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# Hedging with an Edge: Parametric Currency Overlay

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**Abstract.** We propose an optimal currency hedging strategy for global equity investors using currency value, carry, and momentum to proxy for expected currency returns. A benchmark risk constraint ensures the overlay closely mimics a fully hedged portfolio. We compare this with naïve and alternative hedges in a demanding out-of-sample test, with transaction and rebalancing costs and margin requirements. Other hedging methods generally reduce risk but at a cost. Some tend to short currencies with high returns and all incur substantial costs with frictions, mostly margin requirements and equity rebalancing costs. The proposed strategy uses predictable returns to reduce this cost. It produces a statistically significant 17% gain in Sharpe ratio and an annualized Jensen-$\alpha$ of 0.93% versus a fully hedged benchmark. Notably, most of the implementation costs of the strategy would be incurred by the benchmark anyway. This reduces its marginal cost and highlights a specific synergy of integrating hedging with speculation.

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## 1. Introduction

Diversifying equity portfolios internationally is, in principle, desirable. However, this brings with it the unwelcome guest of currency risk, and there is no consensus on how to deal with it.[1] A problem well documented by De Roon et al. (2012) is that hedging tends to short currencies with high expected returns. Nevertheless, the growing literature on currency risk premia suggests it is possible to create informed expectations of currency returns. We argue this knowledge can be used to improve the currency hedging overlay.[2]

Admittedly, incorporating expected returns in an optimization is not self-evident. In fact, it is often found that optimizations perform better if one deliberately ignores information on expected returns (Jorion 1985, 1994, Eun and Resnick 1988, Glen and Jorion 1993). In this paper, we propose addressing this issue by using the parametric portfolio policies (PPPs) of Brandt et al. (2009) to design the optimal currency overlay. PPPs use characteristics to proxy for expected returns (and risk) in the cross-section of assets. Previous studies find PPPs successfully address estimation issues in forming portfolios of equities (Brandt et al. 2009; DeMiguel et al. 2013, 2020), currencies (Kroencke et al. 2013, Barroso and Santa-Clara 2015), and options (Faias and Santa-Clara 2017). On top of robustness, PPPs are also appealing for their flexibility dealing with frictions such as transaction costs and for their economic intuition.

We take advantage of the flexibility of PPPs to introduce a realistic setting with multiple complications faced by institutional asset managers. This includes estimation in real time, transaction costs (both in equities and in currencies), and the need to post collateral for margin requirements in order to engage in forward positions. Furthermore, it is well known that asset managers care strongly about their relative performance (Roll 1992, Sensoy 2009). Recognizing this important feature of the asset management industry, our proposed strategy incorporates a benchmark risk constraint.

Taken all together, these frictions considerably raise the bar for currency hedging. Our starting point is a U.S.-based investor holding a value-weighted international equity portfolio of 11 developed markets over the observation period from 1986 to 2016. To assess and compare the performance of our proposed parametric currency overlay, we examine nine alternative hedging strategies to deal with currency risk, including naïve and sophisticated methods.[3] We find that, after frictions, in this set of alternative hedging methods: (i) none of the approaches significantly outperforms a naïve fully hedged portfolio in terms of Sharpe ratio; (ii) none of the hedged portfolios—neither naïve nor sophisticated—outperforms the unhedged portfolio; and (iii) even the internationally diversified unhedged portfolio underperforms a pure domestic portfolio.

Generally, the single biggest drag on returns for hedged strategies comes from margin requirements. Rebalancing costs because of the need to readjust equity positions also dampen performance. All alternative hedging strategies achieve their intended purpose and reduce risk in terms of portfolio volatility. However, the decline in risk is expensively bought by an over-proportional lowering in economic performance with the result that the corresponding Sharpe ratios deteriorate. Overall, these results support the possibility that the benefits of international diversification are not necessarily evident to investors, an explanation for home bias proposed by Bekaert and Urias (1996).

We apply the PPP method and construct a parametric currency overlay mainly applying three well-known currency characteristics, that is, currency forward discount, foreign exchange (FX) value, and currency momentum. These have been proposed as currency market factors (Pojarliev and Levich 2008) and their effects are well documented in currencies and other asset classes (Menkhoff et al. 2012b, 2017; Koijen et al. 2018). The optimization enforces a tight benchmark risk constraint of 2% (annualized).[4] As a result, our parametric currency overlay assumes a passive underlying portfolio of international equities and combines it with an active currency overlay. This currency overlay has as its first aim to achieve a risk reduction such as the fully hedged value-weighted portfolio, which is henceforth considered as the benchmark portfolio. The optimization procedure has only limited freedom to reduce risk in the least costly manner it can find.

By contrast to the alternative hedging strategies and after all frictions are considered, the parametric currency overlay applying forward discount, currency value and FX momentum jointly creates a statistically significant increase in the Sharpe ratio relative to the benchmark portfolio (from 0.38 to 0.45). This parametric currency overlay produces an annualized alpha of 93 basis points (bps). It is worth mentioning that both measures are observed out-of-sample and after transaction costs. A decomposition exercise reveals this gain is mainly attributable to harvesting the predictability in cross-sectional returns by capturing a speculative component.

The parametric currency overlay does not propose extreme currency positions. Comparing the absolute volume of currency forwards relative to portfolio size with the benchmark, the amount of currency forward increases from 62.82% to 87.53%. In contrast, the hedging method introduced by Campbell et al. (2010) has a volume of 122.08%, and Barroso and Santa-Clara (2015) suggest an aggressive currency positioning of 594%.

A recurrent concern on trading strategies is if their observed gross performance subsists after accounting for frictions (Lesmond et al. 2004). Ironically, we find frictions reinforce the case for combining hedging with speculation. This happens because the fully hedged strategy has total costs related to frictions equivalent to approximately 70% of the costs associated with the parametric currency overlay. However, the parametric currency overlay has higher frictionless returns (114 bps versus –29 bps), which are much higher than the additional transaction costs. This reveals an interesting synergy of superimposing a currency overlay on top of a fully hedged benchmark portfolio. A large proportion of costs is already incurred implicitly by the benchmark, which ignores expected returns at a considerable cost to the investor. Overall, the marginal cost of investing in currency risk premiums is lower for an investor that would alternatively assume positions in currencies for hedging purposes. Additionally, as our optimization process takes several frictions into account, we evaluate the impact of each friction. The tracking error constraint has the largest influence on the optimization followed by opportunity costs of margin requirements and by transaction costs.

Previous studies propose using information on expected returns to improve hedging. De Roon et al. (2003) suggest a *dynamic hedging strategy* that uses (a time series version of) currency carry. Also, De Roon et al. (2012) superimpose a 100% allocation to FX carry on an optimally hedged portfolio. Recently, other studies using PPP and show that combinations of currency carry, FX value, and currency momentum produce very interesting portfolio diversification properties for equity investors (Kroencke et al. 2013, Barroso and Santa-Clara 2015). Particularly close to our study, Kroencke et al. (2013) produce a subset of results with the PPP method, incorporating transaction costs, and controlling for three alternative hedging procedures. Relative to all of these studies, our paper differs by including simultaneously in the definition of the investor problem margin requirements, trading and rebalancing costs. The presence of margin requirements introduces a tradeoff between holding currency positions and earning the stock market risk premium. In our results, the opportunity cost related to these margin requirements is economically important. We also include benchmark risk in the analysis. We find the potential benefits of investing in FX styles can imply deviations from the benchmark most active managers would feel uncomfortable with. Finally, we stress a new rationale for an integrated hedging and speculation approach that builds on recent literature highlighting the nonlinearity of transaction costs (DeMiguel et al. 2020).

Besides currency carry, FX value, and currency momentum, we test two other characteristics in our study: the equity sensitivity of Campbell et al. (2003) and the loading on FX volatility risk of Menkhoff et al. (2012a). We find that equity sensitivity is not statistically significant. Empirically, this characteristic is associated with both higher returns and risk. But the trade-off leaves the investor indifferent. FX volatility risk is already found to be significant.

We test the robustness of our findings to investor domicile, using power utility, adopting an unhedged benchmark instead, different tracking error constraints, alternative investment universes and investment horizon. Generally, these empirical exercises underpinned the robustness of our findings. Most of the related results are therefore available in the Internet appendix.

The paper is organized as follows. Section 2 introduces the methodology and the currency characteristics. Section 3 presents the empirical analysis. It explains the data set, compares hedging methods, and discusses the economic intuition. Section 4 summarizes our findings.

## 2. Currency Overlay

We follow Campbell et al. (2010) and define the portfolio return of an international equity investor as a composition of equity currency return, that is,

$$R_{p,t+1} = \boldsymbol{\omega}_t' \boldsymbol{R}_{t+1}^{E} \cdot (\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) + \boldsymbol{\omega}_t' \boldsymbol{R}_{t+1}^{FX} - \boldsymbol{\Psi}_t' \boldsymbol{R}_{t+1}^{FX}, \quad (1)$$

where $\boldsymbol{R}_{t+1}^{E}$ equals a $(n+1 \times 1)$-vector of gross equity return measured in the local currency, $\boldsymbol{\omega}_t = \{\omega_{c,t}\}_c$ denotes the equity weighting scheme, and $\boldsymbol{R}_{t+1}^{FX}$ equals the currency return defined as

$$\boldsymbol{R}_{t+1}^{FX} = (\boldsymbol{F}_t - \boldsymbol{S}_{t+1}) \div \boldsymbol{S}_t, \quad (2)$$

where $\div$ and $\cdot$ are an element operator and $\boldsymbol{F}_t$ and $\boldsymbol{S}_t$ capture a $(n+1 \times 1)$-vector of one-period forward and FX spot rates at time $t$, respectively. These are expressed as the price of one foreign currency unity (FCU) in domestic currency units (DCU). $\boldsymbol{\Psi}_t$ captures the deviation from the full hedge and has due to construction a cumulative weight of zero.[5] A nonzero $\boldsymbol{\Psi}_t$ means that the portfolio under-hedges some currency exposures and compensates this by over-hedging the others.

To determine the currency exposure $\boldsymbol{\Psi}_t$, such that the portfolio return $R_{t+1}^{h,PF}$ has a minimal risk, Campbell et al. (2010) rewrite Equation (1) with log return and regress

$$\boldsymbol{\omega}_t' \left( \boldsymbol{r}_{t+1}^{e} - \boldsymbol{i}_t \right) = \alpha_t + \tilde{\boldsymbol{\Psi}}_t' \left( \Delta \tilde{\boldsymbol{s}}_{t+1} + \tilde{\boldsymbol{i}}_t - \tilde{\boldsymbol{i}}_t^{1} \right) + \epsilon_t, \quad (3)$$

where $\tilde{\boldsymbol{\Psi}}_t = (\tilde{\Psi}_{2,t}, \ldots, \tilde{\Psi}_{n+1,t})'$, $\boldsymbol{r}_{t+1}^{e} = \log(\boldsymbol{R}_{t+1}^{E})$, $\Delta \boldsymbol{s}_{t+1} = \log(\boldsymbol{S}_{t+1}) - \log(\boldsymbol{S}_t)$, $\boldsymbol{i}_t = \log(1 + \boldsymbol{I}_t)$, $\boldsymbol{i}_t^{1} = \log(1 + I_{1,t}) \mathbb{1}_{n+1\times 1}$, $\mathbb{1}_{n+1\times 1}$ is an unit vector of length $n+1$, and $\boldsymbol{I}_t = (I_{1,t}, \ldots, I_{n+1,t})$ denotes the vector of implied interest rates at which the investor can borrow and lend.[6] By definition, the domestic currency exposure is the negative cumulative sum of all foreign currency exposures, that is, $\Psi_{1,t} = -\sum_{c=2}^{n+1} \Psi_{c,t}$. The currency overlay computed with Regression (3) corresponds to CMV Optimal (CMV Opt) and switches the sign of $\boldsymbol{\Psi}_t$. This method has a constant currency exposure $\boldsymbol{\Psi}_t$ on the entire observation period. Therefore, it is an in-sample (IS) method.

We expand the CMV method in two different ways. First, building on Howard and D'Antonio (1984) and De Jong et al. (1997), we adjust the optimized hedging strategy to maximize Sharpe ratio instead of minimizing risk alone. We call this CMV SR. This adaptation allows a more direct comparison with the PPP method that features an utility[7] sensitive to both expected returns and volatility, whereas CMV is only concerned with risk minimization. CMV SR is optimized numerically over the entire sample picking the best (constant) currency weights to maximize Sharpe ratio. Hence, it is also an in-sample method.

Second, for out-of-sample (OOS) testing, we also estimate $\boldsymbol{\Psi}_t$ using a time-expanding window of initial length $l$[8] (henceforth called CMV TE). For each time step $t$, we determine $\boldsymbol{\Psi}_t$ by Regression (3) based on the available historical sample. Both currency overlays $\boldsymbol{\Psi}_t$ are applied between $t$ and $t+1$.

Toward the end of the sample CMV TE and CMV Opt should have a similar impact on the global equity portfolio, as the currency overlay of CMV TE converges toward CMV Opt. Generally, the economic interpretation of CMV Opt, and CMV TE is that equity has a certain sensitivity to currencies, which is captured by $\Psi_t$. By taking the negative of $\Psi_t$, the FX sensitivity of equity is entirely neutralized. Nevertheless, the currency exposure is estimated from sample data. Consequently, this method faces estimation issues. The true currency exposure is not observable.

### 2.1. Parametric Currency Overlay

We use the method of Brandt et al. (2009)—called parametric portfolio policy—to design a novel approach to create a currency overlay. The innovation of this currency overlay is the direct mapping of currency characteristics and weighting scheme of the related currency overlay. We therefore define $\boldsymbol{\Psi}_t$ as

$$\boldsymbol{\Psi}_t = \frac{1}{n+1} \boldsymbol{\kappa}_t \boldsymbol{\eta} = f(\boldsymbol{\kappa}_t; \boldsymbol{\eta}), \quad (4)$$

where $\boldsymbol{\kappa}_t$ is a $(n+1 \times m)$-matrix, with the result that $\kappa_{ijt}$ captures the characteristic $j$ of currency $i$. $\boldsymbol{\eta} \in \mathbb{R}^m$ is the loading of each characteristic, which has the interpretation of a sensitivity.[9] The term $m$ denotes the number of considered characteristics. To apply parametric portfolio policy, we first derive the portfolio return, which is applied in the optimization process in a second step.

We begin with a reformulation of the basic portfolio log return, that is,

$$r^h_{p,t+1} = \boldsymbol{\omega}'_t\left(\boldsymbol{r}^e_{t+1} + \boldsymbol{i}^1_t - \boldsymbol{i}_t\right) + f\left(\boldsymbol{\kappa}_t;\boldsymbol{\eta}\right)'\left(\Delta \boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}^1_t\right), \quad (5)$$

with the result that $r^h_{p,t+1}$ is an approximation for $\log(R^h_{p,t+1})$. To expand the basic portfolio return by transaction costs (TC), we define these costs for currency $c$ at time $t$ as the bid-ask-spread:

$$TC_{c,t} = \log\left(\frac{F^{ask}_{c,t,t+1} - F^{bid}_{c,t,t+1}}{F^{mid}_{c,t,t+1}}\right),$$

where the indices *ask*, *bid* and *mid* correspond to ask, bid and mid rates, respectively. The assumption is that the investor buys (sells) the forward contract at the ask (bid) rate. The transaction costs are weighted by the absolute amount of currency transactions, that is, $|\omega_{c,t} - f_c(\boldsymbol{\kappa}_t;\boldsymbol{\eta})|$, where $f_c(\boldsymbol{\kappa}_t;\boldsymbol{\eta})$ is the $c^{\text{th}}$ component of $f(\boldsymbol{\kappa}_t;\boldsymbol{\eta})$.

Besides trading costs, entering a forward contract also requires a margin. Often research in optimized currency portfolios implicitly assumes that the margin requirements for forward contracts are zero (Campbell et al. 2010, Barroso and Santa-Clara 2015). However, market standards for institutional investors require margins of between 5% and 15% for liquid currencies.[10]

With margin requirements, the investor faces an additional tradeoff. He needs to decide whether an investment in currencies or equity offers a larger utility. We follow industry standard and reserve the margin requirements equivalent to the foreign currency overlay exposure in a risk-free asset.[11] In order to address the corresponding circularity problem between the margin requirement and the equity investment, we redefine the portfolio weight $\boldsymbol{\Omega}_t(\boldsymbol{\eta}) = \{\Omega_{c,t}(\boldsymbol{\eta})\}_c$ as a function of the raw weight $\boldsymbol{\omega}_t$ and equal to

$$\Omega_{c,t}(\boldsymbol{\eta}) = \left(1 - \frac{\rho}{1+\rho}\sum_{i=2}^{n+1}\left|\omega_{i,t} - f_i\left(\boldsymbol{\kappa}_t;\boldsymbol{\eta}\right)\right|\right)\omega_{c,t}, \forall c. \quad (6)$$

Furthermore, the currency overlay has a direct impact on the equity portfolio, because of the tradeoff that exists between the volume of currency overlay and equity investment. This tradeoff is caused by margin requirements of forward contracts. Therefore, the equity positions have to be rebalanced. We assume constant rebalancing costs equal to $\zeta$. This reflects the cost of a change in weights applied uniformly to the entire equity component (as opposed to different trading intensities at the stock level). The portfolio's rebalancing costs at time $t$ equal

$$RBC_t(\boldsymbol{\eta}) = \sum_{i=1}^{n+1}\left|\Omega_{i,t}(\boldsymbol{\eta}) - \Omega^{hold}_{i,t}(\boldsymbol{\eta})\right|\zeta, \quad (7)$$

and $\Omega^{hold}_{i,t}$ is the previous period portfolio weight adjusted by margins, and the equity return between $t-1$ and $t$, that is,

$$\Omega^{hold}_{i,t}(\boldsymbol{\eta}) = \Omega_{i,t-1}(\boldsymbol{\eta}) \cdot \exp\left(r_{i,t} - r_{p,t}(\boldsymbol{\eta})\right), \quad (8)$$

where $r_{i,t}$ $(r_{p,t}(\boldsymbol{\eta}))$ is the return of asset $i$ (the portfolio return). Thus, the final portfolio return equals

$$\begin{aligned} r_{p,t+1}(\boldsymbol{\eta}) = {} & \boldsymbol{\Omega}_t(\boldsymbol{\eta})'\left(\boldsymbol{r}_{t+1} + \boldsymbol{i}^1_t - \boldsymbol{i}_t\right) + f\left(\boldsymbol{\kappa}_t;\boldsymbol{\eta}\right)'\left(\Delta\boldsymbol{s}_{t+1} + \boldsymbol{i}_\tau - \boldsymbol{i}^1_t\right) \\ & - \left|\boldsymbol{\Omega}_t(\boldsymbol{\eta}) - f\left(\boldsymbol{\kappa}_t;\boldsymbol{\eta}\right)\right|'\boldsymbol{TC}_t - RBC_t(\boldsymbol{\eta}) \end{aligned} \quad (9)$$

and is only dependent on the loading of the considered characteristic. The optimization of the parametric portfolio policy at time $t$ equals

$$\hat{\boldsymbol{\eta}} = \arg\max_{\boldsymbol{\eta}} \frac{1}{t+1}\sum_{\tau=0}^{t} U\left(r_{p,\tau}(\boldsymbol{\eta})\right), \quad (10)$$

and aims to find an optimal loading. Notably, the relevance of a characteristic depends on its ability to increase utility. This makes statistical significance tests in the PPP method distinct from asset pricing tests. For instance, a characteristic associated with higher expected returns can still be irrelevant for utility purposes if it also forecasts higher contributions to risk (and vice versa).[12]

Theoretically, the optimization process of Equation (10) can suggest to allocate the entire capital to currency exposure, such that no equities are purchased. The results in Barroso and Santa-Clara (2015) and Kroencke et al. (2013), for example, show that currency style investing has interesting diversifying properties. As a result, the unconstrained investors assumed in those studies endogenously choose optimal currency overlays with absolute exposures multiple times larger than the value of the underlying portfolio. Therefore, the unconstrained currency overlay dominates the risk-return profile of those portfolios.[13] However, this implies levels of benchmark risk many investors can find excessive.

It is well known that asset management institutions worry about their performance relative to each other. Sensoy (2009) document that 94.6% of U.S. mutual fund managers track the S&P 500 or some other popular equity index. Black and Litterman (1992) argue that even for a manager without an explicit benchmark "[...] his risk is clearly related to the stance of his portfolio relative to the portfolios of his competitors." Therefore, currency positions implying large deviations from the benchmark are risky for asset managers. In our setting, excessive deviations from the benchmark violate the basic idea of the currency overlay. We therefore introduce a tracking error constraint following Jorion (2003, p. 38). We define the tracking error constraint as

$$\sigma\left(r^b_{p,t+1} - r_{p,t+1}(\hat{\boldsymbol{\eta}})\right) \le \mathcal{C},$$

where $r^b_{p,t+1}$ and $r_{p,t+1}(\hat{\boldsymbol{\eta}})$ are the return of the designated benchmark portfolio and optimized portfolio, respectively. The term $\mathcal{C}$ is the threshold of maximal deviation from the benchmark portfolio. Using a benchmark risk constraint is not a standard feature of previous implementations of the PPP method (Brandt et al. 2009, DeMiguel et al. 2013, 2020).

To the best of our knowledge, our study offers the first optimization of a currency overlay simultaneously encompassing trading and rebalancing costs, margin requirements, and benchmark risk constraints. These frictions should make a closed form analytical solution challenging. For instance, referring to trading costs alone, DeMiguel et al. (2015, p. 1444) mention that "[...] the case with multiple risky assets and proportional transaction costs is generally intractable analytically."[14] In this regard, our currency overlay takes advantage of the flexibility of the PPP method. All constraints and frictions are embodied in the definition of the utility maximization problem. The choice variable consists of a low-dimensional vector based on a small set of (hopefully) economically relevant characteristics selected based on previous research.

## 2.2. Currency Characteristics

The effectiveness of the PPP method depends crucially on the economic relevance and robustness of the characteristics used to proxy for expected returns and risk. For expected returns, we draw from the growing asset pricing literature in the FX market while for risk we resort to the hedging literature.

There is no standard asset pricing model in the FX market. Yet, Pojarliev and Levich (2008) propose FX style portfolios formed on carry, value, and trend-following/momentum as risk factors to study the performance of currency managers. These are widely known and popular among investors (Pojarliev and Levich 2013). The economic relevance of these effects in currencies have been documented in a broad set of influential studies.[15] Cross-sections of currency portfolios formed on these signals are, in turn, often used as test assets for newly proposed risk factors. Recent tests recover from the cross section risk premiums for exposure to innovations in FX volatility (Menkhoff et al. 2012a), a systematic high-minus-low carry factor (Lustig et al. 2011), consumption growth risk (Lustig and Verdelhan 2007), external imbalances (Della Corte et al. 2016), or a proxy for global political risk (Filippou et al. 2018). These provide risk-based explanations with economic intuition for the return predictability captured by these signals.[16] We build on this literature and define currency characteristics based on carry, value, and momentum to proxy for expected returns.

- Forward discount ($fd_{qc,bc,t}$) between quoted ($qc$) and base ($bc$) currency at time $t$.

The motivation for this characteristic is that the forward discount is frequently used as a predictor for currency returns (Fama 1976, Wolff 1987). Furthermore, the carry trade strategy purchases (sells) currencies with high (low) forward discounts. The extensive literature on carry trade documents its profitability. Using the forward discount as a characteristic implies that a carry trade is incorporated in the currency overlay.

Next, we define currency characteristics that capture historical return properties. The return characteristics are as follows:

- Currency momentum ($mom_{qc,bc,t}$) is the cumulative currency return over the previous three months at time $t$ between the quoted and base currency. This variable investigates the persistence of currency returns in the short term. There is evidence that a three-month momentum provides persistence at the portfolio level especially for this period. Menkhoff et al. (2012b) demonstrate that a momentum portfolio defined with a longer formation period offers no additional gain.
- Long-term reversal ($rev_{qc,bc,t}$). It quantifies the cumulative real currency depreciation between quoted and base currency. This measure is comparable with the currency value in Asness et al. (2013) and Kroencke et al. (2013). The currency depreciation over the past five years is defined as

$$rev_{qc,bc,t} = \frac{S_{qc,bc,t}CPI_{qc,t-60}CPI_{bc,t}}{S_{qc,bc,t-60}CPI_{qc,t}CPI_{bc,t-60}},$$

where $CPI_{qc,t}$ ($CPI_{bc,t}$) is the consumer price index at time $t$ associated with the quoted (base) currency. A positive value of $rev_{qc,bc,t}$ implies that the quoted currency has a larger real depreciation against the base currency over the past five years, and vice versa, if $rev_{qc,bc,t-60,t}$ is negative.

- Currency sensitivity with respect to the excess equity return, measured in local currency ($\mathcal{S}_{qc,bc,t}$). The sensitivity is calculated following Campbell et al. (2010), by regressing the excess equity return $\mathbb{1}'_{n+1\times 1}\boldsymbol{\omega}_t(\boldsymbol{r}_{t+1} - \boldsymbol{i}_t)$ on a constant and the excess currency return $\Delta \boldsymbol{s}_{qc,bc,t+1} + \boldsymbol{i}_{qc,t} - \boldsymbol{i}_{bc,t}$. This is a natural characteristic to proxy for a currency's contribution to the risk of a portfolio of international equities.

The main difference between our approach and that of Campbell et al. (2010) is that we run this regression on daily spot returns on a rolling window of 60 months.

• Loading of currency volatility innovation ($\beta_{\sigma_{FX}}$). We follow Menkhoff et al. (2012a) and ran the following regression on a rolling window of 60 months, that is,

$$r_{qc,bc,t} = \alpha + \beta_{HML,qc} HML_t + \beta_{\sigma_{FX},qc} \Delta\sigma_t^{FX}, \qquad (11)$$

where $r_{qc,bc,t}$ is the log currency return defined in Equation (2). The term $HML_t$ is the return of the carry trade with one currency pair at time $t$. The term $\sigma_t^{FX}$ is the currency volatility at time $t$ calculated as in Menkhoff et al. (2012a), whereas $\Delta\sigma_t^{FX}$ is the innovation in currency volatility at time $t$, which we proxy the first difference.

Black (1989) and Campbell et al. (2010) derive currency exposures that are independent from the reference currency, and each investor holds an identical currency portfolio. We adapt the method of Brandt et al. (2009) to obtain a currency overlay that is also independent of the reference currency. We are not aware of other studies designing global currency characteristics for a parametric currency overlay in a two-step process.

First, the characteristics are calculated for each possible currency pair. Second, characteristics at the currency pair level are merged into global characteristics for each currency as in Black (1989). The global currency characteristic of each currency $c$ is therefore the mean of the currency characteristic quoted against currency $bc$, that is,

$$\kappa_{c,t}^{global} = \frac{1}{n} \sum_{bc=2}^{n+1} \kappa_{bc,c,t}. \qquad (12)$$

Finally, the global characteristics are cross-sectionally standardized, so that the parametric currency overlay is a zero investment strategy, that is, $\sum_c \kappa_{c,t}^{global} = 0 \forall t$.

## 3. Empirical Analysis

Our empirical analysis is based on a global equity investor holding a value-weighted portfolio of developed economies.[17] We follow Lustig et al. (2011) and define developed economies as Australia, Canada, Denmark, Europe, Japan, New Zealand, Norway, Sweden, Switzerland, the United Kingdom, and the United States. Each economy is represented by the corresponding index of Morgan Stanley Capital International (MSCI). We assume that the global equity investor has a currency risk against the following currencies: Australian dollar, British pound (GBP), Canadian dollar, Danish krona, Euro, Japanese Yen, New Zealand dollar, Norwegian krona, Swiss franc, Swedish krona and U.S. dollar (USD).[18] All exchange rate data are downloaded from Thomson Reuters DataStream. The frequency of the data sample is monthly and spans the period from January 1976 to December 2016. Following Burnside et al. (2010), we use forward rates quoted against USD and GBP, and merge them to obtain the longest samples. We follow Campbell et al. (2010) and use Europe as a proxy for the European monetary union. This implies a look-ahead bias, as in 1976, there was only a small indication of this eventual union. However, from the perspective of a present-day investor, it makes sense to assume such a look-ahead bias, and to regard the European monetary union as one market.

Table 1 reflects the implied interest rates and excess equity returns between 1976 and 2016. The annual implied interest rates vary across the selected countries. Switzerland and Japan have the lowest interest rates of ≈2%, whereas New Zealand and Norway are regarded as high-yield countries with interest rate levels of approximately ≈7%. The excess equity returns are primarily in the range of 5%–10%, with a standard deviation of 15%–24%.[19] Moreover, the significance in correlations between currency return of local currency against USD and equity return in local currency illustrates that currencies and equities are related to each other. Thus, it seems that, from a risk minimization perspective, a neutralization of currency exposure is not efficient (Solnik 1974). Stock returns

**Table 1.** Summary Statistics Stock Market Return

| | Interest rates | | Excess stock returns | | | |
|---|---|---|---|---|---|---|
| | ret | vol | ret | vol | SR | Corr |
| Australia | 6.35 | 2.61 | 7.80 | 17.41 | 0.45 | 0.35*** |
| Canada | 5.28 | 1.92 | 5.58 | 16.00 | 0.35 | 0.42*** |
| Denmark | 5.85 | 2.56 | 7.21 | 18.05 | 0.40 | –0.13* |
| Europe | 5.00 | 1.96 | 6.88 | 15.33 | 0.45 | –0.17*** |
| Japan | 1.69 | 1.38 | 5.33 | 18.36 | 0.29 | –0.16*** |
| New Zealand | 7.40 | 3.23 | –0.64 | 15.42 | –0.04 | 0.15*** |
| Norway | 6.53 | 2.19 | 6.14 | 23.75 | 0.26 | 0.10* |
| Sweden | 6.14 | 2.50 | 10.36 | 22.72 | 0.46 | –0.04 |
| Switzerland | 1.94 | 1.28 | 7.65 | 15.32 | 0.50 | –0.29*** |
| United Kingdom | 6.18 | 2.08 | 6.01 | 16.03 | 0.38 | –0.10* |
| United States | 4.55 | 1.70 | 6.80 | 14.77 | 0.46 | |

*Notes.* Data are from MSCI database. The frequency of the data is monthly. Excess stock returns are quantified in local currency adjusted by the corresponding interest rate. The interest rates are implied interest rates computed from forward discounts quoted against the USD. The USD interest rate equals the treasury bill rate and is from IMF IFS database. FX data are sourced from Thomson Reuters Datastream. ret, vol, and SR stand for annualized return, volatility, and Sharpe Ratio, respectively. Corr refers to the correlation between equity return in local terms and currency return quoted against USD. The sample period is between January 1976 and December 2016.

*$p < 0.10$; **$p < 0.05$; ***$p < 0.01$ significant levels, respectively.

and FX returns are not orthogonal to each other and the existing correlation can potentially be exploited (as Campbell et al. 2010 do).

### 3.1. Alternative Hedging Methods

We examine the performance of an international equity portfolio with different methods to deal with currency risk. Table 2 sets out the summary statistics of the international equity portfolio, following standard currency overlay strategies.[20] The international equity portfolio is kept the same across columns, and the only difference is the method of hedging currency risk. In the following we assume an U.S. investor—the reference currency is therefore the USD,[21] and margin requirements for forward contracts are 15%. The focus of our paper is on international portfolios and how to deal with currency management. Yet, for comparison purposes, we also report in this table the performance of a pure domestic portfolio consisting of U.S. equities. This is an alternative portfolio our assumed investor can choose to avoid dealing with currency risk altogether.

We assume constant, proportional rebalancing costs equal to 50 bps as in Brandt et al. (2009) and De Roon et al. (2001). The overlay strategy unhedged (full hedge) bears (entirely neutralizes) the currency risk. On average, the value-weighted portfolio allocates 37.18% on U.S. equity, 30.31% to European assets, and 16.35% to Japanese securities. Overall, these three economic areas cover approximately 84% of the asset allocation. A U.S. investor therefore holds a nondomestic exposure of 62.82%, which needs to be hedged. To implement a full hedge, he purchases a volume of 62.82% for forward contracts. As margin requirements of 15% are assumed, the investor purchases only a cumulative volume relative to the underlying portfolio of 54.62% ($= 62.82 \cdot \frac{1}{1+0.15}$). Therefore, 8.20% is held in a risk-free asset to back the currency overlay.[22] and the cumulative nondomestic exposure equals 62.82% ($= 54.62\% + 8.20\%$).

The unhedged and fully hedged strategies have a significant impact on the first two momenta. The fully hedged strategy leads to a significant reduction in the excess return and standard deviation. The annual excess return and standard deviation of the fully hedged (unhedged) portfolio equal 4.89% (6.47%) and 12.97% (15.17%), respectively. The neutralization of the currency risk leads to a larger tail risk measured by skewness and kurtosis. This observation is consistent with De Roon et al. (2012).

For subsequent comparisons, we adopt the fully hedged portfolio as the benchmark, and report various performance measures in order to provide a transparent risk-return evaluation. Sharpe ratio, volatility, Jenson-$\alpha$, and certainty equivalent are the measures with more interest. The reason for this selection is that the investor considered in our optimizations has the aim of enhancing the risk-return

**Table 2.** Volatility Minimization with Margin Requirements

| | | | | | | | | Tracking error of 2% | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Domestic | Unhedged | Full Hedge | Uni. Hedge | CMV | CMV SR | CMV TE | CMV | CMV SR | CMV TE |
| ex ret | 7.79 | 6.47 | 4.89 | 5.29 | 3.06 | 7.20 | 2.96 | 3.83 | 5.53 | 3.66 |
| | [0.01] | [0.06] | | [0.04] | [0.01] | [0.04] | [0.00] | [0.01] | [0.04] | [0.00] |
| vol | 15.12 | 15.17 | 12.97 | 13.56 | 10.56 | 16.07 | 10.54 | 11.45 | 13.49 | 11.33 |
| | [0.00] | [0.00] | | [0.39] | [0.00] | [0.00] | [0.00] | [0.02] | [0.44] | [0.01] |
| SR | 0.52 | 0.43 | 0.38 | 0.39 | 0.29 | 0.45 | 0.28 | 0.33 | 0.41 | 0.32 |
| | [0.06] | [0.53] | | [0.47] | [0.08] | [0.47] | [0.03] | [0.11] | [0.28] | [0.04] |
| $\alpha$ | 2.64 | 1.13 | | 0.20 | −0.77 | 1.80 | −0.88 | −0.44 | 0.50 | −0.56 |
| | [0.02] | [0.26] | | [0.37] | [0.14] | [0.18] | [0.08] | [0.16] | [0.17] | [0.09] |
| skew | −0.78 | −0.70 | −0.98 | −0.94 | −0.91 | −0.78 | −1.11 | −0.97 | −0.96 | −1.09 |
| kurt | 5.36 | 4.93 | 5.75 | 5.69 | 5.47 | 5.27 | 6.61 | 5.66 | 5.70 | 6.35 |
| CE | 2.93 | 3.67 | 3.72 | 3.69 | 3.50 | 3.54 | 3.35 | 3.71 | 3.98 | 3.58 |
| Fwd Size | 0.00 | 0.00 | 62.82 | 58.01 | 122.08 | 50.87 | 138.63 | 96.33 | 58.36 | 110.11 |
| TE | 6.27 | 5.58 | | 1.32 | 4.00 | 7.46 | 3.91 | 2.38 | 2.01 | 2.52 |

*Notes.* This table reports the summary statistics of several currency overlay strategies from a U.S. investor's perspective. Domestic stands for a portfolio only invested in the United States. The currency risk of the global equity portfolio is not hedged (Unhedged) or entirely neutralized (Full Hedge). CMV, approach of Campbell et al. (2010) using currencies to minimize the volatility of the global equity portfolio; CMV SR, alternative version of the CMV method aiming to maximize the Sharpe ratio; CMV TE, imitation of CMV using a time expanding window; Uni. Hedge, universal hedging ratio of Black (1989); Ex ret, vol, and SR, annualized excess return, volatility, and Sharpe Ratio, respectively; $\alpha$, annualized Jensen-$\alpha$; Skew and kurt, skewness and kurtosis, respectively. The certainty equivalent (CE) is expressed in annualized percentage points. Fwd Size refers to the volume of the currency overlay strategy, that is, the absolute, cumulative value of $\boldsymbol{\omega} - \boldsymbol{\Psi}$ without domestic currency. TE refers to the tracking error with respect to the benchmark portfolio. The numbers in brackets are the $p$ values examining the differences in excess return (correlated $t$-test), volatility ($f$-test), and Sharpe ratio (boot-TS of Ledoit and Wolf 2008) to the fully hedged portfolio. The standard errors of Jensen-$\alpha$ are adjusted as suggested in White (1980). We assume currency margin requirements of 15%. The sample period is from January 1986 to December 2016.

profile while not adding any risk. Therefore, the ability to deliver a statistically significant improvement in Sharpe ratio without incurring in more risk is an important test in our setting. The Jenson-$\alpha$ is an intuitive measure quantifying the risk-adjusted return relative to the benchmark. The certainty equivalent has the advantage of taking higher moments into consideration. This is a potentially pertinent performance measure as hedging can improve the Sharpe ratio while deteriorating higher-order-moment risk. It also conveniently maps expected returns and volatility into a single measure. In our set of proposed strategies, having a small tracking error acts as a prerequisite as benchmark risk is assumed to be important for asset managers. For methods with similar and small tracking error (close to 2 percentage points (pp)) volatility, and higher-order risk is mostly determined by the benchmark. In these cases, it is adequate to simply compare average returns, as we do below, as a rough but intuitive measure of economic performance.

The universal hedging approach introduced by Black (1989) identifies a hedging ratio for each currency, which is applicable independent of the reference currency. The hedge ratio quantifies relatively to the currency exposure the amount of required forward contracts. The empirical results demonstrate that universal hedging is able to increase the excess return to 5.29% in a statistically significant manner. However, the increase in Sharpe ratio and Jenson-$\alpha$ are both statistically insignificant,[23] and the certainty equivalent declines to 3.69%. These measures show that a universal hedging ratio does not increase the risk-return profile.

The next three columns of Table 2 indicate the impact of the currency overlay strategy following Campbell et al. (2010) (CMV) and their expansions (CMV SR, CMV TE). We set the initial window of CMV TE to a length of 120 months. The idea of the CMV method is to use currencies to minimize the volatility of the international equity portfolio. The average weights allocated to each currency are set out in the Internet appendix. It is primarily the Australian dollar, British pound (GBP), and Canadian dollar that are sold, whereas Danish krona, Euro, Japanese Yen, Swiss franc and USD are purchased.

We find the CMV method recommends extremely large currency positions that are not feasible once margin requirements are taken into account. To study implementable versions of the CMV method, we rescale the currency positions in such a way that the margin requirements are compatible with equity holdings. For example, in the column CMV, the investor holds 15.92% (=$122.08 \cdot (1 - \frac{1}{1+0.15})$) in cash, as collateral for the currency positions, and the remaining 84.08% in the equity portfolio. We find the CMV approach results in a significant reduction in excess return and standard deviation, of approximately 1.83 and 2.41 pp, respectively. Moreover, applying CMV for a mean-variance investor, the risk-return tradeoff also significantly declines with the result that the Sharpe ratio equals 0.29—a reduction of approximately 24%—and provides a negative Jenson-$\alpha$ of −0.77%. The empirical results reveal that the CMV method is able to reduce a portfolio's volatility under margin requirements, and this reduction is robust to OOS estimation uncertainty. However, this decline in risk comes at an expensive price, and the method does not appeal to investors who care about both return and risk. Our results confirm qualitatively the problem De Roon et al. (2012) uncovered with an added concern: The large size of the currency overlay in the CMV method further reduces returns by increasing margin requirements.

Additionally, we adjust the method of Campbell et al. (2010) such that a maximal Sharpe ratio is chased, henceforth called CMV SR. CMV SR follows also an in-sample optimization approach and provides an annual return of 7.20%, Jenson-$\alpha$ equals 1.80%, and Sharpe ratio increases to 0.45. On a first view, these results indicate an enhancement of the risk-return profile. However, this rise is purchased for a significantly 3.10 pp higher portfolio volatility (an increase of 24%) and a smaller certainty equivalent. It also implies a high tracking error of 7.46 pp. It is also worth mentioning that the increase in Sharpe ratio and the positive Jenson-$\alpha$ are not statistically significant. This leads to the conclusion that a Sharpe ratio maximizing currency overlay does not significantly enhance the risk-return profile of the underlying benchmark. Additionally, this approach cannot be applied practically because of the forward-looking bias.

One plausible concern is that the large tracking error caused by the CMV method might affect the comparability with the fully hedged portfolio. We therefore limit the tracking error of CMV, CMV TE, and CMV SR to 2%. The results for these methods are displayed in the last three columns of Table 2. The empirical examination demonstrates that constrained versions of both CMV and CMV TE methods are able to reduce the volatility significantly and with acceptable tracking errors. Simultaneously, the corresponding portfolio return also declines significantly with the result that the related Sharpe ratio is not significantly affected, certainty equivalents are smaller than the benchmark and Jenson-$\alpha$ is negative.

The CMV SR approach is able to increase the Sharpe ratio and certainty equivalent and provides a positive Jenson-$\alpha$. However, none of these enhancements is statistically significant. The measure Fwd Size captures the absolute size of forward currency positions.

Only 58.36% of the overall portfolio size is hedged following a CMV SR approach, whereas on average, 62.82% is allocated to nondomestic currency positions. A closer look at the CMV SR currency overlay shows that the foreign currency positions are underhedged with the result that a cumulative weight on non-USD positions is negative.

Noticeably, none of the methods to manage currency risk achieves a statistically significant increase in Sharpe ratio coupled with volatility at approximately the same level as the fully hedged portfolio, the desirable combination for a currency overlay. As a conclusion, none of the alternative hedging approaches including complete and forward-looking information sets are able to outperform the benchmark significantly.

It is interesting to note that a purely domestic portfolio of U.S. equities does quite well. It has the highest Sharpe ratio over the OOS sample period and also the highest Jensen-$\alpha$ with respect to (w.r.t.) the fully hedged benchmark. Therefore, a U.S. investor does not necessarily see, ex post, a benefit in diversifying internationally in our OOS period. Nevertheless, this essentially reflects the fact that U.S. equities happened to be one of the best performing stock markets over this sample period. The performance of domestic portfolios depends crucially on the (somehow arbitrary) assumed domicile. Investors domiciled in Japan, for example, should have regretted the decision to do without international diversification in the same period. As investors do not know ex ante which undiversified country portfolios will perform better, an internationally diversified portfolio has the benefit of offering a more consistent performance across domiciles.

### 3.2. Parametric Currency Overlay

In comparison, the PPP approach, introduced by Equation (10), provides several potential advantages. The PPP currency overlay is able to optimize several tradeoffs simultaneously. First, there is a tradeoff between volume of currency overlay and investment in risky assets, because of margin requirements. Second, the currency position must provide a higher return than the first-order transaction costs. These two tradeoffs are directly implemented in Equation (10). Third, the currency overlay should not dominate the entire portfolio, that is, the volume of the currency overlay, and hence, the corresponding reserved margin, should have a minor quantity. This last tradeoff is considered indirectly, as the currency overlay should cause a maximal tracking error of 2% annually. Fourth, PPP redefines the asset space to characteristic portfolios, intuitively explainable by previous research, as opposed to the primitive assets themselves (the currencies). Using characteristics known to proxy for expected returns and risk alleviates the well-known problems of noise in the estimation of those moments to infer weights at the asset level (Jorion 1986, Michaud 1989, DeMiguel et al. 2009).

We follow Brandt et al. (2009) and Barroso and Santa-Clara (2015) and set the length of the initial window at 120 months. The OOS observation period is therefore between January 1986 and December 2016. All out-of-sample statistics are constructed in the following way. Suppose the parametric currency overlay should be defined at time $t$ for the trading period from $t$ to $t+1$. Then, the optimization of Equation (10) is applied on the historical window between January 1976 and time $t$ and optimization estimates $\hat{\eta}$ capture the loadings of each characteristic.[24] The characteristics observed at time $t$ are weighted linearly with $\hat{\eta}$, as defined in Equation (4). The result of this calculation is the currency overlay between $t$ and $t+1$. Table 3 sets out the summary statistics and the factor loadings of the international equity portfolio, following a PPP currency overlay strategy. The portfolio performance figures in Table 3 are adjusted for transaction and rebalancing costs.

The first column of Table 3 (Panel A) depicts the portfolio performance of the value weighted, fully hedged portfolio.[25] The next four columns show a parametric currency overlay optimized with currency momentum (mom), forward discount (fd), and long term reversal (rev) characteristics, first separately and then jointly. The joint optimization is henceforth called PPP joint currency overlay. These characteristics are also used in Barroso and Santa-Clara (2015), and rely on extensive evidence of carry, value, and momentum as predictors of currency returns (Lustig and Verdelhan 2007, Menkhoff et al. 2017).

The currency overlay optimizations using a single characteristic at a time reveal that currency momentum, long-term reversal, and forward discount are able to generate a higher OOS return relative to the benchmark. On a stand-alone basis, the forward discount characteristic is able to improve performance with statistical significance as shown by a larger Sharpe ratio, positive information ratio, and positive Jenson-$\alpha$. The certainty equivalent (CE) equals 4.36% and is 0.64 pp higher than the benchmark. This confirms long-standing results in the literature showing that carry earns a significant premium (Burnside et al. 2010, Lustig et al. 2011). In fact, it is, when taken in isolation, the best proxy of expected returns in our study.

The other two currency characteristics are able to increase the risk-return profile but are based on a weaker statistical basis as, for example, the loading (Panel B) is not significant. Menkhoff et al. (2012b, 2017) report stronger momentum and value effects in

**Table 3.** Parametric Currency Overlay

| | Full Hedge | mom | fd | rev | mom, fd, rev | mom, fd, rev, $\mathcal{S}_{FX}$ | mom, fd, rev, $\beta_{\sigma_{FX}}$ | mom, fd, rev, $\mathcal{S}_{FX}$, $\beta_{\sigma_{FX}}$ |
|---|---|---|---|---|---|---|---|---|
| **Panel A: Summary statistic** | | | | | | | | |
| ex ret | 4.89 | 5.05 | 5.66 | 5.56 | 5.74 | 5.53 | 5.70 | 5.59 |
| | | [0.32] | [0.01] | [0.03] | [0.01] | [0.03] | [0.01] | [0.02] |
| vol | 12.97 | 12.66 | 13.09 | 12.69 | 12.89 | 12.96 | 12.88 | 12.94 |
| | | [0.64] | [0.85] | [0.68] | [0.92] | [0.99] | [0.90] | [0.87] |
| SR | 0.38 | 0.40 | 0.43 | 0.44 | 0.45 | 0.43 | 0.44 | 0.43 |
| | | [0.40] | [0.06] | [0.07] | [0.01] | [0.05] | [0.04] | [0.05] |
| IR | | 0.08 | 0.40 | 0.44 | 0.46 | 0.35 | 0.42 | 0.35 |
| | | [0.31] | [0.01] | [0.03] | [0.01] | [0.03] | [0.01] | [0.02] |
| $\alpha$ | | 0.34 | 0.78 | 0.83 | 0.93 | 0.70 | 0.90 | 0.76 |
| | | [0.35] | [0.03] | [0.02] | [0.01] | [0.04] | [0.01] | [0.04] |
| Dom-$\alpha$ | −1.20 | −0.82 | −0.42 | −0.32 | −0.26 | −0.49 | −0.28 | −0.41 |
| | [0.21] | [0.42] | [0.68] | [0.75] | [0.80] | [0.63] | [0.78] | [0.70] |
| skew | −0.98 | −1.03 | −1.09 | −0.93 | −1.06 | −1.06 | −1.07 | −1.07 |
| kurt | 5.75 | 6.15 | 6.46 | 5.64 | 6.22 | 6.19 | 6.23 | 6.16 |
| CE | 3.72 | 4.10 | 4.36 | 4.64 | 4.62 | 4.35 | 4.58 | 4.42 |
| Fwd Size | 62.82 | 82.52 | 92.92 | 80.23 | 87.53 | 90.45 | 94.29 | 97.76 |
| TE | | 2.01 | 1.94 | 1.98 | 1.84 | 1.85 | 1.93 | 1.98 |
| **Panel B: Loadings of characteristics** | | | | | | | | |
| mom | | 0.52 | | | 0.21 | 0.21 | 0.18 | 0.18 |
| | | [0.15] | | | [0.01] | [0.01] | [0.02] | [0.02] |
| fd | | | 0.60 | | 0.44 | 0.44 | 0.43 | 0.43 |
| | | | [0.01] | | [0.00] | [0.00] | [0.00] | [0.00] |
| rev | | | | 0.61 | 0.26 | 0.26 | 0.39 | 0.39 |
| | | | | [0.33] | [0.01] | [0.01] | [0.01] | [0.01] |
| $\mathcal{S}_{FX}$ | | | | | | 0.01 | | 0.01 |
| | | | | | | [0.52] | | [0.50] |
| $\beta_{\sigma_{FX}}$ | | | | | | | 0.20 | 0.20 |
| | | | | | | | [0.04] | [0.04] |
| **Panel C: Return contribution of optimization elements** | | | | | | | | |
| ex ret (Full Hedge) | | 4.89 | 4.89 | 4.89 | 4.89 | 4.89 | 4.89 | 4.89 |
| + CCY over. ret | | 7.15 | 13.78 | 33.01 | 58.70 | 34.53 | 64.94 | 51.39 |
| + TE Cont | | −6.57 | −12.41 | −32.79 | −57.46 | −33.53 | −63.59 | −50.13 |
| + Opp. Costs of Margin | | −0.23 | −0.33 | 0.07 | −0.15 | −0.13 | −0.26 | −0.29 |
| + T-Costs | | −0.19 | −0.28 | 0.38 | −0.24 | −0.21 | −0.28 | −0.28 |
| ex ret | | 5.05 | 5.66 | 5.56 | 5.74 | 5.53 | 5.70 | 5.59 |

*Notes.* This table reports the OOS (panel A) summary statistics of a global equity portfolio following a parametric currency overlay. Each column refers to a different parametric currency overlay. The parametric currency overlay is estimated under a 2% tracking error constraint. Full Hedge refers to the currency overlay strategy neutralizing currency risk entirely. Ex ret, vol, and SR stand for annualized excess return, volatility, and Sharpe Ratio, respectively. Skew and kurt denote skewness and kurtosis, respectively. $\alpha$ is the annualized Jensen-$\alpha$. Dom-$\alpha$ is the Jensen-$\alpha$ with respect to the domestic equity portfolio. The certainty equivalent (CE) is expressed in annualized percentage points. Fwd Size refers to the volume of the currency overlay strategy, that is, the absolute, cumulative value of $\boldsymbol{\omega} - \boldsymbol{\Psi}$ without domestic currency. The numbers in brackets are the $p$ values examining the differences in excess return (correlated $t$-test), volatility ($f$-test), and Sharpe ratio (boot-TS of Ledoit and Wolf 2008) to the fully hedged portfolio. The $p$ value of the information ratio (IR) is computed by a bootstrap method counting the frequency of a negative sign. The standard errors of Jensen-$\alpha$ are adjusted as suggested in White (1980). TE refers to the tracking error with respect to the benchmark portfolio. A margin requirement of 15% is assumed. The economic quantities are measured in USD. The sample period is from January 1986 to December 2016. Panel B illustrates the statistical significance of the examined characteristics in an in-sample test. The displayed coefficient maximizes the quadratic utility over wealth with risk aversion of 1. The numbers in brackets are the corresponding $p$ values. The $p$ values are computed by a bootstrap method, which generates 1,000 random samples and captures the percentage of random samples, where the sign of the estimate varies from the expected sign. Panel C depicts the return contribution of the various optimization elements. The starting point is the excess return of the fully hedged portfolio (ex ret (Full Hedge)). CCY over. ret is the return related to the currency overlay considering no frictions and constraints. TE Cont is the margin return of the currency overlay assuming a tracking error constraint of 2%. Opp. Costs of Margin equals the contribution of the margin requirement with the additional assumption of a 2% tracking error constraint. T-Costs is related to the return which is caused by the additional friction of trading costs for equity and currency forward transactions.

portfolios with large sets of currencies including emerging markets. In our sample these stand-alone effects seem weaker but signs are in conformity with expectations based on these previous studies.

By contrast, the PPP joint currency overlay increases the Sharpe ratio to 0.45 and provides a Jenson-$\alpha$ and information ratio of 0.93% and 0.46, respectively. All three quantities are highly significant.[26] Moreover, certainty equivalent equals 4.62%,[27] which is 0.90 pp higher than the benchmark's CE. This is higher than the certainty equivalent achieved by the fully hedged benchmark or the unhedged portfolio. PPP joint currency overlay achieves this enhancement, as the excess return increases significantly by 0.85 pp; meanwhile, the standard deviation drops marginally. This is in line with previous literature documenting the power of this small set of signals to capture premiums in the currency market (Kroencke et al. 2013, Barroso and Santa-Clara 2015).

Also reported in panel A is the Jenson-$\alpha$ w.r.t. to a domestic portfolio. Noticeably, from the particular perspective of a U.S. investor, ex post, all Jenson-$\alpha$ point estimates of international equity portfolios are negative, regardless of currency management. But none of these estimates is statistically significant. This suggests that there is substantial noise in an ex post comparison of the performance of U.S. equities versus international equities. The PPP using momentum, value, and carry achieves the least negative alpha in the set of international portfolios.

Panel B of Table 3 shows the loading of each currency characteristic. Related to the PPP joint currency overlay, the characteristics momentum, forward discount, and long-term reversal have loadings of 0.21, 0.44, and 0.26, respectively, which are all statistically significant.[28] The significance of all characteristics means that each characteristic improves significantly the currency overlay. As a result of the loading of characteristics, the PPP joint currency overlay overweights on average high yielding currencies like New Zealand dollar by 4.01%, Australian dollar by 3.15% or Norwegian krona by 2.38%, whereas low yielding currencies are underweighted such as Japanese yen by −5.26%, Swiss franc by −4.40%, or USD by −1.73%. Notably, this pattern of weights goes in the opposite direction of equity sensitivity. Nevertheless, the weights at currency level are time varying as the characteristics of each currency change over time, particularly FX momentum.

Panel B also depicts the loadings of the single characteristic optimizations. The loadings between joint and single optimization are sign-wise identical. Each characteristic on a stand-alone basis has a larger loading than on a joint optimization basis, but none shows strong statistical significance. By contrast, the joint optimization already has statistically significant results for all three characteristics. This illustrates the economic importance of using carry, value, and momentum signals in combination.

As all optimizations take several constraints and frictions into consideration, we examine in panel C of Table 3 the return contribution of tracking error, margin requirements, and transaction costs. Our initial point for this analysis is the benchmark excess return, that is, 4.89%. If we add the PPP joint currency overlay without any constrains and frictions, the PPP joint currency overlay adds 58.70 pp, resulting in a total excess return of 63.59%. In a next step, we compute the impact of a 2% tracking error, which leads to a smaller return, that is, the tracking error constraint decreases the portfolio return by −57.46 pp. The PPP joint currency overlay limited by a 2% tracking error has a net return contribution of 1.24 pp. The opportunity costs for margin and trading costs for equity and currency transactions have a negative contribution of −0.15 and −0.24 pp, respectively.

The return contribution of currency momentum and forward discount on a stand-alone are comparable. The stand-alone optimization considering long-term reversal demonstrates a larger return contribution of 33.01 pp. However, the tracking error constraints creates a loss of −32.79 pp, eliminating most of this contribution. Consistently, the test statistic for economic relevance (Table 3, panel B) shows no significance. This means that long-term reversal, on a stand-alone basis, does not contribute economic value in terms of larger investor's utility. This result suggests that an investor should not consider this currency characteristic on a stand-alone basis for a currency overlay optimization.

The three characteristics of the PPP joint currency overlay build on basic properties of currencies. Next, we examine whether an additional fourth currency characteristic is able to provide any improvement.

We add the sensitivity of equity returns ($\mathcal{S}_{FX}$) as another variable to the PPP joint currency overlay. The positive slope (0.01) of $\mathcal{S}_{FX}$ has the opposite sign of what the CMV method recommends, that is, an increase in risk consumption. However, with a $p$ value of 0.52, it does not contribute to an enhanced risk-return profile. This observation is in line with the empirical results of Table 2, that is, the sensitivity of equity returns does not create an enhancement of risk-return profile under margin requirements, transaction, and rebalancing costs. Hence, the risk-return profile is similar to the PPP joint currency overlay. This provides direct evidence that $\mathcal{S}_{FX}$, the only variable used in the CMV method, is not relevant to our investor who cares about the overall risk-return tradeoff. Because of the insignificance of $\mathcal{S}_{FX}$, it seems that only noise is added to the currency overlay, as standard deviation increases and excess return declines.

Subsequent risk-return measures also worsen from an economic and statistical perspective.

The last characteristic added to the PPP joint currency overlay is the sensitivity to innovations in global FX volatility. Menkhoff et al. (2012a) show that high-yield currencies are negatively related to innovations in global FX variation. In times of unexpected high variation, high-yield currencies tend to have bad returns. Conversely, low-yield currencies act as a hedge and generate high returns in times of turmoil. This is proposed as an explanation to the profitability of the carry trade. In our setting, however, we find the loading on FX-volatility risk improves investor mean-variance utility once controlling for carry, value, and momentum as highlighted by the $p$ value of 0.04 shown in panel B.

Last, we also run a currency overlay optimization with all five characteristics jointly. The results do not largely differ from the other optimizations which is why we refrain from a detailed discussion. It is worthy to mention that the risk-return increase is less significant, and the currency overlay is dominated by the characteristics of the PPP joint currency overlay. Moreover, the return contribution displayed in panel C is comparable with the PPP joint currency overlay.

Additionally to this empirical investigation, we execute several robustness tests like an examination of multiple subperiods, different tracking errors, several domestic currencies, alternative investment universe, different margin requirements, and the properties with respect to an investor having a unhedged portfolio as the benchmark instead. We also consider the case of an equal-weighted equity portfolio. All of these results are presented in the online appendix. The main conclusion from all our robustness tests is that the introduced currency overlay provides a significant enhancement of the investor's risk-return profile.

This set of results shows that an optimized currency overlay, incorporating information on expected returns, increases investor utility after a set of pertinent frictions is considered. In the following section, we examine possible economic explanations for this result at two different levels: before and after frictions.

### 3.3. Economic Explanation

In frictionless markets, the set of assets is linear, and a stochastic discount factor exists by absence of arbitrage opportunities (Cochrane 2009). In such a setting, as put by Ang (2014, chapter 14, p. 455): "Assets are bundles of factors, and assets earn returns because of their underlying exposures to factor risks." The parametric portfolio policy approach models asset-level weights as functions of characteristics proxying for these, possibly unknown, factors. This method dispenses the difficult step of estimating the expected returns and covariance matrix of individual assets—a process known to be prone to estimation error. In that sense, it cuts through the veil of individual assets, moving one step closer to the real economic drivers of returns: the risk factors. As such, it is natural to ask if, ex post, simple linear combinations of well-known factor portfolios explain the performance of the PPP joint currency overlay.

Table 4 depicts the results regressing the return of the PPP joint currency overlay on several known currency risk factors. In all regressions, the dependent variable is the return, before transaction costs, of the deviation from the full hedge, which equals $f(\boldsymbol{\kappa}_t;\boldsymbol{\eta})\cdot R^{FX}$. This inner product is a simple linear combination of individual currency excess returns ($R^{FX}$). The average excess return of this overlay in the OOS period is 143 bps.

Following Pojarliev and Levich (2008), we construct standard factors such as carry trade, momentum, and value from our sample of currencies. The factors sort currencies according to these attributes and form equally weighted long-short currency portfolios that buy (sell) the top (bottom) three currencies. We construct similar long-short factors for the innovation of global FX volatility (Menkhoff et al. 2012a), the RX factor of Lustig et al. (2011), and the dollar carry factor of Lustig et al. (2014). Finally, we use the external imbalances factor defined in the set of developed economies from Della Corte et al. (2016).[29]

As all factors are tradable, the intercept can be interpreted as a risk-adjusted return. It is the spread in expected returns between the PPP joint currency overlay and its linear projection onto the factor space spanned by the right-hand-side factors in the regression. Intuitively, under the standard assumption of a linear asset space, investors could either hold the PPP joint currency overlay or its respective mimicking portfolio: a combination of factor portfolios. The intercept measures the economic cost of doing so.

In univariate regression models, all loadings on the currency factors considered are significant at least at a 10% significance level. Also, alpha point estimates are all smaller than the original spread in excess returns. Therefore, part of the mean returns of the strategy are explained by factors. In some cases, more than half of the original spread in returns is explained. Therefore, the PPP joint currency overlay is clearly exposed to standard currency risk factors and these partly explain its returns.

The carry and momentum factors have a high $R^2$ of 0.44 and 0.37, respectively. The remaining univariate regression results reveal that loadings on the respective currency risk factors are significant, too. Nevertheless, these other factors explain relatively little as can be seen from the low $R^2$ and high intercepts of the regressions. As a whole, all intercept

**Table 4.** Regressions

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.53** | 1.03*** | 0.99*** | 1.26*** | 1.26*** | 1.30*** | 1.07*** | 0.56*** | 0.79*** | 0.41** |
| | (0.08) | (0.08) | (0.10) | (0.10) | (0.09) | (0.10) | (0.10) | (0.05) | (0.09) | (0.05) |
| $ca$ | 0.35*** | | | | | | | 0.30*** | | 0.36*** |
| | (0.04) | | | | | | | (0.02) | | (0.03) |
| $mom_{FX}$ | | 0.30*** | | | | | | 0.26*** | | 0.24*** |
| | | (0.04) | | | | | | (0.02) | | (0.03) |
| $va_{FX}$ | | | 0.08* | | | | | 0.02 | | 0.08** |
| | | | (0.04) | | | | | (0.03) | | (0.03) |
| $\Delta\sigma_{FX}$ | | | | −0.02 | | | | | | −0.02 |
| | | | | (0.46) | | | | | | (0.01) |
| $DF$ | | | | | 0.04*** | | | | 0.07*** | 0.09 |
| | | | | | (0.01) | | | | (0.01) | (0.07) |
| $RX$ | | | | | | −0.05*** | | | −0.02* | 0.09 |
| | | | | | | (0.01) | | | (0.01) | (0.07) |
| $IMB$ | | | | | | | 0.08*** | | | −0.01 |
| | | | | | | | (0.02) | | | (0.01) |
| $R^2$ | 0.44 | $0.37 \times 10^{-1}$ | 0.02 | 0.00 | 0.03 | 0.03 | 0.09 | 0.70 | 0.12 | 0.75 |

*Notes.* This table displays the regression result of the pure currency return of the PPP joint currency overlay, that is, $f(\boldsymbol{\kappa},\boldsymbol{\eta}) \cdot R^{FX}$ on various risk factors. The terms $ca$, $mom_{FX}$, and $va_{FX}$ refer to a standard carry trade, currency momentum, and currency value strategy, respectively. Each of these strategies is an equally weighted long-short portfolio with three currency pairs. $\Delta\sigma_{FX}$ is the first difference of currency volatility followed by Menkhoff et al. (2012a). *DF* and *RX* equal the dollar carry factor, and the equal-weighted portfolio of Lustig et al. (2014) and Lustig et al. (2011), respectively. *IMB* refers to the imbalance factor following Della Corte et al. (2016). $\alpha$ is annualized and expressed in percentage points. Standard errors of each regression are adjusted as suggested by White (1980). The sample period is from January 1986 to December 2016. $R^2$ refers to the adjusted $R^2$. The numbers in brackets refer to standard errors.
*$p < 0.10$; **$p < 0.05$; ***$p < 0.01$ significant levels, respectively.

terms of univariate regressions are significant and economically relevant in a range between 0.53% and 1.30%. Therefore, the PPP joint currency overlay has significant abnormal returns w.r.t. all factors.

As the PPP joint currency overlay is a result of three currency characteristics, we run a multidimensional regression with carry trade, currency momentum, and currency value. As expected, this multidimensional regression has a large $R^2$ of 0.70. Yet, the intercept term is still significant and equals 0.56%. This regression result reveals that even a joint currency strategy of carry trade, momentum, and value is not able to fully explain the additional excess return of the PPP joint currency overlay. As this currency overlay is built on exactly the same trading signals, this evidence shows that the weighting scheme (characteristic-weighted) has some marginal information relative to the long-short factors. Barroso and Santa-Clara (2015) find a similar result for carry only, whereby different weighting schemes exploiting the same carry information do not necessarily subsume each other.

We also run a kitchen sink and multidimensional regression for the dollar carry and RX currency factors. As both regression results are comparable with the previous, we refrain from a detailed discussion.

As a whole, the results in Table 4 suggest that a substantial portion of the excess returns of the PPP joint currency overlay can be explained by well-known currency risk factors. However, no combination of risk factors examined is able to explain the return spread entirely.

We now turn to a setting with frictions. Frictions introduce potentially important nonlinearities in the asset space. Equation (5) defines, in a frictionless setting, the return of the currency overlay at time $t+1$ as $f(\boldsymbol{\kappa}_t;\boldsymbol{\eta})'(\Delta\boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}_t^1)$. This is a linear combination of currency returns $(\Delta\boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}_t^1)$. More importantly, the weights $f(\boldsymbol{\kappa}_t;\boldsymbol{\eta})$ are themselves a linear combination of characteristics $(\boldsymbol{\kappa}_t)$ as can be seen from Equation (4). Therefore, the return of the overlay is a linear combination of the return of characteristic portfolios.[30] By contrast, the return in Equation (9), already accounting for frictions, introduces terms that are nonlinear in $\boldsymbol{\eta}$ and $\boldsymbol{\kappa}_t$: $-|\boldsymbol{\Omega}_t(\boldsymbol{\eta}) - f(\boldsymbol{\kappa}_t;\boldsymbol{\eta})|'\boldsymbol{TC}_t - RBC_t(\boldsymbol{\eta})$. For instance, $f(\boldsymbol{\kappa}_t;\boldsymbol{\eta})$ affects $\boldsymbol{\Omega}_t(\boldsymbol{\eta})$ nonlinearly and rebalancing costs, in turn, are also a nonlinear function of this quantity (they depend on the absolute distance between $\boldsymbol{\Omega}_t(\boldsymbol{\eta})$ and inertia weights of equity positions).

At a more intuitive level, performance after costs of a portfolio formed using multiple characteristics depends crucially on the interaction between characteristics. For instance, the momentum signal can recommend buying a low yielding currency that would otherwise be shorted due to the negative carry signal. In this case, both signals offset each other, hence reducing trading costs in a combined strategy. As a result, an integrated portfolio with the same weight on a carry and a momentum characteristic should perform better, after transaction costs, than one equal-weighted combination of portfolios formed on the exact same signals, but with trading costs computed separately.[31] With the additional frictions considered in our study, we find this nonlinearity argument is further reinforced.

To illustrate this, we present in Table 5 a decomposition of the PPP joint strategy into different sources of return accounting for frictions. Examining the frictions one by one allows an assessment of the relative economic importance of each one.

Our starting point is the unhedged excess return, without trading costs, of the value-weighted benchmark. Then, we add the currency ($ret_{FX}$) and the equity ($ret_{EQ}$) deviation returns, which results in the (gross) hedged return. The currency deviation return is defined as the entire return sourced from entering currency forwards. Equity deviation return captures the return of equity de-investment to finance the margin of currency forwards.[32] From these hedged returns, we subtract the transaction costs related to the currency deviation ($TC_{FX}$) and the equity rebalancing trading costs ($TC_{EQ}$).

The unhedged benchmark without trading costs has a return of 6.60%. The currency and equity deviation returns of the fully hedged benchmark equal −57 and −54 bps, respectively. Therefore, the benchmark portfolio incurs on total costs related to frictions of 114 bps. This is the sum of 54 bps (because of margin requirements) with 11 bps (trading costs with currencies) and 49 bps (equity rebalancing costs). The most important cost, in terms of foregone returns, comes from margin requirements. Rebalancing costs are also quite relevant in our results. Intuitively, hedging currency risk creates a wedge between the return of the unhedged equity portfolio and its fully hedged counterpart. For instance, a drop in the value of the U.S. dollar creates, ceteris paribus, a rise in the value of foreign equities denominated in U.S. dollar coupled with a loss in the corresponding currency hedging overlay. This implies a need to liquidate part of the equity holdings to rebalance and cover the loss in the FX position. Such rebalancing costs, often ignored in the currency hedging literature, turn out to be nontrivial for all the hedging methods.

On top of these, the fully hedged creates a currency loss before trading costs of −57 bps. This reflects that the overlay is long USD against a basket of foreign currencies and those have positive average returns over this sample period.

**Table 5.** Return Split

| | *unhedged* | $ret_{FX}$ | $ret_{EQ}$ | *hedged* | $TC_{FX}$ | $TC_{EQ}$ | $ret_{net}$ | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|---|---|---|---|
| Full Hedged | 6.60 | −0.57 | −0.54 | 5.49 | −0.11 | −0.49 | 4.89 | −1.26*** | 0.92*** |
| Uni. Hedge | 6.60 | −0.24 | −0.51 | 5.85 | −0.10 | −0.46 | 5.29 | −0.84*** | 0.78*** |
| CMV | 6.60 | −1.17 | −1.06 | 4.38 | −0.36 | −0.96 | 3.06 | −1.70** | 0.70*** |
| CMV TE2 | 6.60 | −0.93 | −0.83 | 4.84 | −0.25 | −0.76 | 3.83 | −1.53*** | 0.79*** |
| $CMV_{SR}$ TE2 | 6.60 | 0.10 | −0.51 | 6.19 | −0.20 | −0.46 | 5.53 | −0.36** | 0.81*** |
| PPP mimic | 6.60 | 0.39 | −0.66 | 6.34 | −0.20 | −0.63 | 5.51 | −0.59*** | 0.98*** |
| PPP TE2 | 6.60 | 0.76 | −0.69 | 6.67 | −0.25 | −0.69 | 5.74 | | |

*Notes.* This table shows the return composition from an U.S. investor's perspective between January 1986 and December 2016. *unhedged* refers to the excess, value-weighted, and unhedged portfolio return. This return is not adjusted by any trading cost or margin requirement. $ret_{FX}$, and $ret_{EQ}$ equal the return of the currency and equity components, respectively. The currency component refers to the deviation from the unhedged currency approach. The equity component captures the return driven by the de-investment to finance the margin for the currency overlay. The sum of *unhedged*, currency and equity components equals the hedged return. $TC_{FX}$, and $TC_{EQ}$ refer to the transaction costs of the currency and equity component, respectively. The hedged return adjusted by the transaction costs of the currency and equity component equals the net return ($ret_{net}$). The row titled Full Hedged refers to strategy neutralizing currency risk. CMV refers to the approach of Campbell et al. (2010) using currencies to minimize the volatility of the global equity portfolio. CMV TE2 is an imitation of CMV using a time expanding window with a tracking error constraint of 2%. CMV SR corresponds to an alternative CMV method aiming to maximize the Sharpe ratio with a tracking error constraint of 2%. The row Uni. Hedge refers to the universal hedging ratio introduced by Black (1989). PPP TE2 refers to the PPP joint currency overlay following a parametric portfolio policy with a tracking error constraint of 2%. PPP mimic mirrors jointly PPP TE2 by the carry trade, currency momentum, and currency value. A margin requirement of 15% is assumed. The terms $\alpha$ and $\beta$ refer to the regression results between PPP joint currency overlay and the considered currency overlay strategy. For these regressions only the return of the currency overlay is considered.

$^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$ significant levels, respectively.

The difference between the full and the universal hedge introduced by Black (1989) is mainly driven by a reduction of currency losses, as the currency deviation return is −24 bps, which is 33 bps higher.

Turning to the results of CMV, they indicate that a currency sensitivity neutralization increases costs. CMV (CMV TE2) has a currency deviation return of −117 bps (−93 bps). As De Roon et al. (2012) show, the strategy has low expected returns, even before frictions. However, we find this is not the main drag on this strategy's performance. As its volume in currency positions is large, the equity deviation return stands at −106 bps (−83 bps). Additionally, it incurs on higher trading costs in equity and currencies. Overall, margin requirements are the main contributor to these costs.

The PPP joint currency overlay (PPP TE2) has a positive currency deviation return of +76 bps. This is the main driver of its enhanced performance. In comparison with the fully hedged portfolio, transaction costs with currencies and equities are also higher. This is because of a larger currency overlay in terms of absolute volume and, consequently, a larger fraction of capital needs to be pledged in order to finance margins. The currency speculative return provides a much larger contribution, which is not offset by higher transaction costs and equity deinvestment. The total costs related to frictions are −163 bps (= −69 − 25 − 69). This means that the fully hedged portfolio has costs that are approximately 70% (= 114/163) of the PPP join currency overlay. This provides an investor with a fully hedged equity benchmark a compelling additional rationale for currency speculation.

As his benchmark also has costs, the marginal cost of speculation is reduced. Most of the costs would be incurred anyway just for hedging purposes. Notably, this cost saving would not be accessible to a hypothetical investor allocating capital between a fully hedged international equity portfolio and a separate fund following a purely speculative currency investment strategy. The saving rather results from integrated optimal combination of hedging with speculation.

Next, we consider a strategy that mimics the PPP joint currency overlay using the carry, momentum, and value characteristic portfolios defined previously. The weighting scheme is kept constant and equals the multidimensional regression coefficients from Table 4. These weights have hindsight as they would only be known at the end of the sample. Yet, they allow answering the question whether an investor could achieve a similar performance by instructing one currency manager following jointly these FX investment styles. We call this PPP mimic. The main difference between PPP mimic and the PPP joint currency overlay corresponds only to the dynamic weighting scheme of the parametric portfolio policy.[33] This strategy achieves a smaller speculative performance compared with PPP TE2 as $ret_{FX}$ equals 39 bps. After adjusting the return by equity deviation and trading costs, the PPP joint currency overlay has an outperformance of 23 bps. This shows that, ex post, a static currency portfolio (at the characteristics level), with an integrated speculative and hedging components, produces comparable results.

To better compare PPP with CMV, we also compute the return decomposition of CMV SR. This method produces a currency deviation return of 10 bps. Although positive, this is still much less than the 76 bps of PPP TE2. This comparison of gross performance is revealing as it isolates a likely cause of TE2 outperformance. CMV SR, being an in-sample method, achieves, by construction, the best *unconditional* linear combination of currencies available (with hindsight) to maximize the Sharpe ratio. Parametric currency overlay has inherently a *dynamic* weighting at the individual currency level. The fact that PPP TE2 outperforms, in real time, the best ex post static combination of currencies available shows that its outperformance likely comes from taping into the rich information on conditional moments of the cross-section. One take away of our study is the importance of applying a hedge that is conditional on some well-known variables.[34]

Notably, for CMV SR, the costs with frictions are smaller than for PPP TE2. This is driven by the volume of the corresponding currency overlay. However, the PPP joint currency overlay offsets this by providing a currency deviation return, which is 66 bps higher.

In the last two columns of Table 5, we present results of regressions of the currency deviation returns of each method on those of the PPP TE2, which now assumes the role of the benchmark. This depicts that all alternative currency overlays produce negative intercepts of −36 bps or more with significant estimates at least at the 5% level. This result underpins that PPP joint currency overlay outperforms static currency strategies and dominates variance minimization methods.

Jorion (1994) and Kroencke et al. (2013) propose splitting the return of a currency overlay into a hedging and a speculative component. We present the results of such a decomposition exercise in the Internet appendix. It shows that the PPP joint currency overlay captures a significant speculative return, while the hedging component is small. Moreover, all alternative currency overlays have a nonsignificant speculative return.

All in all, the results from this analysis show that the edge of the PPP currency overlay over other methods originates at two levels. First, before frictions, the overlay uses characteristics to capture information on (time-varying) cross-sectional expected returns. As a result, it harvests predictable risk premiums that are now well documented in the FX literature. Second, this edge is amplified after frictions because (i) an integrated use of the predictive signals saves on trading costs and margin requirements and (ii) the marginal cost of currency investing is smaller compared with hedging strategies that also incur on their own costs (and have essentially zero or negative expected returns).

### 3.4. Joint Optimization

Jorion (1994) advocates that the optimal currency overlay should be jointly optimized, that is, the equity weighting scheme and currency overlay are interdependent decisions. We expand our optimization to allow the optimal equity weighting scheme to deviate linearly from the benchmark value-weighted portfolio. For the purpose, we consider the equity characteristics equity momentum, equity volatility, and equity value and apply a linear weighting scheme, that is,

$$\boldsymbol{\omega}_t^D = \boldsymbol{\omega}_t + \eta_{mom}\frac{\boldsymbol{\kappa}_{mom,t}}{n+1} + \eta_{value}\frac{\boldsymbol{\kappa}_{value,t}}{n+1} + \eta_{vol}\frac{\boldsymbol{\kappa}_{vol,t}}{n+1}, \quad (13)$$

where $\boldsymbol{\kappa}_{mom,t}$ equals equity 12-month momentum, $\boldsymbol{\kappa}_{value,t}$ is minus the 60-month average return, both are cross-sectionally standardized, and $\boldsymbol{\kappa}_{vol,t}$ corresponds to the realized volatility of the previous month based on daily observations.

Because of the construction and without additional constraints, the optimization can freely compute the optimal equity and currency allocation, as long as the optimized and benchmark portfolio deviates with a maximal tracking error of 2%. Theoretically, it is possible that the optimization only deviates in the equity or currency overlay dimension.

The OOS results of the joint optimization are displayed in Table 6. The excess return equals 6.02% with a volatility of 12.99%, a Sharpe ratio of 0.46 and a certainty equivalent of 4.81%. These results show that a joint optimization of equity weighting scheme and currency overlay creates an additional 28 bps of return in comparison with the PPP joint currency overlay. The return enhancement shows in the larger Jensen-$\alpha$ (118 bps) and information ratio (0.56).

However, the equity characteristics show little significance. By contrast, comparing the characteristic loading of panel C in Table 6 with those of the PPP joint currency overlay, it shows that the two currency overlays have very similar compositions. As equity characteristics have a relatively small influence, the currency overlay is dominating the result. In fact, the average currency overlay volume equals 92.06%, which is approximately 4.5 pp larger than the PPP joint currency overlay. Therefore, the joint optimization pays an even larger attention to the currency component. This explains why the currency deviation return (Table 6, panel D) increases to 112 bps.

The equity deviation return is –71 bps, almost the same as the PPP joint currency overlay (–69 bps). On a first thought, one would expect that the equity deviation return should increase. After all, the parametric portfolio policy would only put weight to a

**Table 6.** Joint Optimization

| Panel A: Return measures | | | | | |
|---|---|---|---|---|---|
| ex ret | vol | SR | skew | CE | Fwd Size |
| 6.02 | 12.99 | 0.46 | –1.09 | 4.81 | 92.06 |
| **Panel B: Test statistics** | | | | | |
| *t*-test | *f*-test | SR-test | *IR* | $\alpha$ | *TE* |
| 0.00 | 0.97 | 0.00 | 0.56 | 1.18 | 2.01 |
| **Panel C: Loading of characteristics** | | | | | |
| *mom* | *fd* | *rev* | $value_{EQ}$ | $mom_{EQ}$ | *vol* |
| 0.21 | 0.44 | 0.25 | 0.01 | 0.01 | 0.02 |
| [0.01] | [0.00] | [0.01] | [0.49] | [0.24] | [0.17] |
| **Panel D: Return split** | | | | | |
| *unhedged* | $ret_{FX}$ | $ret_{EQ}$ | $TC_{FX}$ | $TC_{EQ}$ | $ret_{net}$ |
| 6.60 | 1.12 | –0.71 | –0.27 | –0.72 | 6.02 |

*Notes.* The table reports the OOS results (panel A) for a global equity portfolio following a joint parametric currency and equity optimization from a U.S. investor's perspective between January 1986 and December 2016. Ex ret, vol, and SR stand for annualized excess return, volatility, and Sharpe Ratio, respectively. Skew denotes skewness. The certainty equivalent (CE) is expressed in annualized percentage points. Fwd Size refers to the volume of the currency overlay strategy, that is, the absolute, cumulative value of $\boldsymbol{\omega} - \boldsymbol{\Psi}$ without domestic currency. Panel B illustrates test statistics of returns. *t*-, *f*-, and SR-test *p*-values refer examining the differences in excess return, volatility and Sharpe ratio, respectively. IR refers to the information ratio with respect to the fully hedged benchmark portfolio. $\alpha$ equals the annualized Jensen-$\alpha$. TE refers to the tracking error with respect to the benchmark portfolio. Panel C displays the coefficient of each characteristic such that the quadratic utility function over wealth with risk aversion of 1 is maximized. *mom*, *fd*, and *rev* are currency characteristics and refer to the currency momentum, forward discount, and long-term reversal, respectively. The equity characteristics are equity value ($value_{EQ}$), equity momentum ($mom_{EQ}$) and equity volatility (*vol*). Panel D illustrates the decomposition of return. *unhedged* refers to the excess, value-weighted, and unhedged portfolio return. This return is not adjusted by any trading costs or margin requirements. $ret_{FX}$, and $ret_{EQ}$ equal the return of the currency and equity component. The currency component refers to the deviation from the unhedged currency approach. The equity component captures the return driven by the deinvestment to finance the margin for the currency overlay. The sum of *unhedged*, currency, and equity components equals the hedged return. $TC_{FX}$, and $TC_{EQ}$ refer to the transaction costs of the currency and equity component, respectively. The hedged return adjusted by the transaction costs of the currency and equity component equals the net return ($ret_{net}$).

characteristic if it is relevant to improve utility. We confirm the equity characteristics and the corresponding deviation in equity weighting create a small positive marginal utility. However, this effect is hidden because the overall volume in equity investment also decreases because of a larger currency overlay. This implies that the opportunity costs due to margin requirements in equity deviation become larger. Therefore, the optimization achieves a similar equity return with a smaller allocation to equities.

Nevertheless, the joint optimization results do not differ substantially from the results with the PPP joint currency overlay, without optimization of the equity component. We conclude from this analysis that the speculative FX component seems more important to increase utility.

## 3.5. Multiperiod Optimization

Transaction costs have a meaningful impact on economic performance in our previous exercises. Lowering rebalancing frequency could offer a plausible and straightforward way to reduce them. Table 7 reports results of optimizations keeping positions fixed, both for equities and currencies, for multiple periods: two, three, and six months. Using PPP for a multiperiod optimization is not a standard feature of the method. In fact, to the best of our knowledge, this is the first application of the method with this kind of analysis.[35]

Regardless of investment horizon, the PPP joint currency overlay shows a significant information ratio relative to the fully hedged benchmark. It also has consistently higher returns and certainty equivalents. However, keeping positions fixed does not increase expected returns, Sharpe ratio, or certainty equivalent of the PPP joint currency overlay relative to our baseline setting with one-month investments in Table 3.

The loadings on characteristics reveal an interesting pattern. Currency momentum becomes irrelevant for longer horizons, a reflection of the short-lived nature of this signal. The economic significance of carry and value, as can be seen by the magnitude of their coefficients, is robust to longer horizons. Strikingly, for six-month horizons, the currency overlay becomes a mere combination of these two drivers of returns.

Noticeably, even when economic significance of the characteristics does not change much with horizon, the standard errors monotonically increase for longer holding periods. This suggests that, for longer horizons, there are less independent nonoverlapping observations for returns and so bootstrap standard errors become larger for all signals.

**Table 7.** Parametric Currency Overlay: Multiperiod Holding

| | Two-period | | Three-period | | Six-period | |
|---|---|---|---|---|---|---|
| | Full Hedge | mom, fd rev | Full Hedge | mom, fd rev | Full Hedge | mom, fd rev |
| **Panel A: Summary statistic** | | | | | | |
| ex ret | 5.13 | 5.92 | 5.21 | 5.90 | 5.30 | 5.89 |
| | | [0.01] | | [0.02] | | [0.04] |
| vol | 12.97 | 12.96 | 12.96 | 13.03 | 12.96 | 13.15 |
| | | [0.99] | | [0.92] | | [0.79] |
| SR | 0.40 | 0.46 | 0.40 | 0.45 | 0.41 | 0.45 |
| | | [0.02] | | [0.10] | | [0.21] |
| IR | | 0.42 | | 0.36 | | 0.31 |
| | | [0.01] | | [0.02] | | [0.04] |
| $\alpha$ | | 0.84 | | 0.71 | | 0.57 |
| | | [0.01] | | [0.00] | | [0.01] |
| skew | −0.98 | −1.07 | −0.98 | −1.03 | −0.98 | −1.04 |
| kurt | 5.73 | 6.31 | 5.76 | 6.27 | 5.74 | 6.27 |
| CE | 3.97 | 4.74 | 4.06 | 4.68 | 4.14 | 4.58 |
| TE | | 1.87 | | 1.89 | | 1.88 |
| **Panel B: Loadings of characteristics** | | | | | | |
| mom | | 0.25 | | 0.17 | | −0.05 |
| | | [0.06] | | [0.23] | | [0.50] |
| | | (0.14) | | (0.19) | | (0.26) |
| fd | | 0.39 | | 0.35 | | 0.47 |
| | | [0.00] | | [0.03] | | [0.06] |
| | | (0.12) | | (0.18) | | (0.24) |
| rev | | 0.31 | | 0.38 | | 0.34 |
| | | [0.04] | | [0.09] | | [0.21] |
| | | (0.14) | | (0.22) | | (0.30) |

*Notes.* This table reports the OOS (panel A) summary statistics of a global equity portfolio following full hedge or a parametric currency overlay with currency momentum, forward discount, and long-term reversal for different holding periods. The holding periods are indicated by the column label. The parametric currency overlay is estimated under a 2% tracking error constraint. Full Hedge refers to the currency overlay strategy neutralizing currency risk entirely. Ex ret, vol, and SR stand for annualized excess return, volatility, and Sharpe Ratio, respectively. Skew and kurt denote skewness and kurtosis, respectively. $\alpha$ is the annualized Jensen-$\alpha$. The certainty equivalent (CE) is expressed in annualized percentage points. Fwd Size refers to the volume of the currency overlay strategy, that is, the absolute, cumulative value of $\boldsymbol{\omega} - \boldsymbol{\Psi}$ without domestic currency. The numbers in brackets are the $p$ values examining the differences in excess return (correlated $t$-test), volatility ($f$-test), and Sharpe ratio (boot-TS of Ledoit and Wolf 2008) to the fully hedged portfolio. The $p$ value of the information ratio (IR) is computed by a bootstrap method counting the frequency of a negative sign. The standard errors of Jensen-$\alpha$ are adjusted as suggested in White (1980). TE refers to the tracking error with respect to the benchmark portfolio. A margin requirement of 15% is assumed. The economic quantities are measured in USD. The sample period is from January 1986 to December 2016. All margins are financed by general collateral repo transactions. Panel B illustrates the statistical significance of the examined characteristics in an in-sample test. The displayed coefficient maximizes the quadratic utility over wealth with risk aversion of 1. The numbers in squared brackets (parentheses) are the corresponding $p$ values (standard errors). The $p$ values and standard errors are computed by a bootstrap method, which generates 1,000 random samples and captures the percentage of random samples, where the sign of the estimate varies from the expected sign.

This exercise suggests that the potential to improve performance with less rebalancing is limited. It essentially implies dropping the momentum signal for its lack of persistence and that is a useful characteristic for monthly horizons. The analysis has the merit of showing that currency value, because of being a more stable signal, becomes an even more interesting characteristic for longer horizons compared with our baseline optimization. This underscores the potential relevance of the findings in Menkhoff et al. (2017) for long-horizon currency investors.

## 4. Conclusion

This study proposes adapting PPPs to construct a value adding currency overlay strategy. The weights are modeled as a function of currency characteristics. The relevance of characteristics is estimated by maximizing the investor's quadratic utility of the associated portfolio returns over a given sample period. The currency characteristics momentum, forward discount and long-term reversal form our PPP joint currency overlay.

Over the entire sample period, from 1986 to 2016, the PPP overlay strategy generates a larger Sharpe ratio, a higher certainty equivalent, and positive Jensen-$\alpha$ and information ratio with respect to the fully hedged portfolio. All enhancements are underpinned by highly significant test statistics. Furthermore, the outperformance is quantified after transaction and rebalancing costs, incorporating margin requirements and in an OOS setting.

To test for robustness, shown in the online appendix, we apply the parametric currency overlay for several subsamples, various reference currencies, tracking error ranging from a passive to an aggressive investor, modified currency universe, various benchmark risks, and levels of margin requirements. As optimal hedging can depend on the utility function (De Roon et al. 2003), we also run tests with a power-utility function sensitive to portfolio skewness and kurtosis. All examinations reveal that the parametric currency overlay enhances the risk-return profile of the designated benchmark portfolios in a robust and statistically significant manner. Moreover, a decomposition exercise reveals significant positive speculative return of the parametric currency overlay.

To provide economic interpretation, a regression analysis reveals that the parametric currency overlay is exposed to standard currency risk factors. However, neither univariate nor multivariate factor models are able to fully capture the expected returns of the proposed currency overlay strategy. We examine the performance of combinations of well-known currency strategies to mimic the parametric currency overlay. We find that all examined mimicking currency strategies leave a significant proportion of returns on the table. This is particularly the case after frictions are taken into consideration. There are notable cost savings by integrating several trading signals in an optimized strategy. We also find that combining currency speculation and FX hedging in one currency overlay reduces the marginal cost of speculation by 70%.

Even an alternative hedging strategy using complete information and a maximal Sharpe ratio approach to construct constant currency overlay provides a smaller excess return than the out-of-sample parametric currency overlay. This shows that, by using characteristics, the proposed strategy is able to dynamically weight currencies while capturing time-varying, asset-level, risk premiums in the cross section.

In a subsequent analysis, we also run a joint optimization of equity weighting and currency overlay. The additional degree of freedom causes a higher excess return; however, the currency overlay remains dominant. To reduce trading costs, we also examine longer holding periods up to six months. We find that our proposed method also enhances performance significantly relative to fully hedged portfolios with longer holding periods.

Overall, the parametric currency overlay provides an apparently viable method to handle currency risk in international equity portfolios. First, relying on previous literature, or some other information set, the investor decides which properties or characteristics are likely to be important for the currency overlay. Second, each property or characteristic is tested and weighted in the overall currency overlay. One of the unique advantages of this applied method is its flexibility. Other methods of estimating exchange rate expected returns or risks (e.g., private forecasts or information from currency financial derivatives) could also be tested as characteristics.

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## Endnotes

[1] There is support in the literature for no hedging (Froot 2019), half hedging (Gastineau 1995), full hedging (Perold and Schulman 1988), or optimized hedging (Campbell et al. 2010), among other methods.

[2] The hedging issue is typically formulated as finding an optimal currency strategy to complement a fixed portfolio of international equities. This is designated as the currency overlay.

[3] The naïve methods are unhedged and full-hedged. The optimized methods are Black (1989) and (variants of) Campbell et al. (2010).

[4] We choose this target after examining fact sheets of global equity investment funds explicitly tracking a MSCI, which are similar to our benchmark. For example, the JP Morgan Global Research Enhanced Index Equity Fund tracks the MSCI world and reports a tracking error close to 2%. The median tracking error of the mutual fund universe is somehow higher at about 9% (Koski and Pontiff 1999, Chen and Pennacchi 2009). In the online appendix, we examine the case of looser constraints. These result in larger gains.

[5] The online appendix covers a section with more details of the derivation of this formula.

[6] All variables marked with $\tilde{}$ do not include the first component, that is, $x_t = (x_{2,t}, \ldots x_{n+1,t})'$, with the result that the component for the reference currency is withdrawn.

[7] In general, utility refers to a particular utility function. If nothing is mentioned, we always refer to a quadratic utility function.

[8] We set $l$ equal to 120 months as for our parametric currency overlay.

[9] Following Brandt et al. (2009), the coefficients on the characteristics are constant over the sample period and considered currencies.

[10] Requirements also vary per currency but that is more relevant for currencies of developing economies not included in our sample.

[11] The margin reserved for the risk-free asset is calculated as follows: Suppose an investor has USD100 to invest in international equities and a margin requirement of $\rho$ =10%. The investor wants to full hedge of the foreign currency exposure. Then, he can only invest USD90.91 as the remaining 10% (USD9.09) is used for the margin. To calculate the available amount to invest, we apply the following logic: 100 – required margin = 100 − 9.09 = 100 − $100/(1+\rho) = (1-\rho/(1+\rho))^{*}100$.

[12] DeMiguel et al. (2020) offer, in annex A.1., a succinct and clear discussion of the similarities and contrasts between PPP and Fama-MacBeth asset pricing regressions.

[13] In unreported results, we estimate the annualized benchmark risk of an unconstrained portfolio akin to Barroso and Santa-Clara (2015) to be at least 26%. Therefore, its behavior differs substantially from the fully hedged equity benchmark.

[14] Several studies provide analytical solutions to complex portfolio optimization problems such as Gârleanu and Pedersen (2013), DeMiguel and Uppal (2005), or DeMiguel et al. (2015). However, a simultaneous analytical treatment of all elements in our setting is not yet available.

[15] See, among others, Okunev and White (2003) and Menkhoff et al. (2012b) for momentum, Burnside et al. (2010) and Lustig et al. (2011) for carry, Menkhoff et al. (2017) for value, and Kroencke et al. (2013) and Asness et al. (2013) for combinations of these styles/factors.

[16] Other studies, in line with the adaptive market hypothesis of Lo (2004), suggest an historical scarcity of speculative capital in currency markets (Jylhä and Suominen 2011, Pojarliev and Levich 2013, Barroso and Santa-Clara 2015) and gradual learning by investors (Neely et al. 2009) also play a role explaining the profitability of FX styles.

[17] Campbell et al. (2010) adopt a reasonable and simplifying assumption of an equal-weighted equity portfolio in their study. In our optimization setting, emphasizing implementation, it is more natural to adopt a value-weighted allocation though. Regardless, the results of PPP are qualitatively similar for the two allocations.

[18] We follow the weighting scheme of the Bank of England (2015) to design an artificial Euro rate before 2000. In a robustness test we vary the currency and equity universe to demonstrate that the results are not driven by a certain selection of equity and currencies.

[19] New Zealand has an excess return of −0.64%. This is likely because of sampling error as the MSCI Standard Index of New Zealand is heavily concentrated in a few stocks.

[20] The online appendix indicates the same summary statistics for other domiciled investors.

[21] In the online appendix, we examine the case for investors domiciled in different economies.

[22] We consider in this paper a simple and conservative approach where an investor pledges a risk-free asset as margin. Alternatively, sophisticated investors can use repo transactions to swap equity for risk-free assets and fulfill margin requirements. These repo transactions are mainly over-the-counter and, have a haircut dependent on both counterparties and the quality of the related collateral. A complete examination of the repo market lies beyond the scope our study, but, in unreported results, we find the PPP joint currency overlay can also improve performance for an hypothetical investor using this market to fund positions.

[23] The test for differences in Sharpe ratio is the bootstrap method, introduced by Ledoit and Wolf (2008). The applied bootstrap method examines two time series for differences in Sharpe ratio. It provides a robust estimate under autocorrelation, heteroscedasticity, and heavy tails.

[24] The loadings of each characteristic is optimized with respect to a quadratic utility function. The online appendix also presents an empirical examination with respect of a power utility function and a constant risk aversion of 5 as in Barroso and Santa-Clara (2015). The empirical results are closely comparable with the following discussion, which is why we excluded this investigation.

[25] The online appendix includes portfolio performances for other domiciled investors.

[26] To determine the $p$ value of the information ratio, we apply the bootstrap method and count the frequency of a negative information ratio. For the Jenson-$\alpha$, we follow White (1980) to adjust the standard errors.

[27] The certainty equivalent is defined via a power utility function with Constant Relative Risk Aversion of 5 as in Barroso and Santa-Clara (2015).

[28] The statistical significance is an in-sample test, based on a bootstrap method with 1,000 random samples, and considers data from January 1976 to December 2016. The $p$ values equal the quantity of the sign opposite to the expected one.

[29] We thank Lucio Sarno for providing these data.

[30] More specifically, one can redefine the asset space as a $m \times 1$ matrix of characteristic-portfolio returns equal to $(1/(1+n))\boldsymbol{\kappa}_t'(\Delta \boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}_t^1)$. Then the return of the currency overlay, in a frictionless market, is simply this matrix premultiplied by $\boldsymbol{\eta}'$.

[31] Barroso and Santa-Clara (2015) first note, in a setting close to ours, that "In practice, the momentum signal reinforces the carry signal for some currency-periods, resulting in higher trading costs, but momentum offsets carry for other currency-periods, decreasing transaction costs. A priori there is no way of telling if a high-cost standalone strategy, such as momentum, actually results in increased costs for the investor. All depends on the interaction between signals." DeMiguel et al. (2020) provide a thorough and more formal treatment of the role of this offsetting effect in portfolios combining multiple signals.

[32] As defined, the margin is held in cash or short-term debt securities and earns a risk-free rate.

[33] In unreported results, we also examine a mimic strategy where the PPP joint currency overlay is tracked by separate carry, momentum,

and long-term reversal portfolios. This means that these three strategies are not allowed to net each other, with the result that offsetting currency positions can coexist. We find such a currency strategy is not optimal, as unnecessarily large margins must be pledged leading to a poor economic result.

[34] This also suggests an alternative path to our line of investigation is to expand the Campbell et al. (2010) method to a conditional setting with state-dependent regression coefficients. Although this is an interesting research question, it is beyond the scope of the present work.

[35] We thank an anonymous referee for this suggestion.

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