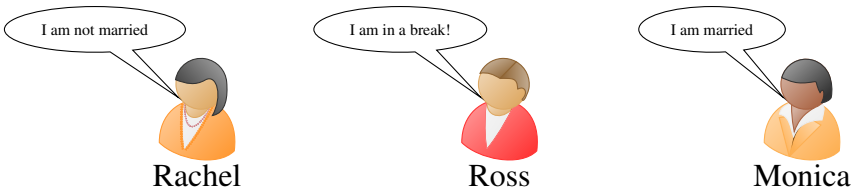
Puzzle 73. Looking at unmarried people

There are three friends staying on the couch in Central Perk: Rachel, Ross, and Monica. Monica is looking at Ross. Ross is looking at Rachel. Monica is married; Rachel is not. Is a married person looking at an unmarried person?



Listing 8.1: Proving that a married person is looking at an unmarried one

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1  formulas(assumptions).
2      married(Monica).
3    -married(Rachel).
4      looking(Monica,Ross).
5      looking(Ross,Rachel).
6  end_of_list.
7
8  formulas(goals).
9   exists x exists y (married(x) & -married(y) & looking(x,y)).
10 end_of_list.
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Solution

A proof by resolution for

$$\exists x\ \exists y\ (married(x) \wedge \neg married(y) \wedge looking(x, y)). \tag{8.1}$$

is obtained with:

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prover9 -f married.in | prooftrans xml renumber | gvizify | dot -Tpdf
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The prover starts by negating the goal, that is clause {2} (see Figure 8.1). On the one hand, resolution introduces clause {4} into {2}, resulting in $\neg married(Ross) \vee married(Rachel)$ (i.e. clause {8}). As we know from {6} that Rachel is not married, the system deduces that Ross is not married (i.e. clause {10}). On the other hand, resolution introduces clause {3} into {2} to infer clause {7}: $\neg married(Monica) \vee married(Ross)$. Knowing that Monica is married (clause {5}), the prover infers that Ross must be married. Obviously, clauses {9} and {10} contradict each other, meaning that the negated goal was a wrong assumption. That is, the theorem was proved.