FRANKLIN'S LECTURES

COMPUTER GRAPHICS & IMAGE PROCESSING

Module 2 Part 1

CST 304

SYLLABUS

Module - 2 (Filled Area Primitives and transformations)

- Filled Area Primitives- Scan line polygon filling, Boundary filling and flood filling. Two dimensional transformations-Translation, Rotation, Scaling, Reflection and Shearing, Composite transformations, Matrix representations & homogeneous coordinates. Basic 3D transformations.

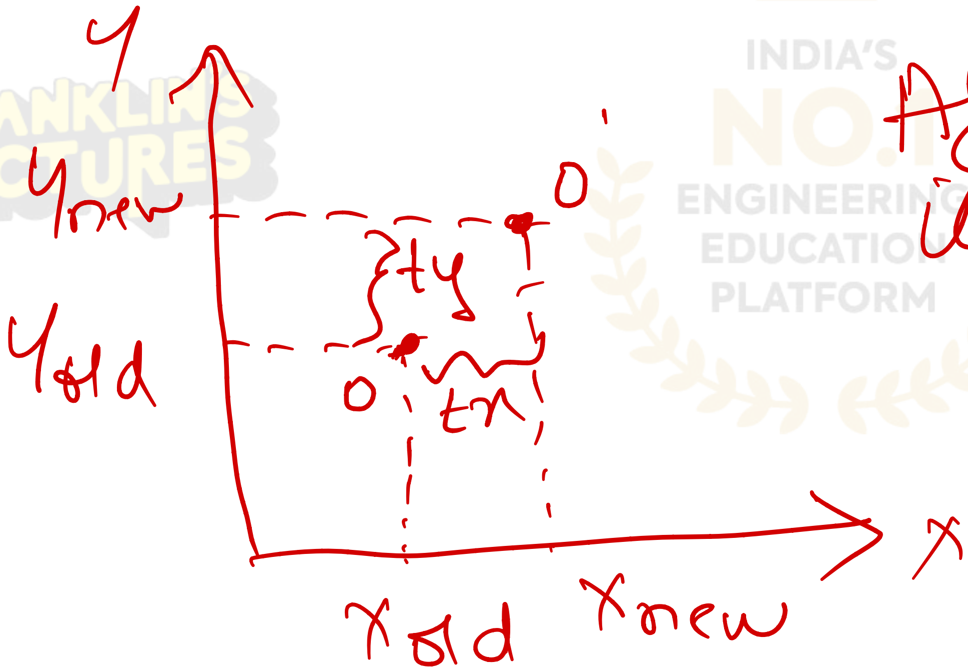
PRACTICAL APPLICATIONS

- Filled area primitives such as scan-line, boundary fill, and flood fill are used to color regions in computer graphics, paint programs, image processing, and game graphics.
- Two-dimensional transformations like translation, rotation, scaling, reflection, and shearing are applied to move, resize, rotate, flip, and distort objects in animations, GUI design, and CAD systems.
- Composite transformations allow multiple transformations to be combined and are widely used in complex animations, object modeling, and robotic motion.

BASIC 2D TRANSFORMATIONS

TRANSLATION

To transfer the object from one place to another



After translation it become = (x_{new}, y_{new})

$$\boxed{\begin{aligned} x_{new} &= x_{old} + t_x \\ y_{new} &= y_{old} + t_y \end{aligned}}$$

Matrix form :-

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

In General ;

$$\boxed{P' = P + T}$$

Q. Translate the given point (2,5) by translation vector (3,3).

$(x_{old}, y_{old}) = (\overset{x}{2}, \overset{y}{5})$

$(T_x, T_y) = (3,3)$

$x_{new} = x_{old} + t_x$

$= 2 + 3 = 5$

$y_{new} = y_{old} + t_y$

$= 5 + 3 = 8$

Translated point $= (5,8)$

Q. Translate a polygon with coordinates A(2,5), B(7,10) and C(10,2) by 3 units in x direction and 4 unit in y direction.

$A(2,5)$

$T_x = 3$

$T_y = 4$

$x_{new} = x_{old} + t_x$

$= 2+3$

$= 5$

$y_{new} = y_{old} + t_y$

$= 5+4$

$= 9$

$A(5,9)$

$B(7,10)$

$T_x = 3$

$T_y = 4$

$x_{new} = 7+3$

$= 10$

$y_{new} = 10+4$

$= 14$

$B(10,14)$

$C(10,2)$

$T_x = 3$

$T_y = 4$

$x_{new} = 10+3$

$= 13$

$y_{new} = 2+4$

$= 6$

$C(13,6)$

Q. Given a circle with radius 10 and centre coordinates (1,4). Apply the translation 5 towards x- axis and 1 towards y-axis. Obtain the loop coordinates without changing the radius.

$(x_{old}, y_{old}) = (1,4)$

$(T_x, T_y) = (5,1)$

$x_{new} = x_{old} + t_x$

$= 1 + 5 = 6$

$y_{new} = y_{old} + t_y$

$= 4 + 1 = 5$

New position $(6,5)$

SCALING

- To change the size of an object, scaling transformation is used

- It is used to increase or reduce the size of an object.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, object size is reduced.
- Sx and Sy

Initial coordinates of the object (x_{old}, y_{old})

Scaling factor for x-axis $= S_x$

Scaling factor for y-axis $= S_y$

New coordinates of the object after scaling $= (x_{new}, y_{new})$

$$x_{new} = x_{old} \times S_x$$

$$y_{new} = y_{old} \times S_y$$

In matrix form,

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \times \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$$

Q. Given a square with coordinate points A (0,3), B(3,3) C(3,0), D (0,0). Apply the scale parameter 2 towards x-axis and 3 towards y-axis and obtain the nexw coordinates.

A (0, 3)

$(S_x, S_y) = (2, 3)$

$x_{new} = x_{old} \times S_x$

$= 0 \times 2$

$= 0$

$y_{new} = y_{old} \times S_y$

$= 3 \times 3$

$= 9$

A' (0, 9)

B (3, 3)

$(S_x, S_y) = (2, 3)$

$x_{new} = 3 \times 2$

$= 6$

$y_{new} = 3 \times 3$

$= 9$

B' (6, 9)

C (3, 0)

$(S_x, S_y) = (2, 3)$

$x_{new} = 3 \times 2$

$= 6$

$y_{new} = 0 \times 3$

$= 0$

C' (6, 0)

D (0, 0)

$(S_x, S_y) = (2, 3)$

$x_{new} = 0 \times 2$

$= 0$

$y_{new} = 0 \times 3$

$= 0$

D' (0, 0)

Q. Find the new coordinate of the triangle (0,0), (1,1), (5,2) after it has been magnified to twice its size.

A (0,0)

B (1,1)

C (5,2)

Scaling factor = 2

A (0,0)

$s = 2$

$x_{new} = x_{old} \times s$

$= 0 \times 2$

$= 0$

$y_{new} = 0 \times 2$

$= 0$

$A'(0,0)$

B (1,1)

$s = 2$

$x_{new} = 1 \times 2$

$= 2$

$y_{new} = 1 \times 2$

$= 2$

$B'(2,2)$

C (5,2)

$s = 2$

$x_{new} = 5 \times 2$

$= 10$

$y_{new} = 2 \times 2$

$= 4$

$C' = (10,4)$

Q. A Square object with the coordinate points P (1, 4), Q (4, 4), R (4, 1), T (1,1). Apply the scaling factor 3 on the X-axis and 4 on the Y-axis. Find out the new coordinates of the square?

$X \cdot S_x$

$P' = (3, 16)$

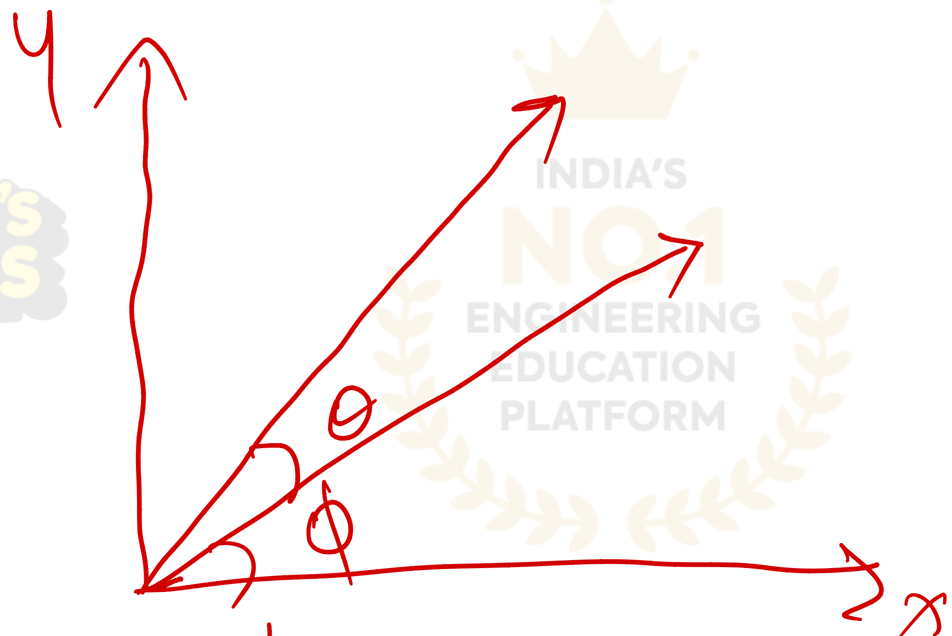
$Q' = (12, 16)$

$R' = (12, 4)$

$T' = (3, 4)$

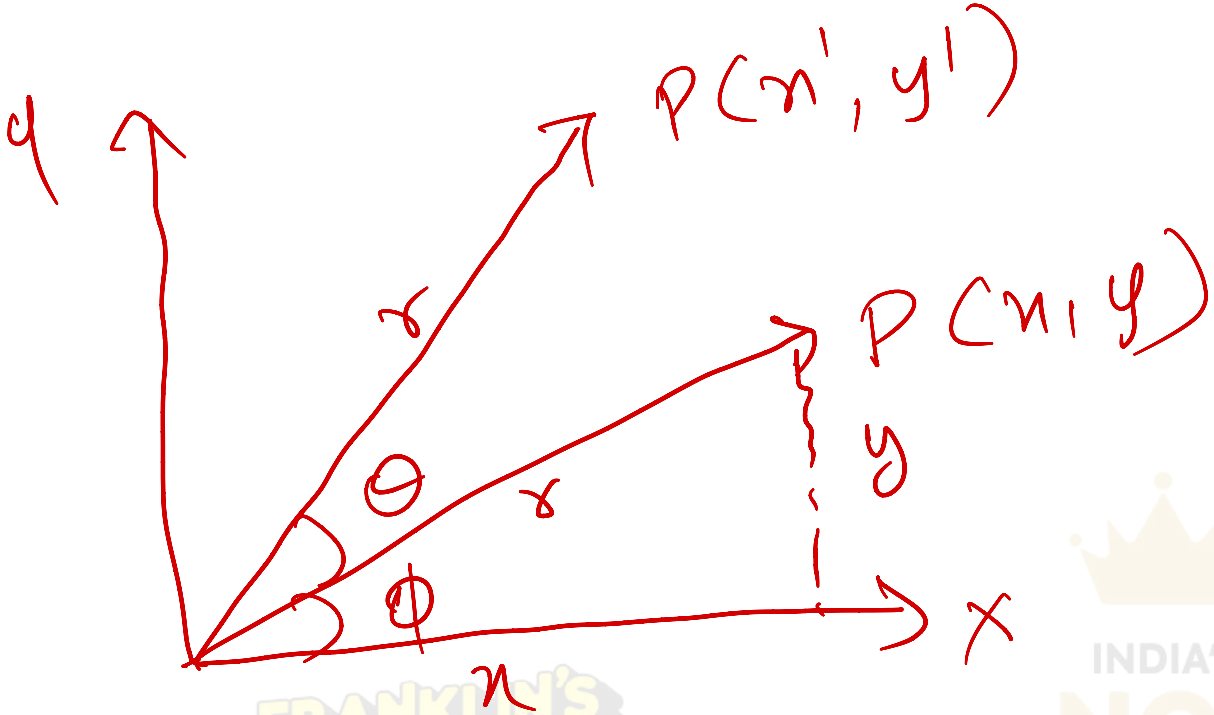
ROTATION

- Repositioning along a circular path



↳ total change $(\phi + \theta)$

- $\theta > 0 \rightarrow$ Rotate anti-clockwise (θ is +ve)
- $\theta < 0 \rightarrow$ Rotate clockwise (θ is -ve)



$$x = r\cos\phi$$

$$y = r\sin\phi$$

$$\cos\phi = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\cos\phi = \frac{x}{r}$$

$$\sin\phi = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin\phi = \frac{y}{r}$$

$$x' = r\cos(\phi+\theta)$$

$$= r\cos\phi\cos\theta - \underbrace{r\sin\phi}\sin\theta$$

$$y' = r\sin(\phi+\theta)$$

$$= \underbrace{r\cos\phi}\sin\theta + r\sin\phi\cos\theta$$

$$\boxed{\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}}$$

$$\boxed{\begin{aligned} \cos(\phi+\theta) &= \cos\phi\cos\theta - \sin\phi\sin\theta \\ \sin(\phi+\theta) &= \cos\phi\sin\theta + \sin\phi\cos\theta \end{aligned}}$$

Matrix Representation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Q. Find the transformed point, P′, caused by rotating P = (5, 1) about the origin through an angle of 90°.

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$P(\overset{x}{5}, \overset{y}{1})$

$\theta = 90^\circ$

$x' = 5\cos 90^\circ - 1\sin 90^\circ$

$= 5 \times 0 - 1 \times 1$

$= \underline{\underline{-1}}$

$y' = 5 \times \sin 90^\circ + 1 \times \cos 90^\circ$

$= 5 \times 1 + 1 \times 0$

$= \underline{\underline{5}}$

New coordinates $(-1, 5)$

Q. Rotate the triangle ABC having coordinates A(1, 2) B(2, 3) and C(4,5) by 60 deg about the origin.

$A(\overset{x}{1}, \overset{y}{2})$, $\theta = 60^\circ$

$x' = x\cos\theta - y\sin\theta$

$= 1 \times \cos 60 - 2 \times \sin 60^\circ$

$= 1 \times \frac{1}{2} - 2 \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} - \frac{2\sqrt{3}}{2}$

$= \frac{1 - 2\sqrt{3}}{2}$

$y' = x\sin\theta + y\cos\theta$

$= 1 \times \sin 60^\circ + 2 \times \cos 60^\circ$

$= 1 \times \frac{\sqrt{3}}{2} + \cancel{2} \times \frac{1}{\cancel{2}}$

$= \frac{\sqrt{3}}{2} + 1$

$= \frac{\sqrt{3}+2}{2}$

$A'\left(\frac{1-2\sqrt{3}}{2}, \frac{\sqrt{3}+2}{2}\right)$

$B(\underset{x}{2}, \underset{y}{3})$

$x' = x\cos\theta - y\sin\theta$

$= 2 \times \cos 60^\circ - 3 \times \sin 60^\circ$

$= \cancel{2} \times \frac{1}{\cancel{2}} - 3 \times \frac{\sqrt{3}}{2}$

$= 1 - \frac{3\sqrt{3}}{2}$

$= \frac{2 - 3\sqrt{3}}{2}$

$y' = x\sin\theta + y\cos\theta$

$= 2 \times \sin 60^\circ + 3 \times \cos 60^\circ$

$= \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} + 3 \times \frac{1}{2}$

$= \sqrt{3} + \frac{3}{2}$

$= \frac{2\sqrt{3} + 3}{2}$

New coordinates $= \left(\frac{2-3\sqrt{3}}{2}, \frac{2\sqrt{3}+3}{2}\right)$

$C(4, 5)$
$\quad\ \ x \quad y$

$x' = x\cos\theta - y\sin\theta$
$= 4 \times \cos 60 - 5 \times \sin 60$
$= \cancel{4}^{2} \times \dfrac{1}{\cancel{2}} - 5 \times \dfrac{\sqrt{3}}{2}$
$= 2 - \dfrac{5\sqrt{3}}{2}$
$= \dfrac{4 - 5\sqrt{3}}{2}$

$y' = x\sin\theta + y\cos\theta$
$= 4 \times \sin 60^\circ + 5 \times \cos 60^\circ$
$= \cancel{4}^{2} \times \dfrac{\sqrt{3}}{\cancel{2}} + 5 \times \dfrac{1}{2}$
$= 2\sqrt{3} + \dfrac{5}{2}$
$= \dfrac{4\sqrt{3} + 5}{2}$

$C'\left(\dfrac{4 - 5\sqrt{3}}{2}, \dfrac{4\sqrt{3} + 5}{2}\right)$

Q. A point (4,3) is rotated clockwise by an angle -45 degree. Find the rotation matrix and the resultant point.

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\theta = -45^\circ$$

$$R(-45^\circ) = \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) \\ \sin(-45^\circ) & \cos(-45^\circ) \end{bmatrix}$$

$$\boxed{\begin{aligned} \cos(-\theta) &= \cos\theta \\ \sin(-\theta) &= -\sin\theta \end{aligned}}$$

$$R = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$x' = \left(\frac{1}{\sqrt{2}} \times 4\right) + \left(\frac{1}{\sqrt{2}} \times 3\right)$$

$$= \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

$$y' = \left(-\frac{1}{\sqrt{2}} \times 4\right) + \left(\frac{1}{\sqrt{2}} \times 3\right)$$

$$= -\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

Resultant point $\left(\frac{7}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Q. Rotate the triangle with vertices A(5, 8) B(12, 10) and C(10,10) about origin with angle 90 degree in anticlockwise direction.

$(x, y) \Rightarrow (-y, x)$

$A(5, 8) \rightarrow (-y, x)$

$= (-8, 5)$

$B(12, 10) \rightarrow (-10, 12)$

$C(10, 10) \rightarrow (-10, 10)$

$x' = x\cos 90^\circ - y\sin 90^\circ$

$= x \times 0 - y \times 1$

$x' = -y$

$y' = x\sin 90^\circ + y\cos 90^\circ$

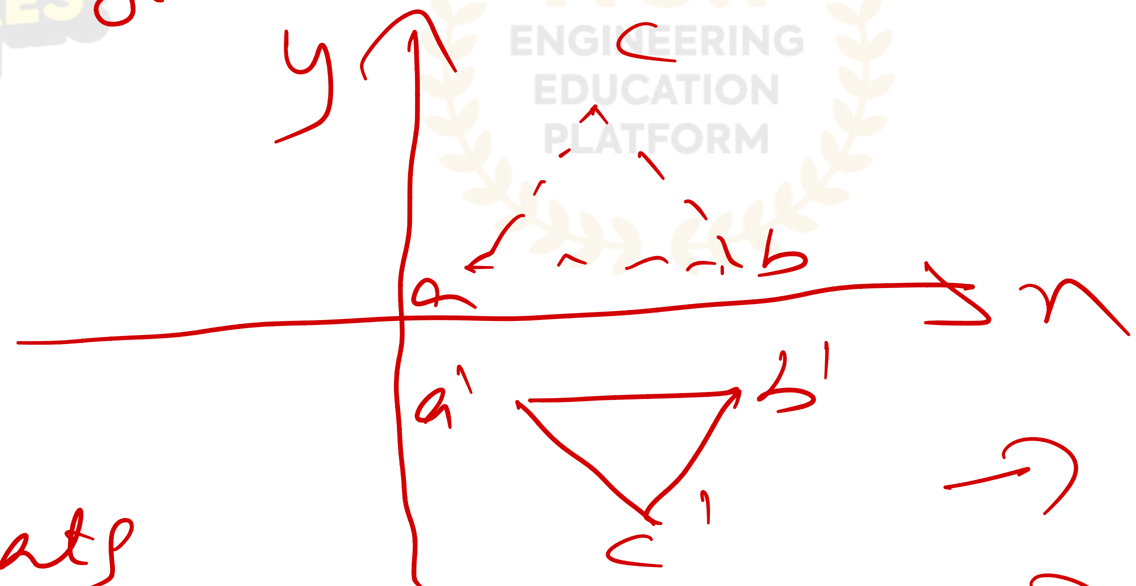
$= x \times 1 + y \times 0$

$y' = x$

REFLECTION

- Reflection is the mirror image of original object. In other words, we can say that it is a rotation operation with 180°. In reflection transformation, the size of the object does not change.

Reflection about x-axis



→ magnitude doesn't change

→ position ✗

→ y-coordinate will change

$$\begin{array}{c|ccc} & x & y & 1 \\ \hline x & & & \\ y & & \left[\begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}\right] & \\ 1 & & & \end{array}$$

$$\downarrow \quad x' \quad y' \quad 1$$

$$\begin{bmatrix} x' \\ y' \\ 1' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

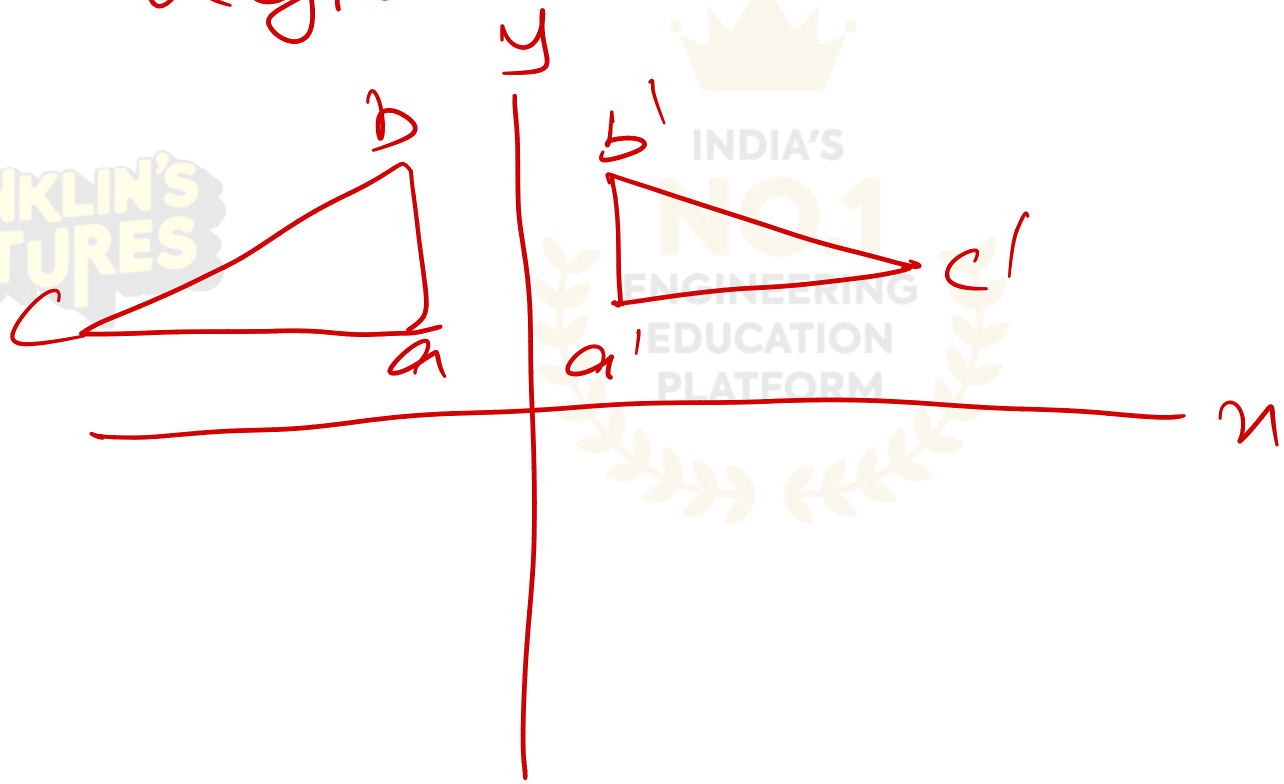
$$x' = 1 \times x + 0 \times y + 0 \times 1$$

$$\boxed{x' = x}$$

$y' = 0 \times x + (-1) \times y + 0 \times 1$

$$\boxed{y' = -y}$$

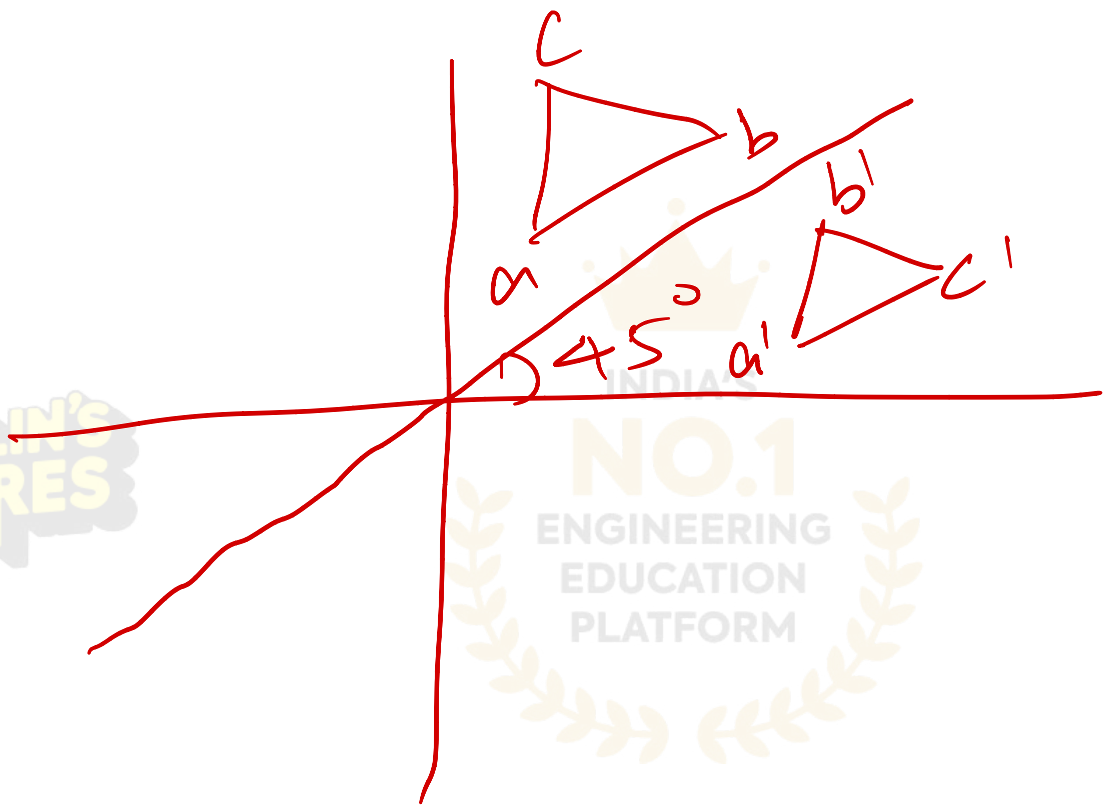
Reflection about y-axis



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{array}{c c} & \begin{array}{ccc} x & y & 1 \end{array} \\ \begin{array}{c} x \\ y \\ 1 \end{array} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} x' &= -x \\ y' &= y \end{aligned}}$$

Reflection about line $y = x$



Matrix :-

$$\begin{array}{c|ccc} & x & y & 1 \\ \hline x & & & \\ y & & \left[\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}\right] & \\ 1 & & & \end{array}$$

⇒ Here magnitude change but sign will not change.

$$\boxed{\begin{aligned} x' &= y \\ y' &= x \end{aligned}}$$

Reflection about $y = -x$

$$\begin{array}{c|ccc} & x & y & 1 \\ \hline x & 0 & -1 & 0 \\ y & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{array}$$

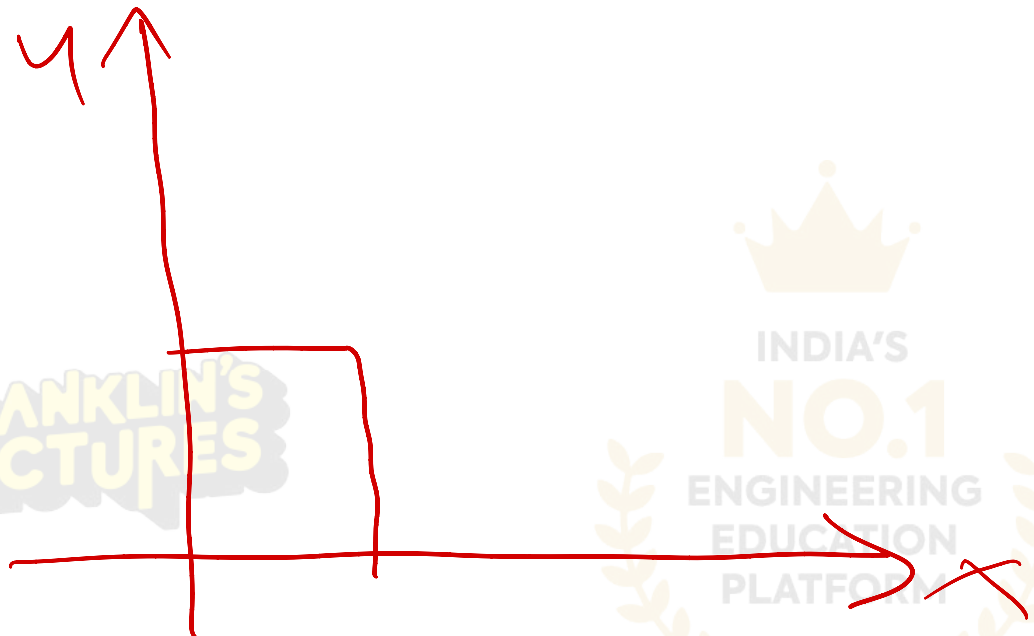
$$\boxed{\begin{aligned} x' &= -y \\ y' &= -x \end{aligned}}$$

SHEAR

- A transformation that slants the shape of an object is called the shear transformation. There are two shear transformations X-Shear and Y-Shear

- Also known as Skewing

X-SHEAR



Original object

object after x-shear

$x' = x + sh_x \cdot y$

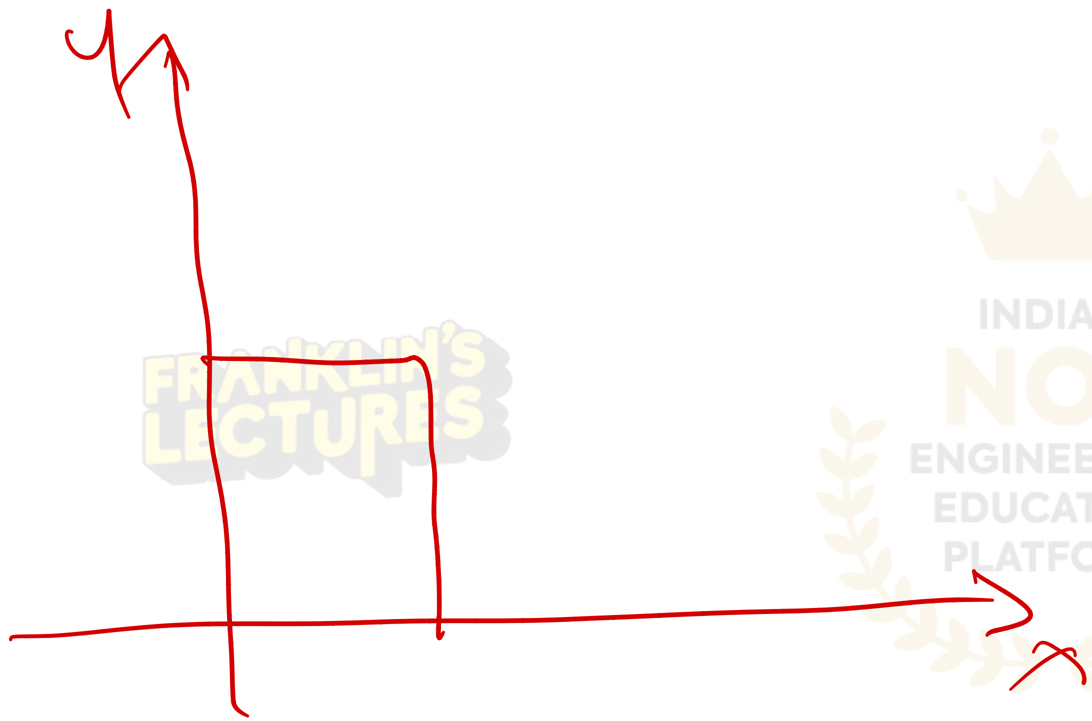
shearing factor along x-axis x y

$y' = y$

matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{array}{c} \\ x \\ y \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & 1 \end{array} \\ \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Y-SHEAR



Original position

object after y-shear.

Matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{array}{c} \\ x \\ y \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & 1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ shy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} y' &= y + shy \cdot x \\ x' &= x \end{aligned}}$$

Q. A triangle with (2,2),(0,0) & (2,0). Apply shearing factor 2 on x-axis and 2 on y- axis. Find out the new coordinates of the triangle

$A(2,2)$
$x \quad y$

x-shear

$x' = x + sh_x \cdot y$

$sh_x = 2$

$= 2 + 2 \times 2$

$= 6$

y-shear

$y' = y + sh_y \cdot x$

$= 2 + 2 \times 2$

$= 6$

$A'(6,6)$

$B(0,0)$

$\quad x \quad y$

$x' = 0 + 2 \times 0$

$= \underline{\underline{0}}$

$y' = 0 + 2 \times 0$

$= 0$

$B'(0,0)$

$C(2,0)$

$\quad x \quad y$

$x' = x + sh_x \cdot y$

$= 2 + 2 \times 0$

$= \underline{\underline{2}}$

$y' = y + sh_y \cdot x$

$= 0 + 2 \times 2$

$= 4$

$C'(2,4)$

THANK YOU