Asymptotic Notations

Asymptotic Notations

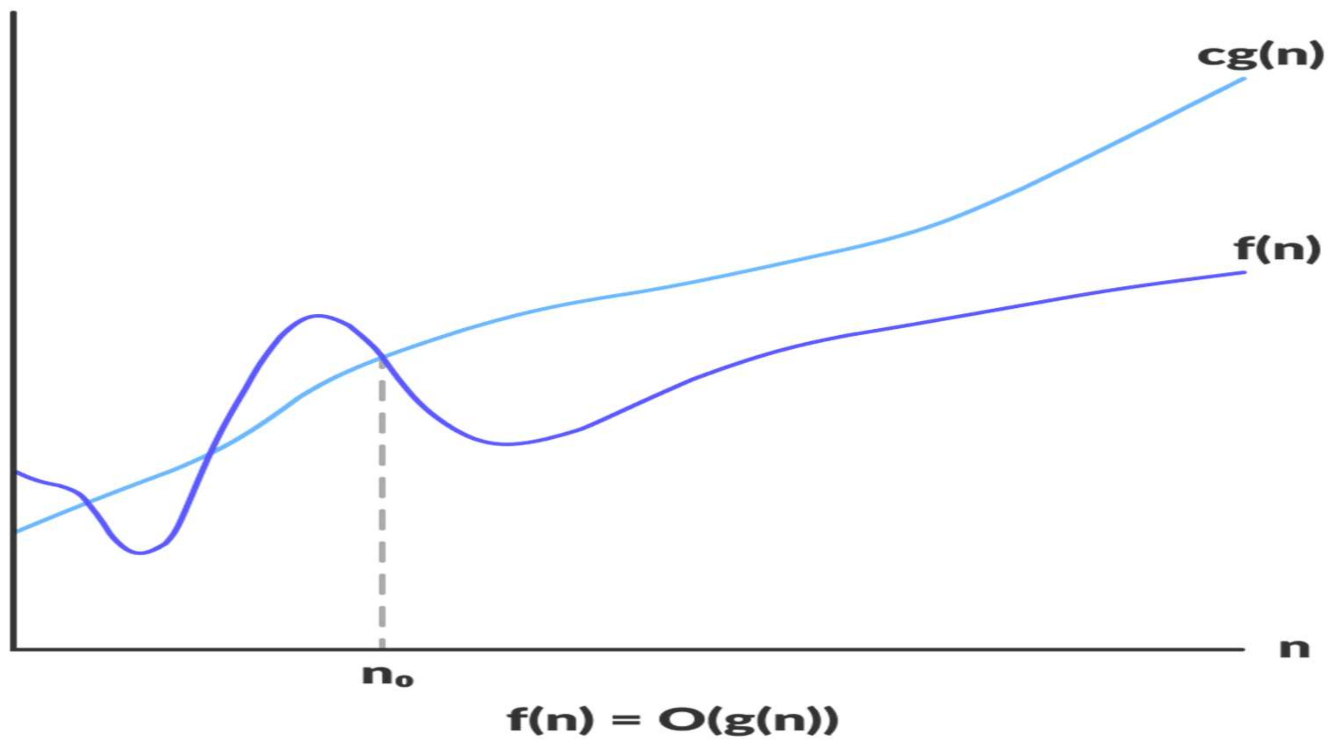
- It is the mathematical notations to represent frequency count.
- 5 types of asymptotic notations
  - **Big Oh (O)**
  - **Omega (Ω)**
  - **Theta (Θ)**
  - **Little Oh (o)**
  - **Little Omega (ω)**

Big Oh (O) Notation

Definition:

The function $f(n) = O(g(n))$ iff there exists 2 positive constants c and n_0 such that

$0 \leq f(n) \leq c\, g(n)$ for all $n \geq n_0$



Big Oh (O) Notation

- It is the measure of longest amount of time taken by an algorithm(Worst case).
- It is asymptotically tight upper bound
- $O(1)$: Computational time is constant
- $O(n)$: Computational time is linear
- $O(n^2)$: Computational time is quadratic
- $O(n^3)$: Computational time is cubic
- $O(2^n)$: Computational time is exponential

Find the O notation of the following function

$f(n) = 3n + 2$

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Here $f(n) = 3n + 2$ $\quad$ $g(n) = n$ $\quad$ $c = 4$

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Here $f(n) = 3n + 2$ $g(n) = n$ $c = 4$

$3n + 2 \leq 4n$ for all $n \geq n_0$

Find the O notation of the following function

$$3n + 2 \leq 4\,n$$

If n=1, LHS=5 , RHS=4, False

Find the O notation of the following function

$$3n + 2 \leq 4\, n$$

If n=1, LHS=5 ,	RHS=4,	False
If n=2, LHS=8 ,	RHS=8,	True

Find the O notation of the following function

$$3n + 2 \leq 4\,n$$

If n=1, LHS=5 ,	RHS=4,	False
If n=2, LHS=8 ,	RHS=8,	True
If n=3, LHS=11 ,	RHS=12,	True

Find the O notation of the following function

$$3n + 2 \leq 4\,n$$

If n=1, LHS=5 ,	RHS=4,	False
If n=2, LHS=8 ,	RHS=8,	True
If n=3, LHS=11 ,	RHS=12,	True
If n=4, LHS=14 ,	RHS=16,	True

Find the O notation of the following function

$$3n + 2 \leq 4\,n$$

If n=1, LHS=5 ,	RHS=4,	False
If n=2, LHS=8 ,	RHS=8,	True
If n=3, LHS=11 ,	RHS=12,	True
If n=4, LHS=14 ,	RHS=16,	True

The above equation is True when $n \geq 2$

Therefore $n_0=2$

Find the O notation of the following function

$f(n) = 3n + 2$

$0 \leq f(n) \leq c\, g(n)$ for all $n \geq n_0$

Here $f(n) = 3n + 2$ $g(n) = n$ $c = 4$ $n_0 = 2$

$3n + 2 \leq 4n$ for all $n \geq 2$

Find the O notation of the following function

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Here $f(n) = 3n + 2$ $\quad g(n) = n$ $\quad c = 4$ $\quad n_0 = 2$

$3n + 2 \leq 4n$ for all $n \geq 2$

Therefore $f(n) = \mathbf{O(g(n))}$

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$3n + 2 \leq 4\ n$ for all $n \geq 2$

Therefore $f(n) = \mathbf{O(g(n))}$

$3n + 2 = \mathbf{O(n)}$

Find the O notation of the following function

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Here $f(n) = 4n^3 + 2n + 3$ $\quad g(n) = n^3$ $\quad c=5$

$4n^3 + 2n + 3 \leq 5\, n^3$ for all $n \geq n_0$

Find the O notation of the following function

$$4n^3 + 2n + 3 \leq 5\, n^3$$

If n=1, LHS=9 ,   RHS=5,   False

Find the O notation of the following function

$$4n^3 + 2n + 3 \le 5\, n^3$$

If n=1, LHS=9 ,	RHS=5,	False
If n=2, LHS=39 ,	RHS=40,	True

Find the O notation of the following function

$$4n^3 + 2n + 3 \leq 5\ n^3$$

If n=1, LHS=9 ,	RHS=5,	False
If n=2, LHS=39 ,	RHS=40,	True
If n=3, LHS=117 ,	RHS=135,	True

Find the O notation of the following function

$$4n^3 + 2n + 3 \leq 5\, n^3$$

If n=1, LHS=9 ,	RHS=5,	False
If n=2, LHS=39 ,	RHS=40,	True
If n=3, LHS=117 ,	RHS=135,	True
If n=4, LHS=267 ,	RHS=320,	True

Find the O notation of the following function

$$4n^3 + 2n + 3 \le 5\, n^3$$

If n=1, LHS=9 ,	RHS=5,	False
If n=2, LHS=39 ,	RHS=40,	True
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If n=4, LHS=267 ,	RHS=320,	True

The above equation is True when $n \ge 2$

Therefore $n_0=2$

Find the O notation of the following function

$f(n) = 4n^3 + 2n + 3$

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Here $f(n) = 4n^3 + 2n + 3$ $g(n) = n^3$ $c=5$ $n_0=2$

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$4n^3 + 2n + 3 \leq 5\, n^3$ for all $n \geq 2$

Therefore $f(n) =$ **O(g(n))**

$4n^3 + 2n + 3 = \mathbf{O(n^3)}$

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$2^{n+1} \leq 2\; 2^n$ for all $n \geq n_0$

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If n=1, LHS=4 ,	RHS=4,	True
If n=2, LHS=8 ,	RHS=8,	True

Find the O notation of the following function

$$2^{n+1} \leq 2\ 2^n$$

If n=1, LHS=4 ,	RHS=4,	True
If n=2, LHS=8 ,	RHS=8,	True
If n=3, LHS=16 ,	RHS=16,	True

Find the O notation of the following function

$$2^{n+1} \leq 2\ 2^n$$

If n=1, LHS=4 ,	RHS=4,	True
If n=2, LHS=8 ,	RHS=8,	True
If n=3, LHS=16 ,	RHS=16,	True

The above equation is True when $n \geq 1$

Therefore $n_0=1$

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Therefore $f(n) = O(g(n))$

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$2^{n+1} = \mathbf{O(2^n)}$

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Here $f(n) = 2^n + 6n^2 + 3n$ $\quad g(n) = 2^n$ $\quad c = 7$

Find the O notation of the following function

$f(n) = 2^n + 6n^2 + 3n$

$0 \leq f(n) \leq c\ g(n)$ for all $n \geq n_0$

Here $f(n) = 2^n + 6n^2 + 3n$ $g(n) = 2^n$ $c=7$ $n_0=5$

$2^n + 6n^2 + 3n \leq 7\ 2^n$ for all $n \geq 5$

Find the O notation of the following function

$f(n) = 2^n + 6n^2 + 3n$

$0 \le f(n) \le c\, g(n)$ for all $n \ge n_0$

Here $f(n) = 2^n + 6n^2 + 3n$ $g(n) = 2^n$ $c=7$ $n_0=5$

$2^n + 6n^2 + 3n \le 7\, 2^n$ for all $n \ge 5$

Therefore $f(n) = O(g(n))$

Find the O notation of the following function

$f(n) = 2^n + 6n^2 + 3n$

$0 \le f(n) \le c\, g(n)$ for all $n \ge n_0$

Here $f(n) = 2^n + 6n^2 + 3n$ $g(n) = 2^n$ $c=7$ $n_0=5$

$2^n + 6n^2 + 3n \le 7\, 2^n$ for all $n \ge 5$

Therefore $f(n) = O(g(n))$

$2^n + 6n^2 + 3n = \mathbf{O(2^n)}$

Is $2^{2n} = O(2^n)$?

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$$0 \leq 2^{2n} \leq c\, 2^n \quad \text{for } n \geq n_0$$

$$2^n\, 2^n \leq c\, 2^n$$

Is $2^{2n} = O(2^n)$?

$0 \le 2^{2n} \le c\, 2^n \quad \text{for } n \ge n_0$

$2^n\, 2^n \le c\, 2^n$

$2^n \le c$

Is $2^{2n} = O(2^n)$?

$$0 \le 2^{2n} \le c\, 2^n \quad \text{for } n \ge n_0$$

$$2^n\, 2^n \le c\, 2^n$$

$$2^n \le c \qquad \text{for } n \ge n_0$$

Is $2^{2n} = O(2^n)$?

$0 \leq 2^{2n} \leq c\, 2^n \quad$ for $n \geq n_0$

$2^n\, 2^n \leq c\, 2^n$

$2^n \leq c \qquad$ for $n \geq n_0$

There is no value for c and n_0 that can make this true.

Therefore $\qquad 2^{2n}$!= $\mathbf{O(2^n)}$

Is $2^{n+1} = O(2^n)$?

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$0 \leq 2^{n+1} \leq c\, 2^n$ for $n \geq n_0$

Is $2^{n+1} = O(2^n)$?

$$0 \leq 2^{n+1} \leq c\, 2^n \text{ for } n \geq n_0$$

$$2 \times 2^n \leq c\, 2^n$$

Is $2^{n+1} = O(2^n)$?

$0 \leq 2^{n+1} \leq c\,2^n$ for $n \geq n_0$

$2 \times 2^n \leq c\,2^n$

$2 \leq c$

Is $2^{n+1} = O(2^n)$?

$0 \leq 2^{n+1} \leq c\, 2^n$ for $n \geq n_0$

$2 \times 2^n \leq c\, 2^n$

$2 \leq c$

$2^{n+1} \leq c\, 2^n$ is True if $c=2$ and $n \geq 1$.

Therefore $2^{n+1} = \mathbf{O(2^n)}$

Is $2^{n+1} = O(2^n)$?

$0 \leq 2^{n+1} \leq c\ 2^n$ for $n \geq n_0$

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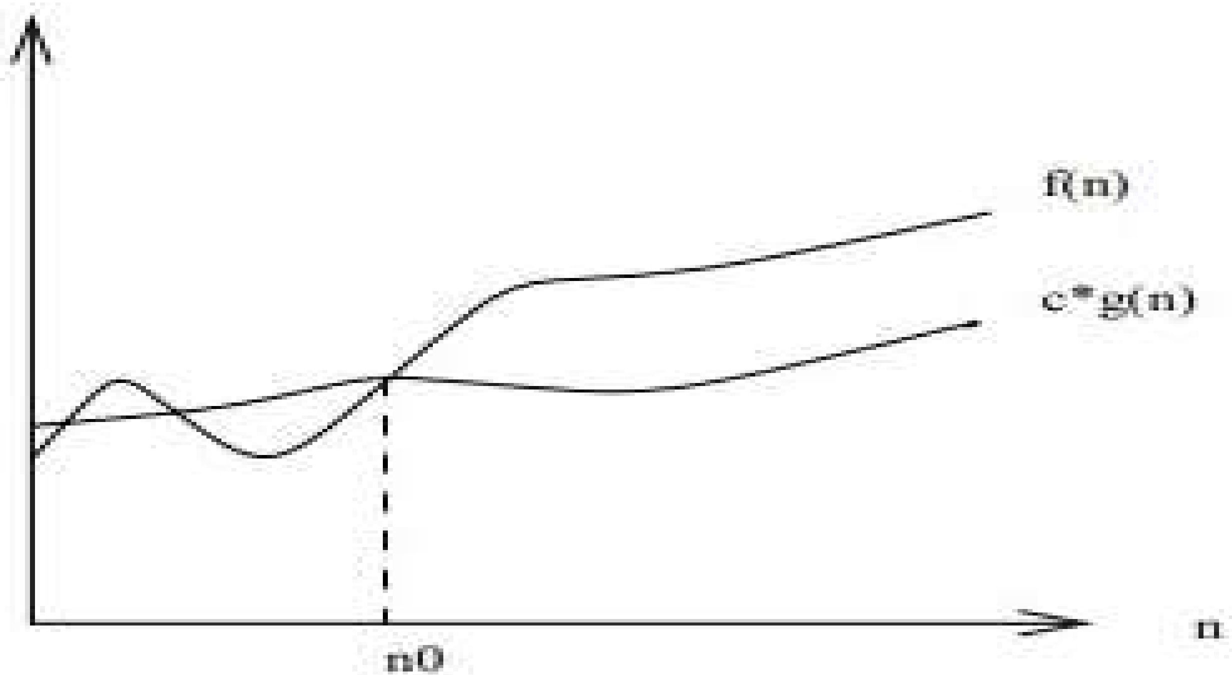
$2^{n+1} \leq c\ 2^n$ is True if $c=2$ and $n \geq 1$.

Therefore $2^{n+1} = \mathbf{O(2^n)}$

Omega (Ω) Notation

Definition:

The function $f(n) = \Omega(g(n))$ iff there exists 2 positive constant c and n_0 such that $f(n) \geq c\, g(n) \geq 0$ for all $n \geq n_0$



Omega (Ω) Notation

- It is the measure of smallest amount of time taken by an algorithm(Best case)
- It is asymptotically tight lower bound

Find the $\boldsymbol{\Omega}$ notation of the following function

$$f(n) = 27\, n^2 + 16n + 25$$

Find the Ω notation of the following function

$f(n) = 27\,n^2 + 16n + 25$

$f(n) \geq c\,g(n) \geq 0$ for all $n \geq n_0$

Find the $\mathbf{\Omega}$ notation of the following function

$f(n) = 27\,n^2 + 16n + 25$

$f(n) \geq c\,g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 27\,n^2 + 16n + 25$ $g(n)= n^2$ $c=27$

Find the **Ω** notation of the following function

$f(n) = 27\,n^2 + 16n + 25$

$f(n) \geq c\,g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 27\,n^2 + 16n + 25$ $\quad g(n)= n^2 \quad c=27$

$27\,n^2 + 16n + 25 \geq 27\,n^2$ $\qquad$ for all $n \geq n_0$

Find the $\boldsymbol{\Omega}$ notation of the following function

$$27\ n^2 + 16n + 25 \geq 27\ n^2$$

Find the Ω notation of the following function

$$27\,n^2 + 16n + 25 \geq 27\,n^2$$

If $n=1$: LHS=68 RHS=27 True

Find the **Ω** notation of the following function

$$27n^2 + 16n + 25 \geq 27n^2$$

If n=1:	LHS=68	RHS=27	True
If n=2:	LHS=165	RHS=108	True

Find the Ω notation of the following function

$$27\,n^2 + 16n + 25 \geq 27\,n^2$$

If n=1:	LHS=68	RHS=27	True
If n=2:	LHS=165	RHS=108	True

This equation is true if $n \geq 1$

Therefore $n_0 = 1$

Find the **Ω** notation of the following function

$f(n) = 27\,n^2 + 16n + 25$

$f(n) \geq c\,g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 27\,n^2 + 16n + 25$ $g(n)= n^2$ $c=27$ $n_0=1$

$27\,n^2 + 16n + 25 \geq 27\,n^2$ for all $n \geq 1$

Find the **Ω** notation of the following function

$f(n) = 27\,n^2 + 16n + 25$

$f(n) \geq c\,g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 27\,n^2 + 16n + 25$ $g(n)= n^2$ $c=27$ $n_0=1$

$27\,n^2 + 16n + 25 \geq 27\,n^2$ for all $n \geq 1$

Therefore $f(n) = \mathbf{\Omega(g(n))}$

Find the $\boldsymbol{\Omega}$ notation of the following function

$f(n) = 27\, n^2 + 16n + 25$

$f(n) \geq c\, g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 27\, n^2 + 16n + 25$ $g(n) = n^2$ $c=27$ $n_0=1$

$27\, n^2 + 16n + 25 \geq 27\, n^2$ for all $n \geq 1$

Therefore $f(n) = \boldsymbol{\Omega(g(n))}$

$\boldsymbol{27\, n^2 + 16n + 25 = \Omega(n^2)}$

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Find the $\boldsymbol{\Omega}$ notation of the following function

$f(n) = 3^n + 6n^2 + 3n$

$f(n) \geq c\ g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 3^n + 6n^2 + 3n$ $\quad g(n)= 3^n \quad c=1$

$3^n + 6n^2 + 3n \geq 1 \times 3^n$ $\qquad$ for all $n \geq n_0$

Find the Ω notation of the following function

$$3^n + 6n^2 + 3n \geq 1 \times 3^n$$

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$$3^n + 6n^2 + 3n \geq 1 \times 3^n$$

If n=1: LHS=12 RHS=3 True

Find the Ω notation of the following function

$$3^n + 6n^2 + 3n \geq 1 \times 3^n$$

If n=1:	LHS=12	RHS=3	True
If n=2:	LHS=39	RHS=9	True

Find the Ω notation of the following function

$$3^n + 6n^2 + 3n \geq 1 \times 3^n$$

If n=1:	LHS=12	RHS=3	True
If n=2:	LHS=39	RHS=9	True

This equation is true if $n \geq 1$

Therefore $n_0 = 1$

Find the Ω notation of the following function

$f(n) = 3^n + 6n^2 + 3n$

$f(n) \geq c\ g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 3^n + 6n^2 + 3n$ $g(n) = 3^n$ $c=1$ $n_0=1$

$3^n + 6n^2 + 3n \geq 1 \times 3^n$ for all $n \geq 1$

Find the **Ω** notation of the following function

$f(n) = 3^n + 6n^2 + 3n$

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$3^n + 6n^2 + 3n \geq 1 \times 3^n$ for all $n \geq 1$

Therefore $f(n) = \mathbf{\Omega(g(n))}$

Find the $\boldsymbol{\Omega}$ notation of the following function

$\mathbf{f(n) = 3^n + 6n^2 + 3n}$

$f(n) \geq c\ g(n) \geq 0$ for all $n \geq n_0$

Here $f(n) = 3^n + 6n^2 + 3n$ $g(n) = 3^n$ $c=1$ $n_0=1$

$3^n + 6n^2 + 3n \geq 1 \times 3^n$ for all $n \geq 1$

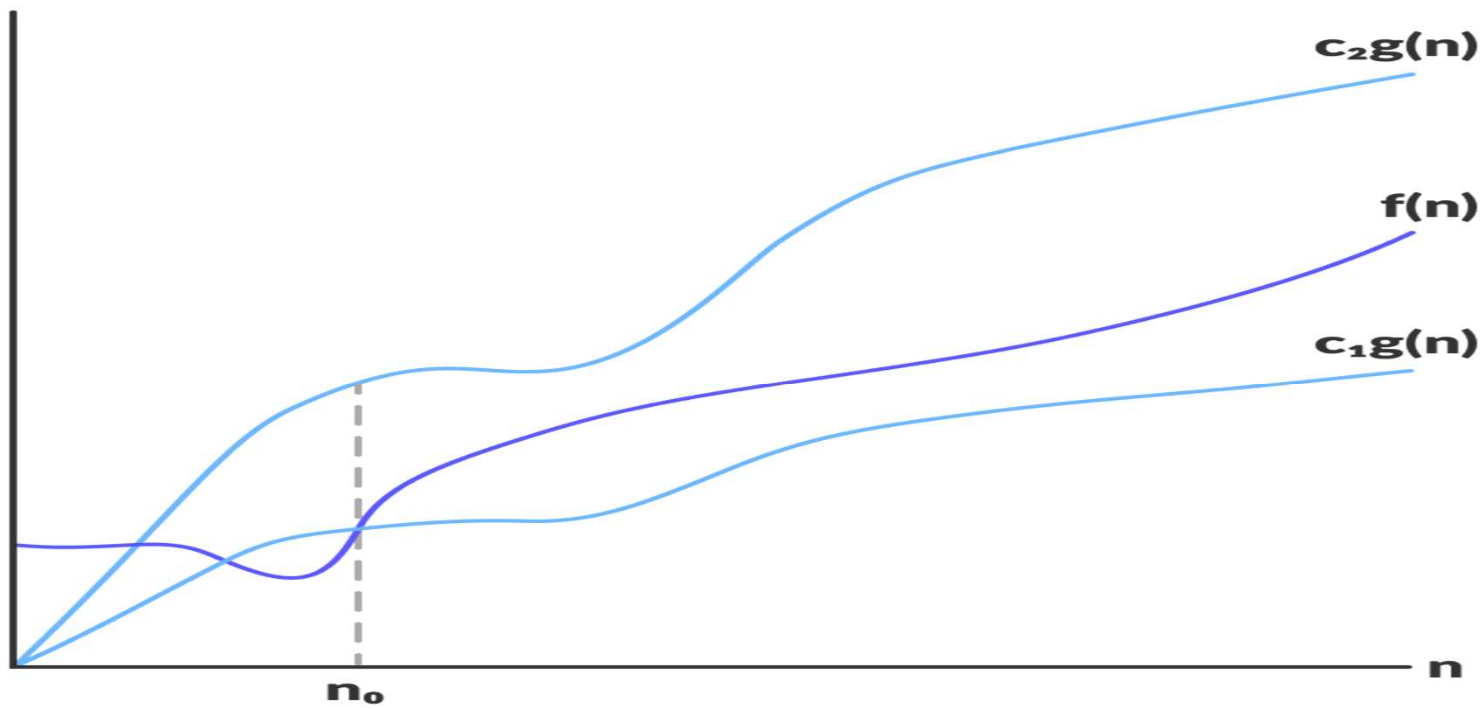
Therefore $f(n) = \boldsymbol{\Omega(g(n))}$

$\mathbf{3^n + 6n^2 + 3n = \Omega(3^n)}$

Theta (Θ) Notation

Definition:

The function $f(n) = \Theta(g(n))$ iff there exists 3 positive constants c_1, c_2 and n_0 such that $0 \leq c_1\, g(n) \leq f(n) \leq c_2\, g(n)$ for all $n \geq n_0$



$f(n) = \Theta(g(n))$

Theta (Θ) Notation

- It is the measure of average amount of time taken by an algorithm(Average case)

Find the ϴ notation of the following function

$f(n) = 3n + 2$

Find the Ɵ notation of the following function

$f(n) = 3n + 2$

$0 \le c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all $n \ge n_0$

Find the ϴ notation of the following function

$f(n) = 3n + 2$

$0 \le c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all $n \ge n_0$

$c_1\, g(n) \le f(n)$ for all $n \ge n_0$

Find the Θ notation of the following function

$f(n) = 3n + 2$

$0 \leq c_1\, g(n) \leq f(n) \leq c_2\, g(n)$ for all $n \geq n_0$

$c_1\, g(n) \leq f(n)$ for all $n \geq n_0$

Here $f(n) = 3n + 2$ $\quad g(n)= n$ $\quad c_1 =3$ $\quad n_0 =1$

$3\, n \leq 3n + 2$ $\quad$ for all $n \geq 1$

Find the ϴ notation of the following function

$f(n) = 3n + 2$

$0 \leq c_1\, g(n) \leq f(n) \leq c_2\, g(n)$ for all $n \geq n_0$

$c_1\, g(n) \leq f(n)$ for all $n \geq n_0$

Here $f(n) = 3n + 2$ $g(n) = n$ $c_1 = 3$ $n_0 = 1$

$3n \leq 3n + 2$ for all $n \geq 1$

$f(n) \leq c_2\, g(n)$ for all $n \geq n_0$

Find the Θ notation of the following function

$f(n) = 3n + 2$

$0 \leq c_1\, g(n) \leq f(n) \leq c_2\, g(n)$ for all $n \geq n_0$

$c_1\, g(n) \leq f(n)$ for all $n \geq n_0$

Here $f(n) = 3n + 2$ $g(n) = n$ $c_1 = 3$ $n_0 = 1$

$3n \leq 3n + 2$ for all $n \geq 1$

$f(n) \leq c_2\, g(n)$ for all $n \geq n_0$

Here $f(n) = 3n + 2$ $g(n) = n$ $c_2 = 4$ $n_0 = 2$

$3n + 2 \leq 4n$ for all $n \geq 2$

Find the Θ notation of the following function

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Here $f(n) = 3n + 2$ $g(n)= n$ $c_1 = 3$ $n_0 = 1$

$3\, n \leq 3n + 2$ for all $n \geq 1$

$f(n) \leq c_2\, g(n)$ for all $n \geq n_0$

Here $f(n) = 3n + 2$ $g(n)= n$ $c_2 = 4$ $n_0 = 2$

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Therefore $f(n) = \mathbf{\Theta(g(n))}$ ➔ **$3n + 2 = \Theta(n)$**

Find the ϴ notation of the following function

$f(n) = 3 \times 2^n + 4n^2 + 5n + 2$

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Here $f(n) = 3 \times 2^n + 4n^2 + 5n + 2$ $g(n) = 2^n$ $c_2 = 10$ $n_0 = 1$

$3 \times 2^n + 4n^2 + 5n + 2 \le 10 \times 2^n$ for all $n \ge 1$

Find the Θ notation of the following function

$f(n) = 3 \times 2^n + 4n^2 + 5n + 2$

$0 \le c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all $n \ge n_0$

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$3 \times 2^n + 4n^2 + 5n + 2 \leq 10 \times 2^n$ for all $n \geq 1$

$3 \times 2^n \leq 3 \times 2^n + 4n^2 + 5n + 2 \leq 10 \times 2^n$ for all $n \geq 1$ $c_1 = 3$ $c_2 = 10$ $n_0 = 1$

Therefore $f(n) = \boldsymbol{\Theta(g(n))}$ $\Rightarrow$ **$3 \times 2^n + 4n^2 + 5n + 2 = \Theta(2^n)$**

Little oh (o) Notation

Definition:

The function $f(n) = o(g(n))$ iff for any positive constant $c>0$, there exists a constant $n_0>0$ such that $0 \leq f(n) < c\, g(n)$ for all $n \geq n_0$

It is asymptotically loose upper bound

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

g(n) becomes arbitrarily large relative to f(n) as n approaches infinity

Little Omega (ω)

Definition:

The function $f(n) = \omega(g(n))$ iff for any positive constant $c>0$, there exists a constant $n_0>0$ such that $f(n) > c\, g(n) \geq 0$ for all $n \geq n_0$

It is asymptotically loose lower bound

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

$f(n)$ becomes arbitrarily large relative to $g(n)$ as n approaches infinity