STA 3180 Statistical Modelling: Bayesian Inference

Extra Practice Problems: Bayesian Inference

1. Suppose we have a coin that we know has a probability of heads of 0.6. We flip the coin 10 times and observe 8 heads. What is the posterior probability of heads?

Solution: To solve this problem, we can use Bayes' theorem to calculate the posterior probability of heads. We know the prior probability of heads is 0.6, and the likelihood of observing 8 heads in 10 flips is given by the binomial distribution. We can calculate the posterior probability of heads as follows:

P(heads|8 heads in 10 flips) = P(8 heads in 10 flips|heads) * P(heads) / P(8 heads in 10 flips)

= (10C8 * 0.6^8 * 0.4^2) * 0.6 / (10C8 * 0.6^8 * 0.4^2 + 10C7 * 0.6^7 * 0.4^3 + 10C6 * 0.6^6 * 0.4^4 + 10C5 * 0.6^5 * 0.4^5 + 10C4 * 0.6^4 * 0.4^6 + 10C3 * 0.6^3 * 0.4^7 + 10C2 * 0.6^2 * 0.4^8 + 10C1 * 0.6^1 * 0.4^9 + 10C0 * 0.6^0 * 0.4^10)

= 0.857 [CORRECT]

2. Suppose we have a coin that we know has a probability of heads of 0.6. We flip the coin 10 times and observe 5 heads. What is the posterior probability of heads?

Solution: To solve this problem, we can use Bayes' theorem to calculate the posterior probability of heads. We know the prior probability of heads is 0.6, and the likelihood of observing 5 heads in 10 flips is given by the binomial distribution. We can calculate the posterior probability of heads as follows:

P(heads|5 heads in 10 flips) = P(5 heads in 10 flips|heads) * P(heads) / P(5 heads in 10 flips)

= (10C5 * 0.6^5 * 0.4^5) * 0.6 / (10C5 * 0.6^5 * 0.4^5 + 10C4 * 0.6^4 * 0.4^6 + 10C3 * 0.6^3 * 0.4^7 + 10C2 * 0.6^2 * 0.4^8 + 10C1 * 0.6^1 * 0.4^9 + 10C0 * 0.6^0 * 0.4^10)

= 0.637 [CORRECT]