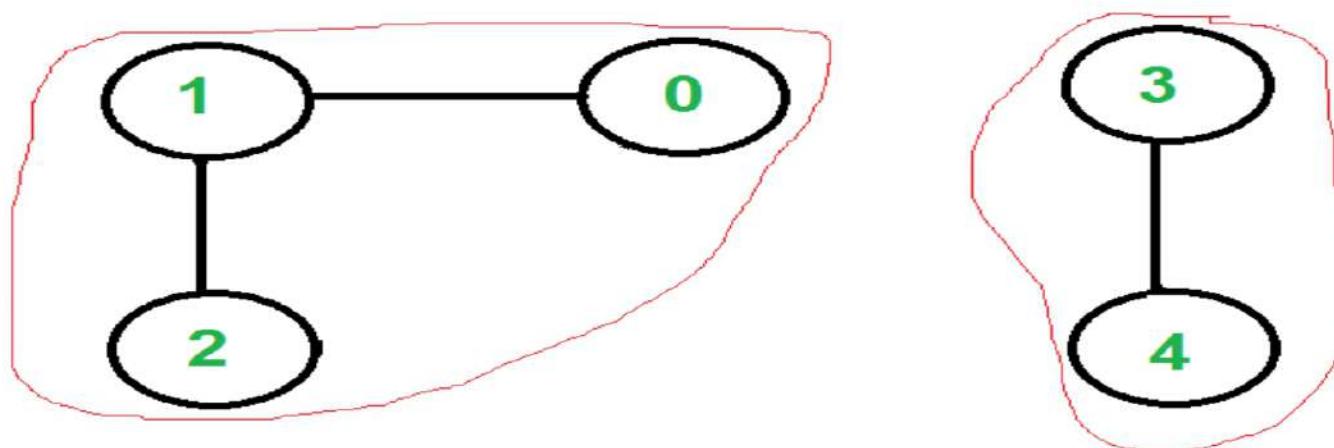


# Strongly Connected Components

# Connected Components

- Connected component of a graph  $G$  is a connected subgraph of  $G$  of maximum size
- A graph may have more than one connected components.

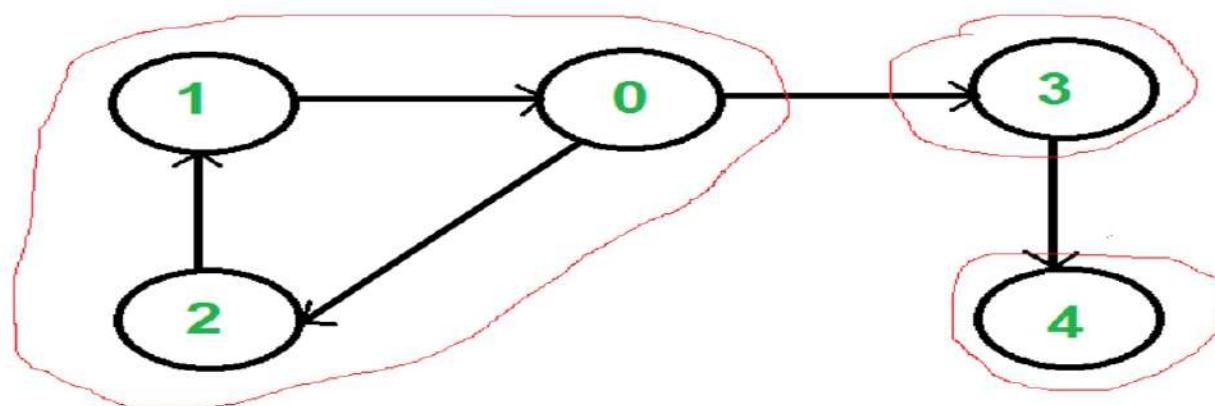


**There are two connected components in above undirected graph**

0 1 2  
3 4

# Strongly Connected Components(SCC)

- Strong Connectivity applies only to directed graphs.
- A strongly connected component of a directed graph is a compliment such that all the vertices in that component is reachable from every other vertex in that component.

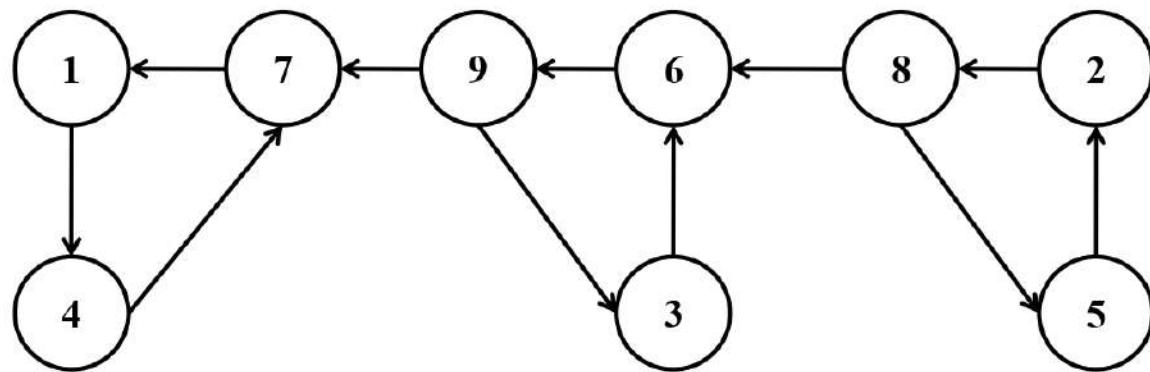


There are 3 SCCs:- {0,1,2}, {3}, {4}

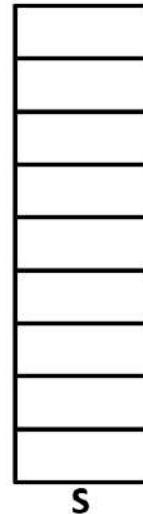
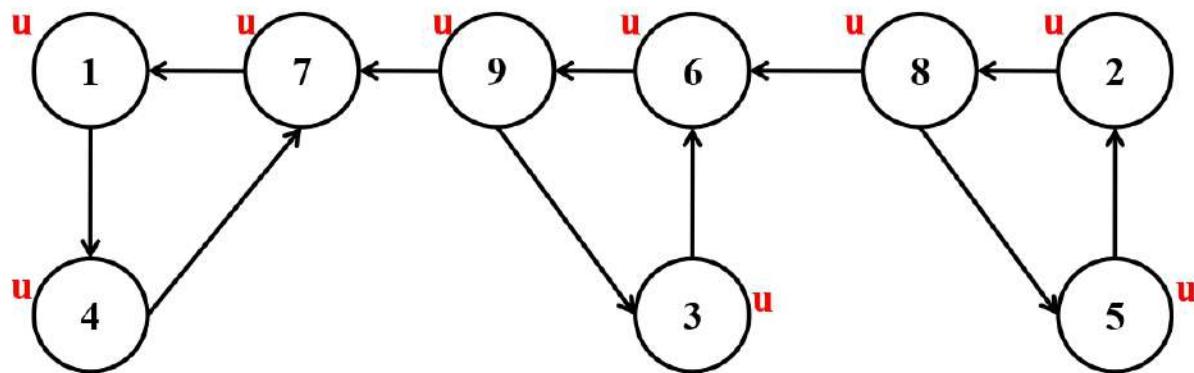
## **Strongly Connected Component Algorithm -Kosaraju's Algorithm**

- It is a 2 Pass algorithm. Steps 1-4 are Pass1. Steps 5-7 are Pass2.
1. Set all vertices of graph G are unvisited.
  2. Create an empty stack S.
  3. Do DFS traversal on unvisited vertices and set it as visited. If a vertex has no unvisited neighbor, push it in to the stack.
  4. Perform the above step until all vertices are visited
  5. Reverse the graph G.
  6. Set all nodes are unvisited.
  7. While S is not Empty
    1. POP one vertex  $v'$
    2. If  $v'$  is not visited
      1. Set  $v'$  as visited
      2. Call DFS( $v'$ ). It will print strongly connected component of  $v'$ .

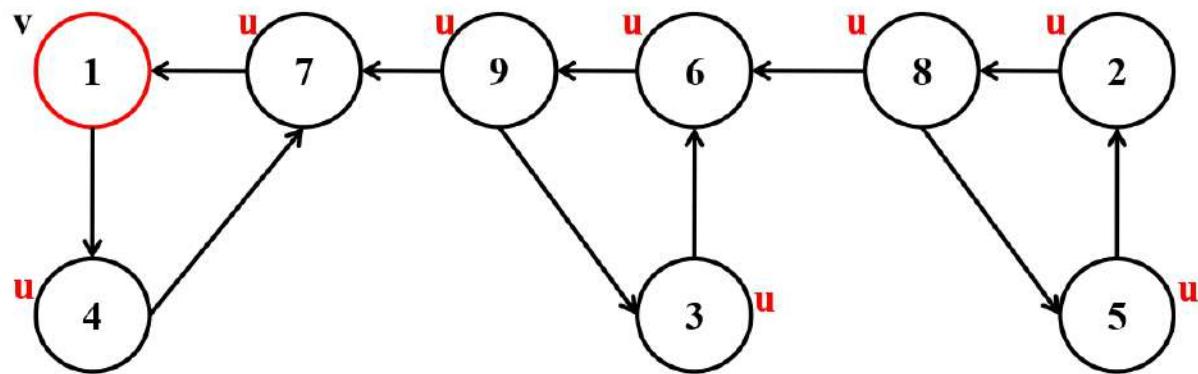
**Find strongly connected components of the digraph using the algorithm showing each step**



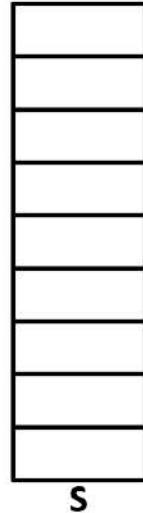
Find strongly connected components of the digraph using the algorithm showing each step



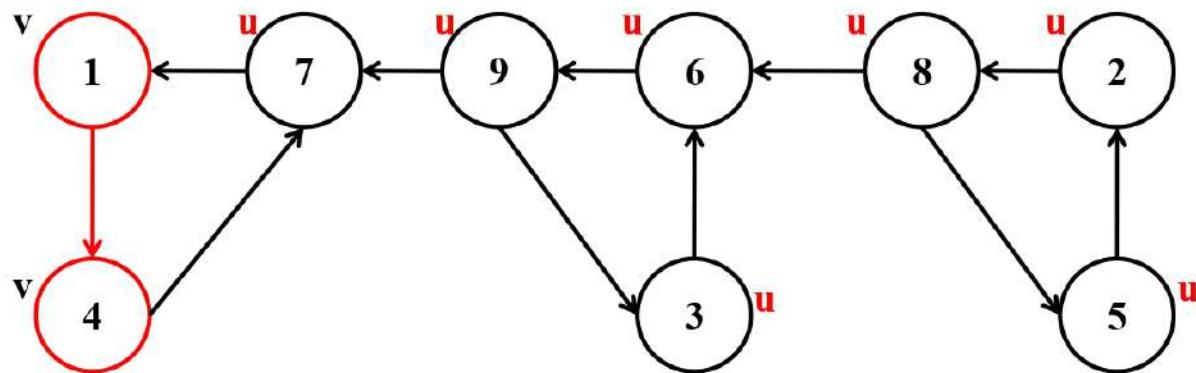
Find strongly connected components of the digraph using the algorithm showing each step



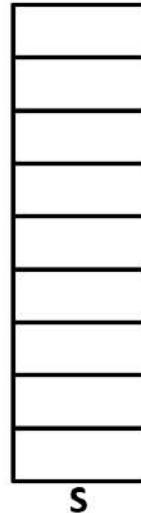
Pass 1  
DFS: 1



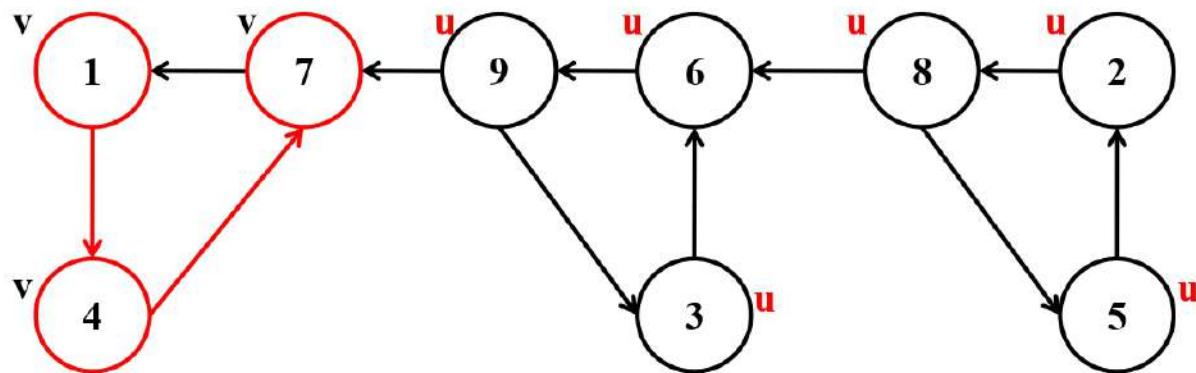
Find strongly connected components of the digraph using the algorithm showing each step



Pass 1  
DFS: 1,4

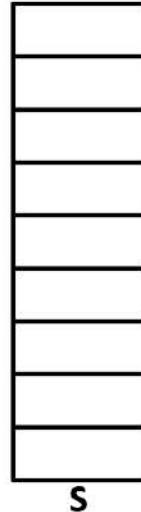


Find strongly connected components of the digraph using the algorithm showing each step

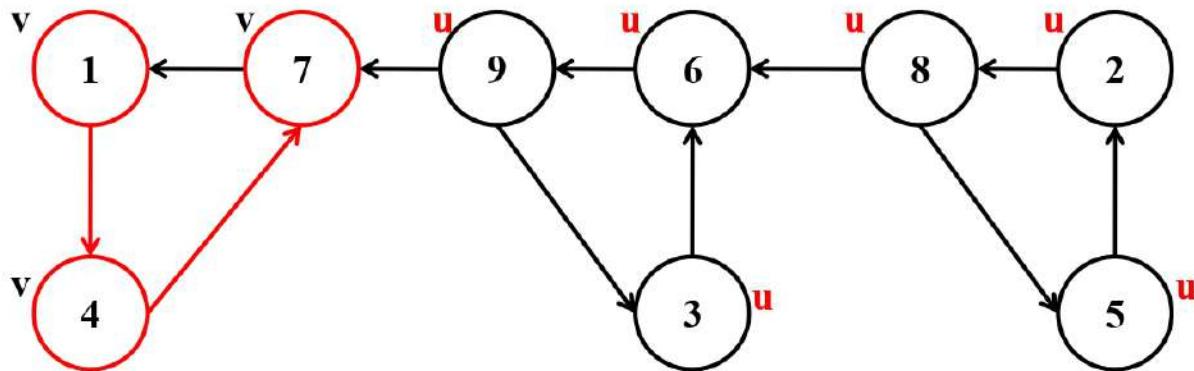


Pass 1

DFS: 1,4,7

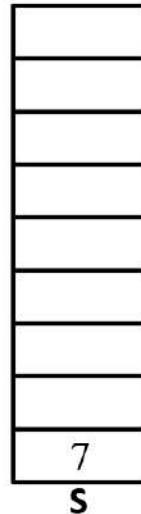


**Find strongly connected components of the digraph using the algorithm showing each step**

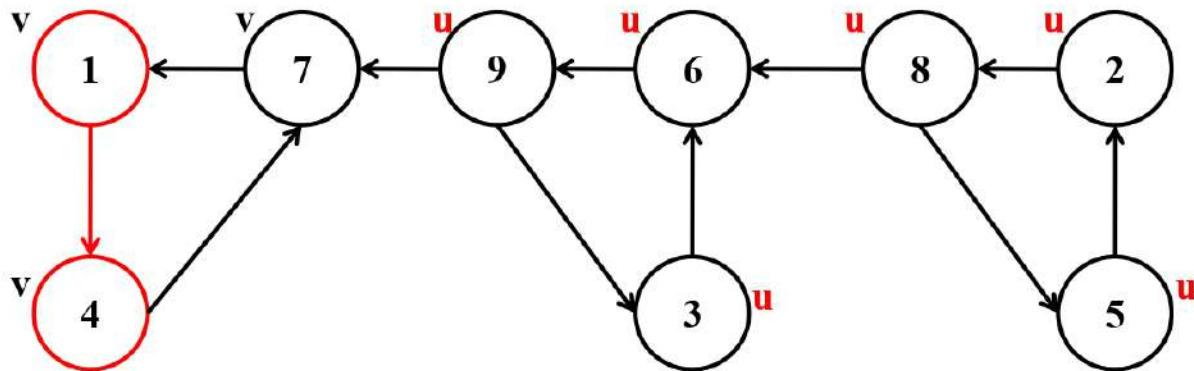


Pass 1

DFS: 1,4,7

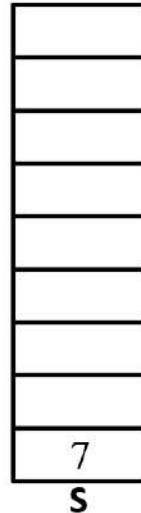


**Find strongly connected components of the digraph using the algorithm showing each step**

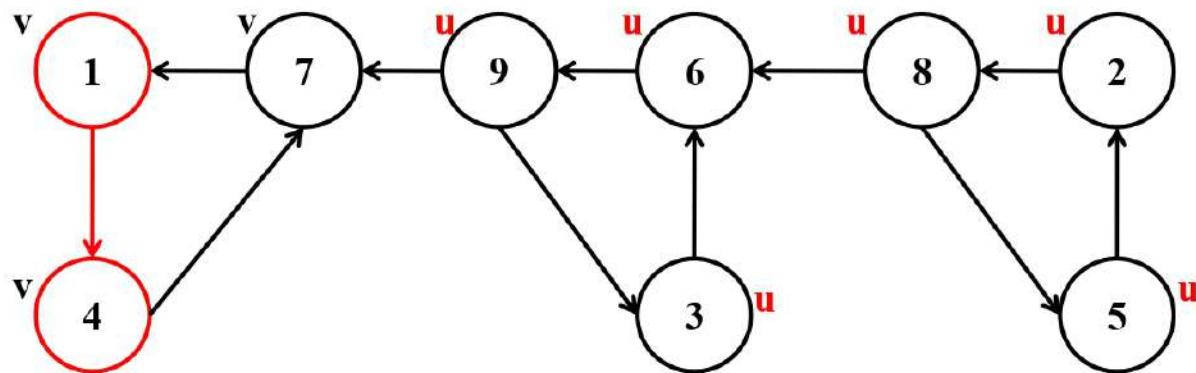


Pass 1

DFS: 1,4,7

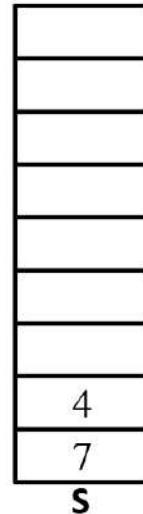


Find strongly connected components of the digraph using the algorithm showing each step

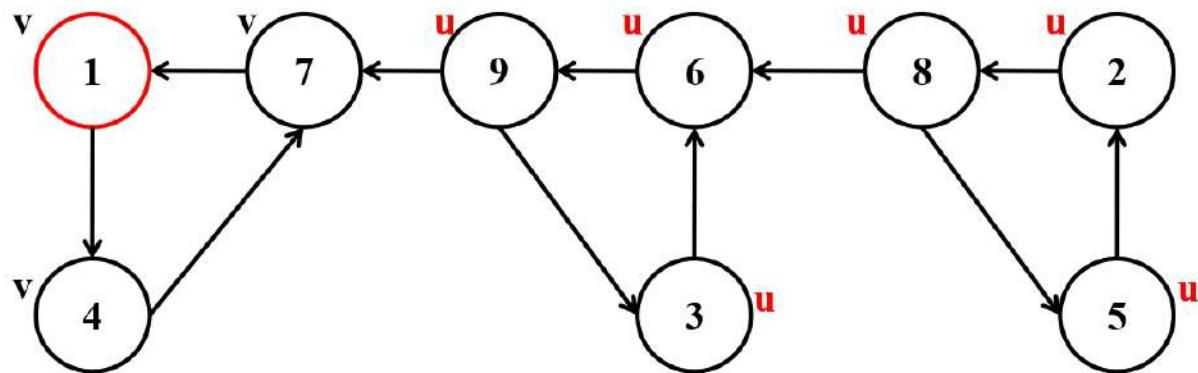


Pass 1

DFS: 1,4,7

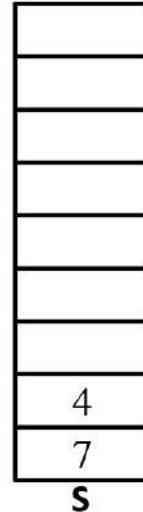


Find strongly connected components of the digraph using the algorithm showing each step

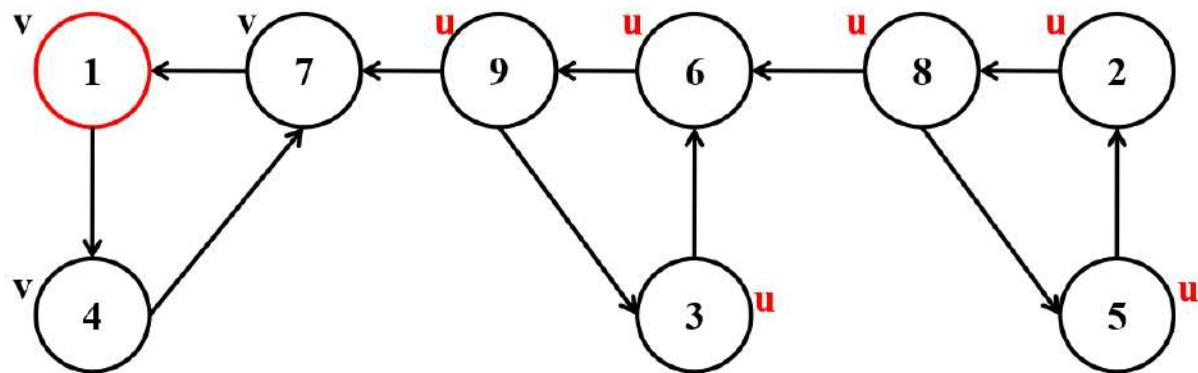


Pass 1

DFS: 1,4,7



Find strongly connected components of the digraph using the algorithm showing each step

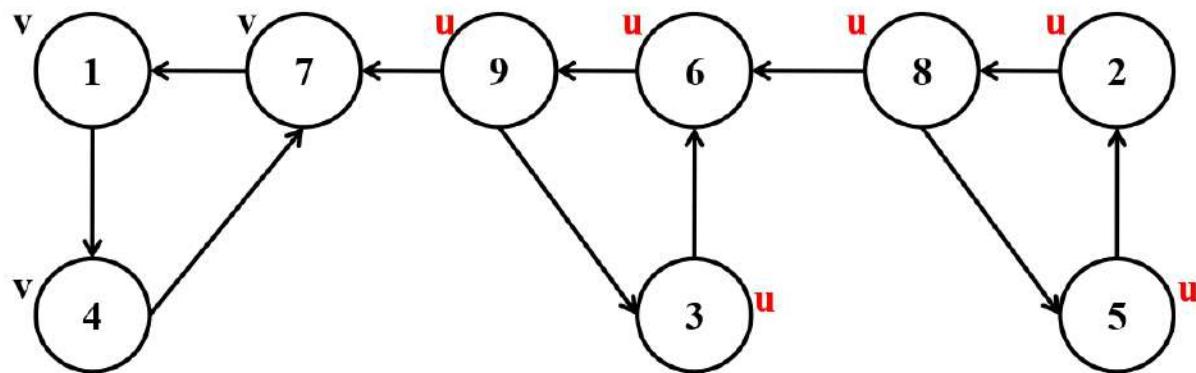


Pass 1

DFS: 1,4,7

1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

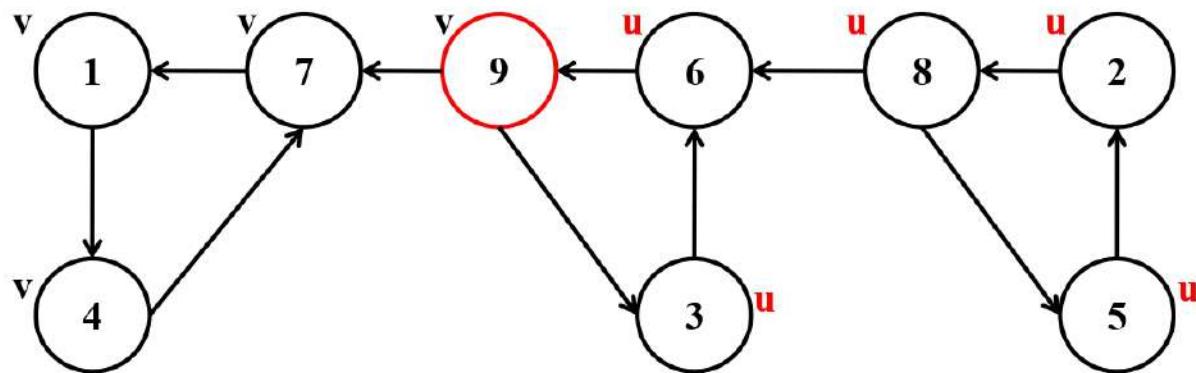


Pass 1

DFS: 1,4,7

1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

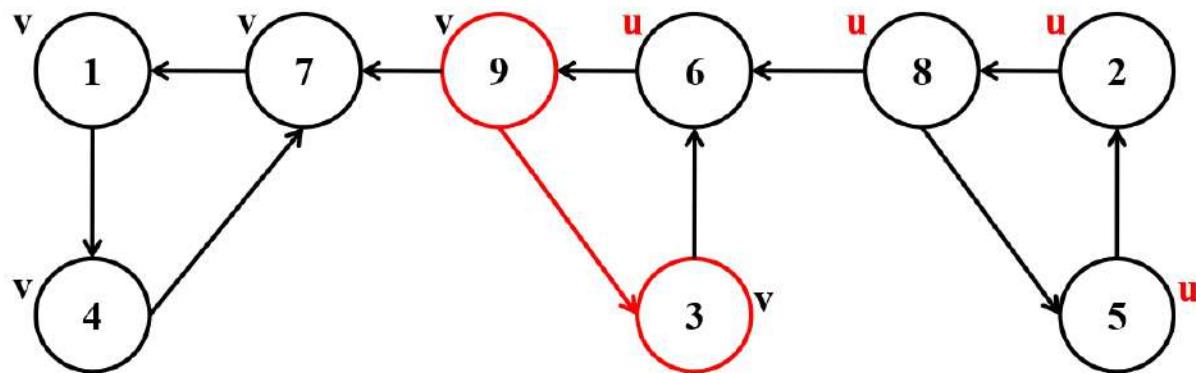


Pass 1

DFS: 1,4,7,9

1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

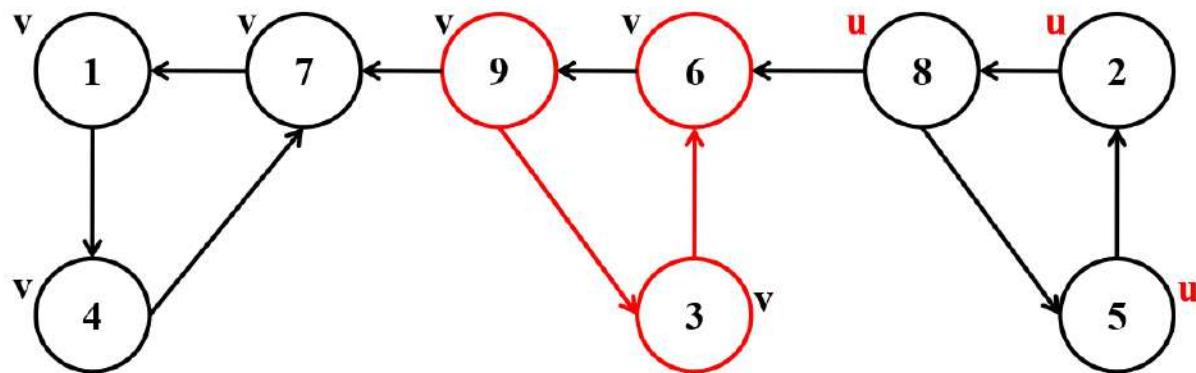


Pass 1

DFS: 1,4,7,9,3

1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

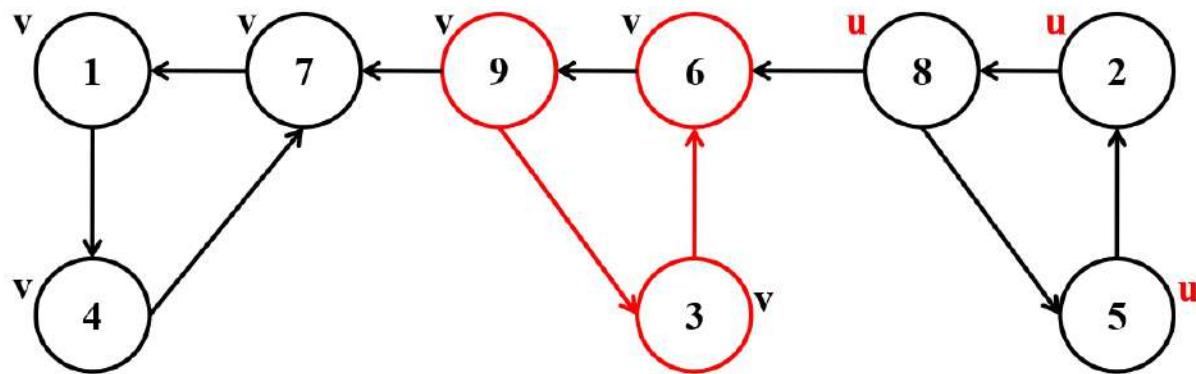


Pass 1

DFS: 1,4,7,9,3,6

1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

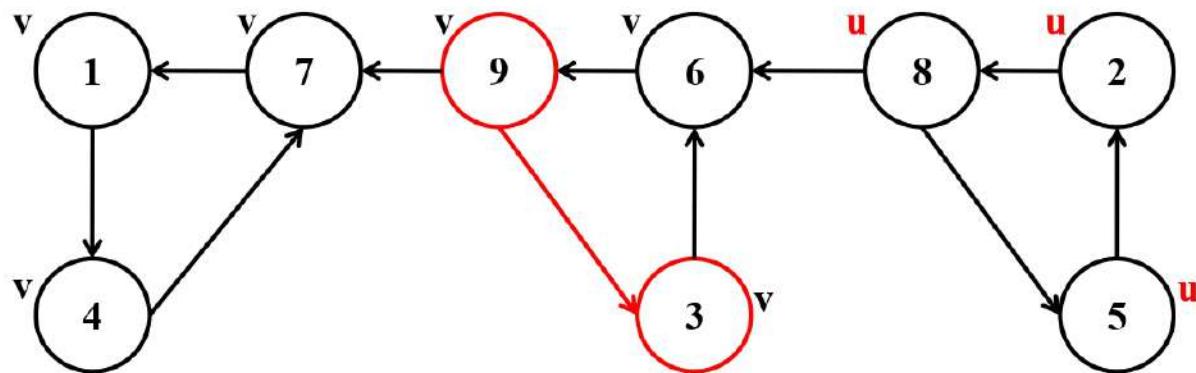


Pass 1

DFS: 1,4,7,9,3,6

6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

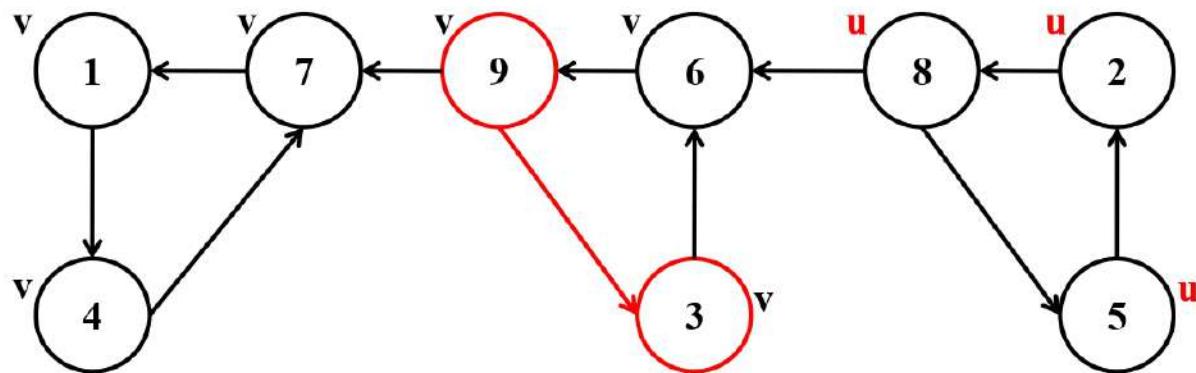


Pass 1

DFS: 1,4,7,9,3,6

6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

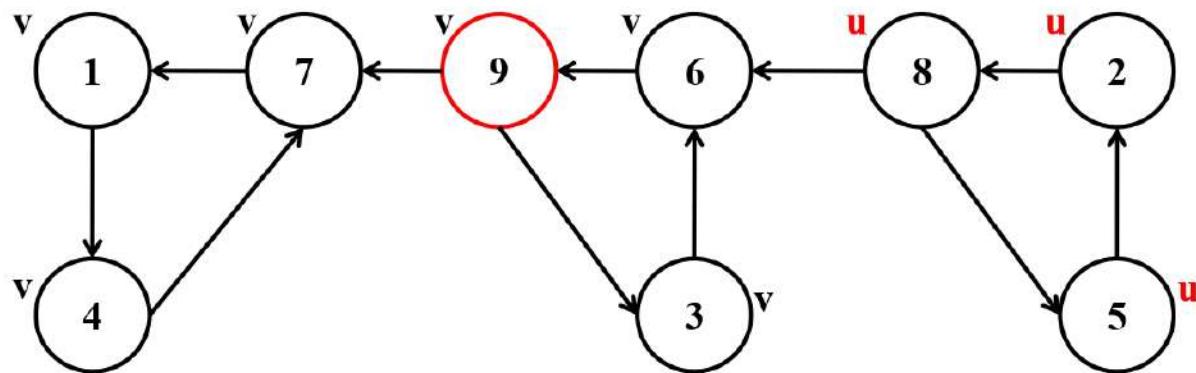


Pass 1

DFS: 1,4,7,9,3,6

3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

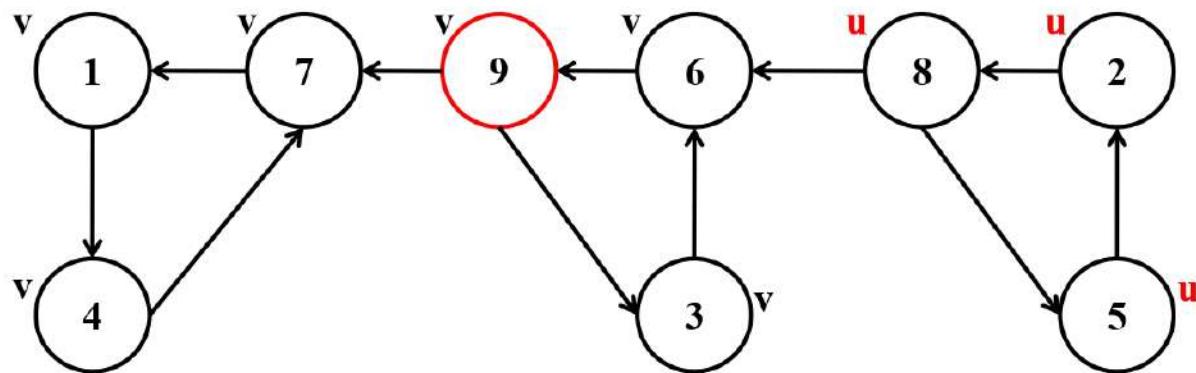


Pass 1

DFS: 1,4,7,9,3,6

3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

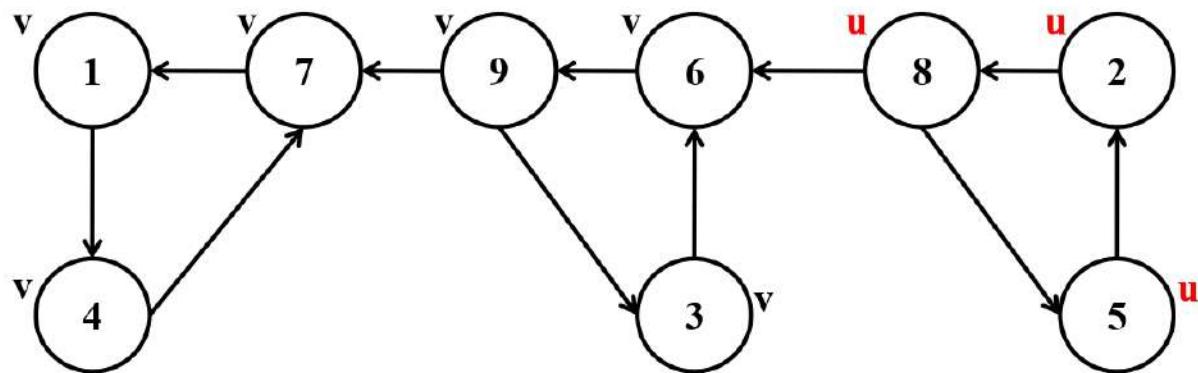


Pass 1

DFS: 1,4,7,9,3,6

9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

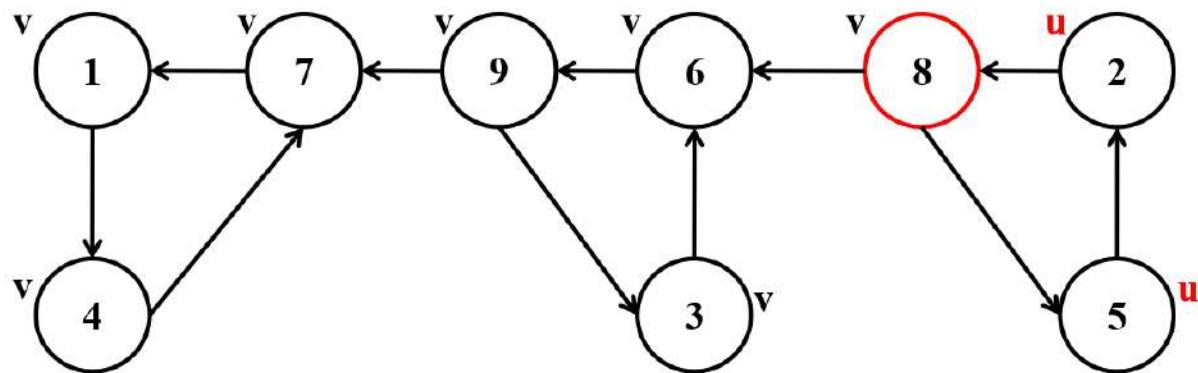


Pass 1

DFS: 1,4,7,9,3,6

9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

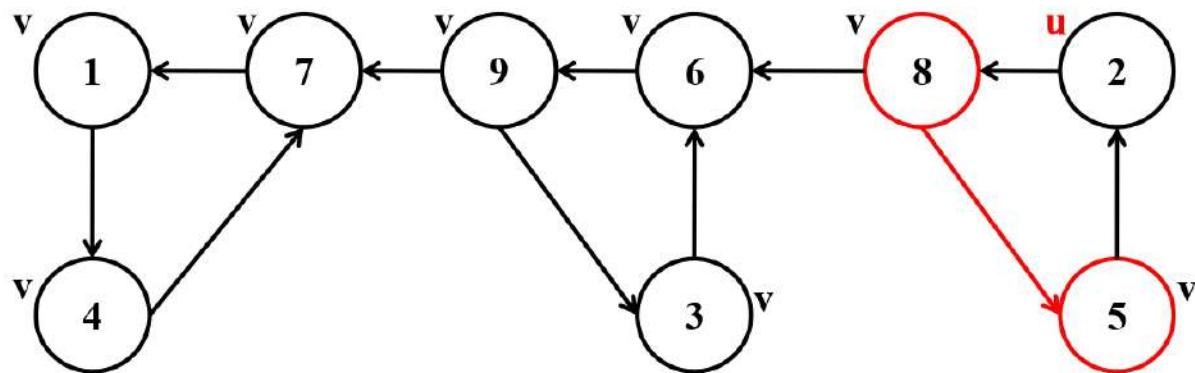


Pass 1

DFS: 1,4,7,9,3,6,8

9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

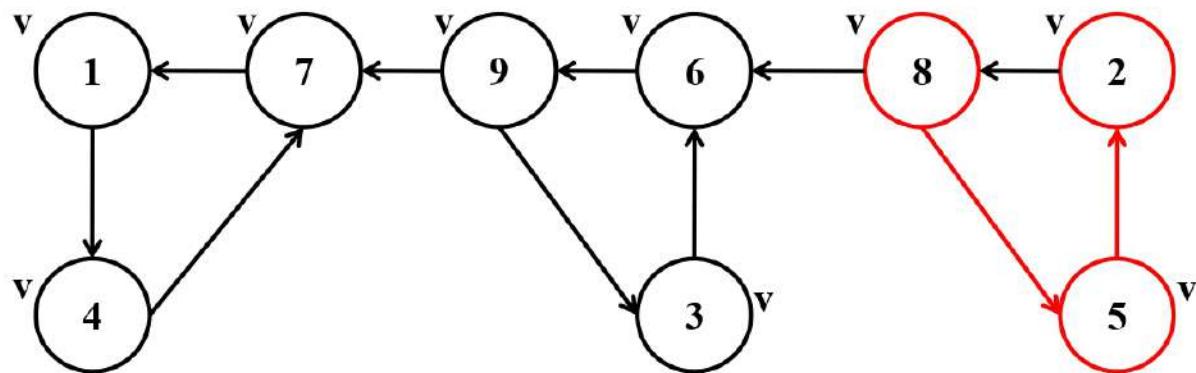


Pass 1

DFS: 1,4,7,9,3,6,8,5

9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

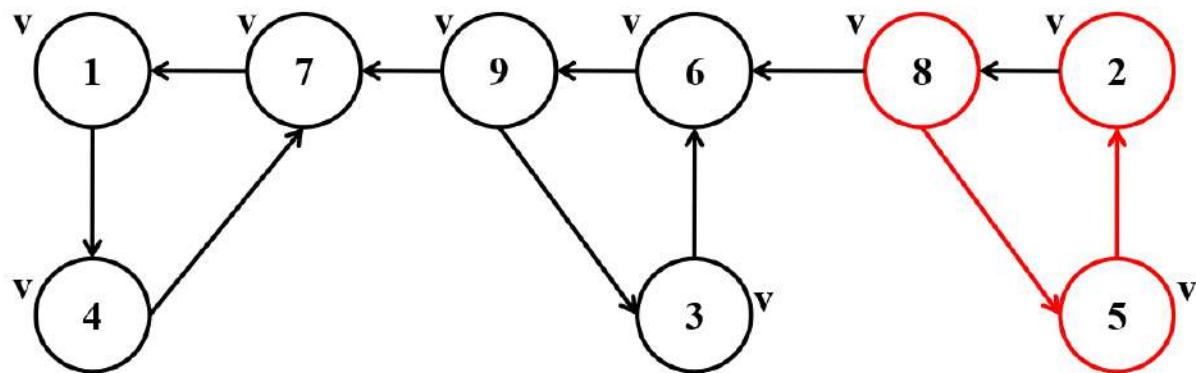


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

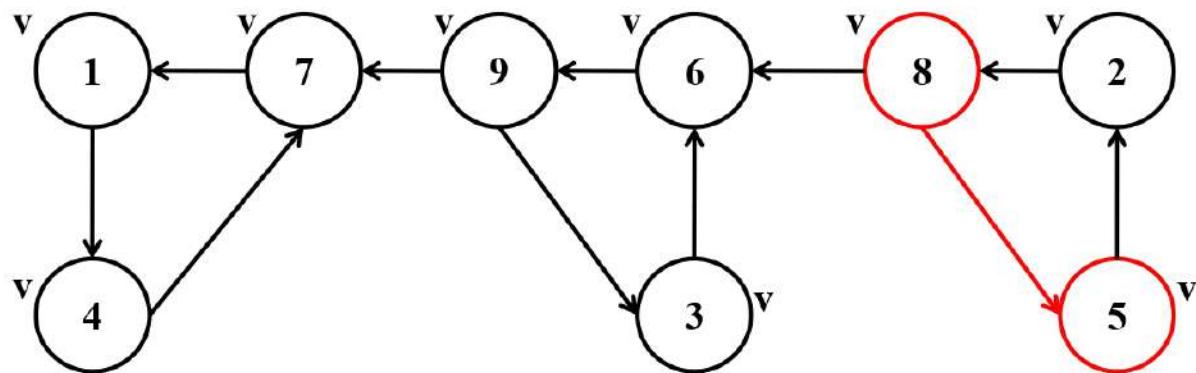


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

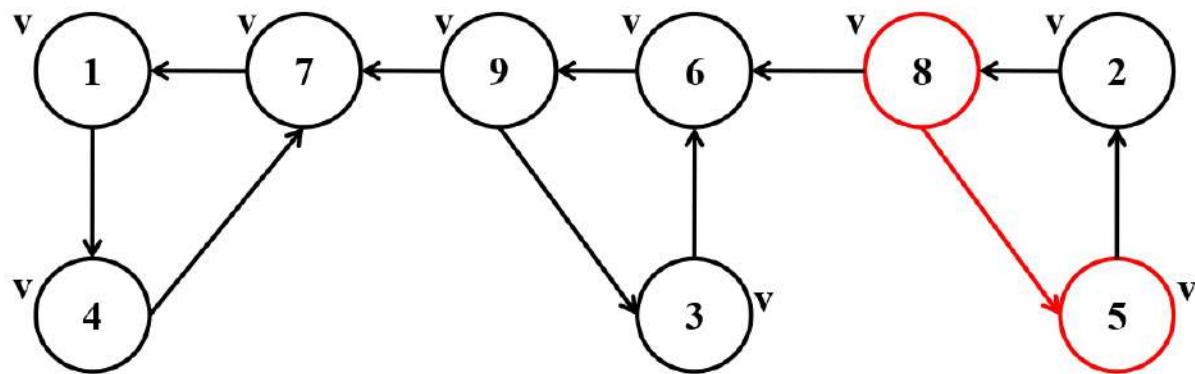


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

2
9
3
6
1
4
7
s

Find strongly connected components of the digraph using the algorithm showing each step

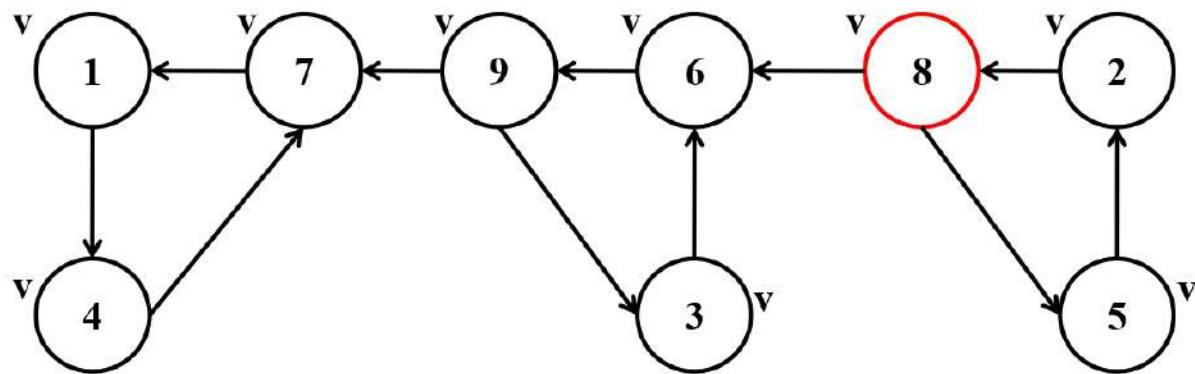


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

5
2
9
3
6
1
4
7
s

Find strongly connected components of the digraph using the algorithm showing each step

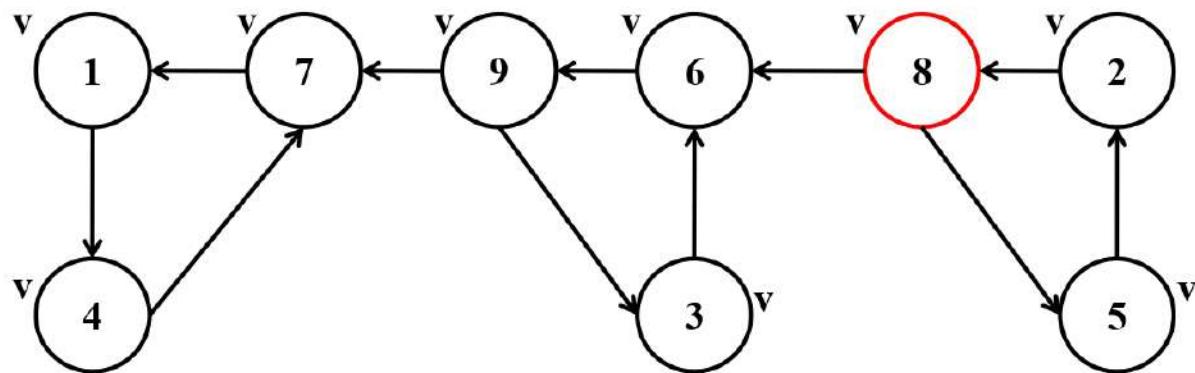


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

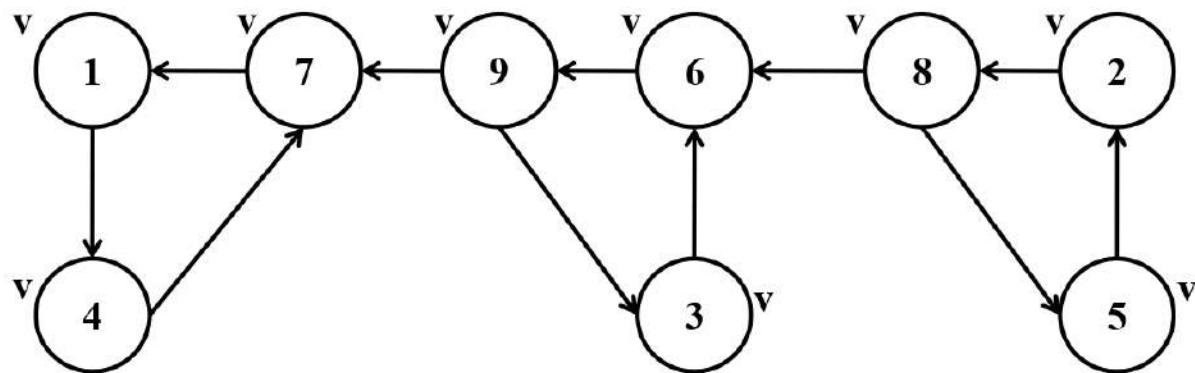


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

8
5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

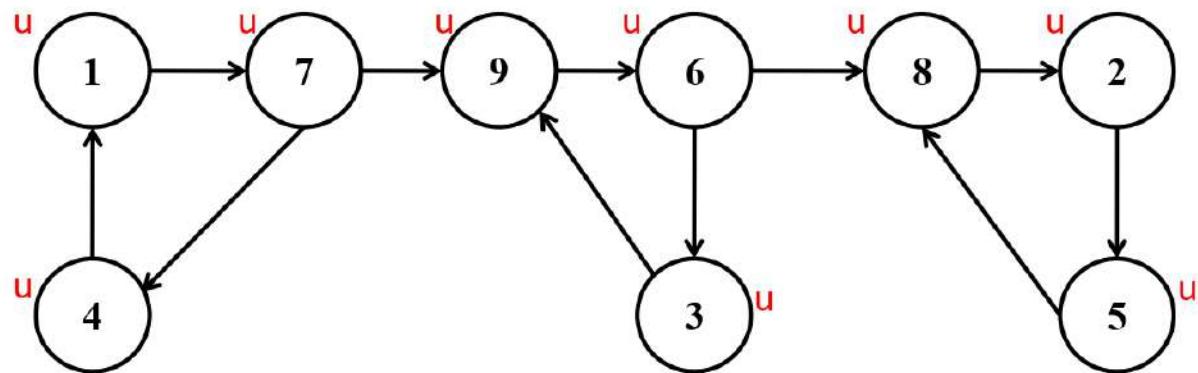


Pass 1

DFS: 1,4,7,9,3,6,8,5,2

8
5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

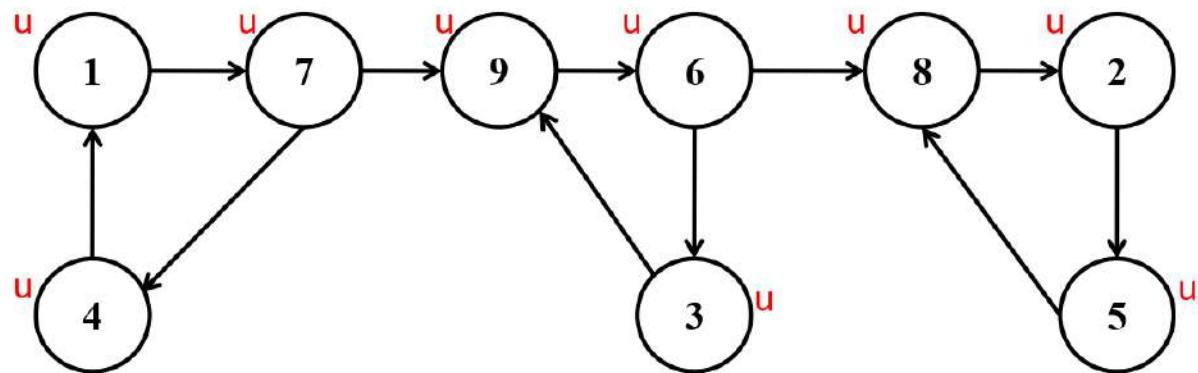


Pass 2

Reverse the graph

8
5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step

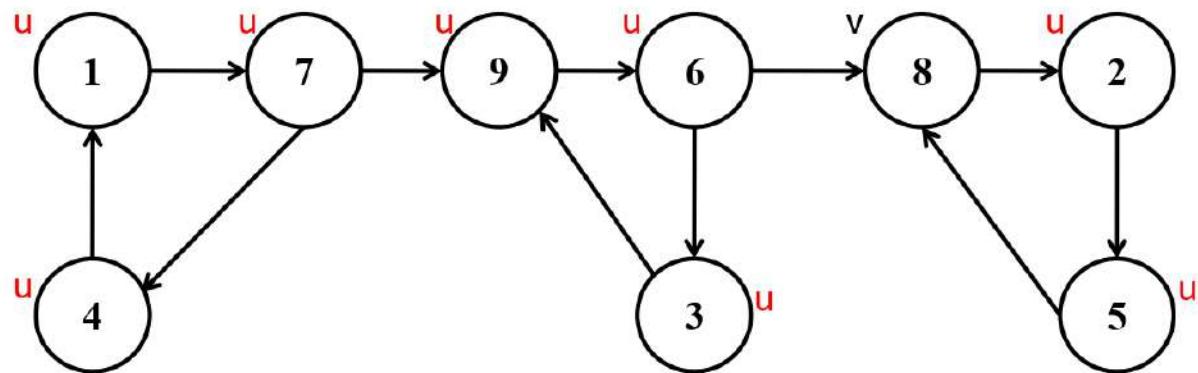


Pass 2

Connected Components:

5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step



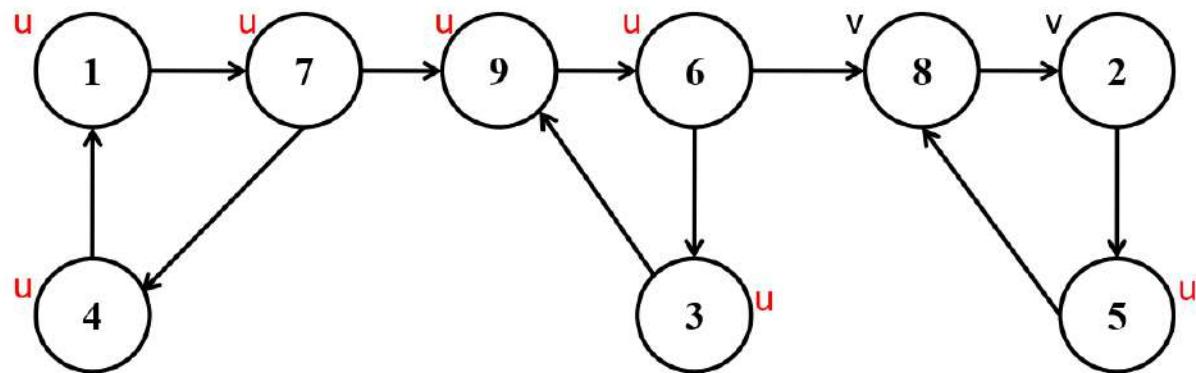
Pass 2

Connected Components:

8

5
2
9
3
6
1
4
7
s

Find strongly connected components of the digraph using the algorithm showing each step



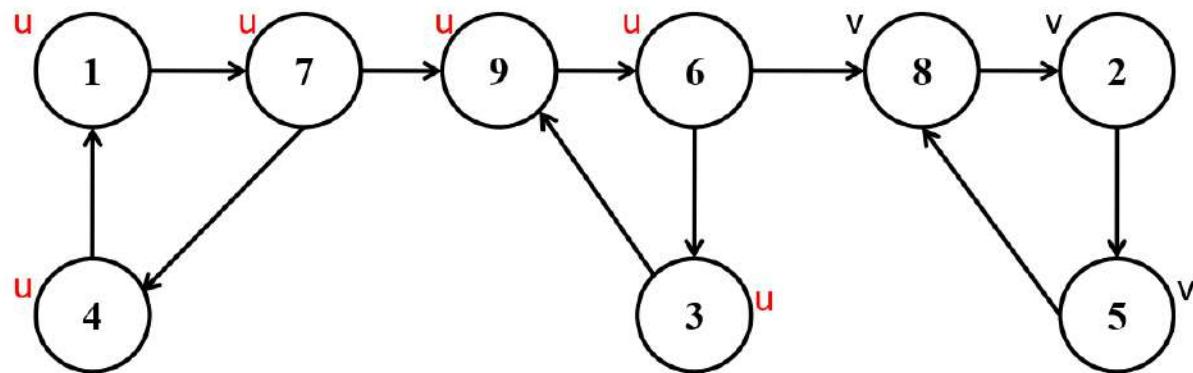
Pass 2

Connected Components:

8-2

5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step



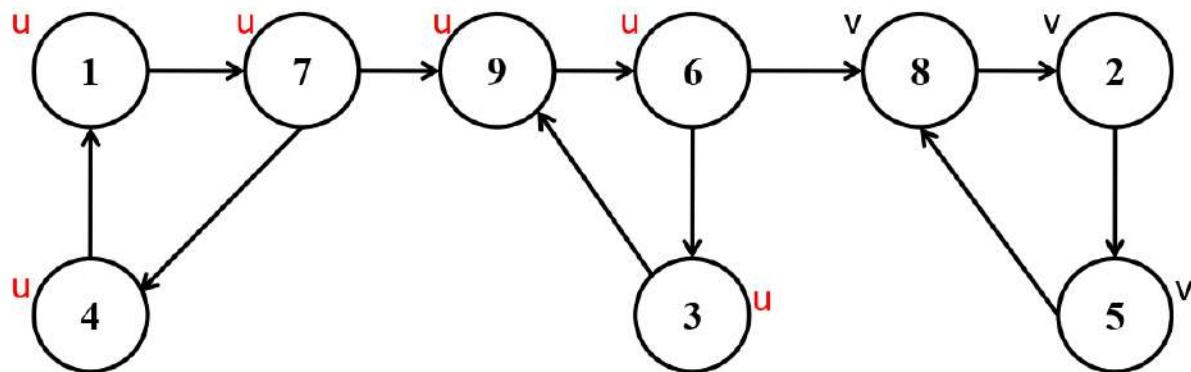
Pass 2

Connected Components:

8-2-5

5
2
9
3
6
1
4
7
S

Find strongly connected components of the digraph using the algorithm showing each step



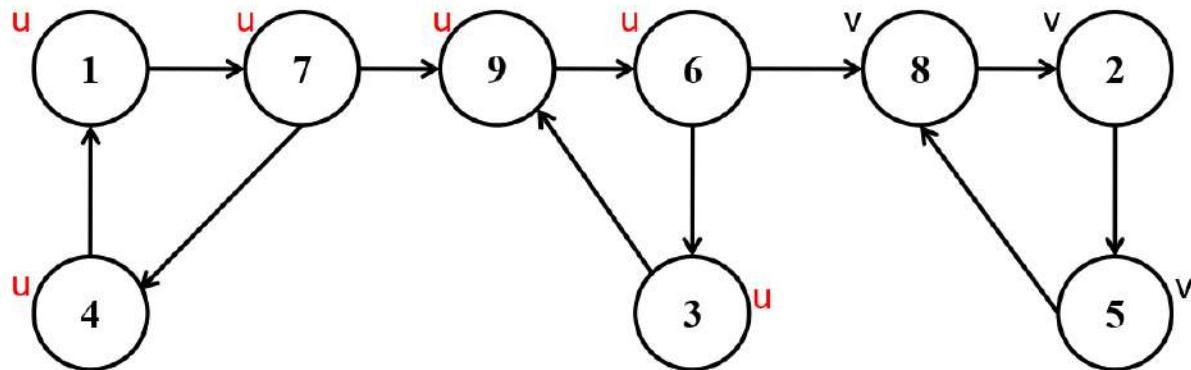
Pass 2

Connected Components:

8-2-5

2
9
3
6
1
4
7
S

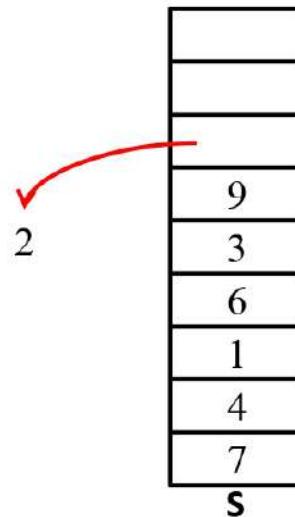
**Find strongly connected components of the digraph using the algorithm showing each step**



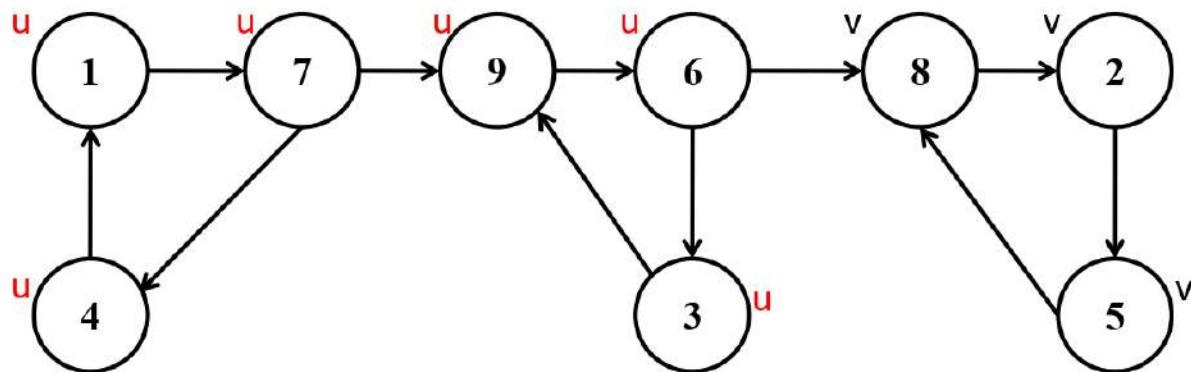
Pass 2

## Connected Components:

8-2-5



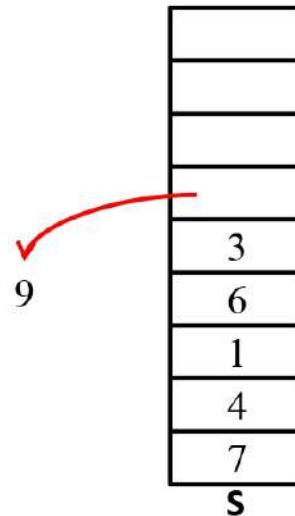
Find strongly connected components of the digraph using the algorithm showing each step



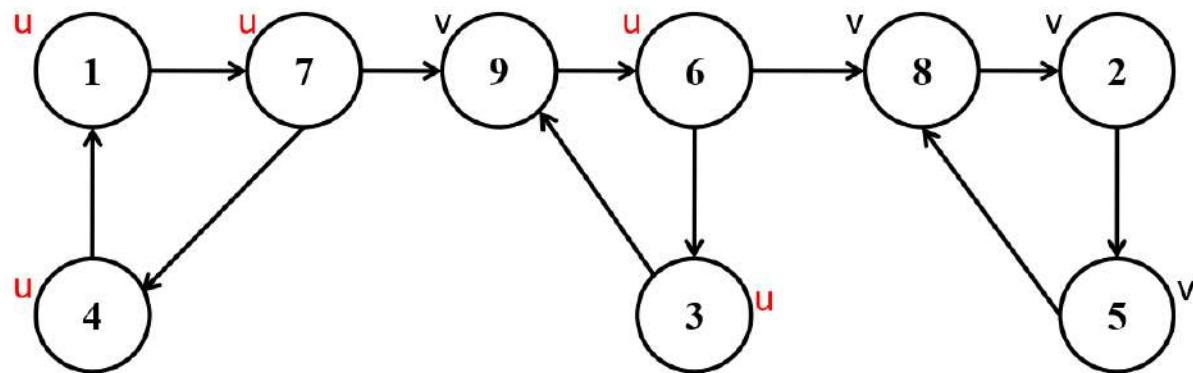
Pass 2

Connected Components:

8-2-5



Find strongly connected components of the digraph using the algorithm showing each step

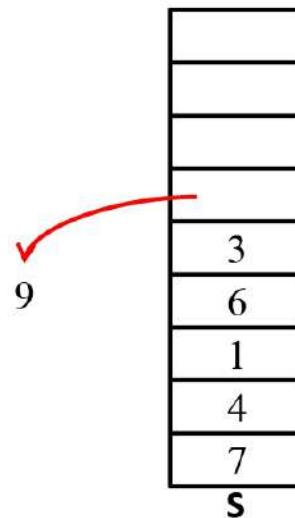


Pass 2

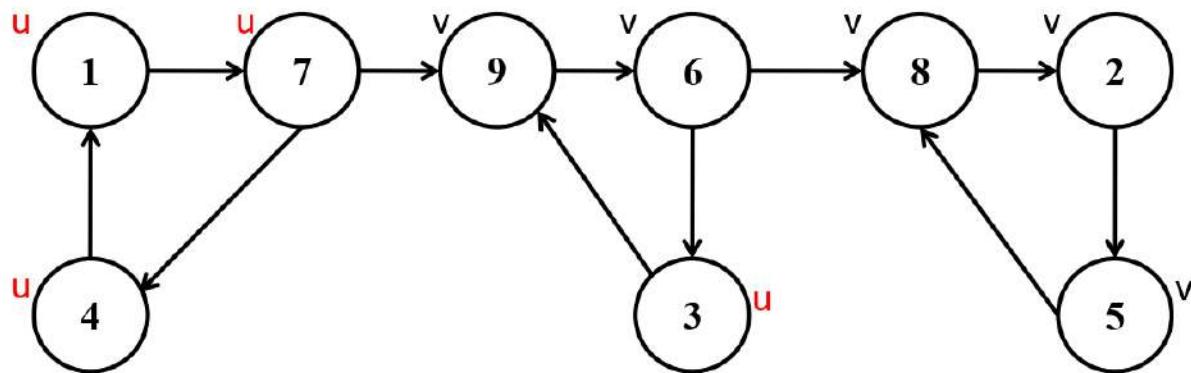
Connected Components:

8-2-5

9



Find strongly connected components of the digraph using the algorithm showing each step

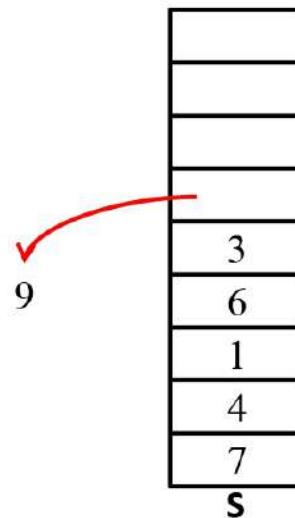


Pass 2

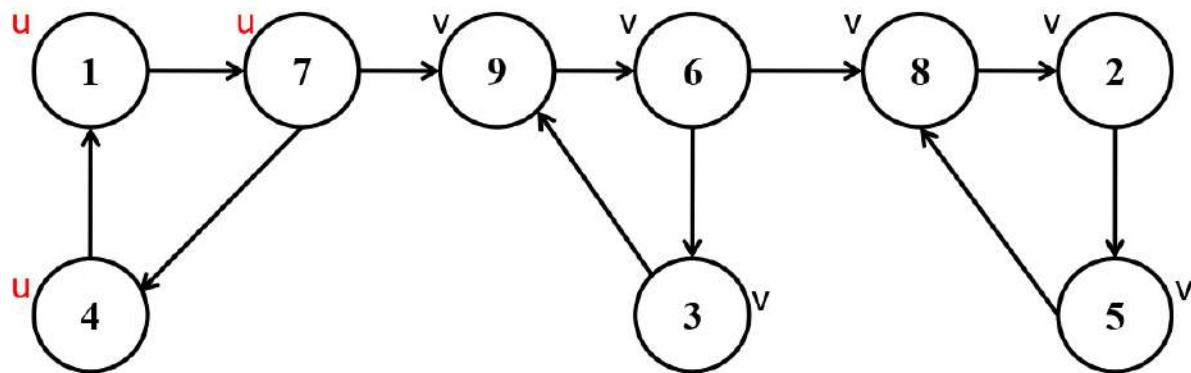
Connected Components:

8-2-5

9-6



Find strongly connected components of the digraph using the algorithm showing each step

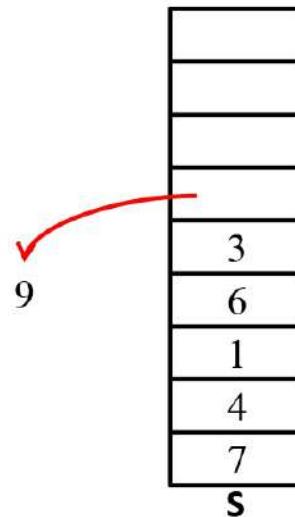


Pass 2

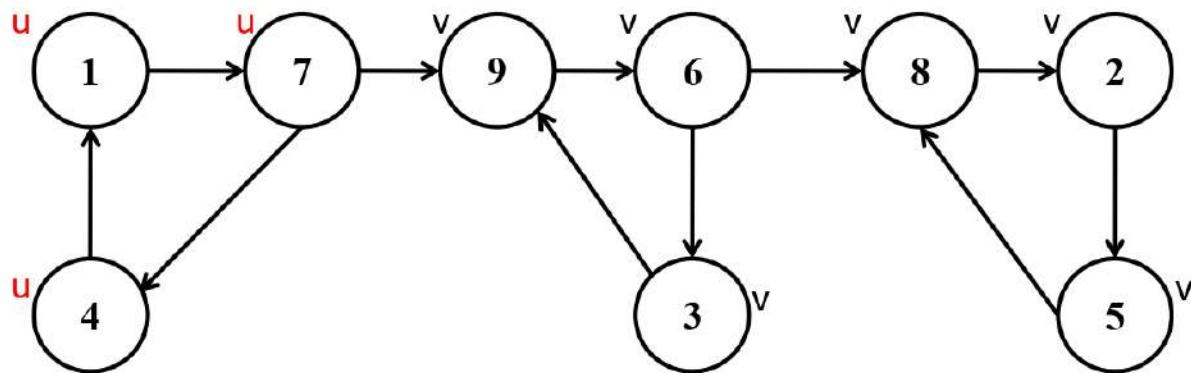
Connected Components:

8-2-5

9-6-3



Find strongly connected components of the digraph using the algorithm showing each step

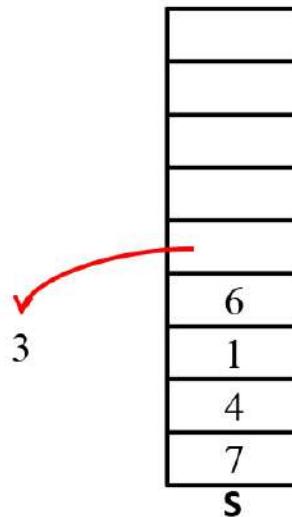


Pass 2

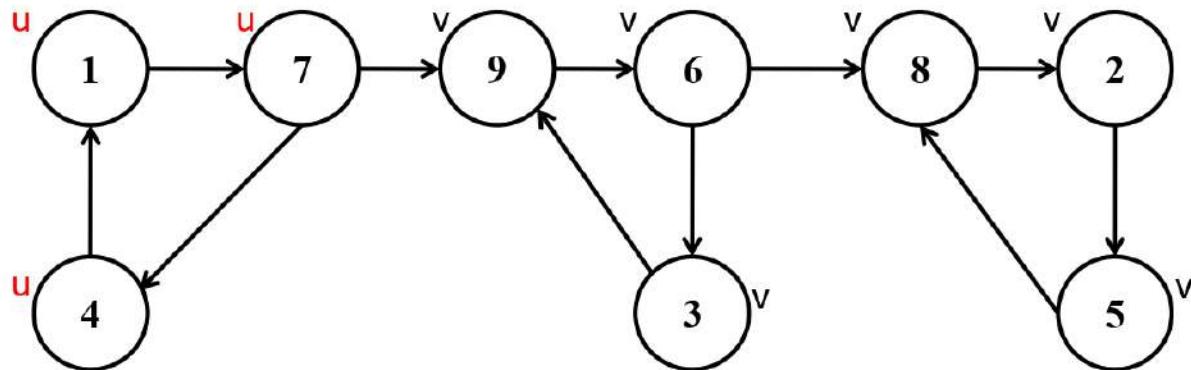
Connected Components:

8-2-5

9-6-3



**Find strongly connected components of the digraph using the algorithm showing each step**

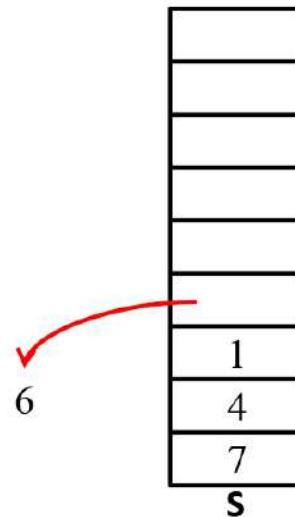


Pass 2

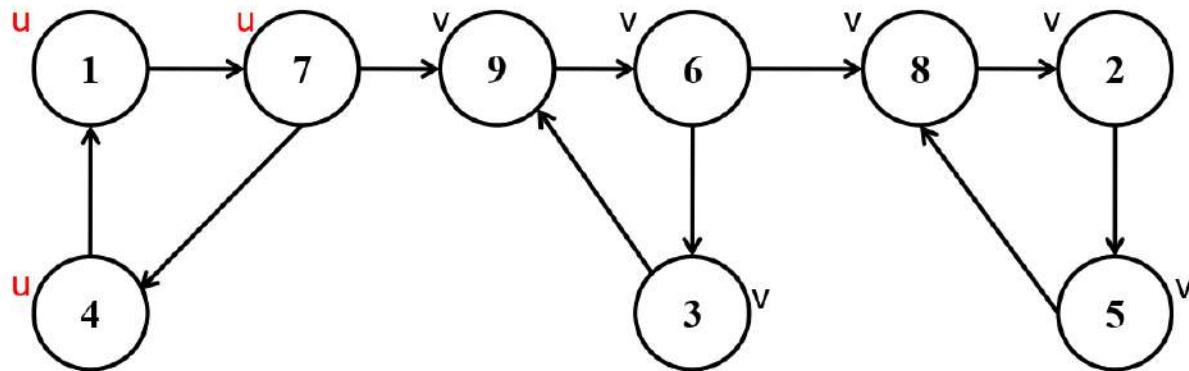
## Connected Components:

8-2-5

9-6-3



**Find strongly connected components of the digraph using the algorithm showing each step**

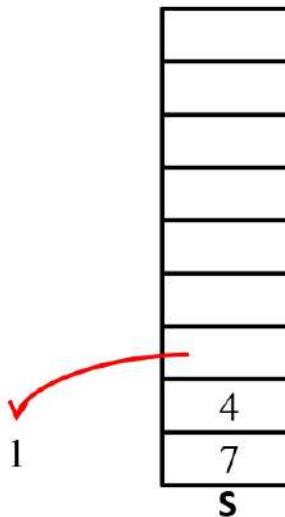


Pass 2

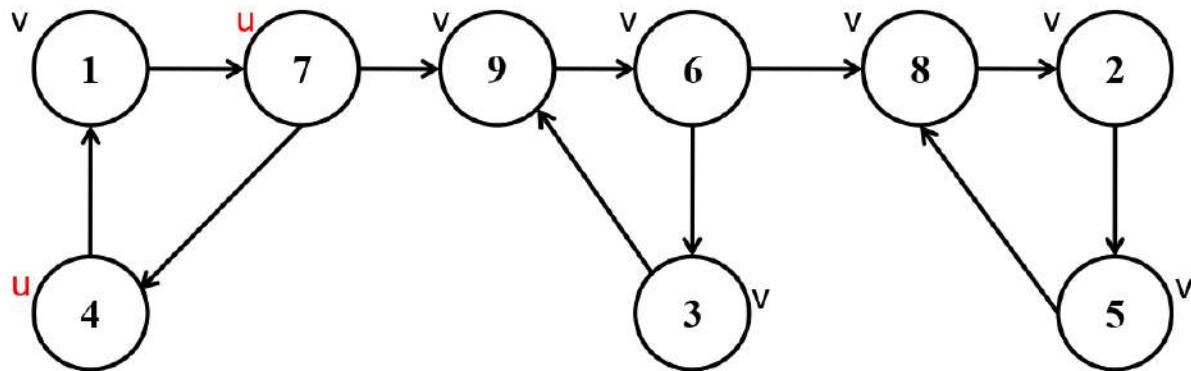
## Connected Components:

8-2-5

9-6-3



**Find strongly connected components of the digraph using the algorithm showing each step**



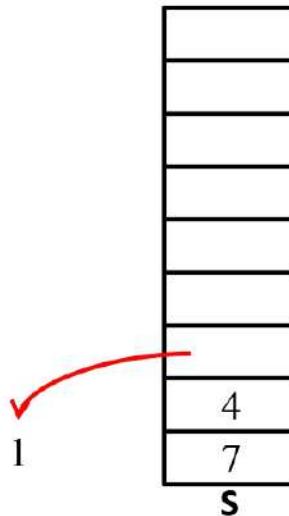
Pass 2

## Connected Components:

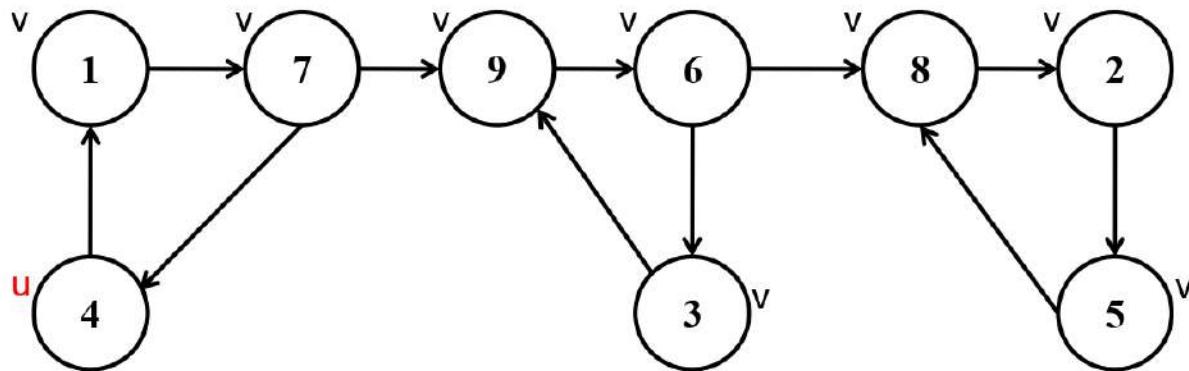
8-2-5

9-6-3

1



**Find strongly connected components of the digraph using the algorithm showing each step**



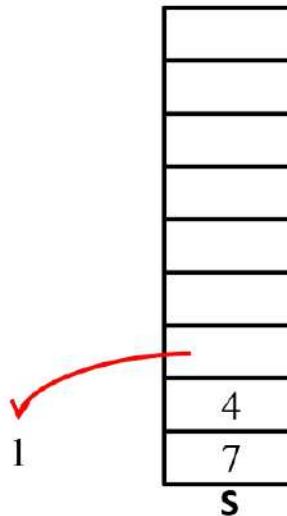
Pass 2

## Connected Components:

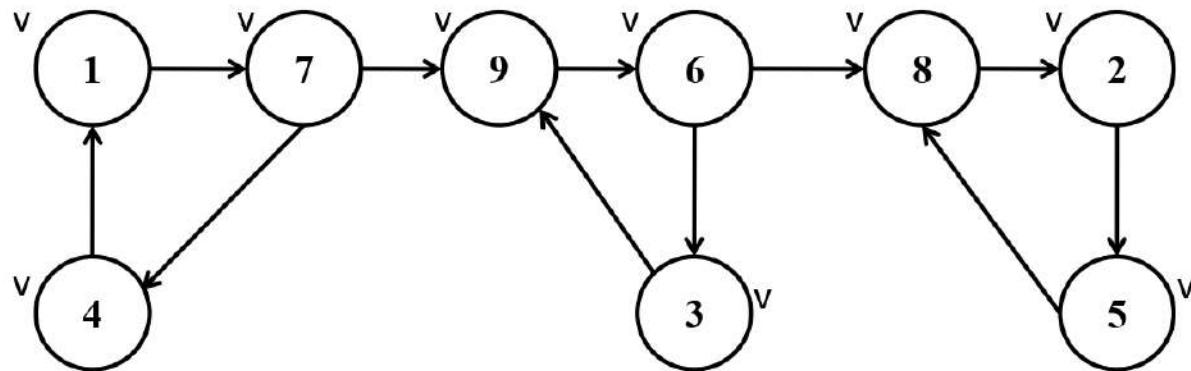
8-2-5

9-6-3

1-7



**Find strongly connected components of the digraph using the algorithm showing each step**



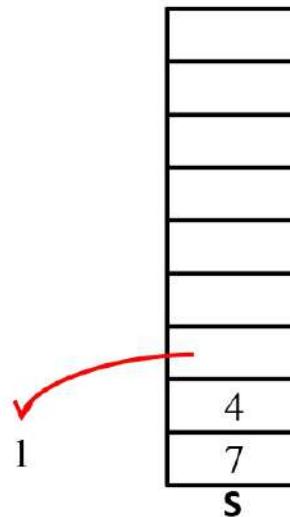
Pass 2

Connected Components:

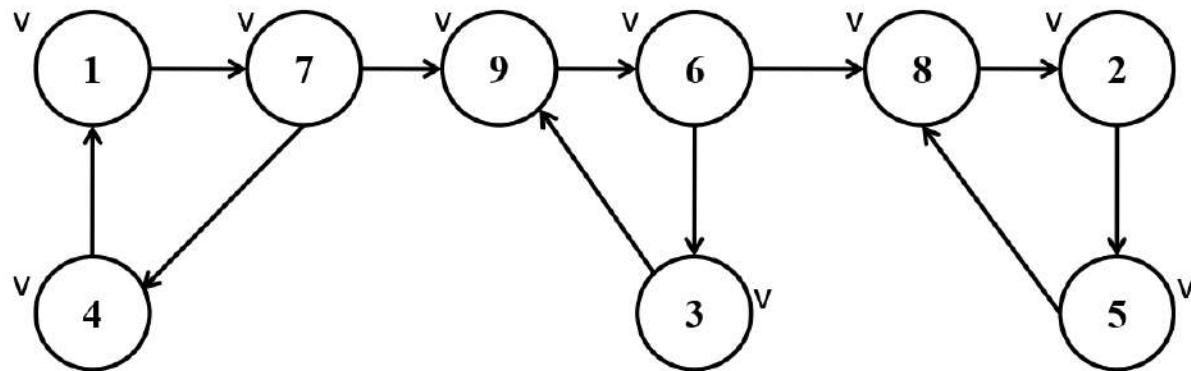
8-2-5

9-6-3

1-7-4



**Find strongly connected components of the digraph using the algorithm showing each step**



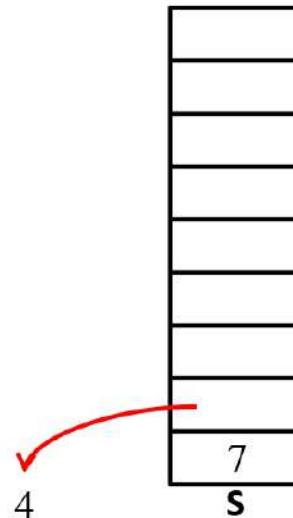
Pass 2

Connected Components:

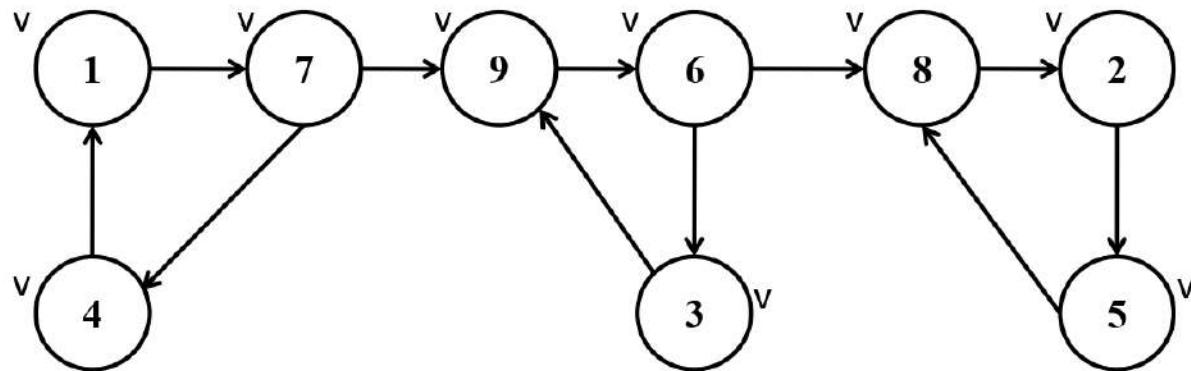
8-2-5

9-6-3

1-7-4



**Find strongly connected components of the digraph using the algorithm showing each step**



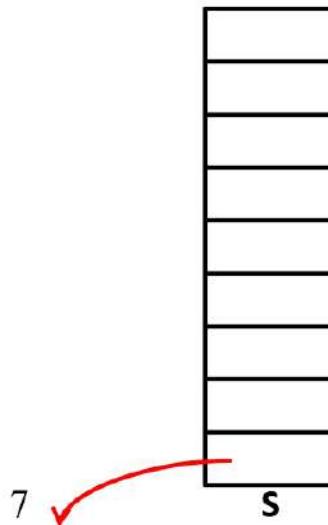
Pass 2

Connected Components:

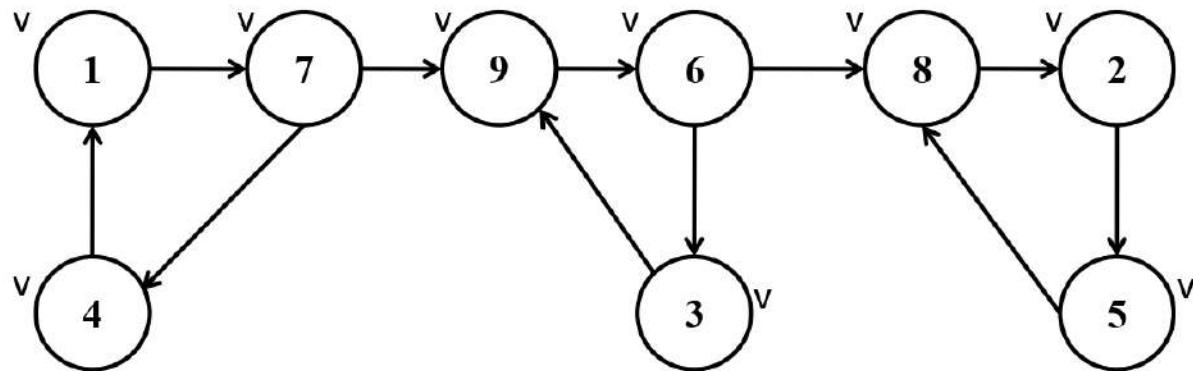
8-2-5

9-6-3

1-7-4



**Find strongly connected components of the digraph using the algorithm showing each step**



Pass 2

Connected Components:

**8-2-5**

**9-6-3**

**1-7-4**



# SCC – Time Complexity

- First pass we did DFS. So the time complexity =  $\mathbf{O(V+E)}$
- Reversal of a graph will take  $\mathbf{O(V+E)}$  time
- Pass 2 will take another  $\mathbf{O(V+E)}$  time
- Total time complexity =  $\mathbf{O(V+E)}$

# **SCC – Application**

- In social networks, a group of people are generally strongly connected (For example, students of a class or any other common place). Many people in these groups generally like some common pages or play common games. The SCC algorithms can be used to find such groups and suggest the commonly liked pages or games to the people in the group.