A: Solutions To Selected Problems

Chapter 1

Section 1.1

1. Answers will vary.
2. An indeterminate form.
3. F
4. The function may approach different values from the left and right, the function may grow without bound, or the function might oscillate.
5. Answers will vary.
6. -1
7. -5
8. Limit does not exist
9. 2
10. 1.5
11. Limit does not exist.
12. Limit does not exist.
13. 7
14. 1
15. Limit does not exist.
16. The limit seems to be exactly 7.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-7
-0.01	-7
0.01	-7
0.1	-7

17. The limit seems to be exactly 9.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	9
-0.01	9
0.01	9
0.1	9

18. The limit is approx. 5.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	4.9
-0.01	4.99
0.01	5.01
0.1	5.1

19. The limit is approx. -0.11.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.114943
-0.01	-0.111483
0.01	-0.110742
0.1	-0.107527

20. The limit is approx. 29.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	29.4
-0.01	29.04
0.01	28.96
0.1	28.6

21. The limit is approx. 0.2.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	0.202027
-0.01	0.2002
0.01	0.1998
0.1	0.198026

22. The limit is approx. -1.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.998334
-0.01	-0.999983
0.01	-0.999983
0.1	-0.998334

23. The limit is approx. 0.005.

h	$\frac{f(a+h)-f(a)}{h}$
-0.1	-0.0499583
-0.01	-0.00499996
0.01	0.00499996
0.1	0.0499583

Section 1.2

1. ε should be given first, and the restriction $|x-a|<\delta$ implies $|f(x)-K|<\varepsilon$, not the other way around.
2. The y–tolerance.
3. T
4. T
5. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-5|<\delta$, $|f(x)-(-2)|<\varepsilon$.
   Consider $|f(x)-(-2)|<\varepsilon$:
$$\begin{gathered}|f(x)+2|<\varepsilon\\ |(3-x)+2|<\varepsilon\\ |5-x|<\varepsilon\\ -\varepsilon<5-x<\varepsilon\\ -\varepsilon<x-5<\varepsilon.\end{gathered}$$
   This implies we can let $\delta=\varepsilon$. Then:
$$\begin{gathered}|x-5|<\delta\\ -\delta<x-5<\delta\\ -\varepsilon<x-5<\varepsilon\\ -\varepsilon<(x-3)-2<\varepsilon\\ -\varepsilon<(-x+3)-(-2)<\varepsilon\\ |3-x-(-2)|<\varepsilon,\end{gathered}$$
   which is what we wanted to prove.
6. Let $\varepsilon>0$ be given. We wish to find $\delta>0$ such that when $|x-3|<\delta$, $|f(x)-6|<\varepsilon$.
   Consider $|f(x)-6|<\varepsilon$, keeping in mind we want to make a statement about $|x-3|$:
$$\begin{gathered}|f(x)-6|<\varepsilon\\ |x^2-3-6|<\varepsilon\\ |x^2-9|<\varepsilon\\ |x-3|\cdot|x+3|<\varepsilon\\ |x-3|<\varepsilon/|x+3|\end{gathered}$$

Since x is near 3, we can safely assume that, for instance, $2 < x < 4$. Thus

$$\begin{gathered} 2+3 < x+3 < 4+3 \\ 5 < x+3 < 7 \\ \frac{1}{7} < \frac{1}{x+3} < \frac{1}{5} \\ \frac{\varepsilon}{7} < \frac{\varepsilon}{x+3} < \frac{\varepsilon}{5} \end{gathered}$$

Let $\delta = \frac{\varepsilon}{7}$. Then:

$$\begin{aligned} |x-3| &< \delta \\ |x-3| &< \frac{\varepsilon}{7} \\ |x-3| &< \frac{\varepsilon}{x+3} \\ |x-3| \cdot |x+3| &< \frac{\varepsilon}{x+3} \cdot |x+3| \end{aligned}$$

Assuming x is near 3, $x+3$ is positive and we can drop the absolute value signs on the right.

$$\begin{gathered} |x-3| \cdot |x+3| < \frac{\varepsilon}{x+3} \cdot (x+3) \\ |x^2-9| < \varepsilon |(x^2-3)-6| < \varepsilon, \end{gathered}$$

which is what we wanted to prove.

7. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-4| < \delta$, $|f(x)-15| < \varepsilon$.
   Consider $|f(x)-15| < \varepsilon$, keeping in mind we want to make a statement about $|x-4|$:

$$\begin{gathered} |f(x)-15| < \varepsilon \\ |x^2+x-5-15| < \varepsilon \\ |x^2+x-20| < \varepsilon \\ |x-4| \cdot |x+5| < \varepsilon \\ |x-4| < \varepsilon/|x+5| \end{gathered}$$

   Since x is near 4, we can safely assume that, for instance, $3 < x < 5$. Thus

$$\begin{gathered} 3+5 < x+5 < 5+5 \\ 8 < x+5 < 10 \\ \frac{1}{8} < \frac{1}{x+5} < \frac{1}{10} \\ \frac{\varepsilon}{8} < \frac{\varepsilon}{x+5} < \frac{\varepsilon}{10} \end{gathered}$$

   Let $\delta = \frac{\varepsilon}{8}$. Then:

$$\begin{aligned} |x-4| &< \delta \\ |x-4| &< \frac{\varepsilon}{8} \\ |x-4| &< \frac{\varepsilon}{x+5} \\ |x-4| \cdot |x+5| &< \frac{\varepsilon}{x+5} \cdot |x+5| \end{aligned}$$

   Assuming x is near 4, $x+4$ is positive and we can drop the absolute value signs on the right.

$$\begin{gathered} |x-4| \cdot |x+5| < \frac{\varepsilon}{x+5} \cdot (x+5) \\ |x^2+x-20| < \varepsilon |(x^2+x-5)-15| < \varepsilon, \end{gathered}$$

   which is what we wanted to prove.

8. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x)-7| < \varepsilon$.
   Consider $|f(x)-7| < \varepsilon$, keeping in mind we want to make a statement about $|x-2|$:

$$\begin{gathered} |f(x)-7| < \varepsilon \\ |x^3-1-7| < \varepsilon \\ |x^3-8| < \varepsilon \\ |x-2| \cdot |x^2+2x+4| < \varepsilon \\ |x-3| < \varepsilon/|x^2+2x+4| \end{gathered}$$

   Since x is near 2, we can safely assume that, for instance, $1 < x < 3$. Thus

$$\begin{gathered} 1^2+2\cdot 1+4 < x^2+2x+4 < 3^2+2\cdot 3+4 \\ 7 < x^2+2x+4 < 19 \\ \frac{1}{19} < \frac{1}{x^2+2x+4} < \frac{1}{7} \\ \frac{\varepsilon}{19} < \frac{\varepsilon}{x^2+2x+4} < \frac{\varepsilon}{7} \end{gathered}$$

   Let $\delta = \frac{\varepsilon}{19}$. Then:

$$\begin{aligned} |x-2| &< \delta \\ |x-2| &< \frac{\varepsilon}{19} \\ |x-2| &< \frac{\varepsilon}{x^2+2x+4} \\ |x-2| \cdot |x^2+2x+4| &< \frac{\varepsilon}{x^2+2x+4} \cdot |x^2+2x+4| \end{aligned}$$

   Assuming x is near 2, x^2+2x+4 is positive and we can drop the absolute value signs on the right.

$$\begin{gathered} |x-2| \cdot |x^2+2x+4| < \frac{\varepsilon}{x^2+2x+4} \cdot (x^2+2x+4) \\ |x^3-8| < \varepsilon |(x^3-1)-7| < \varepsilon, \end{gathered}$$

   which is what we wanted to prove.

9. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-2| < \delta$, $|f(x)-5| < \varepsilon$. However, since $f(x) = 5$, a constant function, the latter inequality is simply $|5-5| < \varepsilon$, which is always true. Thus we can choose any δ we like; we arbitrarily choose $\delta = \varepsilon$.

10. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x-0| < \delta$, $|f(x)-0| < \varepsilon$.
    Consider $|f(x)-0| < \varepsilon$, keeping in mind we want to make a statement about $|x-0|$ (i.e., $|x|$):

$$\begin{gathered} |f(x)-0| < \varepsilon \\ |e^{2x}-1| < \varepsilon \\ -\varepsilon < e^{2x}-1 < \varepsilon \\ 1-\varepsilon < e^{2x} < 1+\varepsilon \\ \ln(1-\varepsilon) < 2x < \ln(1+\varepsilon) \\ \frac{\ln(1-\varepsilon)}{2} < x < \frac{\ln(1+\varepsilon)}{2} \end{gathered}$$

Let $\delta = \min\left\{\left|\frac{\ln(1-\varepsilon)}{2}\right|, \left|\frac{\ln(1+\varepsilon)}{2}\right|\right\}$.

Thus:

$$|x| < \delta$$
$$|x| < \min\left\{\left|\frac{\ln(1-\varepsilon)}{2}\right|, \left|\frac{\ln(1+\varepsilon)}{2}\right|\right\}$$
$$\frac{\ln(1-\varepsilon)}{2} < x < \frac{\ln(1\varepsilon)}{2}$$
$$-\varepsilon < e^{2x} < \varepsilon$$

which is what we wanted to prove.

11. Let $\varepsilon > 0$ be given. We wish to find $\delta > 0$ such that when $|x - 0| < \delta$, $|f(x) - 0| < \varepsilon$. In simpler terms, we want to show that when $|x| < \delta$, $|\sin x| < \varepsilon$.
   Set $\delta = \varepsilon$. We start with assuming that $|x| < \delta$. Using the hint, we have that $|\sin x| < |x| < \delta = \varepsilon$. Hence if $|x| < \delta$, we know immediately that $|\sin x| < \varepsilon$.

Section 1.3

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. As x is near 1, both f and g are near 0, but f is approximately twice the size of g. (I.e., $f(x) \approx 2g(x)$.)
6. 9
7. 6
8. 0
9. Limit does not exist.
10. 3
11. Not possible to know.
12. 3
13. -45
14. 1
15. -1
16. 0
17. π
18. 7
19. $-0.000000015 \approx 0$
20. $1/2$
21. Limit does not exist
22. 64
23. 2
24. 0
25. $\frac{\pi^2+3\pi+5}{5\pi^2-2\pi-3} \approx 0.6064$
26. $\frac{3\pi+1}{1-\pi}$
27. -8
28. -1
29. 10
30. -2
31. $-3/2$
32. $-7/8$
33. 0
34. 0
35. 9
36. 3
37. $5/8$
38. 1
39. $\pi/180$

Section 1.4

1. The function approaches different values from the left and right; the function grows without bound; the function oscillates.
2. F
3. F
4. T
5. (a) 2
   (b) 2
   (c) 2
   (d) 1
   (e) As f is not defined for $x < 0$, this limit is not defined.
   (f) 1
6. (a) 1
   (b) 2
   (c) Does not exist.
   (d) 2
   (e) 0
   (f) As f is not defined for $x < 0$, this limit is not defined.
7. (a) Does not exist.
   (b) Does not exist.
   (c) Does not exist.
   (d) Not defined.
   (e) 0
   (f) 0
8. (a) 2
   (b) 0
   (c) Does not exist.
   (d) 1
9. (a) 2
   (b) 2
   (c) 2
   (d) 2
10. (a) 4
   (b) -4
   (c) Does not exist.
   (d) 0
11. (a) 2
   (b) 2
   (c) 2
   (d) 0
   (e) 2

(f) 2

(g) 2

(h) Not defined

12. (a) $a - 1$
(b) a
(c) Does not exist.
(d) a

13. (a) 2
(b) -4
(c) Does not exist.
(d) 2

14. (a) -1
(b) 1
(c) Does not exist.
(d) 1

15. (a) 0
(b) 0
(c) 0
(d) 0
(e) 2
(f) 2
(g) 2
(h) 2

16. (a) -1
(b) 0
(c) Does not exist.
(d) 0

17. (a) $1 - \cos^2 a = \sin^2 a$
(b) $\sin^2 a$
(c) $\sin^2 a$
(d) $\sin^2 a$

18. (a) 2
(b) 0
(c) Does not exist
(d) 1

19. (a) 4
(b) 4
(c) 4
(d) 3

20. (a) c
(b) c
(c) c
(d) c

21. (a) -1
(b) 1
(c) Does not exist
(d) 0

22. $-3/5$

23. $2/3$

24. 2.5

25. $-1/2$

26. -9

27. $-31/19$

28. $-3/11$

29. $11/81$

Section 1.5

1. Answers will vary.
2. Answers will vary.
3. A root of a function f is a value c such that $f(c) = 0$.
4. Consider the function $h(x) = g(x) - f(x)$, and use the Bisection Method to find a root of h.
5. F
6. T
7. T
8. F
9. F
10. T
11. No; $\lim\limits_{x\to 1} f(x) = 2$, while $f(1) = 1$.
12. No; $\lim\limits_{x\to 1} f(x)$ does not exist.
13. No; $f(1)$ does not exist.
14. Yes
15. Yes
16. Yes
17. (a) No; $\lim\limits_{x\to -2} f(x) \neq f(-2)$
(b) Yes
(c) No; $f(2)$ is not defined.
18. (a) Yes
(b) Yes
19. (a) Yes
(b) No; the left and right hand limits at 1 are not equal.
20. (a) Yes
(b) Yes
21. (a) Yes
(b) No. $\lim_{x\to 8} f(x) = 16/5 \neq f(8) = 5$.
22. $(-\infty, \infty)$
23. $(-\infty, -2] \cup [2, \infty)$
24. $[-1, 1]$
25. $(-\infty, -\sqrt{6}] \cup [\sqrt{6}, \infty)$
26. $(-1, 1)$
27. $(-\infty, \infty)$
28. $(-\infty, \infty)$
29. $(0, \infty)$
30. $(-\infty, \infty)$
31. $(-\infty, 0]$
32. $(-\infty, \infty)$

33. Yes, by the Intermediate Value Theorem.

34. Yes, by the Intermediate Value Theorem. In fact, we can be more specific and state such a value c exists in $(0,2)$, not just in $(-3,7)$.

35. We cannot say; the Intermediate Value Theorem only applies to function values between -10 and 10; as 11 is outside this range, we do not know.

36. We cannot say; the Intermediate Value Theorem only applies to continuous functions. As we do know know if h is continuous, we cannot say.

37. Approximate root is $x = 1.23$. The intervals used are:
$[1, 1.5]$ $[1, 1.25]$ $[1.125, 1.25]$
$[1.1875, 1.25]$ $[1.21875, 1.25]$ $[1.234375, 1.25]$
$[1.234375, 1.2421875]$ $[1.234375, 1.2382813]$

38. Approximate root is $x = 0.52$. The intervals used are:
$[0.5, 0.55]$ $[0.5, 0.525]$ $[0.5125, 0.525]$
$[0.51875, 0.525]$ $[0.521875, 0.525]$

39. Approximate root is $x = 0.69$. The intervals used are:
$[0.65, 0.7]$ $[0.675, 0.7]$ $[0.6875, 0.7]$
$[0.6875, 0.69375]$ $[0.690625, 0.69375]$

40. Approximate root is $x = 0.78$. The intervals used are:
$[0.7, 0.8]$ $[0.75, 0.8]$ $[0.775, 0.8]$
$[0.775, 0.7875]$ $[0.78125, 0.7875]$
(A few more steps show that 0.79 is better as the root is $\pi/4 \approx 0.78539$.)

41. (a) 20
(b) 25
(c) Limit does not exist
(d) 25

42.

x	$f(x)$
-0.81	-2.34129
-0.801	-2.33413
-0.79	-2.32542
-0.799	-2.33254

The top two lines give an approximation of the limit from the left: -2.33. The bottom two lines give an approximation from the right: -2.33 as well.

43. Answers will vary.

Section 1.6

1. F
2. T
3. F
4. T
5. T
6. Answers will vary.
7. Answers will vary.
8. The limit of f as x approaches 7 does not exist, hence f is not continuous. (Note: f could be defined at 7!)

9. (a) ∞
(b) ∞

10. (a) $-\infty$
(b) ∞
(c) ∞
(d) ∞

11. (a) 1
(b) 0
(c) $1/2$
(d) $1/2$

12. (a) Limit does not exist
(b) Limit does not exist

13. (a) Limit does not exist
(b) Limit does not exist

14. (a) 10
(b) ∞

15. Tables will vary.

(a)

x	$f(x)$
2.9	-15.1224
2.99	-159.12
2.999	-1599.12

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

x	$f(x)$
3.1	16.8824
3.01	160.88
3.001	1600.88

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x)$ does not exist.

16. Tables will vary.

(a)

x	$f(x)$
2.9	-335.64
2.99	-30350.6

It seems $\lim_{x\to 3^-} f(x) = -\infty$.

(b)

x	$f(x)$
3.1	-265.61
3.01	-29650.6

It seems $\lim_{x\to 3^+} f(x) = -\infty$.

(c) It seems $\lim_{x\to 3} f(x) = -\infty$.

17. Tables will vary.

(a)

x	$f(x)$
2.9	132.857
2.99	12124.4

It seems $\lim_{x\to 3^-} f(x) = \infty$.

(b)

x	$f(x)$
3.1	108.039
3.01	11876.4

It seems $\lim_{x\to 3^+} f(x) = \infty$.

(c) It seems $\lim_{x\to 3} f(x) = \infty$.

18. Tables will vary.

(a)

x	$f(x)$
2.9	-0.632
2.99	-0.6032
2.999	-0.60032

It seems $\lim_{x\to 3^-} f(x) = -0.6$.

(b)

x	$f(x)$
3.1	-0.5686
3.01	-0.5968
3.001	-0.59968

It seems $\lim_{x\to 3^+} f(x) = -0.6$.

(c) It seems $\lim_{x\to 3} f(x) = -0.6$.

19. Horizontal asymptote at $y = 2$; vertical asymptotes at $x = -5, 4$.
20. Horizontal asymptote at $y = -3/5$; vertical asymptote at $x = 3$.
21. Horizontal asymptote at $y = 0$; vertical asymptotes at $x = -1, 0, 3$.
22. No horizontal asymptote; vertical asymptote at $x = 1$.
23. No horizontal or vertical asymptotes.
24. Horizontal asymptote at $y = -1$; no vertical asymptotes
25. ∞

26. $-\infty$

27. $-\infty$

28. ∞

29. Solution omitted.

30. (a) 2

    (b) -3

    (c) -3

    (d) $1/3$

31. Yes. The only "questionable" place is at $x = 3$, but the left and right limits agree.

32. 1

Chapter 2

Section 2.1

1. T

2. T

3. Answers will vary.

4. Answers will vary.

5. Answers will vary.

6. $f'(x) = 0$

7. $f'(x) = 2$

8. $h'(t) = -3$

9. $g'(x) = 2x$

10. $f''(x) = 6x - 1$

11. $h'(x) = \frac{-1}{x^2}$

12. $r'(s) = \frac{-1}{(s-2)^2}$

13. (a) $y = 6$

    (b) $x = -2$

14. (a) $y = 2x$

    (b) $y = -1/2x$

15. (a) $y = -3x + 4$

    (b) $y = 1/3x + 4$

16. (a) $y = 4(x - 2) + 4$

    (b) $y = -1/4(x - 2) + 4$

17. (a) $y = -7(x + 1) + 8$

    (b) $y = 1/7(x + 1) + 8$

18. (a) $y = -1/4(x + 2) - 1/2$

    (b) $y = 4(x + 2) - 1/2$

19. (a) $y = -1(x - 3) + 1$

    (b) $y = 1(x - 3) + 1$

20. $y = 8.1(x - 3) + 16$

21. $y = -0.99(x - 9) + 1$

22. $y = 7.77(x - 2) + e^2$, or $y = 7.77(x - 2) + 7.39$.

23. $y = -0.05x + 1$

24. (a) Approximations will vary; they should match (c) closely.

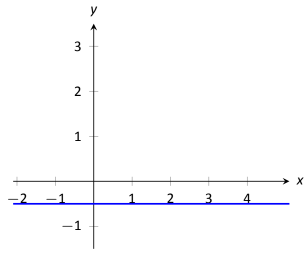
    (b) $f'(x) = 2x$

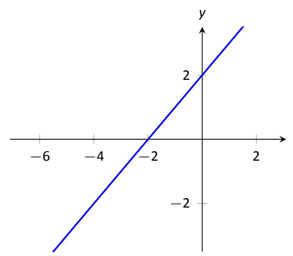
    (c) At $(-1, 0)$, slope is -2. At $(0, -1)$, slope is 0. At $(2, 3)$, slope is 4.

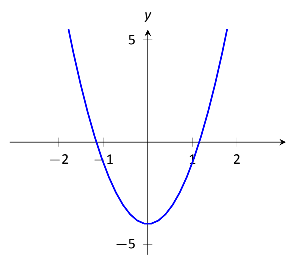
25. (a) Approximations will vary; they should match (c) closely.

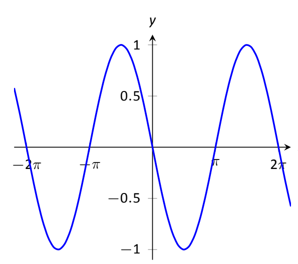
    (b) $f'(x) = -1/(x + 1)^2$

    (c) At $(0, 1)$, slope is -1. At $(1, 0.5)$, slope is $-1/4$.

26. 

27. 

28. 

29. 

30. (a) Approximately on $(-1.5, 1.5)$.

    (b) Approximately on $(-\infty, -1.5) \cup (1.5, \infty)$.

    (c) Approximately at $x = \pm 1.5$.

    (d) On $(-\infty, -1) \cup (0, 1)$.

    (e) On $(-1, 0) \cup (1, \infty)$.

    (f) At $x = \pm 1$.

31. Approximately 24.

32. Approximately 0.54.

33. (a) $(-\infty, \infty)$

    (b) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

    (c) $(-\infty, 5]$

    (d) $[-5, 5]$

34. (a) 1

    (b) 3

    (c) Does not exist

    (d) $(-\infty, -3) \cup (3, \infty)$

Section 2.2

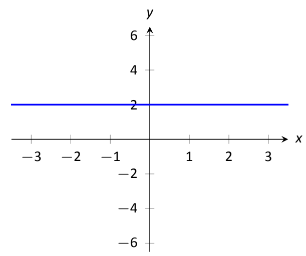
1. Velocity
2. Answers will vary.
3. Linear functions.
4. 12
5. -17
6. 102
7. $f(10.1)$ is likely most accurate, as accuracy is lost the farther from $x = 10$ we go.
8. -4
9. 6
10. decibels per person
11. ft/s^2
12. ft/h
13. (a) thousands of dollars per car
    (b) It is likely that $P(0) < 0$. That is, negative profit for not producing any cars.
14. (a) degrees Fahrenheit per hour
    (b) It is likely that $T'(8) > 0$ since at 8 in the morning, the temperature is likely rising.
    (c) It is very likely that $T(8) > 0$, as at 8 in the morning on July 4, we would expect the temperature to be well above 0.
15. $f(x) = g'(x)$
16. $g(x) = f'(x)$
17. Either $g(x) = f'(x)$ or $f(x) = g'(x)$ is acceptable. The actual answer is $g(x) = f'(x)$, but is very hard to show that $f(x) \neq g'(x)$ given the level of detail given in the graph.
18. $g(x) = f'(x)$
19. $f'(x) = 10x$
20. $f'(x) = 3x^2 - 12x + 12$
21. $f'(\pi) \approx 0$.
22. $f'(9) \approx 0.1667$.

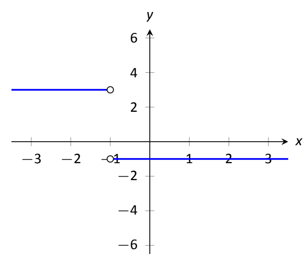
Section 2.3

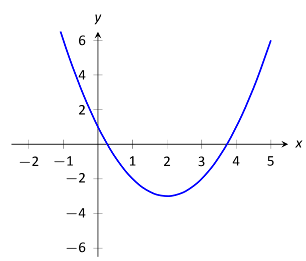
1. Power Rule.
2. $1/x$
3. One answer is $f(x) = 10e^x$.
4. One answer is $f(x) = 10$.
5. $g(x)$ and $h(x)$
6. Answers will vary.
7. One possible answer is $f(x) = 17x - 205$.
8. Answers will vary.
9. $f'(x)$ is a velocity function, and $f''(x)$ is acceleration.
10. lbs/ft^2.
11. $f'(x) = 14x - 5$
12. $g'(x) = 42x^2 + 14x + 11$
13. $m'(t) = 45t^4 - \frac{3}{8}t^2 + 3$
14. $f'(\theta) = 9\cos\theta - 10\sin\theta$
15. $f'(r) = 6e^r$
16. $g'(t) = 40t^3 + \sin t + 7\cos t$
17. $f'(x) = \frac{2}{x} - 1$
18. $p'(s) = s^3 + s^2 + s + 1$
19. $h'(t) = e^t - \cos t + \sin t$
20. $f'(x) = \frac{2}{x}$
21. $f'(x) = 0$
22. $g'(t) = 18t + 6$
23. $g'(x) = 24x^2 - 120x + 150$
24. $f'(x) = -3x^2 + 6x - 3$
25. $f'(x) = 18x - 12$
26. $f'(x) = 6x^5$ $f''(x) = 30x^4$ $f'''(x) = 120x^3$ $f^{(4)}(x) = 360x^2$
27. $g'(x) = -2\sin x$ $g''(x) = -2\cos x$ $g'''(x) = 2\sin x$ $g^{(4)}(x) = 2\cos x$
28. $h'(t) = 2t - e^t$ $h''(t) = 2 - e^t$ $h'''(t) = -e^t$ $h^{(4)}(t) = -e^t$
29. $p'(\theta) = 4\theta^3 - 3\theta^2$ $p''(\theta) = 12\theta^2 - 6\theta$ $p'''(\theta) = 24\theta - 6$ $p^{(4)}(\theta) = 24$
30. $f'(\theta) = \cos\theta + \sin\theta$ $f''(\theta) = -\sin\theta + \cos\theta$ $f'''(\theta) = -\cos\theta - \sin\theta$ $f^{(4)}(\theta) = \sin\theta - \cos\theta$
31. $f'(x) = f''(x) = f'''(x) = f^{(4)}(x) = 0$
32. Tangent line: $y = 2(x - 1)$
    Normal line: $y = -1/2(x - 1)$
33. Tangent line: $y = t + 4$
    Normal line: $y = -t + 4$
34. Tangent line: $y = x - 1$
    Normal line: $y = -x + 1$
35. Tangent line: $y = 4$
    Normal line: $x = \pi/2$
36. Tangent line: $y = \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) - \sqrt{2}$
    Normal line: $y = \frac{-2}{\sqrt{2}}(x - \frac{\pi}{4}) - \sqrt{2}$
37. Tangent line: $y = 2x + 3$
    Normal line: $y = -1/2(x - 5) + 13$
38. The tangent line to $f(x) = e^x$ at $x = 0$ is $y = x + 1$; thus $e^{0.1} \approx y(0.1) = 1.1$.
39. The tangent line to $f(x) = x^4$ at $x = 3$ is $y = 108(x - 3) + 81$; thus $(3.01)^4 \approx y(3.01) = 108(.01) + 81 = 82.08$.

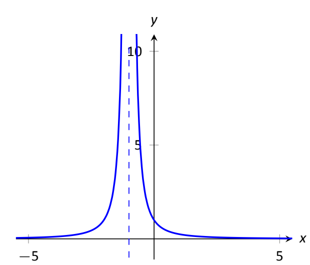
Section 2.4

1. F
2. F
3. T
4. Quotient Rule
5. F
6. Answers will vary.
7. (a) $f'(x) = (x^2 + 3x) + x(2x + 3)$
   (b) $f'(x) = 3x^2 + 6x$
   (c) They are equal.
8. (a) $g'(x) = 4x(5x^3) + 2x^2(15x^2)$
   (b) $g'(x) = 50x^4$
   (c) They are equal.

9. (a) $h'(s) = 2(s+4) + (2s-1)(1)$
   (b) $h'(s) = 4s + 7$
   (c) They are equal.
10. (a) $f'(x) = 2x(3-x^3) + (x^2+5)(-3x^2)$
    (b) $f'(x) = -5x^4 - 15x^2 + 6x$
    (c) They are equal.
11. (a) $f'(x) = \frac{x(2x)-(x^2+3)1}{x^2}$
    (b) $f'(x) = 1 - \frac{3}{x^2}$
    (c) They are equal.
12. (a) $g'(x) = \frac{2x^2(3x^2-4x)-(x^3-2x^2)(4x)}{4x^4}$
    (b) $g'(x) = 1/2$
    (c) They are equal.
13. (a) $h'(s) = \frac{4s^3(0)-3(12s^2)}{16s^6}$
    (b) $h'(s) = -9/4s^{-4}$
    (c) They are equal.
14. (a) $f'(t) = \frac{(t+1)(2t)-(t^2-1)(1)}{(t+1)^2}$
    (b) $f(t) = t-1$ when $t \neq -1$, so $f'(t) = 1$.
    (c) They are equal.
15. (a) $f'(x) = \frac{(x+2)(4x^3+6x^2)-(x^4+2x^3)(1)}{(x+2)^2}$
    (b) $f(x) = x^3$ when $x \neq -2$, so $f'(x) = 3x^2$.
    (c) They are equal.
16. $f'(x) = \sin x + x\cos x$
17. $f'(t) = \frac{-2}{t^3}(\csc t - 4) + \frac{1}{t^2}(-\csc t \cot t)$
18. $g'(x) = \frac{\sqrt{x}(1)-(x+7)(1/2x^{-1/2})}{x} = \frac{1}{2\sqrt{x}} - \frac{7}{2\sqrt{x^3}}$
19. $g'(t) = \frac{(\cos t - 2t^2)(5t^4)-(t^5)(-\sin t - 4t)}{(\cos t - 2t^2)^2}$
20. $h'(x) = -\csc^2 x - e^x$
21. $h'(t) = 14t + 6$
22. $f'(x) = 7$
23. $f'(t) = \frac{1}{5}x^{-4/5}(\sec t + e^t) + \sqrt[5]{t}(\sec t \tan t + e^t)$
24. $f'(x) = \frac{\sin^2(x)+\cos^2(x)+3\cos(x)}{(\cos(x)+3)^2}$
25. $g'(x) = 0$
26. $g'(t) = 12t^2e^t + 4t^3e^t - \cos^2 t + \sin^2 t$
27. $f'(x) = \frac{(3^t+2)(\ln 22^t)-(2^t+3)(\ln 3)}{(3^t+2)^2}$
28. $f'(x) = 2xe^x \tan x = x^2e^x \tan x + x^2e^x \sec^2 x$
29. $g'(x) = 2\sin x \sec x + 2x\cos x \sec x + 2x \sin x \sec x \tan x = 2\tan x + 2x + 2x\tan^2 x = 2\tan x + 2x\sec^2 x$
30. Tangent line: $y = 2x + 2$
    Normal line: $y = -1/2x + 2$
31. Tangent line: $y = -(x - \frac{3\pi}{2}) - \frac{3\pi}{2} = -x$
    Normal line: $y = (x - \frac{3\pi}{2}) - \frac{3\pi}{2} = -x$
32. Tangent line: $y = 4$
    Normal line: $x = 2$
33. Tangent line: $y = -9x - 5$
    Normal line: $y = 1/9x - 5$
34. $x = 3/2$
35. $x = 0$
36. $f'(x)$ is never 0.
37. $x = -2, 0$
38. $f''(x) = 2\cos x - x\sin x$
39. $f^{(4)}(x) = -4\cos x + x\sin x$
40. $f''(x) = \cot^2 x \csc x + \csc^3 x$
41. $f^{(8)} = 0$
42. 

43. 

44. 

45. 

Section 2.5

1. T
2. F
3. F
4. T
5. T
6. $f'(x) = 10(4x^3-x)^9 \cdot (12x^2-1) = (120x^2-10)(4x^3-x)^9$
7. $f'(t) = 15(3t-2)^4$
8. $g'(\theta) = 3(\sin\theta + \cos\theta)^2(\cos\theta - \sin\theta)$
9. $h'(t) = (6t+1)e^{3t^2+t-1}$

10. $f'(x) = 4\left(x + \frac{1}{x}\right)^3 \left(1 - \frac{1}{x^2}\right)$

11. $f'(x) = -3\sin(3x)$

12. $g'(x) = 5\sec^2(5x)$

13. $h'(t) = 8\sin^3(2t)\cos(2t)$

14. $p'(t) = -3\cos^2(t^2 + 3t + 1)\sin(t^2 + 3t + 1)(2t + 3)$

15. $f'(x) = -\tan x$

16. $f'(x) = 2/x$

17. $f'(x) = 2/x$

18. $g'(r) = \ln 4 \cdot 4^r$

19. $g'(t) = -\ln 5 \cdot 5^{\cos t}\sin t$

20. $g'(t) = 0$

21. $m'(w) = \ln(3/2)(3/2)^w$

22. $m'(w) = \frac{2^w\left(\ln 3 \cdot 3^w - \ln 2 \cdot (3^w + 1)\right)}{2^{2w}}$

23. $f'(x) = \frac{2^{x^2}(\ln 3 \cdot 3^x x^2 2x + 1) - (3^{x^2} + x)(\ln 2 \cdot 2^{x^2} 2x)}{2^{2x^2}}$

24. $f'(x) = 5x^2\cos(5x) + 2x\sin(5x)$

25. $g'(t) = 5\cos(t^2+3t)\cos(5t-7) - (2t+3)\sin(t^2+3t)\sin(5t-7)$

26. $g'(t) = 10t\cos\left(\frac{1}{t}\right)e^{5t^2} + \frac{1}{t^2}\sin\left(\frac{1}{t}\right)e^{5t^2}$

27. Tangent line: $y = 0$
    Normal line: $x = 0$

28. Tangent line: $y = 15(t - 1) + 1$
    Normal line: $y = -1/15(t - 1) + 1$

29. Tangent line: $y = -3(\theta - \pi/2) + 1$
    Normal line: $y = 1/3(\theta - \pi/2) + 1$

30. Tangent line: $y = -5e(t + 1) + e$
    Normal line: $y = 1/(5e)(t + 1) + e$

31. In both cases the derivative is the same: $1/x$.

32. In both cases the derivative is the same: k/x.

33. (a) $^\circ$ F/mph
    (b) The sign would be negative; when the wind is blowing at 10 mph, any increase in wind speed will make it feel colder, i.e., a lower number on the Fahrenheit scale.

34. (a) $2xe^x\cot x + x^2e^x\cot x - x^2e^x\csc^2 x$
    (b) $\ln(48)48^x$

Section 2.6

1. Answers will vary.

2. The Chain Rule.

3. T

4. T

5. $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

6. $f'(t) = \frac{-t}{\sqrt{1-t^2}}$

7. $g'(t) = \sqrt{t}\cos t + \frac{\sin t}{2\sqrt{t}}$

8. $h'(x) = 1.5x^{0.5} = 1.5\sqrt{x}$

9. $\frac{dy}{dx} = \frac{-4x^3}{2y+1}$

10. $\frac{dy}{dx} = -\frac{y^{3/5}}{x^{3/5}}$

11. $\frac{dy}{dx} = \sin(x)\sec(y)$

12. $\frac{dy}{dx} = \frac{y}{x}$

13. $\frac{dy}{dx} = \frac{y}{x}$

14. $-\frac{e^x x(x+2)2^{-y}}{\ln|2|}$

15. $-\frac{2\sin(y)\cos(y)}{x}$

16. $-\frac{x}{y^2}$

17. $\frac{1}{2y+2}$

18. $\frac{x^2+2xy^2-y}{2x^2y-x+y^2}$

19. $\frac{-\cos(x)(x+\cos(y))+\sin(x)+y}{\sin(y)(\sin(x)+y)+x+\cos(y)}$

20. $-\frac{x}{y}$

21. $-\frac{2x+y}{2y+x}$

22. In each, $\frac{dy}{dx} = -\frac{y}{x}$.

23. (a) $y = 0$
    (b) $y = -1.859(x - 0.1) + 0.281$

24. (a) $x = 1$
    (b) $y = -\frac{3\sqrt{3}}{8}(x - \sqrt{.6}) + \sqrt{.8} \approx -0.65(x - 0.775) + 0.894$
    (c) $y = 1$

25. (a) $y = 4$
    (b) $y = 0.93(x - 2) + \sqrt[4]{108}$

26. (a) $y = -1/3x + 1$
    (b) $y = 3\sqrt{3}/4$

27. (a) $y = -\frac{1}{\sqrt{3}}\left(x - \frac{7}{2}\right) + \frac{6+3\sqrt{3}}{2}$
    (b) $y = \sqrt{3}\left(x - \frac{4+3\sqrt{3}}{2}\right) + \frac{3}{2}$

28. $\frac{d^2y}{dx^2} = \frac{(2y+1)(-12x^2) + 4x^3\left(2\frac{-4x^3}{2y+1}\right)}{(2y+1)^2}$

29. $\frac{d^2y}{dx^2} = \frac{3}{5}\frac{y^{3/5}}{x^{8/5}} + \frac{3}{5}\frac{1}{yx^{6/5}}$

30. $\frac{d^2y}{dx^2} = \frac{\cos x\cos y + \sin^2 x\tan y}{\cos^2 y}$

31. $\frac{d^2y}{dx^2} = 0$

32. $y' = (1 + x)^{1/x}\left(\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2}\right)$
    Tangent line: $y = (1 - 2\ln 2)(x - 1) + 2$

33. $y' = (2x)^{x^2}\left(2x\ln(2x) + x\right)$
    Tangent line: $y = (2 + 4\ln 2)(x - 1) + 2$

34. $y' = \frac{x^x}{x+1}\left(\ln x + 1 - \frac{1}{x+1}\right)$
    Tangent line: $y = (1/4)(x - 1) + 1/2$

35. $y' = x^{\sin(x)+2}\left(\cos x\ln x + \frac{\sin x + 2}{x}\right)$
    Tangent line: $y = (3\pi^2/4)(x - \pi/2) + (\pi/2)^3$

36. $y' = \frac{x+1}{x+2}\left(\frac{1}{x+1} - \frac{1}{x+2}\right)$
    Tangent line: $y = 1/9(x - 1) + 2/3$

37. $y' = \frac{(x+1)(x+2)}{(x+3)(x+4)}\left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3} - \frac{1}{x+4}\right)$
    Tangent line: $y = 11/72x + 1/6$

Section 2.7

1. F

2. Answers will vary.
3. The point $(10,1)$ lies on the graph of $y=f^{-1}(x)$ (assuming f is invertible).
4. The point $(10,1)$ lies on the graph of $y=f^{-1}(x)$ (assuming f is invertible) and $(f^{-1})'(10)=1/5$.
5. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
6. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
7. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
8. Compose $f(g(x))$ and $g(f(x))$ to confirm that each equals x.
9. $\left(f^{-1}\right)'(20)=\frac{1}{f'(2)}=1/5$
10. $\left(f^{-1}\right)'(7)=\frac{1}{f'(3)}=1/4$
11. $\left(f^{-1}\right)'(\sqrt{3}/2)=\frac{1}{f'(\pi/6)}=1$
12. $\left(f^{-1}\right)'(8)=\frac{1}{f'(1)}=1/6$
13. $\left(f^{-1}\right)'(1/2)=\frac{1}{f'(1)}=-2$
14. $\left(f^{-1}\right)'(6)=\frac{1}{f'(0)}=1/6$
15. $h'(t)=\frac{2}{\sqrt{1-4t^2}}$
16. $f'(t)=\frac{1}{|t|\sqrt{4t^2+1}}$
17. $g'(x)=\frac{2}{1+4x^2}$
18. $f'(x)=\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$
19. $g'(t)=\cos^{-1}(t)\cos(t)-\frac{\sin(t)}{\sqrt{1-t^2}}$
20. $f'(t)=\frac{e^t}{t}+\ln te^t$
21. $h'(x)=\frac{\sin^{-1}(x)+\cos^{-1}(x)}{\sqrt{1-x^2}\cos^{-1}(x)^2}$
22. $g'(x)=\frac{1}{\sqrt{x}(2x+2)}$
23. $f'(x)=-\frac{1}{\sqrt{1-x^2}}$
24. (a) $f(x)=x$, so $f'(x)=1$
    (b) $f'(x)=\cos(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}}=1.$
25. (a) $f(x)=x$, so $f'(x)=1$
    (b) $f'(x)=\cos(\sin^{-1}x)\frac{1}{\sqrt{1-x^2}}=1.$
26. (a) $f(x)=x$, so $f'(x)=1$
    (b) $f'(x)=\frac{1}{1+\tan^2 x}\sec^2 x=1$
27. (a) $f(x)=\sqrt{1-x^2}$, so $f'(x)=\frac{-x}{\sqrt{1-x^2}}$
    (b) $f'(x)=\cos(\cos^{-1}x)(\frac{1}{\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}$
28. $y=\sqrt{2}(x-\sqrt{2}/2)+\pi/4$
29. $y=-4(x-\sqrt{3}/4)+\pi/6$
30. $\frac{dy}{dx}=\frac{y(y-2x)}{x(x-2y)}$
31. $y=-4/5(x-1)+2$
32. $3x^2+1$

Chapter 3

Section 3.1

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. F
6. A: none B: abs. max C: rel. min D: none E: none F: rel. min G: none
7. A: abs. min B: none C: abs. max D: none E: none
8. $f'(0)=0$
9. $f'(0)=0\, f'(2)=0$
10. $f'(\pi/2)=0\, f'(3\pi/2)=0$
11. $f'(0)=0\, f'(3.2)=0\, f'(4)$ is undefined
12. $f'(0)=0$
13. $f'(0)$ is not defined
14. $f'(2)$ is not defined $f'(6)=0$
15. min: $(-0.5, 3.75)$
    max: $(2, 10)$
16. min: $(5, -134.5)$
    max: $(0, 3)$
17. min: $(\pi/4, 3\sqrt{2}/2)$
    max: $(\pi/2, 3)$
18. min: $(0,0)$ and $(\pm 2, 0)$
    max: $(\pm 2\sqrt{2/3}, 16\sqrt{3}/9)$
19. min: $(\sqrt{3}, 2\sqrt{3})$
    max: $(5, 28/5)$
20. min: $(0,0)$
    max: $(5, 5/6)$
21. min: $(\pi, -e^{\pi})$
    max: $(\pi/4, \frac{\sqrt{2}e^{\pi/4}}{2})$
22. min: $(0,0)$ and $(\pi, 0)$
    max: $(3\pi/4, \frac{\sqrt{2}e^{3\pi/4}}{2})$
23. min: $(1,0)$
    max: $(e, 1/e)$
24. min: $(2, 2^{2/3}-2)$
    max: $(8/27, 4/27)$
25. $\frac{dy}{dx}=\frac{y(y-2x)}{x(x-2y)}$
26. $y=-4/5(x-1)+2$
27. $3x^2+1$

Section 3.2

1. Answers will vary.
2. Answers will vary.
3. Any c in $[-1, 1]$ is valid.
4. Rolle's Thm. does not apply.
5. $c=-1/2$
6. $c=-1/2$
7. Rolle's Thm. does not apply.
8. $c=\pi/2$
9. Rolle's Thm. does not apply.

10. Rolle's Theorem does not apply.
11. $c = 0$
12. $c = 5/2$
13. $c = 3/\sqrt{2}$
14. $c = 19/4$
15. The Mean Value Theorem does not apply.
16. $c = 4/\ln 5$
17. $c = \pm\sec^{-1}(2/\sqrt{\pi})$
18. $c = -2/3$
19. $c = \frac{5\pm 7\sqrt{7}}{6}$
20. $c = \frac{\pm\sqrt{\pi^2-4}}{\pi}$
21. Max value of 19 at $x = -2$ and $x = 5$; min value of 6.75 at $x = 1.5$.
22. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)
23. They are the odd, integer valued multiples of $\pi/2$ (such as $0, \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc.)

Section 3.3

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. Increasing
6. Graph and verify.
7. Graph and verify.
8. Graph and verify.
9. Graph and verify.
10. Graph and verify.
11. Graph and verify.
12. Graph and verify.
13. Graph and verify.
14. c.p. at $c = -1$; decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$; rel. min at $x = -1$.
15. c.p. at $c = -2, 0$; increasing on $(-\infty, -2) \cup (0, \infty)$; decreasing on $(-2, 0)$; rel. min at $x = 0$; rel. max at $x = -2$.
16. c.p. at $c = \frac{1}{6}(-1 \pm \sqrt{7})$; decreasing on $(\frac{1}{6}(-1-\sqrt{7}), \frac{1}{6}(-1+\sqrt{7})))$; increasing on $(-\infty, \frac{1}{6}(-1-\sqrt{7})) \cup (\frac{1}{6}(-1+\sqrt{7}), \infty)$; rel. min at $x = \frac{1}{6}(-1+\sqrt{7})$; rel. max at $x = \frac{1}{6}(-1-\sqrt{7})$.
17. c.p. at $c = 1$; increasing on $(-\infty, \infty)$;
18. c.p. at $c = 1$; decreasing on $(1, \infty)$ increasing on $(-\infty, 1)$; rel. max at $x = 1$.
19. c.p. at $c = -1, 0, 1$; decreasing on $(-\infty, -1) \cup (-1, 0)$ increasing on $(0, 1) \cup (1, \infty)$; rel. min at $x = 0$;
20. c.p. at $c = -2, 4$; decreasing on $(-\infty, \infty)$ except $x = -2, 4$
21. c.p. at $c = 2, 6, 0$; decreasing on $(-\infty, 0) \cup (0, 2)$; increasing on $(2, \infty)$; rel. min at $x = 2$;
22. c.p. at $c = -3\pi/4, -\pi.4, \pi/4, 3\pi/4$; decreasing on $(-3\pi/4, -\pi/4) \cup (\pi/4, 3\pi/4)$; increasing on $(-\pi, -3\pi/4) \cup (-\pi/4, \pi/4) \cup (3\pi/4, \pi)$; rel. min at $x = -\pi/4, 3\pi/4$; rel. max at $x = -3\pi/4, \pi/4$.
23. c.p. at $c = -1, 1$ decreasing on $(-1, 1)$ increasing on $(-\infty, -1) \cup (1, \infty)$; rel. min at $x = 1$; rel. max at $x = -1$
24. $c = 1/2$
25. $c = \pm\cos^{-1}(2/\pi)$

Section 3.4

1. Answers will vary.
2. Answers will vary.
3. Yes; Answers will vary.
4. No.
5. Graph and verify.
6. Graph and verify.
7. Graph and verify.
8. Graph and verify.
9. Graph and verify.
10. Graph and verify.
11. Graph and verify.
12. Graph and verify.
13. Graph and verify.
14. Graph and verify.
15. Graph and verify.
16. Possible points of inflection: none; concave up on $(-\infty, \infty)$
17. Possible points of inflection: none; concave down on $(-\infty, \infty)$
18. Possible points of inflection: $x = 0$; concave down on $(-\infty, 0)$; concave up on $(0, \infty)$
19. Possible points of inflection: $x = 1/2$; concave down on $(-\infty, 1/2)$; concave up on $(1/2, \infty)$
20. Possible points of inflection: $x = -2/3, 0$; concave down on $(-2/3, 0)$; concave up on $(-\infty, -2/3) \cup (0, \infty)$
21. Possible points of inflection: $x = (1/3)(2 \pm \sqrt{7})$; concave up on $((1/3)(2-\sqrt{7}), (1/3)(2+\sqrt{7}))$; concave down on $(-\infty, (1/3)(2-\sqrt{7})) \cup ((1/3)(2+\sqrt{7}), \infty)$
22. Possible points of inflection: $x = 1$; concave up on $(-\infty, \infty)$
23. Possible points of inflection: $x = \pm 1/\sqrt{3}$; concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$; concave up on $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$
24. Possible points of inflection: $x = 0, \pm 1$; concave down on $(-\infty, -1) \cup (0, 1)$ concave up on $(-1, 0) \cup (1, \infty)$
25. Possible points of inflection: $x = -\pi/4, 3\pi/4$; concave down on $(-\pi/4, 3\pi/4)$ concave up on $(-\pi, -\pi/4) \cup (3\pi/4, \pi)$
26. Possible points of inflection: $x = -2 \pm \sqrt{2}$; concave down on $(-2-\sqrt{2}, -2+\sqrt{2})$ concave up on $(-\infty, -2-\sqrt{2}) \cup (-2+\sqrt{2}, \infty)$
27. Possible points of inflection: $x = 1/e^{3/2}$; concave down on $(0, 1/e^{3/2})$ concave up on $(1/e^{3/2}, \infty)$
28. Possible points of inflection: $x = \pm 1/\sqrt{2}$; concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$ concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$
29. min: $x = 1$
30. max: $x = -5/2$
31. max: $x = -1/\sqrt{3}$ min: $x = 1/\sqrt{3}$
32.

33. min: $x = 1$
34. max: $x = -1, 2$; min: $x = 1$
35. min: $x = 1$
36. max: $x = 0$
37. critical values: $x = -1, 1$; no max/min
38. max: $x = \pi/4$; min: $x = -3\pi/4$
39. max: $x = -2$; min: $x = 0$
40. min: $x = 1/\sqrt{e}$
41. max: $x = 0$
42. f' has no maximal or minimal value.
43. f' has no maximal or minimal value
44. f' has a minimal value at $x = 0$
45. f' has a minimal value at $x = 1/2$
46. Possible points of inflection: $x = -2/3, 0$; f' has a relative min at: $x = 0$; relative max at: $x = -2/3$
47. f' has a relative max at: $x = (1/3)(2 + \sqrt{7})$ relative min at: $x = (1/3)(2 - \sqrt{7})$
48. f' has no relative extrema
49. f' has a relative max at $x = -1/\sqrt{3}$; relative min at $x = 1/\sqrt{3}$
50. f' has a relative max at $x = 0$
51. f' has a relative min at $x = 3\pi/4$; relative max at $x = -\pi/4$
52. f' has a relative max at $x = -2 - \sqrt{2}$; relative min at $x = -2 + \sqrt{2}$
53. f' has a relative min at $x = 1/\sqrt{e^3} = e^{-3/2}$
54. f' has a relative max at $x = -1/\sqrt{2}$; a relative min at $x = 1/\sqrt{2}$

Section 3.5

1. Answers will vary.
2. Found everywhere.
3. T
4. T
5. T
6. A good sketch will include the x and y intercepts and draw the appropriate line.
7. A good sketch will include the x and y intercepts..
8. Use technology to verify sketch.
9. Use technology to verify sketch.
10. Use technology to verify sketch.
11. Use technology to verify sketch.
12. Use technology to verify sketch.
13. Use technology to verify sketch.
14. Use technology to verify sketch.
15. Use technology to verify sketch.
16. Use technology to verify sketch.
17. Use technology to verify sketch.
18. Use technology to verify sketch.
19. Use technology to verify sketch.
20. Use technology to verify sketch.
21. Use technology to verify sketch.
22. Use technology to verify sketch.
23. Use technology to verify sketch.
24. Use technology to verify sketch.
25. Use technology to verify sketch.
26. Critical point: $x = 0$ Points of inflection: $\pm b/\sqrt{3}$
27. Critical points: $x = \frac{n\pi/2 - b}{a}$, where n is an odd integer Points of inflection: $(n\pi - b)/a$, where n is an integer.
28. Critical point: $x = (a + b)/2$ Points of inflection: none
29. $\frac{dy}{dx} = -x/y$, so the function is increasing in second and fourth quadrants, decreasing in the first and third quadrants.
    $\frac{d^2y}{dx^2} = -1/y - x^2/y^3$, which is positive when $y < 0$ and is negative when $y > 0$. Hence the function is concave down in the first and second quadrants and concave up in the third and fourth quadrants.

Chapter 4

Section 4.1

1. F
2. F
3. $x_0 = 1.5$, $x_1 = 1.5709148$, $x_2 = 1.5707963$, $x_3 = 1.5707963$, $x_4 = 1.5707963$, $x_5 = 1.5707963$
4. $x_0 = 1$, $x_1 = -0.55740772$, $x_2 = 0.065936452$, $x_3 = -0.000095721919$, $x_4 = 2.9235662 * 10^{-13}$, $x_5 = 0$
5. $x_0 = 0$, $x_1 = 2$, $x_2 = 1.2$, $x_3 = 1.0117647$, $x_4 = 1.0000458$, $x_5 = 1$
6. $x_0 = 1.5$, $x_1 = 1.4166667$, $x_2 = 1.4142157$, $x_3 = 1.4142136$, $x_4 = 1.4142136$, $x_5 = 1.4142136$
7. $x_0 = 2$, $x_1 = 0.6137056389$, $x_2 = 0.9133412072$, $x_3 = 0.9961317034$, $x_4 = 0.9999925085$, $x_5 = 1$
8. roots are: $x = -5.156$, $x = -0.369$ and $x = 0.525$
9. roots are: $x = -3.714$, $x = -0.857$, $x = 1$ and $x = 1.571$
10. roots are: $x = -1.013$, $x = 0.988$, and $x = 1.393$
11. roots are: $x = -2.165$, $x = 0$, $x = 0.525$ and $x = 1.813$
12. $x = \pm 0.824$,
13. $x = -0.637$, $x = 1.410$
14. $x = \pm 0.743$
15. $x = \pm 4.493$, $x = 0$
16. The approximations alternate between $x = 1$ and $x = 2$.
17. The approximations alternate between $x = 1$, $x = 2$ and $x = 3$.

Section 4.2

1. T
2. F
3. (a) $5/(2\pi) \approx 0.796$cm/s
   (b) $1/(4\pi) \approx 0.0796$ cm/s
   (c) $1/(40\pi) \approx 0.00796$ cm/s
4. (a) $5/(2\pi) \approx 0.796$cm/s
   (b) $1/(40\pi) \approx 0.00796$ cm/s
   (c) $1/(4000\pi) \approx 0.0000796$ cm/s

5. 49mph
6. 53mph
7. Due to the height of the plane, the gun does not have to rotate very fast.
   (a) 0.0573 rad/s
   (b) 0.0725 rad/s
   (c) In the limit, rate goes to 0.0733 rad/s
8. Due to the height of the plane, the gun does not have to rotate very fast.
   (a) 0.073 rad/s
   (b) 3.66 rad/s (about 1/2 revolution/sec)
   (c) In the limit, rate goes to 7.33 rad/s (more than 1 revolution/sec)
9. (a) 0.04 ft/s
   (b) 0.458 ft/s
   (c) 3.35 ft/s
   (d) Not defined; as the distance approaches 24, the rates approaches ∞.
10. (a) 30.59 ft/min
    (b) 36.1 ft/min
    (c) 301 ft/min
    (d) The boat no longer floats as usual, but is being pulled up by the winch (assuming it has the power to do so).
11. (a) 50.92 ft/min
    (b) 0.509 ft/min
    (c) 0.141 ft/min
    As the tank holds about 523.6ft^3, it will take about 52.36 minutes.
12. (a) 0.63 ft/sec
    (b) 1.6 ft/sec
    About 52 ft.
13. (a) The rope is 80ft long.
    (b) 1.71 ft/sec
    (c) 1.87 ft/sec
    (d) About 34 feet.
14. (a) The balloon is 105ft in the air.
    (b) The balloon is rising at a rate of 17.45ft/min. (Hint: convert all angles to radians.)
15. The cone is rising at a rate of 0.003ft/s.

Section 4.3

1. T
2. F
3. 2500; the two numbers are each 50.
4. The minimum sum is $2\sqrt{500}$; the two numbers are each $\sqrt{500}$.
5. There is no maximum sum; the fundamental equation has only 1 critical value that corresponds to a minimum.
6. The only critical point of the fundamental equation corresponds to a minimum; to find maximum, we check the endpoints.
   If one number is 300, the other number y satisfies $300y = 500$; $y = 5/3$. Thus the sum is $300 + 5/3$.
   The other endpoint, 0, is not feasible as we cannot solve $0 \cdot y = 500$ for y. In fact, if $0 < x < 5/3$, then $x \cdot y = 500$ forces $y > 300$, which is not a feasible solution.
   Hence the maximum sum is $301.\overline{6}$.
7. Area = 1/4, with sides of length $1/\sqrt{2}$.
8. Each pen should be $500/3 \approx 166.67$ feet by 125 feet.
9. The radius should be about 3.84cm and the height should be $2r = 7.67$cm. No, this is not the size of the standard can.
10. The radius should be about 3.2in and the height should be $2r = 6.4$in. As the #10 is not a perfect cylinder (with extra material to aid in stacking, etc.), the dimensions are close enough to assume that minimizing surface area was a consideration.
11. The height and width should be 18 and the length should be 36, giving a volume of $11,664$in^3.
12. $w = 4\sqrt{3}$, $h = 4\sqrt{6}$
13. $5 - 10/\sqrt{39} \approx 3.4$ miles should be run underground, giving a minimum cost of \$374,899.96.
14. The power line should be run directly to the off shore facility, skipping any underground, giving a cost of about \$430,813.
15. The dog should run about 19 feet along the shore before starting to swim.
16. The dog should run about 13 feet along the shore before starting to swim.
17. The largest area is 2 formed by a square with sides of length $\sqrt{2}$.

Section 4.4

1. T
2. T
3. F
4. T
5. Answers will vary.
6. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 2$ and $dx = 0.05$. Thus $dy = .2$; knowing $2^2 = 4$, we have $2.05^2 \approx 4.2$.
7. Use $y = x^2$; $dy = 2x \cdot dx$ with $x = 6$ and $dx = -0.07$. Thus $dy = -0.84$; knowing $6^2 = 36$, we have $5.93^2 \approx 35.16$.
8. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 5$ and $dx = 0.1$. Thus $dy = 7.5$; knowing $5^3 = 125$, we have $5.1^3 \approx 132.5$.
9. Use $y = x^3$; $dy = 3x^2 \cdot dx$ with $x = 7$ and $dx = -0.2$. Thus $dy = -29.4$; knowing $7^3 = 343$, we have $6.8^3 \approx 313.6$.
10. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 16$ and $dx = 0.5$. Thus $dy = .0625$; knowing $\sqrt{16} = 4$, we have $\sqrt{16.5} \approx 4.0625$.
11. Use $y = \sqrt{x}$; $dy = 1/(2\sqrt{x}) \cdot dx$ with $x = 25$ and $dx = -1$. Thus $dy = -0.1$; knowing $\sqrt{25} = 5$, we have $\sqrt{24} \approx 4.9$.
12. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 64$ and $dx = -1$. Thus $dy = -1/48 \approx 0.0208$; we could use $-1/48 \approx -1/50 = -0.02$; knowing $\sqrt[3]{64} = 4$, we have $\sqrt[3]{63} \approx 3.98$.
13. Use $y = \sqrt[3]{x}$; $dy = 1/(3\sqrt[3]{x^2}) \cdot dx$ with $x = 8$ and $dx = 0.5$. Thus $dy = 1/24 \approx 1/25 = 0.04$; knowing $\sqrt[3]{8} = 2$, we have $\sqrt[3]{8.5} \approx 2.04$.
14. Use $y = \sin x$; $dy = \cos x \cdot dx$ with $x = \pi$ and $dx \approx -0.14$. Thus $dy = 0.14$; knowing $\sin \pi = 0$, we have $\sin 3 \approx 0.14$.
15. Use $y = \cos x$; $dy = -\sin x \cdot dx$ with $x = \pi/2 \approx 1.57$ and $dx \approx -0.07$. Thus $dy = 0.07$; knowing $\cos \pi/2 = 0$, we have $\cos 1.5 \approx 0.07$.
16. Use $y = e^x$; $dy = e^x \cdot dx$ with $x = 0$ and $dx = 0.1$. Thus $dy = 0.1$; knowing $e^0 = 1$, we have $e^{0.1} \approx 1.1$.

17. $dy = (2x + 3)dx$
18. $dy = (7x^6 - 5x^4)dx$
19. $dy = \frac{-2}{4x^3} dx$
20. $dy = 2(2x + \sin x)(2 + \cos x)dx$
21. $dy = \left(2xe^{3x} + 3x^2 e^{3x}\right) dx$
22. $dy = \frac{-16}{x^5} dx$
23. $dy = \frac{2(\tan x+1) - 2x \sec^2 x}{(\tan x+1)^2} dx$
24. $dy = \frac{1}{x} dx$
25. $dy = (e^x \sin x + e^x \cos x)dx$
26. $dy = (-\sin(\sin x) \cos x)dx$
27. $dy = \frac{1}{(x+2)^2} dx$
28. $dy = ((\ln 3)3^x \ln x + \frac{3^x}{x})dx$
29. $dy = (\ln x)dx$
30. $dV = \pm 0.157$
31. (a) ± 12.8 feet
    (b) ± 32 feet
32. $\pm 15\pi/8 \approx \pm 5.89\text{in}^2$
33. $\pm 48\text{in}^2$, or $1/3\text{ft}^2$
34. (a) 297.8 feet
    (b) ± 62.3 ft
    (c) $\pm 20.9\%$
35. (a) 298.8 feet
    (b) ± 17.3 ft
    (c) $\pm 5.8\%$
36. (a) 298.9 feet
    (b) ± 8.67 ft
    (c) $\pm 2.9\%$
37. The isosceles triangle setup works the best with the smallest percent error.
38. 1%

Chapter 5

Section 5.1

1. Answers will vary.
2. "an"
3. Answers will vary.
4. opposite; opposite
5. Answers will vary.
6. velocity
7. velocity
8. $3/4x^4 + C$
9. $1/9x^9 + C$
10. $10/3x^3 - 2x + C$
11. $t + C$
12. $s + C$
13. $-1/(3t) + C$
14. $-3/(t) + C$
15. $2\sqrt{x} + C$
16. $\tan\theta + C$
17. $-\cos\theta + C$
18. $\sec x - \csc x + C$
19. $5e^\theta + C$
20. $3^t / \ln 3 + C$
21. $\frac{5^t}{2 \ln 5} + C$
22. $4/3t^3 + 6t^2 + 9t + C$
23. $t^6/6 + t^4/4 - 3t^2 + C$
24. $x^6/6 + C$
25. $e^\pi x + C$
26. $tx + C$
27. (a) $x > 0$
    (b) $1/x$
    (c) $x < 0$
    (d) $1/x$
    (e) $\ln |x| + C$. Explanations will vary.
28. $-\cos x + 3$
29. $5e^x + 5$
30. $x^4 - x^3 + 7$
31. $\tan x + 4$
32. $7^x / \ln 7 + 1 - 49/ \ln 7$
33. $5/2x^2 + 7x + 3$
34. $\frac{7x^3}{6} - \frac{9x}{2} + \frac{40}{3}$
35. $5e^x - 2x$
36. $x - \sin(x) - \pi + 4$
37. $\frac{2x^4 \ln^2(2) + 2^x + x \ln 2)(\ln 32 - 1) + \ln^2(2) \cos(x) - 1 - \ln^2(2)}{\ln^2(2)}$
38. $3x - 2$
39. No answer provided.
40. $dy = (2xe^x \cos x + x^2 e^x \cos x - x^2 e^x \sin x)dx$

Section 5.2

1. Answers will vary.
2. Answers will vary.
3. 0
4. $\int 0^2 (2x + 3)\, dx$
5. (a) 3
   (b) 4
   (c) 3
   (d) 0
   (e) -4
   (f) 9
6. (a) -4
   (b) -5
   (c) -3
   (d) 1

(e) -2
(f) 10

7. (a) 4
(b) 2
(c) 4
(d) 2
(e) 1
(f) 2

8. (a) $-1/2$
(b) 0
(c) $3/2$
(d) $3/2$
(e) $9/2$
(f) $15/2$

9. (a) π
(b) π
(c) 2π
(d) 10π

10. (a) -59
(b) -48
(c) -27
(d) -33

11. (a) $4/\pi$
(b) $-4/\pi$
(c) 0
(d) $2/\pi$

12. (a) 4
(b) 4
(c) -4
(d) -2

13. (a) $40/3$
(b) $26/3$
(c) $8/3$
(d) $38/3$

14. (a) 2ft/s
(b) 2ft
(c) 1.5ft

15. (a) 3ft/s
(b) 9.5ft
(c) 9.5ft

16. (a) 64ft/s
(b) 64ft
(c) $t = 2$
(d) $t = 2 + \sqrt{7} \approx 4.65$ seconds

17. (a) 96ft/s
(b) 6 seconds
(c) 6 seconds
(d) Never; the maximum height is 208ft.

18. 2
19. 5
20. 16
21. Answers can vary; one solution is $a = -2$, $b = 7$
22. 24
23. -7
24. -7
25. Answers can vary; one solution is $a = -11$, $b = 18$
26. $1/4x^4 - 2/3x^3 + 7/2x^2 - 9x + C$
27. $-\cos x - \sin x + \tan x + C$
28. $3/4t^{4/3} - 1/t + 2^t/\ln 2 + C$
29. $\ln|x| + \csc x + C$

Section 5.3

1. limits
2. 14
3. Rectangles.
4. T
5. $2^2 + 3^2 + 4^2 = 29$
6. $-6 - 2 + 2 + 6 + 10 = 10$
7. $0 - 1 + 0 + 1 + 0 = 0$
8. $1 + 1/2 + 1/3 + 1/4 + 1/5 = 137/60$
9. $-1 + 2 - 3 + 4 - 5 + 6 = 3$
10. $1/2 + 1/6 + 1/12 + 1/20 = 4/5$
11. $1 + 1 + 1 + 1 + 1 + 1 = 6$
12. Answers may vary; $\sum_{i=1}^{5} 3i$
13. Answers may vary; $\sum_{i=0}^{8} (i^2 - 1)$
14. Answers may vary; $\sum_{i=1}^{4} \frac{i}{i+1}$
15. Answers may vary; $\sum_{i=0}^{4} (-1)^i e^i$
16. 325
17. 1045
18. $28{,}650$
19. -8525
20. 2050
21. 5050
22. 2870
23. 155
24. $91,225$
25. 24
26. $11,700$
27. 19
28. $59/8$
29. $\pi/3 + \pi/(2\sqrt{3}) \approx 1.954$
30. 8.144
31. 0.388584
32. $496/315 \approx 1.5746$
33. (a) Exact expressions will vary; $\frac{(1+n)^2}{4n^2}$.

(b) $121/400, 10201/40000, 1002001/4000000$

(c) $1/4$

34. (a) Exact expressions will vary; $2+4/n^2$.

    (b) $51/25, 5001/2500, 500001/250000$

    (c) 2

35. (a) 8.

    (b) 8, 8, 8

    (c) 8

36. (a) Exact expressions will vary; $20/3-96/(3n)+64/(3n^2)$.

    (b) $92/25, 3968/625, 103667/15625$

    (c) $20/3$

37. (a) Exact expressions will vary; $100-200/n$.

    (b) $80, 98, 499/5$

    (c) 100

38. (a) Exact expressions will vary; $-1/12(1-1/n^2)$.

    (b) $-33/400, -3333/40000, -333333/4000000$

    (c) $-1/12$

39. $F(x)=5\tan x+4$
40. $F(x)=7\ln|x|+14$
41. $G(t)=4/6t^6-5/4t^4+8t+9$
42. $G(t)=5\cdot 8^t/\ln 8+900$
43. $G(t)=\sin t-\cos t-78$
44. $F(x)=2\sqrt{x}-\pi$

Section 5.4

1. Answers will vary.
2. 0
3. T
4. Answers will vary.
5. 20
6. $28/3$
7. 0
8. 1
9. 1
10. 1
11. $(5-1/5)/\ln 5$
12. $23/2$
13. -4
14. e^3-e
15. $16/3$
16. 4
17. $45/4$
18. $\ln 2$
19. $1/2$
20. $3/8$
21. $1/2$
22. $1/3$
23. $1/4$
24. $1/101$
25. 8
26. 15
27. 0
28. $2-2/\sqrt{3}$
29. Explanations will vary. A sketch will help.
30. $c=2/\sqrt{3}$
31. $c=\pm 2/\sqrt{3}$
32. $c=\ln(e-1)\approx 0.54$
33. $c=64/9\approx 7.1$
34. $2/\pi$
35. $2/pi$
36. 2
37. $16/3$
38. 16
39. $1/(e-1)$
40. -300ft
41. 400ft
42. $1.5/\ln(2)\approx 2.164$miles
43. -1ft
44. $128/5$ft
45. -64ft/s
46. 50ft/s
47. 2ft/s
48. 0ft/s
49. $27/2$
50. 21
51. $9/2$
52. $343/6$
53. $F'(x)=(3x^2+1)\frac{1}{x^3+x}$
54. $F'(x)=3x^{11}$
55. $F'(x)=2x(x^2+2)-(x+2)$
56. $F'(x)=e^x\sin(e^x)-1/x\sin(\ln x)$

Section 5.5

1. F
2. When the antiderivative cannot be computed and when the integrand is unknown.
3. They are superseded by the Trapezoidal Rule; it takes an equal amount of work and is generally more accurate.
4. (a) $3/4$

   (b) $2/3$

   (c) $2/3$

5. (a) 250

   (b) 250

   (c) 250

6. (a) $\frac{1}{4}(1+\sqrt{2})\pi\approx 1.896$

(b) $\frac{1}{6}(1+2\sqrt{2})\pi \approx 2.005$

(c) 2

7. (a) $2+\sqrt{2}+\sqrt{3} \approx 5.15$

(b) $2/3(3+\sqrt{2}+2\sqrt{3}) \approx 5.25$

(c) $16/3 \approx 5.33$

8. (a) 38.5781

(b) $147/4 \approx 36.75$

(c) $147/4 \approx 36.75$

9. (a) 0.2207

(b) 0.2005

(c) $1/5$

10. (a) 0

(b) 0

(c) 0

11. (a) $9/2(1+\sqrt{3}) \approx 12.294$

(b) $3+6\sqrt{3} \approx 13.392$

(c) $9\pi/2 \approx 14.137$

12. Trapezoidal Rule: 0.9006
Simpson's Rule: 0.90452

13. Trapezoidal Rule: 3.0241
Simpson's Rule: 2.9315

14. Trapezoidal Rule: 13.9604
Simpson's Rule: 13.9066

15. Trapezoidal Rule: 3.0695
Simpson's Rule: 3.14295

16. Trapezoidal Rule: 1.1703
Simpson's Rule: 1.1873

17. Trapezoidal Rule: 2.52971
Simpson's Rule: 2.5447

18. Trapezoidal Rule: 1.0803
Simpson's Rule: 1.077

19. Trapezoidal Rule: 3.5472
Simpson's Rule: 3.6133

20. (a) $n=161$ (using max $\left(f''(x)\right)=1$)

(b) $n=12$ (using max $\left(f^{(4)}(x)\right)=1$)

21. (a) $n=150$ (using max $\left(f''(x)\right)=1$)

(b) $n=18$ (using max $\left(f^{(4)}(x)\right)=7$)

22. (a) $n=1004$ (using max $\left(f''(x)\right)=39$)

(b) $n=62$ (using max $\left(f^{(4)}(x)\right)=800$)

23. (a) $n=5591$ (using max $\left(f''(x)\right)=300$)

(b) $n=46$ (using max $\left(f^{(4)}(x)\right)=24$)

24. (a) Area is 30.8667 cm^2.

(b) Area is $308,667$ yd^2.

25. (a) Area is 25.0667 cm^2

(b) Area is 250,667 yd^2

Chapter 6

Section 6.1

1. Chain Rule.
2. T
3. $\frac{1}{8}(x^3-5)^8+C$
4. $\frac{1}{4}(x^2-5x+7)^4+C$
5. $\frac{1}{18}\left(x^2+1\right)^9+C$
6. $\frac{1}{3}(3x^2+7x-1)^6+C$
7. $\frac{1}{2}\ln|2x+7|+C$
8. $\sqrt{2x+3}+C$
9. $\frac{2}{3}(x+3)^{3/2}-6(x+3)^{1/2}+C=\frac{2}{3}(x-6)\sqrt{x+3}+C$
10. $\frac{2}{21}x^{3/2}\left(3x^2-7\right)+C$
11. $2e^{\sqrt{x}}+C$
12. $\frac{2\sqrt{x^5+1}}{5}+C$
13. $-\frac{1}{2x^2}-\frac{1}{x}+C$
14. $\frac{\ln^2(x)}{2}+C$
15. $\frac{\sin^3(x)}{3}+C$
16. $-\frac{1}{6}\sin(3-6x)+C$
17. $-\tan(4-x)+C$
18. $\frac{1}{2}\ln|\sec(2x)+\tan(2x)|+C$
19. $\frac{\tan^3(x)}{3}+C$
20. $\frac{\sin\left(x^2\right)}{2}+C$
21. $\tan(x)-x+C$
22. $\frac{1}{3}e^{3x-1}+C$
23. $\frac{e^{x^3}}{3}+C$
24. $\frac{1}{2}e^{(x-1)^2}+C$
25. $x-e^{-x}+C$
26. $\frac{e^{-3x}}{3}-e^{-x}+C$
27. $\frac{27^x}{\ln 27}+C$
28. $\frac{16^x}{\ln(16)}+C$
29. $\frac{1}{2}\ln^2(x)+C$
30. $\frac{\ln^3(x)}{3}+C$
31. $\frac{1}{6}\ln^2\left(x^3\right)+C$
32. $\frac{1}{2}\ln\left(\ln\left(x^2\right)\right)+C$
33. $\frac{x^2}{2}+3x+\ln|x|)+C$
34. $\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|x|+C$
35. $\frac{x^3}{3}-\frac{x^2}{2}+x-2\ln|x+1|+C$
36. $\frac{1}{2}\left(x^2+10x+20\ln|x-3|\right)+C$
37. $\frac{3}{2}x^2-8x+15\ln|x+1|+C$
38. $\frac{1}{3}\ln\left|x^2+3x+3\right|+\frac{\ln|x|}{3}+C$
39. $\sqrt{7}\tan^{-1}\left(\frac{x}{\sqrt{7}}\right)+C$
40. $3\sin^{-1}\left(\frac{x}{3}\right)+C$

41. $14\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$
42. $\frac{2}{3}\sec^{-1}(|x|/3)+C$
43. $\frac{5}{4}\sec^{-1}(|x|/4)+C$
44. $\frac{1}{2}\sin^{-1}\left(x^2\right)+C$
45. $\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{7}}\right)}{\sqrt{7}}+C$
46. $-2\sin^{-1}\left(\frac{3-x}{4}\right)+C$
47. $-3\sin^{-1}\left(\frac{4-x}{5}\right)+C$
48. $\tan^{-1}\left(\frac{x+3}{5}\right)+C$
49. $-\frac{1}{3(x^3+3)}+C$
50. $\frac{1}{45}(5x^3+5x^2+2)^9+C$
51. $-\sqrt{1-x^2}+C$
52. $-\frac{1}{3}\cot\left(x^3+1\right)+C$
53. $-\frac{2}{3}\cos^{\frac{3}{2}}(x)+C$
54. $\ln|x-5|+C$
55. $\frac{7}{3}\ln|3x+2|+C$
56. $\frac{3x^2}{2}+\ln\left|x^2+3x+5\right|-5x+C$
57. $\ln\left|x^2+7x+3\right|+C$
58. $3\ln\left|3x^2+9x+7\right|+C$
59. $-\frac{x^2}{2}+2\ln\left|x^2-7x+1\right|+7x+C$
60. $\frac{1}{18}\tan^{-1}\left(\frac{x^2}{9}\right)+C$
61. $\tan^{-1}(2x)+C$
62. $\sec^{-1}(|2x|)+C$
63. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$
64. $\frac{3}{2}\ln\left|x^2-2x+10\right|+\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right)+C$
65. $\frac{19}{5}\tan^{-1}\left(\frac{x+6}{5}\right)-\ln\left|x^2+12x+61\right|+C$
66. $\frac{15}{2}\ln\left|x^2-10x+32\right|+x+\frac{41\tan^{-1}\left(\frac{x-5}{\sqrt{7}}\right)}{\sqrt{7}}+C$
67. $\frac{x^2}{2}-\frac{9}{2}\ln\left|x^2+9\right|+C$
68. $\frac{x^2}{2}+3\ln\left|x^2+4x+9\right|-4x+\frac{24\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)}{\sqrt{5}}+C$
69. $-\tan^{-1}(\cos(x))+C$
70. $\tan^{-1}(\sin(x))+C$
71. $\ln|\sec x+\tan x|+C$ (integrand simplifies to $\sec x$)
72. $3\sqrt{x^2-2x-6}+C$
73. $\sqrt{x^2-6x+8}+C$
74. $-\ln 2$
75. $352/15$
76. $2/3$
77. $1/5$
78. $(1-e)/2$
79. $\pi/2$
80. $\pi/2$
81. $\pi/6$

Section 6.2

1. T
2. F
3. Determining which functions in the integrand to set equal to "u" and which to set equal to "dv".
4. $\sin x-x\cos x+C$
5. $-e^{-x}-xe^{-x}+C$
6. $-x^2\cos x+2x\sin x+2\cos x+C$
7. $-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+C$
8. $1/2e^{x^2}+C$
9. $x^3e^x-3x^2e^x+6xe^x-6e^x+C$
10. $-\frac{1}{2}xe^{-2x}-\frac{e^{-2x}}{4}+C$
11. $1/2e^x(\sin x-\cos x)+C$
12. $1/5e^{2x}(\sin x+2\cos x)+C$
13. $1/13e^{2x}(2\sin(3x)-3\cos(3x))+C$
14. $1/10e^{5x}(\sin(5x)+\cos(5x))+C$
15. $-1/2\cos^2 x+C$
16. $\sqrt{1-x^2}+x\sin^{-1}(x)+C$
17. $x\tan^{-1}(2x)-\frac{1}{4}\ln\left|4x^2+1\right|+C$
18. $\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+C$
19. $\sqrt{1-x^2}+x\sin^{-1}x+C$
20. $\frac{1}{2}x^2\ln|x|-\frac{x^2}{4}+C$
21. $-\frac{x^2}{4}+\frac{1}{2}x^2\ln|x|+2x-2x\ln|x|+C$
22. $-\frac{x^2}{4}+\frac{1}{2}x^2\ln|x-1|-\frac{x}{2}-\frac{1}{2}\ln|x-1|+C$
23. $\frac{1}{2}x^2\ln\left(x^2\right)-\frac{x^2}{2}+C$
24. $\frac{1}{3}x^3\ln|x|-\frac{x^3}{9}+C$
25. $2x+x(\ln|x|)^2-2x\ln|x|+C$
26. $2x+x(\ln|x+1|)+(\ln|x+1|)^2-2x\ln|x+1|-2\ln|x+1|+2+C$
27. $x\tan(x)+\ln|\cos(x)|+C$
28. $\ln|\sin(x)|-x\cot(x)+C$
29. $\frac{2}{5}(x-2)^{5/2}+\frac{4}{3}(x-2)^{3/2}+C$
30. $\frac{1}{3}(x^2-2)^{3/2}+C$
31. $\sec x+C$
32. $x\sec x-\ln|\sec x+\tan x|+C$
33. $-x\csc x-\ln|\csc x+\cot x|+C$
34. $1/2x\big(\sin(\ln x)-\cos(\ln x)\big)+C$
35. $2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)+C$
36. $\frac{1}{2}x\ln|x|-\frac{x}{2}+C$
37. $2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}+C$
38. $1/2x^2+C$
39. π
40. $-2/e$
41. 0

42. $\frac{3\pi^2}{2} - 12$

43. $1/2$

44. $6 - 2e$

45. $\frac{3}{4e^2} - \frac{5}{4e^4}$

46. $\frac{1}{2} + \frac{e^\pi}{2}$

47. $1/5\left(e^\pi + e^{-\pi}\right)$

Section 6.3

1. F
2. F
3. F
4. $-\frac{1}{5}\cos^5(x) + C$
5. $\frac{1}{4}\sin^4(x) + C$
6. $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$
7. $\frac{1}{6}\cos^6 x - \frac{1}{4}\cos^4 x + C$
8. $\frac{1}{11}\sin^{11} x - \frac{2}{9}\sin^9 x + \frac{1}{7}\sin^7 x + C$
9. $-\frac{1}{9}\sin^9(x) + \frac{3\sin^7(x)}{7} - \frac{3\sin^5(x)}{5} + \frac{\sin^3(x)}{3} + C$
10. $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$
11. $\frac{1}{2}\left(-\frac{1}{8}\cos(8x) - \frac{1}{2}\cos(2x)\right) + C$
12. $\frac{1}{2}\left(-\frac{1}{3}\cos(3x) + \cos(-x)\right) + C$
13. $\frac{1}{2}\left(\frac{1}{4}\sin(4x) - \frac{1}{10}\sin(10x)\right) + C$
14. $\frac{1}{2}\left(\frac{1}{\pi}\sin(\pi x) - \frac{1}{3\pi}\sin(3\pi x)\right) + C$
15. $\frac{1}{2}\left(\sin(x) + \frac{1}{3}\sin(3x)\right) + C$
16. $\frac{1}{\pi}\sin(\frac{\pi}{2}x) + \frac{1}{3\pi}\sin(\pi x) + C$
17. $\frac{\tan^5(x)}{5} + C$
18. $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$
19. $\frac{\tan^6(x)}{6} + \frac{\tan^4(x)}{4} + C$
20. $\frac{\tan^4(x)}{4} + C$
21. $\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3} + C$
22. $\frac{\sec^9(x)}{9} - \frac{2\sec^7(x)}{7} + \frac{\sec^5(x)}{5} + C$
23. $\frac{1}{3}\tan^3 x - \tan x + x + C$
24. $\frac{1}{4}\tan x\sec^3 x + \frac{3}{8}\left(\sec x\tan x + \ln|\sec x + \tan x|\right) + C$
25. $\frac{1}{2}\left(\sec x\tan x - \ln|\sec x + \tan x|\right) + C$
26. $\frac{1}{4}\tan x\sec^3 x - \frac{1}{8}\left(\sec x\tan x + \ln|\sec x + \tan x|\right) + C$
27. $\frac{2}{5}$
28. 0
29. $32/315$
30. $1/2$
31. $2/3$
32. $1/5$
33. $16/15$

Section 6.4

1. backwards
2. $5\sin\theta$
3. (a) $\tan^2\theta + 1 = \sec^2\theta$
   (b) $9\sec^2\theta$.
4. Because we are considering $a > 0$ and $x = a\sin\theta$, which means $\theta = \sin^{-1}(x/a)$. The arcsine function has a domain of $-\pi/2 \le \theta \le \pi/2$; on this domain, $\cos\theta \ge 0$, so $a\cos\theta$ is always non-negative, allowing us to drop the absolute value signs.
5. $\frac{1}{2}\left(x\sqrt{x^2+1} + \ln|\sqrt{x^2+1} + x|\right) + C$
6. $2\left(\frac{x}{4}\sqrt{x^2+4} + \ln|\frac{\sqrt{x^2+1}}{2} + \frac{x}{2}|\right) + C$
7. $\frac{1}{2}\left(\sin^{-1} x + x\sqrt{1-x^2}\right) + C$
8. $\frac{1}{2}\left(9\sin^{-1}(x/3) + x\sqrt{9-x^2}\right) + C$
9. $\frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln|x + \sqrt{x^2-1}| + C$
10. $\frac{1}{2}x\sqrt{x^2-16} - 8\ln\left|\frac{x}{4} + \frac{\sqrt{x^2-16}}{4}\right| + C$
11. $x\sqrt{x^2+1/4} + \frac{1}{4}\ln|2\sqrt{x^2+1/4} + 2x| + C =$
$\frac{1}{2}x\sqrt{4x^2+1} + \frac{1}{4}\ln|\sqrt{4x^2+1} + 2x| + C$
12. $\frac{1}{6}\sin^{-1}(3x) + \frac{3}{2}\sqrt{1/9-x^2} + C = \frac{1}{6}\sin^{-1}(3x) + \frac{1}{2}\sqrt{1-9x^2} + C$
13. $4\left(\frac{1}{2}x\sqrt{x^2-1/16} - \frac{1}{32}\ln|4x + 4\sqrt{x^2-1/16}|\right) + C =$
$\frac{1}{2}x\sqrt{16x^2-1} - \frac{1}{8}\ln|4x + \sqrt{16x^2-1}| + C$
14. $8\ln\left|\frac{\sqrt{x^2+2}}{\sqrt{2}} + \frac{x}{\sqrt{2}}\right| + C$; with Section 6.6, we can state the answer as $8\sinh^{-1}(x/\sqrt{2}) + C$.
15. $3\sin^{-1}\left(\frac{x}{\sqrt{7}}\right) + C$ (Trig. Subst. is not needed)
16. $5\ln\left|\frac{x}{\sqrt{8}} + \frac{\sqrt{x^2-8}}{\sqrt{8}}\right| + C$
17. $\sqrt{x^2-11} - \sqrt{11}\sec^{-1}(x/\sqrt{11}) + C$
18. $\frac{1}{2}\left(\tan^{-1} x + \frac{x}{x^2+1}\right) + C$
19. $\sqrt{x^2-3} + C$ (Trig. Subst. is not needed)
20. $\frac{1}{8}\sin^{-1} x - \frac{1}{8}x\sqrt{1-x^2}(1-2x^2) + C$
21. $-\frac{1}{\sqrt{x^2+9}} + C$ (Trig. Subst. is not needed)
22. $\frac{5}{2}x\sqrt{x^2-10} + 25\ln\left|\frac{x}{\sqrt{10}} + \frac{\sqrt{x^2-10}}{\sqrt{10}}\right| + C$
23. $\frac{1}{18}\frac{x+2}{x^2+4x+13} + \frac{1}{54}\tan^{-1}\left(\frac{x+2}{2}\right) + C$
24. $\frac{x}{\sqrt{1-x^2}} - \sin^{-1} x + C$
25. $\frac{1}{7}\left(-\frac{\sqrt{5-x^2}}{x} - \sin^{-1}(x/\sqrt{5})\right) + C$
26. $\frac{1}{2}x\sqrt{x^2+3} - \frac{3}{2}\ln\left|\frac{\sqrt{x^2+3}}{\sqrt{3}} + \frac{x}{\sqrt{3}}\right| + C$
27. $\pi/2$
28. $16\sqrt{3} - 8\ln(2+\sqrt{3})$
29. $2\sqrt{2} + 2\ln(1+\sqrt{2})$
30. $\pi/4 + 1/2)$

31. $9\sin^{-1}(1/3)+\sqrt{8}$ Note: the new lower bound is $\theta=\sin^{-1}(-1/3)$ and the new upper bound is $\theta=\sin^{-1}(1/3)$. The final answer comes with recognizing that $\sin^{-1}(-1/3)=-\sin^{-1}(1/3)$ and that $\cos\left(\sin^{-1}(1/3)\right)=\cos\left(\sin^{-1}(-1/3)\right)=\sqrt{8}/3$.

32. $\pi/8$

Section 6.5

1. rational
2. T
3. $\frac{A}{x}+\frac{B}{x-3}$
4. $\frac{A}{x-3}+\frac{B}{x+3}$
5. $\frac{A}{x-\sqrt{7}}+\frac{B}{x+\sqrt{7}}$
6. $\frac{A}{x}+\frac{Bx+C}{x^2+7}$
7. $3\ln|x-2|+4\ln|x+5|+C$
8. $9\ln|x+1|-2\ln|x|+C$
9. $\frac{1}{3}(\ln|x+2|-\ln|x-2|)+C$
10. $\ln|x+5|-\frac{2}{x+5}+C$
11. $-\frac{4}{x+8}-3\ln|x+8|+C$
12. $\frac{5}{x+1}+7\ln|x|+2\ln|x+1|+C$
13. $-\ln|2x-3|+5\ln|x-1|+2\ln|x+3|+C$
14. $-\frac{1}{5}\ln|5x-1|+\frac{2}{3}\ln|3x-1|+\frac{3}{7}\ln|7x+3|+C$
15. $x+\ln|x-1|-\ln|x+2|+C$
16. $\frac{x^2}{2}+x+\frac{125}{9}\ln|x-5|+\frac{64}{9}\ln|x+4|-\frac{35}{2}+C$
17. $2x+C$
18. $\frac{1}{6}\left(-\ln\left|x^2+2x+3\right|+2\ln|x|-\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)\right)+C$
19. $-\frac{3}{2}\ln\left|x^2+4x+10\right|+x+\frac{\tan^{-1}\left(\frac{x+2}{\sqrt{6}}\right)}{\sqrt{6}}+C$
20. $\ln\left|3x^2+5x-1\right|+2\ln|x+1|+C$
21. $2\ln|x-3|+2\ln|x^2+6x+10|-4\tan^{-1}(x+3)+C$
22. $\frac{9}{10}\ln\left|x^2+9\right|+\frac{1}{5}\ln|x+1|-\frac{4}{15}\tan^{-1}\left(\frac{x}{3}\right)+C$
23. $\frac{1}{2}\left(3\ln\left|x^2+2x+17\right|-4\ln|x-7|+\tan^{-1}\left(\frac{x+1}{4}\right)\right)+C$
24. $3\left(\ln\left|x^2-2x+11\right|+\ln|x-9|\right)+3\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{x-1}{\sqrt{10}}\right)+C$
25. $\frac{1}{2}\ln\left|x^2+10x+27\right|+5\ln|x+2|-6\sqrt{2}\tan^{-1}\left(\frac{x+5}{\sqrt{2}}\right)+C$
26. $\ln(2000/243)\approx 2.108$
27. $5\ln(9/4)-\frac{1}{3}\ln(17/2)\approx 3.3413$
28. $-\pi/4+\tan^{-1}3-\ln(11/9)\approx 0.263$
29. $1/8$

Section 6.6

1. Because $\cosh x$ is always positive.
2. The points on the left hand side can be defined as $(-\cosh x,\sinh x)$.
3. $$\begin{aligned}\coth^2 x-\operatorname{csch}^2 x&=\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)^2-\left(\frac{2}{e^x-e^{-x}}\right)^2\\&=\frac{(e^{2x}+2+e^{-2x})-(4)}{e^{2x}-2+e^{-2x}}\\&=\frac{e^{2x}-2+e^{-2x}}{e^{2x}-2+e^{-2x}}\\&=1\end{aligned}$$
4. $$\begin{aligned}\cosh^2 x+\sinh^2 x&=\left(\frac{e^x+e^{-x}}{2}\right)^2+\left(\frac{e^x-e^{-x}}{2}\right)^2\\&=\frac{e^{2x}+2+e^{-2x}}{4}+\frac{e^{2x}-2+e^{-2x}}{4}\\&=\frac{2e^{2x}+2e^{-2x}}{4}\\&=\frac{e^{2x}+e^{-2x}}{2}\\&=\cosh 2x.\end{aligned}$$
5. $$\begin{aligned}\cosh^2 x&=\left(\frac{e^x+e^{-x}}{2}\right)^2\\&=\frac{e^{2x}+2+e^{-2x}}{4}\\&=\frac{1}{2}\frac{(e^{2x}+e^{-2x})+2}{2}\\&=\frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}+1\right)\\&=\frac{\cosh 2x+1}{2}.\end{aligned}$$
6. $$\begin{aligned}\sinh^2 x&=\left(\frac{e^x-e^{-x}}{2}\right)^2\\&=\frac{e^{2x}-2+e^{-2x}}{4}\\&=\frac{1}{2}\frac{(e^{2x}+e^{-2x})-2}{2}\\&=\frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}-1\right)\\&=\frac{\cosh 2x-1}{2}.\end{aligned}$$
7. $$\begin{aligned}\frac{d}{dx}\left[\operatorname{sech} x\right]&=\frac{d}{dx}\left[\frac{2}{e^x+e^{-x}}\right]\\&=\frac{-2(e^x-e^{-x})}{(e^x+e^{-x})^2}\\&=-\frac{2(e^x-e^{-x})}{(e^x+e^{-x})(e^x+e^{-x})}\\&=-\frac{2}{e^x+e^{-x}}\cdot\frac{e^x-e^{-x}}{e^x+e^{-x}}\\&=-\operatorname{sech} x\tanh x\end{aligned}$$
8. $$\begin{aligned}\frac{d}{dx}\left[\coth x\right]&=\frac{d}{dx}\left[\frac{e^x+e^{-x}}{e^x-e^{-x}}\right]\\&=\frac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}\\&=\frac{e^{2x}+e^{-2x}-2-(e^{2x}+e^{-2x}+2)}{(e^x-e^{-x})^2}\\&=-\frac{4}{(e^x-e^{-x})^2}\\&=-\operatorname{csch}^2 x\end{aligned}$$

9.
$$\int \tanh x\, dx = \int \frac{\sinh x}{\cosh x}\, dx$$
Let $u = \cosh x$; $du = (\sinh x)dx$
$$= \int \frac{1}{u}\, du$$
$$= \ln |u| + C$$
$$= \ln(\cosh x) + C.$$

10.
$$\int \coth x\, dx = \int \frac{\cosh x}{\sinh x}\, dx$$
Let $u = \sinh x$; $du = (\cosh x)dx$
$$= \int \frac{1}{u}\, du$$
$$= \ln |u| + C$$
$$= \ln |\sinh x| + C.$$

11. $2\sinh 2x$
12. $2x\sec^2(x^2)$
13. $\coth x$
14. $\sinh^2 x + \cosh^2 x$
15. $x\cosh x$
16. $\frac{-2x}{(x^2)\sqrt{1-x^4}}$
17. $\frac{3}{\sqrt{9x^2+1}}$
18. $\frac{4x}{\sqrt{4x^4-1}}$
19. $\frac{1}{1-(x+5)^2}$
20. $-\csc x$
21. $\sec x$
22. $y = x$
23. $y = 3/4(x - \ln 2) + 5/4$
24. $y = -72/125(x - \ln 3) + 9/25$
25. $y = x$
26. $y = (x - \sqrt{2}) + \cosh^{-1}(\sqrt{2}) \approx (x - 1.414) + 0.881$
27. $1/2\ln(\cosh(2x)) + C$
28. $1/3\sinh(3x - 7) + C$
29. $1/2\sinh^2 x + C$ or $1/2\cosh^2 x + C$
30. $x\sinh(x) - \cosh(x) + C$
31. $x\cosh(x) - \sinh(x) + C$
32. $\begin{cases} \frac{1}{3}\tanh^{-1}\left(\frac{x}{3}\right) + C & x^2 < 9 \\ \frac{1}{3}\coth^{-1}\left(\frac{x}{3}\right) + C & 9 < x^2 \end{cases} = \frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + C$
33. $\cosh^{-1}(x^2/2) + C = \ln(x^2 + \sqrt{x^4 - 4}) + C$
34. $2/3\sinh^{-1} x^{3/2} + C = 2/3\ln(x^{3/2} + \sqrt{x^3 + 1}) + C$
35. $\frac{1}{16}\tan^{-1}(x/2) + \frac{1}{32}\ln|x-2| + \frac{1}{32}\ln|x+2| + C$
36. $\ln x - \ln|x+1| + C$
37. $\tan^{-1}(e^x) + C$
38. $x\sinh^{-1} x - \sqrt{x^2+1} + C$
39. $x\tanh^{-1} x + 1/2\ln|x^2 - 1| + C$
40. $\tan^{-1}(\sinh x) + C$
41. 0
42. $3/2$
43. 2

Section 6.7

1. $0/0, \infty/\infty, 0\cdot\infty, \infty-\infty, 0^0, 1^\infty, \infty^0$
2. F
3. F
4. The base of an expression is approaching 1 while its power is growing without bound.
5. derivatives; limits
6. Answers will vary.
7. Answers will vary.
8. 3
9. $-5/3$
10. -1
11. $-\sqrt{2}/2$
12. 5
13. 0
14. $2/3$
15. a/b
16. ∞
17. $1/2$
18. 0
19. 0
20. 0
21. ∞
22. ∞
23. 0
24. 2
25. -2
26. 0
27. 0
28. 0
29. 0
30. 0
31. ∞
32. ∞
33. ∞
34. 0
35. 0
36. e
37. 1
38. 1
39. 1
40. 1
41. 1
42. 0

43. 1
44. 1
45. 1
46. 1
47. 2
48. $1/2$
49. $-\infty$
50. 1
51. 0
52. 3

Section 6.8

1. The interval of integration is finite, and the integrand is continuous on that interval.
2. converge
3. converges; could also state < 10.
4. $p > 1$
5. $p > 1$
6. $p < 1$
7. $e^5/2$
8. $1/2$
9. $1/3$
10. $\pi/3$
11. $1/\ln 2$
12. diverges
13. diverges
14. $\pi/2$
15. 1
16. diverges
17. diverges
18. diverges
19. diverges
20. diverges
21. diverges
22. $2+2\sqrt{2}$
23. 1
24. $1/2$
25. 0
26. $\pi/2$
27. $-1/4$
28. diverges
29. -1
30. 1
31. diverges
32. $1/2$
33. $1/2$
34. diverges; Limit Comparison Test with $1/x$.
35. converges; Limit Comparison Test with $1/x^{3/2}$.
36. diverges; Limit Comparison Test with $1/x$.
37. converges; Direct Comparison Test with xe^{-x}.
38. converges; Direct Comparison Test with e^{-x}.
39. converges; Direct Comparison Test with xe^{-x}.
40. converges; Direct Comparison Test with $1/(x^2-1)$.
41. diverges; Direct Comparison Test with $x/(x^2+\cos x)$.
42. converges; Direct Comparison Test with $1/e^x$.
43. converges; Limit Comparison Test with $1/e^x$.

Chapter 7

Section 7.1

1. T
2. T
3. Answers will vary.
4. $4\pi+\pi^2 \approx 22.436$
5. $16/3$
6. π
7. π
8. $1/2$
9. $2\sqrt{2}$
10. $1/\ln 4$
11. 4.5
12. $4/3$
13. $2-\pi/2$
14. 8
15. $1/6$
16. $37/12$
17. On regions such as $[\pi/6, 5\pi/6]$, the area is $3\sqrt{3}/2$. On regions such as $[-\pi/2, \pi/6]$, the area is $3\sqrt{3}/4$.
18. 1
19. $5/3$
20. $9/2$
21. $9/4$
22. $1/12(9-2\sqrt{2}) \approx 0.514$
23. 1
24. 5
25. 4
26. 133/20
27. 219,000 ft^2
28. 623,333 ft^2

Section 7.2

1. T
2. Answers will vary.
3. Recall that "dx" does not just "sit there;" it is multiplied by $A(x)$ and represents the thickness of a small slice of the solid. Therefore dx has units of in, giving $A(x)\,dx$ the units of in^3.

4. $48\pi\sqrt{3}/5$ units^3
5. $175\pi/3$ units^3
6. $\pi^2/4$ units^3
7. $\pi/6$ units^3
8. $9\pi/2$ units^3
9. $35\pi/3$ units^3
10. $\pi^2 - 2\pi$ units^3
11. $2\pi/15$ units^3
12. (a) $\pi/2$
    (b) $\pi/2$
    (c) $4\pi/5$
    (d) $8\pi/15$
13. (a) $512\pi/15$
    (b) $256\pi/5$
    (c) $832\pi/15$
    (d) $128\pi/3$
14. (a) $4\pi/3$
    (b) $2\pi/3$
    (c) $4\pi/3$
    (d) $\pi/3$
15. (a) $104\pi/15$
    (b) $64\pi/15$
    (c) $32\pi/5$
16. (a) $\pi^2/2$
    (b) $\pi^2/2 - 4\pi\sinh^{-1}(1)$
    (c) $\pi^2/2 + 4\pi\sinh^{-1}(1)$
17. (a) 8π
    (b) 8π
    (c) $16\pi/3$
    (d) $8\pi/3$
18. Placing the tip of the cone at the origin such that the x-axis runs through the center of the circular base, we have $A(x) = \pi x^2/4$. Thus the volume is $250\pi/3$ units^3.
19. The cross–sections of this cone are the same as the cone in Exercise 18. Thus they have the same volume of $250\pi/3$ units^3.
20. Orient the cone such that the tip is at the origin and the x-axis is perpendicular to the base. The cross–sections of this cone are right, isosceles triangles with side length $2x/5$; thus the cross–sectional areas are $A(x) = 2x^2/25$, giving a volume of $80/3$ units^3.
21. Orient the solid so that the x-axis is parallel to long side of the base. All cross–sections are trapezoids (at the far left, the trapezoid is a square; at the far right, the trapezoid has a top length of 0, making it a triangle). The area of the trapezoid at x is $A(x) = 1/2(-1/2x + 5 + 5)(5) = -5/4x + 25$. The volume is 187.5 units^3.

Section 7.3

1. T
2. F
3. F
4. T
5. $9\pi/2$ units^3
6. $70\pi/3$ units^3
7. $\pi^2 - 2\pi$ units^3
8. $2\pi/15$ units^3
9. $48\pi\sqrt{3}/5$ units^3
10. $350\pi/3$ units^3
11. $\pi^2/4$ units^3
12. $\pi/6$ units^3
13. (a) $4\pi/5$
    (b) $8\pi/15$
    (c) $\pi/2$
    (d) $5\pi/6$
14. (a) $128\pi/3$
    (b) $128\pi/3$
    (c) $512\pi/15$
    (d) $256\pi/5$
15. (a) $4\pi/3$
    (b) $\pi/3$
    (c) $4\pi/3$
    (d) $2\pi/3$
16. (a) $16\pi/3$
    (b) $8\pi/3$
    (c) 8π
17. (a) $2\pi(\sqrt{2} - 1)$
    (b) $2\pi(1 - \sqrt{2} + \sinh^{-1}(1))$
18. (a) $16\pi/3$
    (b) $8\pi/3$
    (c) 8π
    (d) 8π

Section 7.4

1. T
2. F
3. $\sqrt{2}$
4. 6
5. $4/3$
6. 6
7. $109/2$
8. $3/2$
9. $12/5$
10. $79953333/400000 \approx 199.883$
11. $-\ln(2 - \sqrt{3}) \approx 1.31696$
12. $\sinh^{-1} 1$
13. $\int_0^1 \sqrt{1 + 4x^2}\, dx$
14. $\int_0^1 \sqrt{1 + 100x^{18}}\, dx$
15. $\int_0^1 \sqrt{1 + \frac{1}{4x}}\, dx$
16. $\int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$

17. $\int_{-1}^{1} \sqrt{1+\frac{x^2}{1-x^2}}\, dx$

18. $\int_{-3}^{3} \sqrt{1+\frac{x^2}{81-9x^2}}\, dx$

19. $\int_{1}^{2} \sqrt{1+\frac{1}{x^4}}\, dx$

20. $\int_{-\pi/4}^{\pi/4} \sqrt{1+\sec^2 x \tan^2 x}\, dx$

21. 1.4790

22. 1.8377

23. Simpson's Rule fails, as it requires one to divide by 0. However, recognize the answer should be the same as for $y = x^2$; why?

24. 2.1300

25. Simpson's Rule fails.

26. Simpson's Rule fails.

27. 1.4058

28. 1.7625

29. $2\pi \int_0^1 2x\sqrt{5}\, dx = 2\pi\sqrt{5}$

30. $2\pi \int_0^1 x\sqrt{1+4x^2}\, dx = \pi/6(5\sqrt{5}-1)$

31. $2\pi \int_0^1 x^3\sqrt{1+9x^4}\, dx = \pi/27(10\sqrt{10}-1)$

32. $2\pi \int_0^1 \sqrt{x}\sqrt{1+1/(4x)}\, dx = \pi/6(5\sqrt{5}-1)$

33. $2\pi \int_0^1 \sqrt{1-x^2}\sqrt{1+x/(1-x^2)}\, dx = 4\pi$

Section 7.5

1. In SI units, it is one joule, i.e., one Newton–meter, or kg·m/s^2·m. In Imperial Units, it is ft–lb.

2. The same.

3. Smaller.

4. (a) 500 ft–lb
   (b) $100 - 50\sqrt{2} \approx 29.29$ ft–lb

5. (a) 2450 j
   (b) 1568 j

6. (a) $\frac{1}{2} \cdot d \cdot l^2$ ft–lb
   (b) 75 %
   (c) $\ell(1-\sqrt{2}/2) \approx 0.2929\ell$

7. 735 j

8. (a) 756 ft–lb
   (b) 60,000 ft–lb
   (c) Yes, for the cable accounts for about 1% of the total work.

9. 11,100 ft–lb

10. 575 ft–lb

11. 125 ft–lb

12. 0.05 j

13. 12.5 ft–lb

14. 5/3 ft–lb

15. 7/20 j

16. $f \cdot d/2$ j

17. 45 ft–lb

18. 5 ft–lb

19. 953, 284 j

20. (a) 52,929.6 ft–lb
    (b) 18,525.3 ft–lb
    (c) When 3.83 ft of water have been pumped from the tank, leaving about 2.17 ft in the tank.

21. 192,767 ft–lb. Note that the tank is oriented horizontally. Let the origin be the center of one of the circular ends of the tank. Since the radius is 3.75 ft, the fluid is being pumped to $y = 4.75$; thus the distance the gas travels is $h(y) = 4.75 - y$. A differential element of water is a rectangle, with length 20 and width $2\sqrt{3.75^2 - y^2}$. Thus the force required to move that slab of gas is $F(y) = 40 \cdot 45.93 \cdot \sqrt{3.75^2 - y^2}dy$. Total work is $\int_{-3.75}^{3.75} 40 \cdot 45.93 \cdot (4.75 - y)\sqrt{3.75^2 - y^2}\, dy$. This can be evaluated without actual integration; split the integral into $\int_{-3.75}^{3.75} 40 \cdot 45.93 \cdot (4.75)\sqrt{3.75^2 - y^2}\, dy + \int_{-3.75}^{3.75} 40 \cdot 45.93 \cdot (-y)\sqrt{3.75^2 - y^2}\, dy$. The first integral can be evaluated as measuring half the area of a circle; the latter integral can be shown to be 0 without much difficulty. (Use substitution and realize the bounds are both 0.)

22. 212,135 ft–lb

23. (a) approx. 577,000 j
    (b) approx. 399,000 j
    (c) approx 110,000 j (By volume, half of the water is between the base of the cone and a height of 3.9685 m. If one rounds this to 4 m, the work is approx 104,000 j.)

24. 187,214 ft–lb

25. 617,400 j

26. 4,917,150 j

Section 7.6

1. Answers will vary.

2. Answers will vary.

3. 499.2 lb

4. 249.6 lb

5. 6739.2 lb

6. 5241.6 lb

7. 3920.7 lb

8. 15682.8 lb

9. 2496 lb

10. 2496 lb

11. 602.59 lb

12. 291.2 lb

13. (a) 2340 lb
    (b) 5625 lb

14. (a) 1064.96 lb
    (b) 2560 lb

15. (a) 1597.44 lb
    (b) 3840 lb

16. (a) 41.6 lb
    (b) 100 lb

17. (a) 56.42 lb
    (b) 135.62 lb

18. (a) 1123.2 lb
    (b) 2700 lb

19. 5.1 ft

20. 4.1 ft

Chapter 8

Section 8.1

1. Answers will vary.
2. natural
3. Answers will vary.
4. Answers will vary.
5. $2, \frac{8}{3}, \frac{8}{3}, \frac{32}{15}, \frac{64}{45}$
6. $-\frac{3}{2}, \frac{9}{4}, -\frac{27}{8}, \frac{81}{16}, -\frac{243}{32}$
7. $\frac{1}{3}, 2, \frac{81}{5}, \frac{512}{3}, \frac{15625}{7}$
8. $1, 1, 2, 3, 5$
9. $a_n = 3n + 1$
10. $a_n = (-1)^{n+1}\frac{3}{2^{n-1}}$
11. $a_n = 10 \cdot 2^{n-1}$
12. $a_n = 1/(n-1)!$
13. $1/7$
14. $3e^2 - 1$
15. 0
16. e^4
17. diverges
18. converges to $4/3$
19. converges to 0
20. converges to 0
21. diverges
22. converges to 3
23. converges to e
24. converges to 5
25. converges to 0
26. diverges
27. converges to 2
28. converges to 0
29. bounded
30. neither bounded above or below
31. bounded
32. bounded below
33. neither bounded above or below
34. bounded above
35. monotonically increasing
36. monotonically increasing for $n \geq 3$
37. never monotonic
38. monotonically decreasing for $n \geq 3$
39. Let $\{a_n\}$ be given such that $\lim\limits_{n\to\infty} |a_n| = 0$. By the definition of the limit of a sequence, given any $\varepsilon > 0$, there is a m such that for all $n > m$, $|\,|a_n| - 0| < \varepsilon$. Since $|\,|a_n| - 0| = |a_n - 0|$, this directly implies that for all $n > m$, $|a_n - 0| < \varepsilon$, meaning that $\lim\limits_{n\to\infty} a_n = 0$.
40. (a) Left to reader
    (b) $a_n = 1/3^n$ and $b_n = 1/2^n$
41. Left to reader

Section 8.2

1. Answers will vary.
2. Answers will vary.
3. One sequence is the sequence of terms $\{a_i\}$. The other is the sequence of n^{th} partial sums, $\{S_n\} = \{\sum_{i=1}^n a_i\}$.
4. Answers will vary.
5. F
6. (a) $-1, -\frac{1}{2}, -\frac{5}{6}, -\frac{7}{12}, -\frac{47}{60}$
   (b) Plot omitted
7. (a) $1, \frac{5}{4}, \frac{49}{36}, \frac{205}{144}, \frac{5269}{3600}$
   (b) Plot omitted
8. (a) $-1, 0, -1, 0, -1$
   (b) Plot omitted
9. (a) $1, 3, 6, 10, 15$
   (b) Plot omitted
10. (a) $1, \frac{3}{2}, \frac{5}{3}, \frac{41}{24}, \frac{103}{60}$
    (b) Plot omitted
11. (a) $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \frac{121}{243}$
    (b) Plot omitted
12. (a) $-0.9, -0.09, -0.819, -0.1629, -0.75339$
    (b) Plot omitted
13. (a) $0.1, 0.11, 0.111, 0.1111, 0.11111$
    (b) Plot omitted
14. $\lim\limits_{n\to\infty} a_n = 3$; by Theorem 63 the series diverges.
15. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 63 the series diverges.
16. $\lim\limits_{n\to\infty} a_n = \infty$; by Theorem 63 the series diverges.
17. $\lim\limits_{n\to\infty} a_n = 1$; by Theorem 63 the series diverges.
18. $\lim\limits_{n\to\infty} a_n = 1/2$; by Theorem 63 the series diverges.
19. $\lim\limits_{n\to\infty} a_n = e$; by Theorem 63 the series diverges.
20. Converges
21. Converges
22. Diverges
23. Converges
24. Diverges
25. Converges
26. Diverges
27. Converges
28. Diverges
29. Diverges
30. (a) $S_n = \frac{1-(1/4)^n}{3/4}$
    (b) Converges to $4/3$.

31. (a) $S_n = \left(\frac{n(n+1)}{2}\right)^2$

(b) Diverges

32. (a) $S_n = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ -\frac{n}{2} & \text{n is even} \end{cases}$

(b) Diverges

33. (a) $S_n = 5\frac{1-1/2^n}{1/2}$

(b) Converges to 10.

34. (a) $S_n = \frac{1-(1/e)^{n+1}}{1-1/e}$.

(b) Converges to $1/(1-1/e) = e/(e-1)$.

35. (a) $S_n = \frac{1-(-1/3)^n}{4/3}$

(b) Converges to $3/4$.

36. (a) With partial fractions, $a_n = \frac{1}{n} - \frac{1}{n+1}$. Thus $S_n = 1 - \frac{1}{n+1}$.

(b) Converges to 1.

37. (a) With partial fractions, $a_n = \frac{3}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$. Thus $S_n = \frac{3}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$.

(b) Converges to 9/4

38. (a) Use partial fraction decomposition to recognize the telescoping series: $S_n = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) = \frac{n}{2n+1}$.

(b) Converges to $1/2$.

39. (a) $S_n = \ln\big(1/(n+1)\big)$

(b) Diverges (to $-\infty$).

40. (a) $S_n = 1 - \frac{1}{(n+1)^2}$

(b) Converges to 1.

41. (a) $a_n = \frac{1}{n(n+3)}$; using partial fractions, the resulting telescoping sum reduces to $S_n = \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right)$

(b) Converges to 11/18.

42. (a) $a_n = 1/2^n + 1/3^n$ for $n \geq 0$. Thus $S_n = \frac{1-1/2^2}{1/2} + \frac{1-1/3^n}{2/3}$.

(b) Converges to $2 + 3/2 = 7/2$.

43. (a) With partial fractions, $a_n = \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$. Thus $S_n = \frac{1}{2}\left(3/2 - \frac{1}{n} - \frac{1}{n+1}\right)$.

(b) Converges to 3/4.

44. (a) $S_n = \frac{1-(\sin 1)^{n+1}}{1-\sin 1}$

(b) Converges to $\frac{1}{1-\sin 1}$..

45. (a) The n^{th} partial sum of the odd series is $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. The n^{th} partial sum of the even series is $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n}$. Each term of the even series is less than the corresponding term of the odd series, giving us our result.

(b) The n^{th} partial sum of the odd series is $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. The n^{th} partial sum of 1 plus the even series is $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2(n-1)}$. Each term of the even series is now greater than or equal to the corresponding term of the odd series, with equality only on the first term. This gives us the result.

(c) If the odd series converges, the work done in (a) shows the even series converges also. (The sequence of the n^{th} partial sum of the even series is bounded and monotonically increasing.) Likewise, (b) shows that if the even series converges, the odd series will, too. Thus if either series converges, the other does.

Similarly, (a) and (b) can be used to show that if either series diverges, the other does, too.

(d) If both the even and odd series converge, then their sum would be a convergent series. This would imply that the Harmonic Series, their sum, is convergent. It is not. Hence each series diverges.

46. Using partial fractions, we can show that $a_n = \frac{1}{4}\left(\frac{1}{2n-1} + \frac{1}{2n+1}\right)$. The series is effectively twice the sum of the odd terms of the Harmonic Series which was shown to diverge in Exercise 45. Thus this series diverges.

Section 8.3

1. continuous, positive and decreasing
2. F
3. The Integral Test (we do not have a continuous definition of $n!$ yet) and the Limit Comparison Test (same as above, hence we cannot take its derivative).
4. $\sum_{n=0}^{\infty} b_n$ converges; we cannot conclude anything about $\sum_{n=0}^{\infty} c_n$
5. Converges
6. Converges
7. Diverges
8. Diverges
9. Converges
10. Converges
11. Converges
12. Converges
13. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, as $1/(n^2+3n-5) \leq 1/n^2$ for all $n > 1$.
14. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{4^n}$, as $1/(4^n+n^2-n) \leq 1/4^n$ for all $n \geq 1$.
15. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, as $1/n \leq \ln n/n$ for all $n \geq 2$.
16. Converges; compare to $\sum_{n=1}^{\infty} \frac{1}{n!}$, as $1/(n!+n) \leq 1/n!$ for all $n \geq 1$.
17. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$. Since $n = \sqrt{n^2} > \sqrt{n^2-1}$, $1/n \leq 1/\sqrt{n^2-1}$ for all $n \geq 2$.
18. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, as $1/\sqrt{n} \leq 1/(\sqrt{n}-2)$ for all $n \geq 5$.
19. Diverges; compare to $\sum_{n=1}^{\infty} \frac{1}{n}$:
$$\frac{1}{n} = \frac{n^2}{n^3} < \frac{n^2+n+1}{n^3} < \frac{n^2+n+1}{n^3-5},$$
for all $n \geq 1$.

20. Converges; compare to $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n$, as $2^n/(5^n+10) < 2^n/5^n$ for all $n \geq 1$.

21. Diverges; compare to $\sum_{n=1}^{\infty}\frac{1}{n}$. Note that
$$\frac{n}{n^2-1} = \frac{n^2}{n^2-1}\cdot\frac{1}{n} > \frac{1}{n},$$
as $\frac{n^2}{n^2-1} > 1$, for all $n \geq 2$.

22. Converges; compare to $\sum_{n=1}^{\infty}\frac{1}{n^2}$, as $1/(n^2\ln n) \leq 1/n^2$ for all $n \geq 2$.

23. Converges; compare to $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

24. Converges; compare to $\sum_{n=1}^{\infty}\frac{1}{4^n}$.

25. Diverges; compare to $\sum_{n=1}^{\infty}\frac{\ln n}{n}$.

26. Diverges; compare to $\sum_{n=1}^{\infty}\frac{1}{n}$.

27. Diverges; compare to $\sum_{n=1}^{\infty}\frac{1}{n}$.

28. Diverges; compare to $\sum_{n=1}^{\infty}\frac{1}{n}$.

29. Diverges; compare to $\sum_{n=1}^{\infty}\frac{1}{n}$. Just as $\lim_{n\to 0}\frac{\sin n}{n} = 1$, $\lim_{n\to\infty}\frac{\sin(1/n)}{1/n} = 1$.

30. Converges; compare to $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

31. Converges; compare to $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$.

32. Diverges; compare to $\sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$.

33. Converges; Integral Test

34. Converges; Integral Test, p-Series Test, Direct & Limit Comparison Tests can all be used.

35. Diverges; the n^{th} Term Test and Direct Comparison Test can be used.

36. Converges; the Direct Comparison Test can be used with sequence $1/(n-1)!$.

37. Converges; the Direct Comparison Test can be used with sequence $1/3^n$.

38. Diverges; the n^{th} Term Test can be used, along with the Limit Comparison Test (compare with 1/10).

39. Diverges; the n^{th} Term Test can be used, along with the Integral Test.

40. Converges; the Direct Comparison Test can be used with sequence $1/\sqrt{n}$.

41. (a) Converges; use Direct Comparison Test as $\frac{a_n}{n} < n$.
(b) Converges; since original series converges, we know $\lim_{n\to\infty} a_n = 0$. Thus for large n, $a_na_{n+1} < a_n$.
(c) Converges; similar logic to part (b) so $(a_n)^2 < a_n$.
(d) May converge; certainly $na_n > a_n$ but that does not mean it does not converge.
(e) Does not converge, using logic from (b) and n^{th} Term Test.

Section 8.4

1. algebraic, or polynomial.
2. factorial and/or exponential
3. Integral Test, Limit Comparison Test, and Root Test
4. raised to a power
5. Converges
6. Diverges
7. Converges
8. Converges
9. The Ratio Test is inconclusive; the p-Series Test states it diverges.
10. The Ratio Test is inconclusive; the Direct Comparison Test with $1/n^3$ shows it converges.
11. Converges
12. Converges
13. Converges; note the summation can be rewritten as $\sum_{n=1}^{\infty}\frac{2^n n!}{3^n n!}$, from which the Ratio Test can be applied.
14. Converges; rewrite the summation as $\sum_{n=1}^{\infty}\frac{n!}{5^n n!}$ then apply the Ratio Test.
15. Converges
16. Converges
17. Converges
18. Converges
19. Diverges
20. Converges
21. Diverges. The Root Test is inconclusive, but the n^{th}-Term Test shows divergence. (The terms of the sequence approach e^2, not 0, as $n\to\infty$.)
22. Converges
23. Converges
24. Converges
25. Diverges; Limit Comparison Test
26. Converges; Ratio Test
27. Converges; Ratio Test or Limit Comparison Test with $1/3^n$.
28. Converges; Root Test
29. Diverges; n^{th}-Term Test or Limit Comparison Test with 1.
30. Converges; Ratio Test
31. Diverges; Direct Comparison Test with $1/n$
32. Diverges; n^{th}-Term Test (n^{th} term approaches e.)
33. Converges; Root Test
34. Converges; Limit Comparison Test with $1/n^2$ (get common denominator first). It is also a Telescoping Series.

Section 8.5

1. The signs of the terms do not alternate; in the given series, some terms are negative and the others positive, but they do not necessarily alternate.
2. postive, decreasing, 0
3. Many examples exist; one common example is $a_n = (-1)^n/n$.
4. conditionally
5. (a) converges
   (b) converges (p-Series)
   (c) absolute
6. (a) converges
   (b) converges (use Ratio Test)
   (c) absolute
7. (a) diverges (limit of terms is not 0)
   (b) diverges
   (c) n/a; diverges
8. (a) diverges (limit of terms is not 0)
   (b) diverges
   (c) n/a; diverges
9. (a) converges
   (b) diverges (Limit Comparison Test with $1/n$)
   (c) conditional
10. (a) converges
    (b) diverges (Direct Comparison Test with $1/n$)
    (c) conditional
11. (a) diverges (limit of terms is not 0)
    (b) diverges
    (c) n/a; diverges
12. (a) converges
    (b) converges (the sum in the denominator is n^2)
    (c) absolute
13. (a) diverges (terms oscillate between ± 1)
    (b) diverges
    (c) n/a; diverges
14. (a) converges
    (b) diverges (Integral Test)
    (c) conditional
15. (a) converges
    (b) converges (Geometric Series with $r = 2/3$)
    (c) absolute
16. (a) converges
    (b) converges (Geometric Series with $r = 1/e$)
    (c) absolute
17. (a) converges
    (b) converges (Ratio Test)
    (c) absolute
18. (a) converges
    (b) converges (Ratio Test)
    (c) absolute
19. (a) converges
    (b) diverges (p-Series Test with $p = 1/2$)
    (c) conditional
20. (a) converges
    (b) converges (Ratio Test)
    (c) absolute
21. $S_5 = -1.1906$; $S_6 = -0.6767$;
    $$-1.1906 \le \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \le -0.6767$$
22. $S_4 = 0.9459$; $S_5 = 0.9475$;
    $$0.9459 \le \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \le 0.9475$$
23. $S_6 = 0.3681$; $S_7 = 0.3679$;
    $$0.3681 \le \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \le 0.3679$$
24. $S_9 = 0.666016$; $S_{10} = 0.666992$;
    $$0.666016 \le \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \le 0.666992$$
25. $n = 5$
26. $n = 7$
27. Using the theorem, we find $n = 499$ guarantees the sum is within 0.001 of $\pi/4$. (Convergence is actually faster, as the sum is within ε of $\pi/24$ when $n \ge 249$.)
28. $n = 5$ ($(2n)! > 10^8$ when $n \ge 6$)

Section 8.6

1. 1
2. The radius of convergence is a **value** R such that a power series, centered at $x = c$, converges for all values of x in $(c - R, c + R)$. The interval of convergence is an **interval** on which the power series converges; it may differ from $(c - R, c + R)$ only at the endpoints.
3. 5
4. 5
5. $1 + 2x + 4x^2 + 8x^3 + 16x^4$
6. $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16} + \frac{x^5}{25}$
7. $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$
8. $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$
9. (a) $R = \infty$
   (b) $(-\infty, \infty)$
10. (a) $R = 1$
    (b) $(-1, 1)$
11. (a) $R = 1$
    (b) $(2, 4]$
12. (a) $R = \infty$
    (b) $(-\infty, \infty)$
13. (a) $R = 2$
    (b) $(-2, 2)$
14. (a) $R = 10$
    (b) $(-5, 15)$

15. (a) $R = 1/5$
(b) $(4/5, 6/5)$

16. (a) $R = 1/2$
(b) $(-1/2, 1/2)$

17. (a) $R = 1$
(b) $(-1, 1)$

18. (a) $R = 3$
(b) $(-3, 3)$

19. (a) $R = \infty$
(b) $(-\infty, \infty)$

20. (a) $R = 0$
(b) $x = 10$

21. (a) $R = 1$
(b) $[-1, 1]$

22. (a) $R = 1$
(b) $[-3, -1]$

23. (a) $R = 0$
(b) $x = 0$

24. (a) $R = 4$
(b) $x = (-8, 0)$

25. (a) $f'(x) = \sum_{n=1}^{\infty} n^2 x^{n-1}; \quad (-1, 1)$
(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{n}{n+1} x^{n+1}; \quad (-1, 1)$

26. (a) $f'(x) = \sum_{n=1}^{\infty} x^{n-1}; \quad (-1, 1)$
(b) $\int f(x)\,dx = C + \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^{n+1}; \quad [-1, 1]$

27. (a) $f'(x) = \sum_{n=1}^{\infty} \frac{n}{2^n} x^{n-1}; \quad (-2, 2)$
(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{1}{(n+1)2^n} x^{n+1}; \quad [-2, 2)$

28. (a) $f'(x) = \sum_{n=1}^{\infty} n(-3)^n x^{n-1}; \quad (-1/3, 1/3)$
(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-3)^n}{n+1} x^{n+1}; \quad (-1/3, 1/3]$

29. (a) $f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$; $(-\infty, \infty)$
(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}; \quad (-\infty, \infty)$

30. (a) $f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{n-1}}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n!}; \quad (-\infty, \infty)$
(b) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}; \quad (-\infty, \infty)$

31. $1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \frac{27}{8}x^4$

32. $5 + 25x + \frac{125}{2}x^2 + \frac{625}{6}x^3 + \frac{3125}{24}x^4$

33. $1 + x + x^2 + x^3 + x^4$

34. $1 + 2x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4$

35. $0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4$

36. $1 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4$

Section 8.7

1. The Maclaurin polynomial is a special case of Taylor polynomials. Taylor polynomials are centered at a specific x-value; when that x-value is 0, it is a Maclauring polynomial.
2. T
3. $p_2(x) = 6 + 3x - 4x^2$.
4. $f'''(0) = 30$
5. $p_3(x) = 1 - x + \frac{1}{2}x^3 - \frac{1}{6}x^3$
6. $p_8(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7$
7. $p_8(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5$
8. $p_6(x) = \frac{2x^5}{15} + \frac{x^3}{3} + x$
9. $p_4(x) = \frac{2x^4}{3} + \frac{4x^3}{3} + 2x^2 + 2x + 1$
10. $p_4(x) = x^4 + x^3 + x^2 + x + 1$
11. $p_4(x) = x^4 - x^3 + x^2 - x + 1$
12. $p_7(x) = -\frac{x^7}{7} + \frac{x^5}{5} - \frac{x^3}{3} + x$
13. $p_4(x) = 1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4$
14. $p_4(x) =$ $\ln(2) + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{24}(-1+x)^3 - \frac{1}{64}(-1+x)^4$
15. $p_6(x) = \frac{1}{\sqrt{2}} - \frac{-\frac{\pi}{4}+x}{\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^2}{2\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^3}{6\sqrt{2}} + \frac{\left(-\frac{\pi}{4}+x\right)^4}{24\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^5}{120\sqrt{2}} - \frac{\left(-\frac{\pi}{4}+x\right)^6}{720\sqrt{2}}$
16. $p_5(x) = \frac{1}{2} + \frac{1}{2}\sqrt{3}\left(-\frac{\pi}{6}+x\right) - \frac{1}{4}\left(-\frac{\pi}{6}+x\right)^2 - \frac{\left(-\frac{\pi}{6}+x\right)^3}{4\sqrt{3}} + \frac{1}{48}\left(-\frac{\pi}{6}+x\right)^4 + \frac{\left(-\frac{\pi}{6}+x\right)^5}{80\sqrt{3}}$
17. $p_5(x) = \frac{1}{2} - \frac{x-2}{4} + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \frac{1}{32}(x-2)^4 - \frac{1}{64}(x-2)^5$
18. $p_8(x) = 1 - 2(-1+x) + 3(-1+x)^2 - 4(-1+x)^3 + 5(-1+x)^4 - 6(-1+x)^5 + 7(-1+x)^6 - 8(-1+x)^7 + 9(-1+x)^8$
19. $p_3(x) = \frac{1}{2} + \frac{1+x}{2} + \frac{1}{4}(1+x)^2$
20. $p_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{1}{2}\left(\pi^2 - 2\right)(x-\pi)^2$
21. $p_3(x) = x - \frac{x^3}{6}$; $p_3(0.1) = 0.09983$. Error is bounded by $\pm\frac{1}{4!} \cdot 0.1^4 \approx \pm 0.000004167$.
22. $p_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$; $p_4(1) = 13/24 \approx 0.54167$. Error is bounded by $\pm\frac{1}{5!} \cdot 1^5 \approx \pm 0.00833$
23. $p_2(x) = 3 + \frac{1}{6}(-9+x) - \frac{1}{216}(-9+x)^2$; $p_2(10) = 3.16204$. The third derivative of $f(x) = \sqrt{x}$ is bounded on $(8, 11)$ by 0.003. Error is bounded by $\pm\frac{0.003}{3!} \cdot 1^3 = \pm 0.0005$.
24. $p_3(x) = -1 + x - \frac{1}{2}(-1+x)^2 + \frac{1}{3}(-1+x)^3$; $p_3(1.5) = 0.41667$. The fourth derivative of $f(x) = \ln x$ is bounded on $(.9, 2)$ by 10. Error is bounded by $\pm\frac{10}{4!} \cdot .5^4 = \pm 0.026$.

25. The n^{th} derivative of $f(x)=e^x$ is bounded by 3 on intervals containing 0 and 1. Thus $|R_n(1)| \leq \frac{3}{(n+1)!}1^{(n+1)}$. When $n=7$, this is less than 0.0001.

26. The n^{th} derivative of $f(x)=\sqrt{x}$ is bounded by 0.1 on intervals containing 3 and 4. Thus $|R_n(\pi)| \leq \frac{0.1}{(n+1)!}(1)^{(n+1)}$. When $n=4$, this is less than 0.0001.

27. The n^{th} derivative of $f(x)=\cos x$ is bounded by 1 on intervals containing 0 and $\pi/3$. Thus $|R_n(\pi/3)| \leq \frac{1}{(n+1)!}(\pi/3)^{(n+1)}$. When $n=7$, this is less than 0.0001. Since the Maclaurin polynomial of $\cos x$ only uses even powers, we can actually just use $n=6$.

28. The n^{th} derivative of $f(x)=\sin x$ is bounded by 1 on intervals containing 0 and π. Thus $|R_n(\pi)| \leq \frac{1}{(n+1)!}(\pi)^{(n+1)}$. When $n=12$, this is less than 0.0001. Since the Maclaurin polynomial of $\sin x$ only uses odd powers, we can actually just use $n=11$.

29. The n^{th} term is $\frac{1}{n!}x^n$.

30. The n^{th} term is: when n is even, $\frac{(-1)^{n/2}}{n!}x^n$; when n is odd, 0.

31. The n^{th} term is x^n.

32. The n^{th} term is $(-1)^n x^n$.

33. The n^{th} term is $(-1)^n\frac{(x-1)^n}{n}$.

34. $1+x+\dfrac{1}{2}x^2+\dfrac{1}{6}x^3+\dfrac{1}{24}x^4$

35. $3+15x+\dfrac{75}{2}x^2+\dfrac{375}{6}x^3+\dfrac{1875}{24}x^4$

36. $1+2x-2x^2+4x^3-10x^4$

Section 8.8

1. A Taylor polynomial is a **polynomial**, containing a finite number of terms. A Taylor series is a **series**, the summation of an infinite number of terms.

2. Theorem 77, entitled “Function and Taylor Series Equality”

3. All derivatives of e^x are e^x which evaluate to 1 at $x=0$.
The Taylor series starts $1+x+\frac{1}{2}x^2+\frac{1}{3!}x^3+\frac{1}{4!}x^4+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$

4. All derivatives of $\sin x$ are either $\pm\cos x$ or $\pm\sin x$, which evaluate to ±1 or 0 at $x=0$. The Taylor series starts
$0+x+0x^2-\frac{1}{6}x^3+0x^4+\frac{1}{120}x^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$

5. The n^{th} derivative of $1/(1-x)$ is $f^{(n)}(x)=(n)!/(1-x)^{n+1}$, which evaluates to $n!$ at $x=0$.
The Taylor series starts $1+x+x^2+x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}x^n$

6. The derivative of $\tan^{-1}x$ is $1/(1+x^2)$. Taking successive derivatives using the Quotient Rule, the derivatives of $\tan^{-1}x$ fall into two categories in terms of their evaluation at $x=0$.
When n is even, $f^{(n)}(x)=(-1)^{(n-1)/2}\dfrac{p(x)}{(1+x^2)^n}$, where $p(x)$ is a polynomial such that $p(0)=0$. Hence $f^{(n)}(0)=0$ when n is even.
When n is odd, $f^{(n)}(x)=(-1)^{(n-1)/2}\dfrac{p(x)}{(1+x^2)^n}$, where $p(x)$ is a polynomial such that $p(0)=(n-1)!$. Hence
$f^{(n)}(0)=(-1)^{(n-1)/2}(n-1)!$ when n is odd. (The unusual power of (-1) is such that every other odd term is negative.)
The Taylor series starts $x-\frac{1}{3}x^3+\frac{1}{5}x^5+\cdots$; by reindexing to only obtain odd powers of x, we get
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$.

7. The Taylor series starts
$0-(x-\pi/2)+0x^2+\frac{1}{6}(x-\pi/2)^3+0x^4-\frac{1}{120}(x-\pi/2)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-\pi/2)^{2n+1}}{(2n+1)!}$

8. The Taylor series starts
$1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

9. $f^{(n)}(x)=(-1)^n e^{-x}$; at $x=0$, $f^{(n)}(0)=-1$ when n is odd and $f^{(n)}(0)=1$ when n is even.
The Taylor series starts $1-x+\frac{1}{2}x^2-\frac{1}{3!}x^3+\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$.

10. $f^{(n)}(x)=(-1)^{n+1}\frac{(n-1)!}{(1+x)^n}$; at $x=0$, $f^{(n)}(0)=(-1)^{n+1}(n-1)!$
The Taylor series starts $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$;
the Taylor series is $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$.

11. $f^{(n)}(x)=(-1)^{n+1}\frac{n!}{(x+1)^{n+1}}$; at $x=1$, $f^{(n)}(1)=(-1)^{n+1}\frac{n!}{2^{n+1}}$
The Taylor series starts
$\frac{1}{2}+\frac{1}{4}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3\cdots$;
the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{2^{n+1}}$.

12. The derivatives of $\sin x$ are $\pm\cos x$ and $\pm\sin x$; at $x=\pi/4$, these derivatives evaluate to $\pm\sqrt{2}/2$.
The Taylor series starts $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\pi/4)-\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^2}{2}-\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^3}{3!}+\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^4}{4!}+\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^5}{5!}\cdots$. Note how the signs are “even, even, odd, odd, even, even, odd, odd,... We saw signs like these in Example 228 of Section 8.1; one way of producing such signs is to raise (-1) to a special quadratic power. While many possibilities exist, one such quadratic is $(n+3)(n+4)/2$.
Thus the Taylor series is $\displaystyle\sum_{n=0}^{\infty}(-1)^{\frac{(n+3)(n+4)}{2}}\frac{\sqrt{2}}{2}\frac{(x-\pi/4)^n}{n!}$.

13. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\big|R_n(x)\big| \leq \frac{\max\big|f^{(n+1)}(z)\big|}{(n+1)!}\big|x^{(n+1)}\big|,$$
where z is between 0 and x.
If $x>0$, then $z<x$ and $f^{(n+1)}(z)=e^z<e^x$. If $x<0$, then $x<z<0$ and $f^{(n+1)}(z)=e^z<1$. So given a fixed x value, let $M=\max\{e^x,1\}$; $f^{(n)}(z)<M$. This allows us to state
$$\big|R_n(x)\big| \leq \frac{M}{(n+1)!}\big|x^{(n+1)}\big|.$$
For any x, $\displaystyle\lim_{n\to\infty}\frac{M}{(n+1)!}\big|x^{(n+1)}\big|=0$. Thus by the Squeeze Theorem, we conclude that $\displaystyle\lim_{n\to\infty}R_n(x)=0$ for all x, and hence
$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\quad\text{for all } x.$$

14. The following argument is essentially the same as that given for $f(x) = \cos x$ in Example 267.
Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|.$$
Since all derivatives of $\sin x$ are $\pm\cos x$ or $\pm\sin x$, whose magnitudes are bounded by 1, we can state
$$\left|R_n(x)\right| \leq \frac{1}{(n+1)!}\left|x^{(n+1)}\right|.$$
For any x, $\lim\limits_{n\to\infty} \dfrac{x^{n+1}}{(n+1)!} = 0$. Thus by the Squeeze Theorem, we conclude that $\lim\limits_{n\to\infty} R_n(x) = 0$ for all x, and hence
$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \quad \text{for all } x.$$

15. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|(x-1)^{(n+1)}\right|,$$
where z is between 1 and x.
Note that $\left|f^{(n+1)}(x)\right| = \frac{n!}{x^{n+1}}$.
We consider the cases when $x > 1$ and when $x < 1$ separately.
If $x > 1$, then $1 < z < x$ and $f^{(n+1)}(z) = \frac{n!}{z^{n+1}} < n!$. Thus
$$\left|R_n(x)\right| \leq \frac{n!}{(n+1)!}\left|(x-1)^{(n+1)}\right| = \frac{(x-1)^{n+1}}{n+1}.$$
For a fixed x,
$$\lim_{n\to\infty}\frac{(x-1)^{n+1}}{n+1} = 0.$$
If $0 < x < 1$, then $x < z < 1$ and $f^{(n+1)}(z) = \frac{n!}{z^{n+1}} < \frac{n!}{x^{n+1}}$.
Thus
$$\left|R_n(x)\right| \leq \frac{n!/x^{n+1}}{(n+1)!}\left|(x-1)^{(n+1)}\right| = \frac{x^{n+1}}{n+1}(1-x)^{n+1}.$$
Since $0 < x < 1$, $x^{n+1} < 1$ and $(1-x)^{n+1} < 1$. We can then extend the inequality from above to state
$$\left|R_n(x)\right| \leq \frac{x^{n+1}}{n+1}(1-x)^{n+1} < \frac{1}{n+1}.$$
As $n \to \infty$, $1/(n+1) \to 0$. Thus by the Squeeze Theorem, we conclude that $\lim\limits_{n\to\infty} R_n(x) = 0$ for all x, and hence
$$\ln x = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n} \quad \text{for all } 0 < x \leq 2.$$

16. Given a value x, the magnitude of the error term $R_n(x)$ is bounded by
$$\left|R_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x^{(n+1)}\right|,$$
where z is between 0 and x.
Note that $\left|f^{(n+1)}(x)\right| = \frac{(n+1)!}{(1-x)^{n+2}}$.
If $0 < x < 1$, then $0 < z < x$ and
$f^{(n+1)}(z) = \frac{(n+1)!}{(1-z)^{n+2}} < \frac{(n+1)!}{(1-x)^{n+2}}$. Thus
$$\left|R_n(x)\right| \leq \frac{(n+1)!}{(1-x)^{n+2}}\frac{1}{(n+1)!}\left|x^{n+1}\right| = \frac{(x-1)^{n+1}}{n+1}.$$
For a fixed x,
$$\lim_{n\to\infty}\frac{(x-1)^{n+1}}{n+1} = 0,$$
hence
$$\frac{1}{1-x} = \sum_{n=0}^{\infty}x^n \text{ on } (-1,0).$$

17. Given $\cos x = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,
$\cos(-x) = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos x$, as all powers in the series are even.

18. Given $\sin x = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,
$\sin(-x) = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(-x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{-x^{2n+1}}{(2n+1)!} = -\sin x$, as all powers in the series are odd.

19. Given $\sin x = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$,
$\dfrac{d}{dx}\big(\sin x\big) = \dfrac{d}{dx}\left(\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\right) = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos x$. (The summation still starts at $n = 0$ as there was no constant term in the expansion of $\sin x$).

20. Given $\cos x = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$,
$\dfrac{d}{dx}\big(\cos x\big) = \dfrac{d}{dx}\left(\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\right) = \displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{(2n)x^{2n-1}}{(2n)!} = \sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!}$. We can re-index this summation to start at $n = 0$ by replacing n with $n+1$ in the summation:
$$\sum_{n=1}^{\infty}(-1)^n\frac{x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)!}.$$
Note that this series has the opposite sign of the Taylor series for $\sin x$; thus $\frac{d}{dx}(\cos x) = -\sin x$.

21. $1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16} - \dfrac{5x^4}{128}$

22. $1 - \dfrac{x}{2} + \dfrac{3x^2}{8} - \dfrac{5x^3}{16} + \dfrac{35x^4}{128}$

23. $1 + \dfrac{x}{3} - \dfrac{x^2}{9} + \dfrac{5x^3}{81} - \dfrac{10x^4}{243}$

24. $1 + 4x + 6x^2 + 4x^3 + x^4$ (note the series is finite, and the formula still applies)

25. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$.

26. $\displaystyle\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}$.

27. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(2x+3)^{2n+1}}{(2n+1)!}$.

28. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(x/2)^{2n+1}}{(2n+1)}$.

29. $x + x^2 + \dfrac{x^3}{3} - \dfrac{x^5}{30}$

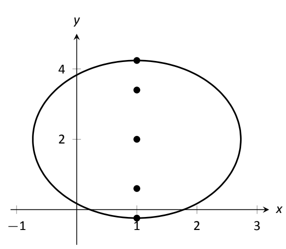
30. $1+\dfrac{x}{2}-\dfrac{5x^2}{8}-\dfrac{3x^3}{16}$

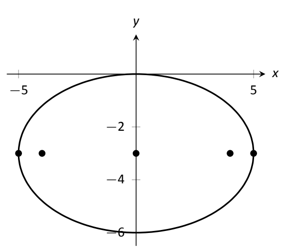
31. $\displaystyle\int_0^{\sqrt{\pi}} \sin\left(x^2\right)\,dx \approx \int_0^{\sqrt{\pi}}\left(x^2-\frac{x^6}{6}+\frac{x^{10}}{120}-\frac{x^{14}}{5040}\right)dx =$ 0.8877

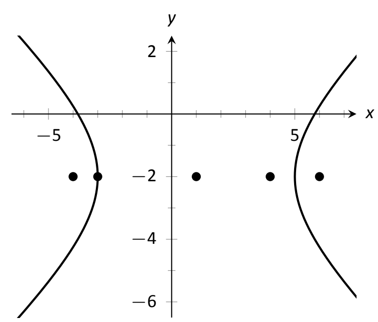
32. $\displaystyle\int_0^{\pi^2/4} \cos\left(\sqrt{x}\right)\,dx \approx \int_0^{\pi^2/4}\left(1-\frac{x}{2}+\frac{x^2}{24}-\frac{x^3}{720}\right)\,dx =$ 1.1412. (Actual answer: $\pi-2$)

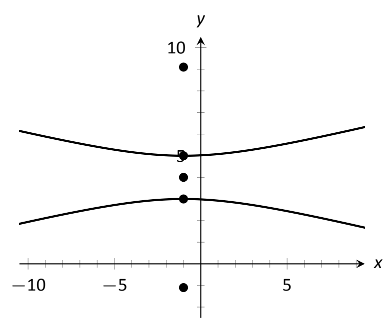
Chapter 9

Section 9.1

1. When defining the conics as the intersections of a plane and a double napped cone, degenerate conics are created when the plane intersects the tips of the cones (usually taken as the origin). Nondegenerate conics are formed when this plane does not contain the origin.
2. Answers will vary.
3. Hyperbola
4. With the equation $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, the ellipse has a horizontal major axis if $a>b$. But the coefficient of the x^2 term is $1/a^2$ (not a^2), so if $1/a^2<1/b^2$, then $a>b$ and the major axis is horizontal.
5. With a horizontal transverse axis, the x^2 term has a positive coefficient; with a vertical transverse axis, the y^2 term has a positive coefficient.
6. $y=\frac{1}{2}(x-3)^2+\frac{3}{2}$
7. $y=\frac{-1}{12}(x+1)^2-1$
8. $x=-\frac{1}{4}(y-5)^2+2$
9. $x=y^2$
10. $y=-\frac{1}{4}(x-1)^2+2$
11. $x=-\frac{1}{12}y^2$
12. $y=4x^2$
13. $x=-\frac{1}{8}(y-3)^2+2$
14. focus: $(0,1)$; directrix: $y=-1$. The point P is 2 units from each.
15. focus: $(5,2)$; directrix: $x=1$. The point P is 10 units from each.
16. 

17. 

18. $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$; foci at $(-1\pm\sqrt{5},2)$; $e=\sqrt{5}/3$
19. $\frac{(x-1)^2}{1/4}+\frac{y^2}{9}=1$; foci at $(1,\pm\sqrt{8.75})$; $e=\sqrt{8.75}/3\approx 0.99$
20. $\frac{x^2}{9}+\frac{y^2}{5}=1$
21. $\frac{(x-2)^2}{25}+\frac{(y-3)^2}{16}=1$
22. $\frac{(x-2)^2}{45}+\frac{y^2}{49}=1$
23. $\frac{(x+1)^2}{9}+\frac{(y-1)^2}{25}=1$
24. $\frac{(x-1)^2}{2}+(y-2)^2=1$
25. $\frac{x^2}{3}+\frac{y^2}{5}=1$
26. $\frac{x^2}{4}+\frac{(y-3)^2}{6}=1$
27. $\frac{(x-2)^2}{4}+\frac{(y-2)^2}{4}=1$
28. (a) $c=\sqrt{12-4}=2\sqrt{2}$.
    (b) The sum of distances for each point is $2\sqrt{12}\approx 6.9282$.
29. $x^2-\frac{y^2}{3}=1$
30. $y^2-\frac{x^2}{24}=1$
31. $\frac{(y-3)^2}{4}-\frac{(x-1)^2}{9}=1$
32. $\frac{(x-1)^2}{9}-\frac{(y-3)^2}{4}=1$
33. 

34. 

35. $\frac{x^2}{4}-\frac{y^2}{5}=1$
36. $\frac{y^2}{4}-\frac{x^2}{5}=1$
37. $\frac{(x-3)^2}{16}-\frac{(y-3)^2}{9}=1$
38. $\frac{(y-3)^2}{9}-\frac{(x-3)^2}{16}=1$
39. $\frac{x^2}{4}-\frac{y^2}{3}=1$
40. $\frac{x^2}{3}-\frac{(y-1)^2}{9}=1$
41. $(y-2)^2-\frac{x^2}{10}=1$
42. $4y^2-\frac{x^2}{4}=1$

43. (a) Solve for c in $e = c/a$: $c = ae$. Thus $a^2e^2 = a^2 - b^2$, and $b^2 = a^2 - a^2e^2$. The result follows.

(b) Mercury: $x^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
Earth: $x^2 + y^2/(0.99986)^2 = 1$
Mars: $x^2/(1.524)^2 + y^2/(1.517)^2 = 1$

(c) Mercury: $(x - 0.08)^2/(0.387)^2 + y^2/(0.3787)^2 = 1$
Earth: $(x - 0.0167)^2 + y^2/(0.99986)^2 = 1$
Mars: $(x - 0.1423)^2/(1.524)^2 + y^2/(1.517)^2 = 1$

44. The sound originated from a point approximately 31m to the left of B and 1340m above it.

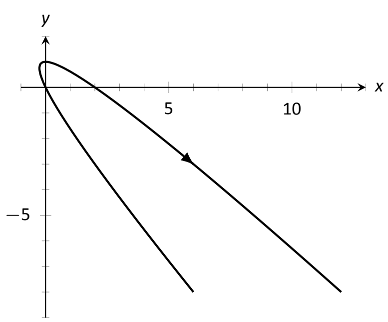
Section 9.2

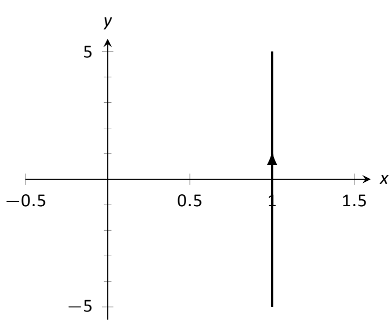
1. T

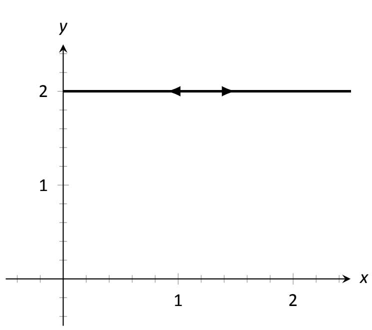
2. orientation

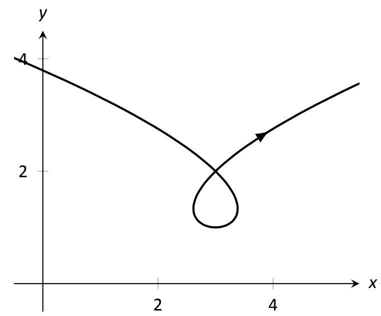
3. rectangular

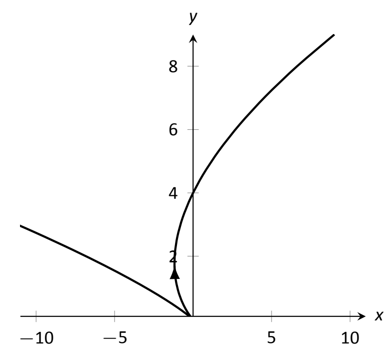
4. Answers will vary.

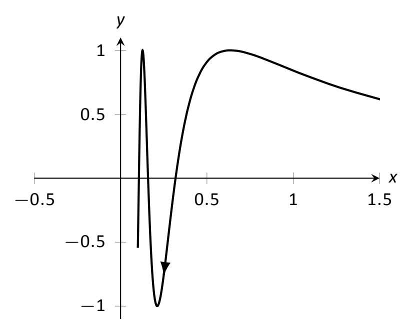
5. 

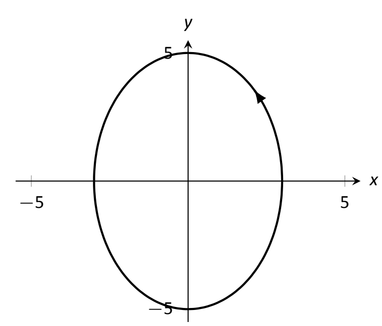
6. 

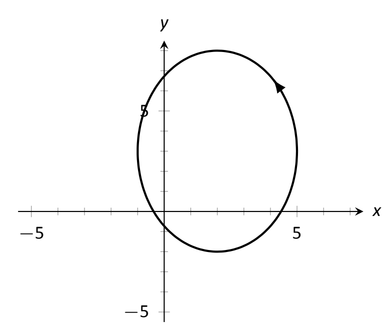
7. 

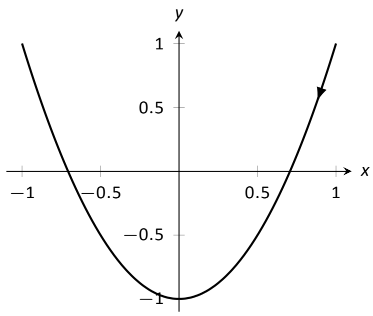
8. 

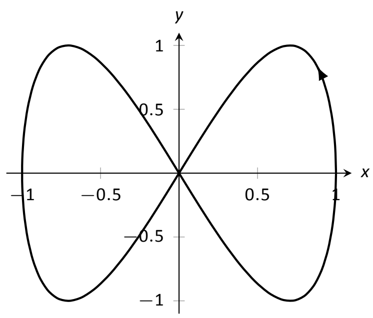
9. 

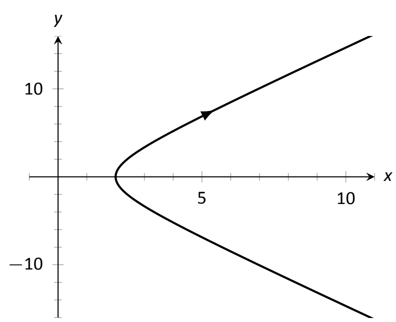
10. 

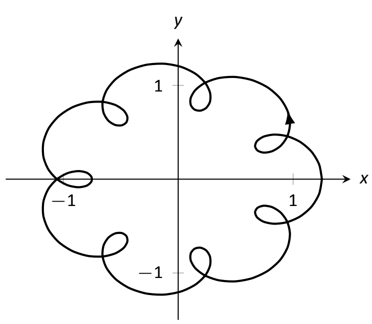
11. 

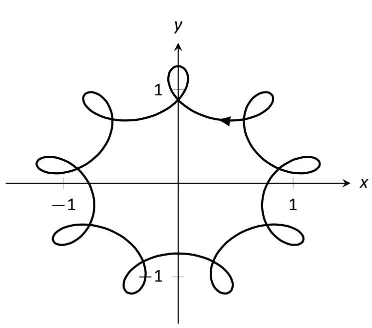
12. 

13. 

14. 

15. 

16. 

17. 

18. (a) Traces the parabola $y = x^2$, moves from left to right.
(b) Traces the parabola $y = x^2$, but only from $-1 \leq x \leq 1$; traces this portion back and forth infinitely.
(c) Traces the parabola $y = x^2$, but only for $0 < x$. Moves left to right.
(d) Traces the parabola $y = x^2$, moves from right to left.

19. (a) Traces a circle of radius 1 counterclockwise once.
(b) Traces a circle of radius 1 counterclockwise over 6 times.
(c) Traces a circle of radius 1 clockwise infinite times.
(d) Traces an arc of a circle of radius 1, from an angle of -1 radians to 1 radian, twice.

20. $y = -1.5x + 8.5$

21. $x^2 - y^2 = 1$

22. $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$

23. $y = x^{3/2}$

24. $y = 2x + 3$

25. $y = x^3 - 3$

26. $y = e^{2x} - 1$

27. $y^2 - x^2 = 1$

28. $x^2 - y^2 = 1$

29. $x = 1 - 2y^2$

30. $y = \frac{b}{a}(x - x_0) + y_0$; line through (x_0, y_0) with slope b/a.

31. $x^2 + y^2 = r^2$; circle centered at $(0, 0)$ with radius r.

32. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$; ellipse centered at (h, k) with horizontal axis of length $2a$ and vertical axis of length $2b$.

33. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$; hyperbola centered at (h, k) with horizontal transverse axis and asymptotes with slope b/a. The parametric equations only give half of the hyperbola. When $a > 0$, the right half; when $a < 0$, the left half.

34. $x = (t + 11)/6$, $y = (t^2 - 97)/12$. At $t = 1$, $x = 2$, $y = -8$.
$y' = 6x - 11$; when $x = 2$, $y' = 1$.

35. $x = \ln t$, $y = t$. At $t = 1$, $x = 0$, $y = 1$.
$y' = e^x$; when $x = 0$, $y' = 1$.

36. $x = \cos^{-1} t$, $y = \sqrt{1 - t^2}$. At $t = 1$, $x = 0$, $y = 0$.
$y' = \cos x$; when $x = 0$, $y' = 1$.

37. $x = 1/(4t^2)$, $y = 1/(2t)$. At $t = 1$, $x = 1/4$, $y = 1/2$.
$y' = 1/(2\sqrt{x})$; when $x = 1/4$, $y' = 1$.

38. $t = \pm 1$

39. $t = -1,\ 2$

40. $t = \pi/2, 3\pi/2$

41. $t = \pi/6, \pi/2, 5\pi/6$

42. $t = -1$

43. $t = 2$

44. $t = \ldots \pi/2,\ 3\pi/2,\ 5\pi/2,\ \ldots$

45. $t = \ldots 0,\ 2\pi,\ 4\pi,\ \ldots$

46. $x = 4t$, $y = -16t^2 + 64t$

47. $x = 50t$, $y = -16t^2 + 64t$

48. $x = 10t$, $y = -16t^2 + 320t$

49. $x = 2\cos t$, $y = -2\sin t$; other answers possible

50. $x = 3\cos(2\pi t) + 1$, $y = 3\sin(2\pi t) + 1$; other answers possible

51. $x = \cos t + 1$, $y = 3\sin t + 3$; other answers possible

52. $x = 5\cos t$, $y = \sqrt{24}\sin t$; other answers possible

53. $x = \pm \sec t + 2$, $y = \sqrt{8}\tan t - 3$; other answers possible

54. $x = 2\tan t$, $y = \pm 6\sec t$; other answers possible

Section 9.3

1. F

2. t

3. F

4. T

5. (a) $\frac{dy}{dx} = 2t$
   (b) Tangent line: $y = 2(x-1)+1$; normal line: $y = -1/2(x-1)+1$

6. (a) $\frac{dy}{dx} = 10\sqrt{t}$
   (b) Tangent line: $y = 20(x-2)+22$; normal line: $y = -1/20(x-2)+22$

7. (a) $\frac{dy}{dx} = \frac{2t+1}{2t-1}$
   (b) Tangent line: $y = 3x+2$; normal line: $y = -1/3x+2$

8. (a) $\frac{dy}{dx} = \frac{3t^2}{2t}$
   (b) $t = 0$: Tangent line: $x = -1$; normal line: $y = 0$
   $t = 1$: Tangent line: $y = x$; normal line: $y = -x$

9. (a) $\frac{dy}{dx} = \csc t$
   (b) $t = \pi/4$: Tangent line: $y = \sqrt{2}(x-\sqrt{2})+1$; normal line: $y = -1/\sqrt{2}(x-\sqrt{2})+1$

10. (a) $\frac{dy}{dx} = -2\cos(2t)\csc t$
    (b) $t = \pi/4$: Tangent line: $y = 1$; normal line: $x = \sqrt{2}/2$

11. (a) $\frac{dy}{dx} = \frac{\cos t\sin(2t)+\sin t\cos(2t)}{-\sin t\sin(2t)+2\cos t\cos(2t)}$
    (b) Tangent line: $y = x - \sqrt{2}$; normal line: $y = -x - \sqrt{2}$

12. (a) $\frac{dy}{dx} = \frac{\sin(t)+10\cos(t)}{\cos(t)-10\sin(t)}$
    (b) Tangent line: $y = -x/10 + e^{\pi/20}$; normal line: $y = 10x + e^{\pi/20}$

13. $t = 0$

14. $t = 0$ (though this uses a one–sided limit, as $x(t)$ is not defined for $t < 0$.

15. $t = -1/2$

16. $t = \pm 1/\sqrt{3}$

17. The graph does not have a horizontal tangent line.

18. $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

19. The solution is non-trivial; use identities $\sin(2t) = 2\sin t\cos t$ and $\cos(2t) = \cos^2 t - \sin^2 t$ to rewrite $g'(t) = 2\sin t(2\cos^2 t - \sin^2 t)$. On $[0, 2\pi]$, $\sin t = 0$ when $t = 0, \pi, 2\pi$, and $2\cos^2 t - \sin^2 t = 0$ when $t = \tan^{-1}(\sqrt{2}),\ \pi \pm \tan^{-1}(\sqrt{2}),\ 2\pi - \tan^{-1}(\sqrt{2})$.

20. $t = \tan^{-1}(-10),\ \tan^{-1}(-10)+\pi$

21. $t_0 = 0$; $\lim_{t\to 0} \frac{dy}{dx} = 0$.

22. $t_0 = 2$; $\lim_{t\to 2} \frac{dy}{dx} = 1$.

23. $t_0 = 1$; $\lim_{t\to 1} \frac{dy}{dx} = \infty$.

24. $t_0 = \ldots, -\pi/2, 0, \pi/2, \pi, \ldots$; $\lim_{t\to 0} \frac{dy}{dx} = 1$.

25. $\frac{d^2y}{dx^2} = 2$; always concave up

26. $\frac{d^2y}{dx^2} = 10$; always concave up

27. $\frac{d^2y}{dx^2} = -\frac{4}{(2t-1)^3}$; concave up on $(-\infty, 1/2)$; concave down on $(1/2, \infty)$.

28. $\frac{d^2y}{dx^2} = \frac{3t^2+1}{4t^3}$; concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.

29. $\frac{d^2y}{dx^2} = -\cot^3 t$; concave up on $(-\infty, 0)$; concave down on $(0, \infty)$.

30. $\frac{d^2y}{dx^2} = \frac{\cos t\sin(2t)+2\sin t\cos(2t)}{(-\sin t\sin(2t)+2\cos t\cos(2t))^2}$; concavity switches at $t = \tan^{-1}(1/\sqrt{2}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{2}),\ \pi + \tan^{-1}(1/\sqrt{2}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{2})$

31. $\frac{d^2y}{dx^2} = \frac{4(13+3\cos(4t))}{(\cos t+3\cos(3t))^3}$, obtained with a computer algebra system; concave up on $\big(-\tan^{-1}(\sqrt{2}/2), \tan^{-1}(\sqrt{2}/2)\big)$, concave down on $\big(-\pi/2, -\tan^{-1}(\sqrt{2}/2)\big) \cup \big(\tan^{-1}(\sqrt{2}/2), \pi/2\big)$

32. $\frac{d^2y}{dx^2} = \frac{1010}{e^{t/10}\left(\cos t - 10\sin t\right)^3}$; concavity switches at $t = \tan^{-1}(1/10) + n\pi$, where n is an integer.

33. $L = 6\pi$

34. On $[0, 2\pi]$, arc length is $L = \sqrt{101}(e^{\pi/5} - 1)$; on $[2\pi, 4\pi]$, $L = \sqrt{101}(e^{2\pi/5} - 1)$.

35. $L = 2\sqrt{34}$

36. $L = 4\sqrt{2} - 2$

37. $L \approx 2.4416$ (actual value: $L = 2.42211$)

38. $L \approx 9.73004$ (actual value: $L = 9.42943$)

39. $L \approx 4.19216$ (actual value: $L = 4.18308$)

40. Formula: $C \approx 25.9062$; Simpson's Rule: $C \approx 25.4786$ (actual value: $C = 25.527$)

41. The answer is 16π for both (of course), but the integrals are different.

42. $8\pi^2$.

43. $SA \approx 8.50101$ (actual value $SA = 8.02851$

44. $SA \approx 1.36751$ (actual value $SA = 1.36707$

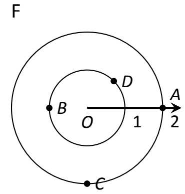
Section 9.4

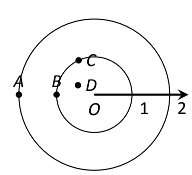
1. Answers will vary.

2. F

3. T

4. F

5. 

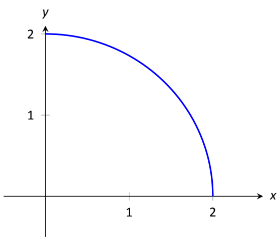
6. 

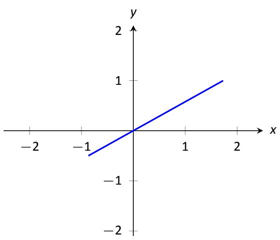
7. $A = P(2.5, \pi/4)$ and $P(-2.5, 5\pi/4)$;
   $B = P(-1, 5\pi/6)$ and $P(1, 11\pi/6)$;
   $C = P(3, 4\pi/3)$ and $P(-3, \pi/3)$;
   $D = P(1.5, 2\pi/3)$ and $P(-1.5, 5\pi/3)$;

8. $A = P(2, \pi/6)$ and $P(-2, -5\pi/6)$;
$B = P(1, -\pi/3)$ and $P(-1, 2\pi/3)$;
$C = P(2, 3\pi/4)$ and $P(-2, -\pi/4)$;
$D = P(2.5, \pi)$ and $P(2.5, -\pi)$;

9. $A = (\sqrt{2}, \sqrt{2})$
$B = (\sqrt{2}, -\sqrt{2})$
$C = P(\sqrt{5}, -0.46)$
$D = P(\sqrt{5}, 2.68)$

10. $A = (-3, 0)$
$B = (-1/2, \sqrt{3}/2)$
$C = P(4, \pi/2)$
$D = P(2, -\pi/3)$

11.


12.


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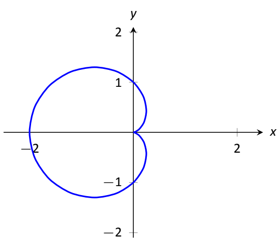

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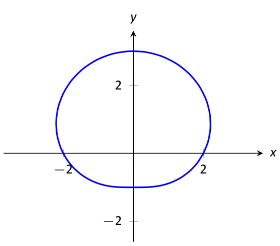

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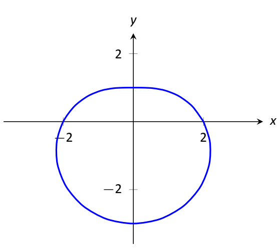

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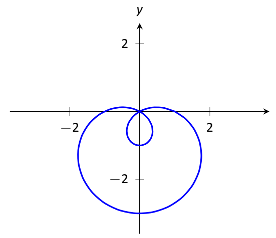

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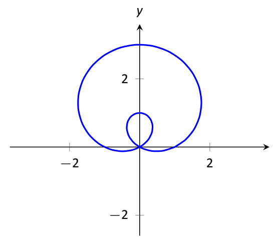

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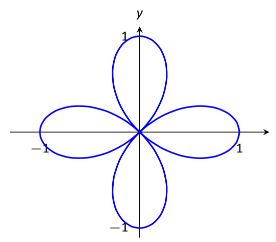

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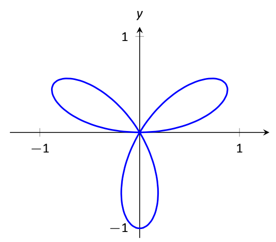

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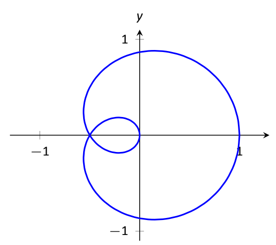

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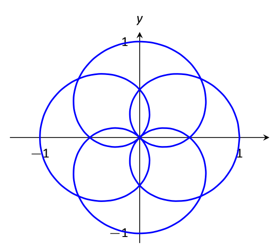

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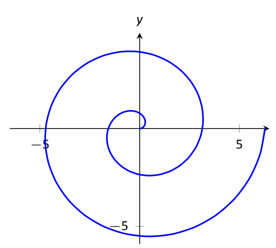
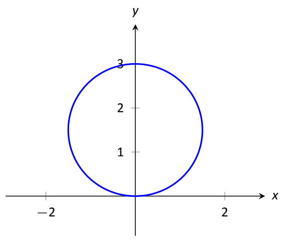
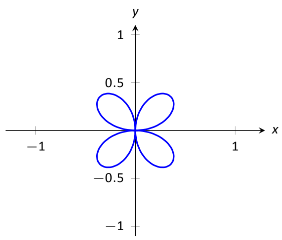
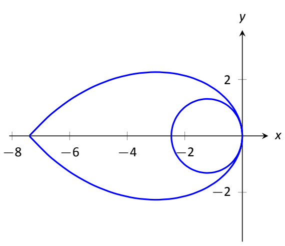
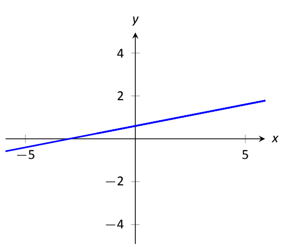
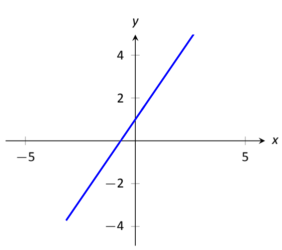


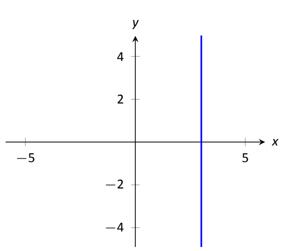
23. 

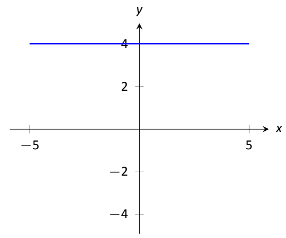
24. 

25. 

26. 

27. 

28. 

29. 

30. $(x-1)^2+y^2=1$

31. $x^2+(y+2)^2=4$

32. $(x-1/2)^2+(y-1/2)^2=1/2$

33. $y=2/5x+7/5$

34. $x=3$

35. $y=4$

36. $x^4+x^2y^2x^2-y^2=0$

37. $x^2+y^2=4$

38. $y=x/\sqrt{3}$

39. $\theta=\pi/4$

40. $r=7/(\sin\theta-4\cos\theta)$

41. $r=5\sec\theta$

42. $r=5\csc\theta$

43. $r=\cos\theta/\sin^2\theta$

44. $r=1/\sqrt[3]{\cos^2\theta\sin\theta}$

45. $r=\sqrt{7}$

46. $r=-2\cos\theta$

47. $P(\sqrt{3}/2,\pi/6)$, $P(0,\pi/2)$, $P(-\sqrt{3}/2,5\pi/6)$

48. $P(1,0)$, $P(0,\pi/2)=P(0,\pi/4)$, $P(-1/2,\pi/3)$

49. $P(0,0)=P(0,\pi/2)$, $P(\sqrt{2},\pi/4)$

50. $P(\sqrt{3}/2,\pi/3)=P(-\sqrt{3}/2,4\pi/3)$,
$P(\sqrt{3}/2,2\pi/3)=P(-\sqrt{3}/2,5\pi/3)$, $P(0,\pi/2)$

51. $P(\sqrt{2}/2,\pi/12)$, $P(-\sqrt{2}/2,5\pi/12)$, $P(\sqrt{2}/2,3\pi/4)$

52. $P(3/2,\pi/3)$, $P(3/2,-\pi/3)$

53. For all points, $r=1$; $\theta=$
$\pi/12,\ 5\pi/12,\ 7\pi/12,\ 11\pi/12,\ 13\pi/12,\ 17\pi/12,\ 19\pi/12,\ 23\pi/12.$

54. $P(0,0)=P(0,3\pi/2)$, $P(1+\sqrt{2}/2,3\pi/4)$, $P(1-\sqrt{2}/2,7\pi/4)$

55. Answers will vary. If m and n do not have any common factors, then an interval of $2n\pi$ is needed to sketch the entire graph.

56. Answers will vary.

Section 9.5

1. Using $x=r\cos\theta$ and $y=r\sin\theta$, we can write $x=f(\theta)\cos\theta$, $y=f(\theta)\sin\theta$.

2. rectangles; sectors of circles

3. (a) $\frac{dy}{dx}=-\cot\theta$

   (b) tangent line: $y=-(x-\sqrt{2}/2)+\sqrt{2}/2$; normal line: $y=x$

4. (a) $\frac{dy}{dx}=1/2(\tan\theta-\cot\theta)$

   (b) tangent line: $y=1/2$; normal line: $x=1/2$

5. (a) $\frac{dy}{dx} = \frac{\cos\theta(1+2\sin\theta)}{\cos^2\theta - \sin\theta(1+\sin\theta)}$

   (b) tangent line: $x = 3\sqrt{3}/4$; normal line: $y = 3/4$

6. (a) $\frac{dy}{dx} = \frac{3\sin^2(t)+(1-3\cos(t))\cos(t)}{3\sin(t)\cos(t)-\sin(t)(1-3\cos(t))}$

   (b) tangent line: $y = \frac{1}{1+3\sqrt{2}}(x + (1/\sqrt{2}+3/2)) + 1/\sqrt{2}+3/2 \approx y = 0.19(x+2.21)+2.21$; normal line: $y = -(1+3\sqrt{2})(x+(1/\sqrt{2}+3/2)) + 1/\sqrt{2}+3/2$

7. (a) $\frac{dy}{dx} = \frac{\theta\cos\theta+\sin\theta}{\cos\theta-\theta\sin\theta}$

   (b) tangent line: $y = -2/\pi x + \pi/2$; normal line: $y = \pi/2x + \pi/2$

8. (a) $\frac{dy}{dx} = \frac{\cos\theta\cos(3\theta)-3\sin\theta\sin(3\theta)}{-\cos(3\theta)\sin\theta-3\cos\theta\sin(3\theta)}$

   (b) tangent line: $y = x/\sqrt{3}$; normal line: $y = -\sqrt{3}x$

9. (a) $\frac{dy}{dx} = \frac{4\sin(t)\cos(4t)+\sin(4t)\cos(t)}{4\cos(t)\cos(4t)-\sin(t)\sin(4t)}$

   (b) tangent line: $y = 5\sqrt{3}(x+\sqrt{3}/4) - 3/4$; normal line: $y = -1/5\sqrt{3}(x+\sqrt{3}/4) - 3/4$

10. (a) $\frac{dy}{dx} = 1$

    (b) tangent line: $y = x+1$; normal line: $y = -x-1$

11. horizontal: $\theta = \pi/2, 3\pi/2$;
    vertical: $\theta = 0, \pi, 2\pi$

12. horizontal: $\theta = 0, \pi/2, \pi$;
    vertical: $\theta = \pi/4, 3\pi/4$

13. horizontal: $\theta = \tan^{-1}(1/\sqrt{5}),\ \pi/2,\ \pi - \tan^{-1}(1/\sqrt{5}),\ \pi + \tan^{-1}(1/\sqrt{5}),\ 3\pi/2,\ 2\pi - \tan^{-1}(1/\sqrt{5})$;
    vertical: $\theta = 0,\ \tan^{-1}(\sqrt{5}),\ \pi - \tan^{-1}(\sqrt{5}),\ \pi,\ \pi + \tan^{-1}(\sqrt{5}),\ 2\pi - \tan^{-1}(\sqrt{5})$

14. horizontal: $\theta = \pi/3,\ 5\pi/3$;
    vertical: $\theta = 0,\ 2\pi/3,\ 4\pi/3,\ 2\pi$
    At $\theta = \pi$, $\frac{dy}{dx} = 0/0$; apply L'Hopital's Rule to find that $\frac{dy}{dx} \to 0$ as $\theta \to \pi$.

15. In polar: $\theta = 0 \ \cong\ \theta = \pi$
    In rectangular: $y = 0$

16. In polar: $\theta = 0,\ \theta = \pi/3,\ \theta = 2\pi/3$.
    In rectangular: $y = 0$, $y = \sqrt{3}x$, and $y = -\sqrt{3}x$.

17. area = 4π

18. area = 25π

19. area = $\pi/12$

20. area = $3\pi/2$

21. area = $\pi - 3\sqrt{3}/2$

22. area = $2\pi + 3\sqrt{3}/2$

23. area = $\pi + 3\sqrt{3}$

24. area = 1

25. area = $\displaystyle\int_{\pi/12}^{\pi/3} \frac{1}{2}\sin^2(3\theta)\,d\theta - \int_{\pi/12}^{\pi/6} \frac{1}{2}\cos^2(3\theta)\,d\theta = \frac{1}{12} + \frac{\pi}{24}$

26. area = $\frac{1}{32}(4\pi - 3\sqrt{3})$

27. area = $\displaystyle\int_0^{5\pi/12} \frac{1}{2}(1-\cos\theta)^2\,d\theta + \int_{5\pi/12}^{\pi/2} \frac{1}{2}(3\cos\theta)^2\,d\theta = \frac{1}{4}(2\pi - \sqrt{6} - \sqrt{2} - 2) \approx 0.105$

28. $x'(\theta) = f'(\theta)\cos\theta - f(\theta)\sin\theta$, $y'(\theta) = f'(\theta)\sin\theta + f(\theta)\cos\theta$. Square each and add; applying the Pythagoran Theorem twice achieves the result.

29. 4π

30. 4π

31. $L \approx 2.2592$; (actual value $L = 2.22748$)

32. $L \approx 7.62933$; (actual value $L = 8$)

33. $SA = 16\pi$

34. $SA = 4\pi$

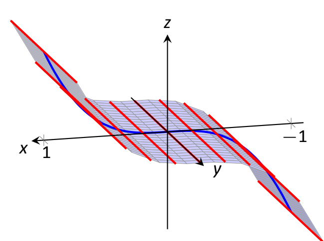
35. $SA = 32\pi/5$

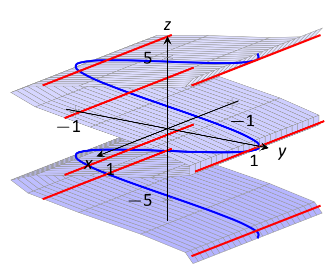
36. $SA = 4\pi^2$

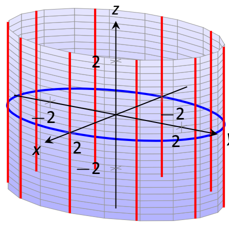
37. $SA = 36\pi$

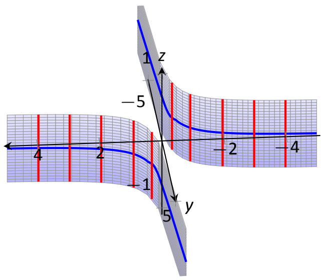
Chapter 10

Section 10.1

1. right hand
2. line; plane
3. curve (a parabola); surface (a cylinder)
4. a hyperbolic paraboloid
5. a hyperboloid of two sheets
6. a hyperboloid of one sheet
7. $||\overline{AB}|| = \sqrt{6}$; $||\overline{BC}|| = \sqrt{17}$; $||\overline{AC}|| = \sqrt{11}$. Yes, it is a right triangle as $||\overline{AB}||^2 + ||\overline{AC}||^2 = ||\overline{BC}||^2$.
8. Yes, as opposite sides have equal length. $||\overline{AB}|| = \sqrt{21} = ||\overline{CD}||$; $||\overline{BC}|| = \sqrt{6} = ||\overline{AD}||$.
9. Center at $(4, -1, 0)$; radius = 3
10. Center at $(-2, 1, 2)$; radius = $\sqrt{5}$
11. Interior of a sphere with radius 1 centered at the origin.
12. Region bounded between the planes $x = 0$ (the $y - z$ coordinate plane) and $x = 3$.
13. The first octant of space; all points (x, y, z) where each of x, y and z are positive. (Analogous to the first quadrant in the plane.)
14. All points in space where the y value is greater than 3; viewing space as often depicted in this text, this is the region "to the right" of the plane $y = 3$ (which is parallel to the $x - z$ coordinate plane.)
15. 

16. 

17. 

18. 

19. $y^2 + z^2 = x^4$

20. $y^2 + z^2 = x^4$

21. $z = (\sqrt{x^2 + y^2})^2 = x^2 + y^2$

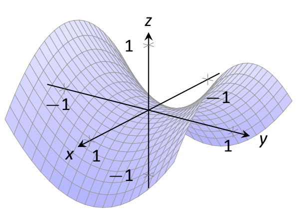
22. $z = \frac{1}{\sqrt{x^2+y^2}}$

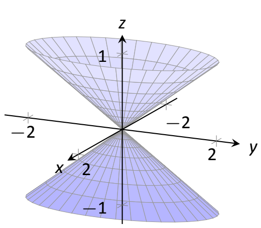
23. (a) $x = y^2 + \dfrac{z^2}{9}$

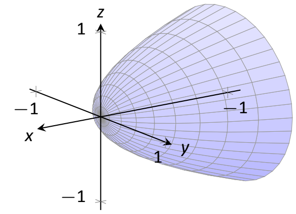
24. (b) $x^2 - y^2 + z^2 = 0$

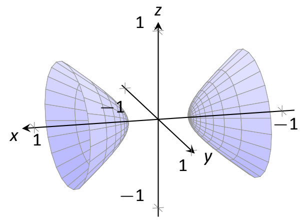
25. (b) $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

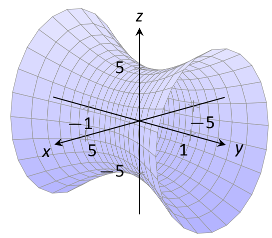
26. (a) $y^2 - x^2 - z^2 = 1$

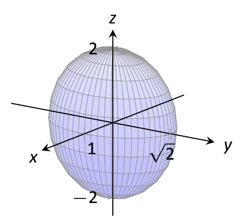
27. 

28. 

29. 

30. 

31. 

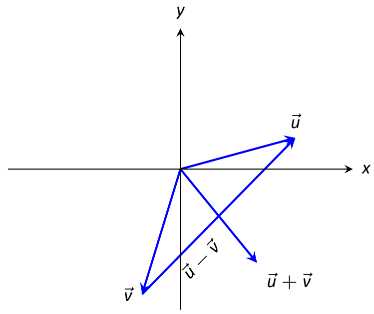
32. 

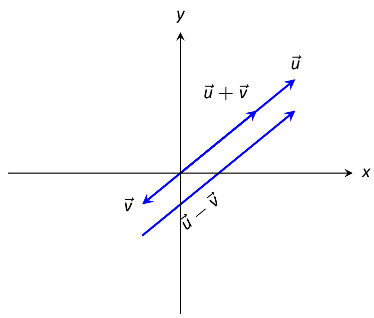
Section 10.2

1. Answers will vary.
2. $(1, 2)$ is a point; $\langle 1, 2\rangle$ is a vector that describes a displacement of 1 unit in the x-direction and 2 units in the y-direction.
3. A vector with magnitude 1.
4. Their respective unit vectors are parallel; unit vectors $\vec{u}_1$ and $\vec{u}_2$ are parallel if $\vec{u}_1 = \pm\vec{u}_2$.
5. It stretches the vector by a factor of 2, and points it in the opposite direction.
6. $\overrightarrow{PQ} = \langle 1, 6\rangle = 1\vec{i} + 6\vec{j}$
7. $\overrightarrow{PQ} = \langle -4, 4\rangle = -4\vec{i} + 4\vec{j}$
8. $\overrightarrow{PQ} = \langle 6, -1, 6\rangle = 6\vec{i} - \vec{j} + 6\vec{k}$
9. $\overrightarrow{PQ} = \langle 2, 2, 0\rangle = 2\vec{i} + 2\vec{j}$
10. (a) $\vec{u} + \vec{v} = \langle 2, -1\rangle$; $\vec{u} - \vec{v} = \langle 0, -3\rangle$; $2\vec{u} - 3\vec{v} = \langle -1, -7\rangle$.
    (c) $\vec{x} = \langle 1/2, 2\rangle$.

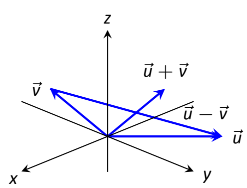
11. (a) $\vec{u}+\vec{v}=\langle 3,2,1\rangle$; $\vec{u}-\vec{v}=\langle -1,0,-3\rangle$; $\pi\vec{u}-\sqrt{2}\vec{v}=\langle \pi-2\sqrt{2}, \pi-\sqrt{2}, -\pi-2\sqrt{2}\rangle$.

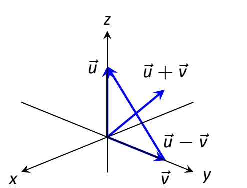
(c) $\vec{x}=\langle -1,0,-3\rangle$.

12. 

13. 

Sketch of $\vec{u}-\vec{v}$ shifted for clarity.

14. 

15. 

16. $||\,\vec{u}\,||=\sqrt{5}$, $||\,\vec{v}\,||=\sqrt{13}$, $||\,\vec{u}+\vec{v}\,||=\sqrt{26}$, $||\,\vec{u}-\vec{v}\,||=\sqrt{10}$

17. $||\,\vec{u}\,||=\sqrt{17}$, $||\,\vec{v}\,||=\sqrt{3}$, $||\,\vec{u}+\vec{v}\,||=\sqrt{14}$, $||\,\vec{u}-\vec{v}\,||=\sqrt{26}$

18. $||\,\vec{u}\,||=\sqrt{5}$, $||\,\vec{v}\,||=3\sqrt{5}$, $||\,\vec{u}+\vec{v}\,||=2\sqrt{5}$, $||\,\vec{u}-\vec{v}\,||=4\sqrt{5}$

19. $||\,\vec{u}\,||=7$, $||\,\vec{v}\,||=35$, $||\,\vec{u}+\vec{v}\,||=42$, $||\,\vec{u}-\vec{v}\,||=28$

20. When $\vec{u}$ and $\vec{v}$ have the same direction. (Note: parallel is not enough.)

21. $\vec{u}=\langle 3/\sqrt{30}, 7/\sqrt{30}\rangle$

22. $\vec{u}=\langle 0.6, 0.8\rangle$

23. $\vec{u}=\langle 1/3, -2/3, 2/3\rangle$

24. $\vec{u}=\langle 1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3}\rangle$

25. $\vec{u}=\langle \cos 50^\circ, \sin 50^\circ\rangle \approx \langle 0.643, 0.766\rangle$.

26. $\vec{u}=\langle \cos 120^\circ, \sin 120^\circ\rangle = \langle -1/2, \sqrt{3}/2\rangle$.

27.
$$\begin{aligned}||\,\vec{u}\,|| &= \sqrt{\sin^2\theta\cos^2\varphi+\sin^2\theta\sin^2\varphi+\cos^2\theta}\\ &= \sqrt{\sin^2\theta(\cos^2\varphi+\sin^2\varphi)+\cos^2\theta}\\ &= \sqrt{\sin^2\theta+\cos^2\theta}\\ &= 1.\end{aligned}$$

28. The force on each chain is $100/\sqrt{3}\approx 57.735$lb.

29. The force on each chain is 100lb.

30. The force on the chain with angle θ is approx. 45.124lb; the force on the chain with angle φ is approx. 59.629lb.

31. The force on each chain is 50lb.

32. $\theta=45^\circ$; the weight is lifted 0.29 ft (about 3.5in).

33. $\theta=5.71^\circ$; the weight is lifted 0.005 ft (about 1/16th of an inch).

34. $\theta=45^\circ$; the weight is lifted 2.93 ft.

35. $\theta=84.29^\circ$; the weight is lifted 9 ft.

Section 10.3

1. Scalar

2. The magnitude of a vectors is the square root of the dot product of a vector with itself; that is, $||\,\vec{v}\,||=\sqrt{\vec{v}\cdot\vec{v}}$.

3. By considering the sign of the dot product of the two vectors. If the dot product is positive, the angle is acute; if the dot product is negative, the angle is obtuse.

4. "Perpendicular" is one answer.

5. -22

6. 33

7. 3

8. 0

9. not defined

10. 0

11. Answers will vary.

12. Answers will vary.

13. $\theta=0.3218\approx 18.43^\circ$

14. $\theta=1.6476\approx 94.4^\circ$

15. $\theta=\pi/4=45^\circ$

16. $\theta=\pi/2=90^\circ$

17. Answers will vary; two possible answers are $\langle -7,4\rangle$ and $\langle 14,-8\rangle$.

18. Answers will vary; two possible answers are $\langle 5,3\rangle$ and $\langle -15,-9\rangle$.

19. Answers will vary; two possible answers are $\langle 1,0,-1\rangle$ and $\langle 4,5,-9\rangle$.

20. Answers will vary; two possible answers are $\langle 2,1,0\rangle$ and $\langle 1,1,1/3\rangle$.

21. $\text{proj}_{\vec{v}}\,\vec{u}=\langle -1/2, 3/2\rangle$.

22. $\text{proj}_{\vec{v}}\,\vec{u}=\langle 2,6\rangle$.

23. $\text{proj}_{\vec{v}}\,\vec{u}=\langle -1/2,-1/2\rangle$.

24. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 0, 0\rangle$.
25. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 1, 2, 3\rangle$.
26. $\text{proj}_{\vec{v}}\,\vec{u} = \langle 4/3, 4/3, 2/3\rangle$.
27. $\vec{u} = \langle -1/2, 3/2\rangle + \langle 3/2, 1/2\rangle$.
28. $\vec{u} = \langle 2, 6\rangle + \langle 3, -1\rangle$.
29. $\vec{u} = \langle -1/2, -1/2\rangle + \langle -5/2, 5/2\rangle$.
30. $\vec{u} = \langle 0, 0\rangle + \langle -3, 2\rangle$.
31. $\vec{u} = \langle 1, 2, 3\rangle + \langle 0, 3, -2\rangle$.
32. $\vec{u} = \langle 4/3, 4/3, 2/3\rangle + \langle 5/3, -7/3, 4/3\rangle$.
33. 1.96lb
34. 5lb
35. 141.42ft–lb
36. 196.96ft–lb
37. 500ft–lb
38. 424.26ft–lb
39. 500ft–lb

Section 10.4

1. vector
2. right hand rule
3. “Perpendicular” is one answer.
4. T
5. Torque
6. $\vec{u} \times \vec{v} = \langle 12, -15, 3\rangle$
7. $\vec{u} \times \vec{v} = \langle 11, 1, -17\rangle$
8. $\vec{u} \times \vec{v} = \langle -5, -31, 27\rangle$
9. $\vec{u} \times \vec{v} = \langle 47, -36, -44\rangle$
10. $\vec{u} \times \vec{v} = \langle 0, -2, 0\rangle$
11. $\vec{u} \times \vec{v} = \langle 0, 0, 0\rangle$
12. $\vec{i} \times \vec{j} = \vec{k}$
13. $\vec{i} \times \vec{k} = -\vec{j}$
14. $\vec{j} \times \vec{k} = \vec{i}$
15. Answers will vary.
16. Answers will vary.
17. 5
18. 21
19. 0
20. 5
21. $\sqrt{14}$
22. $\sqrt{230}$
23. 3
24. 6
25. $5\sqrt{2}/2$
26. $3\sqrt{30}/2$
27. 1
28. $5/2$
29. 7
30. $8\sqrt{7/2}$
31. 2
32. 15
33. $\pm\frac{1}{\sqrt{6}}\langle 1, 1, -2\rangle$
34. $\pm\frac{1}{\sqrt{21}}\langle -2, 1, 4\rangle$
35. $\langle 0, \pm 1, 0\rangle$
36. any vector orthogonal to $\vec{u}$ works (such as $\frac{1}{\sqrt{2}}\langle 1, 0, -1\rangle$).
37. 87.5ft–lb
38. $43.75\sqrt{3} \approx 75.78$ft–lb
39. $200/3 \approx 66.67$ft–lb
40. 11.58ft–lb
41. With $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$, we have
$$\begin{aligned}\vec{u}\cdot(\vec{u}\times\vec{v}) &= \langle u_1, u_2, u_3\rangle \cdot (\langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1\rangle)\\ &= u_1(u_2v_3 - u_3v_2) - u2(u_1v_3 - u_3v_1) + u3(u_1v_2 - u_2v_1)\\ &= 0.\end{aligned}$$
42. With $\vec{u} = \langle u_1, u_2, u_3\rangle$, we have
$$\begin{aligned}\vec{u}\times\vec{u} &= \langle u_2u_3 - u_3u_2, -(u_1u_3 - u_3u_1), u_1u_2 - u_2u_1\rangle)\\ &= \langle 0, 0, 0\rangle\\ &= \vec{0}.\end{aligned}$$

Section 10.5

1. A point on the line and the direction of the line.
2. parallel
3. parallel, skew
4. Answers will vary
5. vector: $\ell(t) = \langle 2, -4, 1\rangle + t\,\langle 9, 2, 5\rangle$
parametric: $x = 2 + 9t, y = -4 + 2t, z = 1 + 5t$
symmetric: $(x - 2)/9 = (y + 4)/2 = (z - 1)/5$
6. vector: $\ell(t) = \langle 6, 1, 7\rangle + t\,\langle -3, 2, 5\rangle$
parametric: $x = 6 - 3t, y = 1 + 2t, z = 7 + 5t$
symmetric: $-(x - 6)/3 = (y - 1)/2 = (z - 7)/5$
7. Answers can vary: vector: $\ell(t) = \langle 2, 1, 5\rangle + t\,\langle 5, -3, -1\rangle$
parametric: $x = 2 + 5t, y = 1 - 3t, z = 5 - t$
symmetric: $(x - 2)/5 = -(y - 1)/3 = -(z - 5)$
8. Answers can vary: vector: $\ell(t) = \langle 1, -2, 3\rangle + t\,\langle 4, 7, 2\rangle$
parametric: $x = 1 + 4t, y = -2 + 7t, z = 3 + 2t$
symmetric: $(x - 1)/4 = (y + 2)/7 = (z - 3)/2$
9. Answers can vary; here the direction is given by $\vec{d_1} \times \vec{d_2}$: vector: $\ell(t) = \langle 0, 1, 2\rangle + t\,\langle -10, 43, 9\rangle$
parametric: $x = -10t, y = 1 + 43t, z = 2 + 9t$
symmetric: $-x/10 = (y - 1)/43 = (z - 2)/9$
10. Answers can vary; here the direction is given by $\vec{d_1} \times \vec{d_2}$: vector: $\ell(t) = \langle 5, 1, 9\rangle + t\,\langle 0, -1, 0\rangle$
parametric: $x = 5, y = 1 - t, z = 9$
symmetric: not defined, as some components of the direction are 0.
11. Answers can vary; here the direction is given by $\vec{d_1} \times \vec{d_2}$: vector: $\ell(t) = \langle 7, 2, -1\rangle + t\,\langle 1, -1, 2\rangle$
parametric: $x = 7 + t, y = 2 - t, z = -1 + 2t$
symmetric: $x - 7 = 2 - y = (z + 1)/2$

12. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$: vector: $\ell(t) = \langle 2, 2, 3\rangle + t\langle 5, -1, -3\rangle$
parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
symmetric: $(x - 2)/5 = -(y - 2) = -(z - 3)/3$

13. vector: $\ell(t) = \langle 1, 1\rangle + t\langle 2, 3\rangle$
parametric: $x = 1 + 2t$, $y = 1 + 3t$
symmetric: $(x - 1)/2 = (y - 1)/3$

14. vector: $\ell(t) = \langle -2, 5\rangle + t\langle 0, 1\rangle$
parametric: $x = -2$, $y = 5 + t$
symmetric: not defined

15. parallel

16. intersecting; $\ell_1(2) = \ell_2(-2) = \langle 12, 3, 7\rangle$

17. intersecting; $\ell_1(3) = \ell_2(4) = \langle 9, -5, 13\rangle$

18. same

19. skew

20. parallel

21. same

22. skew

23. $\sqrt{41}/3$

24. $3\sqrt{2}$

25. $5\sqrt{2}/2$

26. 5

27. $3/\sqrt{2}$

28. 2

29. Since both P and Q are on the line, $\overrightarrow{PQ}$ is parallel to $\vec{d}$. Thus $\overrightarrow{PQ} \times \vec{d} = \vec{0}$, giving a distance of 0.

30. (Note: this solution is easier once one has studied Section 10.6.) Since the two lines intersect, we can state $P_2 = P_1 + a\vec{d}_1 + b\vec{d}_2$ for some scalars a and b. (Here we abuse notation slightly and add points to vectors.) Thus $\overrightarrow{P_1P_2} = a\vec{d}_1 + b\vec{d}_2$. Vector $\vec{c}$ is the cross product of $\vec{d}_1$ and $\vec{d}_2$, hence is orthogonal to both, and hence is orthogonal to $\overrightarrow{P_1P_2}$. Thus $\overrightarrow{P_1P_2} \cdot \vec{c} = 0$, and the distance between lines is 0.

31. The distance formula cannot be used because since $\vec{d}_1$ and $\vec{d}_2$ are parallel, $\vec{c}$ is $\vec{0}$ and we cannot divide by $||\,\vec{0}\,||$.
Since $\vec{d}_1$ and $\vec{d}_2$ are parallel, $\overrightarrow{P_1P_2}$ lies in the plane formed by the two lines. Thus $\overrightarrow{P_1P_2} \times \vec{d}_2$ is orthogonal to this plane, and $\vec{c} = (\overrightarrow{P_1P_2} \times \vec{d}_2) \times \vec{d}_2$ is parallel to the plane, but still orthogonal to both $\vec{d}_1$ and $\vec{d}_2$. We desire the length of the projection of $\overrightarrow{P_1P_2}$ onto $\vec{c}$, which is what the formula provides.

Section 10.6

1. A point in the plane and a normal vector (i.e., a direction orthogonal to the plane).

2. A normal vector is orthogonal to the plane.

3. Answers will vary.

4. Answers will vary.

5. Answers will vary.

6. Answers will vary.

7. Standard form: $3(x - 2) - (y - 3) + 7(z - 4) = 0$
general form: $3x - y + 7z = 31$

8. Standard form: $2(y - 3) + 4(z - 5) = 0$
general form: $2y + 4z = 26$

9. Answers may vary;
Standard form: $8(x - 1) + 4(y - 2) - 4(z - 3) = 0$
general form: $8x + 4y - 4z = 4$

10. Answers may vary;
Standard form: $-5(x - 5) + 3(y - 3) + 2(z - 8) = 0$
general form: $-5x + 3y + 2z = 0$

11. Answers may vary;
Standard form: $-7(x - 2) + 2(y - 1) + (z - 2) = 0$
general form: $-7x + 2y + z = -10$

12. Answers may vary;
Standard form: $3(x - 5) + 3(z - 3) = 0$
general form: $3x + 3z = 24$

13. Answers may vary;
Standard form: $2(x - 1) - (y - 1) = 0$
general form: $2x - y = 1$

14. Answers may vary;
Standard form: $2(x - 1) + (y - 1) - 3(z - 1) = 0$
general form: $2x + y - 3z = 0$

15. Answers may vary;
Standard form: $2(x - 2) - (y + 6) - 4(z - 1) = 0$
general form: $2x - y - 4z = 6$

16. Answers may vary;
Standard form: $4(x - 5) - 2(y - 7) - 2(z - 3) = 0$
general form: $4x - 2y - 2z = 0$

17. Answers may vary;
Standard form: $(x - 5) + (y - 7) + (z - 3) = 0$
general form: $x + y + z = 15$

18. Answers may vary;
Standard form: $4(x - 4) + (y - 1) + (z - 1) = 0$
general form: $4x + y + z = 18$

19. Answers may vary;
Standard form: $3(x + 4) + 8(y - 7) - 10(z - 2) = 0$
general form: $3x + 8y - 10z = 24$

20. Standard form: $x - 1 = 0$
general form: $x = 1$

21. Answers may vary:
$$\ell = \begin{cases} x = 14t \\ y = -1 - 10t \\ z = 2 - 8t \end{cases}$$

22. Answers may vary:
$$\ell = \begin{cases} x = 1 + 20t \\ y = 3 + 2t \\ z = 3.5 - 26t \end{cases}$$

23. $(-3, -7, -5)$

24. $(3, 1, 1)$

25. No point of intersection; the plane and line are parallel.

26. The plane contains the line, so every point on the line is a “point of intersection.”

27. $\sqrt{5/7}$

28. $8/\sqrt{21}$

29. $1/\sqrt{3}$

30. 3

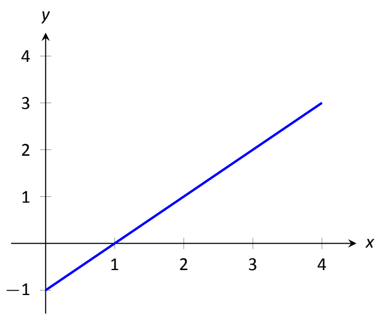
31. If P is any point in the plane, and Q is also in the plane, then $\overrightarrow{PQ}$ lies parallel to the plane and is orthogonal to $\vec{n}$, the normal vector. Thus $\vec{n} \cdot \overrightarrow{PQ} = 0$, giving the distance as 0.

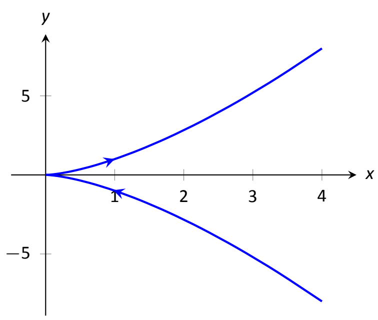
32. The intersecting lines define a plane with normal vector $\vec{n} = \vec{c} = \vec{d}_1 \times \vec{d}_2$. Since points P_1 and P_2 lie in the plane, $\vec{c}$ is orthogonal to $\overrightarrow{P_1P_2}$, hence $\overrightarrow{P_1P_2} \cdot \vec{c} = 0$, giving a distance of 0. Knowing the principles of planes, especially their normal vectors, makes this simpler.

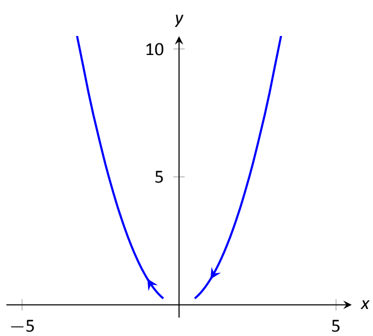
Chapter 11

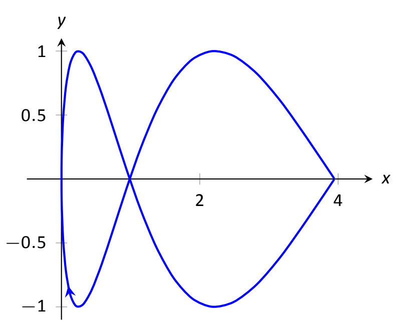
Section 11.1

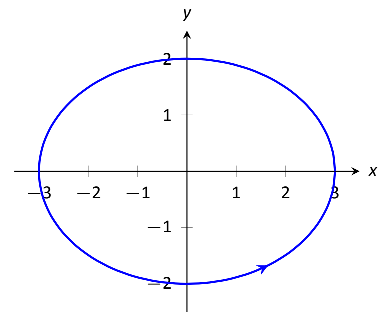
1. parametric equations
2. vectors
3. displacement

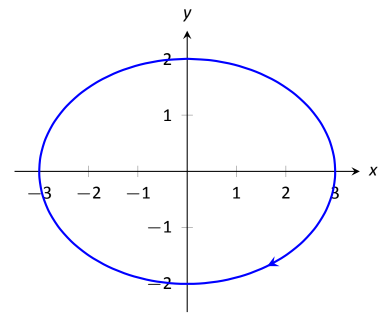
4. 

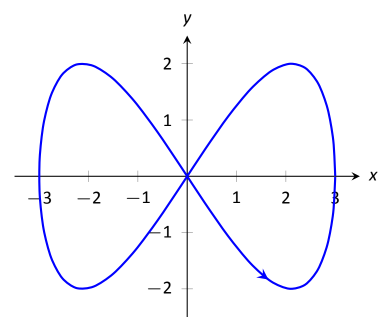
5. 

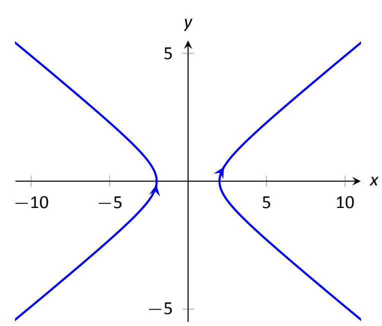
6. 

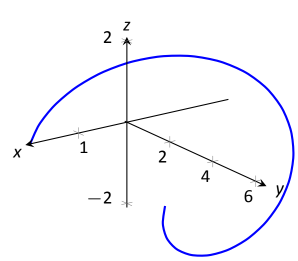
7. 

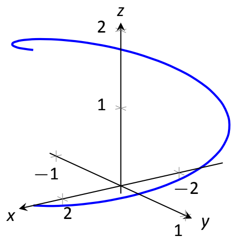
8. 

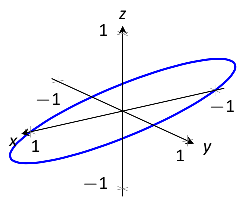
9. 

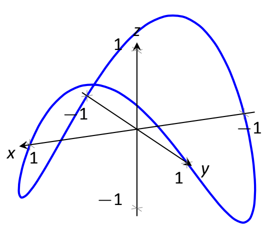
10. 

11. 

12. 

13. 

14. 

15. 

16. $||\vec{r}(t)|| = \sqrt{t^2 + t^4} = |t|\sqrt{t^2+1}$.

17. $||\vec{r}(t)|| = \sqrt{25\cos^2 t + 9\sin^2 t}$.

18. $||\vec{r}(t)|| = \sqrt{4\cos^2 t + 4\sin^2 t + t^2} = \sqrt{t^2+4}$.

19. $||\vec{r}(t)|| = \sqrt{\cos^2 t + t^2 + t^4}$.

20. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 2\cos t + 1, 2\sin t + 2\rangle$.

21. Answers may vary; three solutions are $\vec{r}(t) = \langle 3\sin t + 5, 3\cos t + 5\rangle$, $\vec{r}(t) = \langle -3\cos t + 5, 3\sin t + 5\rangle$ and $\vec{r}(t) = \langle 3\cos t + 5, -3\sin t + 5\rangle$.

22. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 1.5\cos t, 5\sin t\rangle$.

23. Answers may vary, though most direct solutions are $\vec{r}(t) = \langle -3\cos t + 3, 2\sin t - 2\rangle$, $\vec{r}(t) = \langle 3\cos t + 3, -2\sin t - 2\rangle$ and $\vec{r}(t) = \langle 3\sin t + 3, 2\cos t - 2\rangle$.

24. Answers may vary, though most direct solutions are $\vec{r}(t) = \langle t, 5(t-2)+3\rangle$ and $\vec{r}(t) = \langle t+2, 5t+3\rangle$.

25. Answers may vary, though most direct solutions are $\vec{r}(t) = \langle t, -1/2(t-1)+5\rangle$, $\vec{r}(t) = \langle t+1, -1/2t+5\rangle$, $\vec{r}(t) = \langle -2t+1, t+5\rangle$ and $\vec{r}(t) = \langle 2t+1, -t+5\rangle$.

26. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 2\cos t, 2\sin t, 2t\rangle$.

27. Answers may vary, though most direct solution is $\vec{r}(t) = \langle 3\cos(4\pi t), 3\sin(4\pi t), 3t\rangle$.

28. $\langle 1, 0\rangle$

29. $\langle 1, 1\rangle$

30. $\langle 0, 0, 1\rangle$

31. $\langle 1, 2, 7\rangle$

Section 11.2

1. component

2. displacement

3. It is difficult to identify the points on the graphs of $\vec{r}(t)$ and $\vec{r}\,'(t)$ that correspond to each other.

4. $\langle 11, 74, \sin 5\rangle$

5. $\langle e^3, 0\rangle$

6. $\langle 1, e\rangle$

7. $\langle 2t, 1, 0\rangle$

8. $(-\infty, 0)\bigcup(0,\infty)$

9. $(0,\infty)$

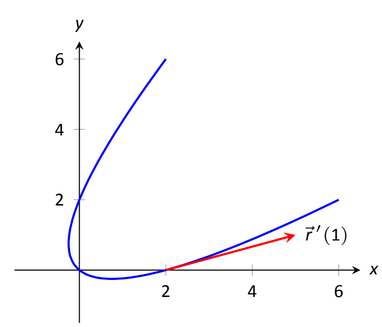
10. $\vec{r}\,'(t) = \langle -\sin t, e^t, 1/t\rangle$

11. $\vec{r}\,'(t) = \langle -1/t^2, 5/(3t+1)^2, \sec^2 t\rangle$

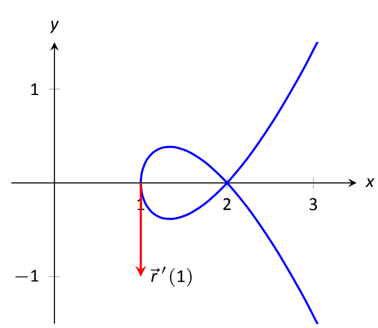
12. $\vec{r}\,'(t) = (2t)\langle \sin t, 2t+5\rangle + (t^2)\langle \cos t, 2\rangle = \langle 2t\sin t + t^2\cos t, 6t^2+10t\rangle$

13. $\vec{r}\,'(t) = \langle 2t, 1\rangle \cdot \langle \sin t, 2t+5\rangle + \langle t^2+1, t-1\rangle \cdot \langle \cos t, 2\rangle = (t^2+1)\cos t + 2t\sin t + 4t + 3$

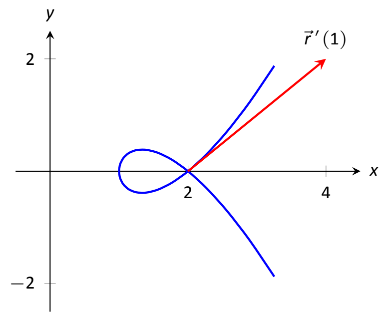
14. $\vec{r}\,'(t) = \langle 2t, 1, 0\rangle \times \langle \sin t, 2t+5, 1\rangle + \langle t^2+1, t-1, 1\rangle \times \langle \cos t, 2, 0\rangle = \langle -1, \cos t - 2t, 6t^2 + 10t + 2 + \cos t - \sin t - t\cos t\rangle$

15. 

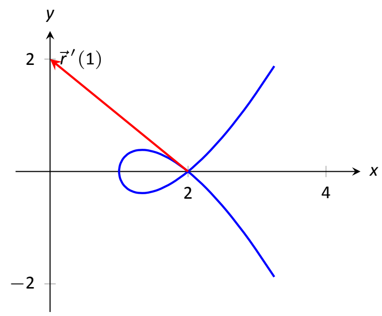
$\vec{r}\,'(t) = \langle 2t+1, 2t-1\rangle$

16. 

$\vec{r}\,'(t) = \langle 2t-2, 3t^2-6t+2\rangle$

17. 

$\vec{r}\,'(t) = \langle 2t, 3t^2 - 1\rangle$

18. 

$\vec{r}\,'(t) = \langle 2t - 4, 3t^2 - 12t + 11\rangle$

19. $\ell(t) = \langle 2, 0\rangle + t\langle 3, 1\rangle$

20. $\ell(t) = \langle 3\sqrt{2}/2, \sqrt{2}/2\rangle + t\langle -3\sqrt{2}/2, \sqrt{2}/2\rangle$

21. $\ell(t) = \langle -3, 0, \pi\rangle + t\langle 0, -3, 1\rangle$

22. $\ell(t) = \langle 1, 0, 0\rangle + t\langle 1, 1, 1\rangle$

23. $t = 0$

24. $t = 1$

25. $\vec{r}(t)$ is not smooth at $t = 3\pi/4 + n\pi$, where n is an integer

26. $t = \pm 1$

27. Both derivatives return $\langle 5t^4, 4t^3 - 3t^2, 3t^2\rangle$.

28. Both derivatives return $2\sin t + t^2\cos t + te^t + 1$.

29. Both derivatives return $\langle 2t - e^t - 1, \cos t - 3t^2, (t^2 + 2t)e^t - (t-1)\cos t - \sin t\rangle$.

30. $\langle \frac{1}{4}t^4, \sin t, te^t - e^t\rangle + \vec{C}$

31. $\langle \tan^{-1} t, \tan t\rangle + \vec{C}$

32. $\langle -2, 0\rangle$

33. $\langle 4, -4\rangle$

34. $\vec{r}(t) = \langle \frac{1}{2}t^2 + 2, -\cos t + 3\rangle$

35. $\vec{r}(t) = \langle \ln|t+1| + 1, -\ln|\cos t| + 2\rangle$

36. $\vec{r}(t) = \langle t^4/12 + t + 4,\ t^3/6 + 2t + 5,\ t^2/2 + 3t + 6\rangle$

37. $\vec{r}(t) = \langle -\cos t + 1, t - \sin t, e^t - t - 1\rangle$

38. $2\sqrt{13}\pi$

39. 10π

40. $\frac{1}{54}\left((22)^{3/2} - 8\right)$

41. $\sqrt{2}(1 - e^{-1})$

42. As $\vec{r}(t)$ has constant length, $\vec{r}(t)\cdot\vec{r}(t) = c^2$ for some constant c. Thus

$$\begin{aligned}
\vec{r}(t)\cdot\vec{r}(t) &= c^2\\
\frac{d}{dt}\big(\vec{r}(t)\cdot\vec{r}(t)\big) &= \frac{d}{dt}\big(c^2\big)\\
\vec{r}\,'(t)\cdot\vec{r}(t) + \vec{r}(t)\cdot\vec{r}\,'(t) &= 0\\
2\vec{r}(t)\cdot\vec{r}\,'(t) &= 0\\
\vec{r}(t)\cdot\vec{r}\,'(t) &= 0.
\end{aligned}$$

Section 11.3

1. Velocity is a vector, indicating an objects direction of travel and its rate of distance change (i.e., its speed). Speed is a scalar.

2. Displacement is a vector, indicating the difference between the starting and ending positions of an object. Distance traveled is a scalar, indicating the arc length of the path followed.

3. The average velocity is found by dividing the displacement by the time traveled – it is a vector. The average speed is found by dividing the distance traveled by the time traveled – it is a scalar.

4. arc length

5. One example is traveling at a constant speed s in a circle, ending at the starting position. Since the displacement is $\vec{0}$, the average velocity is $\vec{0}$, hence $||\,\vec{0}\,|| = 0$. But traveling at constant speed s means the average speed is also $s > 0$.

6. Distance traveled is always greater than or equal to the magnitude of displacement, therefore average speed will always be at least as large as the magnitude of the average velocity.

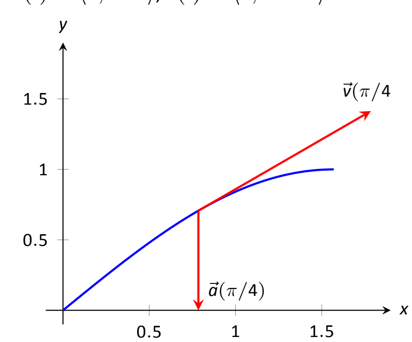
7. $\vec{v}(t) = \langle 2, 5, 0\rangle$, $\vec{a}(t) = \langle 0, 0, 0\rangle$

8. $\vec{v}(t) = \langle 6t - 2, -2t + 1\rangle$, $\vec{a}(t) = \langle 6, -2\rangle$

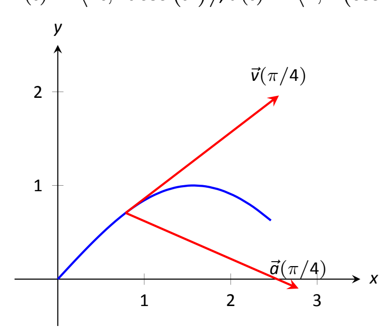
9. $\vec{v}(t) = \langle -\sin t, \cos t\rangle$, $\vec{a}(t) = \langle -\cos t, -\sin t\rangle$

10. $\vec{v}(t) = \langle 1/10, \sin t, \cos t\rangle$, $\vec{a}(t) = \langle 0, \cos t, -\sin t\rangle$

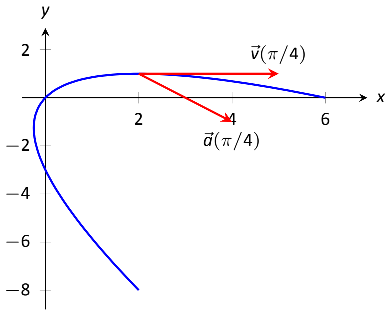
11. $\vec{v}(t) = \langle 1, \cos t\rangle$, $\vec{a}(t) = \langle 0, -\sin t\rangle$



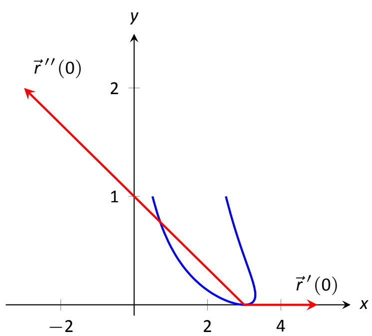
12. $\vec{v}(t) = \langle 2t, 2t\cos(t^2)\rangle$, $\vec{a}(t) = \langle 2, 2\left(\cos(t^2) - 2t^2\sin(t^2)\right)\rangle$



13. $\vec{v}(t) = \langle 2t+1, -2t+2 \rangle$, $\vec{a}(t) = \langle 2, -2 \rangle$



14. $\vec{v}(t) = \left\langle -\frac{2(t^2+3t-1)}{(t^2+1)^2}, 2t \right\rangle$, $\vec{a}(t) = \left\langle \frac{2(2t^3+9t^2-6t-3)}{(t^2+1)^3}, 2 \right\rangle$



15. $\|\vec{v}(t)\| = \sqrt{4t^2+1}$.
Min at $t=0$; Max at $t=\pm 1$.

16. $\|\vec{v}(t)\| = |t|\sqrt{9t^2-12t+8}$.
min: $t=0$; max: $t=-1$

17. $\|\vec{v}(t)\| = 5$.
Speed is constant, so there is no difference between min/max

18. $\|\vec{v}(t)\| = \sqrt{4\sin^2 t + 25\cos^2 t}$.
min: $t=\pi/2,\ 3\pi/2$; max: $t=0,\ 2\pi$

19. $\|\vec{v}(t)\| = |\sec t|\sqrt{\tan^2 t + \sec^2 t}$.
min: $t=0$; max: $t=\pi/4$

20. $\|\vec{v}(t)\| = \sqrt{2-2\sin t}$.
min: $t=\pi/2$; max: $t=3\pi/2$

21. $\|\vec{v}(t)\| = 13$.
speed is constant, so there is no difference between min/max

22. $\|\vec{v}(t)\| = \sqrt{8t^2+3}$.
min: $t=0$; max: $t=1$

23. $\|\vec{v}(t)\| = \sqrt{4t^2+1+t^2/(1-t^2)}$.
min: $t=0$; max: there is no max; speed approaches ∞ as $t\to\pm 1$

24. $\|\vec{v}(t)\| = \sqrt{g^2t^2-(2gv_0\sin\theta)t+v_0^2}$.
min: $t=(v_0\sin\theta)/g$; max: $t=0$, $t=(2v_0\sin\theta)/g$

25. (a) $\vec{r}_1(1) = \langle 1,1 \rangle$; $\vec{r}_2(1) = \langle 1,1 \rangle$
(b) $\vec{v}_1(1) = \langle 1,2 \rangle$; $\|\vec{v}_1(1)\| = \sqrt{5}$; $\vec{a}_1(1) = \langle 0,2 \rangle$
$\vec{v}_2(1) = \langle 2,4 \rangle$; $\|\vec{v}_2(1)\| = 2\sqrt{5}$; $\vec{a}_2(1) = \langle 2,12 \rangle$

26. (a) $\vec{r}_1(\pi/2) = \langle 0,3 \rangle$; $\vec{r}_2(\pi/8) = \langle 0,3 \rangle$
(b) $\vec{v}_1(\pi/2) = \langle -3,0 \rangle$; $\|\vec{v}_1(\pi/2)\| = 3$; $\vec{a}_1(\pi/2) = \langle 0,-3 \rangle$
$\vec{v}_2(\pi/8) = \langle -12,0 \rangle$; $\|\vec{v}_2(\pi/8)\| = 12$;
$\vec{a}_2(\pi/8) = \langle 0,-48 \rangle$

27. (a) $\vec{r}_1(2) = \langle 6,4 \rangle$; $\vec{r}_2(2) = \langle 6,4 \rangle$
(b) $\vec{v}_1(2) = \langle 3,2 \rangle$; $\|\vec{v}_1(2)\| = \sqrt{13}$; $\vec{a}_1(2) = \langle 0,0 \rangle$
$\vec{v}_2(2) = \langle 6,4 \rangle$; $\|\vec{v}_2(2)\| = 2\sqrt{13}$; $\vec{a}_2(2) = \langle 0,0 \rangle$

28. (a) $\vec{r}_1(1) = \langle 1,1 \rangle$; $\vec{r}_2(\pi/2) = \langle 1,1 \rangle$
(b) $\vec{v}_1(1) = \langle 1,1/2 \rangle$; $\|\vec{v}_1(1)\| = \sqrt{5}/2$; $\vec{a}_1(1) = \langle 0,-1/4 \rangle$
$\vec{v}_2(\pi/2) = \langle 0,0 \rangle$; $\|\vec{v}_2(\pi/2)\| = 0$;
$\vec{a}_2(\pi/2) = \langle -1,-1/2 \rangle$

29. $\vec{v}(t) = \langle 2t+1, 3t+2 \rangle$, $\vec{r}(t) = \left\langle t^2+t+5, 3t^2/2+2t-2 \right\rangle$

30. $\vec{v}(t) = \langle 2t-1, 3t-1 \rangle$, $\vec{r}(t) = \left\langle t^2-t+5, 3t^2/2-t-5/2 \right\rangle$

31. $\vec{v}(t) = \langle \sin t, \cos t \rangle$, $\vec{r}(t) = \langle 1-\cos t, \sin t \rangle$

32. $\vec{v}(t) = \langle 10, -32t+50 \rangle$, $\vec{r}(t) = \left\langle 10t, -16t^2+50t \right\rangle$

33. Displacement: $\langle 0,0,6\pi \rangle$; distance traveled: $2\sqrt{13}\pi \approx 22.65$ft; average velocity: $\langle 0,0,3 \rangle$; average speed: $\sqrt{13} \approx 3.61$ft/s

34. Displacement: $\langle -10,0 \rangle$; distance traveled: $5\pi \approx 15.71$ft; average velocity: $\langle -10/\pi, 0 \rangle \approx \langle -3.18, 0 \rangle$; average speed: 5ft/s

35. Displacement: $\langle 0,0 \rangle$; distance traveled: $2\pi \approx 6.28$ft; average velocity: $\langle 0,0 \rangle$; average speed: 1ft/s

36. Displacement: $\langle 10,20,-20 \rangle$; distance traveled: 30ft; average velocity: $\langle 1,2,-2 \rangle$; average speed: 3ft/s

37. At t-values of $\sin^{-1}(9/30)/(4\pi) + n/2 \approx 0.024 + n/2$ seconds, where n is an integer.

38. The stone, while whirling, can be modeled by $\vec{r}(t) = \langle 3\cos(8\pi t), 3\sin(8\pi t) \rangle$.
(a) For t-values $t = \sin^{-1}(3/20)/(8\pi) + n/4 \approx 0.006 + n/4$, where n is an integer.
(b) $\|\vec{r}\,'(t)\| = 24\pi \approx 51.4$ft/s
(c) At $t=0.006$, the stone is approximately 19.77ft from Goliath. Using the formula for projectile motion, we want the angle of elevation that lets a projectile starting at $\langle 0,6 \rangle$ with a initial velocity of 51.4ft/s arrive at $\langle 19.77, 9 \rangle$. The desired angle is 0.27 radians, or 15.69°.

39. (a) Holding the crossbow at an angle of 0.013 radians, $\approx 0.745^\circ$ will hit the target 0.4s later. (Another solution exists, with an angle of 89°, landing 18.75s later, but this is impractical.)
(b) In the .4 seconds the arrow travels, a deer, traveling at 20mph or 29.33ft/s, can travel 11.7ft. So she needs to lead the deer by 11.7ft.

40. The position function of the ball is $\vec{r}(t) = \left\langle (146.67\cos\theta)t, -16t^2 + (146.67\sin\theta)t + 3 \right\rangle$, where θ is the angle of elevation.
(a) With $\theta = 20^\circ$, the ball reaches 310ft from home plate in 2.25 seconds; at this time, the height of the ball is 34.9ft, not enough to clear the Green Monster.
(b) With $\theta = 21^\circ$, the ball reaches 310ft from home plate in 2.26s, with a height of 40ft, clearing the wall.

41. The position function is $\vec{r}(t) = \left\langle 220t, -16t^2+1000 \right\rangle$. The y-component is 0 when $t=7.9$; $\vec{r}(7.9) = \langle 1739.25, 0 \rangle$, meaning the box will travel about 1740ft horizontally before it lands.

42. The position function of the ball is $\vec{r}(t) = \left\langle (v_0\cos\theta)t, -16t^2 + (v_0\sin\theta)t + 6 \right\rangle$, where θ is the angle of elevation and v_0 is the initial ball speed.
(a) With $v_0 = 73.33$ft/s, there are two angles of elevation possible. An angle of $\theta = 9.47^\circ$ delivers the ball in 0.83s, while an angle of 79.57° delivers the ball in 4.5s.
(b) With $\theta = 8^\circ$, the initial speed must be 53.8mph≈ 78.9ft/s.

Section 11.4

1. 1

2. 0
3. $\vec{T}(t)$ and $\vec{N}(t)$.
4. the speed
5. $\vec{T}(t) = \left\langle \frac{4t}{\sqrt{20t^2-4t+1}}, \frac{2t-1}{\sqrt{20t^2-4t+1}} \right\rangle$; $\vec{T}(1) = \langle 4/\sqrt{17}, 1/\sqrt{17} \rangle$
6. $\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+\sin^2 t}}, -\frac{\sin t}{\sqrt{1+\sin^2 t}} \right\rangle$; $\vec{T}(\pi/4) = \left\langle \sqrt{2/3}, -1/\sqrt{3} \right\rangle$
7. $\vec{T}(t) = \frac{\cos t \sin t}{\sqrt{\cos^2 t \sin^2 t}} \langle -\cos t, \sin t \rangle$. (Be careful; this cannot be simplified as just $\langle -\cos t, \sin t \rangle$ as $\sqrt{\cos^2 t \sin^2 t} \neq \cos t \sin t$, but rather $|\cos t \sin t|$.) $\vec{T}(\pi/4) = \langle -\sqrt{2}/2, \sqrt{2}/2 \rangle$
8. $\vec{T}(t) = \langle -\sin t, \cos t \rangle$; $\vec{T}(\pi) = \langle 0, -1 \rangle$
9. $\ell(t) = \langle 2, 0 \rangle + t \langle 4/\sqrt{17}, 1/\sqrt{17} \rangle$; in parametric form,
$$\ell(t) = \left\{ \begin{array}{lcl} x & = & 2 + 4t/\sqrt{17} \\ y & = & t/\sqrt{17} \end{array} \right.$$
10. $\ell(t) = \langle \pi/4, \sqrt{2}/2 \rangle + t \left\langle \sqrt{2/3}, -1/\sqrt{3} \right\rangle$; in parametric form,
$$\ell(t) = \left\{ \begin{array}{lcl} x & = & \pi/4 + \sqrt{2/3}t \\ y & = & \sqrt{2}/2 - t/\sqrt{3} \end{array} \right.$$
11. $\ell(t) = \langle \sqrt{2}/4, \sqrt{2}/4 \rangle + t \langle -\sqrt{2}/2, \sqrt{2}/2 \rangle$; in parametric form,
$$\ell(t) = \left\{ \begin{array}{lcl} x & = & \sqrt{2}/4 - \sqrt{2}t/2 \\ y & = & \sqrt{2}/4 + \sqrt{2}t/2 \end{array} \right.$$
12. $\ell(t) = \langle -1, 0 \rangle + t \langle 0, -1 \rangle$; in parametric form,
$$\ell(t) = \left\{ \begin{array}{lcl} x & = & -1 \\ y & = & -t \end{array} \right.$$
13. $\vec{T}(t) = \langle -\sin t, \cos t \rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t \rangle$
14. $\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}} \right\rangle$; $\vec{N}(t) = \left\langle -\frac{2t}{\sqrt{1+4t^2}}, \frac{1}{\sqrt{1+4t^2}} \right\rangle$
15. $\vec{T}(t) = \left\langle -\frac{\sin t}{\sqrt{4\cos^2 t + \sin^2 t}}, \frac{2\cos t}{\sqrt{4\cos^2 t + \sin^2 t}} \right\rangle$;
$\vec{N}(t) = \left\langle -\frac{2\cos t}{\sqrt{4\cos^2 t + \sin^2 t}}, -\frac{\sin t}{\sqrt{4\cos^2 t + \sin^2 t}} \right\rangle$
16. $\vec{T}(t) = \left\langle \frac{e^t}{\sqrt{e^{2t}+e^{-2t}}}, -\frac{e^{-t}}{\sqrt{e^{2t}+e^{-2t}}} \right\rangle$;
$\vec{N}(t) = \left\langle \frac{e^{-t}}{\sqrt{e^{2t}+e^{-2t}}}, \frac{e^t}{\sqrt{e^{2t}+e^{-2t}}} \right\rangle$
17. (a) Be sure to show work
    (b) $\vec{N}(\pi/4) = \langle -5/\sqrt{34}, -3/\sqrt{34} \rangle$
18. (a) Be sure to show work
    (b) $\vec{N}(1) = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$
19. (a) Be sure to show work
    (b) $\vec{N}(0) = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$
20. (a) Be sure to show work
    (b) $\vec{N}(\pi/4) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$
21. $\vec{T}(t) = \frac{1}{\sqrt{5}} \langle 2, \cos t, -\sin t \rangle$; $\vec{N}(t) = \langle 0, -\sin t, -\cos t \rangle$
22. $\vec{T}(t) = \langle -\sin t, 3/5\cos t, 4/5\cos t \rangle$;
$\vec{N}(t) = \langle -\cos t, -3/5\sin t, -4/5\sin t \rangle$
23. $\vec{T}(t) = \frac{1}{\sqrt{a^2+b^2}} \langle -a\sin t, a\cos t, b \rangle$; $\vec{N}(t) = \langle -\cos t, -\sin t, 0 \rangle$
24. $\vec{T}(t) = \frac{1}{\sqrt{a^2+1}} \langle -a\sin(at), a\cos(at), 1 \rangle$;
$\vec{N}(t) = \langle -\cos t, -\sin t, 0 \rangle$
25. $a_T = \frac{4t}{\sqrt{1+4t^2}}$ and $a_N = \sqrt{4 - \frac{16t^2}{1+4t^2}}$
At $t = 0$, $a_T = 0$ and $a_N = 2$;
At $t = 1$, $a_T = 4/\sqrt{5}$ and $a_N = 2/\sqrt{5}$.
At $t = 0$, all acceleration comes in the form of changing the direction of velocity and not the speed; at $t = 1$, more acceleration comes in changing the speed than in changing direction.
26. $a_T = \frac{-2/t^5}{\sqrt{1+1/t^4}}$ and $a_N = \sqrt{\frac{4}{t^6} - \frac{4/t^{10}}{1+1/t^4}}$
At $t = 1$, $a_T = \sqrt{2}$ and $a_N = -\sqrt{2}$;
At $t = 2$, $a_T = -\frac{1}{4\sqrt{17}}$ and $a_N = \frac{1}{\sqrt{17}}$.
At $t = 1$, acceleration comes from changing speed and changing direction in "equal measure;" at $t = 2$, acceleration is nearly $\vec{0}$ as it is; the low value of a_T shows that the speed is nearly constant and the low value of a_N shows the direction is not changing quickly.
27. $a_T = 0$ and $a_N = 2$
At $t = 0$, $a_T = 0$ and $a_N = 2$;
At $t = \pi/2$, $a_T = 0$ and $a_N = 2$.
The object moves at constant speed, so all acceleration comes from changing direction, hence $a_T = 0$. $\vec{a}(t)$ is always parallel to $\vec{N}(t)$, but twice as long, hence $a_N = 2$.
28. $a_T = 2$ and $a_N = 4t^2$
At $t = \sqrt{\pi/2}$, $a_T = 2$ and $a_N = 2\pi$;
At $t = \sqrt{\pi}$, $a_T = 2$ and $a_N = 4\pi$.
The object moves at increasing speed (increasing at a constant rate of acceleration), hence $a_T = 2$. Since the object is increasing speed yet always traveling in a circle of radius 1, the direction must change more quickly; the amount of acceleration that changes direction increases over time.
29. $a_T = 0$ and $a_N = a$
At $t = 0$, $a_T = 0$ and $a_N = a$;
At $t = \pi/2$, $a_T = 0$ and $a_N = a$.
The object moves at constant speed, meaning that a_T is always 0. The object "rises" along the z-axis at a constant rate, so all acceleration comes in the form of changing direction circling the z-axis. The greater the radius of this circle the greater the acceleration, hence $a_N = a$.
30. $a_T = 0$ and $a_N = 5$
At $t = 0$, $a_T = 0$ and $a_N = 5$;
At $t = \pi/2$, $a_T = 0$ and $a_N = 5$.
The object moves at constant speed, meaning that a_T is always 0. Acceleration is thus always perpendicular to the direction of travel; in this particular case, it is always 5 times the unit vector pointing orthogonal to the direction of travel.

Section 11.5

1. time and/or distance
2. curvature
3. Answers may include lines, circles, helixes
4. Answers will vary; they should mention the circle is tangent to the curve and has the same curvature as the curve at that point.
5. κ
6. a_T is not affected by curvature; the greater the curvature, the larger a_N becomes.
7. $s = 3t$, so $\vec{r}(s) = \langle 2s/3, s/3, -2s/3 \rangle$
8. $s = 7t$, so $\vec{r}(s) = \langle 7\cos(s/7), 7\sin(s/7) \rangle$
9. $s = \sqrt{13}t$, so $\vec{r}(s) = \langle 3\cos(s/\sqrt{13}), 3\sin(s/\sqrt{13}), 2s/\sqrt{13} \rangle$
10. $s = 13t$, so $\vec{r}(s) = \langle 5\cos(s/13), 13\sin(s/13), 12\cos(s/13) \rangle$

11. $\kappa = \frac{|6x|}{\left(1+(3x^2-1)^2\right)^{3/2}}$;
$\kappa(0) = 0$, $\kappa(1/2) = \frac{192}{17\sqrt{17}} \approx 2.74$.

12. $\kappa = \frac{\left|\frac{6x^2-2}{(x^2+1)^3}\right|}{\left(1+\frac{4x^2}{(x^2+1)^4}\right)^{3/2}}$;
$\kappa(0) = 2$, $\kappa(2) = \frac{2750}{641\sqrt{641}} \approx 0.169$.

13. $\kappa = \frac{|\cos x|}{\left(1+\sin^2 x\right)^{3/2}}$;
$\kappa(0) = 1$, $\kappa(\pi/2) = 0$

14. $\kappa = 1$;
$\kappa(0) = 1$, $\kappa(1/2) = 1$

15. $\kappa = \frac{|2\cos t\cos(2t)+4\sin t\sin(2t)|}{\left(4\cos^2(2t)+\sin^2 t\right)^{3/2}}$;
$\kappa(0) = 1/4$, $\kappa(\pi/4) = 8$

16. $\kappa = 2$;
$\kappa(0) = 2$, $\kappa(\pi/3) = 2$

17. $\kappa = \frac{|6t^2+2|}{\left(4t^2+(3t^2-1)^2\right)^{3/2}}$;
$\kappa(0) = 2$, $\kappa(5) = \frac{19}{1394\sqrt{1394}} \approx 0.0004$

18. $\kappa = \frac{|\sec^3 t|}{\left(\sec^4 t+\sec^2 t\tan^2 t\right)^{3/2}}$;
$\kappa(0) = 1$, $\kappa(\pi/6) = \frac{3\sqrt{3}}{5\sqrt{5}} \approx 0.465$

19. $\kappa = 0$;
$\kappa(0) = 0$, $\kappa(1) = 0$

20. $\kappa = \frac{2\sqrt{18t^4+111t^2+17}}{\left(18t^4-26t^2+17\right)^{3/2}}$;
$\kappa(0) = 2/17 \approx 0.117$, $\kappa(1) = 2\sqrt{146}/27 \approx 0.895$

21. $\kappa = \frac{3}{13}$;
$\kappa(0) = 3/13$, $\kappa(\pi/2) = 3/13$

22. $\kappa = \frac{1}{13}$;
$\kappa(0) = 1/13$, $\kappa(\pi/2) = 1/13$

23. maximized at $x = \pm\frac{\sqrt{2}}{\sqrt[4]{5}}$

24. maximized at $x = \ldots - 3\pi/2, -\pi/2, \pi/2, \ldots$

25. maximized at $t = 1/4$

26. maximized at $t = \pm\sqrt{5}$

27. radius of curvature is $5\sqrt{5}/4$.

28. radius of curvature is $5\sqrt{10}$.

29. radius of curvature is 9.

30. radius of curvature is $1/45$.

31. $x^2 + (y-1/2)^2 = 1/4$, or $\vec{c}(t) = \langle 1/2\cos t, 1/2\sin t + 1/2\rangle$

32. $(x-8/3)^2 + y^2 = 1/9$, or $\vec{c}(t) = \left\langle \frac{1}{3}\cos t + \frac{8}{3}, \frac{1}{3}\sin t\right\rangle$

33. $x^2 + (y+8)^2 = 81$, or $\vec{c}(t) = \langle 9\cos t, 9\sin t - 8\rangle$

34. $(x-1/2)^2 + (y-1/2)^2 = 1/2$, or
$\vec{c}(t) = \left\langle \frac{\sqrt{2}}{2}\cos t + \frac{1}{2}, \frac{\sqrt{2}}{2}\sin t + \frac{1}{2}\right\rangle$

Chapter 12

Section 12.1

1. Answers will vary.

2. surface

3. topographical

4. T

5. surface

6. When level curves are close together, it means the function is changing z-values rapidly. When far apart, it changes z-values slowly.

7. domain: $\mathbb{R}^2$
range: $z \geq 2$

8. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

9. domain: $\mathbb{R}^2$
range: $\mathbb{R}$

10. domain: $x \neq 2y$; in set notation, $\{(x,y) \mid x \neq 2y\}$
range: $z \neq 0$

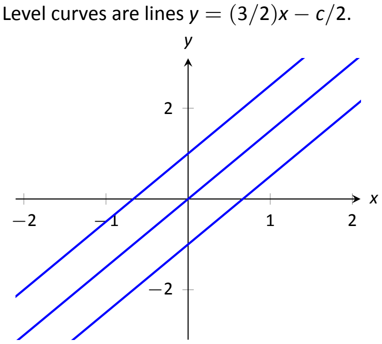
11. domain: $\mathbb{R}^2$
range: $0 < z \leq 1$

12. domain: $\mathbb{R}^2$
range: $-1 \leq z \leq 1$

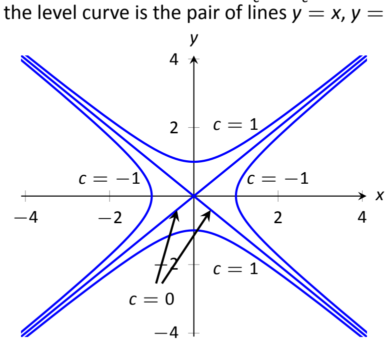
13. domain: $\{(x,y) \mid x^2+y^2 \leq 9\}$, i.e., the domain is the circle and interior of a circle centered at the origin with radius 3.
range: $0 \leq z \leq 3$

14. domain: $\{(x,y) \mid x^2+y^2 \geq 9\}$, i.e., the domain is the exterior of the circle (not including the circle itself) centered at the origin with radius 3.
range: $0 < z < \infty$, or $(0, \infty)$

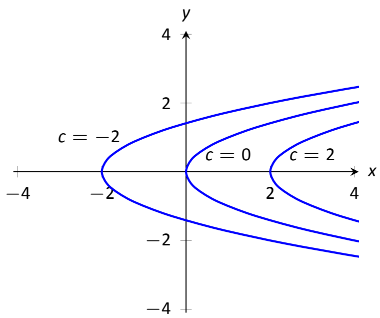
15. Level curves are lines $y = (3/2)x - c/2$.



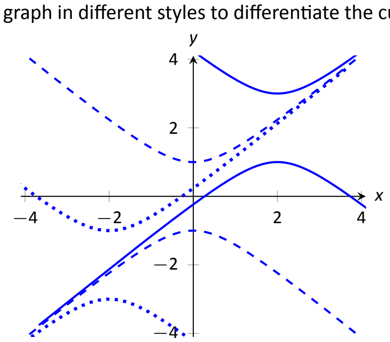
16. Level curves are hyperbolas $\frac{x^2}{c} - \frac{y^2}{c} = 1$, except for $c = 0$, where the level curve is the pair of lines $y = x$, $y = -x$.



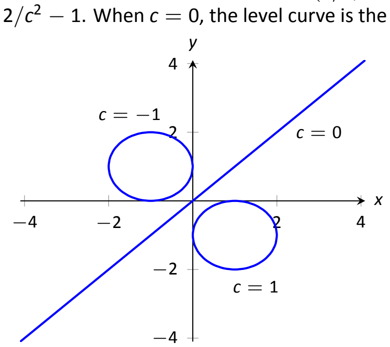
17. Level curves are parabolas $x = y^2 + c$.



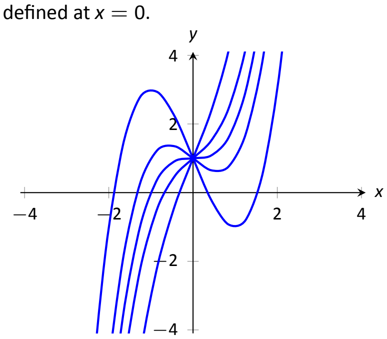
18. Level curves are hyperbolas $(x - c)^2 - (y - c)^2 = 1$, drawn in graph in different styles to differentiate the curves.



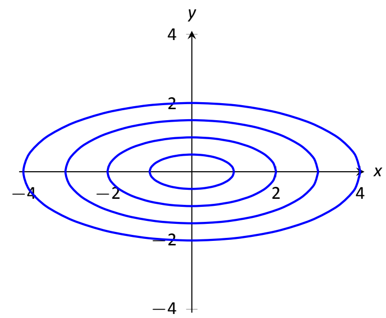
19. Level curves are circles, centered at $(1/c, -1/c)$ with radius $2/c^2 - 1$. When $c = 0$, the level curve is the line $y = x$.



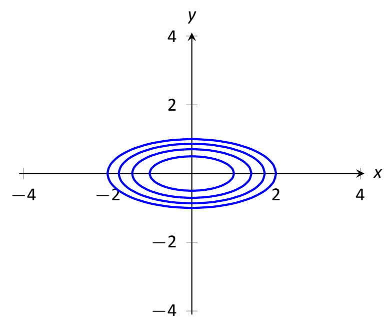
20. Level curves are cubics of the form $y = x^3 + cx + 1$. Note how each curve passes through $(0, 1)$ and that the function is not defined at $x = 0$.



21. Level curves are ellipses of the form $\frac{x^2}{c^2} + \frac{y^2}{c^2/4} = 1$, i.e., $a = c$ and $b = c/2$.



22. Level curves are ellipses of the form $\frac{x^2}{c} + \frac{y^2}{c/4} = 1$, i.e., $a = \sqrt{c}$ and $b = \sqrt{c}/2$.



23. domain: $x + 2y - 4z \neq 0$; the set of points in $\mathbb{R}^3$ NOT in the domain form a plane through the origin.
range: $\mathbb{R}$

24. domain: $x^2 + y^2 + z^2 \neq 1$; the set of points in $\mathbb{R}^3$ NOT in the domain form a sphere of radius 1.
range: $(-\infty, 0) \cup [1, \infty)$

25. domain: $z \geq x^2 - y^2$; the set of points in $\mathbb{R}^3$ above (and including) the hyperbolic paraboloid $z = x^2 - y^2$.
range: $[0, \infty)$

26. domain: $\mathbb{R}^3$
range: $\mathbb{R}$

27. The level surfaces are spheres, centered at the origin, with radius $\sqrt{c}$.

28. The level surfaces are hyperbolic paraboloids of the form $z = x^2 - y^2 + c$; each is shifted up/down by c.

29. The level surfaces are paraboloids of the form $z = \frac{x^2}{c} + \frac{y^2}{c}$; the larger c, the "wider" the paraboloid.

30. The level surfaces are planes through the origin of the form $cx - cy - z = 0$, that is, planes through the origin with normal vector $\langle c, -c, -1\rangle$.

31. The level curves for each surface are similar; for $z = \sqrt{x^2 + 4y^2}$ the level curves are ellipses of the form $\frac{x^2}{c^2} + \frac{y^2}{c^2/4} = 1$, i.e., $a = c$ and $b = c/2$; whereas for $z = x^2 + 4y^2$ the level curves are ellipses of the form $\frac{x^2}{c} + \frac{y^2}{c/4} = 1$, i.e., $a = \sqrt{c}$ and $b = \sqrt{c}/2$. The first set of ellipses are spaced evenly apart, meaning the function grows at a constant rate; the second set of ellipses are more closely spaced together as c grows, meaning the function grows faster and faster as c increases.
The function $z = \sqrt{x^2 + 4y^2}$ can be rewritten as $z^2 = x^2 + 4y^2$, an elliptic cone; the function $z = x^2 + 4y^2$ is a paraboloid, each matching the description above.

Section 12.2

1. Answers will vary.
2. Answers will vary. One answer is "As (x, y) gets close to $(1, 2)$, $f(x, y)$ gets close to 17."
3. Answers will vary.
One possible answer: $\{(x, y)|x^2 + y^2 \leq 1\}$
4. Answers will vary.
One possible answer: $\{(x, y)|y \geq x^2\}$
5. Answers will vary.
One possible answer: $\{(x, y)|x^2 + y^2 < 1\}$
6. Answers will vary.
One possible answer: $\{(x, y)|y > x^2\}$
7. Answers will vary.
interior point: $(1, 3)$
boundary point: $(3, 3)$
S is a closed set
S is bounded
8. Answers will vary.
interior point: $(-5, 28)$
boundary point: $(3, 9)$
S is an open set
S is unbounded
9. Answers will vary.
interior point: none
boundary point: $(0, -1)$
S is a closed set, consisting only of boundary points
S is bounded
10. Answers will vary.
Interior point: $(0, 1)$
Boundary point: $(0, 0)$
S is a closed set, containing all of its boundary points.
S is unbounded.
11. $D = \{(x, y) \mid y \neq 2x\}$; D is an open set.
12. $D = \left\{(x, y) \mid y \geq x^2\right\}$; D is a closed set.
13. $D = \left\{(x, y) \mid y > x^2\right\}$; D is an open set.
14. $D = \{(x, y) \mid (x, y) \neq (0, 0)\}$; D is an open set.
15. (a) Along $y = 0$, the limit is 1.
(b) Along $x = 0$, the limit is -1.
Since the above limits are not equal, the limit does not exist.
16. (a) Along $y = mx$, the limit is $\frac{m+1}{m-1}$.
Since the above limit varies according to what m is used, each limit is different, meaning the overall limit does not exist.
17. (a) Along $y = mx$, the limit is $\dfrac{mx(1-m)}{m^2x+1}$.
(b) Along $x = 0$, the limit is -1.
Since the above limits are not equal, the limit does not exist.
18. (a) Along $y = mx$, the limit is:
$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y} = \lim_{x\to 0} \frac{\sin(x^2)}{mx}$$
apply L'Hôpital's Rule
$$= \lim_{x\to 0} \frac{2x\cos(x^2)}{m} = 0.$$
(b) Along $x = 0$, the limit is:
$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y} = \lim_{x\to 0} \frac{\sin(x^2)}{x^2}.$$
This can be evaluated with L'Hôpital's Rule or from known limits; it is 1.
Since the limits along the lines $y = mx$ are not the same as the limit along $y = x^2$, the overall limit does not exist.
19. (a) Along $y = 2$, the limit is:
$$\lim_{(x,y)\to(1,2)} \frac{x+y-3}{x^2-1} = \lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{1}{x+1} = 1/2.$$
(b) Along $y = x + 1$, the limit is:
$$\lim_{(x,y)\to(1,2)} \frac{x+y-3}{x^2-1} = \lim_{x\to 1} \frac{2(x-1)}{x^2-1} = \lim_{x\to 1} \frac{2}{x+1} = 1.$$
Since the limits along the lines $y = 2$ and $y = x + 1$ differ, the overall limit does not exist.
20. (a) Along $x = \pi$, the limit is:
$$\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y} = \lim_{y\to\pi/2} \frac{0}{\cos y} = 0.$$
(b) Along $y = x - \pi/2$, the limit is:
$$\lim_{(x,y)\to(\pi,\pi/2)} \frac{\sin x}{\cos y} = \lim_{x\to\pi} \frac{\sin x}{\cos(x-\pi/2)}$$
Apply L'Hôpital's Rule:
$$= \lim_{x\to\pi} \frac{\cos x}{\sin(x-\pi/2)} = -1.$$
Since the limits along the lines $x = \pi$ and $y = x - \pi$ differ, the overall limit does not exist.

Section 12.3

1. A constant is a number that is added or subtracted in an expression; a coefficient is a number that is being multiplied by a nonconstant function.
2. Answers will vary; each should include something about treating y as a constant or a coefficient.
3. f_x
4. f_y
5. $f_x = 2xy - 1, f_y = x^2 + 2$
$f_x(1, 2) = 3, f_y(1, 2) = 3$
6. $f_x = 3x^2 - 3, f_y = 2y - 6$
$f_x(-1, 3) = 0, f_y(-1, 3) = 0$
7. $f_x = -\sin x \sin y, f_y = \cos x \cos y$
$f_x(\pi/3, \pi/3) = -3/4, f_y(\pi/3, \pi/3) = 1/4$
8. $f_x = 1/x, f_y = 1/y$
$f_x(-2, -3) = -1/2, f_y(-2, -3) = -1/3$
9. $f_x = 2xy + 6x, f_y = x^2 + 4$
$f_{xx} = 2y + 6, f_{yy} = 0$
$f_{xy} = 2x, f_{yx} = 2x$
10. $f_x = 3x^2 + 6xy + 3y^2, f_y = 3x^2 + 6xy + 3y^2$
$f_{xx} = 6x + 6y, f_{yy} = 6x + 6y$
$f_{xy} = 6x + 6y, f_{yx} = 6x + 6y$

11. $f_x = 1/y, f_y = -x/y^2$
$f_{xx} = 0, f_{yy} = 2x/y^3$
$f_{xy} = -1/y^2, f_{yx} = -1/y^2$

12. $f_x = -4/(x^2y), f_y = -4/(xy^2)$
$f_{xx} = 8/(x^3y), f_{yy} = 8/(xy^3)$
$f_{xy} = 4/(x^2y^2), f_{yx} = 4/(x^2y^2)$

13. $f_x = 2xe^{x^2+y^2}, f_y = 2ye^{x^2+y^2}$
$f_{xx} = 2e^{x^2+y^2} + 4x^2e^{x^2+y^2}, f_{yy} = 2e^{x^2+y^2} + 4y^2e^{x^2+y^2}$
$f_{xy} = 4xye^{x^2+y^2}, f_{yx} = 4xye^{x^2+y^2}$

14. $f_x = e^{x+2y}, f_y = 2e^{x+2y}$
$f_{xx} = e^{x+2y}, f_{yy} = 4e^{x+2y}$
$f_{xy} = 2e^{x+2y}, f_{yx} = 2e^{x+2y}$

15. $f_x = \cos x \cos y, f_y = -\sin x \sin y$
$f_{xx} = -\sin x \cos y, f_{yy} = -\sin x \cos y$
$f_{xy} = -\sin y \cos x, f_{yx} = -\sin y \cos x$

16. $f_x = 3(x+y)^2, f_y = 3(x+y)^2$
$f_{xx} = 6(x+y), f_{yy} = 6(x+y)$
$f_{xy} = 6(x+y), f_{yx} = 6(x+y)$

17. $f_x = -5y^3 \sin(5xy^3), f_y = -15xy^2 \sin(5xy^3)$
$f_{xx} = -25y^6 \cos(5xy^3),$
$f_{yy} = -225x^2y^4 \cos(5xy^3) - 30xy \sin(5xy^3)$
$f_{xy} = -75xy^5 \cos(5xy^3) - 15y^2 \sin(5xy^3),$
$f_{yx} = -75xy^5 \cos(5xy^3) - 15y^2 \sin(5xy^3)$

18. $f_x = 10x \cos(5x^2+2y^3), f_y = 6y^2 \cos(5x^2+2y^3)$
$f_{xx} = 10\cos(5x^2+2y^3) - 100x^2 \sin(5x^2+2y^3),$
$f_{yy} = 12y \cos(5x^2+2y^3) - 36y^4 \sin(5x^2+2y^3)$
$f_{xy} = -60xy^2 \sin(5x^2+2y^3), f_{yx} = -60xy^2 \sin(5x^2+2y^3)$

19. $f_x = \frac{2y^2}{\sqrt{4xy^2+1}}, f_y = \frac{4xy}{\sqrt{4xy^2+1}}$
$f_{xx} = -\frac{4y^4}{\sqrt{4xy^2+1}^3}, f_{yy} = -\frac{16x^2y^2}{\sqrt{4xy^2+1}^3} + \frac{4x}{\sqrt{4xy^2+1}}$
$f_{xy} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}, f_{yx} = -\frac{8xy^3}{\sqrt{4xy^2+1}^3} + \frac{4y}{\sqrt{4xy^2+1}}$

20. $f_x = 2\sqrt{y}, f_y = 5\sqrt{y} + \frac{2x+5y}{2\sqrt{y}}$
$f_{xx} = 0, f_{yy} = \frac{5}{\sqrt{y}} - \frac{2x+5y}{4y^{3/2}}$
$f_{xy} = \frac{1}{\sqrt{y}}, f_{yx} = \frac{1}{\sqrt{y}}$

21. $f_x = -\frac{2x}{(x^2+y^2+1)^2}, f_y = -\frac{2y}{(x^2+y^2+1)^2}$
$f_{xx} = \frac{8x^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}, f_{yy} = \frac{8y^2}{(x^2+y^2+1)^3} - \frac{2}{(x^2+y^2+1)^2}$
$f_{xy} = \frac{8xy}{(x^2+y^2+1)^3}, f_{yx} = \frac{8xy}{(x^2+y^2+1)^3}$

22. $f_x = 5, f_y = -17$
$f_{xx} = 0, f_{yy} = 0$
$f_{xy} = 0, f_{yx} = 0$

23. $f_x = 6x, f_y = 0$
$f_{xx} = 6, f_{yy} = 0$
$f_{xy} = 0, f_{yx} = 0$

24. $f_x = \frac{2x}{(x^2+y)}, f_y = \frac{1}{(x^2+y)}$
$f_{xx} = -\frac{4x^2}{(x^2+y)^2} + \frac{2}{(x^2+y)}, f_{yy} = -\frac{1}{(x^2+y)^2}$
$f_{xy} = -\frac{2x}{(x^2+y)^2}, f_{yx} = -\frac{2x}{(x^2+y)^2}$

25. $f_x = \frac{1}{4xy}, f_y = -\frac{\ln x}{4y^2}$
$f_{xx} = -\frac{1}{4x^2y}, f_{yy} = \frac{\ln x}{2y^3}$
$f_{xy} = -\frac{1}{4xy^2}, f_{yx} = -\frac{1}{4xy^2}$

26. $f_x = 5e^x \sin y, f_y = 5e^x \cos y$
$f_{xx} = 5e^x \sin y, f_{yy} = -5e^x \sin y$
$f_{xy} = 5e^x \cos y, f_{yx} = 5e^x \cos y$

27. $f(x,y) = x \sin y + x + C$, where C is any constant.

28. $f(x,y) = \frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + C$, where C is any constant.

29. $f(x,y) = 3x^2y - 4xy^2 + 2y + C$, where C is any constant.

30. $f(x,y) = \ln(x^2+y^2) + C$, where C is any constant.

31. $f_x = 2xe^{2y-3z}, f_y = 2x^2e^{2y-3z}, f_z = -3x^2e^{2y-3z}$
$f_{yz} = -6x^2e^{2y-3z}, f_{zy} = -6x^2e^{2y-3z}$

32. $f_x = 3x^2y^2 + 3x^2z, f_y = 2x^3y + 2yz, f_z = x^3 + y^2$
$f_{yz} = 2y, f_{zy} = 2y$

33. $f_x = \frac{3}{7y^2z}, f_y = -\frac{6x}{7y^3z}, f_z = -\frac{3x}{7y^2z^2}$
$f_{yz} = \frac{6x}{7y^3z^2}, f_{zy} = \frac{6x}{7y^3z^2}$

34. $f_x = \frac{1}{x}, f_y = \frac{1}{y}, f_z = \frac{1}{z}$
$f_{yz} = 0, f_{zy} = 0$

Section 12.4

1. T
2. T
3. T
4. amount of change
5. $dz = (\sin y + 2x)dx + (x \cos y)dy$
6. $dz = 8x(2x^2+3y)dx + 6(2x^2+3y)dy$
7. $dz = 5dx - 7dy$
8. $dz = (e^{x+y} + xe^{x+y})dx + xe^{x+y}dy$
9. $dz = \frac{x}{\sqrt{x^2+y}}dx + \frac{1}{2\sqrt{x^2+y}}dy$, with $dx = -0.05$ and $dy = .1$. At $(3,7)$, $dz = 3/4(-0.05) + 1/8(.1) = -0.025$, so $f(2.95, 7.1) \approx -0.025 + 4 = 3.975$.
10. $dz = (\cos x \cos y)dx - (\sin x \sin y)dy$, with $dx = 0.1$ and $dy = -0.1$. At $(0,0)$, $dz = 1(.1) - (0)(-0.1) = 0.1$, so $f(0.1, -0.1) \approx 0.1 + 0 = 0.1$.
11. $dz = (2xy - y^2)dx + (x^2 - 2xy)dy$, with $dx = 0.04$ and $dy = 0.06$. At $(2,3)$, $dz = 3(0.04) + (-8)(0.06) = -0.36$, so $f(2.04, 3.06) \approx -0.36 - 6 = -6.36$.
12. $dz = \frac{1}{x-y}dx - \frac{1}{x-y}dy$, with $dx = 0.1$ and $dy = -0.02$. At $(5,4)$, $dz = 1(0.1) + (-1)(-0.02) = 0.12$, so $f(5.1, 3.98) \approx 0.12 + 0 = 0.12$.
13. The total differential of volume is $dV = 4\pi dr + \pi dh$. The coefficient of dr is greater than the coefficient of dh, so the volume is more sensitive to changes in the radius.
14. Distance of the projectile is a function of two variables (leaving $t = 3$): $D(v_0, \theta) = 30\cos\theta$. The total differential of D is $dD = 3\cos\theta dv_0 - 3v_0 \sin\theta d\theta$. The coefficient of $d\theta$ has a much greater magnitude than the coefficient of dv_0, so a small change in the angle of elevation has a much greater effect on distance traveled than a small change in initial velocity.
15. Using trigonometry, $\ell = x\tan\theta$, so $d\ell = \tan\theta dx + x\sec^2\theta d\theta$. With $\theta = 85^\circ$ and $x = 30$, we have $d\ell = 11.43dx + 3949.38d\theta$. The measured length of the wall is much more sensitive to errors in θ than in x. While it can be difficult to compare sensitivities between measuring feet and measuring degrees (it is somewhat like "comparing apples to oranges"), here the coefficients are so different that the result is clear: a small error in degree has a much greater impact than a small error in distance.
16. With $D = n\ell$, the total differential is $dD = \ell\, dn + n\, d\ell$. If one measures with a short tape, n must be large and hence $n\, d\ell$ is going to be greater than when a large tape is used (wherein n will be small).
17. $dw = 2xyz^3\, dx + x^2z^3\, dy + 3x^2yz^2\, dz$
18. $dw = e^x \sin y \ln z\, dx + e^x \cos y \ln z\, dy + e^x \sin y \frac{1}{z}\, dz$

19. $dx = 0.05$, $dy = -0.1$. $dz = 9(.05) + (-2)(-0.1) = 0.65$. So $f(3.5, 0.9) \approx 7 + 0.65 = 7.65$.

20. $dx = -0.12$, $dy = 0.07$. $dz = 2.6(-.12) + (5.1)(0.07) = 0.045$. So $f(-4.12, 2.07) \approx 13 + 0.045 = 13.045$.

21. $dx = 0.5$, $dy = 0.1$, $dz = -0.2$.
$dw = 2(0.5) + (-3)(0.1) + 3.7(-0.2) = -0.04$, so
$f(2.5, 4.1, 4.8) \approx -1 - 0.04 = -1.04$.

22. $dx = 0.1$, $dy = 0.1$, $dz = 0.1$.
$dw = 2(0.1) + (0)(0.1) + (-2)(.1) = 0$, so
$f(3.1, 3.1, 3.1) \approx 5 + 0 = 5$.

Section 12.5

1. Because the parametric equations describe a level curve, z is constant for all t. Therefore $\frac{dz}{dt} = 0$.
2. $g'(x)$
3. $\frac{dx}{dt}$, and $\frac{\partial f}{\partial y}$
4. T
5. F
6. partial
7. (a) $\frac{dz}{dt} = 3(2t) + 4(2) = 6t + 8$.

   (b) At $t = 1$, $\frac{dz}{dt} = 14$.
8. (a) $\frac{dz}{dt} = 2x(1) - 2y(2t) = 2x - 4yt$

   (b) At $t = 1$, $x = 1$, $y = 0$ and $\frac{dz}{dt} = 2$.
9. (a) $\frac{dz}{dt} = 5(-2\sin t) + 2(\cos t) = -10\sin t + 2\cos t$

   (b) At $t = \pi/4$, $\frac{dz}{dt} = -4\sqrt{2}$.
10. (a) $\dfrac{dz}{dt} = \dfrac{1}{1+y^2}(-\sin t) - \dfrac{2xy}{(y^2+1)^2}(\cos t)$.

    (b) At $t = \pi/2$, $x = 0$, $y = 1$, and $\frac{dz}{dt} = -1/2$.
11. (a) $\dfrac{dz}{dt} = 2x(\cos t) + 4y(3\cos t)$.

    (b) At $t = \pi/4$, $x = \sqrt{2}/2$, $y = 3\sqrt{2}/2$, and $\frac{dz}{dt} = 19$.
12. (a) $\frac{dz}{dt} = -\sin x \sin y(\pi) + \cos x \cos y(2\pi)$.

    (b) At $t = 3$, $x = 3\pi$, $y = 13\pi/2$, and $\frac{dz}{dt} = 0$.
13. $t = -4/3$; this corresponds to a minimum
14. $t = 0, \pm\sqrt{3/2}$
15. $t = \tan^{-1}(1/5) + n\pi$, where n is an integer
16. We find that
$$\frac{dz}{dt} = -\frac{2\cos^2 t \sin t}{(1+\sin^2 t)^2} - \frac{\sin t}{1+\sin^2 t}.$$
Setting this equal to 0, finding a common denominator and factoring out $\sin t$, we get
$$\sin t\left(\frac{2\cos^2 t + \sin^2 t + 1}{(1+\sin^2 t)^2}\right) = 0.$$
We have $\sin t = 0$ when $t = \pi n$, where n is an integer. The expression in the parenthesis above is always positive, and hence never equal 0. So all solutions are $t = \pi n$, n is an integer.
17. We find that
$$\frac{dz}{dt} = 38\cos t \sin t.$$
Thus $\frac{dz}{dt} = 0$ when $t = \pi n$ or $\pi n + \pi/2$, where n is any integer.
18. We find that
$$\frac{dz}{dt} = -\pi\sin(\pi t)\sin(2\pi t + \pi/2) + 2\pi\cos(\pi t)\cos(2\pi t + \pi/2).$$
One can "easily" see that when t is an integer, $\sin(\pi t) = 0$ and $\cos(2\pi t + \pi/2) = 0$, hence $\frac{dz}{dt} = 0$ when t is an integer. There are other places where z has a relative max/min that require more work. First, verify that $\sin(2\pi t + \pi/2) = \cos(2\pi t)$, and $\cos(2\pi t + \pi/2) = -\sin(2\pi t)$. This lets us rewrite $\frac{dz}{dt} = 0$ as
$$-\sin(\pi t)\cos(2\pi t) - 2\cos(\pi t)\sin(2\pi t) = 0.$$
By bringing one term to the other side of the equality then dividing, we can get
$$2\tan(2\pi t) = -\tan(\pi t).$$
Using the angle sum/difference formulas found in the back of the book, we know
$$\tan(2\pi t) = \tan(\pi t) + \tan(\pi t) = \frac{\tan(\pi t) + \tan(\pi t)}{1 - \tan^2(\pi t)}.$$
Thus we write
$$2\frac{\tan(\pi t) + \tan(\pi t)}{1 - \tan^2(\pi t)} = -\tan(\pi t).$$
Solving for $\tan^2(\pi t)$, we find
$$\tan^2(\pi t) = 5 \quad \Rightarrow \quad \tan(\pi t) = \pm\sqrt{5},$$
and so
$$\pi t = \tan^{-1}(\pm\sqrt{5}) = \pm\tan^{-1}(\sqrt{5}).$$
Since the period of tangent is π, we can adjust our answer to be
$$\pi t = \pm\tan^{-1}(\sqrt{5}) + n\pi, \text{ where } n \text{ is an integer.}$$
Dividing by π, we find
$$t = \pm\frac{1}{\pi}\tan^{-1}(\sqrt{5}) + n, \text{ where } n \text{ is an integer.}$$
19. (a) $\frac{\partial z}{\partial s} = 2xy(1) + x^2(2) = 2xy + 2x^2$;
$\frac{\partial z}{\partial t} = 2xy(-1) + x^2(4) = -2xy + 4x^2$

    (b) With $s = 1$, $t = 1$, $x = 1$ and $y = 2$. Thus $\frac{\partial z}{\partial s} = 6$ and $\frac{\partial z}{\partial t} = 0$
20. (a) $\frac{\partial z}{\partial s} = -\pi\sin(\pi x + \pi y/2)(t^2) - \frac{1}{2}\pi\sin(\pi x + \pi y/2)(2st) = -\pi\Big(t^2\sin(\pi x + \pi y/2) + st\sin(\pi x + \pi y/2)\Big)$;
$\frac{\partial z}{\partial t} = -\pi\sin(\pi x + \pi y/2)(2st) - \frac{1}{2}\pi\sin(\pi x + \pi y/2)(s^2) = -\pi\Big(2st\sin(\pi x + \pi y/2) + \frac{1}{2}s^2\sin(\pi x + \pi y/2)\Big)$

    (b) With $s = 1$, $t = 1$, $x = 1$ and $y = 1$. Thus $\frac{\partial z}{\partial s} = 2\pi$ and $\frac{\partial z}{\partial t} = 5\pi/2$
21. (a) $\frac{\partial z}{\partial s} = 2x(\cos t) + 2y(\sin t) = 2x\cos t + 2y\sin t$;
$\frac{\partial z}{\partial t} = 2x(-s\sin t) + 2y(s\cos t) = -2xs\sin t + 2ys\cos t$

    (b) With $s = 2$, $t = \pi/4$, $x = \sqrt{2}$ and $y = \sqrt{2}$. Thus $\frac{\partial z}{\partial s} = 4$ and $\frac{\partial z}{\partial t} = 0$
22. (a) $\frac{\partial z}{\partial s} = -2xe^{-(x^2+y^2)}(0) - 2ye^{-(x^2+y^2)}(t^2) = -2yt^2e^{-(x^2+y^2)}$;
$\frac{\partial z}{\partial t} = -2xe^{-(x^2+y^2)}(1) - 2ye^{-(x^2+y^2)}(2st) = -2xe^{-(x^2+y^2)} - 4stye^{-(x^2+y^2)}$

    (b) With $s = 1$, $t = 1$, $x = 1$ and $y = 1$. Thus $\frac{\partial z}{\partial s} = -2/e^2$ and $\frac{\partial z}{\partial t} = -6/e^2$
23. $f_x = 2x\tan y$, $f_y = x^2\sec^2 y$;
$\dfrac{dy}{dx} = -\dfrac{2\tan y}{x\sec^2 y}$

24. $f_x = 4(3x^2+2y^3)^3(6x), f_y = 4(3x^2+2y^3)^3(6y^2)$;
$\dfrac{dy}{dx} = -\dfrac{x}{y^2}$

25. $f_x = \dfrac{(x+y^2)(2x)-(x^2+y)(1)}{(x+y^2)^2}$,
$f_y = \dfrac{(x+y^2)(1)-(x^2+y)(2y)}{(x+y^2)^2}$;
$\dfrac{dy}{dx} = -\dfrac{2x(x+y^2)-(x^2+y)}{x+y^2-2y(x^2+y)}$

26. $f_x = \dfrac{2x+y}{x^2+xy+y^2}, f_y = \dfrac{x+2y}{x^2+xy+y^2}$;
$\dfrac{dy}{dx} = -\dfrac{2x+y}{2y+x}$

Section 12.6

1. A partial derivative is essentially a special case of a directional derivative; it is the directional derivative in the direction of x or y, i.e., $\langle 1,0\rangle$ or $\langle 0,1\rangle$.
2. $\vec{u} = \langle 1,0\rangle$
3. $\vec{u} = \langle 0,1\rangle$
4. orthogonal
5. maximal, or greatest
6. dot
7. $\nabla f = \langle -2xy+y^2+y, -x^2+2xy+x\rangle$
8. $\nabla f = \langle \cos x\cos y, -\sin x\sin y\rangle$
9. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$
10. $\nabla f = \langle -4,3\rangle$
11. $\nabla f = \langle 2x-y-7, 4y-x\rangle$
12. $\nabla f = \langle 2xy^3-2, 3x^2y^2\rangle$
13. $\nabla f = \langle -2xy+y^2+y, -x^2+2xy+x\rangle$; $\nabla f(2,1) = \langle -2,2\rangle$. Be sure to change all directions to unit vectors.
    (a) $2/5$ ($\vec{u} = \langle 3/5, 4/5\rangle$)
    (b) $-2\sqrt{5}$ ($\vec{u} = \langle -1/\sqrt{5}, -2\sqrt{5}\rangle$)
14. $\nabla f = \langle \cos x\cos y, -\sin x\sin y\rangle$; $\nabla f(\frac{\pi}{4},\frac{\pi}{3}) = \left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle$. Be sure to change all directions to unit vectors.
    (a) $\frac{1}{4}(1-\sqrt{3})$ ($\vec{u} = \langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$)
    (b) $\frac{4\sqrt{3}-1}{10\sqrt{2}}$ ($\vec{u} = \langle -3/5, -4/5\rangle$)
15. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$; $\nabla f(1,1) = \langle -2/9, -2/9\rangle$. Be sure to change all directions to unit vectors.
    (a) 0 ($\vec{u} = \langle 1/\sqrt{2}, -1/\sqrt{2}\rangle$)
    (b) $2\sqrt{2}/9$ ($\vec{u} = \langle -1/\sqrt{2}, -1/\sqrt{2}\rangle$)
16. $\nabla f = \langle -4,3\rangle$; $\nabla f(5,2) = \langle -4,3\rangle$. Be sure to change all directions into unit vectors.
    (a) $-9/\sqrt{10}$ ($\vec{u} = \langle 3/\sqrt{10}, 1/\sqrt{10}\rangle$)
    (b) $27/\sqrt{34}$ ($\vec{u} = \langle -3/\sqrt{34}, 5/\sqrt{34}\rangle$)
17. $\nabla f = \langle 2x-y-7, 4y-x\rangle$; $\nabla f(4,1) = \langle 0,0\rangle$.
    (a) 0
    (b) 0
18. $\nabla f = \langle 2xy^3-2, 3x^2y^2\rangle$; $\nabla f(1,1) = \langle 0,3\rangle$ Be sure to change all directions to unit vectors.
    (a) $3/\sqrt{2}$; ($\vec{u} = \langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$)
    (b) 3
19. $\nabla f = \langle -2xy+y^2+y, -x^2+2xy+x\rangle$
    (a) $\nabla f(2,1) = \langle -2,2\rangle$
    (b) $||\nabla f(2,1)|| = ||\langle -2,2\rangle|| = \sqrt{8}$
    (c) $\langle 2,-2\rangle$
    (d) $\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$
20. $\nabla f = \langle \cos x\cos y, -\sin x\sin y\rangle$
    (a) $\nabla f(\frac{\pi}{4},\frac{\pi}{3}) = \left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle$
    (b) $||\nabla f(\frac{\pi}{4},\frac{\pi}{3})|| = \left|\left|\left\langle \frac{1}{2\sqrt{2}}, -\frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle\right|\right| = 1/\sqrt{2}$
    (c) $\left\langle -\frac{1}{2\sqrt{2}}, \frac{1}{2}\sqrt{\frac{3}{2}}\right\rangle$
    (d) $\left\langle \frac{1}{2}\sqrt{\frac{3}{2}}, \frac{1}{2\sqrt{2}}\right\rangle$
21. $\nabla f = \left\langle \frac{-2x}{(x^2+y^2+1)^2}, \frac{-2y}{(x^2+y^2+1)^2}\right\rangle$
    (a) $\nabla f(1,1) = \langle -2/9, -2/9\rangle$.
    (b) $||\nabla f(1,1)|| = ||\langle -2/9, -2/9\rangle|| = 2\sqrt{2}/9$
    (c) $\langle 2/9, 2/9\rangle$
    (d) $\langle 1/\sqrt{2}, -1/\sqrt{2}\rangle$
22. $\nabla f = \langle -4,3\rangle$
    (a) $\nabla f(5,4) = \langle -4,3\rangle$.
    (b) $||\nabla f(5,4)|| = ||\langle -4,3\rangle|| = 5$
    (c) $\langle 4,-3\rangle$
    (d) $\langle 3/5, 4/5\rangle$
23. $\nabla f = \langle 2x-y-7, 4y-x\rangle$
    (a) $\nabla f(4,1) = \langle 0,0\rangle$
    (b) 0
    (c) $\langle 0,0\rangle$
    (d) All directions give a directional derivative of 0.
24. $\nabla f = \langle 2xy^3-2, 3x^2y^2\rangle$
    (a) $\nabla f(1,1) = \langle 0,3\rangle$
    (b) 3
    (c) $\langle 0,-3\rangle$
    (d) $\vec{u} = \langle 1,0\rangle$
25. (a) $\nabla F(x,y,z) = \langle 6xz^3+4y, 4x, 9x^2z^2-6z\rangle$
    (b) $113/\sqrt{3}$
26. (a) $\nabla F(x,y,z) = \langle \cos x\cos ye^z, -\sin x\sin ye^z, \sin x\cos ye^z\rangle$
    (b) $2/3$
27. (a) $\nabla F(x,y,z) = \langle 2xy^2, 2y(x^2-z^2), -2y^2z\rangle$
    (b) 0
28. (a) $\nabla F(x,y,z) =$
    $\left\langle -\frac{4x}{(x^2+y^2+z^2)^2}, -\frac{4y}{(x^2+y^2+z^2)^2}, -\frac{4z}{(x^2+y^2+z^2)^2}\right\rangle$
    (b) 0

Section 12.7

1. Answers will vary. The displacement of the vector is one unit in the x-direction and 3 units in the z-direction, with no change in y. Thus along a line parallel to $\vec{v}$, the change in z is 3 times the change in x – i.e., a "slope" of 3. Specifically, the line in the x-z plane parallel to z has a slope of 3.

2. Answers will vary. Let $\vec{u} = \langle 0.6, 0.8\rangle$; this is a unit vector. The displacement of the vector is one unit in the $\vec{u}$-direction and -2 units in the z-direction. In the plane containing the z-axis and the vector $\vec{u}$, the line parallel to $\vec{v}$ has slope -2.

3. T

4. On a surface through a point, there are many different smooth curves, each with a tangent line at the point. Each of these tangent lines is also "tangent" to the surface. There is not just one tangent line, but many, each in a different direction. Therefore we refer to directional tangent lines, not just *the* tangent line.

5. (a) $\ell_x(t) = \begin{cases} x = 2+t \\ y = 3 \\ z = -48-12t \end{cases}$

   (b) $\ell_y(t) = \begin{cases} x = 2 \\ y = 3+t \\ z = -48-40t \end{cases}$

   (c) $\ell_{\vec{u}}(t) = \begin{cases} x = 2+t/\sqrt{10} \\ y = 3+3t/\sqrt{10} \\ z = -48-66\sqrt{2/5}t \end{cases}$

6. (a) $\ell_x(t) = \begin{cases} x = \pi/3+t \\ y = \pi/6 \\ z = 3/4 - \frac{3\sqrt{3}}{4}t \end{cases}$

   (b) $\ell_y(t) = \begin{cases} x = \pi/3 \\ y = \pi/6+t \\ z = 3/4 + \frac{3\sqrt{3}}{4}t \end{cases}$

   (c) $\ell_{\vec{u}}(t) = \begin{cases} x = \pi/3+t/\sqrt{5} \\ y = \pi/6+2t/\sqrt{5} \\ z = 3/4 + \frac{3\sqrt{3/5}}{4}t \end{cases}$

7. (a) $\ell_x(t) = \begin{cases} x = 4+t \\ y = 2 \\ z = 2+3t \end{cases}$

   (b) $\ell_y(t) = \begin{cases} x = 4 \\ y = 2+t \\ z = 2-5t \end{cases}$

   (c) $\ell_{\vec{u}}(t) = \begin{cases} x = 4+t/\sqrt{2} \\ y = 2+t/\sqrt{2} \\ z = 2-\sqrt{2}t \end{cases}$

8. (a) $\ell_x(t) = \begin{cases} x = 1+t \\ y = 2 \\ z = 3 \end{cases}$

   (b) $\ell_y(t) = \begin{cases} x = 1 \\ y = 2+t \\ z = 3 \end{cases}$

   (c) $\ell_{\vec{u}}(t) = \begin{cases} x = 1+t/\sqrt{2} \\ y = 2+t/\sqrt{2} \\ z = 3 \end{cases}$

9. $\ell_{\vec{n}}(t) = \begin{cases} x = 2-12t \\ y = 3-40t \\ z = -48-t \end{cases}$

10. $\ell_{\vec{n}}(t) = \begin{cases} x = \pi/3 - \frac{3\sqrt{3}}{4}t \\ y = \pi/6 + \frac{3\sqrt{3}}{4}t \\ z = 3/4 - t \end{cases}$

11. $\ell_{\vec{n}}(t) = \begin{cases} x = 4+3t \\ y = 2-5t \\ z = 2-t \end{cases}$

12. $\ell_{\vec{n}}(t) = \begin{cases} x = 1 \\ y = 2 \\ z = 3-t \end{cases}$

13. $(1.425, 1.085, -48.078)$, $(2.575, 4.915, -47.952)$

14. $(-0.195, 1.766, -0.206)$ and $(2.289, -0.719, 1.706)$

15. $(5.014, 0.31, 1.662)$ and $(2.986, 3.690, 2.338)$

16. $(1, 2, 1)$ and $(1, 2, 5)$

17. $-12(x-2) - 40(y-3) - (z+48) = 0$

18. $-\frac{3\sqrt{3}}{4}(x-\pi/3) + \frac{3\sqrt{3}}{4}(y-\pi/6) - (z-3/4) = 0$

19. $3(x-4) - 5(y-2) - (z-2) = 0$ (Note that this tangent plane is the same as the original function, a plane.)

20. $-(z-3) = 0$, or $z = 3$

21. $\nabla F = \langle x/4, y/2, z/8\rangle$; at P, $\nabla F = \left\langle 1/4, \sqrt{2}/2, \sqrt{6}/8\right\rangle$

   (a) $\ell_{\vec{n}}(t) = \begin{cases} x = 1+t/4 \\ y = \sqrt{2}+\sqrt{2}t/2 \\ z = \sqrt{6}+\sqrt{6}t/8 \end{cases}$

   (b) $\frac{1}{4}(x-1) + \frac{\sqrt{2}}{2}(y-\sqrt{2}) + \frac{\sqrt{6}}{8}(z-\sqrt{6}) = 0.$

22. $\nabla F = \left\langle -\frac{x}{2}, -\frac{2y}{9}, 2z\right\rangle$; at P, $\nabla F = \left\langle -2, 2/3, 2\sqrt{5}\right\rangle$

   (a) $\ell_{\vec{n}}(t) = \begin{cases} x = 4-2t \\ y = -3+2t/3 \\ z = \sqrt{5}+2\sqrt{5}t \end{cases}$

   (b) $-2(x-4) + \frac{2}{3}(y+3) + 2\sqrt{5}(z-\sqrt{5}) = 0.$

23. $\nabla F = \left\langle y^2-z^2, 2xy, -2xz\right\rangle$; at P, $\nabla F = \langle 0, 4, 4\rangle$

   (a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2 \\ y = 1+4t \\ z = -1+4t \end{cases}$

   (b) $4(y-1) + 4(z+1) = 0.$

24. $\nabla F = \langle y\cos(xy), x\cos(xy) - z\sin(yz), -y\sin(yz)\rangle$; at P, $\nabla F = \left\langle \frac{\pi}{8\sqrt{3}}, -\sqrt{3}, -\frac{\pi}{8\sqrt{3}}\right\rangle$

   (a) $\ell_{\vec{n}}(t) = \begin{cases} x = 2+\frac{\pi}{8\sqrt{3}}t \\ y = \frac{\pi}{12}-\sqrt{3}t \\ z = 4-\frac{\pi}{8\sqrt{3}}t \end{cases}$

   (b) $\frac{\pi}{8\sqrt{3}}(x-2) - \sqrt{3}(y-\frac{\pi}{12}) - \frac{\pi}{8\sqrt{3}}(z-4) = 0.$

Section 12.8

1. F; it is the "other way around."

2. T

3. T

4. Answers will vary. A good answer will state that we are optimizing a function subject to a constraint, or limit, on the domain of the function. We are looking to maximize/minimize the function while "looking" at only a certain part of the domain.

5. One critical point at $(-4, 2)$; $f_{xx} = 1$ and $D = 4$, so this point corresponds to a relative minimum.

6. One critical point at $(7, -6)$; $D = -5$, so this point corresponds to a saddle point.

7. One critical point at $(6, -3)$; $D = -4$, so this point corresponds to a saddle point.

8. One critical point at $(0,0)$; $f_{xx} = -2$ and $D = 4$, so this point corresponds to a relative maximum.

9. Two critical points: at $(0,-1)$; $f_{xx} = 2$ and $D = -12$, so this point corresponds to a saddle point;
at $(0,1)$, $f_{xx} = 2$ and $D = 12$, so this corresponds to a relative minimum.

10. There are 4 critical points:
$(-1,-2)$, rel. max; $(1,-2)$, saddle point;
$(-1,2)$, saddle point; $(1,2)$, rel. min.,
where $f_{xx} = 2x$ and $D = 4xy$.

11. One critical point at $(0,0)$. $D = -12x^2y^2$, so at $(0,0)$, $D = 0$ and the test is inconclusive. (Some elementary thought shows that it is the absolute minimum.)

12. Six critical points: $f_x = 0$ when $x = -1, 0$ and 1; $f_y = 0$ when $y = -3, 3$. Together, we get the points:
$(-1,-3)$ saddle point; $(-1,3)$ rel. min
$(0,-3)$ rel. max; $(0,3)$ saddle point
$(1,-3)$ saddle point; $(1,3)$ relative min
where $f_{xx} = 12x^2 - 4$ and $D = 24y(3x^2 - 1)$.

13. One critical point: $f_x = 0$ when $x = 3$; $f_y = 0$ when $y = 0$, so one critical point at $(3,0)$, which is a relative maximum, where
$f_{xx} = \frac{y^2-16}{(16-(x-3)^2-y^2)^{3/2}}$ and $D = \frac{16}{(16-(x-3)^2-y^2)^2}$.
Both f_x and f_y are undefined along the circle $(x-3)^2 + y^2 = 16$; at any point along this curve, $f(x,y) = 0$, the absolute minimum of the function.

14. One critical point: $f_x = 0$ when $x = 0$; $f_y = 0$ when $y = 0$, so one critical point at $(0,0)$ (although it should be noted that at $(0,0)$, both f_x and f_y are undefined.) The Second Derivative Test fails at $(0,0)$, with $D = 0$. A graph, or simple calculation, shows that $(0,0)$ is the absolute minimum of f.

15. The triangle is bound by the lines $y = -1$, $y = 2x + 1$ and $y = -2x + 1$.
Along $y = -1$, there is a critical point at $(0,-1)$.
Along $y = 2x + 1$, there is a critical point at $(-3/5, -1/5)$.
Along $y = -2x + 1$, there is a critical point at $(3/5, -1/5)$.
The function f has one critical point, irrespective of the constraint, at $(0, -1/2)$.
Checking the value of f at these four points, along with the three vertices of the triangle, we find the absolute maximum is at $(0,1,3)$ and the absolute minimum is at $(0,-1/2,3/4)$.

16. The region has two "corners" at $(1,1)$ and $(-1,1)$.
Along $y = 1$, there is no critical point.
Along $y = x^2$, there is a critical point at $(5/14, 25/196) \approx (0.357, 0.128)$.
The function f itself has no critical points. Checking the value of f at the corners $(1,1)$, $(-1,1)$ and the critical point $(5/14, 25/196)$, we find the absolute maximum is at $(5/14, 25/196, 25/28) \approx (0.357, 0.128, 0.893)$ and the absolute minimum is at $(-1,1,-12)$.

17. The region has no "corners" or "vertices," just a smooth edge.
To find critical points along the circle $x^2 + y^2 = 4$, we solve for y^2: $y^2 = 4 - x^2$. We can go further and state $y = \pm\sqrt{4-x^2}$.
We can rewrite f as
$f(x) = x^2 + 2x + (4 - x^2) + \sqrt{4-x^2} = 2x + 4 + \sqrt{4-x^2}$. (We will return and use $-\sqrt{4-x^2}$ later.) Solving $f'(x) = 0$, we get $x = \sqrt{2} \Rightarrow y = \sqrt{2}$. $f'(x)$ is also undefined at $x = \pm 2$, where $y = 0$.
Using $y = -\sqrt{4-x^2}$, we rewrite $f(x,y)$ as
$f(x) = 2x + 4 - \sqrt{4-x^2}$. Solving $f'(x) = 0$, we get $x = -\sqrt{2},\ y = -\sqrt{2}$.
The function f itself has a critical point at $(-1,-1)$.
Checking the value of f at $(-1,-1)$, $(\sqrt{2},\sqrt{2})$, $(-\sqrt{2},-\sqrt{2})$, $(2,0)$ and $(-2,0)$, we find the absolute maximum is at $(2,0,8)$ and the absolute minimum is at $(-1,-1,-2)$.

18. The region has two "corners" at $(-1,-1)$ and $(1,1)$.
Along the line $y = x$, $f(x,y)$ becomes $f(x) = 3x - 2x^2$. Along this line, we have a critical point at $(3/4, 3/4)$.
Along the curve $y = x^2 + x - 1$, $f(x,y)$ becomes $f(x) = x^2 + 3x - 3$. There is a critical point along this curve at $(-3/2, -1/4)$. Since $x = -3/2$ lies outside our bounded region, we ignore this critical point.
The function f itself has no critical points.
Checking the value of f at $(-1,-1)$, $(1,1)$, $(3/4,3/4)$, we find the absolute maximum is at $(3/4, 3/4, 9/8)$ and the absolute minimum is at $(-1,-1,-5)$.

Chapter 13

Section 13.1

1. $C(y)$, meaning that instead of being just a constant, like the number 5, it is a function of y, which acts like a constant when taking derivatives with respect to x.

2. iterated integration

3. curve to curve, then from point to point

4. area

5. (a) $18x^2 + 42x - 117$
   (b) -108

6. (a) $2 + \pi^2 \cos y$
   (b) $\pi^2 + \pi$

7. (a) $x^4/2 - x^2 + 2x - 3/2$
   (b) $23/15$

8. (a) $y^4/2 - y^3 + y^2/2$
   (b) $8/15$

9. (a) $\sin^2 y$
   (b) $\pi/2$

10. (a) $x/(1 + x^2)$
    (b) $\frac{1}{2} \ln\left(\frac{5}{2}\right)$

11. $\int_1^4 \int_{-2}^1 dy\,dx$ and $\int_{-2}^1 \int_1^4 dx\,dy$.
area of $R = 9u^2$

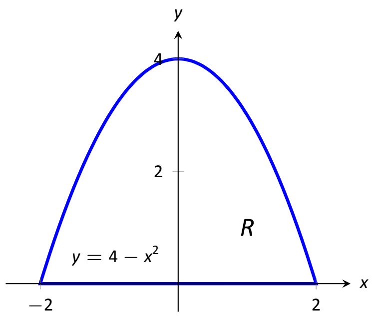
12. $\int_1^4 \int_1^{\frac{2}{3}x+\frac{1}{3}} dy\,dx$ and $\int_1^3 \int_{\frac{3}{2}y-\frac{1}{2}}^4 dx\,dy$.
area of $R = 3u^2$

13. $\int_2^4 \int_{x-1}^{7-x} dy\,dx$. The order $dx\,dy$ needs two iterated integrals as x is bounded above by two different functions. This gives:
$$\int_1^3 \int_2^{y+1} dx\,dy + \int_3^5 \int_2^{7-y} dx\,dy.$$
area of $R = 4u^2$

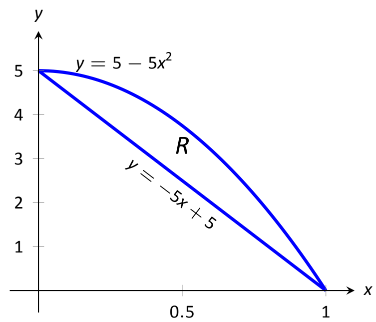
14. $\int_0^{12} \int_{-\sqrt{3x}}^{\sqrt{3x}} dy\,dx$ and $\int_{-6}^{6} \int_{y^2/3}^{12} dx\,dy$
area of $R = 96u^2$

15. $\int_0^1 \int_{x^4}^{\sqrt{x}} dy\,dx$ and $\int_0^1 \int_{y^2}^{\sqrt[4]{y}} dx\,dy$
area of $R = 7/15u^2$

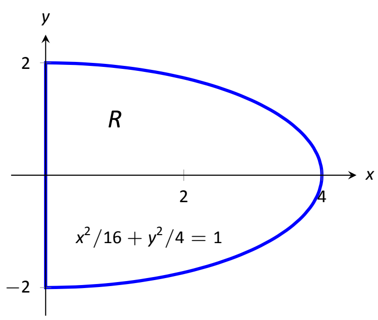
16. $\int_0^2 \int_{x^3}^{4x} dy\,dx$ and $\int_0^8 \int_{y/4}^{\sqrt[3]{y}} dx\,dy$
area of $R = 4u^2$

17. 

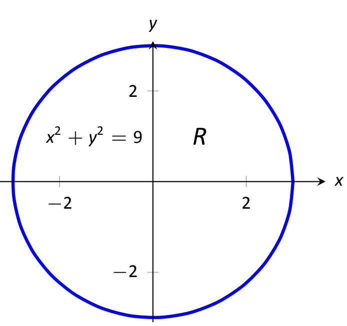
area of $R = \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \,dx\,dy$

18. 

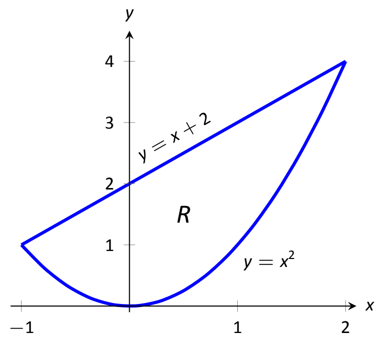
area of $R = \int_0^5 \int_{1-y/5}^{\sqrt{1-y/5}} \,dx\,dy$

19. 

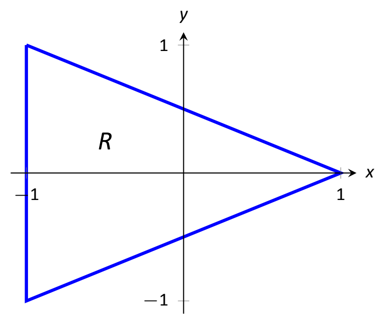
area of $R = \int_0^4 \int_{-\sqrt{4-x^2/4}}^{\sqrt{4-x^2/4}} \,dy\,dx$

20. 

area of $R = \int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \,dx\,dy$

21. 

area of $R = \int_{-1}^2 \int_{x^2}^{x+2} \,dy\,dx$

22. 

area of $R = \int_{-1}^0 \int_0^{2y+1} \,dx\,dy + \int_0^1 \int_0^{1-2y} \,dx\,dy$

Section 13.2

1. volume
2. When switching the order of integration, the bounds integrals must change to reflect the bounds of the region of integration. You cannot merely change the letters x and y in a few places.
3. The double integral gives the signed volume under the surface. Since the surface is always positive, it is always above the x-y plane and hence produces only "positive" volume.
4. No. It means that there is the same amount of signed volume under f and g over R, but the functions could be very different.
5. 6; $\int_{-1}^1 \int_1^2 \left(\frac{x}{y} + 3\right) \,dy\,dx$
6. 4; $\int_0^{\pi} \int_{-\pi/2}^{\pi/2} \left(\sin x \cos y\right) \,dy\,dx$
7. 112/3; $\int_0^2 \int_0^{4-2y} \left(3x^2 - y + 2\right) \,dx\,dy$
8. 76/15; $\int_1^3 \int_0^x \left(x^2y - xy^2\right) \,dy\,dx$
9. 16/5; $\int_{-1}^1 \int_0^{1-x^2} \left(x + y + 2\right) \,dy\,dx$
10. 6561/40; $\int_0^3 \int_{x^2}^{3x} \left(xy^2\right) \,dy\,dx$
11. $\frac{3}{56} = \int_0^1 \int_{x^2}^{\sqrt{x}} x^2y \,dy\,dx = \int_0^1 \int_{y^2}^{\sqrt{y}} x^2y \,dx\,dy.$
12. $\frac{8}{99} = \int_0^1 \int_{x^3}^{\sqrt[3]{x}} x^2y \,dy\,dx = \int_0^1 \int_{y^3}^{\sqrt[3]{y}} x^2y \,dx\,dy.$
13. $0 = \int_{-1}^1 \int_{-1}^1 x^2 - y^2 \,dy\,dx = \int_{-1}^1 \int_{-1}^1 x^2 - y^2 \,dx\,dy.$

14. $e/2-1=\int_0^1\int_0^{y^2} ye^x\,dx\,dy=\int_0^1\int_{\sqrt{x}}^1 ye^x\,dy\,dx.$

15. $6=\int_0^2\int_0^{3-3/2x}\big(6-3x-2y\big)\,dy\,dx=$
$\int_0^3\int_0^{2-2/3y}\big(6-3x-2y\big)\,dx\,dy.$

16. $-\frac{1}{2}e^2+2e-\frac{3}{2}=\int_1^e\int_{\frac{x-1}{e-1}}^{\ln x} e^y\,dy\,dx=\int_0^1\int_{e^y}^{y(e-1)+1} e^y\,dx\,dy.$

17. $0=\int_{-3}^3\int_0^{\sqrt{9-x^2}}\big(x^3y-x\big)\,dy\,dx=$
$\int_0^3\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\big(x^3y-x\big)\,dx\,dy.$

18. $3e-7=\int_0^1\int_{ey}^{e^y}\big(4-3y\big)\,dx\,dy=$
$\int_0^1\int_0^{x/e}\big(4-3y\big)\,dy\,dx+\int_1^e\int_{\ln x}^{x/e}\big(4-3y\big)\,dy\,dx.$

19. Integrating e^{x^2} with respect to x is not possible in terms of elementary functions. $\int_0^2\int_0^{2x} e^{x^2}\,dy\,dx=e^4-1.$

20. Integrating $\cos(y^2)$ with respect to y is not possible in terms of elementary functions. $\int_0^{\sqrt{\pi/2}}\int_0^y\cos(y^2)\,dx\,dy=\frac{1}{2}.$

21. Integrating $\int_y^1\frac{2y}{x^2+y^2}\,dx$ gives $\tan^{-1}(1/y)-\pi/4$; integrating $\tan^{-1}(1/y)$ is hard.
$\int_0^1\int_0^x\frac{2y}{x^2+y^2}\,dy\,dx=\ln 2.$

22. Integrating in the order shown is hard/impossible. By changing the order of integration, we have $\int_1^2\int_{-1}^1\frac{x\tan^2 y}{1+\ln y}\,dx\,dy=0,$ since the integrand is an odd function with respect to x. Thus the iterated integral evaluates to 0.

23. average value of $f=6/2=3$

24. average value of $f=4/\pi^2$

25. average value of $f=\frac{112/3}{4}=28/3$

26. average value of $f=\frac{76/15}{2}=\frac{38}{15}\approx 2.53$

Section 13.3

1. $f\big(r\cos\theta,r\sin\theta\big)$, $r\,dr\,d\theta$

2. Some regions in the x-y plane are easier to describe using polar coordinates than using rectangular coordinates. Also, some integrals are easier to evaluate one the polar substitutions have been made.

3. $\int_0^{2\pi}\int_0^1\big(3r\cos\theta-r\sin\theta+4\big)r\,dr\,d\theta=4\pi$

4. $\int_0^{2\pi}\int_0^2\big(4r\cos\theta+4r\sin\theta\big)r\,dr\,d\theta=0$

5. $\int_0^{\pi}\int_{\cos\theta}^{3\cos\theta}\big(8-r\sin\theta\big)r\,dr\,d\theta=16\pi$

6. $\int_0^{\pi/2}\int_0^{\sin(2\theta)}\big(4\big)r\,dr\,d\theta=\pi/2$

7. $\int_0^{2\pi}\int_1^2\big(\ln(r^2)\big)r\,dr\,d\theta=2\pi\big(\ln 16-3/2\big)$

8. $\int_0^{2\pi}\int_0^1\big(1-r^2\big)r\,dr\,d\theta=\pi/2$

9. $\int_{-\pi/2}^{\pi/2}\int_0^6\big(r^2\cos^2\theta-r^2\sin^2\theta\big)r\,dr\,d\theta=$
$\int_{-\pi/2}^{\pi/2}\int_0^6\big(r^2\cos(2\theta)\big)r\,dr\,d\theta=0$

10. $\int_0^{\pi/4}\int_0^1\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)r\,dr\,d\theta=\ln 2$

11. $\int_{-\pi/2}^{\pi/2}\int_0^5\big(r^2\big)\,dr\,d\theta=125\pi/3$

12. $\int_{\pi/2}^{3\pi/2}\int_0^4\big(2r\sin\theta-r\cos\theta\big)r\,dr\,d\theta=128/3$

13. $\int_0^{\pi/4}\int_0^{\sqrt{8}}\big(r\cos\theta+r\sin\theta\big)r\,dr\,d\theta=16\sqrt{2}/3$

14. $\int_0^{\pi}\int_1^2\big(r\cos\theta+5\big)r\,dr\,d\theta=15\pi/2$

15. (a) This is impossible to integrate with rectangular coordinates as $e^{-(x^2+y^2)}$ does not have an antiderivative in terms of elementary functions.
    (b) $\int_0^{2\pi}\int_0^a re^{r^2}\,dr\,d\theta=\pi(1-e^{-a^2}).$
    (c) $\lim_{a\to\infty}\pi(1-e^{-a^2})=\pi$. This implies that there is a finite volume under the surface $e^{-(x^2+y^2)}$ over the entire x-y plane.

16.
$$\begin{aligned}\iint_R f(x,y)\,dA&=\int_0^{2\pi}\int_0^a\left(h-h\sqrt{\frac{r^2\cos^2\theta}{a^2}+\frac{r^2\sin^2\theta}{a^2}}\right)r\,dr\,d\theta\\&=\int_0^{2\pi}\int_0^a\left(hr-h\frac{r^2}{a}\right)\,dr\,d\theta\\&=\int_0^{2\pi}\left(\frac{1}{2}hr^2-\frac{h}{3a}r^3\right)\Big|_0^a\,d\theta\\&=\int_0^{2\pi}\left(\frac{1}{6}a^2h\right)\,d\theta\\&=\frac{1}{3}\pi a^2h.\end{aligned}$$

Section 13.4

1. Because they are scalar multiples of each other.

2. y

3. “little masses”

4. A collection of individual masses in the plane. Each mass is a point mass, i.e., mass located at a point, not across a region.

5. M_x measures the moment about the x-axis, meaning we need to measure distance from the x-axis. Such measurements are measures in the y-direction.

6. If the lamina is an annulus, the center of mass will likely be in the middle, outside of the region. (See Example 466.)

7. $\bar{x}=5.25$

8. $\bar{x}=1.3$

9. $(\bar{x},\bar{y})=(0,3)$

10. $(\bar{x},\bar{y})=(0,1/3)$

11. $M = 150\text{gm}$;
12. $M = 190\text{gm}$
13. $M = 2\text{lb}$
14. $M = 2/3\text{lb}$
15. $M = 16\pi \approx 50.27\text{kg}$
16. $M = 325\pi/12 \approx 85\text{kg}$
17. $M = 54\pi \approx 169.65\text{lb}$
18. $M = 63\pi \approx 197.92\text{lb}$
19. $M = 150\text{gm}$; $M_y = 600$; $M_x = -75$; $(\bar{x}, \bar{y}) = (4, -0.5)$
20. $M = 190\text{gm}$; $M_y = 850$; $M_x = -315/2$; $(\bar{x}, \bar{y}) = (4.47, -0.83)$
21. $M = 2\text{lb}$; $M_y = 0$; $M_x = 2/3$; $(\bar{x}, \bar{y}) = (0, 1/3)$
22. $M = 2/3\text{lb}$; $M_y = 7/30$; $M_x = 7/30$; $(\bar{x}, \bar{y}) = (0.35, 0.35)$
23. $M = 16\pi \approx 50.27\text{kg}$; $M_y = 4\pi$; $M_x = 4\pi$; $(\bar{x}, \bar{y}) = (1/4, 1/4)$
24. $M = 325\pi/12 \approx 85\text{kg}$; $M_y = 2375/12$; $M_x = 2375/12$; $(\bar{x}, \bar{y}) = (2.33, 2.33)$
25. $M = 54\pi \approx 169.65\text{lb}$; $M_y = 0$; $M_x = 504$; $(\bar{x}, \bar{y}) = (0, 2.97)$
26. $M = 63\pi \approx 197.92\text{lb}$; $M_y = 0$; $M_x = 1215/2$; $(\bar{x}, \bar{y}) = (0, 3.07)$
27. $I_x = 64/3$; $I_y = 64/3$; $I_O = 128/3$
28. $I_x = 16/3$; $I_y = 256/3$; $I_O = 272/3$
29. $I_x = 16/3$; $I_y = 64/3$; $I_O = 80/3$
30. $I_x = 16$; $I_y = 16$; $I_O = 32$

Section 13.5

1. arc length
2. tangent
3. surface areas
4. Technology makes good approximations accessible, if not exact answers.
5. Intuitively, adding h to f only shifts f up (i.e., parallel to the z-axis) and does not change its shape. Therefore it will not change the surface area over R.

   Analytically, $f_x = g_x$ and $f_y = g_y$; therefore, the surface area of each is computed with identical double integrals.
6. Analytically, $g_x = 2f_x$ and $g_y = 2f_y$. The double integral to compute the surface area of f over R is $\iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA$; the double integral to compute the surface area of g over R is $\iint_R \sqrt{1 + 4f_x^2 + 4f_y^2}\, dA$, which is *not* twice the double integral used to calculate the surface area of f.
7. $SA = \int_0^{2\pi} \int_0^{2\pi} \sqrt{1 + \cos^2 x \cos^2 y + \sin^2 x \sin^2 y}\, dx\, dy$
8. $SA = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \sqrt{1 + \dfrac{4x^2 + 4y^2}{(1 + x^2 + y^2)^4}}\, dx\, dy$

   Polar offers simpler bounds:

   $$SA = \int_0^{2\pi} \int_0^{3} r\sqrt{1 + \frac{4r^2}{(1 + r^2)^4}}\, dr\, d\theta$$
9. $SA = \int_{-1}^{1} \int_{-1}^{1} \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy$
10. $SA = \int_{-5}^{5} \int_0^{1} \sqrt{1 + \dfrac{4x^2 e^{2x^2}}{\left(1 + e^{x^2}\right)^4}}\, dy\, dx$
11. $SA = \int_0^{3} \int_{-1}^{1} \sqrt{1 + 9 + 49}\, dx\, dy = 6\sqrt{59} \approx 46.09$
12. $SA = \int_0^{1} \int_0^{1-x} \sqrt{1 + 4 + 4}\, dy\, dx = 18$
13. This is easier in polar:

   $$\begin{aligned} SA &= \int_0^{2\pi} \int_0^{4} r\sqrt{1 + 4r^2\cos^2 t + 4r^2 \sin^2 t}\, dr\, d\theta \\ &= \int_0^{2\pi} \int_0^{4} r\sqrt{1 + 4r^2}\, dr\, d\theta \\ &= \frac{\pi}{6}\left(65\sqrt{65} - 1\right) \approx 273.87 \end{aligned}$$
14. $$\begin{aligned} SA &= \int_0^{1} \int_{-y}^{y} \sqrt{1 + 4 + 64y^2}\, dx\, dy \\ &= \int_0^{1} \left(2y\sqrt{5 + 64y^2}\right) dy \\ &= \frac{1}{96}\left(69\sqrt{69} - 5\sqrt{5}\right) \approx 5.85 \end{aligned}$$
15. $$\begin{aligned} SA &= \int_0^{2} \int_0^{2x} \sqrt{1 + 1 + 4x^2}\, dy\, dx \\ &= \int_0^{2} \left(2x\sqrt{2 + 4x^2}\right) dx \\ &= \frac{26}{3}\sqrt{2} \approx 12.26 \end{aligned}$$
16. $$\begin{aligned} SA &= \int_0^{1} \int_0^{1} \sqrt{1 + x + 9y}\, dx\, dy \\ &= \int_0^{1} \frac{2}{3}\left((9y + 2)^{3/2} - (9y + 1)^{3/2}\right) dy \\ &= \frac{4}{135}\left(121\sqrt{11} - 100\sqrt{10} - 4\sqrt{2} + 1\right) \approx 2.383 \end{aligned}$$
17. This is easier in polar:

   $$\begin{aligned} SA &= \int_0^{2\pi} \int_0^{5} r\sqrt{1 + \frac{4r^2\cos^2 t + 4r^2\sin^2 t}{r^2\sin^2 t + r^2\cos^2 t}}\, dr\, d\theta \\ &= \int_0^{2\pi} \int_0^{5} r\sqrt{5}\, dr\, d\theta \\ &= 25\pi\sqrt{5} \approx 175.62 \end{aligned}$$
18. This is easier in polar:

   $$\begin{aligned} SA &= 2\int_0^{2\pi} \int_0^{5} r\sqrt{1 + \frac{r^2\cos^2 t + r^2\sin^2 t}{25 - r^2\sin^2 t - r^2\cos^2 t}}\, dr\, d\theta \\ &= 2\int_0^{2\pi} \int_0^{5} r\sqrt{\frac{1}{25 - r^2}}\, dr\, d\theta \\ &= 100\pi \approx 314.16 \end{aligned}$$

19. Integrating in polar is easiest considering R:

$$\begin{aligned} SA &= \int_0^{2\pi}\int_0^1 r\sqrt{1+c^2+d^2}\,dr\,d\theta \\ &= \int_0^{2\pi} \frac{1}{2}\left(\sqrt{1+c^2+d^2}\right)\,dy \\ &= \pi\sqrt{1+c^2+d^2}. \end{aligned}$$

The value of h does not matter as it only shifts the plane vertically (i.e., parallel to the z-axis). Different values of h do not create different ellipses in the plane.

Section 13.6

1. surface to surface, curve to curve and point to point
2. One possible answer is "sum up lots of little volumes over D."
3. Answers can vary. From this section we used triple integration to find the volume of a solid region, the mass of a solid, and the center of mass of a solid.
4. δV.
5. $V=\int_{-1}^1\int_{-1}^1 \big(8-x^2-y^2-(2x+y)\big)\,dx\,dy = 88/3$
6. $V=\int_0^2\int_0^3 \big(x^2+y^2-(-x^2-y^2)\big)\,dy\,dx = 52$
7. $V=\int_0^{\pi}\int_0^{x} \big(\cos x\sin y + 2 - \sin x\cos y\big)\,dy\,dx = \pi^2-\pi \approx 6.728$
8. $V=\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \big(6-x^2-y^2-(2x^2+2y^2+3)\big)\,dy\,dx$. Integrating in polar is easier, giving $V=\int_0^{2\pi}\int_0^1 (3-3r^2)r\,dr\,d\theta = 3\pi/2$.
9. dz dy dx: $\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx$
   dz dx dy: $\int_0^1\int_0^{3-3y}\int_0^{2-2x/3-2y} dz\,dy\,dx$
   dy dz dx: $\int_0^3\int_0^{2-2x/3}\int_0^{1-x/3-z/2} dy\,dz\,dx$
   dy dx dz: $\int_0^2\int_0^{3-3z/2}\int_0^{1-x/3-z/2} dy\,dx\,dz$
   dx dz dy: $\int_0^1\int_0^{2-2y}\int_0^{3-3y-3z/2} dx\,dz\,dy$
   dx dy dz: $\int_0^2\int_0^{1-z/2}\int_0^{3-3y-3z/2} dx\,dy\,dz$
   $V=\int_0^3\int_0^{1-x/3}\int_0^{2-2x/3-2y} dz\,dy\,dx = 1.$
10. dz dy dx: $\int_1^3\int_0^2\int_0^{(3-x)/2} dz\,dy\,dx$
    dz dx dy: $\int_0^2\int_1^3\int_0^{(3-x)/2} dz\,dy\,dx$
    dy dz dx: $\int_1^3\int_0^{(3-x)/2}\int_0^2 dy\,dz\,dx$
    dy dx dz: $\int_0^1\int_1^{3-2z}\int_0^2 dy\,dx\,dz$
    dx dz dy: $\int_0^2\int_0^1\int_1^{3-2z} dx\,dz\,dy$
    dx dy dz: $\int_0^1\int_0^2\int_1^{3-2z} dx\,dy\,dz$
    $V=\int_0^1\int_0^2\int_1^{3-2z} dx\,dy\,dz = 2.$
11. dz dy dx: $\int_0^2\int_{-2}^0\int_{y^2/2}^{-y} dz\,dy\,dx$
    dz dx dy: $\int_{-2}^0\int_0^2\int_{y^2/2}^{-y} dz\,dx\,dy$
    dy dz dx: $\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx$
    dy dx dz: $\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dx\,dz$
    dx dz dy: $\int_{-2}^0\int_{y^2/2}^{-y}\int_0^2 dx\,dz\,dy$
    dx dy dz: $\int_0^2\int_{-\sqrt{2z}}^{-z}\int_0^2 dx\,dy\,dz$
    $V=\int_0^2\int_0^2\int_{-\sqrt{2z}}^{-z} dy\,dz\,dx = 4/3.$
12. dz dy dx: $\int_0^3\int_{3x}^9\int_0^{\sqrt{y^2-9x^2}} dz\,dy\,dx$
    dz dx dy: $\int_0^9\int_0^{y/3}\int_0^{\sqrt{y^2-9x^2}} dz\,dy\,dx$
    dy dz dx: $\int_0^3\int_0^{\sqrt{81-9x^2}}\int_{\sqrt{z^2+9x^2}}^9 dy\,dz\,dx$
    dy dx dz: $\int_0^9\int_0^{\sqrt{9-z^2/9}}\int_{\sqrt{z^2+9x^2}}^9 dy\,dx\,dz$
    dx dz dy: $\int_0^9\int_0^y\int_0^{\frac{1}{3}\sqrt{y^2-z^2}} dx\,dz\,dy$
    dx dy dz: $\int_0^9\int_z^9\int_0^{\frac{1}{3}\sqrt{y^2-z^2}} dx\,dy\,dz$
    N/A
13. dz dy dx: $\int_0^2\int_{1-x/2}^1\int_0^{2x+4y-4} dz\,dy\,dx$
    dz dx dy: $\int_0^1\int_{2-2y}^2\int_0^{2x+4y-4} dz\,dy\,dx$
    dy dz dx: $\int_0^2\int_0^{2x}\int_{z/4-x/2+1}^1 dy\,dz\,dx$
    dy dx dz: $\int_0^4\int_{z/2}^2\int_{z/4-x/2+1}^1 dy\,dx\,dz$
    dx dz dy: $\int_0^1\int_0^{4y}\int_{z/2-2y+2}^2 dx\,dz\,dy$
    dx dy dz: $\int_0^4\int_{z/4}^1\int_{z/2-2y+2}^2 dx\,dy\,dz$
    $V=\int_0^4\int_{z/4}^1\int_0^{2y-z/2-2} dx\,dy\,dz = 4/3.$
14. dz dy dx: $\int_{-2}^2\int_0^{4-x^2}\int_0^{2y} dz\,dy\,dx$
    dz dx dy: $\int_0^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_0^{2x+4y-4} dz\,dy\,dx$
    dy dz dx: $\int_{-2}^2\int_0^{8-2x^2}\int_{z/2}^{4-x^2} dy\,dz\,dx$
    dy dx dz: $\int_0^8\int_{-\sqrt{4-z/2}}^{\sqrt{4-z/2}}\int_{z/2}^{4-x^2} dy\,dx\,dz$
    dx dz dy: $\int_0^4\int_0^{2y}\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dz\,dy$
    dx dy dz: $\int_0^8\int_{z/2}^4\int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\,dy\,dz$
    $V=\int_{-2}^2\int_0^{4-x^2}\int_0^{2y} dz\,dy\,dx = 512/15.$
15. dz dy dx: $\int_0^1\int_0^{1-x^2}\int_0^{\sqrt{1-y}} dz\,dy\,dx$
    dz dx dy: $\int_0^1\int_0^{\sqrt{1-y}}\int_0^{\sqrt{1-y}} dz\,dy\,dx$

dy dz dx: $\int_0^1 \int_0^x \int_0^{1-x^2} dy\,dz\,dx + \int_0^1 \int_x^1 \int_0^{1-z^2} dy\,dz\,dx$

dy dx dz: $\int_0^1 \int_0^z \int_0^{1-z^2} dy\,dx\,dz + \int_0^1 \int_z^1 \int_0^{1-x^2} dy\,dx\,dz$

dx dz dy: $\int_0^1 \int_0^{\sqrt{1-y}} \int_0^{\sqrt{1-y}} dx\,dz\,dy$

dx dy dz: $\int_0^1 \int_0^{1-z^2} \int_0^{\sqrt{1-y}} dx\,dy\,dz$

Answers will vary. Neither order is particularly "hard." The order *dz dy dx* requires integrating a square root, so powers can be messy; the order *dy dz dx* requires two triple integrals, but each uses only polynomials.

16. *dz dy dx*: $\int_0^1 \int_0^{3x} \int_0^{1-x} dz\,dy\,dx + \int_0^1 \int_{3x}^3 \int_0^{1-y/3} dz\,dy\,dx$

dz dx dy: $\int_0^3 \int_0^{y/3} \int_0^{1-y/3} dz\,dy\,dx + \int_0^3 \int_{y/3}^1 \int_0^{1-x} dz\,dy\,dx$

dy dz dx: $\int_0^1 \int_0^{1-x} \int_0^{3-3z} dy\,dz\,dx$

dy dx dz: $\int_0^1 \int_0^{1-z} \int_0^{3-3z} dy\,dx\,dz$

dx dz dy: $\int_0^3 \int_0^{1-y/3} \int_0^{1-z} dx\,dz\,dy$

dx dy dz: $\int_0^1 \int_0^{3-3z} \int_0^{1-z} dx\,dy\,dz$

$V = \int_0^1 \int_0^{3-3z} \int_0^{1-z} dx\,dy\,dz = 1.$

17. 8

18. 7/8

19. π

20. 0

21. $M = 10$, $M_{yz} = 15/2$, $M_{xz} = 5/2$, $M_{xy} = 5$; $(\bar{x}, \bar{y}, \bar{z}) = (3/4, 1/4, 1/2)$

22. $M = 4$, $M_{yz} = 20/3$, $M_{xz} = 4$, $M_{xy} = 4/3$; $(\bar{x}, \bar{y}, \bar{z}) = (5/3, 1, 1/3)$

23. $M = 16/5$, $M_{yz} = 16/3$, $M_{xz} = 104/45$, $M_{xy} = 32/9$; $(\bar{x}, \bar{y}, \bar{z}) = (5/3, 13/18, 10/9) \approx (1.67, 0.72, 1.11)$

24. $M = \frac{65{,}536}{15} \approx 208.05$, $M_{yz} = 0$, $M_{xz} = \frac{2{,}097{,}152}{3465} \approx 605.24$, $M_{xy} = \frac{2{,}097{,}152}{3465} \approx 605.24$; $(\bar{x}, \bar{y}, \bar{z}) = (0, 32/11, 32/11) \approx (0, 2.91, 2.91)$