MRCB User Manual

MRCB: An Open-Source Multi-system time-varying Receiver Code Bias Analysis Software

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1 Overview

Multi-system and Multi-frequency Receiver Code Bias Analysis Software (MRCB) is an open-source GNSS data processing package specifically designed for extracting time-varying receiver code biases (RCBs). MRCB is developed in C/C++, which can be easily ported to different operating systems, such as Windows and Linux. It is a post-processing software, which can process multi-frequency data from GPS, Galileo, BDS, GLONASS, and QZSS. Using a least-squares filter (LSF) based on undifferenced and uncombined observations to estimate time-varying RCBs. Furthermore, it requires observation files and broadcast ephemeris, offering a simple operation process. Processing strategies for different data can be configured in the formatted configuration file. The main features of MRCB are as follows:

- Undifferenced and uncombined observations
- Support for both CDMA and FDMA
- Extract time-varying RCBs
- Multi-frequency (dual-frequency and above) data processing
- Least square filter

2 MRCB software

2.1 Structure of MRCB

Generally, the processing can be summarized into three parts.

(1) Data preparation is the first step. In this step, we just need observation file, broadcast ephemeris file. Then, set the parameters in the configuration file.

(2) Data processing is the second step. In this step, MRCB first calculates the satellite

information. Cycle slip detection is then performed using the geometry-free combination and the Melbourne-Wübbena combination. After finishing cycle slip detection, the LSF is employed to estimate unknown parameters. Finally, quality control is performed using the DIA method.

(3) Information output is the third step. In this step, the processing information and estimated parameters such as satellite clock offsets, elevation and azimuth, post-fit code and carrier phase residuals, the SPP positioning results, time-varying RCBs are all output to formatted files. The log file mainly contains the record of satellite eliminations.

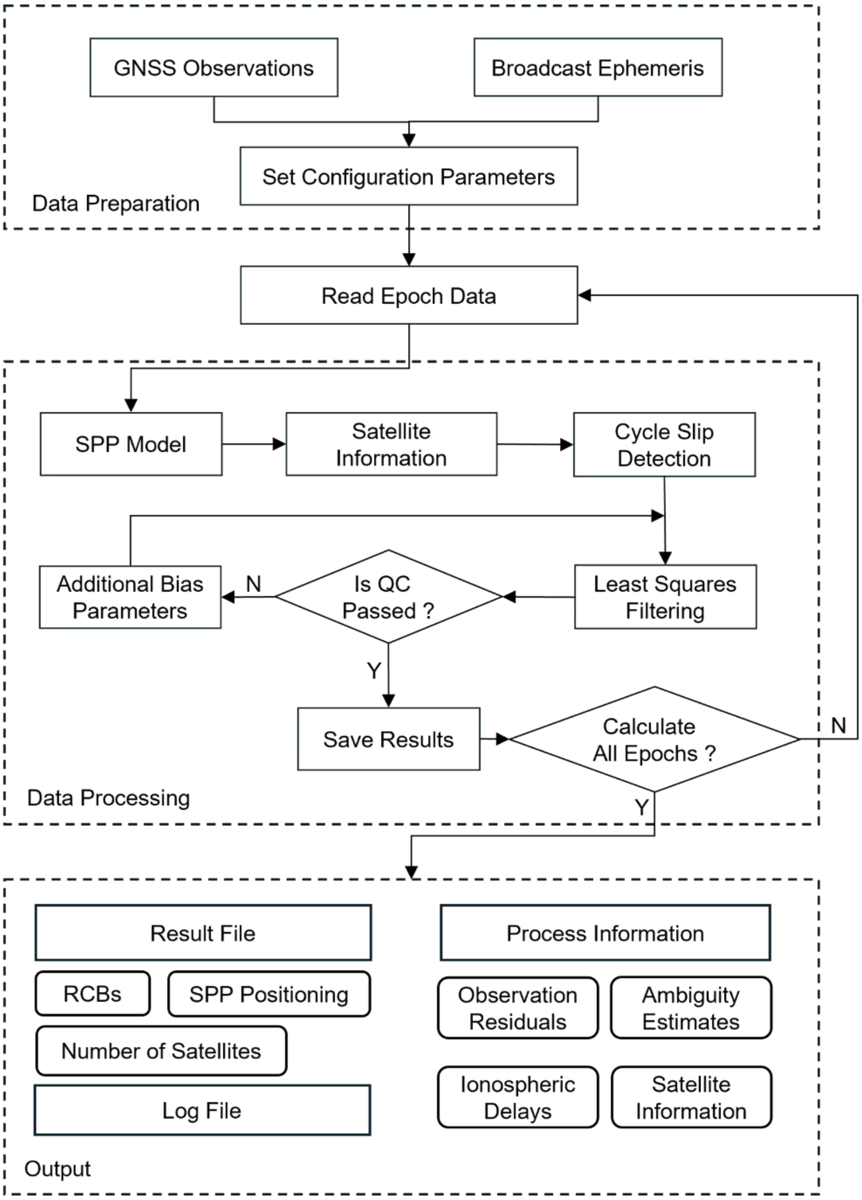

Fig. 1 The processing framework of MRCB software

2.2 Processing strategies

For the processing strategies. The frequency-independent parameters in this study, including the range, receiver clock offsets, satellite clock offsets, and tropospheric delays, were combined into a single parameter, which was estimated as white noise. The ionospheric delays, ambiguity, and time-varying RCBs were also estimated. Detailed information is summarized in Table 1. Complete mathematical derivations are provided in the Appendix for reference.

Table 1 Processing strategies of MRCB

Item	Processing strategies
Satellite constellation	GPS/GLONASS/BDS/Galileo/QZSS
Frequency number	Dual-frequency and above (up to 5 frequencies)
Functional mode	Uncombined model
Sampling rate	30s
Cut-off mask angle	10°
Observation weight	P = $\sin^2(E)$ /σ2(σ2 indicates variance of code or carrier phase) A priori precision of 0.003 and 0.3 m for raw phase and code
Filtering method	Least square filter
Frequency-independent parameters	Estimated as white noise
Ionospheric delays	Estimated as white noise
Phase ambiguities	Estimated as float constants for each satellite and frequency
SCB	Estimated as day constants for each satellite on the third or higher frequency band
RCB	Estimated as white noise
IFB	GLONASS, Estimated as white noise

Note: Ensure at least 3 satellites are available for each processed GNSS system. Because mixed-frequency processing is not yet supported, the selected frequencies must be present in the observations.

Table 2 Frequency priorities for each system

GNSS	1	2	3	4	5
GPS	P1	P2	P5	\	\
GLONASS	R1	R2	\	\	\
Galileo	E1	E5a	E5b	E6	E5
QZSS	P1	P2	P5	\	\
BDS-2	B1I	B3I	B2I	\	\
BDS-3	B1I	B3I	B2b	B1C	B2a

3 Requirements

3.1 Supported platforms

MRCB software was developed in the C/C++ language using the cross-platform CMAKE build system. It can be compiled and executed on both Windows and Linux operating systems. It is recommended to compile and debug MRCB under Visual Studio 2022 in Windows or using GCC/Clang in Linux. Computer configuration requirements are as follows:

Operating system: Linux or Windows

System type: 32 or 64 bit

Memory: at least 512MB

Hard disk space: at least 500MB

CMAKE Version: CMAKE version higher than 3.10

Visual Studio Version: at least version 2022

The software has been tested under Windows 10 and Ubuntu 16.04 or higher, with all tests passed.

3.2 License

This program is an open-source software: you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation.

4 File structure

The project directory structure is as follows.

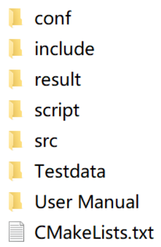

Fig. 2 File list

```
.
├── Your Project Root Directory/
│   ├── conf/
│   │   └── config.dat    # Process configuration file
│   ├── include/
│   │   └── *.h           # Your .h header files
│   ├── result/           # Output result
│   ├── script/
│   │   ├── RCB_day_by_day.m    # Visualizes RCBs for a single day
│   │   └── RCB_Continues.m     # Visualizes RCBs for continues day
│   ├── src/
│   │   └── *.cpp           # Your .cpp source files
│   └── Testdata/
│       ├── day-by-day processing/  # Daily processing test data/
│       │   ├── XXX.o
│       │   └── XXX.n
│       └── continuous processing/  # Continuous processing test data/
│           ├── obs/ # Continuous days observation data from same station
│           └── nav/ # Continuous days broadcast ephemeris from same station
└── User Manual  # User manual documentation
```

Fig. 3 The structure of File

5 Configuration File (config.dat)

The configuration file mainly contains four parts:

a) Input file configuration

b) Output file configuration

c) Processing strategies configuration

d) Station names

It is noted that '#' represents the function of annotation in the configuration file. Users can choose different process options according their realistic needs. Users configure the input and output paths according to their specific requirements.

5.1 Input file configuration

Users need to set an absolute path.

- OBS_PATH: single station observation file path
- NAV_PATH: broadcast ephemeris file path

```
# MRCB_Version 1.0 Configuration File
#===================================================================
# Input File Path

OBS_PATH            =F:\MRCB\Testdata\day-by-day processing\MRO100AUS_R_20242980000_01D_30S_MO.24o      # obs file path
NAV_PATH            =F:\MRCB\Testdata\day-by-day processing\BRDC00IGS_R_20242980000_01D_MN.24n          # nav file path (broadcast ephemeris)
```

Fig. 4 Input file configuration

5.2 Output file configuration

Users need to set an absolute path.

- OUT_PRO: Dir for process information file; file name is auto-generated.
- OUT_PATH: Dir for result file; file name is auto-generated.
- OUT_LOG: Dir for sat log file; file name is auto-generated. record the satellite elimination records

```
#==================================================================
# Output File Path

OUT_PRO     =E:\open_source\result\                                             #processing information file
OUT_PATH    =E:\open_source\result\                                             #result file
OUT_LOG     =E:\open_source\result\                                             #sat Log

#==================================================================
```

Fig. 5 Output file configuration

Note: The slash direction varies by operating system

- **Windows:** Use backslashes (\). Example: C:\Users\username\input\data.obs
- **Linux:** Use forward slashes (/). Example: C:/Users/username/input/data.obs

5.3 Processing strategies configuration

- UseBDS2: Distinguishes between BDS-2 and BDS-3 satellites due to differing RCB variations trends. 0→Uses only BDS-3 satellite data 1→Uses only BDS-2 satellite data
- Frequency: Number of frequency bands to process
- Exclsats: Satellites to be manually excluded
- System: GNSS systems to be processed

```
#==================================================================
# Process Option1
Ephopt              =0                          # Satellite Ephemeris :0->broadcast
FiltMode            =0                          # Filting Mode        :0->Forward Mode
UseBDS2             =0                          # Is Use BDS2         :0->use bds3 , 1->use bds2
Frequency           =2                          # Frequency number
Elmask              =10                         # Satellite elevation mask
Exclsats            =C01 C02 C03 C04 C05        # prn ...
System              =41                         # 1:gps+4:glo+8:gal+16:qzs+32:comp
```

Fig. 6 Processing options 1 configuration

- **Continue:** day-by-day means processes data one day at a time, used to analyze RCB variations within a single day; continuous processing means processes multiple consecutive days together, used to analyze RCB variations across several days.
- OBS_DIR: Observation data folder for multiple consecutive days
- NAV_DIR: Broadcast ephemeris folder for multiple consecutive days

```
# Process Option2
Continue            =0                                                  # 0->day-by-day  1->continuous processing
OBS_DIR             =E:\open_source\Testdata\continuous processing\obs\   # obs dir
NAV_DIR             =E:\open_source\Testdata\continuous processing\nav\   # nav dir

#======================================================================
```

Fig. 7 Processing options 2 configuration

5.4 Station Information

```
#======================================================================
# Station information
StaName          =MRO1                        # Station Name
DOY              =2024298                     # DOY
```

Fig. 8 Station information configuration

6 Compile and Run

6.1 Compile

6.1.1 Windows

Ensure that the complete compilation tool and CMAKE (version 3.10 or higher) is installed.

➢ Extract the files downloaded from GitHub

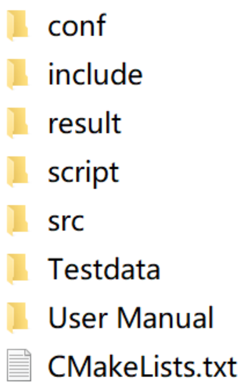

Fig. 9 The extracted files

➢ Search "x64 Native Tools Command Prompt for VS 2022"

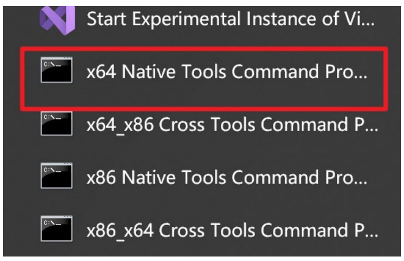

Fig. 10 Open Command Prompt

➢ Navigate to Project Directory and Compile. Execute these commands in sequence

① cd your_project_path ②mkdir build ③cmake -B build ④cmake --build build

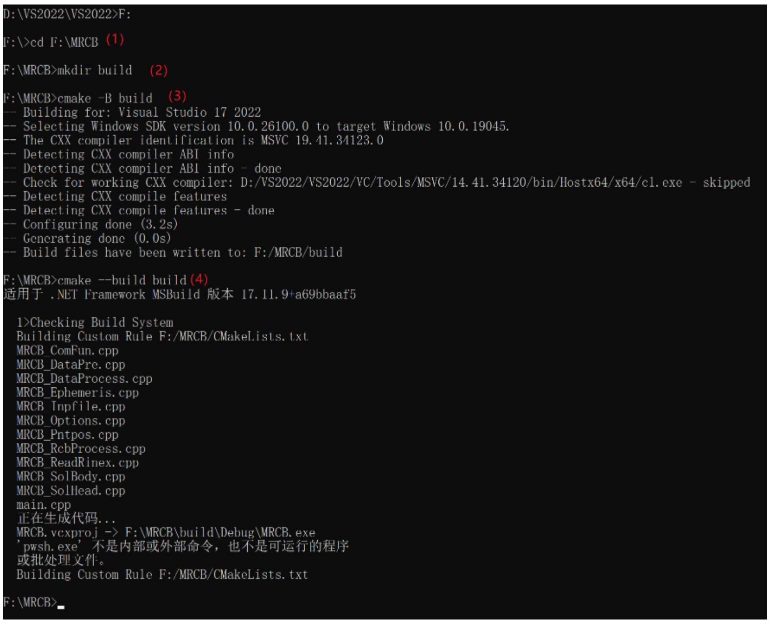

Fig. 11 Finish compile under Windows

6.1.2 Linux

Ensure that the complete compilation tool and CMAKE (version 3.10 or higher) is installed.

Execute these commands in sequence: ① cd your_project_path ② mkdir build ③ cd build ④ cmake .. ⑤ make

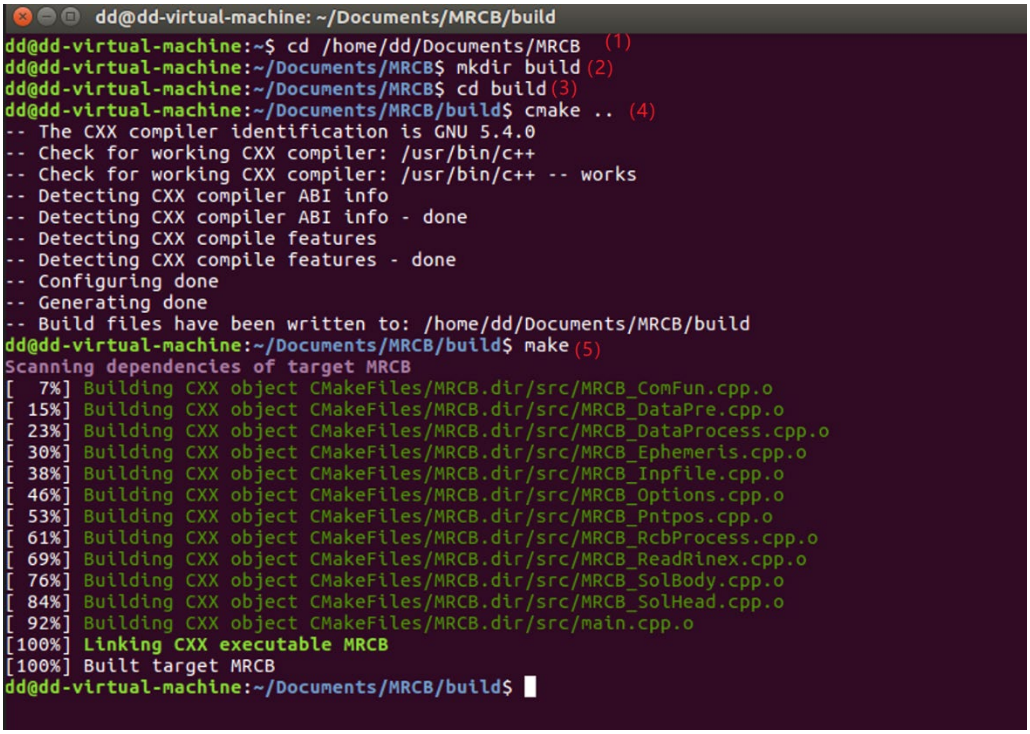

Fig. 12 Finish compile under Linux

6.2 Run

Using Windows as an example, we will introduce the operational process with two examples: the day-by-day process and continuous processing

6.2.1 day-by-day process

（1） Compile code

Reference the **section 6.1.**

（2） Data Preparation

In the **config.dat** file, configure the parameters by setting the input and output paths according to your specific needs and file locations.

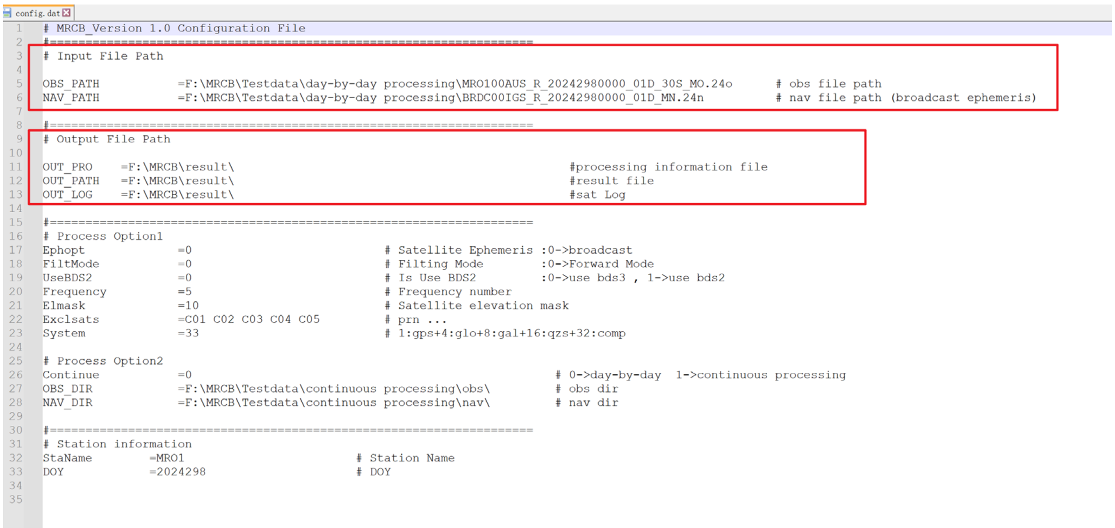

Fig. 13 Paths Requiring Configuration

（3） Run code

Recommend running in the command line: ① cd ./build/Debug ② MRCB.exe

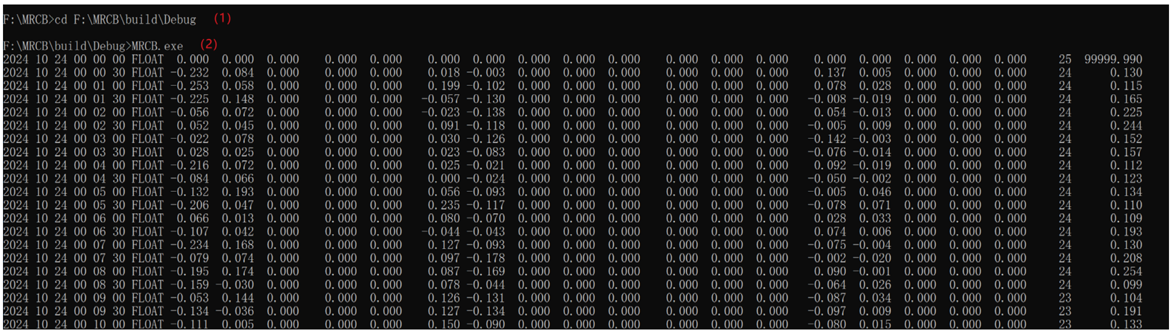

Fig. 14 Runtime Display

（4）Visualization

Two simple visualization scripts are located in the **.\script** directory and require MATLAB (version R2020b or higher) to run.

➢ RCB_day_by_day.m: Visualizes RCBs for a single day. Simply configure the file paths and specify the frequencies and systems to visualize.

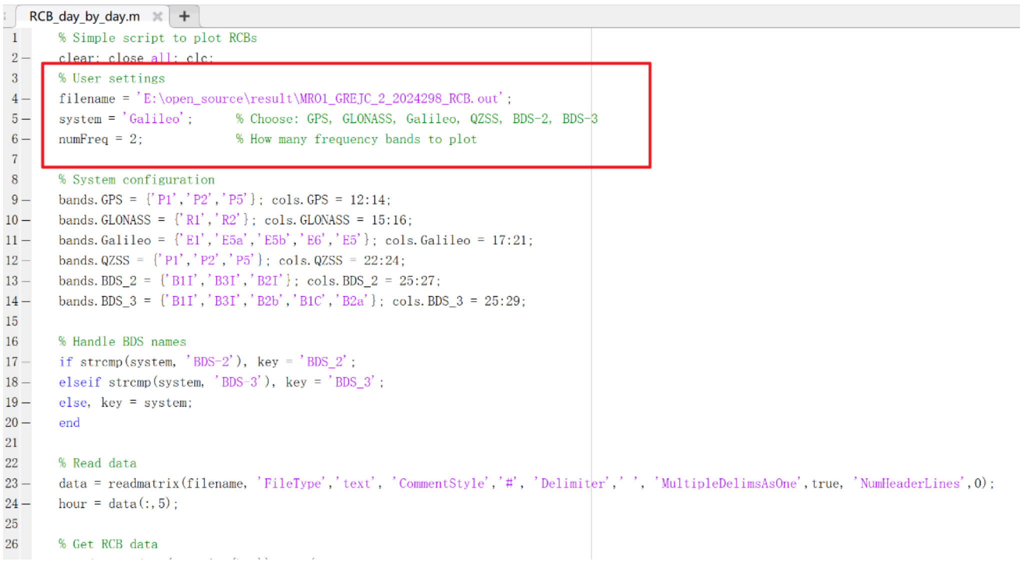

Fig. 15 Visualizes RCBs for a single day

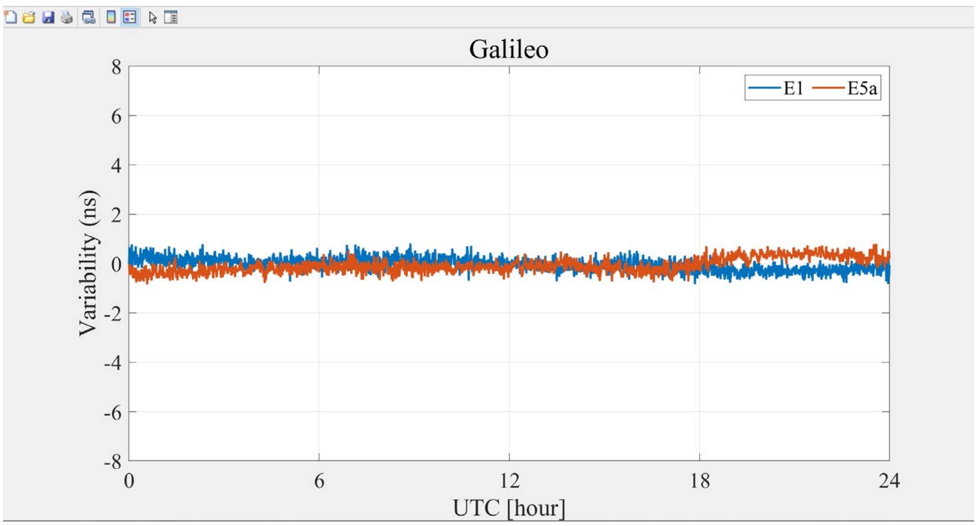

Fig. 16 Visualization Results

6.2.2 continuous processing

（1）Compile code

Reference the **section 6.1.**

（2）Data Preparation

In the **config.dat** file, configure the parameters by setting the input and output paths according to your specific needs and file locations. When use **continuous processing,** the following parameters do not need to be set and can be left as default: OBS_PATH, NAV_PATH, DOY.

Configure the following input and output parameters:

config.dat

```
# MRCB_Version 1.0 Configuration File
#================================================================
# Input File Path

OBS_PATH          =F:\MRCB\Testdata\day-by-day processing\MRO100AUS_R_20242980000_01D_30S_MO.24o      # obs file path
NAV_PATH          =F:\MRCB\Testdata\day-by-day processing\BRDC00IGS_R_20242980000_01D_MN.24n         # nav file path (broadcast ephemeris)

#================================================================
# Output File Path

OUT_PRO     =F:\MRCB\result\                                              #processing information file
OUT_PATH    =F:\MRCB\result\                                              #result file
OUT_LOG     =F:\MRCB\result\                                              #sat Log

#================================================================
# Process Option1
Ephopt             =0                         # Satellite Ephemeris :0->broadcast
FiltMode           =0                         # Filting Mode        :0->Forward Mode
UseBDS2            =0                         # Is Use BDS2         :0->use bds3 , 1->use bds2
Frequency          =5                         # Frequency number
Elmask             =10                        # Satellite elevation mask
Exclsats           =C01 C02 C03 C04 C05       # prn ...
System             =33                        # 1:gps+4:glo+8:gal+16:qzs+32:comp

# Process Option2
Continue           =0                                               # 0->day-by-day  1->continuous processing
OBS_DIR            =F:\MRCB\Testdata\continuous processing\obs\     # obs dir
NAV_DIR            =F:\MRCB\Testdata\continuous processing\nav\     # nav dir

#================================================================
# Station information
StaName         =MRO1                     # Station Name
DOY             =2024298                  # DOY
```

Fig. 17 Paths Requiring Configuration

Note: When using the Continuous processing mode, ensure the observation files in the OBS_DIR directory and navigation files in the NAV_DIR directory follow consistent naming and maintain strict day-to-day correspondence. This ensures the continuous processor can correctly associate observation and navigation data across multiple consecutive days.

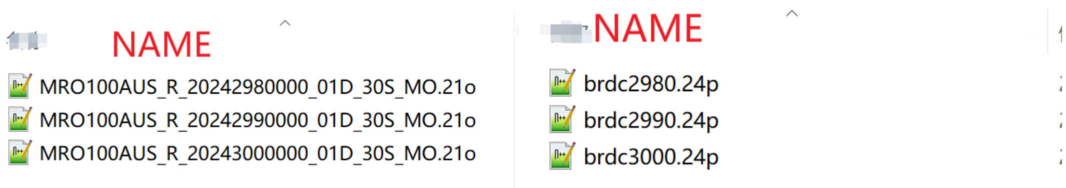

Fig. 18 Example of Correct File Organization

（3）Run code

Recommend running in the command line.

（4）Visualization

➢ RCB_Continues.m: Visualizes RCBs for continues day. Simply configure the file paths and specify the frequencies and systems to visualize.

```
RCB_Continues.m
% Simple script to plot RCBs
clear; close all; clc;
% User settings
filename = 'E:\open_source\result\MRO1_GREJC_2_Continues_RCB.out';
system = 'Galileo';       % Choose: GPS, GLONASS, Galileo, QZSS, BDS-2, BDS-3
numFreq = 2;              % How many frequency bands to plot

% System configuration
bands.GPS = {'P1','P2','P5'}; cols.GPS = 12:14;
bands.GLONASS = {'R1','R2'}; cols.GLONASS = 15:16;
bands.Galileo = {'E1','E5a','E5b','E6','E5'}; cols.Galileo = 17:21;
bands.QZSS = {'P1','P2','P5'}; cols.QZSS = 22:24;
bands.BDS_2 = {'B1I','B3I','B2I'}; cols.BDS_2 = 25:27;
bands.BDS_3 = {'B1I','B3I','B2b','B1C','B2a'}; cols.BDS_3 = 25:29;

% Handle BDS names
if strcmp(system, 'BDS-2'), key = 'BDS_2';
elseif strcmp(system, 'BDS-3'), key = 'BDS_3';
else, key = system;
end

% Read data
data = readmatrix(filename, 'FileType','text', 'CommentStyle','#', 'Delimiter',' ', 'MultipleDelimsAsOne',true, 'NumHeaderLines',0);
hour = data(:,4);
```

Fig. 19 Visualizes RCBs for continues day

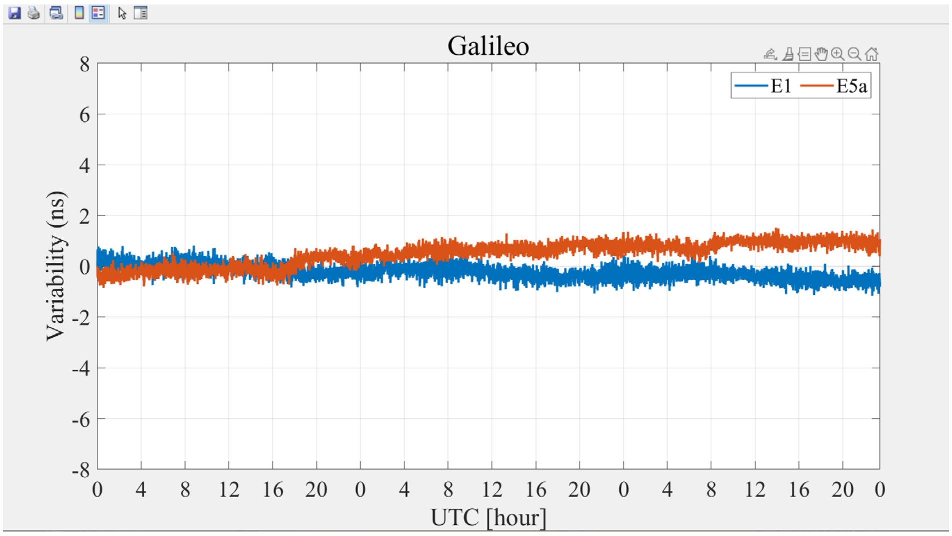

Fig. 20 Visualization Results

6.2.3 Linux Run

If you want run on the Linux system.

Execute these commands in sequence: ./MRCB

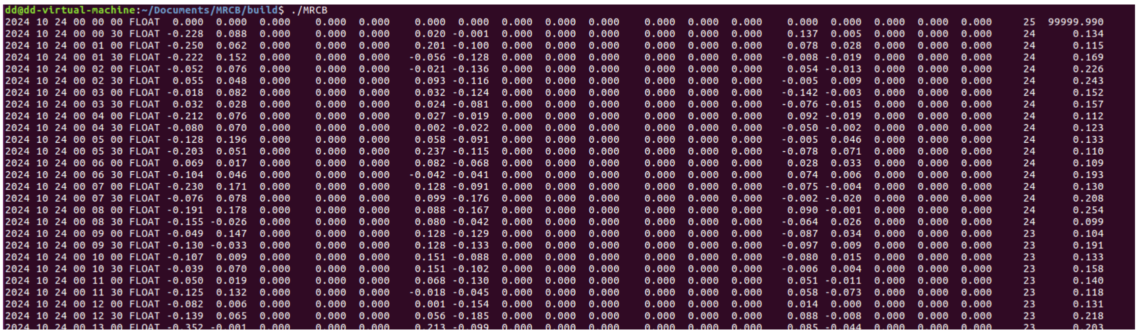

Fig. 21 Run on Linux

7 Output file format description

7.1 Result File

Name: XXXX_GEC_2_YYYYDOY_RCB.out

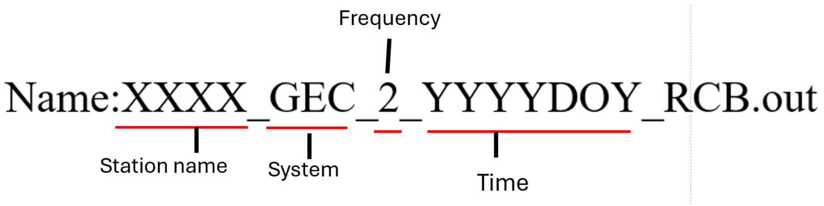

Fig. 22 Structure of output file name

Table 3 Result output file format description

Items		Value Type	Range
Epoch Time (GPST)	Year	char	1-4
	Month	char	6-7
	Day	char	9-10
	Hour	char	12-13
	Minute	char	15-16
	Second	char	18-22
Receiver Position (SPP)	X (m)	double	25-37
	Y (m)	double	41-52
	Z (m)	double	55-67
Number of valid satellites		int	71-72
GDOP		double	76-81

GPS Time-varying RCB	P1	double	86-91
	P2	double	95-100
	P5	double	104-109
GLONASS Time-varying RCB	R1	double	115-120
	R2	double	124-130
Galileo Time-varying RCB	E1	double	136-141
	E5a	double	146-151
	E5b	double	157-162
	E6	double	167-172
	E5	double	177-182
QZSS Time-varying RCB	P1	double	190-195
	P2	double	199-204
	P5	double	208-213
BDS-3 Time-varying RCB	B1I	double	218-223
	B3I	double	230-235
	B2b	double	242-248
	B1C	double	255-260
	B2a	double	266-271
BDS-2 Time-varying RCB	B1I	double	218-223
	B3I	double	230-235
	B2I	double	242-248

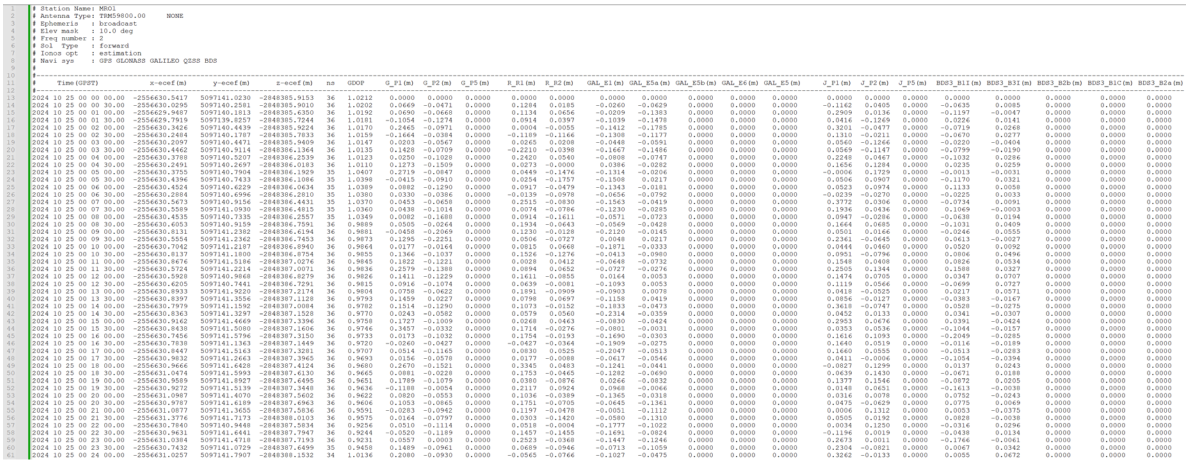

Fig. 23 Screenshot of RCBs result file

7.2 Process Information File

Name: XXXX_GEC_2_YYYYDOY_Inf.pro

Table 4 Process information file description

Items		Value Type	Range
Satellite Sys. & PRN		int	1-4
Satellite Positions	X (m)	double	6-13

	Y (m)	double	23-35
	Z (m)	double	38-51
Satellite Clock Offset	(m)	double	56-67
Ion Delays	(m)	double	75-83
Ambiguity	F1 (cycle)	double	91-99
	F2 (cycle)	double	107-115
	F3 (cycle)	double	124-131
	F4 (cycle)	double	140-147
	F5 (cycle)	double	156-163
Satellite Elevation and Azimuth	Elevation (°)	double	173-179
	Azimuth (°)	double	188-195
post-fit carrier phase residuals	F1 (m)	double	199-211
	F2 (m)	double	215-227
	F3 (m)	double	231-243
	F4 (m)	double	247-259
	F5 (m)	double	263-275
post-fit code residuals	F1 (m)	double	280-292
	F2 (m)	double	296-308
	F3 (m)	double	312-324
	F4 (m)	double	328-340
	F5 (m)	double	344-356
SCB	F3 (m)	double	365-372
	F4 (m)	double	381-388
	F5 (m)	double	397-404

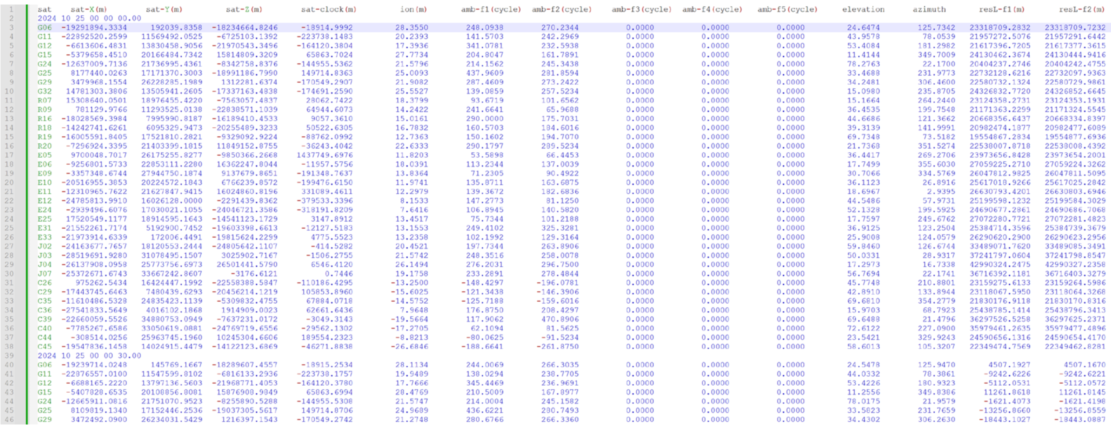

Fig. 24 Screenshot of Process Information file

7.3 Log file

Name: XXXX_GEC_2_YYYYDOY_Sat.log

The log file mainly contains the record of satellite eliminations.

```
2024/10/25 00:00:00.00
Deleter Sat for lack observations, need freq 3:G12(Frq:2)
Deleter Sat for lack observations, need freq 3:G15(Frq:2)
Deleter Sat for lower 10.0 degree:G19(Ele:8.6)
Deleter Sat for lower 10.0 degree:G20(Ele:2.8)
Deleter Sat for lack observations, need freq 3:G29(Frq:2)
Deleter Sat for lower 10.0 degree:E03(Ele:8.8)
Deleter Sat for lower 10.0 degree:C21(Ele:6.6)
Deleter Sat for lower 10.0 degree:C42(Ele:5.1)
Deleter Sat for lower 10.0 degree:C50(Ele:7.4)
2024/10/25 00:00:30.00
Deleter Sat for lack observations, need freq 3:G12(Frq:2)
Deleter Sat for lack observations, need freq 3:G15(Frq:2)
Deleter Sat for lower 10.0 degree:G19(Ele:8.5)
Deleter Sat for lower 10.0 degree:G20(Ele:2.9)
Deleter Sat for lack observations, need freq 3:G29(Frq:2)
Deleter Sat for lower 10.0 degree:E03(Ele:8.9)
Deleter Sat for lower 10.0 degree:C21(Ele:6.5)
Deleter Sat for lower 10.0 degree:C42(Ele:5.1)
Deleter Sat for lower 10.0 degree:C50(Ele:7.3)
2024/10/25 00:01:00.00
Deleter Sat for lack observations, need freq 3:G12(Frq:2)
Deleter Sat for lack observations, need freq 3:G15(Frq:2)
Deleter Sat for lower 10.0 degree:G19(Ele:8.3)
Deleter Sat for lack observations, need freq 3:G29(Frq:2)
Deleter Sat for lower 10.0 degree:E03(Ele:9.0)
Deleter Sat for lower 10.0 degree:C21(Ele:6.4)
Deleter Sat for lower 10.0 degree:C42(Ele:5.0)
Deleter Sat for lower 10.0 degree:C50(Ele:7.2)
2024/10/25 00:01:30.00
Deleter Sat for lack observations, need freq 3:G12(Frq:2)
Deleter Sat for lack observations, need freq 3:G15(Frq:2)
Deleter Sat for lower 10.0 degree:G19(Ele:8.1)
Deleter Sat for lack observations, need freq 3:G29(Frq:2)
Deleter Sat for lower 10.0 degree:E03(Ele:9.0)
Deleter Sat for lower 10.0 degree:C21(Ele:6.3)
Deleter Sat for lower 10.0 degree:C42(Ele:4.9)
Deleter Sat for lower 10.0 degree:C50(Ele:7.1)
```

Fig. 25 Screenshot of Log file

When use Continues processing, the file name become:

XXXX_GEC_2_Continues_RCB.out

XXXX_GEC_2_Continues_Inf.pro

XXXX_GEC_2_Continues_Sat.log

7.4 Execution Interface

GPS GLO GAL QZSS BDS-3/BDS-2 num of sat TqLOM

```
finish read nav file::E:\open_source\Testdata\day-by-day processing\BRDC00IGS_R_20242980000_01D_MN.21n
read obs file header:E:\open_source\Testdata\day-by-day processing\MRO100AUS_R_20242980000_01D_30S_MO.21o
2024 10 24 00 00 00 FLOAT  0.000  0.000  0.000   0.000  0.000   0.000  0.000  0.000  0.000  0.000   0.000  0.000  0.000   0.000  0.000  0.000  0.000  0.000   35  99999.990
2024 10 24 00 00 30 FLOAT -0.234  0.082  0.000  -0.002 -0.117   0.018 -0.003  0.000  0.000  0.000  -0.247  0.014  0.000   0.126 -0.009  0.000  0.000  0.000   34      0.123
2024 10 24 00 01 00 FLOAT -0.255  0.056  0.000   0.158 -0.178   0.199 -0.102  0.000  0.000  0.000  -0.001 -0.020  0.000   0.067  0.014  0.000  0.000  0.000   34      0.101
2024 10 24 00 01 30 FLOAT -0.227  0.146  0.000  -0.128 -0.054  -0.057 -0.131  0.000  0.000  0.000  -0.041  0.074  0.000  -0.019 -0.033  0.000  0.000  0.000   34      0.157
2024 10 24 00 02 00 FLOAT -0.058  0.070  0.000  -0.373 -0.030  -0.023 -0.138  0.000  0.000  0.000   0.150  0.056  0.000   0.043 -0.026  0.000  0.000  0.000   34      0.197
2024 10 24 00 02 30 FLOAT  0.050  0.043  0.000  -0.128 -0.006   0.091 -0.118  0.000  0.000  0.000  -0.138 -0.026  0.000  -0.016 -0.005  0.000  0.000  0.000   34      0.224
2024 10 24 00 03 00 FLOAT -0.024  0.076  0.000  -0.030 -0.030   0.030 -0.127  0.000  0.000  0.000  -0.078  0.012  0.000  -0.153 -0.016  0.000  0.000  0.000   34      0.139
2024 10 24 00 03 30 FLOAT  0.027  0.023  0.000   0.081 -0.003   0.022 -0.083  0.000  0.000  0.000   0.003 -0.061  0.000  -0.087 -0.028  0.000  0.000  0.000   34      0.183
2024 10 24 00 04 00 FLOAT -0.217  0.070  0.000  -0.073 -0.093   0.025 -0.021  0.000  0.000  0.000   0.064  0.043  0.000   0.081 -0.032  0.000  0.000  0.000   34      0.111
2024 10 24 00 04 30 FLOAT -0.085  0.064  0.000   0.117 -0.086  -0.000 -0.024  0.000  0.000  0.000   0.099 -0.019  0.000  -0.061 -0.016  0.000  0.000  0.000   34      0.114
2024 10 24 00 05 00 FLOAT -0.134  0.191  0.000  -0.184 -0.019   0.056 -0.093  0.000  0.000  0.000   0.067 -0.014  0.000  -0.016  0.033  0.000  0.000  0.000   34      0.121
2024 10 24 00 05 30 FLOAT -0.208  0.045  0.000  -0.131  0.003   0.235 -0.118  0.000  0.000  0.000   0.036 -0.031  0.000  -0.089  0.058  0.000  0.000  0.000   34      0.114
2024 10 24 00 06 00 FLOAT  0.064  0.011  0.000  -0.036 -0.119   0.080 -0.071  0.000  0.000  0.000   0.167  0.017  0.000   0.017  0.020  0.000  0.000  0.000   33      0.124
2024 10 24 00 06 30 FLOAT -0.109  0.040  0.000  -0.038 -0.060  -0.044 -0.044  0.000  0.000  0.000  -0.093 -0.074  0.000   0.063 -0.007  0.000  0.000  0.000   33      0.170
2024 10 24 00 07 00 FLOAT -0.236  0.166  0.000   0.007 -0.142   0.127 -0.093  0.000  0.000  0.000  -0.160 -0.073  0.000  -0.086 -0.018  0.000  0.000  0.000   33      0.119
2024 10 24 00 07 30 FLOAT -0.081  0.072  0.000  -0.084 -0.081   0.097 -0.178  0.000  0.000  0.000  -0.097  0.026  0.000  -0.013 -0.033  0.000  0.000  0.000   33      0.198
2024 10 24 00 08 00 FLOAT -0.197  0.172  0.000  -0.042 -0.075   0.087 -0.169  0.000  0.000  0.000   0.048  0.015  0.000   0.079 -0.014  0.000  0.000  0.000   33      0.251
2024 10 24 00 08 30 FLOAT -0.161 -0.032  0.000  -0.092 -0.098   0.078 -0.045  0.000  0.000  0.000  -0.110  0.072  0.000  -0.075  0.012  0.000  0.000  0.000   33      0.127
2024 10 24 00 09 00 FLOAT -0.054  0.142  0.000  -0.117 -0.086   0.126 -0.131  0.000  0.000  0.000   0.194  0.114  0.000  -0.098  0.021  0.000  0.000  0.000   32      0.128
2024 10 24 00 09 30 FLOAT -0.136 -0.039  0.000  -0.082 -0.100   0.126 -0.135  0.000  0.000  0.000  -0.243  0.006  0.000  -0.108 -0.004  0.000  0.000  0.000   32      0.179
2024 10 24 00 10 00 FLOAT -0.112  0.003  0.000  -0.063 -0.214   0.149 -0.090  0.000  0.000  0.000  -0.272  0.069  0.000  -0.091  0.001  0.000  0.000  0.000   32      0.154
2024 10 24 00 10 30 FLOAT -0.045  0.064  0.000  -0.062 -0.087   0.149 -0.104  0.000  0.000  0.000  -0.238 -0.001  0.000  -0.017 -0.009  0.000  0.000  0.000   32      0.148
2024 10 24 00 11 00 FLOAT -0.055  0.013  0.000  -0.012 -0.123   0.067 -0.133  0.000  0.000  0.000   0.010 -0.035  0.000   0.040 -0.024  0.000  0.000  0.000   32      0.148
2024 10 24 00 11 30 FLOAT -0.130  0.127  0.000  -0.062 -0.038  -0.020 -0.047  0.000  0.000  0.000   0.157  0.015  0.000   0.047 -0.086  0.000  0.000  0.000   32      0.105
2024 10 24 00 12 00 FLOAT -0.087 -0.000  0.000  -0.077 -0.229  -0.000 -0.156  0.000  0.000  0.000  -0.149  0.041  0.000   0.003 -0.013  0.000  0.000  0.000   32      0.118
2024 10 24 00 12 30 FLOAT -0.144  0.059  0.000   0.012 -0.152   0.054 -0.187  0.000  0.000  0.000   0.162 -0.008  0.000   0.077 -0.021  0.000  0.000  0.000   32      0.195
2024 10 24 00 13 00 FLOAT -0.358 -0.006  0.000   0.030 -0.041   0.211 -0.101  0.000  0.000  0.000   0.056  0.080  0.000   0.074 -0.057  0.000  0.000  0.000   32      0.211
2024 10 24 00 13 30 FLOAT -0.030 -0.011  0.000   0.009 -0.020   0.017 -0.051  0.000  0.000  0.000   0.047  0.009  0.000  -0.030 -0.085  0.000  0.000  0.000   32      0.110
2024 10 24 00 14 00 FLOAT -0.179  0.111  0.000   0.035 -0.134   0.086 -0.229  0.000  0.000  0.000  -0.063 -0.043  0.000   0.037  0.007  0.000  0.000  0.000   32      0.150
2024 10 24 00 14 30 FLOAT -0.200  0.036  0.000  -0.006 -0.182   0.018 -0.163  0.000  0.000  0.000   0.083  0.002  0.000   0.048  0.017  0.000  0.000  0.000   32      0.099
2024 10 24 00 15 00 FLOAT -0.182  0.051  0.000  -0.053 -0.041   0.118 -0.093  0.000  0.000  0.000  -0.193  0.038  0.000   0.075 -0.047  0.000  0.000  0.000   33      0.229
2024 10 24 00 15 30 FLOAT  0.108  0.149  0.000  -0.089 -0.109   0.089 -0.096  0.000  0.000  0.000  -0.025  0.119  0.000  -0.053 -0.031  0.000  0.000  0.000   33      0.209
2024 10 24 00 16 00 FLOAT -0.083  0.110  0.000   0.004 -0.119   0.084 -0.161  0.000  0.000  0.000  -0.037 -0.003  0.000  -0.128 -0.017  0.000  0.000  0.000   33      0.215
```

Fig. 26 Runtime Display RCBs

Appendix

(1) CDMA RCB Function Model

We commence with the undifferenced and uncombined GNSS observation equations, which are written as (Teunissen and Montenbruck, 2017)

$$\begin{cases} E[P_{r,f}^s(i)] = \rho_r^s(i) + \mu_f \cdot I_{r,1}^s(i) + b_{r,f} - b_f^s \\ E[L_{r,f}^s(i)] = \rho_r^s(i) - \mu_f \cdot I_{r,1}^s(i) + \lambda_f \alpha_{r,f}^s \end{cases} \tag{1}$$

where $E(.)$ denotes the expectation operator. $P_{r,f}^s$ and $L_{r,f}^s$ represent observed-minus-calculated code and phase observations of a satellite s tracked by a receiver r on the frequency f at the epoch i, respectively. The frequency-independent parameters in this, including the range, receiver clock offsets, satellite clock offsets, and tropospheric delays, were combined into a single parameter $\rho_r^s(i)$. the first-order slant ionospheric delay $I_{r,1}^s(i)$ on the first frequency linked to other frequencies by the coefficient $\mu_f = \lambda_f^2/\lambda_1^2$ with λ_f being the wavelength, the RCB $b_{r,f}$ and its satellite counterpart b_f^s, and the float ambiguity $\alpha_{r,f}^s = N_{r,f}^s + \delta_{r,f} - \delta_f^s$ containing the receiver phase bias (RPB) $\delta_{r,f}$, the satellite phase bias δ_f^s, and the integer ambiguity $N_{r,f}^s$. We consider the parameters with an epoch index i as time-varying quantities and assume the parameters without the epoch index to be time constant.

Departing from the time-constant assumption of the RCB, we consider it a time-varying parameter and rewrite Eq. (1) as

$$\begin{cases} E[P_{r,f}^s(i)] = \rho_r^s(i) + \mu_f \cdot I_{r,1}^s(i) + b_{r,f}(1) + \tilde{b}_{r,f}(i) - b_f^s \\ E[L_{r,f}^s(i)] = \rho_r^s(i) - \mu_f \cdot I_{r,1}^s(i) + \lambda_f \alpha_{r,f}^s \end{cases} \tag{2}$$

where

$$\tilde{b}_{r,f}(i) = b_{r,f}(i) - b_{r,f}(1) \tag{3}$$

denotes the time-differenced receiver bias between epoch i and the first epoch, but parameters in Eq. (2) are not uniquely estimable due to the rank deficient problem. For this issue, we turn to the S-system theory (Baarda, 1973; Teunissen, 1985), which constraints a minimum set of parameters and formulates a full-rank model estimating the linear functionals of the original parameters. To make it clear, we first decompose the receiver and satellite code biases on the first two frequencies as

$$(.)_{r,f} = (.)_{r,IF} + \mu_f (.)_{r,GF} \tag{4}$$

where the geometry-free combination of a frequency-dependent quantity is defined as

$$(.)_{GF} = \frac{1}{\mu_2 - \mu_1}[(.)_2 - (.)_1] \tag{5}$$

Inserting Eq. (4) into the code observations equation in Eq. (2) yields

$$\begin{cases} E[P_{r,f}^s(i)] = \rho_r^s(i) + \mu_f I_{r,1}^s(i) + b_{r,IF}(1) + \mu_f b_{r,IF}(1) - \mu_f b_{r,GF}^s + \tilde{b}_{r,f}(i) + \bar{b}_{r,f>2}^s \\ E[L_{r,f}^s(i)] = \rho_r^s(i) - \mu_f I_{r,1}^s(i) + \lambda_f \alpha_{r,f}^s \end{cases} \tag{6}$$

where

$$\bar{b}_{r,f>2}^s = [b_{r,f}(1) - b_{r,IF}(1) - u_f \cdot b_{r,GF}(1)] - (b_{,f}^s - u_f \cdot b_{,GF}^s)$$

denotes the combination of receiver and satellite code biases on the third frequency and above.The rank deficiency elimination for Eq. (6) becomes straightforward, as we can lump the parameters that have common coefficients:

$$E[P_{r,f}^s(i)] = \bar{\rho}_r^s(i) + \mu_f \bar{I}_{r,1}^s(i) + \tilde{b}_{r,f}(i) + \bar{b}_{r,f>2}^s \tag{7}$$

where

$$\begin{cases} \bar{\rho}_r^s(i) = \rho_r^s(i) + b_{r,IF}(1) \\ \bar{I}_{r,1}^s(i) = I_{r,1}^s(i) + b_{r,GF}(1) - b_{,GF}^s \end{cases} \tag{8}$$

denote the frequency-independent parameter and ionospheric delays, respectively. When dealing with the phase observations in Eq. (2), we first replace $\rho_r^s(i)$ and $I_{r,1}^s(i)$ with $\bar{\rho}_r^s(i)$ and $\bar{I}_{r,1}^s(i)$, resulting in additional code biases. By lumping these biases with ambiguities, we obtain the full-rank observation equations as

$$\begin{cases} E[P_{r,f}^s(i)] = \bar{\rho}_r^s(i) + \mu_f \bar{I}_{r,1}^s(i) + \tilde{b}_{r,f}(i) + \bar{b}_{r,f>2}^s \\ E[L_{r,f}^s(i)] = \bar{\rho}_r^s(i) - \mu_f \bar{I}_{r,1}^s(i) + \lambda_f \bar{\alpha}_{r,f}^s \end{cases} \tag{9}$$

where

$$\bar{\alpha}_{r,f}^s = \alpha_{r,f}^s + \left[b_{,IF}^s - b_{r,IF}(1)\right]/\lambda_f - \mu_f \cdot \left[b_{,GF}^s - b_{r,GF}(1)\right]/\lambda_f \tag{10}$$

denote the estimable parameters contain the RCB at the first epoch.

(2) FDMA RCB Function Model

GLONASS raw code and carrier phase observation equations as (Teunissen, 2019)

$$\begin{cases} E[P_{r,f}^s(i)] = \rho_r^s(i) + \mu_f^s \cdot I_{r,1}^s(i) + b_{r,f} - b_f^s + k^s \Delta b_{r,f} \\ E[L_{r,f}^s(i)] = \rho_r^s(i) - \mu_f^s \cdot I_{r,1}^s(i) + \lambda_f^s(\alpha_{r,f}^s + k^s \Delta\delta_{r,f}) \end{cases} \tag{11}$$

different from CDMA observation equations, there exist receiver code and phase biases that are linear functions of the satellite frequency channel number, $\Delta b_{r,f}$ and $\Delta\delta_{r,f}$, respectively. The meanings of the remaining parameters are the same as in Eq. (1).

In particular, GLONASS satellites have different wavelengths, expressed as λ_f^s. Furthermore, the coefficients of the ionospheric parameters μ_f^s are also satellite-dependent. To simplify the coefficients of the ionospheric parameters, they are defined based on the frequency of each GLONASS R1 and R2 satellite as follows:

$$F_f^s = F_f^0 + k^s \Delta F_f \tag{12}$$

where F_f^0 is the center frequency, ΔF_f is the frequency offset, $\Delta F_1 = 9/16$, $\Delta F_2 = 7/16$, k^s is the frequency channel number of satellite s, and they satisfy $|k^s| \leq 13$, $\Delta F_1/F_1^0 = 1/2848$, $\Delta F_2/F_2^0 = 1/2848$. Therefore, the wavelength λ_f^s can be expressed as:

$$\lambda_f^s = \frac{c}{F_f^0 + k^s \Delta F_f} = \frac{\lambda_f^0}{1 + \frac{k^s \Delta F_f}{F_f^0}} = \frac{2848\lambda_f^0}{2848 + k^s} \tag{13}$$

where $\lambda_f^0 = c/F_f^0$ is the wavelength corresponding to the center frequency. According to Equation (13), the coefficients of the ionospheric parameters can be written as:

$$\mu_f^s = \left(\frac{\lambda_f^s}{\lambda_1^s}\right)^2 = \left(\frac{2848\lambda_f^0}{2848 + k^s} \cdot \frac{2848 + k^s}{2848\lambda_1^0}\right)^2 = \mu_f \tag{14}$$

Therefore, Equation (11) can be simplified as:

$$\begin{cases} E[P_{r,f}^s(i)] = \rho_r^s(i) + \mu_f \cdot I_{r,1}^s(i) + b_{r,f} - b_f^s + k^s \Delta b_{r,f} \\ E[L_{r,f}^s(i)] = \rho_r^s(i) - \mu_f \cdot I_{r,1}^s(i) + \lambda_f^s(\alpha_{r,f}^s + k^s \Delta\delta_{r,f}) \end{cases} \tag{15}$$

Departing from the time-constant assumption of the RCB, we consider it a time-varying parameter and rewrite Eq. (14) as:

$$\begin{cases} E[P_{r,f}^s(i)] = \rho_r^s(i) + \mu_f \cdot I_{r,1}^s(i) + b_{r,f}(1) + \tilde{b}_{r,f}(i) - b_f^s + k^s \Delta b_{r,f}(1) + k^s \Delta\bar{b}_{r,f}(i) \\ E[L_{r,f}^s(i)] = \rho_r^s(i) - \mu_f \cdot I_{r,1}^s(i) + \lambda_f^s(\alpha_{r,f}^s + k^s \Delta\delta_{r,f}) \end{cases} \tag{16}$$

where

$$\begin{cases} \tilde{b}_{r,f}(i) = b_{r,f}(i) - b_{r,f}(1) \\ \Delta\bar{b}_{r,f}(i) = \Delta b_{r,f}(i) - \Delta b_{r,f}(1) \end{cases} \tag{17}$$

Note that, apart from the IFBs, the structure of the GLONASS equations is entirely consistent with that of CDMA systems. Therefore, following the rank deficiency elimination procedure used for CDMA, the following equations are obtained:

$$\begin{cases} E[P_{r,f}^s(i)] = \bar{\rho}_r^s(i) + \mu_f \bar{I}_{r,1}^s(i) + \tilde{b}_{r,f}(i) + k^s \Delta\bar{b}_{r,f}(i) \\ E[L_{r,f}^s(i)] = \bar{\rho}_r^s(i) - \mu_f \bar{I}_{r,1}^s(i) + \lambda_f^s(\bar{\alpha}_{r,f}^s + k^s \Delta\delta_{r,f}) \end{cases} \tag{18}$$

where

$$\begin{cases} \bar{\rho}_r^s(i) = \rho_r^s(i) + b_{r,IF}(1) - b_{,IF}^s + k^s \Delta b_{r,IF}(1) \\ \tilde{I}_{r,1}^s(i) = I_{r,1}^s(i) + b_{r,GF}(1) - b_{,GF}^s + k^s \Delta b_{r,GF}(1) \\ \bar{\alpha}_{r,f}^s = \alpha_{r,f}^s + k^s \Delta\delta_{r,f} + \dfrac{\left[b_{,IF}^s - b_{r,IF}(1) - k^s \Delta b_{r,IF}(1)\right]}{\lambda_f^s} - \dfrac{\mu_f \cdot \left[b_{,GF}^s - b_{r,GF}(1) - k^s \Delta b_{r,GF}(1)\right]}{\lambda_f^s} \end{cases}$$

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Contact author

Any suggestions, corrections, and comments on MRCB are sincerely welcomed; please contact us. MRCB will be updated frequently, I hope numerous people can participate in the next update.

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