STA 3180 Statistical Modelling: Bayesian Inference

Lecture Notes on Bayesian Inference for STA 3180 Statistical Modelling

Bayesian Inference is a statistical technique that uses prior knowledge and data to make inferences about unknown parameters. It is based on the Bayes' theorem, which states that the probability of an event occurring given certain conditions is equal to the probability of the conditions given the event has occurred. Bayesian inference can be used to estimate the probability of an event occurring given certain conditions.

Key Concepts:

1. Prior Knowledge: Prior knowledge is information about a parameter that is known before any data is collected. This information can be used to inform Bayesian inference.

2. Posterior Distribution: The posterior distribution is the probability distribution of a parameter after data has been collected. It is calculated using the prior knowledge and data.

3. Bayes' Theorem: Bayes' Theorem states that the probability of an event occurring given certain conditions is equal to the probability of the conditions given the event has occurred. It is used to calculate the posterior distribution.

4. Maximum A Posteriori (MAP) Estimate: The MAP estimate is the most likely value of a parameter given the prior knowledge and data. It is calculated by finding the maximum of the posterior distribution.

Definitions:

1. Prior: A prior is a probability distribution that represents prior knowledge about a parameter.

2. Likelihood: The likelihood is the probability of observing a certain set of data given a certain value of a parameter.

3. Posterior: The posterior is the probability distribution of a parameter after data has been collected.

Rules:

1. Bayes' Theorem: Bayes' Theorem states that the probability of an event occurring given certain conditions is equal to the probability of the conditions given the event has occurred. It is used to calculate the posterior distribution.

2. Maximum A Posteriori (MAP) Estimate: The MAP estimate is the most likely value of a parameter given the prior knowledge and data. It is calculated by finding the maximum of the posterior distribution.

Examples:

1. Suppose we are trying to estimate the probability of a coin landing heads up given that it has landed heads up twice in a row. We can use Bayesian inference to calculate the posterior distribution of the probability of the coin landing heads up. We can use the prior knowledge that the probability of the coin landing heads up is 0.5 and the likelihood that the coin landed heads up twice in a row is 0.25. Using Bayes' Theorem, we can calculate the posterior distribution as 0.8. The MAP estimate of the probability of the coin landing heads up is 0.8.

2. Suppose we are trying to estimate the probability of a patient having a certain disease given their symptoms. We can use Bayesian inference to calculate the posterior distribution of the probability of the patient having the disease. We can use the prior knowledge that the probability of the patient having the disease is 0.1 and the likelihood that the patient has the symptoms is 0.9. Using Bayes' Theorem, we can calculate the posterior distribution as 0.9. The MAP estimate of the probability of the patient having the disease is 0.9.