MAP 4484 Modeling in Mathematical Biology: Disease Dynamics

1. Determine the equilibrium points of a SIR model with parameters $\beta = 0.2$, $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The equilibrium points are $(S^*, I^*, R^*) = (0.8, 0.1, 0.1)$.

2. Find the basic reproduction number (R0) for a SIR model with parameters $\beta = 0.2$, $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The basic reproduction number is $R0 = 2$.

3. Find the stability of the disease-free equilibrium point for a SIR model with parameters $\beta = 0.2$, $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The disease-free equilibrium point is stable if $R0 < 1$, and unstable if $R0 > 1$. In this case, $R0 = 2$, so the disease-free equilibrium point is unstable.

4. Find the endemic equilibrium point for a SIR model with parameters $\beta = 0.2$, $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The endemic equilibrium point is $(S^*, I^*, R^*) = (0.8, 0.1, 0.1)$.

5. Find the maximum number of infected individuals in a SIR model with parameters $\beta = 0.2$, $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The maximum number of infected individuals is $I^* = 0.1$.

6. Find the time to reach the endemic equilibrium point for a SIR model with parameters $\beta = 0.2$, $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The time to reach the endemic equilibrium point is given by $t = \ln(R0)/(\beta - \mu - \gamma)$. In this case, $t = \ln(2)/(0.2 - 0.05 - 0.1) = 5.7$.

7. Find the critical value of the parameter β for a SIR model with parameters $\gamma = 0.1$, and $\mu = 0.05$.

Solution: The critical value of β is given by $\beta c = \mu + \gamma$. In this case, $\beta c = 0.15$.