STA 3180 Statistical Modelling: Markov Chain Monte Carlo

# Markov Chain Monte Carlo (MCMC)

## Introduction

Markov Chain Monte Carlo (MCMC) is a powerful tool used in statistical modelling to sample from a probability distribution. It is a type of computational algorithm that uses random sampling to approximate the posterior distribution of a given model. MCMC algorithms are used in Bayesian statistics to compute the posterior distribution of a given model, and they are also used in frequentist statistics to compute the likelihood of a given model.

## Key Concepts

- **Markov Chains**: A Markov chain is a stochastic process where the future state of the system depends only on the current state and not on the past states.

- **Monte Carlo**: Monte Carlo methods are numerical techniques used to solve problems by randomly sampling from a probability distribution.

- **MCMC Algorithms**: MCMC algorithms are a type of Monte Carlo method used to sample from a probability distribution. They use a Markov chain to explore the space of possible solutions and approximate the posterior distribution of a given model.

## Definitions

- **Markov Chain**: A Markov chain is a stochastic process where the future state of the system depends only on the current state and not on the past states.

- **Monte Carlo**: Monte Carlo methods are numerical techniques used to solve problems by randomly sampling from a probability distribution.

- **MCMC Algorithm**: An MCMC algorithm is a type of Monte Carlo method used to sample from a probability distribution. It uses a Markov chain to explore the space of possible solutions and approximate the posterior distribution of a given model.

- **Metropolis-Hastings Algorithm**: The Metropolis-Hastings algorithm is a type of MCMC algorithm used to sample from a probability distribution. It uses a Markov chain to explore the space of possible solutions and approximate the posterior distribution of a given model.

## Coding Examples

### Example 1: Metropolis-Hastings Algorithm

```
Start of Code
```

```
import numpy as np
def metropolis_hastings(target_dist, initial_state, num_samples):
        # Initialize the chain
        chain = [initial_state]
        for i in range(num_samples):
                # Sample from a proposal distribution
                proposed_state = np.random.normal(chain[-1], 1)
                # Calculate the acceptance probability
                acceptance_prob = min(1,
                target_dist(proposed_state)/target_dist(chain[-1]))
                # Sample from a uniform distribution
                u = np.random.uniform(0, 1)
                # Accept or reject the proposed state
                if u < acceptance_prob:
                        chain.append(proposed_state)
                else:
                        chain.append(chain[-1])
        return chain
End of Code
```

### Example 2: Gibbs Sampling

```
Start of Code
import numpy as np
def gibbs_sampling(target_dist, initial_state, num_samples):
        # Initialize the chain
        chain = [initial_state]
        for i in range(num_samples):
                # Sample from the conditional distributions
                proposed_state = np.random.normal(chain[-1][0], 1),
                np.random.normal(chain[-1][1], 1)
                # Calculate the acceptance probability
                acceptance_prob = min(1,
                target_dist(proposed_state)/target_dist(chain[-1]))
                # Sample from a uniform distribution
                u = np.random.uniform(0, 1)
                # Accept or reject the proposed state
                if u < acceptance_prob:
                        chain.append(proposed_state)
                else:
                        chain.append(chain[-1])
        return chain
End of Code
```

## Practice Multiple Choice Questions

Q1. What is a Markov chain?

A. A stochastic process where the future state of the system depends only on the current state and not on the past states.

Q2. What is Monte Carlo?

A. Monte Carlo methods are numerical techniques used to solve problems by randomly sampling from a probability distribution.

Q3. What is an MCMC algorithm?

A. An MCMC algorithm is a type of Monte Carlo method used to sample from a probability distribution. It uses a Markov chain to explore the space of possible solutions and approximate the posterior distribution of a given model.