MAP 4484 Modeling in Mathematical Biology: Population Dynamics

1. Calculate the growth rate of a population given the following parameters: initial population size (N0) = 100, carrying capacity (K) = 500, and intrinsic growth rate (r) = 0.2.

Solution: The growth rate of the population is calculated using the equation rN(1-N/K). In this case, the growth rate is rN(1-N/K) = 0.2(100)(1-100/500) = 0.08.

2. Determine the equilibrium population size for a population with an initial population size (N0) = 200, carrying capacity (K) = 1000, and intrinsic growth rate (r) = 0.3.

Solution: The equilibrium population size is calculated using the equation N* = K/(1+e^(-r)). In this case, the equilibrium population size is N* = 1000/(1+e^(-0.3)) = 833.

3. Calculate the time it takes for a population to reach its carrying capacity given the following parameters: initial population size (N0) = 50, carrying capacity (K) = 500, and intrinsic growth rate (r) = 0.2.

Solution: The time it takes for a population to reach its carrying capacity is calculated using the equation t = ln(K/N0)/r. In this case, the time it takes for the population to reach its carrying capacity is t = ln(500/50)/0.2 = 5.29.

4. Determine the maximum population size for a population with an initial population size (N0) = 100, carrying capacity (K) = 1000, and intrinsic growth rate (r) = 0.3.

Solution: The maximum population size is calculated using the equation Nmax = K/(1-e^(-r)). In this case, the maximum population size is Nmax = 1000/(1-e^(-0.3)) = 1166.

5. Calculate the time it takes for a population to double in size given the following parameters: initial population size (N0) = 50, carrying capacity (K) = 500, and intrinsic growth rate (r) = 0.2.

Solution: The time it takes for a population to double in size is calculated using the equation t = ln(2)/r. In this case, the time it takes for the population to double in size is t = ln(2)/0.2 = 3.45.