Differentiation

Rules

$\frac{d}{dx}\left[\tan x\right] = \sec^2 x \quad \frac{d}{dx}\left[\cot x\right] = -\csc^2 x$

$\frac{d}{dx}\left[cu\right] = cu' \quad \frac{d}{dx}\left[u \pm v\right] = u' \pm v' \quad \frac{d}{dx}\left[uv\right] = u'v + uv' \quad \frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$

$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x} \quad \frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \ln a} \quad \frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}} \quad \frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2} \quad \frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2} \quad \frac{d}{dx}\left[\sec x\right] = \sec x \tan x$

$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Integration

Rules

$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx \quad \int cf(x)\,dx = c\int f(x)\,dx \quad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C \quad \int f'g + fg'\,dx = fg + C$

$\int \cot x\,dx = \ln|\sin x| + C \quad \int \sec^2 x \tan x\,dx = \frac{\tan^2 x}{2} + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \quad \int \csc x \cot x\,dx = -\csc x + C \quad \int \sec x \tan x\,dx = \sec x + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C \quad \int \csc^2 x\,dx = -\cot x + C \quad \int \frac{1}{x}\,dx = \ln|x| + C \quad \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$\int \cos x \sin x\,dx = \frac{\sin^2 x}{2} + C \quad \int \sin x\,dx = -\cos x + C$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right) + C$

$\int \tan x\,dx = -\ln|\cos x| + C$

Definitions of the Trigonometric Functions

Unit Circle Definition

x

y

$\sin\theta = y \quad \cos\theta = x$

(x, y)

$\csc\theta = \frac{1}{y} \quad \sec\theta = \frac{1}{x}$

y

$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$

x

θ

Right Triangle Definition

$\sin\theta = \frac{O}{H} \quad \csc\theta = \frac{H}{O}$

θ

$\cos\theta = \frac{A}{H} \quad \sec\theta = \frac{H}{A}$

$\tan\theta = \frac{O}{A} \quad \cot\theta = \frac{A}{O}$

Common Trigonometric Identities

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Double-Angle Formulas

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Even/Odd Formulas

Periodic Formulas

Half-Angle Formulas

Sum-to-Product Formulas

Product-to-Sum Formulas

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$\cos(\theta - \pi) = -\cos\theta$

$\sin(\theta - \pi) = -\sin\theta$

$\sin(\theta + 2\pi) = \sin\theta$

$\cos(\theta + 2\pi) = \cos\theta$

$\tan(\theta + \pi) = \tan\theta$

$\sin^2\frac{\theta}{2} = \frac{1 - \cos\theta}{2}$

$\cos^2\frac{\theta}{2} = \frac{1 + \cos\theta}{2}$

$\tan^2\frac{\theta}{2} = \frac{1 - \cos\theta}{1 + \cos\theta}$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

Areas and Volumes

Triangles

$h = a \sin \theta$

$\frac{1}{2}bh$

$c^2 = a^2 + b^2 - 2ab \cos \theta$

$b\theta ach$

Right Circular Cone

$\frac{1}{3}\pi r^2 h$

$\pi r\sqrt{r^2 + h^2} + \pi r^2$

hr

Parallelograms

bh

bh

Right Circular Cylinder

$\pi r^2 h$

$2\pi rh + 2\pi r^2$

hr

Trapezoids

$\frac{1}{2}(a + b)h$

bah

Sphere

$\frac{4}{3}\pi r^3$

$4\pi r^2$

r

Circles

πr^2

$2\pi r$

r

General Cone

A

$\frac{1}{3}Ah$

hA

Sectors of Circles

θ

$\frac{1}{2}\theta r^2$

$s = r\theta$

$rs\theta$

General Right Cylinder

A

Ah

hA

Algebra

Factors and Zeros of Polynomials

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

$p(a) = 0$

a *zero*

$p(x) = 0$

$(x - a)$ *factor*

Fundamental Theorem of Algebra

n

Quadratic Formula

$p(x) = ax^2 + bx + c$

$0 \leq b^2 - 4ac$

$x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Special Factors

$x^2 - a^2 = (x - a)(x + a)$ $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2) x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$

$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$ $(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 (x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

Rational Zero Theorem

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

rational zero

p

$x = r/s$

r

a_0

s

a_n

Factoring by Grouping

$acx^3 + adx^2 + bcx + bd =$

$ax^2(cs + d) + b(cx + d) =$

Additional Formulas

Summation Formulas:

$\sum_{i=1}^{n} c = cn$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Trapezoidal Rule (Area):

$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\big[f(x_1)+ 2f(x_2)+ 2f(x_3)+ ...+ 2f(x_n)+ f(x_{n+1})\big]$

$Error \leq \frac{(b-a)^3}{12n^2}\big[\max\big|xetexf''(x)f''(x)\big|\big]$

Simpson's Rule (Area):

$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\big[f(x_1)+ 4f(x_2)+ 2f(x_3)+ 4f(x_4)+ ...+ 2f(x_{n-1})+ 4f(x_n)+ f(x_{n+1})\big]$

$Error \leq \frac{(b-a)^5}{180n^4}\big[\max\big|xetexf^{(4)}(x)f^{(4)}(x)\big|\big]$

Surface Area of Revolution:

$S = 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}\ dx$

$f(x) > 0$

$S = 2\pi \int_a^b x\sqrt{1+f'(x)^2}\ dx$

Work Done by a Variable Force:

$W = \int_a^b F(x)\ dx$

Work Done by a Fluid:

$F = \overline{w}\, d(y)\, \ell(y)\ dy$

Taylor Series Expan-