1. Prove or disprove the statement "All birds can fly."
2. Prove or disprove the claim $(\forall x, y \in \mathcal{R})[(x-y)^2 > 0]$.
3. Prove that between any two unequal rationals there is a third rational.
4. Explain why proving $\phi \Rightarrow \psi$ and $\psi \Rightarrow \phi$ establishes the truth of $\phi \Leftrightarrow \psi$.
5. Explain why proving $\phi \Rightarrow \psi$ and $(\neg\phi) \Rightarrow (\neg\psi)$ establishes the truth of $\phi \Leftrightarrow \psi$.
6. Prove that if five investors split a payout of \$2M, at least one investor receives at least \$400,000.
7. Prove that $\sqrt{3}$ is irrational.
8. Write down the converses of the following conditional statements:
   (a) If the Dollar falls, the Yuan will rise.
   (b) If $x < y$ then $-y < -x$. (For x, y real numbers.)
   (c) If two triangles are congruent they have the same area.
   (d) The quadratic equation $ax^2 + bx + c = 0$ has a solution whenever $b^2 \geq 4ac$. (Where a, b, c, x denote real numbers and $a \neq 0$.)
   (e) Let $ABCD$ be a quadrilateral. If the opposite sides of $ABCD$ are pairwise equal, then the opposite angles are pairwise equal.
   (f) Let $ABCD$ be a quadilateral. If all four sides of $ABCD$ are equal, then all four angles are equal.
   (g) If n is not divisible by 3 then $n^2 + 5$ is divisible by 3. (For n a natural number.)
9. Discounting the first example, which of the statements in the previous question are true, for which is the converse true, and which are equivalent? Prove your answers.
10. Prove or disprove the statement "An integer n is divisible by 12 if and only if n^3 is divisible by 12."
11. Let r, s be irrationals. For each of the following, say whether the given number is necessarily irrational, and *prove* your answer. (The last one is tricky to do by elementary means. I'll give a solution in Lecture 8, but you should definitely try it first. Give it half an hour of focused thought.)

    1. $r + 3$ 2. $5r$ 3. $r + s$

    4. rs 5. $\sqrt{r}$ 6. r^s
12. Let m and n be integers. Prove that:
    (a) If m and n are even, then $m + n$ is even.
    (b) If m and n are even, then mn is divisible by 4.
    (c) If m and n are odd, then $m + n$ is even.
    (d) If one of m, n is even and the other is odd, then $m + n$ is odd.
    (e) If one of m, n is even and the other is odd, then mn is even.

OPTIONAL PROBLEM

Say whether each of the following is true or false, and support your decision by a *proof*:

(a) There exist real numbers x and y such that $x + y = y$.

(b) $\forall x \exists y (x + y = 0)$ (where x, y are real number variables).

(c) For all integers a, b, c, if a divides bc (no remainder), then either a divides b or a divides c.

(d) For any real numbers x, y, if x is rational and y is irrational, then $x + y$ is irrational.

(e) For any real numbers x, y, if $x + y$ is irrational, then at least one of x, y is irrational.

(f) For any real numbers x, y, if $x + y$ is rational, then at least one of x, y is rational.