Recurrence Relations

Recurrence Relations

- A recurrence is an equation or inequality that describes a function in terms of its values on smaller inputs.
- A **recurrence relation** is a mathematical expression that defines a sequence in terms of its previous terms. In the context of algorithmic analysis, it is often used to model the time complexity of recursive algorithms.

- There are several methods for solving a recurrence relation.
  - ❖ **Iteration Method**
  - ❖ **Substitution Method**
  - ❖ **Recursion tree Method**
  - ❖ **Master's Method**

Iteration Method

- Repeatedly expand the recurrence until a base case is reached, then sum the resulting series.
- Easy to understand and apply.
- Used when the recurrence unfolds into a recognizable series.
- Can become cumbersome for complex recurrences.

Iteration Method

$T(n) = 1 + T(n-1)$

Iteration Method

$T(n) = 1 + T(n-1)$

$= 1 + [1 + T(n-2)] = 2 + T(n-2)$

Iteration Method

$$\mathbf{T(n) = 1 + T(n\text{-}1)}$$

$$= 1 + [1 + T(n-2)] = 2 + T(n-2)$$

$$= 2 + [1 + T(n-3)] = 3 + T(n-3)$$

Iteration Method

$$\begin{aligned}
\mathbf{T(n)} &\mathbf{= 1 + T(n\text{-}1)} \\
&= 1 + [1 + T(n\text{-}2)] = 2 + T(n\text{-}2) \\
&= 2 + [1 + T(n\text{-}3)] = 3 + T(n\text{-}3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k + T(n\text{-}k) \qquad\qquad k^{th} \text{ term}
\end{aligned}$$

Iteration Method

$T(n) = 1 + T(n-1)$

$= 1 + [1 + T(n-2)] = 2 + T(n-2)$

$= 2 + [1 + T(n-3)] = 3 + T(n-3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= k + T(n-k)$ k^{th} term

Assume $n-k = 1 \Rightarrow k = n-1$

Iteration Method

$$\mathbf{T(n) = 1 + T(n\text{-}1)}$$

$$= 1 + [1 + T(n\text{-}2)] = 2 + T(n\text{-}2)$$

$$= 2 + [1 + T(n\text{-}3)] = 3 + T(n\text{-}3)$$

……………………

$= k + T(n\text{-}k)$ k^{th} term

Assume $n\text{-}k = 1 \rightarrow k = n\text{-}1$

$$T(n) = n\text{-}1 + T(1)$$

Iteration Method

$$\mathbf{T(n) = 1 + T(n\text{-}1)}$$

$$= 1 + [1 + T(n\text{-}2)] = 2 + T(n\text{-}2)$$

$$= 2 + [1 + T(n\text{-}3)] = 3 + T(n\text{-}3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= k + T(n\text{-}k) \qquad k^{th} \text{ term}$$

Assume $n\text{-}k = 1 \Rightarrow k = n\text{-}1$

$$T(n) = n\text{-}1 + T(1)$$

$$= O(n) + O(1)$$

Iteration Method

$$\mathbf{T(n) = 1 + T(n\text{-}1)}$$

$$= 1 + [1 + T(n\text{-}2)] = 2 + T(n\text{-}2)$$

$$= 2 + [1 + T(n\text{-}3)] = 3 + T(n\text{-}3)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$= k + T(n\text{-}k) \qquad k^{th} \text{ term}$$

Assume $n\text{-}k = 1 \Rightarrow k = n\text{-}1$

$$T(n) = n\text{-}1 + T(1)$$

$$= O(n) + O(1)$$

$$= \mathbf{O(n)}$$

Iteration Method

$$T(n) = \begin{cases} 2T(n/2)+2 & \text{if } n>2 \\ 1 & \text{if } n=2 \end{cases}$$

Iteration Method

T(n)	**=2T(n/2)+2**	**if n>2**
	=1	**if n=2**

$T(n) = 2 + 2T(n/2)$

Iteration Method

T(n) =2T(n/2)+2 if n>2

=1 if n=2

$T(n) = 2 + 2T(n/2)$

$= 2 + 2[2+2T(n/2^2)] \qquad = 2+2^2+2^2T(n/2^2)$

Iteration Method

$T(n) = 2T(n/2)+2$ if $n>2$

$= 1$ if $n=2$

$T(n) = 2 + 2T(n/2)$

$= 2 + 2[2+2T(n/2^2)] \qquad = 2+2^2+2^2T(n/2^2)$

$= 2+2^2+2^2[2+2T(n/2^3)] \qquad = 2+2^2+2^3+2^3T(n/2^3)$

Iteration Method

$T(n) = 2T(n/2)+2$ if $n>2$

$= 1$ if $n=2$

$T(n) = 2 + 2T(n/2)$

$= 2 + 2[2+2T(n/2^2)] \qquad = 2+2^2+2^2T(n/2^2)$

$= 2+2^2+2^2[2+2T(n/2^3)] \qquad = 2+2^2+2^3+2^3T(n/2^3)$

……………………….

$= 2+2^2+2^3+\ldots\ldots\ldots +2^k + 2^kT(n/2^k)$

Iteration Method

T(n) =2T(n/2)+2 if n>2

=1 if n=2

$$T(n) = 2 + 2T(n/2)$$

$$=2 + 2[2+2T(n/2^2)] \qquad = 2+2^2+2^2T(n/2^2)$$

$$= 2+2^2+2^2[2+2T(n/2^3)] \qquad = 2+2^2+2^3+2^3T(n/2^3)$$

…………………………..

$$= 2+2^2+2^3+\ldots\ldots\ldots +2^k + 2^kT(n/2^k)$$

Assume that $n/2^k=2$

Iteration Method

T(n) =2T(n/2)+2 if n>2

=1 if n=2

$T(n) = 2 + 2T(n/2)$

$=2 + 2[2+2T(n/2^2)] \qquad = 2+2^2+2^2T(n/2^2)$

$= 2+2^2+2^2[2+2T(n/2^3)] \qquad = 2+2^2+2^3+2^3T(n/2^3)$

……………………….

$= 2+2^2+2^3+\ldots\ldots\ldots +2^k + 2^kT(n/2^k)$

Assume that $n/2^k=2$

$T(n) =2[1+2+2^2+2^3+\ldots\ldots\ldots +2^{k-1}] + 2^kT(n/2^k)$

Iteration Method

$T(n) = 2T(n/2)+2$ if $n>2$

$= 1$ if $n=2$

$T(n) = 2 + 2T(n/2)$

$= 2 + 2[2+2T(n/2^2)] \quad = 2+2^2+2^2T(n/2^2)$

$= 2+2^2+2^2[2+2T(n/2^3)] \quad = 2+2^2+2^3+2^3T(n/2^3)$

……………………….

$= 2+2^2+2^3+\ldots\ldots\ldots +2^k + 2^kT(n/2^k)$

Assume that $n/2^k=2$

$T(n) = 2[1+2+2^2+2^3+\ldots\ldots\ldots +2^{k-1}] + 2^kT(n/2^k)$

$= 2[2^k-1] + 2^kT(2) \quad = 2x2^k-2 + 2^k$

Iteration Method

$T(n) = 2T(n/2)+2$ if $n>2$

$= 1$ if $n=2$

$T(n) = 2 + 2T(n/2)$

$= 2 + 2[2+2T(n/2^2)] = 2+2^2+2^2T(n/2^2)$

$= 2+2^2+2^2[2+2T(n/2^3)] = 2+2^2+2^3+2^3T(n/2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= 2+2^2+2^3+\ldots\ldots\ldots +2^k + 2^kT(n/2^k)$

Assume that $n/2^k=2$

$T(n) = 2[1+2+2^2+2^3+\ldots\ldots\ldots +2^{k-1}] + 2^kT(n/2^k)$

$= 2[2^k-1] + 2^kT(2) = 2x2^k-2 + 2^k$

$= 3x2^k-2 = 3x(n/2)-2$

Iteration Method

$$T(n) = \begin{cases} 2T(n/2)+2 & \text{if } n>2 \\ 1 & \text{if } n=2 \end{cases}$$

$T(n) = 2 + 2T(n/2)$

$= 2 + 2[2+2T(n/2^2)] \qquad = 2+2^2+2^2T(n/2^2)$

$= 2+2^2+2^2[2+2T(n/2^3)] \qquad = 2+2^2+2^3+2^3T(n/2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= 2+2^2+2^3+\ldots\ldots\ldots +2^k + 2^kT(n/2^k)$

Assume that $n/2^k=2$

$T(n) = 2[1+2+2^2+2^3+\ldots\ldots\ldots +2^{k-1}] + 2^kT(n/2^k)$

$= 2[2^k-1] + 2^kT(2) \qquad = 2x2^k-2 + 2^k$

$= 3x2^k-2 \qquad = 3x(n/2)-2$

$=$ **$O(n)$**

Iteration Method

$T(n) = 2\ T(n/2) + n$ $\qquad\qquad$ $T(1)=1$

Iteration Method

$T(n) = 2\ T(n/2) + n$ **$T(1)=1$**

$T(n) \quad = n + 2\ T(n/2)$

Iteration Method

$T(n) = 2\ T(n/2) + n$ **$T(1)=1$**

$T(n) \quad = n + 2\ T(n/2)$

$\qquad = n + 2\ [(n/2)+ 2\ T(n/2^2)] = 2n + 2^2 T(n/2^2)$

Iteration Method

T(n) = 2 T(n/2) + n **T(1)=1**

$$
\begin{aligned}
T(n) &= n + 2\,T(n/2) \\
&= n + 2\,[(n/2) + 2\,T(n/2^2)] = 2n + 2^2T(n/2^2) \\
&= 2n + 2^2\,[(n/2^2) + 2T(n/2^3)] = 3n + 2^3T(n/2^3)
\end{aligned}
$$

Iteration Method

$T(n) = 2\,T(n/2) + n$ **$T(1)=1$**

$$
\begin{aligned}
T(n) &= n + 2\,T(n/2) \\
&= n + 2\,[(n/2) + 2\,T(n/2^2)] = 2n + 2^2T(n/2^2) \\
&= 2n + 2^2\,[(n/2^2) + 2T(n/2^3)] = 3n + 2^3T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k\,n + 2^kT(n/2^k)
\end{aligned}
$$

Iteration Method

$T(n) = 2\,T(n/2) + n$ **$T(1)=1$**

$$T(n) = n + 2\,T(n/2)$$

$$= n + 2\,[(n/2) + 2\,T(n/2^2)] = 2n + 2^2T(n/2^2)$$

$$= 2n + 2^2\,[(n/2^2) + 2T(n/2^3)] = 3n + 2^3T(n/2^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= k\,n + 2^kT(n/2^k)$$

Assume that $n/2^k=1$ $\rightarrow$ $2^k=n$ $\rightarrow$ $k=\log_2(n)$

Iteration Method

$T(n) = 2\ T(n/2) + n$ **$T(1)=1$**

$$T(n) = n + 2\ T(n/2)$$

$$= n + 2\ [(n/2) + 2\ T(n/2^2)] = 2n + 2^2 T(n/2^2)$$

$$= 2n + 2^2\ [(n/2^2) + 2T(n/2^3)] = 3n + 2^3 T(n/2^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= k\,n + 2^k T(n/2^k)$$

Assume that $n/2^k=1$ ➔ $2^k=n$ ➔ $k=\log_2(n)$

$$T(n) = n\log_2(n) + nT(1)$$

Iteration Method

$$T(n) = 2\,T(n/2) + n \qquad\qquad T(1)=1$$

$$T(n) = n + 2\,T(n/2)$$

$$= n + 2\,[(n/2) + 2\,T(n/2^2)] = 2n + 2^2T(n/2^2)$$

$$= 2n + 2^2\,[(n/2^2) + 2T(n/2^3)] = 3n + 2^3T(n/2^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= k\,n + 2^kT(n/2^k)$$

Assume that $n/2^k=1$ $\rightarrow$ $2^k=n$ $\rightarrow$ $k=\log_2(n)$

$$T(n) = n\log_2(n) + nT(1)$$

$$= n\log_2(n) + n$$

Iteration Method

$T(n) = 2\,T(n/2) + n$ **$T(1)=1$**

$T(n) \quad = n + 2\,T(n/2)$

$= n + 2\,[(n/2)+ 2\,T(n/2^2)] = 2n + 2^2T(n/2^2)$

$= 2n + 2^2\,[(n/2^2)+2T(n/2^3)]= 3n + 2^3T(n/2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$=k\,n + 2^kT(n/2^k)$

Assume that $n/2^k=1$ $\rightarrow$ $2^k=n$ $\rightarrow$ $k=\log_2(n)$

$T(n) \quad =n\log_2(n) + nT(1)$

$=n\log_2(n) + n$

$=$**$O(n\log_2(n))$**

Iteration Method

$T(n) = 2$	if $n=1$
$T(n) = 2T(n/2) + 2n + 3$	Otherwise

Iteration Method

$T(n) = 2$ **if $n=1$**

$T(n) = 2T(n/2) + 2n + 3$ **Otherwise**

$T(n) \quad = 3 + 2n + 2T(n/2)$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)]
\end{aligned}
$$

Iteration Method

$T(n) = 2$ **if $n=1$**

$T(n) = 2T(n/2) + 2n + 3$ **Otherwise**

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)] \\
&=3+2x3+2x2n+2^2T(n/2^2)
\end{aligned}
$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$T(n) = 3 + 2n + 2T(n/2)$$

$$= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)]$$

$$=3+2x3+2x2n+2^2T(n/2^2)$$

$$=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)]$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)] \\
&=3+2x3+2x2n+2^2T(n/2^2) \\
&=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)] \\
&= 3+2x3+2^2x3+3x2n+2^3T(n/2^3)
\end{aligned}
$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)] \\
&=3+2x3+2x2n+2^2T(n/2^2) \\
&=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)] \\
&= 3+2x3+2^2x3+3x2n+2^3T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= [3+2x3+2^2x3+\ldots. +2^{k-1}x3]+[kx2n]+2^kT(n/2^k)
\end{aligned}
$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)] \\
&= 3+2x3+2x2n+2^2T(n/2^2) \\
&= 3+2x3+2x2n+2^2 [3+2(n/2^2)+2T(n/2^3)] \\
&= 3+2x3+2^2x3+3x2n+2^3T(n/2^3)
\end{aligned}
$$

……………………………………..

$$= [3+2x3+2^2x3+\ldots. +2^{k-1}x3]+[kx2n]+2^kT(n/2^k)$$

Assume that $n/2^k=1$ $\rightarrow$ $2^k=n$ $\rightarrow$ $k=\log_2(n)$

Iteration Method

T(n) = 2 if n=1

T(n) = 2T(n/2) + 2n + 3 Otherwise

$$T(n) = 3 + 2n + 2T(n/2)$$

$$= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)]$$

$$=3+2x3+2x2n+2^2T(n/2^2)$$

$$=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)]$$

$$= 3+2x3+2^2x3+3x2n+2^3T(n/2^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= [3+2x3+2^2x3+\ldots. +2^{k-1}x3]+[kx2n]+2^kT(n/2^k)$$

Assume that $n/2^k=1$ ➔ $2^k=n$ ➔ $k=\log_2(n)$

$$T(n) =3[2^0+2^1+2^2+\ldots+2^{k-1}]+2n\log_2(n) + nT(1)$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)] \\
&=3+2x3+2x2n+2^2T(n/2^2) \\
&=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)] \\
&= 3+2x3+2^2x3+3x2n+2^3T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= [3+2x3+2^2x3+\ldots. +2^{k-1}x3]+[kx2n]+2^kT(n/2^k)
\end{aligned}
$$

Assume that $n/2^k=1$ $\rightarrow$ $2^k=n$ $\rightarrow$ $k=\log_2(n)$

$$
\begin{aligned}
T(n) &=3[2^0+2^1+2^2+\ldots+2^{k-1}]+2n\log_2(n) + nT(1) \\
&=3[(2^k -1)/(2-1)] +2n\log_2(n) + 2n = 3(n-1) +2n\log_2(n) + 2n
\end{aligned}
$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$
\begin{aligned}
T(n) &= 3 + 2n + 2T(n/2) \\
&= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)] \\
&=3+2x3+2x2n+2^2T(n/2^2) \\
&=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)] \\
&= 3+2x3+2^2x3+3x2n+2^3T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= [3+2x3+2^2x3+\ldots. +2^{k-1}x3]+[kx2n]+2^kT(n/2^k)
\end{aligned}
$$

Assume that $n/2^k=1$ ➔ $2^k=n$ ➔ $k=\log_2(n)$

$$
\begin{aligned}
T(n) &=3[2^0+2^1+2^2+\ldots+2^{k-1}]+2n\log_2(n) + nT(1) \\
&=3[(2^k -1)/(2-1)] +2n\log_2(n) + 2n = 3(n-1) +2n\log_2(n) + 2n \\
&= 5n -3 +2n\log_2(n)
\end{aligned}
$$

Iteration Method

$T(n) = 2$ if $n=1$

$T(n) = 2T(n/2) + 2n + 3$ Otherwise

$$T(n) = 3 + 2n + 2T(n/2)$$
$$= 3 + 2n + 2[3+2(n/2)+2T(n/2^2)]$$
$$=3+2x3+2x2n+2^2T(n/2^2)$$
$$=3+2x3+2x2n+2^2\,[3+2(n/2^2)+2T(n/2^3)]$$
$$= 3+2x3+2^2x3+3x2n+2^3T(n/2^3)$$

……………………………………..

$$= [3+2x3+2^2x3+\ldots. +2^{k-1}x3]+[kx2n]+2^kT(n/2^k)$$

Assume that $n/2^k=1$ ➔ $2^k=n$ ➔ $k=\log_2(n)$

$$T(n) =3[2^0+2^1+2^2+\ldots+2^{k-1}]+2n\log_2(n) + nT(1)$$
$$=3[(2^k-1)/(2-1)] +2n\log_2(n) + 2n = 3(n-1) +2n\log_2(n) + 2n$$
$$= 5n -3 +2n\log_2(n) = \mathbf{O(n\log_2(n))}$$

Iteration Method

$T(2^k) = 3T(2^{k-1}) + 1 \qquad\qquad T(1) = 1$

Iteration Method

$\mathbf{T(2^k) = 3T(2^{k-1}) + 1}$ $\mathbf{T(1) = 1}$

$T(2^k) = 1 + 3T(2^{k-1})$

Iteration Method

$T(2^k) = 3T(2^{k-1}) + 1$ $\qquad\qquad$ $T(1) = 1$

$T(2^k) = 1 + 3T(2^{k-1})$

$= 1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

Iteration Method

$T(2^k) = 3T(2^{k-1}) + 1$ **$T(1) = 1$**

$$T(2^k) = 1 + 3T(2^{k-1})$$

$$= 1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$$

$$= 1+3+3^2 [1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3})$$

Iteration Method

$T(2^k) = 3T(2^{k-1}) + 1$ $\qquad$ $T(1) = 1$

$$
\begin{aligned}
T(2^k) &= 1 + 3T(2^{k-1}) \\
&= 1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2}) \\
&= 1+3+3^2\,[1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3}) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= 1+3+3^2+\ldots.. +3^{n-1}+3^nT(2^{k-n})
\end{aligned}
$$

Iteration Method

$T(2^k) = 3T(2^{k-1}) + 1$ $\quad\quad$ $T(1) = 1$

$T(2^k) = 1 + 3T(2^{k-1})$

$= 1+3[1+3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

$= 1+3+3^2[1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3})$

…………………

$= 1+3+3^2+\ldots\ldots+3^{n-1}+3^nT(2^{k-n})$

Assume that $2^{k-n}=1 \rightarrow 2^k/2^n=1 \rightarrow 2^k=2^n \rightarrow k=n$

Iteration Method

$T(2^k) = 3T(2^{k-1}) + 1$ $\qquad$ $T(1) = 1$

$T(2^k) = 1 + 3T(2^{k-1})$

$= 1+3[1+3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

$= 1+3+3^2[1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3})$

…………………

$= 1+3+3^2+\ldots\ldots+3^{n-1}+3^nT(2^{k-n})$

Assume that $2^{k-n}=1 \rightarrow 2^k/2^n=1 \rightarrow 2^k=2^n \rightarrow k=n$

$T(2^k) = 1+3+3^2+\ldots.+3^{n-1}+3^nT(2^{k-n}) = [(3^n-1)/(3-1)]+3^nT(1)$

Iteration Method

$\mathbf{T(2^k) = 3T(2^{k-1}) + 1}$ $\qquad$ $\mathbf{T(1) = 1}$

$T(2^k) \; =1+ 3T(2^{k-1})$

$=1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

$= 1+3+3^2 [1+3T(2^{k-3})]= 1+3+3^2+3^3T(2^{k-3})$

…………………

$= 1+3+3^2+\ldots.. +3^{n-1}+3^nT(2^{k-n})$

Assume that $2^{k-n}=1 \;\rightarrow\; 2^k/2^n=1 \;\rightarrow\; 2^k=2^n \;\rightarrow\; k=n$

$T(2^k) \; = 1+3+3^2+\ldots. +3^{n-1}+3^nT(2^{k-n}) = [(3^n-1)/(3-1)]+3^nT(1)$

$= (3^n/2)-(1/2)+3^n$

Iteration Method

$\mathbf{T(2^k) = 3T(2^{k-1}) + 1}$ $\qquad\qquad$ $\mathbf{T(1) = 1}$

$$T(2^k) = 1 + 3T(2^{k-1})$$

$$= 1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$$

$$= 1+3+3^2 [1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3})$$

$$\dots\dots\dots\dots\dots\dots\dots$$

$$= 1+3+3^2+\dots\dots +3^{n-1}+3^nT(2^{k-n})$$

Assume that $2^{k-n}=1 \rightarrow 2^k/2^n=1 \rightarrow 2^k=2^n \rightarrow k=n$

$$T(2^k) = 1+3+3^2+\dots. +3^{n-1}+3^nT(2^{k-n}) = [(3^n-1)/(3-1)]+3^nT(1)$$

$$= (3^n/2)-(1/2)+3^n$$

$$= (3/2)\ 3^n-(1/2)$$

Iteration Method

$\mathbf{T(2^k) = 3T(2^{k-1}) + 1}$ $\qquad$ $\mathbf{T(1) = 1}$

$T(2^k) = 1 + 3T(2^{k-1})$

$= 1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

$= 1+3+3^2 [1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3})$

…………………

$= 1+3+3^2+\ldots.. +3^{n-1}+3^nT(2^{k-n})$

Assume that $2^{k-n}=1 \Rightarrow 2^k/2^n=1 \Rightarrow 2^k=2^n \Rightarrow k=n$

$T(2^k) = 1+3+3^2+\ldots. +3^{n-1}+3^nT(2^{k-n}) = [(3^n-1)/(3-1)]+3^nT(1)$

$= (3^n/2)-(1/2)+3^n$

$= (3/2)\, 3^n-(1/2)$

$= (3/2)\, 3^k-(1/2)$

Iteration Method

$\mathbf{T(2^k) = 3T(2^{k-1}) + 1 \qquad\qquad T(1) = 1}$

$T(2^k) \;=1+ 3T(2^{k-1})$

$=1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

$= 1+3+3^2 [1+3T(2^{k-3})]= 1+3+3^2+3^3T(2^{k-3})$

……………………

$= 1+3+3^2+\ldots\ldots +3^{n-1}+3^nT(2^{k-n})$

Assume that $2^{k-n}=1 \Rightarrow 2^k/2^n=1 \Rightarrow 2^k=2^n \Rightarrow k=n$

$T(2^k) \;= 1+3+3^2+\ldots. +3^{n-1}+3^nT(2^{k-n}) = [(3^n-1)/(3-1)]+3^nT(1)$

$= (3^n/2)-(1/2)+3^n$

$= (3/2)\, 3^n-(1/2)$

$= (3/2)\, 3^k-(1/2)$

$=\mathbf{O(3^k)}$

Iteration Method

$\mathbf{T(2^k) = 3T(2^{k-1}) + 1} \qquad \mathbf{T(1) = 1}$

$T(2^k) = 1 + 3T(2^{k-1})$

$= 1+3[1+ 3T(2^{k-2})] = 1+3+3^2T(2^{k-2})$

$= 1+3+3^2 [1+3T(2^{k-3})] = 1+3+3^2+3^3T(2^{k-3})$

$\dots\dots\dots\dots\dots\dots\dots$

$= 1+3+3^2+\dots\dots +3^{n-1}+3^nT(2^{k-n})$

Assume that $2^{k-n}=1 \rightarrow 2^k/2^n=1 \rightarrow 2^k=2^n \rightarrow k=n$

$T(2^k) = 1+3+3^2+\dots. +3^{n-1}+3^nT(2^{k-n}) = [(3^n-1)/(3-1)]+3^nT(1)$

$= (3^n/2)-(1/2)+3^n$

$= (3/2)\ 3^n-(1/2)$

$= (3/2)\ 3^k-(1/2)$

$= \mathbf{O(3^k)}$

Iteration Method

$T(n) = T(n/2)+1 \qquad T(1)=1$

Iteration Method

T(n) = T(n/2)+1   T(1)=1

T(n) = 1 + T(n/2)

Iteration Method

$T(n) = T(n/2)+1$ $\quad\quad$ $T(1)=1$

$T(n) = 1 + T(n/2)$

$= 1 + [1+ T(n/2^2)] \quad\quad = 2+ T(n/2^2)$

Iteration Method

$T(n) = T(n/2)+1 \qquad T(1)=1$

$$\begin{aligned} T(n) &= 1 + T(n/2) \\ &= 1 + [1 + T(n/2^2)] &&= 2 + T(n/2^2) \\ &= 2 + [1 + T(n/2^3)] &&= 3 + T(n/2^3) \end{aligned}$$

Iteration Method

$T(n) = T(n/2)+1$ $\quad\quad\quad$ $T(1)=1$

$$
\begin{aligned}
T(n) &= 1 + T(n/2) \\
&= 1 + [1+ T(n/2^2)] && = 2+ T(n/2^2) \\
&= 2+ [1+ T(n/2^3)] && = 3+ T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k + T(n/2^k) && k^{th} \text{ term}
\end{aligned}
$$

Iteration Method

$T(n) = T(n/2)+1$ $\quad\quad$ $T(1)=1$

$$
\begin{aligned}
T(n) &= 1 + T(n/2) \\
&= 1 + [1+ T(n/2^2)] && = 2+ T(n/2^2) \\
&= 2+ [1+ T(n/2^3)] && = 3+ T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k + T(n/2^k) && k^{th} \text{ term}
\end{aligned}
$$

Assume $n/2^k = 1$ ➔ $2^k = n$ ➔ $k = \log_2(n)$

Iteration Method

$$\mathbf{T(n) = T(n/2)+1 \qquad T(1)=1}$$

$$\begin{aligned}
T(n) &= 1 + T(n/2) \\
&= 1 + [1+ T(n/2^2)] && = 2+ T(n/2^2) \\
&= 2+ [1+ T(n/2^3)] && = 3+ T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k + T(n/2^k) && k^{th} \text{ term}
\end{aligned}$$

Assume $n/2^k = 1 \rightarrow 2^k = n \rightarrow k = \log_2(n)$

$$T(n) = \log_2(n) + T(1)$$

Iteration Method

$T(n) = T(n/2)+1$ **$T(1)=1$**

$$
\begin{aligned}
T(n) &= 1 + T(n/2) \\
&= 1 + [1+ T(n/2^2)] && = 2+ T(n/2^2) \\
&= 2+ [1+ T(n/2^3)] && = 3+ T(n/2^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k + T(n/2^k) && k^{th} \text{ term}
\end{aligned}
$$

Assume $n/2^k = 1$ ➔ $2^k = n$ ➔ $k = \log_2(n)$

$$
\begin{aligned}
T(n) &= \log_2(n) + T(1) \\
&= \log_2(n) + 1
\end{aligned}
$$

Iteration Method

$T(n) = T(n/2)+1$ **$T(1)=1$**

$T(n) = 1 + T(n/2)$

$= 1 + [1+ T(n/2^2)]$ $= 2+ T(n/2^2)$

$= 2+ [1+ T(n/2^3)]$ $= 3+ T(n/2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= k + T(n/2^k)$ k^{th} term

Assume $n/2^k = 1$ $\Rightarrow$ $2^k = n$ $\Rightarrow$ $k = \log_2(n)$

$T(n) = \log_2(n) + T(1)$

$= \log_2(n) + 1$

$=$ **$O(\log_2(n))$**

Iteration Method

$T(n) = 4T(n/2)+n^2$ $\qquad$ $T(1)=1$

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

$= 2n^2+4^2[(n/ 2^2)^2 +4T(n/ 2^3)] = 3n^2+4^3T(n/ 2^3)$

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

$= 2n^2+4^2[(n/ 2^2)^2 +4T(n/ 2^3)] = 3n^2+4^3T(n/ 2^3)$

………………………

$= kn^2+4^kT(n/ 2^k)$ k^{th} term

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

$= 2n^2+4^2[(n/ 2^2)^2 +4T(n/ 2^3)] = 3n^2+4^3T(n/ 2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= kn^2+4^kT(n/ 2^k)$ k^{th} term

Assume $n/2^k= 1$ ➔ $2^k = n$ ➔ $k = \log_2(n)$

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

$= 2n^2+4^2[(n/ 2^2)^2 +4T(n/ 2^3)] = 3n^2+4^3T(n/ 2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= kn^2+4^kT(n/ 2^k)$ k^{th} term

Assume $n/2^k= 1$ ➔ $2^k = n$ ➔ $k = \log_2(n)$

$T(n) = n^2\log_2(n) +4^k\, T(1)$

Iteration Method

$T(n) = 4T(n/2)+n^2$ **$T(1)=1$**

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

$= 2n^2+4^2[(n/ 2^2)^2 +4T(n/ 2^3)] = 3n^2+4^3T(n/ 2^3)$

……………………

$= kn^2+4^kT(n/ 2^k)$ k^{th} term

Assume $n/2^k= 1$ ➔ $2^k = n$ ➔ $k = \log_2(n)$

$T(n) = n^2\log_2(n) +4^k T(1)$ $4^k =n^2$

$= n^2\log_2(n) + n^2$

Iteration Method

$\mathbf{T(n) = 4T(n/2)+n^2}$ $\mathbf{T(1)=1}$

$T(n) = n^2 + 4T(n/2)$

$= n^2+4[(n/2)^2 +4T(n/ 2^2)] = 2n^2+4^2T(n/ 2^2)$

$= 2n^2+4^2[(n/ 2^2)^2 +4T(n/ 2^3)] = 3n^2+4^3T(n/ 2^3)$

……………………

$= kn^2+4^kT(n/ 2^k)$ k^{th} term

Assume $n/2^k= 1$ ➔ $2^k = n$ ➔ $k = \log_2(n)$

$T(n) = n^2\log_2(n) +4^k\, T(1)$ $4^k =n^2$

$= n^2\log_2(n) + n^2$

$= \mathbf{O(n^2\log_2(n))}$

Iteration Method

$T(n) = T(n/3)+n$ $\qquad$ $T(1)=1$

Iteration Method

$T(n) = T(n/3)+n$ **$T(1)=1$**

$T(n) = n + T(n/3)$

Iteration Method

$T(n) = T(n/3)+n$ **$T(1)=1$**

$$T(n) = n + T(n/3)$$

$$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$$

Iteration Method

$T(n) = T(n/3)+n$ $\quad$ $T(1)=1$

$$T(n) = n + T(n/3)$$

$$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$$

$$=n+(n/3)+[(n/3^2)+T(n/3^3)]=n+(n/3)+(n/3^2)+T(n/3^3)$$

Iteration Method

T(n) = T(n/3)+n T(1)=1

$$T(n) = n + T(n/3)$$

$$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$$

$$=n+(n/3)+[(n/3^2)+T(n/3^3)]=n+(n/3)+(n/3^2)+T(n/3^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= n+(n/3)+(n/3^2)+ \ldots. +(n/3^{k-1})+T(n/3^k)$$

Iteration Method

T(n) = T(n/3)+n T(1)=1

$T(n) = n + T(n/3)$

$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$

$= n+(n/3)+[(n/3^2)+T(n/3^3)] = n+(n/3)+(n/3^2)+T(n/3^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= n+(n/3)+(n/3^2)+ \ldots. +(n/3^{k-1})+T(n/3^k)$

Assume $n/3^k = 1$ ➔ $3^k = n$ ➔ $k = \log_3(n)$

Iteration Method

$T(n) = T(n/3)+n$ $\qquad$ $T(1)=1$

$T(n) = n + T(n/3)$

$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$

$=n+(n/3)+[(n/3^2)+T(n/3^3)]=n+(n/3)+(n/3^2)+T(n/3^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= n+(n/3)+(n/3^2)+ \ldots. +(n/3^{k-1})+T(n/3^k)$

Assume $n/3^k= 1$ ➔ $3^k = n$ ➔ $k = \log_3(n)$

$T(n) = n[1+(1/3)+(1/3)^2+\ldots.+(1/3)^{k-1}] + T(n/3^k)$

Iteration Method

$T(n) = T(n/3)+n \qquad T(1)=1$

$T(n) = n + T(n/3)$

$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$

$=n+(n/3)+[(n/3^2)+T(n/3^3)]=n+(n/3)+(n/3^2)+T(n/3^3)$

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$

$= n+(n/3)+(n/3^2)+ \ldots. +(n/3^{k-1})+T(n/3^k)$

Assume $n/3^k= 1 \Rightarrow 3^k = n \Rightarrow k = \log_3(n)$

$T(n) = n[1+(1/3)+(1/3)^2+\ldots.+(1/3)^{k-1}] + T(n/3^k)$

$\le n[(1/(1-(1/3))] + T(1)$

Iteration Method

$\mathbf{T(n) = T(n/3)+n \qquad\qquad T(1)=1}$

$T(n) = n + T(n/3)$

$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$

$=n+(n/3)+[(n/3^2)+T(n/3^3)]=n+(n/3)+(n/3^2)+T(n/3^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= n+(n/3)+(n/3^2)+ \;\ldots.\; +(n/3^{k-1})+T(n/3^k)$

Assume $n/3^k= 1 \;\rightarrow\; 3^k = n \;\rightarrow\; k = \log_3(n)$

$T(n) = n[1+(1/3)+(1/3)^2+\ldots.+(1/3)^{k-1}] + T(n/3^k)$

$\leq n[(1/(1-(1/3))] + T(1)$

$=(3/2)n+ T(1) = (3/2)n+ 1$

Iteration Method

$\mathbf{T(n) = T(n/3)+n \qquad T(1)=1}$

$T(n) = n + T(n/3)$

$= n + [(n/3) + T(n/3^2)] = n + (n/3) + T(n/3^2)$

$= n+(n/3)+[(n/3^2)+T(n/3^3)] = n+(n/3)+(n/3^2)+T(n/3^3)$

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$

$= n+(n/3)+(n/3^2)+ \ \ldots \ +(n/3^{k-1})+T(n/3^k)$

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$T(n) = n[1+(1/3)+(1/3)^2+\ldots+(1/3)^{k-1}] + T(n/3^k)$

$\leq n[(1/(1-(1/3))] \quad + T(1)$

$=(3/2)n+ T(1) = (3/2)n+ 1$

$= \mathbf{O(n)}$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

Iteration Method

T(n) = 3T(n/4)+n **T(1)=1**

$T(n) = n + 3T(n/4)$

Iteration Method

$T(n) = 3T(n/4)+n$ $\quad\quad\quad\quad$ $T(1)=1$

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

Iteration Method

T(n) = 3T(n/4)+n **T(1)=1**

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

$$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

$$
\begin{aligned}
T(n) &= n + 3T(n/4) \\
&= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2) \\
&= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)] \\
&= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)
\end{aligned}
$$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

$$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$$

$$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$$

……………………

$$= n + (3/4)n+(3/4)^2n+ \ldots +(3/4)^{k-1}n+3^kT(n/4^k)$$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

$T(n) = n + 3T(n/4)$

$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$

$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$

$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= n + (3/4)n+(3/4)^2n+ \ldots. +(3/4)^{k-1}n+3^kT(n/4^k)$

Assume $n/4^k= 1$ ➔ $4^k = n$ ➔ $k = \log_4(n)$

Iteration Method

$T(n) = 3T(n/4)+n$ $\quad\quad$ $T(1)=1$

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

$$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$$

$$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= n + (3/4)n+(3/4)^2n+ \ \ldots. +(3/4)^{k-1}n+3^kT(n/4^k)$$

Assume $n/4^k= 1 \ \Rightarrow\ 4^k = n \ \Rightarrow\ k = \log_4(n)$

$$T(n) = n[1+(3/4)+(3/4)^2+\ldots.+(3/4)^{k-1}] + 3^kT(1)$$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

$T(n) = n + 3T(n/4)$

$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$

$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$

$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$

……………………

$= n + (3/4)n+(3/4)^2n+ \;\ldots.\; +(3/4)^{k-1}n+3^kT(n/4^k)$

Assume $n/4^k= 1 \rightarrow 4^k = n \rightarrow k = \log_4(n)$

$T(n) = n[1+(3/4)+(3/4)^2+\ldots.+(3/4)^{k-1}] + 3^kT(1)$

$\leq n[(1/(1-(3/4))] + 3^{\log_4(n)} T(1)$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

$$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$$

$$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$$

....

$$= n + (3/4)n+(3/4)^2n+ \ .... +(3/4)^{k-1}n+3^kT(n/4^k)$$

Assume $n/4^k= 1 \rightarrow 4^k = n \rightarrow k = \log_4(n)$

$$T(n) = n[1+(3/4)+(3/4)^2+....+(3/4)^{k-1}] + 3^kT(1)$$

$$\leq n[(1/(1-(3/4))] + 3^{\log_4(n)} T(1)$$

$\mathbf{a^{\log_c(b)} = b^{\log_c(a)}}$

$$=4n+ n^{\log_4(3)}$$

Iteration Method

$T(n) = 3T(n/4)+n$ **$T(1)=1$**

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

$$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$$

$$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= n + (3/4)n+(3/4)^2n+ \ldots. +(3/4)^{k-1}n+3^kT(n/4^k)$$

Assume $n/4^k= 1$ ➔ $4^k = n$ ➔ $k = \log_4(n)$

$$T(n) = n[1+(3/4)+(3/4)^2+\ldots.+(3/4)^{k-1}] + 3^kT(1)$$

$$\leq n[(1/(1-(3/4))] + 3^{\log_4(n)} T(1)$$ **$a^{\log_c(b)} = b^{\log_c(a)}$**

$$=4n+ n^{\log_4(3)} = O(n)+ O(n^{\log_4(3)})$$

Iteration Method

$$\mathbf{T(n) = 3T(n/4)+n \qquad T(1)=1}$$

$$T(n) = n + 3T(n/4)$$

$$= n + 3[(n/4)+3T(n/4^2)] = n + (3/4)n+3^2T(n/4^2)$$

$$= n + (3/4)n+3^2[(n/4)^2+3T(n/4^3)]$$

$$= n + (3/4)n+(3/4)^2n+3^3T(n/4^3)$$

....

$$= n + (3/4)n+(3/4)^2n+ \;\ldots.\; +(3/4)^{k-1}n+3^kT(n/4^k)$$

Assume $n/4^k= 1 \rightarrow 4^k = n \rightarrow k = \log_4(n)$

$$T(n) = n[1+(3/4)+(3/4)^2+\ldots.+(3/4)^{k-1}] + 3^kT(1)$$

$$\leq n[(1/(1-(3/4))] + 3^{\log_4(n)}T(1) \qquad \mathbf{a^{\log_c(b)}=b^{\log_c(a)}}$$

$$=4n+ n^{\log_4(3)} = O(n)+ O(n^{\log_4(3)}) =\mathbf{O(n)}$$

Iteration Method

$T(n) = 4T(n/3)+n$ $T(1)=1$

Iteration Method

$T(n) = 4T(n/3)+n$ **$T(1)=1$**

$T(n) = n + 4T(n/3)$

Iteration Method

$T(n) = 4T(n/3)+n$ $\quad\quad\quad\quad$ $T(1)=1$

$$T(n) = n + 4T(n/3)$$

$$= n + 4[(n/3)+4T(n/3^2)] = n + (4/3)n+4^2T(n/3^2)$$

Iteration Method

$T(n) = 4T(n/3)+n$ **$T(1)=1$**

$T(n) = n + 4T(n/3)$

$= n + 4[(n/3)+4T(n/3^2)] = n + (4/3)n+4^2T(n/3^2)$

$= n + (4/3)n+4^2 [(n/3)^2+4T(n/3^3)]$

Iteration Method

$T(n) = 4T(n/3)+n$ **$T(1)=1$**

$$T(n) = n + 4T(n/3)$$

$$= n + 4[(n/3)+4T(n/3^2)] = n + (4/3)n+4^2T(n/3^2)$$

$$= n + (4/3)n+4^2\,[(n/3)^2+4T(n/3^3)]$$

$$= n + (4/3)n+(4/3)^2n+4^3T(n/3^3)$$

Iteration Method

$T(n) = 4T(n/3)+n$ **$T(1)=1$**

$$
\begin{aligned}
T(n) &= n + 4T(n/3) \\
&= n + 4[(n/3)+4T(n/3^2)] = n + (4/3)n+4^2T(n/3^2) \\
&= n + (4/3)n+4^2\,[(n/3)^2+4T(n/3^3)] \\
&= n + (4/3)n+(4/3)^2n+4^3T(n/3^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= n + (4/3)n+(4/3)^2n+\ \ldots.\ +(4/3)^{k-1}n+4^kT(n/3^k)
\end{aligned}
$$

Iteration Method

$T(n) = 4T(n/3)+n$ **$T(1)=1$**

$$T(n) = n + 4T(n/3)$$

$$= n + 4[(n/3)+4T(n/3^2)] = n + (4/3)n+4^2T(n/3^2)$$

$$= n + (4/3)n+4^2 [(n/3)^2+4T(n/3^3)]$$

$$= n + (4/3)n+(4/3)^2n+4^3T(n/3^3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= n + (4/3)n+(4/3)^2n+ \ \ldots. +(4/3)^{k-1}n+4^kT(n/3^k)$$

Assume $n/3^k= 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

Iteration Method

Assume $n/3^k = 1$ ➔ $3^k = n$ ➔ $k = \log_3(n)$

$$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$$

Iteration Method

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$$

$$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$$

Iteration Method

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$

$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$

$= n[((4^{\log_3 n}/3^{\log_3 n})-1)/(1/3)] + n^{\log_3 4}$

Iteration Method

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$$

$$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$$

$$= n[((4^{\log_3 n}/3^{\log_3 n})-1)/(1/3)] + n^{\log_3 4}$$

$$= 3n[((n^{\log_3 4}/n^{\log_3 3})-1)] + n^{\log_3 4}$$

Iteration Method

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$$

$$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$$

$$= n[((4^{\log_3 n}/3^{\log_3 n})-1)/(1/3)] + n^{\log_3 4}$$

$$= 3n[((n^{\log_3 4}/n^{\log_3 3})-1)] + n^{\log_3 4}$$

$$= 3n[((n^{\log_3 4}/n)-1)] + n^{\log_3 4}$$

Iteration Method

Assume $n/3^k = 1$ ➔ $3^k = n$ ➔ $k = \log_3(n)$

$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$

$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$

$= n[((4^{\log_3 n}/3^{\log_3 n})-1)/(1/3)] + n^{\log_3 4}$

$= 3n[((n^{\log_3 4}/n^{\log_3 3})-1)] + n^{\log_3 4}$

$= 3n[((n^{\log_3 4}/n)-1)] + n^{\log_3 4}$

$= 3n^{\log_3 4} -3n + n^{\log_3 4}$

Iteration Method

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$$

$$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$$

$$= n[((4^{\log_3 n}/3^{\log_3 n})-1)/(1/3)] + n^{\log_3 4}$$

$$= 3n[((n^{\log_3 4}/n^{\log_3 3})-1)] + n^{\log_3 4}$$

$$= 3n[((n^{\log_3 4}/n)-1)] + n^{\log_3 4}$$

$$= 3n^{\log_3 4} - 3n + n^{\log_3 4}$$

$$= 4n^{\log_3 4} - 3n$$

Iteration Method

Assume $n/3^k = 1 \rightarrow 3^k = n \rightarrow k = \log_3(n)$

$T(n) = n[1+(4/3)+(4/3)^2+\ldots.+(4/3)^{k-1}] + 4^k T(1)$

$= n[((4/3)^k-1)/((4/3)-1)] + 4^{\log_3 n}$

$= n[((4^{\log_3 n}/3^{\log_3 n})-1)/(1/3)] + n^{\log_3 4}$

$= 3n[((n^{\log_3 4}/n^{\log_3 3})-1)] + n^{\log_3 4}$

$= 3n[((n^{\log_3 4}/n)-1)] + n^{\log_3 4}$

$= 3n^{\log_3 4} - 3n + n^{\log_3 4}$

$= 4n^{\log_3 4} - 3n$

$= \mathbf{O(n^{\log_3 4})}$

Iteration Method

$T(n) = 3T(n/3)+n$ **$T(1)=1$**

$$
\begin{aligned}
T(n) &= n + 3T(n/3) \\
&= n + 3[(n/3)+3T(n/3^2)] = n + n + 3^2T(n/3^2) \\
&= 2\,n+3^2\,[(n/3^2)+3T(n/3^3)] \\
&= 3\,n + 3^3\,T(n/3^3) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= k\,n + 3^k\,T(n/3^k)
\end{aligned}
$$

Assume $n/3^k= 1 \Rightarrow 3^k = n \Rightarrow k = \log_3(n)$

$$
\begin{aligned}
T(n) &= n\log_3(n) + n\,T(1) \\
&= n\log_3(n) + n \\
&= \mathbf{O(n\log_3(n))}
\end{aligned}
$$

Iteration Method

$\mathbf{T(n) \;\; = 2T(n/2)+n^2}$

$T(n) \;\; = n^2 + 2T(n/2)$

$= n^2 + 2[(n/2)^2+2T(n/2^2)]= n^2+n^2/2 + 2^2\, T(n/2^2)$

$= n^2 + n^2/2 + 2^2\, [(n/2^2)^2+2T(n/2^3)]$

$= n^2+n^2/2 + n^2/2^2 +2^3\, T(n/2^3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= n^2[1+ (1/2) + (1/2)^2 + \ldots. + (1/2)k\text{-}1^2\,] + 2^k\, T(n/2^k)$

Assume $n/2^k= 1 \;\; \Rightarrow \;\; 2^k = n \;\; \Rightarrow \;\; k = \log_2(n)$

$T(n) \;\; = n^2\, [(1-(1/2)^k) / (1-(1/2))] + n\, T(1)$

$= 2\, n^2\, [\, 1-(1/2^k)] + n = 2\, n^2\, [\, 1-(1/2^{\log n})] + n$

$= 2\, n^2\, [\, 1-(1/n^{\log 2})] + n = 2\, n^2\, [\, 1-(1/n)] + n$

$= 2\, n^2 - 2n)+ n = \mathbf{O(n^2)}$

Geometric Progression

a, ar, ar^2, ar^3, ar^4,…,ar^n

a = First term　　　　r = common ratio

n^{th} term = ar^{n-1}

Sum of n term　　$S_n = a+ar+ar^2+ ar^3+\ldots+ar^{n-1}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Infinite Geometric Progression　a, ar, ar^2, ar^3, ……ar^{n-1},…….

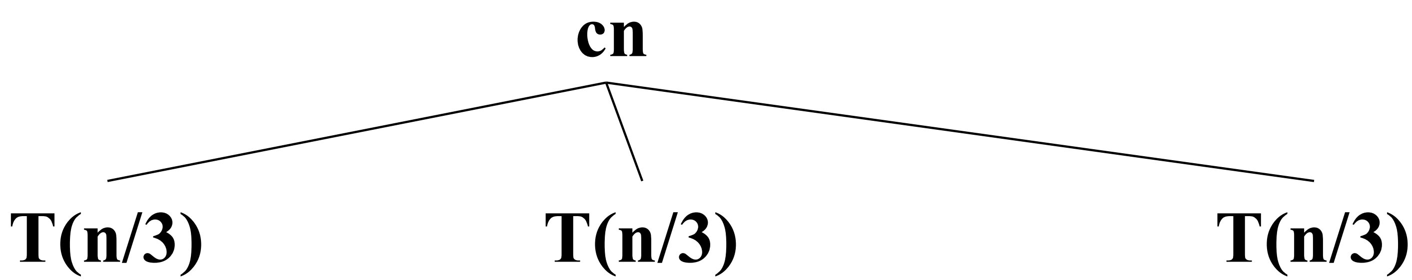
The sum of infinite geometric series $S_\infty = \dfrac{a_1}{1 - r}$

Recursion Tree Method

Recursion Tree Method

$T(n) = 3T(n/3) + cn$

$$T(n) = 3T(n/3) + cn$$



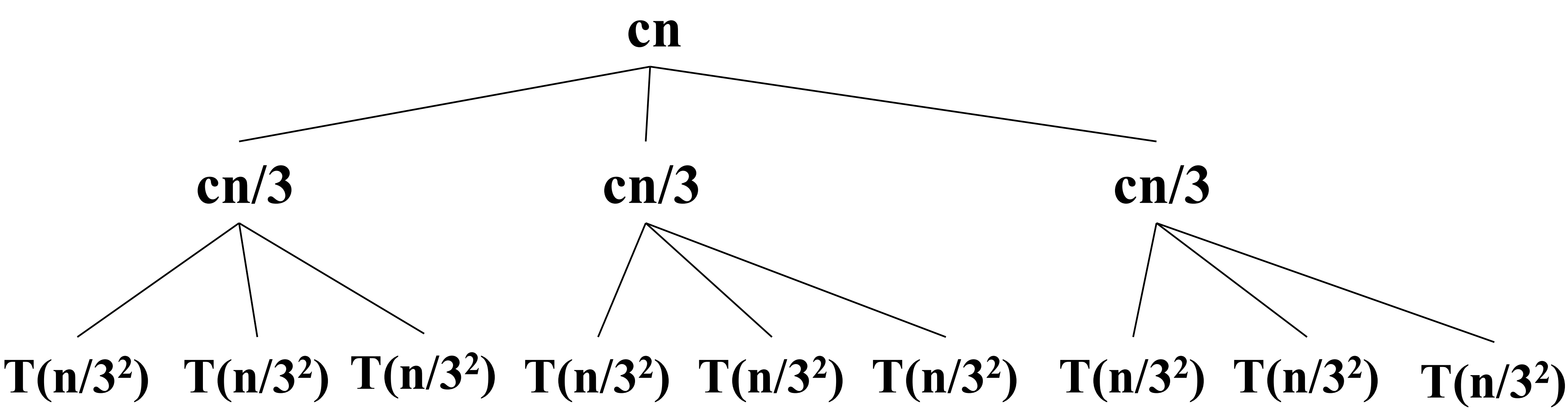
$$T(n) = 3T(n/3) + cn$$

$$cn$$

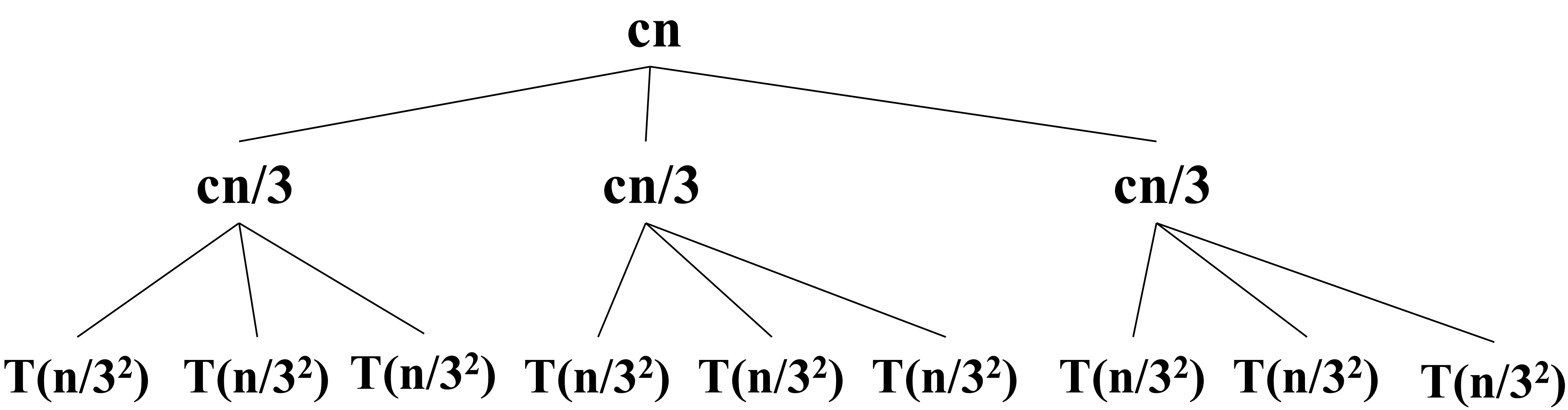
$$T(n/3) \qquad T(n/3) \qquad T(n/3)$$

$$T(n/3) = 3T(n/3^2) + (cn/3)$$

$$T(n/3) = 3T(n/3^2) + (cn/3)$$

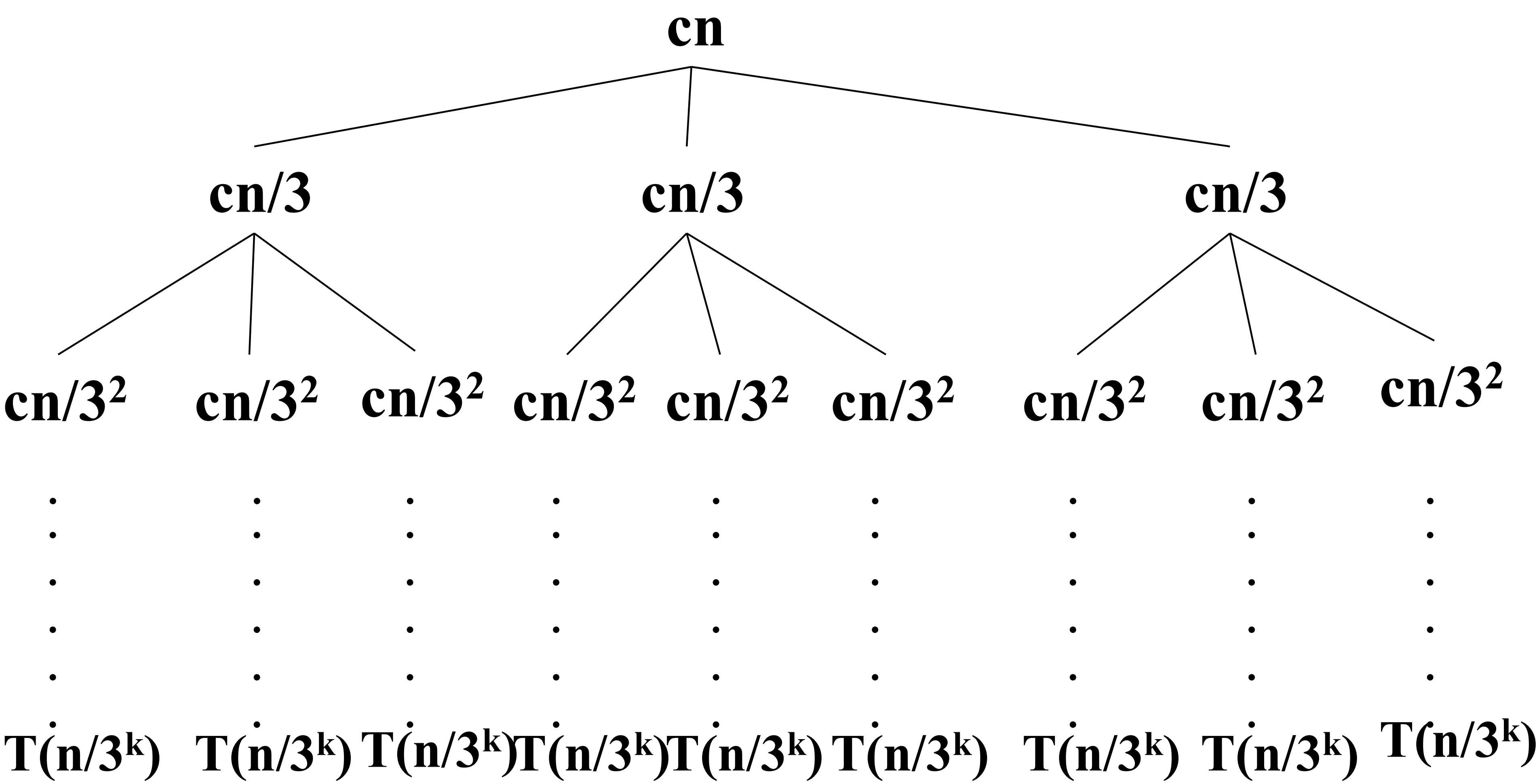


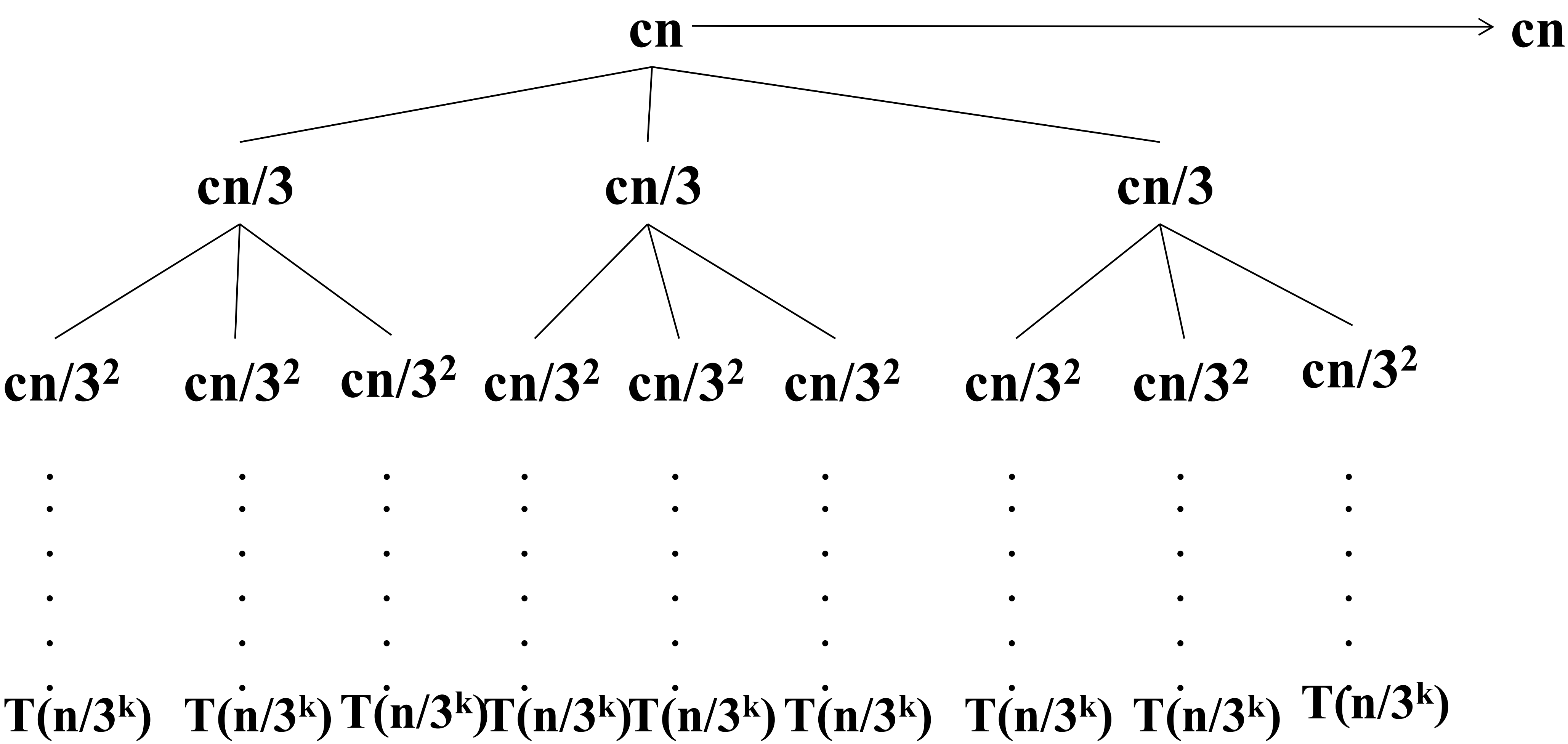
$$T(n/3) = 3T(n/3^2) + (cn/3)$$



$$T(n/3^2) = 3T(n/3^3) + (cn/3^2)$$

$$T(n/3^2) = 3T(n/3^3) + (cn/3^2)$$

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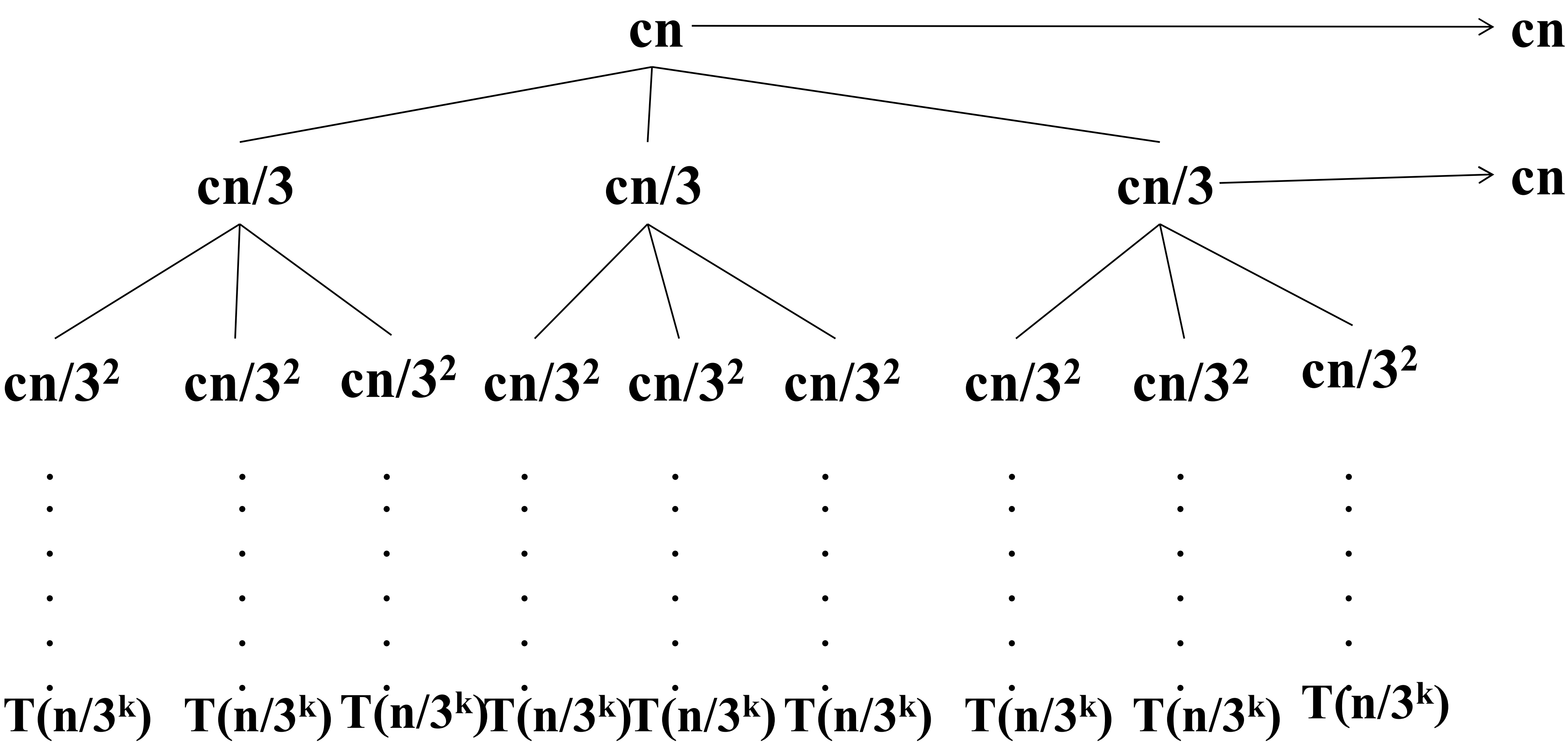
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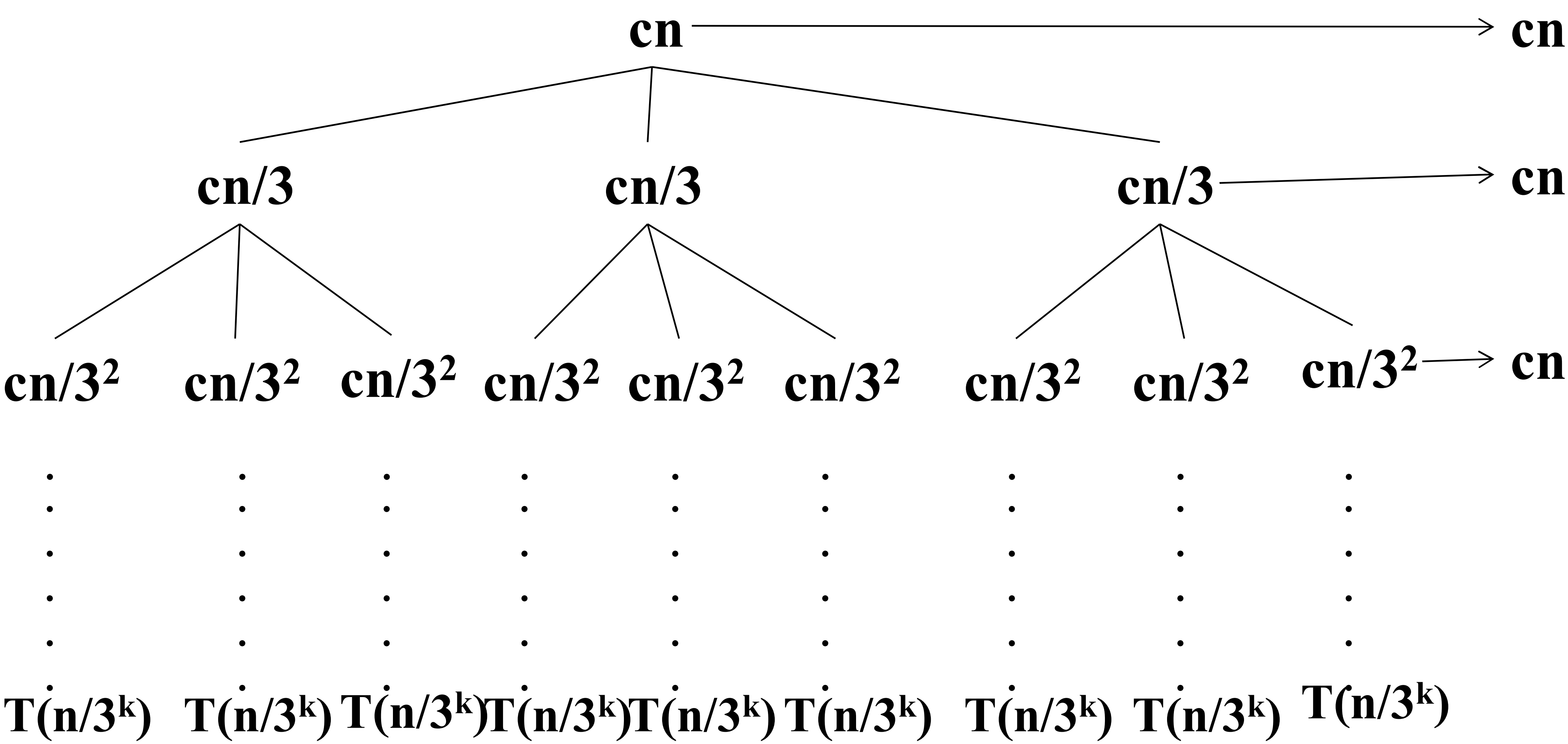

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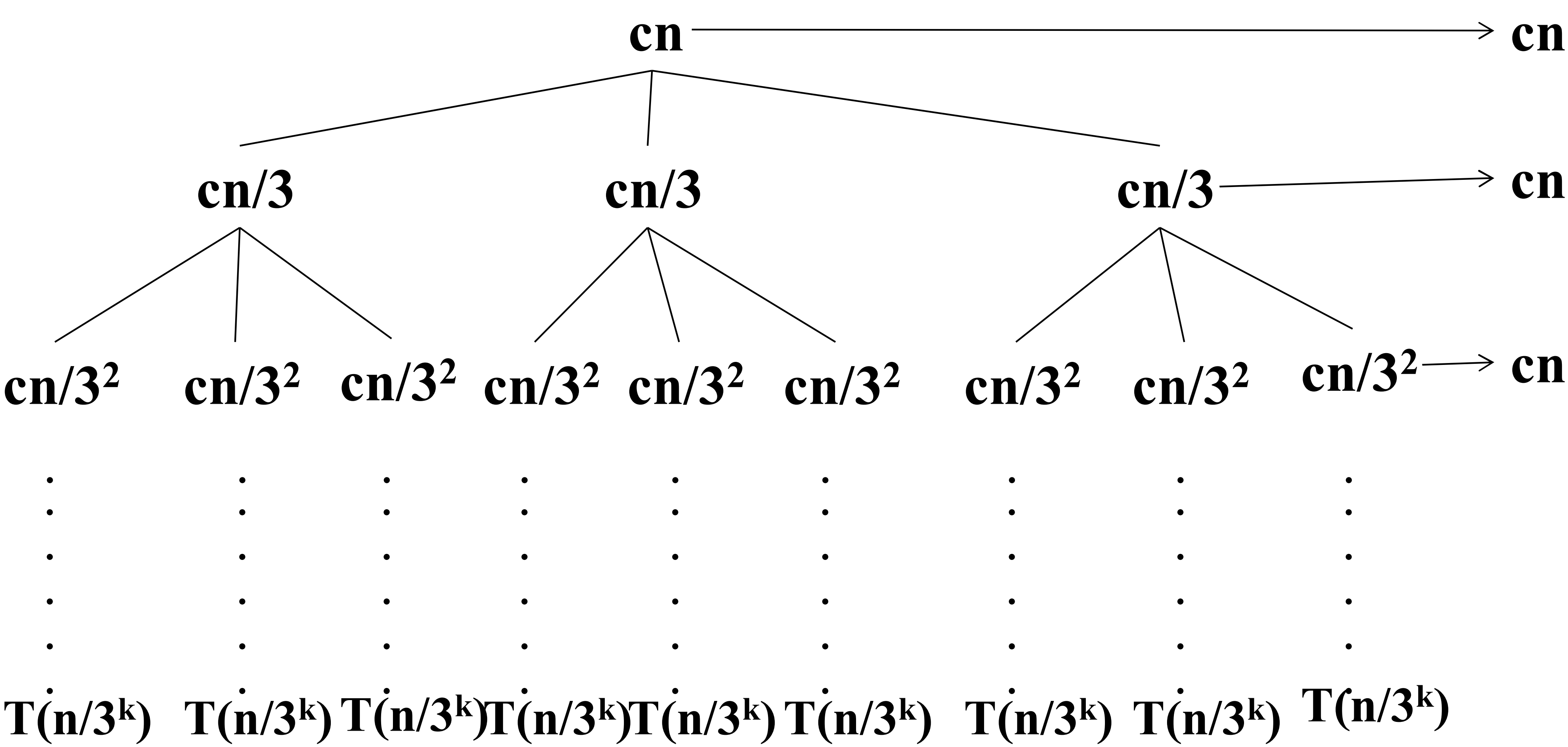
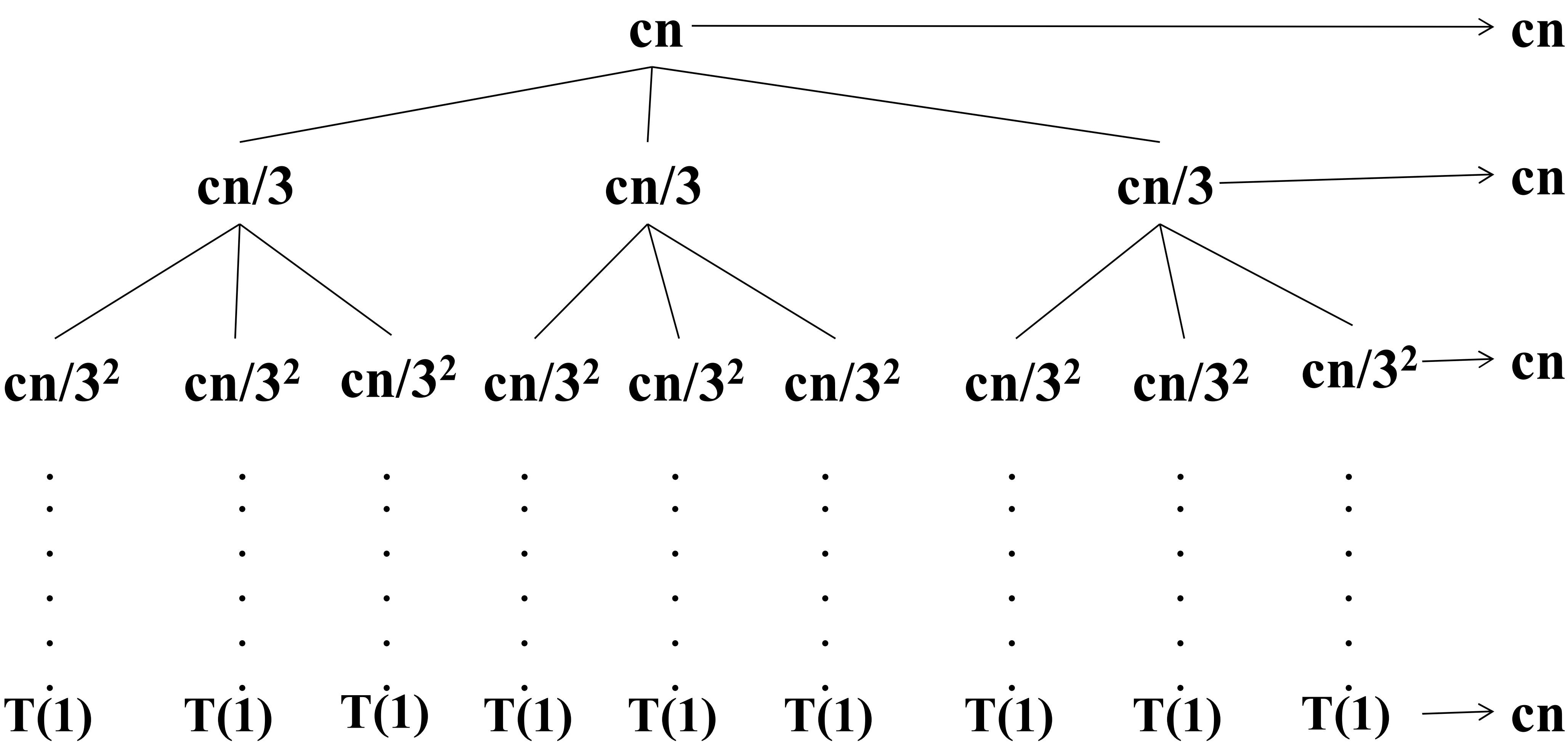


Assume that $n/3^k=1 \Rightarrow 3^k=n \Rightarrow k=\log_3(n)$



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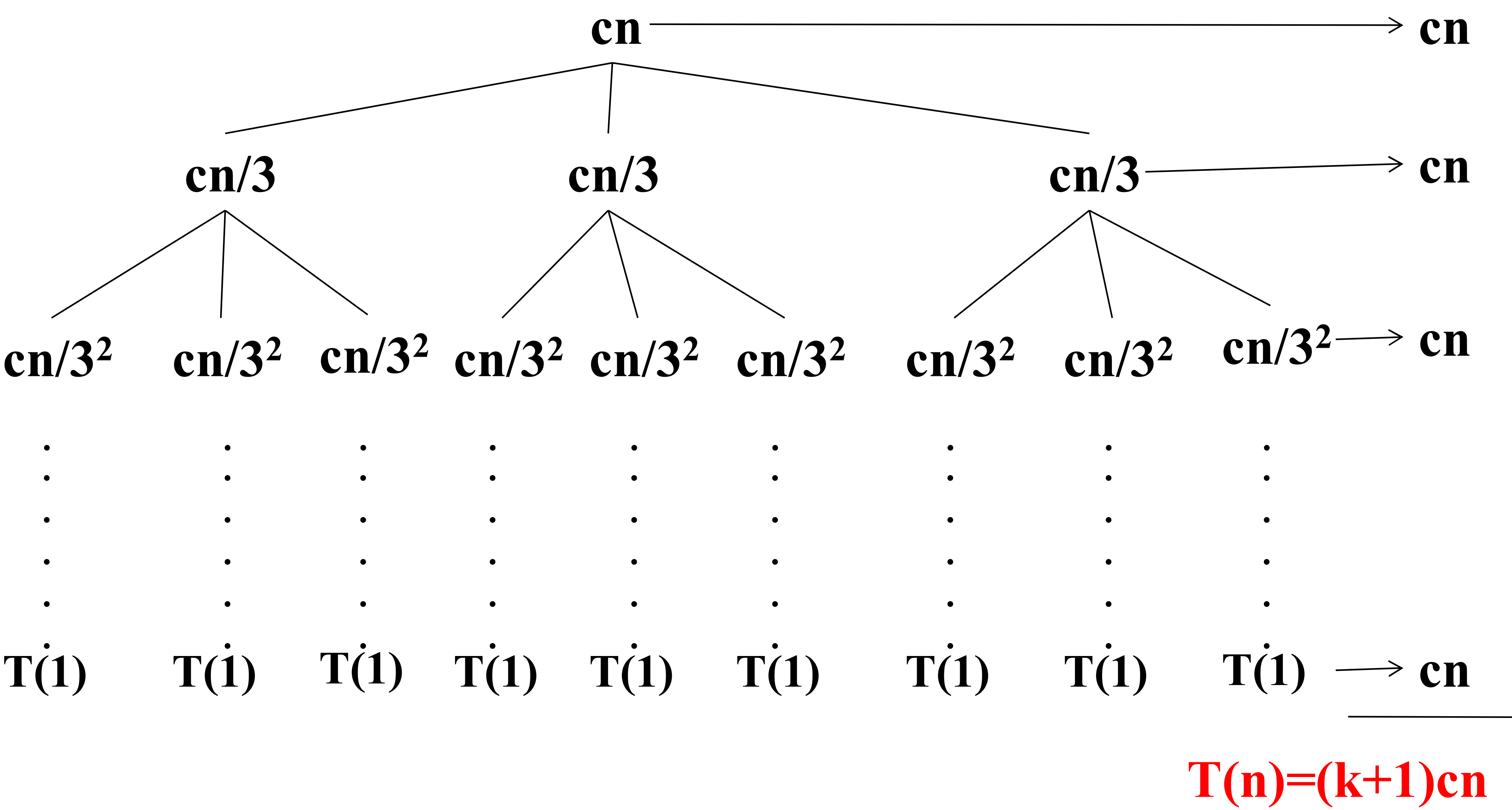
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T(n)=(k+1)cn

Assume that $n/3^k=1 \rightarrow 3^k=n \rightarrow k=\log_3(n)$

$T(n) = (k+1)cn$

Assume that $n/3^k=1 \Rightarrow 3^k=n \Rightarrow k=\log_3(n)$

$$T(n) = (k+1)cn$$
$$=(\log_3(n)+1)cn$$

Assume that $n/3^k=1 \Rightarrow 3^k=n \Rightarrow k=\log_3(n)$

$$
\begin{aligned}
T(n) &= (k+1)cn \\
&=(\log_3(n)+1)cn \\
&=cn\log_3(n)+cn
\end{aligned}
$$

Assume that $n/3^k=1 \Rightarrow 3^k=n \Rightarrow k=\log_3(n)$

$T(n) = (k+1)cn$

$=(\log_3(n)+1)cn$

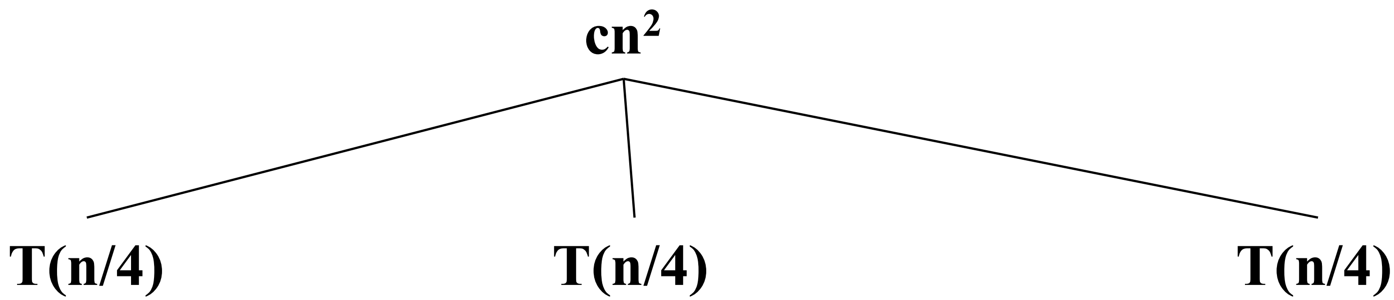
$=cn\log_3(n)+cn$

$=\mathbf{O(n\log_3(n))}$

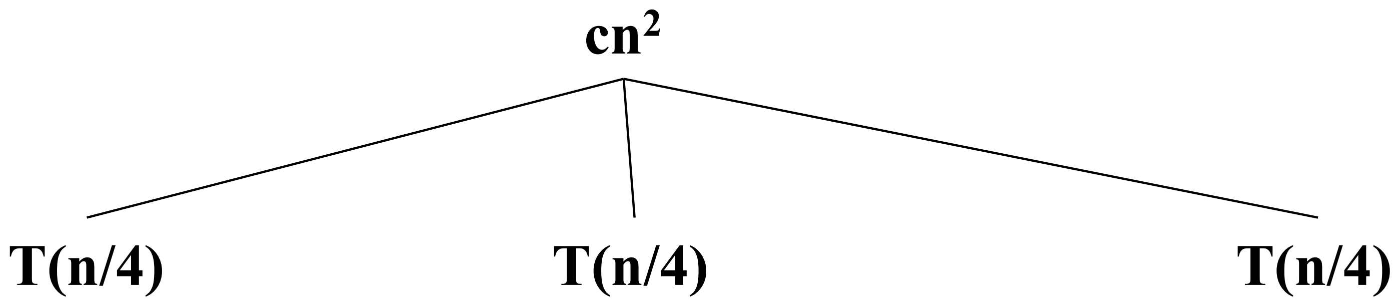
Recursion Tree Method

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 3T(n/4) + cn^2 & \text{Otherwise} \end{cases}$$

$$T(n) = 3T(n/4) + cn^2$$

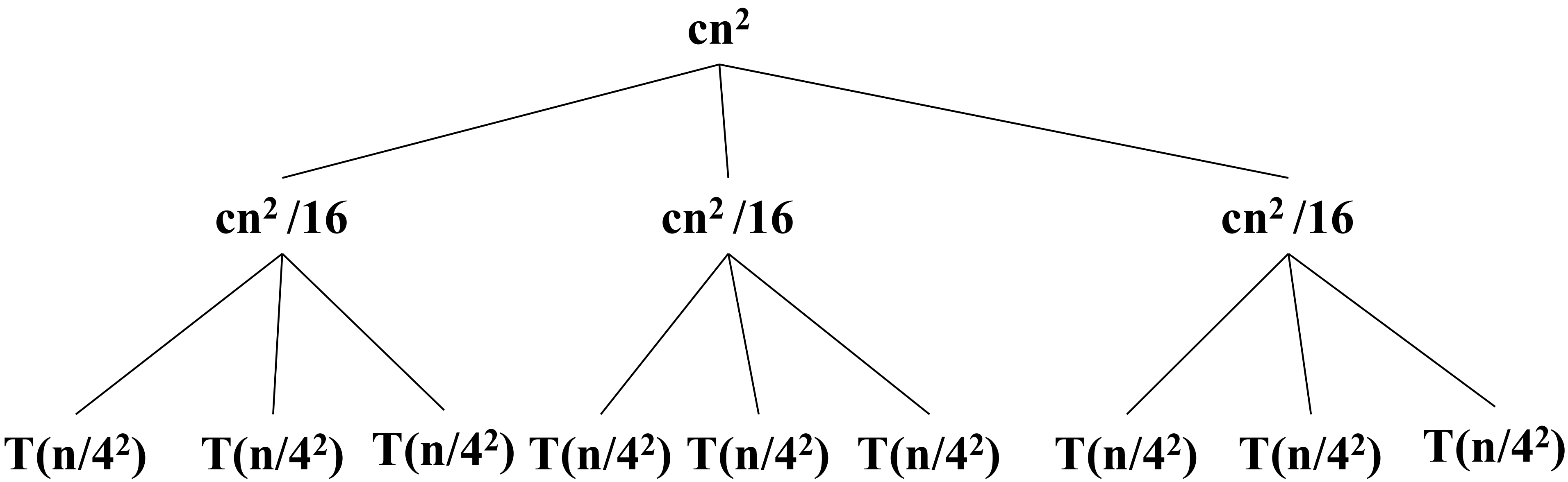


$$T(n) = 3T(n/4) + cn^2$$

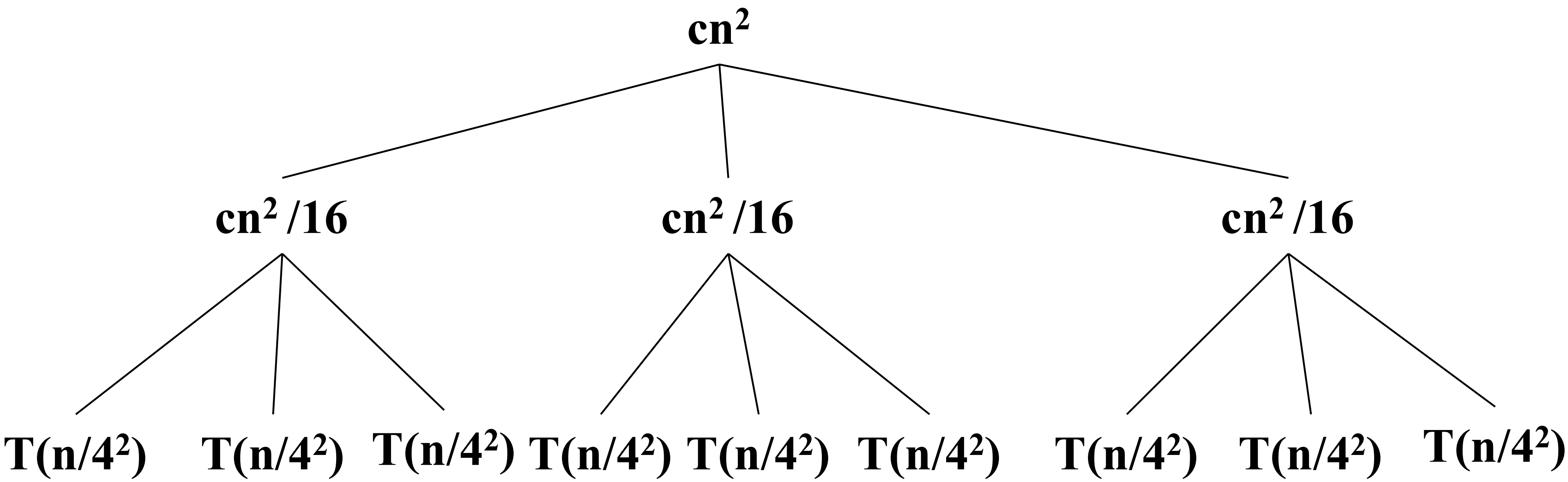


$$T(n/4) = 3T(n/4^2) + cn^2/16$$

$$T(n/4) = 3T(n/4^2) + cn^2/16$$

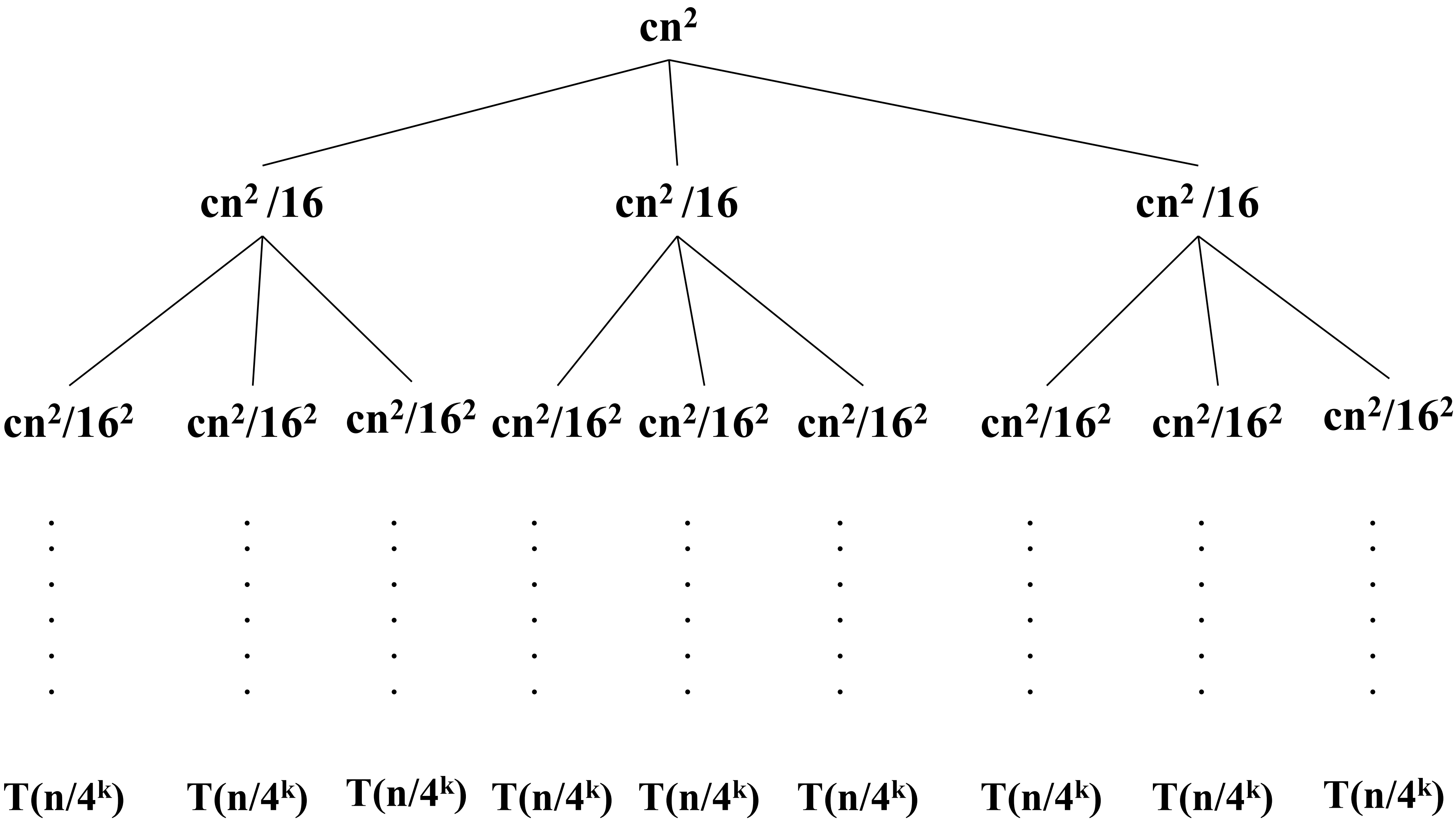


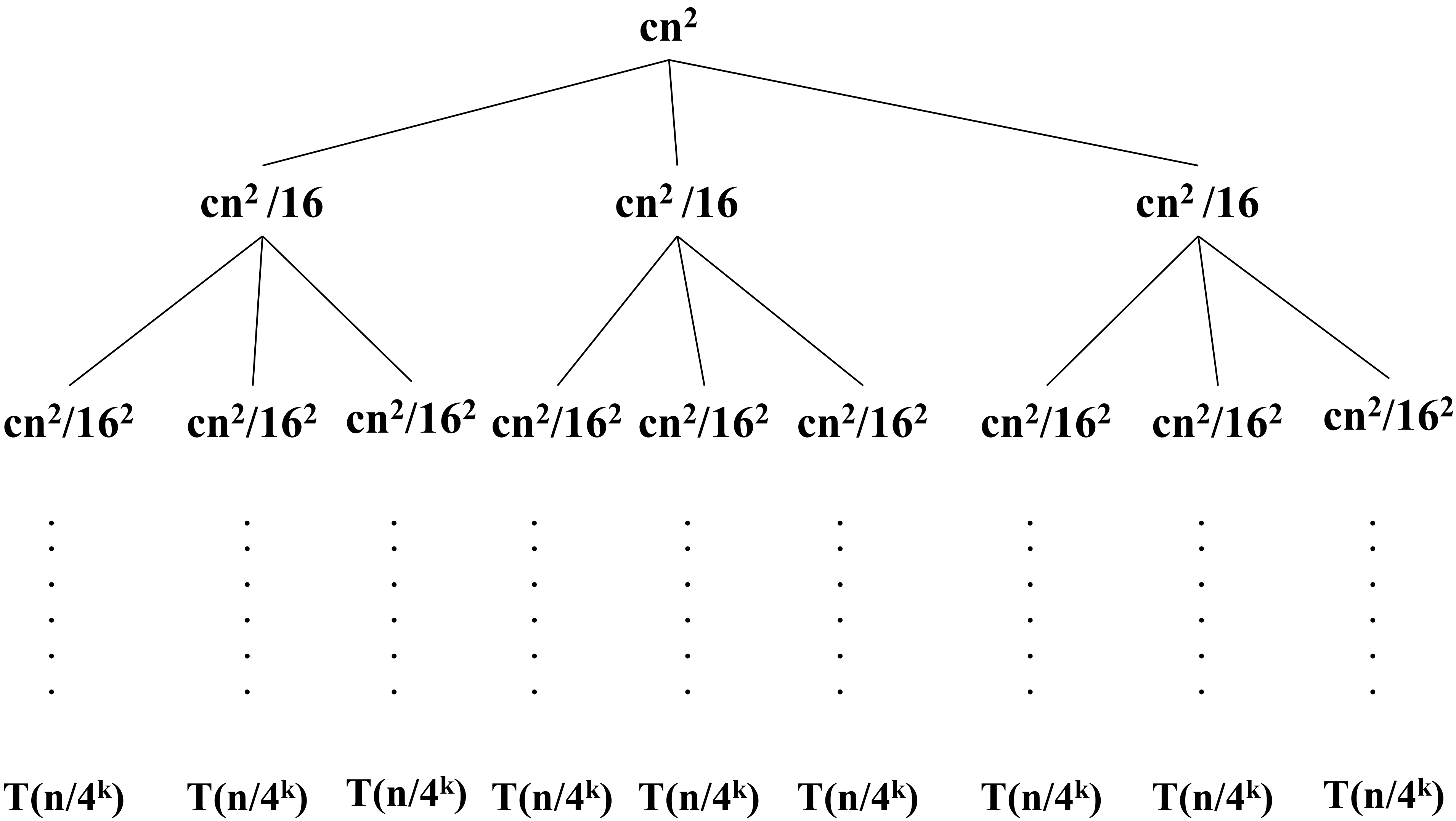
$$T(n/4) = 3T(n/4^2) + cn^2/16$$



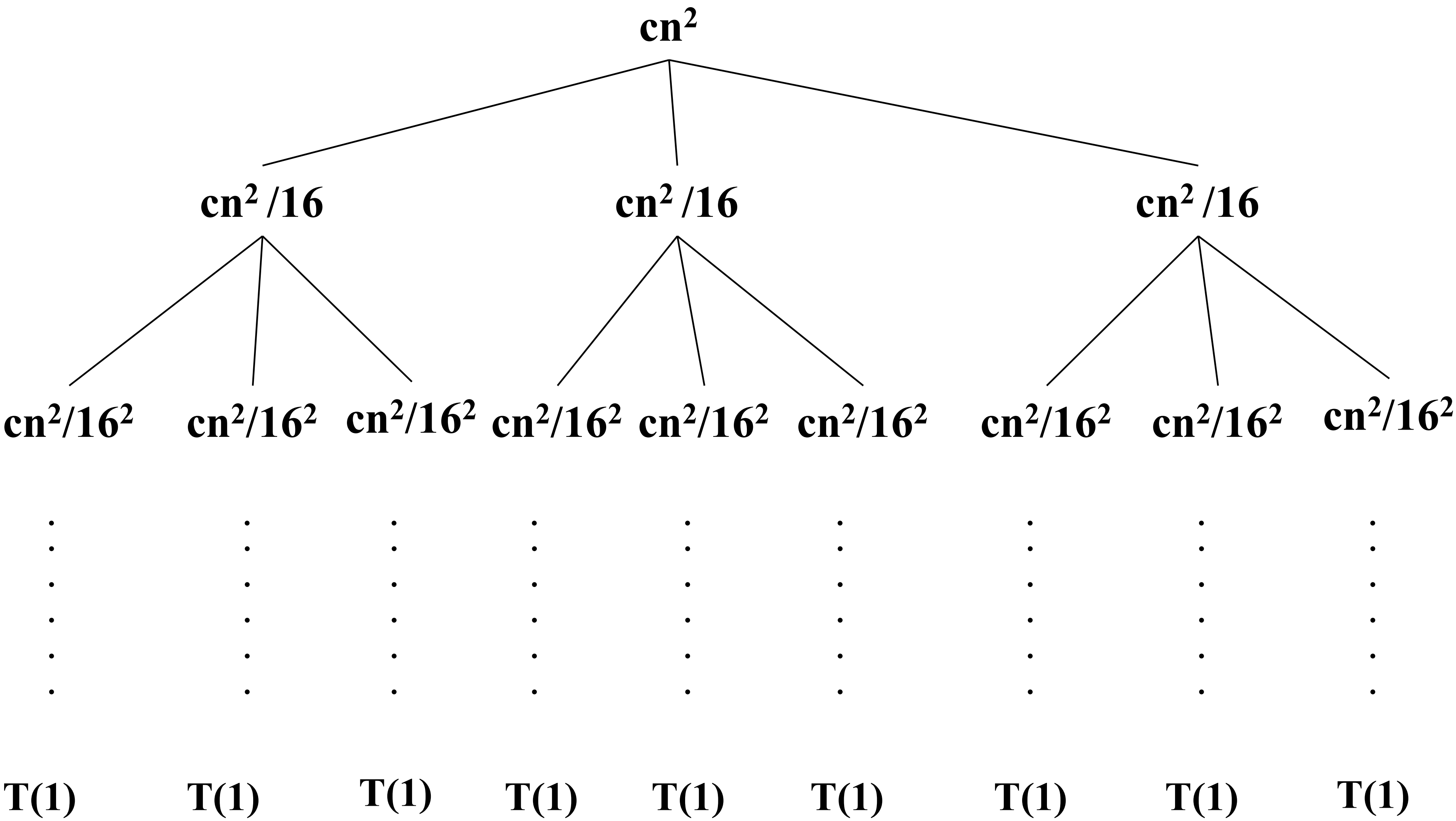
$$T(n/4^2) = 3T(n/4^3) + cn^2/16^2$$

$$T(n/4^2) = 3T(n/4^3) + cn^2/16^2$$

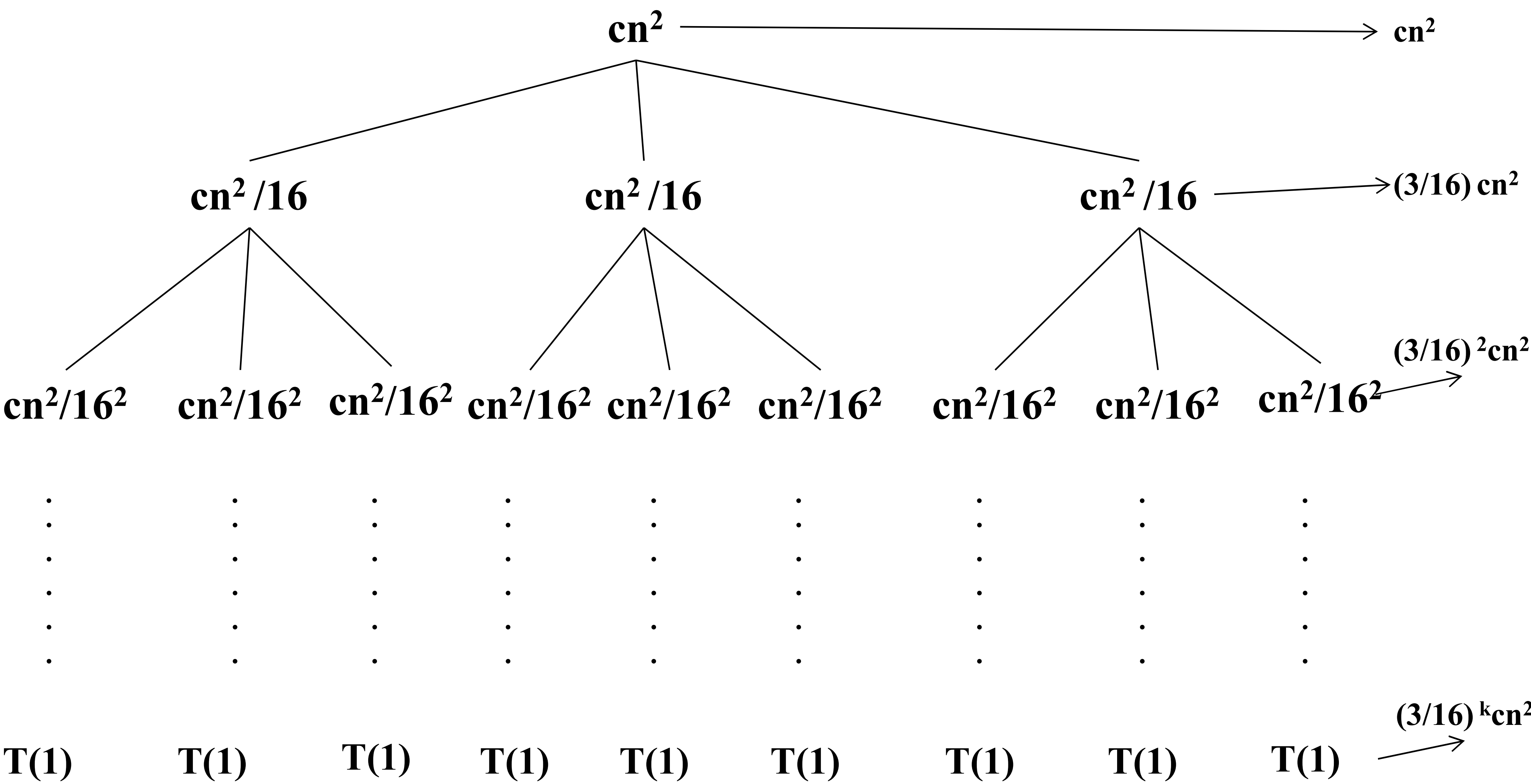




Assume that $n/4^k=1 \rightarrow 4^k=n \rightarrow k=\log_4 n$



Assume that $n/4^k=1$ ➔ $4^k=n$ ➔ $k=\log_4 n$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$$T(n) = cn^2 + (3/16)\, cn^2 + (3/16)^2\, cn^2 + \ldots\ldots + (3/16)^k\, cn^2$$

$$
\begin{aligned}
T(n) &= cn^2 + (3/16)\, cn^2 + (3/16)^2\, cn^2 + \ldots\ldots + (3/16)^k\, cn^2 \\
&= cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= cn^2 + (3/16)\, cn^2 + (3/16)^2\, cn^2 + \ldots\ldots + (3/16)^k\, cn^2 \\
&= cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\leq cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= cn^2 + (3/16)\,cn^2 + (3/16)^2\,cn^2 + \ldots\ldots + (3/16)^k\,cn^2 \\
&= cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\leq cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots] \\
&= cn^2[1/(1 - (3/16))]
\end{aligned}
$$

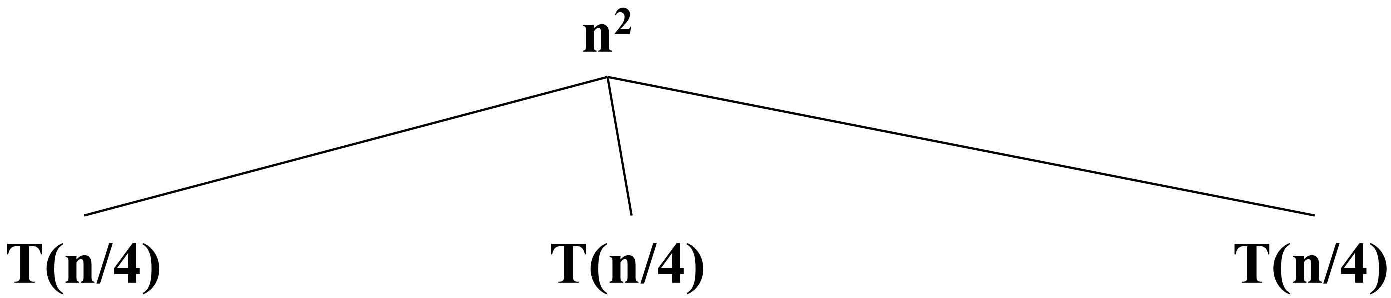
$$
\begin{aligned}
T(n) &= cn^2 + (3/16)\, cn^2 + (3/16)^2\, cn^2 + \ldots\ldots + (3/16)^k\, cn^2 \\
&= cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\le cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots] \\
&= cn^2[1/(1 - (3/16))] \\
&= (16/13)\, cn^2
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= cn^2 + (3/16)\, cn^2 + (3/16)^2\, cn^2 + \ldots\ldots + (3/16)^k\, cn^2 \\
&= cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\le cn^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots] \\
&= cn^2[1/(1- (3/16))] \\
&= (16/13)\, cn^2 \\
&= \mathbf{O(n^2)}
\end{aligned}
$$

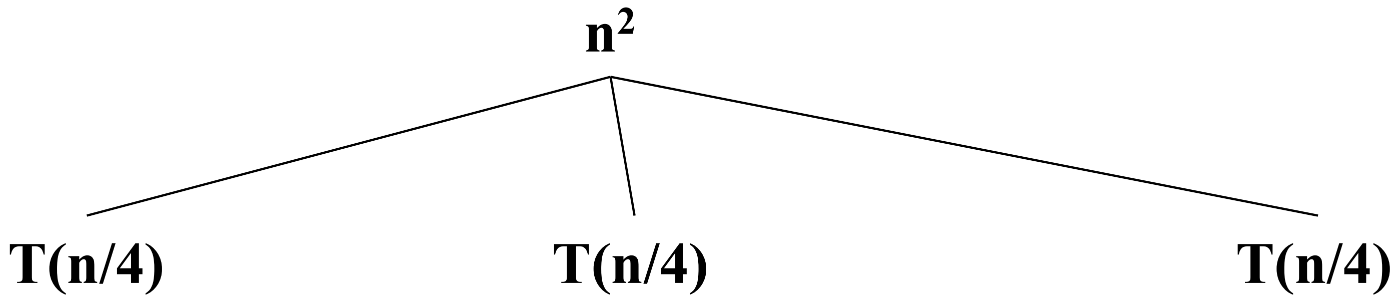
Recursion Tree Method

$T(n) = \quad 3T(n/4) + n^2$

$T(n) = 3T(n/4) + n^2$

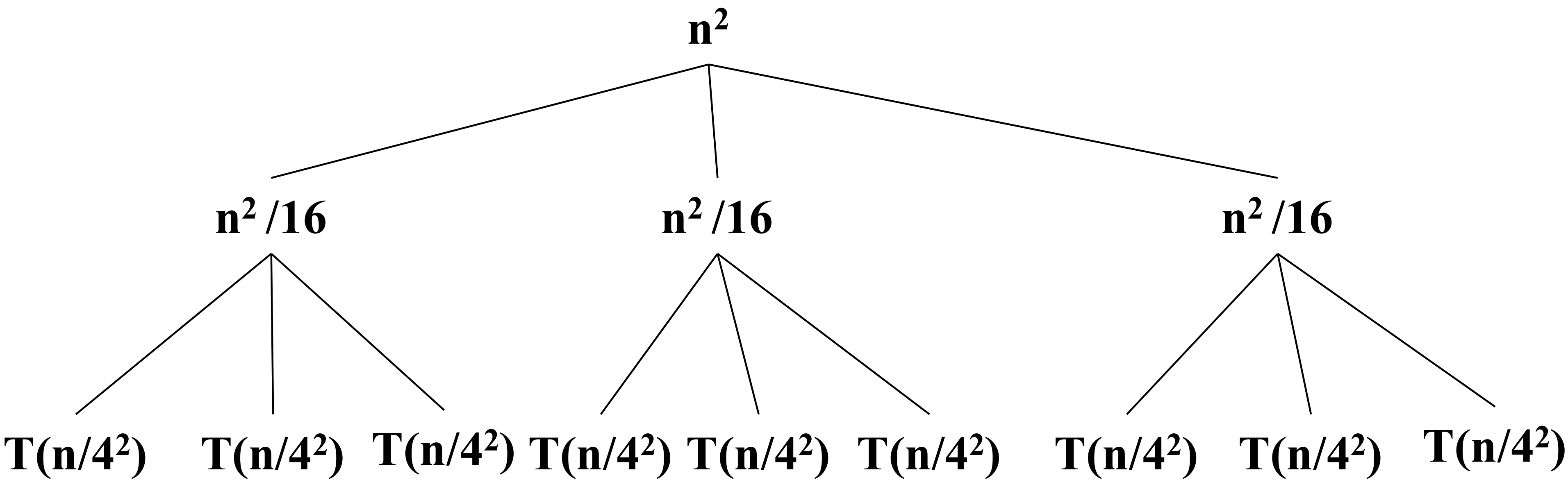


$$T(n) = 3T(n/4) + n^2$$

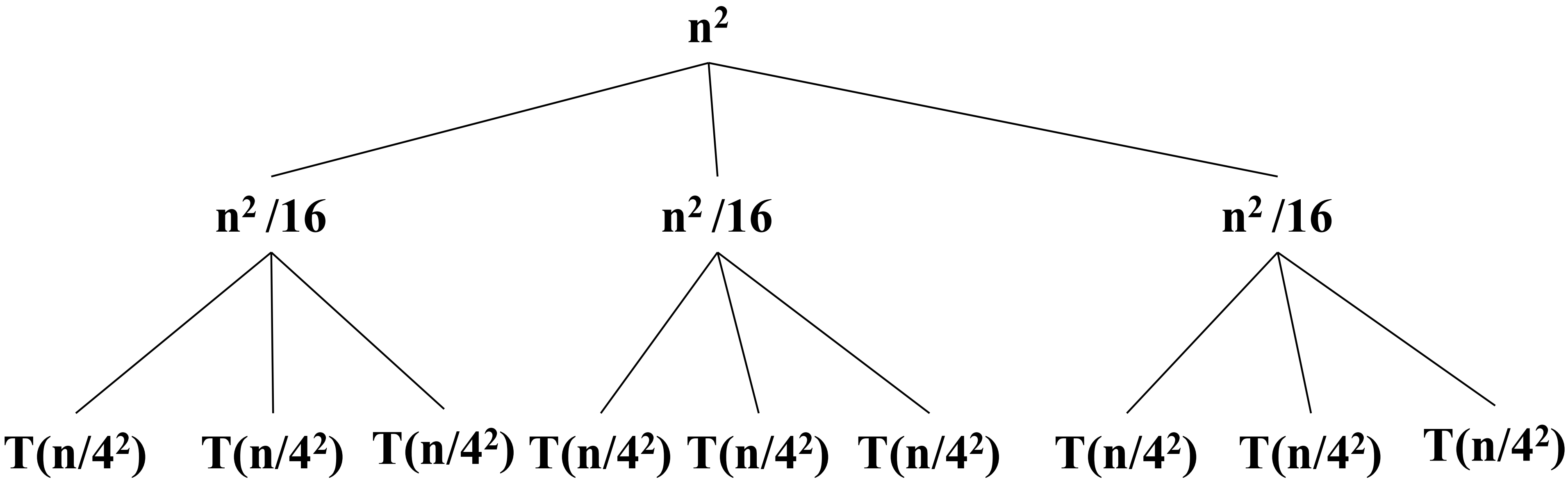


$$T(n/4) = 3T(n/4^2) + n^2/16$$

$T(n/4) = 3T(n/4^2) + n^2/16$

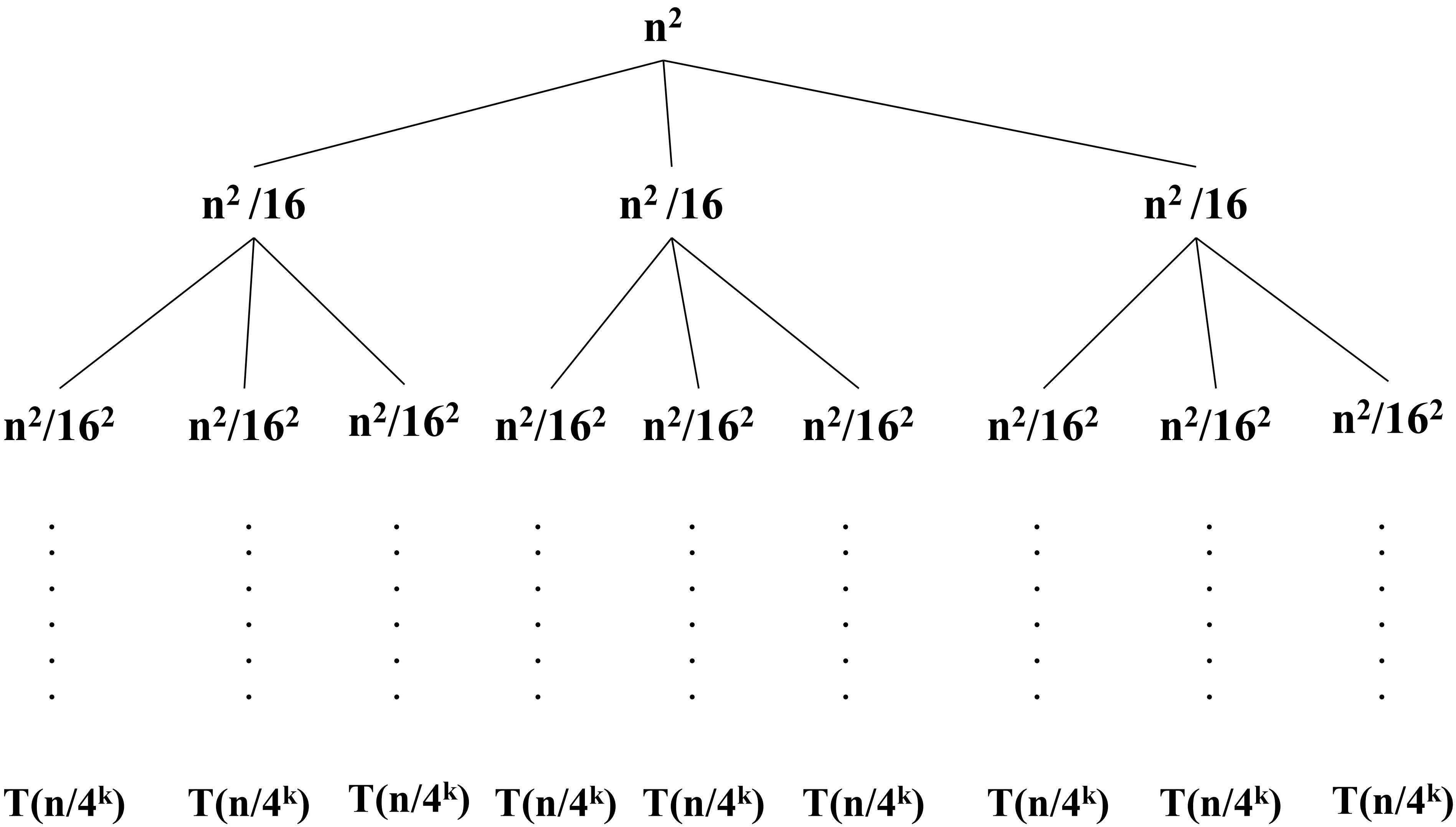


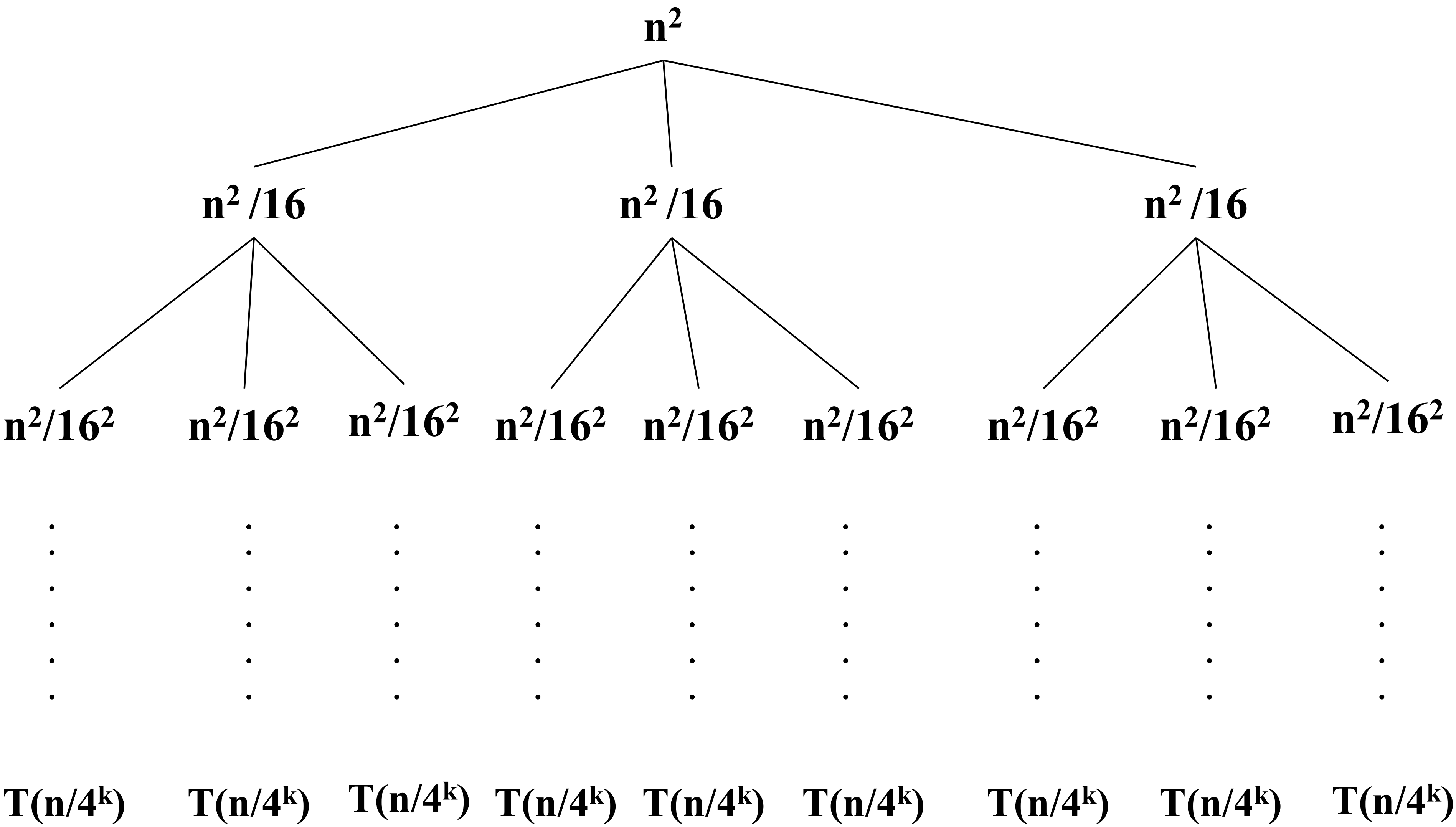
$$T(n/4) = 3T(n/4^2) + n^2/16$$



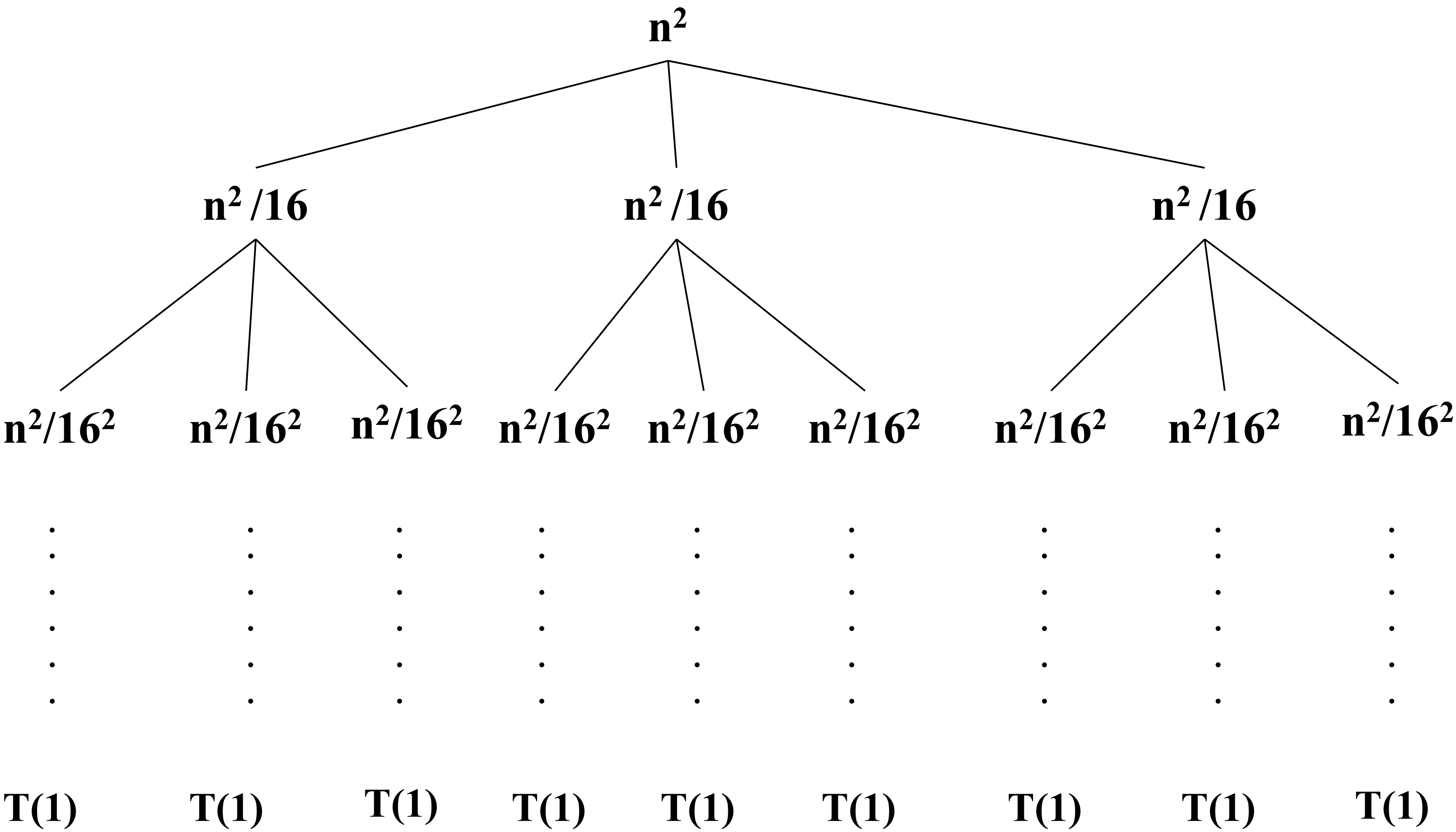
$$T(n^2/4^2) = 3T(n/4^3) + n^2/16^2$$

$$T(n/4^2) = 3T(n/4^3) + n^2/16^2$$

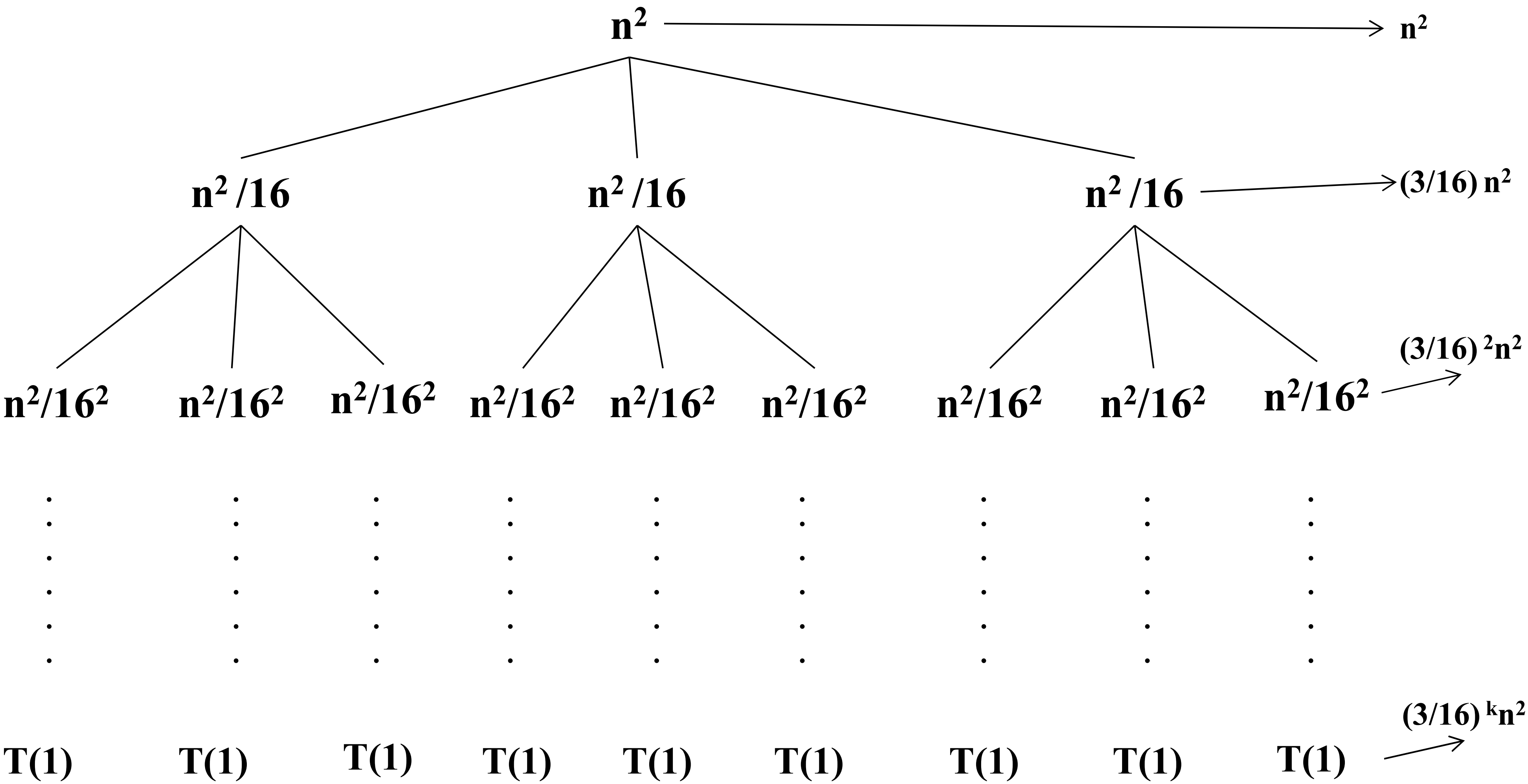




Assume that $n/4^k=1 \rightarrow 4^k=n \rightarrow k=\log_4 n$



Assume that $n/4^k=1$ ➔ $4^k=n$ ➔ $k=\log_4 n$



n2
n2
n2 /16
n2 /16
n2 /16
(3/16) n2
n2/162
(3/16) 2n2
T(1)
(3/16) kn2

$$T(n) \quad = n^2 + (3/16)\, n^2 + (3/16)^2\, n^2 + \ldots\ldots + (3/16)^k\, n^2$$

$$T(n) = n^2 + (3/16)\, n^2 + (3/16)^2\, n^2 + \ldots\ldots + (3/16)^k\, n^2$$
$$= n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k]$$

$$T(n) = n^2 + (3/16)\, n^2 + (3/16)^2\, n^2 + \ldots\ldots + (3/16)^k\, n^2$$
$$= n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k]$$
$$\leq n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots]$$

$$
\begin{aligned}
T(n) &= n^2 + (3/16)\, n^2 + (3/16)^2\, n^2 + \ldots\ldots + (3/16)^k\, n^2 \\
&= n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\leq n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots] \\
&= n^2[1/(1 - (3/16))]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2 + (3/16)\, n^2 + (3/16)^2\, n^2 + \ldots\ldots + (3/16)^k\, n^2 \\
&= n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\le n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots] \\
&= n^2[1/(1- (3/16))] \\
&= (16/13)\, n^2
\end{aligned}
$$

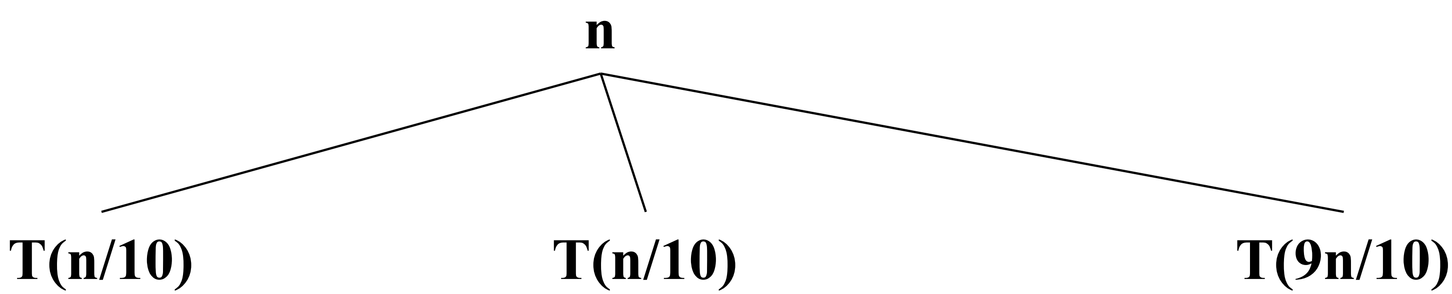
$$
\begin{aligned}
T(n) &= n^2 + (3/16)\, n^2 + (3/16)^2\, n^2 + \ldots\ldots + (3/16)^k\, n^2 \\
&= n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots + (3/16)^k] \\
&\leq n^2[1 + (3/16) + (3/16)^2 + \ldots\ldots\ldots\ldots] \\
&= n^2[1/(1- (3/16))] \\
&= (16/13)\, n^2 \\
&\mathbf{= O(n^2)}
\end{aligned}
$$

Recursion Tree Method

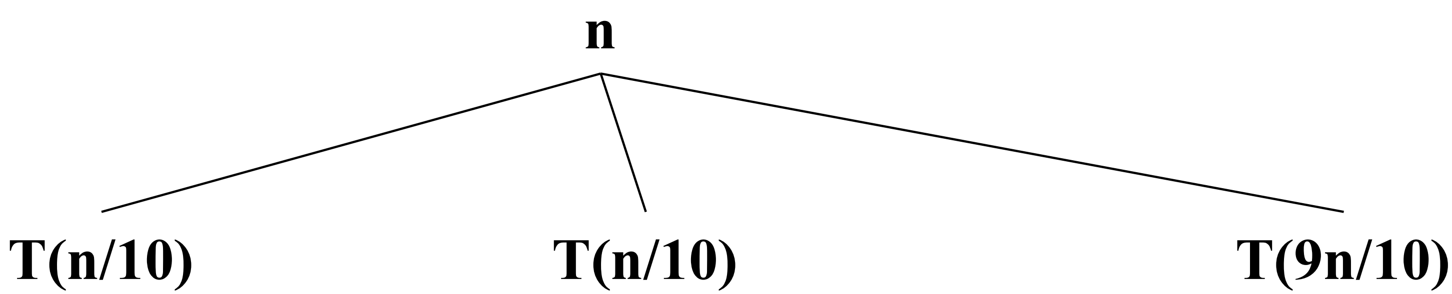
$T(n) = \quad 2T(n/10) + T(9n/10) + n$

Assume constant time for small value of n

$$T(n) = \quad 2T(n/10) + T(9n/10) + n$$



$$T(n) = \quad 2T(n/10) + T(9n/10) + n$$

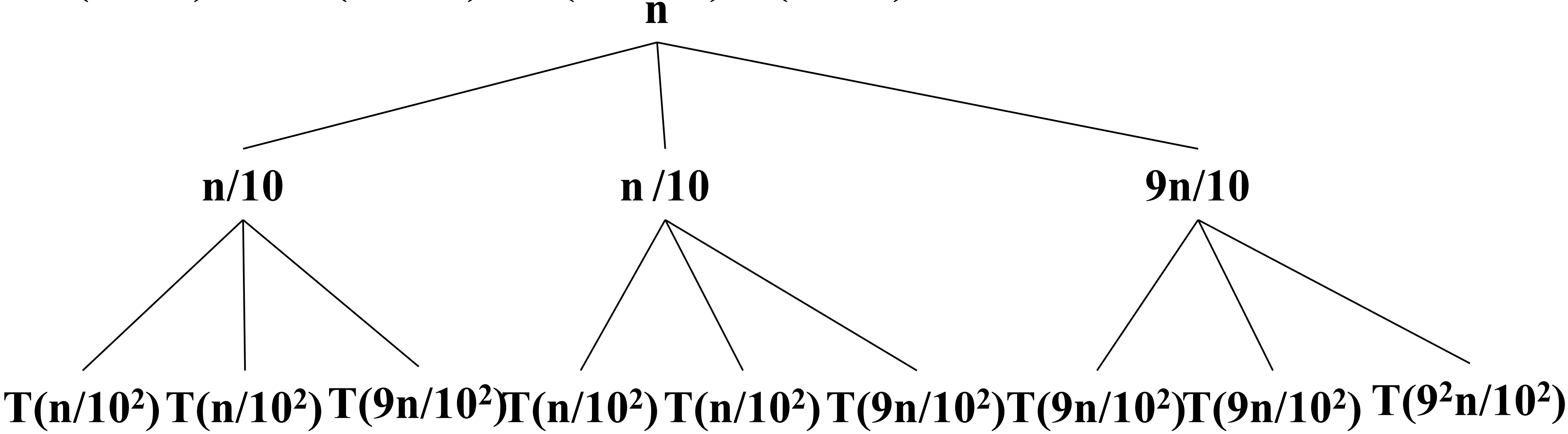


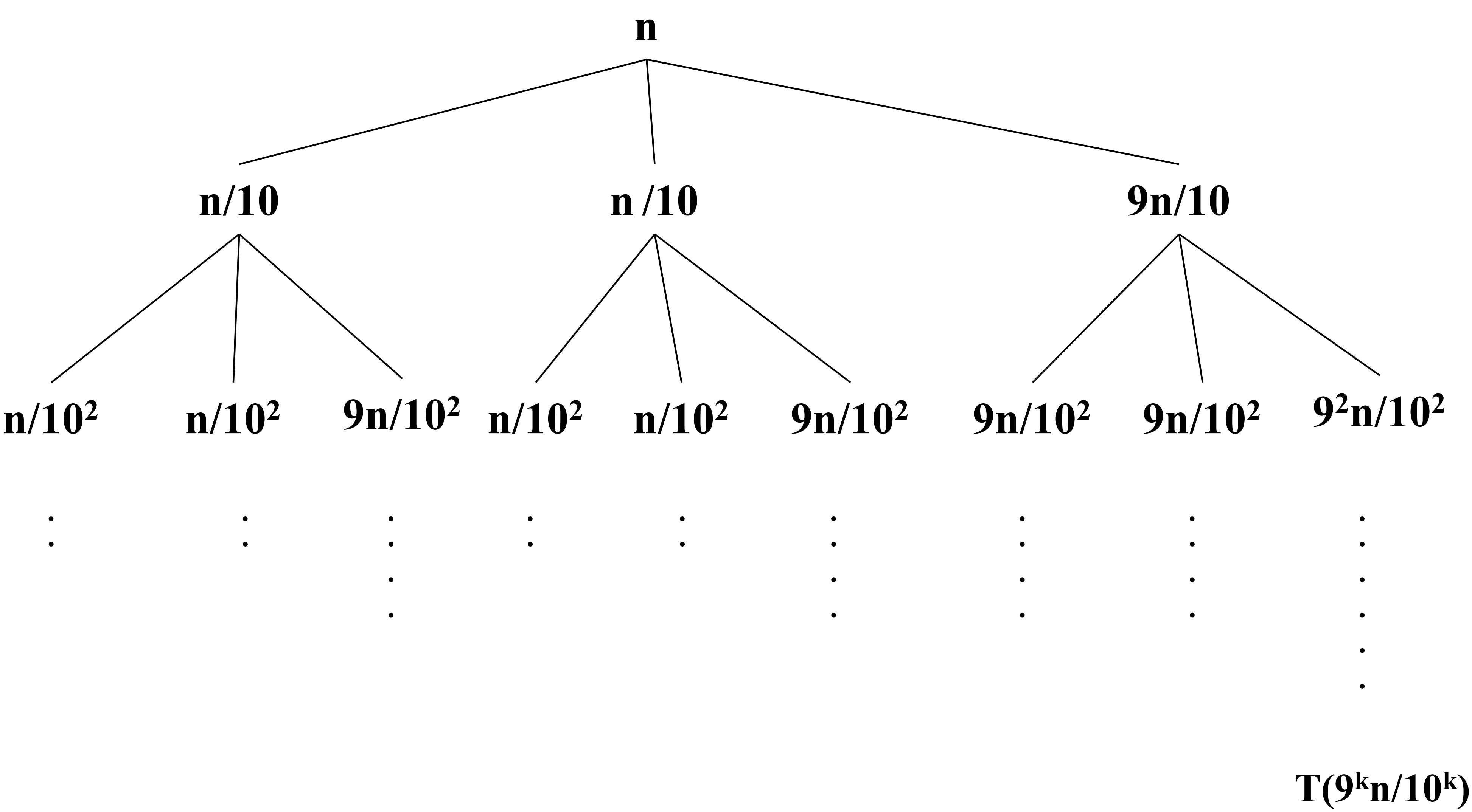
$$T(n/10) = \quad 2T(n/10^2) + T(9n/10^2) + (n/10)$$

$$T(9n/10) = \quad 2T(9n/10^2) + T(9^2n/10^2) + (9n/10)$$

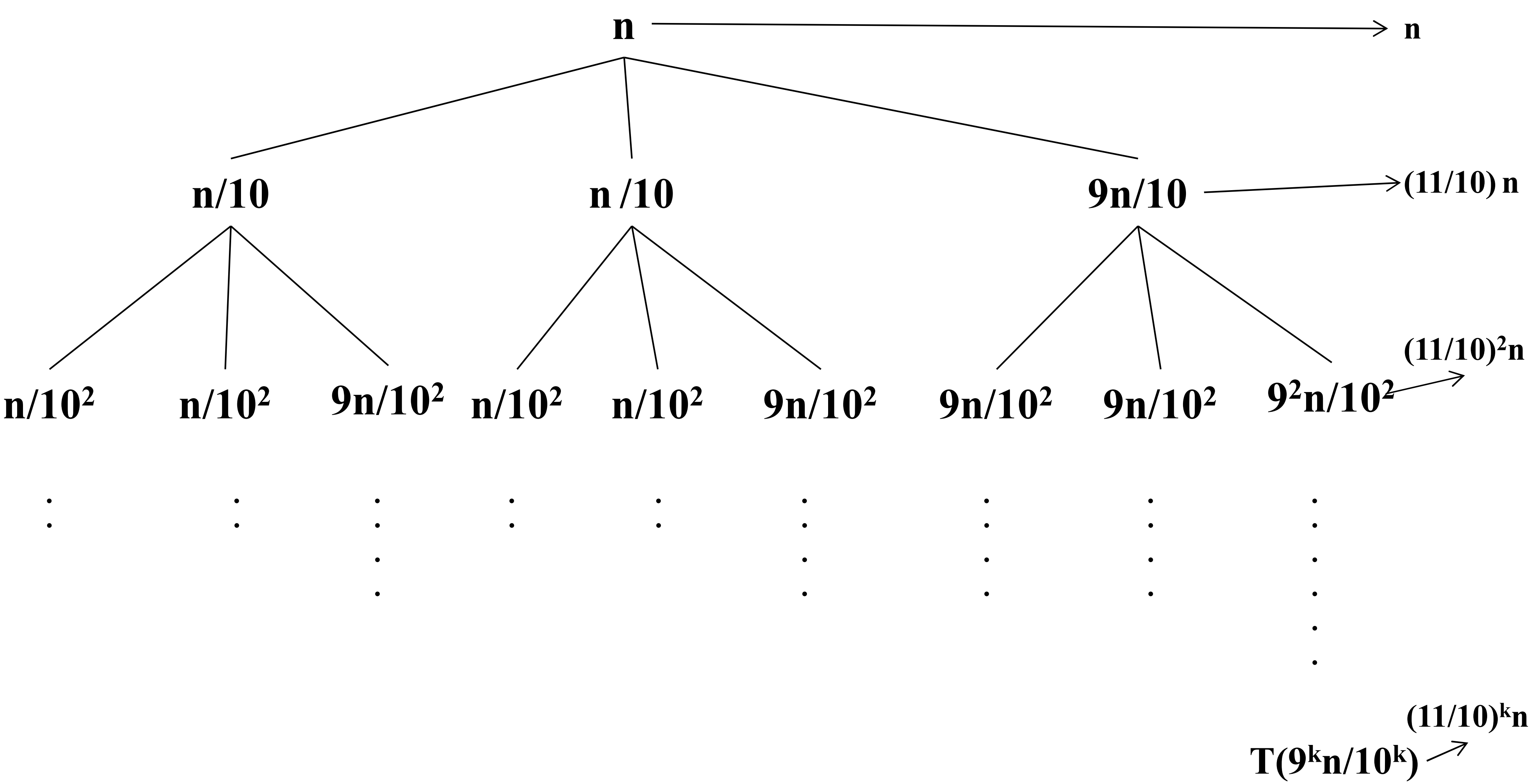
$$T(n/10) = \quad 2T(n/10^2) + T(9n/10^2) + (n/10)$$

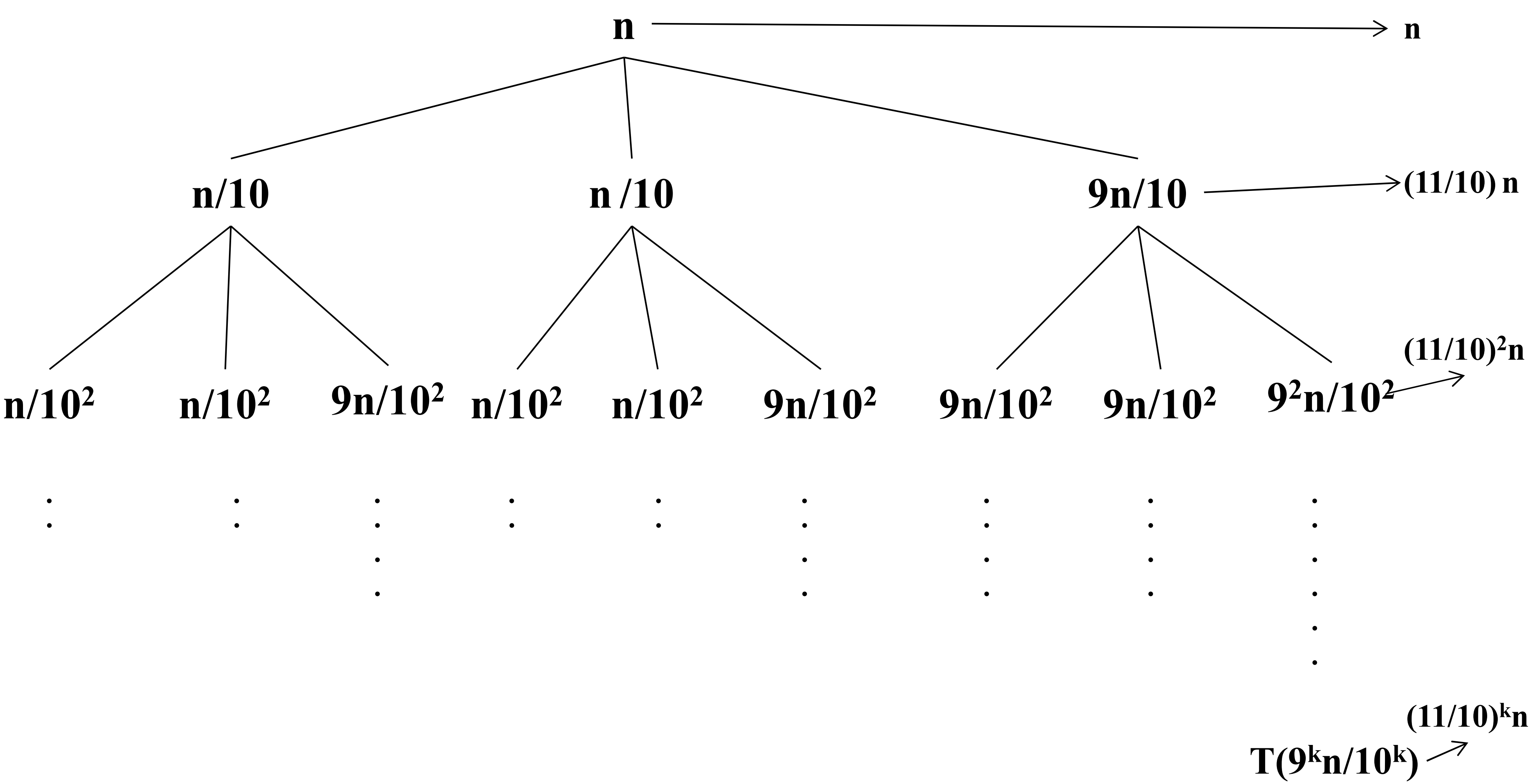
$$T(9n/10) = \quad 2T(9n/10^2) + T(9^2n/10^2) + (9n/10)$$





n
n/10
n /10
9n/10
n/10²
n/10²
9n/10²
n/10²
n/10²
9n/10²
9n/10²
9n/10²
9²n/10²
T(9ᵏn/10ᵏ)


n
n
n/10
n /10
9n/10
(11/10) n
n/10²
n/10²
9n/10²
n/10²
n/10²
9n/10²
9n/10²
9n/10²
9²n/10²
(11/10)²n
(11/10)ᵏn
T(9ᵏn/10ᵏ)



Assume that $9^k n/10^k=1$ ➔ $(10/9)^k=n$ ➔ $k=\log_{(10/9)} n$

$$T(n) = n + (11/10)n + (11/10)^2 n + \ldots\ldots\ldots + (11/10)^k n$$

$$T(n) = n + (11/10)n + (11/10)^2 n + \ldots\ldots\ldots + (11/10)^k n$$

$$= n[1 + (11/10) + (11/10)^2 + \ldots\ldots\ldots + (11/10)^k]$$

$$T(n) = n+(11/10)n+(11/10)^2n+\ldots\ldots..+(11/10)^kn$$

$$= n[1+(11/10)+(11/10)^2+\ldots\ldots..+(11/10)^k]$$

$$= n[((11/10)^{k+1}-1)/((11/10)-1)]$$

$$T(n) = n+(11/10)n+(11/10)^2n+\ldots\ldots..+(11/10)^kn$$

$$=n[1+(11/10)+(11/10)^2+\ldots\ldots..+(11/10)^k]$$

$$=n[((11/10)^{k+1}-1)/((11/10)-1)]$$

$$=10n[\ [(11/10)x(11/10)^{\,k}]\ -1]$$

$$T(n) = n+(11/10)n+(11/10)^2n+\ldots\ldots..+(11/10)^kn$$

$$=n[1+(11/10)+(11/10)^2+\ldots\ldots..+(11/10)^k]$$

$$=n[((11/10)^{k+1}-1)/((11/10)-1)]$$

$$=10n[\ [(11/10)x(11/10)^{\ k}]\ -1]$$

$$=10n[\ [(11/10)x(11/10)^{\ \log_{10/9} n}]\ -1]$$

$$
\begin{aligned}
T(n) &= n+(11/10)n+(11/10)^2 n+\ldots\ldots\ldots+(11/10)^k n \\
&= n[1+(11/10)+(11/10)^2+\ldots\ldots\ldots+(11/10)^k] \\
&= n[((11/10)^{k+1}-1)/((11/10)-1)] \\
&= 10n[\,[(11/10)x(11/10)^{\,k}]\,-1] \\
&= 10n[\,[(11/10)x(11/10)^{\,\log_{10/9} n}]\,-1] \\
&= 10n[\,[(11/10)\ x\ n^{\,\log_{10/9}(11/10)}\,]\,-1]
\end{aligned}
$$

$$T(n) = n+(11/10)n+(11/10)^2 n+\ldots\ldots..+(11/10)^k n$$

$$= n[1+(11/10)+(11/10)^2+\ldots\ldots..+(11/10)^k]$$

$$= n[((11/10)^{k+1}-1)/((11/10)-1)]$$

$$= 10n[\ [(11/10)x(11/10)^{k}] -1]$$

$$= 10n[\ [(11/10)x(11/10)^{\log_{10/9} n}] -1]$$

$$= 10n[\ [(11/10) \text{ x } n^{\log_{10/9}(11/10)}] -1]$$

$$= 10n[\ [(11/10) \text{ x } n^{1}] -1]$$

$$T(n) = n + (11/10)n + (11/10)^2 n + \ldots\ldots\ldots + (11/10)^k n$$

$$= n[1 + (11/10) + (11/10)^2 + \ldots\ldots\ldots + (11/10)^k]$$

$$= n[((11/10)^{k+1} - 1)/((11/10) - 1)]$$

$$= 10n[\ [(11/10) x (11/10)^{k}] - 1]$$

$$= 10n[\ [(11/10) x (11/10)^{\log_{10/9} n}] - 1]$$

$$= 10n[\ [(11/10)\ x\ n^{\log_{10/9}(11/10)}\ ] - 1]$$

$$= 10n[\ [(11/10)\ x\ n^{1}\ ] - 1]$$

$$= 11n^2 - 10n$$

$$T(n) = n + (11/10)n + (11/10)^2 n + \ldots\ldots\ldots + (11/10)^k n$$

$$= n[1 + (11/10) + (11/10)^2 + \ldots\ldots\ldots + (11/10)^k]$$

$$= n[((11/10)^{k+1} - 1)/((11/10) - 1)]$$

$$= 10n[\,[(11/10) x (11/10)^{k}] - 1]$$

$$= 10n[\,[(11/10) x (11/10)^{\log_{10/9} n}] - 1]$$

$$= 10n[\,[(11/10) \text{ x } n^{\log_{10/9}(11/10)}] - 1]$$

$$= 10n[\,[(11/10) \text{ x } n^{1}] - 1]$$

$$= 11n^2 - 10n$$

$$= \mathbf{O(n^2)}$$

Longest path: Rightmost path

Longest Path length:

$9^k n/10^k=1$, where k is the length of the longest path

$(10/9)^k=n$

$\mathbf{k=\log_{(10/9)} n}$

Shortest path: Leftmost path

Shortest Path length:

$n/10^k=1$, where k is the length of the shortest path

$10^k=n$

$\mathbf{k=\log_{10} n}$

Recursion Tree Method

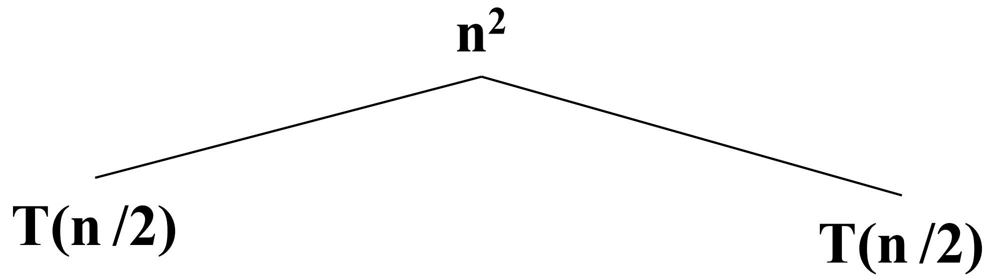
T(n) = 2T(n/2) + n^2

$$T(n) = 2T(n/2) + n^2$$

$$n^2$$

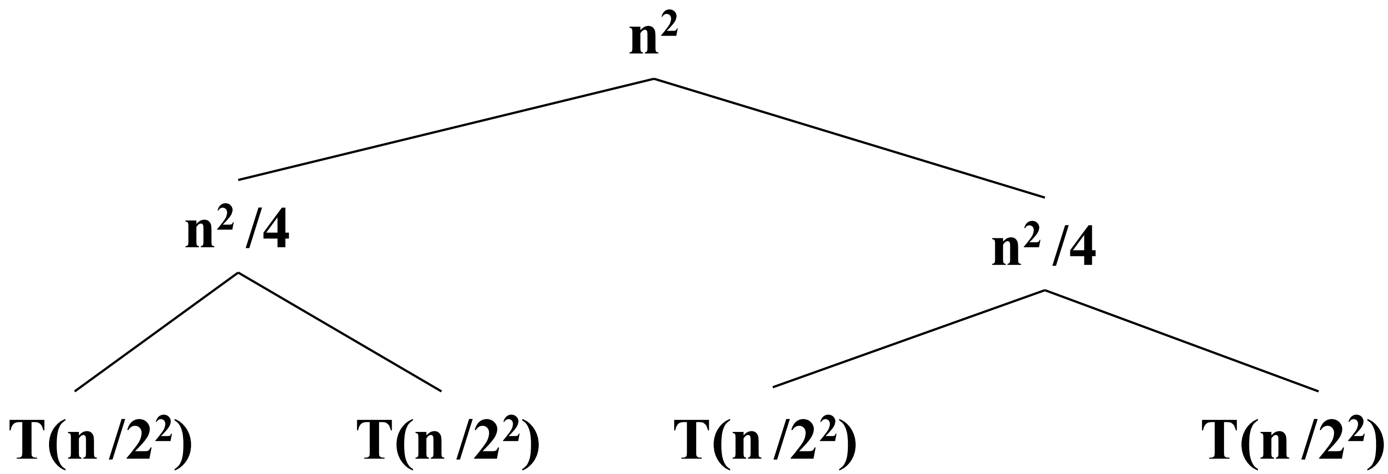
$$T(n/2) \qquad\qquad T(n/2)$$

$$T(n) = 2T(n/2) + n^2$$

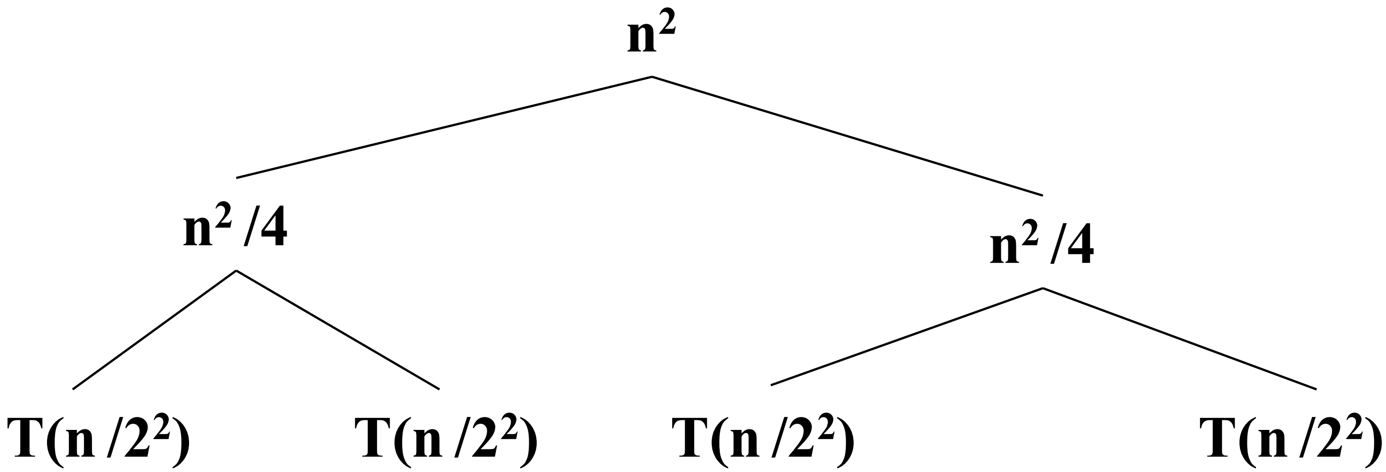


$$T(n/2) = 2T(n/2^2) + (n/2)^2$$

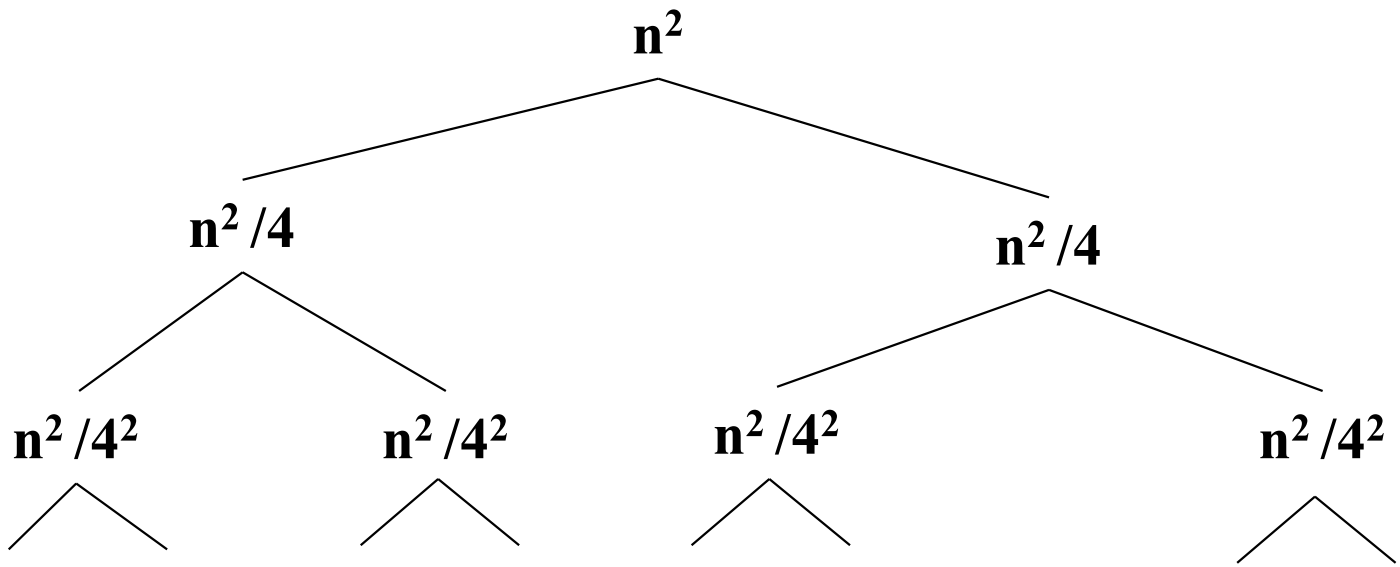
$$T(n/2) = 2T(n/2^2) + (n/2)^2$$

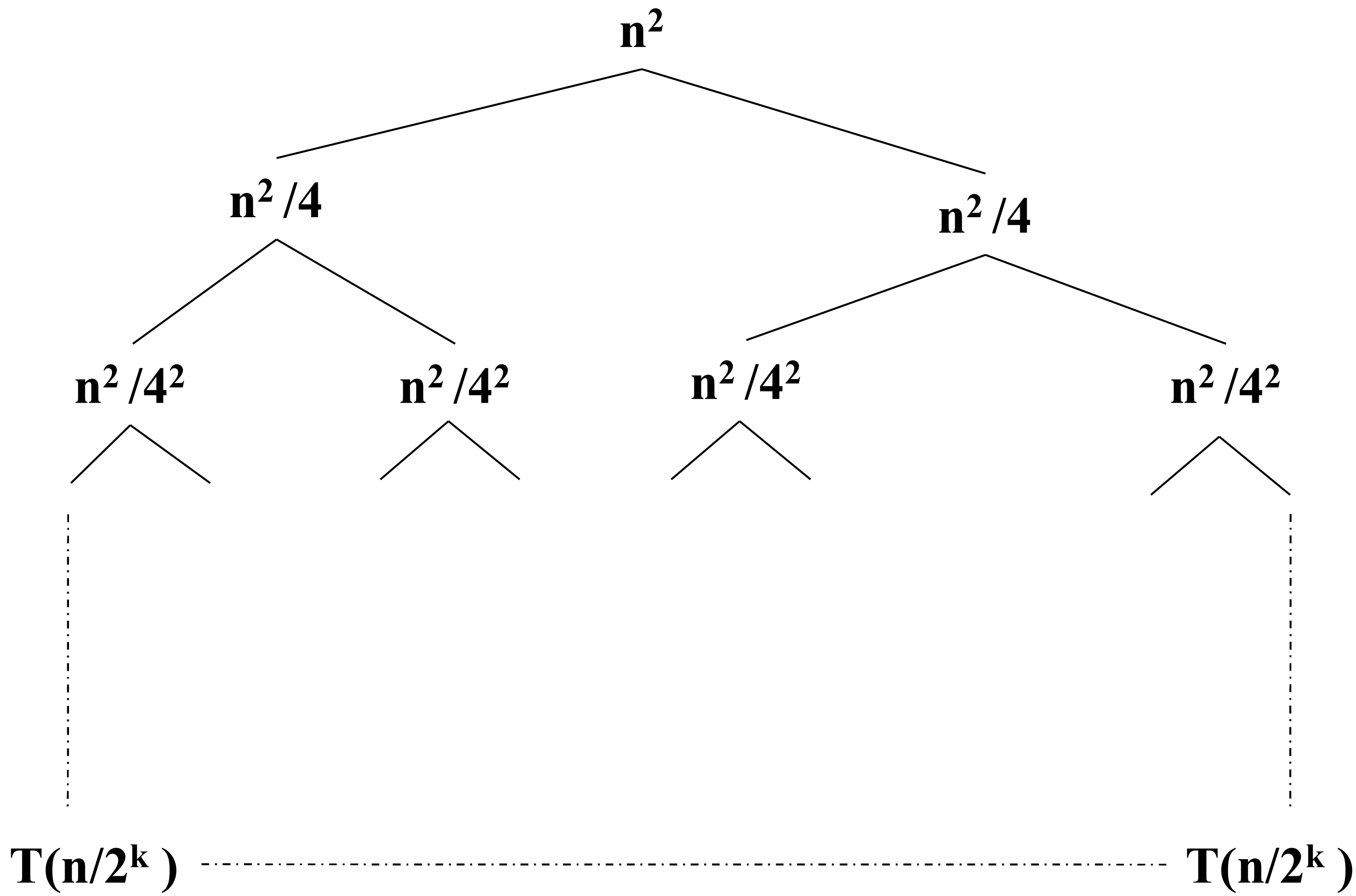


$$T(n/2) = 2T(n/2^2) + (n/2)^2$$

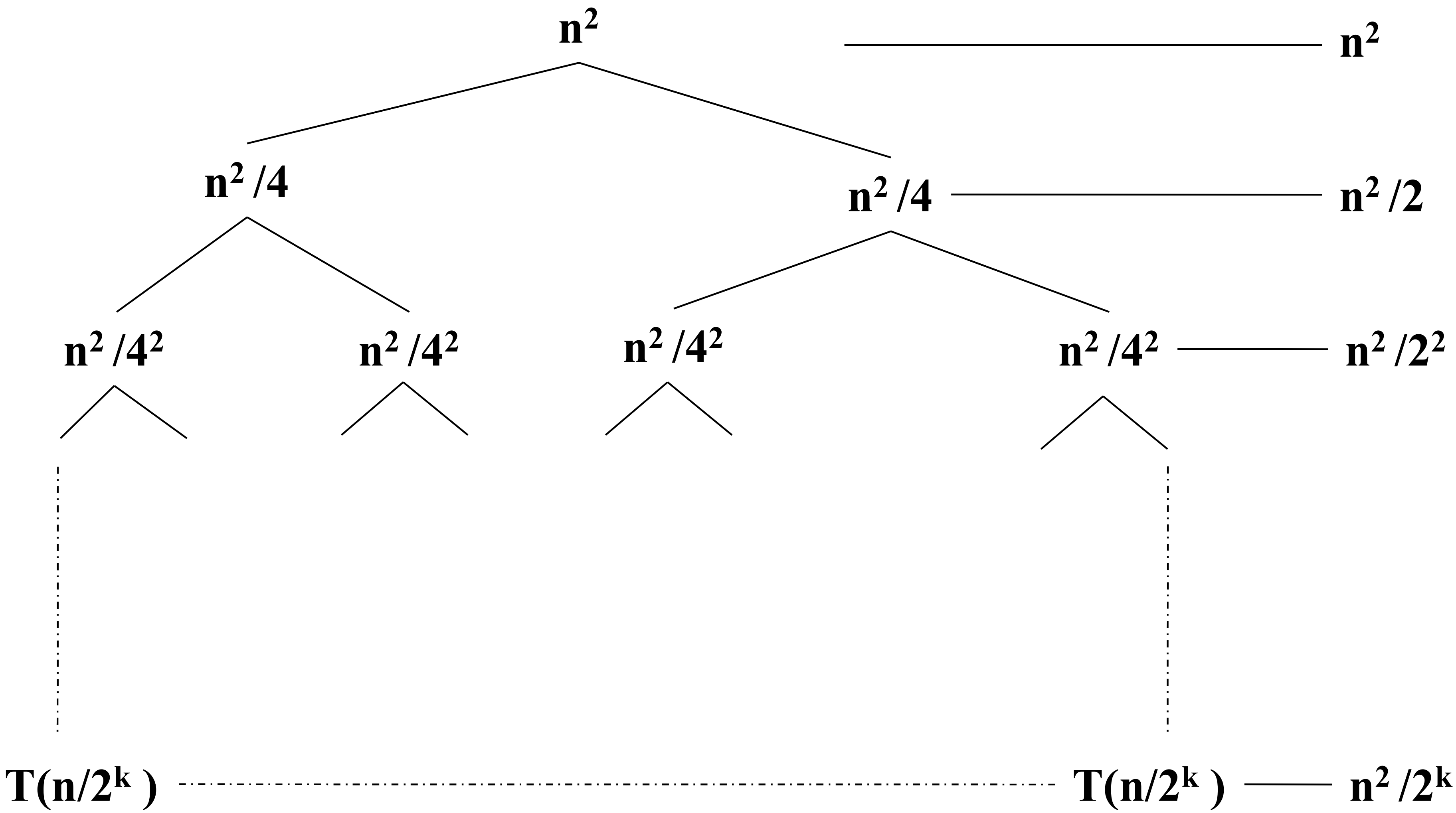


$$T(n/2^2) = 2T(n/2^3) + (n/2^2)^2$$

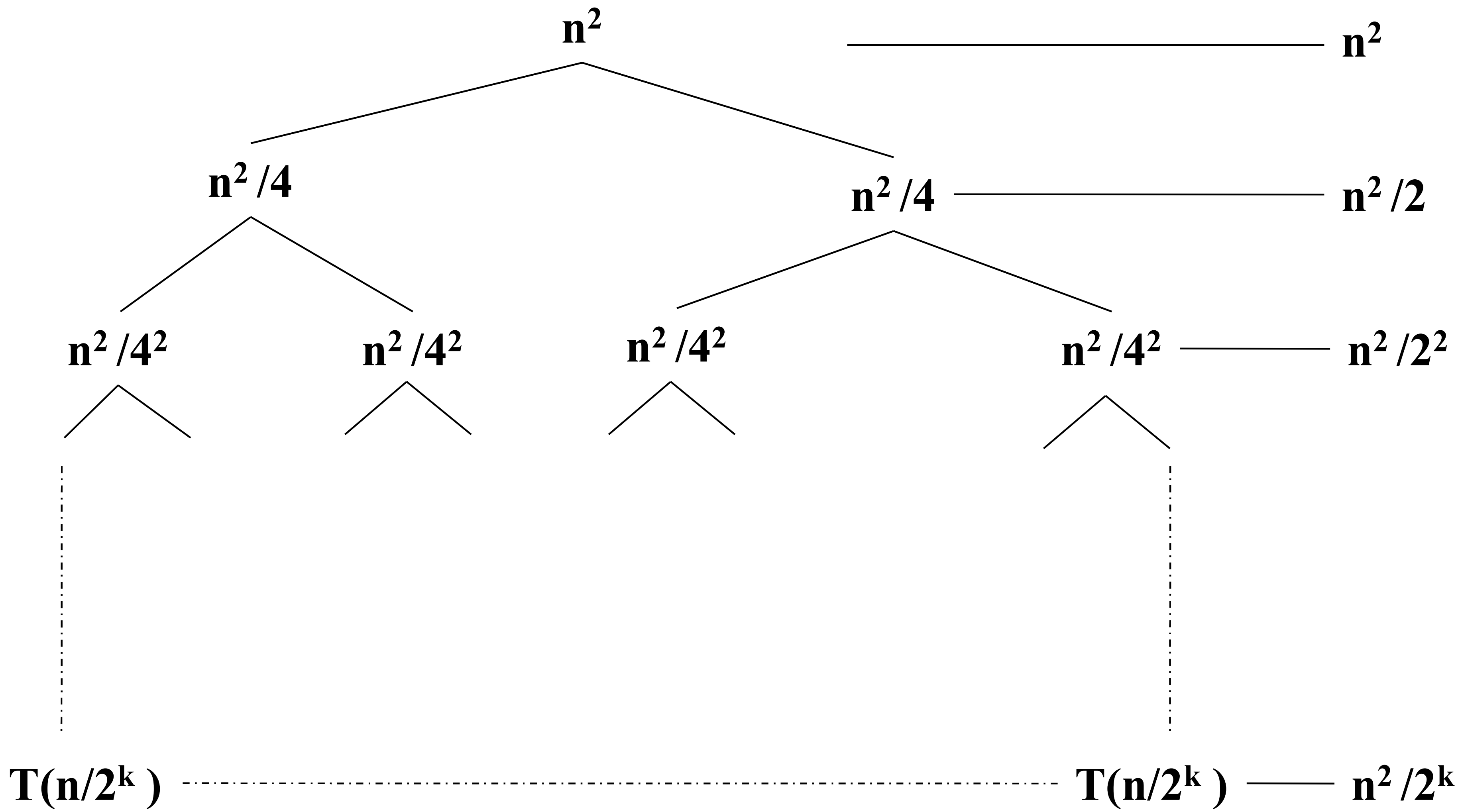

n^2
$n^2/4$
$n^2/4$
$n^2/4^2$
$n^2/4^2$
$n^2/4^2$
$n^2/4^2$



n2
n2 /4
n2 /4
n2 /42
n2 /42
n2 /42
n2 /42
T(n/2k)
T(n/2k)



n^2
n^2
$n^2/4$
$n^2/4$
$n^2/2$
$n^2/4^2$
$n^2/4^2$
$n^2/4^2$
$n^2/4^2$
$n^2/2^2$
$T(n/2^k)$
$T(n/2^k)$
$n^2/2^k$



Assume $n/2^k=1$ ➔ $2^k=n$ ➔ $k=\log_2 n$

$$T(n) = n^2 + (n^2/2) + (n^2/2^2) + \ldots\ldots\ldots + (n^2/2^k)$$

$$T(n) = n^2+(n^2/2)+(n^2/2^2)+\dots\dots\dots+(n^2/2^k)$$

$$= n^2[1+(1/2)+(1/2)^2+\dots\dots+(1/2)^k]$$

$$T(n) = n^2+(n^2/2)+(n^2/2^2)+\ldots\ldots\ldots+(n^2/2^k)$$

$$= n^2[1+(1/2)+(1/2)^2+\ldots\ldots\ldots+(1/2)^k]$$

$$= n^2[[1-(1/2)^{k+1}]/[1-(1/2)]]$$

$$T(n) = n^2+(n^2/2)+(n^2/2^2)+\ldots\ldots..+(n^2/2^k)$$
$$= n^2[1+(1/2)+(1/2)^2+.\ .\ .\ .\ .\ .\ .+(1/2)^k]$$
$$= n^2[\,[1-(1/2)^{k+1}]\,/\,[1-(1/2)]\,]$$
$$= 2n^2[\,1-(1/2)x(1/2)^k]$$

$$
\begin{aligned}
T(n) &= n^2+(n^2/2)+(n^2/2^2)+\ldots\ldots\ldots+(n^2/2^k)\\
&= n^2[1+(1/2)+(1/2)^2+.\ .\ .\ .\ .\ .\ .+(1/2)^k]\\
&= n^2\,[\,[1-(1/2)^{k+1}]\,/\,[1-(1/2)]\,]\\
&= 2n^2\,[\,1-(1/2)x(1/2)^k]\\
&= 2n^2\,[\,1-(1/2)x(1/2^k)]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+(n^2/2)+(n^2/2^2)+\ldots\ldots..+(n^2/2^k) \\
&= n^2[1+(1/2)+(1/2)^2+.\ .\ .\ .\ .\ .\ .+(1/2)^k] \\
&= n^2\,[\,[1-(1/2)^{k+1}]\,/\,[1-(1/2)]\,] \\
&= 2n^2\,[\,1-(1/2)x(1/2)^k] \\
&= 2n^2\,[\,1-(1/2)x(1/2^k)] \\
&= 2n^2\,[\,1-(1/2)x(1/n)]
\end{aligned}
$$

$$T(n) = n^2+(n^2/2)+(n^2/2^2)+\ldots\ldots\ldots+(n^2/2^k)$$

$$= n^2[1+(1/2)+(1/2)^2+\ldots\ldots\ldots+(1/2)^k]$$

$$= n^2[\,[1-(1/2)^{k+1}] / [1-(1/2)]\,]$$

$$= 2n^2[\,1-(1/2)x(1/2)^k]$$

$$= 2n^2[\,1-(1/2)x(1/2^k)]$$

$$= 2n^2[\,1-(1/2)x(1/n)]$$

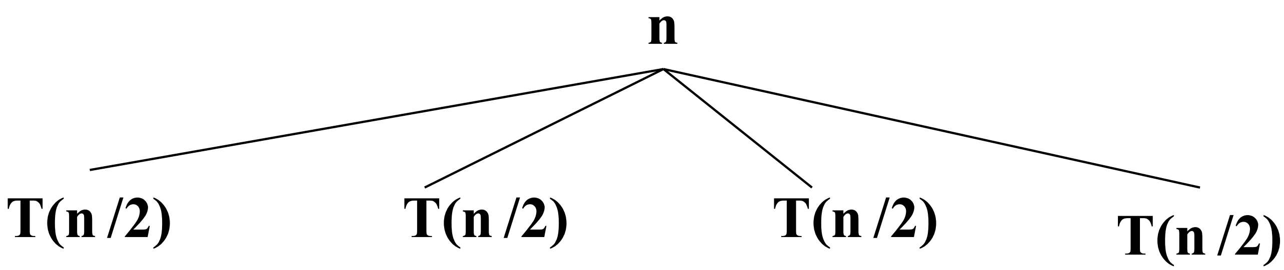
$$= 2n^2 - n$$

$$
\begin{aligned}
T(n) &= n^2+(n^2/2)+(n^2/2^2)+\ldots\ldots\ldots+(n^2/2^k) \\
&= n^2[1+(1/2)+(1/2)^2+.\ .\ .\ .\ .\ .\ .+(1/2)^k] \\
&= n^2\,[\,[1-(1/2)^{k+1}]\,/\,[1-(1/2)]\,] \\
&= 2n^2\,[\,1-(1/2)x(1/2)^k] \\
&= 2n^2\,[\,1-(1/2)x(1/2^k)] \\
&= 2n^2\,[\,1-(1/2)x(1/n)] \\
&= 2n^2-n \\
&\mathbf{= O(n^2)}
\end{aligned}
$$

Recursion Tree Method

$$T(n) = \quad 4T(n/2) + n$$

$T(n) = 4T(n/2) + n$



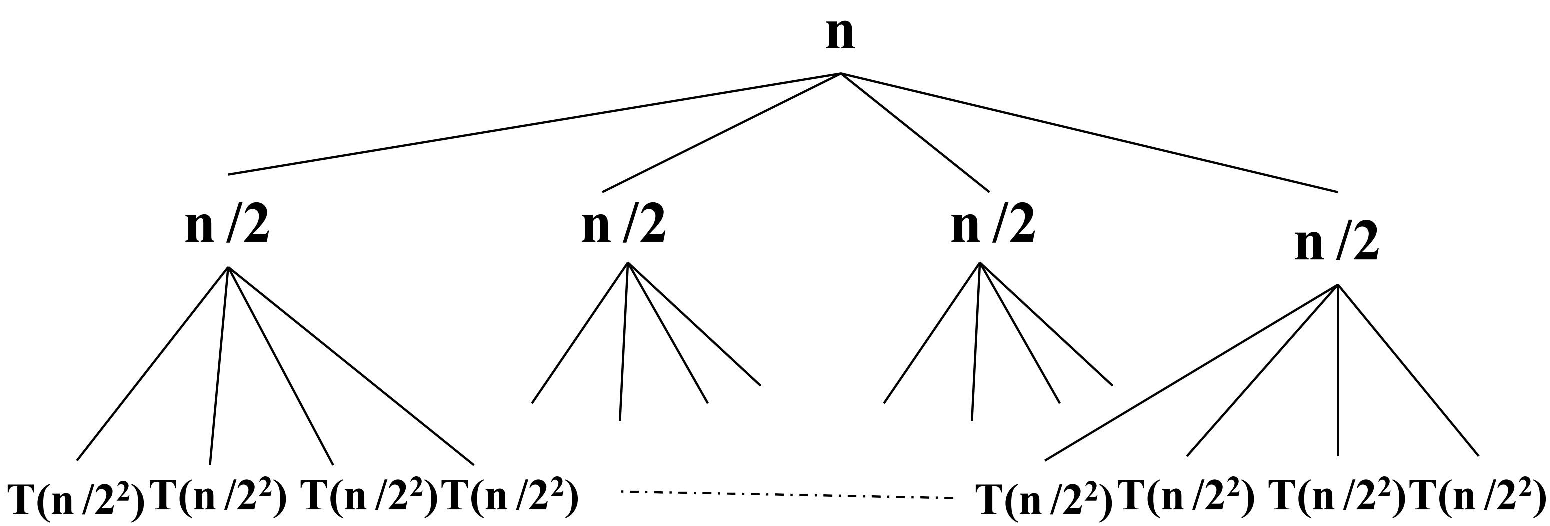
$$T(n) = 4T(n/2) + n$$

n

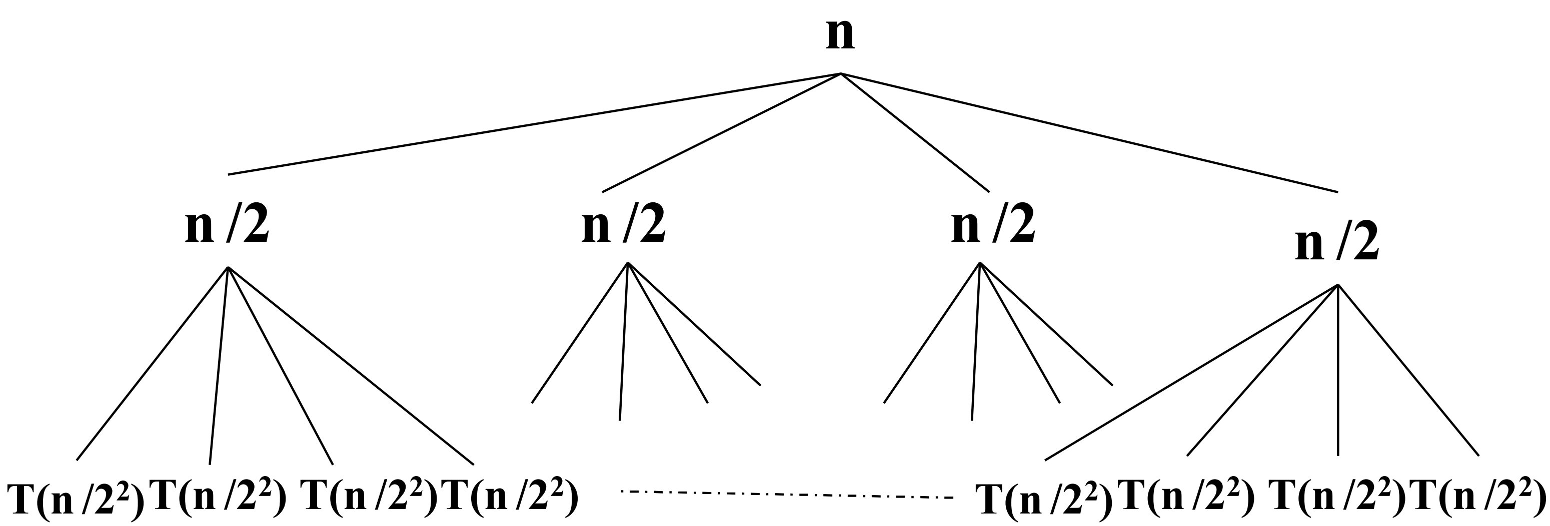
$T(n/2)$ $T(n/2)$ $T(n/2)$ $T(n/2)$

$$T(n/2) = 4T(n/2^2) + n/2$$

$$T(n/2) = 4T(n/2^2) + n/2$$

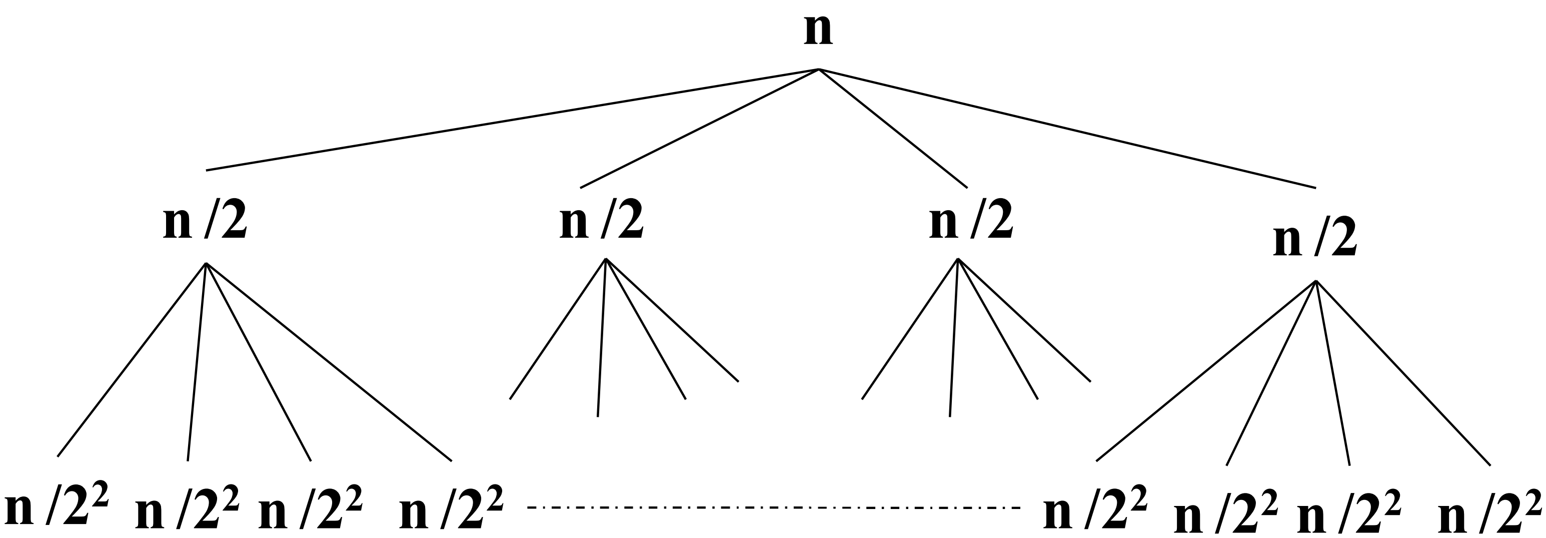


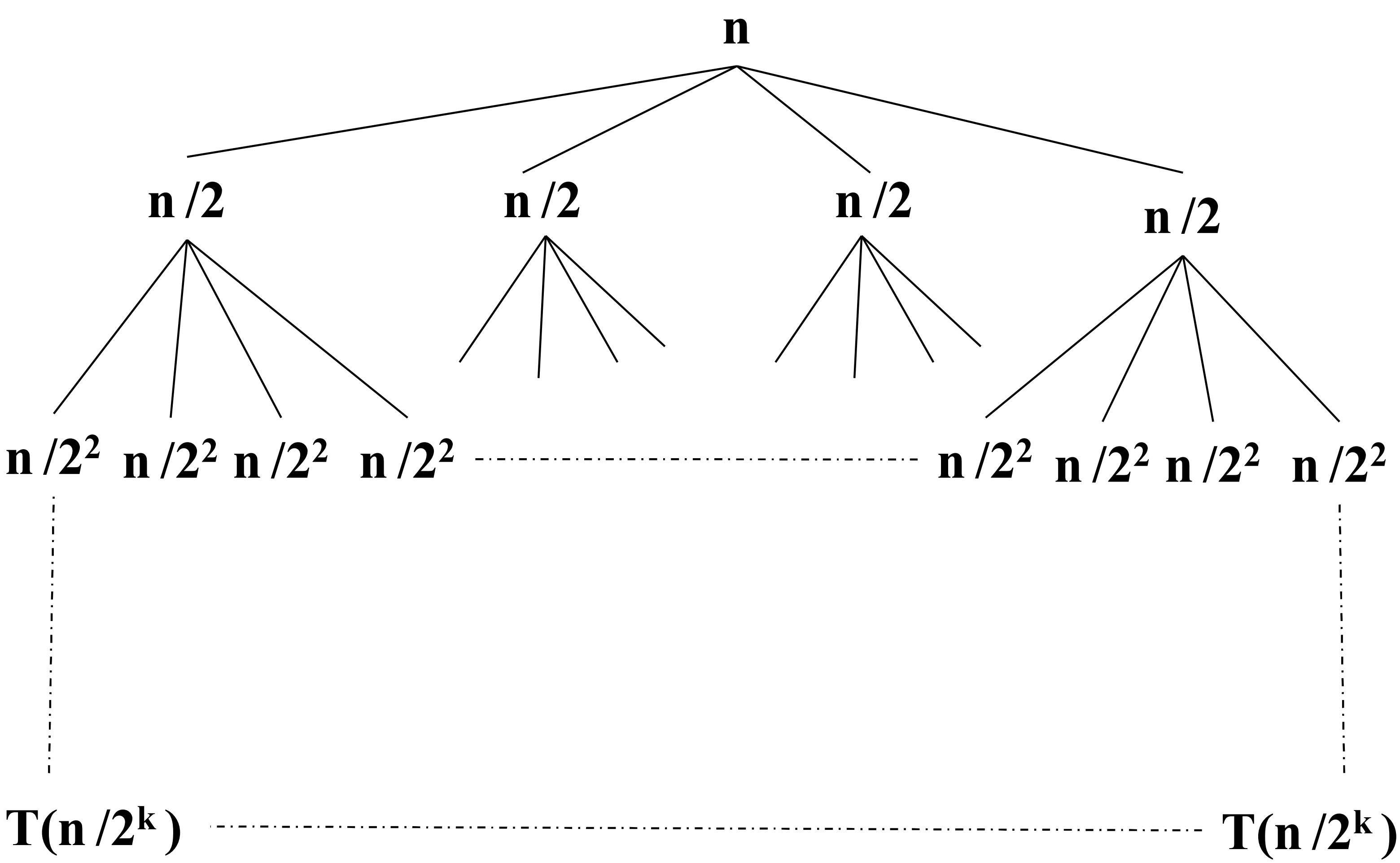
$$T(n/2) = 4T(n/2^2) + n/2$$

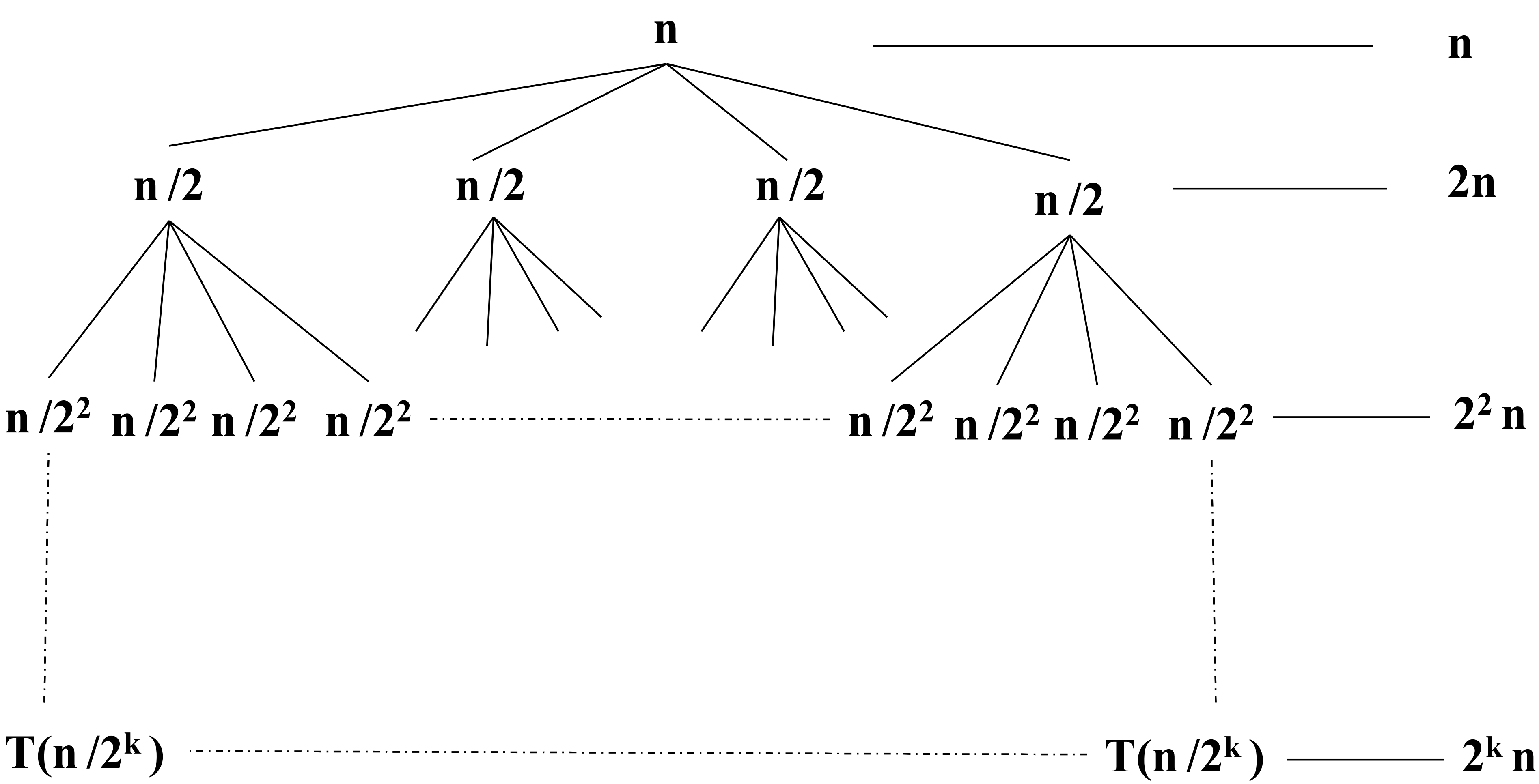


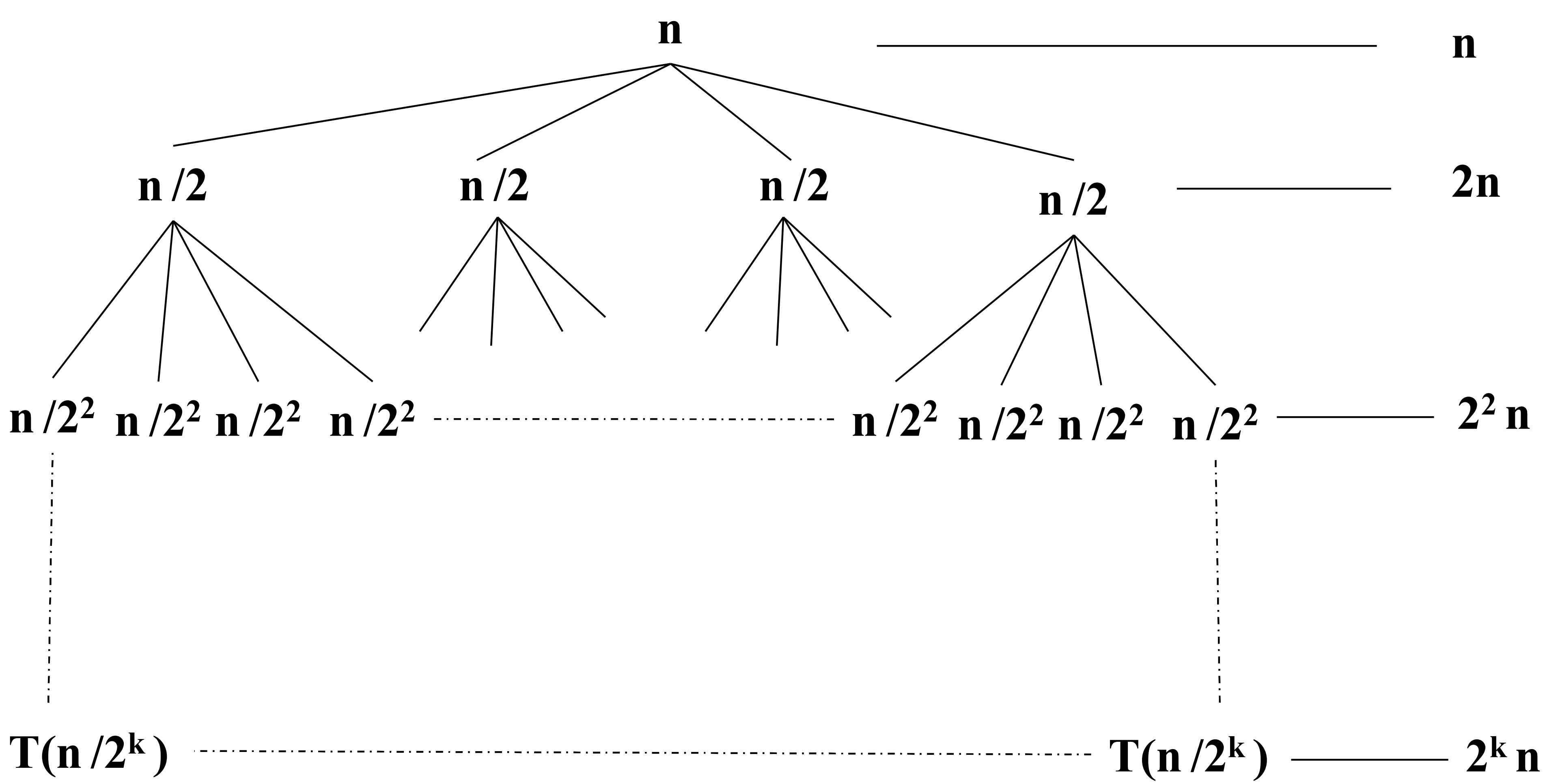
$$T(n/2^2) = 4T(n/2^3) + n/2^2$$

$$T(n/2^2) = 4T(n/2^3) + n/2^2$$




n
n/2
n/2
n/2
n/2
n/2^2
n/2^2
n/2^2
n/2^2
n/2^2
n/2^2
n/2^2
n/2^2
T(n/2^k)
T(n/2^k)


n
n
n /2
n /2
n /2
n /2
2n
n /2^2
n /2^2
n /2^2
n /2^2
n /2^2
n /2^2
n /2^2
n /2^2
2^2 n
T(n /2^k)
T(n /2^k)
2^k n



Assume $n/2^k=1$ ➔ $2^k=n$ ➔ $k=\log_2 n$

$$T(n) = n + 2n + 2^2 n + \ldots\ldots\ldots + 2^k n$$

$$T(n) = n+2n + 2^2 n+\ldots\ldots\ldots+2^k n$$
$$= n[1+2+2^2+.\ .\ .\ .\ .\ .+ 2^k]$$

$$\begin{aligned} T(n) &= n+2n + 2^2 n+\dots\dots+2^k n \\ &= n[1+2+2^2+. \ . \ . \ . \ . \ .+ 2^k] \\ &= n[\ [2^{k+1}-1] \ / \ [2-1]\ ] \end{aligned}$$

$$T(n) = n + 2n + 2^2 n + \ldots\ldots\ldots + 2^k n$$

$$= n[1 + 2 + 2^2 + \ldots\ldots + 2^k]$$

$$= n[\,[2^{k+1} - 1] / [2-1]\,]$$

$$= n\,[2 \text{x} 2^k - 1]$$

$$
\begin{aligned}
T(n) &= n+2n + 2^2 n+\ldots\ldots\ldots+2^k n \\
&= n[1+2+2^2+. \ . \ . \ . \ . \ .+ 2^k] \\
&= n[\ [2^{k+1}-1] \ / \ [2-1]\ ] \\
&= n\ [2x2^k-1] \\
&= n\ [\ 2n-1]
\end{aligned}
$$

$$\begin{aligned}
T(n) &= n+2n + 2^2 n+\dots\dots\dots+2^k n \\
&= n[1+2+2^2+. \ . \ . \ . \ . \ .+ 2^k] \\
&= n[\ [2^{k+1}-1] \ / \ [2-1]\ ] \\
&= n\ [2x2^k-1] \\
&= n\ [\ 2n-1] \\
&= 2n^2 -n
\end{aligned}$$

$$
\begin{aligned}
T(n) &= n+2n + 2^2 n+\ldots\ldots\ldots+2^k n \\
&= n[1+2+2^2+.\;.\;.\;.\;.\;.+ 2^k] \\
&= n[\,[2^{k+1}-1] / [2-1]\,] \\
&= n\,[2\text{x}2^k-1] \\
&= n\,[\,2n-1] \\
&= 2n^2 - n \\
&\mathbf{= O(n^2)}
\end{aligned}
$$

Recursion Tree Method

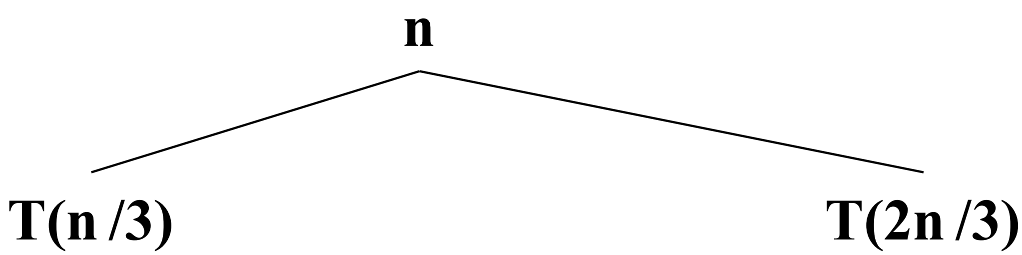
$T(n) = \quad T(n/3) + T(2n/3) + n$

$$T(n) = T(n/3) + T(2n/3) + n$$

n

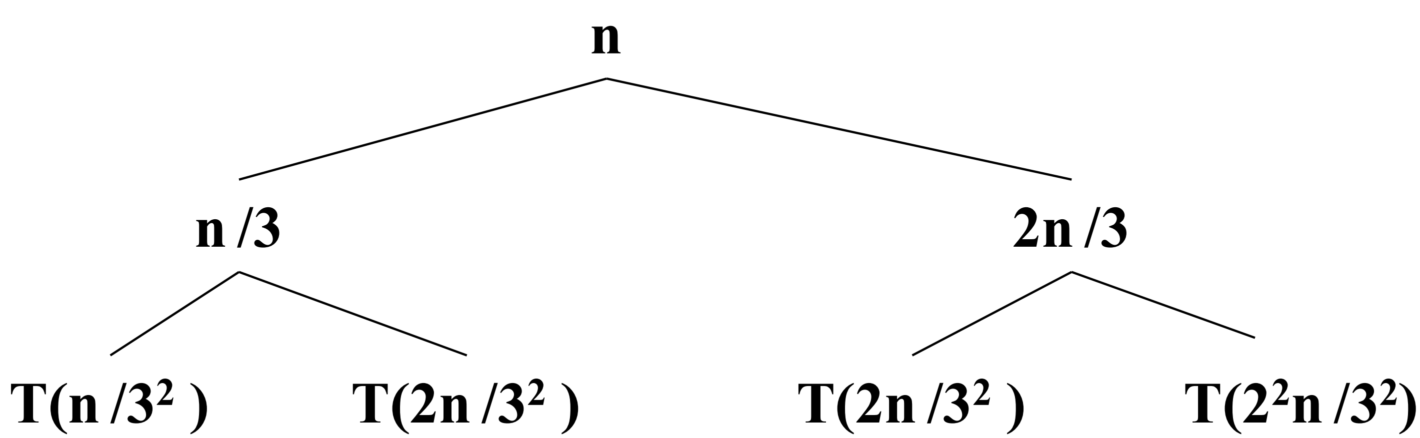
$T(n/3)$ $\qquad\qquad$ $T(2n/3)$

$$T(n) = T(n/3) + T(2n/3) + n$$

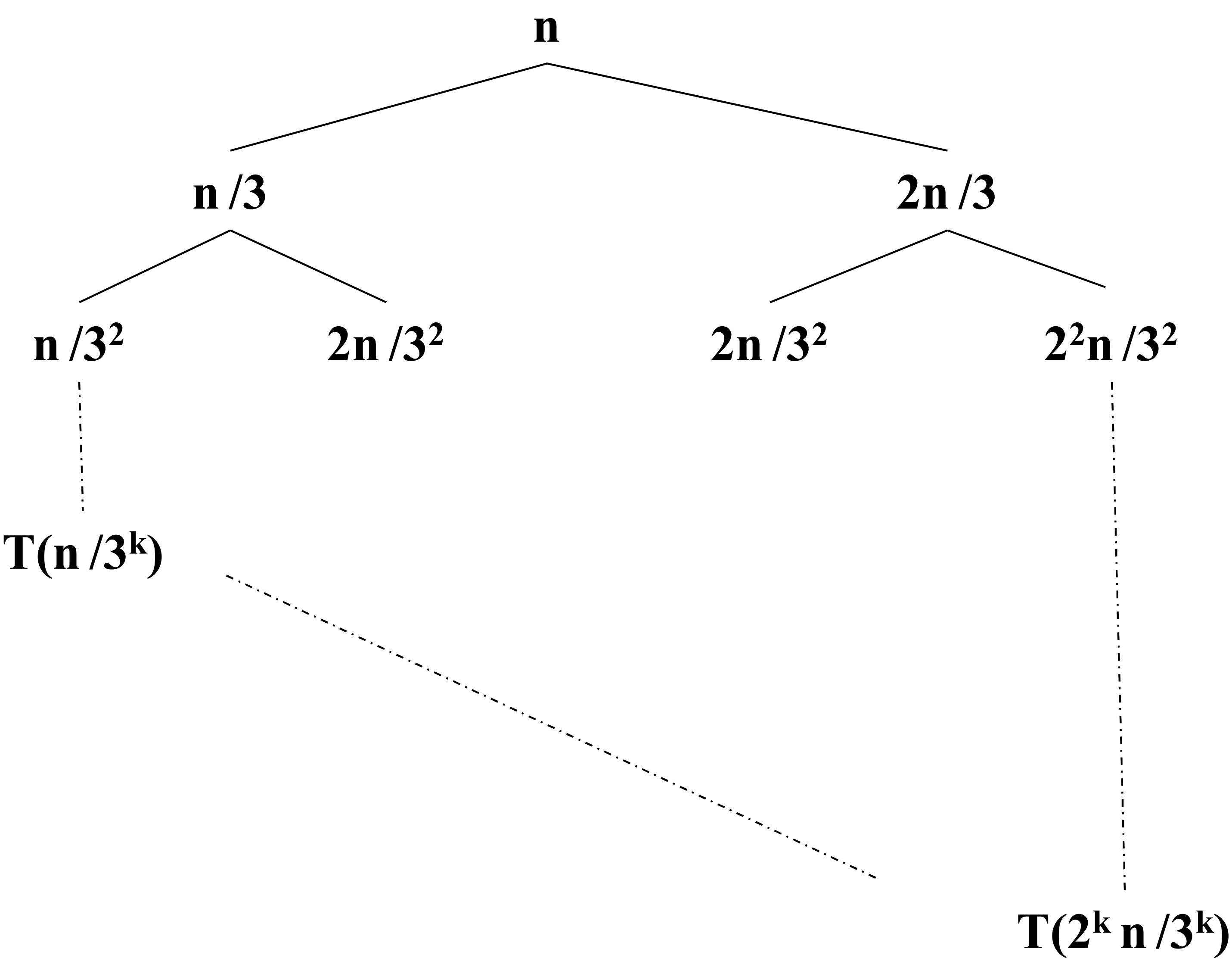


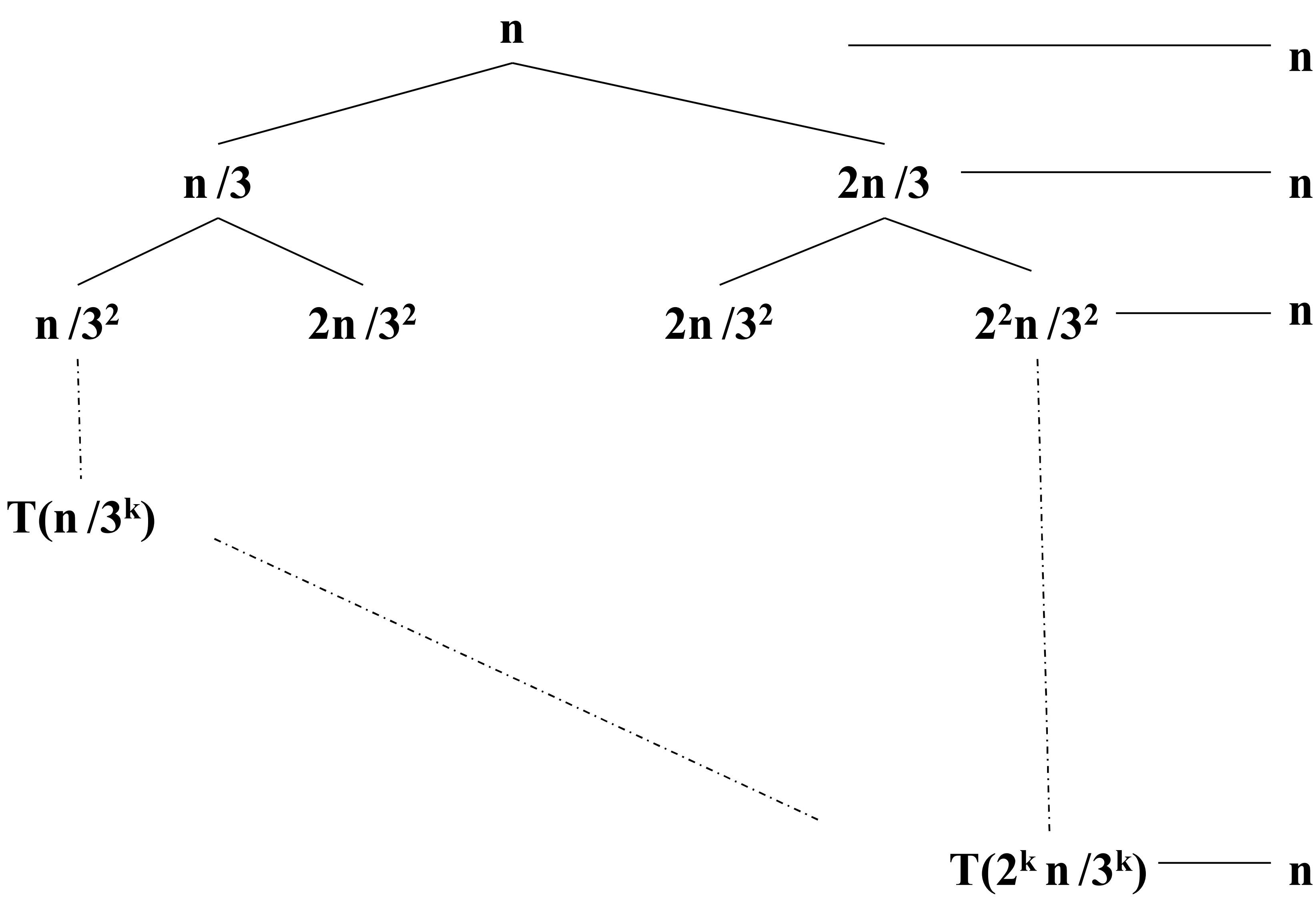
$$T(n/3) = T(n/3^2) + T(2n/3^2) + n/3$$

$$T(2n/3) = T(2n/3^2) + T(2^2n/3^2) + 2n/3$$

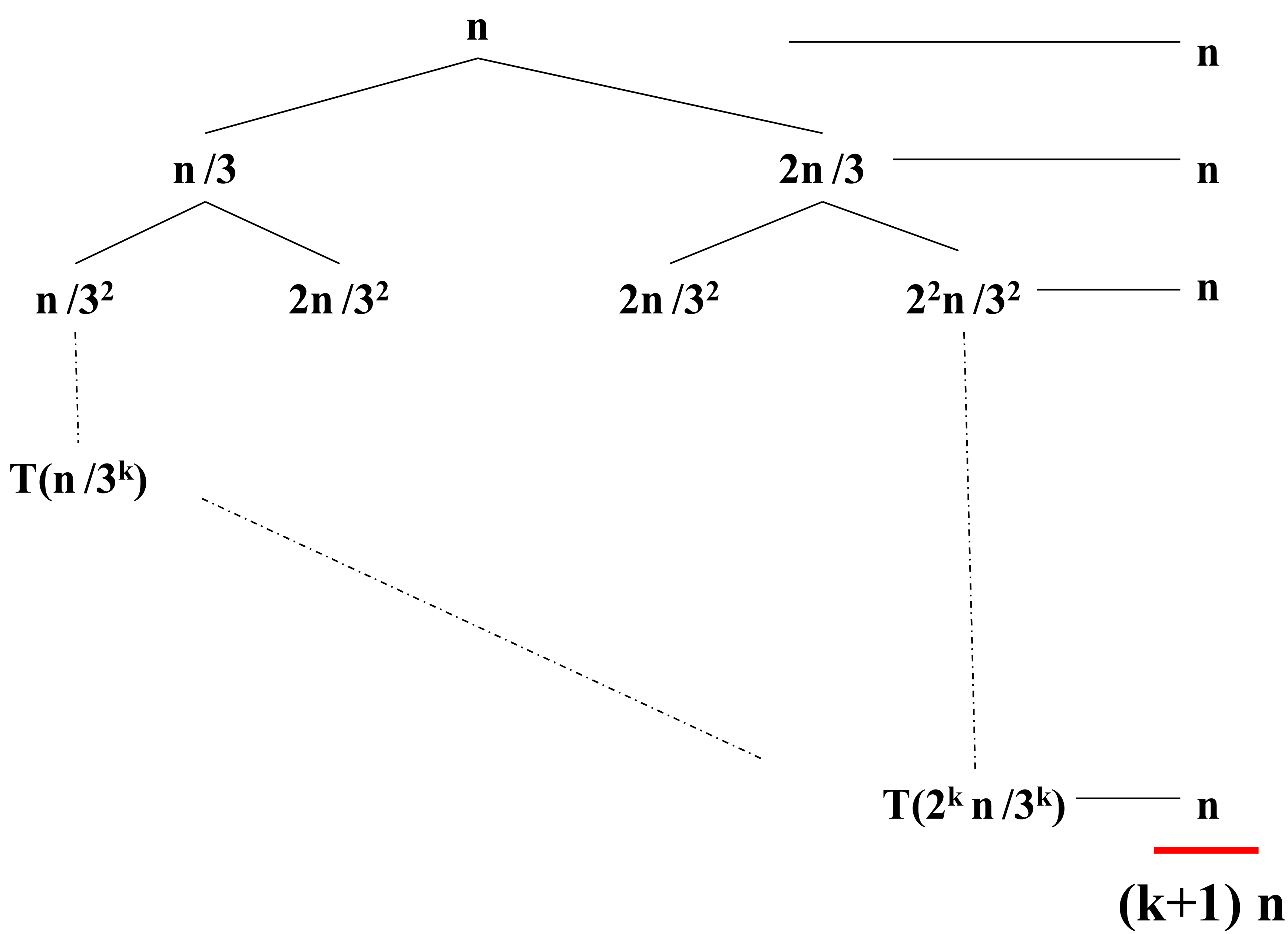


n
n /3
2n /3
T(n /3²)
T(2n /3²)
T(2n /3²)
T(2²n /3²)


n
n /3
2n /3
n /3²
2n /3²
2n /3²
2²n /3²
T(n /3ᵏ)
T(2ᵏ n /3ᵏ)



n
n /3
2n /3
n /3²
2n /3²
2n /3²
2²n /3²
T(n /3ᵏ)
T(2ᵏ n /3ᵏ)
n
n
n
n


n
n
n /3
2n /3
n
n /3²
2n /3²
2n /3²
2²n /3²
n
T(n /3ᵏ)
T(2ᵏ n /3ᵏ)
n
(k+1) n

Assume that $2^k n/3^k = 1 \Rightarrow (3/2)^k = n \Rightarrow k = \log_{(3/2)} n$

Assume that $2^k n/3^k = 1$ $\rightarrow$ $(3/2)^k = n$ $\rightarrow$ $k = \log_{(3/2)} n$

$T(n) = (k+1)\, n$

Assume that $2^k n/3^k = 1 \Rightarrow (3/2)^k = n \Rightarrow k = \log_{(3/2)} n$

$$T(n) = (k+1)\, n$$
$$= (\log_{(3/2)} n + 1)\, n$$

Assume that $2^k n/3^k = 1$ ➔ $(3/2)^k = n$ ➔ $k = \log_{(3/2)} n$

$$
\begin{aligned}
T(n) &= (k+1)\, n \\
&= (\log_{(3/2)} n + 1)\, n \\
&= n \log_{(3/2)} n + n
\end{aligned}
$$

Assume that $2^k n/3^k = 1 \rightarrow (3/2)^k = n \rightarrow k = \log_{(3/2)} n$

$$\begin{aligned} T(n) &= (k+1)\, n \\ &= (\log_{(3/2)} n + 1)\, n \\ &= n \log_{(3/2)} n + n \\ &= \mathbf{O(n \log_{(3/2)} n)} \end{aligned}$$

Longest path: Rightmost path

Longest Path length:

$2^k n/3^k = 1$, where k is the length of the longest path

$(3/2)^k = n$

$\mathbf{k = \log_{(3/2)} n}$

Shortest path: Leftmost path

Shortest Path length:

$n/3^k=1$, where k is the length of the shortest path

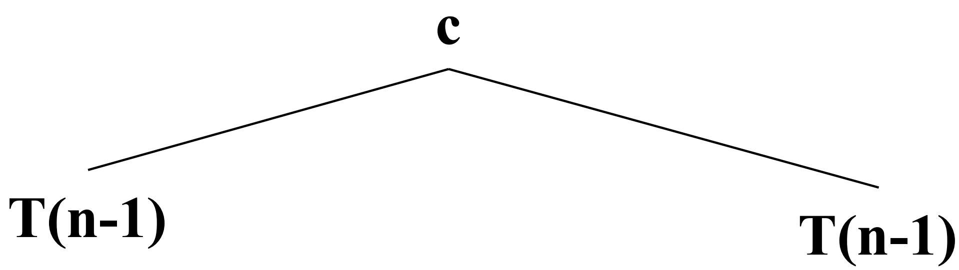
$3^k=n$

$\mathbf{k=\log_3 n}$

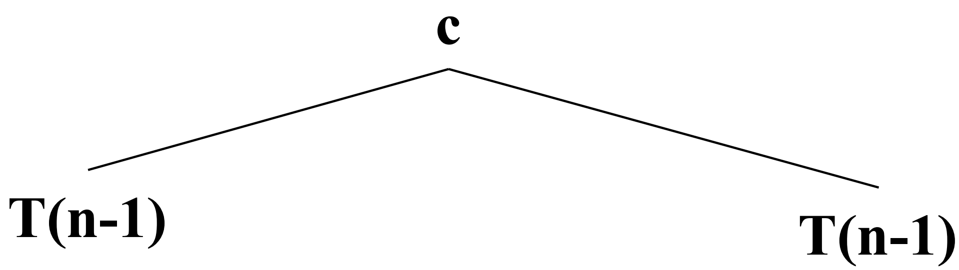
Recursion Tree Method

$$T(n) = \quad 2\,T(n-1) + c$$

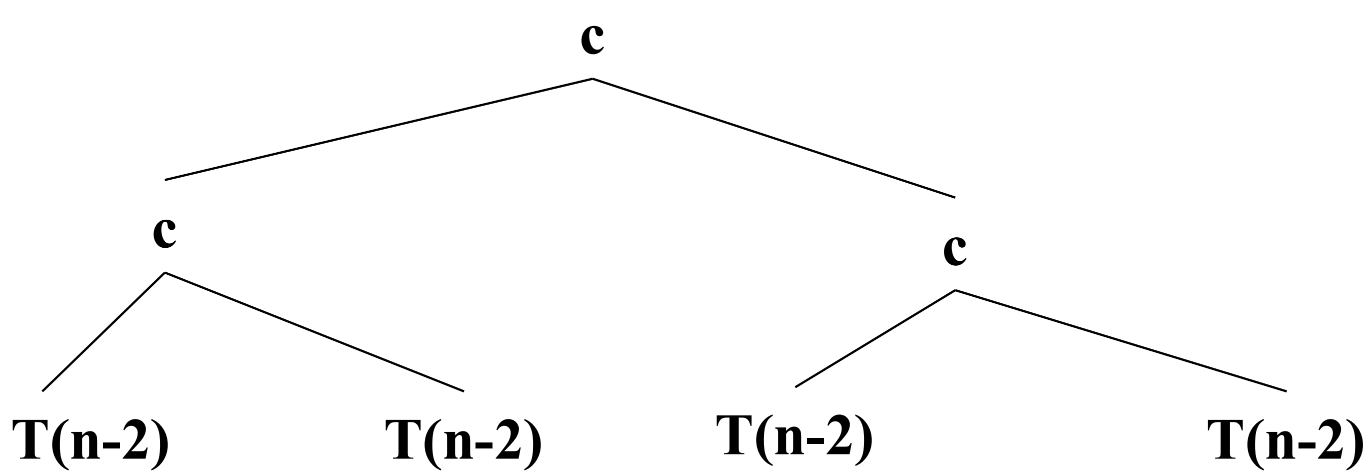
$$T(n) = 2\,T(n-1) + c$$

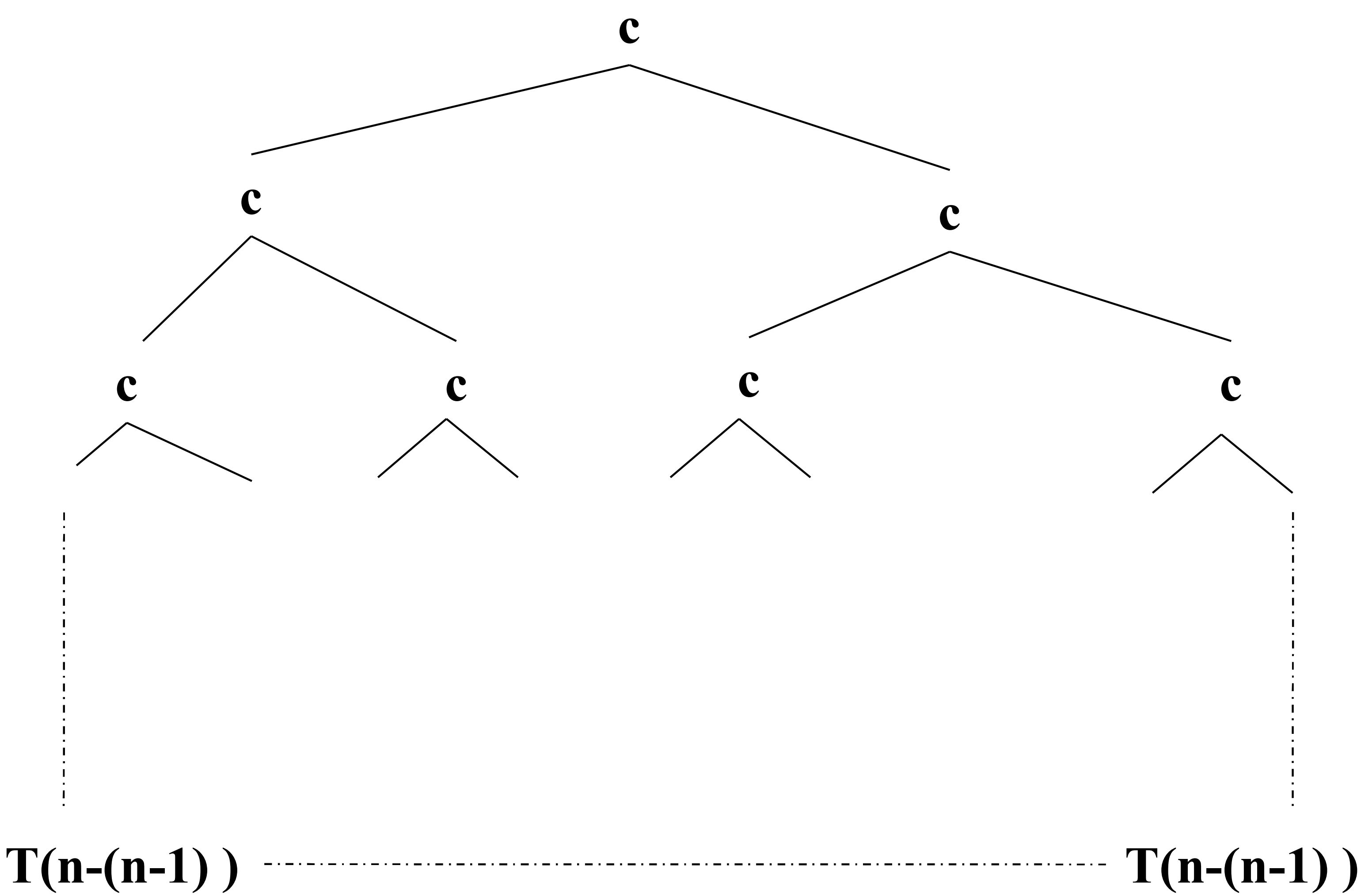


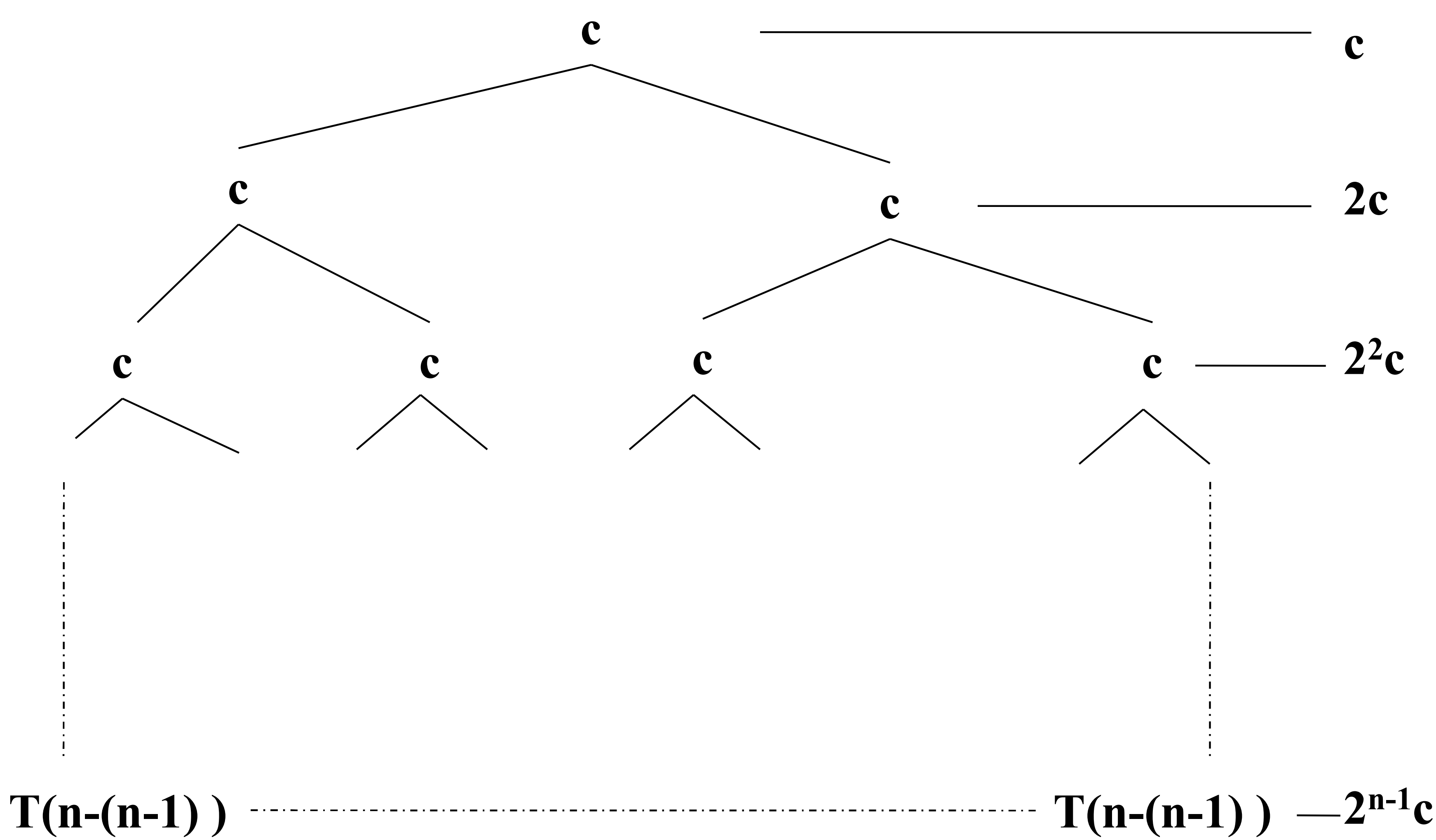
$$T(n) = 2\,T(n-1) + c$$



$$T(n-1) = 2\,T(n-2) + c$$


c
c
c
T(n-2)
T(n-2)
T(n-2)
T(n-2)


c
c
c
c
c
c
c
T(n-(n-1))
T(n-(n-1))


c
c
c
2c
c
c
c
c
2^2c
T(n-(n-1))
T(n-(n-1))
$2^{n-1}c$

$$T(n) = c + 2c + 2^2 c + \ldots\ldots\ldots + 2^{n-1} c$$

$$
\begin{aligned}
T(n) &= c+2c + 2^2 c+\dots\dots\dots+2^{n-1} c \\
&= c[1+2+2^2+.\ .\ .\ .\ .\ .+ 2^{n-1}]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= c+2c+2^2c+\dots\dots\dots+2^{n-1}c \\
&= c[1+2+2^2+\dots\dots+2^{n-1}] \\
&= c[\,[2^n-1]/[2-1]\,]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= c+2c + 2^2 c+\ldots\ldots\ldots+2^{n-1} c \\
&= c[1+2+2^2+\ldots\ldots\ldots+ 2^{n-1}] \\
&= c[\ [2^n-1] / [2-1]\ ] \\
&= c\ [2^n-1]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= c+2c + 2^2 c+\dots\dots\dots+2^{n-1} c \\
&= c[1+2+2^2+\dots\dots+ 2^{n-1}] \\
&= c[\,[2^{n}-1] / [2-1]\,] \\
&= c\,[2^{n}-1] \\
&= c\,2^n - c
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= c+2c + 2^2 c+\ldots\ldots\ldots+2^{n-1} c \\
&= c[1+2+2^2+.\ .\ .\ .\ .\ .+ 2^{n-1}] \\
&= c[\ [2^n-1] / [2-1]\ ] \\
&= c\ [2^n-1] \\
&= c\ 2^n - c \\
&= \mathbf{O(2^n)}
\end{aligned}
$$

Recursion Tree Method

$$T(n) = \quad T(n/2) + n^2$$

$$T(n) = T(n/2) + n^2$$

$$n^2$$

$$|$$

$$T(n/2)$$

$$T(n) = T(n/2) + n^2$$

$$\begin{tikzcd} n^2 \arrow[d, dash] \\ T(n/2) \end{tikzcd}$$

$$T(n/2) = T(n/2^2) + (n/2)^2$$

$$T(n/2) = \quad T(n/2^2) + (n/2)^2$$

$$n^2$$

$$(n/2)^2$$

$$T(n/2^2)$$

$$T(n/2) = \quad T(n/2^2) + (n/2)^2$$

$$n^2$$

$$(n/2)^2$$

$$T(n/2^2)$$

$$T(n/2^2) = \quad T(n/2^3) + (n/2^2)^2$$

$$T(n/2^2) = T(n/2^3) + (n/2^2)^2$$

$$n^2$$

$$(n/2)^2$$

$$(n/2^2)^2$$

$$T(n/2^3)$$

$$T(n/2^2) = T(n/2^3) + (n/2^2)^2$$

n^2

$(n/2)^2$

$(n/2^2)^2$

$T(n/2^k)$

$$T(n/2^2) = T(n/2^3) + (n/2^2)^2$$

n^2	n^2
$(n/2)^2$	$n^2/4$
$(n/2^2)^2$	$n^2/4^2$
$T(n/2^k)$	$n^2/4^k$

$$T(n/2^2) = T(n/2^3) + (n/2^2)^2$$

n^2	n^2
$(n/2)^2$	$n^2/4$
$(n/2^2)^2$	$n^2/4^2$
$\vdots$	
$T(n/2^k)$	$n^2/4^k$

Assume that $n/2^k = 1 \Rightarrow 2^k=n \Rightarrow k=\log_2 n$

$$T(n) = n^2+n^2/4 + n^2/4^2+\ldots\ldots\ldots+ n^2/4^k$$

$$T(n) = n^2+n^2/4 + n^2/4^2+\dots\dots\dots+ n^2/4^k$$

$$= n^2 [1+(1/4)+(1/4)^2+\dots\dots+ (1/4)^k]$$

$$
\begin{aligned}
T(n) &= n^2+n^2/4 + n^2/4^2+\ldots\ldots\ldots+ n^2/4^k \\
&= n^2\,[1+(1/4)+(1/4)^2+.\ .\ .\ .\ .\ .+ (1/4)^{k}] \\
&= n^2[\,[1- (1/4)^{k+1}] / [1-(1/4)]\,]
\end{aligned}
$$

$$\begin{aligned} T(n) &= n^2+n^2/4 + n^2/4^2+\ldots\ldots\ldots+ n^2/4^k \\ &= n^2\,[1+(1/4)+(1/4)^2+.\;.\;.\;.\;.\;.\;.+ (1/4)^{k}] \\ &= n^2[\;[1- (1/4)^{k+1}] \,/\, [1-(1/4)]\;] \\ &= n^2[\;[1- (1/4)\,(1/4)^{k}] \,/\, [3/4]\;] \end{aligned}$$

$$
\begin{aligned}
T(n) &= n^2+n^2/4 + n^2/4^2+\dots\dots\dots+ n^2/4^k \\
&= n^2 [1+(1/4)+(1/4)^2+. \ . \ . \ . \ . \ . \ .+ (1/4)^{k}] \\
&= n^2[\ [1- (1/4)^{k+1}] / [1-(1/4)]\ ] \\
&= n^2[\ [1- (1/4)(1/4)^{k}] / [3/4]\ ] \\
&= (4/3)\ n^2[\ 1- (1/4)(1/4^{\log n})]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+n^2/4 + n^2/4^2+\ldots\ldots\ldots+ n^2/4^k \\
&= n^2 [1+(1/4)+(1/4)^2+\,.\,.\,.\,.\,.\,.\,.+ (1/4)^k] \\
&= n^2[\, [1- (1/4)^{k+1}] \,/\, [1-(1/4)]\,] \\
&= n^2[\, [1- (1/4)\,(1/4)^k] \,/\, [3/4]\,] \\
&= (4/3)\, n^2[\, 1- (1/4)\,(1/4^{\log n})] \\
&= (4/3)\, n^2[\, 1- (1/4)\,(1/n^{\log 4})]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+n^2/4 + n^2/4^2+\dots\dots\dots+ n^2/4^k \\
&= n^2 [1+(1/4)+(1/4)^2+. \ . \ . \ . \ . \ . \ .+ (1/4)^k] \\
&= n^2[\ [1- (1/4)^{k+1}] / [1-(1/4)]\ ] \\
&= n^2[\ [1- (1/4)(1/4)^k] / [3/4]\ ] \\
&= (4/3)\ n^2[\ 1- (1/4)(1/4^{\log n})] \\
&= (4/3)\ n^2[\ 1- (1/4)(1/n^{\log 4})] \\
&= (4/3)\ n^2[\ 1- (1/4)(1/n^2)]
\end{aligned}
$$

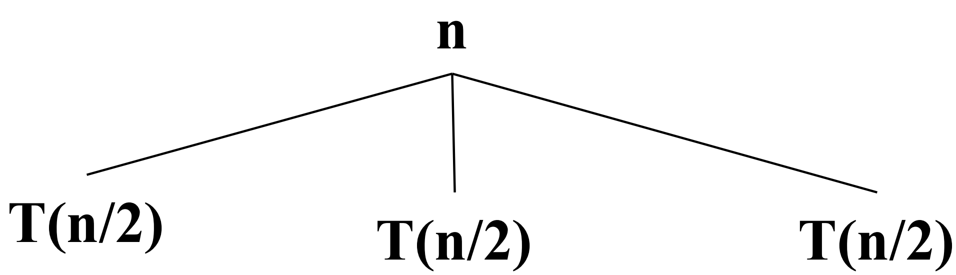
$$
\begin{aligned}
T(n) &= n^2+n^2/4 + n^2/4^2+\ldots\ldots\ldots+ n^2/4^k \\
&= n^2 \,[1+(1/4)+(1/4)^2+. \;. \;. \;. \;. \;. \;.+ (1/4)^{\,k}] \\
&= n^2[\;[1- (1/4)^{\,k+1}] \,/\, [1-(1/4)]\;] \\
&= n^2[\;[1- (1/4)\,(1/4)^{\,k}] \,/\, [3/4]\;] \\
&= (4/3)\; n^2[\; 1- (1/4)\,(1/4^{\,\log n}\;)] \\
&= (4/3)\; n^2[\; 1- (1/4)\,(1/n^{\,\log 4}\;)] \\
&= (4/3)\; n^2[\; 1- (1/4)\,(1/n^2)] \\
&= (4/3)\; n^2- (1/3)
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+n^2/4 + n^2/4^2+\ldots\ldots\ldots+ n^2/4^k \\
&= n^2\,[1+(1/4)+(1/4)^2+.\ .\ .\ .\ .\ .\ .+ (1/4)^{\,k}] \\
&= n^2[\ [1- (1/4)^{\,k+1}] \ /\ [1-(1/4)]\ ] \\
&= n^2[\ [1- (1/4)\,(1/4)^{\,k}] \ /\ [3/4]\ ] \\
&= (4/3)\ n^2[\ 1- (1/4)\,(1/4^{\,\log n}\ )] \\
&= (4/3)\ n^2[\ 1- (1/4)\,(1/n^{\,\log 4}\ )] \\
&= (4/3)\ n^2[\ 1- (1/4)\,(1/n^2)] \\
&= (4/3)\ n^2- (1/3) \\
&= \mathbf{O(n^2)}
\end{aligned}
$$

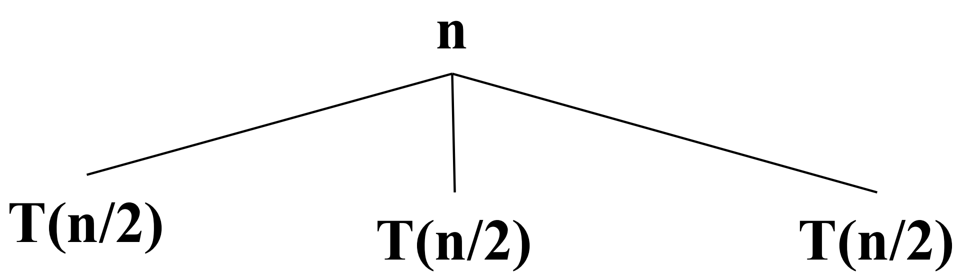
Recursion Tree Method

$T(n) = \quad 3\,T(n/2) + n$

$$T(n) = 3\,T(n/2) + n$$

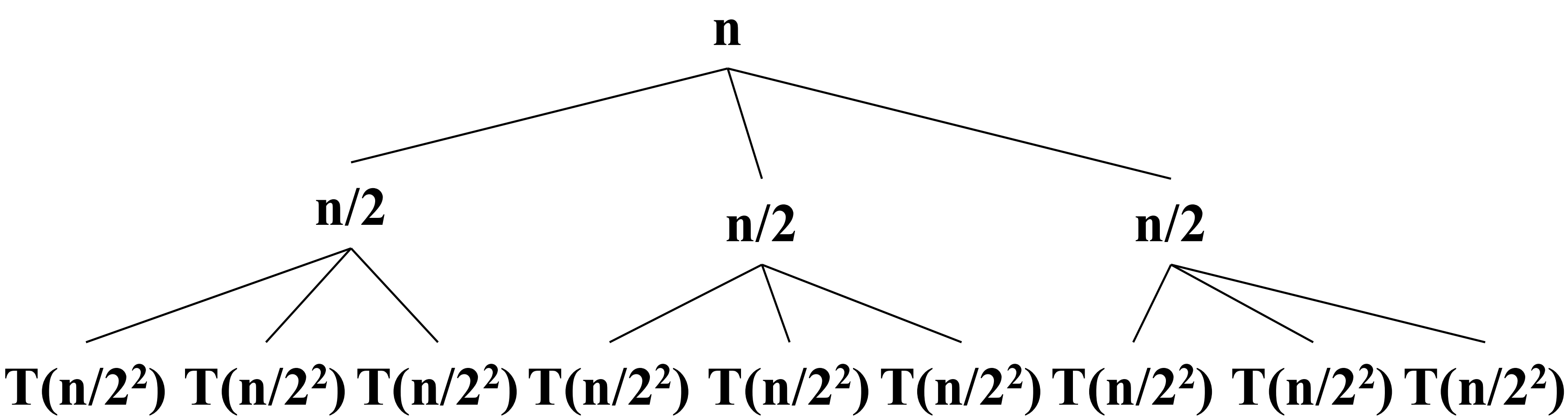


$$T(n) = 3\,T(n/2) + n$$

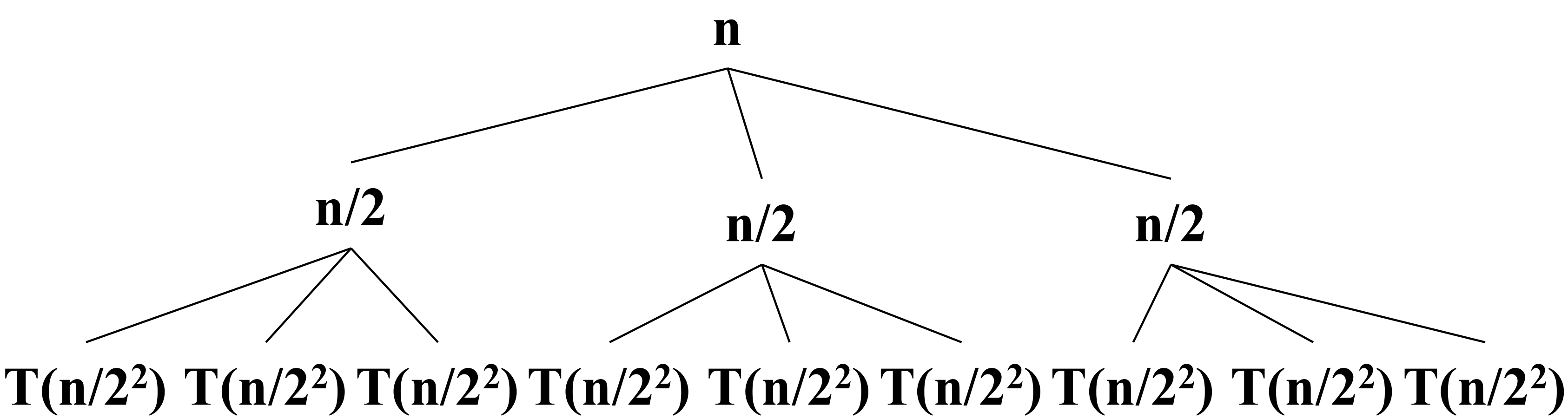


$$T(n/2) = 3\,T(n/2^2) + (n/2)$$

$$T(n/2) = 3\,T(n/2^2) + (n/2)$$

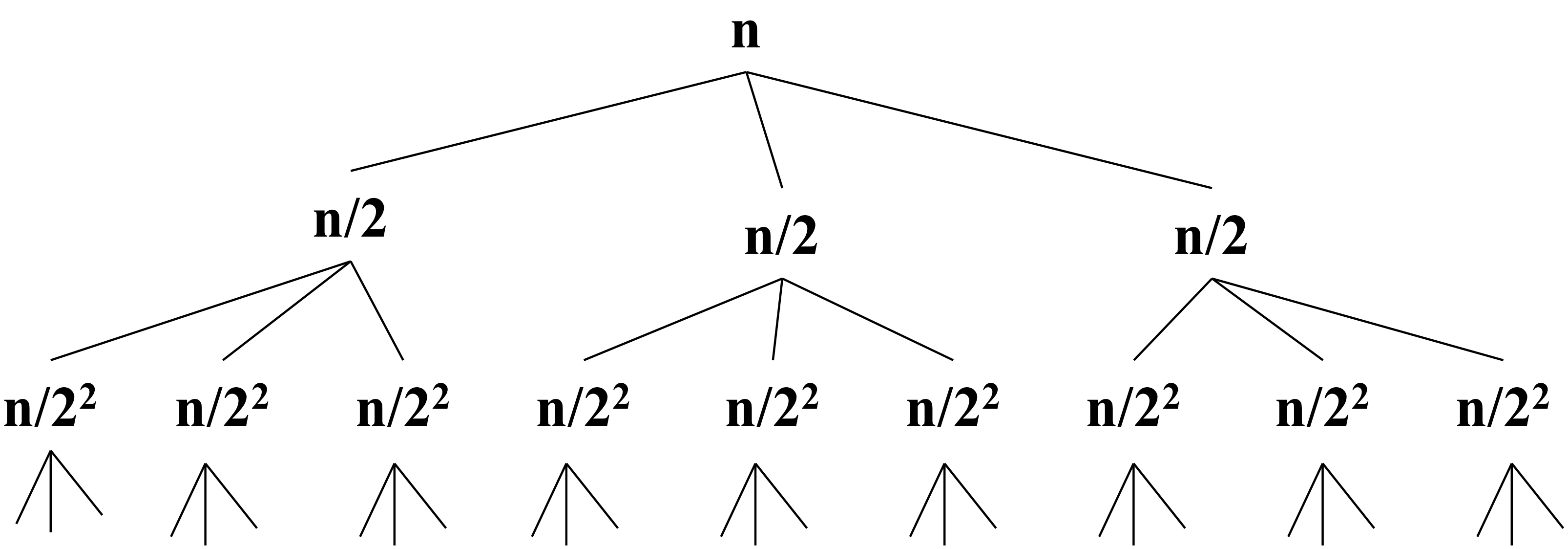


$$T(n/2) = \quad 3\,T(n/2^2) + (n/2)$$

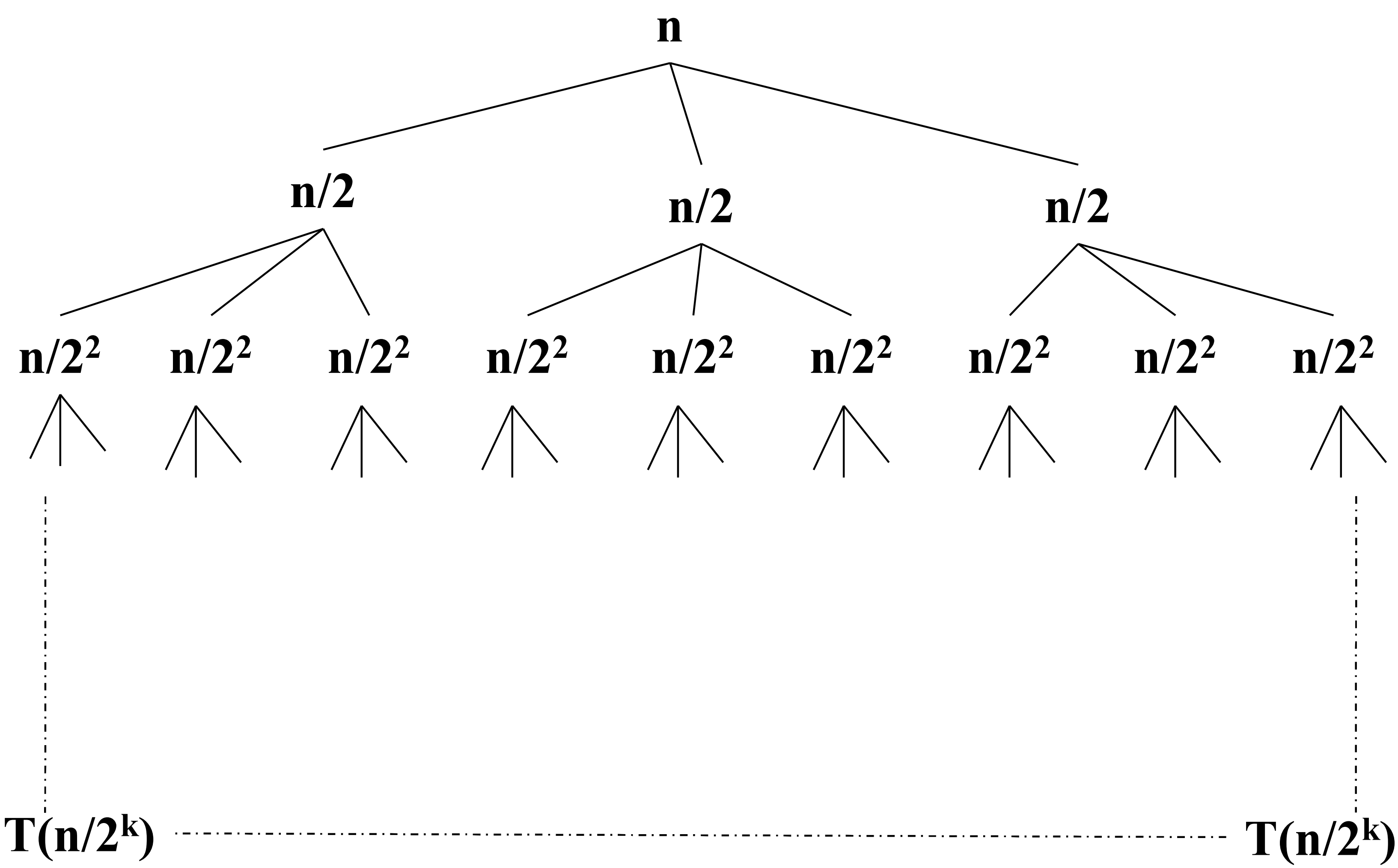


$$T(n/2^2) = \quad 3\,T(n/2^3) + (n/2^2)$$

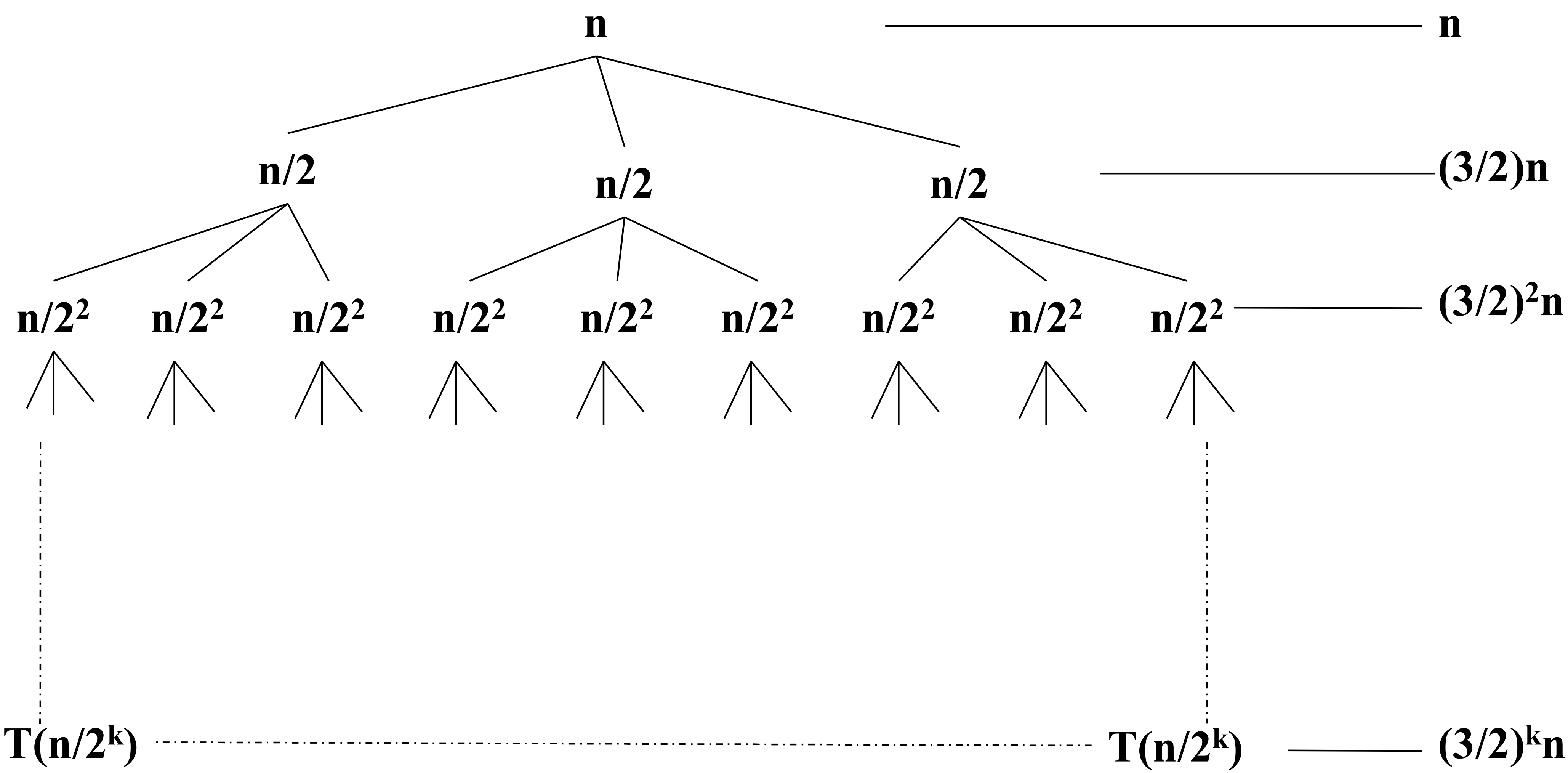
$$T(n/2^2) = \quad 3\,T(n/2^3) + (n/2^2)$$



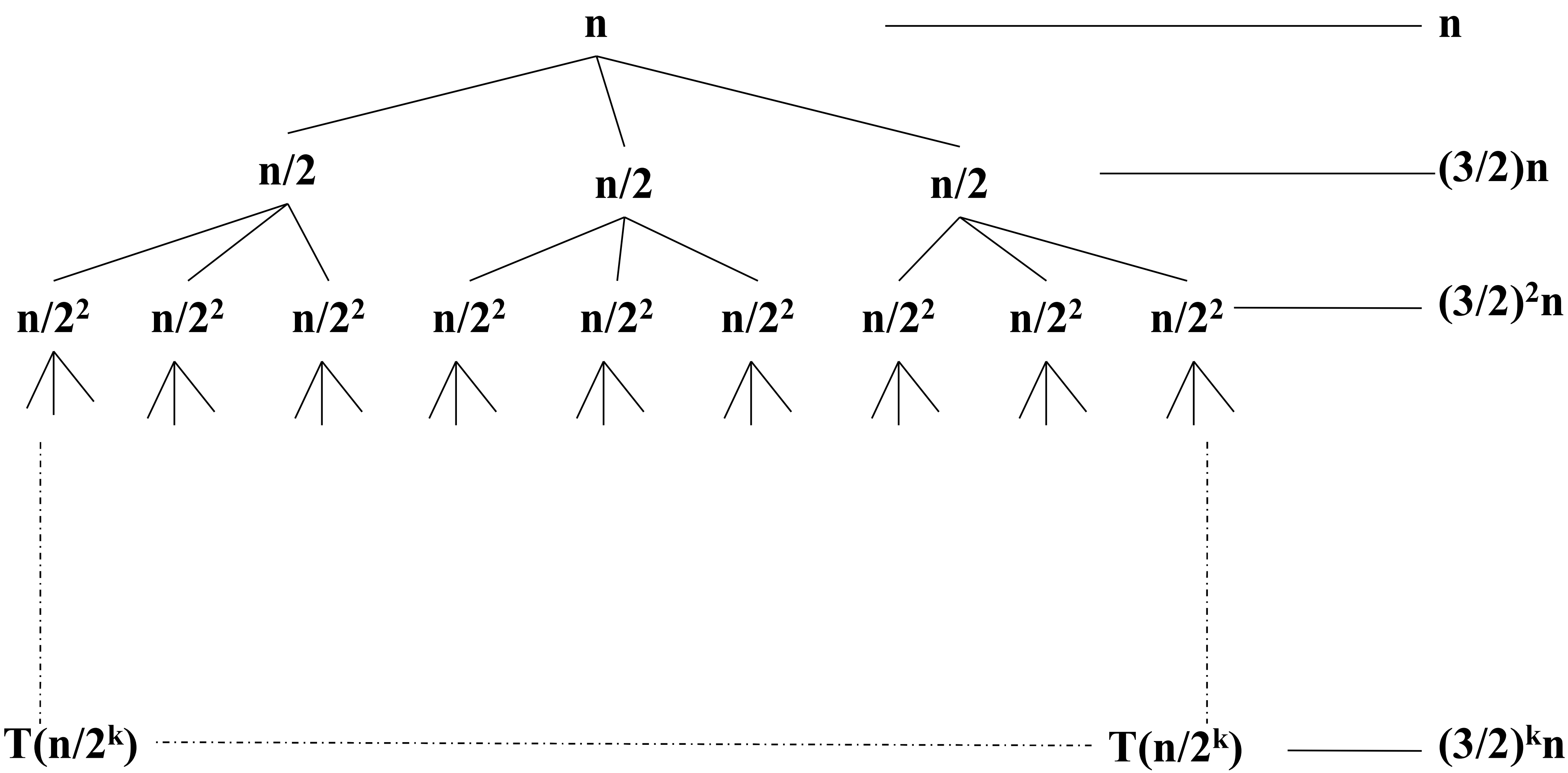
$$T(n/2^2) = 3\,T(n/2^3) + (n/2^2)$$



$$T(n/2^2) = \; 3\,T(n/2^3) + (n/2^2)$$



$$T(n/2^2) = \; 3\,T(n/2^3) + (n/2^2)$$



Assume that $n/2^k = 1$ ➔ $2^k = n$ ➔ $k = \log_2 n$

$$T(n) = n+(3/2)n+ (3/2)^2n+\ldots\ldots\ldots+ (3/2)^kn$$

$$T(n) = n+(3/2)n+(3/2)^2n+\ldots\ldots\ldots+(3/2)^kn$$
$$= n[1+(3/2)+(3/2)^2+\ldots\ldots\ldots+(3/2)^k]$$

$$
\begin{aligned}
T(n) &= n+(3/2)n+(3/2)^2 n+\dots\dots\dots+(3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+\dots\dots\dots+(3/2)^k] \\
&= n[\,[(3/2)^{k+1}-1]\,/\,[(3/2)-1]\,]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n+(3/2)n+(3/2)^2 n+\ldots\ldots\ldots+(3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+.\ .\ .\ .\ .\ .\ .+(3/2)^{k}] \\
&= n[\ [\ (3/2)^{k+1} - 1] \ /\ [(3/2)-1]\ ] \\
&= n[[(3/2)\ (3/2)^{k} - 1] \ /\ [1/2]\ ]
\end{aligned}
$$

$$T(n) = n+(3/2)n+ (3/2)^2 n+ \ldots\ldots..+ (3/2)^k n$$

$$= n[1+(3/2)+(3/2)^2+. \; . \; . \; . \; . \; . \; .+ (3/2)^{k}]$$

$$= n[\; [\; (3/2)^{k+1} - 1] \;/\; [(3/2)-1]\;]$$

$$= n[[(3/2)\; (3/2)^{k} - 1] \;/\; [1/2]\;]$$

$$= 2n[(3/2)\; (3^{k} / 2^{k}) \; - 1]$$

$$
\begin{aligned}
T(n) &= n+(3/2)n+(3/2)^2n+\ldots\ldots..+(3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+.\ .\ .\ .\ .\ .\ .+(3/2)^k] \\
&= n[\,[(3/2)^{k+1}-1]\,/\,[(3/2)-1]\,] \\
&= n[[(3/2)\,(3/2)^k-1]\,/\,[1/2]\,] \\
&= 2n[(3/2)\,(3^k/2^k)\ -1] \\
&= 2n[(3/2)\,(3^{\log_2 n}/2^{\log_2 n})\ -1]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n+(3/2)n+ (3/2)^2 n+\ldots\ldots\ldots+ (3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+.\ .\ .\ .\ .\ .\ .+ (3/2)^{k}] \\
&= n[\ [\ (3/2)^{k+1} - 1] \ /\ [(3/2)-1]\ ] \\
&= n[[(3/2)\ (3/2)^{k} - 1] \ /\ [1/2]\ ] \\
&= 2n[(3/2)\ (3^{k} /2^{k})\ - 1] \\
&= 2n[(3/2)\ (3^{\log_2 n} /2^{\log_2 n})\ - 1] \\
&= 2n[(3/2)\ (n^{\log_2 3} /n^{\log_2 2})\ - 1]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n+(3/2)n+ (3/2)^2 n+\ldots\ldots..+ (3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+.\ .\ .\ .\ .\ .\ .+ (3/2)^{k}] \\
&= n[\ [\ (3/2)^{k+1} - 1] \ /\ [(3/2)-1]\ ] \\
&= n[[(3/2)\ (3/2)^{k} - 1] \ /\ [1/2]\ ] \\
&= 2n[(3/2)\ (3^{k} / 2^{k}) \ - 1] \\
&= 2n[(3/2)\ (3^{\log_2 n} / 2^{\log_2 n}) \ - 1] \\
&= 2n[(3/2)\ (n^{\log_2 3} / n^{\log_2 2}) \ - 1] \\
&= 2n[(3/2)\ (n^{\log_2 3} / n) \ - 1]
\end{aligned}
$$

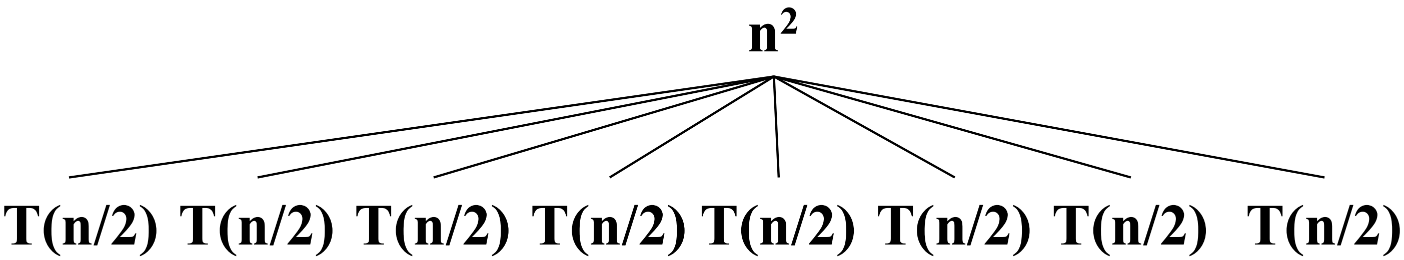
$$
\begin{aligned}
T(n) &= n+(3/2)n+(3/2)^2 n+\ldots\ldots\ldots+(3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+.\ .\ .\ .\ .\ .\ .+(3/2)^{k}] \\
&= n[\ [\ (3/2)^{k+1} - 1] / [(3/2)-1]\ ] \\
&= n[[(3/2)\ (3/2)^{k} - 1] / [1/2]\ ] \\
&= 2n[(3/2)\ (3^{k}/2^{k}) - 1] \\
&= 2n[(3/2)\ (3^{\log_2 n}/2^{\log_2 n}) - 1] \\
&= 2n[(3/2)\ (n^{\log_2 3}/n^{\log_2 2}) - 1] \\
&= 2n[(3/2)\ (n^{\log_2 3}/n) - 1] \\
&= 3n^{\log_2 3} - 2n
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n+(3/2)n+(3/2)^2 n+\ldots\ldots\ldots+(3/2)^k n \\
&= n[1+(3/2)+(3/2)^2+\ldots\ldots\ldots+(3/2)^k] \\
&= n[\,[\,(3/2)^{k+1}-1]\,/\,[(3/2)-1]\,] \\
&= n[[(3/2)\,(3/2)^k-1]\,/\,[1/2]\,] \\
&= 2n[(3/2)\,(3^k/2^k)\ -1] \\
&= 2n[(3/2)\,(3^{\log_2 n}/2^{\log_2 n})\ -1] \\
&= 2n[(3/2)\,(n^{\log_2 3}/n^{\log_2 2})\ -1] \\
&= 2n[(3/2)\,(n^{\log_2 3}/n)\ -1] \\
&= 3n^{\log_2 3}-2n \\
&= \mathbf{O(n^{\log_2 3})}
\end{aligned}
$$

Recursion Tree Method

$T(n) = \quad 8\,T(n/2) + n^2$

$$T(n) = 8\,T(n/2) + n^2$$



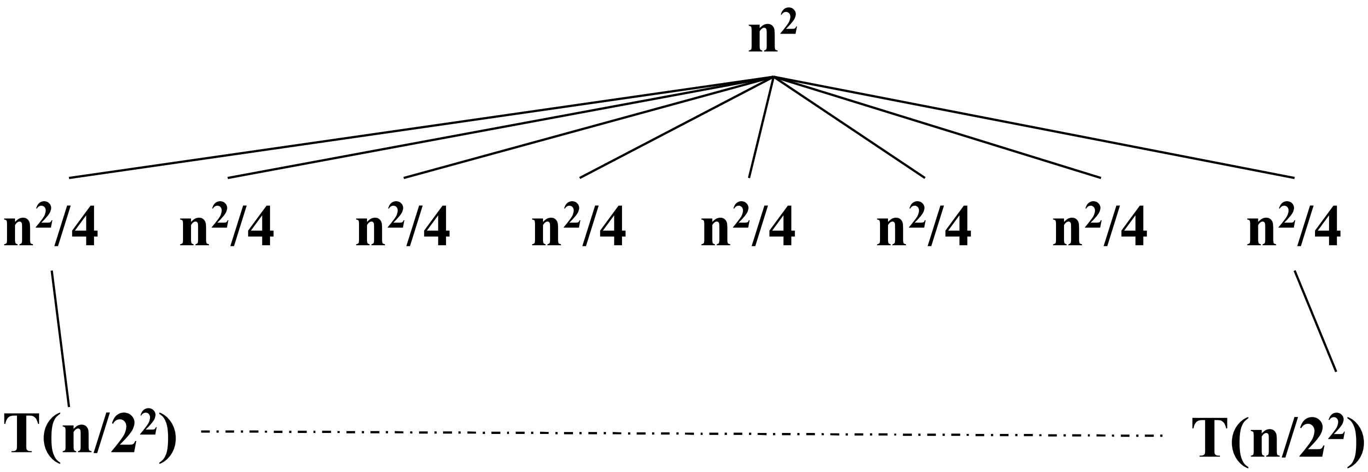
$$T(n) = \quad 8\,T(n/2) + n^2$$

$$n^2$$

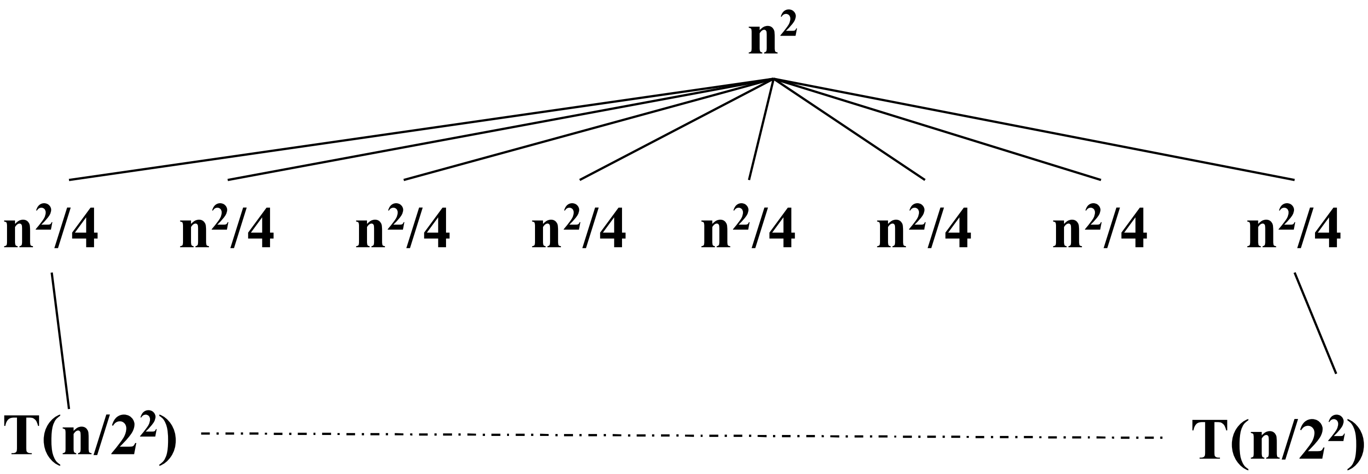
$$T(n/2) \quad T(n/2) \quad T(n/2) \quad T(n/2) \quad T(n/2) \quad T(n/2) \quad T(n/2) \quad T(n/2)$$

$$T(n/2) = \quad 8\,T(n/2^2) + (n/2)^2$$

$$T(n/2) = 8\,T(n/2^2) + (n/2)^2$$

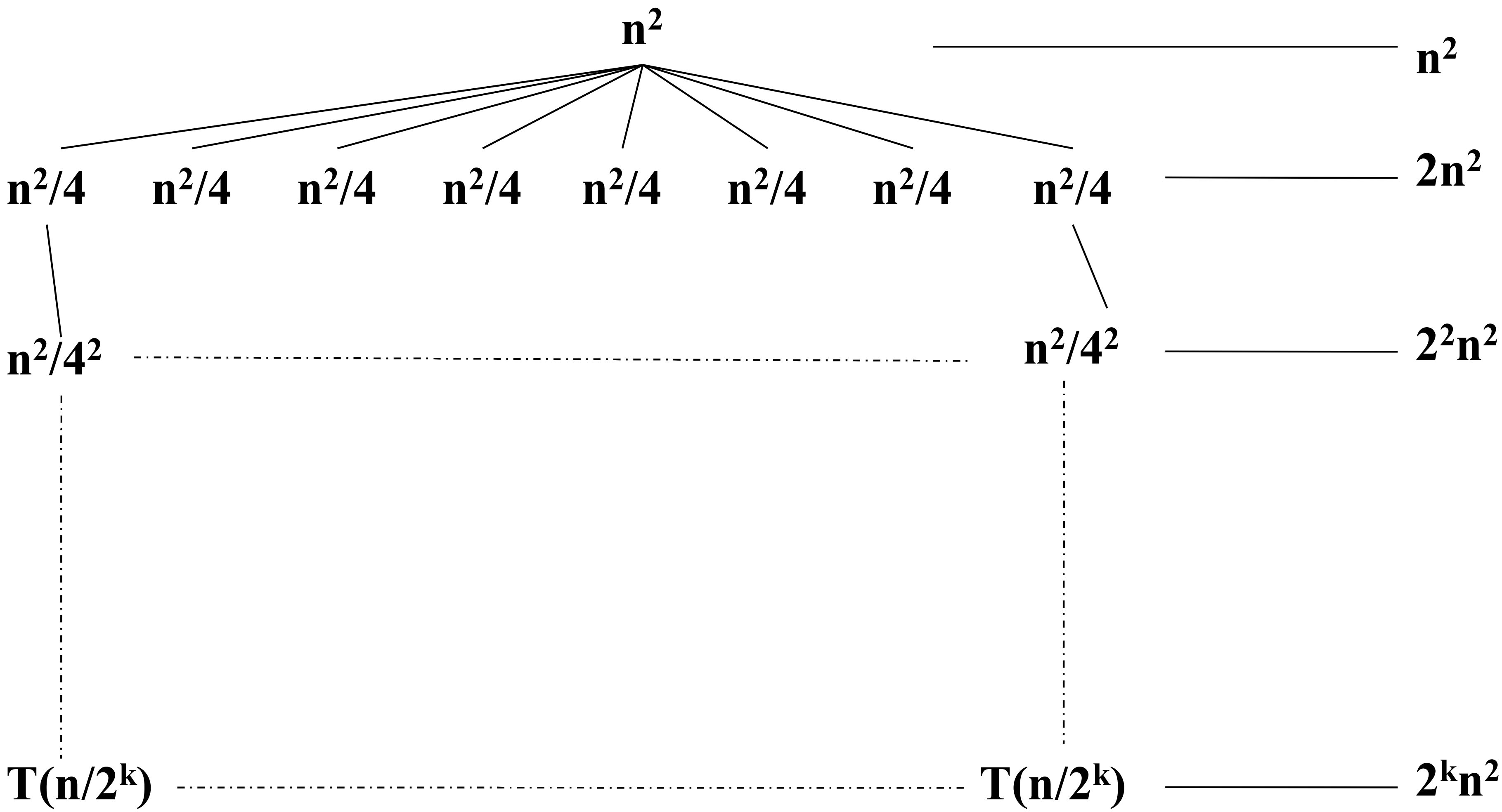


$$T(n/2) = \quad 8\,T(n/2^2) + (n/2)^2$$

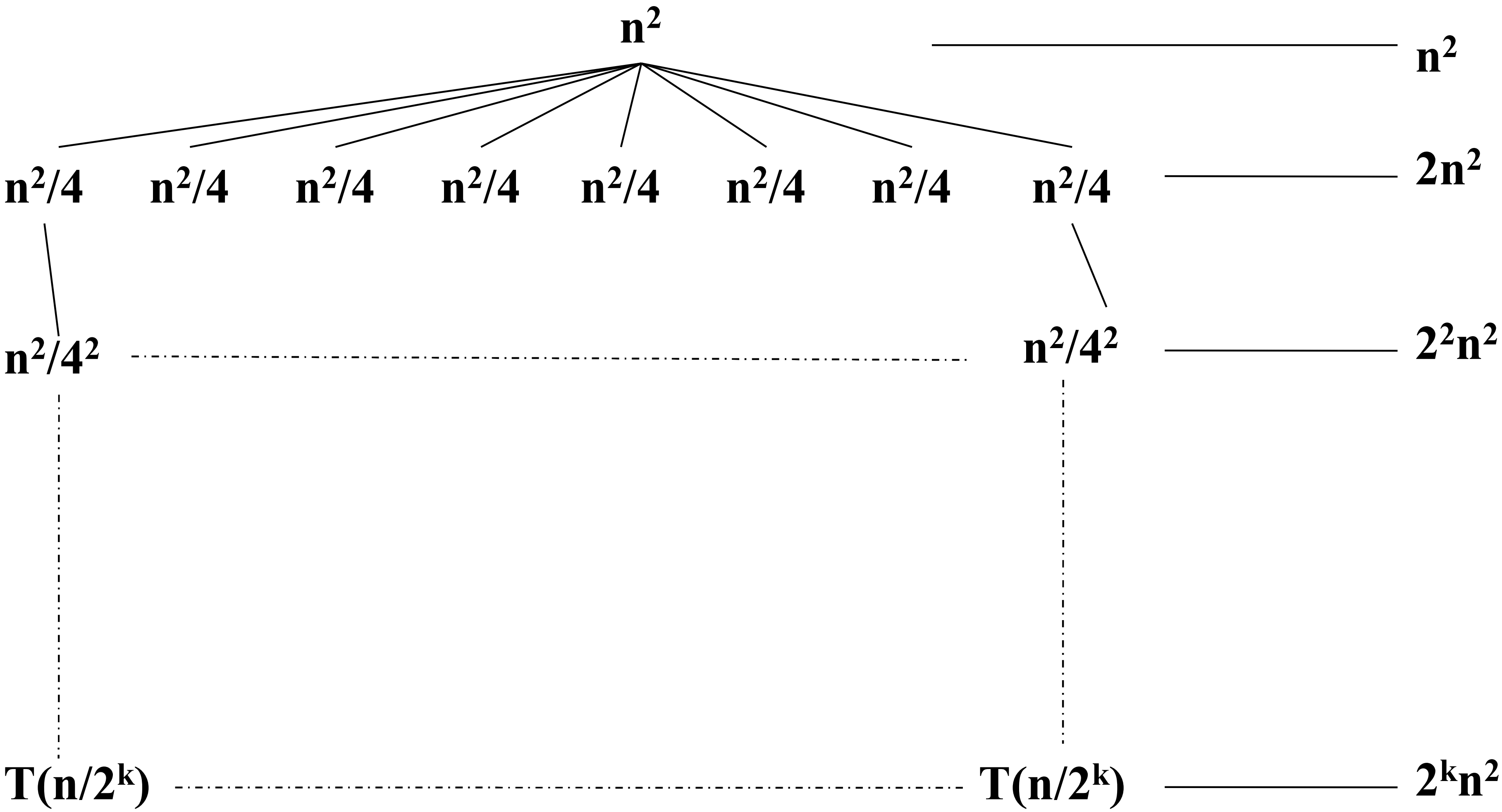


$$T(n/2^2) = \quad 8\,T(n/2^3) + (n/2^2)^2$$

$$T(n/2^2) = \quad 8\,T(n/2^3) + (n/2^2)^2$$



$$T(n/2^2) = \quad 8\,T(n/2^3) + (n/2^2)^2$$



Assume that $n/2^k = 1$ ➔ $2^k = n$ ➔ $k = \log_2 n$

$$T(n) = n^2 + 2n^2 + 2^2 n^2 + \ldots\ldots\ldots + 2^k n^2$$

$$T(n) = n^2+2n^2 + 2^2 n^2+\ldots\ldots\ldots+2^k n^2$$

$$= n^2 [1+2+2^2+. . . . . . .+ 2^k]$$

$$
\begin{aligned}
T(n) &= n^2+2n^2 + 2^2 n^2+\ldots\ldots\ldots+2^k n^2 \\
&= n^2 [1+2+2^2+. . . . . . .+ 2^k] \\
&= n^2 [[2^{k+1}-1] / [2-1]]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+2n^2 + 2^2 n^2+\ldots\ldots\ldots+2^k n^2 \\
&= n^2 [1+2+2^2+. \ . \ . \ . \ . \ .+ 2^k] \\
&= n^2 [[2^{k+1}-1] / [2-1]] \\
&= n^2 [2x2^k-1]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+2n^2 + 2^2 n^2+\ldots\ldots\ldots+2^k n^2 \\
&= n^2 [1+2+2^2+.\ .\ .\ .\ .\ .+ 2^k] \\
&= n^2 [\ [2^{k+1}-1] / [2-1]\ ] \\
&= n^2 [2x2^k-1] \\
&= n^2 [\ 2n-1]
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+2n^2 + 2^2\, n^2+\ldots\ldots\ldots+2^k\, n^2 \\
&= n^2\,[1+2+2^2+.\ .\ .\ .\ .\ .+ 2^{\,k}] \\
&= n^2\,[\ [2^{\,k+1}-1] \,/\, [2-1]\ ] \\
&= n^2\ [2x2^{\,k}-1] \\
&= n^2\,[\ 2n-1] \\
&= 2n^3 – n^2
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= n^2+2n^2 + 2^2 n^2+\dots\dots\dots+2^k n^2 \\
&= n^2 [1+2+2^2+. \; . \; . \; . \; . \; .+ 2^{\,k}] \\
&= n^2 [\; [2^{\,k+1}-1] \,/\, [2-1] \;] \\
&= n^2 \; [2x2^{\,k}-1] \\
&= n^2 [\; 2n-1] \\
&= 2n^3 - n^2 \\
&\mathbf{= O(n^3)}
\end{aligned}
$$

Master's Method

- Apply a direct formula to solve recurrences of a specific form.
- Fast and precise.
- Applicable only to specific forms.

Master's Method

$T(n) = aT(n/b) + \Theta\ (n^k \log^p(n))$

$a \geq 1$ $b > 1$ $k \geq 0$ and p is a real number

1. If $a > b^k$ then $T(n) = \Theta(n^{(\log_b a)})$
2. If $a = b^k$
   a) If $p > -1$ then $T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$
   b) If $p = -1$ then $T(n) = \Theta(n^{(\log_b a)} \log(\log n))$
   c) If $p < -1$ then $T(n) = \Theta(n^{(\log_b a)})$
3. If $a < b^k$
   a) If $p \geq 0$, then $T(n) = \Theta(n^k \log^p(n))$
   b) If $p < 0$, then $T(n) = O(n^k)$

Master's Method

$T(n) = 7T(n/2) + n^2$

$$T(n) = aT(n/b) + \Theta\,(n^k \log^p(n))$$

$a \geq 1$ $b > 1$ $k \geq 0$ and p is a real number

Master's Method

$T(n) = 7T(n/2) + n^2$

$a=7$ $b=2$ $n^2=\Theta(n^2 \log^0(n))$ $k=2$ $p=0$

Master's Method

$T(n) = 7T(n/2) + n^2$

$a=7$ $b=2$ $n^2=\Theta(n^2 \log^0(n))$ $k=2$ $p=0$

$b^k = 2^2 = 4$

Master's Method

$T(n) = 7T(n/2) + n^2$

$a=7$ $b=2$ $n^2=\Theta(n^2 \log^0(n))$ $k=2$ $p=0$

$b^k = 2^2 = 4$

Here $a > b^k$ 1

Master's Method

$T(n) = aT(n/b) + \Theta\ (n^k \log^p(n))$

$a \geq 1$ $b > 1$ $k \geq 0$ and p is a real number

1. If $a > b^k$ then $T(n) = \Theta(n^{(\log_b a)})$
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   a) If $p > -1$ then $T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$
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3. If $a < b^k$
   a) If $p \geq 0$, then $T(n) = \Theta(n^k \log^p(n))$
   b) If $p < 0$, then $T(n) = O(n^k)$

Master's Method

$\mathbf{T(n) = 7T(n/2) + n^2}$

$a=7 \quad b=2 \quad n^2=\Theta(n^2 \log^0(n)) \quad k=2 \quad p=0$

$b^k = 2^2 = 4$

Here $a > b^k$ 1

$T(n) = \Theta(n^{(\log_b a)})$

Master's Method

$$\mathbf{T(n) = 7T(n/2) + n^2}$$

$a=7 \quad b=2 \quad n^2=\Theta(n^2 \log^0(n)) \quad k=2 \quad p=0$

$b^k = 2^2 = 4$

Here $a > b^k$ 1

$T(n) = \Theta(n^{(\log_b a)})$

$= \boldsymbol{\Theta(n^{(\log_2 7)})}$

Master's Method

$T(n) = 2\,T(n/2) + n \log n$

$T(n) = aT(n/b) + \Theta\,(n^k \log^p(n))$

Master's Method

$T(n) = 2\,T(n/2) + n \log n$

$a=2$ $b=2$ $n \log n = \Theta(n^1 \log^1(n))$ $k=1$ $p=1$

Master's Method

T(n) = 2 T(n/2) + n log n

a=2 b=2 n log n =Θ ($n^1 \log^1(n)$) k=1 p=1

$b^k = 2^1 = 2$

Master's Method

$T(n) = 2\,T(n/2) + n \log n$

$a=2$ $b=2$ $n \log n = \Theta(n^1 \log^1(n))$ $k=1$ $p=1$

$b^k = 2^1 = 2$

Here $a = b^k$ and $p > -1$ 2a)

Master's Method

$T(n) = aT(n/b) + \Theta\,(n^k \log^p(n))$

$a \geq 1$ $b > 1$ $k \geq 0$ and p is a real number

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2. If $a = b^k$
   a) If $p > -1$ then $T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$
   b) If $p = -1$ then $T(n) = \Theta(n^{(\log_b a)} \log(\log n))$
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   a) If $p \geq 0$, then $T(n) = \Theta(n^k \log^p(n))$
   b) If $p < 0$, then $T(n) = O(n^k)$

Master's Method

T(n) = 2 T(n/2) + n log n

a=2 b=2 n log n $=\Theta(n^1 \log^1(n))$ k=1 p=1

$b^k = 2^1 = 2$

Here $a = b^k$ and $p > -1$ 2a)

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

Master's Method

T(n) = 2 T(n/2) + n log n

a=2   b=2   n log n $=\Theta\ (n^1 \log^1(n))$ k=1   p=1

$b^k = 2^1 = 2$

Here $a = b^k$ and $p > -1$ 2a)

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

$= \Theta(n^{(\log_2 2)} \log^2(n))$

Master's Method

$T(n) = 2\,T(n/2) + n \log n$

$a=2 \quad b=2 \quad n \log n = \Theta(n^1 \log^1(n)) \quad k=1 \quad p=1$

$b^k = 2^1 = 2$

Here $a = b^k$ and $p > -1$ — **2a)**

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

$= \Theta(n^{(\log_2 2)} \log^2(n))$

$=$ **$\Theta(n\log^2(n))$**

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2$ $b=4$ $\sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n))$ $k=1/2$ $p=0$

Master’s Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2 \quad b=4 \quad \sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n)) \qquad k=1/2 \qquad p=0$

$b^k = 4^{1/2} = 2$

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2 \quad b=4 \quad \sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n)) \qquad k=1/2 \qquad p=0$

$b^k = 4^{1/2} = 2$

Here $a = b^k$ and $p > -1$ 2a)

Master's Method

$T(n) = aT(n/b) + \Theta\ (n^k \log^p(n))$

$a \geq 1$ $b > 1$ $k \geq 0$ and p is a real number

1. If $a > b^k$ then $T(n) = \Theta(n^{(\log_b a)})$
2. If $a = b^k$
   a) If $p > -1$ then $T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$
   b) If $p = -1$ then $T(n) = \Theta(n^{(\log_b a)} \log(\log n))$
   c) If $p < -1$ then $T(n) = \Theta(n^{(\log_b a)})$
3. If $a < b^k$
   a) If $p \geq 0$, then $T(n) = \Theta(n^k \log^p(n))$
   b) If $p < 0$, then $T(n) = O(n^k)$

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2 \quad b=4 \quad \sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n)) \quad k=1/2 \quad p=0$

$b^k = 4^{1/2} = 2$

Here $a = b^k$ and $p > -1$ 2a)

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2 \quad b=4 \quad \sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n)) \qquad k=1/2 \qquad p=0$

$b^k = 4^{1/2} = 2$

Here $a = b^k$ and $p > -1$ 2a)

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

$= \Theta(n^{(\log_4 2)} \log^1(n))$

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2 \quad b=4 \quad \sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n)) \quad k=1/2 \quad p=0$

$b^k = 4^{1/2} = 2$

Here $a = b^k$ and $p > -1$ 2a)

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

$= \Theta(n^{(\log_4 2)} \log^1(n))$

$= \Theta(n^{(1/2)} \log(n))$

Master's Method

$T(n) = 2T(n/4) + \sqrt{n}$

$a=2$ $b=4$ $\sqrt{n} = n^{1/2} = \Theta(n^{1/2} \log^0(n))$ $k=1/2$ $p=0$

$b^k = 4^{1/2} = 2$

Here $a = b^k$ and $p > -1$ — 2a)

$T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$

$= \Theta(n^{(\log_4 2)} \log^1(n))$

$= \Theta(n^{(1/2)} \log(n))$

$= \Theta(\sqrt{n} \log(n))$

Master's Method

$T(n) = 2\,T(n/2) + (n/\log n)$

Master's Method

$T(n) = 2\,T(n/2) + (n/\log n)$

$a=2$ $b=2$ $(n/\log n) = \Theta\,(n^1 \log^{-1}(n))$ $k=1$ $p=-1$

Master's Method

$T(n) = 2\ T(n/2) + (n/\log n)$

$a=2$ $b=2$ $(n/\log n) = \Theta\ (n^1 \log^{-1}(n))$ $k=1$ $p=-1$

$b^k = 2^1 = 2$

Master's Method

$T(n) = 2\,T(n/2) + (n/\log n)$

$a=2$ $b=2$ $(n/\log n) = \Theta(n^1 \log^{-1}(n))$ $k=1$ $p=-1$

$b^k = 2^1 = 2$

Here $a = b^k$ and $p=-1$ 2b)

Master's Method

$T(n) = aT(n/b) + \Theta(n^k \log^p(n))$

$a \geq 1$ $\quad b > 1$ $\quad k \geq 0$ and $\quad$ p is a real number

1. If $a > b^k$ then $T(n) = \Theta(n^{(\log_b a)})$
2. If $a = b^k$
   a) If $p > -1$ then $T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$
   b) If $p = -1$ then $T(n) = \Theta(n^{(\log_b a)} \log(\log n))$
   c) If $p < -1$ then $T(n) = \Theta(n^{(\log_b a)})$
3. If $a < b^k$
   a) If $p \geq 0$, then $T(n) = \Theta(n^k \log^p(n))$
   b) If $p < 0$, then $T(n) = O(n^k)$

Master's Method

T(n) = 2 T(n/2) + (n/log n)

a=2 b=2 (n/log n) $=\Theta(n^1 \log^{-1}(n))$ k=1 p=-1

$b^k = 2^1 = 2$

Here $a = b^k$ and p=-1 2b)

$T(n) = \Theta(n^{(\log_b a)} \log(\log n))$

Master's Method

$T(n) = 2\ T(n/2) + (n/\log n)$

$a=2 \quad b=2 \quad (n/\log n) = \Theta\ (n^1 \log^{-1}(n)) \quad k=1 \quad p=-1$

$b^k = 2^1 = 2$

Here $a = b^k$ and $p=-1$ 2b)

$T(n) = \Theta(n^{(\log_b a)} \log(\log n))$

$= \Theta(n^{(\log_2 2)} \log(\log n))$

Master's Method

$T(n) = 2\ T(n/2) + (n/\log n)$

$a=2 \quad b=2 \quad (n/\log n) = \Theta\ (n^1 \log^{-1}(n)) \quad k=1 \quad p=-1$

$b^k = 2^1 = 2$

Here $a = b^k$ and $p=-1$ 2b)

$T(n) = \Theta(n^{(\log_b a)} \log(\log n))$

$= \Theta(n^{(\log_2 2)} \log(\log n))$

$=$ **$\Theta(n \log(\log n))$**

Master's Method

$T(n) = 3\ T(n/2) + n^2$

Master's Method

$T(n) = 3\ T(n/2) + n^2$

$a=3 \quad b=2 \quad n^2 = \Theta\ (n^2 \log^0(n)) \quad k=2 \quad p=0$

Master's Method

$T(n) = 3\,T(n/2) + n^2$

$a=3$ $b=2$ $n^2 = \Theta(n^2 \log^0(n))$ $k=2$ $p=0$

$b^k = 2^2 = 4$

Master's Method

$T(n) = 3\ T(n/2) + n^2$

$a=3 \quad b=2 \quad n^2 = \Theta(n^2 \log^0(n)) \quad k=2 \quad p=0$

$b^k = 2^2 = 4$

Here $a < b^k$ and $p \geq 0$ — **3a)**

Master's Method

$T(n) = aT(n/b) + \Theta\ (n^k \log^p(n))$

$a \geq 1$ $b > 1$ $k \geq 0$ and p is a real number

1. If $a > b^k$ then $T(n) = \Theta(n^{(\log_b a)})$
2. If $a = b^k$
   a) If $p > -1$ then $T(n) = \Theta(n^{(\log_b a)} \log^{p+1}(n))$
   b) If $p = -1$ then $T(n) = \Theta(n^{(\log_b a)} \log(\log n))$
   c) If $p < -1$ then $T(n) = \Theta(n^{(\log_b a)})$
3. If $a < b^k$
   a) If $p \geq 0$, then $T(n) = \Theta(n^k \log^p(n))$
   b) If $p < 0$, then $T(n) = O(n^k)$

Master's Method

$T(n) = 3\ T(n/2) + n^2$

$a=3$ $b=2$ $n^2 = \Theta(n^2 \log^0(n))$ $k=2$ $p=0$

$b^k = 2^2 = 4$

Here $a < b^k$ and $p \geq 0$ **3a)**

$T(n) = \Theta(n^k \log^p(n))$

Master's Method

$T(n) = 3\ T(n/2) + n^2$

$a=3 \quad b=2 \quad n^2 = \Theta\ (n^2 \log^0(n)) \quad k=2 \quad p=0$

$b^k = 2^2 = 4$

Here $a < b^k$ and $p \geq 0$ — 3a)

$T(n) = \Theta(n^k \log^p(n))$

$\quad = \Theta(n^2 \log^0(n))$

Master's Method

$T(n) = 3\,T(n/2) + n^2$

$a=3$ $b=2$ $n^2 = \Theta(n^2 \log^0(n))$ $k=2$ $p=0$

$b^k = 2^2 = 4$

Here $a < b^k$ and $p \geq 0$ 3a)

$T(n) = \Theta(n^k \log^p(n))$

$= \Theta(n^2 \log^0(n))$

$= \boldsymbol{\Theta(n^2)}$

Master's Method

$T(n) = 2^n T(n/2) + n^n$

As per master's theorem, $a \geq 1$. Here $a=2^n$

So we cannot solve this relation using Master's Method.