MATH 3406 Advanced Linear Algebra: Change of basis

1. What is the standard basis for $\mathbb{R}^3$?

A. $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$
B. $(1, 1, 1)$, $(1, -1, 1)$, $(1, 1, -1)$
C. $(1, 2, 3)$, $(4, 5, 6)$, $(7, 8, 9)$
D. $(-1, 1, 0)$, $(1, 0, 1)$, $(0, -1, 1)$

2. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (y, z, x)$ with respect to the standard basis?

A. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
B. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

3. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x + y + z, x - y + z, -x + y + z)$ with respect to the standard basis?

A. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\-1 & 1 & 1\end{bmatrix}$
B. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
C. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
D. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$

4. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (2x, y, z)$ with respect to the standard basis?

A. $\begin{bmatrix}2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$
C. $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}$
D. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\2 & 0 & 0\end{bmatrix}$

5. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x, y, 0)$ with respect to the standard basis?

A. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}$
B. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

6. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (0, y, z)$ with respect to the standard basis?

A. $\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

7. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x, 0, z)$ with respect to the standard basis?

A. $\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

8. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x, y, z)$ with respect to the standard basis?

A. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$
C. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
D. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$

9. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (y, z, x)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
B. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

10. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x + y + z, x - y + z, -x + y + z)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\-1 & 1 & 1\end{bmatrix}$
B. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
C. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
D. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$

11. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (2x, y, z)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$
C. $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}$
D. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\2 & 0 & 0\end{bmatrix}$

12. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x, y, 0)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}$
B. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

13. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (0, y, z)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

14. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x, 0, z)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
C. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
D. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

15. What is the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(x, y, z) = (x, y, z)$ with respect to the basis $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$?

A. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
B. $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$
C. $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$
D. $\begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$

Answer Key:

1. A
2. A
3. A
4. C
5. A
6. A
7. A
8. A
9. D
10. A
11. D
12. D
13. D
14. C
15. B