

STA 3180 Statistical Modelling: Regression

Extra Practice Problems: Regression

1.

****Question:**** Find the least squares regression line for the following data set: (2,3), (4,5), (6,7).

****Solution:**** To solve this problem, we will use the least squares method to find the equation of the regression line. First, we will calculate the mean of x and y values. The mean of x is 4 and the mean of y is 5. Then, we will calculate the slope of the regression line using the formula: $\text{slope} = (\Sigma(xy) - n \cdot \bar{x} \cdot \bar{y}) / (\Sigma(x^2) - n \cdot \bar{x}^2)$. In this case, $\Sigma(xy) = 2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 = 56$, $\Sigma(x^2) = 2^2 + 4^2 + 6^2 = 56$, $n = 3$, $\bar{x} = 4$, and $\bar{y} = 5$. Plugging these values into the formula, we get $\text{slope} = (56 - 3 \cdot 4 \cdot 5) / (56 - 3 \cdot 4^2) = (56 - 60) / (56 - 48) = -4/8 = -0.5$.

Now, we can calculate the y-intercept of the regression line using the formula: $\text{y-intercept} = \bar{y} - \text{slope} \cdot \bar{x}$. In this case, $\text{y-intercept} = 5 - (-0.5) \cdot 4 = 5 + 2 = 7$.

Therefore, the equation of the least squares regression line is $y = -0.5x + 7$.

[CORRECT]

2.

****Question:**** Find the least squares regression line for the following data set: (2,3), (4,5), (6,7), (8,9).

****Solution:**** To solve this problem, we will use the least squares method to find the equation of the regression line. First, we will calculate the mean of x and y values. The mean of x is 5 and the mean of y is 6. Then, we will calculate the slope of the regression line using the formula: $\text{slope} = (\Sigma(xy) - n \cdot \bar{x} \cdot \bar{y}) / (\Sigma(x^2) - n \cdot \bar{x}^2)$. In this case, $\Sigma(xy) = 2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 + 8 \cdot 9 = 130$, $\Sigma(x^2) = 2^2 + 4^2 + 6^2 + 8^2 = 120$, $n = 4$, $\bar{x} = 5$, and $\bar{y} = 6$. Plugging these values into the formula, we get $\text{slope} = (130 - 4 \cdot 5 \cdot 6) / (120 - 4 \cdot 5^2) = (130 - 120) / (120 - 100) = 10/20 = 0.5$.

Now, we can calculate the y-intercept of the regression line using the formula: $\text{y-intercept} = \bar{y} - \text{slope} \cdot \bar{x}$. In this case, $\text{y-intercept} = 6 - (0.5) \cdot 5 = 6 - 2.5 = 3.5$.

Therefore, the equation of the least squares regression line is $y = 0.5x + 3.5$.

[CORRECT]