COMPUTER GRAPHICS AND IMAGE PROCESSING

Module 2 Part 2

CST304

HOMOGENEOUS COORDINATES

- 2 dimensional with 3 vectors
- Here we perform translation,rotation & scaling to fit the object into proper position
- Two dimensional coordinates positions(x,y) are represented by triple coordinates(x,y,1)

$$\begin{bmatrix} \ \end{bmatrix}_{m \times n} \times \begin{bmatrix} \ \end{bmatrix}_{p \times q}$$

$$n = p$$

TRANSLATION

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x + t_x$$

$$y' = y + t_y$$

ROTATION

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}}$$

SCALING

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} x' &= x \cdot S_x \\ y' &= y \cdot S_y \end{aligned}}$$

2-D COMPOSITE TRANSFORMATION

- The Process of applying several transformation in succession to form one overall transformation

- Matrix multiplication
- Order from right to left

COMPOSITE 2D TRANSLATIONS

FRANKLIN'S LECTURES

- Performing more than one translation. Translation vectors (tx_1, ty_1) and (tx_2, ty_2)

$$P' = \{T(tx_2, ty_2) \cdot T(tx_1, ty_1)\} \cdot P$$

$$\begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & tx_1 + tx_2 \\ 0 & 1 & ty_1 + ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(tx_2, ty_2) \cdot T(tx_1, ty_1) = T(tx_1 + tx_2, ty_1 + ty_2)$$

two successive translations are <u>additive</u>.

COMPOSITE 2D ROTATIONS

$$P' = \{R(\theta_2) \cdot (R(\theta_1) \cdot P\}$$

$$= \{R(\theta_2) \cdot R(\theta_1)\} \cdot P$$

$$\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1 & -(\cos\theta_2\sin\theta_1 + \sin\theta_2\cos\theta_1) & 0 \\ \sin\theta_2\cos\theta_1 + \cos\theta_2\sin\theta_1 & -\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cos(\theta_1 + \theta_2) = \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1$$

$$\begin{bmatrix} \cos(\theta_2+\theta_1) & -\sin(\theta_2+\theta_1) & 0 \\ \sin(\theta_2+\theta_1) & \cos(\theta_2+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta_2) \cdot R(\theta_1) = R(\theta_2+\theta_1)$$

Two successive rotations are additive.

$$P' = R(\theta_1+\theta_2) \cdot P$$

COMPOSITE 2D SCALING

$$\begin{bmatrix} S_{x_2} & 0 & 0 \\ 0 & S_{y_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_{x_1} & 0 & 0 \\ 0 & S_{y_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

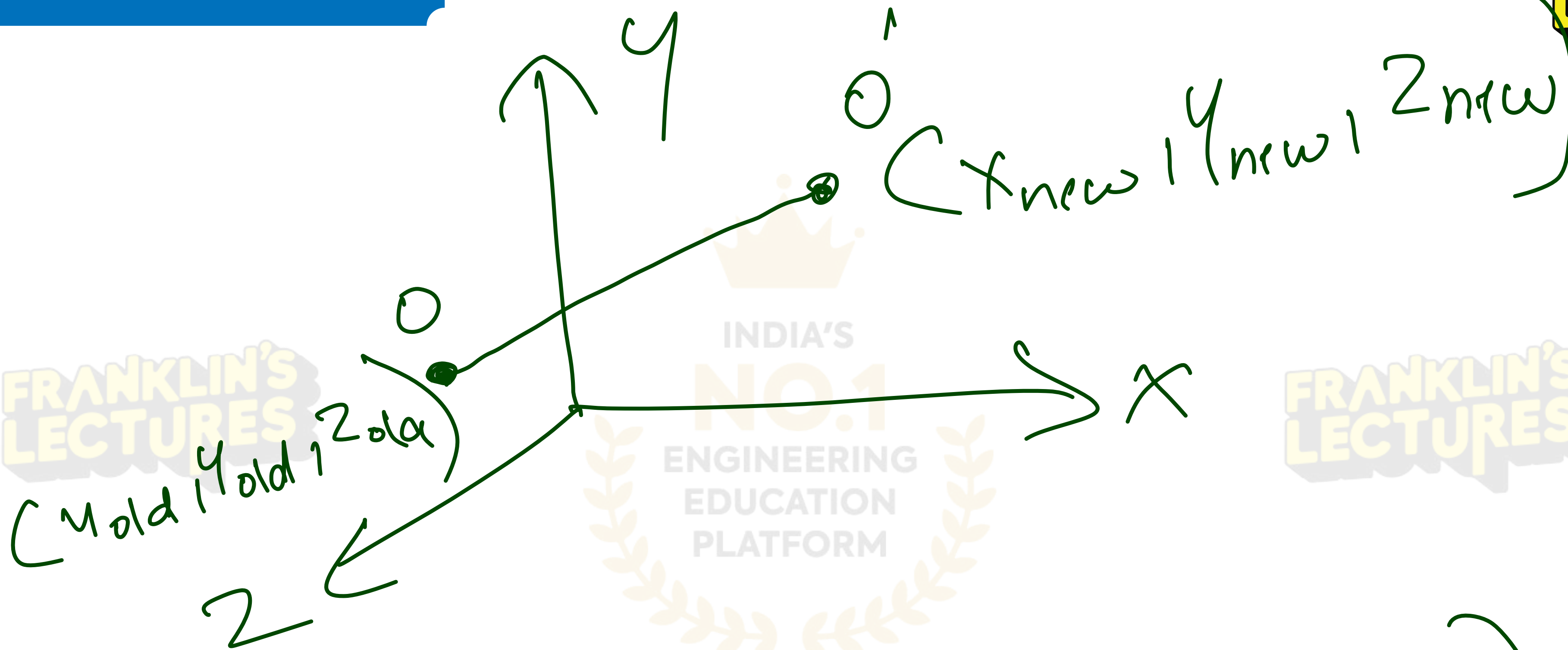
$$= \begin{bmatrix} S_{x_1} \cdot S_{x_2} & 0 & 0 \\ 0 & S_{y_2} \cdot S_{y_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here the resultant matrix as multiplicative.

3D TRANSFORMATION

- When the transformation takes place on a 3D plane.
- It includes z coordinates also

3-D TRANSLATION



Translation vectors = (T_x, T_y, T_z)

$$\boxed{\begin{aligned} x_{new} &= x_{old} + T_x \\ y_{new} &= y_{old} + T_y \\ z_{new} &= z_{old} + T_z \end{aligned}}$$

Now, Homogeneous Representation

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

Q. Given a 3D object with coordinate points A(0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0). Apply the translation with the distance 1 towards X axis, 1 towards Y axis and 2 toward Z axis & obtain the new coordinates of the object.

$T_x = 1$

$T_y = 1$

$T_z = 2$

$A(0, 3, 1)$
$\quad\;\; x \;\; y \;\; z$

$X_{new} = X_{old} + T_x$
$= 0 + 1$
$X_{new} = \underline{\underline{1}}$

$Y_{new} = Y_{old} + T_y$
$= 3 + 1$
$= \underline{\underline{4}}$

$Z_{new} = 1 + 2$
$= 3$

$A'(1, 4, 3)$

$B(3, 3, 2)$
$\;\; x \;\; y \;\; z$

$x_{new} = 3 + 1 = 4$

$y_{new} = 3 + 1 = 4$

$z_{new} = 2 + 2 = 4$

$B'(4, 4, 4)$

$C(3, 0, 0)$
$\quad\ \ x\ \ y\ \ z$

$x_{new} = 3 + 1 = 4$

$y_{new} = 0 + 1 = 1$

$z_{new} = 0 + 2 = 2$

$C'(4, 1, 2)$

$D(0, 0, \underline{\underline{0}})$

$x_{new} = 0 + 1 = 1$

$y_{new} = 0 + 1 = 1$

$z_{new} = 0 + 2 = \underline{\underline{2}}$

$D'(1, 1, 2)$

3-D SCALING

- To change the size of an object, scaling transformation is used

$$x_{new} = x_{old} \times S_x$$

$$y_{new} = y_{old} \times S_y$$

$$z_{new} = z_{old} \times S_z$$

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

Q. Given a 3D object with coordinate points A(0, 3, 3), B(3, 3, 6), C(3, 0, 1), D(0, 0, 0). Apply the scaling parameter 2 towards X axis, 3 towards Y axis and 3 towards Z axis and obtain the new coordinates of the object.

$S_x = 2$

$S_y = 3$

$S_z = 3$

$A(\underset{x}{0}, \underset{y}{3}, \underset{z}{3})$

$x_{new} = x_{old} \times S_x$

$= 0 \times 2$

$= 0$

$y_{new} = y_{old} \times S_y$

$= 3 \times 3$

$= 9$

$z_{new} = z_{old} \times S_z$

$= 3 \times 3$

$= 9$

$A'(0, 9, 9)$

$B(3,3,6)$
$x \; y \; z$

$x_{new} = 3 \times 2$
$= 6$

$y_{new} = 3 \times 3$
$= 9$

$z_{new} = 6 \times 3$
$= 18$

$C(3,0,1)$
$x \; y \; z$

$x_{new} = 3 \times 2$
$= 6$

$y_{new} = 0 \times 3$
$= 0$

$z_{new} = 1 \times 3$
$= 3$

$C'(6,0,3)$

$D(0,0,0)$

$x_{new} = 0 \times 2$
$= 0$

$y_{new} = 0$

$z_{new} = 0$

$D'(0,0,0)$

SCALING WITH RESPECT TO A SELECTED FIXED POINT(XF,YF,ZF)

- Translate the fixed point to the origin.
- Scale the object relative to the coordinate origin
- Translate the fixed point back to its original position.

$$T(x_f, y_f, z_f) \cdot S(s_x, s_y, s_z) \cdot T(-x_f, -y_f, -z_f)$$

$$T(-x_f, -y_f, -z_f) = \begin{bmatrix} 1 & 0 & 0 & -x_f \\ 0 & 1 & 0 & -y_f \\ 0 & 0 & 1 & -z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

$$T(x_f, y_f, z_f) = \begin{bmatrix} 1 & 0 & 0 & x_f \\ 0 & 1 & 0 & y_f \\ 0 & 0 & 1 & z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S \cdot T(-x_f, -y_f, -z_f) = \begin{bmatrix} S_x & 0 & 0 & -S_x \cdot x_f \\ 0 & S_y & 0 & -S_y \cdot y_f \\ 0 & 0 & S_z & -S_z \cdot z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(x_f, y_f, z_f) \cdot (S \cdot T(-x_f, -y_f, -z_f))$$

$$= \begin{bmatrix} S_x & 0 & 0 & x_f \cdot - S_x x_f \\ 0 & S_y & 0 & y_f \cdot - S_y y_f \\ 0 & 0 & S_z & z_f \cdot - S_z z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 & (1-S_x)x_f \\ 0 & S_y & 0 & (1-S_y)y_f \\ 0 & 0 & S_z & (1-S_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3-D ROTATION

- 3D Rotation is a process of rotating an object with respect to an angle in a three-dimensional plane.
- In 3 dimensions, there are 3 possible types of rotation-
- X-axis Rotation
- Y-axis Rotation
- Z-axis Rotation

X-AXIS ROTATION

$$\boxed{\begin{aligned} x_{new} &= x_{old} \\ y_{new} &= y_{old} \cdot \cos\theta - z_{old} \cdot \sin\theta \\ z_{new} &= y_{old} \cdot \sin\theta + z_{old} \cdot \cos\theta \end{aligned}}$$

$$\left.\begin{aligned} x_{new} &= x\cos\theta - y\sin\theta \\ y_{new} &= x\sin\theta + y\cos\theta \end{aligned}\right\} 2D$$

Matrix:

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{array}{cc} & \begin{array}{cccc} x & y & z & 1 \end{array} \\ \begin{array}{c} x \\ y \\ z \\ 1 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

Y-AXIS ROTATION

$$x_{new} = Z_{old} \cdot \sin\theta + X_{old} \cdot \cos\theta$$

$$y_{new} = y_{old}$$

$$Z_{new} = Z_{old} \cdot \cos\theta - X_{old} \cdot \sin\theta$$

2D

$$x_{new} = x\cos\theta - y\sin\theta$$

$$y_{new} = x\sin\theta + y\cos\theta$$

Matrix,

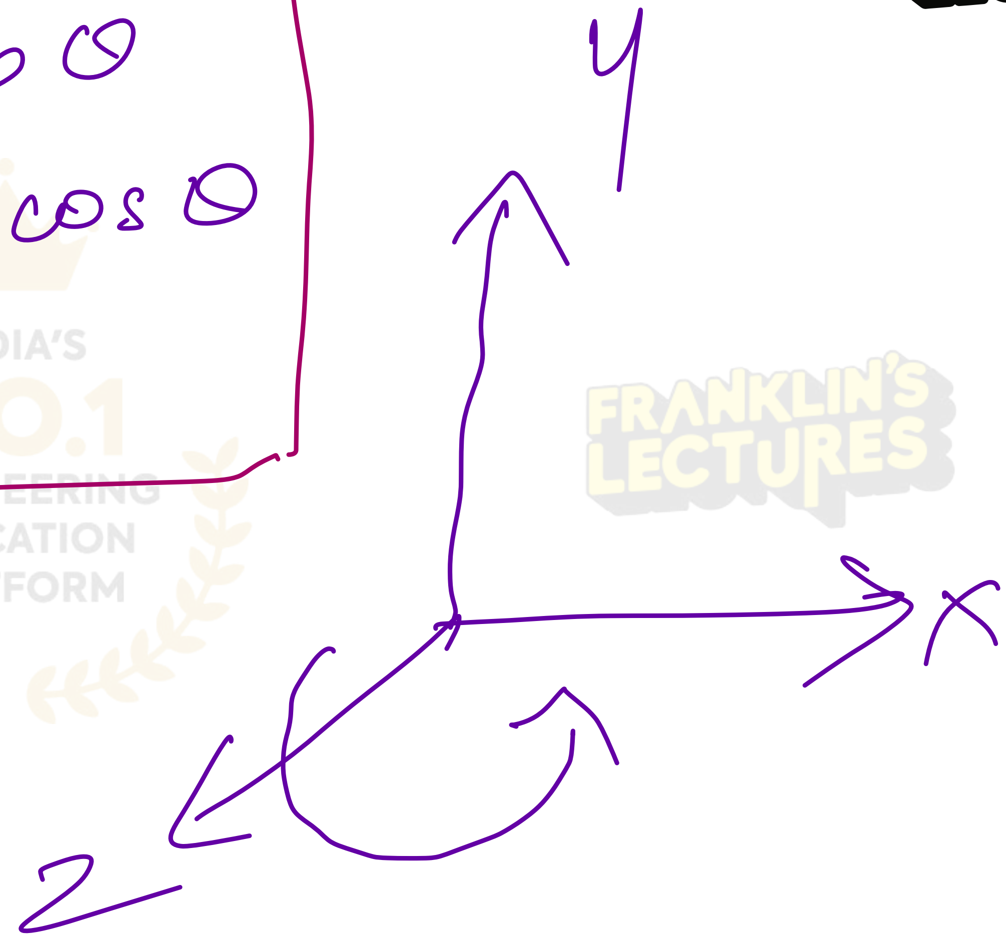
$$
\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix}
=
\begin{array}{c|c}
 & \begin{array}{cccc} x & y & z & 1 \end{array} \\
\begin{array}{c} x \\ y \\ z \\ 1 \end{array} &
\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{array}
\times
\begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}
$$

Z-AXIS ROTATION

$x_{new} = x_{old} \cdot \cos\theta - y_{old} \cdot \sin\theta$

$y_{new} = x_{old} \cdot \sin\theta + y_{old} \cdot \cos\theta$

$z_{new} = z_{old}$



2D,

$x_{new} = x\cos\theta - y\sin\theta$

$y_{new} = x\sin\theta + y\cos\theta$

Matrix Representation

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{array}{cc} & \begin{array}{cccc} x & \quad y & \quad z & \quad 1 \end{array} \\ \begin{array}{c} x \\ y \\ z \\ 1 \end{array} & \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

3-D REFLECTION

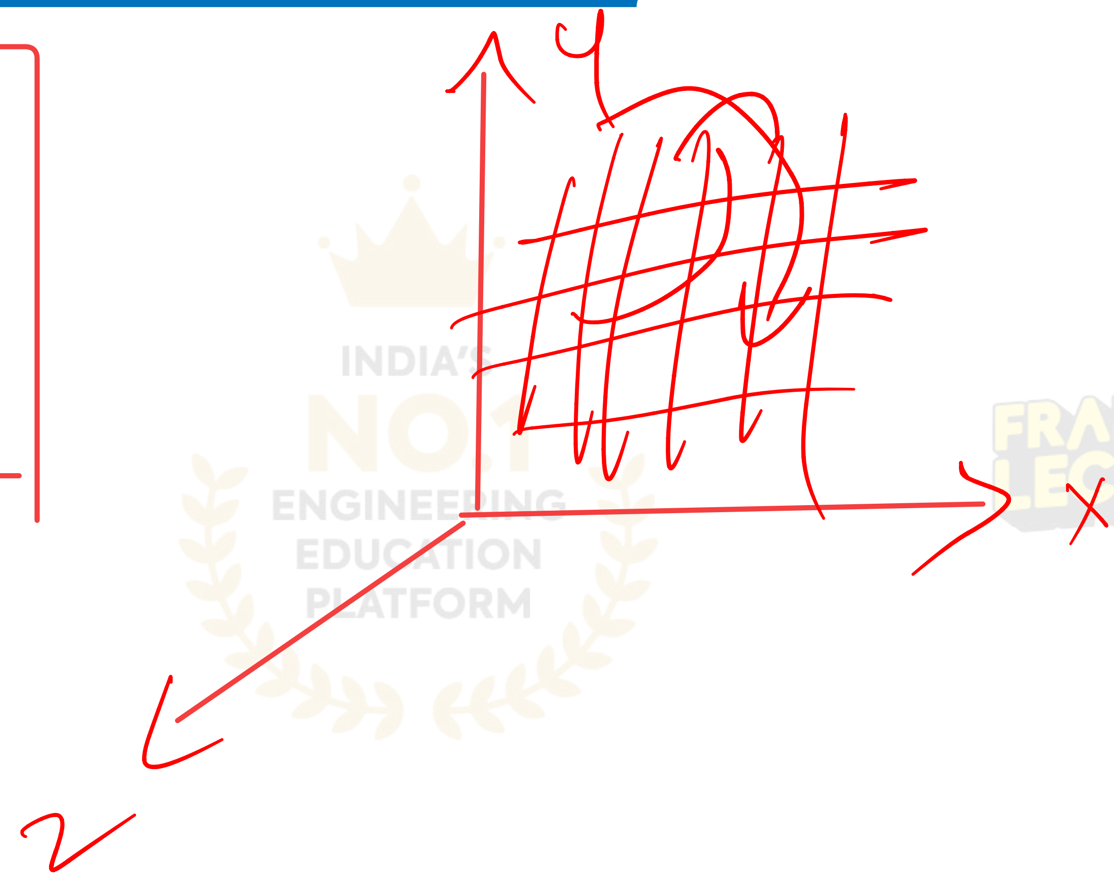
- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of the mirror.
- The size of the reflected object is the same as the original object's size.

REFLECTION RELATIVE TO XY PLANE

$x_{new} = x_{old}$

$y_{new} = y_{old}$

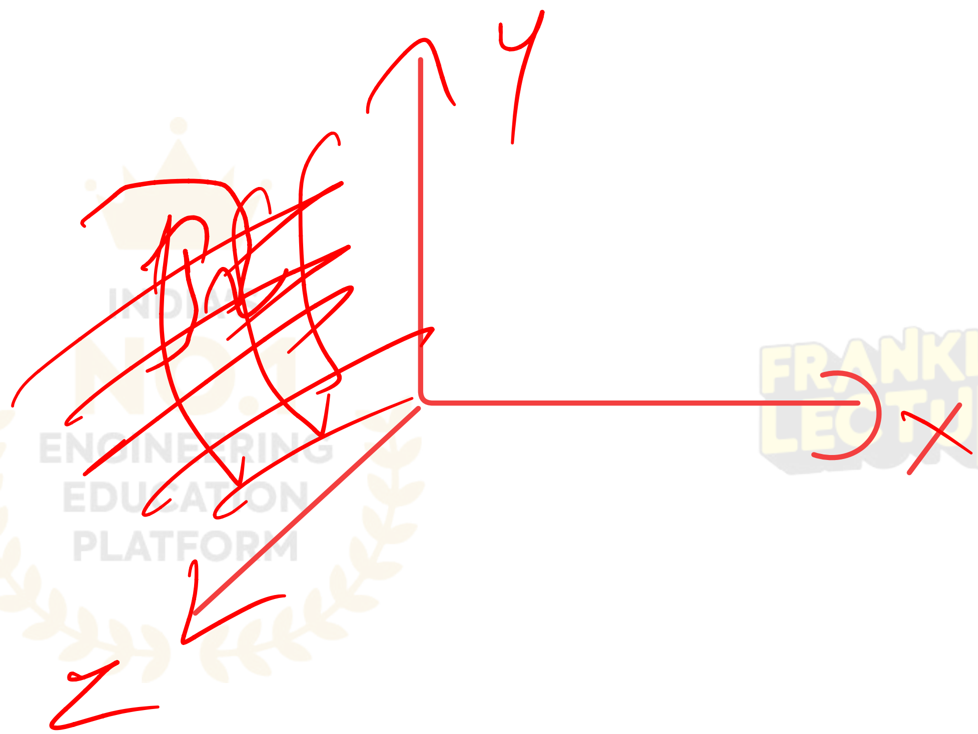
$z_{new} = -z_{old}$



$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

REFLECTION RELATIVE TO YZ PLANE

$$x_{new} = -x_{old}$$
$$y_{new} = y_{old}$$
$$z_{new} = z_{old}$$



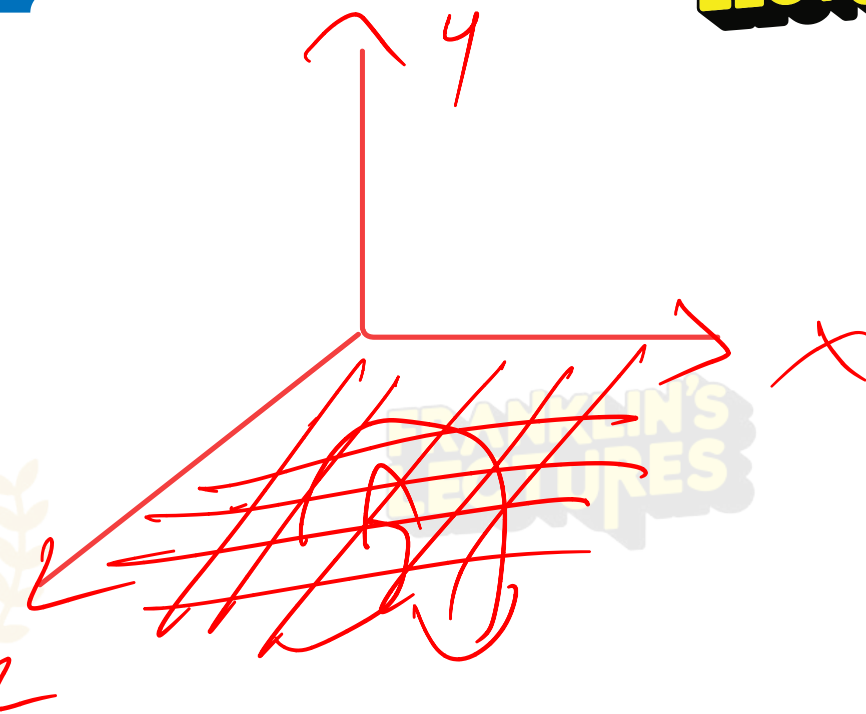
$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

REFLECTION RELATIVE TO XZ PLANE

$x_{new} = x_{old}$

$y_{new} = -y_{old}$

$z_{new} = z_{old}$



$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

3D SHEAR

- In a three dimensional plane, the object shape can be changed along the X direction, Y direction as well as Z direction.

SHEARING IN X-AXIS

$$x_{new} = x_{old}$$

$$y_{new} = y_{old} + sh_y \times x_{old}$$

$$z_{new} = z_{old} + sh_z \times x_{old}$$

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{array}{c c} & \begin{array}{cccc} x & y & z & \end{array} \\ \begin{array}{c} x \\ y \\ z \\ \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_y & 1 & 0 & 0 \\ sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

SHEARING IN Y-AXIS

$$x_{new} = x_{old} + sh_x \times y_{old}$$

$$y_{new} = y_{old}$$

$$z_{new} = z_{old} + sh_z \times y_{old}$$

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

SHEARING IN Z-AXIS

$$x_{new} = x_{old} + sh_x \times z_{old}$$

$$y_{new} = y_{old} + sh_y \times z_{old}$$

$$z_{new} = z_{old}$$

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \\ z_{old} \\ 1 \end{bmatrix}$$

Q. List the fundamental types of 3D transformations in computer graphics.

(3,APRIL 2025)

1) Translation
2) Rotation
3) Scaling
4) Reflection
5) Shearing

Q. List the steps for general pivot point rotation.

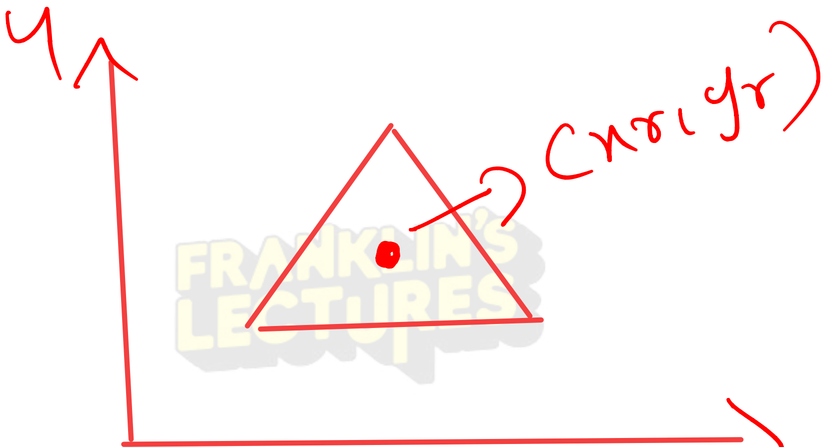
(3,MAY 2024)

Two arbitrary points (x_r, y_r) → Pivot points

Step 1 → Translate the object so that pivot point position is moved to the coordinate origin.

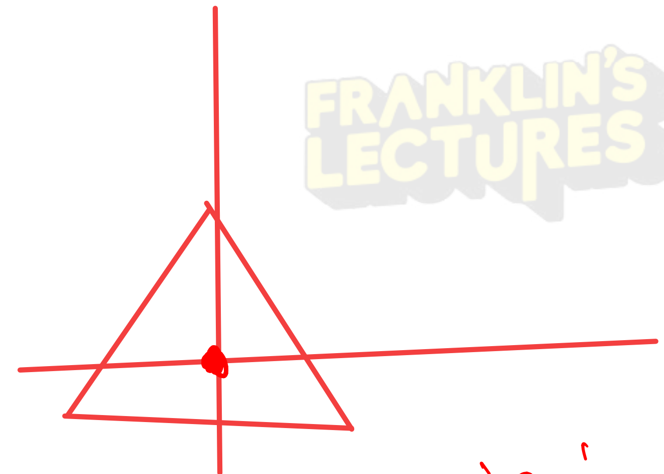
Step 2 → Rotate the object about the coordinate origin.

Step 3 → Translate the object so that pivot point is returned to original position



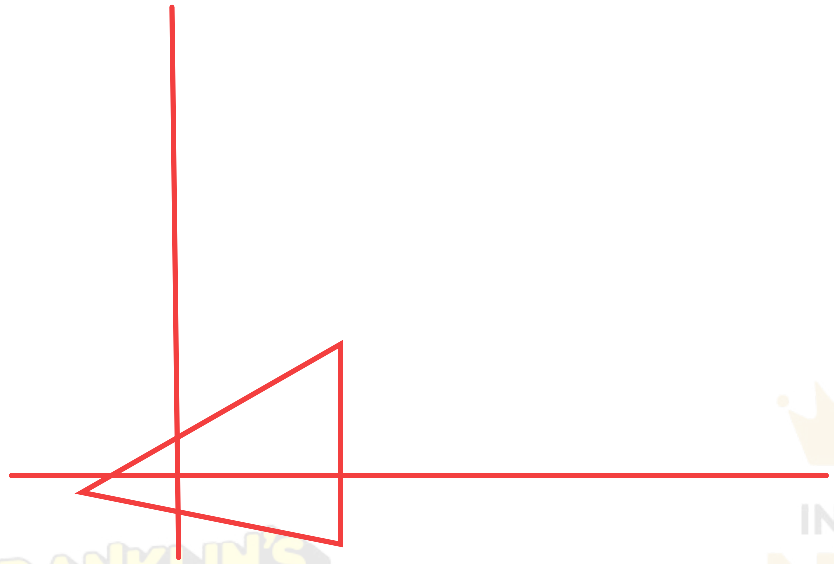
Original position

Step 1

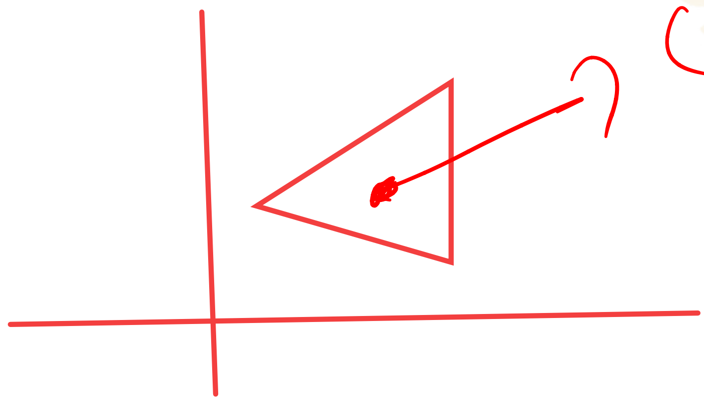


About origin

Step 2



Step 3 → Rotate the object



(x, y)

Q. Prove that the multiplication of transformation matrices for each of the following sequence of operations is commutative :

i) Any two successive translations.

ii) Any two successive scaling operations.

(6,may 2024)

(i) $T_1 = T(t_{x1}, t_{y1})$

$T_2 = T(t_{x2}, t_{y2})$

$$T_1 \cdot T_2 = \begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & (tx_1 + tx_2) \\ 0 & 1 & (ty_1 + ty_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 . T_1 = \begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & (tx_2 + tx_1) \\ 0 & 1 & (ty_2 + ty_1) \\ 0 & 0 & 1 \end{bmatrix}$$

$T_1 . T_2 = T_2 . T_1$, so all is commutative

Q. Explain three-dimensional reflection based on zy,xy & xz planes Also, give the transformation matrices.

(6,june 2022)

Already discussed

THANK YOU