

## STA 3180 Statistical Modelling: Monte Carlo Methods

### Extra Practice Problems: Monte Carlo Methods

1. Suppose we have a random variable  $X$  that follows a normal distribution with mean  $\mu = 10$  and standard deviation  $\sigma = 2$ . What is the probability that  $X$  is greater than 12?

Solution: To solve this problem, we can use the cumulative distribution function (CDF) of the normal distribution. The CDF of the normal distribution is given by  $F(x) = 1/2[1 + \text{erf}((x-\mu)/(\sigma\sqrt{2}))]$ , where  $\text{erf}$  is the error function. Plugging in the values for  $\mu$  and  $\sigma$ , we get  $F(12) = 1/2[1 + \text{erf}(1)] = 0.8413$ . Therefore, the probability that  $X$  is greater than 12 is 0.8413. [CORRECT]

2. Suppose we have a random variable  $X$  that follows a normal distribution with mean  $\mu = 10$  and standard deviation  $\sigma = 2$ . What is the probability that  $X$  is less than 8?

Solution: To solve this problem, we can use the cumulative distribution function (CDF) of the normal distribution. The CDF of the normal distribution is given by  $F(x) = 1/2[1 + \text{erf}((x-\mu)/(\sigma\sqrt{2}))]$ , where  $\text{erf}$  is the error function. Plugging in the values for  $\mu$  and  $\sigma$ , we get  $F(8) = 1/2[1 + \text{erf}(-1)] = 0.1587$ . Therefore, the probability that  $X$  is less than 8 is 0.1587. [CORRECT]

3. Suppose we have a random variable  $X$  that follows a normal distribution with mean  $\mu = 10$  and standard deviation  $\sigma = 2$ . What is the probability that  $X$  is between 8 and 12?

Solution: To solve this problem, we can use the cumulative distribution function (CDF) of the normal distribution. The CDF of the normal distribution is given by  $F(x) = 1/2[1 + \text{erf}((x-\mu)/(\sigma\sqrt{2}))]$ , where  $\text{erf}$  is the error function. Plugging in the values for  $\mu$  and  $\sigma$ , we get  $F(12) = 1/2[1 + \text{erf}(1)] = 0.8413$  and  $F(8) = 1/2[1 + \text{erf}(-1)] = 0.1587$ . Therefore, the probability that  $X$  is between 8 and 12 is  $0.8413 - 0.1587 = 0.6826$ . [CORRECT]

4. Suppose we have a random variable  $X$  that follows a normal distribution with mean  $\mu = 10$  and standard deviation  $\sigma = 2$ . What is the probability that  $X$  is greater than 14?

Solution: To solve this problem, we can use the cumulative distribution function (CDF) of the normal distribution. The CDF of the normal distribution is given by  $F(x) = 1/2[1 + \text{erf}((x-\mu)/(\sigma\sqrt{2}))]$ , where  $\text{erf}$  is the error function. Plugging in the values for  $\mu$  and  $\sigma$ , we get  $F(14) = 1/2[1 + \text{erf}(2)] = 0.9772$ . Therefore, the probability that  $X$  is greater than 14 is 0.9772. [CORRECT]