IMPROVING OPTIMIZATION

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Mean-variance optimization provides a systematic way to allocate wealth among assets that results in a portfolio with the highest expected return for a given level of risk. The technique requires forecasts of asset returns, variances, and cross-correlations as inputs. Quadratic programming is used to determine a set of weights that characterize the optimal portfolio.[1]

It is possible to create optimal portfolios that satisfy constraints reflecting regulation or judgment or investor preference for certain portfolio characteristics. Other techniques such as chance-constrained programming, safety first, and maximization of a more general utility function are either not easily available or not as widely understood and accepted as mean-variance.

While the mean-variance framework is theoretically appealing and easily implementable, some limitations restrict its use in practice. Various authors have discussed the problems associated with the use of the technique, including Bawa, Brown, and Klein [1979] and Michaud [1989]. The major problem is the need to come up with reasonable estimates of asset returns, variances, and cross-correlations, and the fact that the optimization "maximizes errors" in these estimates.

This article highlights two pitfalls that users of MV optimization should be aware of. The first deals with the sensitivity of the optimal portfolio weights to errors in the input parameters. Small errors in input parameters can lead to large changes in the composition of the optimal portfolio. We examine how these errors affect portfolio composition.

The second problem deals with "near-optimal" portfolios — portfolios that are similar to the optimal portfolio in expected return and risk but very different in composition. Michaud [1989] alludes to this problem but does not examine the extent of changes in portfolio composition. This article illustrates the size of the problem in a simple three-asset framework.

Rather than abandon the mean-variance approach, we discuss three techniques that address the estimation error problem and make the mean-variance approach more useful. The first involves the use of sensible constraints on portfolio weights. The second involves the use of Stein estimation to adjust the inputs. The third technique, which is a variation of the second, is Bayesian estimation.

Of the three, the first one is easiest to use. When used in conjunction with one or more of these techniques, mean-variance does live up to its promise. It is especially useful when the asset universe is large, making it difficult to keep track of the risk, returns, and interrelations among assets.

The assets considered are the S&P 500 Index, Lehman Brothers Long Term Government Bond Index, and one-month Treasury bills.[2] We use eleven years of monthly data (1980-1990) to compute historical mean returns and the covariance matrix. In practice, historical estimates may be improved through fundamental research to reflect current market conditions.

SENSITIVITY TO INPUT ERRORS

In order to examine the sensitivity to input errors we assume that the historical means and covariances are the "true" inputs. We compute a base mean-variance-optimal portfolio using these

EXHIBIT 1
ESTIMATES OF EXPECTED RETURNS, STANDARD DEVIATIONS, AND CROSS-CORRELATIONS (JANUARY 1980-DECEMBER 1990)

	Stocks	Bonds	T-Bills
Mean	1.323	1.027	0.729
Standard Deviation	4.793	3.984	0.219
Correlations:			
Stocks	1.000	0.341	-0.081
Bonds		1.00	0.050
T-Bills			1.000
Optimal Asset Weights*	58.1	22.8	19.1

*For a moderate risk tolerance.

inputs. Next, we artificially induce errors in the true inputs by changing them slightly, then compute the optimal portfolio based on these error-tainted inputs. We then compute the portfolio turnover involved in switching from the base portfolio to the new optimal portfolio.

Exhibit 1 shows the inputs to the optimization and the optimal portfolio for an investor with a moderate risk tolerance.[3] The analysis is repeated for conservative and aggressive risk tolerances.

The effect of changes in the expected returns is examined first. We increase the expected return for one asset by 5% from the base level and compute the turnover involved in switching to the new optimal portfolio.[4] We increase, decrease, or leave unchanged the expected return for each asset, and compute the resulting turnover from the base. We then compute the maximum and average turnover for the different combinations of input changes.[5]

How does the size of the error in inputs influence portfolio allocations? To examine this we induce errors of different sizes in the inputs and repeat the analysis. We let the size of the error increase from 5% to 50% in steps of 5%. Note that the turnover is always computed with respect to the base portfolio. In order to isolate the influence of changes in the expected returns, the variances and covariances are not changed.

Exhibit 2 shows the average turnover and Exhibit 3 the maximum turnover for errors in the expected returns. Surprisingly, a 5% error in the expected return estimates can lead to a turnover of over 18%.

So how likely is a 5% error in a return forecast? There is a greater than 98% probability that at least one of the true mean return values will differ by more than 5% from the sample mean.[6] The average turnover will exceed 10% and the maximum potential turnover will exceed 18% with a 98.5% probabil-

EXHIBIT 2
AVERAGE TURNOVER FOR DIFFERENT PERCENTAGE ERRORS IN MEANS, VARIANCES, AND COVARIANCES

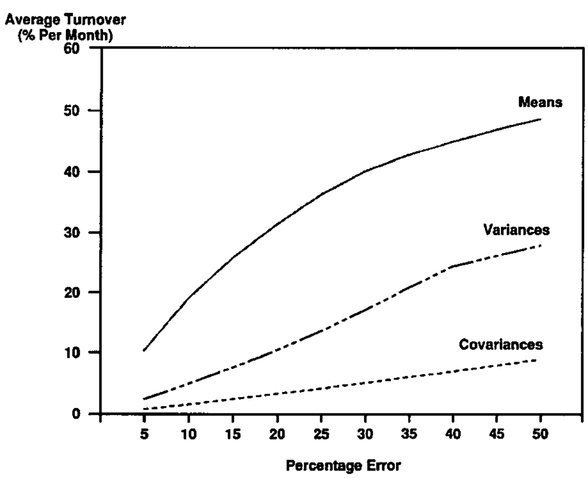
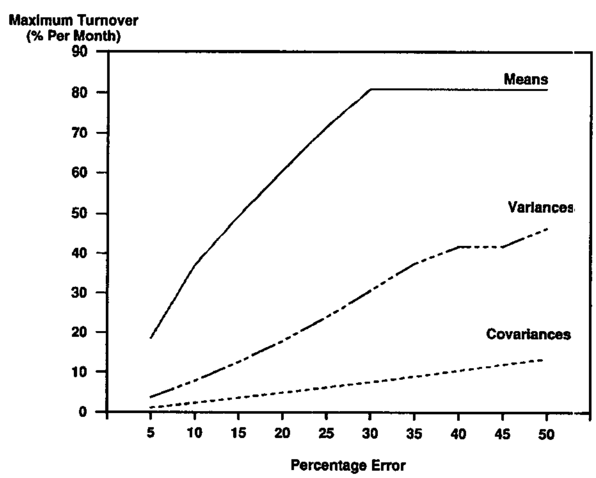


EXHIBIT 3
MAXIMUM TURNOVER FOR DIFFERENT PERCENTAGE ERRORS IN MEANS, VARIANCES, AND COVARIANCES



ity! The probability that the true means will deviate less than 10% from the sample means is 6.14%.[7] As expected, the average turnover increases as the size of the errors in the means increases.

Estimation errors are inevitable as the sample mean invariably differs from the true underlying mean. The more observations one uses to compute the sample mean, the more likely it is to be closer to the true mean. It may appear that using sixty or seventy years of data would improve the input estimates, but in that case one runs into the "non-stationarity" problem: the process that generates returns may change fundamentally over time.[8]

If small errors in expected returns can alter portfolio composition dramatically, is the same true for errors in variances and covariances? To answer this question, we artificially induce errors in the true values of the variances and covariances and see how the portfolio composition changes. We first induce errors in the variances, leaving the means and covariances at their true values. The size of the errors is increased gradually as in the case of expected returns.[9]

As Exhibits 2 and 3 show, the turnover is lower for changes in variances than for changes in means. The average turnover for a 5% change in means, 10.42, is over four times the average turnover for a 5% change in variances, 2.47.

The turnover is even lower for errors in covariances than it is for errors in variances. For a 10% change, the turnover is 19.03% for the case of means, 5.01% for variances, and 1.63% for covariances. Clearly, mean-variance-optimal portfolios are more sensitive to errors in expected returns than to errors in variances, and are more sensitive to errors in variances than to errors in covariances.

These results are consistent with those of Kallberg and Ziemba [1984] and Chopra and Ziemba [1993]. These authors examine the average cash equivalent loss from the use of estimated data rather than the true values.[10] Kallberg and Ziemba find that estimation errors in the means are about ten times as important as estimation errors in the covariance matrix. Chopra and Ziemba find that, for an investor with a moderate risk tolerance, errors in means are eleven times as damaging as errors in variances and errors in variances are twice as damaging as errors in covariances. Best and Grauer [1991] demonstrate the extreme sensitivity of MV-optimal portfolio weights to changes in means.

The result that, for a given percentage change, the portfolio turnover is greatest for changes in means, and least for changes in covariances, continues to hold at conservative and aggressive risk tolerances. Chopra and Ziemba [1993] find that an increase in risk tolerance magnifies the relative cash equivalent losses for errors in means, variances, and covariances. At a high risk tolerance, errors in means are over twenty-one times as important as errors in variances and over fifty-six times as important as errors in covariances.

The average turnover does not vary directly with risk tolerance. In general, the turnover is greater for a moderate risk tolerance than for extremely high and low risk tolerances.

Turnover is an important consideration for active portfolio management strategies (such as tactical asset allocation) because a high portfolio turnover generates high transaction costs, which can render an active strategy inferior to a passive strategy. For a moderate risk tolerance, the average turnover for estimation errors in means is about two to four times the average turnover for estimation errors in variances and about five to thirteen times the average turnover for estimation errors in covariances. (The results for high and low risk tolerances are similar.) Thus, it is more important to minimize estimation errors in means than in the variances and covariances.

NEAR-OPTIMAL PORTFOLIOS

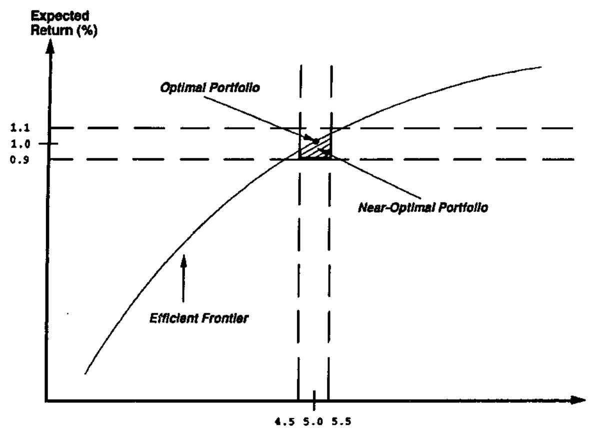
Can portfolios that are similar in risk and expected return differ in composition? The unexpected answer is that portfolios with very different asset weights can have very similar risk and return characteristics. An important implication of this for portfolio rebalancing is that an investor may be better off switching to a near-optimal portfolio that entails a lower transaction cost than moving to an optimal portfolio that requires a large turnover.

We examine all possible combinations of weights (in increments of 1%) for the three assets and compute portfolio expected returns and standard deviations. If the return and standard deviation of a portfolio are both within plus or minus 10% of those of the base portfolio, it is considered a near-optimal portfolio.[11]

The shaded region in Exhibit 4 represents the near-optimal portfolios. We also examine near-optimal portfolios for high and low risk tolerances.

There are literally hundreds of near-optimal portfolios. Restricting portfolio weights to integers, there are 1,182 distinct near-optimal portfolios for a

EXHIBIT 4
NEAR-OPTIMAL PORTFOLIO WITHIN A 10% WINDOW AROUND THE OPTIMAL PORTFOLIO



moderate risk tolerance.

Exhibit 5 lists the near-optimal portfolios that have the highest or lowest weights in each asset category. Rows 1 and 2 show the near-optimal portfolios with the lowest and highest weights in equities; rows 3 and 4 show those for bonds; rows 5 and 6, those for cash.

The weight given to equities varies between 9% and 73%; the weight in bonds varies between 0% and 84%, and the weight in cash varies from 1% to 39%. This shows that portfolios quite similar to the optimal in terms of expected return and risk can be quite dissimilar in composition.

The turnover of switching from the optimal portfolio to a near-optimal portfolio can be as high as 61%, as in row 4 for the portfolio with the highest weight in bonds.[12] This has important implications for portfolio management using mean-variance optimization.

If a small change in one of the inputs leads to an optimal portfolio recommendation that entails a large turnover, the investor should examine near-optimal portfolios to see if there is one that results in a low turnover. Rebalancing to a near-optimal portfolio would reduce the transaction cost while providing the investor with close-to-desired risk and expected return. This is especially important for high-turnover strategies such as tactical asset allocation.

To determine how turnover varies with the size of the window (in expected return-standard deviation space) around the optimal portfolio, we examine the maximum turnover for windows that vary from plus or minus 2% through 20% in steps of 2%. Exhibit 6 shows the portfolio weights that entailed the highest turnover for each window size and the corresponding mean and standard deviation for a moderate risk tolerance. Exhibit 7 shows the maximum possible turnover for windows of different sizes around the optimal portfolio. The turnover first increases rapidly and then somewhat slowly with the size of the window.

The results for conservative and aggressive levels of risk tolerance are similar to, although less dramatic than, those for a moderate risk tolerance. At the conservative end, the maximum turnover for a window size of 10% around the optimal is 33%. The corresponding turnover at the aggressive end is 21%.

EXHIBIT 5
NEAR-OPTIMAL PORTFOLIOS WITH EXTREME WEIGHTS IN INDIVIDUAL ASSET CLASSES*

Number	Weight in Stocks (%)	Weight in Bonds (%)	Weight in Cash (%)	Portfolio Return	Portfolio Risk	Turnover from Base (%)
1	**9**	83	8	1.03	3.48	60
2	**73**	2	25	1.17	3.52	21
3	73	**0**	27	1.16	3.50	23
4	9	**84**	7	1.03	3.52	61
5	27	72	**1**	1.10	3.53	49
6	61	0	**39**	1.09	2.92	23
Base	58	23	19	1.14	3.21	---

*Extreme weights in each asset class are bold-faced.

EXHIBIT 6
MAXIMUM POSSIBLE TURNOVER FOR PORTFOLIOS LYING WITHIN A SMALL WINDOW AROUND THE OPTIMAL PORTFOLIO

% from the Optimal	Maximum Turnover (%)	Weight in Stocks (%)	Weight in Bonds (%)	Weight in Cash (%)	Expected Return	Standard Deviation
2	29	40	52	8	1.12	3.27
4	41	30	64	6	1.10	3.33
6	49	22	72	6	1.07	3.38
8	56	15	79	6	1.05	3.46
10	61	9	84	7	1.03	3.52
12	66	2	89	9	1.01	3.58
14	68	0	91	9	1.00	3.63
16	70	0	93	7	1.01	3.71
18	72	0	95	5	1.01	3.79
20	73	0	96	4	1.01	3.38

For the aggressive investor, the optimal portfolio is almost 80% in equity, the asset with the highest expected return. The optimal portfolio is not diversified and behaves like equity. On the other extreme, for the conservative investor, the optimal portfolio is about 60% in cash, the asset with the lowest variance, and again is not too diversified.

Even in these extreme cases, the individual asset weights can vary considerably without the portfolio moving too far from the optimal portfolio. Because most institutional investors hold fairly diversified portfolios, near-optimal portfolios will exhibit a larger variation than for the extreme conservative and aggressive cases.

Thus, portfolios generated by the mean-variance framework are not precise. One can use other criteria (such as minimizing the transaction cost of making the transition) to choose portfolios that are almost like the optimal portfolio.

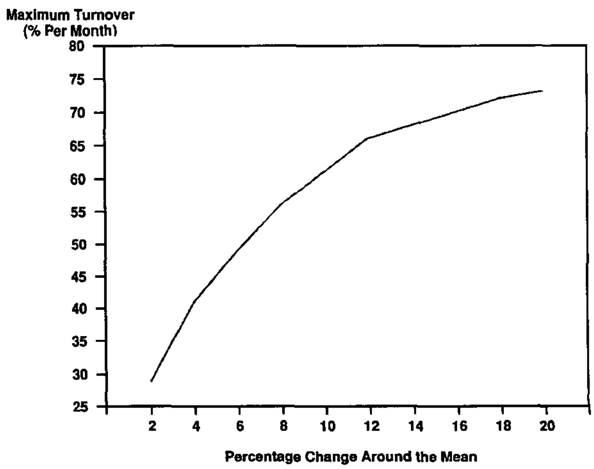
We have shown that mean-variance-optimal portfolios are not stable with respect to small errors in the inputs, and can be very similar in terms of expected return and risk to portfolios with very dissimilar composition. This is not a reason to abandon the mean-variance framework.

IMPROVEMENTS TO MEAN-VARIANCE

There are several ways to improve the performance of mean-variance optimization by reducing its sensitivity to errors. Several researchers have pointed out that the optimization "maximizes errors" — that is, the estimation errors in the inputs are magnified by the optimization (see Michaud [1989]). If an asset's expected return is estimated with a positive estimation error, the asset is likely to be overweighted in the optimal portfolio.

The expected return of the optimal portfolio is then inflated in two ways. First, the asset's true return is less than the estimated return. Second, the asset is overweighted in the portfolio. The optimiza-

EXHIBIT 7
MAXIMUM TURNOVER WITHIN WINDOWS OF DIFFERENT SIZES AROUND THE OPTIMAL PORTFOLIO



tion treats the inputs as if they are known with certainty. In practice, the inputs are estimates based on historical and other information and are thus subject to estimation error. The estimation error degrades the performance of the optimizer.

Constraining Asset Weights

Imposing reasonable constraints on asset weights is one way to reduce the optimization's tendency to maximize errors. Frost and Savarino [1988] use a simulation to highlight the improvement in portfolio performance that results from limiting the proportion of wealth invested in a single security. In the absence of constraints, positive estimation error in the return of an asset will induce an overallocation to that asset. Constraints on asset weights will limit the allocation to that asset.

We illustrate the benefits of "sensible" constraints in the three-asset framework with a simulation over time. We compare the performance over time of two portfolios: one with no constraints on asset weights, and the other where weights are not allowed to vary too far from the 60% equity-40% bonds benchmark.

We use a sixty-month rolling estimation period to compute mean returns, variances, and covariances that are used as inputs to the optimization. The optimal allocation is held for the month following the sixty-month estimation period, and the process is repeated by rolling forward one month at a time.[13] This yields out-of-sample portfolio allocations for the seventy-two-month interval from January 1985 through December 1990.

An unconstrained base portfolio for a moderate risk tolerance is formed as a reference for the constrained portfolio. Given that a portfolio of 60% equities and 40% bonds is a common benchmark, we examine the impact of constraints that prevent the MV-optimal portfolio from deviating too far from the 60% equities, 40% bonds, and 0% cash benchmark.

Specifically, we constrain the weight for each asset to lie within a 20% band of the 60-40-0 benchmark.[14] We also examine the performance over time of portfolios that lie within 10% and 30% bands. Short sales are not permitted.

Exhibit 8 shows the summary statistics for the unconstrained and the three constrained portfolios. The constrained portfolios dominate the unconstrained portfolio; each has a higher mean return and lower risk and turnover than the unconstrained portfolio. Constraints at 20% around the 60-40-0 benchmark increase the minimum portfolio return from -21.52 to -15.97, while reducing the maximum return only marginally (from 9.96 to 9.92). These results do not include transaction costs (which would further improve the performance of the constrained portfolio, because the unconstrained portfolio has a higher average turnover).

Investment managers often intuitively impose constraints on portfolio weights without realizing that they are effectively controlling for forecast errors. Our results show that such constraints can improve portfolio performance considerably.

While constraints on weights reduce the impact of errors, they also can reduce the potential gains from information that is not currently reflected in the market price. Thus, even if an asset return is correctly forecast as high, the allocation to that asset will not exceed the level determined by the constraint, hurting the performance of the portfolio.

This study shows that the benefits of imposing sensible constraints outweigh the costs. Each constrained portfolio has a higher realized mean return and lower risk and turnover than the unconstrained portfolio. Any reasonable constraints are

EXHIBIT 8
PERFORMANCE OVER TIME OF CONSTRAINED AND UNCONSTRAINED OPTIMAL PORTFOLIOS

Portfolio Type	Mean Return	Standard Deviation	Minimum Return	Maximum Return	Average Turnover (%)
Constrained (10%)	1.21	3.78	-13.20	9.53	2.17
Constrained (20%)	1.14	3.86	-15.97	9.92	4.26
Constrained (30%)	1.08	3.99	-18.75	9.96	6.16
Unconstrained	0.97	4.18	-21.52	9.96	9.56

better than the results of using unconstrained optimization.[15] Frost and Savarino [1988] show that constraints improve portfolio performance at the individual security level.

James-Stein Estimation

Another approach that reduces the impact of estimation errors is the use of James-Stein estimators. (See Efron and Morris [1977] for a non-technical overview of Stein estimation. A more rigorous discussion can be found in Efron and Morris [1973, 1975].) The basic idea behind this technique is that objects that are "similar" should behave in a similar manner over the long run. Thus, if assets are considered to belong to the same basket, their characteristics (returns, variances, and cross-correlations) are likely to be similar.

The technique involves shrinking individual expected returns for assets within the same class toward a global expected return for that asset class. Similar adjustments can be made to the variances and covariances, although, as shown above, estimation errors in variances and covariances do not degrade portfolio performance as much as errors in means.

Chopra, Hensel, and Turner [1993] illustrate the improvement in portfolio performance to be gained from using Stein estimators. They use monthly observations from January 1980 through December 1990 for a sample of sixteen international assets: six equity indexes, five fixed-income indexes, and five cash indexes. A rolling sixty-month period is used to estimate mean returns and the covariance matrix.

The performance of a mean-variance-optimal portfolio based on the original means is compared with one in which the means are replaced by Stein estimates. Each equity (bond) mean return is replaced by the average of individual equity (bond) means. This represents complete shrinkage to the global mean, an extreme form of Stein estimation. An additional justification for the adjustment is that equity (bond) means are not statistically different across countries.

The authors find that, over time, the mean-variance portfolio with Stein adjusted inputs outperformed the portfolio with unadjusted inputs. The portfolio with Stein inputs had a higher return, lower risk, and lower turnover than the one with unadjusted inputs. Portfolio performance improved further when correlations were adjusted in addition to the means.

Bayesian Estimation

A technique similar in principle to James-Stein estimation is Bayesian estimation. This involves combining the prior beliefs about asset characteristics with recent asset performance to come up with better estimates of its characteristics.

Klein and Bawa [1976] discuss the cases of "non-informative" and "informative" priors. A "non-informative" prior means that the investor has no beliefs about the asset before observing its behavior. If more data are available for some assets than for others, however, the investor may have more faith in some parameter estimates. This would be a case of an "informative" prior.

Frost and Savarino [1986] examine the "all assets are identical" informed prior for individual stocks. The informative prior that all securities have identical expected returns, variances, and pairwise correlation coefficients is used to draw the parameter estimates for individual securities toward the average parameter estimates for all securities. The amount that each parameter is drawn toward its grand mean depends upon the degree to which the sample was consistent with the informative prior. In a simulation, this empirical Bayes method selects portfolios that dominate other portfolios based on a non-informative prior and historical sample estimates.

Summary of Improvements

Of the three techniques, constraining asset weights is the easiest to understand and implement. Stein estimation is somewhat more complex, as it requires decisions on what assets are similar and what shrinkage factor to use to draw individual parameters toward global parameters. Bayesian estimation is theoretically more complex than Stein estimation and is therefore more difficult to implement.

CONCLUSION

Mean-variance is a widely accepted and easy-to-use framework for allocating wealth among assets. It permits customization of portfolios to suit individual investment objectives. Yet two pitfalls are that 1) portfolios that are very close to the optimal portfolio in terms of risk and expected return can be very different in composition, and 2) the composition of the optimal portfolio is very sensitive to small changes in the input parameters. As Michaud [1989] points out, "The fundamental problem is that the level of mathematical sophistication of the optimization algorithm is far greater than the level of information in the input forecasts."

Of the three techniques that can be used to modify the inputs or adjust the optimization procedure to minimize the impact of estimation errors, constraining asset weights is the easiest to implement. Tests of this technique in a three-asset framework show that the optimal portfolio with sensible constraints has a higher mean return and lower risk than an unconstrained optimal portfolio. When used in conjunction with one of the three techniques, mean-variance is still a powerful tool.

ENDNOTES

[1]Mathematically, the problem can be stated as

$$\text{maximize} \sum_{i=1}^{n} E[r_i]x_i - \frac{1}{t}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j E[\sigma_{ij}]$$

where E $[r_i]$ is the expected return, x_i is the proportion of the portfolio in asset i, t is the risk tolerance of the investor, and $E[\sigma_{ij}]$ is the predicted covariance. See Markowitz [1987] for a thorough review of mean-variance optimization.

[2]Stock and bond data are provided by Ibbotson Associates. The T-bill data are provided by Salomon Brothers.

[3]The risk tolerance measures the investor's desired tradeoff between extra return and extra risk (variance). The greater the risk tolerance, the more risk an investor is willing to take for a little extra return. Under fairly general input assumptions, a risk tolerance of 50 describes the typical portfolio allocations of large U.S. pension funds. Risk tolerances of most major U.S. pension plans lie between 40 and 60. Risk tolerances of 25, 50, and 75 characterize extremely conservative, moderate, and extremely aggressive investors, respectively.

[4]The turnover is equal to the dollar amount of assets purchased (or sold) as a percentage of the total portfolio value. For example, if the mean return for stocks is 1% per month, we increase it to 1.05%. If construction of the new optimal portfolio involves moving 15% of the total wealth from bonds to stocks, the turnover is 15%.

[5]With three assets and three different values of each mean (unchanged, or up or down by 5%), there are twenty-seven different combinations of means.

[6]Statistically, the probability that the true mean returns will deviate less than 5% from the sample mean return is only 1.5%. This probability is based on the assumption that the assets have a trivariate normal distribution. For a large enough sample from a multivariate normal distribution, the sample means are themselves normally distributed with a covariance matrix equal to the sample covariance matrix divided by the number of observations. Using the parameter estimates from Exhibit 1 and the density function for a trivariate normal distribution, we integrate numerically between the appropriate limits to compute the probability.

[7]These probabilities are valid only if the data come from a trivariate normal distribution. While the univariate distributions of stock and bond returns appear to be bell-shaped, this is not true of the distribution of T-bill returns. Although the probabilities will not be exact in this case, they should be reasonable approximations.

[8]The parameters of a stationary distribution — for example, means, variances, and covariances — do not change over time. Non-stationarity occurs when the parameters or the distribution itself change over time. Hsu [1984] provides evidence that the behavior of stock returns is evolutionary rather than stationary. His conclusion about stock market risk is that "like a barometer, it reflects the general investment climate and the influence of special political-economic events."

[9]Each variance is either left unchanged, or increased or decreased by k% from its base level, and the turnover computed. k is varied from 5 through 50 in steps of 5. Because there are three variances, and each can take one of three values, there are twenty-seven distinct combinations of variances.

[10]The cash equivalent of a risky portfolio is the certain amount of cash that provides the same utility as the risky portfolio.

[11]Thus, if the optimal portfolio has an expected return of 1% per month and a standard deviation of 5%, portfolios with expected returns between 0.90% and 1.10% and standard deviations between 4.50% and 5.50% are considered near-optimal portfolios. These portfolios lie within a 10% window (in expected return-standard deviation space) around the optimal portfolio.

[12]Since the near-optimal portfolios considered here have integer weights, the weights of the optimal base portfolio are rounded off to the nearest integer to compute the turnover.

[13]For example, the portfolio to be held over January 1985 is computed on the basis of the sixty-month period January 1980 through December 1984. At any point, the optimal portfolio is based solely on information available prior to that point.

[14]The weight in equity can vary between 40% and 80%, the weight in bonds can vary between 20% and 60%, and the weight in cash can vary between 0% and 20%.

[15]Tighter constraints around the 60-40-0 benchmark led to a better performance over our sample period. This does not imply that tighter constraints will always lead to better performance. The important point is that any reasonable constraints are preferable to no constraints.

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