t - 2)^5$

02 05 ex 08 9. $g(\theta) = (\sin\theta + \cos\theta)^3$

02 05 ex 09 10. $h(t) = e^{3t^2+t-1}$

02 05 ex 10 11. $f(x) = \left(x + \frac{1}{x}\right)^4$

02 05 ex 11 12. $f(x) = \cos(3x)$

02 05 ex 12 13. $g(x) = \tan(5x)$

02 05 ex 13 14. $h(t) = \sin^4(2t)$

02 05 ex 14 15. $p(t) = \cos^3(t^2 + 3t + 1)$

02 05 ex 15 16. $f(x) = \ln(\cos x)$

02 05 ex 16 17. $f(x) = \ln(x^2)$

02 05 ex 17 18. $f(x) = 2\ln(x)$

02 05 ex 18 19. $g(r) = 4^r$

02 05 ex 19 20. $g(t) = 5^{\cos t}$

02 05 ex 20 21. $g(t) = 15^2$

02 05 ex 21 22. $m(w) = \dfrac{3^w}{2^w}$

02 05 ex 42 23. $h(t) = \dfrac{2^t + 3}{3^t + 2}$

02 05 ex 22 24. $m(w) = \dfrac{3^w + 1}{2^w}$

02 05 ex 23 25. $f(x) = \dfrac{3^{x^2} + x}{2^{x^2}}$

02 05 ex 24 26. $f(x) = x^2 \sin(5x)$

02 05 ex 25 27. $g(t) = \cos(t^2 + 3t)\sin(5t - 7)$

02 05 ex 26 28. $g(t) = \cos(\frac{1}{t})e^{5t^2}$

02 05 exset 02 **In Exercises 29 – 32, find the equations of tangent and normal lines to the graph of the function at the given point. Note: the functions here are the same as in Exercises 7 through 10.**

02 05 ex 27 29. $f(x) = (4x^3 - x)^{10}$ at $x = 0$

02 05 ex 28 30. $f(t) = (3t - 2)^5$ at $t = 1$

02 05 ex 29 31. $g(\theta) = (\sin\theta + \cos\theta)^3$ at $\theta = \pi/2$

02 05 ex 30 32. $h(t) = e^{3t^2+t-1}$ at $t = -1$

02 05 ex 31 33. Compute $\frac{d}{dx}\big(\ln(kx)\big)$ two ways:

(a) Using the Chain Rule, and

(b) by first using the logarithm rule $\ln(ab) = \ln a + \ln b$, then taking the derivative.

02 05 ex 32 34. Compute $\frac{d}{dx}\big(\ln(x^k)\big)$ two ways:

(a) Using the Chain Rule, and

(b) by first using the logarithm rule $\ln(a^p) = p\ln a$, then taking the derivative.

## *Review*

02 05 ex 33 35. The "wind chill factor" is a measurement of how cold it "feels" during cold, windy weather. Let $W(w)$ be the wind chill factor, in degrees Fahrenheit, when it is 25°F outside with a wind of $w$ mph.

(a) What are the units of $W'(w)$?

(b) What would you expect the sign of $W'(10)$ to be?

02 05 ex 34 36. Find the derivatives of the following functions.

(a) $f(x) = x^2 e^x \cot x$

(b) $g(x) = 2^x 3^x 4^x$

# Exercises 2.6

## Terms and Concepts

02 06 ex 01 1. In your own words, explain the difference between implicit functions and explicit functions.

02 06 ex 02 2. Implicit differentiation is based on what other differentiation rule?

02 06 ex 03 3. T/F: Implicit differentiation can be used to find the derivative of $y = \sqrt{x}$.

02 06 ex 04 4. T/F: Implicit differentiation can be used to find the derivative of $y = x^{3/4}$.

## Problems

02 06 exset 01 **In Exercises 5 – 12, compute the derivative of the given function.**

02 05 ex 50 5. $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

02 05 ex 51 6. $f(x) = \sqrt[3]{x} + x^{2/3}$

02 06 ex 06 7. $f(t) = \sqrt{1 - t^2}$

02 06 ex 07 8. $g(t) = \sqrt{t}\sin t$

02 06 ex 08 9. $h(x) = x^{1.5}$

02 05 ex 52 10. $f(x) = x^{\pi} + x^{1.9} + \pi^{1.9}$

02 05 ex 40 11. $g(x) = \dfrac{x+7}{\sqrt{x}}$

02 05 ex 41 12. $f(t) = \sqrt[5]{t}(\sec t + e^t)$

02 06 exset 02 **In Exercises 13 – 25, find $\dfrac{dy}{dx}$ using implicit differentiation.**

02 06 ex 09 13. $x^4 + y^2 + y = 7$

02 06 ex 10 14. $x^{2/5} + y^{2/5} = 1$

02 06 ex 11 15. $\cos(x) + \sin(y) = 1$

02 06 ex 12 16. $\dfrac{x}{y} = 10$

02 06 ex 13 17. $\dfrac{y}{x} = 10$

02 06 ex 14 18. $x^2 e^2 + 2^y = 5$

02 06 ex 15 19. $x^2 \tan y = 50$

02 06 ex 16 20. $(3x^2 + 2y^3)^4 = 2$

02 06 ex 17 21. $(y^2 + 2y - x)^2 = 200$

02 06 ex 18 22. $\dfrac{x^2 + y}{x + y^2} = 17$

02 06 ex 19 23. $\dfrac{\sin(x) + y}{\cos(y) + x} = 1$

02 06 ex 21 24. $\ln(x^2 + y^2) = e$

02 06 ex 22 25. $\ln(x^2 + xy + y^2) = 1$

02 06 ex 20 26. Show that $\dfrac{dy}{dx}$ is the same for each of the following implicitly defined functions.

(a) $xy = 1$

(b) $x^2y^2 = 1$

(c) $\sin(xy) = 1$

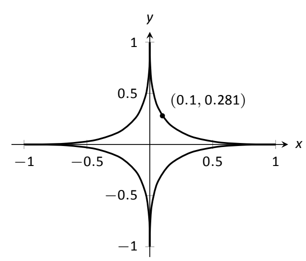
(d) $\ln(xy) = 1$

02 06 exset 03 **In Exercises 27 – 31, find the equation of the tangent line to the graph of the implicitly defined function at the indicated points. As a visual aid, each function is graphed.**

02 06 ex 23 27. $x^{2/5} + y^{2/5} = 1$

(a) At $(1, 0)$.

(b) At $(0.1, 0.281)$ (which does not *exactly* lie on the curve, but is very close).

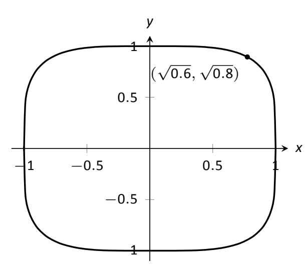



02 06 ex 24 28. $x^4 + y^4 = 1$

(a) At $(1, 0)$.

(b) At $(\sqrt{0.6}, \sqrt{0.8})$.

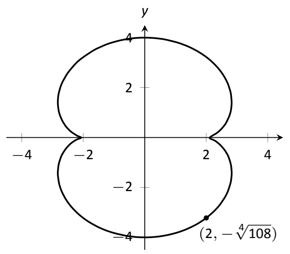
(c) At $(0, 1)$.



02 06 ex 25 29. $(x^2+y^2-4)^3 = 108y^2$

(a) At $(0, 4)$.

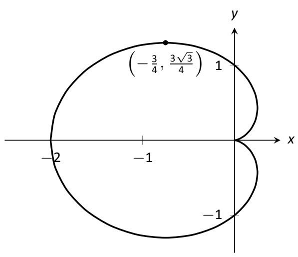
(b) At $(2, -\sqrt[4]{108})$.




02 06 ex 26 30. $(x^2+y^2+x)^2 = x^2+y^2$

(a) At $(0, 1)$.

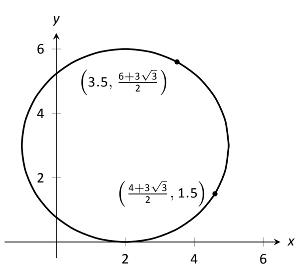
(b) At $\left(-\dfrac{3}{4}, \dfrac{3\sqrt{3}}{4}\right)$.




02 06 ex 27 31. $(x-2)^2+(y-3)^2 = 9$

(a) At $\left(\dfrac{7}{2}, \dfrac{6+3\sqrt{3}}{2}\right)$.

(b) At $\left(\dfrac{4+3\sqrt{3}}{2}, \dfrac{3}{2}\right)$.




02 06 exset 04 **In Exercises 32 – 35, an implicitly defined function is given. Find $\dfrac{d^2y}{dx^2}$. Note: these are the same problems used in Exercises 13 through 16.**

02 06 ex 28 32. $x^4+y^2+y = 7$

02 06 ex 29 33. $x^{2/5}+y^{2/5} = 1$

02 06 ex 30 34. $\cos x + \sin y = 1$

02 06 ex 31 35. $\dfrac{x}{y} = 10$

02 06 exset 05 **In Exercises 36 – 41, use logarithmic differentiation to find $\dfrac{dy}{dx}$, then find the equation of the tangent line at the indicated $x$–value.**

02 06 ex 32 36. $y = (1+x)^{1/x}, \quad x = 1$

02 06 ex 33 37. $y = (2x)^{x^2}, \quad x = 1$

02 06 ex 34 38. $y = \dfrac{x^x}{x+1}, \quad x = 1$

02 06 ex 35 39. $y = x^{\sin(x)+2}, \quad x = \pi/2$

02 06 ex 36 40. $y = \dfrac{x+1}{x+2}, \quad x = 1$

02 06 ex 37 41. $y = \dfrac{(x+1)(x+2)}{(x+3)(x+4)}, \quad x = 0$

# Exercises 2.7

## Terms and Concepts

02 07 ex 01 1. T/F: Every function has an inverse.

02 07 ex 02 2. In your own words explain what it means for a function to be "one to one."

02 07 ex 03 3. If $(1, 10)$ lies on the graph of $y = f(x)$, what can be said about the graph of $y = f^{-1}(x)$?

02 07 ex 04 4. If $(1, 10)$ lies on the graph of $y = f(x)$ and $f'(1) = 5$, what can be said about $y = f^{-1}(x)$?

## Problems

02 07 exset 01 **In Exercises 5 – 8, verify that the given functions are inverses.**

02 07 ex 05 5. $f(x) = 2x + 6$ and $g(x) = \frac{1}{2}x - 3$

02 07 ex 06 6. $f(x) = x^2 + 6x + 11$, $x \geq 3$ and $g(x) = \sqrt{x-2} - 3$, $x \geq 2$

02 07 ex 07 7. $f(x) = \dfrac{3}{x-5}$, $x \neq 5$ and $g(x) = \dfrac{3+5x}{x}$, $x \neq 0$

02 07 ex 08 8. $f(x) = \dfrac{x+1}{x-1}$, $x \neq 1$ and $g(x) = f(x)$

02 07 exset 02 **In Exercises 9 – 14, an invertible function $f(x)$ is given along with a point that lies on its graph. Using Theorem 22, evaluate $\left(f^{-1}\right)'(x)$ at the indicated value.**

02 07 ex 09 9. $f(x) = 5x + 10$
Point$= (2, 20)$
Evaluate $\left(f^{-1}\right)'(20)$

02 07 ex 10 10. $f(x) = x^2 - 2x + 4$, $x \geq 1$
Point$= (3, 7)$
Evaluate $\left(f^{-1}\right)'(7)$

02 07 ex 11 11. $f(x) = \sin 2x$, $-\pi/4 \leq x \leq \pi/4$
Point$= (\pi/6, \sqrt{3}/2)$
Evaluate $\left(f^{-1}\right)'(\sqrt{3}/2)$

02 07 ex 12 12. $f(x) = x^3 - 6x^2 + 15x - 2$
Point$= (1, 8)$
Evaluate $\left(f^{-1}\right)'(8)$

02 07 ex 13 13. $f(x) = \dfrac{1}{1+x^2}$, $x \geq 0$
Point$= (1, 1/2)$
Evaluate $\left(f^{-1}\right)'(1/2)$

02 07 ex 14 14. $f(x) = 6e^{3x}$
Point$= (0, 6)$
Evaluate $\left(f^{-1}\right)'(6)$

02 07 exset 03 **In Exercises 15 – 24, compute the derivative of the given function.**

02 07 ex 15 15. $h(t) = \sin^{-1}(2t)$

02 07 ex 16 16. $f(t) = \sec^{-1}(2t)$

02 07 ex 17 17. $g(x) = \tan^{-1}(2x)$

02 07 ex 18 18. $f(x) = x\sin^{-1} x$

02 07 ex 19 19. $g(t) = \sin t \cos^{-1} t$

02 07 ex 20 20. $f(t) = \ln te^t$

02 07 ex 21 21. $h(x) = \dfrac{\sin^{-1} x}{\cos^{-1} x}$

02 07 ex 22 22. $g(x) = \tan^{-1}(\sqrt{x})$

02 07 ex 23 23. $f(x) = \sec^{-1}(1/x)$

02 07 ex 24 24. $f(x) = \sin(\sin^{-1} x)$

02 07 exset 04 **In Exercises 25 – 27, compute the derivative of the given function in two ways:**

**(a) By simplifying first, then taking the derivative, and**

**(b) by using the Chain Rule first then simplifying.**

**Verify that the two answers are the same.**

02 07 ex 24 25. $f(x) = \sin(\sin^{-1} x)$

02 07 ex 25 26. $f(x) = \tan^{-1}(\tan x)$

02 07 ex 26 27. $f(x) = \sin(\cos^{-1} x)$

02 07 exset 05 **In Exercises 28 – 29, find the equation of the line tangent to the graph of $f$ at the indicated $x$ value.**

02 07 ex 27 28. $f(x) = \sin^{-1} x$ at $x = \frac{\sqrt{2}}{2}$

02 07 ex 28 29. $f(x) = \cos^{-1}(2x)$ at $x = \frac{\sqrt{3}}{4}$

## Review

02 07 ex 29 30. Find $\frac{dy}{dx}$, where $x^2y - y^2x = 1$.

02 07 ex 30 31. Find the equation of the line tangent to the graph of $x^2 + y^2 + xy = 7$ at the point $(1, 2)$.

02 07 ex 31 32. Let $f(x) = x^3 + x$.
Evaluate $\displaystyle\lim_{s \to 0} \frac{f(x+s) - f(x)}{s}$.

# Exercises 3.1

## *Terms and Concepts*

03 01 ex 01 1. Describe what an "extreme value" of a function is in your own words.

03 01 ex 02 2. Sketch the graph of a function $f$ on $(-1, 1)$ that has both a maximum and minimum value.

03 01 ex 03 3. Describe the difference between absolute and relative maxima in your own words.

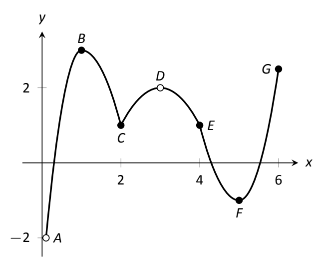
03 01 ex 04 4. Sketch the graph of a function $f$ where $f$ has a relative maximum at $x = 1$ and $f'(1)$ is undefined.

03 01 ex 05 5. T/F: If $c$ is a critical value of a function $f$, then $f$ has either a relative maximum or relative minimum at $x = c$.

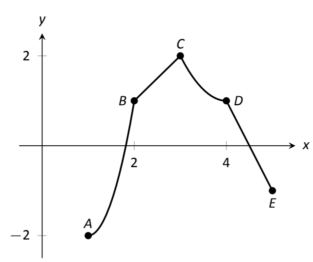
## *Problems*

03 01 exset 01 **In Exercises 6 – 7, identify each of the marked points as being an absolute maximum or minimum, a relative maximum or minimum, or none of the above.**

03 01 ex 06 6.

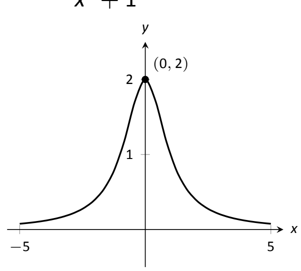



03 01 ex 07 7.

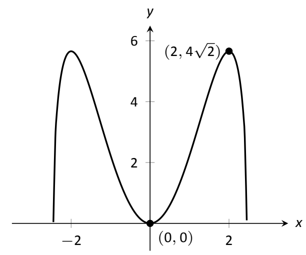



03 01 exset 02 **In Exercises 8 – 14, evaluate $f'(x)$ at the points indicated in the graph.**

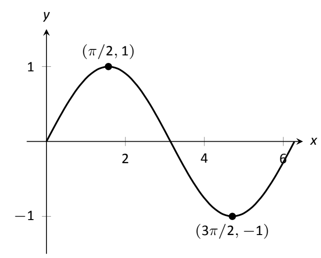
03 01 ex 08 8. $f(x) = \dfrac{2}{x^2 + 1}$



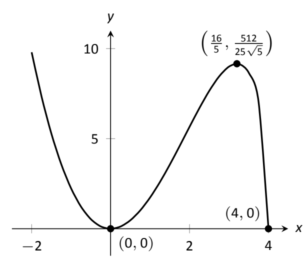

03 01 ex 09 9. $f(x) = x^2\sqrt{6 - x^2}$



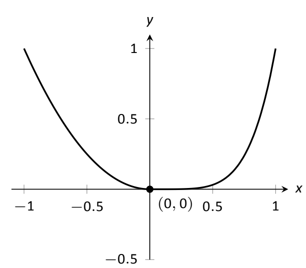

03 01 ex 10 10. $f(x) = \sin x$



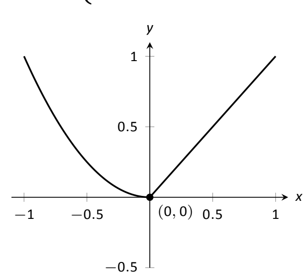

03 01 ex 11 11. $f(x) = x^2\sqrt{4 - x}$



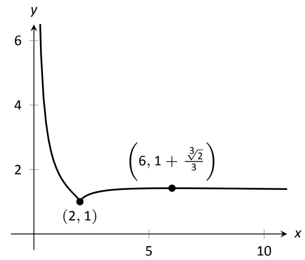

03 01 ex 12 12. $f(x) = \begin{cases} x^2 & x \leq 0 \\ x^5 & x > 0 \end{cases}$




03 01 ex 13 13. $f(x) = \begin{cases} x^2 & x \leq 0 \\ x & x > 0 \end{cases}$



03 01 ex 14 14. $f(x) = \dfrac{(x-2)^{2/3}}{x}$




03 01 exset 03 **In Exercises 15 – 24, find the extreme values of the function on the given interval.**

03 01 ex 15 15. $f(x) = x^2 + x + 4 \quad \text{on} \quad [-1, 2]$.

03 01 ex 16 16. $f(x) = x^3 - \dfrac{9}{2}x^2 - 30x + 3 \quad \text{on} \quad [0, 6]$.

03 01 ex 17 17. $f(x) = 3\sin x \quad \text{on} \quad [\pi/4, 2\pi/3]$.

03 01 ex 18 18. $f(x) = x^2\sqrt{4 - x^2} \quad \text{on} \quad [-2, 2]$.

03 01 ex 19 19. $f(x) = x + \dfrac{3}{x} \quad \text{on} \quad [1, 5]$.

03 01 ex 20 20. $f(x) = \dfrac{x^2}{x^2+5} \quad \text{on} \quad [-3, 5]$.

03 01 ex 21 21. $f(x) = e^x \cos x \quad \text{on} \quad [0, \pi]$.

03 01 ex 22 22. $f(x) = e^x \sin x \quad \text{on} \quad [0, \pi]$.

03 01 ex 23 23. $f(x) = \dfrac{\ln x}{x} \quad \text{on} \quad [1, 4]$.

03 01 ex 24 24. $f(x) = x^{2/3} - x \quad \text{on} \quad [0, 2]$.

## *Review*

02 07 ex 29 25. Find $\frac{dy}{dx}$, where $x^2y - y^2x = 1$.

02 07 ex 30 26. Find the equation of the line tangent to the graph of $x^2 + y^2 + xy = 7$ at the point $(1, 2)$.

02 07 ex 31 27. Let $f(x) = x^3 + x$.

Evaluate $\displaystyle\lim_{s \to 0} \frac{f(x+s) - f(x)}{s}$.

# Exercises 3.2

## *Terms and Concepts*

03 02 ex 01 1. Explain in your own words what the Mean Value Theorem states.

03 02 ex 02 2. Explain in your own words what Rolle's Theorem states.

## *Problems*

03 02 exset 01 **In Exercises 3 – 10, a function $f(x)$ and interval $[a, b]$ are given. Check if Rolle's Theorem can be applied to $f$ on $[a, b]$; if so, find $c$ in $[a, b]$ such that $f'(c) = 0$.**

03 02 ex 03 3. $f(x) = 6$ on $[-1, 1]$.

03 02 ex 04 4. $f(x) = 6x$ on $[-1, 1]$.

03 02 ex 05 5. $f(x) = x^2 + x - 6$ on $[-3, 2]$.

03 02 ex 06 6. $f(x) = x^2 + x - 2$ on $[-3, 2]$.

03 02 ex 07 7. $f(x) = x^2 + x$ on $[-2, 2]$.

03 02 ex 08 8. $f(x) = \sin x$ on $[\pi/6, 5\pi/6]$.

03 02 ex 09 9. $f(x) = \cos x$ on $[0, \pi]$.

03 02 ex 10 10. $f(x) = \dfrac{1}{x^2 - 2x + 1}$ on $[0, 2]$.

03 02 exset 02 **In Exercises 11 – 20, a function $f(x)$ and interval $[a, b]$ are given. Check if the Mean Value Theorem can be applied to $f$ on $[a, b]$; if so, find a value $c$ in $[a, b]$ guaranteed by the Mean Value Theorem.**

03 02 ex 11 11. $f(x) = x^2 + 3x - 1$ on $[-2, 2]$.

03 02 ex 12 12. $f(x) = 5x^2 - 6x + 8$ on $[0, 5]$.

03 02 ex 13 13. $f(x) = \sqrt{9 - x^2}$ on $[0, 3]$.

03 02 ex 14 14. $f(x) = \sqrt{25 - x}$ on $[0, 9]$.

03 02 ex 20 15. $f(x) = \dfrac{x^2 - 9}{x^2 - 1}$ on $[0, 2]$.

03 02 ex 15 16. $f(x) = \ln x$ on $[1, 5]$.

03 02 ex 16 17. $f(x) = \tan x$ on $[-\pi/4, \pi/4]$.

03 02 ex 17 18. $f(x) = x^3 - 2x^2 + x + 1$ on $[-2, 2]$.

03 02 ex 18 19. $f(x) = 2x^3 - 5x^2 + 6x + 1$ on $[-5, 2]$.

03 02 ex 19 20. $f(x) = \sin^{-1} x$ on $[-1, 1]$.

## *Review*

03 02 ex 21 21. Find the extreme values of $f(x) = x^2 - 3x + 9$ on $[-2, 5]$.

03 02 ex 22 22. Describe the critical points of $f(x) = \cos x$.

03 02 ex 23 23. Describe the critical points of $f(x) = \tan x$.

# Exercises 3.3

## *Terms and Concepts*

03 03 ex 01 1. In your own words describe what it means for a function to be increasing.

03 03 ex 02 2. What does a decreasing function "look like"?

03 03 ex 03 3. Sketch a graph of a function on $[0, 2]$ that is increasing but not strictly increasing.

03 03 ex 04 4. Give an example of a function describing a situation where it is "bad" to be increasing and "good" to be decreasing.

03 03 ex 05 5. A function $f$ has derivative $f'(x) = (\sin x + 2)e^{x^2+1}$, where $f'(x) > 1$ for all $x$. Is $f$ increasing, decreasing, or can we not tell from the given information?

## *Problems*

03 03 exset 01 **In Exercises 6 – 13, a function $f(x)$ is given.**

**(a) Compute $f'(x)$.**

**(b) Graph $f$ and $f'$ on the same axes (using technology is permitted) and verify Theorem 29.**

03 03 ex 06 6. $f(x) = 2x + 3$

03 03 ex 07 7. $f(x) = x^2 - 3x + 5$

03 03 ex 08 8. $f(x) = \cos x$

03 03 ex 09 9. $f(x) = \tan x$

03 03 ex 10 10. $f(x) = x^3 - 5x^2 + 7x - 1$

03 03 ex 11 11. $f(x) = 2x^3 - x^2 + x - 1$

03 03 ex 12 12. $f(x) = x^4 - 5x^2 + 4$

03 03 ex 13 13. $f(x) = \dfrac{1}{x^2 + 1}$

03 03 exset 02 **In Exercises 14 – 23, a function $f(x)$ is given.**

**(a) Give the domain of $f$.**

**(b) Find the critical numbers of $f$.**

**(c) Create a number line to determine the intervals on which $f$ is increasing and decreasing.**

**(d) Use the First Derivative Test to determine whether each critical point is a relative maximum, minimum, or neither.**

03 03 ex 14 14. $f(x) = x^2 + 2x - 3$

03 03 ex 15 15. $f(x) = x^3 + 3x^2 + 3$

03 03 ex 16 16. $f(x) = 2x^3 + x^2 - x + 3$

03 03 ex 17 17. $f(x) = x^3 - 3x^2 + 3x - 1$

03 03 ex 18 18. $f(x) = \dfrac{1}{x^2 - 2x + 2}$

03 03 ex 19 19. $f(x) = \dfrac{x^2 - 4}{x^2 - 1}$

03 03 ex 20 20. $f(x) = \dfrac{x}{x^2 - 2x - 8}$

03 03 ex 21 21. $f(x) = \dfrac{(x - 2)^{2/3}}{x}$

03 03 ex 22 22. $f(x) = \sin x \cos x$ on $(-\pi, \pi)$.

03 03 ex 23 23. $f(x) = x^5 - 5x$

## *Review*

03 03 ex 24 24. Consider $f(x) = x^2 - 3x + 5$ on $[-1, 2]$; find $c$ guaranteed by the Mean Value Theorem.

03 03 ex 25 25. Consider $f(x) = \sin x$ on $[-\pi/2, \pi/2]$; find $c$ guaranteed by the Mean Value Theorem.

# Exercises 3.4

## *Terms and Concepts*

03 04 ex 01 1. Sketch a graph of a function $f(x)$ that is concave up on $(0,1)$ and is concave down on $(1,2)$.

03 04 ex 02 2. Sketch a graph of a function $f(x)$ that is:

(a) Increasing, concave up on $(0,1)$,

(b) increasing, concave down on $(1,2)$,

(c) decreasing, concave down on $(2,3)$ and

(d) increasing, concave down on $(3,4)$.

03 04 ex 03 3. Is is possible for a function to be increasing and concave down on $(0,\infty)$ with a horizontal asymptote of $y=1$? If so, give a sketch of such a function.

03 04 ex 04 4. Is is possible for a function to be increasing and concave up on $(0,\infty)$ with a horizontal asymptote of $y=1$? If so, give a sketch of such a function.

## *Problems*

03 04 exset 01 **In Exercises 5 – 15, a function $f(x)$ is given.**

**(a) Compute $f''(x)$.**

**(b) Graph $f$ and $f''$ on the same axes (using technology is permitted) and verify Theorem 31.**

03 04 ex 05 5. $f(x) = -7x+3$

03 04 ex 06 6. $f(x) = -4x^2+3x-8$

03 04 ex 07 7. $f(x) = 4x^2+3x-8$

03 04 ex 08 8. $f(x) = x^3-3x^2+x-1$

03 04 ex 09 9. $f(x) = -x^3+x^2-2x+5$

03 04 ex 10 10. $f(x) = \cos x$

03 04 ex 11 11. $f(x) = \sin x$

03 04 ex 12 12. $f(x) = \tan x$

03 04 ex 13 13. $f(x) = \dfrac{1}{x^2+1}$

03 04 ex 14 14. $f(x) = \dfrac{1}{x}$

03 04 ex 15 15. $f(x) = \dfrac{1}{x^2}$

03 04 exset 02 **In Exercises 16 – 28, a function $f(x)$ is given.**

**(a) Find the possible points of inflection of $f$.**

**(b) Create a number line to determine the intervals on which $f$ is concave up or concave down.**

03 04 ex 16 16. $f(x) = x^2-2x+1$

03 04 ex 17 17. $f(x) = -x^2-5x+7$

03 04 ex 18 18. $f(x) = x^3-x+1$

03 04 ex 19 19. $f(x) = 2x^3-3x^2+9x+5$

03 04 ex 20 20. $f(x) = \dfrac{x^4}{4}+\dfrac{x^3}{3}-2x+3$

03 04 ex 21 21. $f(x) = -3x^4+8x^3+6x^2-24x+2$

03 04 ex 22 22. $f(x) = x^4-4x^3+6x^2-4x+1$

03 04 ex 23 23. $f(x) = \dfrac{1}{x^2+1}$

03 04 ex 24 24. $f(x) = \dfrac{x}{x^2-1}$

03 04 ex 25 25. $f(x) = \sin x+\cos x$ on $(-\pi,\pi)$

03 04 ex 26 26. $f(x) = x^2e^x$

03 04 ex 27 27. $f(x) = x^2\ln x$

03 04 ex 28 28. $f(x) = e^{-x^2}$

03 04 exset 03 **In Exercises 29 – 41, a function $f(x)$ is given. Find the critical points of $f$ and use the Second Derivative Test, when possible, to determine the relative extrema. (Note: these are the same functions as in Exercises 16 – 28.)**

03 04 ex 29 29. $f(x) = x^2-2x+1$

03 04 ex 30 30. $f(x) = -x^2-5x+7$

03 04 ex 31 31. $f(x) = x^3-x+1$

03 04 ex 32 32. $f(x) = 2x^3-3x^2+9x+5$

03 04 ex 33 33. $f(x) = \dfrac{x^4}{4}+\dfrac{x^3}{3}-2x+3$

03 04 ex 34 34. $f(x) = -3x^4+8x^3+6x^2-24x+2$

03 04 ex 35 35. $f(x) = x^4-4x^3+6x^2-4x+1$

03 04 ex 36 36. $f(x) = \dfrac{1}{x^2+1}$

03 04 ex 37 37. $f(x) = \dfrac{x}{x^2 - 1}$

03 04 ex 38 38. $f(x) = \sin x + \cos x$ on $(-\pi, \pi)$

03 04 ex 39 39. $f(x) = x^2 e^x$

03 04 ex 40 40. $f(x) = x^2 \ln x$

03 04 ex 41 41. $f(x) = e^{-x^2}$

03 04 exset 04 **In Exercises 42 – 54, a function $f(x)$ is given. Find the $x$ values where $f'(x)$ has a relative maximum or minimum. (Note: these are the same functions as in Exercises 16 – 28.)**

03 04 ex 42 42. $f(x) = x^2 - 2x + 1$

03 04 ex 43 43. $f(x) = -x^2 - 5x + 7$

03 04 ex 44 44. $f(x) = x^3 - x + 1$

03 04 ex 45 45. $f(x) = 2x^3 - 3x^2 + 9x + 5$

03 04 ex 46 46. $f(x) = \dfrac{x^4}{4} + \dfrac{x^3}{3} - 2x + 3$

03 04 ex 47 47. $f(x) = -3x^4 + 8x^3 + 6x^2 - 24x + 2$

03 04 ex 48 48. $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$

03 04 ex 49 49. $f(x) = \dfrac{1}{x^2 + 1}$

03 04 ex 50 50. $f(x) = \dfrac{x}{x^2 - 1}$

03 04 ex 51 51. $f(x) = \sin x + \cos x$ on $(-\pi, \pi)$

03 04 ex 52 52. $f(x) = x^2 e^x$

03 04 ex 53 53. $f(x) = x^2 \ln x$

03 04 ex 54 54. $f(x) = e^{-x^2}$

# Exercises 3.5

## *Terms and Concepts*

03 05 ex 01 1. Why is sketching curves by hand beneficial even though technology is ubiquitous?

03 05 ex 02 2. What does "ubiquitous" mean?

03 05 ex 03 3. T/F: When sketching graphs of functions, it is useful to find the critical points.

03 05 ex 04 4. T/F: When sketching graphs of functions, it is useful to find the possible points of inflection.

03 05 ex 05 5. T/F: When sketching graphs of functions, it is useful to find the horizontal and vertical asymptotes.

## *Problems*

03 05 exset 01 **In Exercises 6 – 11, practice using Key Idea 4 by applying the principles to the given functions with familiar graphs.**

03 05 ex 06 6. $f(x) = 2x + 4$

03 05 ex 07 7. $f(x) = -x^2 + 1$

03 05 ex 08 8. $f(x) = \sin x$

03 05 ex 09 9. $f(x) = e^x$

03 05 ex 10 10. $f(x) = \dfrac{1}{x}$

03 05 ex 11 11. $f(x) = \dfrac{1}{x^2}$

03 05 exset 02 **In Exercises 12 – 25, sketch a graph of the given function using Key Idea 4. Show all work; check your answer with technology.**

03 05 ex 12 12. $f(x) = x^3 - 2x^2 + 4x + 1$

03 05 ex 13 13. $f(x) = -x^3 + 5x^2 - 3x + 2$

03 05 ex 14 14. $f(x) = x^3 + 3x^2 + 3x + 1$

03 05 ex 15 15. $f(x) = x^3 - x^2 - x + 1$

03 05 ex 16 16. $f(x) = (x-2)\ln(x-2)$

03 05 ex 17 17. $f(x) = (x-2)^2\ln(x-2)$

03 05 ex 18 18. $f(x) = \dfrac{x^2-4}{x^2}$

03 05 ex 19 19. $f(x) = \dfrac{x^2-4x+3}{x^2-6x+8}$

03 05 ex 20 20. $f(x) = \dfrac{x^2-2x+1}{x^2-6x+8}$

03 05 ex 21 21. $f(x) = x\sqrt{x+1}$

03 05 ex 22 22. $f(x) = x^2e^x$

03 05 ex 23 23. $f(x) = \sin x\cos x$ on $[-\pi, \pi]$

03 05 ex 24 24. $f(x) = (x-3)^{2/3} + 2$

03 05 ex 25 25. $f(x) = \dfrac{(x-1)^{2/3}}{x}$

03 05 exset 03 **In Exercises 26 – 28, a function with the parameters *a* and *b* are given. Describe the critical points and possible points of inflection of *f* in terms of *a* and *b*.**

03 05 ex 26 26. $f(x) = \dfrac{a}{x^2+b^2}$

03 05 ex 27 27. $f(x) = \sin(ax+b)$

03 05 ex 28 28. $f(x) = (x-a)(x-b)$

03 05 ex 29 29. Given $x^2 + y^2 = 1$, use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Use this information to justify the sketch of the unit circle.

# Exercises 4.1

## *Terms and Concepts*

04 01 ex 01 1. T/F: Given a function $f(x)$, Newton's Method produces an exact solution to $f(x) = 0$.

04 01 ex 02 2. T/F: In order to get a solution to $f(x) = 0$ accurate to $d$ places after the decimal, at least $d + 1$ iterations of Newtons' Method must be used.

## *Problems*

04 01 exset 01 **In Exercises 3 – 7, the roots of $f(x)$ are known or are easily found. Use 5 iterations of Newton's Method with the given initial approximation to approximate the root. Compare it to the known value of the root.**

04 01 ex 03 3. $f(x) = \cos x$, $x_0 = 1.5$

04 01 ex 04 4. $f(x) = \sin x$, $x_0 = 1$

04 01 ex 05 5. $f(x) = x^2 + x - 2$, $x_0 = 0$

04 01 ex 06 6. $f(x) = x^2 - 2$, $x_0 = 1.5$

04 01 ex 07 7. $f(x) = \ln x$, $x_0 = 2$

04 01 exset 02 **In Exercises 8 – 11, use Newton's Method to approximate all roots of the given functions accurate to 3 places after the decimal. If an interval is given, find only the roots that lie in that interval. Use technology to obtain good initial approximations.**

04 01 ex 09 8. $f(x) = x^3 + 5x^2 - x - 1$

04 01 ex 10 9. $f(x) = x^4 + 2x^3 - 7x^2 - x + 5$

04 01 ex 11 10. $f(x) = x^{17} - 2x^{13} - 10x^8 + 10$ on $(-2, 2)$

04 01 ex 12 11. $f(x) = x^2 \cos x + (x - 1)\sin x$ on $(-3, 3)$

04 01 exset 03 **In Exercises 12 – 15, use Newton's Method to approximate when the given functions are equal, accurate to 3 places after the decimal. Use technology to obtain good initial approximations.**

04 01 ex 13 12. $f(x) = x^2$, $g(x) = \cos x$

04 01 ex 14 13. $f(x) = x^2 - 1$, $g(x) = \sin x$

04 01 ex 15 14. $f(x) = e^{x^2}$, $g(x) = \cos x$

04 01 ex 16 15. $f(x) = x$, $g(x) = \tan x$ on $[-6, 6]$

04 01 ex 17 16. Why does Newton's Method fail in finding a root of $f(x) = x^3 - 3x^2 + x + 3$ when $x_0 = 1$?

04 01 ex 18 17. Why does Newton's Method fail in finding a root of $f(x) = -17x^4 + 130x^3 - 301x^2 + 156x + 156$ when $x_0 = 1$?

# Exercises 4.2

## *Terms and Concepts*

04 02 ex 01 1. T/F: Implicit differentiation is often used when solving "related rates" type problems.

04 02 ex 02 2. T/F: A study of related rates is part of the standard police officer training.

## *Problems*

04 02 ex 03 3. Water flows onto a flat surface at a rate of 5cm$^3$/s forming a circular puddle 10mm deep. How fast is the radius growing when the radius is:

   (a) 1 cm?
   (b) 10 cm?
   (c) 100 cm?

04 02 ex 04 4. A circular balloon is inflated with air flowing at a rate of 10cm$^3$/s. How fast is the radius of the balloon increasing when the radius is:

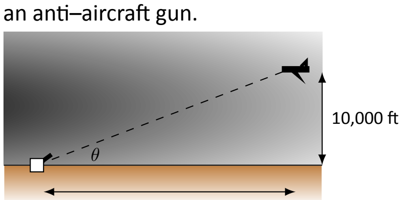
   (a) 1 cm?
   (b) 10 cm?
   (c) 100 cm?

04 02 ex 05 5. Consider the traffic situation introduced in Example 100. How fast is the "other car" traveling if the officer and the other car are each 1/2 mile from the intersection, the other car is traveling *due west*, the officer is traveling north at 50mph, and the radar reading is $-80$mph?

04 02 ex 06 6. Consider the traffic situation introduced in Example 100. Calculate how fast the "other car" is traveling in each of the following situations.

   (a) The officer is traveling due north at 50mph and is 1/2 mile from the intersection, while the other car is 1 mile from the intersection traveling west and the radar reading is $-80$mph?
   (b) The officer is traveling due north at 50mph and is 1 mile from the intersection, while the other car is 1/2 mile from the intersection traveling west and the radar reading is $-80$mph?

04 02 ex 07 7. An F-22 aircraft is flying at 500mph with an elevation of 10,000ft on a straight–line path that will take it directly over an anti–aircraft gun.




How fast must the gun be able to turn to accurately track the aircraft when the plane is:

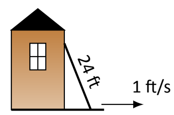
   (a) 1 mile away?
   (b) 1/5 mile away?
   (c) Directly overhead?

04 02 ex 08 8. An F-22 aircraft is flying at 500mph with an elevation of 100ft on a straight–line path that will take it directly over an anti–aircraft gun as in Exercise 7 (note the lower elevation here).

How fast must the gun be able to turn to accurately track the aircraft when the plane is:

   (a) 1000 feet away?
   (b) 100 feet away?
   (c) Directly overhead?

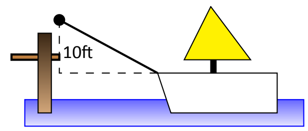
04 02 ex 09 9. A 24ft. ladder is leaning against a house while the base is pulled away at a constant rate of 1ft/s.




At what rate is the top of the ladder sliding down the side of the house when the base is:

   (a) 1 foot from the house?
   (b) 10 feet from the house?
   (c) 23 feet from the house?
   (d) 24 feet from the house?

04 02 ex 10 10. A boat is being pulled into a dock at a constant rate of 30ft/min by a winch located 10ft above the deck of the boat.




At what rate is the boat approaching the dock when the boat is:

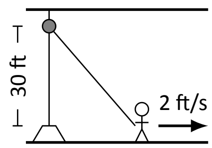
   (a) 50 feet out?
   (b) 15 feet out?
   (c) 1 foot from the dock?
   (d) What happens when the length of rope pulling in the boat is less than 10 feet long?

04 02 ex 11 11. An inverted cylindrical cone, 20ft deep and 10ft across at the top, is being filled with water at a rate of 10ft$^3$/min. At what rate is the water rising in the tank when the depth of the water is:

   (a) 1 foot?
   (b) 10 feet?
   (c) 19 feet?

How long will the tank take to fill when starting at empty?

04 02 ex 12 12. A rope, attached to a weight, goes up through a pulley at the ceiling and back down to a worker. The man holds the rope at the same height as the connection point between rope and weight.




Suppose the man stands directly next to the weight (i.e., a total rope length of 60 ft) and begins to walk away at a rate of 2ft/s. How fast is the weight rising when the man has walked:

(a) 10 feet?

(b) 40 feet?

How far must the man walk to raise the weight all the way to the pulley?

04 02 ex 13 13. Consider the situation described in Exercise 12. Suppose the man starts 40ft from the weight and begins to walk away at a rate of 2ft/s.

(a) How long is the rope?

(b) How fast is the weight rising after the man has walked 10 feet?

(c) How fast is the weight rising after the man has walked 40 feet?

(d) How far must the man walk to raise the weight all the way to the pulley?

04 02 ex 14 14. A hot air balloon lifts off from ground rising vertically. From 100 feet away, a 5' woman tracks the path of the balloon. When her sightline with the balloon makes a $45^\circ$ angle with the horizontal, she notes the angle is increasing at about $5^\circ$/min.

(a) What is the elevation of the balloon?

(b) How fast is it rising?

04 02 ex 15 15. A company that produces landscaping materials is dumping sand into a conical pile. The sand is being poured at a rate of $5\text{ft}^3$/sec; the physical properties of the sand, in conjunction with gravity, ensure that the cone's height is roughly 2/3 the length of the diameter of the circular base.

How fast is the cone rising when it has a height of 30 feet?

# Exercises 4.3

## *Terms and Concepts*

04 03 ex 01 1. T/F: An "optimization problem" is essentially an "extreme values" problem in a "story problem" setting.

04 03 ex 02 2. T/F: This section teaches one to find the extreme values of function that have more than one variable.

## *Problems*

04 03 ex 03 3. Find the maximum product of two numbers (not necessarily integers) that have a sum of 100.

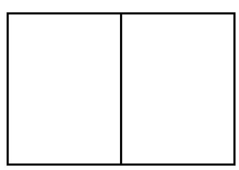
04 03 ex 04 4. Find the minimum sum of two numbers whose product is 500.

04 03 ex 05 5. Find the maximum sum of two numbers whose product is 500.

04 03 ex 06 6. Find the maximum sum of two numbers, each of which is in $[0, 300]$ whose product is 500.

04 03 ex 07 7. Find the maximal area of a right triangle with hypotenuse of length 1.

04 03 ex 08 8. A rancher has 1000 feet of fencing in which to construct adjacent, equally sized rectangular pens. What dimensions should these pens have to maximize the enclosed area?



04 03 ex 09 9. A standard soda can is roughly cylindrical and holds 355cm$^3$ of liquid. What dimensions should the cylinder be to minimize the material needed to produce the can? Based on your dimensions, determine whether or not the standard can is produced to minimize the material costs.

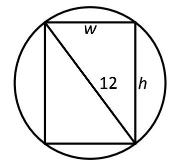
04 03 ex 10 10. Find the dimensions of a cylindrical can with a volume of 206in$^3$ that minimizes the surface area.

The "#10 can"is a standard sized can used by the restaurant industry that holds about 206in$^3$ with a diameter of 6 2/16in and height of 7in. Does it seem these dimensions where chosen with minimization in mind?

04 03 ex 11 11. The United States Postal Service charges more for boxes whose combined length and girth exceeds 108" (the "length" of a package is the length of its longest side; the girth is the perimeter of the cross section, i.e., $2w + 2h$).

What is the maximum volume of a package with a square cross section ($w = h$) that does not exceed the 108" standard?

04 03 ex 12 12. The strength $S$ of a wooden beam is directly proportional to its cross sectional width $w$ and the square of its height $h$; that is, $S = kwh^2$ for some constant $k$.




Given a circular log with diameter of 12 inches, what sized beam can be cut from the log with maximum strength?

04 03 ex 13 13. A power line is to be run to an offshore facility in the manner described in Example 104. The offshore facility is 2 miles at sea and 5 miles along the shoreline from the power plant. It costs \$50,000 per mile to lay a power line underground and \$80,000 to run the line underwater.

How much of the power line should be run underground to minimize the overall costs?

04 03 ex 14 14. A power line is to be run to an offshore facility in the manner described in Example 104. The offshore facility is 5 miles at sea and 2 miles along the shoreline from the power plant. It costs \$50,000 per mile to lay a power line underground and \$80,000 to run the line underwater.

How much of the power line should be run underground to minimize the overall costs?

04 03 ex 15 15. A woman throws a stick into a lake for her dog to fetch; the stick is 20 feet down the shore line and 15 feet into the water from there. The dog may jump directly into the water and swim, or run along the shore line to get closer to the stick before swimming. The dog runs about 22ft/s and swims about 1.5ft/s.

How far along the shore should the dog run to minimize the time it takes to get to the stick? (Hint: the figure from Example 104 can be useful.)

04 03 ex 16 16. A woman throws a stick into a lake for her dog to fetch; the stick is 15 feet down the shore line and 30 feet into the water from there. The dog may jump directly into the water and swim, or run along the shore line to get closer to the stick before swimming. The dog runs about 22ft/s and swims about 1.5ft/s.

How far along the shore should the dog run to minimize the time it takes to get to the stick? *(Google "calculus dog" to learn more about a dog's ability to minimize times.)*

04 03 ex 17 17. What are the dimensions of the rectangle with largest area that can be drawn inside the unit circle?

# Exercises 4.4

## *Terms and Concepts*

04 04 ex 01 1. T/F: Given a differentiable function $y = f(x)$, we are generally free to choose a value for $dx$, which then determines the value of $dy$.

04 04 ex 02 2. T/F: The symbols "$dx$" and "$\Delta x$" represent the same concept.

04 04 ex 03 3. T/F: The symbols "$dy$" and "$\Delta y$" represent the same concept.

04 04 ex 04 4. T/F: Differentials are important in the study of integration.

04 04 ex 05 5. How are differentials and tangent lines related?

## *Problems*

04 04 exset 01 **In Exercises 6 – 17, use differentials to approximate the given value by hand.**

04 04 ex 06 6. $2.05^2$

04 04 ex 07 7. $5.93^2$

04 04 ex 08 8. $5.1^3$

04 04 ex 09 9. $6.8^3$

04 04 ex 10 10. $\sqrt{16.5}$

04 04 ex 11 11. $\sqrt{24}$

04 04 ex 12 12. $\sqrt[3]{63}$

04 04 ex 13 13. $\sqrt[3]{8.5}$

04 04 ex 14 14. $\sin 3$

04 04 ex 15 15. $\cos 1.5$

04 04 ex 16 16. $e^{0.1}$

04 04 exset 02 **In Exercises 17 – 29, compute the differential $dy$.**

04 04 ex 17 17. $y = x^2 + 3x - 5$

04 04 ex 18 18. $y = x^7 - x^5$

04 04 ex 19 19. $y = \dfrac{1}{4x^2}$

04 04 ex 20 20. $y = (2x + \sin x)^2$

04 04 ex 21 21. $y = x^2 e^{3x}$

04 04 ex 22 22. $y = \dfrac{4}{x^4}$

04 04 ex 23 23. $y = \dfrac{2x}{\tan x + 1}$

04 04 ex 24 24. $y = \ln(5x)$

04 04 ex 25 25. $y = e^x \sin x$

04 04 ex 26 26. $y = \cos(\sin x)$

04 04 ex 27 27. $y = \dfrac{x+1}{x+2}$

04 04 ex 28 28. $y = 3^x \ln x$

04 04 ex 29 29. $y = x \ln x - x$

04 04 ex 30 30. A set of plastic spheres are to be made with a diameter of 1cm. If the manufacturing process is accurate to 1mm, what is the propagated error in volume of the spheres?

04 04 ex 31 31. The distance, in feet, a stone drops in $t$ seconds is given by $d(t) = 16t^2$. The depth of a hole is to be approximated by dropping a rock and listening for it to hit the bottom. What is the propagated error if the time measurement is accurate to $2/10^{\text{ths}}$ of a second and the measured time is:

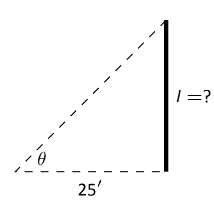
(a) 2 seconds?
(b) 5 seconds?

04 04 ex 32 32. What is the propagated error in the measurement of the cross sectional area of a circular log if the diameter is measured at $15''$, accurate to $1/4''$?

04 04 ex 33 33. A wall is to be painted that is $8'$ high and is measured to be $10'$, $7''$ long. Find the propagated error in the measurement of the wall's surface area if the measurement is accurate to $1/2''$.

04 04 exset 03 **Exercises 34 – 38 explore some issues related to surveying in which distances are approximated using other measured distances and measured angles. *(Hint: Convert all angles to radians before computing.)***

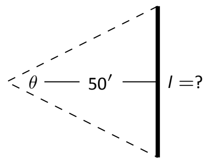
04 04 ex 35 34. The length $l$ of a long wall is to be approximated. The angle $\theta$, as shown in the diagram (not to scale), is measured to be $85.2^\circ$, accurate to $1^\circ$. Assume that the triangle formed is a right triangle.




(a) What is the measured length $l$ of the wall?
(b) What is the propagated error?
(c) What is the percent error?

04 04 ex 36 35. Answer the questions of Exercise 34, but with a measured angle of $71.5^\circ$, accurate to $1^\circ$, measured from a point $100'$ from the wall.

04 04 ex 34 36. The length $l$ of a long wall is to be calculated by measuring the angle $\theta$ shown in the diagram (not to scale). Assume the formed triangle is an isosceles triangle. The measured angle is $143^\circ$, accurate to $1^\circ$.




(a) What is the measured length of the wall?

(b) What is the propagated error?

(c) What is the percent error?

04 04 ex 37 37. The length of the walls in Exercises 34 – 36 are essentially the same. Which setup gives the most accurate result?

04 04 ex 38 38. Consider the setup in Exercises 36. This time, assume the angle measurement of $143^\circ$ is exact but the measured $50'$ from the wall is accurate to $6''$. What is the approximate percent error?

# Exercises 5.1

## *Terms and Concepts*

05 01 ex 01 1. Define the term “antiderivative” in your own words.

05 01 ex 02 2. Is it more accurate to refer to “the” antiderivative of $f(x)$ or “an” antiderivative of $f(x)$?

05 01 ex 03 3. Use your own words to define the indefinite integral of $f(x)$.

05 01 ex 04 4. Fill in the blanks: “Inverse operations do the ________ things in the ________ order.”

05 01 ex 05 5. What is an “initial value problem”?

05 01 ex 06 6. The derivative of a position function is a ________ function.

05 01 ex 07 7. The antiderivative of an acceleration function is a ________ function.

## *Problems*

05 01 exset 01 **In Exercises 8 – 26, evaluate the given indefinite integral.**

05 01 ex 08 8. $\displaystyle\int 3x^3\,dx$

05 01 ex 09 9. $\displaystyle\int x^8\,dx$

05 01 ex 10 10. $\displaystyle\int (10x^2 - 2)\,dx$

05 01 ex 11 11. $\displaystyle\int dt$

05 01 ex 12 12. $\displaystyle\int 1\,ds$

05 01 ex 13 13. $\displaystyle\int \frac{1}{3t^2}\,dt$

05 01 ex 14 14. $\displaystyle\int \frac{3}{t^2}\,dt$

05 01 ex 15 15. $\displaystyle\int \frac{1}{\sqrt{x}}\,dx$

05 01 ex 16 16. $\displaystyle\int \sec^2\theta\,d\theta$

05 01 ex 17 17. $\displaystyle\int \sin\theta\,d\theta$

05 01 ex 18 18. $\displaystyle\int (\sec x\tan x + \csc x\cot x)\,dx$

05 01 ex 19 19. $\displaystyle\int 5e^\theta\,d\theta$

05 01 ex 20 20. $\displaystyle\int 3^t\,dt$

05 01 ex 21 21. $\displaystyle\int \frac{5^t}{2}\,dt$

05 01 ex 22 22. $\displaystyle\int (2t+3)^2\,dt$

05 01 ex 23 23. $\displaystyle\int (t^2+3)(t^3-2t)\,dt$

05 01 ex 24 24. $\displaystyle\int x^2x^3\,dx$

05 01 ex 25 25. $\displaystyle\int e^\pi\,dx$

05 01 ex 26 26. $\displaystyle\int a\,dx$

05 01 ex 39 27. This problem investigates why Theorem 35 states that $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$.

(a) What is the domain of $y = \ln x$?

(b) Find $\frac{d}{dx}\big(\ln x\big)$.

(c) What is the domain of $y = \ln(-x)$?

(d) Find $\frac{d}{dx}\big(\ln(-x)\big)$.

(e) You should find that $1/x$ has two types of antiderivatives, depending on whether $x > 0$ or $x < 0$. In one expression, give a formula for $\displaystyle\int \frac{1}{x}\,dx$ that takes these different domains into account, and explain your answer.

05 01 exset 02 **In Exercises 28 – 38, find $f(x)$ described by the given initial value problem.**

05 01 ex 28 28. $f'(x) = \sin x$ and $f(0) = 2$

05 01 ex 29 29. $f'(x) = 5e^x$ and $f(0) = 10$

05 01 ex 30 30. $f'(x) = 4x^3 - 3x^2$ and $f(-1) = 9$

05 01 ex 31 31. $f'(x) = \sec^2 x$ and $f(\pi/4) = 5$

05 01 ex 32 32. $f'(x) = 7^x$ and $f(2) = 1$

05 01 ex 33 33. $f''(x) = 5$ and $f'(0) = 7, f(0) = 3$

05 01 ex 34 34. $f''(x) = 7x$ and $f'(1) = -1, f(1) = 10$

05 01 ex 35 35. $f''(x) = 5e^x$ and $f'(0) = 3, f(0) = 5$

05 01 ex 36 36. $f''(\theta) = \sin\theta$ and $f'(\pi) = 2, f(\pi) = 4$

05 01 ex 37 37. $f''(x) = 24x^2 + 2^x - \cos x$ and $f'(0) = 5, f(0) = 0$

05 01 ex 38 38. $f''(x) = 0$ and $f'(1) = 3, f(1) = 1$

## *Review*

05 01 ex 40 39. Use information gained from the first and second derivatives to sketch $f(x) = \dfrac{1}{e^x + 1}$.

05 01 ex 41 40. Given $y = x^2 e^x \cos x$, find $dy$.

# Exercises 5.2

## *Terms and Concepts*

05 02 ex 01 1. What is "total signed area"?

05 02 ex 02 2. What is "displacement"?

05 02 ex 03 3. What is $\int_3^3 \sin x\, dx$?

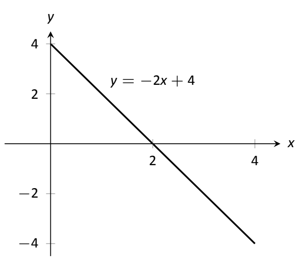
05 02 ex 04 4. Give a single definite integral that has the same value as

$$\int_0^1 (2x+3)\, dx + \int_1^2 (2x+3)\, dx.$$

## *Problems*

05 02 exset 01 **In Exercises 5 – 9, a graph of a function $f(x)$ is given. Using the geometry of the graph, evaluate the definite integrals.**

05 02 ex 05 5.




(a) $\int_0^1 (-2x+4)\, dx$

(b) $\int_0^2 (-2x+4)\, dx$

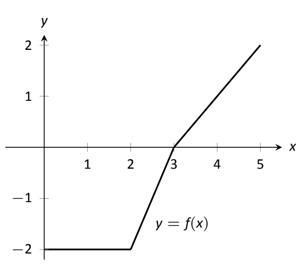
(c) $\int_0^3 (-2x+4)\, dx$

(d) $\int_1^3 (-2x+4)\, dx$

(e) $\int_2^4 (-2x+4)\, dx$

(f) $\int_0^1 (-6x+12)\, dx$

05 02 ex 06 6.




(a) $\int_0^2 f(x)\, dx$

(b) $\int_0^3 f(x)\, dx$

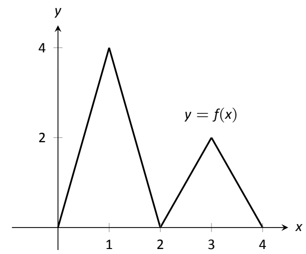
(c) $\int_0^5 f(x)\, dx$

(d) $\int_2^5 f(x)\, dx$

(e) $\int_5^3 f(x)\, dx$

(f) $\int_0^3 -2f(x)\, dx$

05 02 ex 07 7.




(a) $\int_0^2 f(x)\, dx$

(b) $\int_2^4 f(x)\, dx$

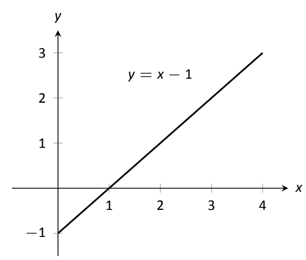
(c) $\int_2^4 2f(x)\, dx$

(d) $\int_0^1 4x\, dx$

(e) $\int_2^3 (2x-4)\, dx$

(f) $\int_2^3 (4x-8)\, dx$

05 02 ex 08 8.




(a) $\int_0^1 (x-1)\, dx$

(b) $\int_0^2 (x-1)\, dx$

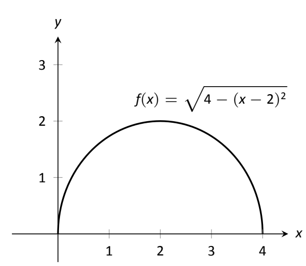
(c) $\int_0^3 (x-1)\, dx$

(d) $\int_2^3 (x-1)\, dx$

(e) $\int_1^4 (x-1)\, dx$

(f) $\int_1^4 \big((x-1)+1\big)\, dx$

05 02 ex 09 9.



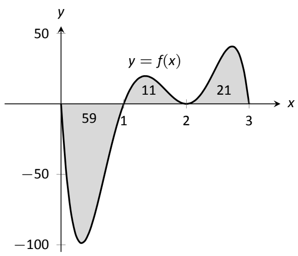

(a) $\int_0^2 f(x)\, dx$

(b) $\int_2^4 f(x)\, dx$

(c) $\int_0^4 f(x)\, dx$

(d) $\int_0^4 5f(x)\, dx$

05 02 exset 02 **In Exercises 10 – 13, a graph of a function $f(x)$ is given; the numbers inside the shaded regions give the area of that region. Evaluate the definite integrals using this area information.**

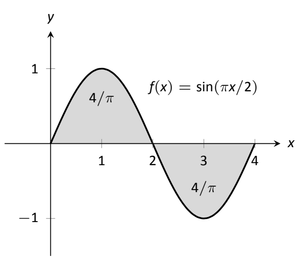
05 02 ex 10 10. 


(a) $\displaystyle\int_0^1 f(x)\,dx$

(b) $\displaystyle\int_0^2 f(x)\,dx$

(c) $\displaystyle\int_0^3 f(x)\,dx$

(d) $\displaystyle\int_1^2 -3f(x)\,dx$

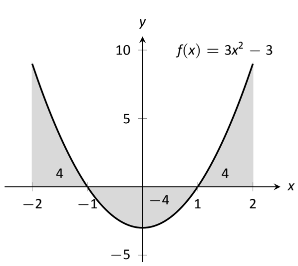
05 02 ex 11 11. 


(a) $\displaystyle\int_0^2 f(x)\,dx$

(b) $\displaystyle\int_2^4 f(x)\,dx$

(c) $\displaystyle\int_0^4 f(x)\,dx$

(d) $\displaystyle\int_0^1 f(x)\,dx$

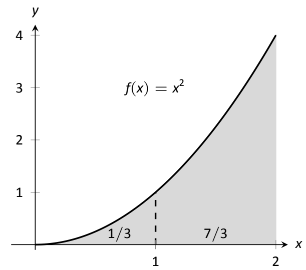
05 02 ex 12 12. 


(a) $\displaystyle\int_{-2}^{-1} f(x)\,dx$

(b) $\displaystyle\int_1^2 f(x)\,dx$

(c) $\displaystyle\int_{-1}^1 f(x)\,dx$

(d) $\displaystyle\int_0^1 f(x)\,dx$

05 02 ex 13 13. 

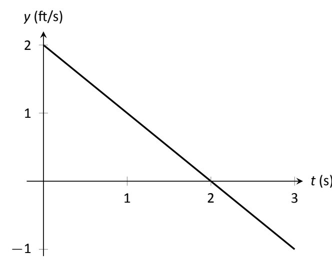

(a) $\displaystyle\int_0^2 5x^2\,dx$

(b) $\displaystyle\int_0^2 (x^2+3)\,dx$

(c) $\displaystyle\int_1^3 (x-1)^2\,dx$

(d) $\displaystyle\int_2^4 \left((x-2)^2+5\right)\,dx$

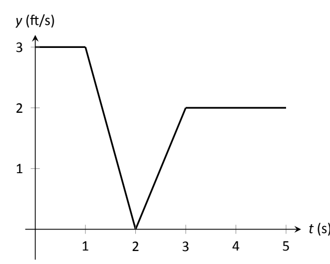
05 02 exset 03 **In Exercises 14 – 15, a graph of the velocity function of an object moving in a straight line is given. Answer the questions based on that graph.**

05 02 ex 14 14. 


(a) What is the object's maximum velocity?

(b) What is the object's maximum displacement?

(c) What is the object's total displacement on $[0, 3]$?

05 02 ex 15 15. 


(a) What is the object's maximum velocity?

(b) What is the object's maximum displacement?

(c) What is the object's total displacement on $[0, 5]$?

05 02 ex 16 16. An object is thrown straight up with a velocity, in ft/s, given by $v(t) = -32t + 64$, where $t$ is in seconds, from a height of 48 feet.

(a) What is the object's maximum velocity?

(b) What is the object's maximum displacement?

(c) When does the maximum displacement occur?

(d) When will the object reach a height of 0? (Hint: find when the displacement is $-48$ft.)

05 02 ex 17 17. An object is thrown straight up with a velocity, in ft/s, given by $v(t) = -32t + 96$, where $t$ is in seconds, from a height of 64 feet.

(a) What is the object's initial velocity?

(b) When is the object's displacement 0?

(c) How long does it take for the object to return to its initial height?

(d) When will the object reach a height of 210 feet?

05 02 exset 04 **In Exercises 18 – 21, let**

- $\int_0^2 f(x)\,dx = 5$,
- $\int_0^3 f(x)\,dx = 7$,
- $\int_0^2 g(x)\,dx = -3$, **and**
- $\int_2^3 g(x)\,dx = 5$.

**Use these values to evaluate the given definite integrals.**

05 02 ex 18 18. $\int_0^2 \big(f(x) + g(x)\big)\,dx$

05 02 ex 19 19. $\int_0^3 \big(f(x) - g(x)\big)\,dx$

05 02 ex 20 20. $\int_2^3 \big(3f(x) + 2g(x)\big)\,dx$

05 02 ex 21 21. Find values for $a$ and $b$ such that
$$\int_0^3 \big(af(x) + bg(x)\big)\,dx = 0$$

05 02 exset 05 **In Exercises 22 – 25, let**

- $\int_0^3 s(t)\,dt = 10$,
- $\int_3^5 s(t)\,dt = 8$,
- $\int_3^5 r(t)\,dt = -1$, **and**
- $\int_0^5 r(t)\,dt = 11$.

**Use these values to evaluate the given definite integrals.**

05 02 ex 22 22. $\int_0^3 \big(s(t) + r(t)\big)\,dt$

05 02 ex 23 23. $\int_5^0 \big(s(t) - r(t)\big)\,dt$

05 02 ex 24 24. $\int_3^3 \big(\pi s(t) - 7r(t)\big)\,dt$

05 02 ex 25 25. Find values for $a$ and $b$ such that
$$\int_0^5 \big(ar(t) + bs(t)\big)\,dt = 0$$

## *Review*

05 02 exset 06 **In Exercises 26 – 29, evaluate the given indefinite integral.**

05 02 ex 26 26. $\int \big(x^3 - 2x^2 + 7x - 9\big)\,dx$

05 02 ex 27 27. $\int \big(\sin x - \cos x + \sec^2 x\big)\,dx$

05 02 ex 28 28. $\int \big(\sqrt[3]{t} + \frac{1}{t^2} + 2^t\big)\,dt$

05 02 ex 29 29. $\int \left(\frac{1}{x} - \csc x \cot x\right)\,dx$

# Exercises 5.3

## *Terms and Concepts*

05 03 ex 01 1. A fundamental calculus technique is to use _________ to refine approximations to get an exact answer.

05 03 ex 02 2. What is the upper bound in the summation $\sum_{i=7}^{14}(48i - 201)$?

05 03 ex 03 3. This section approximates definite integrals using what geometric shape?

05 03 ex 04 4. T/F: A sum using the Right Hand Rule is an example of a Riemann Sum.

## *Problems*

05 03 exset 01 **In Exercises 5 – 11, write out each term of the summation and compute the sum.**

05 03 ex 05 5. $\displaystyle\sum_{i=2}^{4} i^2$

05 03 ex 06 6. $\displaystyle\sum_{i=-1}^{3}(4i-2)$

05 03 ex 07 7. $\displaystyle\sum_{i=-2}^{2}\sin(\pi i/2)$

05 03 ex 08 8. $\displaystyle\sum_{i=1}^{5}\frac{1}{i}$

05 03 ex 09 9. $\displaystyle\sum_{i=1}^{6}(-1)^i i$

05 03 ex 10 10. $\displaystyle\sum_{i=1}^{4}\left(\frac{1}{i}-\frac{1}{i+1}\right)$

05 03 ex 11 11. $\displaystyle\sum_{i=0}^{5}(-1)^i\cos(\pi i)$

05 03 exset 02 **In Exercises 12 – 15, write each sum in summation notation.**

05 03 ex 12 12. $3+6+9+12+15$

05 03 ex 13 13. $-1+0+3+8+15+24+35+48+63$

05 03 ex 14 14. $\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}+\dfrac{4}{5}$

05 03 ex 15 15. $1-e+e^2-e^3+e^4$

05 03 exset 03 **In Exercises 16 – 22, evaluate the summation using Theorem 37.**

05 03 ex 16 16. $\displaystyle\sum_{i=1}^{25} i$

05 03 ex 17 17. $\displaystyle\sum_{i=1}^{10}(3i^2-2i)$

05 03 ex 18 18. $\displaystyle\sum_{i=1}^{15}(2i^3-10)$

05 03 ex 19 19. $\displaystyle\sum_{i=1}^{10}(-4i^3+10i^2-7i+11)$

05 03 ex 20 20. $\displaystyle\sum_{i=1}^{10}(i^3-3i^2+2i+7)$

05 03 ex 21 21. $1+2+3+\ldots+99+100$

05 03 ex 22 22. $1+4+9+\ldots+361+400$

05 03 exset 04 **Theorem 37 states**

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{k} a_i + \sum_{i=k+1}^{n} a_i \text{ , so}$$

$$\sum_{i=k+1}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{k} a_i \text{ .}$$

**Use this fact, along with other parts of Theorem 37, to evaluate the summations given in Exercises 23 – 26.**

05 03 ex 23 23. $\displaystyle\sum_{i=11}^{20} i$

05 03 ex 24 24. $\displaystyle\sum_{i=16}^{25} i^3$

05 03 ex 25 25. $\displaystyle\sum_{i=7}^{12} 4$

05 03 ex 26 26. $\displaystyle\sum_{i=5}^{10} 4i^3$

05 03 exset 05 **In Exercises 27 – 32, a definite integral $\int_a^b f(x)\,dx$ is given.**

**(a) Graph $f(x)$ on $[a,b]$.**

**(b) Add to the sketch rectangles using the provided rule.**

**(c) Approximate $\int_a^b f(x)\,dx$ by summing the areas of the rectangles.**

05 03 ex 27 27. $\int_{-3}^{3} x^2\,dx$, with 6 rectangles using the Left Hand Rule.

05 03 ex 28 28. $\int_0^2 (5-x^2)\,dx$, with 4 rectangles using the Midpoint Rule.

05 03 ex 29 29. $\int_0^{\pi} \sin x\,dx$, with 6 rectangles using the Right Hand Rule.

05 03 ex 30 30. $\int_0^3 2^x\,dx$, with 5 rectangles using the Left Hand Rule.

05 03 ex 31 31. $\int_1^2 \ln x\,dx$, with 3 rectangles using the Midpoint Rule.

05 03 ex 32 32. $\int_1^9 \frac{1}{x}\,dx$, with 4 rectangles using the Right Hand Rule.

05 03 exset 06 **In Exercises 33 – 38, a definite integral $\int_a^b f(x)\,dx$ is given. As demonstrated in Examples 123 and 124, do the following.**

**(a) Find a formula to approximate $\int_a^b f(x)\,dx$ using $n$ subintervals and the provided rule.**

**(b) Evaluate the formula using $n = 10, 100$ and $1,000$.**

**(c) Find the limit of the formula, as $n \to \infty$, to find the exact value of $\int_a^b f(x)\,dx$.**

05 03 ex 33 33. $\int_0^1 x^3\,dx$, using the Right Hand Rule.

05 03 ex 34 34. $\int_{-1}^1 3x^2\,dx$, using the Left Hand Rule.

05 03 ex 35 35. $\int_{-1}^3 (3x-1)\,dx$, using the Midpoint Rule.

05 03 ex 36 36. $\int_1^4 (2x^2-3)\,dx$, using the Left Hand Rule.

05 03 ex 37 37. $\int_{-10}^{10} (5-x)\,dx$, using the Right Hand Rule.

05 03 ex 38 38. $\int_0^1 (x^3-x^2)\,dx$, using the Right Hand Rule.

## *Review*

05 03 exset 07 **In Exercises 39 – 44, find an antiderivative of the given function.**

05 03 ex 39 39. $f(x) = 5\sec^2 x$

05 03 ex 40 40. $f(x) = \frac{7}{x}$

05 03 ex 41 41. $g(t) = 4t^5 - 5t^3 + 8$

05 03 ex 42 42. $g(t) = 5\cdot 8^t$

05 03 ex 43 43. $g(t) = \cos t + \sin t$

05 03 ex 44 44. $f(x) = \frac{1}{\sqrt{x}}$

# Exercises 5.4

## *Terms and Concepts*

05 04 ex 01 1. How are definite and indefinite integrals related?

05 04 ex 02 2. What constant of integration is most commonly used when evaluating definite integrals?

05 04 ex 03 3. T/F: If $f$ is a continuous function, then $F(x) = \int_a^x f(t)\,dt$ is also a continuous function.

05 04 ex 04 4. The definite integral can be used to find "the area under a curve." Give two other uses for definite integrals.

## *Problems*

05 04 exset 01 **In Exercises 5 – 28, evaluate the definite integral.**

05 04 ex 05 5. $\int_1^3 (3x^2 - 2x + 1)\,dx$

05 04 ex 06 6. $\int_0^4 (x-1)^2\,dx$

05 04 ex 07 7. $\int_{-1}^1 (x^3 - x^5)\,dx$

05 04 ex 08 8. $\int_{\pi/2}^{\pi} \cos x\,dx$

05 04 ex 09 9. $\int_0^{\pi/4} \sec^2 x\,dx$

05 04 ex 10 10. $\int_1^e \frac{1}{x}\,dx$

05 04 ex 11 11. $\int_{-1}^1 5^x\,dx$

05 04 ex 12 12. $\int_{-2}^{-1} (4 - 2x^3)\,dx$

05 04 ex 13 13. $\int_0^{\pi} (2\cos x - 2\sin x)\,dx$

05 04 ex 14 14. $\int_1^3 e^x\,dx$

05 04 ex 15 15. $\int_0^4 \sqrt{t}\,dt$

05 04 ex 16 16. $\int_9^{25} \frac{1}{\sqrt{t}}\,dt$

05 04 ex 17 17. $\int_1^8 \sqrt[3]{x}\,dx$

05 04 ex 18 18. $\int_1^2 \frac{1}{x}\,dx$

05 04 ex 19 19. $\int_1^2 \frac{1}{x^2}\,dx$

05 04 ex 20 20. $\int_1^2 \frac{1}{x^3}\,dx$

05 04 ex 21 21. $\int_0^1 x\,dx$

05 04 ex 22 22. $\int_0^1 x^2\,dx$

05 04 ex 23 23. $\int_0^1 x^3\,dx$

05 04 ex 24 24. $\int_0^1 x^{100}\,dx$

05 04 ex 25 25. $\int_{-4}^4 dx$

05 04 ex 26 26. $\int_{-10}^{-5} 3\,dx$

05 04 ex 27 27. $\int_{-2}^2 0\,dx$

05 04 ex 29 28. $\int_{\pi/6}^{\pi/3} \csc x \cot x\,dx$

05 04 ex 28 29. Explain why:

(a) $\int_{-1}^1 x^n\,dx = 0$, when $n$ is a positive, odd integer, and

(b) $\int_{-1}^1 x^n\,dx = 2\int_0^1 x^n\,dx$ when $n$ is a positive, even integer.

05 04 exset 02 **In Exercises 30 – 33, find a value $c$ guaranteed by the Mean Value Theorem.**

05 04 ex 30 30. $\int_0^2 x^2\,dx$

05 04 ex 31 31. $\int_{-2}^2 x^2\,dx$

05 04 ex 32 32. $\int_0^1 e^x\,dx$

05 04 ex 33 33. $\int_0^{16} \sqrt{x}\,dx$

05 04 exset 03 **In Exercises 34 – 39, find the average value of the function on the given interval.**

05 04 ex 34 34. $f(x) = \sin x$ on $[0, \pi/2]$

05 04 ex 35 35. $y = \sin x$ on $[0, \pi]$

05 04 ex 36 36. $y = x$ on $[0, 4]$

05 04 ex 37 37. $y = x^2$ on $[0, 4]$

05 04 ex 38 38. $y = x^3$ on $[0, 4]$

05 04 ex 39 39. $g(t) = 1/t$ on $[1, e]$

05 04 exset 04 **In Exercises 40 – 44, a velocity function of an object moving along a straight line is given. Find the displacement of the object over the given time interval.**

05 04 ex 40 40. $v(t) = -32t + 20\text{ft/s}$ on $[0, 5]$

05 04 ex 41 41. $v(t) = -32t + 200\text{ft/s}$ on $[0, 10]$

05 04 ex 42 42. $v(t) = 2^t\text{mph}$ on $[-1, 1]$

05 04 ex 43 43. $v(t) = \cos t$ ft/s on $[0, 3\pi/2]$

05 04 ex 44 44. $v(t) = \sqrt[4]{t}$ ft/s on $[0, 16]$

05 04 exset 05 **In Exercises 45 – 48, an acceleration function of an object moving along a straight line is given. Find the change of the object's velocity over the given time interval.**

05 04 ex 45 45. $a(t) = -32\text{ft/s}^2$ on $[0, 2]$

05 04 ex 46 46. $a(t) = 10\text{ft/s}^2$ on $[0, 5]$

05 04 ex 47 47. $a(t) = t\ \text{ft/s}^2$ on $[0, 2]$

05 04 ex 48 48. $a(t) = \cos t\ \text{ft/s}^2$ on $[0, \pi]$

05 04 exset 06 **In Exercises 49 – 52, sketch the given functions and find the area of the enclosed region.**

05 04 ex 51 49. $y = 2x$, $y = 5x$, and $x = 3$.

05 04 ex 52 50. $y = -x + 1$, $y = 3x + 6$, $x = 2$ and $x = -1$.

05 04 ex 53 51. $y = x^2 - 2x + 5$, $y = 5x - 5$.

05 04 ex 54 52. $y = 2x^2 + 2x - 5$, $y = x^2 + 3x + 7$.

05 04 exset 07 **In Exercises 53 – 56, find $F'(x)$.**

05 04 ex 55 53. $F(x) = \displaystyle\int_2^{x^3+x} \frac{1}{t}\, dt$

05 04 ex 56 54. $F(x) = \displaystyle\int_{x^3}^{0} t^3\, dt$

05 04 ex 57 55. $F(x) = \displaystyle\int_{x}^{x^2} (t+2)\, dt$

05 04 ex 58 56. $F(x) = \displaystyle\int_{\ln x}^{e^x} \sin t\, dt$

# Exercises 5.5

## *Terms and Concepts*

05 05 ex 23 1. T/F: Simpson's Rule is a method of approximating antiderivatives.

05 05 ex 24 2. What are the two basic situations where approximating the value of a definite integral is necessary?

05 05 ex 25 3. Why are the Left and Right Hand Rules rarely used?

## *Problems*

05 05 exset 02 **In Exercises 4 – 11, a definite integral is given.**

**(a) Approximate the definite integral with the Trapezoidal Rule and $n = 4$.**

**(b) Approximate the definite integral with Simpson's Rule and $n = 4$.**

**(c) Find the exact value of the integral.**

05 05 ex 03 4. $\int_{-1}^{1} x^2\, dx$

05 05 ex 04 5. $\int_{0}^{10} 5x\, dx$

05 05 ex 05 6. $\int_{0}^{\pi} \sin x\, dx$

05 05 ex 06 7. $\int_{0}^{4} \sqrt{x}\, dx$

05 05 ex 07 8. $\int_{0}^{3} (x^3 + 2x^2 - 5x + 7)\, dx$

05 05 ex 08 9. $\int_{0}^{1} x^4\, dx$

05 05 ex 09 10. $\int_{0}^{2\pi} \cos x\, dx$

05 05 ex 10 11. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

05 05 exset 03 **In Exercises 12 – 19, approximate the definite integral with the Trapezoidal Rule and Simpson's Rule, with $n = 6$.**

05 05 ex 11 12. $\int_{0}^{1} \cos\left(x^2\right) dx$

05 05 ex 12 13. $\int_{-1}^{1} e^{x^2}\, dx$

05 05 ex 13 14. $\int_{0}^{5} \sqrt{x^2 + 1}\, dx$

05 05 ex 14 15. $\int_{0}^{\pi} x \sin x\, dx$

05 05 ex 15 16. $\int_{0}^{\pi/2} \sqrt{\cos x}\, dx$

05 05 ex 16 17. $\int_{1}^{4} \ln x\, dx$

05 05 ex 17 18. $\int_{-1}^{1} \frac{1}{\sin x + 2}\, dx$

05 05 ex 18 19. $\int_{0}^{6} \frac{1}{\sin x + 2}\, dx$

05 05 exset 04 **In Exercises 20 – 23, find $n$ such that the error in approximating the given definite integral is less than $0.0001$ when using:**

**(a) the Trapezoidal Rule**

**(b) Simpson's Rule**

05 05 ex 19 20. $\int_{0}^{\pi} \sin x\, dx$

05 05 ex 20 21. $\int_{1}^{4} \frac{1}{\sqrt{x}}\, dx$

05 05 ex 21 22. $\int_{0}^{\pi} \cos\left(x^2\right) dx$

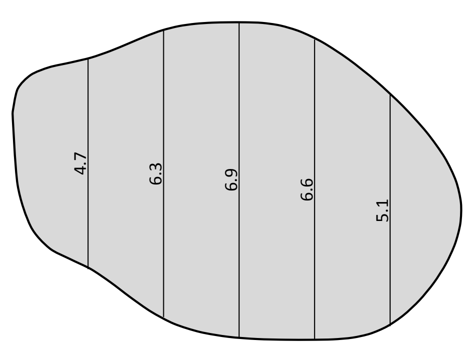
05 05 ex 22 23. $\int_{0}^{5} x^4\, dx$

05 05 exset 01 **In Exercises 24 – 25, a region is given. Find the area of the region using Simpson's Rule:**

**(a) where the measurements are in centimeters, taken in 1 cm increments, and**

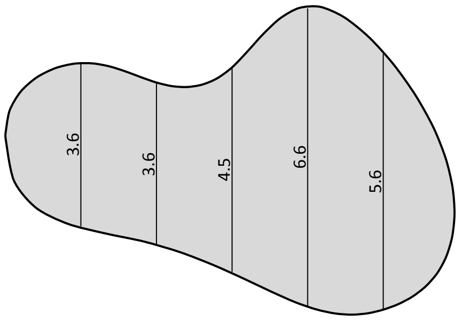
**(b) where the measurements are in hundreds of yards, taken in 100 yd increments.**

05 05 ex 01 24.



05 05 ex 02

25.



# Exercises 6.1

## *Terms and Concepts*

06 01 ex 01 1. Substitution "undoes" what derivative rule?

06 01 ex 02 2. T/F: One can use algebra to rewrite the integrand of an integral to make it easier to evaluate.

## *Problems*

06 01 exset 01 **In Exercises 3 – 14, evaluate the indefinite integral to develop an understanding of Substitution.**

06 01 ex 03 3. $\displaystyle\int 3x^2\left(x^3-5\right)^7\,dx$

06 01 ex 04 4. $\displaystyle\int (2x-5)\left(x^2-5x+7\right)^3\,dx$

06 01 ex 05 5. $\displaystyle\int x\left(x^2+1\right)^8\,dx$

06 01 ex 06 6. $\displaystyle\int (12x+14)\left(3x^2+7x-1\right)^5\,dx$

06 01 ex 11 7. $\displaystyle\int \frac{1}{2x+7}dx$

06 01 ex 12 8. $\displaystyle\int \frac{1}{\sqrt{2x+3}}dx$

06 01 ex 13 9. $\displaystyle\int \frac{x}{\sqrt{x+3}}dx$

06 01 ex 17 10. $\displaystyle\int \frac{x^3-x}{\sqrt{x}}dx$

06 01 ex 18 11. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

06 01 ex 19 12. $\displaystyle\int \frac{x^4}{\sqrt{x^5+1}}dx$

06 01 ex 20 13. $\displaystyle\int \frac{\frac{1}{x}+1}{x^2}dx$

06 01 ex 21 14. $\displaystyle\int \frac{\ln(x)}{x}dx$

06 01 exset 02 **In Exercises 15 – 23, use Substitution to evaluate the indefinite integral involving trigonometric functions.**

06 01 ex 08 15. $\displaystyle\int \sin^2(x)\cos(x)dx$

06 01 ex 09 16. $\displaystyle\int \cos(3-6x)dx$

06 01 ex 10 17. $\displaystyle\int \sec^2(4-x)dx$

06 01 ex 16 18. $\displaystyle\int \sec(2x)dx$

06 01 ex 22 19. $\displaystyle\int \tan^2(x)\sec^2(x)dx$

06 01 ex 25 20. $\displaystyle\int x\cos\left(x^2\right)\,dx$

06 01 ex 27 21. $\displaystyle\int \tan^2(x)dx$

06 01 ex 82 22. $\displaystyle\int \cot x\,dx$. Do not just refer to Theorem 45 for the answer; justify it through Substitution.

06 01 ex 83 23. $\displaystyle\int \csc x\,dx$. Do not just refer to Theorem 45 for the answer; justify it through Substitution.

06 01 exset 03 **In Exercises 24 – 30, use Substitution to evaluate the indefinite integral involving exponential functions.**

06 01 ex 29 24. $\displaystyle\int e^{3x-1}dx$

06 01 ex 30 25. $\displaystyle\int e^{x^3}x^2dx$

06 01 ex 31 26. $\displaystyle\int e^{x^2-2x+1}(x-1)dx$

06 01 ex 32 27. $\displaystyle\int \frac{e^x+1}{e^x}dx$

06 01 ex 33 28. $\displaystyle\int \frac{e^x-e^{-x}}{e^{2x}}dx$

06 01 ex 34 29. $\displaystyle\int 3^{3x}dx$

06 01 ex 35 30. $\displaystyle\int 4^{2x}dx$

06 01 exset 04 **In Exercises 31 – 34, use Substitution to evaluate the indefinite integral involving logarithmic functions.**

06 01 ex 36 31. $\displaystyle\int \frac{\ln x}{x}dx$

06 01 ex 37 32. $\displaystyle\int \frac{\left(\ln x\right)^2}{x}dx$

06 01 ex 38 33. $\displaystyle\int \frac{\ln\left(x^3\right)}{x}dx$

06 01 ex 39 34. $\int \frac{1}{x\ln\left(x^2\right)}dx$

06 01 exset 05 **In Exercises 35 – 40, use Substitution to evaluate the indefinite integral involving rational functions.**

06 01 ex 40 35. $\int \frac{x^2+3x+1}{x}dx$

06 01 ex 41 36. $\int \frac{x^3+x^2+x+1}{x}dx$

06 01 ex 42 37. $\int \frac{x^3-1}{x+1}dx$

06 01 ex 43 38. $\int \frac{x^2+2x-5}{x-3}dx$

06 01 ex 44 39. $\int \frac{3x^2-5x+7}{x+1}dx$

06 01 ex 45 40. $\int \frac{x^2+2x+1}{x^3+3x^2+3x}dx$

06 01 exset 06 **In Exercises 41 – 50, use Substitution to evaluate the indefinite integral involving inverse trigonometric functions.**

06 01 ex 50 41. $\int \frac{7}{x^2+7}dx$

06 01 ex 51 42. $\int \frac{3}{\sqrt{9-x^2}}dx$

06 01 ex 52 43. $\int \frac{14}{\sqrt{5-x^2}}dx$

06 01 ex 53 44. $\int \frac{2}{x\sqrt{x^2-9}}dx$

06 01 ex 54 45. $\int \frac{5}{\sqrt{x^4-16x^2}}dx$

06 01 ex 55 46. $\int \frac{x}{\sqrt{1-x^4}}dx$

06 01 ex 60 47. $\int \frac{1}{x^2-2x+8}dx$

06 01 ex 61 48. $\int \frac{2}{\sqrt{-x^2+6x+7}}dx$

06 01 ex 62 49. $\int \frac{3}{\sqrt{-x^2+8x+9}}dx$

06 01 ex 63 50. $\int \frac{5}{x^2+6x+34}dx$

06 01 exset 07 **In Exercises 51 – 75, evaluate the indefinite integral.**

06 01 ex 14 51. $\int \frac{x^2}{\left(x^3+3\right)^2}dx$

06 01 ex 07 52. $\int \left(3x^2+2x\right)\left(5x^3+5x^2+2\right)^8\,dx$

06 01 ex 15 53. $\int \frac{x}{\sqrt{1-x^2}}dx$

06 01 ex 26 54. $\int x^2\csc^2\left(x^3+1\right)dx$

06 01 ex 28 55. $\int \sin(x)\sqrt{\cos(x)}dx$

06 01 ex 23 56. $\int \frac{1}{x-5}dx$

06 01 ex 24 57. $\int \frac{7}{3x+2}dx$

06 01 ex 46 58. $\int \frac{3x^3+4x^2+2x-22}{x^2+3x+5}dx$

06 01 ex 47 59. $\int \frac{2x+7}{x^2+7x+3}dx$

06 01 ex 48 60. $\int \frac{9(2x+3)}{3x^2+9x+7}dx$

06 01 ex 49 61. $\int \frac{-x^3+14x^2-46x-7}{x^2-7x+1}dx$

06 01 ex 56 62. $\int \frac{x}{x^4+81}dx$

06 01 ex 57 63. $\int \frac{2}{4x^2+1}dx$

06 01 ex 58 64. $\int \frac{1}{x\sqrt{4x^2-1}}dx$

06 01 ex 59 65. $\int \frac{1}{\sqrt{16-9x^2}}dx$

06 01 ex 64 66. $\int \frac{3x-2}{x^2-2x+10}dx$

06 01 ex 65 67. $\int \frac{7-2x}{x^2+12x+61}dx$

06 01 ex 66 68. $\int \frac{x^2+5x-2}{x^2-10x+32}dx$

06 01 ex 67 69. $\int \frac{x^3}{x^2+9}dx$

06 01 ex 68 70. $\int \frac{x^3-x}{x^2+4x+9}dx$

06 01 ex 69 71. $\int \frac{\sin(x)}{\cos^2(x)+1}dx$

06 01 ex 70 72. $\displaystyle\int \frac{\cos(x)}{\sin^2(x)+1}dx$

06 01 ex 71 73. $\displaystyle\int \frac{\cos(x)}{1-\sin^2(x)}dx$

06 01 ex 72 74. $\displaystyle\int \frac{3x-3}{\sqrt{x^2-2x-6}}dx$

06 01 ex 73 75. $\displaystyle\int \frac{x-3}{\sqrt{x^2-6x+8}}dx$

06 01 exset 08 **In Exercises 76 – 83, evaluate the definite integral.**

06 01 ex 74 76. $\displaystyle\int_1^3 \frac{1}{x-5}dx$

06 01 ex 75 77. $\displaystyle\int_2^6 x\sqrt{x-2}dx$

06 01 ex 76 78. $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\, dx$

06 01 ex 77 79. $\displaystyle\int_0^1 2x(1-x^2)^4\, dx$

06 01 ex 78 80. $\displaystyle\int_{-2}^{-1} (x+1)e^{x^2+2x+1}\, dx$

06 01 ex 79 81. $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\, dx$

06 01 ex 80 82. $\displaystyle\int_2^4 \frac{1}{x^2-6x+10}\, dx$

06 01 ex 81 83. $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\, dx$

# Exercises 6.2

## *Terms and Concepts*

06 02 ex 01 1. T/F: Integration by Parts is useful in evaluating integrands that contain products of functions.

06 02 ex 02 2. T/F: Integration by Parts can be thought of as the “opposite of the Chain Rule.”

06 02 ex 03 3. For what is “LIATE” useful?

## *Problems*

06 02 exset 01 **In Exercises 4 – 33, evaluate the given indefinite integral.**

06 02 ex 04 4. $\int x \sin x \, dx$

06 02 ex 05 5. $\int x e^{-x} \, dx$

06 02 ex 06 6. $\int x^2 \sin x \, dx$

06 02 ex 07 7. $\int x^3 \sin x \, dx$

06 02 ex 08 8. $\int x e^{x^2} \, dx$

06 02 ex 09 9. $\int x^3 e^x \, dx$

06 02 ex 10 10. $\int x e^{-2x} \, dx$

06 02 ex 11 11. $\int e^x \sin x \, dx$

06 02 ex 12 12. $\int e^{2x} \cos x \, dx$

06 02 ex 13 13. $\int e^{2x} \sin(3x) \, dx$

06 02 ex 14 14. $\int e^{5x} \cos(5x) \, dx$

06 02 ex 15 15. $\int \sin x \cos x \, dx$

06 02 ex 16 16. $\int \sin^{-1} x \, dx$

06 02 ex 17 17. $\int \tan^{-1}(2x) \, dx$

06 02 ex 18 18. $\int x \tan^{-1} x \, dx$

06 02 ex 19 19. $\int \sin^{-1} x \, dx$

06 02 ex 22 20. $\int x \ln x \, dx$

06 02 ex 23 21. $\int (x-2) \ln x \, dx$

06 02 ex 24 22. $\int x \ln(x-1) \, dx$

06 02 ex 25 23. $\int x \ln(x^2) \, dx$

06 02 ex 26 24. $\int x^2 \ln x \, dx$

06 02 ex 27 25. $\int (\ln x)^2 \, dx$

06 02 ex 28 26. $\int (\ln(x+1))^2 \, dx$

06 02 ex 29 27. $\int x \sec^2 x \, dx$

06 02 ex 30 28. $\int x \csc^2 x \, dx$

06 02 ex 31 29. $\int x\sqrt{x-2} \, dx$

06 02 ex 32 30. $\int x\sqrt{x^2-2} \, dx$

06 02 ex 35 31. $\int \sec x \tan x \, dx$

06 02 ex 33 32. $\int x \sec x \tan x \, dx$

06 02 ex 34 33. $\int x \csc x \cot x \, dx$

06 02 exset 02 **In Exercises 34 – 38, evaluate the indefinite integral after first making a substitution.**

06 02 ex 36 34. $\int \sin(\ln x) \, dx$

06 02 ex 37 35. $\int \sin(\sqrt{x}) \, dx$

06 02 ex 38 36. $\int \ln(\sqrt{x}) \, dx$

06 02 ex 39 37. $\int e^{\sqrt{x}} \, dx$

06 02 ex 40 38. $\int e^{\ln x}\, dx$

06 02 exset 03 **In Exercises 39 – 47, evaluate the definite integral. Note: the corresponding indefinite integrals appear in Exercises 4 – 12.**

06 02 ex 41 39. $\int_0^{\pi} x \sin x\, dx$

06 02 ex 42 40. $\int_{-1}^{1} xe^{-x}\, dx$

06 02 ex 43 41. $\int_{-\pi/4}^{\pi/4} x^2 \sin x\, dx$

06 02 ex 44 42. $\int_{-\pi/2}^{\pi/2} x^3 \sin x\, dx$

06 02 ex 45 43. $\int_0^{\sqrt{\ln 2}} xe^{x^2}\, dx$

06 02 ex 46 44. $\int_0^1 x^3 e^x\, dx$

06 02 ex 47 45. $\int_1^2 xe^{-2x}\, dx$

06 02 ex 48 46. $\int_0^{\pi} e^x \sin x\, dx$

06 02 ex 49 47. $\int_{-\pi/2}^{\pi/2} e^{2x} \cos x\, dx$

# Exercises 6.3

## *Terms and Concepts*

06 03 ex 01 1. T/F: $\int \sin^2 x \cos^2 x\, dx$ cannot be evaluated using the techniques described in this section since both powers of $\sin x$ and $\cos x$ are even.

06 03 ex 02 2. T/F: $\int \sin^3 x \cos^3 x\, dx$ cannot be evaluated using the techniques described in this section since both powers of $\sin x$ and $\cos x$ are odd.

06 03 ex 03 3. T/F: This section addresses how to evaluate indefinite integrals such as $\int \sin^5 x \tan^3 x\, dx$.

## *Problems*

06 03 exset 01 **In Exercises 4 – 26, evaluate the indefinite integral.**

06 03 ex 04 4. $\int \sin x \cos^4 x\, dx$

06 03 ex 05 5. $\int \sin^3 x \cos x\, dx$

06 03 ex 06 6. $\int \sin^3 x \cos^2 x\, dx$

06 03 ex 07 7. $\int \sin^3 x \cos^3 x\, dx$

06 03 ex 08 8. $\int \sin^6 x \cos^5 x\, dx$

06 03 ex 09 9. $\int \sin^2 x \cos^7 x\, dx$

06 03 ex 10 10. $\int \sin^2 x \cos^2 x\, dx$

06 03 ex 11 11. $\int \sin(5x)\cos(3x)\, dx$

06 03 ex 12 12. $\int \sin(x)\cos(2x)\, dx$

06 03 ex 13 13. $\int \sin(3x)\sin(7x)\, dx$

06 03 ex 14 14. $\int \sin(\pi x)\sin(2\pi x)\, dx$

06 03 ex 15 15. $\int \cos(x)\cos(2x)\, dx$

06 03 ex 16 16. $\int \cos\left(\frac{\pi}{2}x\right)\cos(\pi x)\, dx$

06 03 ex 17 17. $\int \tan^4 x \sec^2 x\, dx$

06 03 ex 18 18. $\int \tan^2 x \sec^4 x\, dx$

06 03 ex 19 19. $\int \tan^3 x \sec^4 x\, dx$

06 03 ex 20 20. $\int \tan^3 x \sec^2 x\, dx$

06 03 ex 21 21. $\int \tan^3 x \sec^3 x\, dx$

06 03 ex 22 22. $\int \tan^5 x \sec^5 x\, dx$

06 03 ex 23 23. $\int \tan^4 x\, dx$

06 03 ex 24 24. $\int \sec^5 x\, dx$

06 03 ex 25 25. $\int \tan^2 x \sec x\, dx$

06 03 ex 26 26. $\int \tan^2 x \sec^3 x\, dx$

06 03 exset 02 **In Exercises 27 – 33, evaluate the definite integral. Note: the corresponding indefinite integrals appear in the previous set.**

06 03 ex 27 27. $\int_0^{\pi} \sin x \cos^4 x\, dx$

06 03 ex 28 28. $\int_{-\pi}^{\pi} \sin^3 x \cos x\, dx$

06 03 ex 29 29. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^7 x\, dx$

06 03 ex 30 30. $\int_0^{\pi/2} \sin(5x)\cos(3x)\, dx$

06 03 ex 31 31. $\int_{-\pi/2}^{\pi/2} \cos(x)\cos(2x)\, dx$

06 03 ex 32 32. $\int_0^{\pi/4} \tan^4 x \sec^2 x\, dx$

06 03 ex 33 33. $\int_{-\pi/4}^{\pi/4} \tan^2 x \sec^4 x\, dx$

# Exercises 6.4

## Terms and Concepts

06 04 ex 01 1. Fill in the blank: Partial Fraction Decomposition is a method of rewriting ________ functions.

06 04 ex 02 2. T/F: It is sometimes necessary to use polynomial division before using Partial Fraction Decomposition.

06 04 ex 03 3. Decompose $\dfrac{1}{x^2-3x}$ without solving for the coefficients, as done in Example 181.

06 04 ex 04 4. Decompose $\dfrac{7-x}{x^2-9}$ without solving for the coefficients, as done in Example 181.

06 04 ex 05 5. Decompose $\dfrac{x-3}{x^2-7}$ without solving for the coefficients, as done in Example 181.

06 04 ex 06 6. Decompose $\dfrac{2x+5}{x^3+7x}$ without solving for the coefficients, as done in Example 181.

## Problems

06 04 exset 01 **In Exercises 7 – 25, evaluate the indefinite integral.**

06 04 ex 07 7. $\displaystyle\int \frac{7x+7}{x^2+3x-10}\,dx$

06 04 ex 08 8. $\displaystyle\int \frac{7x-2}{x^2+x}\,dx$

06 04 ex 09 9. $\displaystyle\int \frac{-4}{3x^2-12}\,dx$

06 04 ex 10 10. $\displaystyle\int \frac{x+7}{(x+5)^2}\,dx$

06 04 ex 11 11. $\displaystyle\int \frac{-3x-20}{(x+8)^2}\,dx$

06 04 ex 12 12. $\displaystyle\int \frac{9x^2+11x+7}{x(x+1)^2}\,dx$

06 04 ex 13 13. $\displaystyle\int \frac{-12x^2-x+33}{(x-1)(x+3)(3-2x)}\,dx$

06 04 ex 14 14. $\displaystyle\int \frac{94x^2-10x}{(7x+3)(5x-1)(3x-1)}\,dx$

06 04 ex 15 15. $\displaystyle\int \frac{x^2+x+1}{x^2+x-2}\,dx$

06 04 ex 16 16. $\displaystyle\int \frac{x^3}{x^2-x-20}\,dx$

06 04 ex 17 17. $\displaystyle\int \frac{2x^2-4x+6}{x^2-2x+3}\,dx$

06 04 ex 18 18. $\displaystyle\int \frac{1}{x^3+2x^2+3x}\,dx$

06 04 ex 19 19. $\displaystyle\int \frac{x^2+x+5}{x^2+4x+10}\,dx$

06 04 ex 20 20. $\displaystyle\int \frac{12x^2+21x+3}{(x+1)(3x^2+5x-1)}\,dx$

06 04 ex 21 21. $\displaystyle\int \frac{6x^2+8x-4}{(x-3)(x^2+6x+10)}\,dx$

06 04 ex 22 22. $\displaystyle\int \frac{2x^2+x+1}{(x+1)(x^2+9)}\,dx$

06 04 ex 23 23. $\displaystyle\int \frac{x^2-20x-69}{(x-7)(x^2+2x+17)}\,dx$

06 04 ex 24 24. $\displaystyle\int \frac{9x^2-60x+33}{(x-9)(x^2-2x+11)}\,dx$

06 04 ex 25 25. $\displaystyle\int \frac{6x^2+45x+121}{(x+2)(x^2+10x+27)}\,dx$

06 04 exset 02 **In Exercises 26 – 29, evaluate the definite integral.**

06 04 ex 26 26. $\displaystyle\int_1^2 \frac{8x+21}{(x+2)(x+3)}\,dx$

06 04 ex 27 27. $\displaystyle\int_0^5 \frac{14x+6}{(3x+2)(x+4)}\,dx$

06 04 ex 28 28. $\displaystyle\int_{-1}^1 \frac{x^2+5x-5}{(x-10)(x^2+4x+5)}\,dx$

06 04 ex 29 29. $\displaystyle\int_0^1 \frac{x}{(x+1)(x^2+2x+1)}\,dx$

# Exercises 6.5

## *Terms and Concepts*

06 05 ex 01 1. In Key Idea 16, the equation $\int \tanh x\, dx = \ln(\cosh x) + C$ is given. Why is "$\ln|\cosh x|$" not used – i.e., why are absolute values not necessary?

06 05 ex 02 2. The hyperbolic functions are used to define points on the right hand portion of the hyperbola $x^2 - y^2 = 1$, as shown in Figure 6.13. How can we use the hyperbolic functions to define points on the left hand portion of the hyperbola?

## *Problems*

06 05 exset 01 **In Exercises 3 – 10, verify the given identity using Definition 23, as done in Example 186.**

06 05 ex 03 3. $\coth^2 x - \operatorname{csch}^2 x = 1$

06 05 ex 04 4. $\cosh 2x = \cosh^2 x + \sinh^2 x$

06 05 ex 05 5. $\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

06 05 ex 06 6. $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

06 05 ex 07 7. $\dfrac{d}{dx}\left[\operatorname{sech} x\right] = -\operatorname{sech} x \tanh x$

06 05 ex 08 8. $\dfrac{d}{dx}\left[\coth x\right] = -\operatorname{csch}^2 x$

06 05 ex 09 9. $\int \tanh x\, dx = \ln(\cosh x) + C$

06 05 ex 10 10. $\int \coth x\, dx = \ln|\sinh x| + C$

06 05 exset 02 **In Exercises 11 – 21, find the derivative of the given function.**

06 05 ex 11 11. $f(x) = \cosh 2x$

06 05 ex 12 12. $f(x) = \tanh(x^2)$

06 05 ex 13 13. $f(x) = \ln(\sinh x)$

06 05 ex 14 14. $f(x) = \sinh x \cosh x$

06 05 ex 15 15. $f(x) = x \sinh x - \cosh x$

06 05 ex 16 16. $f(x) = \operatorname{sech}^{-1}(x^2)$

06 05 ex 19 17. $f(x) = \sinh^{-1}(3x)$

06 05 ex 20 18. $f(x) = \cosh^{-1}(2x^2)$

06 05 ex 21 19. $f(x) = \tanh^{-1}(x + 5)$

06 05 ex 17 20. $f(x) = \tanh^{-1}(\cos x)$

06 05 ex 18 21. $f(x) = \cosh^{-1}(\sec x)$

06 05 exset 03 **In Exercises 22 – 26, find the equation of the line tangent to the function at the given $x$-value.**

06 05 ex 22 22. $f(x) = \sinh x$ at $x = 0$

06 05 ex 23 23. $f(x) = \cosh x$ at $x = \ln 2$

06 05 ex 24 24. $f(x) = \operatorname{sech}^2 x$ at $x = \ln 3$

06 05 ex 25 25. $f(x) = \sinh^{-1} x$ at $x = 0$

06 05 ex 26 26. $f(x) = \cosh^{-1} x$ at $x = \sqrt{2}$

06 05 exset 04 **In Exercises 27 – 40, evaluate the given indefinite integral.**

06 05 ex 27 27. $\int \tanh(2x)\, dx$

06 05 ex 28 28. $\int \cosh(3x - 7)\, dx$

06 05 ex 29 29. $\int \sinh x \cosh x\, dx$

06 02 ex 20 30. $\int x \cosh x\, dx$

06 02 ex 21 31. $\int x \sinh x\, dx$

06 05 ex 30 32. $\int \dfrac{1}{9 - x^2}\, dx$

06 05 ex 31 33. $\int \dfrac{2x}{\sqrt{x^4 - 4}}\, dx$

06 05 ex 32 34. $\int \dfrac{\sqrt{x}}{\sqrt{1 + x^3}}\, dx$

06 05 ex 33 35. $\int \dfrac{1}{x^4 - 16}\, dx$

06 05 ex 34 36. $\int \dfrac{1}{x^2 + x}\, dx$

06 05 ex 35 37. $\int \dfrac{e^x}{e^{2x} + 1}\, dx$

06 05 ex 36 38. $\int \sinh^{-1} x\, dx$

06 05 ex 37 39. $\int \tanh^{-1} x\, dx$

06 05 ex 38 40. $\displaystyle\int \operatorname{sech} x\, dx$ (Hint: mutiply by $\frac{\cosh x}{\cosh x}$; set $u = \sinh x$.)

06 05 exset 05 **In Exercises 41 – 43, evaluate the given definite integral.**

06 05 ex 39 41. $\displaystyle\int_{-1}^{1} \sinh x\, dx$

06 05 ex 40 42. $\displaystyle\int_{-\ln 2}^{\ln 2} \cosh x\, dx$

06 05 ex 41 43. $\displaystyle\int_{0}^{1} \tanh^{-1} x\, dx$

# Exercises 6.6

## *Terms and Concepts*

06 06 ex 01 1. List the different indeterminate forms described in this section.

06 06 ex 02 2. T/F: l'Hôpital's Rule provides a faster method of computing derivatives.

06 06 ex 03 3. T/F: l'Hôpital's Rule states that $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)}{g'(x)}$.

06 06 ex 04 4. Explain what the indeterminate form "$1^\infty$" means.

06 06 ex 05 5. Fill in the blanks: The Quotient Rule is applied to $\dfrac{f(x)}{g(x)}$ when taking ________; l'Hôpital's Rule is applied when taking certain ________.

06 06 ex 06 6. Create (but do not evaluate!) a limit that returns "$\infty^0$".

06 06 ex 07 7. Create a function $f(x)$ such that $\lim\limits_{x\to 1} f(x)$ returns "$0^0$".

## *Problems*

06 06 exset 01 **In Exercises 8 – 52, evaluate the given limit.**

06 06 ex 08 8. $\displaystyle\lim_{x\to 1} \frac{x^2+x-2}{x-1}$

06 06 ex 09 9. $\displaystyle\lim_{x\to 2} \frac{x^2+x-6}{x^2-7x+10}$

06 06 ex 10 10. $\displaystyle\lim_{x\to \pi} \frac{\sin x}{x-\pi}$

06 06 ex 11 11. $\displaystyle\lim_{x\to \pi/4} \frac{\sin x - \cos x}{\cos(2x)}$

06 06 ex 12 12. $\displaystyle\lim_{x\to 0} \frac{\sin(5x)}{x}$

06 06 ex 13 13. $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{x+2}$

06 06 ex 14 14. $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$

06 06 ex 15 15. $\displaystyle\lim_{x\to 0} \frac{\sin(ax)}{\sin(bx)}$

06 06 ex 16 16. $\displaystyle\lim_{x\to 0^+} \frac{e^x-1}{x^2}$

06 06 ex 17 17. $\displaystyle\lim_{x\to 0^+} \frac{e^x-x-1}{x^2}$

06 06 ex 18 18. $\displaystyle\lim_{x\to 0^+} \frac{x-\sin x}{x^3-x^2}$

06 06 ex 19 19. $\displaystyle\lim_{x\to \infty} \frac{x^4}{e^x}$

06 06 ex 20 20. $\displaystyle\lim_{x\to \infty} \frac{\sqrt{x}}{e^x}$

06 06 ex 21 21. $\displaystyle\lim_{x\to \infty} \frac{e^x}{\sqrt{x}}$

06 06 ex 22 22. $\displaystyle\lim_{x\to \infty} \frac{e^x}{2^x}$

06 06 ex 23 23. $\displaystyle\lim_{x\to \infty} \frac{e^x}{3^x}$

06 06 ex 24 24. $\displaystyle\lim_{x\to 3} \frac{x^3-5x^2+3x+9}{x^3-7x^2+15x-9}$

06 06 ex 25 25. $\displaystyle\lim_{x\to -2} \frac{x^3+4x^2+4x}{x^3+7x^2+16x+12}$

06 06 ex 26 26. $\displaystyle\lim_{x\to \infty} \frac{\ln x}{x}$

06 06 ex 27 27. $\displaystyle\lim_{x\to \infty} \frac{\ln(x^2)}{x}$

06 06 ex 28 28. $\displaystyle\lim_{x\to \infty} \frac{\left(\ln x\right)^2}{x}$

06 06 ex 29 29. $\displaystyle\lim_{x\to 0^+} x\cdot \ln x$

06 06 ex 30 30. $\displaystyle\lim_{x\to 0^+} \sqrt{x}\cdot \ln x$

06 06 ex 52 31. $\displaystyle\lim_{x\to 0^+} xe^{1/x}$

06 06 ex 31 32. $\displaystyle\lim_{x\to \infty} x^3-x^2$

06 06 ex 32 33. $\displaystyle\lim_{x\to \infty} \sqrt{x}-\ln x$

06 06 ex 33 34. $\displaystyle\lim_{x\to -\infty} xe^x$

06 06 ex 34 35. $\displaystyle\lim_{x\to 0^+} \frac{1}{x^2}e^{-1/x}$

06 06 ex 35 36. $\displaystyle\lim_{x\to 0^+} (1+x)^{1/x}$

06 06 ex 36 37. $\displaystyle\lim_{x\to 0^+} (2x)^x$

06 06 ex 37 38. $\displaystyle\lim_{x\to 0^+} (2/x)^x$

06 06 ex 38 39. $\displaystyle\lim_{x\to 0^+} (\sin x)^x$ Hint: use the Squeeze Theorem.

06 06 ex 39 40. $\lim_{x\to 1^+} (1-x)^{1-x}$

06 06 ex 40 41. $\lim_{x\to\infty} (x)^{1/x}$

06 06 ex 41 42. $\lim_{x\to\infty} (1/x)^{x}$

06 06 ex 42 43. $\lim_{x\to 1^+} (\ln x)^{1-x}$

06 06 ex 43 44. $\lim_{x\to\infty} (1+x)^{1/x}$

06 06 ex 44 45. $\lim_{x\to\infty} (1+x^2)^{1/x}$

06 06 ex 45 46. $\lim_{x\to\pi/2} \tan x \cos x$

06 06 ex 46 47. $\lim_{x\to\pi/2} \tan x \sin(2x)$

06 06 ex 47 48. $\lim_{x\to 1^+} \frac{1}{\ln x} - \frac{1}{x-1}$

06 06 ex 48 49. $\lim_{x\to 3^+} \frac{5}{x^2-9} - \frac{x}{x-3}$

06 06 ex 49 50. $\lim_{x\to\infty} x\tan(1/x)$

06 06 ex 50 51. $\lim_{x\to\infty} \frac{(\ln x)^3}{x}$

06 06 ex 51 52. $\lim_{x\to 1} \frac{x^2+x-2}{\ln x}$

# Exercises 6.7

## Terms and Concepts

06 07 ex 03 1. The definite integral was defined with what two stipulations?

06 07 ex 01 2. If $\displaystyle\lim_{b\to\infty}\int_0^b f(x)\,dx$ exists, then the integral $\displaystyle\int_0^\infty f(x)\,dx$ is said to ________________.

06 07 ex 02 3. If $\displaystyle\int_1^\infty f(x)\,dx = 10$, and $0 \le g(x) \le f(x)$ for all $x$, then we know that $\displaystyle\int_1^\infty g(x)\,dx$ ________________.

06 07 ex 04 4. For what values of $p$ will $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge?

06 07 ex 05 5. For what values of $p$ will $\displaystyle\int_{10}^\infty \frac{1}{x^p}\,dx$ converge?

06 07 ex 06 6. For what values of $p$ will $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ converge?

## Problems

06 07 exset 01 **In Exercises 7 – 33, evaluate the given improper integral.**

06 07 ex 07 7. $\displaystyle\int_0^\infty e^{5-2x}\,dx$

06 07 ex 08 8. $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$

06 07 ex 09 9. $\displaystyle\int_1^\infty x^{-4}\,dx$

06 07 ex 10 10. $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+9}\,dx$

06 07 ex 11 11. $\displaystyle\int_{-\infty}^0 2^x\,dx$

06 07 ex 12 12. $\displaystyle\int_{-\infty}^0 \left(\frac{1}{2}\right)^x\,dx$

06 07 ex 13 13. $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

06 07 ex 14 14. $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+4}\,dx$

06 07 ex 15 15. $\displaystyle\int_2^\infty \frac{1}{(x-1)^2}\,dx$

06 07 ex 16 16. $\displaystyle\int_1^2 \frac{1}{(x-1)^2}\,dx$

06 07 ex 17 17. $\displaystyle\int_2^\infty \frac{1}{x-1}\,dx$

06 07 ex 18 18. $\displaystyle\int_1^2 \frac{1}{x-1}\,dx$

06 07 ex 43 19. $\displaystyle\int_{-1}^1 \frac{1}{x}\,dx$

06 07 ex 19 20. $\displaystyle\int_1^3 \frac{1}{x-2}\,dx$

06 07 ex 20 21. $\displaystyle\int_0^\pi \sec^2 x\,dx$

06 07 ex 21 22. $\displaystyle\int_{-2}^1 \frac{1}{\sqrt{|x|}}\,dx$

06 07 ex 22 23. $\displaystyle\int_0^\infty xe^{-x}\,dx$

06 07 ex 23 24. $\displaystyle\int_0^\infty xe^{-x^2}\,dx$

06 07 ex 24 25. $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\,dx$

06 07 ex 25 26. $\displaystyle\int_{-\infty}^\infty \frac{1}{e^x+e^{-x}}\,dx$

06 07 ex 26 27. $\displaystyle\int_0^1 x\ln x\,dx$

06 07 ex 27 28. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$

06 07 ex 28 29. $\displaystyle\int_0^1 \ln x\,dx$

06 07 ex 29 30. $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$

06 07 ex 30 31. $\displaystyle\int_1^\infty \frac{\ln x}{\sqrt{x}}\,dx$

06 07 ex 31 32. $\displaystyle\int_0^\infty e^{-x}\sin x\,dx$

06 07 ex 32 33. $\displaystyle\int_0^\infty e^{-x}\cos x\,dx$

06 07 exset 02 **In Exercises 34 – 43, use the Direct Comparison Test or the Limit Comparison Test to determine whether the given definite integral converges or diverges. Clearly state what test is being used and what function the integrand is being compared to.**

06 07 ex 33 34. $\displaystyle\int_{10}^{\infty} \frac{3}{\sqrt{3x^2+2x-5}}\,dx$

06 07 ex 34 35. $\displaystyle\int_{2}^{\infty} \frac{4}{\sqrt{7x^3-x}}\,dx$

06 07 ex 35 36. $\displaystyle\int_{0}^{\infty} \frac{\sqrt{x+3}}{\sqrt{x^3-x^2+x+1}}\,dx$

06 07 ex 36 37. $\displaystyle\int_{1}^{\infty} e^{-x}\ln x\,dx$

06 07 ex 37 38. $\displaystyle\int_{5}^{\infty} e^{-x^2+3x+1}\,dx$

06 07 ex 38 39. $\displaystyle\int_{0}^{\infty} \frac{\sqrt{x}}{e^x}\,dx$

06 07 ex 39 40. $\displaystyle\int_{2}^{\infty} \frac{1}{x^2+\sin x}\,dx$

06 07 ex 42 41. $\displaystyle\int_{0}^{\infty} \frac{x}{x^2+\cos x}\,dx$

06 07 ex 40 42. $\displaystyle\int_{0}^{\infty} \frac{1}{x+e^x}\,dx$

06 07 ex 41 43. $\displaystyle\int_{0}^{\infty} \frac{1}{e^x-x}\,dx$

# Exercises 6.8

## *Terms and Concepts*

06 08 ex 01 1. Trigonometric Substitution works on the same principles as Integration by Substitution, though it can feel "_________".

06 08 ex 02 2. If one uses Trigonometric Substitution on an integrand containing $\sqrt{25-x^2}$, then one should set $x =$_________.

06 08 ex 03 3. Consider the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$.

   (a) What identity is obtained when both sides are divided by $\cos^2\theta$?

   (b) Use the new identity to simplify $9\tan^2\theta + 9$.

06 08 ex 04 4. Why does Key Idea 13(a) state that $\sqrt{a^2-x^2} = a\cos\theta$, and not $|a\cos\theta|$?

## *Problems*

06 08 exset 01 **In Exercises 5 – 16, apply Trigonometric Substitution to evaluate the indefinite integrals.**

06 08 ex 05 5. $\displaystyle\int \sqrt{x^2+1}\,dx$

06 08 ex 06 6. $\displaystyle\int \sqrt{x^2+4}\,dx$

06 08 ex 07 7. $\displaystyle\int \sqrt{1-x^2}\,dx$

06 08 ex 08 8. $\displaystyle\int \sqrt{9-x^2}\,dx$

06 08 ex 09 9. $\displaystyle\int \sqrt{x^2-1}\,dx$

06 08 ex 10 10. $\displaystyle\int \sqrt{x^2-16}\,dx$

06 08 ex 11 11. $\displaystyle\int \sqrt{4x^2+1}\,dx$

06 08 ex 12 12. $\displaystyle\int \sqrt{1-9x^2}\,dx$

06 08 ex 13 13. $\displaystyle\int \sqrt{16x^2-1}\,dx$

06 08 ex 14 14. $\displaystyle\int \frac{8}{\sqrt{x^2+2}}\,dx$

06 08 ex 15 15. $\displaystyle\int \frac{3}{\sqrt{7-x^2}}\,dx$

06 08 ex 16 16. $\displaystyle\int \frac{5}{\sqrt{x^2-8}}\,dx$

06 08 exset 02 **In Exercises 17 – 26, evaluate the indefinite integrals. Some may be evaluated without Trigonometric Substitution.**

06 08 ex 18 17. $\displaystyle\int \frac{\sqrt{x^2-11}}{x}\,dx$

06 08 ex 20 18. $\displaystyle\int \frac{1}{(x^2+1)^2}\,dx$

06 08 ex 17 19. $\displaystyle\int \frac{x}{\sqrt{x^2-3}}\,dx$

06 08 ex 24 20. $\displaystyle\int x^2\sqrt{1-x^2}\,dx$

06 08 ex 22 21. $\displaystyle\int \frac{x}{(x^2+9)^{3/2}}\,dx$

06 08 ex 19 22. $\displaystyle\int \frac{5x^2}{\sqrt{x^2-10}}\,dx$

06 08 ex 21 23. $\displaystyle\int \frac{1}{(x^2+4x+13)^2}\,dx$

06 08 ex 23 24. $\displaystyle\int x^2(1-x^2)^{-3/2}\,dx$

06 08 ex 25 25. $\displaystyle\int \frac{\sqrt{5-x^2}}{7x^2}\,dx$

06 08 ex 26 26. $\displaystyle\int \frac{x^2}{\sqrt{x^2+3}}\,dx$

06 08 exset 03 **In Exercises 27 – 32, evaluate the definite integrals by making the proper trigonometric substitution *and* changing the bounds of integration. (Note: each of the corresponding indefinite integrals has appeared previously in this Exercise set.)**

06 08 ex 27 27. $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\,dx$

06 08 ex 28 28. $\displaystyle\int_{4}^{8} \sqrt{x^2-16}\,dx$

06 08 ex 29 29. $\displaystyle\int_{0}^{2} \sqrt{x^2+4}\,dx$

06 08 ex 30 30. $\displaystyle\int_{-1}^{1} \frac{1}{(x^2+1)^2}\,dx$

06 08 ex 31 31. $\displaystyle\int_{-1}^{1} \sqrt{9-x^2}\,dx$

06 08 ex 32 32. $\displaystyle\int_{-1}^{1} x^2\sqrt{1-x^2}\,dx$

# Exercises 7.1

## *Terms and Concepts*

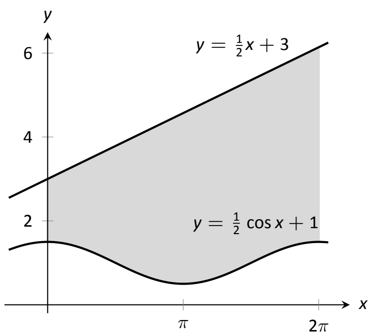
07 01 ex 01 1. T/F: The area between curves is always positive.

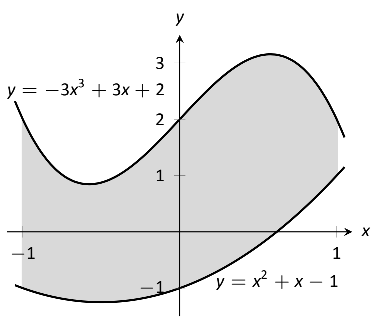
07 01 ex 02 2. T/F: Calculus can be used to find the area of basic geometric shapes.

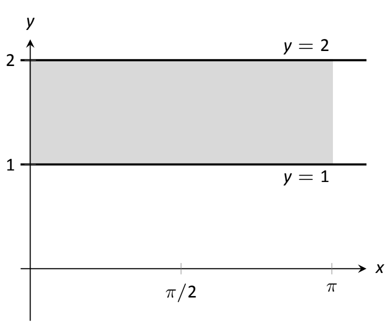
07 01 ex 03 3. In your own words, describe how to find the total area enclosed by $y = f(x)$ and $y = g(x)$.

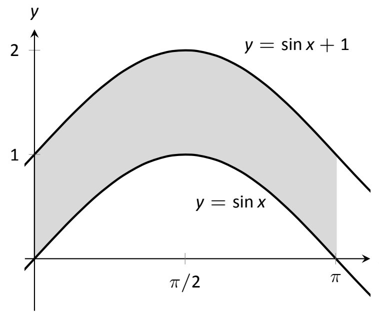
## *Problems*

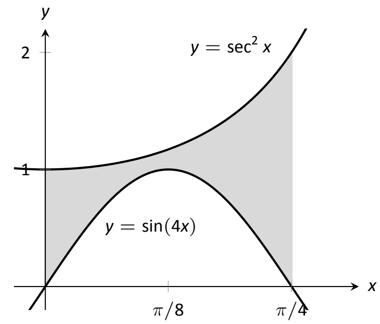
07 01 exset 01 **In Exercises 4 – 10, find the area of the shaded region in the given graph.**

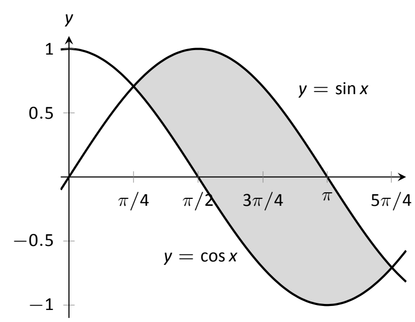
07 01 ex 04 4. 

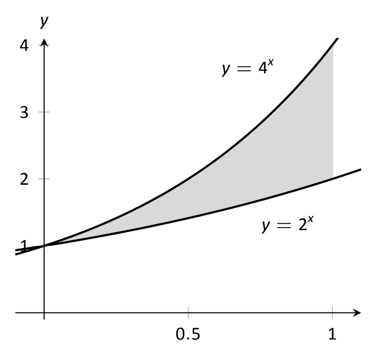

07 01 ex 05 5. 


07 01 ex 10 6. 


07 01 ex 06 7. 


07 01 ex 07 8. 


07 01 ex 08 9. 


07 01 ex 09 10. 


07 01 exset 02 **In Exercises 11 – 16, find the total area enclosed by the functions *f* and *g*.**

07 01 ex 11 11. $f(x) = 2x^2 + 5x - 3$, $g(x) = x^2 + 4x - 1$

07 01 ex 12 12. $f(x) = x^2 - 3x + 2$, $g(x) = -3x + 3$

07 01 ex 13 13. $f(x) = \sin x$, $g(x) = 2x/\pi$

07 01 ex 14 14. $f(x) = x^3 - 4x^2 + x - 1$, $g(x) = -x^2 + 2x - 4$

07 01 ex 15 15. $f(x) = x, g(x) = \sqrt{x}$

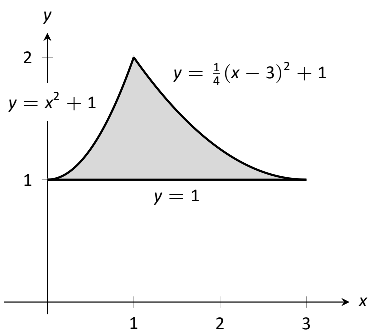
07 01 ex 16 16. $f(x) = -x^3 + 5x^2 + 2x + 1, g(x) = 3x^2 + x + 3$

07 01 ex 17 17. The functions $f(x) = \cos(2x)$ and $g(x) = \sin x$ intersect infinitely many times, forming an infinite number of repeated, enclosed regions. Find the areas of these regions.

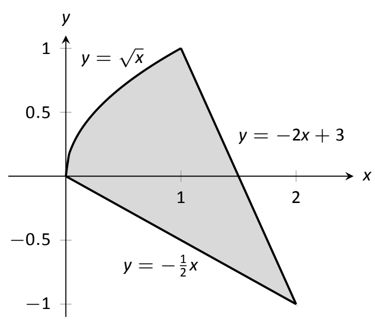
07 01 exset 03 **In Exercises 18 – 22, find the area of the enclosed region in two ways:**

1. **by treating the boundaries as functions of $x$, and**
2. **by treating the boundaries as functions of $y$.**

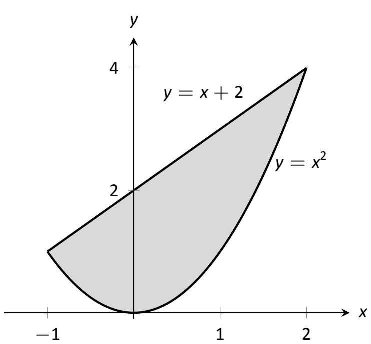
07 01 ex 18 18.



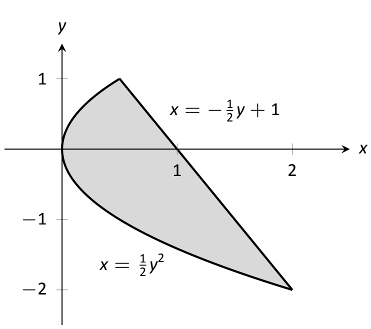

07 01 ex 19 19.



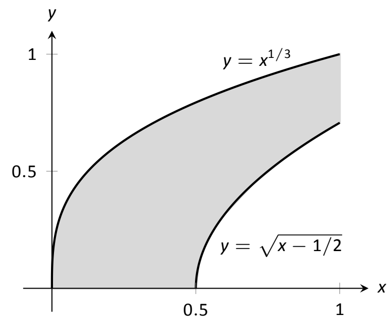

07 01 ex 20 20.




07 01 ex 21 21.




07 01 ex 22 22.




07 01 exset 04 **In Exercises 23 – 26, find the area triangle formed by the given three points.**

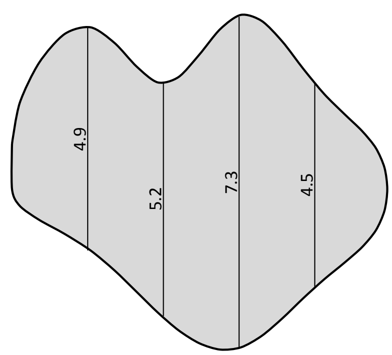
07 01 ex 23 23. $(1, 1)$, $(2, 3)$, and $(3, 3)$

07 01 ex 24 24. $(-1, 1)$, $(1, 3)$, and $(2, -1)$

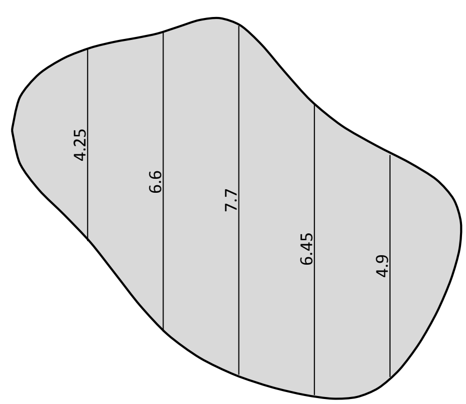
07 01 ex 25 25. $(1, 1)$, $(3, 3)$, and $(3, 3)$

07 01 ex 26 26. $(0, 0)$, $(2, 5)$, and $(5, 2)$

07 01 ex 29 27. Use the Trapezoidal Rule to approximate the area of the pictured lake whose lengths, in hundreds of feet, are measured in 100-foot increments.




07 01 ex 30 28. Use Simpson's Rule to approximate the area of the pictured lake whose lengths, in hundreds of feet, are measured in 200-foot increments.



# Exercises 7.2

## Terms and Concepts

07 02 ex 01 1. T/F: A solid of revolution is formed by revolving a shape around an axis.

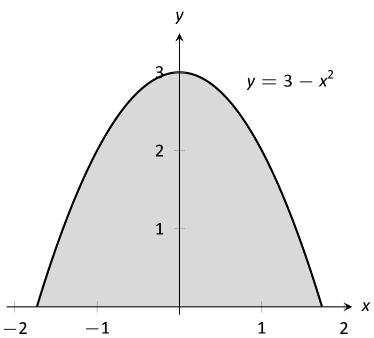
07 02 ex 02 2. In your own words, explain how the Disk and Washer Methods are related.

07 02 ex 03 3. Explain the how the units of volume are found in the integral of Theorem 54: if $A(x)$ has units of in$^2$, how does $\int A(x)\,dx$ have units of in$^3$?

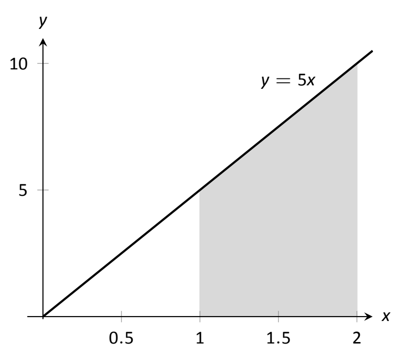
## Problems

07 02 exset 01 **In Exercises 4 – 7, a region of the Cartesian plane is shaded. Use the Disk/Washer Method to find the volume of the solid of revolution formed by revolving the region about the *x*-axis.**

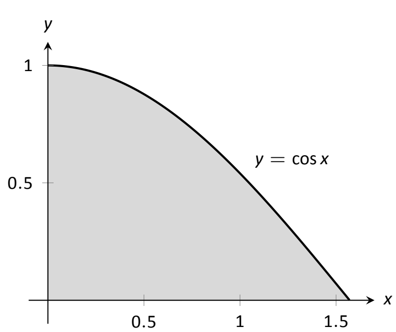
07 02 ex 05 4.



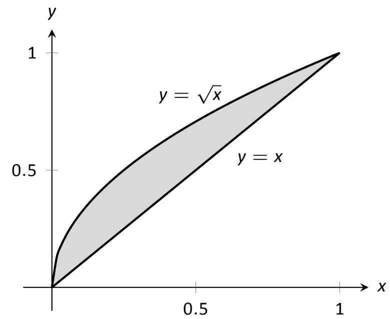

07 02 ex 06 5.




07 02 ex 07 6.

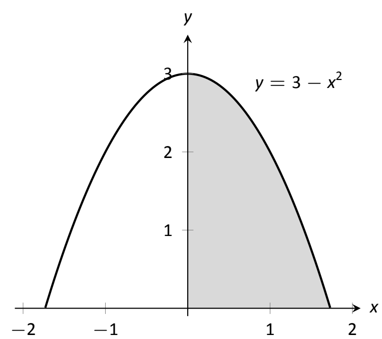



07 02 ex 04 7.

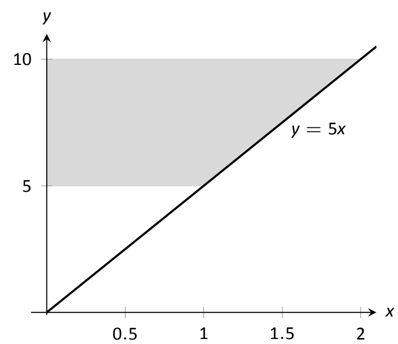



07 02 exset 02 **In Exercises 8 – 11, a region of the Cartesian plane is shaded. Use the Disk/Washer Method to find the volume of the solid of revolution formed by revolving the region about the *y*-axis.**

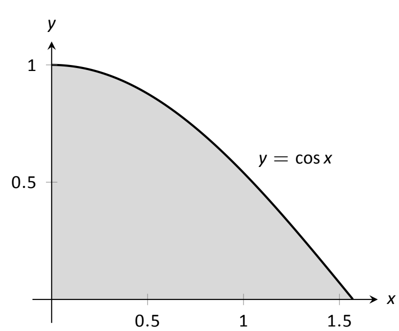
07 02 ex 09 8.




07 02 ex 10 9.



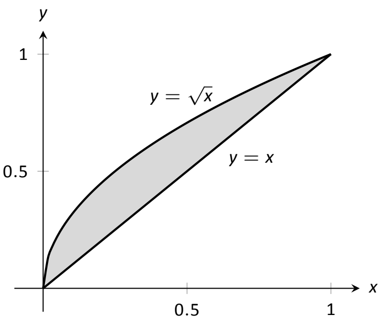

07 02 ex 11 10.




(Hint: Integration By Parts will be necessary, twice. First let $u = \arccos^2 x$, then let $u = \arccos x$.)

07 02 ex 08

11.




07 02 exset 03

**In Exercises 12 – 17, a region of the Cartesian plane is described. Use the Disk/Washer Method to find the volume of the solid of revolution formed by rotating the region about each of the given axes.**

07 02 ex 12

12. Region bounded by: $y = \sqrt{x}$, $y = 0$ and $x = 1$.
Rotate about:

(a) the $x$-axis
(b) $y = 1$
(c) the $y$-axis
(d) $x = 1$

07 02 ex 13

13. Region bounded by: $y = 4 - x^2$ and $y = 0$.
Rotate about:

(a) the $x$-axis
(b) $y = 4$
(c) $y = -1$
(d) $x = 2$

07 02 ex 14

14. The triangle with vertices $(1,1)$, $(1,2)$ and $(2,1)$.
Rotate about:

(a) the $x$-axis
(b) $y = 2$
(c) the $y$-axis
(d) $x = 1$

07 02 ex 15

15. Region bounded by $y = x^2 - 2x + 2$ and $y = 2x - 1$.
Rotate about:

(a) the $x$-axis
(b) $y = 1$
(c) $y = 5$

07 02 ex 16

16. Region bounded by $y = 1/\sqrt{x^2+1}$, $x = -1$, $x = 1$ and the $x$-axis.
Rotate about:

(a) the $x$-axis
(b) $y = 1$
(c) $y = -1$

07 02 ex 17

17. Region bounded by $y = 2x$, $y = x$ and $x = 2$.
Rotate about:

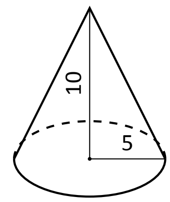
(a) the $x$-axis
(b) $y = 4$
(c) the $y$-axis
(d) $x = 2$

07 02 exset 04

**In Exercises 18 – 21, a solid is described. Orient the solid along the $x$-axis such that a cross-sectional area function $A(x)$ can be obtained, then apply Theorem 54 to find the volume of the solid.**

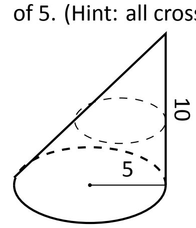
07 02 ex 18

18. A right circular cone with height of 10 and base radius of 5.



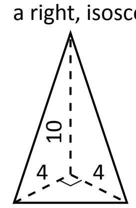

07 02 ex 19

19. A skew right circular cone with height of 10 and base radius of 5. (Hint: all cross-sections are circles.)



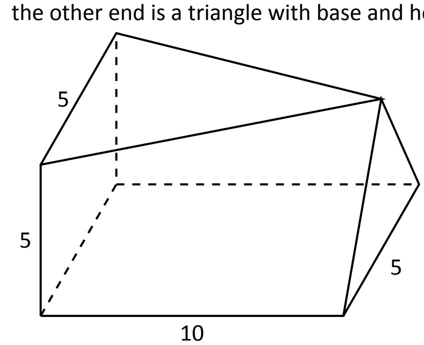

07 02 ex 20

20. A right triangular cone with height of 10 and whose base is a right, isosceles triangle with side length 4.




07 02 ex 21

21. A solid with length 10 with a rectangular base and triangular top, wherein one end is a square with side length 5 and the other end is a triangle with base and height of 5.



# Exercises 7.3

## *Terms and Concepts*

07 02 ex 01 1. T/F: A solid of revolution is formed by revolving a shape around an axis.

07 03 ex 01 2. T/F: The Shell Method can only be used when the Washer Method fails.

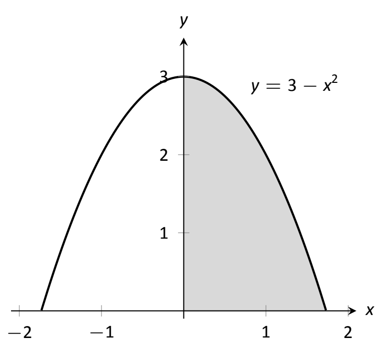
07 03 ex 02 3. T/F: The Shell Method works by integrating cross–sectional areas of a solid.

07 03 ex 03 4. T/F: When finding the volume of a solid of revolution that was revolved around a vertical axis, the Shell Method integrates with respect to $x$.

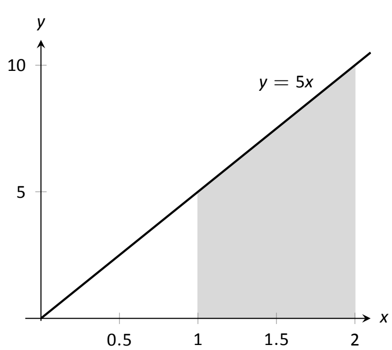
## *Problems*

07 03 exset 02 **In Exercises 5 – 8, a region of the Cartesian plane is shaded. Use the Shell Method to find the volume of the solid of revolution formed by revolving the region about the *y*-axis.**

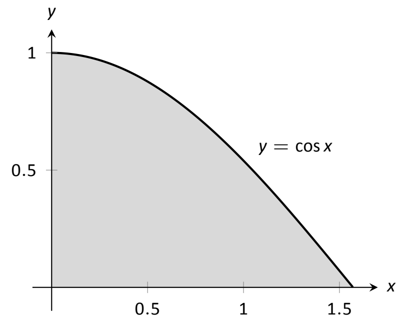
07 02 ex 09 5.



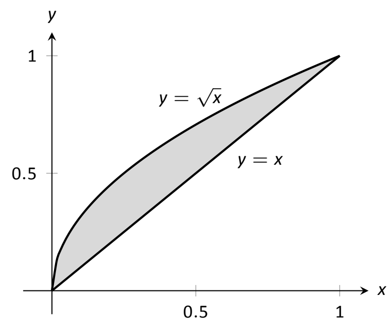

07 03 ex 06 6.




07 03 ex 11 7.

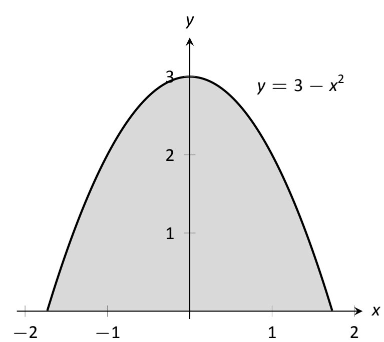



07 02 ex 08 8.

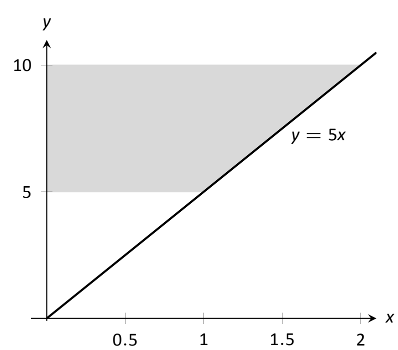



07 03 exset 01 **In Exercises 9 – 12, a region of the Cartesian plane is shaded. Use the Shell Method to find the volume of the solid of revolution formed by revolving the region about the *x*-axis.**

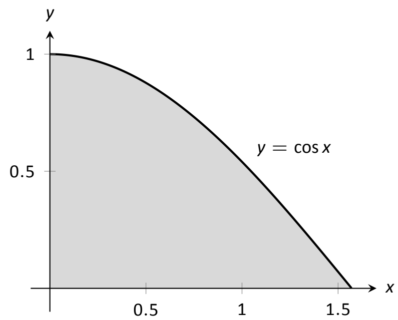
07 02 ex 05 9.



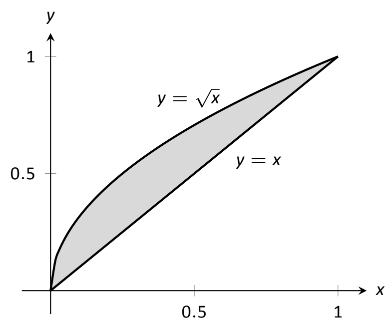

07 03 ex 10 10.



07 03 ex 20 11.




07 02 ex 04 12.




07 03 exset 03 **In Exercises 13 – 18, a region of the Cartesian plane is described. Use the Shell Method to find the volume of the solid of revolution formed by rotating the region about each of the given axes.**

07 03 ex 12 13. Region bounded by: $y = \sqrt{x}$, $y = 0$ and $x = 1$.
Rotate about:

(a) the $y$-axis
(b) $x = 1$
(c) the $x$-axis
(d) $y = 1$

07 03 ex 13 14. Region bounded by: $y = 4 - x^2$ and $y = 0$.
Rotate about:

(a) $x = 2$
(b) $x = -2$
(c) the $x$-axis
(d) $y = 4$

07 03 ex 14 15. The triangle with vertices $(1, 1)$, $(1, 2)$ and $(2, 1)$.
Rotate about:

(a) the $y$-axis
(b) $x = 1$
(c) the $x$-axis
(d) $y = 2$

07 03 ex 15 16. Region bounded by $y = x^2 - 2x + 2$ and $y = 2x - 1$.
Rotate about:

(a) the $y$-axis
(b) $x = 1$
(c) $x = -1$

07 03 ex 16 17. Region bounded by $y = 1/\sqrt{x^2 + 1}$, $x = 1$ and the $x$ and $y$-axes.
Rotate about:

(a) the $y$-axis
(b) $x = 1$

07 03 ex 17 18. Region bounded by $y = 2x$, $y = x$ and $x = 2$.
Rotate about:

(a) the $y$-axis
(b) $x = 2$
(c) the $x$-axis
(d) $y = 4$

# Exercises 7.4

## *Terms and Concepts*

07 04 ex 01 1. T/F: The integral formula for computing Arc Length was found by first approximating arc length with straight line segments.

07 04 ex 02 2. T/F: The integral formula for computing Arc Length includes a square–root, meaning the integration is probably easy.

## *Problems*

07 04 exset 01 **In Exercises 3 – 12, find the arc length of the function on the given interval.**

07 04 ex 11 3. $f(x) = x$ on $[0, 1]$.

07 04 ex 12 4. $f(x) = \sqrt{8}x$ on $[-1, 1]$.

07 04 ex 03 5. $f(x) = \frac{1}{3}x^{3/2} - x^{1/2}$ on $[0, 1]$.

07 04 ex 04 6. $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$ on $[1, 4]$.

07 04 ex 05 7. $f(x) = 2x^{3/2} - \frac{1}{6}\sqrt{x}$ on $[0, 9]$.

07 04 ex 06 8. $f(x) = \cosh x$ on $[-\ln 2, \ln 2]$.

07 04 ex 07 9. $f(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ on $[0, \ln 5]$.

07 04 ex 08 10. $f(x) = \frac{1}{12}x^5 + \frac{1}{5x^3}$ on $[.1, 1]$.

07 04 ex 09 11. $f(x) = \ln\big(\sin x\big)$ on $[\pi/6, \pi/2]$.

07 04 ex 10 12. $f(x) = \ln\big(\cos x\big)$ on $[0, \pi/4]$.

07 04 exset 02 **In Exercises 13 – 20, set up the integral to compute the arc length of the function on the given interval. Do not evaluate the integral.**

07 04 ex 13 13. $f(x) = x^2$ on $[0, 1]$.

07 04 ex 14 14. $f(x) = x^{10}$ on $[0, 1]$.

07 04 ex 15 15. $f(x) = \sqrt{x}$ on $[0, 1]$.

07 04 ex 16 16. $f(x) = \ln x$ on $[1, e]$.

07 04 ex 17 17. $f(x) = \sqrt{1 - x^2}$ on $[-1, 1]$. (Note: this describes the top half of a circle with radius 1.)

07 04 ex 18 18. $f(x) = \sqrt{1 - x^2/9}$ on $[-3, 3]$. (Note: this describes the top half of an ellipse with a major axis of length 6 and a minor axis of length 2.)

07 04 ex 19 19. $f(x) = \frac{1}{x}$ on $[1, 2]$.

07 04 ex 20 20. $f(x) = \sec x$ on $[-\pi/4, \pi/4]$.

07 04 exset 03 **In Exercises 21 – 28, use Simpson's Rule, with $n = 4$, to approximate the arc length of the function on the given interval. Note: these are the same problems as in Exercises 13–20.**

07 04 ex 21 21. $f(x) = x^2$ on $[0, 1]$.

07 04 ex 22 22. $f(x) = x^{10}$ on $[0, 1]$.

07 04 ex 23 23. $f(x) = \sqrt{x}$ on $[0, 1]$. (Note: $f'(x)$ is not defined at $x = 0$.)

07 04 ex 24 24. $f(x) = \ln x$ on $[1, e]$.

07 04 ex 25 25. $f(x) = \sqrt{1 - x^2}$ on $[-1, 1]$. (Note: $f'(x)$ is not defined at the endpoints.)

07 04 ex 26 26. $f(x) = \sqrt{1 - x^2/9}$ on $[-3, 3]$. (Note: $f'(x)$ is not defined at the endpoints.)

07 04 ex 27 27. $f(x) = \frac{1}{x}$ on $[1, 2]$.

07 04 ex 28 28. $f(x) = \sec x$ on $[-\pi/4, \pi/4]$.

07 04 exset 04 **In Exercises 29 – 33, find the surface area of the described solid of revolution.**

07 04 ex 29 29. The solid formed by revolving $y = 2x$ on $[0, 1]$ about the $x$-axis.

07 04 ex 30 30. The solid formed by revolving $y = x^2$ on $[0, 1]$ about the $y$-axis.

07 04 ex 31 31. The solid formed by revolving $y = x^3$ on $[0, 1]$ about the $x$-axis.

07 04 ex 32 32. The solid formed by revolving $y = \sqrt{x}$ on $[0, 1]$ about the $x$-axis.

07 04 ex 33 33. The sphere formed by revolving $y = \sqrt{1 - x^2}$ on $[-1, 1]$ about the $x$-axis.

# Exercises 7.5

## *Terms and Concepts*

07 05 ex 01 1. What are the typical units of work?

07 05 ex 02 2. If a man has a mass of 80 kg on Earth, will his mass on the moon be bigger, smaller, or the same?

07 05 ex 03 3. If a woman weighs 130 lb on Earth, will her weight on the moon be bigger, smaller, or the same?

## *Problems*

07 05 ex 04 4. A 100 ft rope, weighing 0.1 lb/ft, hangs over the edge of a tall building.

   (a) How much work is done pulling the entire rope to the top of the building?

   (b) How much rope is pulled in when half of the total work is done?

07 05 ex 05 5. A 50 m rope, with a mass density of 0.2 kg/m, hangs over the edge of a tall building.

   (a) How much work is done pulling the entire rope to the top of the building?

   (b) How much work is done pulling in the first 20 m?

07 05 ex 06 6. A rope of length $\ell$ ft hangs over the edge of tall cliff. (Assume the cliff is taller than the length of the rope.) The rope has a weight density of $d$ lb/ft.

   (a) How much work is done pulling the entire rope to the top of the cliff?

   (b) What percentage of the total work is done pulling in the first half of the rope?

   (c) How much rope is pulled in when half of the total work is done?

07 05 ex 07 7. A 20 m rope with mass density of 0.5 kg/m hangs over the edge of a 10 m building. How much work is done pulling the rope to the top?

07 05 ex 08 8. A crane lifts a 2,000 lb load vertically 30 ft with a 1" cable weighing 1.68 lb/ft.

   (a) How much work is done lifting the cable alone?

   (b) How much work is done lifting the load alone?

   (c) Could one conclude that the work done lifting the cable is negligible compared to the work done lifting the load?

07 05 ex 09 9. A 100 lb bag of sand is lifted uniformly 120 ft in one minute. Sand leaks from the bag at a rate of 1/4 lb/s. What is the total work done in lifting the bag?

07 05 ex 10 10. A box weighing 2 lb lifts 10 lb of sand vertically 50 ft. A crack in the box allows the sand to leak out such that 9 lb of sand is in the box at the end of the trip. Assume the sand leaked out at a uniform rate. What is the total work done in lifting the box and sand?

07 05 ex 11 11. A force of 1000 lb compresses a spring 3 in. How much work is performed in compressing the spring?

07 05 ex 12 12. A force of 2 N stretches a spring 5 cm. How much work is performed in stretching the spring?

07 05 ex 13 13. A force of 50 lb compresses a spring from a natural length of 18 in to 12 in. How much work is performed in compressing the spring?

07 05 ex 14 14. A force of 20 lb stretches a spring from a natural length of 6 in to 8 in. How much work is performed in stretching the spring?

07 05 ex 15 15. A force of 7 N stretches a spring from a natural length of 11 cm to 21 cm. How much work is performed in stretching the spring from a length of 16 cm to 21 cm?

07 05 ex 16 16. A force of $f$ N stretches a spring $d$ m from its natural length. How much work is performed in stretching the spring?

07 05 ex 17 17. A 20 lb weight is attached to a spring. The weight rests on the spring, compressing the spring from a natural length of 1 ft to 6 in.

   How much work is done in lifting the box 1.5 ft (i.e, the spring will be stretched 1 ft beyond its natural length)?

07 05 ex 18 18. A 20 lb weight is attached to a spring. The weight rests on the spring, compressing the spring from a natural length of 1 ft to 6 in.

   How much work is done in lifting the box 6 in (i.e, bringing the spring back to its natural length)?

07 05 ex 19 19. A 5 m tall cylindrical tank with radius of 2 m is filled with 3 m of gasoline, with a mass density of 737.22 kg/m$^3$. Compute the total work performed in pumping all the gasoline to the top of the tank.

07 05 ex 20 20. A 6 ft cylindrical tank with a radius of 3 ft is filled with water, which has a weight density of 62.4 lb/ft$^3$. The water is to be pumped to a point 2 ft above the top of the tank.

   (a) How much work is performed in pumping all the water from the tank?

   (b) How much work is performed in pumping 3 ft of water from the tank?

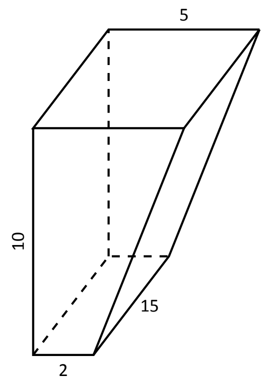
   (c) At what point is 1/2 of the total work done?

07 05 ex 21 21. A gasoline tanker is filled with gasoline with a weight density of 45.93 lb/ft$^3$. The dispensing valve at the base is jammed shut, forcing the operator to empty the tank via

pumping the gas to a point 1 ft above the top of the tank. Assume the tank is a perfect cylinder, 20 ft long with a diameter of 7.5 ft. How much work is performed in pumping all the gasoline from the tank?

07 05 ex 22

22. A fuel oil storage tank is 10 ft deep with trapezoidal sides, 5 ft at the top and 2 ft at the bottom, and is 15 ft wide (see diagram below). Given that fuel oil weighs 55.46 lb/ft$^3$, find the work performed in pumping all the oil from the tank to a point 3 ft above the top of the tank.




07 05 ex 23

23. A conical water tank is 5 m deep with a top radius of 3 m. (This is similar to Example 224.) The tank is filled with pure water, with a mass density of 1000 kg/m$^3$.

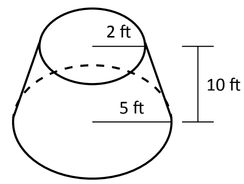
    (a) Find the work performed in pumping all the water to the top of the tank.

    (b) Find the work performed in pumping the top 2.5 m of water to the top of the tank.

    (c) Find the work performed in pumping the top half of the water, by volume, to the top of the tank.

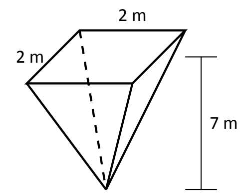
07 05 ex 24

24. A water tank has the shape of a truncated cone, with dimensions given below, and is filled with water with a weight density of 62.4 lb/ft$^3$. Find the work performed in pumping all water to a point 1 ft above the top of the tank.



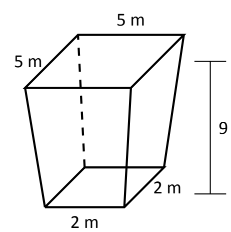

07 05 ex 25

25. A water tank has the shape of an inverted pyramid, with dimensions given below, and is filled with water with a mass density of 1000 kg/m$^3$. Find the work performed in pumping all water to a point 5 m above the top of the tank.




07 05 ex 26

26. A water tank has the shape of an truncated, inverted pyramid, with dimensions given below, and is filled with water with a mass density of 1000 kg/m$^3$. Find the work performed in pumping all water to a point 1 m above the top of the tank.



# Exercises 7.6

## *Terms and Concepts*

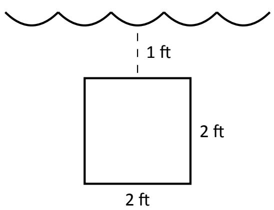
07 06 ex 01 1. State in your own words Pascal's Principle.

07 06 ex 02 2. State in your own words how pressure is different from force.

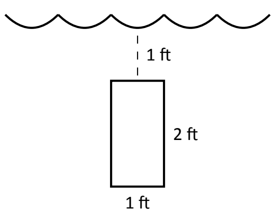
## *Problems*

07 06 exset 01 **In Exercises 3 – 12, find the fluid force exerted on the given plate, submerged in water with a weight density of 62.4 lb/ft$^3$.**

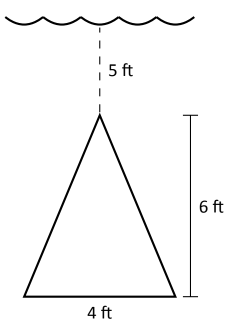
07 06 ex 03 3.



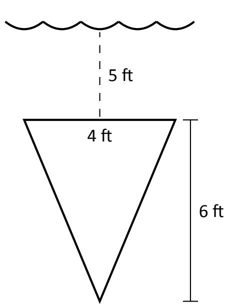

07 06 ex 06 4.



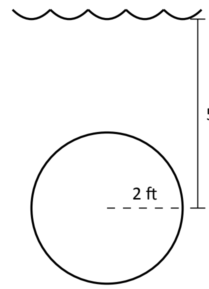

07 06 ex 13 5.



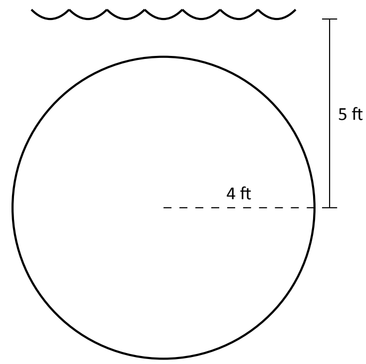

07 06 ex 14 6.



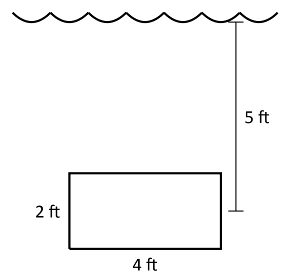

07 06 ex 15 7.



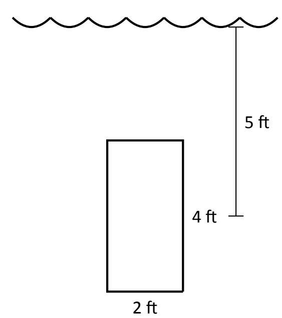

07 06 ex 16 8.



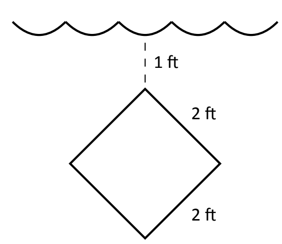

07 06 ex 17 9.



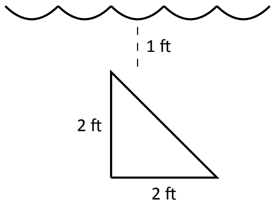

07 06 ex 18 10.




07 06 ex 04 11.



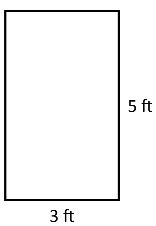
07 06 ex 05 12.



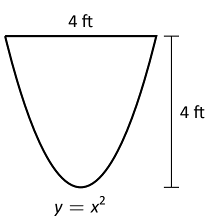

07 06 exset 02 **In Exercises 13 – 18, the side of a container is pictured. Find the fluid force exerted on this plate when the container is full of:**

1. **water, with a weight density of 62.4 lb/ft$^3$, and**
2. **concrete, with a weight density of 150 lb/ft$^3$.**

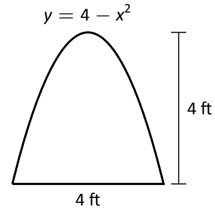
07 06 ex 07 13.



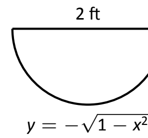

07 06 ex 08 14.



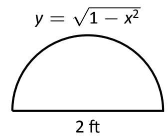

07 06 ex 09 15.



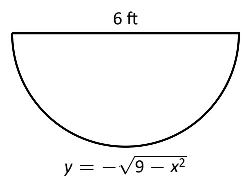

07 06 ex 10 16.




07 06 ex 11 17.




07 06 ex 12 18.




07 06 ex 19 19. How deep must the center of a vertically oriented circular plate with a radius of 1 ft be submerged in water, with a weight density of 62.4 lb/ft$^3$, for the fluid force on the plate to reach 1,000 lb?

07 06 ex 20 20. How deep must the center of a vertically oriented square plate with a side length of 2 ft be submerged in water, with a weight density of 62.4 lb/ft$^3$, for the fluid force on the plate to reach 1,000 lb?

# Exercises 8.1

## *Terms and Concepts*

08 01 ex 01 1. Use your own words to define a *sequence.*

08 01 ex 02 2. The domain of a sequence is the ________ numbers.

08 01 ex 03 3. Use your own words to describe the *range* of a sequence.

08 01 ex 04 4. Describe what it means for a sequence to be *bounded*.

## *Problems*

08 01 exset 01 **In Exercises 5 – 8, give the first five terms of the given sequence.**

08 01 ex 05 5. $\{a_n\} = \left\{\dfrac{4^n}{(n+1)!}\right\}$

08 01 ex 06 6. $\{b_n\} = \left\{\left(-\dfrac{3}{2}\right)^n\right\}$

08 01 ex 07 7. $\{c_n\} = \left\{-\dfrac{n^{n+1}}{n+2}\right\}$

08 01 ex 08 8. $\{d_n\} = \left\{\dfrac{1}{\sqrt{5}}\left(\left(\dfrac{1+\sqrt{5}}{2}\right)^n - \left(\dfrac{1-\sqrt{5}}{2}\right)^n\right)\right\}$

08 01 exset 02 **In Exercises 9 – 12, determine the $n^{\text{th}}$ term of the given sequence.**

08 01 ex 09 9. $4, 7, 10, 13, 16, \ldots$

08 01 ex 10 10. $3,\ -\dfrac{3}{2},\ \dfrac{3}{4},\ -\dfrac{3}{8},\ \ldots$

08 01 ex 11 11. $10,\ 20,\ 40,\ 80,\ 160,\ \ldots$

08 01 ex 12 12. $1, 1,\ \dfrac{1}{2},\ \dfrac{1}{6},\ \dfrac{1}{24},\ \dfrac{1}{120},\ \ldots$

08 01 exset 06 **In Exercises 13 – 16, use the following information to determine the limit of the given sequences.**

- $\{a_n\} = \left\{\dfrac{2^n - 20}{2^n}\right\}$; $\quad \lim\limits_{n\to\infty} a_n = 1$
- $\{b_n\} = \left\{\left(1+\dfrac{2}{n}\right)^n\right\}$; $\quad \lim\limits_{n\to\infty} b_n = e^2$
- $\{c_n\} = \{\sin(3/n)\}$; $\quad \lim\limits_{n\to\infty} c_n = 0$

08 01 ex 35 13. $\{a_n\} = \left\{\dfrac{2^n - 20}{7\cdot 2^n}\right\}$

08 01 ex 36 14. $\{a_n\} = \{3b_n - a_n\}$

08 01 ex 37 15. $\{a_n\} = \left\{\sin(3/n)\left(1+\dfrac{2}{n}\right)^n\right\}$

08 01 ex 38 16. $\{a_n\} = \left\{\left(1+\dfrac{2}{n}\right)^{2n}\right\}$

08 01 exset 03 **In Exercises 17 – 28, determine whether the sequence converges or diverges. If convergent, give the limit of the sequence.**

08 01 ex 13 17. $\{a_n\} = \left\{(-1)^n \dfrac{n}{n+1}\right\}$

08 01 ex 14 18. $\{a_n\} = \left\{\dfrac{4n^2 - n + 5}{3n^2 + 1}\right\}$

08 01 ex 15 19. $\{a_n\} = \left\{\dfrac{4^n}{5^n}\right\}$

08 01 ex 16 20. $\{a_n\} = \left\{\dfrac{n-1}{n} - \dfrac{n}{n-1}\right\},\ n \geq 2$

08 01 ex 17 21. $\{a_n\} = \{\ln(n)\}$

08 01 ex 18 22. $\{a_n\} = \left\{\dfrac{3n}{\sqrt{n^2+1}}\right\}$

08 01 ex 19 23. $\{a_n\} = \left\{\left(1+\dfrac{1}{n}\right)^n\right\}$

08 01 ex 26 24. $\{a_n\} = \left\{5 - \dfrac{1}{n}\right\}$

08 01 ex 27 25. $\{a_n\} = \left\{\dfrac{(-1)^{n+1}}{n}\right\}$

08 01 ex 28 26. $\{a_n\} = \left\{\dfrac{1.1^n}{n}\right\}$

08 01 ex 29 27. $\{a_n\} = \left\{\dfrac{2n}{n+1}\right\}$

08 01 ex 30 28. $\{a_n\} = \left\{(-1)^n \dfrac{n^2}{2^n - 1}\right\}$

08 01 exset 04 **In Exercises 29 – 34, determine whether the sequence is bounded, bounded above, bounded below, or none of the above.**

08 01 ex 20 29. $\{a_n\} = \{\sin n\}$

08 01 ex 21 30. $\{a_n\} = \{\tan n\}$

08 01 ex 22 31. $\{a_n\} = \left\{(-1)^n \dfrac{3n-1}{n}\right\}$

08 01 ex 23 32. $\{a_n\} = \left\{\dfrac{3n^2 - 1}{n}\right\}$

08 01 ex 24 33. $\{a_n\} = \{n\cos n\}$

08 01 ex 25 34. $\{a_n\} = \{2^n - n!\}$

08 01 exset 05 **In Exercises 35 – 38, determine whether the sequence is monotonically increasing or decreasing. If it is not, determine if there is an $m$ such that it is monotonic for all $n \geq m$.**

08 01 ex 31 35. $\{a_n\} = \left\{\dfrac{n}{n+2}\right\}$

08 01 ex 32 36. $\{a_n\} = \left\{\dfrac{n^2 - 6n + 9}{n}\right\}$

08 01 ex 33 37. $\{a_n\} = \left\{(-1)^n \dfrac{1}{n^3}\right\}$

08 01 ex 34 38. $\{a_n\} = \left\{\dfrac{n^2}{2^n}\right\}$

08 01 ex 39 39. Prove Theorem 56; that is, use the definition of the limit of a sequence to show that if $\lim\limits_{n\to\infty} |a_n| = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.

08 01 ex 40 40. Let $\{a_n\}$ and $\{b_n\}$ be sequences such that $\lim\limits_{n\to\infty} a_n = L$ and $\lim\limits_{n\to\infty} b_n = K$.

   (a) Show that if $a_n < b_n$ for all $n$, then $L \leq K$.

   (b) Give an example where $L = K$.

08 01 ex 41 41. Prove the Squeeze Theorem for sequences: Let $\{a_n\}$ and $\{b_n\}$ be such that $\lim\limits_{n\to\infty} a_n = L$ and $\lim\limits_{n\to\infty} b_n = L$, and let $\{c_n\}$ be such that $a_n \leq c_n \leq b_n$ for all $n$. Then $\lim\limits_{n\to\infty} c_n = L$

# Exercises 8.2

## *Terms and Concepts*

08 02 ex 01 1. Use your own words to describe how sequences and series are related.

08 02 ex 02 2. Use your own words to define a *partial sum*.

08 02 ex 25 3. Given a series $\sum_{n=1}^{\infty} a_n$, describe the two sequences related to the series that are important.

08 02 ex 03 4. Use your own words to explain what a geometric series is.

08 02 ex 04 5. T/F: If $\{a_n\}$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is also convergent.

## *Problems*

08 02 exset 01 **In Exercises 6 – 13, a series $\sum_{n=1}^{\infty} a_n$ is given.**

**(a) Give the first 5 partial sums of the series.**

**(b) Give a graph of the first 5 terms of $a_n$ and $S_n$ on the same axes.**

08 02 ex 05 6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

08 02 ex 06 7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

08 02 ex 07 8. $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

08 02 ex 08 9. $\displaystyle\sum_{n=1}^{\infty} n$

08 02 ex 09 10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

08 02 ex 10 11. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$

08 02 ex 11 12. $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{9}{10}\right)^n$

08 02 ex 12 13. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$

08 02 exset 03 **In Exercises 14 – 19, use Theorem 63 to show the given series diverges.**

08 02 ex 28 14. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{n(n+2)}$

08 02 ex 29 15. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

08 02 ex 30 16. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

08 02 ex 31 17. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n - n^5}{5^n + n^5}$

08 02 ex 33 18. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}}$

08 02 ex 34 19. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

08 02 exset 04 **In Exercises 20 – 29, state whether the given series converges or diverges.**

08 02 ex 37 20. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

08 02 ex 38 21. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{5^n}$

08 02 ex 39 22. $\displaystyle\sum_{n=0}^{\infty} \frac{6^n}{5^n}$

08 02 ex 40 23. $\displaystyle\sum_{n=1}^{\infty} n^{-4}$

08 02 ex 41 24. $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}$

08 02 ex 42 25. $\displaystyle\sum_{n=1}^{\infty} \frac{10}{n!}$

08 02 ex 46 26. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n!} + \frac{1}{n}\right)$

08 02 ex 43 27. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(2n+8)^2}$

08 02 ex 44 28. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$

08 02 ex 45 29. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

08 02 exset 02 **In Exercises 30 – 44, a series is given.**

**(a) Find a formula for $S_n$, the $n^{\text{th}}$ partial sum of the series.**

**(b) Determine whether the series converges or diverges. If it converges, state what it converges to.**

08 02 ex 13 30. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4^n}$

08 02 ex 14 31. $1^3 + 2^3 + 3^3 + 4^3 + \cdots$

08 02 ex 15 32. $\displaystyle\sum_{n=1}^{\infty} (-1)^n n$

08 02 ex 16 33. $\displaystyle\sum_{n=0}^{\infty} \frac{5}{2^n}$

08 02 ex 32 34. $\displaystyle\sum_{n=1}^{\infty} e^{-n}$

08 02 ex 17 35. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} + \cdots$

08 02 ex 26 36. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

08 02 ex 27 37. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$

08 02 ex 18 38. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

08 02 ex 19 39. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

08 02 ex 20 40. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

08 02 ex 21 41. $\frac{1}{1\cdot 4} + \frac{1}{2\cdot 5} + \frac{1}{3\cdot 6} + \frac{1}{4\cdot 7} + \cdots$

08 02 ex 22 42. $2 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{9}\right) + \left(\frac{1}{8} + \frac{1}{27}\right) + \cdots$

08 02 ex 35 43. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

08 02 ex 36 44. $\displaystyle\sum_{n=0}^{\infty} \big(\sin 1\big)^n$

08 02 ex 24 45. Break the Harmonic Series into the sum of the odd and even terms:

$$\sum_{n=1}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{2n-1} + \sum_{n=1}^{\infty} \frac{1}{2n}.$$

The goal is to show that each of the series on the right diverge.

(a) Show why $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1} > \sum_{n=1}^{\infty} \frac{1}{2n}$.
(Compare each $n^{\text{th}}$ partial sum.)

(b) Show why $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1} < 1 + \sum_{n=1}^{\infty} \frac{1}{2n}$

(c) Explain why (a) and (b) demonstrate that the series of odd terms is convergent, if, and only if, the series of even terms is also convergent. (That is, show both converge or both diverge.)

(d) Explain why knowing the Harmonic Series is divergent determines that the even and odd series are also divergent.

08 02 ex 23 46. Show the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)}$ diverges.

# Exercises 8.3

## *Terms and Concepts*

08 03 ex 01 1. In order to apply the Integral Test to a sequence $\{a_n\}$, the function $a(n) = a_n$ must be ______, ______ and ______.

08 03 ex 02 2. T/F: The Integral Test can be used to determine the sum of a convergent series.

08 03 ex 03 3. What test(s) in this section do not work well with factorials?

08 03 ex 04 4. Suppose $\displaystyle\sum_{n=0}^{\infty} a_n$ is convergent, and there are sequences $\{b_n\}$ and $\{c_n\}$ such that $b_n \leq a_n \leq c_n$ for all $n$. What can be said about the series $\displaystyle\sum_{n=0}^{\infty} b_n$ and $\displaystyle\sum_{n=0}^{\infty} c_n$?

## *Problems*

08 03 exset 01 **In Exercises 5 – 12, use the Integral Test to determine the convergence of the given series.**

08 03 ex 06 5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$

08 03 ex 07 6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

08 03 ex 08 7. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

08 03 ex 09 8. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

08 03 ex 10 9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

08 03 ex 11 10. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

08 03 ex 12 11. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

08 03 ex 05 12. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

08 03 exset 02 **In Exercises 13 – 22, use the Direct Comparison Test to determine the convergence of the given series; state what series is used for comparison.**

08 03 ex 13 13. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+3n-5}$

08 03 ex 14 14. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n+n^2-n}$

08 03 ex 15 15. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

08 03 ex 16 16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!+n}$

08 03 ex 17 17. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$

08 03 ex 18 18. $\displaystyle\sum_{n=5}^{\infty} \frac{1}{\sqrt{n}-2}$

08 03 ex 19 19. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^3-5}$

08 03 ex 20 20. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^n+10}$

08 03 ex 21 21. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2-1}$

08 03 ex 22 22. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$

08 03 exset 03 **In Exercises 23 – 32, use the Limit Comparison Test to determine the convergence of the given series; state what series is used for comparison.**

08 03 ex 23 23. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2-3n+5}$

08 03 ex 24 24. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n-n^2}$

08 03 ex 25 25. $\displaystyle\sum_{n=4}^{\infty} \frac{\ln n}{n-3}$

08 03 ex 26 26. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$

08 03 ex 27 27. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

08 03 ex 28 28. $\displaystyle\sum_{n=1}^{\infty} \frac{n-10}{n^2+10n+10}$

08 03 ex 29 29. $\displaystyle\sum_{n=1}^{\infty} \sin\left(1/n\right)$

08 03 ex 30 30. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{n^3-5}$

08 03 ex 31 31. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{n^2+17}$

08 03 ex 32 32. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+100}$

08 03 exset 04 **In Exercises 33 – 40, determine the convergence of the given series. State the test used; more than one test may be appropriate.**

08 03 ex 33 33. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

08 03 ex 34 34. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+5)^3}$

08 03 ex 35 35. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

08 03 ex 36 36. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n!}$

08 03 ex 37 37. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n+n}$

08 03 ex 38 38. $\displaystyle\sum_{n=1}^{\infty} \frac{n-2}{10n+5}$

08 03 ex 39 39. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

08 03 ex 40 40. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(1/n)}{\sqrt{n}}$

08 03 ex 41 41. Given that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, state which of the following series converges, may converge, or does not converge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} a_n a_{n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} (a_n)^2$

(d) $\displaystyle\sum_{n=1}^{\infty} n a_n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n}$

# Exercises 8.4

## Terms and Concepts

08 04 ex 01

1. The Ratio Test is not effective when the terms of a sequence only contain ______ functions.

08 04 ex 02

2. The Ratio Test is most effective when the terms of a sequence contains ______ and/or ______ functions.

08 04 ex 03

3. What three convergence tests do not work well with terms containing factorials?

08 04 ex 04

4. The Root Test works particularly well on series where each term is ______ to a ______.

## Problems

08 04 exset 01

**In Exercises 5 – 14, determine the convergence of the given series using the Ratio Test. If the Ratio Test is inconclusive, state so and determine convergence with another test.**

08 04 ex 05

5. $\displaystyle\sum_{n=0}^{\infty} \frac{2n}{n!}$

08 04 ex 06

6. $\displaystyle\sum_{n=0}^{\infty} \frac{5^n - 3n}{4^n}$

08 04 ex 07

7. $\displaystyle\sum_{n=0}^{\infty} \frac{n!10^n}{(2n)!}$

08 04 ex 14

8. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n + n^4}{7^n + n^2}$

08 04 ex 08

9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

08 04 ex 09

10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^3 + 7}$

08 04 ex 10

11. $\displaystyle\sum_{n=1}^{\infty} \frac{10 \cdot 5^n}{7^n - 3}$

08 04 ex 11

12. $\displaystyle\sum_{n=1}^{\infty} n \cdot \left(\frac{3}{5}\right)^n$

08 04 ex 12

13. $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots 2n}{3 \cdot 6 \cdot 9 \cdot 12 \cdots 3n}$

08 04 ex 13

14. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{5 \cdot 10 \cdot 15 \cdots (5n)}$

08 04 exset 02

**In Exercises 15 – 24, determine the convergence of the given series using the Root Test. If the Root Test is inconclusive, state so and determine convergence with another test.**

08 04 ex 15

15. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+5}{3n+11}\right)^n$

08 04 ex 16

16. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{.9n^2 - n - 3}{n^2 + n + 3}\right)^n$

08 04 ex 17

17. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^2}{3^n}$

08 04 ex 19

18. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

08 04 ex 18

19. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2 2^{n+1}}$

08 04 ex 20

20. $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+7}}{7^n}$

08 04 ex 21

21. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2 - n}{n^2 + n}\right)^n$

08 04 ex 22

22. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$

08 04 ex 23

23. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\left(\ln n\right)^n}$

08 04 ex 24

24. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\left(\ln n\right)^n}$

08 04 exset 03

**In Exercises 25 – 34, determine the convergence of the given series. State the test used; more than one test may be appropriate.**

08 04 ex 25

25. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 4n - 2}{n^3 + 4n^2 - 3n + 7}$

08 04 ex 26

26. $\displaystyle\sum_{n=1}^{\infty} \frac{n^4 4^n}{n!}$

08 04 ex 27

27. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n + n}$

08 04 ex 29

28. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^n}$

08 04 ex 28

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 4n + 1}}$

08 04 ex 30

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n!n!n!}{(3n)!}$

08 04 ex 31

31. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln n}$

08 04 ex 32

32. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+2}{n+1}\right)^n$

08 04 ex 33

33. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{(\ln n)^n}$

08 04 ex 34

34. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

# Exercises 8.5

## *Terms and Concepts*

08 05 ex 01 1. Why is $\sum_{n=1}^{\infty} \sin n$ not an alternating series?

08 05 ex 02 2. A series $\sum_{n=1}^{\infty} (-1)^n a_n$ converges when $\{a_n\}$ is ______, ______ and $\lim_{n\to\infty} a_n =$ ______.

08 05 ex 03 3. Give an example of a series where $\sum_{n=0}^{\infty} a_n$ converges but $\sum_{n=0}^{\infty} |a_n|$ does not.

08 05 ex 04 4. The sum of a ______ convergent series can be changed by rearranging the order of its terms.

## *Problems*

08 05 exset 01 **In Exercises 5 – 20, an alternating series $\sum_{n=i}^{\infty} a_n$ is given.**

**(a) Determine if the series converges or diverges.**

**(b) Determine if $\sum_{n=0}^{\infty} |a_n|$ converges or diverges.**

**(c) If $\sum_{n=0}^{\infty} a_n$ converges, determine if the convergence is conditional or absolute.**

08 05 ex 05 5. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

08 05 ex 06 6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n!}}$

08 05 ex 07 7. $\sum_{n=0}^{\infty} (-1)^n \frac{n+5}{3n-5}$

08 05 ex 08 8. $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^2}$

08 05 ex 09 9. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{3n+5}{n^2-3n+1}$

08 05 ex 10 10. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n + 1}$

08 05 ex 11 11. $\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$

08 05 ex 12 12. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+3+5+\cdots+(2n-1)}$

08 05 ex 13 13. $\sum_{n=1}^{\infty} \cos(\pi n)$

08 05 ex 14 14. $\sum_{n=1}^{\infty} \frac{\sin\left((n+1/2)\pi\right)}{n \ln n}$

08 05 ex 15 15. $\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$

08 05 ex 16 16. $\sum_{n=0}^{\infty} (-e)^{-n}$

08 05 ex 17 17. $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n!}$

08 05 ex 18 18. $\sum_{n=0}^{\infty} (-1)^n 2^{-n^2}$

08 05 ex 19 19. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

08 05 ex 20 20. $\sum_{n=1}^{\infty} \frac{(-1000)^n}{n!}$

08 05 exset 02 **Let $S_n$ be the $n^{\text{th}}$ partial sum of a series. In Exercises 21 – 24, a convergent alternating series is given and a value of $n$. Compute $S_n$ and $S_{n+1}$ and use these values to find bounds on the sum of the series.**

08 05 ex 21 21. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}, \quad n = 5$

08 05 ex 22 22. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}, \quad n = 4$

08 05 ex 23 23. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}, \quad n = 6$

08 05 ex 24 24. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n, \quad n = 9$

08 05 exset 03 **In Exercises 25 – 28, a convergent alternating series is given along with its sum and a value of $\varepsilon$. Use Theorem 71 to find $n$ such that the $n^{\text{th}}$ partial sum of the series is within $\varepsilon$ of the sum of the series.**

08 05 ex 25 25. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}, \quad \varepsilon = 0.001$

08 05 ex 26

26. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}, \quad \varepsilon = 0.0001$

08 05 ex 27

27. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}, \quad \varepsilon = 0.001$

08 05 ex 28

28. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = \cos 1, \quad \varepsilon = 10^{-8}$

# Exercises 8.6

## *Terms and Concepts*

08 06 ex 01 1. We adopt the convenction that $x^0 =$______, regardless of the value of $x$.

08 06 ex 02 2. What is the difference between the radius of convergence and the interval of convergence?

08 06 ex 03 3. If the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is 5, what is the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} n \cdot a_n x^{n-1}$?

08 06 ex 04 4. If the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is 5, what is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} (-1)^n a_n x^n$?

## *Problems*

08 06 exset 01 **In Exercises 5 – 8, write out the sum of the first 5 terms of the given power series.**

08 06 ex 05 5. $\displaystyle\sum_{n=0}^{\infty} 2^n x^n$

08 06 ex 06 6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$

08 06 ex 07 7. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} x^n$

08 06 ex 08 8. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

08 06 exset 02 **In Exercises 9 – 24, a power series is given.**

**(a) Find the radius of convergence.**

**(b) Find the interval of convergence.**

08 06 ex 09 9. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!} x^n$

08 06 ex 10 10. $\displaystyle\sum_{n=0}^{\infty} n x^n$

08 06 ex 11 11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n}$

08 06 ex 12 12. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+4)^n}{n!}$

08 06 ex 13 13. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n}$

08 06 ex 14 14. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-5)^n}{10^n}$

08 06 ex 15 15. $\displaystyle\sum_{n=0}^{\infty} 5^n (x-1)^n$

08 06 ex 16 16. $\displaystyle\sum_{n=0}^{\infty} (-2)^n x^n$

08 06 ex 17 17. $\displaystyle\sum_{n=0}^{\infty} \sqrt{n} x^n$

08 06 ex 18 18. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{3^n} x^n$

08 06 ex 19 19. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!} (x-5)^n$

08 06 ex 20 20. $\displaystyle\sum_{n=0}^{\infty} (-1)^n n! (x-10)^n$

08 06 ex 21 21. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

08 06 ex 22 22. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^3}$

08 06 ex 23 23. $\displaystyle\sum_{n=0}^{\infty} n! \left(\frac{x}{10}\right)^n$

08 06 ex 24 24. $\displaystyle\sum_{n=0}^{\infty} n^2 \left(\frac{x+4}{4}\right)^n$

08 06 exset 03 **In Exercises 25 – 30, a function $f(x) = \displaystyle\sum_{n=0}^{\infty} a_n x^n$ is given.**

**(a) Give a power series for $f'(x)$ and its interval of convergence.**

**(b) Give a power series for $\int f(x)\,dx$ and its interval of convergence.**

08 06 ex 25 25. $\displaystyle\sum_{n=0}^{\infty} n x^n$

08 06 ex 26 26. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

08 06 ex 27 27. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

08 06 ex 28 28. $\sum_{n=0}^{\infty}(-3x)^n$

08 06 ex 29 29. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

08 06 ex 30 30. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$

08 06 exset 04 **In Exercises 31 – 36, give the first 5 terms of the series that is a solution to the given differential equation.**

08 06 ex 31 31. $y' = 3y, \quad y(0) = 1$

08 06 ex 32 32. $y' = 5y, \quad y(0) = 5$

08 06 ex 33 33. $y' = y^2, \quad y(0) = 1$

08 06 ex 34 34. $y' = y + 1, \quad y(0) = 1$

08 06 ex 35 35. $y'' = -y, \quad y(0) = 0, y'(0) = 1$

08 06 ex 36 36. $y'' = 2y, \quad y(0) = 1, y'(0) = 1$

# Exercises 8.7

## *Terms and Concepts*

08 07 ex 30 1. What is the difference between a Taylor polynomial and a Maclaurin polynomial?

08 07 ex 33 2. T/F: In general, $p_n(x)$ approximates $f(x)$ better and better as $n$ gets larger.

08 07 ex 31 3. For some function $f(x)$, the Maclaurin polynomial of degree 4 is $p_4(x) = 6 + 3x - 4x^2 + 5x^3 - 7x^4$. What is $p_2(x)$?

08 07 ex 32 4. For some function $f(x)$, the Maclaurin polynomial of degree 4 is $p_4(x) = 6 + 3x - 4x^2 + 5x^3 - 7x^4$. What is $f'''(0)$?

## *Problems*

08 07 exset 01 **In Exercises 5 – 12, find the Maclaurin polynomial of degree $n$ for the given function.**

08 07 ex 01 5. $f(x) = e^{-x}, \quad n = 3$

08 07 ex 02 6. $f(x) = \sin x, \quad n = 8$

08 07 ex 03 7. $f(x) = x \cdot e^x, \quad n = 5$

08 07 ex 04 8. $f(x) = \tan x, \quad n = 6$

08 07 ex 05 9. $f(x) = e^{2x}, \quad n = 4$

08 07 ex 06 10. $f(x) = \dfrac{1}{1-x}, \quad n = 4$

08 07 ex 07 11. $f(x) = \dfrac{1}{1+x}, \quad n = 4$

08 07 ex 08 12. $f(x) = \dfrac{1}{1+x}, \quad n = 7$

08 07 exset 02 **In Exercises 13 – 20, find the Taylor polynomial of degree $n$, at $x = c$, for the given function.**

08 07 ex 09 13. $f(x) = \sqrt{x}, \quad n = 4, \quad c = 1$

08 07 ex 10 14. $f(x) = \ln(x+1), \quad n = 4, \quad c = 1$

08 07 ex 11 15. $f(x) = \cos x, \quad n = 6, \quad c = \pi/4$

08 07 ex 12 16. $f(x) = \sin x, \quad n = 5, \quad c = \pi/6$

08 07 ex 13 17. $f(x) = \dfrac{1}{x}, \quad n = 5, \quad c = 2$

08 07 ex 14 18. $f(x) = \dfrac{1}{x^2}, \quad n = 8, \quad c = 1$

08 07 ex 15 19. $f(x) = \dfrac{1}{x^2+1}, \quad n = 3, \quad c = -1$

08 07 ex 16 20. $f(x) = x^2 \cos x, \quad n = 2, \quad c = \pi$

08 07 exset 03 **In Exercises 21 – 24, approximate the function value with the indicated Taylor polynomial and give approximate bounds on the error.**

08 07 ex 17 21. Approximate $\sin 0.1$ with the Maclaurin polynomial of degree 3.

08 07 ex 18 22. Approximate $\cos 1$ with the Maclaurin polynomial of degree 4.

08 07 ex 19 23. Approximate $\sqrt{10}$ with the Taylor polynomial of degree 2 centered at $x = 9$.

08 07 ex 20 24. Approximate $\ln 1.5$ with the Taylor polynomial of degree 3 centered at $x = 1$.

08 07 exset 04 **Exercises 25 – 28 ask for an $n$ to be found such that $p_n(x)$ approximates $f(x)$ within a certain bound of accuracy.**

08 07 ex 21 25. Find $n$ such that the Maclaurin polynomial of degree $n$ of $f(x) = e^x$ approximates $e$ within 0.0001 of the actual value.

08 07 ex 24 26. Find $n$ such that the Taylor polynomial of degree $n$ of $f(x) = \sqrt{x}$, centered at $x = 4$, approximates $\sqrt{3}$ within 0.0001 of the actual value.

08 07 ex 22 27. Find $n$ such that the Maclaurin polynomial of degree $n$ of $f(x) = \cos x$ approximates $\cos \pi/3$ within 0.0001 of the actual value.

08 07 ex 23 28. Find $n$ such that the Maclaurin polynomial of degree $n$ of $f(x) = \sin x$ approximates $\cos \pi$ within 0.0001 of the actual value.

08 07 exset 05 **In Exercises 29 – 33, find the $n^{\text{th}}$ term of the indicated Taylor polynomial.**

08 07 ex 25 29. Find a formula for the $n^{\text{th}}$ term of the Maclaurin polynomial for $f(x) = e^x$.

08 07 ex 26 30. Find a formula for the $n^{\text{th}}$ term of the Maclaurin polynomial for $f(x) = \cos x$.

08 07 ex 27 31. Find a formula for the $n^{\text{th}}$ term of the Maclaurin polynomial for $f(x) = \dfrac{1}{1-x}$.

08 07 ex 28 32. Find a formula for the $n^{\text{th}}$ term of the Maclaurin polynomial for $f(x) = \dfrac{1}{1+x}$.

08 07 ex 29 33. Find a formula for the $n^{\text{th}}$ term of the Taylor polynomial for $f(x) = \ln x$.

08 07 exset 06 **In Exercises 34 – 36, approximate the solution to the given differential equation with a degree 4 Maclaurin polynomial.**

08 07 ex 34 34. $y' = y, \qquad y(0) = 1$

08 07 ex 35 35. $y' = 5y, \qquad y(0) = 3$

08 07 ex 36 36. $y' = \dfrac{2}{y}, \qquad y(0) = 1$

# Exercises 8.8

## *Terms and Concepts*

08 08 ex 01 1. What is the difference between a Taylor polynomial and a Taylor series?

08 08 ex 02 2. What theorem must we use to show that a function is equal to its Taylor series?

## *Problems*

08 08 exset 01 **Key Idea 32 gives the $n^{\text{th}}$ term of the Taylor series of common functions. In Exercises 3 – 6, verify the formula given in the Key Idea by finding the first few terms of the Taylor series of the given function and identifying a pattern.**

08 08 ex 06 3. $f(x) = e^x; \quad c = 0$

08 08 ex 03 4. $f(x) = \sin x; \quad c = 0$

08 08 ex 07 5. $f(x) = 1/(1-x); \quad c = 0$

08 08 ex 08 6. $f(x) = \tan^{-1} x; \quad c = 0$

08 08 exset 02 **In Exercises 7 – 12, find a formula for the $n^{\text{th}}$ term of the Taylor series of $f(x)$, centered at $c$, by finding the coefficients of the first few powers of $x$ and looking for a pattern. (The formulas for several of these are found in Key Idea 32; show work verifying these formula.)**

08 08 ex 04 7. $f(x) = \cos x; \quad c = \pi/2$

08 08 ex 05 8. $f(x) = 1/x; \quad c = 1$

08 08 ex 09 9. $f(x) = e^{-x}; \quad c = 0$

08 08 ex 10 10. $f(x) = \ln(1+x); \quad c = 0$

08 08 ex 11 11. $f(x) = x/(x+1); \quad c = 1$

08 08 ex 12 12. $f(x) = \sin x; \quad c = \pi/4$

08 08 exset 03 **In Exercises 13 – 16, show that the Taylor series for $f(x)$, as given in Key Idea 32, is equal to $f(x)$ by applying Theorem 77; that is, show $\lim\limits_{n\to\infty} R_n(x) = 0$.**

08 08 ex 14 13. $f(x) = e^x$

08 08 ex 13 14. $f(x) = \sin x$

08 08 ex 15 15. $f(x) = \ln x$

08 08 ex 16 16. $f(x) = 1/(1-x)$ (show equality only on $(-1, 0)$)

08 08 exset 04 **In Exercises 17 – 20, use the Taylor series given in Key Idea 32 to verify the given identity.**

08 08 ex 17 17. $\cos(-x) = \cos x$

08 08 ex 18 18. $\sin(-x) = -\sin x$

08 08 ex 19 19. $\frac{d}{dx}\big(\sin x\big) = \cos x$

08 08 ex 20 20. $\frac{d}{dx}\big(\cos x\big) = -\sin x$

08 08 exset 06 **In Exercises 21 – 24, write out the first 5 terms of the Binomial series with the given $k$-value.**

08 08 ex 27 21. $k = 1/2$

08 08 ex 28 22. $k = -1/2$

08 08 ex 29 23. $k = 1/3$

08 08 ex 30 24. $k = 4$

08 08 exset 05 **In Exercises 25 – 30, use the Taylor series given in Key Idea 32 to create the Taylor series of the given functions.**

08 08 ex 21 25. $f(x) = \cos\left(x^2\right)$

08 08 ex 22 26. $f(x) = e^{-x}$

08 08 ex 23 27. $f(x) = \sin\left(2x+3\right)$

08 08 ex 24 28. $f(x) = \tan^{-1}\left(x/2\right)$

08 08 ex 25 29. $f(x) = e^x \sin x$ (only find the first 4 terms)

08 08 ex 26 30. $f(x) = (1+x)^{1/2}\cos x$ (only find the first 4 terms)

08 08 exset 07 **In Exercises 31 – 32, approximate the value of the given definite integral by using the first 4 nonzero terms of the integrand's Taylor series.**

08 08 ex 31 31. $\displaystyle\int_0^{\sqrt{\pi}} \sin\left(x^2\right)\,dx$

08 08 ex 32 32. $\displaystyle\int_0^{\pi^2/4} \cos\left(\sqrt{x}\right)\,dx$

# Exercises 9.1

## *Terms and Concepts*

09 01 ex 01 1. What is the difference between degenerate and nondegenerate conics?

09 01 ex 03 2. Use your own words to explain what the eccentricity of an ellipse measures.

09 01 ex 02 3. What has the largest eccentricity: an ellipse or a hyperbola?

09 01 ex 44 4. Explain why the following is true: "If the coefficient of the $x^2$ term in the equation of an ellipse in standard form is smaller than the coefficient of the $y^2$ term, then the ellipse has a horizontal major axis."

09 01 ex 45 5. Explain how one can quickly look at the equation of a hyperbola in standard form and determine whether the transverse axis is horizontal or vertical.

## *Problems*

09 01 exset 01 **In Exercises 6 – 13, find the equation of the parabola defined by the given information. Sketch the parabola.**

09 01 ex 04 6. Focus: $(3, 2)$; directrix: $y = 1$

09 01 ex 05 7. Focus: $(-1, -4)$; directrix: $y = 2$

09 01 ex 06 8. Focus: $(1, 5)$; directrix: $x = 3$

09 01 ex 07 9. Focus: $(1/4, 0)$; directrix: $x = -1/4$

09 01 ex 08 10. Focus: $(1, 1)$; vertex: $(1, 2)$

09 01 ex 09 11. Focus: $(-3, 0)$; vertex: $(0, 0)$

09 01 ex 10 12. Vertex: $(0, 0)$; directrix: $y = -1/16$

09 01 ex 11 13. Vertex: $(2, 3)$; directrix: $x = 4$

09 01 exset 02 **In Exercises 14 – 15, the equation of a parabola and a point on its graph are given. Find the focus and directrix of the parabola, and verify that the given point is equidistant from the focus and directrix.**

09 01 ex 12 14. $y = \frac{1}{4}x^2$, $P = (2, 1)$

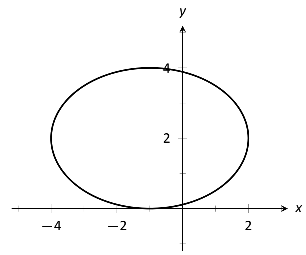
09 01 ex 13 15. $x = \frac{1}{8}(y-2)^2 + 3$, $P = (11, 10)$

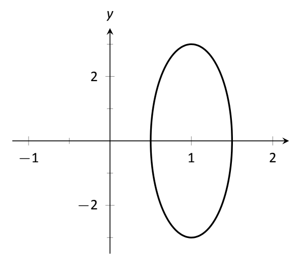
09 01 exset 03 **In Exercises 16 – 17, sketch the ellipse defined by the given equation. Label the center, foci and vertices.**

09 01 ex 18 16. $\dfrac{(x-1)^2}{3} + \dfrac{(y-2)^2}{5} = 1$

09 01 ex 19 17. $\dfrac{1}{25}x^2 + \dfrac{1}{9}(y+3)^2 = 1$

09 01 exset 05 **In Exercises 18 – 19, find the equation of the ellipse shown in the graph. Give the location of the foci and the eccentricity of the ellipse.**

09 01 ex 20 18. 


09 01 ex 21 19. 


09 01 exset 04 **In Exercises 20 – 23, find the equation of the ellipse defined by the given information. Sketch the elllipse.**

09 01 ex 14 20. Foci: $(\pm 2, 0)$; vertices: $(\pm 3, 0)$

09 01 ex 15 21. Foci: $(-1, 3)$ and $(5, 3)$; vertices: $(-3, 3)$ and $(7, 3)$

09 01 ex 16 22. Foci: $(2, \pm 2)$; vertices: $(2, \pm 7)$

09 01 ex 17 23. Focus: $(-1, 5)$; vertex: $(-1, -4)$; center: $(-1, 1)$

09 01 exset 06 **In Exercises 24 – 27, write the equation of the given ellipse in standard form.**

09 01 ex 22 24. $x^2 - 2x + 2y^2 - 8y = -7$

09 01 ex 23 25. $5x^2 + 3y^2 = 15$

09 01 ex 24 26. $3x^2 + 2y^2 - 12y + 6 = 0$

09 01 ex 25 27. $x^2 + y^2 - 4x - 4y + 4 = 0$

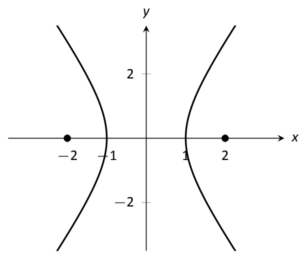
09 01 ex 26 28. Consider the ellipse given by $\dfrac{(x-1)^2}{4} + \dfrac{(y-3)^2}{12} = 1$.

(a) Verify that the foci are located at $(1, 3 \pm 2\sqrt{2})$.

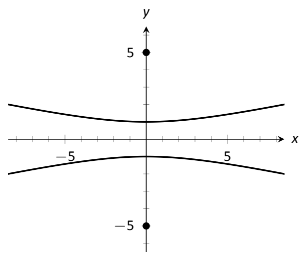
(b) The points $P_1 = (2, 6)$ and $P_2 = (1+\sqrt{2}, 3+\sqrt{6}) \approx (2.414, 5.449)$ lie on the ellipse. Verify that the sum of distances from each point to the foci is the same.

09 01 exset 07 **In Exercises 29 – 32, find the equation of the hyperbola shown in the graph.**

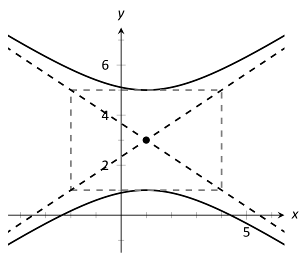
09 01 ex 27 29.



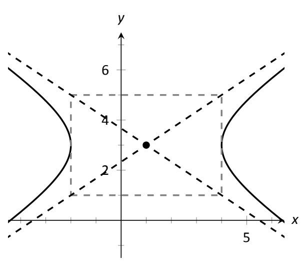

09 01 ex 28 30.




09 01 ex 29 31.




09 01 ex 30 32.




09 01 exset 08 **In Exercises 33 – 34, sketch the hyperbola defined by the given equation. Label the center and foci.**

09 01 ex 31 33. $\dfrac{(x-1)^2}{16} - \dfrac{(y+2)^2}{9} = 1$

09 01 ex 32 34. $(y-4)^2 - \dfrac{(x+1)^2}{25} = 1$

09 01 exset 09 **In Exercises 35 – 38, find the equation of the hyperbola defined by the given information. Sketch the hyperbola.**

09 01 ex 33 35. Foci: $(\pm 3, 0)$; vertices: $(\pm 2, 0)$

09 01 ex 34 36. Foci: $(0, \pm 3)$; vertices: $(0, \pm 2)$

09 01 ex 35 37. Foci: $(-2, 3)$ and $(8, 3)$; vertices: $(-1, 3)$ and $(7, 3)$

09 01 ex 36 38. Foci: $(3, -2)$ and $(3, 8)$; vertices: $(3, 0)$ and $(3, 6)$

09 01 exset 10 **In Exercises 39 – 42, write the equation of the hyperbola in standard form.**

09 01 ex 37 39. $3x^2 - 4y^2 = 12$

09 01 ex 38 40. $3x^2 - y^2 + 2y = 10$

09 01 ex 39 41. $x^2 - 10y^2 + 40y = 30$

09 01 ex 40 42. $(4y - x)(4y + x) = 4$

09 01 ex 42 43. Johannes Kepler discovered that the planets of our solar system have elliptical orbits with the Sun at one focus. The Earth's elliptical orbit is used as a standard unit of distance; the distance from the center of Earth's elliptical orbit to one vertex is 1 Astronomical Unit, or A.U.

The following table gives information about the orbits of three planets.

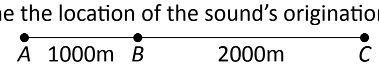
| | Distance from center to vertex | eccentricity |
|---|---|---|
| Mercury | 0.387 A.U. | 0.2056 |
| Earth | 1 A.U. | 0.0167 |
| Mars | 1.524 A.U. | 0.0934 |

(a) In an ellipse, knowing $c^2 = a^2 - b^2$ and $e = c/a$ allows us to find $b$ in terms of $a$ and $e$. Show $b = a\sqrt{1-e^2}$.

(b) For each planet, find equations of their elliptical orbit of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (This places the center at $(0, 0)$, but the Sun is in a different location for each planet.)

(c) Shift the equations so that the Sun lies at the origin. Plot the three elliptical orbits.

09 01 ex 43 44. A loud sound is recorded at three stations that lie on a line as shown in the figure below. Station $A$ recorded the sound 1 second after Station $B$, and Station $C$ recorded the sound 3 seconds after $B$. Using the speed of sound as 340m/s, determine the location of the sound's origination.



# Exercises 9.2

## *Terms and Concepts*

09 02 ex 01 1. T/F: When sketching the graph of parametric equations, the $x$ and $y$ values are found separately, then plotted together.

09 02 ex 02 2. The direction in which a graph is "moving" is called the ____ of the graph.

09 02 ex 03 3. An equation written as $y = f(x)$ is written in ____ form.

09 02 ex 04 4. Create parametric equations $x = f(t)$, $y = g(t)$ and sketch their graph. Explain any interesting features of your graph based on the functions $f$ and $g$.

## *Problems*

09 02 exset 01 **In Exercises 5 – 8, sketch the graph of the given parametric equations by hand, making a table of points to plot. Be sure to indicate the orientation of the graph.**

09 02 ex 05 5. $x = t^2 + t, \quad y = 1 - t^2, \quad -3 \leq t \leq 3$

09 02 ex 06 6. $x = 1, \quad y = 5\sin t, \quad -\pi/2 \leq t \leq \pi/2$

09 02 ex 07 7. $x = t^2, \quad y = 2, \quad -2 \leq t \leq 2$

09 02 ex 08 8. $x = t^3 - t + 3, \quad y = t^2 + 1, \quad -2 \leq t \leq 2$

09 02 exset 02 **In Exercises 9 – 17, sketch the graph of the given parametric equations; using a graphing utility is advisable. Be sure to indicate the orientation of the graph.**

09 02 ex 09 9. $x = t^3 - 2t^2, \quad y = t^2, \quad -2 \leq t \leq 3$

09 02 ex 10 10. $x = 1/t, \quad y = \sin t, \quad 0 < t \leq 10$

09 02 ex 11 11. $x = 3\cos t, \quad y = 5\sin t, \quad 0 \leq t \leq 2\pi$

09 02 ex 12 12. $x = 3\cos t + 2, \quad y = 5\sin t + 3, \quad 0 \leq t \leq 2\pi$

09 02 ex 13 13. $x = \cos t, \quad y = \cos(2t), \quad 0 \leq t \leq \pi$

09 02 ex 14 14. $x = \cos t, \quad y = \sin(2t), \quad 0 \leq t \leq 2\pi$

09 02 ex 15 15. $x = 2\sec t, \quad y = 3\tan t, \quad -\pi/2 < t < \pi/2$

09 02 ex 16 16. $x = \cos t + \frac{1}{4}\cos(8t), \quad y = \sin t + \frac{1}{4}\sin(8t), \quad 0 \leq t \leq 2\pi$

09 02 ex 17 17. $x = \cos t + \frac{1}{4}\sin(8t), \quad y = \sin t + \frac{1}{4}\cos(8t), \quad 0 \leq t \leq 2\pi$

09 02 exset 03 **In Exercises 18 – 19, four sets of parametric equations are given. Describe how their graphs are similar and different. Be sure to discuss orientation and ranges.**

09 02 ex 18 18.

(a) $x = t \quad y = t^2, \quad -\infty < t < \infty$

(b) $x = \sin t \quad y = \sin^2 t, \quad -\infty < t < \infty$

(c) $x = e^t \quad y = e^{2t}, \quad -\infty < t < \infty$

(d) $x = -t \quad y = t^2, \quad -\infty < t < \infty$

09 02 ex 19 19.

(a) $x = \cos t \quad y = \sin t, \quad 0 \leq t \leq 2\pi$

(b) $x = \cos(t^2) \quad y = \sin(t^2), \quad 0 \leq t \leq 2\pi$

(c) $x = \cos(1/t) \quad y = \sin(1/t), \quad 0 < t < 1$

(d) $x = \cos(\cos t) \quad y = \sin(\cos t), \quad 0 \leq t \leq 2\pi$

09 02 exset 04 **In Exercises 20 – 29, eliminate the parameter in the given parametric equations.**

09 02 ex 27 20. $x = 2t + 5, \quad y = -3t + 1$

09 02 ex 20 21. $x = \sec t, \quad y = \tan t$

09 02 ex 21 22. $x = 4\sin t + 1, \quad y = 3\cos t - 2$

09 02 ex 22 23. $x = t^2, \quad y = t^3$

09 02 ex 23 24. $x = \dfrac{1}{t+1}, \quad y = \dfrac{3t+5}{t+1}$

09 02 ex 24 25. $x = e^t, \quad y = e^{3t} - 3$

09 02 ex 25 26. $x = \ln t, \quad y = t^2 - 1$

09 02 ex 26 27. $x = \cot t, \quad y = \csc t$

09 02 ex 28 28. $x = \cosh t, \quad y = \sinh t$

09 02 ex 29 29. $x = \cos(2t), \quad y = \sin t$

09 02 exset 05 **In Exercises 30 – 33, eliminate the parameter in the given parametric equations. Describe the curve defined by the parametric equations based on its rectangular form.**

09 02 ex 30 30. $x = at + x_0, \quad y = bt + y_0$

09 02 ex 31 31. $x = r\cos t, \quad y = r\sin t$

09 02 ex 32 32. $x = a\cos t + h, \quad y = b\sin t + k$

09 02 ex 33 33. $x = a\sec t + h, \quad y = b\tan t + k$

09 02 exset 06 **In Exercises 34 – 37, find parametric equations for the given rectangular equation using the parameter $t = \dfrac{dy}{dx}$. Verify that at $t = 1$, the point on the graph has a tangent line with slope of 1.**

09 02 ex 34 34. $y = 3x^2 - 11x + 2$

09 02 ex 35 35. $y = e^x$

09 02 ex 36 36. $y = \sin x$ on $[0, \pi]$

09 02 ex 37 37. $y = \sqrt{x}$ on $[0, \infty)$

09 02 exset 07 **In Exercises 38 – 41, find the values of $t$ where the graph of the parametric equations crosses itself.**

09 02 ex 38 38. $x = t^3 - t + 3, \quad y = t^2 - 3$

09 02 ex 41 39. $x = t^3 - 4t^2 + t + 7, \quad y = t^2 - t$

09 02 ex 39 40. $x = \cos t, \quad y = \sin(2t)$ on $[0, 2\pi]$

09 02 ex 40 41. $x = \cos t \cos(3t), \quad y = \sin t \cos(3t)$ on $[0, \pi]$

09 02 exset 08 **In Exercises 42 – 45, find the value(s) of $t$ where the curve defined by the parametric equations is not smooth.**

09 02 ex 42 42. $x = t^3 + t^2 - t, \quad y = t^2 + 2t + 3$

09 02 ex 43 43. $x = t^2 - 4t, \quad y = t^3 - 2t^2 - 4t$

09 02 ex 44 44. $x = \cos t, \quad y = 2\cos t$

09 02 ex 45 45. $x = 2\cos t - \cos(2t), \quad y = 2\sin t - \sin(2t)$

09 02 exset 09 **In Exercises 46 – 54, find parametric equations that describe the given situation.**

09 02 ex 46 46. A projectile is fired from a height of 0ft, landing 16ft away in 4s.

09 02 ex 47 47. A projectile is fired from a height of 0ft, landing 200ft away in 4s.

09 02 ex 48 48. A projectile is fired from a height of 0ft, landing 200ft away in 20s.

09 02 ex 49 49. A circle of radius 2, centered at the origin, that is traced clockwise once on $[0, 2\pi]$.

09 02 ex 50 50. A circle of radius 3, centered at $(1, 1)$, that is traced once counter–clockwise on $[0, 1]$.

09 02 ex 51 51. An ellipse centered at $(1, 3)$ with vertical major axis of length 6 and minor axis of length 2.

09 02 ex 52 52. An ellipse with foci at $(\pm 1, 0)$ and vertices at $(\pm 5, 0)$.

09 02 ex 53 53. A hyperbola with foci at $(5, -3)$ and $(-1, -3)$, and with vertices at $(1, -3)$ and $(3, -3)$.

09 02 ex 54 54. A hyperbola with vertices at $(0, \pm 6)$ and asymptotes $y = \pm 3x$.

# Exercises 9.3

## *Terms and Concepts*

09 03 ex 01 1. T/F: Given parametric equations $x = f(t)$ and $y = g(t)$, $\frac{dy}{dx} = f'(t)/g'(t)$, as long as $g'(t) \neq 0$.

09 03 ex 02 2. Given parametric equations $x = f(t)$ and $y = g(t)$, the derivative $\frac{dy}{dx}$ as given in Key Idea 37 is a function of ________?

09 03 ex 03 3. T/F: Given parametric equations $x = f(t)$ and $y = g(t)$, to find $\frac{d^2y}{dx^2}$, one simply computes $\frac{d}{dt}\left(\frac{dy}{dx}\right)$.

09 03 ex 04 4. T/F: If $\frac{dy}{dx} = 0$ at $t = t_0$, then the normal line to the curve at $t = t_0$ is a vertical line.

## *Problems*

09 03 exset 01 **In Exercises 5 – 12, parametric equations for a curve are given.**

**(a) Find $\dfrac{dy}{dx}$.**

**(b) Find the equations of the tangent and normal line(s) at the point(s) given.**

**(c) Sketch the graph of the parametric functions along with the found tangent and normal lines.**

09 03 ex 05 5. $x = t, y = t^2; \quad t = 1$

09 03 ex 06 6. $x = \sqrt{t}, y = 5t + 2; \quad t = 4$

09 03 ex 07 7. $x = t^2 - t, y = t^2 + t; \quad t = 1$

09 03 ex 08 8. $x = t^2 - 1, y = t^3 - t; \quad t = 0$ and $t = 1$

09 03 ex 09 9. $x = \sec t, y = \tan t$ on $(-\pi/2, \pi/2); \quad t = \pi/4$

09 03 ex 10 10. $x = \cos t, y = \sin(2t)$ on $[0, 2\pi]; \quad t = \pi/4$

09 03 ex 11 11. $x = \cos t \sin(2t), y = \sin t \sin(2t)$ on $[0, 2\pi]; \quad t = 3\pi/4$

09 03 ex 12 12. $x = e^{t/10} \cos t, y = e^{t/10} \sin t; \quad t = \pi/2$

09 03 exset 02 **In Exercises 13 – 20, find $t$-values where the curve defined by the given parametric equations has a horizontal tangent line. Note: these are the same equations as in Exercises 5 – 12.**

09 03 ex 13 13. $x = t, y = t^2$

09 03 ex 14 14. $x = \sqrt{t}, y = 5t + 2$

09 03 ex 15 15. $x = t^2 - t, y = t^2 + t$

09 03 ex 16 16. $x = t^2 - 1, y = t^3 - t$

09 03 ex 17 17. $x = \sec t, y = \tan t$ on $(-\pi/2, \pi/2)$

09 03 ex 18 18. $x = \cos t, y = \sin(2t)$ on $[0, 2\pi]$

09 03 ex 19 19. $x = \cos t \sin(2t), y = \sin t \sin(2t)$ on $[0, 2\pi]$

09 03 ex 20 20. $x = e^{t/10} \cos t, y = e^{t/10} \sin t$

09 03 exset 03 **In Exercises 21 – 24, find $t = t_0$ where the graph of the given parametric equations is not smooth, then find $\displaystyle\lim_{t \to t_0} \frac{dy}{dx}$.**

09 03 ex 21 21. $x = \dfrac{1}{t^2 + 1}, \quad y = t^3$

09 03 ex 22 22. $x = -t^3 + 7t^2 - 16t + 13, \quad y = t^3 - 5t^2 + 8t - 2$

09 03 ex 23 23. $x = t^3 - 3t^2 + 3t - 1, \quad y = t^2 - 2t + 1$

09 03 ex 43 24. $x = \cos^2 t, \quad y = 1 - \sin^2 t$

09 03 exset 04 **In Exercises 25 – 32, parametric equations for a curve are given. Find $\frac{d^2y}{dx^2}$, then determine the intervals on which the graph of the curve is concave up/down. Note: these are the same equations as in Exercises 5 – 12.**

09 03 ex 24 25. $x = t, \quad y = t^2$

09 03 ex 25 26. $x = \sqrt{t}, \quad y = 5t + 2$

09 03 ex 26 27. $x = t^2 - t, \quad y = t^2 + t$

09 03 ex 27 28. $x = t^2 - 1, \quad y = t^3 - t$

09 03 ex 28 29. $x = \sec t, \quad y = \tan t$ on $(-\pi/2, \pi/2)$

09 03 ex 29 30. $x = \cos t, \quad y = \sin(2t)$ on $[0, 2\pi]$

09 03 ex 44 31. $x = \cos t \sin(2t), \quad y = \sin t \sin(2t)$ on $[-\pi/2, \pi/2]$

09 03 ex 30 32. $x = e^{t/10} \cos t, \quad y = e^{t/10} \sin t$

09 03 exset 05 **In Exercises 33 – 36, find the arc length of the graph of the parametric equations on the given interval(s).**

09 03 ex 32 33. $x = -3\sin(2t), \quad y = 3\cos(2t)$ on $[0, \pi]$

09 03 ex 31 34. $x = e^{t/10} \cos t, \quad y = e^{t/10} \sin t$ on $[0, 2\pi]$ and $[2\pi, 4\pi]$

09 03 ex 33 35. $x = 5t + 2, \quad y = 1 - 3t$ on $[-1, 1]$

09 03 ex 34 36. $x = 2t^{3/2}, \quad y = 3t$ on $[0, 1]$

09 03 exset 06 **In Exercises 37 – 40, numerically approximate the given arc length.**

09 03 ex 35 37. Approximate the arc length of one petal of the rose curve $x = \cos t \cos(2t), \quad y = \sin t \cos(2t)$ using Simpson's Rule and $n = 4$.

09 03 ex 36 38. Approximate the arc length of the "bow tie curve" $x = \cos t, \quad y = \sin(2t)$ using Simpson's Rule and $n = 6$.

09 03 ex 37 39. Approximate the arc length of the parabola $x = t^2 - t$, $y = t^2 + t$ on $[-1, 1]$ using Simpson's Rule and $n = 4$.

09 03 ex 38 40. A common approximate of the circumference of an ellipse given by $x = a\cos t, \quad y = b\sin t$ is $C \approx 2\pi\sqrt{\dfrac{a^2+b^2}{2}}$. Use this formula to approximate the circumference of $x = 5\cos t, \quad y = 3\sin t$ and compare this to the approximation given by Simpson's Rule and $n = 6$.

09 03 exset 07 **In Exercises 41 – 44, a solid of revolution is described. Find or approximate its surface area as specified.**

09 03 ex 39 41. Find the surface area of the sphere formed by rotating the circle $x = 2\cos t, \quad y = 2\sin t$ about:

(a) the $x$-axis and

(b) the $y$-axis.

09 03 ex 40 42. Find the surface area of the torus (or "donut") formed by rotating the circle $x = \cos t + 2, \quad y = \sin t$ about the $y$-axis.

09 03 ex 41 43. Approximate the surface area of the solid formed by rotating the "upper right half" of the bow tie curve $x = \cos t$, $y = \sin(2t)$ on $[0, \pi/2]$ about the $x$-axis, using Simpson's Rule and $n = 4$.

09 03 ex 42 44. Approximate the surface area of the solid formed by rotating the one petal of the rose curve $x = \cos t\cos(2t)$, $y = \sin t\cos(2t)$ on $[0, \pi/4]$ about the $x$-axis, using Simpson's Rule and $n = 4$.

# Exercises 9.4

## *Terms and Concepts*

09 04 ex 01 1. In your own words, describe how to plot the polar point $P(r, \theta)$.

09 04 ex 02 2. T/F: When plotting a point with polar coordinate $P(r, \theta)$, $r$ must be positive.

09 04 ex 03 3. T/F: Every point in the Cartesian plane can be represented by a polar coordinate.

09 04 ex 04 4. T/F: Every point in the Cartesian plane can be represented uniquely by a polar coordinate.

## *Problems*

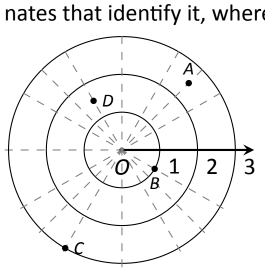
09 04 ex 05 5. Plot the points with the given polar coordinates.

(a) $A = P(2, 0)$ (b) $B = P(1, \pi)$ (c) $C = P(-2, \pi/2)$ (d) $D = P(1, \pi/4)$

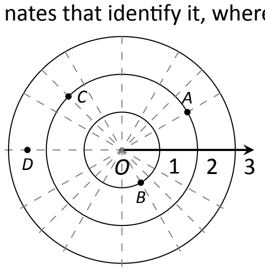
09 04 ex 06 6. Plot the points with the given polar coordinates.

(a) $A = P(2, 3\pi)$ (b) $B = P(1, -\pi)$ (c) $C = P(1, 2)$ (d) $D = P(1/2, 5\pi/6)$

09 04 ex 07 7. For each of the given points give two sets of polar coordinates that identify it, where $0 \leq \theta \leq 2\pi$.




09 04 ex 08 8. For each of the given points give two sets of polar coordinates that identify it, where $-\pi \leq \theta \leq \pi$.




09 04 ex 09 9. Convert each of the following polar coordinates to rectangular, and each of the following rectangular coordinates to polar.

(a) $A = P(2, \pi/4)$ (b) $B = P(2, -\pi/4)$ (c) $C = (2, -1)$ (d) $D = (-2, 1)$

09 04 ex 10 10. Convert each of the following polar coordinates to rectangular, and each of the following rectangular coordinates to polar.

(a) $A = P(3, \pi)$ (b) $B = P(1, 2\pi/3)$ (c) $C = (0, 4)$ (d) $D = (1, -\sqrt{3})$

09 04 exset 01 **In Exercises 11 – 29, graph the polar function on the given interval.**

09 04 ex 11 11. $r = 2, \quad 0 \leq \theta \leq \pi/2$

09 04 ex 12 12. $\theta = \pi/6, \quad -1 \leq r \leq 2$

09 04 ex 13 13. $r = 1 - \cos\theta, \quad [0, 2\pi]$

09 04 ex 14 14. $r = 2 + \sin\theta, \quad [0, 2\pi]$

09 04 ex 15 15. $r = 2 - \sin\theta, \quad [0, 2\pi]$

09 04 ex 16 16. $r = 1 - 2\sin\theta, \quad [0, 2\pi]$

09 04 ex 17 17. $r = 1 + 2\sin\theta, \quad [0, 2\pi]$

09 04 ex 18 18. $r = \cos(2\theta), \quad [0, 2\pi]$

09 04 ex 19 19. $r = \sin(3\theta), \quad [0, \pi]$

09 04 ex 20 20. $r = \cos(\theta/3), \quad [0, 3\pi]$

09 04 ex 21 21. $r = \cos(2\theta/3), \quad [0, 6\pi]$

09 04 ex 22 22. $r = \theta/2, \quad [0, 4\pi]$

09 04 ex 23 23. $r = 3\sin(\theta), \quad [0, \pi]$

09 04 ex 24 24. $r = \cos\theta\sin\theta, \quad [0, 2\pi]$

09 04 ex 25 25. $r = \theta^2 - (\pi/2)^2, \quad [-\pi, \pi]$

09 04 ex 26 26. $r = \dfrac{3}{5\sin\theta - \cos\theta}, \quad [0, 2\pi]$

09 04 ex 27 27. $r = \dfrac{-2}{3\cos\theta - 2\sin\theta}, \quad [0, 2\pi]$

09 04 ex 28 28. $r = 3\sec\theta, \quad (-\pi/2, \pi/2)$

09 04 ex 29 29. $r = 3\csc\theta, \quad (0, \pi)$

09 04 exset 02 **In Exercises 30 – 38, convert the polar equation to a rectangular equation.**

09 04 ex 30 30. $r = 2\cos\theta$

09 04 ex 31 31. $r = -4\sin\theta$

09 04 ex 32 32. $r = \cos\theta + \sin\theta$

09 04 ex 33 33. $r = \dfrac{7}{5\sin\theta - 2\cos\theta}$

09 04 ex 34 34. $r = \dfrac{3}{\cos\theta}$

09 04 ex 35 35. $r = \dfrac{4}{\sin\theta}$

09 04 ex 36 36. $r = \tan\theta$

09 04 ex 37 37. $r = 2$

09 04 ex 38 38. $\theta = \pi/6$

09 04 exset 03 **In Exercises 39 – 46, convert the rectangular equation to a polar equation.**

09 04 ex 39 39. $y = x$

09 04 ex 40 40. $y = 4x + 7$

09 04 ex 41 41. $x = 5$

09 04 ex 42 42. $y = 5$

09 04 ex 43 43. $x = y^2$

09 04 ex 44 44. $x^2y = 1$

09 04 ex 45 45. $x^2 + y^2 = 7$

09 04 ex 46 46. $(x+1)^2 + y^2 = 1$

09 04 exset 04 **In Exercises 47 – 54, find the points of intersection of the polar graphs.**

09 04 ex 47 47. $r = \sin(2\theta)$ and $r = \cos\theta$ on $[0, \pi]$

09 04 ex 48 48. $r = \cos(2\theta)$ and $r = \cos\theta$ on $[0, \pi]$

09 04 ex 49 49. $r = 2\cos\theta$ and $r = 2\sin\theta$ on $[0, \pi]$

09 04 ex 50 50. $r = \sin\theta$ and $r = \sqrt{3} + 3\sin\theta$ on $[0, 2\pi]$

09 04 ex 51 51. $r = \sin(3\theta)$ and $r = \cos(3\theta)$ on $[0, \pi]$

09 04 ex 52 52. $r = 3\cos\theta$ and $r = 1 + \cos\theta$ on $[-\pi, \pi]$

09 04 ex 53 53. $r = 1$ and $r = 2\sin(2\theta)$ on $[0, 2\pi]$

09 04 ex 54 54. $r = 1 - \cos\theta$ and $r = 1 + \sin\theta$ on $[0, 2\pi]$

09 04 ex 55 55. Pick a integer value for $n$, where $n \neq 2, 3$, and use technology to plot $r = \sin\left(\frac{m}{n}\theta\right)$ for three different integer values of $m$. Sketch these and determine a minimal interval on which the entire graph is shown.

09 04 ex 56 56. Create your own polar function, $r = f(\theta)$ and sketch it. Describe why the graph looks as it does.

# Exercises 9.5

## *Terms and Concepts*

09 05 ex 01 1. Given polar equation $r = f(\theta)$, how can one create parametric equations of the same curve?

09 05 ex 02 2. With rectangular coordinates, it is natural to approximate area with ________; with polar coordinates, it is natural to approximate area with ________.

## *Problems*

09 05 exset 01 **In Exercises 3 – 10, find:**

**(a)** $\dfrac{dy}{dx}$

**(b) the equation of the tangent and normal lines to the curve at the indicated $\theta$–value.**

09 05 ex 03 3. $r = 1; \quad \theta = \pi/4$

09 05 ex 04 4. $r = \cos\theta; \quad \theta = \pi/4$

09 05 ex 05 5. $r = 1 + \sin\theta; \quad \theta = \pi/6$

09 05 ex 10 6. $r = 1 - 3\cos\theta; \quad \theta = 3\pi/4$

09 05 ex 06 7. $r = \theta; \quad \theta = \pi/2$

09 05 ex 07 8. $r = \cos(3\theta); \quad \theta = \pi/6$

09 05 ex 08 9. $r = \sin(4\theta); \quad \theta = \pi/3$

09 05 ex 09 10. $r = \dfrac{1}{\sin\theta - \cos\theta}; \quad \theta = \pi$

09 05 exset 02 **In Exercises 11 – 14, find the values of $\theta$ in the given interval where the graph of the polar function has horizontal and vertical tangent lines.**

09 05 ex 11 11. $r = 3; \quad [0, 2\pi]$

09 05 ex 12 12. $r = 2\sin\theta; \quad [0, \pi]$

09 05 ex 13 13. $r = \cos(2\theta); \quad [0, 2\pi]$

09 05 ex 14 14. $r = 1 + \cos\theta; \quad [0, 2\pi]$

09 05 exset 03 **In Exercises 15 – 16, find the equation of the lines tangent to the graph at the pole.**

09 05 ex 15 15. $r = \sin\theta; \quad [0, \pi]$

09 05 ex 16 16. $r = \sin(3\theta); \quad [0, \pi]$

09 05 exset 04 **In Exercises 17 – 27, find the area of the described region.**

09 05 ex 17 17. Enclosed by the circle: $r = 4\sin\theta$

09 05 ex 20 18. Enclosed by the circle $r = 5$

09 05 ex 18 19. Enclosed by one petal of $r = \sin(3\theta)$

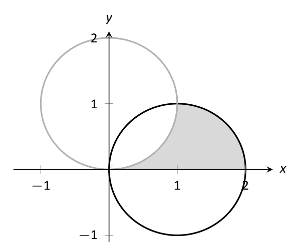
09 05 ex 19 20. Enclosed by the cardiod $r = 1 - \sin\theta$

09 05 ex 21 21. Enclosed by the inner loop of the limaçon $r = 1 + 2\cos t$

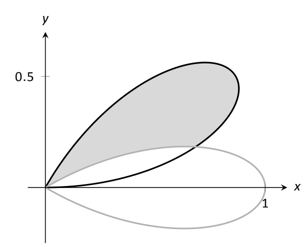
09 05 ex 22 22. Enclosed by the outer loop of the limaçon $r = 1 + 2\cos t$ (including area enclosed by the inner loop)

09 05 ex 23 23. Enclosed between the inner and outer loop of the limaçon $r = 1 + 2\cos t$

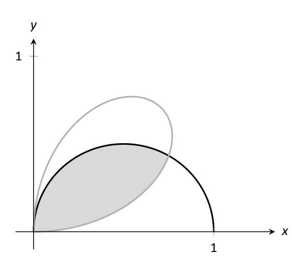
09 05 ex 25 24. Enclosed by $r = 2\cos\theta$ and $r = 2\sin\theta$, as shown:



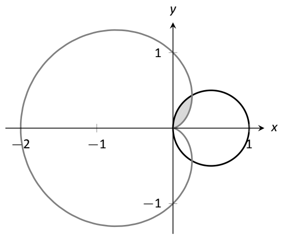

09 05 ex 26 25. Enclosed by $r = \cos(3\theta)$ and $r = \sin(3\theta)$, as shown:




09 05 ex 24 26. Enclosed by $r = \cos\theta$ and $r = \sin(2\theta)$, as shown:



09 05 ex 27 27. Enclosed by $r = \cos\theta$ and $r = 1 - \cos\theta$, as shown.




09 05 exset 05 **In Exercises 28 – 32, answer the questions involving arc length.**

09 05 ex 28 28. Let $x(\theta) = f(\theta)\cos\theta$ and $y(\theta) = f(\theta)\sin\theta$. Show, as suggested by the text, that

$$x'(\theta)^2 + y'(\theta)^2 = f'(\theta)^2 + f(\theta)^2.$$

09 05 ex 29 29. Use the arc length formula to compute the arc length of the circle $r = 2$.

09 05 ex 30 30. Use the arc length formula to compute the arc length of the circle $r = 4\sin\theta$.

09 05 ex 31 31. Approximate the arc length of one petal of the rose curve $r = \sin(3\theta)$ with Simpson's Rule and $n = 4$.

09 05 ex 32 32. Approximate the arc length of the cardiod $r = 1 + \cos\theta$ with Simpson's Rule and $n = 6$.

09 05 exset 06 **In Exercises 33 – 37, answer the questions involving surface area.**

09 05 ex 33 33. Use Key Idea 44 to find the surface area of the sphere formed by revolving the circle $r = 2$ about the initial ray.

09 05 ex 34 34. Use Key Idea 44 to find the surface area of the sphere formed by revolving the circle $r = 2\cos\theta$ about the initial ray.

09 05 ex 35 35. Find the surface area of the solid formed by revolving the cardiod $r = 1 + \cos\theta$ about the initial ray.

09 05 ex 36 36. Find the surface area of the solid formed by revolving the circle $r = 2\cos\theta$ about the line $\theta = \pi/2$.

09 05 ex 37 37. Find the surface area of the solid formed by revolving the line $r = 3\sec\theta$, $-\pi/4 \leq \theta \leq \pi/4$, about the line $\theta = \pi/2$.

# Exercises 10.1

## *Terms and Concepts*

10 01 ex 08 1. Axes drawn in space must conform to the ________ ________ rule.

10 01 ex 01 2. In the plane, the equation $x = 2$ defines a ________; in space, $x = 2$ defines a ________.

10 01 ex 02 3. In the plane, the equation $y = x^2$ defines a ________; in space, $y = x^2$ defines a ________.

10 01 ex 03 4. Which quadric surface looks like a Pringles® chip?

10 01 ex 04 5. Consider the hyperbola $x^2 - y^2 = 1$ in the plane. If this hyperbola is rotated about the $x$-axis, what quadric surface is formed?

10 01 ex 05 6. Consider the hyperbola $x^2 - y^2 = 1$ in the plane. If this hyperbola is rotated about the $y$-axis, what quadric surface is formed?

## *Problems*

10 01 ex 06 7. The points $A = (1, 4, 2)$, $B = (2, 6, 3)$ and $C = (4, 3, 1)$ form a triangle in space. Find the distances between each pair of points and determine if the triangle is a right triangle.

10 01 ex 07 8. The points $A = (1, 1, 3)$, $B = (3, 2, 7)$, $C = (2, 0, 8)$ and $D = (0, -1, 4)$ form a quadrilateral $ABCD$ in space. Is this a parallelogram?

10 01 ex 09 9. Find the center and radius of the sphere defined by $x^2 - 8x + y^2 + 2y + z^2 + 8 = 0$.

10 01 ex 10 10. Find the center and radius of the sphere defined by $x^2 + y^2 + z^2 + 4x - 2y - 4z + 4 = 0$.

10 01 exset 01 **In Exercises 11 – 14, describe the region in space defined by the inequalities.**

10 01 ex 11 11. $x^2 + y^2 + z^2 < 1$

10 01 ex 12 12. $0 \leq x \leq 3$

10 01 ex 13 13. $x \geq 0,\ y \geq 0,\ z \geq 0$

10 01 ex 14 14. $y \geq 3$

10 01 exset 02 **In Exercises 15 – 18, sketch the cylinder in space.**

10 01 ex 15 15. $z = x^3$

10 01 ex 16 16. $y = \cos z$

10 01 ex 17 17. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

10 01 ex 18 18. $y = \dfrac{1}{x}$

10 01 exset 05 **In Exercises 19 – 22, give the equation of the surface of revolution described.**

10 01 ex 29 19. Revolve $z = \dfrac{1}{1 + y^2}$ about the $y$-axis.

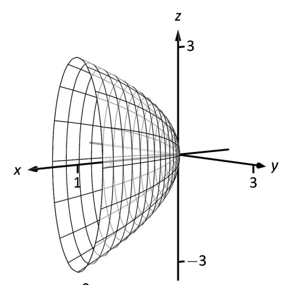
10 01 ex 30 20. Revolve $y = x^2$ about the $x$-axis.

10 01 ex 31 21. Revolve $z = x^2$ about the $z$-axis.

10 01 ex 32 22. Revolve $z = 1/x$ about the $z$-axis.

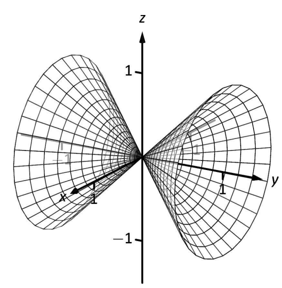
10 01 exset 03 **In Exercises 23 – 26, a quadric surface is sketched. Determine which of the given equations best fits the graph.**

10 01 ex 19 23.



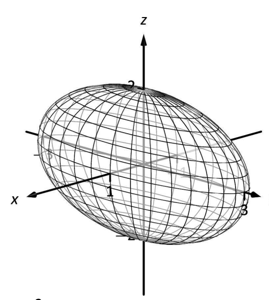

(a) $x = y^2 + \dfrac{z^2}{9}$ (b) $x = y^2 + \dfrac{z^2}{3}$

10 01 ex 20 24.



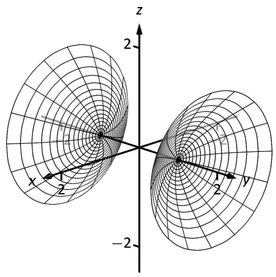

(a) $x^2 - y^2 - z^2 = 0$ (b) $x^2 - y^2 + z^2 = 0$

10 01 ex 21 25.




(a) $x^2 + \dfrac{y^2}{3} + \dfrac{z^2}{2} = 1$ (b) $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

10 01 ex 22 26.




(a) $y^2 - x^2 - z^2 = 1$ (b) $y^2 + x^2 - z^2 = 1$

10 01 exset 04 **In Exercises 27 – 32, sketch the quadric surface.**

10 01 ex 28 27. $z - y^2 + x^2 = 0$

10 01 ex 24 28. $z^2 = x^2 + \dfrac{y^2}{4}$

10 01 ex 23 29. $x = -y^2 - z^2$

10 01 ex 27 30. $16x^2 - 16y^2 - 16z^2 = 1$

10 01 ex 26 31. $\dfrac{x^2}{9} - y^2 + \dfrac{z^2}{25} = 1$

10 01 ex 25 32. $4x^2 + 2y^2 + z^2 = 4$

# Exercises 10.2

## *Terms and Concepts*

10 02 ex 01 1. Name two different things that cannot be described with just one number, but rather need 2 or more numbers to fully describe them.

10 02 ex 02 2. What is the difference between $(1,2)$ and $\langle 1,2\rangle$?

10 02 ex 03 3. What is a unit vector?

10 02 ex 04 4. What does it mean for two vectors to be parallel?

10 02 ex 05 5. What effect does multiplying a vector by $-2$ have?

## *Problems*

10 02 exset 01 **In Exercises 6 – 9, points $P$ and $Q$ are given. Write the vector $\overrightarrow{PQ}$ in component form and using the standard unit vectors.**

10 02 ex 06 6. $P=(2,-1)$, $\quad Q=(3,5)$

10 02 ex 07 7. $P=(3,2)$, $\quad Q=(7,-2)$

10 02 ex 08 8. $P=(0,3,-1)$, $\quad Q=(6,2,5)$

10 02 ex 09 9. $P=(2,1,2)$, $\quad Q=(4,3,2)$

10 02 ex 10 10. Let $\vec{u}=\langle 1,-2\rangle$ and $\vec{v}=\langle 1,1\rangle$.

(a) Find $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, $2\vec{u}-3\vec{v}$.

(b) Sketch the above vectors on the same axes, along with $\vec{u}$ and $\vec{v}$.

(c) Find $\vec{x}$ where $\vec{u}+\vec{x}=2\vec{v}-\vec{x}$.

10 02 ex 11 11. Let $\vec{u}=\langle 1,1,-1\rangle$ and $\vec{v}=\langle 2,1,2\rangle$.

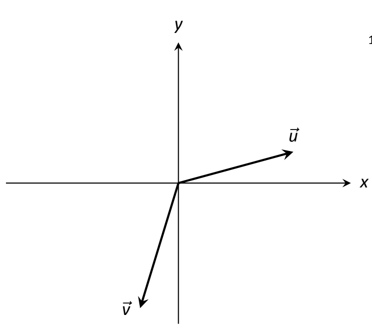
(a) Find $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, $\pi\vec{u}-\sqrt{2}\vec{v}$.

(b) Sketch the above vectors on the same axes, along with $\vec{u}$ and $\vec{v}$.

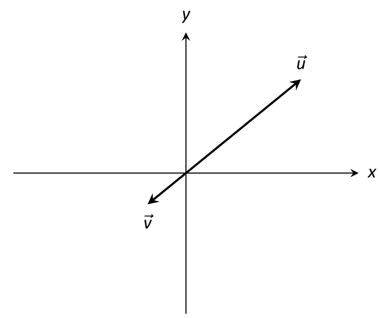
(c) Find $\vec{x}$ where $\vec{u}+\vec{x}=\vec{v}+2\vec{x}$.

10 02 exset 02 **In Exercises 12 – 15, sketch $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ on the same axes.**

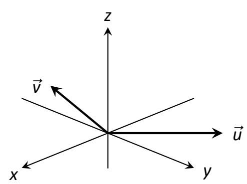
10 02 ex 12 12.



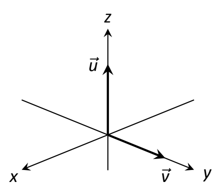

10 02 ex 13 13.




10 02 ex 14 14.




10 02 ex 15 15.




10 02 exset 03 **In Exercises 16 – 19, find $||\,\vec{u}\,||$, $||\,\vec{v}\,||$, $||\,\vec{u}+\vec{v}\,||$ and $||\,\vec{u}-\vec{v}\,||$.**

10 02 ex 16 16. $\vec{u}=\langle 2,1\rangle$, $\quad \vec{v}=\langle 3,-2\rangle$

10 02 ex 17 17. $\vec{u}=\langle -3,2,2\rangle$, $\quad \vec{v}=\langle 1,-1,1\rangle$

10 02 ex 18 18. $\vec{u}=\langle 1,2\rangle$, $\quad \vec{v}=\langle -3,-6\rangle$

10 02 ex 19 19. $\vec{u}=\langle 2,-3,6\rangle$, $\quad \vec{v}=\langle 10,-15,30\rangle$

10 02 ex 20 20. Under what conditions is $||\,\vec{u}\,||+||\,\vec{v}\,||=||\,\vec{u}+\vec{v}\,||$?

10 02 exset 04 **In Exercises 21 – 24, find the unit vector $\vec{u}$ in the direction of $\vec{v}$.**

10 02 ex 21 21. $\vec{v}=\langle 3,7\rangle$

10 02 ex 22 22. $\vec{v}=\langle 6,8\rangle$

10 02 ex 23 23. $\vec{v}=\langle 1,-2,2\rangle$

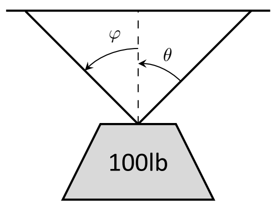
10 02 ex 24 24. $\vec{v}=\langle 2,-2,2\rangle$

10 02 ex 25 25. Find the unit vector in the first quadrant of $\mathbb{R}^2$ that makes a $50^\circ$ angle with the *x*-axis.

10 02 ex 26 26. Find the unit vector in the second quadrant of $\mathbb{R}^2$ that makes a $30^\circ$ angle with the *y*-axis.

10 02 ex 27 27. Verify, from Key Idea 48, that $\vec{u} = \langle \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\rangle$ is a unit vector for all angles $\theta$ and $\varphi$.

10 02 exset 05 **A weight of 100lb is suspended from two chains, making angles with the vertical of $\theta$ and $\varphi$ as shown in the figure below.**




**In Exercises 28 – 31, angles $\theta$ and $\varphi$ are given. Find the force applied to each chain.**

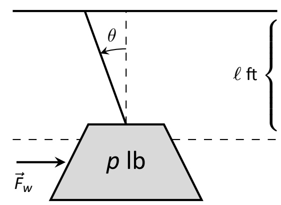
10 02 ex 28 28. $\theta = 30^\circ$, $\varphi = 30^\circ$

10 02 ex 29 29. $\theta = 60^\circ$, $\varphi = 60^\circ$

10 02 ex 30 30. $\theta = 20^\circ$, $\varphi = 15^\circ$

10 02 ex 31 31. $\theta = 0^\circ$, $\varphi = 0^\circ$

10 02 exset 06 **A weight of $p$lb is suspended from a chain of length $\ell$ while a constant force of $\vec{F}_w$ pushes the weight to the right, making an angle of $\theta$ with the vertical, as shown in the figure below.**




**In Exercises 32 – 35, a force $\vec{F}_w$ and length $\ell$ are given. Find the angle $\theta$ and the height the weight is lifted as it moves to the right.**

10 02 ex 32 32. $\vec{F}_w = 1$lb, $\ell = 1$ft, $p = 1$lb

10 02 ex 33 33. $\vec{F}_w = 1$lb, $\ell = 1$ft, $p = 10$lb

10 02 ex 34 34. $\vec{F}_w = 1$lb, $\ell = 10$ft, $p = 1$lb

10 02 ex 35 35. $\vec{F}_w = 10$lb, $\ell = 10$ft, $p = 1$lb

# Exercises 10.3

## *Terms and Concepts*

10 03 ex 01 1. The dot product of two vectors is a ________, not a vector.

10 03 ex 02 2. How are the concepts of the dot product and vector magnitude related?

10 03 ex 03 3. How can one quickly tell if the angle between two vectors is acute or obtuse?

10 03 ex 04 4. Give a synonym for "orthogonal."

## *Problems*

10 03 exset 01 **In Exercises 5 – 10, find the dot product of the given vectors.**

10 03 ex 05 5. $\vec{u} = \langle 2, -4 \rangle$, $\vec{v} = \langle 3, 7 \rangle$

10 03 ex 06 6. $\vec{u} = \langle 5, 3 \rangle$, $\vec{v} = \langle 6, 1 \rangle$

10 03 ex 07 7. $\vec{u} = \langle 1, -1, 2 \rangle$, $\vec{v} = \langle 2, 5, 3 \rangle$

10 03 ex 08 8. $\vec{u} = \langle 3, 5, -1 \rangle$, $\vec{v} = \langle 4, -1, 7 \rangle$

10 03 ex 09 9. $\vec{u} = \langle 1, 1 \rangle$, $\vec{v} = \langle 1, 2, 3 \rangle$

10 03 ex 10 10. $\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle 0, 0, 0 \rangle$

10 03 ex 11 11. Create your own vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$ and show that $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$.

10 03 ex 12 12. Create your own vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ and scalar $c$ and show that $c(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (c\vec{v})$.

10 03 exset 02 **In Exercises 13 – 16, find the measure of the angle between the two vectors in both radians and degrees.**

10 03 ex 13 13. $\vec{u} = \langle 1, 1 \rangle$, $\vec{v} = \langle 1, 2 \rangle$

10 03 ex 14 14. $\vec{u} = \langle -2, 1 \rangle$, $\vec{v} = \langle 3, 5 \rangle$

10 03 ex 15 15. $\vec{u} = \langle 8, 1, -4 \rangle$, $\vec{v} = \langle 2, 2, 0 \rangle$

10 03 ex 16 16. $\vec{u} = \langle 1, 7, 2 \rangle$, $\vec{v} = \langle 4, -2, 5 \rangle$

10 03 exset 03 **In Exercises 17 – 20, a vector $\vec{v}$ is given. Give two vectors that are orthogonal to $\vec{v}$.**

10 03 ex 17 17. $\vec{v} = \langle 4, 7 \rangle$

10 03 ex 18 18. $\vec{v} = \langle -3, 5 \rangle$

10 03 ex 19 19. $\vec{v} = \langle 1, 1, 1 \rangle$

10 03 ex 20 20. $\vec{v} = \langle 1, -2, 3 \rangle$

10 03 exset 04 **In Exercises 21 – 26, vectors $\vec{u}$ and $\vec{v}$ are given. Find $\text{proj}_{\vec{v}}\,\vec{u}$, the orthogonal projection of $\vec{u}$ onto $\vec{v}$, and sketch all three vectors on the same axes.**

10 03 ex 21 21. $\vec{u} = \langle 1, 2 \rangle$, $\vec{v} = \langle -1, 3 \rangle$

10 03 ex 22 22. $\vec{u} = \langle 5, 5 \rangle$, $\vec{v} = \langle 1, 3 \rangle$

10 03 ex 25 23. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 1, 1 \rangle$

10 03 ex 26 24. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 2, 3 \rangle$

10 03 ex 23 25. $\vec{u} = \langle 1, 5, 1 \rangle$, $\vec{v} = \langle 1, 2, 3 \rangle$

10 03 ex 24 26. $\vec{u} = \langle 3, -1, 2 \rangle$, $\vec{v} = \langle 2, 2, 1 \rangle$

10 03 exset 05 **In Exercises 27 – 32, vectors $\vec{u}$ and $\vec{v}$ are given. Write $\vec{u}$ as the sum of two vectors, one of which is parallel to $\vec{v}$ and one of which is perpendicular to $\vec{v}$. Note: these are the same pairs of vectors as found in Exercises 21 – 26.**

10 03 ex 27 27. $\vec{u} = \langle 1, 2 \rangle$, $\vec{v} = \langle -1, 3 \rangle$

10 03 ex 28 28. $\vec{u} = \langle 5, 5 \rangle$, $\vec{v} = \langle 1, 3 \rangle$

10 03 ex 29 29. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 1, 1 \rangle$

10 03 ex 30 30. $\vec{u} = \langle -3, 2 \rangle$, $\vec{v} = \langle 2, 3 \rangle$

10 03 ex 31 31. $\vec{u} = \langle 1, 5, 1 \rangle$, $\vec{v} = \langle 1, 2, 3 \rangle$

10 03 ex 32 32. $\vec{u} = \langle 3, -1, 2 \rangle$, $\vec{v} = \langle 2, 2, 1 \rangle$

10 03 ex 33 33. A 10lb box sits on a ramp that rises 4ft over a distance of 20ft. How much force is required to keep the box from sliding down the ramp?

10 03 ex 34 34. A 10lb box sits on a 15ft ramp that makes a $30^\circ$ angle with the horizontal. How much force is required to keep the box from sliding down the ramp?

10 03 ex 35 35. How much work is performed in moving a box horizontally 10ft with a force of 20lb applied at an angle of $45^\circ$ to the horizontal?

10 03 ex 36 36. How much work is performed in moving a box horizontally 10ft with a force of 20lb applied at an angle of $10^\circ$ to the horizontal?

10 03 ex 37 37. How much work is performed in moving a box up the length of a ramp that rises 2ft over a distance of 10ft, with a force of 50lb applied horizontally?

10 03 ex 38 38. How much work is performed in moving a box up the length of a ramp that rises 2ft over a distance of 10ft, with a force of 50lb applied at an angle of $45^\circ$ to the horizontal?

10 03 ex 39 39. How much work is performed in moving a box up the length of a 10ft ramp that makes a $5^\circ$ angle with the horizontal, with 50lb of force applied in the direction of the ramp?

# Exercises 10.4

## *Terms and Concepts*

10 04 ex 01 1. The cross product of two vectors is a ________, not a scalar.

10 04 ex 02 2. One can visualize the direction of $\vec{u} \times \vec{v}$ using the ________ ________ ________.

10 03 ex 04 3. Give a synonym for "orthogonal."

10 04 ex 03 4. T/F: A fundamental principle of the cross product is that $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$.

10 04 ex 04 5. ________ is a measure of the turning force applied to an object.

## *Problems*

10 04 exset 01 **In Exercises 6 – 14, vectors $\vec{u}$ and $\vec{v}$ are given. Compute $\vec{u} \times \vec{v}$ and show this is orthogonal to both $\vec{u}$ and $\vec{v}$.**

10 04 ex 07 6. $\vec{u} = \langle 3, 2, -2 \rangle, \quad \vec{v} = \langle 0, 1, 5 \rangle$

10 04 ex 08 7. $\vec{u} = \langle 5, -4, 3 \rangle, \quad \vec{v} = \langle 2, -5, 1 \rangle$

10 04 ex 05 8. $\vec{u} = \langle 4, -5, -5 \rangle, \quad \vec{v} = \langle 3, 3, 4 \rangle$

10 04 ex 06 9. $\vec{u} = \langle -4, 7, -10 \rangle, \quad \vec{v} = \langle 4, 4, 1 \rangle$

10 04 ex 09 10. $\vec{u} = \langle 1, 0, 1 \rangle, \quad \vec{v} = \langle 5, 0, 7 \rangle$

10 04 ex 10 11. $\vec{u} = \langle 1, 5, -4 \rangle, \quad \vec{v} = \langle -2, -10, 8 \rangle$

10 04 ex 11 12. $\vec{u} = \vec{i}, \quad \vec{v} = \vec{j}$

10 04 ex 12 13. $\vec{u} = \vec{i}, \quad \vec{v} = \vec{k}$

10 04 ex 13 14. $\vec{u} = \vec{j}, \quad \vec{v} = \vec{k}$

10 04 ex 14 15. Pick any vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ and show that $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$.

10 04 ex 15 16. Pick any vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ and show that $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$.

10 04 exset 02 **In Exercises 17 – 20, the magnitudes of vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ are given, along with the angle $\theta$ between them. Use this information to find the magnitude of $\vec{u} \times \vec{v}$.**

10 04 ex 16 17. $||\,\vec{u}\,|| = 2, \quad ||\,\vec{v}\,|| = 5, \quad \theta = 30^\circ$

10 04 ex 17 18. $||\,\vec{u}\,|| = 3, \quad ||\,\vec{v}\,|| = 7, \quad \theta = \pi/2$

10 04 ex 18 19. $||\,\vec{u}\,|| = 3, \quad ||\,\vec{v}\,|| = 4, \quad \theta = \pi$

10 04 ex 19 20. $||\,\vec{u}\,|| = 2, \quad ||\,\vec{v}\,|| = 5, \quad \theta = 5\pi/6$

10 04 exset 03 **In Exercises 21 – 24, find the area of the parallelogram defined by the given vectors.**

10 04 ex 20 21. $\vec{u} = \langle 1, 1, 2 \rangle, \quad \vec{v} = \langle 2, 0, 3 \rangle$

10 04 ex 21 22. $\vec{u} = \langle -2, 1, 5 \rangle, \quad \vec{v} = \langle -1, 3, 1 \rangle$

10 04 ex 22 23. $\vec{u} = \langle 1, 2 \rangle, \quad \vec{v} = \langle 2, 1 \rangle$

10 04 ex 23 24. $\vec{u} = \langle 2, 0 \rangle, \quad \vec{v} = \langle 0, 3 \rangle$

10 04 exset 04 **In Exercises 25 – 28, find the area of the triangle with the given vertices.**

10 04 ex 24 25. Vertices: $(0, 0, 0)$, $(1, 3, -1)$ and $(2, 1, 1)$.

10 04 ex 25 26. Vertices: $(5, 2, -1)$, $(3, 6, 2)$ and $(1, 0, 4)$.

10 04 ex 26 27. Vertices: $(1, 1)$, $(1, 3)$ and $(2, 2)$.

10 04 ex 27 28. Vertices: $(3, 1)$, $(1, 2)$ and $(4, 3)$.

10 04 exset 05 **In Exercises 29 – 30, find the area of the quadrilateral with the given vertices. (Hint: break the quadrilateral into 2 triangles.)**

10 04 ex 28 29. Vertices: $(0, 0)$, $(1, 2)$, $(3, 0)$ and $(4, 3)$.

10 04 ex 29 30. Vertices: $(0, 0, 0)$, $(2, 1, 1)$, $(-1, 2, -8)$ and $(1, -1, 5)$.

10 04 exset 06 **In Exercises 31 – 32, find the volume of the parallelepiped defined by the given vectors.**

10 04 ex 30 31. $\vec{u} = \langle 1, 1, 1 \rangle, \quad \vec{v} = \langle 1, 2, 3 \rangle, \quad \vec{w} = \langle 1, 0, 1 \rangle$

10 04 ex 31 32. $\vec{u} = \langle -1, 2, 1 \rangle, \quad \vec{v} = \langle 2, 2, 1 \rangle, \quad \vec{w} = \langle 3, 1, 3 \rangle$

10 04 exset 07 **In Exercises 33 – 36, find a unit vector orthogonal to both $\vec{u}$ and $\vec{v}$.**

10 04 ex 35 33. $\vec{u} = \langle 1, 1, 1 \rangle, \quad \vec{v} = \langle 2, 0, 1 \rangle$

10 04 ex 32 34. $\vec{u} = \langle 1, -2, 1 \rangle, \quad \vec{v} = \langle 3, 2, 1 \rangle$

10 04 ex 33 35. $\vec{u} = \langle 5, 0, 2 \rangle, \quad \vec{v} = \langle -3, 0, 7 \rangle$

10 04 ex 34 36. $\vec{u} = \langle 1, -2, 1 \rangle, \quad \vec{v} = \langle -2, 4, -2 \rangle$

10 04 ex 36 37. A bicycle rider applies 150lb of force, straight down, onto a pedal that extends 7in horizontally from the crankshaft. Find the magnitude of the torque applied to the crankshaft.

10 04 ex 37 38. A bicycle rider applies 150lb of force, straight down, onto a pedal that extends 7in from the crankshaft, making a $30^\circ$ angle with the horizontal. Find the magnitude of the torque applied to the crankshaft.

10 04 ex 38 39. To turn a stubborn bolt, 80lb of force is applied to a 10in wrench. What is the maximum amount of torque that can be applied to the bolt?

10 04 ex 39 40. To turn a stubborn bolt, 80lb of force is applied to a 10in wrench in a confined space, where the direction of applied force makes a $10^\circ$ angle with the wrench. How much torque is subsequently applied to the wrench?

10 04 ex 40 41. Show, using the definition of the Cross Product, that $\vec{u}\cdot(\vec{u}\times\vec{v}) = 0$; that is, that $\vec{u}$ is orthogonal to the cross product of $\vec{u}$ and $\vec{v}$.

10 04 ex 41 42. Show, using the definition of the Cross Product, that $\vec{u}\times\vec{u} = \vec{0}$.

# Exercises 10.5

## *Terms and Concepts*

10 05 ex 01 1. To find an equation of a line, what two pieces of information are needed?

10 05 ex 02 2. Two distinct lines in the plane can intersect or be ________.

10 05 ex 03 3. Two distinct lines in space can intersect, be ________ or be ________.

10 05 ex 04 4. Use your own words to describe what it means for two lines in space to be skew.

## *Problems*

10 05 exset 01 **In Exercises 5 – 14, write the vector, parametric and symmetric equations of the lines described.**

10 05 ex 05 5. Passes through $P = (2, -4, 1)$, parallel to $\vec{d} = \langle 9, 2, 5\rangle$.

10 05 ex 06 6. Passes through $P = (6, 1, 7)$, parallel to $\vec{d} = \langle -3, 2, 5\rangle$.

10 05 ex 07 7. Passes through $P = (2, 1, 5)$ and $Q = (7, -2, 4)$.

10 05 ex 08 8. Passes through $P = (1, -2, 3)$ and $Q = (5, 5, 5)$.

10 05 ex 09 9. Passes through $P = (0, 1, 2)$ and orthogonal to both $\vec{d_1} = \langle 2, -1, 7\rangle$ and $\vec{d_2} = \langle 7, 1, 3\rangle$.

10 05 ex 10 10. Passes through $P = (5, 1, 9)$ and orthogonal to both $\vec{d_1} = \langle 1, 0, 1\rangle$ and $\vec{d_2} = \langle 2, 0, 3\rangle$.

10 05 ex 11 11. Passes through the point of intersection of $\vec{\ell_1}(t)$ and $\vec{\ell_2}(t)$ and orthogonal to both lines, where
$\vec{\ell_1}(t) = \langle 2, 1, 1\rangle + t\langle 5, 1, -2\rangle$ and
$\vec{\ell_2}(t) = \langle -2, -1, 2\rangle + t\langle 3, 1, -1\rangle$.

10 05 ex 12 12. Passes through the point of intersection of $\ell_1(t)$ and $\ell_2(t)$ and orthogonal to both lines, where

$$\ell_1 = \begin{cases} x = t \\ y = -2 + 2t \\ z = 1 + t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 2 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}.$$

10 05 ex 13 13. Passes through $P = (1, 1)$, parallel to $\vec{d} = \langle 2, 3\rangle$.

10 05 ex 14 14. Passes through $P = (-2, 5)$, parallel to $\vec{d} = \langle 0, 1\rangle$.

10 05 exset 02 **In Exercises 15 – 22, determine if the described lines are the same line, parallel lines, intersecting or skew lines. If intersecting, give the point of intersection.**

10 05 ex 15 15. $\vec{\ell_1}(t) = \langle 1, 2, 1\rangle + t\langle 2, -1, 1\rangle$,
$\vec{\ell_2}(t) = \langle 3, 3, 3\rangle + t\langle -4, 2, -2\rangle$.

10 05 ex 16 16. $\vec{\ell_1}(t) = \langle 2, 1, 1\rangle + t\langle 5, 1, 3\rangle$,
$\vec{\ell_2}(t) = \langle 14, 5, 9\rangle + t\langle 1, 1, 1\rangle$.

10 05 ex 17 17. $\vec{\ell_1}(t) = \langle 3, 4, 1\rangle + t\langle 2, -3, 4\rangle$,
$\vec{\ell_2}(t) = \langle -3, 3, -3\rangle + t\langle 3, -2, 4\rangle$.

10 05 ex 18 18. $\vec{\ell_1}(t) = \langle 1, 1, 1\rangle + t\langle 3, 1, 3\rangle$,
$\vec{\ell_2}(t) = \langle 7, 3, 7\rangle + t\langle 6, 2, 6\rangle$.

10 05 ex 19 19. $$\ell_1 = \begin{cases} x = 1 + 2t \\ y = 3 - 2t \\ z = t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 3 - t \\ y = 3 + 5t \\ z = 2 + 7t \end{cases}$$

10 05 ex 20 20. $$\ell_1 = \begin{cases} x = 1.1 + 0.6t \\ y = 3.77 + 0.9t \\ z = -2.3 + 1.5t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 3.11 + 3.4t \\ y = 2 + 5.1t \\ z = 2.5 + 8.5t \end{cases}$$

10 05 ex 21 21. $$\ell_1 = \begin{cases} x = 0.2 + 0.6t \\ y = 1.33 - 0.45t \\ z = -4.2 + 1.05t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 0.86 + 9.2t \\ y = 0.835 - 6.9t \\ z = -3.045 + 16.1t \end{cases}$$

10 05 ex 22 22. $$\ell_1 = \begin{cases} x = 0.1 + 1.1t \\ y = 2.9 - 1.5t \\ z = 3.2 + 1.6t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 4 - 2.1t \\ y = 1.8 + 7.2t \\ z = 3.1 + 1.1t \end{cases}$$

10 05 exset 03 **In Exercises 23 – 26, find the distance from the point to the line.**

10 05 ex 23 23. $P = (1, 1, 1)$, $\quad \vec{\ell}(t) = \langle 2, 1, 3\rangle + t\langle 2, 1, -2\rangle$

10 05 ex 24 24. $P = (2, 5, 6)$, $\quad \vec{\ell}(t) = \langle -1, 1, 1\rangle + t\langle 1, 0, 1\rangle$

10 05 ex 25 25. $P = (0, 3)$, $\quad \vec{\ell}(t) = \langle 2, 0\rangle + t\langle 1, 1\rangle$

10 05 ex 26 26. $P = (1, 1)$, $\quad \vec{\ell}(t) = \langle 4, 5\rangle + t\langle -4, 3\rangle$

10 05 exset 04 **In Exercises 27 – 28, find the distance between the two lines.**

10 05 ex 27 27. $\vec{\ell_1}(t) = \langle 1, 2, 1\rangle + t\langle 2, -1, 1\rangle$,
$\vec{\ell_2}(t) = \langle 3, 3, 3\rangle + t\langle 4, 2, -2\rangle$.

10 05 ex 28 28. $\vec{\ell_1}(t) = \langle 0, 0, 1\rangle + t\langle 1, 0, 0\rangle$,
$\vec{\ell_2}(t) = \langle 0, 0, 3\rangle + t\langle 0, 1, 0\rangle$.

10 05 exset 05 **Exercises 29 – 31 explore special cases of the distance formulas found in Key Idea 50.**

10 05 ex 29 29. Let $Q$ be a point on the line $\ell(t)$. Show why the distance formula correctly gives the distance from the point to the line as 0.

10 05 ex 30 30. Let lines $\ell_1(t)$ and $\ell_2(t)$ be intersecting lines. Show why the distance formula correctly gives the distance between these lines as 0.

10 05 ex 31

31. Let lines $\ell_1(t)$ and $\ell_2(t)$ be parallel.

 (a) Show why the distance formula for distance between lines cannot be used as stated to find the distance between the lines.

 (b) Show why letting $c = (\overrightarrow{P_1P_2} \times \vec{d}_2) \times \vec{d}_2$ allows one to the use the formula.

 (c) Show how one can use the formula for the distance between a point and a line to find the distance between parallel lines.

# Exercises 10.6

## *Terms and Concepts*

10 06 ex 01 1. In order to find the equation of a plane, what two pieces of information must one have?

10 06 ex 02 2. What is the relationship between a plane and one of its normal vectors?

## *Problems*

10 06 exset 01 **In Exercises 3 – 6, give any two points in the given plane.**

10 06 ex 03 3. $2x - 4y + 7z = 2$

10 06 ex 04 4. $3(x+2) + 5(y-9) - 4z = 0$

10 06 ex 05 5. $x = 2$

10 06 ex 06 6. $4(y+2) - (z-6) = 0$

10 06 exset 02 **In Exercises 7 – 20, give the equation of the described plane in standard and general forms.**

10 06 ex 07 7. Passes through $(2,3,4)$ and has normal vector $\vec{n} = \langle 3,-1,7\rangle$.

10 06 ex 08 8. Passes through $(1,3,5)$ and has normal vector $\vec{n} = \langle 0,2,4\rangle$.

10 06 ex 09 9. Passes through the points $(1,2,3)$, $(3,-1,4)$ and $(1,0,1)$.

10 06 ex 10 10. Passes through the points $(5,3,8)$, $(6,4,9)$ and $(3,3,3)$.

10 06 ex 11 11. Contains the intersecting lines $\ell_1(t) = \langle 2,1,2\rangle + t\langle 1,2,3\rangle$ and $\ell_2(t) = \langle 2,1,2\rangle + t\langle 2,5,4\rangle$.

10 06 ex 12 12. Contains the intersecting lines $\ell_1(t) = \langle 5,0,3\rangle + t\langle -1,1,1\rangle$ and $\ell_2(t) = \langle 1,4,7\rangle + t\langle 3,0,-3\rangle$.

10 06 ex 13 13. Contains the parallel lines $\ell_1(t) = \langle 1,1,1\rangle + t\langle 1,2,3\rangle$ and $\ell_2(t) = \langle 1,1,2\rangle + t\langle 1,2,3\rangle$.

10 06 ex 14 14. Contains the parallel lines $\ell_1(t) = \langle 1,1,1\rangle + t\langle 4,1,3\rangle$ and $\ell_2(t) = \langle 2,2,2\rangle + t\langle 4,1,3\rangle$.

10 06 ex 15 15. Contains the point $(2,-6,1)$ and the line
$$\ell(t) = \begin{cases} x = 2+5t \\ y = 2+2t \\ z = -1+2t \end{cases}$$

10 06 ex 16 16. Contains the point $(5,7,3)$ and the line
$$\ell(t) = \begin{cases} x = t \\ y = t \\ z = t \end{cases}$$

10 06 ex 17 17. Contains the point $(5,7,3)$ and is orthogonal to the line $\ell(t) = \langle 4,5,6\rangle + t\langle 1,1,1\rangle$.

10 06 ex 18 18. Contains the point $(4,1,1)$ and is orthogonal to the line
$$\ell(t) = \begin{cases} x = 4+4t \\ y = 1+1t \\ z = 1+1t \end{cases}$$

10 06 ex 19 19. Contains the point $(-4,7,2)$ and is parallel to the plane $3(x-2) + 8(y+1) - 10z = 0$.

10 06 ex 20 20. Contains the point $(1,2,3)$ and is parallel to the plane $x = 5$.

10 06 exset 03 **In Exercises 21 – 22, give the equation of the line that is the intersection of the given planes.**

10 06 ex 21 21. $p1:\ 3(x-2) + (y-1) + 4z = 0$, and $p2:\ 2(x-1) - 2(y+3) + 6(z-1) = 0$.

10 06 ex 22 22. $p1:\ 5(x-5) + 2(y+2) + 4(z-1) = 0$, and $p2:\ 3x - 4(y-1) + 2(z-1) = 0$.

10 06 exset 05 **In Exercises 23 – 26, find the point of intersection between the line and the plane.**

10 06 ex 30 23. line: $\langle 5,1,-1\rangle + t\langle 2,2,1\rangle$, plane: $5x - y - z = -3$

10 06 ex 31 24. line: $\langle 4,1,0\rangle + t\langle 1,0,-1\rangle$, plane: $3x + y - 2z = 8$

10 06 ex 28 25. line: $\langle 1,2,3\rangle + t\langle 3,5,-1\rangle$, plane: $3x - 2y - z = 4$

10 06 ex 29 26. line: $\langle 1,2,3\rangle + t\langle 3,5,-1\rangle$, plane: $3x - 2y - z = -4$

10 06 exset 04 **In Exercises 27 – 30, find the given distances.**

10 06 ex 23 27. The distance from the point $(1,2,3)$ to the plane $3(x-1) + (y-2) + 5(z-2) = 0$.

10 06 ex 24 28. The distance from the point $(2,6,2)$ to the plane $2(x-1) - y + 4(z+1) = 0$.

10 06 ex 25 29. The distance between the parallel planes $x + y + z = 0$ and $(x-2) + (y-3) + (z+4) = 0$

10 06 ex 26 30. The distance between the parallel planes $2(x-1)+2(y+1)+(z-2)=0$ and $2(x-3)+2(y-1)+(z-3)=0$

10 06 ex 27 31. Show why if the point $Q$ lies in a plane, then the distance formula correctly gives the distance from the point to the plane as 0.

10 06 ex 32 32. How is Exercise 30 in Section 10.5 easier to answer once we have an understanding of planes?

# Exercises 11.1

## *Terms and Concepts*

11 01 ex 01 1. Vector–valued functions are closely related to ________ of graphs.

11 01 ex 02 2. When sketching vector–valued functions, technically one isn't graphing points, but rather ________.

11 01 ex 31 3. It can be useful to think of ________ as a vector that points from a starting position to an ending position.

## *Problems*

11 01 exset 01 **In Exercises 4 – 11, sketch the vector–valued function on the given interval.**

11 01 ex 03 4. $\vec{r}(t) = \langle t^2, t^2 - 1 \rangle$, for $-2 \le t \le 2$.

11 01 ex 04 5. $\vec{r}(t) = \langle t^2, t^3 \rangle$, for $-2 \le t \le 2$.

11 01 ex 05 6. $\vec{r}(t) = \langle 1/t, 1/t^2 \rangle$, for $-2 \le t \le 2$.

11 01 ex 06 7. $\vec{r}(t) = \langle \frac{1}{10}t^2, \sin t \rangle$, for $-2\pi \le t \le 2\pi$.

11 01 ex 07 8. $\vec{r}(t) = \langle \frac{1}{10}t^2, \sin t \rangle$, for $-2\pi \le t \le 2\pi$.

11 01 ex 08 9. $\vec{r}(t) = \langle 3\sin(\pi t), 2\cos(\pi t) \rangle$, on $[0, 2]$.

11 01 ex 09 10. $\vec{r}(t) = \langle 3\cos t, 2\sin(2t) \rangle$, on $[0, 2\pi]$.

11 01 ex 10 11. $\vec{r}(t) = \langle 2\sec t, \tan t \rangle$, on $[-\pi, \pi]$.

11 01 exset 02 **In Exercises 12 – 15, sketch the vector–valued function on the given interval in $\mathbb{R}^3$. Technology may be useful in creating the sketch.**

11 01 ex 11 12. $\vec{r}(t) = \langle 2\cos t, t, 2\sin t \rangle$, on $[0, 2\pi]$.

11 01 ex 12 13. $\vec{r}(t) = \langle 3\cos t, \sin t, t/\pi \rangle$ on $[0, 2\pi]$.

11 01 ex 13 14. $\vec{r}(t) = \langle \cos t, \sin t, \sin t \rangle$ on $[0, 2\pi]$.

11 01 ex 14 15. $\vec{r}(t) = \langle \cos t, \sin t, \sin(2t) \rangle$ on $[0, 2\pi]$.

11 01 exset 03 **In Exercises 16 – 19, find $|| \vec{r}(t) ||$.**

11 01 ex 15 16. $\vec{r}(t) = \langle t, t^2 \rangle$.

11 01 ex 16 17. $\vec{r}(t) = \langle 5\cos t, 3\sin t \rangle$.

11 01 ex 17 18. $\vec{r}(t) = \langle 2\cos t, 2\sin t, t \rangle$.

11 01 ex 18 19. $\vec{r}(t) = \langle \cos t, t, t^2 \rangle$.

11 01 exset 04 **In Exercises 20 – 27, create a vector–valued function whose graph matches the given description.**

11 01 ex 19 20. A circle of radius 2, centered at $(1, 2)$, traced counter–clockwise once on $[0, 2\pi]$.

11 01 ex 20 21. A circle of radius 3, centered at $(5, 5)$, traced clockwise once on $[0, 2\pi]$.

11 01 ex 21 22. An ellipse, centered at $(0, 0)$ with vertical major axis of length 10 and minor axis of length 3, traced once counter–clockwise on $[0, 2\pi]$.

11 01 ex 22 23. An ellipse, centered at $(3, -2)$ with horizontal major axis of length 6 and minor axis of length 4, traced once clockwise on $[0, 2\pi]$.

11 01 ex 23 24. A line through $(2, 3)$ with a slope of 5.

11 01 ex 24 25. A line through $(1, 5)$ with a slope of $-1/2$.

11 01 ex 25 26. A vertically oriented helix with radius of 2 that starts at $(2, 0, 0)$ and ends at $(2, 0, 4\pi)$ after 1 revolution on $[0, 2\pi]$.

11 01 ex 26 27. A vertically oriented helix with radius of 3 that starts at $(3, 0, 0)$ and ends at $(3, 0, 3)$ after 2 revolutions on $[0, 1]$.

11 01 exset 05 **In Exercises 28 – 31, find the average rate of change of $\vec{r}(t)$ on the given interval.**

11 01 ex 27 28. $\vec{r}(t) = \langle t, t^2 \rangle$ on $[-2, 2]$.

11 01 ex 28 29. $\vec{r}(t) = \langle t, t + \sin t \rangle$ on $[0, 2\pi]$.

11 01 ex 29 30. $\vec{r}(t) = \langle 3\cos t, 2\sin t, t \rangle$ on $[0, 2\pi]$.

11 01 ex 30 31. $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ on $[-1, 3]$.

# Exercises 11.2

## *Terms and Concepts*

11 02 ex 01 1. Limits, derivatives and integrals of vector–valued functions are all evaluated ________–wise.

11 02 ex 02 2. The definite integral of a rate of change function gives ________.

11 02 ex 03 3. Why is it generally not useful to graph both $\vec{r}(t)$ and $\vec{r}\,'(t)$ on the same axes?

## *Problems*

11 02 exset 01 **In Exercises 4 – 7, evaluate the given limit.**

11 02 ex 04 4. $\lim\limits_{t\to 5} \left\langle 2t+1, 3t^2-1, \sin t\right\rangle$

11 02 ex 05 5. $\lim\limits_{t\to 3} \left\langle e^t, \dfrac{t^2-9}{t+3}\right\rangle$

11 02 ex 06 6. $\lim\limits_{t\to 0} \left\langle \dfrac{t}{\sin t}, (1+t)^{\frac{1}{t}}\right\rangle$

11 02 ex 07 7. $\lim\limits_{h\to 0} \dfrac{\vec{r}(t+h)-\vec{r}(t)}{h}$, where $\vec{r}(t) = \left\langle t^2, t, 1\right\rangle$.

11 02 exset 02 **In Exercises 8 – 9, identify the interval(s) on which $\vec{r}(t)$ is continuous.**

11 02 ex 08 8. $\vec{r}(t) = \left\langle t^2, 1/t\right\rangle$

11 02 ex 09 9. $\vec{r}(t) = \left\langle \cos t, e^t, \ln t\right\rangle$

11 02 exset 03 **In Exercises 10 – 14, find the derivative of the given function.**

11 02 ex 10 10. $\vec{r}(t) = \left\langle \cos t, e^t, \ln t\right\rangle$

11 02 ex 11 11. $\vec{r}(t) = \left\langle \dfrac{1}{t}, \dfrac{2t-1}{3t+1}, \tan t\right\rangle$

11 02 ex 12 12. $\vec{r}(t) = (t^2)\left\langle \sin t, 2t+5\right\rangle$

11 02 ex 13 13. $\vec{r}(t) = \left\langle t^2+1, t-1\right\rangle \cdot \left\langle \sin t, 2t+5\right\rangle$

11 02 ex 14 14. $\vec{r}(t) = \left\langle t^2+1, t-1, 1\right\rangle \times \left\langle \sin t, 2t+5, 1\right\rangle$

11 02 exset 04 **In Exercises 15 – 18, find $\vec{r}\,'(t)$. Sketch $\vec{r}(t)$ and $\vec{r}\,'(1)$, with the initial point of $\vec{r}\,'(1)$ at $\vec{r}(1)$.**

11 02 ex 15 15. $\vec{r}(t) = \left\langle t^2+t, t^2-t\right\rangle$

11 02 ex 16 16. $\vec{r}(t) = \left\langle t^2-2t+2, t^3-3t^2+2t\right\rangle$

11 02 ex 17 17. $\vec{r}(t) = \left\langle t^2+1, t^3-t\right\rangle$

11 02 ex 18 18. $\vec{r}(t) = \left\langle t^2-4t+5, , t^3-6t^2+11t-6\right\rangle$

11 02 exset 05 **In Exercises 19 – 22, give the equation of the line tangent to the graph of $\vec{r}(t)$ at the given $t$ value.**

11 02 ex 19 19. $\vec{r}(t) = \left\langle t^2+t, t^2-t\right\rangle$ at $t=1$.

11 02 ex 20 20. $\vec{r}(t) = \left\langle 3\cos t, \sin t\right\rangle$ at $t=\pi/4$.

11 02 ex 21 21. $\vec{r}(t) = \left\langle 3\cos t, 3\sin t, t\right\rangle$ at $t=\pi$.

11 02 ex 22 22. $\vec{r}(t) = \left\langle e^t, \tan t, t\right\rangle$ at $t=0$.

11 02 exset 06 **In Exercises 23 – 26, find the value(s) of $t$ for which $\vec{r}(t)$ is not smooth.**

11 02 ex 23 23. $\vec{r}(t) = \left\langle \cos t, \sin t - t\right\rangle$

11 02 ex 24 24. $\vec{r}(t) = \left\langle t^2-2t+1, t^3+t^2-5t+3\right\rangle$

11 02 ex 25 25. $\vec{r}(t) = \left\langle \cos t - \sin t, \sin t - \cos t, \cos(4t)\right\rangle$

11 02 ex 26 26. $\vec{r}(t) = \left\langle t^3-3t+2, -\cos(\pi t), \sin^2(\pi t)\right\rangle$

11 02 exset 07 **Exercises 27 – 29 ask you to verify parts of Theorem 92. In each let $f(t) = t^3$, $\vec{r}(t) = \left\langle t^2, t-1, 1\right\rangle$ and $\vec{s}(t) = \left\langle \sin t, e^t, t\right\rangle$. Compute the various derivatives as indicated.**

11 02 ex 27 27. Simplify $f(t)\vec{r}(t)$, then find its derivative; show this is the same as $f\,'(t)\vec{r}(t) + f(t)\vec{r}\,'(t)$.

11 02 ex 28 28. Simplify $\vec{r}(t)\cdot\vec{s}(t)$, then find its derivative; show this is the same as $\vec{r}\,'(t)\cdot\vec{s}(t) + \vec{r}(t)\cdot\vec{s}\,'(t)$.

11 02 ex 29 29. Simplify $\vec{r}(t)\times\vec{s}(t)$, then find its derivative; show this is the same as $\vec{r}\,'(t)\times\vec{s}(t) + \vec{r}(t)\times\vec{s}\,'(t)$.

11 02 exset 08 **In Exercises 30 – 33 , evaluate the given definite or indefinite integral.**

11 02 ex 30 30. $\displaystyle\int \left\langle t^3, \cos t, te^t\right\rangle\,dt$

11 02 ex 31 31. $\displaystyle\int \left\langle \frac{1}{1+t^2}, \sec^2 t\right\rangle\,dt$

11 02 ex 32 32. $\displaystyle\int_0^{\pi} \left\langle -\sin t, \cos t\right\rangle\,dt$

11 02 ex 33 33. $\displaystyle\int_{-2}^{2} \left\langle 2t+1, 2t-1\right\rangle\,dt$

11 02 exset 09 **In Exercises 34 – 37 , solve the given initial value problems.**

11 02 ex 34 34. Find $\vec{r}(t)$, given that $\vec{r}\,'(t) = \left\langle t, \sin t\right\rangle$ and $\vec{r}(0) = \left\langle 2, 2\right\rangle$.

11 02 ex 35 35. Find $\vec{r}(t)$, given that $\vec{r}\,'(t) = \langle 1/(t+1), \tan t\rangle$ and $\vec{r}(0) = \langle 1, 2\rangle$.

11 02 ex 36 36. Find $\vec{r}(t)$, given that $\vec{r}\,''(t) = \left\langle t^2, t, 1\right\rangle$, $\vec{r}\,'(0) = \langle 1, 2, 3\rangle$ and $\vec{r}(0) = \langle 4, 5, 6\rangle$.

11 02 ex 37 37. Find $\vec{r}(t)$, given that $\vec{r}\,''(t) = \left\langle \cos t, \sin t, e^t\right\rangle$, $\vec{r}\,'(0) = \langle 0, 0, 0\rangle$ and $\vec{r}(0) = \langle 0, 0, 0\rangle$.

11 02 exset 10 **In Exercises 38 – 41 , find the arc length of $\vec{r}(t)$ on the indicated interval.**

11 02 ex 38 38. $\vec{r}(t) = \langle 2\cos t, 2\sin t, 3t\rangle$ on $[0, 2\pi]$.

11 02 ex 39 39. $\vec{r}(t) = \langle 5\cos t, 3\sin t, 4\sin t\rangle$ on $[0, 2\pi]$.

11 02 ex 40 40. $\vec{r}(t) = \left\langle t^3, t^2, t^3\right\rangle$ on $[0, 1]$.

11 02 ex 41 41. $\vec{r}(t) = \left\langle e^{-t}\cos t, e^{-t}\sin t\right\rangle$ on $[0, 1]$.

11 02 ex 42 42. Prove Theorem 93; that is, show if $\vec{r}(t)$ has constant length and is differentiable, then $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$. (Hint: use the Product Rule to compute $\frac{d}{dt}\big(\vec{r}(t)\cdot\vec{r}(t)\big)$.)

# Exercises 11.3

## *Terms and Concepts*

11 03 ex 01 1. How is *velocity* different from *speed*?

11 03 ex 02 2. What is the difference between *displacement* and *distance traveled*?

11 03 ex 03 3. What is the difference between *average velocity* and *average speed*?

11 03 ex 04 4. *Distance traveled* is the same as ________ ________, just viewed in a different context.

11 03 ex 05 5. Describe a scenario where an object's average speed is a large number, but the magnitude of the average velocity is not a large number.

11 03 ex 24 6. Explain why it is not possible to have an average velocity with a large magnitude but a small average speed.

## *Problems*

11 03 exset 01 **In Exercises 7 – 10 , a position function $\vec{r}(t)$ is given. Find $\vec{v}(t)$ and $\vec{a}(t)$.**

11 03 ex 06 7. $\vec{r}(t) = \langle 2t+1, 5t-2, 7\rangle$

11 03 ex 07 8. $\vec{r}(t) = \left\langle 3t^2-2t+1, -t^2+t+14\right\rangle$

11 03 ex 08 9. $\vec{r}(t) = \langle \cos t, \sin t\rangle$

11 03 ex 09 10. $\vec{r}(t) = \langle t/10, -\cos t, \sin t\rangle$

11 03 exset 02 **In Exercises 11 – 14 , a position function $\vec{r}(t)$ is given. Sketch $\vec{r}(t)$ on the indicated interval. Find $\vec{v}(t)$ and $\vec{a}(t)$, then add $\vec{v}(t_0)$ and $\vec{a}(t_0)$ to your sketch, with their initial points at $\vec{r}(t_0)$, for the given value of $t_0$.**

11 03 ex 11 11. $\vec{r}(t) = \langle t, \sin t\rangle$ on $[0, \pi/2]$; $t_0 = \pi/4$

11 03 ex 12 12. $\vec{r}(t) = \left\langle t^2, \sin t^2\right\rangle$ on $[0, \pi/2]$; $t_0 = \sqrt{\pi/4}$

11 03 ex 13 13. $\vec{r}(t) = \left\langle t^2+t, -t^2+2t\right\rangle$ on $[-2, 2]$; $t_0 = 1$

11 03 ex 10 14. $\vec{r}(t) = \left\langle \dfrac{2t+3}{t^2+1}, t^2\right\rangle$ on $[-1, 1]$; $t_0 = 0$

11 03 exset 03 **In Exercises 15 – 24 , a position function $\vec{r}(t)$ of an object is given. Find the speed of the object in terms of $t$, and find where the speed is minimized/maximized on the indicated interval.**

11 03 ex 14 15. $\vec{r}(t) = \left\langle t^2, t\right\rangle$ on $[-1, 1]$

11 03 ex 19 16. $\vec{r}(t) = \left\langle t^2, t^2-t^3\right\rangle$ on $[-1, 1]$

11 03 ex 15 17. $\vec{r}(t) = \langle 5\cos t, 5\sin t\rangle$ on $[0, 2\pi]$

11 03 ex 16 18. $\vec{r}(t) = \langle 2\cos t, 5\sin t\rangle$ on $[0, 2\pi]$

11 03 ex 17 19. $\vec{r}(t) = \langle \sec t, \tan t\rangle$ on $[0, \pi/4]$

11 03 ex 18 20. $\vec{r}(t) = \langle t+\cos t, 1-\sin t\rangle$ on $[0, 2\pi]$

11 03 ex 20 21. $\vec{r}(t) = \langle 12t, 5\cos t, 5\sin t\rangle$ on $[0, 4\pi]$

11 03 ex 21 22. $\vec{r}(t) = \left\langle t^2-t, t^2+t, t\right\rangle$ on $[0, 1]$

11 03 ex 22 23. $\vec{r}(t) = \left\langle t, t^2, \sqrt{1-t^2}\right\rangle$ on $[-1, 1]$

11 03 ex 23 24. **Projectile Motion:** $\vec{r}(t) = \left\langle (v_0\cos\theta)t, -\dfrac{1}{2}gt^2+(v_0\sin\theta)t\right\rangle$ on $\left[0, \dfrac{2v_0\sin\theta}{g}\right]$

11 03 exset 04 **In Exercises 25 – 28 , position functions $\vec{r}_1(t)$ and $\vec{r}_2(s)$ for two objects are given that follow the same path on the respective intervals.**

**(a) Show that the positions are the same at the indicated $t_0$ and $s_0$ values; i.e., show $\vec{r}_1(t_0) = \vec{r}_2(s_0)$.**

**(b) Find the velocity, speed and acceleration of the two objects at $t_0$ and $s_0$, respectively.**

11 03 ex 25 25. $\vec{r}_1(t) = \left\langle t, t^2\right\rangle$ on $[0, 1]$; $t_0 = 1$
$\vec{r}_2(s) = \left\langle s^2, s^4\right\rangle$ on $[0, 1]$; $s_0 = 1$

11 03 ex 26 26. $\vec{r}_1(t) = \langle 3\cos t, 3\sin t\rangle$ on $[0, 2\pi]$; $t_0 = \pi/2$
$\vec{r}_2(s) = \langle 3\cos(4s), 3\sin(4s)\rangle$ on $[0, \pi/2]$; $s_0 = \pi/8$

11 03 ex 27 27. $\vec{r}_1(t) = \langle 3t, 2t\rangle$ on $[0, 2]$; $t_0 = 2$
$\vec{r}_2(s) = \langle 6t-6, 4t-4\rangle$ on $[1, 2]$; $s_0 = 2$

11 03 ex 28 28. $\vec{r}_1(t) = \left\langle t, \sqrt{t}\right\rangle$ on $[0, 1]$; $t_0 = 1$
$\vec{r}_2(s) = \left\langle \sin t, \sqrt{\sin t}\right\rangle$ on $[0, \pi/2]$; $s_0 = \pi/2$

11 03 exset 05 **In Exercises 29 – 32 , find the position function of an object given its acceleration and initial velocity and position.**

11 03 ex 29 29. $\vec{a}(t) = \langle 2, 3\rangle$; $\quad \vec{v}(0) = \langle 1, 2\rangle$, $\quad \vec{r}(0) = \langle 5, -2\rangle$

11 03 ex 30 30. $\vec{a}(t) = \langle 2, 3\rangle$; $\quad \vec{v}(1) = \langle 1, 2\rangle$, $\quad \vec{r}(1) = \langle 5, -2\rangle$

11 03 ex 31 31. $\vec{a}(t) = \langle \cos t, -\sin t\rangle$; $\quad \vec{v}(0) = \langle 0, 1\rangle$, $\quad \vec{r}(0) = \langle 0, 0\rangle$

11 03 ex 32 32. $\vec{a}(t) = \langle 0, -32\rangle$; $\quad \vec{v}(0) = \langle 10, 50\rangle$, $\quad \vec{r}(0) = \langle 0, 0\rangle$

11 03 exset 06 **In Exercises 33 – 36 , find the displacement, distance traveled, average velocity and average speed of the described object on the given interval.**

11 03 ex 39 33. An object with position function $\vec{r}(t) = \langle 2\cos t, 2\sin t, 3t\rangle$, where distances are measured in feet and time is in seconds, on $[0, 2\pi]$.

11 03 ex 41 34. An object with position function $\vec{r}(t) = \langle 5\cos t, -5\sin t\rangle$, where distances are measured in feet and time is in seconds, on $[0, \pi]$.

11 03 ex 40 35. An object with velocity function $\vec{v}(t) = \langle \cos t, \sin t\rangle$, where distances are measured in feet and time is in seconds, on $[0, 2\pi]$.

11 03 ex 42 36. An object with velocity function $\vec{v}(t) = \langle 1, 2, -1\rangle$, where distances are measured in feet and time is in seconds, on $[0, 10]$.

11 03 exset 07 **Exercises 37 – 42 ask you to solve a variety of problems based on the principles of projectile motion.**

11 03 ex 33 37. A boy whirls a ball, attached to a 3ft string, above his head in a counter–clockwise circle. The ball makes 2 revolutions per second.

At what *t*-values should the boy release the string so that the ball heads directly for a tree standing 10ft in front of him?

11 03 ex 34 38. David faces Goliath with only a stone in a 3ft sling, which he whirls above his head at 4 revolutions per second. They stand 20ft apart.

(a) At what *t*-values must David release the stone in his sling in order to hit Goliath?

(b) What is the speed at which the stone is traveling when released?

(c) Assume David releases the stone from a height of 6ft and Goliath's forehead is 9ft above the ground. What angle of elevation must David apply to the stone to hit Goliath's head?

11 03 ex 35 39. A hunter aims at a deer which is 40 yards away. Her crossbow is at a height of 5ft, and she aims for a spot on the deer 4ft above the ground. The crossbow fires her arrows at 300ft/s.

(a) At what angle of elevation should she hold the crossbow to hit her target?

(b) If the deer is moving perpendicularly to her line of sight at a rate of 20mph, by approximately how much should she lead the deer in order to hit it in the desired location?

11 03 ex 36 40. A baseball player hits a ball at 100mph, with an initial height of 3ft and an angle of elevation of $20^\circ$, at Boston's Fenway Park. The ball flies towards the famed "Green Monster," a wall 37ft high located 310ft from home plate.

(a) Show that as hit, the ball hits the wall.

(b) Show that if the angle of elevation is $21^\circ$, the ball clears the Green Monster.

11 03 ex 37 41. A Cessna flies at 1000ft at 150mph and drops a box of supplies to the professor (and his wife) on an island. Ignoring wind resistance, how far horizontally will the supplies travel before they land?

11 03 ex 38 42. A football quarterback throws a pass from a height of 6ft, intending to hit his receiver 20yds away at a height of 5ft.

(a) If the ball is thrown at a rate of 50mph, what angle of elevation is needed to hit his intended target?

(b) If the ball is thrown at with an angle of elevation of $8^\circ$, what initial ball speed is needed to hit his target?

# Exercises 11.4

## Terms and Concepts

11 04 ex 01 1. If $\vec{T}(t)$ is a unit tangent vector, what is $\|\ \vec{T}(t)\ \|$?

11 04 ex 02 2. If $\vec{N}(t)$ is a unit normal vector, what is $\vec{N}(t)\cdot\vec{r}\,'(t)$?

11 04 ex 03 3. The acceleration vector $\vec{a}(t)$ lies in the plane defined by what two vectors?

11 04 ex 04 4. $a_{\text{T}}$ measures how much the acceleration is affecting the ________ of an object.

## Problems

11 04 exset 01 **In Exercises 5 – 8 , given $\vec{r}(t)$, find $\vec{T}(t)$ and evaluate it at the indicated value of $t$.**

11 04 ex 05 5. $\vec{r}(t)=\langle 2t^2, t^2-t\rangle, \quad t=1$

11 04 ex 06 6. $\vec{r}(t)=\langle t, \cos t\rangle, \quad t=\pi/4$

11 04 ex 07 7. $\vec{r}(t)=\langle \cos^3 t, \sin^3 t\rangle, \quad t=\pi/4$

11 04 ex 08 8. $\vec{r}(t)=\langle \cos t, \sin t\rangle, \quad t=\pi$

11 04 exset 02 **In Exercises 9 – 12 , find the equation of the line tangent to the curve at the indicated $t$-value using the unit tangent vector. Note: these are the same problems as in Exercises 5 – 8.**

11 04 ex 09 9. $\vec{r}(t)=\langle 2t^2, t^2-t\rangle, \quad t=1$

11 04 ex 10 10. $\vec{r}(t)=\langle t, \cos t\rangle, \quad t=\pi/4$

11 04 ex 11 11. $\vec{r}(t)=\langle \cos^3 t, \sin^3 t\rangle, \quad t=\pi/4$

11 04 ex 12 12. $\vec{r}(t)=\langle \cos t, \sin t\rangle, \quad t=\pi$

11 04 exset 03 **In Exercises 13 – 16 , find $\vec{N}(t)$ using Definition 75. Confirm the result using Theorem 97.**

11 04 ex 13 13. $\vec{r}(t)=\langle 3\cos t, 3\sin t\rangle$

11 04 ex 14 14. $\vec{r}(t)=\langle t, t^2\rangle$

11 04 ex 15 15. $\vec{r}(t)=\langle \cos t, 2\sin t\rangle$

11 04 ex 16 16. $\vec{r}(t)=\langle e^t, e^{-t}\rangle$

11 04 exset 06 **In Exercises 17 – 20 , a position function $\vec{r}(t)$ is given along with its unit tangent vector $\vec{T}(t)$ evaluated at $t=a$, for some value of $a$.**

**(a) Confirm that $\vec{T}(a)$ is as stated.**

**(b) Using a graph of $\vec{r}(t)$ and Theorem 97, find $\vec{N}(a)$.**

11 04 ex 27 17. $\vec{r}(t)=\langle 3\cos t, 5\sin t\rangle; \quad \vec{T}(\pi/4)=\left\langle -\frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}}\right\rangle.$

11 04 ex 28 18. $\vec{r}(t)=\left\langle t, \frac{1}{t^2+1}\right\rangle; \quad \vec{T}(1)=\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\rangle.$

11 04 ex 29 19. $\vec{r}(t)=(1+2\sin t)\langle \cos t, \sin t\rangle; \quad \vec{T}(0)=\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle.$

11 04 ex 30 20. $\vec{r}(t)=\langle \cos^3 t, \sin^3 t\rangle; \quad \vec{T}(\pi/4)=\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle.$

11 04 exset 04 **In Exercises 21 – 24 , find $\vec{N}(t)$.**

11 04 ex 17 21. $\vec{r}(t)=\langle 4t, 2\sin t, 2\cos t\rangle$

11 04 ex 18 22. $\vec{r}(t)=\langle 5\cos t, 3\sin t, 4\sin t\rangle$

11 04 ex 19 23. $\vec{r}(t)=\langle a\cos t, a\sin t, bt\rangle; \quad a>0$

11 04 ex 20 24. $\vec{r}(t)=\langle \cos(at), \sin(at), t\rangle$

11 04 exset 05 **In Exercises 25 – 30 , find $a_{\text{T}}$ and $a_{\text{N}}$ given $\vec{r}(t)$. Sketch $\vec{r}(t)$ on the indicated interval, and comment on the relative sizes of $a_{\text{T}}$ and $a_{\text{N}}$ at the indicated $t$ values.**

11 04 ex 21 25. $\vec{r}(t)=\langle t, t^2\rangle$ on $[-1,1]$; consider $t=0$ and $t=1$.

11 04 ex 22 26. $\vec{r}(t)=\langle t, 1/t\rangle$ on $(0,4]$; consider $t=1$ and $t=2$.

11 04 ex 23 27. $\vec{r}(t)=\langle 2\cos t, 2\sin t\rangle$ on $[0,2\pi]$; consider $t=0$ and $t=\pi/2$.

11 04 ex 24 28. $\vec{r}(t)=\langle \cos(t^2), \sin(t^2)\rangle$ on $(0,2\pi]$; consider $t=\sqrt{\pi/2}$ and $t=\sqrt{\pi}$.

11 04 ex 25 29. $\vec{r}(t)=\langle a\cos t, a\sin t, bt\rangle$ on $[0,2\pi]$, where $a,b>0$; consider $t=0$ and $t=\pi/2$.

11 04 ex 26 30. $\vec{r}(t)=\langle 5\cos t, 4\sin t, 3\sin t\rangle$ on $[0,2\pi]$; consider $t=0$ and $t=\pi/2$.

# Exercises 11.5

## *Terms and Concepts*

11 05 ex 01 1. It is common to describe position in terms of both ________ and/or ________.

11 05 ex 02 2. A measure of the "curviness" of a curve is ________.

11 05 ex 03 3. Give two shapes with constant curvature.

11 05 ex 04 4. Describe in your own words what an "osculating circle" is.

11 05 ex 05 5. Complete the identity: $\vec{T}\,'(s) =$ ________ $\vec{N}(s)$.

11 05 ex 06 6. Given a position function $\vec{r}(t)$, how are $a_T$ and $a_N$ affected by the curvature?

## *Problems*

11 05 exset 01 **In Exercises 7 – 10 , a position function $\vec{r}(t)$ is given, where $t = 0$ corresponds to the initial position. Find the arc length parameter $s$, and rewrite $\vec{r}(t)$ in terms of $s$; that is, find $\vec{r}(s)$.**

11 05 ex 07 7. $\vec{r}(t) = \langle 2t, t, -2t \rangle$

11 05 ex 08 8. $\vec{r}(t) = \langle 7\cos t, 7\sin t \rangle$

11 05 ex 09 9. $\vec{r}(t) = \langle 3\cos t, 3\sin t, 2t \rangle$

11 05 ex 10 10. $\vec{r}(t) = \langle 5\cos t, 13\sin t, 12\cos t \rangle$

11 05 exset 02 **In Exercises 11 – 22 , a curve $C$ is described along with 2 points on $C$.**

**(a) Using a sketch, determine at which of these points the curvature is greater.**

**(b) Find the curvature $\kappa$ of $C$, and evaluate $\kappa$ at each of the 2 given points.**

11 05 ex 11 11. $C$ is defined by $y = x^3 - x$; points given at $x = 0$ and $x = 1/2$.

11 05 ex 12 12. $C$ is defined by $y = \dfrac{1}{x^2+1}$; points given at $x = 0$ and $x = 2$.

11 05 ex 14 13. $C$ is defined by $y = \cos x$; points given at $x = 0$ and $x = \pi/2$.

11 05 ex 13 14. $C$ is defined by $y = \sqrt{1-x^2}$ on $(-1, 1)$; points given at $x = 0$ and $x = 1/2$.

11 05 ex 15 15. $C$ is defined by $\vec{r}(t) = \langle \cos t, \sin(2t) \rangle$; points given at $t = 0$ and $t = \pi/4$.

11 05 ex 16 16. $C$ is defined by $\vec{r}(t) = \langle \cos^2 t, \sin t\cos t \rangle$; points given at $t = 0$ and $t = \pi/3$.

11 05 ex 17 17. $C$ is defined by $\vec{r}(t) = \langle t^2 - 1, t^3 - t \rangle$; points given at $t = 0$ and $t = 5$.

11 05 ex 18 18. $C$ is defined by $\vec{r}(t) = \langle \tan t, \sec t \rangle$; points given at $t = 0$ and $t = \pi/6$.

11 05 ex 22 19. $C$ is defined by $\vec{r}(t) = \langle 4t+2, 3t-1, 2t+5 \rangle$; points given at $t = 0$ and $t = 1$.

11 05 ex 21 20. $C$ is defined by $\vec{r}(t) = \langle t^3 - t, t^3 - 4, t^2 - 1 \rangle$; points given at $t = 0$ and $t = 1$.

11 05 ex 19 21. $C$ is defined by $\vec{r}(t) = \langle 3\cos t, 3\sin t, 2t \rangle$; points given at $t = 0$ and $t = \pi/2$.

11 05 ex 20 22. $C$ is defined by $\vec{r}(t) = \langle 5\cos t, 13\sin t, 12\cos t \rangle$; points given at $t = 0$ and $t = \pi/2$.

11 05 exset 03 **In Exercises 23 – 26 , find the value of $x$ or $t$ where curvature is maximized.**

11 05 ex 24 23. $y = \dfrac{1}{6}x^3$

11 05 ex 25 24. $y = \sin x$

11 05 ex 23 25. $\vec{r}(t) = \langle t^2 + 2t, 3t - t^2 \rangle$

11 05 ex 26 26. $\vec{r}(t) = \langle t, 4/t, 3/t \rangle$

11 05 exset 04 **In Exercises 27 – 30 , find the radius of curvature at the indicated value.**

11 05 ex 27 27. $y = \tan x$, at $x = \pi/4$

11 05 ex 28 28. $y = x^2 + x - 3$, at $x = \pi/4$

11 05 ex 29 29. $\vec{r}(t) = \langle \cos t, \sin(3t) \rangle$, at $t = 0$

11 05 ex 30 30. $\vec{r}(t) = \langle 5\cos(3t), t \rangle$, at $t = 0$

11 05 exset 05 **In Exercises 31 – 34 , find the equation of the osculating circle to the curve at the indicated $t$-value.**

11 05 ex 31 31. $\vec{r}(t) = \langle t, t^2 \rangle$, at $t = 0$

11 05 ex 32 32. $\vec{r}(t) = \langle 3\cos t, \sin t \rangle$, at $t = 0$

11 05 ex 33 33. $\vec{r}(t) = \langle 3\cos t, \sin t \rangle$, at $t = \pi/2$

11 05 ex 34 34. $\vec{r}(t) = \langle t^2 - t, t^2 + t \rangle$, at $t = 0$

# Exercises 12.1

## *Terms and Concepts*

12 01 ex 01 1. Give two examples (other than those given in the text) of "real world" functions that require more than one input.

12 01 ex 02 2. The graph of a function of two variables is a ________.

12 01 ex 03 3. Most people are familiar with the concept of level curves in the context of ________ maps.

12 01 ex 04 4. T/F: Along a level curve, the output of a function does not change.

12 01 ex 05 5. The analogue of a level curve for functions of three variables is a level ________.

12 01 ex 06 6. What does it mean when level curves are close together? Far apart?

## *Problems*

12 01 exset 01 **In Exercises 7 – 14, give the domain and range of the multivariable function.**

12 01 ex 07 7. $f(x,y)=x^2+y^2+2$

12 01 ex 08 8. $f(x,y)=x+2y$

12 01 ex 09 9. $f(x,y)=x-2y$

12 01 ex 10 10. $f(x,y)=\dfrac{1}{x+2y}$

12 01 ex 11 11. $f(x,y)=\dfrac{1}{x^2+y^2+1}$

12 01 ex 12 12. $f(x,y)=\sin x\cos y$

12 01 ex 13 13. $f(x,y)=\sqrt{9-x^2-y^2}$

12 01 ex 14 14. $f(x,y)=\dfrac{1}{\sqrt{x^2+y^2-9}}$

12 01 exset 02 **In Exercises 15 – 22, describe in words and sketch the level curves for the function and given $c$ values.**

12 01 ex 15 15. $f(x,y)=3x-2y;\ c=-2,0,2$

12 01 ex 16 16. $f(x,y)=x^2-y^2;\ c=-1,0,1$

12 01 ex 17 17. $f(x,y)=x-y^2;\ c=-2,0,2$

12 01 ex 18 18. $f(x,y)=\dfrac{1-x^2-y^2}{2y-2x};\ c=-2,0,2$

12 01 ex 19 19. $f(x,y)=\dfrac{2x-2y}{x^2+y^2+1};\ c=-1,0,1$

12 01 ex 20 20. $f(x,y)=\dfrac{y-x^3-1}{x};\ c=-3,-1,0,1,3$

12 01 ex 21 21. $f(x,y)=\sqrt{x^2+4y^2};\ c=1,2,3,4$

12 01 ex 22 22. $f(x,y)=x^2+4y^2;\ c=1,2,3,4$

12 01 exset 03 **In Exercises 23 – 26, give the domain and range of the functions of three variables.**

12 01 ex 24 23. $f(x,y,z)=\dfrac{x}{x+2y-4z}$

12 01 ex 25 24. $f(x,y,z)=\dfrac{1}{1-x^2-y^2-z^2}$

12 01 ex 26 25. $f(x,y,z)=\sqrt{z-x^2+y^2}$

12 01 ex 27 26. $f(x,y,z)=z^2\sin x\cos y$

12 01 exset 04 **In Exercises 27 – 30, describe the level surfaces of the given functions of three variables.**

12 01 ex 28 27. $f(x,y,z)=x^2+y^2+z^2$

12 01 ex 29 28. $f(x,y,z)=z-x^2+y^2$

12 01 ex 30 29. $f(x,y,z)=\dfrac{x^2+y^2}{z}$

12 01 ex 31 30. $f(x,y,z)=\dfrac{z}{x-y}$

12 01 ex 23 31. Compare the level curves of Exercises 21 and 22. How are they similar, and how are they different? Each surface is a quadric surface; describe how the level curves are consistent with what we know about each surface.

# Exercises 12.2

## *Terms and Concepts*

12 02 ex 01 1. Describe in your own words the difference between boundary and interior point of a set.

12 02 ex 02 2. Use your own words to describe (informally) what $\displaystyle\lim_{(x,y)\to(1,2)} f(x,y) = 17$ means.

12 02 ex 17 3. Give an example of a closed, bounded set.

12 02 ex 18 4. Give an example of a closed, unbounded set.

12 02 ex 19 5. Give an example of a open, bounded set.

12 02 ex 20 6. Give an example of a open, unbounded set.

## *Problems*

12 02 exset 01 **In Exercises 7 – 10, a set $S$ is given.**

**(a) Give one boundary point and one interior point, when possible, of $S$.**

**(b) State whether $S$ is open, closed, or neither.**

**(c) State whether $S$ is bounded or unbounded.**

12 02 ex 03 7. $S = \left\{(x,y) \;\middle|\; \dfrac{(x-1)^2}{4} + \dfrac{(y-3)^2}{9} \leq 1\right\}$

12 02 ex 04 8. $S = \left\{(x,y) \mid y \neq x^2\right\}$

12 02 ex 05 9. $S = \left\{(x,y) \mid x^2 + y^2 = 1\right\}$

12 02 ex 06 10. $S = \left\{(x,y) | y > \sin x\right\}$

12 02 exset 02 **In Exercises 11 – 14:**

**(a) Find the domain $D$ of the given function.**

**(b) State whether $D$ is an open or closed set.**

**(c) State whether $D$ is bounded or unbounded.**

12 02 ex 09 11. $f(x,y) = \sqrt{9 - x^2 - y^2}$

12 02 ex 07 12. $f(x,y) = \sqrt{y - x^2}$

12 02 ex 08 13. $f(x,y) = \dfrac{1}{\sqrt{y - x^2}}$

12 02 ex 10 14. $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$

12 02 exset 03 **In Exercises 15 – 20, a limit is given. Evaluate the limit along the paths given, then state why these results show the given limit does not exist.**

12 02 ex 12 15. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

(a) Along the path $y = 0$.

(b) Along the path $x = 0$.

12 02 ex 11 16. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x + y}{x - y}$

(a) Along the path $y = mx$.

12 02 ex 13 17. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy - y^2}{y^2 + x}$

(a) Along the path $y = mx$.

(b) Along the path $x = 0$.

12 02 ex 14 18. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y}$

(a) Along the path $y = mx$.

(b) Along the path $y = x^2$.

12 02 ex 15 19. $\displaystyle\lim_{(x,y)\to(1,2)} \frac{x + y - 3}{x^2 - 1}$

(a) Along the path $y = 2$.

(b) Along the path $y = x + 1$.

12 02 ex 16 20. $\displaystyle\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y}$

(a) Along the path $x = \pi$.

(b) Along the path $y = x - \pi/2$.

# Exercises 12.3

## Terms and Concepts

12 03 ex 01 1. What is the difference between a constant and a coefficient?

12 03 ex 02 2. Given a function $z = f(x, y)$, explain in your own words how to compute $f_x$.

12 03 ex 03 3. In the mixed partial fraction $f_{xy}$, which is computed first, $f_x$ or $f_y$?

12 03 ex 04 4. In the mixed partial fraction $\dfrac{\partial^2 f}{\partial x \partial y}$, which is computed first, $f_x$ or $f_y$?

## Problems

12 03 exset 01 **In Exercises 5 – 8, evaluate $f_x(x, y)$ and $f_y(x, y)$ at the indicated point.**

12 03 ex 05 5. $f(x, y) = x^2y - x + 2y + 3$ at $(1, 2)$

12 03 ex 06 6. $f(x, y) = x^3 - 3x + y^2 - 6y$ at $(-1, 3)$

12 03 ex 07 7. $f(x, y) = \sin y \cos x$ at $(\pi/3, \pi/3)$

12 03 ex 08 8. $f(x, y) = \ln(xy)$ at $(-2, -3)$

12 03 exset 02 **In Exercises 9 – 26, find $f_x$, $f_y$, $f_{xx}$, $f_{yy}$, $f_{xy}$ and $f_{yx}$.**

12 03 ex 09 9. $f(x, y) = x^2y + 3x^2 + 4y - 5$

12 03 ex 10 10. $f(x, y) = y^3 + 3xy^2 + 3x^2y + x^3$

12 03 ex 13 11. $f(x, y) = \dfrac{x}{y}$

12 03 ex 14 12. $f(x, y) = \dfrac{4}{xy}$

12 03 ex 15 13. $f(x, y) = e^{x^2+y^2}$

12 03 ex 16 14. $f(x, y) = e^{x+2y}$

12 03 ex 17 15. $f(x, y) = \sin x \cos y$

12 03 ex 18 16. $f(x, y) = (x + y)^3$

12 03 ex 11 17. $f(x, y) = \cos(5xy^3)$

12 03 ex 12 18. $f(x, y) = \sin(5x^2 + 2y^3)$

12 03 ex 19 19. $f(x, y) = \sqrt{4xy^2 + 1}$

12 03 ex 20 20. $f(x, y) = (2x + 5y)\sqrt{y}$

12 03 ex 21 21. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$

12 03 ex 22 22. $f(x, y) = 5x - 17y$

12 03 ex 23 23. $f(x, y) = 3x^2 + 1$

12 03 ex 24 24. $f(x, y) = \ln(x^2 + y)$

12 03 ex 25 25. $f(x, y) = \dfrac{\ln x}{4y}$

12 03 ex 26 26. $f(x, y) = 5e^x \sin y + 9$

12 03 exset 03 **In Exercises 27 – 30, form a function $z = f(x, y)$ such that $f_x$ and $f_y$ match those given.**

12 03 ex 27 27. $f_x = \sin y + 1, \quad f_y = x \cos y$

12 03 ex 28 28. $f_x = x + y, \quad f_y = x + y$

12 03 ex 29 29. $f_x = 6xy - 4y^2, \quad f_y = 3x^2 - 8xy + 2$

12 03 ex 30 30. $f_x = \dfrac{2x}{x^2 + y^2}, \quad f_y = \dfrac{2y}{x^2 + y^2}$

12 03 exset 04 **In Exercises 31 – 34, find $f_x$, $f_y$, $f_z$, $f_{yz}$ and $f_{zy}$.**

12 03 ex 31 31. $f(x, y, z) = x^2e^{2y-3z}$

12 03 ex 32 32. $f(x, y, z) = x^3y^2 + x^3z + y^2z$

12 03 ex 33 33. $f(x, y, z) = \dfrac{3x}{7y^2z}$

12 03 ex 34 34. $f(x, y, z) = \ln(xyz)$

# Exercises 12.4

## Terms and Concepts

12 04 ex 01 1. T/F: If $f(x,y)$ is differentiable on $S$, the $f$ is continuous on $S$.

12 04 ex 02 2. T/F: If $f_x$ and $f_y$ are continuous on $S$, then $f$ is differentiable on $S$.

12 04 ex 03 3. T/F: If $z = f(x,y)$ is differentiable, then the change in $z$ over small changes $dx$ and $dy$ in $x$ and $y$ is approximately $dz$.

12 04 ex 04 4. Finish the sentence: "The new $z$-value is approximately the old $z$-value plus the approximate ________."

## Problems

12 04 exset 01 **In Exercises 5 – 8, find the total differential $dz$.**

12 04 ex 05 5. $z = x\sin y + x^2$

12 04 ex 06 6. $z = (2x^2 + 3y)^2$

12 04 ex 07 7. $z = 5x - 7y$

12 04 ex 08 8. $z = xe^{x+y}$

12 04 exset 02 **In Exercises 9 – 12, a function $z = f(x,y)$ is given. Give the indicated approximation using the total differential.**

12 04 ex 09 9. $f(x,y) = \sqrt{x^2 + y}$. Approximate $f(2.95, 7.1)$ knowing $f(3,7) = 4$.

12 04 ex 10 10. $f(x,y) = \sin x\cos y$. Approximate $f(0.1, -0.1)$ knowing $f(0,0) = 0$.

12 04 ex 11 11. $f(x,y) = x^2y - xy^2$. Approximate $f(2.04, 3.06)$ knowing $f(2,3) = -6$.

12 04 ex 12 12. $f(x,y) = \ln(x-y)$. Approximate $f(5.1, 3.98)$ knowing $f(5,4) = 0$.

12 04 exset 04 **Exercises 13 – 16 ask a variety of questions dealing with approximating error and sensitivity analysis.**

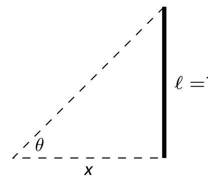
12 04 ex 15 13. A cylindrical storage tank is to be 2ft tall with a radius of 1ft. Is the volume of the tank more sensitive to changes in the radius or the height?

12 04 ex 16 14. **Projectile Motion:** The $x$-value of an object moving under the principles of projectile motion is $x(\theta, v_0, t) = (v_0\cos\theta)t$. A particular projectile is fired with an initial velocity of $v_0 = 250$ft/s and an angle of elevation of $\theta = 60^\circ$. It travels a distance of 375ft in 3 seconds.

Is the projectile more sensitive to errors in initial speed or angle of elevation?

12 04 ex 17 15. The length $\ell$ of a long wall is to be approximated. The angle $\theta$, as shown in the diagram (not to scale), is measured to be 85°, and the distance $x$ is measured to be 30'. Assume that the triangle formed is a right triangle.

Is the measurement of the length of $\ell$ more sensitive to errors in the measurement of $x$ or in $\theta$?




12 04 ex 18 16. It is "common sense" that it is far better to measure a long distance with a long measuring tape rather than a short one. A measured distance $D$ can be viewed as the product of the length $\ell$ of a measuring tape times the number $n$ of times it was used. For instance, using a 3' tape 10 times gives a length of 30'. To measure the same distance with a 12' tape, we would use the tape 2.5 times. (I.e., $30 = 12 \times 2.5$.) Thus $D = n\ell$.

Suppose each time a measurement is taken with the tape, the recorded distance is within 1/16" of the actual distance. (I.e., $d\ell = 1/16'' \approx 0.005$ft). Using differentials, show why common sense proves correct in that it is better to use a long tape to measure long distances.

12 04 exset 05 **In Exercises 17 – 18, find the total differential $dw$.**

12 04 ex 19 17. $w = x^2yz^3$

12 04 ex 20 18. $w = e^x\sin y\ln z$

12 04 exset 03 **In Exercises 19 – 22, use the information provided and the total differential to make the given approximation.**

12 04 ex 13 19. $f(3,1) = 7$, $f_x(3,1) = 9$, $f_y(3,1) = -2$. Approximate $f(3.05, 0.9)$.

12 04 ex 14 20. $f(-4,2) = 13$, $f_x(-4,2) = 2.6$, $f_y(-4,2) = 5.1$. Approximate $f(-4.12, 2.07)$.

12 04 ex 21 21. $f(2,4,5) = -1$, $f_x(2,4,5) = 2$, $f_y(2,4,5) = -3$, $f_z(2,4,5) = 3.7$. Approximate $f(2.5, 4.1, 4.8)$.

12 04 ex 22 22. $f(3,3,3) = 5$, $f_x(3,3,3) = 2$, $f_y(3,3,3) = 0$, $f_z(3,3,3) = -2$. Approximate $f(3.1, 3.1, 3.1)$.

# Exercises 12.5

## *Terms and Concepts*

12 05 ex 01 1. What is the difference between a directional derivative and a partial derivative?

12 05 ex 02 2. For what $\vec{u}$ is $D_{\vec{u}}f = f_x$?

12 05 ex 03 3. For what $\vec{u}$ is $D_{\vec{u}}f = f_y$?

12 05 ex 04 4. The gradient is ________ to level curves.

12 05 ex 05 5. The gradient points in the direction of ________ increase.

12 05 ex 06 6. It is generally more informative to view the directional derivative not as the result of a limit, but rather as the result of a ________ product.

## *Problems*

12 05 exset 01 **In Exercises 7 – 12, a function $z = f(x, y)$ is given. Find $\nabla f$.**

12 05 ex 07 7. $f(x, y) = -x^2y + xy^2 + xy$

12 05 ex 08 8. $f(x, y) = \sin x \cos y$

12 05 ex 09 9. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$

12 05 ex 10 10. $f(x, y) = -4x + 3y$

12 05 ex 19 11. $f(x, y) = x^2 + 2y^2 - xy - 7x$

12 05 ex 22 12. $f(x, y) = x^2y^3 - 2x$

12 05 exset 02 **In Exercises 13 – 18, a function $z = f(x, y)$ and a point $P$ are given. Find the directional derivative of $f$ in the indicated directions. Note: these are the same functions as in Exercises 7 through 12.**

12 05 ex 11 13. $f(x, y) = -x^2y + xy^2 + xy$, $P = (2, 1)$

(a) In the direction of $\vec{v} = \langle 3, 4\rangle$

(b) In the direction toward the point $Q = (1, -1)$.

12 05 ex 12 14. $f(x, y) = \sin x \cos y$, $P = \left(\dfrac{\pi}{4}, \dfrac{\pi}{3}\right)$

(a) In the direction of $\vec{v} = \langle 1, 1\rangle$.

(b) In the direction toward the point $Q = (0, 0)$.

12 05 ex 13 15. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$, $P = (1, 1)$.

(a) In the direction of $\vec{v} = \langle 1, -1\rangle$.

(b) In the direction toward the point $Q = (-2, -2)$.

12 05 ex 14 16. $f(x, y) = -4x + 3y$, $P = (5, 2)$

(a) In the direction of $\vec{v} = \langle 3, 1\rangle$ .

(b) In the direction toward the point $Q = (2, 7)$.

12 05 ex 20 17. $f(x, y) = x^2 + 2y^2 - xy - 7x$, $P = (4, 1)$

(a) In the direction of $\vec{v} = \langle -2, 5\rangle$

(b) In the direction toward the point $Q = (4, 0)$.

12 05 ex 23 18. $f(x, y) = x^2y^3 - 2x$, $P = (1, 1)$

(a) In the direction of $\vec{v} = \langle 3, 3\rangle$

(b) In the direction toward the point $Q = (1, 2)$.

12 05 exset 03 **In Exercises 19 – 24, a function $z = f(x, y)$ and a point $P$ are given.**

**(a) Find the direction of maximal increase of $f$ at $P$.**

**(b) What is the maximal value of $D_{\vec{u}}f$ at $P$?**

**(c) Find the direction of minimal increase of $f$ at $P$.**

**(d) Give a direction $\vec{u}$ such that $D_{\vec{u}}f = 0$ at $P$.**

**Note: these are the same functions and points as in Exercises 13 through 18.**

12 05 ex 15 19. $f(x, y) = -x^2y + xy^2 + xy$, $P = (2, 1)$

12 05 ex 16 20. $f(x, y) = \sin x \cos y$, $P = \left(\dfrac{\pi}{4}, \dfrac{\pi}{3}\right)$

12 05 ex 17 21. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$, $P = (1, 1)$.

12 05 ex 18 22. $f(x, y) = -4x + 3y$, $P = (5, 4)$.

12 05 ex 21 23. $f(x, y) = x^2 + 2y^2 - xy - 7x$, $P = (4, 1)$

12 05 ex 24 24. $f(x, y) = x^2y^3 - 2x$, $P = (1, 1)$

12 05 exset 04 **In Exercises 25 – 28, a function $w = F(x, y, z)$, a vector $\vec{v}$ and a point $P$ are given.**

**(a) Find $\nabla F(x, y, z)$.**

**(b) Find $D_{\vec{u}}F$ at $P$.**

12 05 ex 25 25. $F(x, y, z) = 3x^2z^3 + 4xy - 3z^2$, $\vec{v} = \langle 1, 1, 1\rangle$, $P = (3, 2, 1)$

12 05 ex 26 26. $F(x, y, z) = \sin(x)\cos(y)e^z$, $\vec{v} = \langle 2, 2, 1\rangle$, $P = (0, 0, 0)$

12 05 ex 27 27. $F(x, y, z) = x^2y^2 - y^2z^2$, $\vec{v} = \langle -1, 7, 3\rangle$, $P = (1, 0, -1)$

12 05 ex 28 28. $F(x, y, z) = \dfrac{2}{x^2 + y^2 + z^2}$, $\vec{v} = \langle 1, 1, -2\rangle$, $P = (1, 1, 1)$

# Exercises 12.6

## Terms and Concepts

12 06 ex 01 1. Explain how the vector $\vec{v} = \langle 1, 0, 3\rangle$ can be thought of as having a "slope" of 3.

12 06 ex 02 2. Explain how the vector $\vec{v} = \langle 0.6, 0.8, -2\rangle$ can be thought of as having a "slope" of $-2$.

12 06 ex 03 3. T/F: Let $z = f(x, y)$ be differentiable at $P$. If $\vec{n}$ is a normal vector to the tangent plane of $f$ at $P$, then $\vec{n}$ is orthogonal to $f_x$ and $f_y$ at $P$.

12 06 ex 04 4. Explain in your own words why we do not refer to *the* tangent line to a surface at a point, but rather to *directional* tangent line*s* to a surface at a point.

## Problems

12 06 exset 01 **In Exercises 5 – 8, a function $z = f(x, y)$, a vector $\vec{v}$ and a point $P$ are given. Give the parametric equations of the following directional tangent lines to $f$ at $P$:**

**(a)** $\ell_x(t)$

**(b)** $\ell_y(t)$

**(c)** $\ell_{\vec{u}}(t)$**, where $\vec{u}$ is the unit vector in the direction of $\vec{v}$.**

12 06 ex 05 5. $f(x, y) = 2x^2y - 4xy^2$, $\vec{v} = \langle 1, 3\rangle$, $P = (2, 3)$.

12 06 ex 06 6. $f(x, y) = 3\cos x \sin y$, $\vec{v} = \langle 1, 2\rangle$, $P = (\pi/3, \pi/6)$.

12 06 ex 07 7. $f(x, y) = 3x - 5y$, $\vec{v} = \langle 1, 1\rangle$, $P = (4, 2)$.

12 06 ex 08 8. $f(x, y) = x^2 - 2x - y^2 + 4y$, $\vec{v} = \langle 1, 1\rangle$, $P = (1, 2)$.

12 06 exset 02 **In Exercises 9 – 12, a function $z = f(x, y)$ and a point $P$ are given. Find the equation of the normal line to $f$ at $P$. Note: these are the same functions as in Exercises 5 – 8.**

12 06 ex 09 9. $f(x, y) = 2x^2y - 4xy^2$, $P = (2, 3)$.

12 06 ex 10 10. $f(x, y) = 3\cos x \sin y$, $P = (\pi/3, \pi/6)$.

12 06 ex 11 11. $f(x, y) = 3x - 5y$, $P = (4, 2)$.

12 06 ex 12 12. $f(x, y) = x^2 - 2x - y^2 + 4y$, $P = (1, 2)$.

12 06 exset 03 **In Exercises 13 – 16, a function $z = f(x, y)$ and a point $P$ are given. Find the two points that are 2 units from the surface $f$ at $P$. Note: these are the same functions as in Exercises 5 – 8.**

12 06 ex 13 13. $f(x, y) = 2x^2y - 4xy^2$, $P = (2, 3)$.

12 06 ex 14 14. $f(x, y) = 3\cos x \sin y$, $P = (\pi/3, \pi/6)$.

12 06 ex 15 15. $f(x, y) = 3x - 5y$, $P = (4, 2)$.

12 06 ex 16 16. $f(x, y) = x^2 - 2x - y^2 + 4y$, $P = (1, 2)$.

12 06 exset 04 **In Exercises 17 – 20, a function $z = f(x, y)$ and a point $P$ are given. Find the equation of the tangent plane to $f$ at $P$. Note: these are the same functions as in Exercises 5 – 8.**

12 06 ex 17 17. $f(x, y) = 2x^2y - 4xy^2$, $P = (2, 3)$.

12 06 ex 18 18. $f(x, y) = 3\cos x \sin y$, $P = (\pi/3, \pi/6)$.

12 06 ex 19 19. $f(x, y) = 3x - 5y$, $P = (4, 2)$.

12 06 ex 20 20. $f(x, y) = x^2 - 2x - y^2 + 4y$, $P = (1, 2)$.

12 06 exset 05 **In Exercises 21 – 24, an implicitly defined function of $x$, $y$ and $z$ is given along with a point $P$ that lies on the surface. Use the gradient $\nabla F$ to:**

**(a) find the equation of the normal line to the surface at $P$, and**

**(b) find the equation of the plane tangent to the surface at $P$.**

12 06 ex 21 21. $\dfrac{x^2}{8} + \dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$, at $P = (1, \sqrt{2}, \sqrt{6})$

12 06 ex 22 22. $z^2 - \dfrac{x^2}{4} - \dfrac{y^2}{9} = 0$, at $P = (4, -3, \sqrt{5})$

12 06 ex 23 23. $xy^2 - xz^2 = 0$, at $P = (2, 1, -1)$

12 06 ex 24 24. $\sin(xy) + \cos(yz) = 0$, at $P = (2, \pi/12, 4)$

# Exercises 12.7

## *Terms and Concepts*

12 07 ex 01 1. T/F: Theorem 114 states that if $f$ has a critical point at $P$, then $f$ has a relative extrema at $P$.

12 07 ex 02 2. T/F: A point $P$ is a critical point of $f$ if $f_x$ and $f_y$ are both 0 at $P$.

12 07 ex 03 3. T/F: A point $P$ is a critical point of $f$ if $f_x$ or $f_y$ are undefined at $P$.

12 07 ex 04 4. Explain what it means to "solve a constrained optimization" problem.

## *Problems*

12 07 exset 01 **In Exercises 5 – 14, find the critical points of the given function. Use the Second Derivative Test to determine if each critical point corresponds to a relative maximum, minimum, or saddle point.**

12 07 ex 07 5. $f(x,y) = \frac{1}{2}x^2 + 2y^2 - 8y + 4x$

12 07 ex 05 6. $f(x,y) = x^2 + 4x + y^2 - 9y + 3xy$

12 07 ex 06 7. $f(x,y) = x^2 + 3y^2 - 6y + 4xy$

12 07 ex 08 8. $f(x,y) = \dfrac{1}{x^2 + y^2 + 1}$

12 07 ex 09 9. $f(x,y) = x^2 + y^3 - 3y + 1$

12 07 ex 12 10. $f(x,y) = \frac{1}{3}x^3 - x + \frac{1}{3}y^3 - 4y$

12 07 ex 11 11. $f(x,y) = x^2y^2$

12 07 ex 10 12. $f(x,y) = x^4 - 2x^2 + y^3 - 27y - 15$

12 07 ex 17 13. $f(x,y) = \sqrt{16 - (x-3)^2 - y^2}$

12 07 ex 18 14. $f(x,y) = \sqrt{x^2 + y^2}$

12 07 exset 02 **In Exercises 15 – 18, find the absolute maximum and minimum of the function subject to the given constraint.**

12 07 ex 13 15. $f(x,y) = x^2 + y^2 + y + 1$, constrained to the triangle with vertices $(0,1)$, $(-1,-1)$ and $(1,-1)$.

12 07 ex 14 16. $f(x,y) = 5x - 7y$, constrained to the region bounded by $y = x^2$ and $y = 1$.

12 07 ex 15 17. $f(x,y) = x^2 + 2x + y^2 + 2y$, constrained to the region bounded by the circle $x^2 + y^2 = 4$.

12 07 ex 16 18. $f(x,y) = 3y - 2x^2$, constrained to the region bounded by the parabola $y = x^2 + x - 1$ and the line $y = x$.

# Exercises 12.8

## Terms and Concepts

12 08 ex 01 1. Let a level curve of $z = f(x, y)$ be described by $x = g(t)$, $y = h(t)$. Explain why $\frac{dz}{dt} = 0$.

12 08 ex 02 2. Fill in the blank: The single variable Chain Rule states $\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)\cdot$________.

12 08 ex 03 3. Fill in the blank: The Multivariable Chain Rule states $\dfrac{df}{dt} = \dfrac{\partial f}{\partial x} \cdot$ ________ $+$ ________ $\cdot \dfrac{dy}{dt}$.

12 08 ex 04 4. If $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$, we can substitute and write $z$ as an explicit function of $t$.
T/F: Using the Multivariable Chain Rule to find $\frac{dz}{dt}$ is sometimes easier than first substituting and then taking the derivative.

12 08 ex 05 5. T/F: The Multivariable Chain Rule is only useful when all the related functions are known explicitly.

12 08 ex 06 6. The Multivariable Chain Rule allows us to compute implicit derivatives easily by just computing two ________ derivatives.

## Problems

12 08 exset 01 **In Exercises 7 – 12, functions $z = f(x, y)$, $x = g(t)$ and $y = h(t)$ are given.**

**(a) Use the Multivariable Chain Rule to compute $\dfrac{dz}{dt}$.**

**(b) Evaluate $\dfrac{dz}{dt}$ at the indicated $t$-value.**

12 08 ex 07 7. $z = 3x + 4y, \quad x = t^2, \quad y = 2t; \quad t = 1$

12 08 ex 19 8. $z = x^2 - y^2, \quad x = t, \quad y = t^2 - 1; \quad t = 1$

12 08 ex 20 9. $z = 5x + 2y, \quad x = 2\cos t + 1, \quad y = \sin t - 3; \quad t = \pi/4$

12 08 ex 09 10. $z = \dfrac{x}{y^2 + 1}, \quad x = \cos t, \quad y = \sin t; \quad t = \pi/2$

12 08 ex 10 11. $z = x^2 + 2y^2, \quad x = \sin t, \quad y = 3\sin t; \quad t = \pi/4$

12 08 ex 08 12. $z = \cos x \sin y, \quad x = \pi t, \quad y = 2\pi t + \pi/2; \quad t = 3$

12 08 exset 02 **In Exercises 13 – 18, functions $z = f(x, y)$, $x = g(t)$ and $y = h(t)$ are given. Find the values of $t$ where $\frac{dz}{dt} = 0$. Note: these are the same surfaces/curves as found in Exercises 7 – 12.**

12 08 ex 11 13. $z = 3x + 4y, \quad x = t^2, \quad y = 2t$

12 08 ex 21 14. $z = x^2 - y^2, \quad x = t, \quad y = t^2 - 1$

12 08 ex 22 15. $z = 5x + 2y, \quad x = 2\cos t + 1, \quad y = \sin t - 3$

12 08 ex 13 16. $z = \dfrac{x}{y^2 + 1}, \quad x = \cos t, \quad y = \sin t$

12 08 ex 14 17. $z = x^2 + 2y^2, \quad x = \sin t, \quad y = 3\sin t$

12 08 ex 12 18. $z = \cos x \sin y, \quad x = \pi t, \quad y = 2\pi t + \pi/2$

12 08 exset 03 **In Exercises 19 – 22, functions $z = f(x, y)$, $x = g(s, t)$ and $y = h(s, t)$ are given.**

**(a) Use the Multivariable Chain Rule to compute $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.**

**(b) Evaluate $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ at the indicated $s$ and $t$ values.**

12 08 ex 15 19. $z = x^2 y, \quad x = s - t, \quad y = 2s + 4t; \quad s = 1, t = 0$

12 08 ex 16 20. $z = \cos\left(\pi x + \frac{\pi}{2} y\right), \quad x = st^2, \quad y = s^2 t; \quad s = 1, t = 1$

12 08 ex 17 21. $z = x^2 + y^2, \quad x = s\cos t, \quad y = s\sin t; \quad s = 2, t = \pi/4$

12 08 ex 18 22. $z = e^{-(x^2+y^2)}, \quad x = t, \quad y = st^2; \quad s = 1, t = 1$

12 08 exset 04 **In Exercises 23 – 26, find $\dfrac{dy}{dx}$ using Implicit Differentiation and Theorem 109.**

12 08 ex 23 23. $x^2 \tan y = 50$

12 08 ex 24 24. $(3x^2 + 2y^3)^4 = 2$

12 08 ex 25 25. $\dfrac{x^2 + y}{x + y^2} = 17$

12 08 ex 26 26. $\ln(x^2 + xy + y^2) = 1$

12 08 exset 05 **In Exercises 27 – 30, find $\dfrac{dz}{dt}$, or $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$, using the supplied information.**

12 08 ex 27 27. $\dfrac{\partial z}{\partial x} = 2, \quad \dfrac{\partial z}{\partial y} = 1, \quad \dfrac{dx}{dt} = 4, \quad \dfrac{dy}{dt} = -5$

12 08 ex 28 28. $\dfrac{\partial z}{\partial x} = 1, \quad \dfrac{\partial z}{\partial y} = -3, \quad \dfrac{dx}{dt} = 6, \quad \dfrac{dy}{dt} = 2$

12 08 ex 29 29. $\dfrac{\partial z}{\partial x} = -4, \quad \dfrac{\partial z}{\partial y} = 9,$
$\dfrac{\partial x}{\partial s} = 5, \quad \dfrac{\partial x}{\partial t} = 7, \quad \dfrac{\partial y}{\partial s} = -2, \quad \dfrac{\partial y}{\partial t} = 6$

12 08 ex 30 30. $\dfrac{\partial z}{\partial x} = 2, \quad \dfrac{\partial z}{\partial y} = 1,$
$\dfrac{\partial x}{\partial s} = -2, \quad \dfrac{\partial x}{\partial t} = 3, \quad \dfrac{\partial y}{\partial s} = 2, \quad \dfrac{\partial y}{\partial t} = -1$

# Exercises 13.1

## *Terms and Concepts*

13 01 ex 01

1. When integrating $f_x(x, y)$ with respect to $x$, the constant of integration $C$ is really which: $C(x)$ or $C(y)$? What does this mean?

13 01 ex 02

2. Integrating an integral is called ________ ________.

13 01 ex 03

3. When evaluating an iterated integral, we integrate from ________ to ________, then from ________ to ________.

13 01 ex 04

4. One understanding of an iterated integral is that $\int_a^b \int_{g_1(x)}^{g_2(x)} dy\, dx$ gives the ________ of a plane region.

## *Problems*

13 01 exset 01

**In Exercises 5 – 10, evaluate the integral and subsequent iterated integral.**

13 01 ex 09

5.

   (a) $\displaystyle\int_2^5 \left(6x^2 + 4xy - 3y^2\right) dy$

   (b) $\displaystyle\int_{-3}^{-2} \int_2^5 \left(6x^2 + 4xy - 3y^2\right) dy\, dx$

13 01 ex 10

6.

   (a) $\displaystyle\int_0^{\pi} \left(2x\cos y + \sin x\right) dx$

   (b) $\displaystyle\int_0^{\pi/2} \int_0^{\pi} \left(2x\cos y + \sin x\right) dx\, dy$

13 01 ex 05

7.

   (a) $\displaystyle\int_1^{x} \left(x^2y - y + 2\right) dy$

   (b) $\displaystyle\int_0^{2} \int_1^{x} \left(x^2y - y + 2\right) dy\, dx$

13 01 ex 06

8.

   (a) $\displaystyle\int_y^{y^2} \left(x - y\right) dx$

   (b) $\displaystyle\int_{-1}^{1} \int_y^{y^2} \left(x - y\right) dx\, dy$

13 01 ex 07

9.

   (a) $\displaystyle\int_0^{y} \left(\cos x \sin y\right) dx$

   (b) $\displaystyle\int_0^{\pi} \int_0^{y} \left(\cos x \sin y\right) dx\, dy$

13 01 ex 08

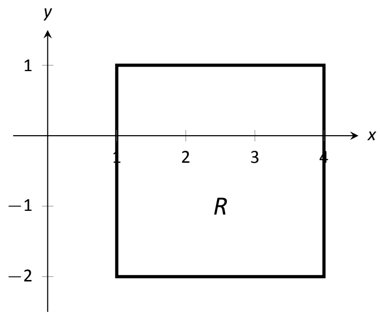
10.

   (a) $\displaystyle\int_0^{x} \left(\frac{1}{1+x^2}\right) dy$

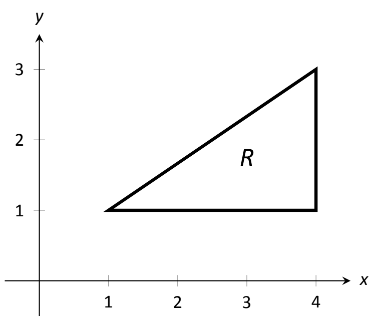
   (b) $\displaystyle\int_1^{2} \int_0^{x} \left(\frac{1}{1+x^2}\right) dy\, dx$

13 01 exset 02 **In Exercises 11 – 16, a graph of a planar region $R$ is given. Give the iterated integrals, with both orders of integration $dy\,dx$ and $dx\,dy$, that give the area of $R$. Evaluate one of the iterated integrals to find the area.**

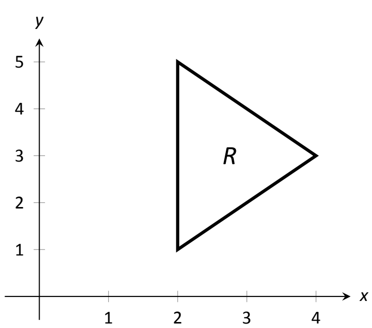
13 01 ex 11 11.



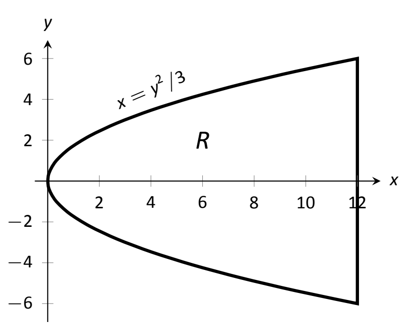

13 01 ex 12 12.



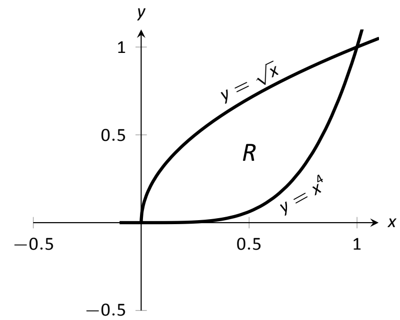

13 01 ex 13 13.



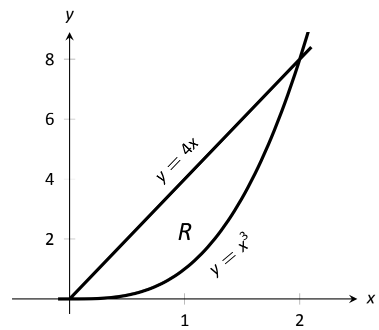

13 01 ex 14 14.




13 01 ex 15 15.




13 01 ex 16 16.




13 01 exset 03 **In Exercises 17 – 22, iterated integrals are given that compute the area of a region $R$ in the $x$-$y$ plane. Sketch the region $R$, and give the iterated integral(s) that give the area of $R$ with the opposite order of integration.**

13 01 ex 18 17. $\displaystyle\int_{-2}^{2}\int_{0}^{4-x^2} dy\,dx$

13 01 ex 20 18. $\displaystyle\int_{0}^{1}\int_{5-5x}^{5-5x^2} dy\,dx$

13 01 ex 19 19. $\displaystyle\int_{-2}^{2}\int_{0}^{2\sqrt{4-y^2}} dx\,dy$

13 01 ex 17 20. $\displaystyle\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dy\,dx$

13 01 ex 22 21. $\displaystyle\int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy + \int_{1}^{4}\int_{y-2}^{\sqrt{y}} dx\,dy$

13 01 ex 21 22. $\displaystyle\int_{-1}^{1}\int_{(x-1)/2}^{(1-x)/2} dy\,dx$

# Exercises 13.2

## *Terms and Concepts*

13 02 ex 01 1. An integral can be interpreted as giving the signed area over an interval; a double integral can be interpreted as giving the signed ________ over a region.

13 02 ex 02 2. Explain why the following statement is false: "Fubini's Theorem states that $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\; dy\; dx = \int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y)\, dx\, dy$."

13 02 ex 03 3. Explain why if $f(x,y) > 0$ over a region $R$, then $\iint_R f(x,y)\; dA > 0$.

13 02 ex 04 4. If $\iint_R f(x,y)\; dA = \iint_R g(x,y)\; dA$, does this imply $f(x,y) = g(x,y)$?

## *Problems*

13 02 exset 01 **In Exercises 5 – 10,**

**(a) Evaluate the given iterated integral, and**

**(b) rewrite the integral using the other order of integration.**

13 02 ex 05 5. $\displaystyle\int_1^2 \int_{-1}^1 \left(\frac{x}{y}+3\right)\; dx\, dy$

13 02 ex 06 6. $\displaystyle\int_{-\pi/2}^{\pi/2} \int_0^{\pi} (\sin x \cos y)\; dx\, dy$

13 02 ex 07 7. $\displaystyle\int_0^4 \int_0^{-x/2+2} \left(3x^2-y+2\right)\; dy\, dx$

13 02 ex 08 8. $\displaystyle\int_1^3 \int_y^3 \left(x^2y-xy^2\right)\; dx\, dy$

13 02 ex 09 9. $\displaystyle\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} (x+y+2)\; dx\, dy$

13 02 ex 10 10. $\displaystyle\int_0^9 \int_{y/3}^{\sqrt{y}} \left(xy^2\right)\; dx\, dy$

13 02 exset 02 **In Exercises 11 – 18:**

**(a) Sketch the region $R$ given by the problem.**

**(b) Set up the iterated integrals, in both orders, that evaluate the given double integral for the described region $R$.**

**(c) Evaluate one of the iterated integrals to find the signed volume under the surface $z = f(x,y)$ over the region $R$.**

13 02 ex 11 11. $\displaystyle\iint_R x^2y\; dA$, where $R$ is bounded by $y=\sqrt{x}$ and $y=x^2$.

13 02 ex 12 12. $\displaystyle\iint_R x^2y\; dA$, where $R$ is bounded by $y=\sqrt[3]{x}$ and $y=x^3$.

13 02 ex 13 13. $\displaystyle\iint_R x^2 - y^2\; dA$, where $R$ is the rectangle with corners $(-1,-1)$, $(1,-1)$, $(1,1)$ and $(-1,1)$.

13 02 ex 14 14. $\displaystyle\iint_R ye^x\; dA$, where $R$ is bounded by $x=0$, $x=y^2$ and $y=1$.

13 02 ex 15 15. $\displaystyle\iint_R \left(6-3x-2y\right)\; dA$, where $R$ is bounded by $x=0$, $y=0$ and $3x+2y=6$.

13 02 ex 16 16. $\displaystyle\iint_R e^y\; dA$, where $R$ is bounded by $y=\ln x$ and $y=\dfrac{1}{e-1}(x-1)$.

13 02 ex 17 17. $\displaystyle\iint_R \left(x^3y-x\right)\; dA$, where $R$ is the half of the circle $x^2+y^2=9$ in the first and second quadrants.

13 02 ex 18 18. $\displaystyle\iint_R \left(4-3y\right)\; dA$, where $R$ is bounded by $y=0$, $y=x/e$ and $y=\ln x$.

13 02 exset 03 **In Exercises 19 – 22, state why it is difficult/impossible to integrate the iterated integral in the given order of integration. Change the order of integration and evaluate the new iterated integral.**

13 02 ex 19 19. $\displaystyle\int_0^4 \int_{y/2}^2 e^{x^2}\; dx\, dy$

13 02 ex 20 20. $\displaystyle\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \cos\left(y^2\right)\; dy\, dx$

13 02 ex 21 21. $\displaystyle\int_0^1 \int_y^1 \frac{2y}{x^2+y^2}\; dx\, dy$

13 02 ex 22 22. $\displaystyle\int_{-1}^1 \int_1^2 \frac{x\tan^2 y}{1+\ln y}\; dy\, dx$

13 02 exset 04 **In Exercises 23 – 26, find the average value of $f$ over the region $R$. Notice how these functions and regions are related to the iterated integrals given in Exercises 5 – 8.**

13 02 ex 23 23. $f(x,y)=\dfrac{x}{y}+3$; $R$ is the rectangle with opposite corners $(-1,1)$ and $(1,2)$.

13 02 ex 24 24. $f(x,y)=\sin x\cos y$; $R$ is bounded by $x=0$, $x=\pi$, $y=-\pi/2$ and $y=\pi/2$.

13 02 ex 25 25. $f(x,y)=3x^2-y+2$; $R$ is bounded by the lines $y=0$, $y=2-x/2$ and $x=0$.

13 02 ex 26 26. $f(x,y)=x^2y-xy^2$; $R$ is bounded by $y=x$, $y=1$ and $x=3$.

# Exercises 13.3

## *Terms and Concepts*

13 03 ex 01

1. When evaluating $\iint_R f(x,y)\ dA$ using polar coordinates, $f(x,y)$ is replaced with ________ and $dA$ is replaced with ________.

13 03 ex 02

2. Why would one be interested in evaluating a double integral with polar coordinates?

## *Problems*

13 03 exset 01

**In Exercises 3 – 10, a function $f(x,y)$ is given and a region $R$ of the $x$-$y$ plane is described. Set up and evaluate $\iint_R f(x,y)\ dA$ using polar coordinates.**

13 03 ex 03

3. $f(x,y) = 3x - y + 4$; $R$ is the region enclosed by the circle $x^2 + y^2 = 1$.

13 03 ex 04

4. $f(x,y) = 4x + 4y$; $R$ is the region enclosed by the circle $x^2 + y^2 = 4$.

13 03 ex 05

5. $f(x,y) = 8 - y$; $R$ is the region enclosed by the circles with polar equations $r = \cos\theta$ and $r = 3\cos\theta$.

13 03 ex 06

6. $f(x,y) = 4$; $R$ is the region enclosed by the petal of the rose curve $r = \sin(2\theta)$ in the first quadrant.

13 03 ex 07

7. $f(x,y) = \ln\left(x^2 + y^2\right)$; $R$ is the annulus enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

13 03 ex 08

8. $f(x,y) = 1 - x^2 - y^2$; $R$ is the region enclosed by the circle $x^2 + y^2 = 1$.

13 03 ex 09

9. $f(x,y) = x^2 - y^2$; $R$ is the region enclosed by the circle $x^2 + y^2 = 36$ in the first and fourth quadrants.

13 03 ex 10

10. $f(x,y) = (x - y)/(x + y)$; $R$ is the region enclosed by the lines $y = x$, $y = 0$ and the circle $x^2 + y^2 = 1$ in the first quadrant.

13 03 exset 02

**In Exercises 11 – 14, an iterated integral in rectangular coordinates is given. Rewrite the integral using polar coordinates and evaluate the new double integral.**

13 03 ex 11

11. $\displaystyle\int_0^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{x^2+y^2}\ dy\,dx$

13 03 ex 12

12. $\displaystyle\int_{-4}^4 \int_{-\sqrt{16-y^2}}^{0} \left(2y - x\right)\ dx\,dy$

13 03 ex 13

13. $\displaystyle\int_0^2 \int_y^{\sqrt{8-y^2}} \left(x + y\right)\ dx\,dy$

13 03 ex 14

14. $\displaystyle\int_{-2}^{-1} \int_0^{\sqrt{4-x^2}} \left(x+5\right)\ dy\,dx + \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} \left(x+5\right)\ dy\,dx + \int_1^2 \int_0^{\sqrt{4-x^2}} \left(x+5\right)\ dy\,dx$

    **Hint:** draw the region of each integral carefully and see how they all connect.

13 03 exset 03

**In Exercises 15 – 16, special double integrals are presented that are especially well suited for evaluation in polar coordinates.**

13 03 ex 15

15. Consider $\displaystyle\iint_R e^{-(x^2+y^2)}\ dA$.

    (a) Why is this integral difficult to evaluate in rectangular coordinates, regardless of the region $R$?

    (b) Let $R$ be the region bounded by the circle of radius $a$ centered at the origin. Evaluate the double integral using polar coordinates.

    (c) Take the limit of your answer from (b), as $a \to \infty$. What does this imply about the volume under the surface of $e^{-(x^2+y^2)}$ over the entire $x$-$y$ plane?

13 03 ex 16

16. The surface of a right circular cone with height $h$ and base radius $a$ can be described by the equation $f(x,y) = h - h\sqrt{\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2}}$, where the tip of the cone lies at $(0,0,h)$ and the circular base lies in the $x$-$y$ plane, centered at the origin.

    Confirm that the volume of a right circular cone with height $h$ and base radius $a$ is $V = \dfrac{1}{3}\pi a^2 h$ by evaluating $\displaystyle\iint_R f(x,y)\ dA$ in polar coordinates.

# Exercises 13.4

## Terms and Concepts

13 04 ex 01 1. Why is it easy to use "mass" and "weight" interchangeably, even though they are different measures?

13 04 ex 02 2. Given a point $(x, y)$, the value of $x$ is a measure of distance from the ________-axis.

13 04 ex 03 3. We can think of $\iint_R \; dm$ as meaning "sum up lots of ________"

13 04 ex 04 4. What is a "discrete planar system?"

13 04 ex 05 5. Why does $M_x$ use $\iint_R y\delta(x,y)\,dA$ instead of $\iint_R x\delta(x,y)\,dA$; that is, why do we use "$y$" and not "$x$"?

13 04 ex 06 6. Describe a situation where the center of mass of a lamina does not lie within the region of the lamina itself.

## Problems

13 04 exset 01 **In Exercises 7 – 10, point masses are given along a line or in the plane. Find the center of mass $\overline{x}$ or $(\overline{x}, \overline{y})$, as appropriate. (All masses are in grams and distances are in cm.)**

13 04 ex 07 7. $m_1 = 4$ at $x = 1$; $\quad m_2 = 3$ at $x = 3$; $\quad m_3 = 5$ at $x = 10$

13 04 ex 08 8. $m_1 = 2$ at $x = -3$; $\quad m_2 = 2$ at $x = -1$;
$m_3 = 3$ at $x = 0$; $\quad m_4 = 3$ at $x = 7$

13 04 ex 10 9. $m_1 = 2$ at $(-2, -2)$; $\quad m_2 = 2$ at $(2, -2)$;
$m_3 = 20$ at $(0, 4)$

13 04 ex 09 10. $m_1 = 1$ at $(-1, -1)$; $\quad m_2 = 2$ at $(-1, 1)$;
$m_3 = 2$ at $(1, 1)$; $\quad m_4 = 1$ at $(1, -1)$

13 04 exset 02 **In Exercises 11 – 18, find the mass/weight of the lamina described by the region $R$ in the plane and its density function $\delta(x, y)$.**

13 04 ex 11 11. $R$ is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = 5$gm/cm$^2$

13 04 ex 12 12. $R$ is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = (x + y^2)$gm/cm$^2$

13 04 ex 13 13. $R$ is the triangle with corners $(-1, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = 2$lb/in$^2$

13 04 ex 14 14. $R$ is the triangle with corners $(0, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = (x^2 + y^2 + 1)$lb/in$^2$

13 04 ex 15 15. $R$ is the circle centered at the origin with radius 2; $\delta(x, y) = (x + y + 4)$kg/m$^2$

13 04 ex 16 16. $R$ is the circle sector bounded by $x^2 + y^2 = 25$ in the first quadrant; $\delta(x, y) = (\sqrt{x^2 + y^2} + 1)$kg/m$^2$

13 04 ex 17 17. $R$ is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = 4$lb/ft$^2$

13 04 ex 18 18. $R$ is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = \sqrt{x^2 + y^2}$lb/ft$^2$

13 04 exset 03 **In Exercises 19 – 26, find the center of mass of the lamina described by the region $R$ in the plane and its density function $\delta(x, y)$.**
**Note: these are the same lamina as in Exercises 11 – 18.**

13 04 ex 19 19. $R$ is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = 5$gm/cm$^2$

13 04 ex 20 20. $R$ is the rectangle with corners $(1, -3)$, $(1, 2)$, $(7, 2)$ and $(7, -3)$; $\delta(x, y) = (x + y^2)$gm/cm$^2$

13 04 ex 21 21. $R$ is the triangle with corners $(-1, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = 2$lb/in$^2$

13 04 ex 22 22. $R$ is the triangle with corners $(0, 0)$, $(1, 0)$, and $(0, 1)$; $\delta(x, y) = (x^2 + y^2 + 1)$lb/in$^2$

13 04 ex 23 23. $R$ is the circle centered at the origin with radius 2; $\delta(x, y) = (x + y + 4)$kg/m$^2$

13 04 ex 24 24. $R$ is the circle sector bounded by $x^2 + y^2 = 25$ in the first quadrant; $\delta(x, y) = (\sqrt{x^2 + y^2} + 1)$kg/m$^2$

13 04 ex 25 25. $R$ is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = 4$lb/ft$^2$

13 04 ex 26 26. $R$ is the annulus in the first and second quadrants bounded by $x^2 + y^2 = 9$ and $x^2 + y^2 = 36$; $\delta(x, y) = \sqrt{x^2 + y^2}$lb/ft$^2$

13 04 exset 04 **The *moment of inertia* $I$ is a measure of the tendency of a lamina to resist rotating about an axis or continue to rotate about an axis. $I_x$ is the moment of inertia about the $x$-axis, $I_x$ is the moment of inertia about the $x$-axis, and $I_O$ is the moment of inertia about the origin. These are computed as follows:**

- $I_x = \displaystyle\iint_R y^2\,dm$
- $I_y = \displaystyle\iint_R x^2\,dm$
- $I_O = \displaystyle\iint_R \left(x^2 + y^2\right)\,dm$

**In Exercises 27 – 30, a lamina corresponding to a planar region $R$ is given with a mass of 16 units. For each, compute $I_x$, $I_y$ and $I_O$.**

13 04 ex 27 27. $R$ is the $4 \times 4$ square with corners at $(-2, -2)$ and $(2, 2)$ with density $\delta(x, y) = 1$.

13 04 ex 28 28. $R$ is the $8 \times 2$ rectangle with corners at $(-4, -1)$ and $(4, 1)$ with density $\delta(x, y) = 1$.

13 04 ex 29 29. $R$ is the $4 \times 2$ rectangle with corners at $(-2, -1)$ and $(2, 1)$ with density $\delta(x, y) = 2$.

13 04 ex 30 30. $R$ is the circle with radius 2 centered at the origin with density $\delta(x, y) = 4/\pi$.

# Exercises 13.5

## Terms and Concepts

13 05 ex 01

1. "Surface area" is analogous to what previously studied concept?

13 05 ex 02

2. To approximate the area of a small portion of a surface, we computed the area of its ________ plane.

13 05 ex 03

3. We interpret $\iint_R dS$ as "sum up lots of little ________ ________."

13 05 ex 04

4. Why is it important to know how to set up a double integral to compute surface area, even if the resulting integral is hard to evaluate?

13 05 ex 18

5. Why do $z = f(x, y)$ and $z = g(x, y) = f(x, y) + h$, for some real number $h$, have the same surface area over a region $R$?

13 05 ex 19

6. Let $z = f(x, y)$ and $z = g(x, y) = 2f(x, y)$. Why is the surface area of $g$ over a region $R$ not twice the surface area of $f$ over $R$?

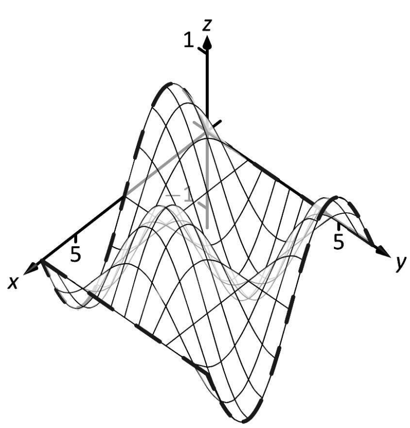
## Problems

13 05 exset 01

**In Exercises 7 – 10, *set up* the iterated integral that computes the surface area of the given surface over the region $R$.**

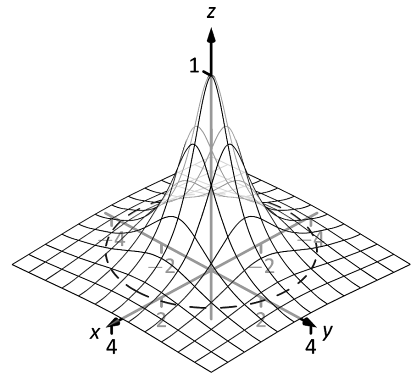
13 05 ex 05

7. $f(x, y) = \sin x \cos y$; $R$ is the rectangle with bounds $0 \leq x \leq 2\pi$, $0 \leq y \leq 2\pi$.



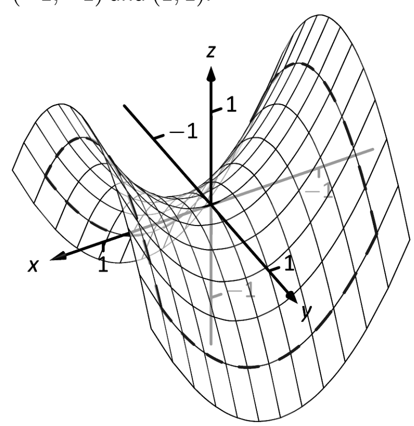

13 05 ex 06

8. $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$; $R$ is the circle $x^2 + y^2 = 9$.



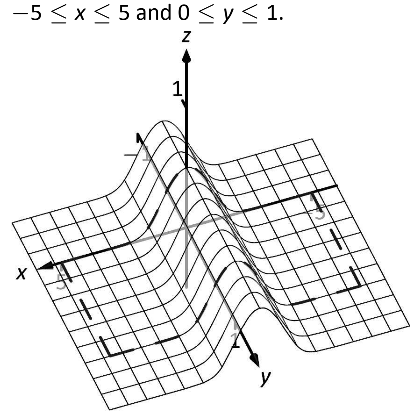

13 05 ex 07

9. $f(x, y) = x^2 - y^2$; $R$ is the rectangle with opposite corners $(-1, -1)$ and $(1, 1)$.




13 05 ex 08

10. $f(x, y) = \dfrac{1}{e^{x^2} + 1}$; $R$ is the rectangle bounded by $-5 \leq x \leq 5$ and $0 \leq y \leq 1$.




13 05 exset 02

**In Exercises 11 – 19, find the area of the given surface over the region $R$.**

13 05 ex 09

11. $f(x, y) = 3x - 7y + 2$; $R$ is the rectangle with opposite corners $(-1, 0)$ and $(1, 3)$.

13 05 ex 10

12. $f(x, y) = 2x + 2y + 2$; $R$ is the triangle with corners $(0, 0)$, $(1, 0)$ and $(0, 1)$.

13 05 ex 11 13. $f(x,y) = x^2 + y^2 + 10$; $R$ is the circle $x^2 + y^2 = 16$.

13 05 ex 14 14. $f(x,y) = -2x + 4y^2 + 7$ over $R$, the triangle bounded by $y = -x$, $y = x$, $0 \le y \le 1$.

13 05 ex 15 15. $f(x,y) = x^2 + y$ over $R$, the triangle bounded by $y = 2x$, $y = 0$ and $x = 2$.

13 05 ex 16 16. $f(x,y) = \frac{2}{3}x^{3/2} + 2y^{3/2}$ over $R$, the rectangle with opposite corners $(0,0)$ and $(1,1)$.

13 05 ex 12 17. $f(x,y) = 10 - 2\sqrt{x^2 + y^2}$ over $R$, the circle $x^2 + y^2 = 25$. (This is the cone with height 10 and base radius 5; be sure to compare you result with the known formula.)

13 05 ex 13 18. Find the surface area of the sphere with radius 5 by doubling the surface area of $f(x,y) = \sqrt{25 - x^2 - y^2}$ over $R$, the circle $x^2 + y^2 = 25$. (Be sure to compare you result with the known formula.)

19. Find the surface area of the ellipse formed by restricting the plane $f(x,y) = cx + dy + h$ to the region $R$, the circle $x^2 + y^2 = 1$, where $c$, $d$ and $h$ are some constants. Your answer should be given in terms of $c$ and $d$; why does the value of $h$ not matter?

# Exercises 13.6

## Terms and Concepts

13 06 ex 01 1. The strategy for establishing bounds for triple integrals is "________ to ________, ________ to ________ and ________ to ________."

13 06 ex 02 2. Give an informal interpretation of what "$\iiint_D dV$" means.

13 06 ex 23 3. Give two uses of triple integration.

13 06 ex 24 4. If an object has a constant density $\delta$ and a volume $V$, what is its mass?

## Problems

13 06 exset 01 **In Exercises 5 – 8, two surfaces $f_1(x, y)$ and $f_2(x, y)$ and a region $R$ in the $x$, $y$ plane are given. Set up and evaluate the double integral that finds the volume between these surfaces over $R$.**

13 06 ex 03 5. $f_1(x, y) = 8 - x^2 - y^2$, $f_2(x, y) = 2x + y$;
$R$ is the square with corners $(-1, -1)$ and $(1, 1)$.

13 06 ex 04 6. $f_1(x, y) = x^2 + y^2$, $f_2(x, y) = -x^2 - y^2$;
$R$ is the square with corners $(0, 0)$ and $(2, 3)$.

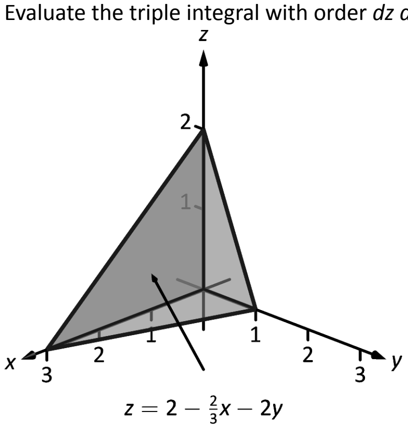
13 06 ex 05 7. $f_1(x, y) = \sin x \cos y$, $f_2(x, y) = \cos x \sin y + 2$;
$R$ is the triangle with corners $(0, 0)$, $(\pi, 0)$ and $(\pi, \pi)$.

13 06 ex 06 8. $f_1(x, y) = 2x^2 + 2y^2 + 3$, $f_2(x, y) = 6 - x^2 - y^2$;
$R$ is the circle $x^2 + y^2 = 1$.

13 06 exset 02 **In Exercises 9 – 16, a domain $D$ is described by its bounding surfaces, along with a graph. Set up the triple integrals that give the volume of $D$ in all 6 orders of integration, and find the volume of $D$ by evaluating the indicated triple integral.**

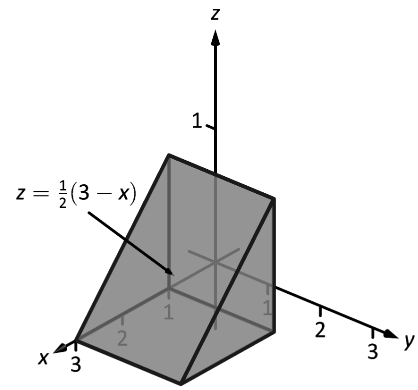
13 06 ex 07 9. $D$ is bounded by the coordinate planes and $z = 2 - 2x/3 - 2y$.

Evaluate the triple integral with order $dz\,dy\,dx$.



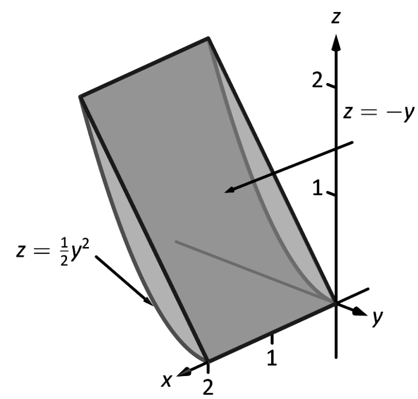

13 06 ex 08 10. $D$ is bounded by the planes $y = 0$, $y = 2$, $x = 1$, $z = 0$ and $z = (3 - x)/2$.

Evaluate the triple integral with order $dx\,dy\,dz$.



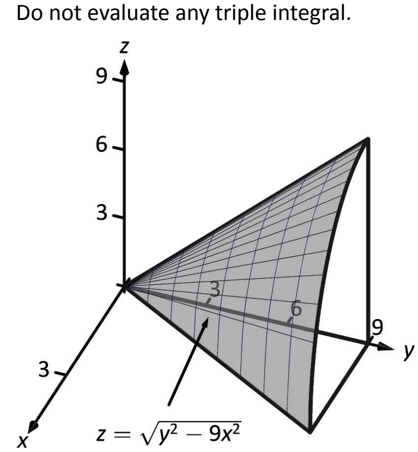

13 06 ex 09 11. $D$ is bounded by the planes $x = 0$, $x = 2$, $z = -y$ and by $z = y^2/2$.

Evaluate the triple integral with the order $dy\,dz\,dx$.



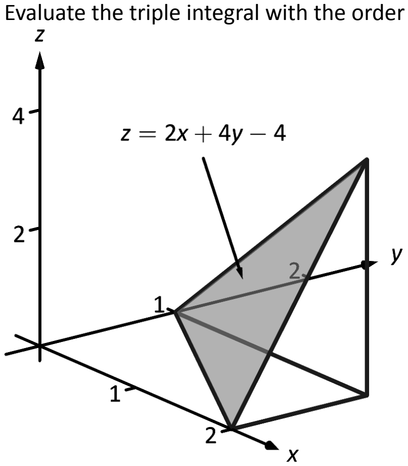

13 06 ex 10 12. $D$ is bounded by the planes $z = 0$, $y = 9$, $x = 0$ and by $z = \sqrt{y^2 - 9x^2}$.

Do not evaluate any triple integral.



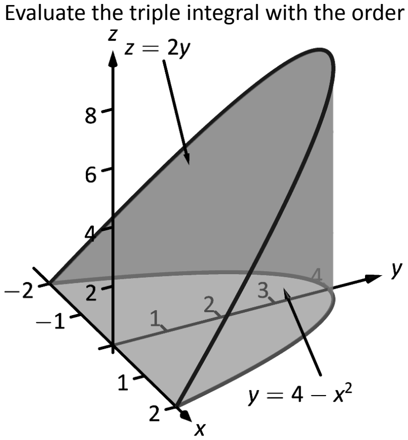
13 06 ex 11 13. $D$ is bounded by the planes $x = 2$, $y = 1$, $z = 0$ and $z = 2x + 4y - 4$.

Evaluate the triple integral with the order $dx\,dy\,dz$.

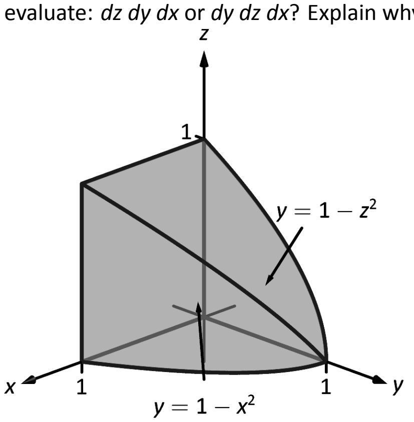



13 06 ex 12 14. $D$ is bounded by the plane $z = 2y$ and by $y = 4 - x^2$.

Evaluate the triple integral with the order $dz\,dy\,dx$.

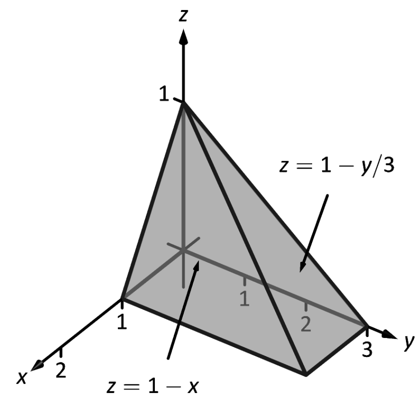



13 06 ex 13 15. $D$ is bounded by the coordinate planes and by $y = 1 - x^2$ and $y = 1 - z^2$.
Do not evaluate any triple integral. Which order is easier to evaluate: $dz\,dy\,dx$ or $dy\,dz\,dx$? Explain why.




13 06 ex 14 16. $D$ is bounded by the coordinate planes and by $z = 1 - y/3$ and $z = 1 - x$.

Evaluate the triple integral with order $dx\,dy\,dz$.




13 06 exset 04 **In Exercises 17 – 20, evaluate the triple integral.**

13 06 ex 19 17. $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^{\pi}\int_0^{\pi}\big(\cos x\sin y\sin z\big)\,dz\,dy\,dx$

13 06 ex 20 18. $\displaystyle\int_0^1\int_0^x\int_0^{x+y}\big(x + y + z\big)\,dz\,dy\,dx$

13 06 ex 22 19. $\displaystyle\int_0^{\pi}\int_0^1\int_0^z\big(\sin(yz)\big)\,dx\,dy\,dz$

13 06 ex 21 20. $\displaystyle\int_{\pi}^{\pi^2}\int_x^{x^3}\int_{-y^2}^{y^2}\left(z\frac{x^2y + y^2x}{e^{x^2+y^2}}\right)dz\,dy\,dx$

13 06 exset 03 **In Exercises 21 – 24, find the center of mass of the solid represented by the indicated space region $D$ with density function $\delta(x, y, z)$.**

13 06 ex 15 21. $D$ is bounded by the coordinate planes and $z = 2 - 2x/3 - 2y$; $\quad \delta(x, y, z) = 10\text{gm/cm}^3$.
(Note: this is the same region as used in Exercise 9.)

13 06 ex 16 22. $D$ is bounded by the planes $y = 0$, $y = 2$, $x = 1$, $z = 0$ and $z = (3 - x)/2$; $\quad \delta(x, y, z) = 2\text{gm/cm}^3$.
(Note: this is the same region as used in Exercise 10.)

13 06 ex 17 23. $D$ is bounded by the planes $x = 2$, $y = 1$, $z = 0$ and $z = 2x + 4y - 4$; $\quad \delta(x, y, z) = x^2\text{lb/in}^3$.
(Note: this is the same region as used in Exercise 13.)

13 06 ex 18 24. $D$ is bounded by the plane $z = 2y$ and by $y = 4 - x^2$. $\delta(x, y, z) = y^2\text{lb/in}^3$.
(Note: this is the same region as used in Exercise 14.)

# A: Solutions To Selected Problems

01 01 ex 19 1. Answers will vary.

01 01 ex 20 2. An indeterminate form.

01 01 ex 21 3. F

01 01 ex 22 4. The function may approach different values from the left and right, the function may grow without bound, or the function might oscillate.

01 01 ex 23 5. Answers will vary.

01 01 ex 01 6. $-1$

01 01 ex 02 7. $-5$

01 01 ex 03 8. Limit does not exist

01 01 ex 04 9. 2

01 01 ex 05 10. 1.5

01 01 ex 06 11. Limit does not exist.

01 01 ex 07 12. Limit does not exist.

01 01 ex 08 13. 7

01 01 ex 09 14. 1

01 01 ex 10 15. Limit does not exist.

01 01 ex 11 16.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | $-7$ |
| $-0.01$ | $-7$ |
| $0.01$ | $-7$ |
| $0.1$ | $-7$ |

The limit seems to be exactly 7.

01 01 ex 12 17.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | 9 |
| $-0.01$ | 9 |
| $0.01$ | 9 |
| $0.1$ | 9 |

The limit seems to be exactly 9.

01 01 ex 13 18.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | 4.9 |
| $-0.01$ | 4.99 |
| $0.01$ | 5.01 |
| $0.1$ | 5.1 |

The limit is approx. 5.

01 01 ex 14 19.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | $-0.114943$ |
| $-0.01$ | $-0.111483$ |
| $0.01$ | $-0.110742$ |
| $0.1$ | $-0.107527$ |

The limit is approx. $-0.11$.

01 01 ex 15 20.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | 29.4 |
| $-0.01$ | 29.04 |
| $0.01$ | 28.96 |
| $0.1$ | 28.6 |

The limit is approx. 29.

01 01 ex 16 21.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | 0.202027 |
| $-0.01$ | 0.2002 |
| $0.01$ | 0.1998 |
| $0.1$ | 0.198026 |

The limit is approx. 0.2.

01 01 ex 17 22.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | $-0.998334$ |
| $-0.01$ | $-0.999983$ |
| $0.01$ | $-0.999983$ |
| $0.1$ | $-0.998334$ |

The limit is approx. $-1$.

01 01 ex 18 23.

| $h$ | $\frac{f(a+h)-f(a)}{h}$ |
|---|---|
| $-0.1$ | $-0.0499583$ |
| $-0.01$ | $-0.00499996$ |
| $0.01$ | $0.00499996$ |
| $0.1$ | $0.0499583$ |

The limit is approx. 0.005.

**Section 1.2**

01 02 ex 01 1. $\varepsilon$ should be given first, and the restriction $|x-a|<\delta$ implies $|f(x)-K|<\varepsilon$, not the other way around.

01 02 ex 02 2. The $y$–tolerance.

01 02 ex 09 3. T

01 02 ex 10 4. T

01 02 ex 04 5. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-5|<\delta$, $|f(x)-(-2)|<\varepsilon$.
Consider $|f(x)-(-2)|<\varepsilon$:

$$\begin{gathered}|f(x)+2|<\varepsilon\\ |(3-x)+2|<\varepsilon\\ |5-x|<\varepsilon\\ -\varepsilon<5-x<\varepsilon\\ -\varepsilon<x-5<\varepsilon.\end{gathered}$$

This implies we can let $\delta=\varepsilon$. Then:

$$\begin{gathered}|x-5|<\delta\\ -\delta<x-5<\delta\\ -\varepsilon<x-5<\varepsilon\\ -\varepsilon<(x-3)-2<\varepsilon\\ -\varepsilon<(-x+3)-(-2)<\varepsilon\\ |3-x-(-2)|<\varepsilon,\end{gathered}$$

which is what we wanted to prove.

01 02 ex 05 6. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-3|<\delta$, $|f(x)-6|<\varepsilon$.
Consider $|f(x)-6|<\varepsilon$, keeping in mind we want to make a statement about $|x-3|$:

$$\begin{gathered}|f(x)-6|<\varepsilon\\ |x^2-3-6|<\varepsilon\\ |x^2-9|<\varepsilon\\ |x-3|\cdot|x+3|<\varepsilon\\ |x-3|<\varepsilon/|x+3|\end{gathered}$$

Since $x$ is near 3, we can safely assume that, for instance, $2 < x < 4$. Thus

$$\begin{aligned} 2+3 < x+3 &< 4+3 \\ 5 < x+3 &< 7 \\ \frac{1}{7} < \frac{1}{x+3} &< \frac{1}{5} \\ \frac{\varepsilon}{7} < \frac{\varepsilon}{x+3} &< \frac{\varepsilon}{5} \end{aligned}$$

Let $\delta = \frac{\varepsilon}{7}$. Then:

$$\begin{aligned} |x-3| &< \delta \\ |x-3| &< \frac{\varepsilon}{7} \\ |x-3| &< \frac{\varepsilon}{x+3} \\ |x-3|\cdot|x+3| &< \frac{\varepsilon}{x+3}\cdot|x+3| \end{aligned}$$

Assuming $x$ is near 3, $x+3$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned} |x-3|\cdot|x+3| &< \frac{\varepsilon}{x+3}\cdot(x+3) \\ |x^2-9| &< \varepsilon \\ |(x^2-3)-6| &< \varepsilon, \end{aligned}$$

which is what we wanted to prove.

01 02 ex 11 7. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-4| < \delta$, $|f(x)-15| < \varepsilon$.

Consider $|f(x)-15| < \varepsilon$, keeping in mind we want to make a statement about $|x-4|$:

$$\begin{aligned} |f(x)-15| &< \varepsilon \\ |x^2+x-5-15| &< \varepsilon \\ |x^2+x-20| &< \varepsilon \\ |x-4|\cdot|x+5| &< \varepsilon \\ |x-4| &< \varepsilon/|x+5| \end{aligned}$$

Since $x$ is near 4, we can safely assume that, for instance, $3 < x < 5$. Thus

$$\begin{aligned} 3+5 < x+5 &< 5+5 \\ 8 < x+5 &< 10 \\ \frac{1}{10} < \frac{1}{x+5} &< \frac{1}{8} \\ \frac{\varepsilon}{10} < \frac{\varepsilon}{x+5} &< \frac{\varepsilon}{8} \end{aligned}$$

Let $\delta = \frac{\varepsilon}{10}$. Then:

$$\begin{aligned} |x-4| &< \delta \\ |x-4| &< \frac{\varepsilon}{10} \\ |x-4| &< \frac{\varepsilon}{x+5} \\ |x-4|\cdot|x+5| &< \frac{\varepsilon}{x+5}\cdot|x+5| \end{aligned}$$

Assuming $x$ is near 4, $x+5$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned} |x-4|\cdot|x+5| &< \frac{\varepsilon}{x+5}\cdot(x+5) \\ |x^2+x-20| &< \varepsilon \\ |(x^2+x-5)-15| &< \varepsilon, \end{aligned}$$

which is what we wanted to prove.

01 02 ex 06 8. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x)-7| < \varepsilon$.

Consider $|f(x)-7| < \varepsilon$, keeping in mind we want to make a statement about $|x-2|$:

$$\begin{aligned} |f(x)-7| &< \varepsilon \\ |x^3-1-7| &< \varepsilon \\ |x^3-8| &< \varepsilon \\ |x-2|\cdot|x^2+2x+4| &< \varepsilon \\ |x-3| &< \varepsilon/|x^2+2x+4| \end{aligned}$$

Since $x$ is near 2, we can safely assume that, for instance, $1 < x < 3$. Thus

$$\begin{aligned} 1^2+2\cdot1+4 < x^2+2x+4 &< 3^2+2\cdot3+4 \\ 7 < x^2+2x+4 &< 19 \\ \frac{1}{19} < \frac{1}{x^2+2x+4} &< \frac{1}{7} \\ \frac{\varepsilon}{19} < \frac{\varepsilon}{x^2+2x+4} &< \frac{\varepsilon}{7} \end{aligned}$$

Let $\delta = \frac{\varepsilon}{19}$. Then:

$$\begin{aligned} |x-2| &< \delta \\ |x-2| &< \frac{\varepsilon}{19} \\ |x-2| &< \frac{\varepsilon}{x^2+2x+4} \\ |x-2|\cdot|x^2+2x+4| &< \frac{\varepsilon}{x^2+2x+4}\cdot|x^2+2x+4| \end{aligned}$$

Assuming $x$ is near 2, $x^2+2x+4$ is positive and we can drop the absolute value signs on the right.

$$\begin{aligned} |x-2|\cdot|x^2+2x+4| &< \frac{\varepsilon}{x^2+2x+4}\cdot(x^2+2x+4) \\ |x^3-8| &< \varepsilon \\ |(x^3-1)-7| &< \varepsilon, \end{aligned}$$

which is what we wanted to prove.

01 02 ex 03 9. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x)-5| < \varepsilon$. However, since $f(x) = 5$, a constant function, the latter inequality is simply $|5-5| < \varepsilon$, which is always true. Thus we can choose any $\delta$ we like; we arbitrarily choose $\delta = \varepsilon$.

01 02 ex 07 10. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-0| < \delta$, $|f(x)-0| < \varepsilon$.

Consider $|f(x)-0| < \varepsilon$, keeping in mind we want to make a statement about $|x-0|$ (i.e., $|x|$):

$$\begin{gathered}
|f(x)-0| < \varepsilon \\
|e^{2x}-1| < \varepsilon \\
-\varepsilon < e^{2x}-1 < \varepsilon \\
1-\varepsilon < e^{2x} < 1+\varepsilon \\
\ln(1-\varepsilon) < 2x < \ln(1+\varepsilon) \\
\frac{\ln(1-\varepsilon)}{2} < x < \frac{\ln(1+\varepsilon)}{2}
\end{gathered}$$

Let $\delta = \min\left\{\left|\frac{\ln(1-\varepsilon)}{2}\right|, \frac{\ln(1+\varepsilon)}{2}\right\} = \frac{\ln(1+\varepsilon)}{2}$.

Thus:

$$\begin{gathered}
|x| < \delta \\
|x| < \frac{\ln(1+\varepsilon)}{2} < \left|\frac{\ln(1-\varepsilon)}{2}\right| \\
\frac{\ln(1-\varepsilon)}{2} < x < \frac{\ln(1+\varepsilon)}{2} \\
\ln(1-\varepsilon) < 2x < \ln(1+\varepsilon) \\
1-\varepsilon < e^{2x} < 1+\varepsilon \\
-\varepsilon < e^{2x}-1 < \varepsilon \\
|e^{2x}-1-(0)| < \varepsilon,
\end{gathered}$$

which is what we wanted to prove.

01 02 ex 08 11. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-0| < \delta$, $|f(x)-0| < \varepsilon$. In simpler terms, we want to show that when $|x| < \delta$, $|\sin x| < \varepsilon$.

Set $\delta = \varepsilon$. We start with assuming that $|x| < \delta$. Using the hint, we have that $|\sin x| < |x| < \delta = \varepsilon$. Hence if $|x| < \delta$, we know immediately that $|\sin x| < \varepsilon$.

**Section 1.3**

01 03 ex 01 1. Answers will vary.

01 03 ex 02 2. Answers will vary.

01 03 ex 03 3. Answers will vary.

01 03 ex 04 4. Answers will vary.

01 03 ex 05 5. As $x$ is near 1, both $f$ and $g$ are near 0, but $f$ is approximately twice the size of $g$. (I.e., $f(x) \approx 2g(x)$.)

01 03 ex 06 6. 9

01 03 ex 07 7. 6

01 03 ex 08 8. 0

01 03 ex 09 9. Limit does not exist.

01 03 ex 10 10. 3

01 03 ex 11 11. Not possible to know.

01 03 ex 12 12. 3

01 03 ex 13 13. $-45$

01 03 ex 14 14. 1

01 03 ex 15 15. $-1$

01 03 ex 16 16. 0

01 03 ex 17 17. $\pi$

01 03 ex 18 18. 7

01 03 ex 21 19. $-0.000000015 \approx 0$

01 03 ex 22 20. $1/2$

01 03 ex 23 21. Limit does not exist

01 03 ex 24 22. 64

01 03 ex 25 23. 2

01 03 ex 26 24. 0

01 03 ex 20 25. $\frac{\pi^2+3\pi+5}{5\pi^2-2\pi-3} \approx 0.6064$

01 03 ex 19 26. $\frac{3\pi+1}{1-\pi}$

01 03 ex 27 27. $-8$

01 03 ex 28 28. $-1$

01 03 ex 29 29. 10

01 03 ex 30 30. $-2$

01 03 ex 31 31. $-3/2$

01 03 ex 32 32. $-7/8$

01 03 ex 38 33. 0

01 03 ex 40 34. 0

01 03 ex 42 35. 1

01 03 ex 41 36. 9

01 03 ex 33 37. 3

01 03 ex 34 38. $5/8$

01 03 ex 35 39. 1

01 03 ex 36 40. $\pi/180$

**Section 1.4**

01 04 ex 01 1. The function approaches different values from the left and right; the function grows without bound; the function oscillates.

01 04 ex 02 2. F

01 04 ex 03 3. F

01 04 ex 04 4. T

01 04 ex 05 5.

(a) 2

(b) 2

(c) 2

(d) 1

(e) As $f$ is not defined for $x < 0$, this limit is not defined.

(f) 1

01 04 ex 06 6.

(a) 1
(b) 2
(c) Does not exist.
(d) 2
(e) 0
(f) As $f$ is not defined for $x < 0$, this limit is not defined.

01 04 ex 07 7.

(a) Does not exist.
(b) Does not exist.
(c) Does not exist.
(d) Not defined.
(e) 0
(f) 0

01 04 ex 08 8.

(a) 2
(b) 0
(c) Does not exist.
(d) 1

01 04 ex 09 9.

(a) 2
(b) 2
(c) 2
(d) 2

01 04 ex 10 10.

(a) 4
(b) $-4$
(c) Does not exist.
(d) 0

01 04 ex 11 11.

(a) 2
(b) 2
(c) 2
(d) 0
(e) 2
(f) 2
(g) 2
(h) Not defined

01 04 ex 12 12.

(a) $a - 1$
(b) $a$
(c) Does not exist.
(d) $a$

01 04 ex 13 13.

(a) 2
(b) $-4$
(c) Does not exist.
(d) 2

01 04 ex 14 14.

(a) $-1$
(b) 1
(c) Does not exist.
(d) 1

01 04 ex 15 15.

(a) 0
(b) 0
(c) 0
(d) 0
(e) 2
(f) 2
(g) 2
(h) 2

01 04 ex 16 16.

(a) $-1$
(b) 0
(c) Does not exist.
(d) 0

01 04 ex 17 17.

(a) $1 - \cos^2 a = \sin^2 a$
(b) $\sin^2 a$
(c) $\sin^2 a$
(d) $\sin^2 a$

01 04 ex 18 18.

(a) 2
(b) 0
(c) Does not exist
(d) 1

01 04 ex 19 19.

(a) 4
(b) 4
(c) 4
(d) 3

01 04 ex 20 20.

(a) $c$
(b) $c$
(c) $c$
(d) $c$

01 04 ex 21 21.

(a) $-1$

(b) 1

(c) Does not exist

(d) 0

01 04 ex 22 22. $-3/5$

01 04 ex 23 23. $2/3$

01 04 ex 24 24. 2.5

01 04 ex 26 25. $-9$

01 04 ex 27 26. $-1.63$

**Section 1.5**

01 05 ex 06 1. Answers will vary.

01 05 ex 08 2. Answers will vary.

01 05 ex 09 3. A root of a function $f$ is a value $c$ such that $f(c) = 0$.

01 05 ex 10 4. Consider the function $h(x) = g(x) - f(x)$, and use the Bisection Method to find a root of $h$.

01 05 ex 01 5. F

01 05 ex 02 6. T

01 05 ex 03 7. T

01 05 ex 04 8. F

01 05 ex 05 9. F

01 05 ex 07 10. T

01 05 ex 11 11. No; $\lim\limits_{x\to 1} f(x) = 2$, while $f(1) = 1$.

01 05 ex 12 12. No; $\lim\limits_{x\to 1} f(x)$ does not exist.

01 05 ex 13 13. No; $f(1)$ does not exist.

01 05 ex 14 14. Yes

01 05 ex 15 15. Yes

01 05 ex 16 16. Yes

01 05 ex 17 17.

(a) No; $\lim\limits_{x\to -2} f(x) \neq f(-2)$

(b) Yes

(c) No; $f(2)$ is not defined.

01 05 ex 18 18.

(a) Yes

(b) Yes

01 05 ex 19 19.

(a) Yes

(b) No; the left and right hand limits at 1 are not equal.

01 05 ex 20 20.

(a) Yes

(b) Yes

01 05 ex 21 21.

(a) Yes

(b) No. $\lim_{x\to 8} f(x) = 16/5 \neq f(8) = 5$.

01 05 ex 22 22. $(-\infty, \infty)$

01 05 ex 23 23. $(-\infty, -2] \cup [2, \infty)$

01 05 ex 24 24. $[-1, 1]$

01 05 ex 25 25. $(-\infty, -\sqrt{6}] \cup [\sqrt{6}, \infty)$

01 05 ex 26 26. $(-1, 1)$

01 05 ex 27 27. $(-\infty, \infty)$

01 05 ex 28 28. $(-\infty, \infty)$

01 05 ex 29 29. $(0, \infty)$

01 05 ex 30 30. $(-\infty, \infty)$

01 05 ex 31 31. $(-\infty, 0]$

01 05 ex 32 32. $(-\infty, \infty)$

01 05 ex 33 33. Yes, by the Intermediate Value Theorem.

01 05 ex 34 34. Yes, by the Intermediate Value Theorem. In fact, we can be more specific and state such a value $c$ exists in $(0, 2)$, not just in $(-3, 7)$.

01 05 ex 35 35. We cannot say; the Intermediate Value Theorem only applies to function values between $-10$ and 10; as 11 is outside this range, we do not know.

01 05 ex 36 36. We cannot say; the Intermediate Value Theorem only applies to continuous functions. As we do know know if $h$ is continuous, we cannot say.

01 05 ex 37 37. Approximate root is $x = 1.23$. The intervals used are:
$[1, 1.5]$ $[1, 1.25]$ $[1.125, 1.25]$
$[1.1875, 1.25]$ $[1.21875, 1.25]$ $[1.234375, 1.25]$
$[1.234375, 1.2421875]$ $[1.234375, 1.2382813]$

01 05 ex 38 38. Approximate root is $x = 0.52$. The intervals used are:
$[0.5, 0.55]$ $[0.5, 0.525]$ $[0.5125, 0.525]$
$[0.51875, 0.525]$ $[0.521875, 0.525]$

01 05 ex 39 39. Approximate root is $x = 0.69$. The intervals used are:
$[0.65, 0.7]$ $[0.675, 0.7]$ $[0.6875, 0.7]$
$[0.6875, 0.69375]$ $[0.690625, 0.69375]$

01 05 ex 40 40. Approximate root is $x = 0.78$. The intervals used are:
$[0.7, 0.8]$ $[0.75, 0.8]$ $[0.775, 0.8]$
$[0.775, 0.7875]$ $[0.78125, 0.7875]$
(A few more steps show that 0.79 is better as the root is $\pi/4 \approx 0.78539$.)

01 05 ex 41 41.

(a) 20

(b) 25

(c) Limit does not exist

(d) 25

01 05 ex 42 42.

| $x$ | $f(x)$ |
|---|---|
| $-0.81$ | $-2.34129$ |
| $-0.801$ | $-2.33413$ |
| $-0.79$ | $-2.32542$ |
| $-0.799$ | $-2.33254$ |

The top two lines give an approximation of the limit from the left: $-2.33$. The bottom two lines give an approximation from the right: $-2.33$ as well.

01 05 ex 43 43. Answers will vary.

## Section 1.6

01 06 ex 01 1. F

01 06 ex 02 2. T

01 06 ex 03 3. F

01 06 ex 04 4. T

01 06 ex 05 5. T

01 06 ex 06 6. Answers will vary.

01 06 ex 07 7. Answers will vary.

01 06 ex 08 8. The limit of $f$ as $x$ approaches 7 does not exist, hence $f$ is not continuous. (Note: $f$ could be defined at 7!)

01 06 ex 09 9.

(a) $\infty$

(b) $\infty$

01 06 ex 10 10.

(a) $-\infty$

(b) $\infty$

(c) Limit does not exist

(d) $\infty$

(e) $\infty$

(f) $\infty$

01 06 ex 11 11.

(a) 1

(b) 0

(c) $1/2$

(d) $1/2$

01 06 ex 12 12.

(a) Limit does not exist

(b) Limit does not exist

01 06 ex 13 13.

(a) Limit does not exist

(b) Limit does not exist

01 06 ex 40 14.

(a) 10

(b) $\infty$

01 06 ex 14 15. Tables will vary.

(a)

| $x$ | $f(x)$ |
|---|---|
| 2.9 | $-15.1224$ |
| 2.99 | $-159.12$ |
| 2.999 | $-1599.12$ |

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

| $x$ | $f(x)$ |
|---|---|
| 3.1 | 16.8824 |
| 3.01 | 160.88 |
| 3.001 | 1600.88 |

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x)$ does not exist.

01 06 ex 15 16. Tables will vary.

(a)

| $x$ | $f(x)$ |
|---|---|
| 2.9 | $-335.64$ |
| 2.99 | $-30350.6$ |

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

| $x$ | $f(x)$ |
|---|---|
| 3.1 | $-265.61$ |
| 3.01 | $-29650.6$ |

It seems $\lim_{x\to 3^+} f(x) = -\infty$.

(c) It seems $\lim_{x\to 3} f(x) = -\infty$.

01 06 ex 16 17. Tables will vary.

(a)

| $x$ | $f(x)$ |
|---|---|
| 2.9 | 132.857 |
| 2.99 | 12124.4 |

It seems $\lim_{x\to 3^-} f(x) = \infty$.

(b)

| $x$ | $f(x)$ |
|---|---|
| 3.1 | 108.039 |
| 3.01 | 11876.4 |

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x) = \infty$.

01 06 ex 17 18. Tables will vary.

(a)

| $x$ | $f(x)$ |
|---|---|
| 2.9 | $-0.632$ |
| 2.99 | $-0.6032$ |
| 2.999 | $-0.60032$ |

It seems $\lim_{x\to 3^-} f(x) = -0.6$.

(b)

| $x$ | $f(x)$ |
|---|---|
| 3.1 | $-0.5686$ |
| 3.01 | $-0.5968$ |
| 3.001 | $-0.59968$ |

It seems $\lim_{x\to 3^+} f(x) = -0.6$.

(c) It seems $\lim_{x\to 3} f(x) = -0.6$.

01 06 ex 18 19. Horizontal asymptote at $y = 2$; vertical asymptotes at $x = -5, 4$.

01 06 ex 19 20. Horizontal asymptote at $y = -3/5$; vertical asymptote at $x = 3$.

01 06 ex 20 21. Horizontal asymptote at $y = 0$; vertical asymptotes at $x = -1, 0$.

01 06 ex 21 22. No horizontal asymptote; vertical asymptote at $x = 1$.

01 06 ex 22 23. No horizontal or vertical asymptotes.

01 06 ex 41 24. Horizontal asymptote at $y = -1$; no vertical asymptotes

01 06 ex 23 25. $\infty$

01 06 ex 24 26. $-\infty$

01 06 ex 25 27. $-\infty$

01 06 ex 26 28. $\infty$

01 06 ex 27 29. Solution omitted.

01 06 ex 28 30.

(a) 2

(b) $-3$

(c) $-3$

(d) $1/3$

01 06 ex 29 31. Yes. The only “questionable” place is at $x = 3$, but the left and right limits agree.

01 06 ex 30 32. 1

# Chapter 2

### Section 2.1

02 01 ex 01 1. T

02 01 ex 02 2. T

02 01 ex 03 3. Answers will vary.

02 01 ex 04 4. Answers will vary.

02 01 ex 05 5. Answers will vary.

02 01 ex 10 6. $f'(x) = 0$

02 01 ex 11 7. $f'(x) = 2$

02 01 ex 12 8. $f'(t) = -3$

02 01 ex 13 9. $g'(x) = 2x$

02 01 ex 14 10. $f''(x) = 6x - 1$

02 01 ex 15 11. $r'(x) = \frac{-1}{x^2}$

02 01 ex 16 12. $r'(s) = \frac{-1}{(s-2)^2}$

02 01 ex 17 13.

(a) $y = 6$

(b) $x = -2$

02 01 ex 18 14.

(a) $y = 2x$

(b) $y = -1/2x$

02 01 ex 19 15.

(a) $y = -3x + 4$

(b) $y = 1/3(x - 7) - 17$

02 01 ex 20 16.

(a) $y = 4(x - 2) + 4$

(b) $y = -1/4(x - 2) + 4$

02 01 ex 21 17.

(a) $y = -7(x + 1) + 8$

(b) $y = 1/7(x + 1) + 8$

02 01 ex 22 18.

(a) $y = -1/4(x + 2) - 1/2$

(b) $y = 4(x + 2) - 1/2$

02 01 ex 23 19.

(a) $y = -1(x - 3) + 1$

(b) $y = 1(x - 3) + 1$

02 01 ex 06 20. $y = 8.1(x - 3) + 16$

02 01 ex 07 21. $y = -0.099(x - 9) + 1$

02 01 ex 08 22. $y = 7.77(x - 2) + e^2$, or $y = 7.77(x - 2) + 7.39$.

02 01 ex 09 23. $y = -0.05x + 1$

02 01 ex 24 24.

(a) Approximations will vary; they should match (c) closely.

(b) $f'(x) = 2x$

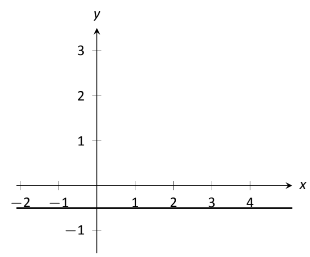
(c) At $(-1, 0)$, slope is $-2$. At $(0, -1)$, slope is 0. At $(2, 3)$, slope is 4.

02 01 ex 25 25.

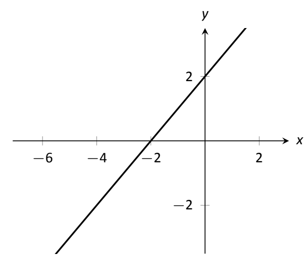
(a) Approximations will vary; they should match (c) closely.

(b) $f'(x) = -1/(x + 1)^2$

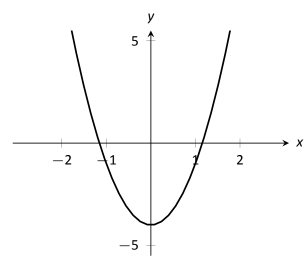
(c) At $(0, 1)$, slope is $-1$. At $(1, 0.5)$, slope is $-1/4$.



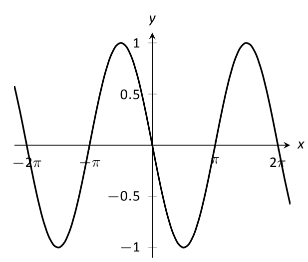

02 01 ex 26 26.




02 01 ex 27 27.




02 01 ex 28 28.




02 01 ex 29 29.

02 01 ex 30 30.

(a) Approximately on $(-1.5, 1.5)$.

(b) Approximately on $(-\infty, -1.5) \cup (1.5, \infty)$.

(c) Approximately at $x = \pm 1.5$.

(d) On $(-\infty, -1) \cup (0, 1)$.

(e) On $(-1, 0) \cup (1, \infty)$.

(f) At $x = \pm 1$.

02 01 ex 31 31. Approximately 24.

02 01 ex 32 32. Approximately 0.54.

02 01 ex 33 33.

(a) $(-\infty, \infty)$

(b) $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

(c) $(-\infty,5]$

(d) $[-5,5]$

02 01 ex 34 34.

(a) 1

(b) 3

(c) Does not exist

(d) $(-\infty,-3)\cup(3,\infty)$

**Section 2.2**

02 02 ex 01 1. Velocity

02 02 ex 02 2. Answers will vary.

02 02 ex 03 3. Linear functions.

02 02 ex 04 4. 12

02 02 ex 05 5. $-17$

02 02 ex 06 6. 102

02 02 ex 07 7. $f(10.1)$ is likely most accurate, as accuracy is lost the farther from $x=10$ we go.

02 02 ex 08 8. $-4$

02 02 ex 09 9. 6

02 02 ex 10 10. decibels per person

02 02 ex 11 11. ft/s$^2$

02 02 ex 12 12. ft/h

02 02 ex 13 13.

(a) thousands of dollars per car

(b) It is likely that $P(0)<0$. That is, negative profit for not producing any cars.

02 02 ex 14 14.

(a) degrees Fahrenheit per hour

(b) It is likely that $T'(8)>0$ since at 8 in the morning, the temperature is likely rising.

(c) It is very likely that $T(8)>0$, as at 8 in the morning on July 4, we would expect the temperature to be well above 0.

02 02 ex 15 15. $f(x)=g'(x)$

02 02 ex 16 16. $g(x)=f'(x)$

02 02 ex 17 17. Either $g(x)=f'(x)$ or $f(x)=g'(x)$ is acceptable. The actual answer is $g(x)=f'(x)$, but is very hard to show that $f(x)\neq g'(x)$ given the level of detail given in the graph.

02 02 ex 18 18. $g(x)=f'(x)$

02 02 ex 19 19. $f'(x)=10x$

02 02 ex 20 20. $f'(x)=3x^2-12x+12$

02 02 ex 21 21. $f'(\pi)\approx 0.$

02 02 ex 22 22. $f'(9)\approx 0.1667.$

**Section 2.3**

02 03 ex 01 1. Power Rule.

02 03 ex 02 2. $1/x$

02 03 ex 03 3. One answer is $f(x)=10e^x$.

02 03 ex 04 4. One answer is $f(x)=10$.

02 03 ex 05 5. $g(x)$ and $h(x)$

02 03 ex 06 6. Answers will vary.

02 03 ex 07 7. One possible answer is $f(x)=17x-205$.

02 03 ex 08 8. Answers will vary.

02 03 ex 09 9. $f'(x)$ is a velocity function, and $f''(x)$ is acceleration.

02 03 ex 10 10. lbs/ft$^2$.

02 03 ex 11 11. $f'(x)=14x-5$

02 03 ex 12 12. $g'(x)=42x^2+14x+11$

02 03 ex 13 13. $m'(t)=45t^4-\frac{3}{8}t^2+3$

02 03 ex 14 14. $f'(\theta)=9\cos\theta-10\sin\theta$

02 03 ex 15 15. $f'(r)=6e^r$

02 03 ex 16 16. $g'(t)=40t^3+\sin t+7\cos t$

02 03 ex 17 17. $f'(x)=\frac{2}{x}-1$

02 03 ex 18 18. $p'(s)=s^3+s^2+s+1$

02 03 ex 19 19. $h'(t)=e^t-\cos t+\sin t$

02 03 ex 20 20. $f'(x)=\frac{2}{x}$

02 03 ex 21 21. $f'(t)=0$

02 03 ex 22 22. $g'(t)=18t+6$

02 03 ex 23 23. $g'(x)=24x^2-120x+150$

02 03 ex 24 24. $f'(x)=-3x^2+6x-3$

02 03 ex 25 25. $f'(x)=18x-12$

02 03 ex 50 26.

02 03 ex 26 27. $f'(x)=6x^5$ $f''(x)=30x^4$ $f'''(x)=120x^3$ $f^{(4)}(x)=360x^2$

02 03 ex 27 28. $g'(x)=-2\sin x$ $g''(x)=-2\cos x$ $g'''(x)=2\sin x$ $g^{(4)}(x)=2\cos x$

02 03 ex 28 29. $h'(t)=2t-e^t$ $h''(t)=2-e^t$ $h'''(t)=-e^t$ $h^{(4)}(t)=-e^t$

02 03 ex 29 30. $p'(\theta)=4\theta^3-3\theta^2$ $p''(\theta)=12\theta^2-6\theta$ $p'''(\theta)=24\theta-6$ $p^{(4)}(\theta)=24$

02 03 ex 30 31. $f'(\theta)=\cos\theta+\sin\theta$ $f''(\theta)=-\sin\theta+\cos\theta$ $f'''(\theta)=-\cos\theta-\sin\theta$ $f^{(4)}(\theta)=\sin\theta-\cos\theta$

02 03 ex 31 32. $f'(x)=f''(x)=f'''(x)=f^{(4)}(x)=0$

02 03 ex 32 33. Tangent line: $y=2(x-1)$
Normal line: $y=-1/2(x-1)$

02 03 ex 33 34. Tangent line: $y=t+4$
Normal line: $y=-t+4$

02 03 ex 34 35. Tangent line: $y=x-1$
Normal line: $y=-x+1$

02 03 ex 35 36. Tangent line: $y=4$
Normal line: $x=\pi/2$

02 03 ex 36 37. Tangent line: $y=\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})-\sqrt{2}$
Normal line: $y=\frac{-2}{\sqrt{2}}(x-\frac{\pi}{4})-\sqrt{2}$

02 03 ex 37 38. Tangent line: $y = 2x + 3$
Normal line: $y = -1/2(x - 5) + 13$

02 03 ex 39 39. The tangent line to $f(x) = e^x$ at $x = 0$ is $y = x + 1$; thus $e^{0.1} \approx y(0.1) = 1.1$.

02 03 ex 40 40. The tangent line to $f(x) = x^4$ at $x = 3$ is $y = 108(x - 3) + 81$; thus $(3.01)^4 \approx y(3.01) = 108(.01) + 81 = 82.08$.

**Section 2.4**

02 04 ex 01 1. F

02 04 ex 02 2. F

02 04 ex 03 3. T

02 04 ex 04 4. Quotient Rule

02 04 ex 05 5. F

02 04 ex 06 6. Answers will vary.

02 04 ex 07 7.
(a) $f'(x) = (x^2 + 3x) + x(2x + 3)$
(b) $f'(x) = 3x^2 + 6x$
(c) They are equal.

02 04 ex 08 8.
(a) $g'(x) = 4x(5x^3) + 2x^2(15x^2)$
(b) $g'(x) = 50x^4$
(c) They are equal.

02 04 ex 09 9.
(a) $h'(s) = 2(s + 4) + (2s - 1)(1)$
(b) $h'(s) = 4s + 7$
(c) They are equal.

02 04 ex 10 10.
(a) $f'(x) = 2x(3 - x^3) + (x^2 + 5)(-3x^2)$
(b) $f'(x) = -5x^4 - 15x^2 + 6x$
(c) They are equal.

02 04 ex 11 11.
(a) $f'(x) = \frac{x(2x) - (x^2+3)1}{x^2}$
(b) $f'(x) = 1 - \frac{3}{x^2}$
(c) They are equal.

02 04 ex 12 12.
(a) $g'(x) = \frac{2x^2(3x^2-4x) - (x^3-2x^2)(4x)}{4x^4}$
(b) $g'(x) = 1/2$
(c) They are equal.

02 04 ex 13 13.
(a) $h'(s) = \frac{4s^3(0) - 3(12s^2)}{16s^6}$
(b) $h'(s) = -9/4s^{-4}$
(c) They are equal.

02 04 ex 14 14.
(a) $f'(t) = \frac{(t+1)(2t) - (t^2-1)(1)}{(t+1)^2}$
(b) $f(t) = t - 1$ when $t \neq -1$, so $f'(t) = 1$.
(c) They are equal.

02 04 ex 16 15. $f'(x) = \sin x + x\cos x$

02 04 ex 17 16. $f'(t) = \frac{-2}{t^3}(\csc t - 4) + \frac{1}{t^2}(-\csc t \cot t)$

02 04 ex 18 17. $g'(x) = \frac{-12}{(x-5)^2}$

02 04 ex 19 18. $g'(t) = \frac{(\cos t - 2t^2)(5t^4) - (t^5)(-\sin t - 4t)}{(\cos t - 2t^2)^2}$

02 04 ex 20 19. $h'(x) = -\csc^2 x - e^x$

02 04 ex 21 20. $h'(t) = 14t + 6$

02 04 ex 15 21.
(a) $f'(x) = \frac{(x+2)(4x^3+6x^2) - (x^4+2x^3)(1)}{(x+2)^2}$
(b) $f(x) = x^3$ when $x \neq -2$, so $f'(x) = 3x^2$.
(c) They are equal.

02 04 ex 22 22. $f'(x) = 7$

02 04 ex 23 23. $f'(t) = 5t^4(\sec t + e^t) + t^5(\sec t \tan t + e^t)$

02 04 ex 46 24. $f'(x) = \frac{\sin^2(x) + \cos^2(x) + 3\cos(x)}{(\cos(x)+3)^2}$

02 04 ex 24 25. $g'(x) = 0$

02 04 ex 25 26. $g'(t) = 12t^2e^t + 4t^3e^t - \cos^2 t + \sin^2 t$

02 04 ex 47 27. $f'(x) = \frac{(t^2\cos t + 2)(2t\sin t + t^2\cos t) - (t^2\sin t + 3)(2t\cos t - t^2\sin t)}{(t^2\cos t + 2)^2}$

02 04 ex 26 28. $f'(x) = 2xe^x\tan x = x^2e^x\tan x + x^2e^x\sec^2 x$

02 04 ex 27 29. $g'(x) = 2\sin x\sec x + 2x\cos x\sec x + 2x\sin x\sec x\tan x = 2\tan x + 2x + 2x\tan^2 x = 2\tan x + 2x\sec^2 x$

02 04 ex 28 30. Tangent line: $y = 2x + 2$
Normal line: $y = -1/2x + 2$

02 04 ex 29 31. Tangent line: $y = -(x - \frac{3\pi}{2}) - \frac{3\pi}{2} = -x$
Normal line: $y = (x - \frac{3\pi}{2}) - \frac{3\pi}{2} = x - 3\pi$

02 04 ex 30 32. Tangent line: $y = 4$
Normal line: $x = 2$

02 04 ex 31 33. Tangent line: $y = -9x - 5$
Normal line: $y = 1/9x - 5$

02 04 ex 32 34. $x = 3/2$

02 04 ex 33 35. $x = 0$

02 04 ex 34 36. $f'(x)$ is never 0.

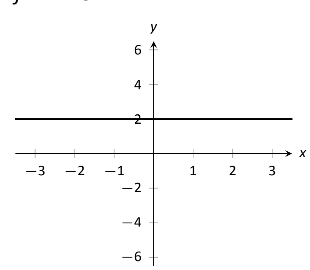
02 04 ex 35 37. $x = -2, 0$

02 04 ex 36 38. $f''(x) = 2\cos x - x\sin x$

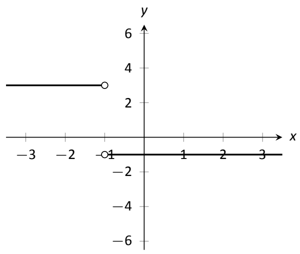
02 04 ex 37 39. $f^{(4)}(x) = -4\cos x + x\sin x$

02 04 ex 38 40. $f''(x) = \cot^2 x\csc x + \csc^3 x$

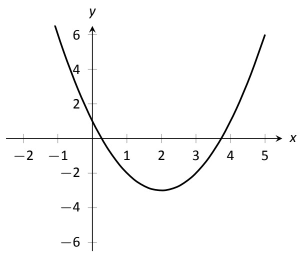
02 04 ex 39 41. $f^{(8)} = 0$



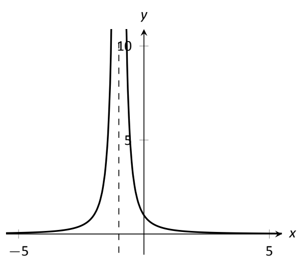

02 04 ex 42 42.



02 04 ex 43 43.



02 04 ex 44 44.



02 04 ex 45 45.

**Section 2.5**

02 05 ex 05 1. T

02 05 ex 01 2. F

02 05 ex 02 3. F

02 05 ex 03 4. T

02 05 ex 04 5. T

02 05 ex 53 6. T

02 05 ex 06 7. $f'(x) = 10(4x^3 - x)^9 \cdot (12x^2 - 1) = (120x^2 - 10)(4x^3 - x)^9$

02 05 ex 07 8. $f'(t) = 15(3t-2)^4$

02 05 ex 08 9. $g'(\theta) = 3(\sin\theta + \cos\theta)^2(\cos\theta - \sin\theta)$

02 05 ex 09 10. $h'(t) = (6t+1)e^{3t^2+t-1}$

02 05 ex 10 11. $f'(x) = 4\left(x + \frac{1}{x}\right)^3\left(1 - \frac{1}{x^2}\right)$

02 05 ex 11 12. $f'(x) = -3\sin(3x)$

02 05 ex 12 13. $g'(x) = 5\sec^2(5x)$

02 05 ex 13 14. $h'(t) = 8\sin^3(2t)\cos(2t)$

02 05 ex 14 15. $p'(t) = -3\cos^2(t^2+3t+1)\sin(t^2+3t+1)(2t+3)$

02 05 ex 15 16. $f'(x) = -\tan x$

02 05 ex 16 17. $f'(x) = 2/x$

02 05 ex 17 18. $f'(x) = 2/x$

02 05 ex 18 19. $g'(r) = \ln 4 \cdot 4^r$

02 05 ex 19 20. $g'(t) = -\ln 5 \cdot 5^{\cos t}\sin t$

02 05 ex 20 21. $g'(t) = 0$

02 05 ex 21 22. $m'(w) = \ln(3/2)(3/2)^w$

02 05 ex 42 23. $f'(x) = \frac{(3^t+2)\left((\ln 2)2^t\right) - (2^t+3)\left((\ln 3)3^t\right)}{(3^t+2)^2}$

02 05 ex 22 24. $m'(w) = \frac{2^w\left(\ln 3 \cdot 3^w - \ln 2 \cdot (3^w+1)\right)}{2^{2w}}$

02 05 ex 23 25. $f'(x) = \frac{2^{x^2}(\ln 3 \cdot 3^x x^2 2x + 1) - (3^{x^2}+x)(\ln 2 \cdot 2^{x^2} 2x)}{2^{2x^2}}$

02 05 ex 24 26. $f'(x) = 5x^2\cos(5x) + 2x\sin(5x)$

02 05 ex 25 27. $g'(t) =$
$5\cos(t^2+3t)\cos(5t-7) - (2t+3)\sin(t^2+3t)\sin(5t-7)$

02 05 ex 26 28. $g'(t) = 10t\cos(\frac{1}{t})e^{5t^2} + \frac{1}{t^2}\sin(\frac{1}{t})e^{5t^2}$

02 05 ex 27 29. Tangent line: $y = 0$
Normal line: $x = 0$

02 05 ex 28 30. Tangent line: $y = 15(t-1)+1$
Normal line: $y = -1/15(t-1)+1$

02 05 ex 29 31. Tangent line: $y = -3(\theta - \pi/2)+1$
Normal line: $y = 1/3(\theta - \pi/2)+1$

02 05 ex 30 32. Tangent line: $y = -5e(t+1)+e$
Normal line: $y = 1/(5e)(t+1)+e$

02 05 ex 31 33. In both cases the derivative is the same: $1/x$.

02 05 ex 32 34. In both cases the derivative is the same: $k/x$.

02 05 ex 33 35.

(a) $^\circ$ F/mph

(b) The sign would be negative; when the wind is blowing at 10 mph, any increase in wind speed will make it feel colder, i.e., a lower number on the Fahrenheit scale.

02 05 ex 34 36.

(a) $2xe^x\cot x + x^2e^x\cot x - x^2e^x\csc^2 x$

(b) $\ln(48)48^x$

**Section 2.6**

02 06 ex 01 1. Answers will vary.

02 06 ex 02 2. The Chain Rule.

02 06 ex 03 3. T

02 06 ex 04 4. T

02 05 ex 50 5. $f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$

02 05 ex 51 6. $f'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-1/3} = \frac{1}{3\sqrt[3]{x^2}} + \frac{2}{3\sqrt[3]{x}}$

02 06 ex 06 7. $f'(t) = \frac{-t}{\sqrt{1-t^2}}$

02 06 ex 07 8. $g'(t) = \sqrt{t}\cos t + \frac{\sin t}{2\sqrt{t}}$

02 06 ex 08 9. $h'(x) = 1.5x^{0.5} = 1.5\sqrt{x}$

02 05 ex 52 10. $f'(x) = \pi x^{\pi-1} + 1.9x^{0.9}$

02 05 ex 40 11. $g'(x) = \frac{\sqrt{x}(1) - (x+7)(1/2x^{-1/2})}{x} = \frac{1}{2\sqrt{x}} - \frac{7}{2\sqrt{x^3}}$

02 05 ex 41 12. $f'(t) = \frac{1}{5}x^{-4/5}(\sec t + e^t) + \sqrt[5]{t}(\sec t\tan t + e^t)$

02 06 ex 09 13. $\frac{dy}{dx} = \frac{-4x^3}{2y+1}$

02 06 ex 10 14. $\frac{dy}{dx} = -\frac{y^{3/5}}{x^{3/5}}$

02 06 ex 11 15. $\frac{dy}{dx} = \sin(x)\sec(y)$

02 06 ex 12 16. $\frac{dy}{dx} = \frac{y}{x}$

02 06 ex 13 17. $\frac{dy}{dx} = \frac{y}{x}$

02 06 ex 14 18. $-\frac{e^x x(x+2)2^{-y}}{\ln|2|}$

02 06 ex 15 19. $-\frac{2\sin(y)\cos(y)}{x}$

02 06 ex 16 20. $-\frac{x}{y^2}$

02 06 ex 17 21. $\frac{1}{2y+2}$

02 06 ex 18 22. $\frac{x^2+2xy^2-y}{2x^2y-x+y^2}$

02 06 ex 19 23. $\frac{-\cos(x)(x+\cos(y))+\sin(x)+y}{\sin(y)(\sin(x)+y)+x+\cos(y)}$

02 06 ex 21 24. $-\frac{x}{y}$

02 06 ex 22 25. $-\frac{2x+y}{2y+x}$

02 06 ex 20 26. In each, $\frac{dy}{dx} = -\frac{y}{x}$.

02 06 ex 23 27.

(a) $y = 0$

(b) $y = -1.859(x-0.1)+0.281$

02 06 ex 24 28.

(a) $x = 1$

(b) $y = -\frac{3\sqrt{3}}{8}(x-\sqrt{.6})+\sqrt{.8} \approx -0.65(x-0.775)+0.894$

(c) $y = 1$

02 06 ex 25 29.

(a) $y = 4$

(b) $y = 0.93(x-2)+\sqrt[4]{108}$

02 06 ex 26 30.

(a) $y = -1/3x+1$

(b) $y = 3\sqrt{3}/4$

02 06 ex 27 31.

(a) $y = -\frac{1}{\sqrt{3}}(x-\frac{7}{2})+\frac{6+3\sqrt{3}}{2}$

(b) $y = \sqrt{3}(x-\frac{4+3\sqrt{3}}{2})+\frac{3}{2}$

02 06 ex 28 32. $\frac{d^2y}{dx^2} = \frac{(2y+1)(-12x^2)+4x^3\left(2\frac{-4x^3}{2y+1}\right)}{(2y+1)^2}$

02 06 ex 29 33. $\frac{d^2y}{dx^2} = \frac{3}{5}\frac{y^{3/5}}{x^{8/5}} + \frac{3}{5}\frac{1}{yx^{6/5}}$

02 06 ex 30 34. $\frac{d^2y}{dx^2} = \frac{\cos x\cos y+\sin^2 x\tan y}{\cos^2 y}$

02 06 ex 31 35. $\frac{d^2y}{dx^2} = 0$

02 06 ex 32 36. $y' = (1+x)^{1/x}\left(\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2}\right)$
Tangent line: $y = (1-2\ln 2)(x-1)+2$

02 06 ex 33 37. $y' = (2x)^{x^2}\left(2x\ln(2x)+x\right)$
Tangent line: $y = (2+4\ln 2)(x-1)+2$

02 06 ex 34 38. $y' = \frac{x^x}{x+1}\left(\ln x+1-\frac{1}{x+1}\right)$
Tangent line: $y = (1/4)(x-1)+1/2$

02 06 ex 35 39. $y' = x^{\sin(x)+2}\left(\cos x\ln x+\frac{\sin x+2}{x}\right)$
Tangent line: $y = (3\pi^2/4)(x-\pi/2)+(\pi/2)^3$

02 06 ex 36 40. $y' = \frac{x+1}{x+2}\left(\frac{1}{x+1}-\frac{1}{x+2}\right)$
Tangent line: $y = 1/9(x-1)+2/3$

02 06 ex 37 41. $y' = \frac{(x+1)(x+2)}{(x+3)(x+4)}\left(\frac{1}{x+1}+\frac{1}{x+2}-\frac{1}{x+3}-\frac{1}{x+4}\right)$
Tangent line: $y = 11/72x+1/6$

**Section 2.7**

02 07 ex 01 1. F

02 07 ex 02 2. Answers will vary.

02 07 ex 03 3. The point $(10, 1)$ lies on the graph of $y = f^{-1}(x)$ (assuming $f$ is invertible).

02 07 ex 04 4. The point $(10, 1)$ lies on the graph of $y = f^{-1}(x)$ (assuming $f$ is invertible) and $(f^{-1})'(10) = 1/5$.

02 07 ex 05 5. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals $x$.

02 07 ex 06 6. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals $x$.

02 07 ex 07 7. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals $x$.

02 07 ex 08 8. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals $x$.

02 07 ex 09 9. $\left(f^{-1}\right)'(20) = \frac{1}{f'(2)} = 1/5$

02 07 ex 10 10. $\left(f^{-1}\right)'(7) = \frac{1}{f'(3)} = 1/4$

02 07 ex 11 11. $\left(f^{-1}\right)'(\sqrt{3}/2) = \frac{1}{f'(\pi/6)} = 1$

02 07 ex 12 12. $\left(f^{-1}\right)'(8) = \frac{1}{f'(1)} = 1/6$

02 07 ex 13 13. $\left(f^{-1}\right)'(1/2) = \frac{1}{f'(1)} = -2$

02 07 ex 14 14. $\left(f^{-1}\right)'(6) = \frac{1}{f'(0)} = 1/6$

02 07 ex 15 15. $h'(t) = \frac{2}{\sqrt{1-4t^2}}$

02 07 ex 16 16. $f'(t) = \frac{1}{|t|\sqrt{4t^2+1}}$

02 07 ex 17 17. $g'(x) = \frac{2}{1+4x^2}$

02 07 ex 18 18. $f'(x) = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$

02 07 ex 19 19. $g'(t) = \cos^{-1}(t)\cos(t) - \frac{\sin(t)}{\sqrt{1-t^2}}$

02 07 ex 20 20. $f'(t) = \frac{e^t}{t} + \ln t e^t$

02 07 ex 21 21. $h'(x) = \frac{\sin^{-1}(x)+\cos^{-1}(x)}{\sqrt{1-x^2}\cos^{-1}(x)^2}$

02 07 ex 22 22. $g'(x) = \frac{1}{\sqrt{x}(2x+2)}$

02 07 ex 23 23. $f'(x) = -\frac{1}{\sqrt{1-x^2}}$

02 07 ex 24 24.

(a) $f(x) = x$, so $f'(x) = 1$

(b) $f'(x) = \cos(\sin^{-1} x)\frac{1}{\sqrt{1-x^2}} = 1.$

02 07 ex 24 25.

(a) $f(x) = x$, so $f'(x) = 1$

(b) $f'(x) = \cos(\sin^{-1} x)\frac{1}{\sqrt{1-x^2}} = 1.$

02 07 ex 25 26.

(a) $f(x) = x$, so $f'(x) = 1$

(b) $f'(x) = \frac{1}{1+\tan^2 x}\sec^2 x = 1$

02 07 ex 26 27.

(a) $f(x) = \sqrt{1-x^2}$, so $f'(x) = \frac{-x}{\sqrt{1-x^2}}$

(b) $f'(x) = \cos(\cos^{-1} x)(\frac{1}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$

02 07 ex 27 28. $y = \sqrt{2}(x - \sqrt{2}/2) + \pi/4$

02 07 ex 28 29. $y = -4(x - \sqrt{3}/4) + \pi/6$

02 07 ex 29 30. $\frac{dy}{dx} = \frac{y(y-2x)}{x(x-2y)}$

02 07 ex 30 31. $y = -4/5(x-1) + 2$

02 07 ex 31 32. $3x^2 + 1$

# Chapter 3

### Section 3.1

03 01 ex 01 1. Answers will vary.

03 01 ex 02 2. Answers will vary.

03 01 ex 03 3. Answers will vary.

03 01 ex 04 4. Answers will vary.

03 01 ex 05 5. F

03 01 ex 06 6. $A$: none $B$: abs. max $C$: rel. min $D$: none $E$: none $F$: rel. min $G$: none

03 01 ex 07 7. $A$: abs. min $B$: none $C$: abs. max $D$: none $E$: none

03 01 ex 08 8. $f'(0) = 0$

03 01 ex 09 9. $f'(0) = 0\, f'(2) = 0$

03 01 ex 10 10. $f'(\pi/2) = 0\, f'(3\pi/2) = 0$

03 01 ex 11 11. $f'(0) = 0\, f'(3.2) = 0\, f'(4)$ is undefined

03 01 ex 12 12. $f'(0) = 0$

03 01 ex 13 13. $f'(0)$ is not defined

03 01 ex 14 14. $f'(2)$ is not defined $f'(6) = 0$

03 01 ex 15 15. min: $(-0.5, 3.75)$
max: $(2, 10)$

03 01 ex 16 16. min: $(5, -134.5)$
max: $(0, 3)$

03 01 ex 17 17. min: $(\pi/4, 3\sqrt{2}/2)$
max: $(\pi/2, 3)$

03 01 ex 18 18. min: $(0, 0)$ and $(\pm 2, 0)$
max: $(\pm 2\sqrt{2/3}, 16\sqrt{3}/9)$

03 01 ex 19 19. min: $(\sqrt{3}, 2\sqrt{3})$
max: $(5, 28/5)$

03 01 ex 20 20. min: $(0, 0)$
max: $(5, 5/6)$

03 01 ex 21 21. min: $(\pi, -e^{\pi})$
max: $(\pi/4, \frac{\sqrt{2}e^{\pi/4}}{2})$

03 01 ex 22 22. min: $(0, 0)$ and $(\pi, 0)$
max: $(3\pi/4, \frac{\sqrt{2}e^{3\pi/4}}{2})$

03 01 ex 23 23. min: $(1, 0)$
max: $(e, 1/e)$

03 01 ex 24 24. min: $(2, 2^{2/3} - 2)$
max: $(8/27, 4/27)$

02 07 ex 29 25. $\frac{dy}{dx} = \frac{y(y-2x)}{x(x-2y)}$

02 07 ex 30 26. $y = -4/5(x-1) + 2$

02 07 ex 31 27. $3x^2 + 1$

### Section 3.2

03 02 ex 01 1. Answers will vary.

03 02 ex 02 2. Answers will vary.

03 02 ex 03 3. Any $c$ in $[-1, 1]$ is valid.

03 02 ex 04 4. Rolle's Thm. does not apply.

03 02 ex 05 5. $c = -1/2$

03 02 ex 06 6. $c = -1/2$

03 02 ex 07 7. Rolle's Thm. does not apply.

03 02 ex 08 8. $c = \pi/2$

03 02 ex 09 9. Rolle's Thm. does not apply.

03 02 ex 10 10. Rolle's Theorem does not apply.

03 02 ex 11 11. $c = 0$

03 02 ex 12 12. $c = 5/2$

03 02 ex 13 13. $c = 3/\sqrt{2}$

03 02 ex 14 14. $c = 19/4$

03 02 ex 20 15. The Mean Value Theorem does not apply.

03 02 ex 15 16. $c = 4/\ln 5$

03 02 ex 16 17. $c = \pm\sec^{-1}(2/\sqrt{\pi})$

03 02 ex 17 18. $c = -2/3$

03 02 ex 18 19. $c = \frac{5\pm 7\sqrt{7}}{6}$

03 02 ex 19 20. $c = \frac{\pm\sqrt{\pi^2-4}}{\pi}$

03 02 ex 21 21. Max value of 19 at $x = -2$ and $x = 5$; min value of 6.75 at $x = 1.5$.

03 02 ex 22 22. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)

03 02 ex 23 23. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)

### Section 3.3

03 03 ex 01 1. Answers will vary.

03 03 ex 02 2. Answers will vary.

03 03 ex 03 3. Answers will vary.

03 03 ex 04 4. Answers will vary.

03 03 ex 05 5. Increasing

03 03 ex 06 6. Graph and verify.

03 03 ex 07 7. Graph and verify.

03 03 ex 08 8. Graph and verify.

03 03 ex 09 9. Graph and verify.

03 03 ex 10 10. Graph and verify.

03 03 ex 11 11. Graph and verify.

03 03 ex 12 12. Graph and verify.

03 03 ex 13 13. Graph and verify.

03 03 ex 14 14. domain: $(-\infty, \infty)$
c.p. at $c = -1$;
decreasing on $(-\infty, -1)$;
increasing on $(-1, \infty)$;
rel. min at $x = -1$.

03 03 ex 15 15. domain=$(-\infty, \infty)$
c.p. at $c = -2, 0$;
increasing on $(-\infty, -2) \cup (0, \infty)$;
decreasing on $(-2, 0)$;
rel. min at $x = 0$;
rel. max at $x = -2$.

03 03 ex 16 16. domain=$(-\infty, \infty)$
c.p. at $c = \frac{1}{6}(-1 \pm \sqrt{7})$;
decreasing on $(\frac{1}{6}(-1 - \sqrt{7}), \frac{1}{6}(-1 + \sqrt{7})))$;
increasing on $(-\infty, \frac{1}{6}(-1 - \sqrt{7})) \cup (\frac{1}{6}(-1 + \sqrt{7}), \infty)$;
rel. min at $x = \frac{1}{6}(-1 + \sqrt{7})$;
rel. max at $x = \frac{1}{6}(-1 - \sqrt{7})$.

03 03 ex 17 17. domain=$(-\infty, \infty)$
c.p. at $c = 1$;
increasing on $(-\infty, \infty)$;

03 03 ex 18 18. domain=$(-\infty, \infty)$
c.p. at $c = 1$;
decreasing on $(1, \infty)$
increasing on $(-\infty, 1)$;
rel. max at $x = 1$.

03 03 ex 19 19. domain=$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
c.p. at $c = 0$;
decreasing on $(-\infty, -1) \cup (-1, 0)$;
increasing on $(0, 1) \cup (1, \infty)$;
rel. min at $x = 0$;

03 03 ex 20 20. domain=$(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$
no c.p.;
decreasing on entire domain, $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$

03 03 ex 21 21. domain=$(-\infty, 0) \cup (0, \infty)$;
c.p. at $c = 2, 6$;
decreasing on $(-\infty, 0) \cup (0, 2) \cup (6, \infty)$;
increasing on $(2, 6)$;
rel. min at $x = 2$; rel. max at $x = 6$.

03 03 ex 22 22. domain=$(-\infty, \infty)$
c.p. at $c = -3\pi/4, -\pi/4, \pi/4, 3\pi/4$;
decreasing on $(-3\pi/4, -\pi/4) \cup (\pi/4, 3\pi/4)$;
increasing on $(-\pi, -3\pi/4) \cup (-\pi/4, \pi/4) \cup (3\pi/4, \pi)$;
rel. min at $x = -\pi/4, 3\pi/4$;
rel. max at $x = -3\pi/4, \pi/4$.

03 03 ex 23 23. domain = $(-\infty, \infty)$;
c.p. at $c = -1, 1$;
decreasing on $(-1, 1)$;
increasing on $(-\infty, -1) \cup (1, \infty)$;
rel. min at $x = 1$;
rel. max at $x = -1$

03 03 ex 24 24. $c = 1/2$

03 03 ex 25 25. $c = \pm \cos^{-1}(2/\pi)$

**Section 3.4**

03 04 ex 01 1. Answers will vary.

03 04 ex 02 2. Answers will vary.

03 04 ex 03 3. Yes; Answers will vary.

03 04 ex 04 4. No.

03 04 ex 05 5. Graph and verify.

03 04 ex 06 6. Graph and verify.

03 04 ex 07 7. Graph and verify.

03 04 ex 08 8. Graph and verify.

03 04 ex 09 9. Graph and verify.

03 04 ex 10 10. Graph and verify.

03 04 ex 11 11. Graph and verify.

03 04 ex 12 12. Graph and verify.

03 04 ex 13 13. Graph and verify.

03 04 ex 14 14. Graph and verify.

03 04 ex 15 15. Graph and verify.

03 04 ex 16 16. Possible points of inflection: none; concave up on $(-\infty, \infty)$

03 04 ex 17 17. Possible points of inflection: none; concave down on $(-\infty, \infty)$

03 04 ex 18 18. Possible points of inflection: $x = 0$; concave down on $(-\infty, 0)$; concave up on $(0, \infty)$

03 04 ex 19 19. Possible points of inflection: $x = 1/2$; concave down on $(-\infty, 1/2)$; concave up on $(1/2, \infty)$

03 04 ex 20 20. Possible points of inflection: $x = -2/3, 0$; concave down on $(-2/3, 0)$; concave up on $(-\infty, -2/3) \cup (0, \infty)$

03 04 ex 21 21. Possible points of inflection: $x = (1/3)(2 \pm \sqrt{7})$; concave up on $((1/3)(2 - \sqrt{7}), (1/3)(2 + \sqrt{7}))$; concave down on $(-\infty, (1/3)(2 - \sqrt{7})) \cup ((1/3)(2 + \sqrt{7}), \infty)$

03 04 ex 22 22. Possible points of inflection: $x = 1$; concave up on $(-\infty, \infty)$

03 04 ex 23 23. Possible points of inflection: $x = \pm 1/\sqrt{3}$; concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$; concave up on $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$

03 04 ex 24 24. Possible points of inflection: $x = 0, \pm 1$; concave down on $(-\infty, -1) \cup (0, 1)$ concave up on $(-1, 0) \cup (1, \infty)$

03 04 ex 25 25. Possible points of inflection: $x = -\pi/4, 3\pi/4$; concave down on $(-\pi/4, 3\pi/4)$ concave up on $(-\pi, -\pi/4) \cup (3\pi/4, \pi)$

03 04 ex 26 26. Possible points of inflection: $x = -2 \pm \sqrt{2}$; concave down on $(-2-\sqrt{2}, -2+\sqrt{2})$ concave up on $(-\infty, -2-\sqrt{2}) \cup (-2+\sqrt{2}, \infty)$

03 04 ex 27 27. Possible points of inflection: $x = 1/e^{3/2}$; concave down on $(0, 1/e^{3/2})$ concave up on $(1/e^{3/2}, \infty)$

03 04 ex 28 28. Possible points of inflection: $x = \pm 1/\sqrt{2}$; concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$ concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$

03 04 ex 29 29. min: $x = 1$

03 04 ex 30 30. max: $x = -5/2$

03 04 ex 31 31. max: $x = -1/\sqrt{3}$ min: $x = 1/\sqrt{3}$

03 04 ex 32 32.

03 04 ex 33 33. min: $x = 1$

03 04 ex 34 34. max: $x = -1, 2$; min: $x = 1$

03 04 ex 35 35. min: $x = 1$

03 04 ex 36 36. max: $x = 0$

03 04 ex 37 37. critical values: $x = -1, 1$; no max/min

03 04 ex 38 38. max: $x = \pi/4$; min: $x = -3\pi/4$

03 04 ex 39 39. max: $x = -2$; min: $x = 0$

03 04 ex 40 40. min: $x = 1/\sqrt{e}$

03 04 ex 41 41. max: $x = 0$

03 04 ex 42 42. $f'$ has no maximal or minimal value.

03 04 ex 43 43. $f'$ has no maximal or minimal value

03 04 ex 44 44. $f'$ has a minimal value at $x = 0$

03 04 ex 45 45. $f'$ has a minimal value at $x = 1/2$

03 04 ex 46 46. Possible points of inflection: $x = -2/3, 0$; $f'$ has a relative min at: $x = 0$ ; relative max at: $x = -2/3$

03 04 ex 47 47. $f'$ has a relative max at: $x = (1/3)(2+\sqrt{7})$ relative min at: $x = (1/3)(2-\sqrt{7})$

03 04 ex 48 48. $f'$ has no relative extrema

03 04 ex 49 49. $f'$ has a relative max at $x = -1/\sqrt{3}$; relative min at $x = 1/\sqrt{3}$

03 04 ex 50 50. $f'$ has a relative max at $x = 0$

03 04 ex 51 51. $f'$ has a relative min at $x = 3\pi/4$; relative max at $x = -\pi/4$

03 04 ex 52 52. $f'$ has a relative max at $x = -2-\sqrt{2}$; relative min at $x = -2+\sqrt{2}$

03 04 ex 53 53. $f'$ has a relative min at $x = 1/\sqrt{e^3} = e^{-3/2}$

03 04 ex 54 54. $f'$ has a relative max at $x = -1/\sqrt{2}$; a relative min at $x = 1/\sqrt{2}$

### Section 3.5

03 05 ex 01 1. Answers will vary.

03 05 ex 02 2. Found everywhere.

03 05 ex 03 3. T

03 05 ex 04 4. T

03 05 ex 05 5. T

03 05 ex 06 6. A good sketch will include the $x$ and $y$ intercepts and draw the appropriate line.

03 05 ex 07 7. A good sketch will include the $x$ and $y$ intercepts..

03 05 ex 08 8. Use technology to verify sketch.

03 05 ex 09 9. Use technology to verify sketch.

03 05 ex 10 10. Use technology to verify sketch.

03 05 ex 11 11. Use technology to verify sketch.

03 05 ex 12 12. Use technology to verify sketch.

03 05 ex 13 13. Use technology to verify sketch.

03 05 ex 14 14. Use technology to verify sketch.

03 05 ex 15 15. Use technology to verify sketch.

03 05 ex 16 16. Use technology to verify sketch.

03 05 ex 17 17. Use technology to verify sketch.

03 05 ex 18 18. Use technology to verify sketch.

03 05 ex 19 19. Use technology to verify sketch.

03 05 ex 20 20. Use technology to verify sketch.

03 05 ex 21 21. Use technology to verify sketch.

03 05 ex 22 22. Use technology to verify sketch.

03 05 ex 23 23. Use technology to verify sketch.

03 05 ex 24 24. Use technology to verify sketch.

03 05 ex 25 25. Use technology to verify sketch.

03 05 ex 26 26. Critical point: $x = 0$ Points of inflection: $\pm b/\sqrt{3}$

03 05 ex 27 27. Critical points: $x = \frac{n\pi/2-b}{a}$, where $n$ is an odd integer
Points of inflection: $(n\pi - b)/a$, where $n$ is an integer.

03 05 ex 28 28. Critical point: $x = (a+b)/2$ Points of inflection: none

03 05 ex 29 29. $\frac{dy}{dx} = -x/y$, so the function is increasing in second and fourth quadrants, decreasing in the first and third quadrants.
$\frac{d^2y}{dx^2} = -1/y - x^2/y^3$, which is positive when $y < 0$ and is negative when $y > 0$. Hence the function is concave down in the first and second quadrants and concave up in the third and fourth quadrants.

# Chapter 4

### Section 4.1

04 01 ex 01 1. F

04 01 ex 02 2. F

04 01 ex 03 3. $x_0 = 1.5$, $x_1 = 1.5709148$, $x_2 = 1.5707963$, $x_3 = 1.5707963$, $x_4 = 1.5707963$, $x_5 = 1.5707963$

04 01 ex 04 4. $x_0 = 1$, $x_1 = -0.55740772$, $x_2 = 0.065936452$, $x_3 = -0.000095721919$, $x_4 = 2.9235662 * 10^{-13}$, $x_5 = 0$

04 01 ex 05 5. $x_0 = 0$, $x_1 = 2$, $x_2 = 1.2$, $x_3 = 1.0117647$, $x_4 = 1.0000458$, $x_5 = 1$

04 01 ex 06 6. $x_0 = 1.5$, $x_1 = 1.4166667$, $x_2 = 1.4142157$, $x_3 = 1.4142136$, $x_4 = 1.4142136$, $x_5 = 1.4142136$

04 01 ex 07 7. $x_0 = 2$, $x_1 = 0.6137056389$, $x_2 = 0.9133412072$, $x_3 = 0.9961317034$, $x_4 = 0.9999925085$, $x_5 = 1$

04 01 ex 09 8. roots are: $x = -5.156$, $x = -0.369$ and $x = 0.525$

04 01 ex 10 9. roots are: $x = -3.714$, $x = -0.857$, $x = 1$ and $x = 1.571$

04 01 ex 11 10. roots are: $x = -1.013$, $x = 0.988$, and $x = 1.393$

04 01 ex 12 11. roots are: $x = -2.165$, $x = 0$, $x = 0.525$ and $x = 1.813$

04 01 ex 13 12. $x = \pm 0.824$,

04 01 ex 14 13. $x = -0.637$, $x = 1.410$

04 01 ex 15 14. $x = \pm 0.743$

04 01 ex 16 15. $x = \pm 4.493$, $x = 0$

04 01 ex 17 16. The approximations alternate between $x = 1$ and $x = 2$.

04 01 ex 18 17. The approximations alternate between $x = 1$, $x = 2$ and $x = 3$.

**Section 4.2**

04 02 ex 01 1. T

04 02 ex 02 2. F

04 02 ex 03 3.

(a) $5/(2\pi) \approx 0.796$cm/s
(b) $1/(4\pi) \approx 0.0796$ cm/s
(c) $1/(40\pi) \approx 0.00796$ cm/s

04 02 ex 04 4.

(a) $5/(2\pi) \approx 0.796$cm/s
(b) $1/(40\pi) \approx 0.00796$ cm/s
(c) $1/(4000\pi) \approx 0.0000796$ cm/s

04 02 ex 05 5. 63.14mph

04 02 ex 06 6.

(a) 64.44 mph
(b) 78.89 mph

04 02 ex 07 7. Due to the height of the plane, the gun does not have to rotate very fast.

(a) 0.0573 rad/s
(b) 0.0725 rad/s
(c) In the limit, rate goes to 0.0733 rad/s

04 02 ex 08 8. Due to the height of the plane, the gun does not have to rotate very fast.

(a) 0.073 rad/s
(b) 3.66 rad/s (about 1/2 revolution/sec)
(c) In the limit, rate goes to 7.33 rad/s (more than 1 revolution/sec)

04 02 ex 09 9.

(a) 0.04 ft/s
(b) 0.458 ft/s
(c) 3.35 ft/s
(d) Not defined; as the distance approaches 24, the rates approaches $\infty$.

04 02 ex 10 10.

(a) 30.59 ft/min
(b) 36.1 ft/min
(c) 301 ft/min
(d) The boat no longer floats as usual, but is being pulled up by the winch (assuming it has the power to do so).

04 02 ex 11 11.

(a) 50.92 ft/min
(b) 0.509 ft/min
(c) 0.141 ft/min

As the tank holds about 523.6ft$^3$, it will take about 52.36 minutes.

04 02 ex 12 12.

(a) 0.63 ft/sec
(b) 1.6 ft/sec

About 52 ft.

04 02 ex 13 13.

(a) The rope is 80ft long.
(b) 1.71 ft/sec
(c) 1.87 ft/sec
(d) About 34 feet.

04 02 ex 14 14.

(a) The balloon is 105ft in the air.
(b) The balloon is rising at a rate of 17.45ft/min. (Hint: convert all angles to radians.)

04 02 ex 15 15. The cone is rising at a rate of 0.003ft/s.

**Section 4.3**

04 03 ex 01 1. T

04 03 ex 02 2. F

04 03 ex 03 3. 2500; the two numbers are each 50.

04 03 ex 04 4. The minimum sum is $2\sqrt{500}$; the two numbers are each $\sqrt{500}$.

04 03 ex 05 5. There is no maximum sum; the fundamental equation has only 1 critical value that corresponds to a minimum.

04 03 ex 06 6. The only critical point of the fundamental equation corresponds to a minimum; to find maximum, we check the endpoints.

If one number is 300, the other number $y$ satisfies $300y = 500$; $y = 5/3$. Thus the sum is $300 + 5/3$.

The other endpoint, 0, is not feasible as we cannot solve $0 \cdot y = 500$ for $y$. In fact, if $0 < x < 5/3$, then $x \cdot y = 500$ forces $y > 300$, which is not a feasible solution.

Hence the maximum sum is $301.\overline{6}$.

04 03 ex 07 7. Area = 1/4, with sides of length $1/\sqrt{2}$.

04 03 ex 08 8. Each pen should be $500/3 \approx 166.67$ feet by 125 feet.

04 03 ex 09 9. The radius should be about 3.84cm and the height should be $2r = 7.67$cm. No, this is not the size of the standard can.

04 03 ex 10 10. The radius should be about 3.2in and the height should be $2r = 6.4$in. As the #10 is not a perfect cylinder (with extra material to aid in stacking, etc.), the dimensions are close enough to assume that minimizing surface area was a consideration.

04 03 ex 11 11. The height and width should be 18 and the length should be 36, giving a volume of $11,664\text{in}^3$.

04 03 ex 12 12. $w = 4\sqrt{3}$, $h = 4\sqrt{6}$

04 03 ex 13 13. $5 - 10/\sqrt{39} \approx 3.4$ miles should be run underground, giving a minimum cost of \$374,899.96.

04 03 ex 14 14. The power line should be run directly to the off shore facility, skipping any underground, giving a cost of about \$430,813.

04 03 ex 15 15. The dog should run about 19 feet along the shore before starting to swim.

04 03 ex 16 16. The dog should run about 13 feet along the shore before starting to swim.

04 03 ex 17 17. The largest area is 2 formed by a square with sides of length $\sqrt{2}$.

**Section 4.4**

04 04 ex 01 1. T

04 04 ex 02 2. T

04 04 ex 03 3. F

04 04 ex 04 4. T

04 04 ex 05 5. Answers will vary.

04 04 ex 06 6. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 2$ and $dx = 0.05$. Thus $dy = .2$; knowing $2^2 = 4$, we have $2.05^2 \approx 4.2$.

04 04 ex 07 7. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 6$ and $dx = -0.07$. Thus $dy = -0.84$; knowing $6^2 = 36$, we have $5.93^2 \approx 35.16$.

04 04 ex 08 8. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 5$ and $dx = 0.1$. Thus $dy = 7.5$; knowing $5^3 = 125$, we have $5.1^3 \approx 132.5$.

04 04 ex 09 9. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 7$ and $dx = -0.2$. Thus $dy = -29.4$; knowing $7^3 = 343$, we have $6.8^3 \approx 313.6$.

04 04 ex 10 10. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 16$ and $dx = 0.5$. Thus $dy = .0625$; knowing $\sqrt{16} = 4$, we have $\sqrt{16.5} \approx 4.0625$.

04 04 ex 11 11. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 25$ and $dx = -1$. Thus $dy = -0.1$; knowing $\sqrt{25} = 5$, we have $\sqrt{24} \approx 4.9$.

04 04 ex 12 12. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 64$ and $dx = -1$. Thus $dy = -1/48 \approx 0.0208$; we could use $-1/48 \approx -1/50 = -0.02$; knowing $\sqrt[3]{64} = 4$, we have $\sqrt[3]{63} \approx 3.98$.

04 04 ex 13 13. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 8$ and $dx = 0.5$. Thus $dy = 1/24 \approx 1/25 = 0.04$; knowing $\sqrt[3]{8} = 2$, we have $\sqrt[3]{8.5} \approx 2.04$.

04 04 ex 14 14. Use $y = \sin x$; $dy = \cos x \cdot dx$ with $x = \pi$ and $dx \approx -0.14$. Thus $dy = 0.14$; knowing $\sin \pi = 0$, we have $\sin 3 \approx 0.14$.

04 04 ex 15 15. Use $y = \cos x$; $dy = -\sin x \cdot dx$ with $x = \pi/2 \approx 1.57$ and $dx \approx -0.07$. Thus $dy = 0.07$; knowing $\cos \pi/2 = 0$, we have $\cos 1.5 \approx 0.07$.

04 04 ex 16 16. Use $y = e^x$; $dy = e^x \cdot dx$ with $x = 0$ and $dx = 0.1$. Thus $dy = 0.1$; knowing $e^0 = 1$, we have $e^{0.1} \approx 1.1$.

04 04 ex 17 17. $dy = (2x + 3)dx$

04 04 ex 18 18. $dy = (7x^6 - 5x^4)dx$

04 04 ex 19 19. $dy = \frac{-2}{4x^3}dx$

04 04 ex 20 20. $dy = 2(2x + \sin x)(2 + \cos x)dx$

04 04 ex 21 21. $dy = \left(2xe^{3x} + 3x^2e^{3x}\right)dx$

04 04 ex 22 22. $dy = \frac{-16}{x^5}dx$

04 04 ex 23 23. $dy = \frac{2(\tan x + 1) - 2x\sec^2 x}{(\tan x + 1)^2}dx$

04 04 ex 24 24. $dy = \frac{1}{x}dx$

04 04 ex 25 25. $dy = (e^x \sin x + e^x \cos x)dx$

04 04 ex 26 26. $dy = (-\sin(\sin x)\cos x)dx$

04 04 ex 27 27. $dy = \frac{1}{(x+2)^2}dx$

04 04 ex 28 28. $dy = ((\ln 3)3^x \ln x + \frac{3^x}{x})dx$

04 04 ex 29 29. $dy = (\ln x)dx$

04 04 ex 30 30. $dV = \pm 0.157$

04 04 ex 31 31.

(a) $\pm 12.8$ feet

(b) $\pm 32$ feet

04 04 ex 32 32. $\pm 15\pi/8 \approx \pm 5.89\text{in}^2$

04 04 ex 33 33. $\pm 48\text{in}^2$, or $1/3\text{ft}^2$

04 04 ex 35 34.

(a) 297.8 feet

(b) $\pm 62.3$ ft

(c) $\pm 20.9\%$

04 04 ex 36 35.

(a) 298.8 feet

(b) $\pm 17.3$ ft

(c) $\pm 5.8\%$

04 04 ex 34 36.

(a) 298.9 feet

(b) $\pm 8.67$ ft

(c) $\pm 2.9\%$

04 04 ex 37 37. The isosceles triangle setup works the best with the smallest percent error.

04 04 ex 38 38. 1%

# Chapter 5

**Section 5.1**

05 01 ex 01 1. Answers will vary.

05 01 ex 02 2. "an"

05 01 ex 03 3. Answers will vary.

05 01 ex 04 4. opposite; opposite

05 01 ex 05 5. Answers will vary.

05 01 ex 06 6. velocity

05 01 ex 07 7. velocity

05 01 ex 08 8. $3/4x^4 + C$

05 01 ex 09 9. $1/9x^9 + C$

05 01 ex 10 10. $10/3x^3 - 2x + C$

05 01 ex 11 11. $t + C$

05 01 ex 12 12. $s + C$

05 01 ex 13 13. $-1/(3t) + C$

05 01 ex 14 14. $-3/(t) + C$

05 01 ex 15 15. $2\sqrt{x} + C$

05 01 ex 16 16. $\tan\theta + C$

05 01 ex 17 17. $-\cos\theta + C$

05 01 ex 18 18. $\sec x - \csc x + C$

05 01 ex 19 19. $5e^\theta + C$

05 01 ex 20 20. $3^t/\ln 3 + C$

05 01 ex 21 21. $\frac{5^t}{2\ln 5} + C$

05 01 ex 22 22. $4/3t^3 + 6t^2 + 9t + C$

05 01 ex 23 23. $t^6/6 + t^4/4 - 3t^2 + C$

05 01 ex 24 24. $x^6/6 + C$

05 01 ex 25 25. $e^\pi x + C$

05 01 ex 26 26. $ax + C$

05 01 ex 39 27.

(a) $x > 0$

(b) $1/x$

(c) $x < 0$

(d) $1/x$

(e) $\ln|x| + C$. Explanations will vary.

05 01 ex 28 28. $-\cos x + 3$

05 01 ex 29 29. $5e^x + 5$

05 01 ex 30 30. $x^4 - x^3 + 7$

05 01 ex 31 31. $\tan x + 4$

05 01 ex 32 32. $7^x/\ln 7 + 1 - 49/\ln 7$

05 01 ex 33 33. $5/2x^2 + 7x + 3$

05 01 ex 34 34. $\frac{7x^3}{6} - \frac{9x}{2} + \frac{40}{3}$

05 01 ex 35 35. $5e^x - 2x$

05 01 ex 36 36. $\theta - \sin(\theta) - \pi + 4$

05 01 ex 37 37. $\frac{2x^4\ln^2(2)+2^x+x\ln 2)(\ln 32-1)+\ln^2(2)\cos(x)-1-\ln^2(2)}{\ln^2(2)}$

05 01 ex 38 38. $3x - 2$

05 01 ex 40 39. No answer provided.

05 01 ex 41 40. $dy = (2xe^x\cos x + x^2e^x\cos x - x^2e^x\sin x)dx$

**Section 5.2**

05 02 ex 01 1. Answers will vary.

05 02 ex 02 2. Answers will vary.

05 02 ex 03 3. 0

05 02 ex 04 4. $\int 0^2(2x+3)\,dx$

05 02 ex 05 5.

(a) 3

(b) 4

(c) 3

(d) 0

(e) $-4$

(f) 9

05 02 ex 06 6.

(a) $-4$

(b) $-5$

(c) $-3$

(d) 1

(e) $-2$

(f) 10

05 02 ex 07 7.

(a) 4

(b) 2

(c) 4

(d) 2

(e) 1

(f) 2

05 02 ex 08 8.

(a) $-1/2$

(b) 0

(c) $3/2$

(d) $3/2$

(e) $9/2$

(f) $15/2$

05 02 ex 09 9.

(a) $\pi$

(b) $\pi$

(c) $2\pi$

(d) $10\pi$

05 02 ex 10 10.

(a) $-59$

(b) $-48$

(c) $-27$

(d) $-33$

05 02 ex 11 11.

(a) $4/\pi$
(b) $-4/\pi$
(c) $0$
(d) $2/\pi$

05 02 ex 12 12.

(a) $4$
(b) $4$
(c) $-4$
(d) $-2$

05 02 ex 13 13.

(a) $40/3$
(b) $26/3$
(c) $8/3$
(d) $38/3$

05 02 ex 14 14.

(a) 2ft/s
(b) 2ft
(c) 1.5ft

05 02 ex 15 15.

(a) 3ft/s
(b) 9.5ft
(c) 9.5ft

05 02 ex 16 16.

(a) 64ft/s
(b) 64ft
(c) $t = 2$
(d) $t = 2 + \sqrt{7} \approx 4.65$ seconds

05 02 ex 17 17.

(a) 96ft/s
(b) 6 seconds
(c) 6 seconds
(d) Never; the maximum height is 208ft.

05 02 ex 18 18. $2$

05 02 ex 19 19. $5$

05 02 ex 20 20. $16$

05 02 ex 21 21. Answers can vary; one solution is $a = -2$, $b = 7$

05 02 ex 22 22. $24$

05 02 ex 23 23. $-7$

05 02 ex 24 24. $-7$

05 02 ex 25 25. Answers can vary; one solution is $a = -11$, $b = 18$

05 02 ex 26 26. $1/4x^4 - 2/3x^3 + 7/2x^2 - 9x + C$

05 02 ex 27 27. $-\cos x - \sin x + \tan x + C$

05 02 ex 28 28. $3/4t^{4/3} - 1/t + 2^t/\ln 2 + C$

05 02 ex 29 29. $\ln|x| + \csc x + C$

**Section 5.3**

05 03 ex 01 1. limits

05 03 ex 02 2. 14

05 03 ex 03 3. Rectangles.

05 03 ex 04 4. T

05 03 ex 05 5. $2^2 + 3^2 + 4^2 = 29$

05 03 ex 06 6. $-6 - 2 + 2 + 6 + 10 = 10$

05 03 ex 07 7. $0 - 1 + 0 + 1 + 0 = 0$

05 03 ex 08 8. $1 + 1/2 + 1/3 + 1/4 + 1/5 = 137/60$

05 03 ex 09 9. $-1 + 2 - 3 + 4 - 5 + 6 = 3$

05 03 ex 10 10. $1/2 + 1/6 + 1/12 + 1/20 = 4/5$

05 03 ex 11 11. $1 + 1 + 1 + 1 + 1 + 1 = 6$

05 03 ex 12 12. Answers may vary; $\sum_{i=1}^{5} 3i$

05 03 ex 13 13. Answers may vary; $\sum_{i=0}^{8} (i^2 - 1)$

05 03 ex 14 14. Answers may vary; $\sum_{i=1}^{4} \frac{i}{i+1}$

05 03 ex 15 15. Answers may vary; $\sum_{i=0}^{4} (-1)^i e^i$

05 03 ex 16 16. 325

05 03 ex 17 17. 1045

05 03 ex 18 18. 28,650

05 03 ex 19 19. $-8525$

05 03 ex 20 20. 2050

05 03 ex 21 21. 5050

05 03 ex 22 22. 2870

05 03 ex 23 23. 155

05 03 ex 24 24. $91,225$

05 03 ex 25 25. 24

05 03 ex 26 26. $11,700$

05 03 ex 27 27. 19

05 03 ex 28 28. $59/8$

05 03 ex 29 29. $\pi/3 + \pi/(2\sqrt{3}) \approx 1.954$

05 03 ex 30 30. 8.144

05 03 ex 31 31. 0.388584

05 03 ex 32 32. $496/315 \approx 1.5746$

05 03 ex 33 33.

(a) Exact expressions will vary; $\frac{(1+n)^2}{4n^2}$.
(b) $121/400$, $10201/40000$, $1002001/4000000$
(c) $1/4$

05 03 ex 34 34.

(a) Exact expressions will vary; $2 + 4/n^2$.
(b) $51/25$, $5001/2500$, $500001/250000$
(c) $2$

05 03 ex 35 35.

(a) 8.

(b) 8, 8, 8

(c) 8

05 03 ex 36 36.

(a) Exact expressions will vary; $20/3 - 96/(3n) + 64/(3n^2)$.

(b) $92/25, 3968/625, 103667/15625$

(c) $20/3$

05 03 ex 37 37.

(a) Exact expressions will vary; $100 - 200/n$.

(b) $80, 98, 499/5$

(c) 100

05 03 ex 38 38.

(a) Exact expressions will vary; $-1/12(1 - 1/n^2)$.

(b) $-33/400, -3333/40000, -333333/4000000$

(c) $-1/12$

05 03 ex 39 39. $F(x) = 5\tan x + 4$

05 03 ex 40 40. $F(x) = 7\ln|x| + 14$

05 03 ex 41 41. $G(t) = 4/6t^6 - 5/4t^4 + 8t + 9$

05 03 ex 42 42. $G(t) = 5 \cdot 8^t/\ln 8 + 900$

05 03 ex 43 43. $G(t) = \sin t - \cos t - 78$

05 03 ex 44 44. $F(x) = 2\sqrt{x} - \pi$

**Section 5.4**

05 04 ex 01 1. Answers will vary.

05 04 ex 02 2. 0

05 04 ex 03 3. T

05 04 ex 04 4. Answers will vary.

05 04 ex 05 5. 20

05 04 ex 06 6. $28/3$

05 04 ex 07 7. 0

05 04 ex 08 8. 1

05 04 ex 09 9. 1

05 04 ex 10 10. 1

05 04 ex 11 11. $(5 - 1/5)/\ln 5$

05 04 ex 12 12. $23/2$

05 04 ex 13 13. $-4$

05 04 ex 14 14. $e^3 - e$

05 04 ex 15 15. $16/3$

05 04 ex 16 16. 4

05 04 ex 17 17. $45/4$

05 04 ex 18 18. $\ln 2$

05 04 ex 19 19. $1/2$

05 04 ex 20 20. $3/8$

05 04 ex 21 21. $1/2$

05 04 ex 22 22. $1/3$

05 04 ex 23 23. $1/4$

05 04 ex 24 24. $1/101$

05 04 ex 25 25. 8

05 04 ex 26 26. 15

05 04 ex 27 27. 0

05 04 ex 29 28. $2 - 2/\sqrt{3}$

05 04 ex 28 29. Explanations will vary. A sketch will help.

05 04 ex 30 30. $c = 2/\sqrt{3}$

05 04 ex 31 31. $c = \pm 2/\sqrt{3}$

05 04 ex 32 32. $c = \ln(e - 1) \approx 0.54$

05 04 ex 33 33. $c = 64/9 \approx 7.1$

05 04 ex 34 34. $2/\pi$

05 04 ex 35 35. $2/pi$

05 04 ex 36 36. 2

05 04 ex 37 37. $16/3$

05 04 ex 38 38. 16

05 04 ex 39 39. $1/(e - 1)$

05 04 ex 40 40. $-300$ft

05 04 ex 41 41. 400ft

05 04 ex 42 42. $1.5/\ln(2) \approx 2.164$miles

05 04 ex 43 43. $-1$ft

05 04 ex 44 44. $128/5$ft

05 04 ex 45 45. $-64$ft/s

05 04 ex 46 46. 50ft/s

05 04 ex 47 47. 2ft/s

05 04 ex 48 48. 0ft/s

05 04 ex 51 49. $27/2$

05 04 ex 52 50. 21

05 04 ex 53 51. $9/2$

05 04 ex 54 52. $343/6$

05 04 ex 55 53. $F'(x) = (3x^2 + 1)\frac{1}{x^3 + x}$

05 04 ex 56 54. $F'(x) = 3x^{11}$

05 04 ex 57 55. $F'(x) = 2x(x^2 + 2) - (x + 2)$

05 04 ex 58 56. $F'(x) = e^x \sin(e^x) - 1/x \sin(\ln x)$

**Section 5.5**

05 05 ex 23 1. F

05 05 ex 24 2. When the antiderivative cannot be computed and when the integrand is unknown.

05 05 ex 25 3. They are superseded by the Trapezoidal Rule; it takes an equal amount of work and is generally more accurate.

05 05 ex 03 4.

(a) $3/4$

(b) $2/3$

(c) $2/3$

05 05 ex 04 5.

(a) 250

(b) 250

(c) 250

05 05 ex 05 6.

(a) $\frac{1}{4}(1+\sqrt{2})\pi \approx 1.896$

(b) $\frac{1}{6}(1+2\sqrt{2})\pi \approx 2.005$

(c) 2

05 05 ex 06 7.

(a) $2+\sqrt{2}+\sqrt{3} \approx 5.15$

(b) $2/3(3+\sqrt{2}+2\sqrt{3}) \approx 5.25$

(c) $16/3 \approx 5.33$

05 05 ex 07 8.

(a) 38.5781

(b) $147/4 \approx 36.75$

(c) $147/4 \approx 36.75$

05 05 ex 08 9.

(a) 0.2207

(b) 0.2005

(c) $1/5$

05 05 ex 09 10.

(a) 0

(b) 0

(c) 0

05 05 ex 10 11.

(a) $9/2(1+\sqrt{3}) \approx 12.294$

(b) $3+6\sqrt{3} \approx 13.392$

(c) $9\pi/2 \approx 14.137$

05 05 ex 11 12. Trapezoidal Rule: 0.9006
Simpson's Rule: 0.90452

05 05 ex 12 13. Trapezoidal Rule: 3.0241
Simpson's Rule: 2.9315

05 05 ex 13 14. Trapezoidal Rule: 13.9604
Simpson's Rule: 13.9066

05 05 ex 14 15. Trapezoidal Rule: 3.0695
Simpson's Rule: 3.14295

05 05 ex 15 16. Trapezoidal Rule: 1.1703
Simpson's Rule: 1.1873

05 05 ex 16 17. Trapezoidal Rule: 2.52971
Simpson's Rule: 2.5447

05 05 ex 17 18. Trapezoidal Rule: 1.0803
Simpson's Rule: 1.077

05 05 ex 18 19. Trapezoidal Rule: 3.5472
Simpson's Rule: 3.6133

05 05 ex 19 20.

(a) $n = 161$ (using $\max\left(f''(x)\right) = 1$)

(b) $n = 12$ (using $\max\left(f^{(4)}(x)\right) = 1$)

05 05 ex 20 21.

(a) $n = 150$ (using $\max\left(f''(x)\right) = 1$)

(b) $n = 18$ (using $\max\left(f^{(4)}(x)\right) = 7$)

05 05 ex 21 22.

(a) $n = 1004$ (using $\max\left(f''(x)\right) = 39$)

(b) $n = 62$ (using $\max\left(f^{(4)}(x)\right) = 800$)

05 05 ex 22 23.

(a) $n = 5591$ (using $\max\left(f''(x)\right) = 300$)

(b) $n = 46$ (using $\max\left(f^{(4)}(x)\right) = 24$)

05 05 ex 01 24.

(a) Area is 30.8667 cm$^2$.

(b) Area is $308,667$ yd$^2$.

05 05 ex 02 25.

(a) Area is 25.0667 cm$^2$

(b) Area is 250,667 yd$^2$

# Chapter 6

### Section 6.1

06 01 ex 01 1. Chain Rule.

06 01 ex 02 2. T

06 01 ex 03 3. $\frac{1}{8}(x^3-5)^8 + C$

06 01 ex 04 4. $\frac{1}{4}(x^2-5x+7)^4 + C$

06 01 ex 05 5. $\frac{1}{18}\left(x^2+1\right)^9 + C$

06 01 ex 06 6. $\frac{1}{3}(3x^2+7x-1)^6 + C$

06 01 ex 11 7. $\frac{1}{2}\ln|2x+7| + C$

06 01 ex 12 8. $\sqrt{2x+3} + C$

06 01 ex 13 9. $\frac{2}{3}(x+3)^{3/2} - 6(x+3)^{1/2} + C = \frac{2}{3}(x-6)\sqrt{x+3} + C$

06 01 ex 17 10. $\frac{2}{21}x^{3/2}\left(3x^2-7\right) + C$

06 01 ex 18 11. $2e^{\sqrt{x}} + C$

06 01 ex 19 12. $\frac{2\sqrt{x^5+1}}{5} + C$

06 01 ex 20 13. $-\frac{1}{2x^2} - \frac{1}{x} + C$

06 01 ex 21 14. $\frac{\ln^2(x)}{2} + C$

06 01 ex 08 15. $\frac{\sin^3(x)}{3} + C$

06 01 ex 09 16. $-\frac{1}{6}\sin(3-6x) + C$

06 01 ex 10 17. $-\tan(4-x) + C$

06 01 ex 16 18. $\frac{1}{2}\ln|\sec(2x)+\tan(2x)| + C$

06 01 ex 22 19. $\frac{\tan^3(x)}{3} + C$

06 01 ex 25 20. $\frac{\sin\left(x^2\right)}{2} + C$

06 01 ex 27 21. $\tan(x) - x + C$

06 01 ex 82 22. The key is to rewrite $\cot x$ as $\cos x/\sin x$, and let $u = \sin x$.

06 01 ex 83 23. The key is to multiply $\csc x$ by 1 in the form $(\csc x + \cot x)/(\csc x + \cot x)$.

06 01 ex 29 24. $\frac{1}{3}e^{3x-1} + C$

06 01 ex 30 25. $\frac{e^{x^3}}{3} + C$

06 01 ex 31 26. $\frac{1}{2}e^{(x-1)^2} + C$

06 01 ex 32 27. $x - e^{-x} + C$

06 01 ex 33 28. $\frac{e^{-3x}}{3} - e^{-x} + C$

06 01 ex 34 29. $\frac{27^x}{\ln 27} + C$

06 01 ex 35 30. $\frac{16^x}{\ln(16)} + C$

06 01 ex 36 31. $\frac{1}{2}\ln^2(x) + C$

06 01 ex 37 32. $\frac{\left(\ln x\right)^3}{3} + C$

06 01 ex 38 33. $\frac{1}{6}\ln^2\left(x^3\right) + C$

06 01 ex 39 34. $\frac{1}{2}\ln\left(\ln\left(x^2\right)\right) + C$

06 01 ex 40 35. $\frac{x^2}{2} + 3x + \ln|x| + C$

06 01 ex 41 36. $\frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x| + C$

06 01 ex 42 37. $\frac{x^3}{3} - \frac{x^2}{2} + x - 2\ln|x+1| + C$

06 01 ex 43 38. $\frac{1}{2}\left(x^2 + 10x + 20\ln|x-3|\right) + C$

06 01 ex 44 39. $\frac{3}{2}x^2 - 8x + 15\ln|x+1| + C$

06 01 ex 45 40. $\frac{1}{3}\ln\left|x^2 + 3x + 3\right| + \frac{\ln|x|}{3} + C$

06 01 ex 50 41. $\sqrt{7}\tan^{-1}\left(\frac{x}{\sqrt{7}}\right) + C$

06 01 ex 51 42. $3\sin^{-1}\left(\frac{x}{3}\right) + C$

06 01 ex 52 43. $14\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

06 01 ex 53 44. $\frac{2}{3}\sec^{-1}(|x|/3) + C$

06 01 ex 54 45. $\frac{5}{4}\sec^{-1}(|x|/4) + C$

06 01 ex 55 46. $\frac{1}{2}\sin^{-1}\left(x^2\right) + C$

06 01 ex 60 47. $\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{7}}\right)}{\sqrt{7}} + C$

06 01 ex 61 48. $2\sin^{-1}\left(\frac{x-3}{4}\right) + C$

06 01 ex 62 49. $3\sin^{-1}\left(\frac{x-4}{5}\right) + C$

06 01 ex 63 50. $\tan^{-1}\left(\frac{x+3}{5}\right) + C$

06 01 ex 14 51. $-\frac{1}{3\left(x^3+3\right)} + C$

06 01 ex 07 52. $\frac{1}{45}(5x^3 + 5x^2 + 2)^9 + C$

06 01 ex 15 53. $-\sqrt{1-x^2} + C$

06 01 ex 26 54. $-\frac{1}{3}\cot\left(x^3 + 1\right) + C$

06 01 ex 28 55. $-\frac{2}{3}\cos^{\frac{3}{2}}(x) + C$

06 01 ex 23 56. $\ln|x-5| + C$

06 01 ex 24 57. $\frac{7}{3}\ln|3x+2| + C$

06 01 ex 46 58. $\frac{3x^2}{2} + \ln\left|x^2 + 3x + 5\right| - 5x + C$

06 01 ex 47 59. $\ln\left|x^2 + 7x + 3\right| + C$

06 01 ex 48 60. $3\ln\left|3x^2 + 9x + 7\right| + C$

06 01 ex 49 61. $-\frac{x^2}{2} + 2\ln\left|x^2 - 7x + 1\right| + 7x + C$

06 01 ex 56 62. $\frac{1}{18}\tan^{-1}\left(\frac{x^2}{9}\right) + C$

06 01 ex 57 63. $\tan^{-1}(2x) + C$

06 01 ex 58 64. $\sec^{-1}(|2x|) + C$

06 01 ex 59 65. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

06 01 ex 64 66. $\frac{3}{2}\ln\left|x^2 - 2x + 10\right| + \frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + C$

06 01 ex 65 67. $\frac{19}{5}\tan^{-1}\left(\frac{x+6}{5}\right) - \ln\left|x^2 + 12x + 61\right| + C$

06 01 ex 66 68. $\frac{15}{2}\ln\left|x^2 - 10x + 32\right| + x + \frac{41\tan^{-1}\left(\frac{x-5}{\sqrt{7}}\right)}{\sqrt{7}} + C$

06 01 ex 67 69. $\frac{x^2}{2} - \frac{9}{2}\ln\left|x^2 + 9\right| + C$

06 01 ex 68 70. $\frac{x^2}{2} + 3\ln\left|x^2 + 4x + 9\right| - 4x + \frac{24\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)}{\sqrt{5}} + C$

06 01 ex 69 71. $-\tan^{-1}(\cos(x)) + C$

06 01 ex 70 72. $\tan^{-1}(\sin(x)) + C$

06 01 ex 71 73. $\ln|\sec x + \tan x| + C$ (integrand simplifies to $\sec x$)

06 01 ex 72 74. $3\sqrt{x^2 - 2x - 6} + C$

06 01 ex 73 75. $\sqrt{x^2 - 6x + 8} + C$

06 01 ex 74 76. $-\ln 2$

06 01 ex 75 77. $352/15$

06 01 ex 76 78. $2/3$

06 01 ex 77 79. $1/5$

06 01 ex 78 80. $(1-e)/2$

06 01 ex 79 81. $\pi/2$

06 01 ex 80 82. $\pi/2$

06 01 ex 81 83. $\pi/6$

**Section 6.2**

06 02 ex 01 1. T

06 02 ex 02 2. F

06 02 ex 03 3. Determining which functions in the integrand to set equal to "$u$" and which to set equal to "$dv$".

06 02 ex 04 4. $\sin x - x\cos x + C$

06 02 ex 05 5. $-e^{-x} - xe^{-x} + C$

06 02 ex 06 6. $-x^2\cos x + 2x\sin x + 2\cos x + C$

06 02 ex 07 7. $-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

06 02 ex 08 8. $1/2e^{x^2} + C$

06 02 ex 09 9. $x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

06 02 ex 10 10. $-\frac{1}{2}xe^{-2x} - \frac{e^{-2x}}{4} + C$

06 02 ex 11 11. $1/2e^x(\sin x - \cos x) + C$

06 02 ex 12 12. $1/5e^{2x}(\sin x + 2\cos x) + C$

06 02 ex 13 13. $1/13e^{2x}(2\sin(3x) - 3\cos(3x)) + C$

06 02 ex 14 14. $1/10e^{5x}(\sin(5x) + \cos(5x)) + C$

06 02 ex 15 15. $-1/2\cos^2 x + C$

06 02 ex 16 16. $\sqrt{1-x^2}+x\sin^{-1}(x)+C$

06 02 ex 17 17. $x\tan^{-1}(2x)-\frac{1}{4}\ln\left|4x^2+1\right|+C$

06 02 ex 18 18. $\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+C$

06 02 ex 19 19. $\sqrt{1-x^2}+x\sin^{-1}x+C$

06 02 ex 22 20. $\frac{1}{2}x^2\ln|x|-\frac{x^2}{4}+C$

06 02 ex 23 21. $-\frac{x^2}{4}+\frac{1}{2}x^2\ln|x|+2x-2x\ln|x|+C$

06 02 ex 24 22. $-\frac{x^2}{4}+\frac{1}{2}x^2\ln|x-1|-\frac{x}{2}-\frac{1}{2}\ln|x-1|+C$

06 02 ex 25 23. $\frac{1}{2}x^2\ln\left(x^2\right)-\frac{x^2}{2}+C$

06 02 ex 26 24. $\frac{1}{3}x^3\ln|x|-\frac{x^3}{9}+C$

06 02 ex 27 25. $2x+x\left(\ln|x|\right)^2-2x\ln|x|+C$

06 02 ex 28 26. $2x+x\left(\ln|x+1|\right)+\left(\ln|x+1|\right)^2-$
$2x\ln|x+1|-2\ln|x+1|+2+C$

06 02 ex 29 27. $x\tan(x)+\ln|\cos(x)|+C$

06 02 ex 30 28. $\ln|\sin(x)|-x\cot(x)+C$

06 02 ex 31 29. $\frac{2}{5}(x-2)^{5/2}+\frac{4}{3}(x-2)^{3/2}+C$

06 02 ex 32 30. $\frac{1}{3}(x^2-2)^{3/2}+C$

06 02 ex 35 31. $\sec x+C$

06 02 ex 33 32. $x\sec x-\ln|\sec x+\tan x|+C$

06 02 ex 34 33. $-x\csc x-\ln|\csc x+\cot x|+C$

06 02 ex 36 34. $1/2x\big(\sin(\ln x)-\cos(\ln x)\big)+C$

06 02 ex 37 35. $2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)+C$

06 02 ex 38 36. $\frac{1}{2}x\ln|x|-\frac{x}{2}+C$

06 02 ex 39 37. $2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}+C$

06 02 ex 40 38. $1/2x^2+C$

06 02 ex 41 39. $\pi$

06 02 ex 42 40. $-2/e$

06 02 ex 43 41. $0$

06 02 ex 44 42. $\frac{3\pi^2}{2}-12$

06 02 ex 45 43. $1/2$

06 02 ex 46 44. $6-2e$

06 02 ex 47 45. $\frac{3}{4e^2}-\frac{5}{4e^4}$

06 02 ex 48 46. $\frac{1}{2}+\frac{e^\pi}{2}$

06 02 ex 49 47. $1/5\left(e^\pi+e^{-\pi}\right)$

**Section 6.3**

06 03 ex 01 1. F

06 03 ex 02 2. F

06 03 ex 03 3. F

06 03 ex 04 4. $-\frac{1}{5}\cos^5(x)+C$

06 03 ex 05 5. $\frac{1}{4}\sin^4(x)+C$

06 03 ex 06 6. $\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x+C$

06 03 ex 07 7. $\frac{1}{6}\cos^6x-\frac{1}{4}\cos^4x+C$

06 03 ex 08 8. $\frac{1}{11}\sin^{11}x-\frac{2}{9}\sin^9x+\frac{1}{7}\sin^7x+C$

06 03 ex 09 9. $-\frac{1}{9}\sin^9(x)+\frac{3\sin^7(x)}{7}-\frac{3\sin^5(x)}{5}+\frac{\sin^3(x)}{3}+C$

06 03 ex 10 10. $\frac{x}{8}-\frac{1}{32}\sin(4x)+C$

06 03 ex 11 11. $\frac{1}{2}\left(-\frac{1}{8}\cos(8x)-\frac{1}{2}\cos(2x)\right)+C$

06 03 ex 12 12. $\frac{1}{2}\left(-\frac{1}{3}\cos(3x)+\cos(-x)\right)+C$

06 03 ex 13 13. $\frac{1}{2}\left(\frac{1}{4}\sin(4x)-\frac{1}{10}\sin(10x)\right)+C$

06 03 ex 14 14. $\frac{1}{2}\left(\frac{1}{\pi}\sin(\pi x)-\frac{1}{3\pi}\sin(3\pi x)\right)+C$

06 03 ex 15 15. $\frac{1}{2}\left(\sin(x)+\frac{1}{3}\sin(3x)\right)+C$

06 03 ex 16 16. $\frac{1}{\pi}\sin(\frac{\pi}{2}x)+\frac{1}{3\pi}\sin(\pi x)+C$

06 03 ex 17 17. $\frac{\tan^5(x)}{5}+C$

06 03 ex 18 18. $\frac{1}{5}\tan^5x+\frac{1}{3}\tan^3x+C$

06 03 ex 19 19. $\frac{\tan^6(x)}{6}+\frac{\tan^4(x)}{4}+C$

06 03 ex 20 20. $\frac{\tan^4(x)}{4}+C$

06 03 ex 21 21. $\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}+C$

06 03 ex 22 22. $\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}+C$

06 03 ex 23 23. $\frac{1}{3}\tan^3x-\tan x+x+C$

06 03 ex 24 24. $\frac{1}{4}\tan x\sec^3x+\frac{3}{8}\left(\sec x\tan x+\ln|\sec x+\tan x|\right)+C$

06 03 ex 25 25. $\frac{1}{2}\left(\sec x\tan x-\ln|\sec x+\tan x|\right)+C$

06 03 ex 26 26. $\frac{1}{4}\tan x\sec^3x-\frac{1}{8}\left(\sec x\tan x+\ln|\sec x+\tan x|\right)+C$

06 03 ex 27 27. $\frac{2}{5}$

06 03 ex 28 28. $0$

06 03 ex 29 29. $32/315$

06 03 ex 30 30. $1/2$

06 03 ex 31 31. $2/3$

06 03 ex 32 32. $1/5$

06 03 ex 33 33. $16/15$

**Section 6.4**

06 04 ex 01 1. rational

06 04 ex 02 2. T

06 04 ex 03 3. $\frac{A}{x}+\frac{B}{x-3}$

06 04 ex 04 4. $\frac{A}{x-3}+\frac{B}{x+3}$

06 04 ex 05 5. $\frac{A}{x-\sqrt{7}}+\frac{B}{x+\sqrt{7}}$

06 04 ex 06 6. $\frac{A}{x}+\frac{Bx+C}{x^2+7}$

06 04 ex 07 7. $3\ln|x-2|+4\ln|x+5|+C$

06 04 ex 08 8. $9\ln|x+1|-2\ln|x|+C$

06 04 ex 09 9. $\frac{1}{3}(\ln|x+2|-\ln|x-2|)+C$

06 04 ex 10 10. $\ln|x+5|-\frac{2}{x+5}+C$

06 04 ex 11 11. $-\frac{4}{x+8}-3\ln|x+8|+C$

06 04 ex 12 12. $\frac{5}{x+1}+7\ln|x|+2\ln|x+1|+C$

06 04 ex 13 13. $-\ln|2x-3|+5\ln|x-1|+2\ln|x+3|+C$

06 04 ex 14 14. $-\frac{1}{5}\ln|5x-1|+\frac{2}{3}\ln|3x-1|+\frac{3}{7}\ln|7x+3|+C$

06 04 ex 15 15. $x+\ln|x-1|-\ln|x+2|+C$

06 04 ex 16 16. $\frac{x^2}{2}+x+\frac{125}{9}\ln|x-5|+\frac{64}{9}\ln|x+4|-\frac{35}{2}+C$

06 04 ex 17 17. $2x+C$

06 04 ex 18 18. $\frac{1}{6}\left(-\ln\left|x^2+2x+3\right|+2\ln|x|-\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)\right)+C$

06 04 ex 19 19. $-\frac{3}{2}\ln\left|x^2+4x+10\right|+x+\frac{\tan^{-1}\left(\frac{x+2}{\sqrt{6}}\right)}{\sqrt{6}}+C$

06 04 ex 20 20. $\ln\left|3x^2+5x-1\right|+2\ln|x+1|+C$

06 04 ex 21 21. $2\ln|x-3|+2\ln\left|x^2+6x+10\right|-4\tan^{-1}(x+3)+C$

06 04 ex 22 22. $\frac{9}{10}\ln\left|x^2+9\right|+\frac{1}{5}\ln|x+1|-\frac{4}{15}\tan^{-1}\left(\frac{x}{3}\right)+C$

06 04 ex 23 23. $\frac{1}{2}\left(3\ln\left|x^2+2x+17\right|-4\ln|x-7|+\tan^{-1}\left(\frac{x+1}{4}\right)\right)+C$

06 04 ex 24 24.
$3\left(\ln\left|x^2-2x+11\right|+\ln|x-9|\right)+3\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{x-1}{\sqrt{10}}\right)+C$

06 04 ex 25 25.
$\frac{1}{2}\ln\left|x^2+10x+27\right|+5\ln|x+2|-6\sqrt{2}\tan^{-1}\left(\frac{x+5}{\sqrt{2}}\right)+C$

06 04 ex 26 26. $\ln(2000/243)\approx 2.108$

06 04 ex 27 27. $5\ln(9/4)-\frac{1}{3}\ln(17/2)\approx 3.3413$

06 04 ex 28 28. $-\pi/4+\tan^{-1}3-\ln(11/9)\approx 0.263$

06 04 ex 29 29. $1/8$

**Section 6.5**

06 05 ex 01 1. Because $\cosh x$ is always positive.

06 05 ex 02 2. The points on the left hand side can be defined as $(-\cosh x,\sinh x)$.

06 05 ex 03 3.
$$\begin{aligned}\coth^2 x-\operatorname{csch}^2 x&=\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)^2-\left(\frac{2}{e^x-e^{-x}}\right)^2\\&=\frac{\left(e^{2x}+2+e^{-2x}\right)-(4)}{e^{2x}-2+e^{-2x}}\\&=\frac{e^{2x}-2+e^{-2x}}{e^{2x}-2+e^{-2x}}\\&=1\end{aligned}$$

06 05 ex 04 4.
$$\begin{aligned}\cosh^2 x+\sinh^2 x&=\left(\frac{e^x+e^{-x}}{2}\right)^2+\left(\frac{e^x-e^{-x}}{2}\right)^2\\&=\frac{e^{2x}+2+e^{-2x}}{4}+\frac{e^{2x}-2+e^{-2x}}{4}\\&=\frac{2e^{2x}+2e^{-2x}}{4}\\&=\frac{e^{2x}+e^{-2x}}{2}\\&=\cosh 2x.\end{aligned}$$

06 05 ex 05 5.
$$\begin{aligned}\cosh^2 x&=\left(\frac{e^x+e^{-x}}{2}\right)^2\\&=\frac{e^{2x}+2+e^{-2x}}{4}\\&=\frac{1}{2}\,\frac{\left(e^{2x}+e^{-2x}\right)+2}{2}\\&=\frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}+1\right)\end{aligned}$$

06 05 ex 06 6.
$$\begin{aligned}\sinh^2 x&=\left(\frac{e^x-e^{-x}}{2}\right)^2\\&=\frac{e^{2x}-2+e^{-2x}}{4}\\&=\frac{1}{2}\,\frac{\left(e^{2x}+e^{-2x}\right)-2}{2}\\&=\frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}-1\right)\\&=\frac{\cosh 2x-1}{2}.\end{aligned}$$

06 05 ex 07 7.
$$\begin{aligned}\frac{d}{dx}\left[\operatorname{sech} x\right]&=\frac{d}{dx}\left[\frac{2}{e^x+e^{-x}}\right]\\&=\frac{-2(e^x-e^{-x})}{(e^x+e^{-x})^2}\\&=-\frac{2(e^x-e^{-x})}{(e^x+e^{-x})(e^x+e^{-x})}\\&=-\frac{2}{e^x+e^{-x}}\cdot\frac{e^x-e^{-x}}{e^x+e^{-x}}\\&=-\operatorname{sech} x\tanh x\end{aligned}$$

06 05 ex 08 8.
$$\begin{aligned}\frac{d}{dx}\left[\coth x\right]&=\frac{d}{dx}\left[\frac{e^x+e^{-x}}{e^x-e^{-x}}\right]\\&=\frac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}\\&=\frac{e^{2x}+e^{-2x}-2-\left(e^{2x}+e^{-2x}+2\right)}{(e^x-e^{-x})^2}\\&=-\frac{4}{(e^x-e^{-x})^2}\\&=-\operatorname{csch}^2 x\end{aligned}$$

06 05 ex 09 9. $\displaystyle\int\tanh x\,dx=\int\frac{\sinh x}{\cosh x}\,dx$
Let $u=\cosh x$; $du=(\sinh x)dx$
$$\begin{aligned}&=\int\frac{1}{u}\,du\\&=\ln|u|+C\\&=\ln(\cosh x)+C.\end{aligned}$$

06 05 ex 10 10. $\displaystyle\int\coth x\,dx=\int\frac{\cosh x}{\sinh x}\,dx$
Let $u=\sinh x$; $du=(\cosh x)dx$
$$\begin{aligned}&=\int\frac{1}{u}\,du\\&=\ln|u|+C\\&=\ln|\sinh x|+C.\end{aligned}$$

06 05 ex 11 11. $2\sinh 2x$

06 05 ex 12 12. $2x\sec^2(x^2)$

06 05 ex 13 13. $\coth x$

06 05 ex 14 14. $\sinh^2 x+\cosh^2 x$

06 05 ex 15 15. $x\cosh x$

06 05 ex 16 16. $\frac{-2x}{(x^2)\sqrt{1-x^4}}$

06 05 ex 19 17. $\frac{3}{\sqrt{9x^2+1}}$

06 05 ex 20 18. $\frac{4x}{\sqrt{4x^4-1}}$

06 05 ex 21 19. $\frac{1}{1-(x+5)^2}$

06 05 ex 17 20. $-\csc x$

06 05 ex 18 21. $\sec x$

06 05 ex 22 22. $y = x$

06 05 ex 23 23. $y = 3/4(x - \ln 2) + 5/4$

06 05 ex 24 24. $y = -72/125(x - \ln 3) + 9/25$

06 05 ex 25 25. $y = x$

06 05 ex 26 26. $y = (x - \sqrt{2}) + \cosh^{-1}(\sqrt{2}) \approx (x - 1.414) + 0.881$

06 05 ex 27 27. $1/2 \ln(\cosh(2x)) + C$

06 05 ex 28 28. $1/3 \sinh(3x - 7) + C$

06 05 ex 29 29. $1/2 \sinh^2 x + C$ or $1/2 \cosh^2 x + C$

06 02 ex 20 30. $x \sinh(x) - \cosh(x) + C$

06 02 ex 21 31. $x \cosh(x) - \sinh(x) + C$

06 05 ex 30 32. $\begin{cases} \frac{1}{3}\tanh^{-1}\left(\frac{x}{3}\right) + C & x^2 < 9 \\ \frac{1}{3}\coth^{-1}\left(\frac{x}{3}\right) + C & 9 < x^2 \end{cases} = \frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + C$

06 05 ex 31 33. $\cosh^{-1}(x^2/2) + C = \ln(x^2 + \sqrt{x^4 - 4}) + C$

06 05 ex 32 34. $2/3 \sinh^{-1} x^{3/2} + C = 2/3 \ln(x^{3/2} + \sqrt{x^3 + 1}) + C$

06 05 ex 33 35. $\frac{1}{16}\tan^{-1}(x/2) + \frac{1}{32}\ln|x-2| + \frac{1}{32}\ln|x+2| + C$

06 05 ex 34 36. $\ln x - \ln|x+1| + C$

06 05 ex 35 37. $\tan^{-1}(e^x) + C$

06 05 ex 36 38. $x \sinh^{-1} x - \sqrt{x^2+1} + C$

06 05 ex 37 39. $x \tanh^{-1} x + 1/2 \ln|x^2 - 1| + C$

06 05 ex 38 40. $\tan^{-1}(\sinh x) + C$

06 05 ex 39 41. 0

06 05 ex 40 42. $3/2$

06 05 ex 41 43. 2

**Section 6.6**

06 06 ex 01 1. $0/0, \infty/\infty, 0 \cdot \infty, \infty - \infty, 0^0, 1^\infty, \infty^0$

06 06 ex 02 2. F

06 06 ex 03 3. F

06 06 ex 04 4. The base of an expression is approaching 1 while its power is growing without bound.

06 06 ex 05 5. derivatives; limits

06 06 ex 06 6. Answers will vary.

06 06 ex 07 7. Answers will vary.

06 06 ex 08 8. 3

06 06 ex 09 9. $-5/3$

06 06 ex 10 10. $-1$

06 06 ex 11 11. $-\sqrt{2}/2$

06 06 ex 12 12. 5

06 06 ex 13 13. 0

06 06 ex 14 14. $2/3$

06 06 ex 15 15. $a/b$

06 06 ex 16 16. $\infty$

06 06 ex 17 17. $1/2$

06 06 ex 18 18. 0

06 06 ex 19 19. 0

06 06 ex 20 20. 0

06 06 ex 21 21. $\infty$

06 06 ex 22 22. $\infty$

06 06 ex 23 23. 0

06 06 ex 24 24. 2

06 06 ex 25 25. $-2$

06 06 ex 26 26. 0

06 06 ex 27 27. 0

06 06 ex 28 28. 0

06 06 ex 29 29. 0

06 06 ex 30 30. 0

06 06 ex 52 31. $\infty$

06 06 ex 31 32. $\infty$

06 06 ex 32 33. $\infty$

06 06 ex 33 34. 0

06 06 ex 34 35. 0

06 06 ex 35 36. $e$

06 06 ex 36 37. 1

06 06 ex 37 38. 1

06 06 ex 38 39. 1

06 06 ex 39 40. 1

06 06 ex 40 41. 1

06 06 ex 41 42. 0

06 06 ex 42 43. 1

06 06 ex 43 44. 1

06 06 ex 44 45. 1

06 06 ex 45 46. 1

06 06 ex 46 47. 2

06 06 ex 47 48. $1/2$

06 06 ex 48 49. $-\infty$

06 06 ex 49 50. 1

06 06 ex 50 51. 0

06 06 ex 51 52. 3

**Section 6.7**

06 07 ex 03 1. The interval of integration is finite, and the integrand is continuous on that interval.

06 07 ex 01 2. converge

06 07 ex 02 3. converges; could also state $< 10$.

06 07 ex 04 4. $p > 1$

06 07 ex 05 5. $p > 1$

06 07 ex 06 6. $p < 1$

06 07 ex 07 7. $e^5/2$

06 07 ex 08 8. $1/2$

06 07 ex 09 9. $1/3$

06 07 ex 10 10. $\pi/3$

06 07 ex 11 11. $1/\ln 2$

06 07 ex 12 12. diverges

06 07 ex 13 13. diverges

06 07 ex 14 14. $\pi/2$

06 07 ex 15 15. 1

06 07 ex 16 16. diverges

06 07 ex 17 17. diverges

06 07 ex 18 18. diverges

06 07 ex 43 19. diverges

06 07 ex 19 20. diverges

06 07 ex 20 21. diverges

06 07 ex 21 22. $2 + 2\sqrt{2}$

06 07 ex 22 23. 1

06 07 ex 23 24. $1/2$

06 07 ex 24 25. 0

06 07 ex 25 26. $\pi/2$

06 07 ex 26 27. $-1/4$

06 07 ex 27 28. diverges

06 07 ex 28 29. $-1$

06 07 ex 29 30. 1

06 07 ex 30 31. diverges

06 07 ex 31 32. $1/2$

06 07 ex 32 33. $1/2$

06 07 ex 33 34. diverges; Limit Comparison Test with $1/x$.

06 07 ex 34 35. converges; Limit Comparison Test with $1/x^{3/2}$.

06 07 ex 35 36. diverges; Limit Comparison Test with $1/x$.

06 07 ex 36 37. converges; Direct Comparison Test with $xe^{-x}$.

06 07 ex 37 38. converges; Direct Comparison Test with $e^{-x}$.

06 07 ex 38 39. converges; Direct Comparison Test with $xe^{-x}$.

06 07 ex 39 40. converges; Direct Comparison Test with $1/(x^2-1)$.

06 07 ex 42 41. diverges; Direct Comparison Test with $x/(x^2+\cos x)$.

06 07 ex 40 42. converges; Direct Comparison Test with $1/e^x$.

06 07 ex 41 43. converges; Limit Comparison Test with $1/e^x$.

**Section 6.8**

06 08 ex 01 1. backwards

06 08 ex 02 2. $5\sin\theta$

06 08 ex 03 3.

  (a) $\tan^2\theta + 1 = \sec^2\theta$

  (b) $9\sec^2\theta$.

06 08 ex 04 4. Because we are considering $a > 0$ and $x = a\sin\theta$, which means $\theta = \sin^{-1}(x/a)$. The arcsine function has a domain of $-\pi/2 \leq \theta \leq \pi/2$; on this domain, $\cos\theta \geq 0$, so $a\cos\theta$ is always non-negative, allowing us to drop the absolute value signs.

06 08 ex 05 5. $\frac{1}{2}\left(x\sqrt{x^2+1} + \ln|\sqrt{x^2+1}+x|\right) + C$

06 08 ex 06 6. $2\left(\frac{x}{4}\sqrt{x^2+4} + \ln\left|\frac{\sqrt{x^2+1}}{2} + \frac{x}{2}\right|\right) + C$

06 08 ex 07 7. $\frac{1}{2}\left(\sin^{-1}x + x\sqrt{1-x^2}\right) + C$

06 08 ex 08 8. $\frac{1}{2}\left(9\sin^{-1}(x/3) + x\sqrt{9-x^2}\right) + C$

06 08 ex 09 9. $\frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln|x+\sqrt{x^2-1}| + C$

06 08 ex 10 10. $\frac{1}{2}x\sqrt{x^2-16} - 8\ln\left|\frac{x}{4} + \frac{\sqrt{x^2-16}}{4}\right| + C$

06 08 ex 11 11. $x\sqrt{x^2+1/4} + \frac{1}{4}\ln|2\sqrt{x^2+1/4}+2x| + C =$
$\frac{1}{2}x\sqrt{4x^2+1} + \frac{1}{4}\ln|\sqrt{4x^2+1}+2x| + C$

06 08 ex 12 12. $\frac{1}{6}\sin^{-1}(3x) + \frac{3}{2}\sqrt{1/9-x^2} + C =$
$\frac{1}{6}\sin^{-1}(3x) + \frac{1}{2}\sqrt{1-9x^2} + C$

06 08 ex 13 13. $4\left(\frac{1}{2}x\sqrt{x^2-1/16} - \frac{1}{32}\ln|4x+4\sqrt{x^2-1/16}|\right) + C =$
$\frac{1}{2}x\sqrt{16x^2-1} - \frac{1}{8}\ln|4x+\sqrt{16x^2-1}| + C$

06 08 ex 14 14. $8\ln\left|\frac{\sqrt{x^2+2}}{\sqrt{2}} + \frac{x}{\sqrt{2}}\right| + C$; with Section 6.6, we can state the answer as $8\sinh^{-1}(x/\sqrt{2}) + C$.

06 08 ex 15 15. $3\sin^{-1}\left(\frac{x}{\sqrt{7}}\right) + C$ (Trig. Subst. is not needed)

06 08 ex 16 16. $5\ln\left|\frac{x}{\sqrt{8}} + \frac{\sqrt{x^2-8}}{\sqrt{8}}\right| + C$

06 08 ex 18 17. $\sqrt{x^2-11} - \sqrt{11}\sec^{-1}(x/\sqrt{11}) + C$

06 08 ex 20 18. $\frac{1}{2}\left(\tan^{-1}x + \frac{x}{x^2+1}\right) + C$

06 08 ex 17 19. $\sqrt{x^2-3} + C$ (Trig. Subst. is not needed)

06 08 ex 24 20. $\frac{1}{8}\sin^{-1}x - \frac{1}{8}x\sqrt{1-x^2}(1-2x^2) + C$

06 08 ex 22 21. $-\frac{1}{\sqrt{x^2+9}} + C$ (Trig. Subst. is not needed)

06 08 ex 19 22. $\frac{5}{2}x\sqrt{x^2-10} + 25\ln\left|\frac{x}{\sqrt{10}} + \frac{\sqrt{x^2-10}}{\sqrt{10}}\right| + C$

06 08 ex 21 23. $\frac{1}{18}\frac{x+2}{x^2+4x+13} + \frac{1}{54}\tan^{-1}\left(\frac{x+2}{2}\right) + C$

06 08 ex 23 24. $\frac{x}{\sqrt{1-x^2}} - \sin^{-1}x + C$

06 08 ex 25 25. $\frac{1}{7}\left(-\frac{\sqrt{5-x^2}}{x} - \sin^{-1}(x/\sqrt{5})\right) + C$

06 08 ex 26 26. $\frac{1}{2}x\sqrt{x^2+3} - \frac{3}{2}\ln\left|\frac{\sqrt{x^2+3}}{\sqrt{3}} + \frac{x}{\sqrt{3}}\right| + C$

06 08 ex 27 27. $\pi/2$

06 08 ex 28 28. $16\sqrt{3} - 8\ln(2+\sqrt{3})$

06 08 ex 29 29. $2\sqrt{2} + 2\ln(1+\sqrt{2})$

06 08 ex 30 30. $\pi/4 + 1/2)$

06 08 ex 31 31. $9\sin^{-1}(1/3) + \sqrt{8}$ Note: the new lower bound is $\theta = \sin^{-1}(-1/3)$ and the new upper bound is $\theta = \sin^{-1}(1/3)$. The final answer comes with recognizing that $\sin^{-1}(-1/3) = -\sin^{-1}(1/3)$ and that $\cos\left(\sin^{-1}(1/3)\right) = \cos\left(\sin^{-1}(-1/3)\right) = \sqrt{8}/3$.

06 08 ex 32 32. $\pi/8$

# Chapter 7

### Section 7.1

07 01 ex 01 1. T

07 01 ex 02 2. T

07 01 ex 03 3. Answers will vary.

07 01 ex 04 4. $4\pi + \pi^2 \approx 22.436$

07 01 ex 05 5. $16/3$

07 01 ex 10 6. $\pi$

07 01 ex 06 7. $\pi$

07 01 ex 07 8. $1/2$

07 01 ex 08 9. $2\sqrt{2}$

07 01 ex 09 10. $1/\ln 4$

07 01 ex 11 11. 4.5

07 01 ex 12 12. $4/3$

07 01 ex 13 13. $2 - \pi/2$

07 01 ex 14 14. 8

07 01 ex 15 15. $1/6$

07 01 ex 16 16. $37/12$

07 01 ex 17 17. On regions such as $[\pi/6, 5\pi/6]$, the area is $3\sqrt{3}/2$. On regions such as $[-\pi/2, \pi/6]$, the area is $3\sqrt{3}/4$.

07 01 ex 18 18. 1

07 01 ex 19 19. $5/3$

07 01 ex 20 20. $9/2$

07 01 ex 21 21. $9/4$

07 01 ex 22 22. $1/12(9 - 2\sqrt{2}) \approx 0.514$

07 01 ex 23 23. 1

07 01 ex 24 24. 5

07 01 ex 25 25. 4

07 01 ex 26 26. 133/20

07 01 ex 29 27. 219,000 $\text{ft}^2$

07 01 ex 30 28. 623,333 $\text{ft}^2$

### Section 7.2

07 02 ex 01 1. T

07 02 ex 02 2. Answers will vary.

07 02 ex 03 3. Recall that "*dx*" does not just "sit there;" it is multiplied by $A(x)$ and represents the thickness of a small slice of the solid. Therefore *dx* has units of in, giving $A(x)\,dx$ the units of $\text{in}^3$.

07 02 ex 05 4. $48\pi\sqrt{3}/5$ units$^3$

07 02 ex 06 5. $175\pi/3$ units$^3$

07 02 ex 07 6. $\pi^2/4$ units$^3$

07 02 ex 04 7. $\pi/6$ units$^3$

07 02 ex 09 8. $9\pi/2$ units$^3$

07 02 ex 10 9. $35\pi/3$ units$^3$

07 02 ex 11 10. $\pi^2 - 2\pi$ units$^3$

07 02 ex 08 11. $2\pi/15$ units$^3$

07 02 ex 12 12.

(a) $\pi/2$
(b) $5\pi/6$
(c) $4\pi/5$
(d) $8\pi/15$

07 02 ex 13 13.

(a) $512\pi/15$
(b) $256\pi/5$
(c) $832\pi/15$
(d) $128\pi/3$

07 02 ex 14 14.

(a) $4\pi/3$
(b) $2\pi/3$
(c) $4\pi/3$
(d) $\pi/3$

07 02 ex 15 15.

(a) $104\pi/15$
(b) $64\pi/15$
(c) $32\pi/5$

07 02 ex 16 16.

(a) $\pi^2/2$
(b) $\pi^2/2 - 4\pi\sinh^{-1}(1)$
(c) $\pi^2/2 + 4\pi\sinh^{-1}(1)$

07 02 ex 17 17.

(a) $8\pi$
(b) $8\pi$
(c) $16\pi/3$
(d) $8\pi/3$

07 02 ex 18 18. Placing the tip of the cone at the origin such that the *x*-axis runs through the center of the circular base, we have $A(x) = \pi x^2/4$. Thus the volume is $250\pi/3$ units$^3$.

07 02 ex 19 19. The cross–sections of this cone are the same as the cone in Exercise 18. Thus they have the same volume of $250\pi/3$ units$^3$.

07 02 ex 20 20. Orient the cone such that the tip is at the origin and the $x$-axis is perpendicular to the base. The cross–sections of this cone are right, isosceles triangles with side length $2x/5$; thus the cross–sectional areas are $A(x) = 2x^2/25$, giving a volume of $80/3$ units$^3$.

07 02 ex 21 21. Orient the solid so that the $x$-axis is parallel to long side of the base. All cross–sections are trapezoids (at the far left, the trapezoid is a square; at the far right, the trapezoid has a top length of 0, making it a triangle). The area of the trapezoid at $x$ is $A(x) = 1/2(-1/2x + 5 + 5)(5) = -5/4x + 25$. The volume is 187.5 units$^3$.

**Section 7.3**

07 02 ex 01 1. T

07 03 ex 01 2. F

07 03 ex 02 3. F

07 03 ex 03 4. T

07 02 ex 09 5. $9\pi/2$ units$^3$

07 03 ex 06 6. $70\pi/3$ units$^3$

07 03 ex 11 7. $\pi^2 - 2\pi$ units$^3$

07 02 ex 08 8. $2\pi/15$ units$^3$

07 02 ex 05 9. $48\pi\sqrt{3}/5$ units$^3$

07 03 ex 10 10. $350\pi/3$ units$^3$

07 03 ex 20 11. $\pi^2/4$ units$^3$

07 02 ex 04 12. $\pi/6$ units$^3$

07 03 ex 12 13.

(a) $4\pi/5$

(b) $8\pi/15$

(c) $\pi/2$

(d) $5\pi/6$

07 03 ex 13 14.

(a) $128\pi/3$

(b) $128\pi/3$

(c) $512\pi/15$

(d) $256\pi/5$

07 03 ex 14 15.

(a) $4\pi/3$

(b) $\pi/3$

(c) $4\pi/3$

(d) $2\pi/3$

07 03 ex 15 16.

(a) $16\pi/3$

(b) $8\pi/3$

(c) $8\pi$

07 03 ex 16 17.

(a) $2\pi(\sqrt{2} - 1)$

(b) $2\pi(1 - \sqrt{2} + \sinh^{-1}(1))$

07 03 ex 17 18.

(a) $16\pi/3$

(b) $8\pi/3$

(c) $8\pi$

(d) $8\pi$

**Section 7.4**

07 04 ex 01 1. T

07 04 ex 02 2. F

07 04 ex 11 3. $\sqrt{2}$

07 04 ex 12 4. 6

07 04 ex 03 5. $4/3$

07 04 ex 04 6. 6

07 04 ex 05 7. $109/2$

07 04 ex 06 8. $3/2$

07 04 ex 07 9. $12/5$

07 04 ex 08 10. $79953333/400000 \approx 199.883$

07 04 ex 09 11. $-\ln(2 - \sqrt{3}) \approx 1.31696$

07 04 ex 10 12. $\sinh^{-1} 1$

07 04 ex 13 13. $\int_0^1 \sqrt{1 + 4x^2}\, dx$

07 04 ex 14 14. $\int_0^1 \sqrt{1 + 100x^{18}}\, dx$

07 04 ex 15 15. $\int_0^1 \sqrt{1 + \frac{1}{4x}}\, dx$

07 04 ex 16 16. $\int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$

07 04 ex 17 17. $\int_{-1}^1 \sqrt{1 + \frac{x^2}{1-x^2}}\, dx$

07 04 ex 18 18. $\int_{-3}^3 \sqrt{1 + \frac{x^2}{81-9x^2}}\, dx$

07 04 ex 19 19. $\int_1^2 \sqrt{1 + \frac{1}{x^4}}\, dx$

07 04 ex 20 20. $\int_{-\pi/4}^{\pi/4} \sqrt{1 + \sec^2 x \tan^2 x}\, dx$

07 04 ex 21 21. 1.4790

07 04 ex 22 22. 1.8377

07 04 ex 23 23. Simpson's Rule fails, as it requires one to divide by 0. However, recognize the answer should be the same as for $y = x^2$; why?

07 04 ex 24 24. 2.1300

07 04 ex 25 25. Simpson's Rule fails.

07 04 ex 26 26. Simpson's Rule fails.

07 04 ex 27 27. 1.4058

07 04 ex 28 28. 1.7625

07 04 ex 29 29. $2\pi \int_0^1 2x\sqrt{5}\, dx = 2\pi\sqrt{5}$

07 04 ex 30 30. $2\pi \int_0^1 x\sqrt{1 + 4x^2}\, dx = \pi/6(5\sqrt{5} - 1)$

07 04 ex 31 31. $2\pi \int_0^1 x^3\sqrt{1 + 9x^4}\, dx = \pi/27(10\sqrt{10} - 1)$

07 04 ex 32 32. $2\pi \int_0^1 \sqrt{x}\sqrt{1 + 1/(4x)}\, dx = \pi/6(5\sqrt{5} - 1)$

07 04 ex 33 33. $2\pi\int_0^1 \sqrt{1-x^2}\sqrt{1+x/(1-x^2)}\,dx = 4\pi$

**Section 7.5**

07 05 ex 01 1. In SI units, it is one joule, i.e., one Newton–meter, or kg·m/s$^2$·m. In Imperial Units, it is ft–lb.

07 05 ex 02 2. The same.

07 05 ex 03 3. Smaller.

07 05 ex 04 4.

(a) 500 ft–lb

(b) $100 - 50\sqrt{2} \approx 29.29$ ft–lb

07 05 ex 05 5.

(a) 2450 j

(b) 1568 j

07 05 ex 06 6.

(a) $\frac{1}{2}\cdot d\cdot l^2$ ft–lb

(b) 75 %

(c) $\ell(1-\sqrt{2}/2) \approx 0.2929\ell$

07 05 ex 07 7. 735 j

07 05 ex 08 8.

(a) 756 ft–lb

(b) 60,000 ft–lb

(c) Yes, for the cable accounts for about 1% of the total work.

07 05 ex 09 9. 11,100 ft–lb

07 05 ex 10 10. 575 ft–lb

07 05 ex 11 11. 125 ft–lb

07 05 ex 12 12. 0.05 j

07 05 ex 13 13. 12.5 ft–lb

07 05 ex 14 14. 5/3 ft–lb

07 05 ex 15 15. 0.2625 = 21/80 j

07 05 ex 16 16. $f\cdot d/2$ j

07 05 ex 17 17. 45 ft–lb

07 05 ex 18 18. 5 ft–lb

07 05 ex 19 19. $953,284$ j

07 05 ex 20 20.

(a) 52,929.6 ft–lb

(b) 18,525.3 ft–lb

(c) When 3.83 ft of water have been pumped from the tank, leaving about 2.17 ft in the tank.

07 05 ex 21 21. 192,767 ft–lb. Note that the tank is oriented horizontally. Let the origin be the center of one of the circular ends of the tank. Since the radius is 3.75 ft, the fluid is being pumped to $y = 4.75$; thus the distance the gas travels is $h(y) = 4.75 - y$. A differential element of water is a rectangle, with length 20 and width $2\sqrt{3.75^2 - y^2}$. Thus the force required to move that slab of gas is $F(y) = 40\cdot 45.93\cdot\sqrt{3.75^2-y^2}dy$. Total work is $\int_{-3.75}^{3.75} 40\cdot 45.93\cdot(4.75-y)\sqrt{3.75^2-y^2}\,dy$. This can be evaluated without actual integration; split the integral into $\int_{-3.75}^{3.75} 40\cdot 45.93\cdot(4.75)\sqrt{3.75^2-y^2}\,dy + \int_{-3.75}^{3.75} 40\cdot 45.93\cdot(-y)\sqrt{3.75^2-y^2}\,dy$. The first integral can be evaluated as measuring half the area of a circle; the latter integral can be shown to be 0 without much difficulty. (Use substitution and realize the bounds are both 0.)

07 05 ex 22 22. 212,135 ft–lb

07 05 ex 23 23.

(a) approx. 577,000 j

(b) approx. 399,000 j

(c) approx 110,000 j (By volume, half of the water is between the base of the cone and a height of 3.9685 m. If one rounds this to 4 m, the work is approx 104,000 j.)

07 05 ex 24 24. 187,214 ft–lb

07 05 ex 25 25. 617,400 j

07 05 ex 26 26. 4,917,150 j

**Section 7.6**

07 06 ex 01 1. Answers will vary.

07 06 ex 02 2. Answers will vary.

07 06 ex 03 3. 499.2 lb

07 06 ex 06 4. 249.6 lb

07 06 ex 13 5. 6739.2 lb

07 06 ex 14 6. 5241.6 lb

07 06 ex 15 7. 3920.7 lb

07 06 ex 16 8. 15682.8 lb

07 06 ex 17 9. 2496 lb

07 06 ex 18 10. 2496 lb

07 06 ex 04 11. 602.59 lb

07 06 ex 05 12. 291.2 lb

07 06 ex 07 13.

(a) 2340 lb

(b) 5625 lb

07 06 ex 08 14.

(a) 1064.96 lb

(b) 2560 lb

07 06 ex 09 15.

(a) 1597.44 lb

(b) 3840 lb

07 06 ex 10 16.

(a) 41.6 lb

(b) 100 lb

07 06 ex 11 17.

(a) 56.42 lb

(b) 135.62 lb

07 06 ex 12 18.

(a) 1123.2 lb

(b) 2700 lb

07 06 ex 19 19. 5.1 ft

07 06 ex 20 20. 4.1 ft

# Chapter 8

### Section 8.1

08 01 ex 01 1. Answers will vary.

08 01 ex 02 2. natural

08 01 ex 03 3. Answers will vary.

08 01 ex 04 4. Answers will vary.

08 01 ex 05 5. $2, \frac{8}{3}, \frac{8}{3}, \frac{32}{15}, \frac{64}{45}$

08 01 ex 06 6. $-\frac{3}{2}, \frac{9}{4}, -\frac{27}{8}, \frac{81}{16}, -\frac{243}{32}$

08 01 ex 07 7. $\frac{1}{3}, 2, \frac{81}{5}, \frac{512}{3}, \frac{15625}{7}$

08 01 ex 08 8. $1, 1, 2, 3, 5$

08 01 ex 09 9. $a_n = 3n + 1$

08 01 ex 10 10. $a_n = (-1)^{n+1}\frac{3}{2^{n-1}}$

08 01 ex 11 11. $a_n = 10 \cdot 2^{n-1}$

08 01 ex 12 12. $a_n = 1/(n-1)!$

08 01 ex 35 13. $1/7$

08 01 ex 36 14. $3e^2 - 1$

08 01 ex 37 15. $0$

08 01 ex 38 16. $e^4$

08 01 ex 13 17. diverges

08 01 ex 14 18. converges to $4/3$

08 01 ex 15 19. converges to $0$

08 01 ex 16 20. converges to $0$

08 01 ex 17 21. diverges

08 01 ex 18 22. converges to $3$

08 01 ex 19 23. converges to $e$

08 01 ex 26 24. converges to $5$

08 01 ex 27 25. converges to $0$

08 01 ex 28 26. diverges

08 01 ex 29 27. converges to $2$

08 01 ex 30 28. converges to $0$

08 01 ex 20 29. bounded

08 01 ex 21 30. neither bounded above or below

08 01 ex 22 31. bounded

08 01 ex 23 32. bounded below

08 01 ex 24 33. neither bounded above or below

08 01 ex 25 34. bounded above

08 01 ex 31 35. monotonically increasing

08 01 ex 32 36. monotonically increasing for $n \geq 3$

08 01 ex 33 37. never monotonic

08 01 ex 34 38. monotonically decreasing for $n \geq 3$

08 01 ex 39 39. Let $\{a_n\}$ be given such that $\lim\limits_{n\to\infty} |a_n| = 0$. By the definition of the limit of a sequence, given any $\varepsilon > 0$, there is a $m$ such that for all $n > m$, $|\,|a_n| - 0| < \varepsilon$. Since $|\,|a_n| - 0| = |a_n - 0|$, this directly implies that for all $n > m$, $|a_n - 0| < \varepsilon$, meaning that $\lim\limits_{n\to\infty} a_n = 0$.

08 01 ex 40 40.

(a) Left to reader

(b) $a_n = 1/3^n$ and $b_n = 1/2^n$

08 01 ex 41 41. Left to reader

### Section 8.2

08 02 ex 01 1. Answers will vary.

08 02 ex 02 2. Answers will vary.

08 02 ex 25 3. One sequence is the sequence of terms $\{a_{\}}$. The other is the sequence of $n^{\text{th}}$ partial sums, $\{S_n\} = \{\sum_{i=1}^n a_i\}$.

08 02 ex 03 4. Answers will vary.

08 02 ex 04 5. F

08 02 ex 05 6.

(a) $-1, -\frac{1}{2}, -\frac{5}{6}, -\frac{7}{12}, -\frac{47}{60}$

(b) Plot omitted

08 02 ex 06 7.

(a) $1, \frac{5}{4}, \frac{49}{36}, \frac{205}{144}, \frac{5269}{3600}$

(b) Plot omitted

08 02 ex 07 8.

(a) $-1, 0, -1, 0, -1$

(b) Plot omitted

08 02 ex 08 9.

(a) $1, 3, 6, 10, 15$

(b) Plot omitted

08 02 ex 09 10.

(a) $1, \frac{3}{2}, \frac{5}{3}, \frac{41}{24}, \frac{103}{60}$

(b) Plot omitted

08 02 ex 10 11.

(a) $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \frac{121}{243}$

(b) Plot omitted

08 02 ex 11 12.

(a) $-0.9, -0.09, -0.819, -0.1629, -0.75339$

(b) Plot omitted

08 02 ex 12 13.

(a) $0.1, 0.11, 0.111, 0.1111, 0.11111$

(b) Plot omitted

08 02 ex 28 14. $\lim\limits_{n\to\infty} a_n = 3$; by Theorem 63 the series diverges.

08 02 ex 29 15. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 63 the series diverges.

08 02 ex 30 16. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 63 the series diverges.

08 02 ex 31 17. $\lim\limits_{n\to\infty} a_n = 1$; by Theorem 63 the series diverges.

08 02 ex 33 18. $\lim\limits_{n\to\infty} a_n = 1/2$; by Theorem 63 the series diverges.

08 02 ex 34 19. $\lim\limits_{n\to\infty} a_n = e$; by Theorem 63 the series diverges.

08 02 ex 37 20. Converges

08 02 ex 38 21. Converges

08 02 ex 39 22. Diverges

08 02 ex 40 23. Converges

08 02 ex 41 24. Diverges

08 02 ex 42 25. Converges

08 02 ex 46 26. Diverges

08 02 ex 43 27. Converges

08 02 ex 44 28. Diverges

08 02 ex 45 29. Diverges

08 02 ex 13 30.

(a) $S_n = \frac{1-(1/4)^n}{3/4}$

(b) Converges to $4/3$.

08 02 ex 14 31.

(a) $S_n = \left(\frac{n(n+1)}{2}\right)^2$

(b) Diverges

08 02 ex 15 32.

(a) $S_n = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ -\frac{n}{2} & \text{n is even} \end{cases}$

(b) Diverges

08 02 ex 16 33.

(a) $S_n = 5\frac{1-1/2^n}{1/2}$

(b) Converges to 10.

08 02 ex 32 34.

(a) $S_n = \frac{1-(1/e)^{n+1}}{1-1/e}$.

(b) Converges to $1/(1-1/e) = e/(e-1)$.

08 02 ex 17 35.

(a) $S_n = \frac{1-(-1/3)^n}{4/3}$

(b) Converges to $3/4$.

08 02 ex 26 36.

(a) With partial fractions, $a_n = \frac{1}{n} - \frac{1}{n+1}$. Thus $S_n = 1 - \frac{1}{n+1}$.

(b) Converges to 1.

08 02 ex 27 37.

(a) With partial fractions, $a_n = \frac{3}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$. Thus $S_n = \frac{3}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$.

(b) Converges to 9/4

08 02 ex 18 38.

(a) Use partial fraction decomposition to recognize the telescoping series: $S_n = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) = \frac{n}{2n+1}$.

(b) Converges to $1/2$.

08 02 ex 19 39.

(a) $S_n = \ln\big(1/(n+1)\big)$

(b) Diverges (to $-\infty$).

08 02 ex 20 40.

(a) $S_n = 1 - \frac{1}{(n+1)^2}$

(b) Converges to 1.

08 02 ex 21 41.

(a) $a_n = \frac{1}{n(n+3)}$; using partial fractions, the resulting telescoping sum reduces to $S_n = \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right)$

(b) Converges to $11/18$.

08 02 ex 22 42.

(a) $a_n = 1/2^n + 1/3^n$ for $n \geq 0$. Thus $S_n = \frac{1-1/2^2}{1/2} + \frac{1-1/3^n}{2/3}$.

(b) Converges to $2 + 3/2 = 7/2$.

08 02 ex 35 43.

(a) With partial fractions, $a_n = \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$. Thus $S_n = \frac{1}{2}\left(3/2 - \frac{1}{n} - \frac{1}{n+1}\right)$.

(b) Converges to 3/4.

08 02 ex 36 44.

(a) $S_n = \frac{1-(\sin 1)^{n+1}}{1-\sin 1}$

(b) Converges to $\frac{1}{1-\sin 1}$..

08 02 ex 24 45.

(a) The $n^{\text{th}}$ partial sum of the odd series is $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. The $n^{\text{th}}$ partial sum of the even series is $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n}$. Each term of the even series is less than the corresponding term of the odd series, giving us our result.

(b) The $n^{\text{th}}$ partial sum of the odd series is $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. The $n^{\text{th}}$ partial sum of 1 plus the even series is $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2(n-1)}$. Each term of the even series is now greater than or equal to the corresponding term of the odd series, with equality only on the first term. This gives us the result.

(c) If the odd series converges, the work done in (a) shows the even series converges also. (The sequence of the $n^{\text{th}}$ partial sum of the even series is bounded and monotonically increasing.) Likewise, (b) shows that if the even series converges, the odd series will, too. Thus if either series converges, the other does.

Similarly, (a) and (b) can be used to show that if either series diverges, the other does, too.

(d) If both the even and odd series converge, then their sum would be a convergent series. This would imply that the Harmonic Series, their sum, is convergent. It is not. Hence each series diverges.

08 02 ex 23 46. Using partial fractions, we can show that $a_n = \frac{1}{4}\left(\frac{1}{2n-1} + \frac{1}{2n+1}\right)$. The series is effectively twice the sum of the odd terms of the Harmonic Series which was shown to diverge in Exercise 45. Thus this series diverges.

**Section 8.3**

08 03 ex 01 1. continuous, positive and decreasing

08 03 ex 02 2. F

08 03 ex 03 3. The Integral Test (we do not have a continuous definition of $n!$ yet) and the Limit Comparison Test (same as above, hence we cannot take its derivative).

08 03 ex 04 4. $\sum_{n=0}^{\infty} b_n$ converges; we cannot conclude anything about $\sum_{n=0}^{\infty} c_n$

08 03 ex 06 5. Converges

08 03 ex 07 6. Converges

08 03 ex 08 7. Diverges

08 03 ex 09 8. Diverges

08 03 ex 10 9. Converges

08 03 ex 11 10. Converges

08 03 ex 12 11. Converges

08 03 ex 05 12. Converges

08 03 ex 13 13. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, as $1/(n^2+3n-5) \le 1/n^2$ for all $n > 1$.

08 03 ex 14 14. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{4^n}$, as $1/(4^n+n^2-n) \le 1/4^n$ for all $n \ge 1$.

08 03 ex 15 15. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, as $1/n \le \ln n/n$ for all $n \ge 2$.

08 03 ex 16 16. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n!}$, as $1/(n!+n) \le 1/n!$ for all $n \ge 1$.

08 03 ex 17 17. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$. Since $n = \sqrt{n^2} > \sqrt{n^2-1}$, $1/n \le 1/\sqrt{n^2-1}$ for all $n \ge 2$.

08 03 ex 18 18. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, as $1/\sqrt{n} \le 1/(\sqrt{n}-2)$ for all $n \ge 5$.

08 03 ex 19 19. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$:

$$\frac{1}{n} = \frac{n^2}{n^3} < \frac{n^2+n+1}{n^3} < \frac{n^2+n+1}{n^3-5},$$

for all $n \ge 1$.

08 03 ex 20 20. Converges; compare to $\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$, as $2^n/(5^n+10) < 2^n/5^n$ for all $n \ge 1$.

08 03 ex 21 21. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$. Note that

$$\frac{n}{n^2-1} = \frac{n^2}{n^2-1} \cdot \frac{1}{n} > \frac{1}{n},$$

as $\frac{n^2}{n^2-1} > 1$, for all $n \ge 2$.

08 03 ex 22 22. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, as $1/(n^2 \ln n) \le 1/n^2$ for all $n \ge 2$.

08 03 ex 23 23. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

08 03 ex 24 24. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{4^n}$.

08 03 ex 25 25. Diverges; compare to $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

08 03 ex 26 26. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

08 03 ex 27 27. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

08 03 ex 28 28. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

08 03 ex 29 29. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$. Just as $\lim_{n\to 0} \frac{\sin n}{n} = 1$, $\lim_{n\to\infty} \frac{\sin(1/n)}{1/n} = 1$.

08 03 ex 30 30. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

08 03 ex 31 31. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

08 03 ex 32 32. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$.

08 03 ex 33 33. Converges; Integral Test

08 03 ex 34 34. Converges; Integral Test, $p$-Series Test, Direct & Limit Comparison Tests can all be used.

08 03 ex 35 35. Diverges; the $n^{\text{th}}$ Term Test and Direct Comparison Test can be used.

08 03 ex 36 36. Converges; the Direct Comparison Test can be used with sequence $1/(n-1)!$.

08 03 ex 37 37. Converges; the Direct Comparison Test can be used with sequence $1/3^n$.

08 03 ex 38 38. Diverges; the $n^{\text{th}}$ Term Test can be used, along with the Limit Comparison Test (compare with 1/10).

08 03 ex 39 39. Diverges; the $n^{\text{th}}$ Term Test can be used, along with the Integral Test.

08 03 ex 40 40. Converges; the Direct Comparison Test can be used with sequence $1/\sqrt{n}$.

08 03 ex 41 41.

(a) Converges; use Direct Comparison Test as $\frac{a_n}{n} < n$.

(b) Converges; since original series converges, we know $\lim_{n\to\infty} a_n = 0$. Thus for large $n$, $a_n a_{n+1} < a_n$.

(c) Converges; similar logic to part (b) so $(a_n)^2 < a_n$.

(d) May converge; certainly $na_n > a_n$ but that does not mean it does not converge.

(e) Does not converge, using logic from (b) and $n^{th}$ Term Test.

**Section 8.4**

08 04 ex 01 1. algebraic, or polynomial.

08 04 ex 02 2. factorial and/or exponential

08 04 ex 03 3. Integral Test, Limit Comparison Test, and Root Test

08 04 ex 04 4. raised to a power

08 04 ex 05 5. Converges

08 04 ex 06 6. Diverges

08 04 ex 07 7. Converges

08 04 ex 14 8. Converges

08 04 ex 08 9. The Ratio Test is inconclusive; the $p$-Series Test states it diverges.

08 04 ex 09 10. The Ratio Test is inconclusive; the Direct Comparison Test with $1/n^3$ shows it converges.

08 04 ex 10 11. Converges

08 04 ex 11 12. Converges

08 04 ex 12 13. Converges; note the summation can be rewritten as $\sum_{n=1}^{\infty}\frac{2^n n!}{3^n n!}$, from which the Ratio Test can be applied.

08 04 ex 13 14. Converges; rewrite the summation as $\sum_{n=1}^{\infty}\frac{n!}{5^n n!}$ then apply the Ratio Test.

08 04 ex 15 15. Converges

08 04 ex 16 16. Converges

08 04 ex 17 17. Converges

08 04 ex 19 18. Converges

08 04 ex 18 19. Diverges

08 04 ex 20 20. Converges

08 04 ex 21 21. Diverges. The Root Test is inconclusive, but the $n^{\text{th}}$-Term Test shows divergence. (The terms of the sequence approach $e^2$, not 0, as $n \to \infty$.)

08 04 ex 22 22. Converges

08 04 ex 23 23. Converges

08 04 ex 24 24. Converges

08 04 ex 25 25. Diverges; Limit Comparison Test

08 04 ex 26 26. Converges; Ratio Test

08 04 ex 27 27. Converges; Ratio Test or Limit Comparison Test with $1/3^n$.

08 04 ex 29 28. Converges; Root Test

08 04 ex 28 29. Diverges; $n^{\text{th}}$-Term Test or Limit Comparison Test with 1.

08 04 ex 30 30. Converges; Ratio Test

08 04 ex 31 31. Diverges; Direct Comparison Test with $1/n$

08 04 ex 32 32. Diverges; $n^{\text{th}}$-Term Test ($n^{\text{th}}$ term approaches $e$.)

08 04 ex 33 33. Converges; Root Test

08 04 ex 34 34. Converges; Limit Comparison Test with $1/n^2$ (get common denominator first). It is also a Telescoping Series.

**Section 8.5**

08 05 ex 01 1. The signs of the terms do not alternate; in the given series, some terms are negative and the others positive, but they do not necessarily alternate.

08 05 ex 02 2. postive, decreasing, 0

08 05 ex 03 3. Many examples exist; one common example is $a_n = (-1)^n/n$.

08 05 ex 04 4. conditionally

08 05 ex 05 5.

(a) converges

(b) converges ($p$-Series)

(c) absolute

08 05 ex 06 6.

(a) converges

(b) converges (use Ratio Test)

(c) absolute

08 05 ex 07 7.

(a) diverges (limit of terms is not 0)

(b) diverges

(c) n/a; diverges

08 05 ex 08 8.

(a) diverges (limit of terms is not 0)

(b) diverges

(c) n/a; diverges

08 05 ex 09 9.

(a) converges

(b) diverges (Limit Comparison Test with $1/n$)

(c) conditional

08 05 ex 10 10.

(a) converges

(b) diverges (Direct Comparison Test with $1/n$)

(c) conditional

08 05 ex 11 11.

(a) diverges (limit of terms is not 0)

(b) diverges

(c) n/a; diverges

08 05 ex 12 12.

(a) converges

(b) converges (the sum in the denominator is $n^2$)

(c) absolute

08 05 ex 13 13.

(a) diverges (terms oscillate between $\pm 1$)

(b) diverges

(c) n/a; diverges

08 05 ex 14 14.

(a) converges

(b) diverges (Integral Test)

(c) conditional

08 05 ex 15 15.

(a) converges

(b) converges (Geometric Series with $r = 2/3$)

(c) absolute

08 05 ex 16 16.

(a) converges

(b) converges (Geometric Series with $r = 1/e$)

(c) absolute

08 05 ex 17 17.

(a) converges

(b) converges (Ratio Test)

(c) absolute

08 05 ex 18 18.

(a) converges

(b) converges (Ratio Test)

(c) absolute

08 05 ex 19 19.

(a) converges

(b) diverges ($p$-Series Test with $p = 1/2$)

(c) conditional

08 05 ex 20 20.

(a) converges

(b) converges (Ratio Test)

(c) absolute

08 05 ex 21 21. $S_5 = -1.1906$; $S_6 = -0.6767$;
$$-1.1906 \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \leq -0.6767$$

08 05 ex 22 22. $S_4 = 0.9459$; $S_5 = 0.9475$;
$$0.9459 \leq \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \leq 0.9475$$

08 05 ex 23 23. $S_6 = 0.3681$; $S_7 = 0.3679$;
$$0.3681 \leq \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \leq 0.3679$$

08 05 ex 24 24. $S_9 = 0.666016$; $S_{10} = 0.666992$;
$$0.666016 \leq \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \leq 0.666992$$

08 05 ex 25 25. $n = 5$

08 05 ex 26 26. $n = 7$

08 05 ex 27 27. Using the theorem, we find $n = 499$ guarantees the sum is within 0.001 of $\pi/4$. (Convergence is actually faster, as the sum is within $\varepsilon$ of $\pi/24$ when $n \geq 249$.)

08 05 ex 28 28. $n = 5$ ($(2n)! > 10^8$ when $n \geq 6$)

**Section 8.6**

08 06 ex 01 1. 1

08 06 ex 02 2. The radius of convergence is a **value** $R$ such that a power series, centered at $x = c$, converges for all values of $x$ in $(c - R, c + R)$. The interval of convergence is an **interval** on which the power series converges; it may differ from $(c - R, c + R)$ only at the endpoints.

08 06 ex 03 3. 5

08 06 ex 04 4. 5

08 06 ex 05 5. $1 + 2x + 4x^2 + 8x^3 + 16x^4$

08 06 ex 06 6. $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16} + \frac{x^5}{25}$

08 06 ex 07 7. $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

08 06 ex 08 8. $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$

08 06 ex 09 9.

(a) $R = \infty$

(b) $(-\infty, \infty)$

08 06 ex 10 10.

(a) $R = 1$

(b) $(-1, 1)$

08 06 ex 11 11.

(a) $R = 1$

(b) $(2, 4]$

08 06 ex 12 12.

(a) $R = \infty$

(b) $(-\infty, \infty)$

08 06 ex 13 13.

(a) $R = 2$

(b) $(-2, 2)$

08 06 ex 14 14.

(a) $R = 10$

(b) $(-5, 15)$

08 06 ex 15 15.

(a) $R = 1/5$

(b) $(4/5, 6/5)$

08 06 ex 16 16.

(a) $R = 1/2$

(b) $(-1/2, 1/2)$

08 06 ex 17 17.

(a) $R = 1$

(b) $(-1, 1)$

08 06 ex 18 18.

(a) $R = 3$

(b) $(-3, 3)$

08 06 ex 19 19.

(a) $R = \infty$

(b) $(-\infty, \infty)$

08 06 ex 20 20.

(a) $R = 0$

(b) $x = 10$

08 06 ex 21 21.

(a) $R = 1$

(b) $[-1, 1]$

08 06 ex 22 22.

(a) $R = 1$

(b) $[-3, -1]$

08 06 ex 23 23.

(a) $R = 0$

(b) $x = 0$

08 06 ex 24 24.

(a) $R = 4$

(b) $x = (-8, 0)$

08 06 ex 25 25.

(a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} n^2 x^{n-1}; \quad (-1, 1)$

(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{n}{n+1} x^{n+1}; \quad (-1, 1)$

08 06 ex 26 26.

(a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} x^{n-1}; \quad (-1, 1)$

(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^{n+1}; \quad [-1, 1]$

08 06 ex 27 27.

(a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n} x^{n-1}; \quad (-2, 2)$

(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{1}{(n+1)2^n} x^{n+1}; \quad [-2, 2)$

08 06 ex 28 28.

(a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} n(-3)^n x^{n-1}; \quad (-1/3, 1/3)$

(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-3)^n}{n+1} x^{n+1}; \quad (-1/3, 1/3]$

08 06 ex 29 29.

(a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$; $(-\infty, \infty)$

(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}; \quad (-\infty, \infty)$

08 06 ex 30 30.

(a) $f'(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n!}$; $(-\infty, \infty)$

(b) $\displaystyle\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}; \quad (-\infty, \infty)$

08 06 ex 31 31. $1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4$

08 06 ex 32 32. $5 + 25x + \frac{125}{2}x^2 + \frac{625}{6}x^3 + \frac{3125}{24}x^4$

08 06 ex 33 33. $1 + x + x^2 + x^3 + x^4$

08 06 ex 34 34. $1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4$

08 06 ex 35 35. $0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4$

08 06 ex 36 36. $1 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4$

**Section 8.7**

08 07 ex 30 1. The Maclaurin polynomial is a special case of Taylor polynomials. Taylor polynomials are centered at a specific $x$-value; when that $x$-value is 0, it is a Maclauring polynomial.

08 07 ex 33 2. T

08 07 ex 31 3. $p_2(x) = 6 + 3x - 4x^2$.

08 07 ex 32 4. $f'''(0) = 30$

08 07 ex 01 5. $p_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

08 07 ex 02 6. $p_8(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7$

08 07 ex 03 7. $p_8(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5$

08 07 ex 04 8. $p_6(x) = \frac{2x^5}{15} + \frac{x^3}{3} + x$

08 07 ex 05 9. $p_4(x) = \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1$

08 07 ex 06 10. $p_4(x) = x^4 + x^3 + x^2 + x + 1$

08 07 ex 07 11. $p_4(x) = x^4 - x^3 + x^2 - x + 1$

08 07 ex 08 12. $p_7(x) = -\frac{x^7}{7} + \frac{x^5}{5} - \frac{x^3}{3} + x$

08 07 ex 09 13. $p_4(x) = 1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4$

08 07 ex 10 14. $p_4(x) = \ln(2) + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{24}(-1+x)^3 - \frac{1}{64}(-1+x)^4$

08 07 ex 11 15. $p_6(x) = \frac{1}{\sqrt{2}} - \frac{-\frac{\pi}{4}+x}{\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^2}{2\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^3}{6\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^4}{24\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^5}{120\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^6}{720\sqrt{2}}$

08 07 ex 12 16. $p_5(x) = \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(-\frac{\pi}{6}+x\right) - \frac{1}{4}\left(-\frac{\pi}{6}+x\right)^2 - \frac{\left(-\frac{\pi}{6}+x\right)^3}{4\sqrt{3}} + \frac{1}{48}\left(-\frac{\pi}{6}+x\right)^4 + \frac{\left(-\frac{\pi}{6}+x\right)^5}{80\sqrt{3}}$

08 07 ex 13 17. $p_5(x) = \frac{1}{2} - \frac{x-2}{4} + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \frac{1}{32}(x-2)^4 - \frac{1}{64}(x-2)^5$

08 07 ex 14 18. $p_8(x) = 1 - 2(-1+x) + 3(-1+x)^2 - 4(-1+x)^3 + 5(-1+x)^4 - 6(-1+x)^5 + 7(-1+x)^6 - 8(-1+x)^7 + 9(-1+x)^8$

08 07 ex 15 19. $p_3(x) = \frac{1}{2} + \frac{1+x}{2} + \frac{1}{4}(1+x)^2$

08 07 ex 16 20. $p_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{1}{2}\left(\pi^2 - 2\right)(x-\pi)^2$

08 07 ex 17 21. $p_3(x) = x - \frac{x^3}{6}$; $p_3(0.1) = 0.09983$. Error is bounded by $\pm\frac{1}{4!} \cdot 0.1^4 \approx \pm 0.000004167$.

08 07 ex 18 22. $p_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$; $p_4(1) = 13/24 \approx 0.54167$. Error is bounded by $\pm\frac{1}{5!} \cdot 1^5 \approx \pm 0.00833$

08 07 ex 19 23. $p_2(x) = 3 + \frac{1}{6}(-9+x) - \frac{1}{216}(-9+x)^2$; $p_2(10) = 3.16204$. The third derivative of $f(x) = \sqrt{x}$ is bounded on $(8, 11)$ by 0.003. Error is bounded by $\pm\frac{0.003}{3!} \cdot 1^3 = \pm 0.0005$.

08 07 ex 20 24. $p_3(x) = -1 + x - \frac{1}{2}(-1+x)^2 + \frac{1}{3}(-1+x)^3$; $p_3(1.5) = 0.41667$. The fourth derivative of $f(x) = \ln x$ is bounded on $(.9, 2)$ by 10. Error is bounded by $\pm\frac{10}{4!} \cdot .5^4 = \pm 0.026$.

08 07 ex 21 25. The $n^{\text{th}}$ derivative of $f(x) = e^x$ is bounded by 3 on intervals containing 0 and 1. Thus $|R_n(1)| \leq \frac{3}{(n+1)!}1^{(n+1)}$. When $n = 7$, this is less than 0.0001.

08 07 ex 24 26. The $n^{\text{th}}$ derivative of $f(x) = \sqrt{x}$ is bounded by 0.1 on intervals containing 3 and 4. Thus $|R_n(\pi)| \leq \frac{0.1}{(n+1)!}(1)^{(n+1)}$. When $n = 4$, this is less than 0.0001.

08 07 ex 22 27. The $n^{\text{th}}$ derivative of $f(x) = \cos x$ is bounded by 1 on intervals containing 0 and $\pi/3$. Thus $|R_n(\pi/3)| \leq \frac{1}{(n+1)!}(\pi/3)^{(n+1)}$. When $n = 7$, this is less than 0.0001. Since the Maclaurin polynomial of $\cos x$ only uses even powers, we can actually just use $n = 6$.

08 07 ex 23 28. The $n^{\text{th}}$ derivative of $f(x) = \sin x$ is bounded by 1 on intervals containing 0 and $\pi$. Thus $|R_n(\pi)| \leq \frac{1}{(n+1)!}(\pi)^{(n+1)}$. When $n = 12$, this is less than 0.0001. Since the Maclaurin polynomial of $\sin x$ only uses odd powers, we can actually just use $n = 11$.

08 07 ex 25 29. The $n^{\text{th}}$ term is $\frac{1}{n!}x^n$.

08 07 ex 26 30. The $n^{\text{th}}$ term is: when $n$ is even, $\frac{(-1)^{n/2}}{n!}x^n$; when $n$ is odd, 0.

08 07 ex 27 31. The $n^{\text{th}}$ term is $x^n$.

08 07 ex 28 32. The $n^{\text{th}}$ term is $(-1)^n x^n$.

08 07 ex 29 33. The $n^{\text{th}}$ term is $(-1)^n \frac{(x-1)^n}{n}$.

08 07 ex 34 34. $1 + x + \dfrac{1}{2}x^2 + \dfrac{1}{6}x^3 + \dfrac{1}{24}x^4$

08 07 ex 35 35. $3 + 15x + \dfrac{75}{2}x^2 + \dfrac{375}{6}x^3 + \dfrac{1875}{24}x^4$

08 07 ex 36 36. $1 + 2x - 2x^2 + 4x^3 - 10x^4$

**Section 8.8**

08 08 ex 01 1. A Taylor polynomial is a **polynomial**, containing a finite number of terms. A Taylor series is a **series**, the summation of an infinite number of terms.

08 08 ex 02 2. Theorem 77, entitled "Function and Taylor Series Equality"

08 08 ex 16 3. All derivatives of $e^x$ are $e^x$ which evaluate to 1 at $x = 0$.

The Taylor series starts $1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$;

the Taylor series is $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

08 08 ex 03 4. All derivatives of $\sin x$ are either $\pm\cos x$ or $\pm\sin x$, which evaluate to $\pm 1$ or 0 at $x = 0$. The Taylor series starts $0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4 + \frac{1}{120}x^5$;

the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

08 08 ex 07 5. The $n^{\text{th}}$ derivative of $1/(1-x)$ is $f^{(n)}(x) = (n)!/(1-x)^{n+1}$, which evaluates to $n!$ at $x = 0$.

The Taylor series starts $1 + x + x^2 + x^3 + \cdots$;

the Taylor series is $\displaystyle\sum_{n=0}^{\infty} x^n$

08 08 ex 08 6. The derivative of $\tan^{-1} x$ is $1/(1+x^2)$. Taking successive derivatives using the Quotient Rule, the derivatives of $\tan^{-1} x$ fall into two categories in terms of their evaluation at $x = 0$.

When $n$ is even, $f^{(n)}(x) = (-1)^{(n-1)/2}\dfrac{p(x)}{(1+x^2)^n}$, where $p(x)$ is a polynomial such that $p(0) = 0$. Hence $f^{(n)}(0) = 0$ when $n$ is even.

When $n$ is odd, $f^{(n)}(x) = (-1)^{(n-1)/2}\dfrac{p(x)}{(1+x^2)^n}$, where $p(x)$ is a polynomial such that $p(0) = (n-1)!$. Hence $f^{(n)}(0) = (-1)^{(n-1)/2}(n-1)!$ when $n$ is odd. (The unusual power of $(-1)$ is such that every other odd term is negative.)

The Taylor series starts $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots$; by reindexing to only obtain odd powers of $x$, we get

the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$.

08 08 ex 04 7. The Taylor series starts $0 - (x-\pi/2) + 0x^2 + \frac{1}{6}(x-\pi/2)^3 + 0x^4 - \frac{1}{120}(x-\pi/2)^5$;

the Taylor series is $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi/2)^{2n+1}}{(2n+1)!}$

08 08 ex 05 8. The Taylor series starts
$1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

08 08 ex 09 9. $f^{(n)}(x)=(-1)^ne^{-x}$; at $x=0$, $f^{(n)}(0)=-1$ when $n$ is odd and $f^{(n)}(0)=1$ when $n$ is even.
The Taylor series starts $1-x+\frac{1}{2}x^2-\frac{1}{3!}x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$.

08 08 ex 10 10. $f^{(n)}(x)=(-1)^{n+1}\frac{(n-1)!}{(1+x)^n}$; at $x=0$,
$f^{(n)}(0)=(-1)^{n+1}(n-1)!$
The Taylor series starts $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$;
the Taylor series is $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$.

08 08 ex 11 11. $f^{(n)}(x)=(-1)^{n+1}\frac{n!}{(x+1)^{n+1}}$; at $x=1$,
$f^{(n)}(1)=(-1)^{n+1}\frac{n!}{2^{n+1}}$
The Taylor series starts
$\frac{1}{2}+\frac{1}{4}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{2^{n+1}}$.

08 08 ex 12 12. The derivatives of $\sin x$ are $\pm\cos x$ and $\pm\sin x$; at $x=\pi/4$, these derivatives evaluate to $\pm\sqrt{2}/2$.
The Taylor series starts $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\pi/4)-\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^2}{2}-\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^3}{3!}+\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^4}{4!}+\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^5}{5!}\cdots$.
Note how the signs are "even, even, odd, odd, even, even, odd, odd,... We saw signs like these in Example 230 of Section 8.1; one way of producing such signs is to raise $(-1)$ to a special quadratic power. While many possibilities exist, one such quadratic is $(n+3)(n+4)/2$.
Thus the Taylor series is
$\displaystyle\sum_{n=0}^{\infty}(-1)^{\frac{(n+3)(n+4)}{2}}\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^n}{n!}$.

08 08 ex 14 13. Given a value $x$, the magnitude of the error term $R_n(x)$ is bounded by

$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$

where $z$ is between 0 and $x$.
If $x>0$, then $z<x$ and $f^{(n+1)}(z)=e^z<e^x$. If $x<0$, then $x<z<0$ and $f^{(n+1)}(z)=e^z<1$. So given a fixed $x$ value, let $M=\max\{e^x,1\}$; $f^{(n)}(z)<M$. This allows us to state

$$\left|R_n(x)\right|\leq\frac{M}{(n+1)!}\left|x^{(n+1)}\right|.$$

For any $x$, $\displaystyle\lim_{n\to\infty}\frac{M}{(n+1)!}\left|x^{(n+1)}\right|=0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all $x$, and hence

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\quad\text{for all } x.$$

08 08 ex 13 14. The following argument is essentially the same as that given for $f(x)=\cos x$ in Example 269.
Given a value $x$, the magnitude of the error term $R_n(x)$ is bounded by

$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|.$$

Since all derivatives of $\sin x$ are $\pm\cos x$ or $\pm\sin x$, whose magnitudes are bounded by 1, we can state

$$\left|R_n(x)\right|\leq\frac{1}{(n+1)!}\left|x^{(n+1)}\right|.$$

For any $x$, $\displaystyle\lim_{n\to\infty}\frac{x^{n+1}}{(n+1)!}=0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all $x$, and hence

$$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\quad\text{for all } x.$$

08 08 ex 15 15. Given a value $x$, the magnitude of the error term $R_n(x)$ is bounded by

$$\left|R_n(x)\right|\leq\frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|(x-1)^{(n+1)}\right|,$$

where $z$ is between 1 and $x$.
Note that $\left|f^{(n+1)}(x)\right|=\frac{n!}{x^{n+1}}$.
We consider the cases when $x>1$ and when $x<1$ separately.
If $x>1$, then $1<z<x$ and $f^{(n+1)}(z)=\frac{n!}{z^{n+1}}<n!$. Thus

$$\left|R_n(x)\right|\leq\frac{n!}{(n+1)!}\left|(x-1)^{(n+1)}\right|=\frac{(x-1)^{n+1}}{n+1}.$$

For a fixed $x$,

$$\lim_{n\to\infty}\frac{(x-1)^{n+1}}{n+1}=0.$$

If $0<x<1$, then $x<z<1$ and $f^{(n+1)}(z)=\frac{n!}{z^{n+1}}<\frac{n!}{x^{n+1}}$. Thus

$$\left|R_n(x)\right|\leq\frac{n!/x^{n+1}}{(n+1)!}\left|(x-1)^{(n+1)}\right|=\frac{x^{n+1}}{n+1}(1-x)^{n+1}.$$

Since $0<x<1$, $x^{n+1}<1$ and $(1-x)^{n+1}<1$. We can then extend the inequality from above to state

$$\left|R_n(x)\right|\leq\frac{x^{n+1}}{n+1}(1-x)^{n+1}<\frac{1}{n+1}.$$

As $n\to\infty$, $1/(n+1)\to 0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all $x$, and hence

$$\ln x=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}\quad\text{for all } 0<x\leq 2.$$

08 08 ex 16 16. Given a value $x$, the magnitude of the error term $R_n(x)$ is bounded by

$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$

where $z$ is between 0 and $x$.
Note that $\left|f^{(n+1)}(x)\right| = \frac{(n+1)!}{(1-x)^{n+2}}$.
If $0 < x < 1$, then $0 < z < x$ and $f^{(n+1)}(z) = \frac{(n+1)!}{(1-z)^{n+2}} < \frac{(n+1)!}{(1-x)^{n+2}}$. Thus

$$\left|R_n(x)\right| \leq \frac{(n+1)!}{(1-x)^{n+2}}\frac{1}{(n+1)!}\left|x^{n+1}\right| = \frac{(x-1)^{n+1}}{n+1}.$$

For a fixed $x$,

$$\lim_{n\to\infty}\frac{(x-1)^{n+1}}{n+1} = 0,$$

hence

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ on } (-1,0).$$

08 08 ex 17 17. Given $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,

$\cos(-x) = \sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos x$, as all powers in the series are even.

08 08 ex 18 18. Given $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,

$\sin(-x) = \sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{-x^{2n+1}}{(2n+1)!} = -\sin x$, as all powers in the series are odd.

08 08 ex 19 19. Given $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,

$\frac{d}{dx}\big(\sin x\big) = \frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\right) = \sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos x$. (The summation still starts at $n=0$ as there was no constant term in the expansion of $\sin x$).

08 08 ex 20 20. Given $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,

$\frac{d}{dx}\big(\cos x\big) = \frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\right) = \sum_{n=1}^{\infty}(-1)^n\frac{(2n)x^{2n-1}}{(2n)!} = \sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!}$. We can re-index this summation to start at $n=0$ by replacing $n$ with $n+1$ in the summation:

$$\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}$$

Note that this series has the opposite sign of the Taylor series for $\sin x$; thus $\frac{d}{dx}(\cos x) = -\sin x$.

08 08 ex 27 21. $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}$

08 08 ex 28 22. $1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}+\frac{35x^4}{128}$

08 08 ex 29 23. $1+\frac{x}{3}-\frac{x^2}{9}+\frac{5x^3}{81}-\frac{10x^4}{243}$

08 08 ex 30 24. $1+4x+6x^2+4x^3+x^4$ (note the series is finite, and the formula still applies)

08 08 ex 21 25. $\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$.

08 08 ex 22 26. $\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}$.

08 08 ex 23 27. $\sum_{n=0}^{\infty}(-1)^n\frac{(2x+3)^{2n+1}}{(2n+1)!}$.

08 08 ex 24 28. $\sum_{n=0}^{\infty}(-1)^n\frac{(x/2)^{2n+1}}{(2n+1)}$.

08 08 ex 25 29. $x+x^2+\frac{x^3}{3}-\frac{x^5}{30}$

08 08 ex 26 30. $1+\frac{x}{2}-\frac{5x^2}{8}-\frac{3x^3}{16}$

08 08 ex 31 31. $\int_0^{\sqrt{\pi}}\sin\left(x^2\right)\,dx \approx \int_0^{\sqrt{\pi}}\left(x^2-\frac{x^6}{6}+\frac{x^{10}}{120}-\frac{x^{14}}{5040}\right)dx = 0.8877$

08 08 ex 32 32. $\int_0^{\pi^2/4}\cos\left(\sqrt{x}\right)\,dx \approx \int_0^{\pi^2/4}\left(1-\frac{x}{2}+\frac{x^2}{24}-\frac{x^3}{720}\right)\,dx = 1.1412$. (Actual answer: $\pi-2$)

# Chapter 9

### Section 9.1

09 01 ex 01 1. When defining the conics as the intersections of a plane and a double napped cone, degenerate conics are created when the plane intersects the tips of the cones (usually taken as the origin). Nondegenerate conics are formed when this plane does not contain the origin.

09 01 ex 03 2. Answers will vary.

09 01 ex 02 3. Hyperbola

09 01 ex 44 4. With the equation $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, the ellipse has a horizontal major axis if $a>b$. But the coefficient of the $x^2$ term is $1/a^2$ (not $a^2$), so if $1/a^2<1/b^2$, then $a>b$ and the major axis is horizontal.

09 01 ex 45 5. With a horizontal transverse axis, the $x^2$ term has a positive coefficient; with a vertical transverse axis, the $y^2$ term has a positive coefficient.

09 01 ex 04 6. $y=\frac{1}{2}(x-3)^2+\frac{3}{2}$

09 01 ex 05 7. $y=\frac{-1}{12}(x+1)^2-1$

09 01 ex 06 8. $x=-\frac{1}{4}(y-5)^2+2$

09 01 ex 07 9. $x=y^2$

09 01 ex 08 10. $y=-\frac{1}{4}(x-1)^2+2$

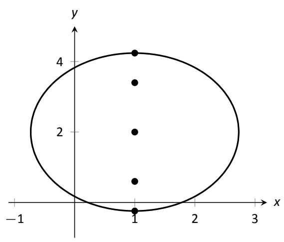
09 01 ex 09 11. $x = -\frac{1}{12}y^2$

09 01 ex 10 12. $y = 4x^2$

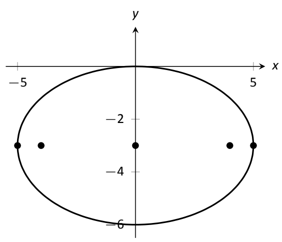
09 01 ex 11 13. $x = -\frac{1}{8}(y-3)^2 + 2$

09 01 ex 12 14. focus: $(0, 1)$; directrix: $y = -1$. The point $P$ is 2 units from each.

09 01 ex 13 15. focus: $(5, 2)$; directrix: $x = 1$. The point $P$ is 10 units from each.




09 01 ex 18 16.




09 01 ex 19 17.

09 01 ex 20 18. $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$; foci at $(-1 \pm \sqrt{5}, 2)$; $e = \sqrt{5}/3$

09 01 ex 21 19. $\frac{(x-1)^2}{1/4} + \frac{y^2}{9} = 1$; foci at $(1, \pm\sqrt{8.75})$; $e = \sqrt{8.75}/3 \approx 0.99$

09 01 ex 14 20. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

09 01 ex 15 21. $\frac{(x-2)^2}{25} + \frac{(y-3)^2}{16} = 1$

09 01 ex 16 22. $\frac{(x-2)^2}{45} + \frac{y^2}{49} = 1$

09 01 ex 17 23. $\frac{(x+1)^2}{9} + \frac{(y-1)^2}{25} = 1$

09 01 ex 22 24. $\frac{(x-1)^2}{2} + (y-2)^2 = 1$

09 01 ex 23 25. $\frac{x^2}{3} + \frac{y^2}{5} = 1$

09 01 ex 24 26. $\frac{x^2}{4} + \frac{(y-3)^2}{6} = 1$

09 01 ex 25 27. $\frac{(x-2)^2}{4} + \frac{(y-2)^2}{4} = 1$

09 01 ex 26 28.

(a) $c = \sqrt{12 - 4} = 2\sqrt{2}$.

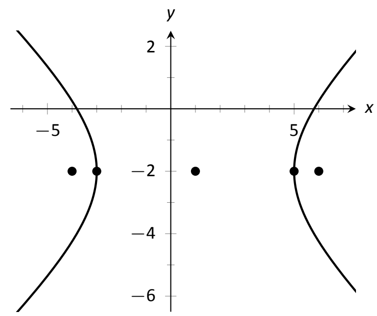
(b) The sum of distances for each point is $2\sqrt{12} \approx 6.9282$.

09 01 ex 27 29. $x^2 - \frac{y^2}{3} = 1$

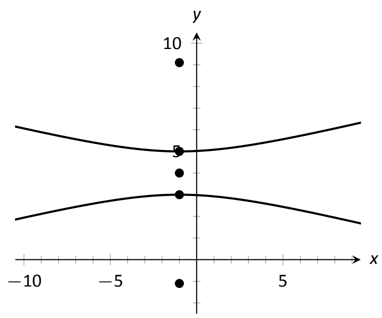
09 01 ex 28 30. $y^2 - \frac{x^2}{24} = 1$

09 01 ex 29 31. $\frac{(y-3)^2}{4} - \frac{(x-1)^2}{9} = 1$

09 01 ex 30 32. $\frac{(x-1)^2}{9} - \frac{(y-3)^2}{4} = 1$




09 01 ex 31 33.




09 01 ex 32 34.

09 01 ex 33 35. $\frac{x^2}{4} - \frac{y^2}{5} = 1$

09 01 ex 34 36. $\frac{y^2}{4} - \frac{x^2}{5} = 1$

09 01 ex 35 37. $\frac{(x-3)^2}{16} - \frac{(y-3)^2}{9} = 1$

09 01 ex 36 38. $\frac{(y-3)^2}{9} - \frac{(x-3)^2}{16} = 1$

09 01 ex 37 39. $\frac{x^2}{4} - \frac{y^2}{3} = 1$

09 01 ex 38 40. $\frac{x^2}{3} - \frac{(y-1)^2}{9} = 1$

09 01 ex 39 41. $(y-2)^2 - \frac{x^2}{10} = 1$

09 01 ex 40 42. $4y^2 - \frac{x^2}{4} = 1$

09 01 ex 42 43.

(a) Solve for $c$ in $e = c/a$: $c = ae$. Thus $a^2e^2 = a^2 - b^2$, and $b^2 = a^2 - a^2e^2$. The result follows.

(b) Mercury: $x^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
Earth: $x^2 + y^2/(0.99986)^2 = 1$
Mars: $x^2/(1.524)^2 + y^2/(1.517)^2 = 1$

(c) Mercury: $(x - 0.08)^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
Earth: $(x - 0.0167)^2 + y^2/(0.99986)^2 = 1$
Mars: $(x - 0.1423)^2/(1.524)^2 + y^2/(1.517)^2 = 1$

09 01 ex 43 44. The sound originated from a point approximately 31m to the left of $B$ and 1340m above it.

**Section 9.2**

09 02 ex 01 1. T

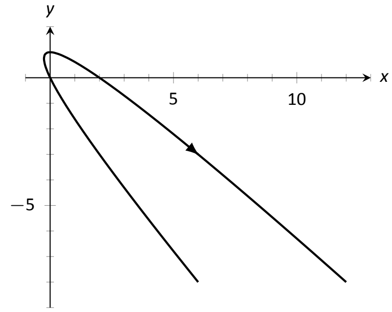
09 02 ex 02 2. orientation

09 02 ex 03 3. rectangular

09 02 ex 04 4. Answers will vary.

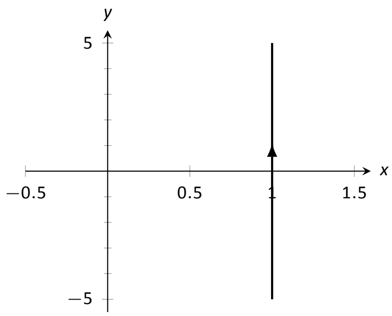
09 02 ex 05

5.



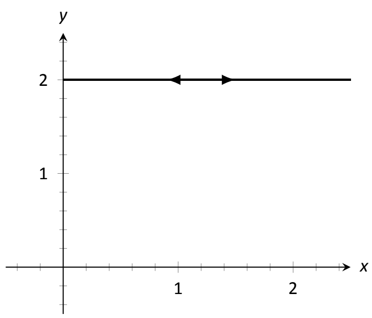

09 02 ex 06

6.



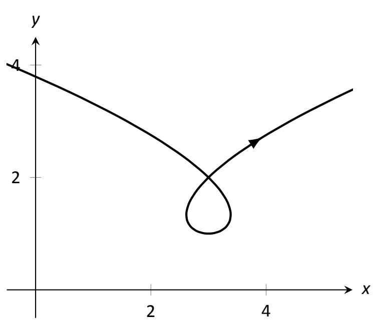

09 02 ex 07

7.



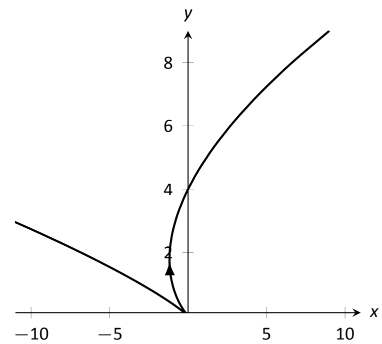

09 02 ex 08

8.



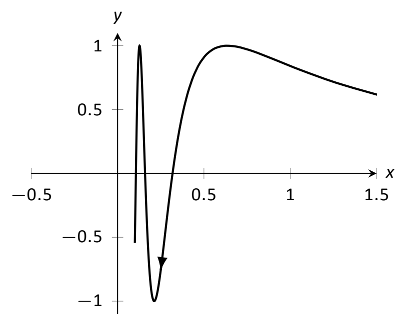

09 02 ex 09

9.



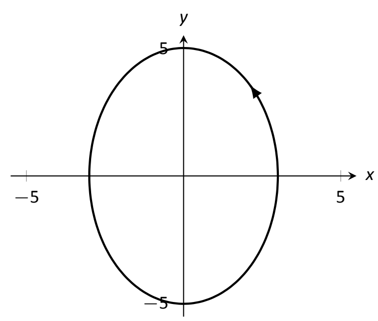

09 02 ex 10

10.



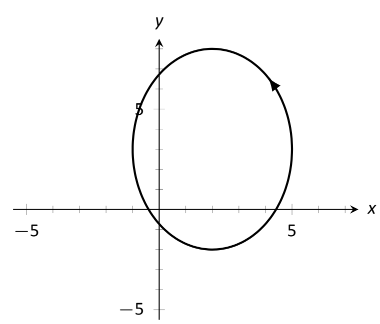

09 02 ex 11

11.



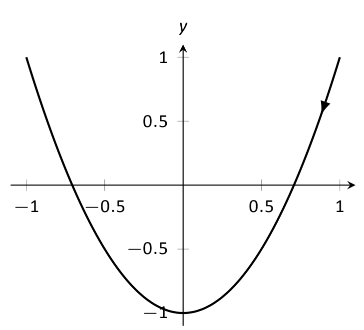

09 02 ex 12

12.

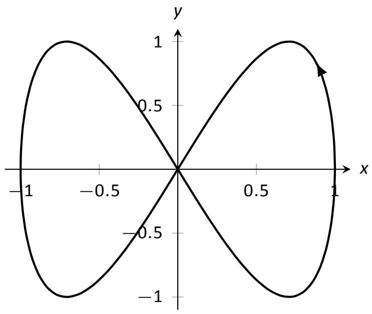



09 02 ex 13

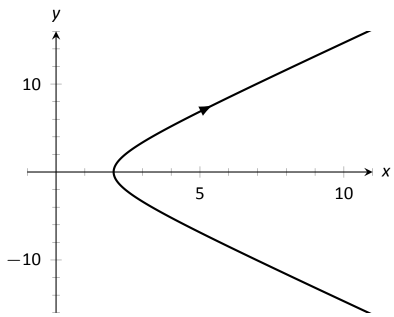
13.



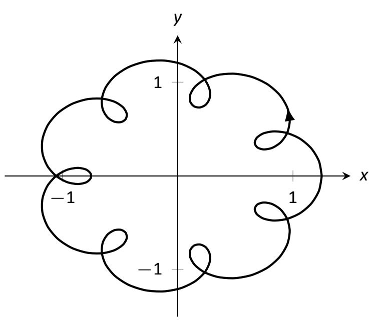
09 02 ex 14 14.



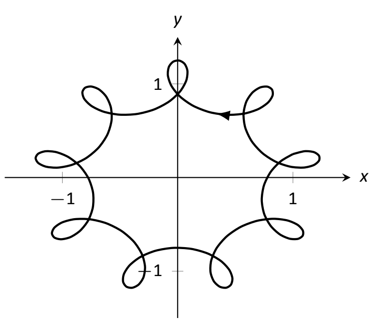

09 02 ex 15 15.




09 02 ex 16 16.




09 02 ex 17 17.




09 02 ex 18 18.

(a) Traces the parabola $y = x^2$, moves from left to right.

(b) Traces the parabola $y = x^2$, but only from $-1 \le x \le 1$; traces this portion back and forth infinitely.

(c) Traces the parabola $y = x^2$, but only for $0 < x$. Moves left to right.

(d) Traces the parabola $y = x^2$, moves from right to left.

09 02 ex 19 19.

(a) Traces a circle of radius 1 counterclockwise once.

(b) Traces a circle of radius 1 counterclockwise over 6 times.

(c) Traces a circle of radius 1 clockwise infinite times.

(d) Traces an arc of a circle of radius 1, from an angle of -1 radians to 1 radian, twice.

09 02 ex 27 20. $y = -1.5x + 8.5$

09 02 ex 20 21. $x^2 - y^2 = 1$

09 02 ex 21 22. $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$

09 02 ex 22 23. $y = x^{3/2}$

09 02 ex 23 24. $y = 2x + 3$

09 02 ex 24 25. $y = x^3 - 3$

09 02 ex 25 26. $y = e^{2x} - 1$

09 02 ex 26 27. $y^2 - x^2 = 1$

09 02 ex 28 28. $x^2 - y^2 = 1$

09 02 ex 29 29. $x = 1 - 2y^2$

09 02 ex 30 30. $y = \frac{b}{a}(x - x_0) + y_0$; line through $(x_0, y_0)$ with slope $b/a$.

09 02 ex 31 31. $x^2 + y^2 = r^2$; circle centered at $(0, 0)$ with radius $r$.

09 02 ex 32 32. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$; ellipse centered at $(h, k)$ with horizontal axis of length $2a$ and vertical axis of length $2b$.

09 02 ex 33 33. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$; hyperbola centered at $(h, k)$ with horizontal transverse axis and asymptotes with slope $b/a$. The parametric equations only give half of the hyperbola. When $a > 0$, the right half; when $a < 0$, the left half.

09 02 ex 34 34. $x = (t + 11)/6$, $y = (t^2 - 97)/12$. At $t = 1$, $x = 2$, $y = -8$.
$y' = 6x - 11$; when $x = 2$, $y' = 1$.

09 02 ex 35 35. $x = \ln t$, $y = t$. At $t = 1$, $x = 0$, $y = 1$.
$y' = e^x$; when $x = 0$, $y' = 1$.

09 02 ex 36 36. $x = \cos^{-1} t$, $y = \sqrt{1 - t^2}$. At $t = 1$, $x = 0$, $y = 0$.
$y' = \cos x$; when $x = 0$, $y' = 1$.

09 02 ex 37 37. $x = 1/(4t^2)$, $y = 1/(2t)$. At $t = 1$, $x = 1/4$, $y = 1/2$.
$y' = 1/(2\sqrt{x})$; when $x = 1/4$, $y' = 1$.

09 02 ex 38 38. $t = \pm 1$

09 02 ex 41 39. $t = -1,\ 2$

09 02 ex 39 40. $t = \pi/2, 3\pi/2$

09 02 ex 40 41. $t = \pi/6, \pi/2, 5\pi/6$

09 02 ex 42 42. $t = -1$

09 02 ex 43 43. $t = 2$

09 02 ex 44 44. $t = \ldots \pi/2,\ 3\pi/2,\ 5\pi/2,\ \ldots$

09 02 ex 45 45. $t = \ldots 0,\ 2\pi,\ 4\pi,\ \ldots$

09 02 ex 46 46. $x = 4t$, $y = -16t^2 + 64t$

09 02 ex 47 47. $x = 50t$, $y = -16t^2 + 64t$

09 02 ex 48 48. $x = 10t$, $y = -16t^2 + 320t$

09 02 ex 49 49. $x = 2\cos t$, $y = -2\sin t$; other answers possible

09 02 ex 50 50. $x = 3\cos(2\pi t) + 1$, $y = 3\sin(2\pi t) + 1$; other answers possible

09 02 ex 51 51. $x = \cos t + 1$, $y = 3\sin t + 3$; other answers possible

09 02 ex 52 52. $x = 5\cos t$, $y = \sqrt{24}\sin t$; other answers possible

09 02 ex 53 53. $x = \pm\sec t + 2$, $y = \sqrt{8}\tan t - 3$; other answers possible

09 02 ex 54 54. $x = 2\tan t$, $y = \pm 6\sec t$; other answers possible

**Section 9.3**

09 03 ex 01 1. F

09 03 ex 02 2. $t$

09 03 ex 03 3. F

09 03 ex 04 4. T

09 03 ex 05 5.
  (a) $\frac{dy}{dx} = 2t$
  (b) Tangent line: $y = 2(x-1) + 1$; normal line: $y = -1/2(x-1) + 1$

09 03 ex 06 6.
  (a) $\frac{dy}{dx} = 10\sqrt{t}$
  (b) Tangent line: $y = 20(x-2) + 22$; normal line: $y = -1/20(x-2) + 22$

09 03 ex 07 7.
  (a) $\frac{dy}{dx} = \frac{2t+1}{2t-1}$
  (b) Tangent line: $y = 3x + 2$; normal line: $y = -1/3x + 2$

09 03 ex 08 8.
  (a) $\frac{dy}{dx} = \frac{3t^2}{2t}$
  (b) $t = 0$: Tangent line: $x = -1$; normal line: $y = 0$
  $t = 1$: Tangent line: $y = x$; normal line: $y = -x$

09 03 ex 09 9.
  (a) $\frac{dy}{dx} = \csc t$
  (b) $t = \pi/4$: Tangent line: $y = \sqrt{2}(x - \sqrt{2}) + 1$; normal line: $y = -1/\sqrt{2}(x - \sqrt{2}) + 1$

09 03 ex 10 10.
  (a) $\frac{dy}{dx} = -2\cos(2t)\csc t$
  (b) $t = \pi/4$: Tangent line: $y = 1$; normal line: $x = \sqrt{2}/2$

09 03 ex 11 11.
  (a) $\frac{dy}{dx} = \frac{\cos t\sin(2t) + \sin t\cos(2t)}{-\sin t\sin(2t) + 2\cos t\cos(2t)}$
  (b) Tangent line: $y = x - \sqrt{2}$; normal line: $y = -x - \sqrt{2}$

09 03 ex 12 12.
  (a) $\frac{dy}{dx} = \frac{\sin(t) + 10\cos(t)}{\cos(t) - 10\sin(t)}$
  (b) Tangent line: $y = -x/10 + e^{\pi/20}$; normal line: $y = 10x + e^{\pi/20}$

09 03 ex 13 13. $t = 0$

09 03 ex 14 14. $t = 0$ (though this uses a one–sided limit, as $x(t)$ is not defined for $t < 0$.

09 03 ex 15 15. $t = -1/2$

09 03 ex 16 16. $t = \pm 1/\sqrt{3}$

09 03 ex 17 17. The graph does not have a horizontal tangent line.

09 03 ex 18 18. $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

09 03 ex 19 19. The solution is non-trivial; use identities $\sin(2t) = 2\sin t\cos t$ and $\cos(2t) = \cos^2 t - \sin^2 t$ to rewrite $g'(t) = 2\sin t(2\cos^2 t - \sin^2 t)$. On $[0, 2\pi]$, $\sin t = 0$ when $t = 0, \pi, 2\pi$, and $2\cos^2 t - \sin^2 t = 0$ when $t = \tan^{-1}(\sqrt{2}),\ \pi \pm \tan^{-1}(\sqrt{2}),\ 2\pi - \tan^{-1}(\sqrt{2})$.

09 03 ex 20 20. $t = \tan^{-1}(-10),\ \tan^{-1}(-10) + \pi$

09 03 ex 21 21. $t_0 = 0$; $\lim_{t\to 0} \frac{dy}{dx} = 0$.

09 03 ex 22 22. $t_0 = 2$; $\lim_{t\to 2} \frac{dy}{dx} = 1$.

09 03 ex 23 23. $t_0 = 1$; $\lim_{t\to 1} \frac{dy}{dx} = \infty$.

09 03 ex 43 24. $t_0 = \ldots, -\pi/2, 0, \pi/2, \pi, \ldots$; $\lim_{t\to 0} \frac{dy}{dx} = 1$.

09 03 ex 24 25. $\frac{d^2y}{dx^2} = 2$; always concave up

09 03 ex 25 26. $\frac{d^2y}{dx^2} = 10$; always concave up

09 03 ex 26 27. $\frac{d^2y}{dx^2} = -\frac{4}{(2t-1)^3}$; concave up on $(-\infty, 1/2)$; concave down on $(1/2, \infty)$.

09 03 ex 27 28. $\frac{d^2y}{dx^2} = \frac{3t^2+1}{4t^3}$; concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.

09 03 ex 28 29. $\frac{d^2y}{dx^2} = -\cot^3 t$; concave up on $(-\infty, 0)$; concave down on $(0, \infty)$.

09 03 ex 29 30. $\frac{d^2y}{dx^2} = \frac{\cos t\sin(2t) + 2\sin t\cos(2t)}{(-\sin t\sin(2t) + 2\cos t\cos(2t))^2}$; concavity switches at $t = \tan^{-1}(1/\sqrt{2}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{2}),\ \pi + \tan^{-1}(1/\sqrt{2}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{2})$

09 03 ex 44 31. $\frac{d^2y}{dx^2} = \frac{4(13+3\cos(4t))}{(\cos t + 3\cos(3t))^3}$, obtained with a computer algebra system; concave up on $\big(-\tan^{-1}(\sqrt{2}/2), \tan^{-1}(\sqrt{2}/2)\big)$, concave down on $\big(-\pi/2, -\tan^{-1}(\sqrt{2}/2)\big) \cup \big(\tan^{-1}(\sqrt{2}/2), \pi/2\big)$

09 03 ex 30 32. $\frac{d^2y}{dx^2} = \frac{1010}{e^{t/10}\left(\cos t - 10\sin t\right)^3}$; concavity switches at $t = \tan^{-1}(1/10) + n\pi$, where $n$ is an integer.

09 03 ex 32 33. $L = 6\pi$

09 03 ex 31 34. On $[0, 2\pi]$, arc length is $L = \sqrt{101}(e^{\pi/5} - 1)$; on $[2\pi, 4\pi]$, $L = \sqrt{101}(e^{2\pi/5} - 1)$.

09 03 ex 33 35. $L = 2\sqrt{34}$

09 03 ex 34 36. $L = 4\sqrt{2} - 2$

09 03 ex 35 37. $L \approx 2.4416$ (actual value: $L = 2.42211$)

09 03 ex 36 38. $L \approx 9.73004$ (actual value: $L = 9.42943$)

09 03 ex 37 39. $L \approx 4.19216$ (actual value: $L = 4.18308$)

09 03 ex 38 40. Formula: $C \approx 25.9062$; Simpson's Rule: $C \approx 25.4786$ (actual value: $C = 25.527$)

09 03 ex 39 41. The answer is $16\pi$ for both (of course), but the integrals are different.

09 03 ex 40 42. $8\pi^2$.

09 03 ex 41 43. $SA \approx 8.50101$ (actual value $SA = 8.02851$

09 03 ex 42 44. $SA \approx 1.36751$ (actual value $SA = 1.36707$

**Section 9.4**

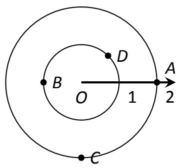
09 04 ex 01 1. Answers will vary.

09 04 ex 02 2. F

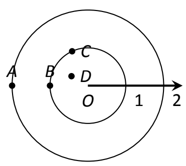
09 04 ex 03 3. T

09 04 ex 04 4. F

09 04 ex 05 5.




09 04 ex 06 6.



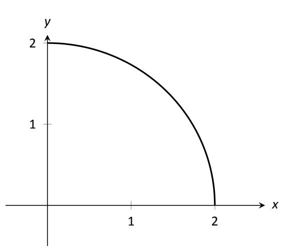

09 04 ex 07 7. $A = P(2.5, \pi/4)$ and $P(-2.5, 5\pi/4)$;
$B = P(-1, 5\pi/6)$ and $P(1, 11\pi/6)$;
$C = P(3, 4\pi/3)$ and $P(-3, \pi/3)$;
$D = P(1.5, 2\pi/3)$ and $P(-1.5, 5\pi/3)$;

09 04 ex 08 8. $A = P(2, \pi/6)$ and $P(-2, -5\pi/6)$;
$B = P(1, -\pi/3)$ and $P(-1, 2\pi/3)$;
$C = P(2, 3\pi/4)$ and $P(-2, -\pi/4)$;
$D = P(2.5, \pi)$ and $P(2.5, -\pi)$;

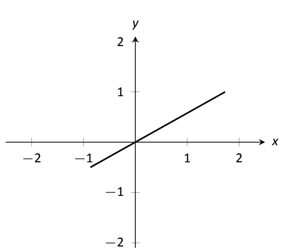
09 04 ex 09 9. $A = (\sqrt{2}, \sqrt{2})$
$B = (\sqrt{2}, -\sqrt{2})$
$C = P(\sqrt{5}, -0.46)$
$D = P(\sqrt{5}, 2.68)$

09 04 ex 10 10. $A = (-3, 0)$
$B = (-1/2, \sqrt{3}/2)$
$C = P(4, \pi/2)$
$D = P(2, -\pi/3)$

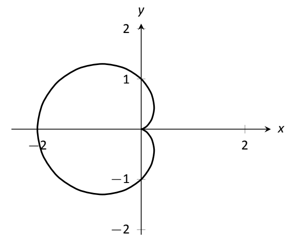
09 04 ex 11 11.



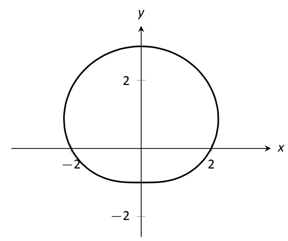
09 04 ex 12 12.



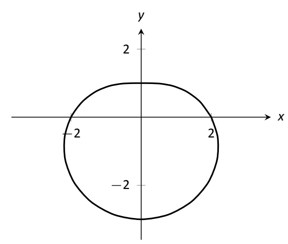
09 04 ex 13 13.



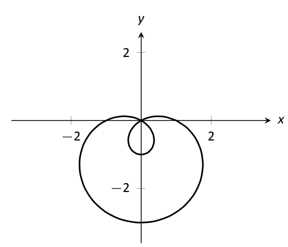
09 04 ex 14 14.



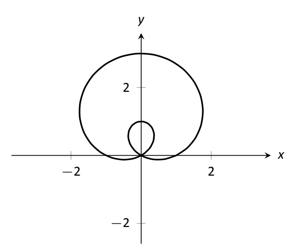
09 04 ex 15 15.



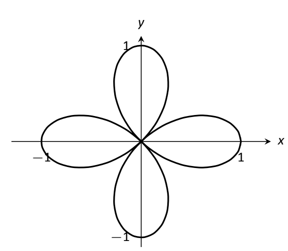
09 04 ex 16 16.



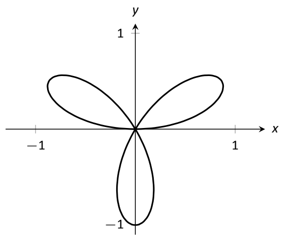
09 04 ex 17 17.



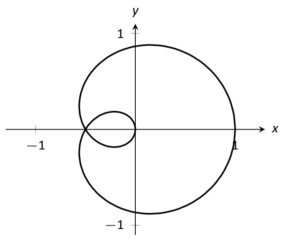
09 04 ex 18 18.



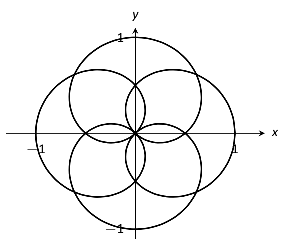
09 04 ex 19 19.



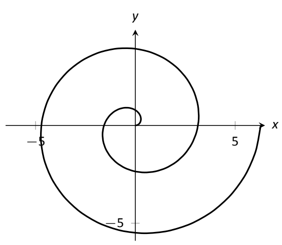

09 04 ex 20 20.



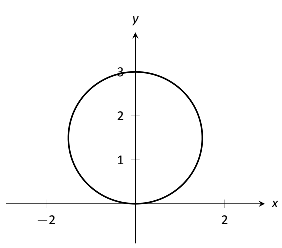

09 04 ex 21 21.



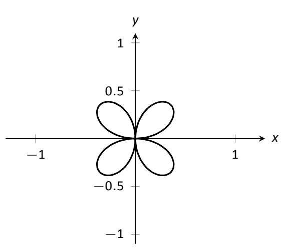

09 04 ex 22 22.



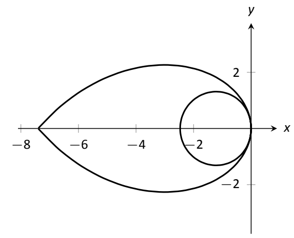

09 04 ex 23 23.



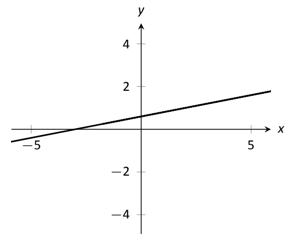

09 04 ex 24 24.



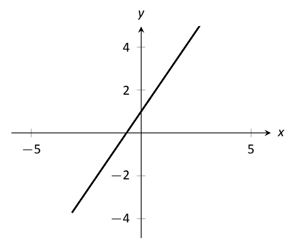

09 04 ex 25 25.



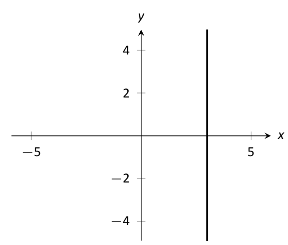

09 04 ex 26 26.



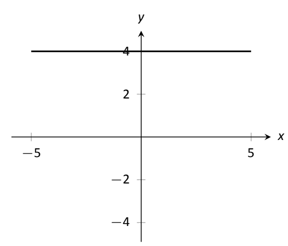

09 04 ex 27 27.




09 04 ex 28 28.




09 04 ex 29 29.




09 04 ex 30 30. $(x-1)^2+y^2=1$

09 04 ex 31 31. $x^2+(y+2)^2=4$

09 04 ex 32 32. $(x-1/2)^2+(y-1/2)^2=1/2$

09 04 ex 33 33. $y=2/5x+7/5$

09 04 ex 34 34. $x=3$

09 04 ex 35 35. $y=4$

09 04 ex 36 36. $x^4+x^2y^2x^2-y^2=0$

09 04 ex 37 37. $x^2+y^2=4$

09 04 ex 38 38. $y=x/\sqrt{3}$

09 04 ex 39 39. $\theta=\pi/4$

09 04 ex 40 40. $r = 7/(\sin\theta - 4\cos\theta)$

09 04 ex 41 41. $r = 5\sec\theta$

09 04 ex 42 42. $r = 5\csc\theta$

09 04 ex 43 43. $r = \cos\theta/\sin^2\theta$

09 04 ex 44 44. $r = 1/\sqrt[3]{\cos^2\theta\sin\theta}$

09 04 ex 45 45. $r = \sqrt{7}$

09 04 ex 46 46. $r = -2\cos\theta$

09 04 ex 47 47. $P(\sqrt{3}/2, \pi/6)$, $P(0, \pi/2)$, $P(-\sqrt{3}/2, 5\pi/6)$

09 04 ex 48 48. $P(1, 0)$, $P(0, \pi/2) = P(0, \pi/4)$, $P(-1/2, \pi/3)$

09 04 ex 49 49. $P(0, 0) = P(0, \pi/2)$, $P(\sqrt{2}, \pi/4)$

09 04 ex 50 50. $P(\sqrt{3}/2, \pi/3) = P(-\sqrt{3}/2, 4\pi/3)$, $P(\sqrt{3}/2, 2\pi/3) = P(-\sqrt{3}/2, 5\pi/3)$, $P(0, \pi/2)$

09 04 ex 51 51. $P(\sqrt{2}/2, \pi/12)$, $P(-\sqrt{2}/2, 5\pi/12)$, $P(\sqrt{2}/2, 3\pi/4)$

09 04 ex 52 52. $P(3/2, \pi/3)$, $P(3/2, -\pi/3)$

09 04 ex 53 53. For all points, $r = 1$; $\theta = \pi/12,\ 5\pi/12,\ 7\pi/12,\ 11\pi/12,\ 13\pi/12,\ 17\pi/12,\ 19\pi/12,\ 23\pi/12.$

09 04 ex 54 54. $P(0, 0) = P(0, 3\pi/2)$, $P(1 + \sqrt{2}/2, 3\pi/4)$, $P(1 - \sqrt{2}/2, 7\pi/4)$

09 04 ex 55 55. Answers will vary. If $m$ and $n$ do not have any common factors, then an interval of $2n\pi$ is needed to sketch the entire graph.

09 04 ex 56 56. Answers will vary.

**Section 9.5**

09 05 ex 01 1. Using $x = r\cos\theta$ and $y = r\sin\theta$, we can write $x = f(\theta)\cos\theta$, $y = f(\theta)\sin\theta$.

09 05 ex 02 2. rectangles; sectors of circles

09 05 ex 03 3.
   (a) $\frac{dy}{dx} = -\cot\theta$
   (b) tangent line: $y = -(x - \sqrt{2}/2) + \sqrt{2}/2$; normal line: $y = x$

09 05 ex 04 4.
   (a) $\frac{dy}{dx} = 1/2(\tan\theta - \cot\theta)$
   (b) tangent line: $y = 1/2$; normal line: $x = 1/2$

09 05 ex 05 5.
   (a) $\frac{dy}{dx} = \frac{\cos\theta(1+2\sin\theta)}{\cos^2\theta - \sin\theta(1+\sin\theta)}$
   (b) tangent line: $x = 3\sqrt{3}/4$; normal line: $y = 3/4$

09 05 ex 10 6.
   (a) $\frac{dy}{dx} = \frac{3\sin^2(t)+(1-3\cos(t))\cos(t)}{3\sin(t)\cos(t)-\sin(t)(1-3\cos(t))}$
   (b) tangent line: $y = \frac{1}{1+3\sqrt{2}}(x + (1/\sqrt{2} + 3/2)) + 1/\sqrt{2} + 3/2 \approx$ $y = 0.19(x + 2.21) + 2.21$; normal line: $y = -(1+3\sqrt{2})(x + (1/\sqrt{2} + 3/2)) + 1/\sqrt{2} + 3/2$

09 05 ex 06 7.
   (a) $\frac{dy}{dx} = \frac{\theta\cos\theta + \sin\theta}{\cos\theta - \theta\sin\theta}$
   (b) tangent line: $y = -2/\pi x + \pi/2$; normal line: $y = \pi/2x + \pi/2$

09 05 ex 07 8.
   (a) $\frac{dy}{dx} = \frac{\cos\theta\cos(3\theta) - 3\sin\theta\sin(3\theta)}{-\cos(3\theta)\sin\theta - 3\cos\theta\sin(3\theta)}$
   (b) tangent line: $y = x/\sqrt{3}$; normal line: $y = -\sqrt{3}x$

09 05 ex 08 9.
   (a) $\frac{dy}{dx} = \frac{4\sin(t)\cos(4t)+\sin(4t)\cos(t)}{4\cos(t)\cos(4t)-\sin(t)\sin(4t)}$
   (b) tangent line: $y = 5\sqrt{3}(x + \sqrt{3}/4) - 3/4$; normal line: $y = -1/5\sqrt{3}(x + \sqrt{3}/4) - 3/4$

09 05 ex 09 10.
   (a) $\frac{dy}{dx} = 1$
   (b) tangent line: $y = x + 1$; normal line: $y = -x - 1$

09 05 ex 11 11. horizontal: $\theta = \pi/2, 3\pi/2$;
vertical: $\theta = 0, \pi, 2\pi$

09 05 ex 12 12. horizontal: $\theta = 0, \pi/2, \pi$;
vertical: $\theta = \pi/4, 3\pi/4$

09 05 ex 13 13. horizontal:
$\theta = \tan^{-1}(1/\sqrt{5}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{5}),\ \pi + \tan^{-1}(1/\sqrt{5}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{5})$;
vertical: $\theta = 0,\ \tan^{-1}(\sqrt{5}),\ \pi - \tan^{-1}(\sqrt{5}),\ \pi,\ \pi + \tan^{-1}(\sqrt{5}),\ 2\pi - \tan^{-1}(\sqrt{5})$

09 05 ex 14 14. horizontal: $\theta = \pi/3,\ 5\pi/3$;
vertical: $\theta = 0,\ 2\pi/3,\ 4\pi/3,\ 2\pi$
At $\theta = \pi$, $\frac{dy}{dx} = 0/0$; apply L'Hopital's Rule to find that $\frac{dy}{dx} \to 0$ as $\theta \to \pi$.

09 05 ex 15 15. In polar: $\theta = 0 \ \cong\ \theta = \pi$
In rectangular: $y = 0$

09 05 ex 16 16. In polar: $\theta = 0,\ \theta = \pi/3,\ \theta = 2\pi/3$.
In rectangular: $y = 0$, $y = \sqrt{3}x$, and $y = -\sqrt{3}x$.

09 05 ex 17 17. area = $4\pi$

09 05 ex 20 18. area = $25\pi$

09 05 ex 18 19. area = $\pi/12$

09 05 ex 19 20. area = $3\pi/2$

09 05 ex 21 21. area = $\pi - 3\sqrt{3}/2$

09 05 ex 22 22. area = $2\pi + 3\sqrt{3}/2$

09 05 ex 23 23. area = $\pi + 3\sqrt{3}$

09 05 ex 25 24. area = 1

09 05 ex 26 25. area = $\displaystyle\int_{\pi/12}^{\pi/3} \frac{1}{2}\sin^2(3\theta)\,d\theta - \int_{\pi/12}^{\pi/6} \frac{1}{2}\cos^2(3\theta)\,d\theta = \frac{1}{12} + \frac{\pi}{24}$

09 05 ex 24 26. area = $\frac{1}{32}(4\pi - 3\sqrt{3})$

09 05 ex 27 27. area = $\displaystyle\int_0^{\pi/3} \frac{1}{2}(1-\cos\theta)^2\,d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\cos\theta)^2\,d\theta = \frac{7\pi}{24} - \frac{\sqrt{3}}{2} \approx 0.0503$

09 05 ex 28 28. $x'(\theta) = f'(\theta)\cos\theta - f(\theta)\sin\theta$, $y'(\theta) = f'(\theta)\sin\theta + f(\theta)\cos\theta$. Square each and add; applying the Pythagoran Theorem twice achieves the result.

09 05 ex 29 29. $4\pi$

09 05 ex 30 30. $4\pi$

09 05 ex 31 31. $L \approx 2.2592$; (actual value $L = 2.22748$)

09 05 ex 32 32. $L \approx 7.62933$; (actual value $L = 8$)

09 05 ex 33 33. $SA = 16\pi$

09 05 ex 34 34. $SA = 4\pi$

09 05 ex 35 35. $SA = 32\pi/5$

09 05 ex 36 36. $SA = 4\pi^2$

09 05 ex 37 37. $SA = 36\pi$

# Chapter 10

**Section 10.1**

10 01 ex 08 1. right hand

10 01 ex 01 2. line; plane

10 01 ex 02 3. curve (a parabola); surface (a cylinder)

10 01 ex 03 4. a hyperbolic paraboloid

10 01 ex 04 5. a hyperboloid of two sheets

10 01 ex 05 6. a hyperboloid of one sheet

10 01 ex 06 7. $|| \overline{AB} || = \sqrt{6}$; $|| \overline{BC} || = \sqrt{17}$; $|| \overline{AC} || = \sqrt{11}$. Yes, it is a right triangle as $|| \overline{AB} ||^2 + || \overline{AC} ||^2 = || \overline{BC} ||^2$.

10 01 ex 07 8. Yes, as opposite sides have equal length. $|| \overline{AB} || = \sqrt{21} = || \overline{CD} ||$; $|| \overline{BC} || = \sqrt{6} = || \overline{AD} ||$.

10 01 ex 09 9. Center at $(4, -1, 0)$; radius = 3

10 01 ex 10 10. Center at $(-2, 1, 2)$; radius = $\sqrt{5}$

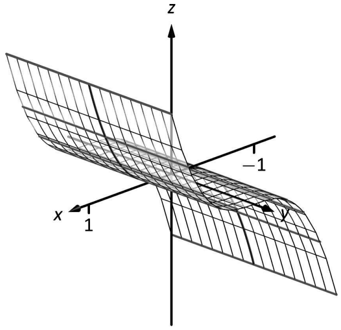
10 01 ex 11 11. Interior of a sphere with radius 1 centered at the origin.

10 01 ex 12 12. Region bounded between the planes $x = 0$ (the $y - z$ coordinate plane) and $x = 3$.

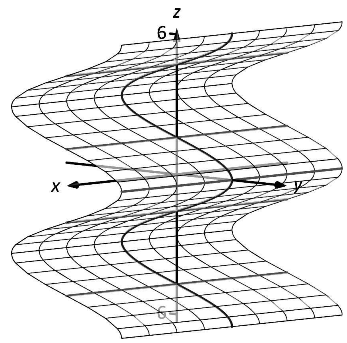
10 01 ex 13 13. The first octant of space; all points $(x, y, z)$ where each of $x$, $y$ and $z$ are positive. (Analogous to the first quadrant in the plane.)

10 01 ex 14 14. All points in space where the $y$ value is greater than 3; viewing space as often depicted in this text, this is the region "to the right" of the plane $y = 3$ (which is parallel to the $x - z$ coordinate plane.)

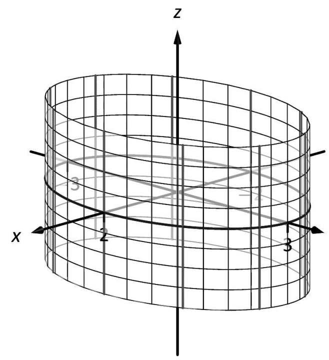
10 01 ex 15 15.



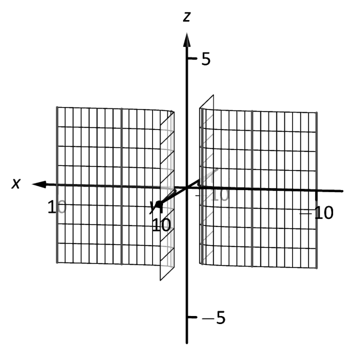

10 01 ex 16 16.




10 01 ex 17 17.




10 01 ex 18 18.




10 01 ex 29 19. $y^2 + z^2 = x^4$

10 01 ex 30 20. $y^2 + z^2 = x^4$

10 01 ex 31 21. $z = (\sqrt{x^2 + y^2})^2 = x^2 + y^2$

10 01 ex 32 22. $z = \frac{1}{\sqrt{x^2+y^2}}$

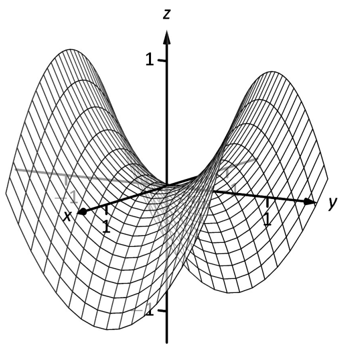
10 01 ex 19 23. (a) $x = y^2 + \dfrac{z^2}{9}$

10 01 ex 20 24. (b) $x^2 - y^2 + z^2 = 0$

10 01 ex 21 25. (b) $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

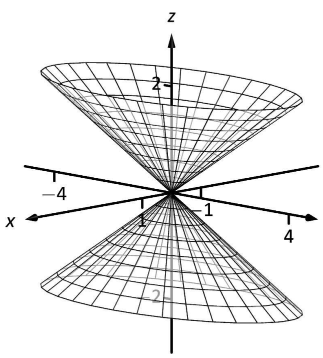
10 01 ex 22 26. (a) $y^2 - x^2 - z^2 = 1$

10 01 ex 28 27.



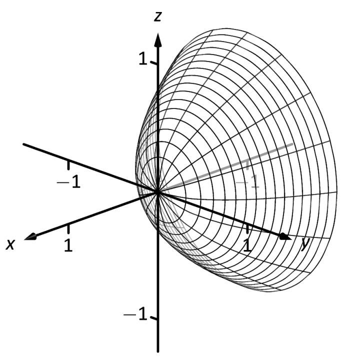

10 01 ex 24

28.



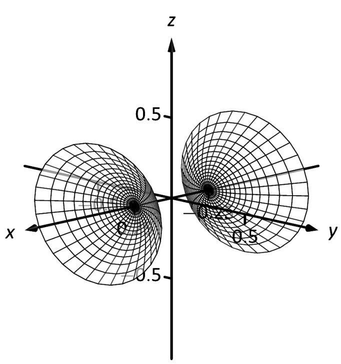

10 01 ex 23

29.



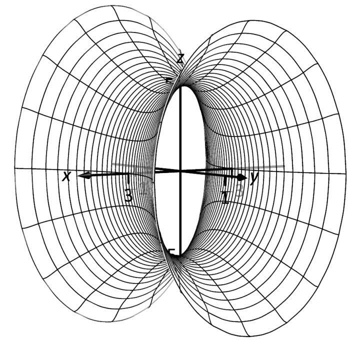

10 01 ex 27

30.



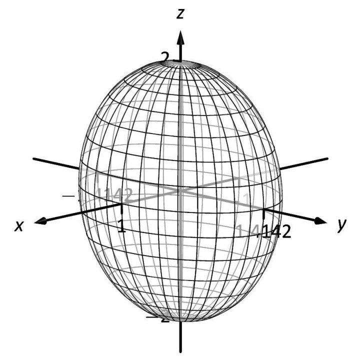

10 01 ex 26

31.




10 01 ex 25

32.




**Section 10.2**

10 02 ex 01

1. Answers will vary.

10 02 ex 02

2. $(1, 2)$ is a point; $\langle 1, 2\rangle$ is a vector that describes a displacement of 1 unit in the *x*-direction and 2 units in the *y*-direction.

10 02 ex 03

3. A vector with magnitude 1.

10 02 ex 04

4. Their respective unit vectors are parallel; unit vectors $\vec{u}_1$ and $\vec{u}_2$ are parallel if $\vec{u}_1 = \pm\vec{u}_2$.

10 02 ex 05

5. It stretches the vector by a factor of 2, and points it in the opposite direction.

10 02 ex 06

6. $\overrightarrow{PQ} = \langle 1, 6\rangle = 1\vec{i} + 6\vec{j}$

10 02 ex 07

7. $\overrightarrow{PQ} = \langle -4, 4\rangle = -4\vec{i} + 4\vec{j}$

10 02 ex 08

8. $\overrightarrow{PQ} = \langle 6, -1, 6\rangle = 6\vec{i} - \vec{j} + 6\vec{k}$

10 02 ex 09

9. $\overrightarrow{PQ} = \langle 2, 2, 0\rangle = 2\vec{i} + 2\vec{j}$

10 02 ex 10

10.

(a) $\vec{u} + \vec{v} = \langle 2, -1\rangle$; $\vec{u} - \vec{v} = \langle 0, -3\rangle$; $2\vec{u} - 3\vec{v} = \langle -1, -7\rangle$.

(c) $\vec{x} = \langle 1/2, 2\rangle$.

10 02 ex 11

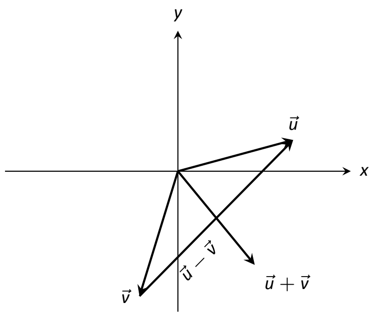
11.

(a) $\vec{u} + \vec{v} = \langle 3, 2, 1\rangle$; $\vec{u} - \vec{v} = \langle -1, 0, -3\rangle$; $\pi\vec{u} - \sqrt{2}\vec{v} = \left\langle \pi - 2\sqrt{2}, \pi - \sqrt{2}, -\pi - 2\sqrt{2}\right\rangle$.

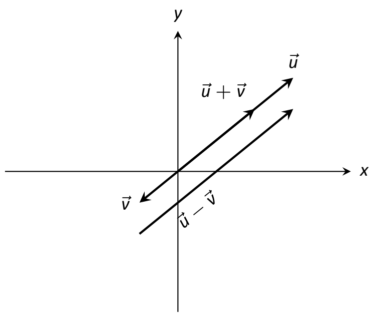
(c) $\vec{x} = \langle -1, 0, -3\rangle$.

10 02 ex 12

12.

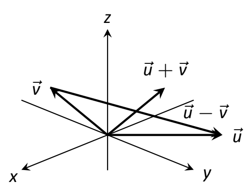



10 02 ex 13 13.

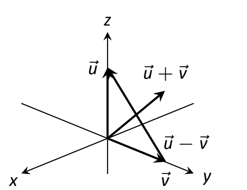



Sketch of $\vec u - \vec v$ shifted for clarity.

10 02 ex 14 14.




10 02 ex 15 15.




10 02 ex 16 16. $||\,\vec u\,|| = \sqrt{5}$, $||\,\vec v\,|| = \sqrt{13}$, $||\,\vec u + \vec v\,|| = \sqrt{26}$, $||\,\vec u - \vec v\,|| = \sqrt{10}$

10 02 ex 17 17. $||\,\vec u\,|| = \sqrt{17}$, $||\,\vec v\,|| = \sqrt{3}$, $||\,\vec u + \vec v\,|| = \sqrt{14}$, $||\,\vec u - \vec v\,|| = \sqrt{26}$

10 02 ex 18 18. $||\,\vec u\,|| = \sqrt{5}$, $||\,\vec v\,|| = 3\sqrt{5}$, $||\,\vec u + \vec v\,|| = 2\sqrt{5}$, $||\,\vec u - \vec v\,|| = 4\sqrt{5}$

10 02 ex 19 19. $||\,\vec u\,|| = 7$, $||\,\vec v\,|| = 35$, $||\,\vec u + \vec v\,|| = 42$, $||\,\vec u - \vec v\,|| = 28$

10 02 ex 20 20. When $\vec u$ and $\vec v$ have the same direction. (Note: parallel is not enough.)

10 02 ex 21 21. $\vec u = \left\langle 3/\sqrt{30}, 7/\sqrt{30}\right\rangle$

10 02 ex 22 22. $\vec u = \langle 0.6, 0.8\rangle$

10 02 ex 23 23. $\vec u = \langle 1/3, -2/3, 2/3\rangle$

10 02 ex 24 24. $\vec u = \left\langle 1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3}\right\rangle$

10 02 ex 25 25. $\vec u = \langle \cos 50^\circ, \sin 50^\circ\rangle \approx \langle 0.643, 0.766\rangle$.

10 02 ex 26 26. $\vec u = \langle \cos 120^\circ, \sin 120^\circ\rangle = \left\langle -1/2, \sqrt{3}/2\right\rangle$.

10 02 ex 27 27.

$$\begin{aligned}
||\,\vec u\,|| &= \sqrt{\sin^2\theta\cos^2\varphi + \sin^2\theta\sin^2\varphi + \cos^2\theta} \\
&= \sqrt{\sin^2\theta(\cos^2\varphi + \sin^2\varphi) + \cos^2\theta} \\
&= \sqrt{\sin^2\theta + \cos^2\theta} \\
&= 1.
\end{aligned}$$

10 02 ex 28 28. The force on each chain is $100/\sqrt{3} \approx 57.735$lb.

10 02 ex 29 29. The force on each chain is 100lb.

10 02 ex 30 30. The force on the chain with angle $\theta$ is approx. 45.124lb; the force on the chain with angle $\varphi$ is approx. 59.629lb.

10 02 ex 31 31. The force on each chain is 50lb.

10 02 ex 32 32. $\theta = 45^\circ$; the weight is lifted 0.29 ft (about 3.5in).

10 02 ex 33 33. $\theta = 5.71^\circ$; the weight is lifted 0.005 ft (about 1/16th of an inch).

10 02 ex 34 34. $\theta = 45^\circ$; the weight is lifted 2.93 ft.

10 02 ex 35 35. $\theta = 84.29^\circ$; the weight is lifted 9 ft.

**Section 10.3**

10 03 ex 01 1. Scalar

10 03 ex 02 2. The magnitude of a vectors is the square root of the dot product of a vector with itself; that is, $||\,\vec v\,|| = \sqrt{\vec v \cdot \vec v}$.

10 03 ex 03 3. By considering the sign of the dot product of the two vectors. If the dot product is positive, the angle is acute; if the dot product is negative, the angle is obtuse.

10 03 ex 04 4. "Perpendicular" is one answer.

10 03 ex 05 5. $-22$

10 03 ex 06 6. 33

10 03 ex 07 7. 3

10 03 ex 08 8. 0

10 03 ex 09 9. not defined

10 03 ex 10 10. 0

10 03 ex 11 11. Answers will vary.

10 03 ex 12 12. Answers will vary.

10 03 ex 13 13. $\theta = 0.3218 \approx 18.43^\circ$

10 03 ex 14 14. $\theta = 1.6476 \approx 94.4^\circ$

10 03 ex 15 15. $\theta = \pi/4 = 45^\circ$

10 03 ex 16 16. $\theta = \pi/2 = 90^\circ$

10 03 ex 17 17. Answers will vary; two possible answers are $\langle -7, 4\rangle$ and $\langle 14, -8\rangle$.

10 03 ex 18 18. Answers will vary; two possible answers are $\langle 5, 3\rangle$ and $\langle -15, -9\rangle$.

10 03 ex 19 19. Answers will vary; two possible answers are $\langle 1, 0, -1\rangle$ and $\langle 4, 5, -9\rangle$.

10 03 ex 20 20. Answers will vary; two possible answers are $\langle 2, 1, 0\rangle$ and $\langle 1, 1, 1/3\rangle$.

10 03 ex 21 21. $\text{proj}_{\vec{v}}\,\vec{u} = \langle -1/2, 3/2\rangle$.

10 03 ex 22 22. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 2, 6\rangle$.

10 03 ex 25 23. $\text{proj}_{\vec{v}}\,\vec{u} = \langle -1/2, -1/2\rangle$.

10 03 ex 26 24. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 0, 0\rangle$.

10 03 ex 23 25. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 1, 2, 3\rangle$.

10 03 ex 24 26. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 4/3, 4/3, 2/3\rangle$.

10 03 ex 27 27. $\vec{u} = \langle -1/2, 3/2\rangle + \langle 3/2, 1/2\rangle$.

10 03 ex 28 28. $\vec{u} = \langle 2, 6\rangle + \langle 3, -1\rangle$.

10 03 ex 29 29. $\vec{u} = \langle -1/2, -1/2\rangle + \langle -5/2, 5/2\rangle$.

10 03 ex 30 30. $\vec{u} = \langle 0, 0\rangle + \langle -3, 2\rangle$.

10 03 ex 31 31. $\vec{u} = \langle 1, 2, 3\rangle + \langle 0, 3, -2\rangle$.

10 03 ex 32 32. $\vec{u} = \langle 4/3, 4/3, 2/3\rangle + \langle 5/3, -7/3, 4/3\rangle$.

10 03 ex 33 33. 1.96lb

10 03 ex 34 34. 5lb

10 03 ex 35 35. 141.42ft–lb

10 03 ex 36 36. 196.96ft–lb

10 03 ex 37 37. 500ft–lb

10 03 ex 38 38. 424.26ft–lb

10 03 ex 39 39. 500ft–lb

**Section 10.4**

10 04 ex 01 1. vector

10 04 ex 02 2. right hand rule

10 03 ex 04 3. "Perpendicular" is one answer.

10 04 ex 03 4. T

10 04 ex 04 5. Torque

10 04 ex 07 6. $\vec{u}\times\vec{v} = \langle 12, -15, 3\rangle$

10 04 ex 08 7. $\vec{u}\times\vec{v} = \langle 11, 1, -17\rangle$

10 04 ex 05 8. $\vec{u}\times\vec{v} = \langle -5, -31, 27\rangle$

10 04 ex 06 9. $\vec{u}\times\vec{v} = \langle 47, -36, -44\rangle$

10 04 ex 09 10. $\vec{u}\times\vec{v} = \langle 0, -2, 0\rangle$

10 04 ex 10 11. $\vec{u}\times\vec{v} = \langle 0, 0, 0\rangle$

10 04 ex 11 12. $\vec{i}\times\vec{j} = \vec{k}$

10 04 ex 12 13. $\vec{i}\times\vec{k} = -\vec{j}$

10 04 ex 13 14. $\vec{j}\times\vec{k} = \vec{i}$

10 04 ex 14 15. Answers will vary.

10 04 ex 15 16. Answers will vary.

10 04 ex 16 17. 5

10 04 ex 17 18. 21

10 04 ex 18 19. 0

10 04 ex 19 20. 5

10 04 ex 20 21. $\sqrt{14}$

10 04 ex 21 22. $\sqrt{230}$

10 04 ex 22 23. 3

10 04 ex 23 24. 6

10 04 ex 24 25. $5\sqrt{2}/2$

10 04 ex 25 26. $3\sqrt{30}/2$

10 04 ex 26 27. 1

10 04 ex 27 28. $5/2$

10 04 ex 28 29. 7

10 04 ex 29 30. $8\sqrt{7/2}$

10 04 ex 30 31. 2

10 04 ex 31 32. 15

10 04 ex 35 33. $\pm\frac{1}{\sqrt{6}}\langle 1, 1, -2\rangle$

10 04 ex 32 34. $\pm\frac{1}{\sqrt{21}}\langle -2, 1, 4\rangle$

10 04 ex 33 35. $\langle 0, \pm 1, 0\rangle$

10 04 ex 34 36. any vector orthogonal to $\vec{u}$ works (such as $\frac{1}{\sqrt{2}}\langle 1, 0, -1\rangle$).

10 04 ex 36 37. 87.5ft–lb

10 04 ex 37 38. $43.75\sqrt{3} \approx 75.78$ft–lb

10 04 ex 38 39. $200/3 \approx 66.67$ft–lb

10 04 ex 39 40. 11.58ft–lb

10 04 ex 40 41. With $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$, we have

$$\begin{aligned}\vec{u}\cdot(\vec{u}\times\vec{v}) &= \langle u_1, u_2, u_3\rangle \cdot (\langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1\rangle)\\ &= u_1(u_2v_3 - u_3v_2) - u2(u_1v_3 - u_3v_1) + u3(u_1v_2 - u_2v_1)\\ &= 0.\end{aligned}$$

10 04 ex 41 42. With $\vec{u} = \langle u_1, u_2, u_3\rangle$, we have

$$\begin{aligned}\vec{u}\times\vec{u} &= \langle u_2u_3 - u_3u_2, -(u_1u_3 - u_3u_1), u_1u_2 - u_2u_1\rangle\\ &= \langle 0, 0, 0\rangle\\ &= \vec{0}.\end{aligned}$$

**Section 10.5**

10 05 ex 01 1. A point on the line and the direction of the line.

10 05 ex 02 2. parallel

10 05 ex 03 3. parallel, skew

10 05 ex 04 4. Answers will vary

10 05 ex 05 5. vector: $\ell(t) = \langle 2, -4, 1\rangle + t\,\langle 9, 2, 5\rangle$
parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$
symmetric: $(x-2)/9 = (y+4)/2 = (z-1)/5$

10 05 ex 06 6. vector: $\ell(t) = \langle 6, 1, 7\rangle + t\,\langle -3, 2, 5\rangle$
parametric: $x = 6 - 3t$, $y = 1 + 2t$, $z = 7 + 5t$
symmetric: $-(x-6)/3 = (y-1)/2 = (z-7)/5$

10 05 ex 07 7. Answers can vary: vector: $\ell(t) = \langle 2, 1, 5\rangle + t\,\langle 5, -3, -1\rangle$
parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$
symmetric: $(x-2)/5 = -(y-1)/3 = -(z-5)$

10 05 ex 08 8. Answers can vary: vector: $\ell(t) = \langle 1, -2, 3\rangle + t\,\langle 4, 7, 2\rangle$
parametric: $x = 1 + 4t$, $y = -2 + 7t$, $z = 3 + 2t$
symmetric: $(x-1)/4 = (y+2)/7 = (z-3)/2$

10 05 ex 09 9. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\ell(t) = \langle 0, 1, 2\rangle + t\,\langle -10, 43, 9\rangle$
parametric: $x = -10t$, $y = 1 + 43t$, $z = 2 + 9t$
symmetric: $-x/10 = (y-1)/43 = (z-2)/9$

10 05 ex 10 10. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\ell(t) = \langle 5, 1, 9\rangle + t\,\langle 0, -1, 0\rangle$
parametric: $x = 5$, $y = 1 - t$, $z = 9$
symmetric: not defined, as some components of the direction are 0.

10 05 ex 11 11. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\ell(t) = \langle 7, 2, -1\rangle + t\,\langle 1, -1, 2\rangle$
parametric: $x = 7 + t$, $y = 2 - t$, $z = -1 + 2t$
symmetric: $x - 7 = 2 - y = (z+1)/2$

10 05 ex 12 12. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\ell(t) = \langle 2, 2, 3\rangle + t\,\langle 5, -1, -3\rangle$
parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
symmetric: $(x-2)/5 = -(y-2) = -(z-3)/3$

10 05 ex 13 13. vector: $\ell(t) = \langle 1, 1\rangle + t\,\langle 2, 3\rangle$
parametric: $x = 1 + 2t$, $y = 1 + 3t$
symmetric: $(x-1)/2 = (y-1)/3$

10 05 ex 14 14. vector: $\ell(t) = \langle -2, 5\rangle + t\,\langle 0, 1\rangle$
parametric: $x = -2$, $y = 5 + t$
symmetric: not defined

10 05 ex 15 15. parallel

10 05 ex 16 16. intersecting; $\ell_1(2) = \ell_2(-2) = \langle 12, 3, 7\rangle$

10 05 ex 17 17. intersecting; $\vec{\ell}_1(3) = \vec{\ell}_2(4) = \langle 9, -5, 13\rangle$

10 05 ex 18 18. same

10 05 ex 19 19. skew

10 05 ex 20 20. parallel

10 05 ex 21 21. same

10 05 ex 22 22. skew

10 05 ex 23 23. $\sqrt{41}/3$

10 05 ex 24 24. $3\sqrt{2}$

10 05 ex 25 25. $5\sqrt{2}/2$

10 05 ex 26 26. 5

10 05 ex 27 27. $3/\sqrt{2}$

10 05 ex 28 28. 2

10 05 ex 29 29. Since both $P$ and $Q$ are on the line, $\overrightarrow{PQ}$ is parallel to $\vec{d}$. Thus $\overrightarrow{PQ} \times \vec{d} = \vec{0}$, giving a distance of 0.

10 05 ex 30 30. (Note: this solution is easier once one has studied Section 10.6.) Since the two lines intersect, we can state $P_2 = P_1 + a\vec{d}_1 + b\vec{d}_2$ for some scalars $a$ and $b$. (Here we abuse notation slightly and add points to vectors.) Thus $\overrightarrow{P_1P_2} = a\vec{d}_1 + b\vec{d}_2$. Vector $\vec{c}$ is the cross product of $\vec{d}_1$ and $\vec{d}_2$, hence is orthogonal to both, and hence is orthogonal to $\overrightarrow{P_1P_2}$. Thus $\overrightarrow{P_1P_2} \cdot \vec{c} = 0$, and the distance between lines is 0.

10 05 ex 31 31.

(a) The distance formula cannot be used because since $\vec{d}_1$ and $\vec{d}_2$ are parallel, $\vec{c}$ is $\vec{0}$ and we cannot divide by $||\,\vec{0}\,||$.

(b) Since $\vec{d}_1$ and $\vec{d}_2$ are parallel, $\overrightarrow{P_1P_2}$ lies in the plane formed by the two lines. Thus $\overrightarrow{P_1P_2} \times \vec{d}_2$ is orthogonal to this plane, and $\vec{c} = (\overrightarrow{P_1P_2} \times \vec{d}_2) \times \vec{d}_2$ is parallel to the plane, but still orthogonal to both $\vec{d}_1$ and $\vec{d}_2$. We desire the length of the projection of $\overrightarrow{P_1P_2}$ onto $\vec{c}$, which is what the formula provides.

(c) Since the lines are parallel, one can measure the distance between the lines at any location on either line (just as to find the distance between straight railroad tracks, one can use a measuring tape anywhere along the track, not just at one specific place.) Let $P = P_1$ and $Q = P_2$ as given by the equations of the lines, and apply the formula for distance between a point and a line.

**Section 10.6**

10 06 ex 01 1. A point in the plane and a normal vector (i.e., a direction orthogonal to the plane).

10 06 ex 02 2. A normal vector is orthogonal to the plane.

10 06 ex 03 3. Answers will vary.

10 06 ex 04 4. Answers will vary.

10 06 ex 05 5. Answers will vary.

10 06 ex 06 6. Answers will vary.

10 06 ex 07 7. Standard form: $3(x-2) - (y-3) + 7(z-4) = 0$
general form: $3x - y + 7z = 31$

10 06 ex 08 8. Standard form: $2(y-3) + 4(z-5) = 0$
general form: $2y + 4z = 26$

10 06 ex 09 9. Answers may vary;
Standard form: $8(x-1) + 4(y-2) - 4(z-3) = 0$
general form: $8x + 4y - 4z = 4$

10 06 ex 10 10. Answers may vary;
Standard form: $-5(x-5) + 3(y-3) + 2(z-8) = 0$
general form: $-5x + 3y + 2z = 0$

10 06 ex 11 11. Answers may vary;
Standard form: $-7(x-2) + 2(y-1) + (z-2) = 0$
general form: $-7x + 2y + z = -10$

10 06 ex 12 12. Answers may vary;
Standard form: $3(x-5) + 3(z-3) = 0$
general form: $3x + 3z = 24$

10 06 ex 13 13. Answers may vary;
Standard form: $2(x-1) - (y-1) = 0$
general form: $2x - y = 1$

10 06 ex 14 14. Answers may vary;
Standard form: $2(x-1) + (y-1) - 3(z-1) = 0$
general form: $2x + y - 3z = 0$

10 06 ex 15 15. Answers may vary;
Standard form: $2(x-2) - (y+6) - 4(z-1) = 0$
general form: $2x - y - 4z = 6$

10 06 ex 16 16. Answers may vary;
Standard form: $4(x-5) - 2(y-7) - 2(z-3) = 0$
general form: $4x - 2y - 2z = 0$

10 06 ex 17 17. Answers may vary;
Standard form: $(x-5)+(y-7)+(z-3)=0$
general form: $x+y+z=15$

10 06 ex 18 18. Answers may vary;
Standard form: $4(x-4)+(y-1)+(z-1)=0$
general form: $4x+y+z=18$

10 06 ex 19 19. Answers may vary;
Standard form: $3(x+4)+8(y-7)-10(z-2)=0$
general form: $3x+8y-10z=24$

10 06 ex 20 20. Standard form: $x-1=0$
general form: $x=1$

10 06 ex 21 21. Answers may vary:
$$\ell=\begin{cases} x=14t \\ y=-1-10t \\ z=2-8t \end{cases}$$

10 06 ex 22 22. Answers may vary:
$$\ell=\begin{cases} x=1+20t \\ y=3+2t \\ z=3.5-26t \end{cases}$$

10 06 ex 30 23. $(-3,-7,-5)$

10 06 ex 31 24. $(3,1,1)$

10 06 ex 28 25. No point of intersection; the plane and line are parallel.

10 06 ex 29 26. The plane contains the line, so every point on the line is a "point of intersection."

10 06 ex 23 27. $\sqrt{5/7}$

10 06 ex 24 28. $8/\sqrt{21}$

10 06 ex 25 29. $1/\sqrt{3}$

10 06 ex 26 30. 3

10 06 ex 27 31. If $P$ is any point in the plane, and $Q$ is also in the plane, then $\overrightarrow{PQ}$ lies parallel to the plane and is orthogonal to $\vec{n}$, the normal vector. Thus $\vec{n}\cdot\overrightarrow{PQ}=0$, giving the distance as 0.

10 06 ex 32 32. The intersecting lines define a plane with normal vector $\vec{n}=\vec{c}=\vec{d}_1\times\vec{d}_2$. Since points $P_1$ and $P_2$ lie in the plane, $\vec{c}$ is orthogonal to $\overrightarrow{P_1P_2}$, hence $\overrightarrow{P_1P_2}\cdot\vec{c}=0$, giving a distance of 0. Knowing the principles of planes, especially their normal vectors, makes this simpler.

# Chapter 11

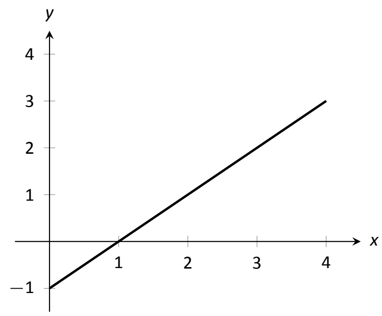
### Section 11.1

11 01 ex 01 1. parametric equations

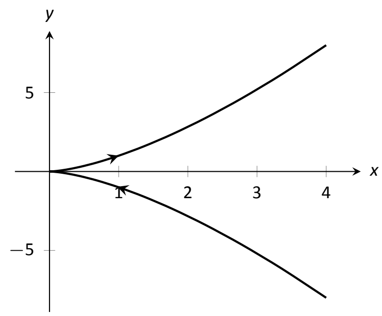
11 01 ex 02 2. vectors

11 01 ex 31 3. displacement

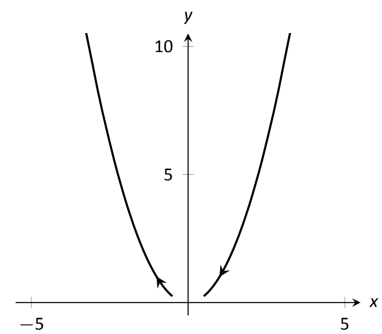
11 01 ex 03 4.



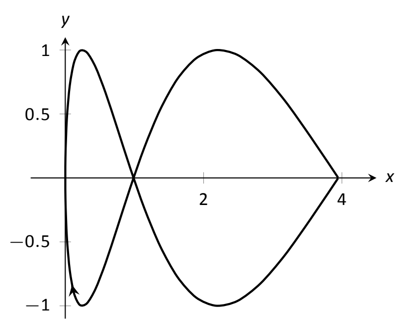

11 01 ex 04 5.



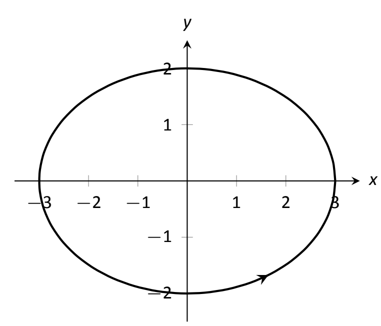

11 01 ex 05 6.



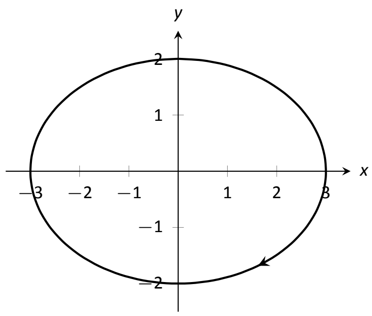

11 01 ex 06 7.



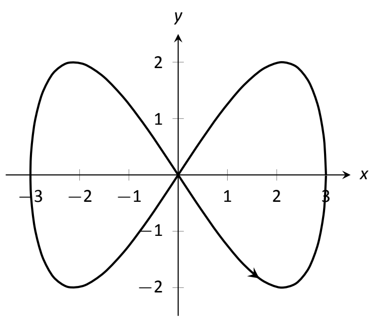

11 01 ex 07 8.



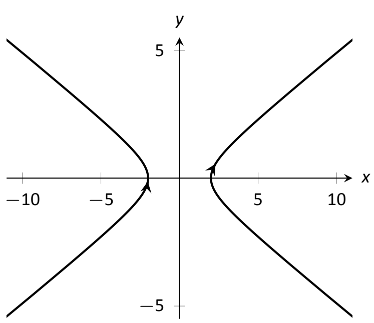

11 01 ex 08 9.



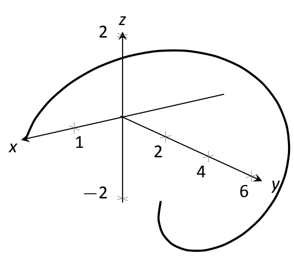

11 01 ex 09 10.



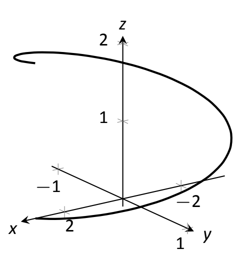

11 01 ex 10 11.



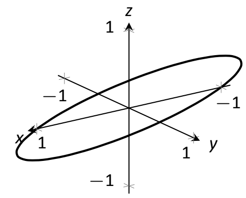

11 01 ex 11 12.



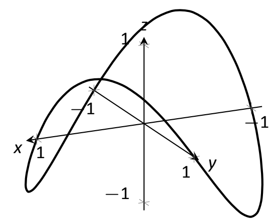

11 01 ex 12 13.




11 01 ex 13 14.




11 01 ex 14 15.




11 01 ex 15 16. $||\,\vec{r}(t)\,|| = \sqrt{t^2+t^4} = |t|\sqrt{t^2+1}$.

11 01 ex 16 17. $||\,\vec{r}(t)\,|| = \sqrt{25\cos^2 t + 9\sin^2 t}$.

11 01 ex 17 18. $||\,\vec{r}(t)\,|| = \sqrt{4\cos^2 t + 4\sin^2 t + t^2} = \sqrt{t^2+4}$.

11 01 ex 18 19. $||\,\vec{r}(t)\,|| = \sqrt{\cos^2 t + t^2 + t^4}$.

11 01 ex 19 20. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 2\cos t + 1, 2\sin t + 2\rangle$.

11 01 ex 20 21. Answers may vary; three solutions are
$\vec{r}(t) = \langle 3\sin t + 5, 3\cos t + 5\rangle$,
$\vec{r}(t) = \langle -3\cos t + 5, 3\sin t + 5\rangle$ and
$\vec{r}(t) = \langle 3\cos t + 5, -3\sin t + 5\rangle$.

11 01 ex 21 22. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 1.5\cos t, 5\sin t\rangle$.

11 01 ex 22 23. Answers may vary, though most direct solutions are
$\vec{r}(t) = \langle -3\cos t + 3, 2\sin t - 2\rangle$,
$\vec{r}(t) = \langle 3\cos t + 3, -2\sin t - 2\rangle$ and
$\vec{r}(t) = \langle 3\sin t + 3, 2\cos t - 2\rangle$.

11 01 ex 23 24. Answers may vary, though most direct solutions are
$\vec{r}(t) = \langle t, 5(t-2) + 3\rangle$ and
$\vec{r}(t) = \langle t+2, 5t+3\rangle$.

11 01 ex 24 25. Answers may vary, though most direct solutions are
$\vec{r}(t) = \langle t, -1/2(t-1) + 5\rangle$,
$\vec{r}(t) = \langle t+1, -1/2t + 5\rangle$,
$\vec{r}(t) = \langle -2t+1, t+5\rangle$ and
$\vec{r}(t) = \langle 2t+1, -t+5\rangle$.

11 01 ex 25 26. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 2\cos t, 2\sin t, 2t\rangle$.

11 01 ex 26 27. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 3\cos(4\pi t), 3\sin(4\pi t), 3t\rangle$.

11 01 ex 27 28. $\langle 1, 0\rangle$

11 01 ex 28 29. $\langle 1, 1\rangle$

11 01 ex 29 30. $\langle 0, 0, 1\rangle$

11 01 ex 30 31. $\langle 1, 2, 7\rangle$

**Section 11.2**

11 02 ex 01 1. component

11 02 ex 02 2. displacement

11 02 ex 03 3. It is difficult to identify the points on the graphs of $\vec{r}(t)$ and $\vec{r}\,'(t)$ that correspond to each other.

11 02 ex 04 4. $\langle 11, 74, \sin 5\rangle$

11 02 ex 05 5. $\langle e^3, 0\rangle$

11 02 ex 06 6. $\langle 1, e\rangle$

11 02 ex 07 7. $\langle 2t, 1, 0\rangle$

11 02 ex 08 8. $(-\infty, 0)\bigcup(0,\infty)$

11 02 ex 09 9. $(0,\infty)$

11 02 ex 10 10. $\vec{r}\,'(t) = \langle -\sin t, e^t, 1/t\rangle$

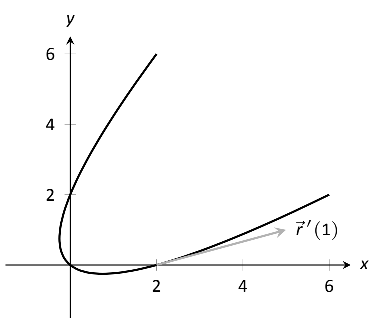
11 02 ex 11 11. $\vec{r}\,'(t) = \langle -1/t^2, 5/(3t+1)^2, \sec^2 t\rangle$

11 02 ex 12 12. $\vec{r}\,'(t) = (2t)\langle \sin t, 2t+5\rangle + (t^2)\langle \cos t, 2\rangle = \langle 2t\sin t + t^2\cos t, 6t^2+10t\rangle$

11 02 ex 13 13. $\vec{r}\,'(t) = \langle 2t, 1\rangle\cdot\langle \sin t, 2t+5\rangle + \langle t^2+1, t-1\rangle\cdot\langle \cos t, 2\rangle = (t^2+1)\cos t + 2t\sin t + 4t + 3$

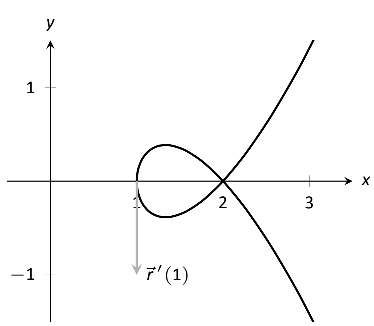
11 02 ex 14 14. $\vec{r}\,'(t) = \langle 2t, 1, 0\rangle \times \langle \sin t, 2t+5, 1\rangle + \langle t^2+1, t-1, 1\rangle \times \langle \cos t, 2, 0\rangle = \langle -1, \cos t - 2t, 6t^2+10t+2+\cos t - \sin t - t\cos t\rangle$

11 02 ex 15 15.



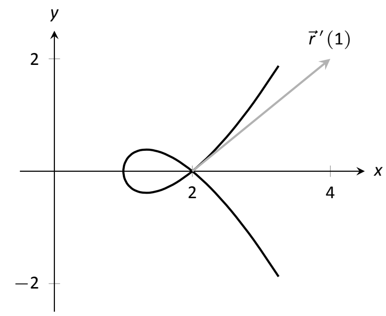

$\vec{r}\,'(t) = \langle 2t+1, 2t-1\rangle$

11 02 ex 16 16.



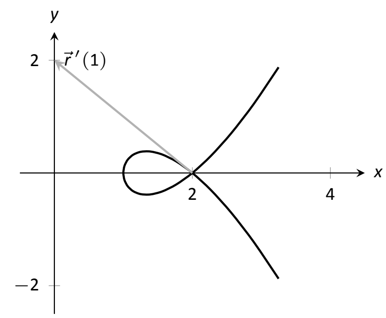

$\vec{r}\,'(t) = \langle 2t-2, 3t^2-6t+2\rangle$

11 02 ex 17 17.




$\vec{r}\,'(t) = \langle 2t, 3t^2-1\rangle$

11 02 ex 18 18.




$\vec{r}\,'(t) = \langle 2t-4, 3t^2-12t+11\rangle$

11 02 ex 19 19. $\ell(t) = \langle 2, 0\rangle + t\langle 3, 1\rangle$

11 02 ex 20 20. $\ell(t) = \langle 3\sqrt{2}/2, \sqrt{2}/2\rangle + t\langle -3\sqrt{2}/2, \sqrt{2}/2\rangle$

11 02 ex 21 21. $\ell(t) = \langle -3, 0, \pi\rangle + t\langle 0, -3, 1\rangle$

11 02 ex 22 22. $\ell(t) = \langle 1, 0, 0\rangle + t\langle 1, 1, 1\rangle$

11 02 ex 23 23. $t = 2n\pi$, where $n$ is an integer; so $t = \ldots -4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots$

11 02 ex 24 24. $t = 1$

11 02 ex 25 25. $\vec{r}(t)$ is not smooth at $t = 3\pi/4 + n\pi$, where $n$ is an integer

11 02 ex 26 26. $t = \pm 1$

11 02 ex 27 27. Both derivatives return $\langle 5t^4, 4t^3-3t^2, 3t^2\rangle$.

11 02 ex 28 28. Both derivatives return $2\sin t + t^2\cos t + te^t + 1$.

11 02 ex 29 29. Both derivatives return $\langle 2t - e^t - 1, \cos t - 3t^2, (t^2+2t)e^t - (t-1)\cos t - \sin t\rangle$.

11 02 ex 30 30. $\langle \frac{1}{4}t^4, \sin t, te^t - e^t\rangle + \vec{C}$

11 02 ex 31 31. $\langle \tan^{-1} t, \tan t\rangle + \vec{C}$

11 02 ex 32 32. $\langle -2, 0\rangle$

11 02 ex 33 33. $\langle 4, -4\rangle$

11 02 ex 34 34. $\vec{r}(t) = \langle \frac{1}{2}t^2+2, -\cos t + 3\rangle$

11 02 ex 35 35. $\vec{r}(t) = \langle \ln|t+1|+1, -\ln|\cos t| + 2\rangle$

11 02 ex 36 36. $\vec{r}(t) = \langle t^4/12 + t + 4,\ t^3/6 + 2t + 5,\ t^2/2 + 3t + 6\rangle$

11 02 ex 37 37. $\vec{r}(t) = \langle -\cos t + 1, t - \sin t, e^t - t - 1\rangle$

11 02 ex 38 38. $2\sqrt{13}\pi$

11 02 ex 39 39. $10\pi$

11 02 ex 40 40. $\frac{1}{54}\left((22)^{3/2} - 8\right)$

11 02 ex 41 41. $\sqrt{2}(1 - e^{-1})$

11 02 ex 42 42. As $\vec{r}(t)$ has constant length, $\vec{r}(t)\cdot\vec{r}(t)=c^2$ for some constant $c$. Thus

$$\begin{aligned}\vec{r}(t)\cdot\vec{r}(t)&=c^2\\ \frac{d}{dt}\big(\vec{r}(t)\cdot\vec{r}(t)\big)&=\frac{d}{dt}\big(c^2\big)\\ \vec{r}\,'(t)\cdot\vec{r}(t)+\vec{r}(t)\cdot\vec{r}\,'(t)&=0\\ 2\vec{r}(t)\cdot\vec{r}\,'(t)&=0\\ \vec{r}(t)\cdot\vec{r}\,'(t)&=0.\end{aligned}$$

### Section 11.3

11 03 ex 01 1. Velocity is a vector, indicating an objects direction of travel and its rate of distance change (i.e., its speed). Speed is a scalar.

11 03 ex 02 2. Displacement is a vector, indicating the difference between the starting and ending positions of an object. Distance traveled is a scalar, indicating the arc length of the path followed.

11 03 ex 03 3. The average velocity is found by dividing the displacement by the time traveled – it is a vector. The average speed is found by dividing the distance traveled by the time traveled – it is a scalar.

11 03 ex 04 4. arc length

11 03 ex 05 5. One example is traveling at a constant speed $s$ in a circle, ending at the starting position. Since the displacement is $\vec{0}$, the average velocity is $\vec{0}$, hence $||\,\vec{0}\,||=0$. But traveling at constant speed $s$ means the average speed is also $s>0$.

11 03 ex 24 6. Distance traveled is always greater than or equal to the magnitude of displacement, therefore average speed will always be at least as large as the magnitude of the average velocity.

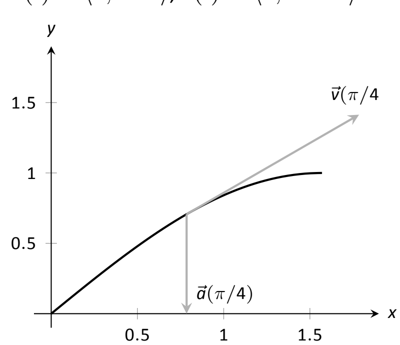
11 03 ex 06 7. $\vec{v}(t)=\langle 2,5,0\rangle$, $\vec{a}(t)=\langle 0,0,0\rangle$

11 03 ex 07 8. $\vec{v}(t)=\langle 6t-2,-2t+1\rangle$, $\vec{a}(t)=\langle 6,-2\rangle$

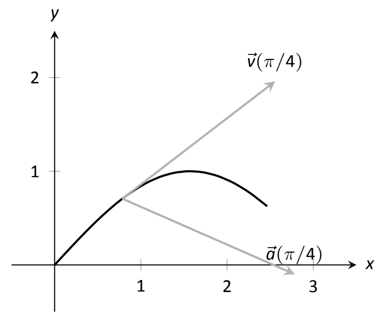
11 03 ex 08 9. $\vec{v}(t)=\langle -\sin t,\cos t\rangle$, $\vec{a}(t)=\langle -\cos t,-\sin t\rangle$

11 03 ex 09 10. $\vec{v}(t)=\langle 1/10,\sin t,\cos t\rangle$, $\vec{a}(t)=\langle 0,\cos t,-\sin t\rangle$

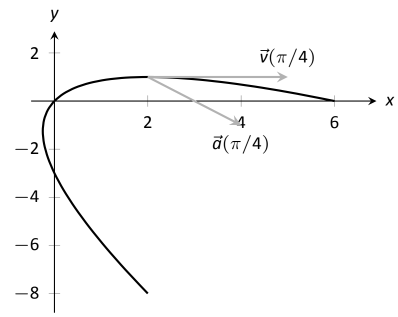
11 03 ex 11 11. $\vec{v}(t)=\langle 1,\cos t\rangle$, $\vec{a}(t)=\langle 0,-\sin t\rangle$



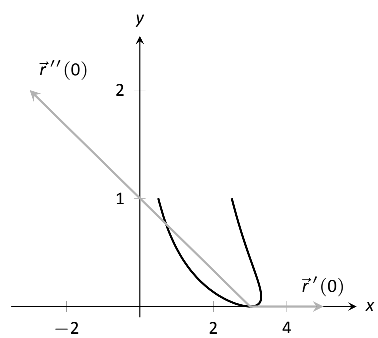

11 03 ex 12 12. $\vec{v}(t)=\big\langle 2t,2t\cos\big(t^2\big)\big\rangle$, $\vec{a}(t)=\big\langle 2,2\big(\cos\big(t^2\big)-2t^2\sin\big(t^2\big)\big)\big\rangle$




11 03 ex 13 13. $\vec{v}(t)=\langle 2t+1,-2t+2\rangle$, $\vec{a}(t)=\langle 2,-2\rangle$




11 03 ex 10 14. $\vec{v}(t)=\left\langle -\frac{2\left(t^2+3\,t-1\right)}{\left(t^2+1\right)^2},2t\right\rangle$, $\vec{a}(t)=\left\langle \frac{2\left(2t^3+9t^2-6t-3\right)}{\left(t^2+1\right)^3},2\right\rangle$




11 03 ex 14 15. $||\,\vec{v}(t)\,||=\sqrt{4t^2+1}$.
Min at $t=0$; Max at $t=\pm 1$.

11 03 ex 19 16. $||\,\vec{v}(t)\,||=|t|\sqrt{9t^2-12t+8}$.
min: $t=0$; max: $t=-1$

11 03 ex 15 17. $||\,\vec{v}(t)\,||=5$.
Speed is constant, so there is no difference between min/max

11 03 ex 16 18. $||\,\vec{v}(t)\,||=\sqrt{4\sin^2 t+25\cos^2 t}$.
min: $t=\pi/2,\ 3\pi/2$; max: $t=0,\ 2\pi$

11 03 ex 17 19. $||\,\vec{v}(t)\,||=|\sec t|\sqrt{\tan^2 t+\sec^2 t}$.
min: $t=0$; max: $t=\pi/4$

11 03 ex 18 20. $||\,\vec{v}(t)\,||=\sqrt{2-2\sin t}$.
min: $t=\pi/2$; max: $t=3\pi/2$

11 03 ex 20 21. $||\,\vec{v}(t)\,||=13$.
speed is constant, so there is no difference between min/max

11 03 ex 21 22. $||\,\vec{v}(t)\,||=\sqrt{8t^2+3}$.
min: $t=0$; max: $t=1$

11 03 ex 22 23. $||\vec{v}(t)|| = \sqrt{4t^2 + 1 + t^2/(1-t^2)}$.
min: $t = 0$; max: there is no max; speed approaches $\infty$ as $t \to \pm 1$

11 03 ex 23 24. $||\vec{v}(t)|| = \sqrt{g^2t^2 - (2gv_0 \sin\theta)t + v_0^2}$.
min: $t = (v_0 \sin\theta)/g$; max: $t = 0$, $t = (2v_0 \sin\theta)/g$

11 03 ex 25 25.

(a) $\vec{r}_1(1) = \langle 1, 1\rangle$; $\vec{r}_2(1) = \langle 1, 1\rangle$

(b) $\vec{v}_1(1) = \langle 1, 2\rangle$; $||\vec{v}_1(1)|| = \sqrt{5}$; $\vec{a}_1(1) = \langle 0, 2\rangle$
$\vec{v}_2(1) = \langle 2, 4\rangle$; $||\vec{v}_2(1)|| = 2\sqrt{5}$; $\vec{a}_2(1) = \langle 2, 12\rangle$

11 03 ex 26 26.

(a) $\vec{r}_1(\pi/2) = \langle 0, 3\rangle$; $\vec{r}_2(\pi/8) = \langle 0, 3\rangle$

(b) $\vec{v}_1(\pi/2) = \langle -3, 0\rangle$; $||\vec{v}_1(\pi/2)|| = 3$;
$\vec{a}_1(\pi/2) = \langle 0, -3\rangle$
$\vec{v}_2(\pi/8) = \langle -12, 0\rangle$; $||\vec{v}_2(\pi/8)|| = 12$;
$\vec{a}_2(\pi/8) = \langle 0, -48\rangle$

11 03 ex 27 27.

(a) $\vec{r}_1(2) = \langle 6, 4\rangle$; $\vec{r}_2(2) = \langle 6, 4\rangle$

(b) $\vec{v}_1(2) = \langle 3, 2\rangle$; $||\vec{v}_1(2)|| = \sqrt{13}$; $\vec{a}_1(2) = \langle 0, 0\rangle$
$\vec{v}_2(2) = \langle 6, 4\rangle$; $||\vec{v}_2(2)|| = 2\sqrt{13}$; $\vec{a}_2(2) = \langle 0, 0\rangle$

11 03 ex 28 28.

(a) $\vec{r}_1(1) = \langle 1, 1\rangle$; $\vec{r}_2(\pi/2) = \langle 1, 1\rangle$

(b) $\vec{v}_1(1) = \langle 1, 1/2\rangle$; $||\vec{v}_1(1)|| = \sqrt{5}/2$;
$\vec{a}_1(1) = \langle 0, -1/4\rangle$
$\vec{v}_2(\pi/2) = \langle 0, 0\rangle$; $||\vec{v}_2(\pi/2)|| = 0$;
$\vec{a}_2(\pi/2) = \langle -1, -1/2\rangle$

11 03 ex 29 29. $\vec{v}(t) = \langle 2t+1, 3t+2\rangle$,
$\vec{r}(t) = \left\langle t^2 + t + 5, 3t^2/2 + 2t - 2\right\rangle$

11 03 ex 30 30. $\vec{v}(t) = \langle 2t-1, 3t-1\rangle$,
$\vec{r}(t) = \left\langle t^2 - t + 5, 3t^2/2 - t - 5/2\right\rangle$

11 03 ex 31 31. $\vec{v}(t) = \langle \sin t, \cos t\rangle$, $\vec{r}(t) = \langle 1 - \cos t, \sin t\rangle$

11 03 ex 32 32. $\vec{v}(t) = \langle 10, -32t + 50\rangle$, $\vec{r}(t) = \left\langle 10t, -16t^2 + 50t\right\rangle$

11 03 ex 39 33. Displacement: $\langle 0, 0, 6\pi\rangle$; distance traveled: $2\sqrt{13}\pi \approx 22.65$ft; average velocity: $\langle 0, 0, 3\rangle$; average speed: $\sqrt{13} \approx 3.61$ft/s

11 03 ex 41 34. Displacement: $\langle -10, 0\rangle$; distance traveled: $5\pi \approx 15.71$ft; average velocity: $\langle -10/\pi, 0\rangle \approx \langle -3.18, 0\rangle$; average speed: 5ft/s

11 03 ex 40 35. Displacement: $\langle 0, 0\rangle$; distance traveled: $2\pi \approx 6.28$ft; average velocity: $\langle 0, 0\rangle$; average speed: 1ft/s

11 03 ex 42 36. Displacement: $\langle 10, 20, -20\rangle$; distance traveled: 30ft; average velocity: $\langle 1, 2, -2\rangle$; average speed: 3ft/s

11 03 ex 33 37. At $t$-values of $\sin^{-1}(9/30)/(4\pi) + n/2 \approx 0.024 + n/2$ seconds, where $n$ is an integer.

11 03 ex 34 38. The stone, while whirling, can be modeled by $\vec{r}(t) = \langle 3\cos(8\pi t), 3\sin(8\pi t)\rangle$.

(a) For $t$-values $t = \sin^{-1}(3/20)/(8\pi) + n/4 \approx 0.006 + n/4$, where $n$ is an integer.

(b) $||\vec{r}\,'(t)|| = 24\pi \approx 51.4$ft/s

(c) At $t = 0.006$, the stone is approximately 19.77ft from Goliath. Using the formula for projectile motion, we want the angle of elevation that lets a projectile starting at $\langle 0, 6\rangle$ with a initial velocity of 51.4ft/s arrive at $\langle 19.77, 9\rangle$. The desired angle is 0.27 radians, or $15.69^\circ$.

11 03 ex 35 39.

(a) Holding the crossbow at an angle of 0.013 radians, $\approx 0.745^\circ$ will hit the target 0.4s later. (Another solution exists, with an angle of $89^\circ$, landing 18.75s later, but this is impractical.)

(b) In the .4 seconds the arrow travels, a deer, traveling at 20mph or 29.33ft/s, can travel 11.7ft. So she needs to lead the deer by 11.7ft.

11 03 ex 36 40. The position function of the ball is $\vec{r}(t) = \left\langle (146.67\cos\theta)t, -16t^2 + (146.67\sin\theta)t + 3\right\rangle$, where $\theta$ is the angle of elevation.

(a) With $\theta = 20^\circ$, the ball reaches 310ft from home plate in 2.25 seconds; at this time, the height of the ball is 34.9ft, not enough to clear the Green Monster.

(b) With $\theta = 21^\circ$, the ball reaches 310ft from home plate in 2.26s, with a height of 40ft, clearing the wall.

11 03 ex 37 41. The position function is $\vec{r}(t) = \left\langle 220t, -16t^2 + 1000\right\rangle$. The $y$-component is 0 when $t = 7.9$; $\vec{r}(7.9) = \langle 1739.25, 0\rangle$, meaning the box will travel about 1740ft horizontally before it lands.

11 03 ex 38 42. The position function of the ball is $\vec{r}(t) = \left\langle (v_0\cos\theta)t, -16t^2 + (v_0\sin\theta)t + 6\right\rangle$, where $\theta$ is the angle of elevation and $v_0$ is the initial ball speed.

(a) With $v_0 = 73.33$ft/s, there are two angles of elevation possible. An angle of $\theta = 9.47^\circ$ delivers the ball in 0.83s, while an angle of $79.57^\circ$ delivers the ball in 4.5s.

(b) With $\theta = 8^\circ$, the initial speed must be 53.8mph$\approx$ 78.9ft/s.

**Section 11.4**

11 04 ex 01 1. 1

11 04 ex 02 2. 0

11 04 ex 03 3. $\vec{T}(t)$ and $\vec{N}(t)$.

11 04 ex 04 4. the speed

11 04 ex 05 5. $\vec{T}(t) = \left\langle \frac{4t}{\sqrt{20t^2-4t+1}}, \frac{2t-1}{\sqrt{20t^2-4t+1}} \right\rangle$;
$\vec{T}(1) = \left\langle 4/\sqrt{17}, 1/\sqrt{17}\right\rangle$

11 04 ex 06 6. $\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+\sin^2 t}}, -\frac{\sin t}{\sqrt{1+\sin^2 t}} \right\rangle$;
$\vec{T}(\pi/4) = \left\langle \sqrt{2/3}, -1/\sqrt{3}\right\rangle$

11 04 ex 07 7. $\vec{T}(t) = \frac{\cos t \sin t}{\sqrt{\cos^2 t \sin^2 t}} \langle -\cos t, \sin t\rangle$. (Be careful; this cannot be simplified as just $\langle -\cos t, \sin t\rangle$ as $\sqrt{\cos^2 t \sin^2 t} \neq \cos t \sin t$, but rather $|\cos t \sin t|$.)
$\vec{T}(\pi/4) = \left\langle -\sqrt{2}/2, \sqrt{2}/2\right\rangle$

11 04 ex 08 8. $\vec{T}(t) = \langle -\sin t, \cos t\rangle$; $\vec{T}(\pi) = \langle 0, -1\rangle$

11 04 ex 09 9. $\ell(t) = \langle 2, 0\rangle + t\,\langle 4/\sqrt{17}, 1/\sqrt{17}\rangle$; in parametric form,

$$\ell(t) = \begin{cases} x = 2 + 4t/\sqrt{17} \\ y = t/\sqrt{17} \end{cases}$$

11 04 ex 10 10. $\ell(t) = \langle \pi/4, \sqrt{2}/2\rangle + t\left\langle \sqrt{2/3}, -1/\sqrt{3}\right\rangle$; in parametric form,

$$\ell(t) = \begin{cases} x = \pi/4 + \sqrt{2/3}t \\ y = \sqrt{2}/2 - t/\sqrt{3} \end{cases}$$

11 04 ex 11 11. $\ell(t) = \langle \sqrt{2}/4, \sqrt{2}/4\rangle + t\,\langle -\sqrt{2}/2, \sqrt{2}/2\rangle$; in parametric form,

$$\ell(t) = \begin{cases} x = \sqrt{2}/4 - \sqrt{2}t/2 \\ y = \sqrt{2}/4 + \sqrt{2}t/2 \end{cases}$$

11 04 ex 12 12. $\ell(t) = \langle -1, 0\rangle + t\,\langle 0, -1\rangle$; in parametric form,

$$\ell(t) = \begin{cases} x = -1 \\ y = -t \end{cases}$$

11 04 ex 13 13. $\vec{T}(t) = \langle -\sin t, \cos t\rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t\rangle$

11 04 ex 14 14. $\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}}\right\rangle$; $\vec{N}(t) = \left\langle -\frac{2t}{\sqrt{1+4t^2}}, \frac{1}{\sqrt{1+4t^2}}\right\rangle$

11 04 ex 15 15. $\vec{T}(t) = \left\langle -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}}, \frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}}\right\rangle$;
$\vec{N}(t) = \left\langle -\frac{2\cos t}{\sqrt{4\cos^2 t+\sin^2 t}}, -\frac{\sin t}{\sqrt{4\cos^2 t+\sin^2 t}}\right\rangle$

11 04 ex 16 16. $\vec{T}(t) = \left\langle \frac{e^t}{\sqrt{e^{2t}+e^{-2t}}}, -\frac{e^{-t}}{\sqrt{e^{2t}+e^{-2t}}}\right\rangle$;
$\vec{N}(t) = \left\langle \frac{e^{-t}}{\sqrt{e^{2t}+e^{-2t}}}, \frac{e^t}{\sqrt{e^{2t}+e^{-2t}}}\right\rangle$

11 04 ex 27 17.

(a) Be sure to show work

(b) $\vec{N}(\pi/4) = \langle -5/\sqrt{34}, -3/\sqrt{34}\rangle$

11 04 ex 28 18.

(a) Be sure to show work

(b) $\vec{N}(1) = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$

11 04 ex 29 19.

(a) Be sure to show work

(b) $\vec{N}(0) = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$

11 04 ex 30 20.

(a) Be sure to show work

(b) $\vec{N}(\pi/4) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$

11 04 ex 17 21. $\vec{T}(t) = \frac{1}{\sqrt{5}}\langle 2, \cos t, -\sin t\rangle$; $\vec{N}(t) = \langle 0, -\sin t, -\cos t\rangle$

11 04 ex 18 22. $\vec{T}(t) = \langle -\sin t, 3/5\cos t, 4/5\cos t\rangle$;
$\vec{N}(t) = \langle -\cos t, -3/5\sin t, -4/5\sin t\rangle$

11 04 ex 19 23. $\vec{T}(t) = \frac{1}{\sqrt{a^2+b^2}}\langle -a\sin t, a\cos t, b\rangle$;
$\vec{N}(t) = \langle -\cos t, -\sin t, 0\rangle$

11 04 ex 20 24. $\vec{T}(t) = \frac{1}{\sqrt{a^2+1}}\langle -a\sin(at), a\cos(at), 1\rangle$;
$\vec{N}(t) = \langle -\cos t, -\sin t, 0\rangle$

11 04 ex 21 25. $a_{\mathrm{T}} = \frac{4t}{\sqrt{1+4t^2}}$ and $a_{\mathrm{N}} = \sqrt{4 - \frac{16t^2}{1+4t^2}}$
At $t = 0$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 2$;
At $t = 1$, $a_{\mathrm{T}} = 4/\sqrt{5}$ and $a_{\mathrm{N}} = 2/\sqrt{5}$.
At $t = 0$, all acceleration comes in the form of changing the direction of velocity and not the speed; at $t = 1$, more acceleration comes in changing the speed than in changing direction.

11 04 ex 22 26. $a_{\mathrm{T}} = \frac{-2/t^5}{\sqrt{1+1/t^4}}$ and $a_{\mathrm{N}} = \sqrt{\frac{4}{t^6} - \frac{4/t^{10}}{1+1/t^4}}$
At $t = 1$, $a_{\mathrm{T}} = \sqrt{2}$ and $a_{\mathrm{N}} = -\sqrt{2}$;
At $t = 2$, $a_{\mathrm{T}} = -\frac{1}{4\sqrt{17}}$ and $a_{\mathrm{N}} = \frac{1}{\sqrt{17}}$.
At $t = 1$, acceleration comes from changing speed and changing direction in "equal measure;" at $t = 2$, acceleration is nearly $\vec{0}$ as it is; the low value of $a_{\mathrm{T}}$ shows that the speed is nearly constant and the low value of $a_{\mathrm{N}}$ shows the direction is not changing quickly.

11 04 ex 23 27. $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 2$
At $t = 0$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 2$;
At $t = \pi/2$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 2$.
The object moves at constant speed, so all acceleration comes from changing direction, hence $a_{\mathrm{T}} = 0$. $\vec{a}(t)$ is always parallel to $\vec{N}(t)$, but twice as long, hence $a_{\mathrm{N}} = 2$.

11 04 ex 24 28. $a_{\mathrm{T}} = 2$ and $a_{\mathrm{N}} = 4t^2$
At $t = \sqrt{\pi/2}$, $a_{\mathrm{T}} = 2$ and $a_{\mathrm{N}} = 2\pi$;
At $t = \sqrt{\pi}$, $a_{\mathrm{T}} = 2$ and $a_{\mathrm{N}} = 4\pi$.
The object moves at increasing speed (increasing at a constant rate of acceleration), hence $a_{\mathrm{T}} = 2$. Since the object is increasing speed yet always traveling in a circle of radius 1, the direction must change more quickly; the amount of acceleration that changes direction increases over time.

11 04 ex 25 29. $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = a$
At $t = 0$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = a$;
At $t = \pi/2$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = a$.
The object moves at constant speed, meaning that $a_{\mathrm{T}}$ is always 0. The object "rises" along the $z$-axis at a constant rate, so all acceleration comes in the form of changing direction circling the $z$-axis. The greater the radius of this circle the greater the acceleration, hence $a_{\mathrm{N}} = a$.

11 04 ex 26 30. $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 5$
At $t = 0$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 5$;
At $t = \pi/2$, $a_{\mathrm{T}} = 0$ and $a_{\mathrm{N}} = 5$.
The object moves at constant speed, meaning that $a_{\mathrm{T}}$ is always 0. Acceleration is thus always perpendicular to the direction of travel; in this particular case, it is always 5 times the unit vector pointing orthogonal to the direction of travel.

**Section 11.5**

11 05 ex 01 1. time and/or distance

11 05 ex 02 2. curvature

11 05 ex 03 3. Answers may include lines, circles, helixes

11 05 ex 04 4. Answers will vary; they should mention the circle is tangent to the curve and has the same curvature as the curve at that point.

11 05 ex 05 5. $\kappa$

11 05 ex 06 6. $a_T$ is not affected by curvature; the greater the curvature, the larger $a_N$ becomes.

11 05 ex 07 7. $s = 3t$, so $\vec{r}(s) = \langle 2s/3, s/3, -2s/3\rangle$

11 05 ex 08 8. $s = 7t$, so $\vec{r}(s) = \langle 7\cos(s/7), 7\sin(s/7)\rangle$

11 05 ex 09 9. $s = \sqrt{13}t$, so $\vec{r}(s) = \left\langle 3\cos(s/\sqrt{13}), 3\sin(s/\sqrt{13}), 2s/\sqrt{13}\right\rangle$

11 05 ex 10 10. $s = 13t$, so $\vec{r}(s) = \langle 5\cos(s/13), 13\sin(s/13), 12\cos(s/13)\rangle$

11 05 ex 11 11. $\kappa = \frac{|6x|}{\left(1+(3x^2-1)^2\right)^{3/2}}$; $\kappa(0) = 0$, $\kappa(1/2) = \frac{192}{17\sqrt{17}} \approx 2.74$.

11 05 ex 12 12. $\kappa = \frac{\left|\frac{6x^2-2}{(x^2+1)^3}\right|}{\left(1+\frac{4x^2}{(x^2+1)^4}\right)^{3/2}}$; $\kappa(0) = 2$, $\kappa(2) = \frac{2750}{641\sqrt{641}} \approx 0.169$.

11 05 ex 14 13. $\kappa = \frac{|\cos x|}{\left(1+\sin^2 x\right)^{3/2}}$; $\kappa(0) = 1$, $\kappa(\pi/2) = 0$

11 05 ex 13 14. $\kappa = 1$; $\kappa(0) = 1$, $\kappa(1/2) = 1$

11 05 ex 15 15. $\kappa = \frac{|2\cos t\cos(2t)+4\sin t\sin(2t)|}{\left(4\cos^2(2t)+\sin^2 t\right)^{3/2}}$; $\kappa(0) = 1/4$, $\kappa(\pi/4) = 8$

11 05 ex 16 16. $\kappa = 2$; $\kappa(0) = 2$, $\kappa(\pi/3) = 2$

11 05 ex 17 17. $\kappa = \frac{|6t^2+2|}{\left(4t^2+(3t^2-1)^2\right)^{3/2}}$; $\kappa(0) = 2$, $\kappa(5) = \frac{19}{1394\sqrt{1394}} \approx 0.0004$

11 05 ex 18 18. $\kappa = \frac{|\sec^3 t|}{\left(\sec^4 t+\sec^2 t\tan^2 t\right)^{3/2}}$; $\kappa(0) = 1$, $\kappa(\pi/6) = \frac{3\sqrt{3}}{5\sqrt{5}} \approx 0.465$

11 05 ex 22 19. $\kappa = 0$; $\kappa(0) = 0$, $\kappa(1) = 0$

11 05 ex 21 20. $\kappa = \frac{2\sqrt{18t^4+15t^2+1}}{\left(18t^4-2t^2+1\right)^{3/2}}$; $\kappa(0) = 2$, $\kappa(1) = 2\sqrt{2}/17 \approx 0.166378$

11 05 ex 19 21. $\kappa = \frac{3}{13}$; $\kappa(0) = 3/13$, $\kappa(\pi/2) = 3/13$

11 05 ex 20 22. $\kappa = \frac{1}{13}$; $\kappa(0) = 1/13$, $\kappa(\pi/2) = 1/13$

11 05 ex 24 23. maximized at $x = \pm\frac{\sqrt{2}}{\sqrt[4]{5}}$

11 05 ex 25 24. maximized at $x = \ldots - 3\pi/2, -\pi/2, \pi/2, \ldots$

11 05 ex 23 25. maximized at $t = 1/4$

11 05 ex 26 26. maximized at $t = \pm\sqrt{5}$

11 05 ex 27 27. radius of curvature is $5\sqrt{5}/4$.

11 05 ex 28 28. radius of curvature is $5\sqrt{10}$.

11 05 ex 29 29. radius of curvature is 9.

11 05 ex 30 30. radius of curvature is $1/45$.

11 05 ex 31 31. $x^2 + (y - 1/2)^2 = 1/4$, or $\vec{c}(t) = \langle 1/2\cos t, 1/2\sin t + 1/2\rangle$

11 05 ex 32 32. $(x - 8/3)^2 + y^2 = 1/9$, or $\vec{c}(t) = \left\langle \frac{1}{3}\cos t + \frac{8}{3}, \frac{1}{3}\sin t\right\rangle$

11 05 ex 33 33. $x^2 + (y + 8)^2 = 81$, or $\vec{c}(t) = \langle 9\cos t, 9\sin t - 8\rangle$

11 05 ex 34 34. $(x - 1/2)^2 + (y - 1/2)^2 = 1/2$, or $\vec{c}(t) = \left\langle \frac{\sqrt{2}}{2}\cos t + \frac{1}{2}, \frac{\sqrt{2}}{2}\sin t + \frac{1}{2}\right\rangle$

# Chapter 12

### Section 12.1

12 01 ex 01 1. Answers will vary.

12 01 ex 02 2. surface

12 01 ex 03 3. topographical

12 01 ex 04 4. T

12 01 ex 05 5. surface

12 01 ex 06 6. When level curves are close together, it means the function is changing $z$-values rapidly. When far apart, it changes $z$-values slowly.

12 01 ex 07 7. domain: $\mathbb{R}^2$
range: $z \geq 2$

12 01 ex 08 8. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

12 01 ex 09 9. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

12 01 ex 10 10. domain: $x \neq 2y$; in set notation, $\{(x, y) \mid x \neq 2y\}$
range: $z \neq 0$

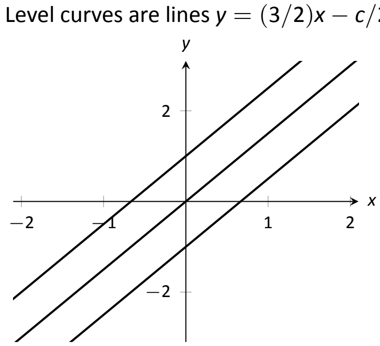
12 01 ex 11 11. domain: $\mathbb{R}^2$
range: $0 < z \leq 1$

12 01 ex 12 12. domain: $\mathbb{R}^2$
range: $-1 \leq z \leq 1$

12 01 ex 13 13. domain: $\{(x, y) \mid x^2 + y^2 \leq 9\}$, i.e., the domain is the circle and interior of a circle centered at the origin with radius 3.
range: $0 \leq z \leq 3$

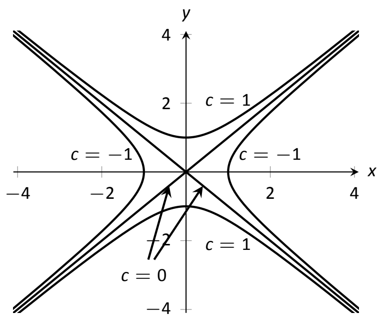
12 01 ex 14 14. domain: $\{(x, y) \mid x^2 + y^2 \geq 9\}$, i.e., the domain is the exterior of the circle (not including the circle itself) centered at the origin with radius 3.
range: $0 < z < \infty$, or $(0, \infty)$

12 01 ex 15 15. Level curves are lines $y = (3/2)x - c/2$.



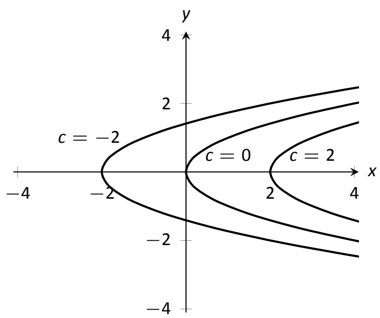
12 01 ex 16

16. Level curves are hyperbolas $\frac{x^2}{c} - \frac{y^2}{c} = 1$, except for $c = 0$, where the level curve is the pair of lines $y = x$, $y = -x$.



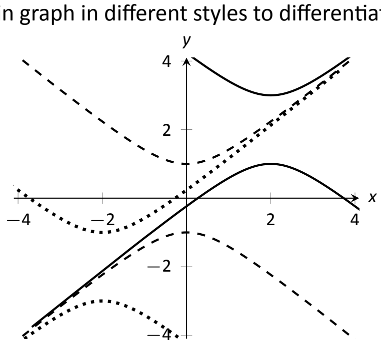

12 01 ex 17

17. Level curves are parabolas $x = y^2 + c$.



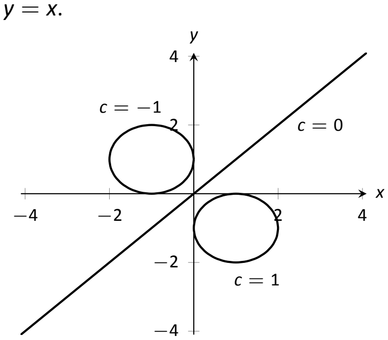

12 01 ex 18

18. Level curves are hyperbolas $(x-c)^2 - (y-c)^2 = 1$, drawn in graph in different styles to differentiate the curves.



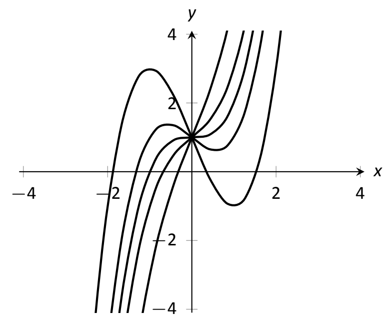

12 01 ex 19

19. Level curves are circles, centered at $(1/c, -1/c)$ with radius $2/c^2 - 1$. When $c = 0$, the level curve is the line $y = x$.



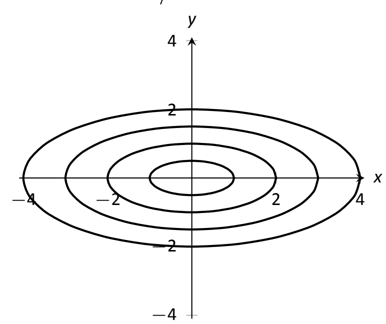

12 01 ex 20

20. Level curves are cubics of the form $y = x^3 + cx + 1$. Note how each curve passes through $(0, 1)$ and that the function is not defined at $x = 0$.



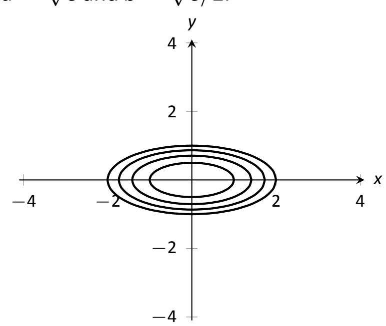

12 01 ex 21

21. Level curves are ellipses of the form $\frac{x^2}{c^2} + \frac{y^2}{c^2/4} = 1$, i.e., $a = c$ and $b = c/2$.




12 01 ex 22

22. Level curves are ellipses of the form $\frac{x^2}{c} + \frac{y^2}{c/4} = 1$, i.e., $a = \sqrt{c}$ and $b = \sqrt{c}/2$.




12 01 ex 24

23. domain: $x + 2y - 4z \neq 0$; the set of points in $\mathbb{R}^3$ NOT in the domain form a plane through the origin.
range: $\mathbb{R}$

12 01 ex 25

24. domain: $x^2 + y^2 + z^2 \neq 1$; the set of points in $\mathbb{R}^3$ NOT in the domain form a sphere of radius 1.
range: $(-\infty, 0) \cup [1, \infty)$

12 01 ex 26

25. domain: $z \geq x^2 - y^2$; the set of points in $\mathbb{R}^3$ above (and including) the hyperbolic paraboloid $z = x^2 - y^2$.
range: $[0, \infty)$

12 01 ex 27

26. domain: $\mathbb{R}^3$
range: $\mathbb{R}$

12 01 ex 28

27. The level surfaces are spheres, centered at the origin, with radius $\sqrt{c}$.

12 01 ex 29

28. The level surfaces are hyperbolic paraboloids of the form $z = x^2 - y^2 + c$; each is shifted up/down by $c$.

12 01 ex 30

29. The level surfaces are paraboloids of the form $z = \frac{x^2}{c} + \frac{y^2}{c}$; the larger $c$, the "wider" the paraboloid.

12 01 ex 31 30. The level surfaces are planes through the origin of the form $cx - cy - z = 0$, that is, planes through the origin with normal vector $\langle c, -c, -1\rangle$.

12 01 ex 23 31. The level curves for each surface are similar; for $z = \sqrt{x^2 + 4y^2}$ the level curves are ellipses of the form $\frac{x^2}{c^2} + \frac{y^2}{c^2/4} = 1$, i.e., $a = c$ and $b = c/2$; whereas for $z = x^2 + 4y^2$ the level curves are ellipses of the form $\frac{x^2}{c} + \frac{y^2}{c/4} = 1$, i.e., $a = \sqrt{c}$ and $b = \sqrt{c}/2$. The first set of ellipses are spaced evenly apart, meaning the function grows at a constant rate; the second set of ellipses are more closely spaced together as $c$ grows, meaning the function grows faster and faster as $c$ increases.

The function $z = \sqrt{x^2 + 4y^2}$ can be rewritten as $z^2 = x^2 + 4y^2$, an elliptic cone; the function $z = x^2 + 4y^2$ is a paraboloid, each matching the description above.

**Section 12.2**

12 02 ex 01 1. Answers will vary.

12 02 ex 02 2. Answers will vary. One answer is "As $(x, y)$ gets close to $(1, 2)$, $f(x, y)$ gets close to 17."

12 02 ex 17 3. Answers will vary.
One possible answer: $\{(x, y) | x^2 + y^2 \leq 1\}$

12 02 ex 18 4. Answers will vary.
One possible answer: $\{(x, y) | y \geq x^2\}$

12 02 ex 19 5. Answers will vary.
One possible answer: $\{(x, y) | x^2 + y^2 < 1\}$

12 02 ex 20 6. Answers will vary.
One possible answer: $\{(x, y) | y > x^2\}$

12 02 ex 03 7.

(a) Answers will vary.
interior point: $(1, 3)$
boundary point: $(3, 3)$

(b) $S$ is a closed set

(c) $S$ is bounded

12 02 ex 04 8.

(a) Answers will vary.
interior point: $(-5, 28)$
boundary point: $(3, 9)$

(b) $S$ is an open set

(c) $S$ is unbounded

12 02 ex 05 9.

(a) Answers will vary.
interior point: none
boundary point: $(0, -1)$

(b) $S$ is a closed set, consisting only of boundary points

(c) $S$ is bounded

12 02 ex 06 10.

(a) Answers will vary.
Interior point: $(0, 1)$
Boundary point: $(0, 0)$

(b) $S$ is a closed set, containing all of its boundary points.

(c) $S$ is unbounded.

12 02 ex 09 11.

(a) $D = \{(x, y) \mid 9 - x^2 - y^2 \geq 0\}$.

(b) $D$ is a closed set.

(c) $D$ is bounded.

12 02 ex 07 12.

(a) $D = \{(x, y) \mid y \geq x^2\}$.

(b) $D$ is a closed set.

(c) $D$ is unbounded.

12 02 ex 08 13.

(a) $D = \{(x, y) \mid y > x^2\}$.

(b) $D$ is an open set.

(c) $D$ is unbounded.

12 02 ex 10 14.

(a) $D = \{(x, y) \mid (x, y) \neq (0, 0)\}$.

(b) $D$ is an open set.

(c) $D$ is unbounded.

12 02 ex 12 15.

(a) Along $y = 0$, the limit is 1.

(b) Along $x = 0$, the limit is $-1$.

Since the above limits are not equal, the limit does not exist.

12 02 ex 11 16.

(a) Along $y = mx$, the limit is $\frac{m+1}{m-1}$.

Since the above limit varies according to what $m$ is used, each limit is different, meaning the overall limit does not exist.

12 02 ex 13 17.

(a) Along $y = mx$, the limit is $\dfrac{mx(1 - m)}{m^2x + 1}$.

(b) Along $x = 0$, the limit is $-1$.

Since the above limits are not equal, the limit does not exist.

12 02 ex 14 18.

(a) Along $y = mx$, the limit is:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y} = \lim_{x\to 0} \frac{\sin(x^2)}{mx}$$

apply L'Hôpital's Rule

$$= \lim_{x\to 0} \frac{2x\cos(x^2)}{m} = 0.$$

(b) Along $x = 0$, the limit is:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y} = \lim_{x\to 0} \frac{\sin(x^2)}{x^2}.$$

This can be evaluated with L'Hôpital's Rule or from known limits; it is 1.

Since the limits along the lines $y = mx$ are not the same as the limit along $y = x^2$, the overall limit does not exist.

12 02 ex 15 19.

(a) Along $y = 2$, the limit is:

$$\begin{aligned}\lim_{(x,y)\to(1,2)} \frac{x+y-3}{x^2-1} &= \lim_{x\to 1} \frac{x-1}{x^2-1} \\ &= \lim_{x\to 1} \frac{1}{x+1} \\ &= 1/2.\end{aligned}$$

(b) Along $y = x + 1$, the limit is:

$$\begin{aligned}\lim_{(x,y)\to(1,2)} \frac{x+y-3}{x^2-1} &= \lim_{x\to 1} \frac{2(x-1)}{x^2-1} \\ &= \lim_{x\to 1} \frac{2}{x+1} \\ &= 1.\end{aligned}$$

Since the limits along the lines $y = 2$ and $y = x + 1$ differ, the overall limit does not exist.

12 02 ex 16 20.

(a) Along $x = \pi$, the limit is:

$$\begin{aligned}\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y} &= \lim_{y\to\pi/2} \frac{0}{\cos y} \\ &= 0.\end{aligned}$$

(b) Along $y = x - \pi/2$, the limit is:

$$\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y} = \lim_{x\to\pi} \frac{\sin x}{\cos(x-\pi/2)}$$

Apply L'Hôpital's Rule:

$$\begin{aligned}&= \lim_{x\to\pi} \frac{\cos x}{\sin(x-\pi/2)} \\ &= -1.\end{aligned}$$

Since the limits along the lines $x = \pi$ and $y = x - \pi/2$ differ, the overall limit does not exist.

**Section 12.3**

12 03 ex 01 1. A constant is a number that is added or subtracted in an expression; a coefficient is a number that is being multiplied by a nonconstant function.

12 03 ex 02 2. Answers will vary; each should include something about treating $y$ as a constant or a coefficient.

12 03 ex 03 3. $f_x$

12 03 ex 04 4. $f_y$

12 03 ex 05 5. $f_x = 2xy - 1, f_y = x^2 + 2$
$f_x(1,2) = 3, f_y(1,2) = 3$

12 03 ex 06 6. $f_x = 3x^2 - 3, f_y = 2y - 6$
$f_x(-1,3) = 0, f_y(-1,3) = 0$

12 03 ex 07 7. $f_x = -\sin x \sin y, f_y = \cos x \cos y$
$f_x(\pi/3,\pi/3) = -3/4, f_y(\pi/3,\pi/3) = 1/4$

12 03 ex 08 8. $f_x = 1/x, f_y = 1/y$
$f_x(-2,-3) = -1/2, f_y(-2,-3) = -1/3$

12 03 ex 09 9. $f_x = 2xy + 6x, f_y = x^2 + 4$
$f_{xx} = 2y + 6, f_{yy} = 0$
$f_{xy} = 2x, f_{yx} = 2x$

12 03 ex 10 10. $f_x = 3x^2 + 6xy + 3y^2, f_y = 3x^2 + 6xy + 3y^2$
$f_{xx} = 6x + 6y, f_{yy} = 6x + 6y$
$f_{xy} = 6x + 6y, f_{yx} = 6x + 6y$

12 03 ex 13 11. $f_x = 1/y, f_y = -x/y^2$
$f_{xx} = 0, f_{yy} = 2x/y^3$
$f_{xy} = -1/y^2, f_{yx} = -1/y^2$

12 03 ex 14 12. $f_x = -4/(x^2y), f_y = -4/(xy^2)$
$f_{xx} = 8/(x^3y), f_{yy} = 8/(xy^3)$
$f_{xy} = 4/(x^2y^2), f_{yx} = 4/(x^2y^2)$

12 03 ex 15 13. $f_x = 2xe^{x^2+y^2}, f_y = 2ye^{x^2+y^2}$
$f_{xx} = 2e^{x^2+y^2} + 4x^2e^{x^2+y^2}, f_{yy} = 2e^{x^2+y^2} + 4y^2e^{x^2+y^2}$
$f_{xy} = 4xye^{x^2+y^2}, f_{yx} = 4xye^{x^2+y^2}$

12 03 ex 16 14. $f_x = e^{x+2y}, f_y = 2e^{x+2y}$
$f_{xx} = e^{x+2y}, f_{yy} = 4e^{x+2y}$
$f_{xy} = 2e^{x+2y}, f_{yx} = 2e^{x+2y}$

12 03 ex 17 15. $f_x = \cos x \cos y, f_y = -\sin x \sin y$
$f_{xx} = -\sin x \cos y, f_{yy} = -\sin x \cos y$
$f_{xy} = -\sin y \cos x, f_{yx} = -\sin y \cos x$

12 03 ex 18 16. $f_x = 3(x+y)^2, f_y = 3(x+y)^2$
$f_{xx} = 6(x+y), f_{yy} = 6(x+y)$
$f_{xy} = 6(x+y), f_{yx} = 6(x+y)$

12 03 ex 11 17. $f_x = -5y^3\sin(5xy^3), f_y = -15xy^2\sin(5xy^3)$
$f_{xx} = -25y^6\cos(5xy^3)$,
$f_{yy} = -225x^2y^4\cos(5xy^3) - 30xy\sin(5xy^3)$
$f_{xy} = -75xy^5\cos(5xy^3) - 15y^2\sin(5xy^3)$,
$f_{yx} = -75xy^5\cos(5xy^3) - 15y^2\sin(5xy^3)$

12 03 ex 12 18. $f_x = 10x\cos(5x^2 + 2y^3), f_y = 6y^2\cos(5x^2 + 2y^3)$
$f_{xx} = 10\cos(5x^2 + 2y^3) - 100x^2\sin(5x^2 + 2y^3)$,
$f_{yy} = 12y\cos(5x^2 + 2y^3) - 36y^4\sin(5x^2 + 2y^3)$
$f_{xy} = -60xy^2\sin(5x^2 + 2y^3), f_{yx} = -60xy^2\sin(5x^2 + 2y^3)$

12 03 ex 19 19. $f_x = \frac{2y^2}{\sqrt{4xy^2+1}}, f_y = \frac{4xy}{\sqrt{4xy^2+1}}$
$f_{xx} = -\frac{4y^4}{\sqrt{4xy^2+1}^3}, f_{yy} = -\frac{16x^2y^2}{\sqrt{4xy^2+1}^3} + \frac{4x}{\sqrt{4xy^2+1}}$
$f_{xy} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}, f_{yx} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}$

12 03 ex 20 20. $f_x = 2\sqrt{y}, f_y = 5\sqrt{y} + \frac{2x+5y}{2\sqrt{y}}$
$f_{xx} = 0, f_{yy} = \frac{5}{\sqrt{y}} - \frac{2x+5y}{4y^{3/2}}$
$f_{xy} = \frac{1}{\sqrt{y}}, f_{yx} = \frac{1}{\sqrt{y}}$

12 03 ex 21 21. $f_x = -\frac{2x}{(x^2+y^2+1)^2}, f_y = -\frac{2y}{(x^2+y^2+1)^2}$
$f_{xx} = \frac{8x^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}, f_{yy} = \frac{8y^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}$
$f_{xy} = \frac{8xy}{(x^2+y^2+1)^3}, f_{yx} = \frac{8xy}{(x^2+y^2+1)^3}$

12 03 ex 22 22. $f_x = 5, f_y = -17$
$f_{xx} = 0, f_{yy} = 0$
$f_{xy} = 0, f_{yx} = 0$

12 03 ex 23 23. $f_x = 6x, f_y = 0$
$f_{xx} = 6, f_{yy} = 0$
$f_{xy} = 0, f_{yx} = 0$

12 03 ex 24 24. $f_x = \frac{2x}{(x^2+y)}, f_y = \frac{1}{(x^2+y)}$
$f_{xx} = -\frac{4x^2}{(x^2+y)^2} + \frac{2}{(x^2+y)}, f_{yy} = -\frac{1}{(x^2+y)^2}$
$f_{xy} = -\frac{2x}{(x^2+y)^2}, f_{yx} = -\frac{2x}{(x^2+y)^2}$

12 03 ex 25 25. $f_x = \frac{1}{4xy}, f_y = -\frac{\ln x}{4y^2}$
$f_{xx} = -\frac{1}{4x^2y}, f_{yy} = \frac{\ln x}{2y^3}$
$f_{xy} = -\frac{1}{4xy^2}, f_{yx} = -\frac{1}{4xy^2}$

12 03 ex 26 26. $f_x = 5e^x \sin y, f_y = 5e^x \cos y$
$f_{xx} = 5e^x \sin y, f_{yy} = -5e^x \sin y$
$f_{xy} = 5e^x \cos y, f_{yx} = 5e^x \cos y$

12 03 ex 27 27. $f(x, y) = x \sin y + x + C$, where $C$ is any constant.

12 03 ex 28 28. $f(x, y) = \frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + C$, where $C$ is any constant.

12 03 ex 29 29. $f(x, y) = 3x^2y - 4xy^2 + 2y + C$, where $C$ is any constant.

12 03 ex 30 30. $f(x, y) = \ln(x^2 + y^2) + C$, where $C$ is any constant.

12 03 ex 31 31. $f_x = 2xe^{2y-3z}, f_y = 2x^2e^{2y-3z}, f_z = -3x^2e^{2y-3z}$
$f_{yz} = -6x^2e^{2y-3z}, f_{zy} = -6x^2e^{2y-3z}$

12 03 ex 32 32. $f_x = 3x^2y^2 + 3x^2z, f_y = 2x^3y + 2yz, f_z = x^3 + y^2$
$f_{yz} = 2y, f_{zy} = 2y$

12 03 ex 33 33. $f_x = \frac{3}{7y^2z}, f_y = -\frac{6x}{7y^3z}, f_z = -\frac{3x}{7y^2z^2}$
$f_{yz} = \frac{6x}{7y^3z^2}, f_{zy} = \frac{6x}{7y^3z^2}$

12 03 ex 34 34. $f_x = \frac{1}{x}, f_y = \frac{1}{y}, f_z = \frac{1}{z}$
$f_{yz} = 0, f_{zy} = 0$

**Section 12.4**

12 04 ex 01 1. T

12 04 ex 02 2. T

12 04 ex 03 3. T

12 04 ex 04 4. amount of change

12 04 ex 05 5. $dz = (\sin y + 2x)dx + (x \cos y)dy$

12 04 ex 06 6. $dz = 8x(2x^2 + 3y)dx + 6(2x^2 + 3y)dy$

12 04 ex 07 7. $dz = 5dx - 7dy$

12 04 ex 08 8. $dz = (e^{x+y} + xe^{x+y})dx + xe^{x+y}dy$

12 04 ex 09 9. $dz = \frac{x}{\sqrt{x^2+y}}dx + \frac{1}{2\sqrt{x^2+y}}dy$, with $dx = -0.05$ and $dy = .1$. At $(3, 7)$, $dz = 3/4(-0.05) + 1/8(.1) = -0.025$, so $f(2.95, 7.1) \approx -0.025 + 4 = 3.975$.

12 04 ex 10 10. $dz = (\cos x \cos y)dx - (\sin x \sin y)dy$, with $dx = 0.1$ and $dy = -0.1$. At $(0, 0)$, $dz = 1(.1) - (0)(-0.1) = 0.1$, so $f(0.1, -0.1) \approx 0.1 + 0 = 0.1$.

12 04 ex 11 11. $dz = (2xy - y^2)dx + (x^2 - 2xy)dy$, with $dx = 0.04$ and $dy = 0.06$. At $(2, 3)$, $dz = 3(0.04) + (-8)(0.06) = -0.36$, so $f(2.04, 3.06) \approx -0.36 - 6 = -6.36$.

12 04 ex 12 12. $dz = \frac{1}{x-y}dx - \frac{1}{x-y}dy$, with $dx = 0.1$ and $dy = -0.02$. At $(5, 4)$, $dz = 1(0.1) + (-1)(-0.02) = 0.12$, so $f(5.1, 3.98) \approx 0.12 + 0 = 0.12$.

12 04 ex 15 13. The total differential of volume is $dV = 4\pi dr + \pi dh$. The coefficient of $dr$ is greater than the coefficient of $dh$, so the volume is more sensitive to changes in the radius.

12 04 ex 16 14. Distance of the projectile is a function of two variables (leaving $t = 3$): $D(v_0, \theta) = 30 \cos \theta$. The total differential of $D$ is $dD = 3 \cos \theta dv_0 - 3v_0 \sin \theta d\theta$. The coefficient of $d\theta$ has a much greater magnitude than the coefficient of $dv_0$, so a small change in the angle of elevation has a much greater effect on distance traveled than a small change in initial velocity.

12 04 ex 17 15. Using trigonometry, $\ell = x \tan \theta$, so $d\ell = \tan \theta dx + x \sec^2 \theta d\theta$. With $\theta = 85^\circ$ and $x = 30$, we have $d\ell = 11.43dx + 3949.38d\theta$. The measured length of the wall is much more sensitive to errors in $\theta$ than in $x$. While it can be difficult to compare sensitivities between measuring feet and measuring degrees (it is somewhat like “comparing apples to oranges”), here the coefficients are so different that the result is clear: a small error in degree has a much greater impact than a small error in distance.

12 04 ex 18 16. With $D = n\ell$, the total differential is $dD = \ell\, dn + n\, d\ell$. If one measures with a short tape, $n$ must be large and hence $n\, d\ell$ is going to be greater than when a large tape is used (wherein $n$ will be small).

12 04 ex 19 17. $dw = 2xyz^3\, dx + x^2z^3\, dy + 3x^2yz^2\, dz$

12 04 ex 20 18. $dw = e^x \sin y \ln z\, dx + e^x \cos y \ln z\, dy + e^x \sin y \frac{1}{z}\, dz$

12 04 ex 13 19. $dx = 0.05, dy = -0.1$.
$dz = 9(.05) + (-2)(-0.1) = 0.65$. So
$f(3.05, 0.9) \approx 7 + 0.65 = 7.65$.

12 04 ex 14 20. $dx = -0.12, dy = 0.07$.
$dz = 2.6(-.12) + (5.1)(0.07) = 0.045$. So
$f(-4.12, 2.07) \approx 13 + 0.045 = 13.045$.

12 04 ex 21 21. $dx = 0.5, dy = 0.1, dz = -0.2$.
$dw = 2(0.5) + (-3)(0.1) + 3.7(-0.2) = -0.04$, so
$f(2.5, 4.1, 4.8) \approx -1 - 0.04 = -1.04$.

12 04 ex 22 22. $dx = 0.1, dy = 0.1, dz = 0.1$.
$dw = 2(0.1) + (0)(0.1) + (-2)(.1) = 0$, so
$f(3.1, 3.1, 3.1) \approx 5 + 0 = 5$.

**Section 12.5**

12 05 ex 01 1. A partial derivative is essentially a special case of a directional derivative; it is the directional derivative in the direction of $x$ or $y$, i.e., $\langle 1, 0\rangle$ or $\langle 0, 1\rangle$.

12 05 ex 02 2. $\vec{u} = \langle 1, 0\rangle$

12 05 ex 03 3. $\vec{u} = \langle 0, 1\rangle$

12 05 ex 04 4. orthogonal

12 05 ex 05 5. maximal, or greatest

12 05 ex 06 6. dot

12 05 ex 07 7. $\nabla f = \left\langle -2xy + y^2 + y, -x^2 + 2xy + x\right\rangle$

12 05 ex 08 8. $\nabla f = \langle \cos x \cos y, -\sin x \sin y\rangle$

12 05 ex 09 9. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$

12 05 ex 10 10. $\nabla f = \langle -4, 3\rangle$

12 05 ex 19 11. $\nabla f = \langle 2x - y - 7, 4y - x\rangle$

12 05 ex 22 12. $\nabla f = \left\langle 2xy^3 - 2, 3x^2y^2\right\rangle$

12 05 ex 11 13. $\nabla f = \left\langle -2xy + y^2 + y, -x^2 + 2xy + x\right\rangle$;
$\nabla f(2, 1) = \langle -2, 2\rangle$. Be sure to change all directions to unit vectors.

(a) $2/5$ ($\vec{u} = \langle 3/5, 4/5\rangle$)

(b) $-2\sqrt{5}$ ($\vec{u} = \left\langle -1/\sqrt{5}, -2\sqrt{5} \right\rangle$)

12 05 ex 12 14. $\nabla f = \langle \cos x \cos y, -\sin x \sin y \rangle$; $\nabla f(\frac{\pi}{4}, \frac{\pi}{3}) = \left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}} \right\rangle$. Be sure to change all directions to unit vectors.

(a) $\frac{1}{4}(1-\sqrt{3})$ ($\vec{u} = \left\langle 1/\sqrt{2}, 1/\sqrt{2} \right\rangle$)

(b) $\frac{4\sqrt{3}-1}{10\sqrt{2}}$ ($\vec{u} = \langle -3/5, -4/5 \rangle$)

12 05 ex 13 15. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2} \right\rangle$; $\nabla f(1,1) = \langle -2/9, -2/9 \rangle$. Be sure to change all directions to unit vectors.

(a) $0$ ($\vec{u} = \left\langle 1/\sqrt{2}, -1/\sqrt{2} \right\rangle$)

(b) $2\sqrt{2}/9$ ($\vec{u} = \left\langle -1/\sqrt{2}, -1/\sqrt{2} \right\rangle$)

12 05 ex 14 16. $\nabla f = \langle -4, 3 \rangle$; $\nabla f(5,2) = \langle -4, 3 \rangle$. Be sure to change all directions into unit vectors.

(a) $-9/\sqrt{10}$ ($\vec{u} = \left\langle 3/\sqrt{10}, 1/\sqrt{10} \right\rangle$)

(b) $27/\sqrt{34}$ ($\vec{u} = \left\langle -3/\sqrt{34}, 5/\sqrt{34} \right\rangle$)

12 05 ex 20 17. $\nabla f = \langle 2x - y - 7, 4y - x \rangle$; $\nabla f(4,1) = \langle 0, 0 \rangle$.

(a) 0

(b) 0

12 05 ex 23 18. $\nabla f = \left\langle 2xy^3 - 2, 3x^2y^2 \right\rangle$; $\nabla f(1,1) = \langle 0, 3 \rangle$ Be sure to change all directions to unit vectors.

(a) $3/\sqrt{2}$; ($\vec{u} = \left\langle 1/\sqrt{2}, 1/\sqrt{2} \right\rangle$)

(b) 3

12 05 ex 15 19. $\nabla f = \left\langle -2xy + y^2 + y, -x^2 + 2xy + x \right\rangle$

(a) $\nabla f(2,1) = \langle -2, 2 \rangle$

(b) $|| \nabla f(2,1) || = || \langle -2, 2 \rangle || = \sqrt{8}$

(c) $\langle 2, -2 \rangle$

(d) $\left\langle 1/\sqrt{2}, 1/\sqrt{2} \right\rangle$

12 05 ex 16 20. $\nabla f = \langle \cos x \cos y, -\sin x \sin y \rangle$

(a) $\nabla f(\frac{\pi}{4}, \frac{\pi}{3}) = \left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}} \right\rangle$

(b) $|| \nabla f(\frac{\pi}{4}, \frac{\pi}{3}) || = || \left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}} \right\rangle || = 1/\sqrt{2}$

(c) $\left\langle -\frac{1}{2\sqrt{2}}, \frac{1}{2}\sqrt{\frac{3}{2}} \right\rangle$

(d) $\left\langle \frac{1}{2}\sqrt{\frac{3}{2}}, \frac{1}{2\sqrt{2}} \right\rangle$

12 05 ex 17 21. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2} \right\rangle$

(a) $\nabla f(1,1) = \langle -2/9, -2/9 \rangle$.

(b) $|| \nabla f(1,1) || = || \langle -2/9, -2/9 \rangle || = 2\sqrt{2}/9$

(c) $\langle 2/9, 2/9 \rangle$

(d) $\left\langle 1/\sqrt{2}, -1/\sqrt{2} \right\rangle$

12 05 ex 18 22. $\nabla f = \langle -4, 3 \rangle$

(a) $\nabla f(5,4) = \langle -4, 3 \rangle$.

(b) $|| \nabla f(5,4) || = || \langle -4, 3 \rangle || = 5$

(c) $\langle 4, -3 \rangle$

(d) $\langle 3/5, 4/5 \rangle$

12 05 ex 21 23. $\nabla f = \langle 2x - y - 7, 4y - x \rangle$

(a) $\nabla f(4,1) = \langle 0, 0 \rangle$

(b) 0

(c) $\langle 0, 0 \rangle$

(d) All directions give a directional derivative of 0.

12 05 ex 24 24. $\nabla f = \left\langle 2xy^3 - 2, 3x^2y^2 \right\rangle$

(a) $\nabla f(1,1) = \langle 0, 3 \rangle$

(b) 3

(c) $\langle 0, -3 \rangle$

(d) $\vec{u} = \langle 1, 0 \rangle$

12 05 ex 25 25.

(a) $\nabla F(x,y,z) = \left\langle 6xz^3 + 4y, 4x, 9x^2z^2 - 6z \right\rangle$

(b) $113/\sqrt{3}$

12 05 ex 26 26.

(a) $\nabla F(x,y,z) = \left\langle \cos x \cos ye^z, -\sin x \sin ye^z, \sin x \cos ye^z \right\rangle$

(b) $2/3$

12 05 ex 27 27.

(a) $\nabla F(x,y,z) = \left\langle 2xy^2, 2y(x^2 - z^2), -2y^2z \right\rangle$

(b) 0

12 05 ex 28 28.

(a) $\nabla F(x,y,z) = \left\langle -\frac{4x}{(x^2+y^2+z^2)^2}, -\frac{4y}{(x^2+y^2+z^2)^2}, -\frac{4z}{(x^2+y^2+z^2)^2} \right\rangle$

(b) 0

**Section 12.6**

12 06 ex 01 1. Answers will vary. The displacement of the vector is one unit in the $x$-direction and 3 units in the $z$-direction, with no change in $y$. Thus along a line parallel to $\vec{v}$, the change in $z$ is 3 times the change in $x$ – i.e., a "slope" of 3. Specifically, the line in the $x$-$z$ plane parallel to $z$ has a slope of 3.

12 06 ex 02 2. Answers will vary. Let $\vec{u} = \langle 0.6, 0.8 \rangle$; this is a unit vector. The displacement of the vector is one unit in the $\vec{u}$-direction and $-2$ units in the $z$-direction. In the plane containing the $z$-axis and the vector $\vec{u}$, the line parallel to $\vec{v}$ has slope $-2$.

12 06 ex 03 3. T

12 06 ex 04 4. On a surface through a point, there are many different smooth curves, each with a tangent line at the point. Each of these tangent lines is also "tangent" to the surface. There is not just one tangent line, but many, each in a different direction. Therefore we refer to directional tangent lines, not just *the* tangent line.

12 06 ex 05 5.

(a) $\ell_x(t) = \begin{cases} x = 2 + t \\ y = 3 \\ z = -48 - 12t \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = 2 \\ y = 3 + t \\ z = -48 - 40t \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = 2 + t/\sqrt{10} \\ y = 3 + 3t/\sqrt{10} \\ z = -48 - 66\sqrt{2/5}t \end{cases}$

12 06 ex 06 6.

(a) $\ell_x(t) = \begin{cases} x = \pi/3 + t \\ y = \pi/6 \\ z = 3/4 - \frac{3\sqrt{3}}{4}t \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = \pi/3 \\ y = \pi/6 + t \\ z = 3/4 + \frac{3\sqrt{3}}{4}t \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = \pi/3 + t/\sqrt{5} \\ y = \pi/6 + 2t/\sqrt{5} \\ z = 3/4 + \frac{3\sqrt{3/5}}{4}t \end{cases}$

12 06 ex 07 7.

(a) $\ell_x(t) = \begin{cases} x = 4 + t \\ y = 2 \\ z = 2 + 3t \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = 4 \\ y = 2 + t \\ z = 2 - 5t \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = 4 + t/\sqrt{2} \\ y = 2 + t/\sqrt{2} \\ z = 2 - \sqrt{2}t \end{cases}$

12 06 ex 08 8.

(a) $\ell_x(t) = \begin{cases} x = 1 + t \\ y = 2 \\ z = 3 \end{cases}$

(b) $\ell_y(t) = \begin{cases} x = 1 \\ y = 2 + t \\ z = 3 \end{cases}$

(c) $\ell_{\vec{u}}(t) = \begin{cases} x = 1 + t/\sqrt{2} \\ y = 2 + t/\sqrt{2} \\ z = 3 \end{cases}$

12 06 ex 09 9. $\ell_{\vec{n}}(t) = \begin{cases} x = 2 - 12t \\ y = 3 - 40t \\ z = -48 - t \end{cases}$

12 06 ex 10 10. $\ell_{\vec{n}}(t) = \begin{cases} x = \pi/3 - \frac{3\sqrt{3}}{4}t \\ y = \pi/6 + \frac{3\sqrt{3}}{4}t \\ z = 3/4 - t \end{cases}$

12 06 ex 11 11. $\ell_{\vec{n}}(t) = \begin{cases} x = 4 + 3t \\ y = 2 - 5t \\ z = 2 - t \end{cases}$

12 06 ex 12 12. $\ell_{\vec{n}}(t) = \begin{cases} x = 1 \\ y = 2 \\ z = 3 - t \end{cases}$

12 06 ex 13 13. $(1.425, 1.085, -48.078)$, $(2.575, 4.915, -47.952)$

12 06 ex 14 14. $(-0.195, 1.766, -0.206)$ and $(2.289, -0.719, 1.706)$

12 06 ex 15 15. $(5.014, 0.31, 1.662)$ and $(2.986, 3.690, 2.338)$

12 06 ex 16 16. $(1, 2, 1)$ and $(1, 2, 5)$

12 06 ex 17 17. $-12(x - 2) - 40(y - 3) - (z + 48) = 0$

12 06 ex 18 18. $-\frac{3\sqrt{3}}{4}(x - \pi/3) + \frac{3\sqrt{3}}{4}(y - \pi/6) - (z - 3/4) = 0$

12 06 ex 19 19. $3(x - 4) - 5(y - 2) - (z - 2) = 0$ (Note that this tangent plane is the same as the original function, a plane.)

12 06 ex 20 20. $-(z - 3) = 0$, or $z = 3$

12 06 ex 21 21. $\nabla F = \langle x/4, y/2, z/8\rangle$; at $P$, $\nabla F = \langle 1/4, \sqrt{2}/2, \sqrt{6}/8\rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 1 + t/4 \\ y = \sqrt{2} + \sqrt{2}t/2 \\ z = \sqrt{6} + \sqrt{6}t/8 \end{cases}$

(b) $\frac{1}{4}(x - 1) + \frac{\sqrt{2}}{2}(y - \sqrt{2}) + \frac{\sqrt{6}}{8}(z - \sqrt{6}) = 0.$

12 06 ex 22 22. $\nabla F = \langle -\frac{x}{2}, -\frac{2y}{9}, 2z\rangle$; at $P$, $\nabla F = \langle -2, 2/3, 2\sqrt{5}\rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 4 - 2t \\ y = -3 + 2t/3 \\ z = \sqrt{5} + 2\sqrt{5}t \end{cases}$

(b) $-2(x - 4) + \frac{2}{3}(y + 3) + 2\sqrt{5}(z - \sqrt{5}) = 0.$

12 06 ex 23 23. $\nabla F = \langle y^2 - z^2, 2xy, -2xz\rangle$; at $P$, $\nabla F = \langle 0, 4, 4\rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2 \\ y = 1 + 4t \\ z = -1 + 4t \end{cases}$

(b) $4(y - 1) + 4(z + 1) = 0.$

12 06 ex 24 24. $\nabla F = \langle y\cos(xy), x\cos(xy) - z\sin(yz), -y\sin(yz)\rangle$; at $P$, $\nabla F = \left\langle \frac{\pi}{8\sqrt{3}}, -\sqrt{3}, -\frac{\pi}{8\sqrt{3}}\right\rangle$

(a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2 + \frac{\pi}{8\sqrt{3}}t \\ y = \frac{\pi}{12} - \sqrt{3}t \\ z = 4 - \frac{\pi}{8\sqrt{3}}t \end{cases}$

(b) $\frac{\pi}{8\sqrt{3}}(x - 2) - \sqrt{3}(y - \frac{\pi}{12}) - \frac{\pi}{8\sqrt{3}}(z - 4) = 0.$

**Section 12.7**

12 07 ex 01 1. F; it is the "other way around."

12 07 ex 02 2. T

12 07 ex 03 3. T

12 07 ex 04 4. Answers will vary. A good answer will state that we are optimizing a function subject to a constraint, or limit, on the domain of the function. We are looking to maximize/minimize the function while "looking" at only a certain part of the domain.

12 07 ex 07 5. One critical point at $(-4, 2)$; $f_{xx} = 1$ and $D = 4$, so this point corresponds to a relative minimum.

12 07 ex 05 6. One critical point at $(7, -6)$; $D = -5$, so this point corresponds to a saddle point.

12 07 ex 06 7. One critical point at $(6, -3)$; $D = -4$, so this point corresponds to a saddle point.

12 07 ex 08 8. One critical point at $(0, 0)$; $f_{xx} = -2$ and $D = 4$, so this point corresponds to a relative maximum.

12 07 ex 09 9. Two critical points: at $(0, -1)$; $f_{xx} = 2$ and $D = -12$, so this point corresponds to a saddle point;
at $(0, 1)$, $f_{xx} = 2$ and $D = 12$, so this corresponds to a relative minimum.

12 07 ex 12 10. There are 4 critical points:
$(-1,-2)$, rel. max; $(1,-2)$, saddle point;
$(-1,2)$, saddle point; $(1,2)$, rel. min.,
where $f_{xx} = 2x$ and $D = 4xy$.

12 07 ex 11 11. One critical point at $(0,0)$. $D = -12x^2y^2$, so at $(0,0)$, $D = 0$ and the test is inconclusive. (Some elementary thought shows that it is the absolute minimum.)

12 07 ex 10 12. Six critical points: $f_x = 0$ when $x = -1, 0$ and $1$; $f_y = 0$ when $y = -3, 3$. Together, we get the points:
$(-1,-3)$ saddle point; $(-1,3)$ rel. min
$(0,-3)$ rel. max; $(0,3)$ saddle point
$(1,-3)$ saddle point; $(1,3)$ relative min
where $f_{xx} = 12x^2 - 4$ and $D = 24y(3x^2 - 1)$.

12 07 ex 17 13. One critical point: $f_x = 0$ when $x = 3$; $f_y = 0$ when $y = 0$, so one critical point at $(3,0)$, which is a relative maximum, where $f_{xx} = \frac{y^2-16}{(16-(x-3)^2-y^2)^{3/2}}$ and $D = \frac{16}{(16-(x-3)^2-y^2)^2}$.
Both $f_x$ and $f_y$ are undefined along the circle $(x-3)^2 + y^2 = 16$; at any point along this curve, $f(x,y) = 0$, the absolute minimum of the function.

12 07 ex 18 14. One critical point: $f_x = 0$ when $x = 0$; $f_y = 0$ when $y = 0$, so one critical point at $(0,0)$ (although it should be noted that at $(0,0)$, both $f_x$ and $f_y$ are undefined.) The Second Derivative Test fails at $(0,0)$, with $D = 0$. A graph, or simple calculation, shows that $(0,0)$ is the absolute minimum of $f$.

12 07 ex 13 15. The triangle is bound by the lines $y = -1$, $y = 2x + 1$ and $y = -2x + 1$.
Along $y = -1$, there is a critical point at $(0,-1)$.
Along $y = 2x + 1$, there is a critical point at $(-3/5, -1/5)$.
Along $y = -2x + 1$, there is a critical point at $(3/5, -1/5)$.
The function $f$ has one critical point, irrespective of the constraint, at $(0, -1/2)$.
Checking the value of $f$ at these four points, along with the three vertices of the triangle, we find the absolute maximum is at $(0,1,3)$ and the absolute minimum is at $(0,-1/2,3/4)$.

12 07 ex 14 16. The region has two "corners" at $(1,1)$ and $(-1,1)$.
Along $y = 1$, there is no critical point.
Along $y = x^2$, there is a critical point at $(5/14, 25/196) \approx (0.357, 0.128)$.
The function $f$ itself has no critical points. Checking the value of $f$ at the corners $(1,1)$, $(-1,1)$ and the critical point $(5/14, 25/196)$, we find the absolute maximum is at $(5/14, 25/196, 25/28) \approx (0.357, 0.128, 0.893)$ and the absolute minimum is at $(-1,1,-12)$.

12 07 ex 15 17. The region has no "corners" or "vertices," just a smooth edge.
To find critical points along the circle $x^2 + y^2 = 4$, we solve for $y^2$: $y^2 = 4 - x^2$. We can go further and state $y = \pm\sqrt{4-x^2}$.
We can rewrite $f$ as
$f(x) = x^2 + 2x + (4 - x^2) + \sqrt{4-x^2} = 2x + 4 + \sqrt{4-x^2}$.
(We will return and use $-\sqrt{4-x^2}$ later.) Solving $f'(x) = 0$, we get $x = \sqrt{2} \Rightarrow y = \sqrt{2}$. $f'(x)$ is also undefined at $x = \pm 2$, where $y = 0$.
Using $y = -\sqrt{4-x^2}$, we rewrite $f(x,y)$ as $f(x) = 2x + 4 - \sqrt{4-x^2}$. Solving $f'(x) = 0$, we get $x = -\sqrt{2},\ y = -\sqrt{2}$.
The function $f$ itself has a critical point at $(-1,-1)$.
Checking the value of $f$ at $(-1,-1)$, $(\sqrt{2}, \sqrt{2})$, $(-\sqrt{2}, -\sqrt{2})$, $(2,0)$ and $(-2,0)$, we find the absolute maximum is at $(2,0,8)$ and the absolute minimum is at $(-1,-1,-2)$.

12 07 ex 16 18. The region has two "corners" at $(-1,-1)$ and $(1,1)$.
Along the line $y = x$, $f(x,y)$ becomes $f(x) = 3x - 2x^2$.
Along this line, we have a critical point at $(3/4, 3/4)$.
Along the curve $y = x^2 + x - 1$, $f(x,y)$ becomes $f(x) = x^2 + 3x - 3$. There is a critical point along this curve at $(-3/2, -1/4)$. Since $x = -3/2$ lies outside our bounded region, we ignore this critical point.
The function $f$ itself has no critical points.
Checking the value of $f$ at $(-1,-1)$, $(1,1)$, $(3/4,3/4)$, we find the absolute maximum is at $(3/4, 3/4, 9/8)$ and the absolute minimum is at $(-1,-1,-5)$.

**Section 12.8**

12 08 ex 01 1. Because the parametric equations describe a level curve, $z$ is constant for all $t$. Therefore $\frac{dz}{dt} = 0$.

12 08 ex 02 2. $g'(x)$

12 08 ex 03 3. $\frac{dx}{dt}$, and $\frac{\partial f}{\partial y}$

12 08 ex 04 4. T

12 08 ex 05 5. F

12 08 ex 06 6. partial

12 08 ex 07 7.
   (a) $\frac{dz}{dt} = 3(2t) + 4(2) = 6t + 8$.
   (b) At $t = 1$, $\frac{dz}{dt} = 14$.

12 08 ex 19 8.
   (a) $\frac{dz}{dt} = 2x(1) - 2y(2t) = 2x - 4yt$
   (b) At $t = 1$, $x = 1$, $y = 0$ and $\frac{dz}{dt} = 2$.

12 08 ex 20 9.
   (a) $\frac{dz}{dt} = 5(-2\sin t) + 2(\cos t) = -10\sin t + 2\cos t$
   (b) At $t = \pi/4$, $\frac{dz}{dt} = -4\sqrt{2}$.

12 08 ex 09 10.
   (a) $\dfrac{dz}{dt} = \dfrac{1}{1+y^2}(-\sin t) - \dfrac{2xy}{(y^2+1)^2}(\cos t)$.
   (b) At $t = \pi/2$, $x = 0$, $y = 1$, and $\frac{dz}{dt} = -1/2$.

12 08 ex 10 11.
   (a) $\dfrac{dz}{dt} = 2x(\cos t) + 4y(3\cos t)$.
   (b) At $t = \pi/4$, $x = \sqrt{2}/2$, $y = 3\sqrt{2}/2$, and $\frac{dz}{dt} = 19$.

12 08 ex 08 12.
   (a) $\frac{dz}{dt} = -\sin x \sin y(\pi) + \cos x \cos y(2\pi)$.
   (b) At $t = 3$, $x = 3\pi$, $y = 13\pi/2$, and $\frac{dz}{dt} = 0$.

12 08 ex 11 13. $t = -4/3$; this corresponds to a minimum

12 08 ex 21 14. $t = 0, \pm\sqrt{3/2}$

12 08 ex 22 15. $t = \tan^{-1}(1/5) + n\pi$, where $n$ is an integer

12 08 ex 13 16. We find that

$$\frac{dz}{dt} = -\frac{2\cos^2 t \sin t}{(1+\sin^2 t)^2} - \frac{\sin t}{1+\sin^2 t}.$$

Setting this equal to 0, finding a common denominator and factoring out $\sin t$, we get

$$\sin t\left(\frac{2\cos^2 t + \sin^2 t + 1}{(1+\sin^2 t)^2}\right) = 0.$$

We have $\sin t = 0$ when $t = \pi n$, where $n$ is an integer. The expression in the parenthesis above is always positive, and hence never equal 0. So all solutions are $t = \pi n$, n is an integer.

12 08 ex 14 17. We find that

$$\frac{dz}{dt} = 38\cos t \sin t.$$

Thus $\frac{dz}{dt} = 0$ when $t = \pi n$ or $\pi n + \pi/2$, where $n$ is any integer.

12 08 ex 12 18. We find that

$$\frac{dz}{dt} = -\pi\sin(\pi t)\sin(2\pi t+\pi/2) + 2\pi\cos(\pi t)\cos(2\pi t+\pi/2).$$

One can "easily" see that when $t$ is an integer, $\sin(\pi t) = 0$ and $\cos(2\pi t + \pi/2) = 0$, hence $\frac{dz}{dt} = 0$ when $t$ is an integer. There are other places where $z$ has a relative max/min that require more work. First, verify that $\sin(2\pi t + \pi/2) = \cos(2\pi t)$, and $\cos(2\pi t + \pi/2) = -\sin(2\pi t)$. This lets us rewrite $\frac{dz}{dt} = 0$ as

$$-\sin(\pi t)\cos(2\pi t) - 2\cos(\pi t)\sin(2\pi t) = 0.$$

By bringing one term to the other side of the equality then dividing, we can get

$$2\tan(2\pi t) = -\tan(\pi t).$$

Using the angle sum/difference formulas found in the back of the book, we know

$$\tan(2\pi t) = \tan(\pi t) + \tan(\pi t) = \frac{\tan(\pi t) + \tan(\pi t)}{1 - \tan^2(\pi t)}.$$

Thus we write

$$2\frac{\tan(\pi t) + \tan(\pi t)}{1 - \tan^2(\pi t)} = -\tan(\pi t).$$

Solving for $\tan^2(\pi t)$, we find

$$\tan^2(\pi t) = 5 \quad \Rightarrow \quad \tan(\pi t) = \pm\sqrt{5},$$

and so

$$\pi t = \tan^{-1}(\pm\sqrt{5}) = \pm\tan^{-1}(\sqrt{5}).$$

Since the period of tangent is $\pi$, we can adjust our answer to be

$$\pi t = \pm\tan^{-1}(\sqrt{5}) + n\pi, \text{ where } n \text{ is an integer.}$$

Dividing by $\pi$, we find

$$t = \pm\frac{1}{\pi}\tan^{-1}(\sqrt{5}) + n, \text{ where } n \text{ is an integer.}$$

12 08 ex 15 19.

(a) $\frac{\partial z}{\partial s} = 2xy(1) + x^2(2) = 2xy + 2x^2$;
$\frac{\partial z}{\partial t} = 2xy(-1) + x^2(4) = -2xy + 4x^2$

(b) With $s = 1$, $t = 0$, $x = 1$ and $y = 2$. Thus $\frac{\partial z}{\partial s} = 6$ and $\frac{\partial z}{\partial t} = 0$

12 08 ex 16 20.

(a) $\frac{\partial z}{\partial s} = -\pi\sin(\pi x + \pi y/2)(t^2) - \frac{1}{2}\pi\sin(\pi x + \pi y/2)(2st) = -\pi\Big(t^2\sin(\pi x + \pi y/2) + st\sin(\pi x + \pi y/2)\Big)$;
$\frac{\partial z}{\partial t} = -\pi\sin(\pi x + \pi y/2)(2st) - \frac{1}{2}\pi\sin(\pi x + \pi y/2)(s^2) = -\pi\Big(2st\sin(\pi x + \pi y/2) + \frac{1}{2}s^2\sin(\pi x + \pi y/2)\Big)$

(b) With $s = 1$, $t = 1$, $x = 1$ and $y = 1$. Thus $\frac{\partial z}{\partial s} = 2\pi$ and $\frac{\partial z}{\partial t} = 5\pi/2$

12 08 ex 17 21.

(a) $\frac{\partial z}{\partial s} = 2x(\cos t) + 2y(\sin t) = 2x\cos t + 2y\sin t$;
$\frac{\partial z}{\partial t} = 2x(-s\sin t) + 2y(s\cos t) = -2xs\sin t + 2ys\cos t$

(b) With $s = 2$, $t = \pi/4$, $x = \sqrt{2}$ and $y = \sqrt{2}$. Thus $\frac{\partial z}{\partial s} = 4$ and $\frac{\partial z}{\partial t} = 0$

12 08 ex 18 22.

(a) $\frac{\partial z}{\partial s} = -2xe^{-(x^2+y^2)}(0) - 2ye^{-(x^2+y^2)}(t^2) = -2yt^2e^{-(x^2+y^2)}$;
$\frac{\partial z}{\partial t} = -2xe^{-(x^2+y^2)}(1) - 2ye^{-(x^2+y^2)}(2st) = -2xe^{-(x^2+y^2)} - 4stye^{-(x^2+y^2)}$

(b) With $s = 1$, $t = 1$, $x = 1$ and $y = 1$. Thus $\frac{\partial z}{\partial s} = -2/e^2$ and $\frac{\partial z}{\partial t} = -6/e^2$

12 08 ex 23 23. $f_x = 2x\tan y$, $f_y = x^2\sec^2 y$;
$$\frac{dy}{dx} = -\frac{2\tan y}{x\sec^2 y}$$

12 08 ex 24 24. $f_x = 4(3x^2 + 2y^3)^3(6x)$, $f_y = 4(3x^2 + 2y^3)^3(6y^2)$;
$$\frac{dy}{dx} = -\frac{x}{y^2}$$

12 08 ex 25 25. $f_x = \dfrac{(x+y^2)(2x) - (x^2+y)(1)}{(x+y^2)^2}$,
$f_y = \dfrac{(x+y^2)(1) - (x^2+y)(2y)}{(x+y^2)^2}$;
$$\frac{dy}{dx} = -\frac{2x(x+y^2) - (x^2+y)}{x + y^2 - 2y(x^2+y)}$$

12 08 ex 26 26. $f_x = \dfrac{2x+y}{x^2+xy+y^2}$, $f_y = \dfrac{x+2y}{x^2+xy+y^2}$;
$$\frac{dy}{dx} = -\frac{2x+y}{2y+x}$$

12 08 ex 27 27. $\frac{dz}{dt} = 2(4) + 1(-5) = 3$.

12 08 ex 28 28. $\frac{dz}{dt} = 1(6) + (-3)(2) = 0$.

12 08 ex 29 29. $\frac{\partial z}{\partial s} = -4(5) + 9(-2) = -38$,
$\frac{\partial z}{\partial t} = -4(7) + 9(6) = 26$.

12 08 ex 30 30. $\frac{\partial z}{\partial s} = 2(-2) + 1(2) = -2$,
$\frac{\partial z}{\partial t} = 2(3) + 1(-1) = 5$.

# Chapter 13

## Section 13.1

13 01 ex 01 1. $C(y)$, meaning that instead of being just a constant, like the number 5, it is a function of $y$, which acts like a constant when taking derivatives with respect to $x$.

13 01 ex 02 2. iterated integration

13 01 ex 03 3. curve to curve, then from point to point

13 01 ex 04 4. area

13 01 ex 09 5.
   (a) $18x^2 + 42x - 117$
   (b) $-108$

13 01 ex 10 6.
   (a) $2 + \pi^2 \cos y$
   (b) $\pi^2 + \pi$

13 01 ex 05 7.
   (a) $x^4/2 - x^2 + 2x - 3/2$
   (b) $23/15$

13 01 ex 06 8.
   (a) $y^4/2 - y^3 + y^2/2$
   (b) $8/15$

13 01 ex 07 9.
   (a) $\sin^2 y$
   (b) $\pi/2$

13 01 ex 08 10.
   (a) $x/(1+x^2)$
   (b) $\frac{1}{2}\ln\left(\frac{5}{2}\right)$

13 01 ex 11 11. $\int_1^4 \int_{-2}^1 dy\,dx$ and $\int_{-2}^1 \int_1^4 dx\,dy$.
area of $R = 9u^2$

13 01 ex 12 12. $\int_1^4 \int_1^{\frac{2}{3}x+\frac{1}{3}} dy\,dx$ and $\int_1^3 \int_{\frac{3}{2}y-\frac{1}{2}}^4 dx\,dy$.
area of $R = 3u^2$

13 01 ex 13 13. $\int_2^4 \int_{x-1}^{7-x} dy\,dx$. The order $dx\,dy$ needs two iterated integrals as $x$ is bounded above by two different functions. This gives:

$$\int_1^3 \int_2^{y+1} dx\,dy + \int_3^5 \int_2^{7-y} dx\,dy.$$

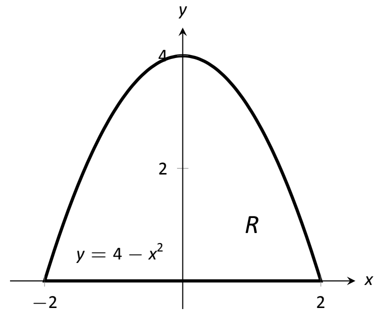
area of $R = 4u^2$

13 01 ex 14 14. $\int_0^{12} \int_{-\sqrt{3x}}^{\sqrt{3x}} dy\,dx$ and $\int_{-6}^6 \int_{y^2/3}^{12} dx\,dy$
area of $R = 96u^2$

13 01 ex 15 15. $\int_0^1 \int_{x^4}^{\sqrt{x}} dy\,dx$ and $\int_0^1 \int_{y^2}^{\sqrt[4]{y}} dx\,dy$
area of $R = 7/15u^2$

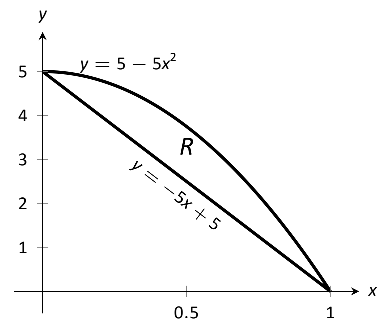
13 01 ex 16 16. $\int_0^2 \int_{x^3}^{4x} dy\,dx$ and $\int_0^8 \int_{y/4}^{\sqrt[3]{y}} dx\,dy$
area of $R = 4u^2$

13 01 ex 18 17.



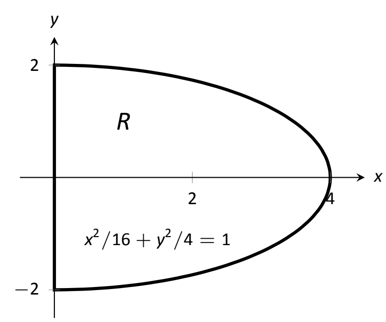

area of $R = \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dy$

13 01 ex 20 18.



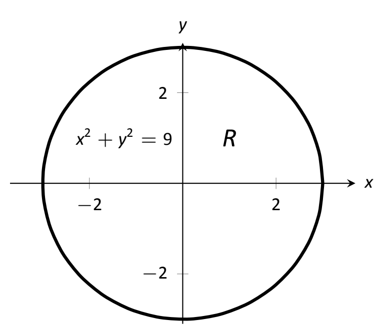

area of $R = \int_0^5 \int_{1-y/5}^{\sqrt{1-y/5}} dx\,dy$

13 01 ex 19 19.



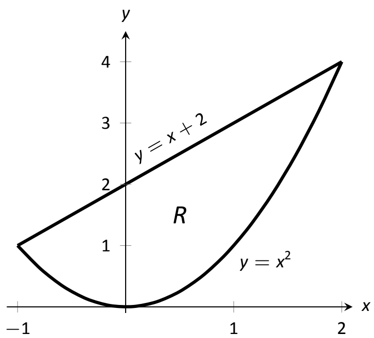

area of $R = \int_0^4 \int_{-\sqrt{4-x^2/4}}^{\sqrt{4-x^2/4}} dy\,dx$

13 01 ex 17 20.



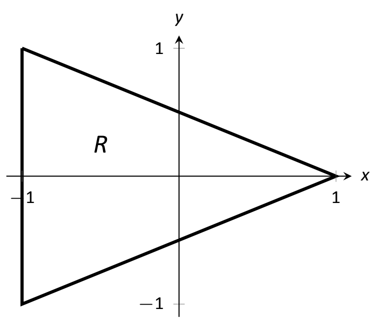

area of $R = \int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} dx\,dy$

13 01 ex 22 21.




area of $R = \int_{-1}^{2}\int_{x^2}^{x+2} dy\,dx$

13 01 ex 21 22.




area of $R = \int_{-1}^{0}\int_{0}^{2y+1} dx\,dy + \int_{0}^{1}\int_{0}^{1-2y} dx\,dy$

**Section 13.2**

13 02 ex 01 1. volume

13 02 ex 02 2. When switching the order of integration, the bounds integrals must change to reflect the bounds of the region of integration. You cannot merely change the letters $x$ and $y$ in a few places.

13 02 ex 03 3. The double integral gives the signed volume under the surface. Since the surface is always positive, it is always above the $x$-$y$ plane and hence produces only "positive" volume.

13 02 ex 04 4. No. It means that there is the same amount of signed volume under $f$ and $g$ over $R$, but the functions could be very different.

13 02 ex 05 5. 6; $\int_{-1}^{1}\int_{1}^{2}\left(\frac{x}{y}+3\right)\,dy\,dx$

13 02 ex 06 6. 4; $\int_{0}^{\pi}\int_{-\pi/2}^{\pi/2}(\sin x\cos y)\,dy\,dx$

13 02 ex 07 7. 112/3; $\int_{0}^{2}\int_{0}^{4-2y}\left(3x^2-y+2\right)\,dx\,dy$

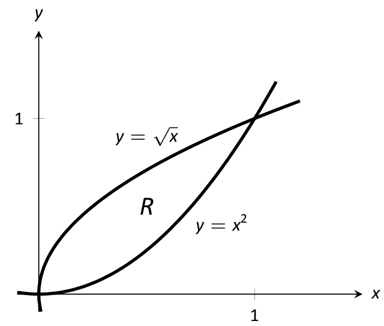
13 02 ex 08 8. 76/15; $\int_{1}^{3}\int_{1}^{x}\left(x^2y-xy^2\right)\,dy\,dx$

13 02 ex 09 9. 16/5; $\int_{-1}^{1}\int_{0}^{1-x^2}(x+y+2)\,dy\,dx$

13 02 ex 10 10. 6561/40; $\int_{0}^{3}\int_{x^2}^{3x}\left(xy^2\right)\,dy\,dx$

13 02 ex 11 11.

(a)

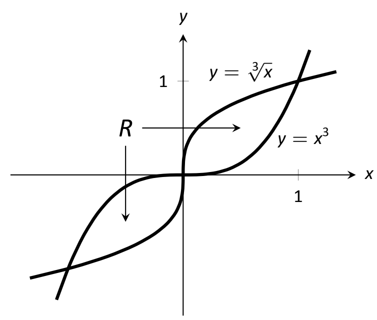



(b) $\int_{0}^{1}\int_{x^2}^{\sqrt{x}} x^2y\,dy\,dx = \int_{0}^{1}\int_{y^2}^{\sqrt{y}} x^2y\,dx\,dy.$

(c) $\frac{3}{56}$

13 02 ex 12 12.

(a)

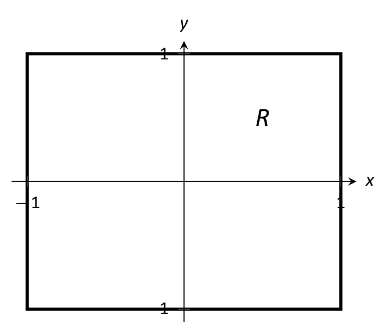



(b) $\int_{0}^{1}\int_{x^3}^{\sqrt[3]{x}} x^2y\,dy\,dx + \int_{-1}^{0}\int_{\sqrt[3]{x}}^{x^3} x^2y\,dy\,dx$
$= \int_{0}^{1}\int_{y^3}^{\sqrt[3]{y}} x^2y\,dx\,dy + \int_{-1}^{0}\int_{\sqrt[3]{y}}^{y^3} x^2y\,dx\,dy.$

(c) 0

13 02 ex 13 13.

(a)

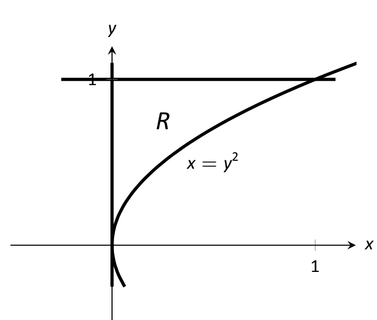



(b) $\int_{-1}^{1}\int_{-1}^{1} x^2-y^2\,dy\,dx = \int_{-1}^{1}\int_{-1}^{1} x^2-y^2\,dx\,dy.$

(c) 0

13 02 ex 14 14.

(a)

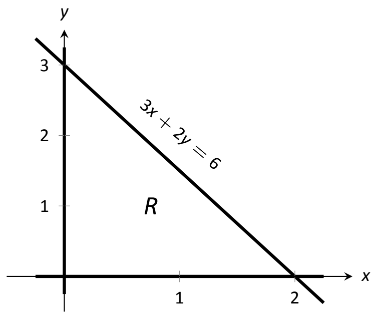


(b) $\int_0^1 \int_0^{y^2} ye^x \, dx \, dy = \int_0^1 \int_{\sqrt{x}}^1 ye^x \, dy \, dx.$

(c) $e/2 - 1$

13 02 ex 15 15.

(a)



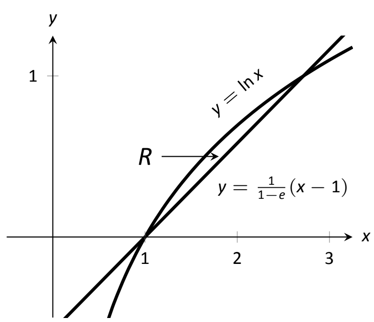

(b)

(c) $\int_0^2 \int_0^{3-3/2x} \big(6 - 3x - 2y\big) \, dy \, dx =$
$\int_0^3 \int_0^{2-2/3y} \big(6 - 3x - 2y\big) \, dx \, dy.$

(d) 6

13 02 ex 16 16.

(a)



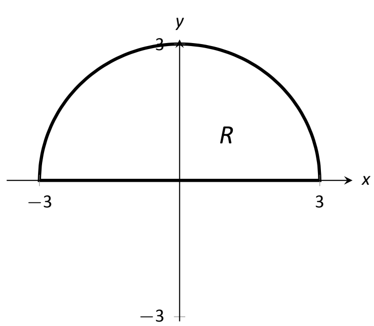

(b)

(c) $\int_1^e \int_{\frac{x-1}{e-1}}^{\ln x} e^y \, dy \, dx = \int_0^1 \int_{e^y}^{y(e-1)+1} e^y \, dx \, dy.$

(d) $-\frac{1}{2}e^2 + 2e - \frac{3}{2}$

13 02 ex 17 17.

(a)

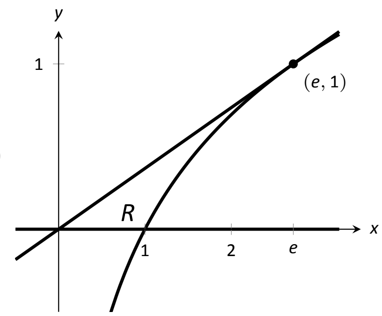



(b) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \big(x^3y - x\big) \, dy \, dx =$
$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \big(x^3y - x\big) \, dx \, dy.$

(c) 0

13 02 ex 18 18.

(a)




(b) $\int_0^1 \int_{ey}^{e^y} \big(4 - 3y\big) \, dx \, dy =$
$\int_0^1 \int_0^{x/e} \big(4 - 3y\big) \, dy \, dx + \int_1^e \int_{\ln x}^{x/e} \big(4 - 3y\big) \, dy \, dx.$

(c) $3e - 7$

13 02 ex 19 19. Integrating $e^{x^2}$ with respect to $x$ is not possible in terms of elementary functions. $\int_0^2 \int_0^{2x} e^{x^2} \, dy \, dx = e^4 - 1.$

13 02 ex 20 20. Integrating $\cos(y^2)$ with respect to $y$ is not possible in terms of elementary functions.
$\int_0^{\sqrt{\pi/2}} \int_0^y \cos(y^2) \, dx \, dy = \frac{1}{2}.$

13 02 ex 21 21. Integrating $\int_y^1 \frac{2y}{x^2 + y^2} \, dx$ gives $\tan^{-1}(1/y) - \pi/4$; integrating $\tan^{-1}(1/y)$ is hard.
$\int_0^1 \int_0^x \frac{2y}{x^2 + y^2} \, dy \, dx = \ln 2.$

13 02 ex 22 22. Integrating in the order shown is hard/impossible. By changing the order of integration, we have
$\int_1^2 \int_{-1}^1 \frac{x \tan^2 y}{1 + \ln y} \, dx \, dy = 0$, since the integrand is an odd function with respect to $x$. Thus the iterated integral evaluates to 0.

13 02 ex 23 23. average value of $f = 6/2 = 3$

13 02 ex 24 24. average value of $f = 4/\pi^2$

13 02 ex 25 25. average value of $f = \frac{112/3}{4} = 28/3$

13 02 ex 26 26. average value of $f = \frac{76/15}{2} = \frac{38}{15} \approx 2.53$

**Section 13.3**

13 03 ex 01 1. $f\big(r\cos\theta, r\sin\theta\big)$, $r \, dr \, d\theta$

13 03 ex 02 2. Some regions in the $x$-$y$ plane are easier to describe using polar coordinates than using rectangular coordinates. Also, some integrals are easier to evaluate one the polar substitutions have been made.

13 03 ex 03 3. $\int_0^{2\pi} \int_0^1 \big(3r\cos\theta - r\sin\theta + 4\big) r \, dr \, d\theta = 4\pi$

13 03 ex 04 4. $\int_0^{2\pi} \int_0^2 \big(4r\cos\theta + 4r\sin\theta\big) r \, dr \, d\theta = 0$

13 03 ex 05 5. $\int_0^{\pi} \int_{\cos\theta}^{3\cos\theta} \big(8 - r\sin\theta\big) r \, dr \, d\theta = 16\pi$

13 03 ex 06 6. $\int_0^{\pi/2} \int_0^{\sin(2\theta)} \big(4\big) r \, dr \, d\theta = \pi/2$

13 03 ex 07 7. $\displaystyle\int_0^{2\pi}\int_1^2 \big(\ln(r^2)\big)r\,dr\,d\theta = 2\pi\big(\ln 16 - 3/2\big)$

13 03 ex 08 8. $\displaystyle\int_0^{2\pi}\int_0^1 \big(1-r^2\big)r\,dr\,d\theta = \pi/2$

13 03 ex 09 9. $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^6 \big(r^2\cos^2\theta - r^2\sin^2\theta\big)r\,dr\,d\theta =$
$\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^6 \big(r^2\cos(2\theta)\big)r\,dr\,d\theta = 0$

13 03 ex 10 10. $\displaystyle\int_0^{\pi/4}\int_0^1 \left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right) r\,dr\,d\theta = \ln 2$

13 03 ex 11 11. $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^5 \big(r^2\big)\,dr\,d\theta = 125\pi/3$

13 03 ex 12 12. $\displaystyle\int_{\pi/2}^{3\pi/2}\int_0^4 \big(2r\sin\theta - r\cos\theta\big)r\,dr\,d\theta = 128/3$

13 03 ex 13 13. $\displaystyle\int_0^{\pi/4}\int_0^{\sqrt{8}} \big(r\cos\theta + r\sin\theta\big)r\,dr\,d\theta = 16\sqrt{2}/3$

13 03 ex 14 14. $\displaystyle\int_0^{\pi}\int_1^2 \big(r\cos\theta + 5\big)r\,dr\,d\theta = 15\pi/2$

13 03 ex 15 15.

(a) This is impossible to integrate with rectangular coordinates as $e^{-(x^2+y^2)}$ does not have an antiderivative in terms of elementary functions.

(b) $\displaystyle\int_0^{2\pi}\int_0^a re^{r^2}\,dr\,d\theta = \pi(1-e^{-a^2})$.

(c) $\displaystyle\lim_{a\to\infty}\pi(1-e^{-a^2}) = \pi$. This implies that there is a finite volume under the surface $e^{-(x^2+y^2)}$ over the entire $x$-$y$ plane.

13 03 ex 16 16.

$$\begin{aligned}\iint_R f(x,y)\,dA &= \int_0^{2\pi}\int_0^a \left(h - h\sqrt{\frac{r^2\cos^2\theta}{a^2}+\frac{r^2\sin^2\theta}{a^2}}\right) r\,dr\,d\theta\\ &= \int_0^{2\pi}\int_0^a \left(hr - h\frac{r^2}{a}\right)\,dr\,d\theta\\ &= \int_0^{2\pi}\left(\frac{1}{2}hr^2 - \frac{h}{3a}r^3\right)\Big|_0^a\,d\theta\\ &= \int_0^{2\pi}\left(\frac{1}{6}a^2h\right)\,d\theta\\ &= \frac{1}{3}\pi a^2 h.\end{aligned}$$

**Section 13.4**

13 04 ex 01 1. Because they are scalar multiples of each other.

13 04 ex 02 2. $y$

13 04 ex 03 3. "little masses"

13 04 ex 04 4. A collection of individual masses in the plane. Each mass is a point mass, i.e., mass located at a point, not across a region.

13 04 ex 05 5. $M_x$ measures the moment about the $x$-axis, meaning we need to measure distance from the $x$-axis. Such measurements are measures in the $y$-direction.

13 04 ex 06 6. If the lamina is an annulus, the center of mass will likely be in the middle, outside of the region. (See Example 469.)

13 04 ex 07 7. $\bar{x} = 5.25$

13 04 ex 08 8. $\bar{x} = 1.3$

13 04 ex 10 9. $(\bar{x},\bar{y}) = (0,3)$

13 04 ex 09 10. $(\bar{x},\bar{y}) = (0,1/3)$

13 04 ex 11 11. $M = 150$gm;

13 04 ex 12 12. $M = 190$gm

13 04 ex 13 13. $M = 2$lb

13 04 ex 14 14. $M = 2/3$lb

13 04 ex 15 15. $M = 16\pi \approx 50.27$kg

13 04 ex 16 16. $M = 325\pi/12 \approx 85$kg

13 04 ex 17 17. $M = 54\pi \approx 169.65$lb

13 04 ex 18 18. $M = 63\pi \approx 197.92$lb

13 04 ex 19 19. $M = 150$gm; $M_y = 600$; $M_x = -75$; $(\bar{x},\bar{y}) = (4,-0.5)$

13 04 ex 20 20. $M = 190$gm; $M_y = 850$; $M_x = -315/2$; $(\bar{x},\bar{y}) = (4.47,-0.83)$

13 04 ex 21 21. $M = 2$lb; $M_y = 0$; $M_x = 2/3$; $(\bar{x},\bar{y}) = (0,1/3)$

13 04 ex 22 22. $M = 2/3$lb; $M_y = 7/30$; $M_x = 7/30$; $(\bar{x},\bar{y}) = (0.35,0.35)$

13 04 ex 23 23. $M = 16\pi \approx 50.27$kg; $M_y = 4\pi$; $M_x = 4\pi$; $(\bar{x},\bar{y}) = (1/4,1/4)$

13 04 ex 24 24. $M = 325\pi/12 \approx 85$kg; $M_y = 2375/12$; $M_x = 2375/12$; $(\bar{x},\bar{y}) = (2.33,2.33)$

13 04 ex 25 25. $M = 54\pi \approx 169.65$lb; $M_y = 0$; $M_x = 504$; $(\bar{x},\bar{y}) = (0,2.97)$

13 04 ex 26 26. $M = 63\pi \approx 197.92$lb; $M_y = 0$; $M_x = 1215/2$; $(\bar{x},\bar{y}) = (0,3.07)$

13 04 ex 27 27. $I_x = 64/3$; $I_y = 64/3$; $I_O = 128/3$

13 04 ex 28 28. $I_x = 16/3$; $I_y = 256/3$; $I_O = 272/3$

13 04 ex 29 29. $I_x = 16/3$; $I_y = 64/3$; $I_O = 80/3$

13 04 ex 30 30. $I_x = 16$; $I_y = 16$; $I_O = 32$

**Section 13.5**

13 05 ex 01 1. arc length

13 05 ex 02 2. tangent

13 05 ex 03 3. surface areas

13 05 ex 04 4. Technology makes good approximations accessible, if not exact answers.

13 05 ex 18 5. Intuitively, adding $h$ to $f$ only shifts $f$ up (i.e., parallel to the $z$-axis) and does not change its shape. Therefore it will not change the surface area over $R$.

Analytically, $f_x = g_x$ and $f_y = g_y$; therefore, the surface area of each is computed with identical double integrals.

13 05 ex 19 6. Analytically, $g_x = 2f_x$ and $g_y = 2f_y$. The double integral to compute the surface area of $f$ over $R$ is $\displaystyle\iint_R \sqrt{1+f_x^2+f_y^2}\,dA$; the double integral to compute the surface area of $g$ over $R$ is $\displaystyle\iint_R \sqrt{1+4f_x^2+4f_y^2}\,dA$, which is *not* twice the double integral used to calculate the surface area of $f$.

13 05 ex 05 7. $SA = \int_0^{2\pi}\int_0^{2\pi}\sqrt{1+\cos^2 x\cos^2 y+\sin^2 x\sin^2 y}\,dx\,dy$

13 05 ex 06 8. $SA = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\sqrt{1+\dfrac{4x^2+4y^2}{(1+x^2+y^2)^4}}\,dx\,dy$
Polar offers simpler bounds:
$$SA = \int_0^{2\pi}\int_0^3 r\sqrt{1+\frac{4r^2}{(1+r^2)^4}}\,dr\,d\theta$$

13 05 ex 07 9. $SA = \int_{-1}^{1}\int_{-1}^{1}\sqrt{1+4x^2+4y^2}\,dx\,dy$

13 05 ex 08 10. $SA = \int_{-5}^{5}\int_0^1\sqrt{1+\dfrac{4x^2e^{2x^2}}{\left(1+e^{x^2}\right)^4}}\,dy\,dx$

13 05 ex 09 11. $SA = \int_0^3\int_{-1}^{1}\sqrt{1+9+49}\,dx\,dy = 6\sqrt{59}\approx 46.09$

13 05 ex 10 12. $SA = \int_0^1\int_0^{1-x}\sqrt{1+4+4}\,dy\,dx = 18$

13 05 ex 11 13. This is easier in polar:
$$\begin{aligned}SA &= \int_0^{2\pi}\int_0^4 r\sqrt{1+4r^2\cos^2 t+4r^2\sin^2 t}\,dr\,d\theta\\ &= \int_0^{2\pi}\int_0^4 r\sqrt{1+4r^2}\,dr\,d\theta\\ &= \frac{\pi}{6}\left(65\sqrt{65}-1\right)\approx 273.87\end{aligned}$$

13 05 ex 14 14.
$$\begin{aligned}SA &= \int_0^1\int_{-y}^{y}\sqrt{1+4+64y^2}\,dx\,dy\\ &= \int_0^1\left(2y\sqrt{5+64y^2}\right)dy\\ &= \frac{1}{96}\left(69\sqrt{69}-5\sqrt{5}\right)\approx 5.85\end{aligned}$$

13 05 ex 15 15.
$$\begin{aligned}SA &= \int_0^2\int_0^{2x}\sqrt{1+1+4x^2}\,dy\,dx\\ &= \int_0^2\left(2x\sqrt{2+4x^2}\right)dx\\ &= \frac{26}{3}\sqrt{2}\approx 12.26\end{aligned}$$

13 05 ex 16 16.
$$\begin{aligned}SA &= \int_0^1\int_0^1\sqrt{1+x+9y}\,dx\,dy\\ &= \int_0^1\frac{2}{3}\left((9y+2)^{3/2}-(9y+1)^{3/2}\right)dy\\ &= \frac{4}{135}\left(121\sqrt{11}-100\sqrt{10}-4\sqrt{2}+1\right)\approx 2.383\end{aligned}$$

13 05 ex 12 17. This is easier in polar:
$$\begin{aligned}SA &= \int_0^{2\pi}\int_0^5 r\sqrt{1+\frac{4r^2\cos^2 t+4r^2\sin^2 t}{r^2\sin^2 t+r^2\cos^2 t}}\,dr\,d\theta\\ &= \int_0^{2\pi}\int_0^5 r\sqrt{5}\,dr\,d\theta\\ &= 25\pi\sqrt{5}\approx 175.62\end{aligned}$$

13 05 ex 13 18. This is easier in polar:
$$\begin{aligned}SA &= 2\int_0^{2\pi}\int_0^5 r\sqrt{1+\frac{r^2\cos^2 t+r^2\sin^2 t}{25-r^2\sin^2 t-r^2\cos^2 t}}\,dr\,d\theta\\ &= 2\int_0^{2\pi}\int_0^5 r\sqrt{\frac{1}{25-r^2}}\,dr\,d\theta\\ &= 100\pi\approx 314.16\end{aligned}$$

13 05 ex 17 19. Integrating in polar is easiest considering $R$:
$$\begin{aligned}SA &= \int_0^{2\pi}\int_0^1 r\sqrt{1+c^2+d^2}\,dr\,d\theta\\ &= \int_0^{2\pi}\frac{1}{2}\left(\sqrt{1+c^2+d^2}\right)dy\\ &= \pi\sqrt{1+c^2+d^2}.\end{aligned}$$
The value of $h$ does not matter as it only shifts the plane vertically (i.e., parallel to the $z$-axis). Different values of $h$ do not create different ellipses in the plane.

**Section 13.6**

13 06 ex 01 1. surface to surface, curve to curve and point to point

13 06 ex 02 2. One possible answer is "sum up lots of little volumes over $D$."

13 06 ex 23 3. Answers can vary. From this section we used triple integration to find the volume of a solid region, the mass of a solid, and the center of mass of a solid.

13 06 ex 24 4. $\delta V$.

13 06 ex 03 5. $V=\int_{-1}^1\int_{-1}^1\left(8-x^2-y^2-(2x+y)\right)dx\,dy = 88/3$

13 06 ex 04 6. $V=\int_0^2\int_0^3\left(x^2+y^2-(-x^2-y^2)\right)dy\,dx = 52$

13 06 ex 05 7. $V=\int_0^\pi\int_0^x\left(\cos x\sin y+2-\sin x\cos y\right)dy\,dx = \pi^2-\pi\approx 6.728$

13 06 ex 06 8. $V=\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\left(6-x^2-y^2-(2x^2+2y^2+3)\right)dy\,dx$.
Integrating in polar is easier, giving
$V=\int_0^{2\pi}\int_0^1\left(3-3r^2\right)r\,dr\,d\theta = 3\pi/2$.

13 06 ex 07 9. $dz\,dy\,dx$: $\displaystyle\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y}dz\,dy\,dx$
$dz\,dx\,dy$: $\displaystyle\int_0^1\int_0^{3-3y}\int_0^{2-2x/3-2y}dz\,dx\,dy$
$dy\,dz\,dx$: $\displaystyle\int_0^3\int_0^{2-2x/3}\int_0^{1-x/3-z/2}dy\,dz\,dx$
$dy\,dx\,dz$: $\displaystyle\int_0^2\int_0^{3-3z/2}\int_0^{1-x/3-z/2}dy\,dx\,dz$

dx dz dy: $\int_0^1\int_0^{2-2y}\int_0^{3-3y-3z/2} dx\,dz\,dy$

dx dy dz: $\int_0^2\int_0^{1-z/2}\int_0^{3-3y-3z/2} dx\,dy\,dz$

$V=\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx=1.$

13 06 ex 08 10. dz dy dx: $\int_1^3\int_0^2\int_0^{(3-x)/2} dz\,dy\,dx$

dz dx dy: $\int_0^2\int_1^3\int_0^{(3-x)/2} dz\,dx\,dy$

dy dz dx: $\int_1^3\int_0^{(3-x)/2}\int_0^2 dy\,dz\,dx$

dy dx dz: $\int_0^1\int_1^{3-2z}\int_0^2 dy\,dx\,dz$

dx dz dy: $\int_0^2\int_0^1\int_1^{3-2z} dx\,dz\,dy$

dx dy dz: $\int_0^1\int_0^2\int_1^{3-2z} dx\,dy\,dz$

$V=\int_0^1\int_0^2\int_1^{3-2z} dx\,dy\,dz=2.$

13 06 ex 09 11. dz dy dx: $\int_0^2\int_{-2}^0\int_{y^2/2}^{-y} dz\,dy\,dx$

dz dx dy: $\int_{-2}^0\int_0^2\int_{y^2/2}^{-y} dz\,dx\,dy$

dy dz dx: $\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx$

dy dx dz: $\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dx\,dz$

dx dz dy: $\int_{-2}^0\int_{y^2/2}^{-y}\int_0^2 dx\,dz\,dy$

dx dy dz: $\int_0^2\int_{-\sqrt{2z}}^{-z}\int_0^2 dx\,dy\,dz$

$V=\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx=4/3.$

13 06 ex 10 12. dz dy dx: $\int_0^3\int_{3x}^9\int_0^{\sqrt{y^2-9x^2}} dz\,dy\,dx$

dz dx dy: $\int_0^9\int_0^{y/3}\int_0^{\sqrt{y^2-9x^2}} dz\,dx\,dy$

dy dz dx: $\int_0^3\int_0^{\sqrt{81-9x^2}}\int_{\sqrt{z^2+9x^2}}^9 dy\,dz\,dx$

dy dx dz: $\int_0^9\int_0^{\sqrt{9-z^2/9}}\int_{\sqrt{z^2+9x^2}}^9 dy\,dx\,dz$

dx dz dy: $\int_0^9\int_0^y\int_0^{\frac{1}{3}\sqrt{y^2-z^2}} dx\,dz\,dy$

dx dy dz: $\int_0^9\int_z^9\int_0^{\frac{1}{3}\sqrt{y^2-z^2}} dx\,dy\,dz$

13 06 ex 11 13. dz dy dx: $\int_0^2\int_{1-x/2}^1\int_0^{2x+4y-4} dz\,dy\,dx$

dz dx dy: $\int_0^1\int_{2-2y}^2\int_0^{2x+4y-4} dz\,dx\,dy$

dy dz dx: $\int_0^2\int_0^{2x}\int_{z/4-x/2+1}^1 dy\,dz\,dx$

dy dx dz: $\int_0^4\int_{z/2}^2\int_{z/4-x/2+1}^1 dy\,dx\,dz$

dx dz dy: $\int_0^1\int_0^{4y}\int_{z/2-2y+2}^2 dx\,dz\,dy$

dx dy dz: $\int_0^4\int_{z/4}^1\int_{z/2-2y+2}^2 dx\,dy\,dz$

$V=\int_0^4\int_{z/4}^1\int_{2y-z/2-2}^2 dx\,dy\,dz=4/3.$

13 06 ex 12 14. dz dy dx: $\int_{-2}^2\int_0^{4-x^2}\int_0^{2y} dz\,dy\,dx$

dz dx dy: $\int_0^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_0^{2x+4y-4} dz\,dx\,dy$

dy dz dx: $\int_{-2}^2\int_0^{8-2x^2}\int_{z/2}^{4-x^2} dy\,dz\,dx$

dy dx dz: $\int_0^8\int_{-\sqrt{4-z/2}}^{\sqrt{4-z/2}}\int_{z/2}^{4-x^2} dy\,dx\,dz$

dx dz dy: $\int_0^4\int_0^{2y}\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dz\,dy$

dx dy dz: $\int_0^8\int_{z/2}^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dy\,dz$

$V=\int_{-2}^2\int_0^{4-x^2}\int_0^{2y} dz\,dy\,dx=512/15.$

13 06 ex 13 15. dz dy dx: $\int_0^1\int_0^{1-x^2}\int_0^{\sqrt{1-y}} dz\,dy\,dx$

dz dx dy: $\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dz\,dx\,dy$

dy dz dx: $\int_0^1\int_0^x\int_0^{1-x^2} dy\,dz\,dx+\int_0^1\int_x^1\int_0^{1-z^2} dy\,dz\,dx$

dy dx dz: $\int_0^1\int_0^z\int_0^{1-z^2} dy\,dx\,dz+\int_0^1\int_z^1\int_0^{1-x^2} dy\,dx\,dz$

dx dz dy: $\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dx\,dz\,dy$

dx dy dz: $\int_0^1\int_0^{1-z^2}\int_0^{\sqrt{1-y}} dx\,dy\,dz$

Answers will vary. Neither order is particularly "hard." The order *dz dy dx* requires integrating a square root, so powers can be messy; the order *dy dz dx* requires two triple integrals, but each uses only polynomials.

13 06 ex 14 16. dz dy dx:

$\int_0^1\int_0^{3x}\int_0^{1-x} dz\,dy\,dx+\int_0^1\int_{3x}^3\int_0^{1-y/3} dz\,dy\,dx$

dz dx dy:

$\int_0^3\int_0^{y/3}\int_0^{1-y/3} dz\,dy\,dx+\int_0^3\int_{y/3}^1\int_0^{1-x} dz\,dx\,dy$

dy dz dx: $\int_0^1\int_0^{1-x}\int_0^{3-3z} dy\,dz\,dx$

dy dx dz: $\int_0^1\int_0^{1-z}\int_0^{3-3z} dy\,dx\,dz$

dx dz dy: $\int_0^3\int_0^{1-y/3}\int_0^{1-z} dx\,dz\,dy$

dx dy dz: $\int_0^1\int_0^{3-3z}\int_0^{1-z} dx\,dy\,dz$

$V=\int_0^1\int_0^{3-3z}\int_0^{1-z} dx\,dy\,dz=1.$

13 06 ex 19 17. 8

13 06 ex 20 18. 7/8

13 06 ex 22 19. $\pi$

13 06 ex 21 20. 0

13 06 ex 15 21. $M = 10$, $M_{yz} = 15/2$, $M_{xz} = 5/2$, $M_{xy} = 5$; $(\bar{x}, \bar{y}, \bar{z}) = (3/4, 1/4, 1/2)$

13 06 ex 16 22. $M = 4$, $M_{yz} = 20/3$, $M_{xz} = 4$, $M_{xy} = 4/3$; $(\bar{x}, \bar{y}, \bar{z}) = (5/3, 1, 1/3)$

13 06 ex 17 23. $M = 16/5$, $M_{yz} = 16/3$, $M_{xz} = 104/45$, $M_{xy} = 32/9$; $(\bar{x}, \bar{y}, \bar{z}) = (5/3, 13/18, 10/9) \approx (1.67, 0.72, 1.11)$

13 06 ex 18 24. $M = \frac{65,536}{15} \approx 208.05$, $M_{yz} = 0$, $M_{xz} = \frac{2,097,152}{3465} \approx 605.24$, $M_{xy} = \frac{2,097,152}{3465} \approx 605.24$; $(\bar{x}, \bar{y}, \bar{z}) = (0, 32/11, 32/11) \approx (0, 2.91, 2.91)$