

1. Show that  $\neg[\exists x A(x)]$  is equivalent to  $\forall x[\neg A(x)]$ .
2. Prove that the following statement is false:

*There is an even prime bigger than 2*

3. Translate the following sentences into symbolic form using quantifiers. In each case the assumed domain is given in parentheses.
  - (a) All students like pizza. (All people)
  - (b) One of my friends does not have a car. (All people)
  - (c) Some elephants do not like muffins. (All animals)
  - (d) Every triangle is isosceles. (All geometric figures)
  - (e) Some of the students in the class are not here today. (All people)
  - (f) Everyone loves somebody. (All people)
  - (g) Nobody loves everybody. (All people)
  - (h) If a man comes, all the women will leave. (All people)
  - (i) All people are tall or short. (All people)
  - (j) All people are tall or all people are short. (All people)
  - (k) Not all precious stones are beautiful. (All stones)
  - (l) Nobody loves me. (All people)
  - (m) At least one American snake is poisonous. (All snakes)
  - (n) At least one American snake is poisonous. (All animals)
4. Negate each of the symbolic statements you wrote in the last question, putting your answers in positive form. Then express each negation in natural, idiomatic English.
5. Which of the following are true? The domain for each is given in parentheses.
  - (a)  $\exists x(2x + 3 = 5x + 1)$  (Natural numbers)
  - (b)  $\exists x(x^2 = 2)$  (Rational numbers)
  - (c)  $\forall x \exists y(y = x^2)$  (Real numbers)
  - (d)  $\forall x \exists y(y = x^2)$  (Natural numbers)
  - (e)  $\forall x \exists y \forall z(xy = xz)$  (Real numbers)
  - (f)  $\forall x \exists y \forall z(xy = xz)$  (Prime numbers)
  - (g)  $\forall x[x < 0 \Rightarrow \exists y(y^2 = x)]$  (Real numbers)
  - (h)  $\forall x[x < 0 \Rightarrow \exists y(y^2 = x)]$  (Positive real numbers)
6. Negate each of the statements in the last question, putting your answers in positive form.
7. Negate the following statements and put each answer into positive form:
  - (a)  $(\forall x \in \mathcal{N})(\exists y \in \mathcal{N})(x + y = 1)$
  - (b)  $(\forall x > 0)(\exists y < 0)(x + y = 0)$  (where  $x, y$  are real number variables)
  - (c)  $\exists x(\forall \epsilon > 0)(-\epsilon < x < \epsilon)$  (where  $x, \epsilon$  are real number variables)
  - (d)  $(\forall x \in \mathcal{N})(\forall y \in \mathcal{N})(\exists z \in \mathcal{N})(x + y = z^2)$

8. Give a negation (in positive form) of the famous “Abraham Lincoln sentence” which we met in the previous assignment: “You may fool all the people some of the time, you can even fool some of the people all of the time, but you cannot fool all of the people all the time.”
9. The standard definition of a real function  $f$  being *continuous at a point*  $x = a$  is

$$(\forall \epsilon > 0)(\exists \delta > 0)(\forall x)[|x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon]$$

Write down a formal definition for  $f$  being *discontinuous at*  $a$ . Your definition should be in positive form.