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Source: *Journal of Accounting Research*, Autumn, 1992, Vol. 30, No. 2 (Autumn, 1992), pp. 185-209

Published by: Wiley on behalf of Accounting Research Center, Booth School of Business, University of Chicago

Stable URL: https://www.jstor.org/stable/2491123

Journal of Accounting Research
Vol. 30 No. 2 Autumn 1992
*Printed in U.S.A.*

# A Nonlinear Model of Security Price Responses to Unexpected Earnings

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## 1. Introduction

This study presents evidence that the marginal response of stock price to unexpected earnings *declines* as the absolute magnitude of unexpected earnings increases. Most previous studies assume a linear relation between unexpected returns (*UR*) and unexpected earnings (*UE*). The *constant* marginal response of prices to earnings in linear models is typically referred to as the earnings response coefficient (*ERC*) and estimated as the slope coefficient from simple linear regression of *UR* on *UE*. Relative to the linear model, a nonlinear approach provides both significantly higher explanatory power and a richer explanation for differences between *ERC*s and price–earnings ratios.

The nonlinear relation described in this paper rests on the premise that the absolute value of unexpected earnings is negatively correlated with earnings persistence. Valuation theory suggests that analysts and investors should place greater emphasis on forecasting high-persistence earnings than low-persistence earnings, because a given amount of the former has a greater valuation impact than the same amount of the

*University of Texas at Austin. We wish to acknowledge helpful suggestions from Rowland Atiase, Linda Bamber, Anne Clem, Dick Dietrich, Ross Jennings, Robert Lipe, Chris Olsen, Byung Ro, Richard Sansing, Karen Smith, Reed Smith, Bob Thompson, workshop participants at the University of Southern California, and an anonymous referee. We also wish to thank Jim Ohlson for valuable discussions and Dave Mest for programming assistance. Analysts' forecasts are from *I/B/E/S* Inc. Project funding was provided by Arthur Andersen & Co. and the Graduate School of Business at the University of Texas.

latter.[1] If additional forecasting effort increases accuracy, forecasts of high-persistence earnings components will be more accurate than those of low-persistence components. As a result, both the average persistence of unexpected earnings components and the related price response will decrease as the absolute magnitude of unexpected earnings increases.[2]

The role of persistence can be illustrated by comparing the values of an unexpected dollar of permanent earnings and transitory earnings. At a 10% discount rate, an unexpected dollar of permanent earnings produces an $11 price reaction: $1 of current earnings plus the present value of an infinite $1 stream of future earnings. In contrast, an unexpected dollar of transitory earnings changes firm value by only $1. In general, valuation models predict that transitory earnings surprises have an empirical *ERC* of one, while the *ERC* of permanent earnings is one plus the inverse of the discount rate. Accordingly, we assume that analysts and investors are relatively uninterested in transitory earnings because the trading profits that could be earned from private foreknowledge of a dollar of transitory earnings are smaller than the profits from private foreknowledge of a dollar of permanent earnings; therefore, the ratio of transitory to permanent components increases as the forecast error increases.

Price-irrelevant *UE* components may also increase with unexpected earnings. For example, investors may be aware of the difference between an asset's book value and its market value before it is sold. The greater this difference, the greater the incentive to discover it before accounting recognition of the gain or loss. Thus, gains and losses from asset sales or liability repayments may be included in the researcher's proxy for unexpected earnings but have no effect on security prices.[3]

In sum, transitory earnings surprises should have less impact on security prices than permanent earnings surprises. If permanent earnings are more accurately forecast (on average) than transitory earnings, then transitory earnings surprises will be concentrated in the tails of the *UE* distribution and the *UR–UE* relation will be nonlinear. Specifically, we predict that the earnings–return relation is *S*-shaped; i.e., convex for bad news and concave for good news. We obtain substantially higher explanatory power for a nonlinear *S*-shaped model than for the traditional linear model. The nonlinearity is consistently observed over time and over different functional forms (which permit, but do not require,

[1] See, for example, Miller and Rock [1985], Kormendi and Lipe [1987], and Ohlson [1991].

[2] To the extent that low-persistence components are more susceptible to earnings management, they may be more difficult to forecast, an effect which would reinforce the nonlinearity we posit.

[3] Price-irrelevant earnings create measurement error in the researcher's proxy for "true" (valuation-relevant) unexpected earnings. Beaver, Lambert, and Morse [1980], among others, discuss the effects of measurement errors on linear-model *ERC*s.

*S* shapes) and is robust across different proxies for the independent variable.[4]

In addition to improvements in statistical fit, the nonlinear model offers a new perspective on the difference between linear-model *ERC*s and price–earnings ratios (*PE*s). Our explanation for this difference is that greater weights are placed on transitory and price-irrelevant earnings in linear-model *ERC* estimators than in *PE* calculations. Specifically, we find that small earnings surprises are valued as permanent shifts, and that approximately 80% of earnings surprises are capitalized at a rate greater than 1:1. The largest absolute earnings surprises have marginal responses less than one, which suggests that there are substantial price-irrelevant components in the marginal dollar of the largest earnings surprises.

We also investigate the relation between marginal price responses and unexpected earnings after controlling for characteristics found in previous research to be associated with cross-sectional differences in *ERC*s, including the ratio of market value to book value of common equity and time-series estimates of earnings persistence, earnings predictability, and systematic risk.[5] Our results indicate that the explanatory power of the absolute value of *UE* for marginal responses is undiminished after controlling for these characteristics. In particular, our model allows marginal responses to vary over time for each firm because investors may assign each earnings surprise a unique persistence measure that depends on the absolute magnitude of the surprise. We estimate the "average" marginal price response for a cross-section of equal earnings surprises. In contrast, most prior studies estimate firm-specific "average" persistence from the time-series behavior of earnings and firm-specific "average" *ERC*s from linear regressions. Our results indicate that *ERC*s are more sensitive to forecast error magnitude than to firm-specific average persistence. We conclude that perceived persistence is more closely related to the magnitude of the earnings forecast error than to time-series estimates of firm-specific persistence.

Our results have implications for research on cross-sectional and time-series differences in *ERC*s. A linear model could yield significant explanatory power for variables which merely stratify firms by earnings forecast error magnitude. Recent research includes Dhaliwal, Lee, and Fargher's [1991] examination of *ERC*s for high- versus low-leverage firms and Collins and DeAngelo's [1990] investigation of *ERC*s before and after proxy contests. If either characteristic is associated with

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[4] Additional support for an *S*-shaped relation is reported by Das and Lev [1991] for annual unexpected returns and annual earnings changes and by Jennings, Mest, and Thompson [1992] for annual unexpected returns and several components of annual unexpected earnings. Specification tests in Cheng, Hopwood, and McKeown [1992] reject the linear functional form for the *UR–UE* relation. Beneish and Harvey's [1991] analysis of time-series regressions also indicates nonlinearities in the earnings–return relation.

[5] See, for example, Kormendi and Lipe [1987], Collins and Kothari [1989], Easton and Zmijewski [1989], and Lipe [1990].

forecast error magnitude, their research may reveal more about the time series of earnings or earnings management than about the market's response to earnings announcements. The potential research problems precipitated by the nonlinearity described here could be mitigated by using linear regressions after discarding large $|UE|$ observations, estimating a nonlinear model, or matching treatment and control firms on unexpected earnings.

Section 2 describes the data and research method. Primary tests of the nonlinear relation are reported in section 3, while section 4 considers the effects of systematic risk, market-to-book value of common equity, and two time-series properties of earnings: average earnings persistence and average earnings predictability. Section 5 summarizes our results and their research implications.

## 2. *Research Design*

### 2.1 SAMPLE SELECTION

Cross-sectional regressions are run on quarterly samples of at least 500 firms with the following data on the 1988 *Compustat* quarterly industrial file, the 1988 *CRSP* daily returns file, and the June 1989 *IBES* tape: (1) *Compustat*—earnings announcement date for the current and previous quarters, price per share at the end of the previous quarter, and primary earnings per share and per-share adjustment factor for the current quarter and the same fiscal quarter of the preceding year; (2) *CRSP*—daily returns from three days following the preceding quarter's earnings announcement through two days following the current announcement; and (3) *IBES*—actual and median analyst forecast *EPS* in the last month of the fiscal quarter. The pooled sample contains 12,381 observations from 1984:3 through 1987:3.

Our independent variable is a financial analyst forecast error:

$$UE_{iq} = (E_{iq} - F_{iq}) \;/\; P_{iq-1},$$

where $E_{iq}$ is actual quarterly earnings per share reported by *IBES* for firm $i$ in quarter $q$, $F_{iq}$ is the median *IBES* analyst forecast in the last month of firm $i$'s fiscal quarter $q$, and $P_{iq-1}$ is the price per share of firm $i$'s common stock on the last day of quarter $q$–1.[6]

[6] The price at the beginning of the current fiscal quarter ($P_{iq-1}$) is selected as the deflator because it precedes the cumulative abnormal return window (the dependent variable). Reported results are not qualitatively different from results obtained with $P_{iq}$ as the deflator. The forecast in the last month of the fiscal quarter ($F_{iq}$) is selected because it follows the announcement of the previous quarter's earnings, for most firm-quarters, and it precedes the announcement of the current quarter's earnings, for all firm-quarters. For 14 observations, the absolute per-share forecast error exceeds the price per share. For five of those observations, *IBES* $E_{iq}$ differs from actual earnings reported by both the *Wall Street Journal* and *Compustat*. Accordingly, we use *Compustat* actual to calculate *UE* in those five instances.

The unexpected return for firm $i$ in quarter $q$ (denoted $UR_{iq}$) is the sum of daily abnormal returns for the period beginning two days after the previous quarterly earnings announcement and ending one day after the current announcement. The abnormal return for firm $i$ on day $t$ is the firm's day-$t$ return less the day-$t$ equally weighted mean return on a *CRSP* beta-matched portfolio.

We select long windows because we do not have the exact dates of analysts' forecasts. We assume that analysts update their forecasts of quarter $q$'s earnings after quarter $q - 1$'s earnings are announced and before quarter $q$'s earnings are announced. If our window is longer than necessary to capture the market's reaction to *UE*, our $R^2$s will be depressed.[7] This cost seems acceptable, since to err in the other direction (a short return period that excludes a portion of the market reaction) would bias *ERC*s downward. To ensure that the *S*-curve is also descriptive of short-window returns, we replicate all tests with unexpected returns for the four-day period beginning two days before the current announcement. We observe an *S*-shaped relation for this window as well. Linear-model *ERC*s for four-day returns are about half the *ERC*s for quarterly returns, which suggests that the four-day window is too short to capture all of the price reaction to our measure of *UE*.

We also run the same tests (not reported) on an expanded sample of 40,728 observations that meet criteria (1) and (2). For this sample, earnings expectations are based on a seasonal random walk (*SRW*) in primary *EPS* before extraordinary items and discontinued operations.[8] The *SRW* results also support an *S*-shaped relation between *UR* and *UE*.

## 2.2 REGRESSION MODEL AND HYPOTHESIS

The estimator $\hat{\gamma}_1$ from the following simple linear regression illustrates the relation between *ERC*s and the absolute magnitude of unexpected earnings:

$$UR_i = \gamma_0 + \gamma_1 \; UE_i + error_i. \tag{1}$$

[7] The mean quarterly Spearman correlation between *UR* and *UE* is .2722 for our sample (in which *UR* is accumulated from the previous announcement through the current announcement) and is .167 in Philbrick and Ricks' [1991] sample (for *UR* accumulated over the three days centered on the announcement day). This difference could also be due to the difference in the accuracies of earnings forecasts. The mean absolute forecast accuracy in our sample of 12,381 observations from 13 quarters is .0159, almost twice as large as the .0087 mean absolute forecast accuracy reported by Philbrick and Ricks [1991] for 4,770 observations from 10 quarters.

[8] Brooks and Buckmaster [1976] and Freeman, Ohlson, and Penman [1982] report that the persistence of *SRW* innovations ($\Delta E$) declines with the absolute value of $\Delta E$. An *S*-shaped relation between *UR* and $\Delta E$ is consistent with Beaver, Clarke, and Wright's [1979] analysis of annual *UR* and annual $\Delta E$ for 25 portfolios ranked on $\Delta E$; for example, the ratio of mean *UR* to mean $\Delta E$ decreases monotonically from .709 to .111 for their six worst-news portfolios and from .443 to .158 for their six best-news portfolios.

If the dependent and independent variables are scaled to have means of zero, the *OLS* slope coefficient is:

$$\hat{\gamma}_1 = [\Sigma_i\, (UR_i\; UE_i)] \;/\; [\Sigma_i\, (UE_i)^2]$$
$$= \Sigma_i\, w_i\, (UR_i / UE_i), \qquad (2)$$

where the weights $w_i = UE_i^{\,2} / \Sigma_i\, (UE_i)^2$ and $\Sigma_i\, w_i = 1$.

Equation (2) indicates that $\hat{\gamma}_1$ is a weighted average price response per unit of unexpected earnings, where weights increase at an increasing rate with the absolute value of *UE*. Accordingly, an *ERC* from a linear model is not necessarily a good measure of the price response to small earnings surprises. If earnings persistence declines as the magnitude of unexpected earnings increases, *ERC*s from linear regressions would predominantly reflect the effects of transitory, rather than permanent, earnings. We argue that *ERC*s from traditional linear regressions underestimate the value of permanent earnings surprises because a linear model heavily weights high-magnitude, low-value transitory earnings at the expense of low-magnitude, high-value permanent earnings. In contrast to prior empirical research, our nonlinear model allows marginal price responses to decline as the absolute value of unexpected earnings increases.

Our hypothesis calls for a model with a positive first derivative for all values of *UE*, a positive second derivative when *UE* is negative, and a negative second derivative when *UE* is positive. A parsimonious model that meets all three criteria is the inverse tangent (arctan):

$$UR_i = a_0 + a_1 \arctan\,(a_2\; UE_i) + error_i, \qquad (3)$$

where $a_0$, $a_1$, and $a_2$ are the regression parameters.[9] For equation (3), the first derivative of unexpected returns with respect to unexpected earnings is $a_1 a_2 / (1 + a_2^2\, UE^2)$. The maximum marginal response of $a_1 a_2$ occurs as *UE* approaches zero; therefore, we expect to find $\hat{a}_1 \hat{a}_2 > 0$. In addition, if $\hat{a}_1 \hat{a}_2 > 0$, equation (3) is convex for negative *UE* and concave for positive *UE*.

Figure 1 graphs price responses predicted by three hypothetical versions of equation (3). Unexpected returns are .03 arctan(400 *UE*) for the solid line, .04 arctan(300 *UE*) for the long-dashed line, and .10 arctan(10 *UE*) for the short-dashed line. In our sample, 76% of the observations fall between ±1% of firm value, and 91% of our sample observations fall within 3% of firm value, which is the *UE* range depicted in figure 1.

[9] Results (not reported) for piecewise-linear and modified-quadratic models and the following "logistic" model:

$$UR = b_0 + b_1 \;/\; [1 + \exp(b_2\; UE)],$$

are similar to the arctan results reported in section 3.

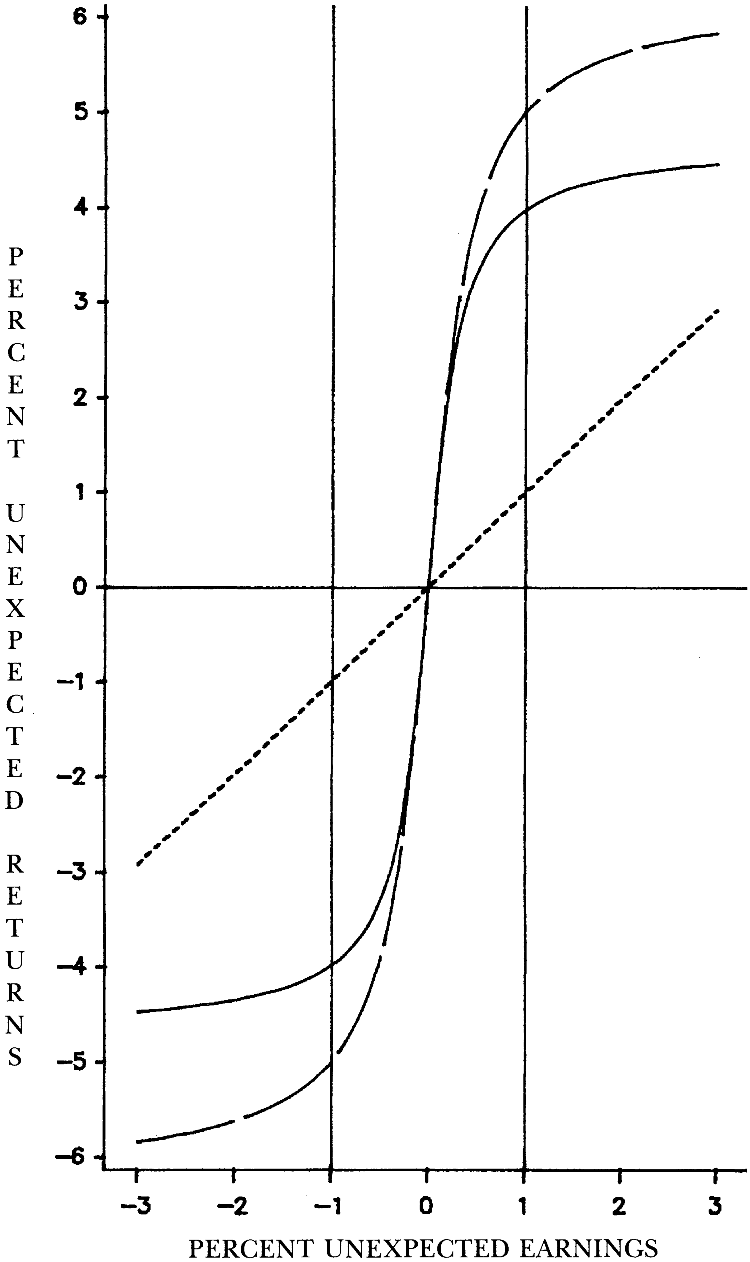


FIG. 1.—Hypothetical nonlinear functions relating unexpected returns (*UR*) to price-deflated unexpected earnings (*UE*). Unexpected returns are .04 arctan(300 *UE*) for the long-dashed line, .03 arctan(400 *UE*) for the solid line, and .10 arctan(10 *UE*) for the short-dashed line.

For the solid line, the marginal response approaches 12 (.03 × 400) as unexpected earnings approach zero; and $\partial UR/\partial UE = 1$ when unexpected earnings are .83% of firm value. At a discount rate of .09, a permanent earnings innovation would have a marginal response of approximately 12 (1 + 1/.09). Transitory earnings have an *ERC* of one regardless of the discount rate. Accordingly, the solid line in figure 1 models very small earnings surprises as permanent earnings at a .09 discount rate. When unexpected earnings reach .83% of firm value, figure 1 treats the marginal dollar of earnings as transitory. For $|UE| > .83\%$, marginal price responses are less than one, which would imply that a significant portion of these large earnings surprises are price-irrelevant.

Comparison of the long-dashed line with the solid line in figure 1 illustrates how varying $a_1$ and $a_2$ affects price responses. These two lines have equal marginal responses of 12 at $UE = 0$ because the product of $a_1$ and $a_2$ is 12 in both functions. The solid line employs a greater value of $a_2$ and has a narrower range of relatively large price responses than the long-dashed line. At $UE = 0$, $\partial UR/\partial UE = a_1 a_2$ and both coefficients have 1:1 effects on the marginal response. For $|UE| > 0$ and $a_2 > 1$, an increase in $a_2$ causes more growth in the marginal response function's denominator ( $1 + a_2^2 UE^2$) than in its numerator ($a_1 a_2$); but a given percentage change in $a_1$ continues to effect an equal percentage change in marginal responses. The greater the value of $a_2$, the more quickly the denominator of $\partial UR/\partial UE$ increases as $|UE|$ increases; therefore, the departure from linearity (i.e., the decrease in $\partial UR/\partial UE$ as $|UE|$ increases) is controlled by $a_2$.

The short dashes demonstrate that regression results for equation (3) could approximate results from equation (1) if the data are consistent with linearity. For absolute values of unexpected earnings less than 3% of firm value, the short-dashed line looks like a linear model with $ERC = 1$; the marginal response is 1.00 at $UE = 0$, .99 at $UE = .01$, and .92 at $UE = .03$.

## 3. *Results: Evidence of Nonlinearity*

Our hypothesis is that the explanatory power of the linear model (equation (1)) is less than that of the arctan model (equation (3)), and we test it by running cross-sectional regressions for each model in 13 fiscal quarters. We also compare the explanatory power of each model (estimated over the entire *UE* range) within segments of the $|UE|$ domain to determine *where* the explanatory powers of the models differ.

### 3.1 REGRESSION RESULTS

Regression results for the linear and nonlinear models are summarized in table 1. Estimated coefficients and adjusted $R^2$s for the 13 cross-sectional samples are reported in panel A. Coefficients for equation (3) are estimated by the Gauss-Newton iterative method (see

Judge et al. [1985]). The objective of this method is to locate the value of $a_2$ that maximizes the explained sum of squares from the following linear regression:

$$UR = a_0 + a_1 \, X + error,$$

where $X \equiv \arctan(a_2 \, UE)$. Our parameter estimates are not sensitive to starting values (except that when the initial value of $a_2$ is less than zero, both $\hat{a}_1$ and $\hat{a}_2$ are negative). We report only positive values of $\hat{a}_1$ and $\hat{a}_2$ because $\hat{a}_1$ and $\hat{a}_2$ have the same sign in each regression and because:

$$a_1 \arctan(a_2 \, UE) \equiv -a_1 \arctan(-a_2 \, UE).$$

The adjusted $R^2$ for the nonlinear model is higher than the adjusted $R^2$ for the linear model in 12 of the 13 quarters, and its average explanatory power of about 8.0% is almost four times as great as the average explanatory power of the linear model (2.1%). For the nonlinear model, estimated marginal responses are positive in every quarter (i.e., $\hat{a}_1 \hat{a}_2 > 0$), and both $\hat{a}_1$ and $\hat{a}_2$ are significantly positive (at the .05 level for a one-tailed test) in ten of the regressions. Linear-model *ERC*s are positive and significant (at the .05 level) in nine quarters, positive but insignificant in two quarters, and negative in two quarters. Linear-model results (not reported) improve in all quarters when we exclude the 346 observations with $|UE| > .10$ (less than 3% of the sample); all 13 slope coefficients become significantly positive (at the .01 level); the range is 1.02 to 2.67.

To determine whether the *S*-shape is driven by a few high-magnitude observations, panel B of table 1 compares results from equations (1) and (3) for a series of pooled samples in which progressively less extreme observations are eliminated. The first pair of regressions in panel B includes all the observations in panel A. Coefficients for the pooled regression are similar to mean coefficients from the cross-sectional regressions. The linear model's *ERC* is .11 in panel B (compared to a mean of .28 for panel A); the nonlinear model's marginal price response at $UE = 0$ is 12.5 in the pooled regression and averages 14.0 in the cross-sectional regressions. As in the cross-sectional samples, the nonlinear model has higher explanatory power than the linear model for the pooled sample.

The next three lines of panel B report results from pooled regressions on samples that omit unexpected earnings with absolute values greater than .10, .05, and .01, respectively. For the linear model, *ERC*s increase monotonically as the sample is restricted to smaller absolute values of unexpected earnings: from $ERC = 1.66$ for the 12,035 observations with $|UE| \leq .10$ to $ERC = 6.89$ for the 9,363 observations with $|UE| \leq .01$. Consistent with an *S*-shaped relation, *ERC*s from linear regressions increase as high-magnitude observations are discarded. In contrast, excluding a few high-magnitude $|UE|$ observations has little effect on the nonlinear model's parameters. A comparison of $\hat{a}_1$ and $\hat{a}_2$ in the

**TABLE 1**

*Linear and Nonlinear Regression Results**

| Sample | Number of Observations | Linear Model (equation (1)): $UR_i = \gamma_0 + \gamma_1 UE_i + error_i$ | | | Nonlinear Model (equation (3)): $UR_i = a_0 + a_1 \arctan(a_2 UE_i) + error_i$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | Adj. $R^2$ | $\hat{a}_0$ | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{a}_1 \times \hat{a}_2$ | Adj. $R^2$ |
| **Panel A: Cross-Sectional Regressions** | | | | | | | | | |
| 1984:3 | 787 | .015[a] | .155 | .001 | .016[a] | .020[a] | 428.8 | 8.6 | .027 |
| 1984:4 | 859 | -.001 | .027 | .000 | .007[c] | .040[a] | 388.2[a] | 15.5 | .114 |
| 1985:1 | 944 | .016[a] | .345[a] | .027 | .027[a] | .049[a] | 310.6[a] | 15.2 | .168 |
| 1985:2 | 1,012 | .007[c] | .373[a] | .022 | .017[a] | .049[a] | 205.1[a] | 10.1 | .094 |
| 1985:3 | 1,017 | -.002 | .213[a] | .015 | .007[c] | .033[a] | 243.8[b] | 8.1 | .057 |
| 1985:4 | 950 | .001 | .526[a] | .062 | .003 | .044[a] | 216.9[b] | 9.5 | .065 |
| 1986:1 | 946 | -.007[c] | .516[a] | .029 | .001 | .052[a] | 248.1[a] | 12.9 | .121 |
| 1986:2 | 1,013 | .001 | .128[b] | .004 | .010[b] | .056[a] | 294.4[a] | 16.5 | .089 |
| 1986:3 | 1,065 | -.004 | -.162 | .002 | .002 | .032[a] | 354.5[b] | 11.3 | .045 |
| 1986:4 | 1,005 | .002 | .080[b] | .004 | .012[a] | .038[a] | 360.4[b] | 13.7 | .068 |
| 1987:1 | 985 | -.013[a] | .675[a] | .018 | -.013[a] | .032[a] | 341.1[c] | 10.9 | .037 |
| 1987:2 | 932 | .001 | -.353 | .042 | .007 | .026[a] | 1315.4 | 34.2 | .042 |
| 1987:3 | 866 | .011 | .830[a] | .042 | .015[a] | .075[a] | 199.9[a] | 15.0 | .106 |
| Mean | | .002 | .283 | .021 | .009 | .042 | 377.5 | 14.0 | .080 |

**TABLE 1**—*continued*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Panel B: Pooled Cross-Sectional Time-Series Regressions** | | | | | | | | | |
| $\|UE\| < \infty$ | 12,381 | .001 | .11[a] | .004 | .008[a] | .040[a] | 310.1[a] | 12.5 | .072 |
| $\|UE\| \leq .1$ | 12,035 | .006[a] | 1.66[a] | .039 | .009[a] | .042[a] | 284.8[a] | 12.0 | .075 |
| $\|UE\| \leq .05$ | 11,680 | .008[a] | 2.83[a] | .052 | .009[a] | .042[a] | 280.1[a] | 11.9 | .075 |
| $\|UE\| \leq .01$ | 9,363 | .012[a] | 6.89[a] | .049 | .012[a] | .036[a] | 384.1[a] | 13.7 | .053 |
| $\|UE\| \leq .005$ | 7,361 | .012[a] | 10.09[a] | .038 | .012[a] | 10.08[d] | 1.0[d] | 10.1[d] | .038 |
| $\|UE\| \leq .001$ | 2,768 | .016[a] | 17.22[a] | .007 | .016[a] | 17.22[d] | 1.0[d] | 17.2[d] | .007 |

*This table reports linear and nonlinear regression results for 13 quarterly cross-sectional samples (panel A) and for 6 pooled cross-sectional time-series samples (panel B).

Variable definitions:

$UR_i$: Unexpected return for firm $i$ is the abnormal return beginning two days after the end of the previous quarterly earnings announcement and ending one day after the announcement of earnings for the current fiscal quarter, which is designated in the first column of panel A.

$UE_i$: Unexpected earnings for firm $i$ in quarter $q$ are announced earnings per share minus the median *IBES* forecast in the last month of the quarter, divided by the per-share price at the beginning of the quarter.

Estimated marginal price response ($\partial UR/\partial UE$) is $\hat{\gamma}_1$ for the linear model and $\hat{a}_1\hat{a}_2 / [1 + (\hat{a}_2 UE)^2]$ for the nonlinear model.

[a,b,c]Significance levels of $a = .01$, $b = .05$, $c = .1$ indicate two-tailed tests for intercepts and one-tailed tests for other regression coefficients, which are predicted to be greater than zero.

[d]Estimated coefficients depend on preselected initial values. Since the $UR$–$UE$ relation is approximately linear, the solution algorithm stops its search with the first $(a_1, a_2)$ pair with predicted values of $UR$ for equation (3) near the predicted values of $UR$ from equation (1). Reported results are for initial values of $(a_1, a_2) = (\hat{\gamma}_1, 1.0)$. Estimated coefficients for other regressions in panel B and for all regressions in panel A are not sensitive to initial values.

first and fourth rows of panel B indicates that regression coefficients from the 76% of the observations with lowest $|UE|$ magnitude are similar to the coefficients from the full sample. For example, the marginal response at $UE = 0$ is 12.5 in the full sample and is 13.7 in the sample trimmed to observations with $|UE| \leq .01$. In general, the first four nonlinear regressions in panel B provide comparable marginal responses at all values of unexpected earnings—e.g., at $UE = .01$, $\partial UR/\partial UE$ is 1.2 in the full sample and .9 in the sample restricted to $|UE| \leq .01$.

The last two lines of panel B report regression results for samples that exclude $|UE|$ greater than .005 and .001, respectively. Nonlinear parameters are not unique for these two samples. In each case, the solution algorithm stops its search at the first ($a_1$, $a_2$) pair with predicted *UR* sufficiently close to the predictions from the linear regression over the same sample. Specification tests (discussed below) indicate that predictions from the linear model and the arctan model are not significantly different when the *UR–UE* relation is estimated on $|UE|$ trimmed to .5% of firm value (59% of the sample). In general, we would not expect to reject the linear model when observations are restricted to any narrow segment of *UE*—i.e., we would expect the *UR–UE* relation to be approximately linear for any *UE* neighborhood but not for the entire *UE* population.

Panel B of table 1 also reports the adjusted $R^2$ for each regression. For the linear model, adjusted $R^2$s increase from .4 to 3.9% when the 346 observations with $|UE| > .1$ are eliminated. That is, eliminating observations from the top-three $|UE|$ percentiles increases $R^2$ by 875%, but further deletions increase explanatory power at a much lower rate. For the nonlinear model, the full sample and the first two trimmed samples have adjusted $R^2$s of approximately .07. For the two most drastically trimmed samples, the linear and nonlinear models have equal $R^2$s. For both models, trimming beyond the .05 $|UE|$ cutoff lowers $R^2$. For example, the explanatory power of the nonlinear model drops 90% from the full sample to the sample with $|UE| \leq .001$. For a well-specified model, we would expect *UE* to explain less of the variation in *UR* as the sample is restricted to smaller earnings surprises.

Specification tests are reported in panel A of table 2. The Davidson and MacKinnon [1981] *J* statistic pits nonnested alternative specifications against each other (1) by posing the linear model as the null hypothesis against a specific alternative function, the arctan function, and (2) by setting the arctan model as the null against the linear function. If the correct specification falls between the two alternatives, the Davidson-MacKinnon *J* test may reject both hypotheses. The column headed "$H_0$: Linear" reports that the linear model is rejected relative to the arctan model (at the .01 level or better) in each of the 13 quarters. The column headed "$H_0$: Nonlinear" indicates that the arctan model is rejected relative to the linear model at the .01 level in three quarters and at the .05 level in one additional quarter. We repeat the

**TABLE 2**

*Specification Tests for Linear and Nonlinear Regressions**

| Sample | Davidson-MacKinnon $J$ Test | | Wilcoxon Signed-Rank $t$ Test for: | |
|---|---|---|---|---|
| | $H_0$: Linear $H_a$: Nonlinear | $H_0$: Nonlinear $H_a$: Linear | Squared Errors | Absolute Errors |
| **Panel A: Cross-Sectional Regressions** | | | | |
| 1984:3 | 4.78 | −1.38 | 2.04 | 2.02 |
| 1984:4 | 10.44 | −.82 | 3.22 | 2.09 |
| 1985:1 | 11.87 | .45 | 6.04 | 5.97 |
| 1985:2 | 8.82 | .73 | 3.97 | 2.98 |
| 1985:3 | 6.61 | 1.56 | 3.75 | 2.96 |
| 1985:4 | 5.18 | 5.71 | 2.72 | 2.53 |
| 1986:1 | 9.61 | .68 | 4.66 | 3.97 |
| 1986:2 | 9.70 | −.78 | 5.12 | 4.86 |
| 1986:3 | 8.41 | 4.91 | 2.06 | 1.32 |
| 1986:4 | 8.09 | .64 | 3.22 | 2.50 |
| 1987:1 | 4.82 | 1.74 | 3.34 | 3.02 |
| 1987:2 | 8.59 | 9.01 | 4.50 | 3.90 |
| 1987:3 | 8.39 | 1.56 | 4.32 | 4.32 |
| **Panel B: Pooled Cross-Sectional Time-Series Regressions** | | | | |
| $\|UE\| < \infty$ | 29.69 | 1.76 | 13.42 | 11.81 |
| $\|UE\| \leq .1$ | 21.19 | .56 | 9.35 | 7.92 |
| $\|UE\| \leq .05$ | 16.30 | .87 | 7.30 | 6.26 |
| $\|UE\| \leq .01$ | 6.24 | .58 | 2.82 | 2.08 |
| $\|UE\| \leq .005$ | —[a] | — | — | — |
| $\|UE\| \leq .001$ | — | — | — | — |

*This table reports parametric and nonparametric specification tests for the linear versus nonlinear regressions in table 1. Regression models and variables are described in table 1.

The Davidson-MacKinnon $J$ test is a two-part test: (1) $J > 2.33$ (1.65) in the second column of the table indicates that the linear relation is rejected at the .01 (.05) level when the alternative hypothesis is $UR$ predictions from equation (3); and (2) $J > 2.33$ (1.65) in the third column of the table indicates that the nonlinear relation is rejected at the .01 (.05) level when the alternative hypothesis is $UR$ predictions from equation (1).

The Wilcoxon signed-rank tests are computed from the ranks of the following differences in squared and absolute residuals, respectively:

$$(UR_i - \hat{\gamma}_0 - \hat{\gamma}_1 UE_i)^2 - [UR_i - \hat{a}_0 - \hat{a}_1 \arctan(\hat{a}_2 UE_i)]^2$$

and:

$$|(UR_i - \hat{\gamma}_0 - \hat{\gamma}_1 UE_i)| - |UR_i - \hat{a}_0 - \hat{a}_1 \arctan(\hat{a}_2 UE_i)|.$$

Positive $t$ statistics indicate that the nonlinear model is more accurate than the linear model. For two-tailed tests, $t$ statistics greater than 2.58 (1.96) are significant at the .01 (.05) level.

[a]A dash (—) indicates that predicted values from the linear and nonlinear models are too close to be distinguished by the test algorithm.

$J$ test on cross-sectional regressions with $|UE| \leq .10$ and reject the linear model in favor of the arctan model in all 13 quarters at the .01 level; we reject the arctan model in favor of the linear model in one quarter at the .01 level and in no additional quarters at the .05 level. Overall, the parametric $J$ test consistently rejects the linear model in favor of the nonlinear model, and rejects the nonlinear model in favor of the linear model much less frequently.

Nonparametric tests of relative explanatory power also support the nonlinear model. Wilcoxon signed-rank statistics for matched pairs of squared errors and absolute errors are reported in the last two columns of table 2. The positive $t$ statistics indicate that squared (absolute) residuals from the linear model are significantly greater than the matched residuals from the nonlinear model in 13 (12) quarters at the .01 level.

Panel B of table 2 reports Davidson-MacKinnon $J$ tests and Wilcoxon tests for the six pairs of pooled regressions in panel B of table 1. The arctan function is a significantly better specification than the linear function for the first four samples. In the samples with $|UE|$ no greater than .005 and .001, there is no significant difference in explanatory power between the two models. In fact, statistical tests are not feasible for the last two samples because their predicted values are too close to be distinguished by algorithms for the $J$ and Wilcoxon tests. In sum, the results in table 2 support our hypothesis that the relation between unexpected returns and unexpected earnings is nonlinear and *S*-shaped, except for small *UE* intervals.

### 3.2 EXPLANATORY POWER WITHIN $|UE|$ SEGMENTS

Table 3 compares the squared residuals from the linear and nonlinear cross-sectional regressions in panel A of table 1. The analysis is based on a measure of pooled explanatory power (*PEP*) for segments of the $|UE|$ domain. For each model, *PEP* is the explained sum of squares divided by the total sum of squares and is calculated for observations meeting the criterion listed in the first column of each row in table 3. For the linear model:

$$PEP(L) = 1 - [\Sigma_q \Sigma_i \, (UR_{iq} - \hat{\gamma}_{0q} - \hat{\gamma}_{1q} \, UE_{iq})^2] \; / \; [\Sigma_q \Sigma_i \, (UR_{iq} - \hat{\gamma}_{0q})^2].$$

For the nonlinear model:

$$PEP(N) = 1 - \{\Sigma_q \Sigma_i \, [UR_{iq} - \hat{a}_{0q} - \hat{a}_{1q} \arctan(\hat{a}_{2q} UE_{iq})]^2\} \; / \; [\Sigma_q \Sigma_i (UR_{iq} - \hat{a}_{0q})^2].$$

The Wilcoxon signed-rank test in the last column is computed from the ranks of the differences in squared residuals—i.e., the ranks of:

$$(UR_{iq} - \hat{\gamma}_{0q} - \hat{\gamma}_{1q} \, UE_{iq})^2 - [UR_{iq} - \hat{a}_{0q} - \hat{a}_{1q} \arctan(\hat{a}_{2q} UE_{iq})]^2.$$

The last row of table 3 reports pooled squared residuals for all 12,381 observations in the 13 quarterly regressions.[10] The positive $t$ statistic in the last column indicates that the squared residuals from the linear

[10] Pooled squared residuals are approximately equal to the mean adjusted $R^2$s from the 13 cross-sectional regressions. In table 1, panel A, the mean $R^2$s are .021 and .080 for the linear and nonlinear models, respectively. The pooled squared residuals reported on the last line of table 3 are .023 and .080, respectively. The pooled squared residuals are based on equally weighted residuals, but the weights on residuals in the mean $R^2$s vary inversely with the number of observations in each cross-sectional regression.

## TABLE 3

*Pooled Explanatory Power of Linear and Nonlinear Cross-Sectional Models for Various Absolute Magnitudes of Unexpected Earnings**

| Range of Unexpected Earnings (*UE*) | Number of Observations (Percentage of Total Sample) | | Pooled Explanatory Power (*PEP*) within *UE* Domain: Linear Model | Pooled Explanatory Power (*PEP*) within *UE* Domain: Nonlinear Model | Ratio of Nonlinear *PEP* to Linear *PEP* | Wilcoxon Signed-Rank *t* |
|---|---|---|---|---|---|---|
| $\lvert UE \rvert \le .001$ | 2,768 | (22%) | .0002 | .0095 | 47.5 | 6.08 |
| $.001 < \lvert UE \rvert \le .005$ | 4,593 | (37%) | .0027 | .0547 | 20.3 | 7.29 |
| $.005 < \lvert UE \rvert \le .01$ | 2,002 | (16%) | .0084 | .1027 | 12.2 | 5.98 |
| $.01 < \lvert UE \rvert \le .05$ | 2,317 | (19%) | .0196 | .1280 | 6.5 | 8.65 |
| $.05 < \lvert UE \rvert \le .1$ | 355 | (3%) | .0338 | .0912 | 2.7 | 2.37 |
| $.1 < \lvert UE \rvert$ | 346 | (3%) | .1484 | .0631 | .4 | .22 |
| All Observations | 12,381 | (100%) | .0225 | .0796 | 3.5 | 13.58 |

*This table compares the explanatory power of the linear ($L$) and nonlinear ($N$) models for the cross-sectional regressions reported in panel A of table 1. The regressions are estimated on untrimmed data, and pooled explanatory power ($PEP$) is reported for $|UE|$ segments described in the first column of each row. Formulas for pooled explanatory power are:

$$PEP(L) = 1 - [\Sigma_q \Sigma_i (UR_{iq} - \hat{\gamma}_{0q} - \hat{\gamma}_{1q}\, UE_{iq})^2] \,/\, [\Sigma_q \Sigma_i (UR_{iq} - \hat{\gamma}_{0q})^2]$$

and:

$$PEP(N) = 1 - \{\Sigma_q \Sigma_i [UR_{iq} - \hat{a}_{0q} - \hat{a}_{1q} \arctan(\hat{a}_{2q} UE_{iq})]^2\} \,/\, [\Sigma_q \Sigma_i (UR_{iq} - \hat{a}_{0q})^2],$$

where $\hat{\gamma}$ and $\hat{a}$ are cross-sectional regression parameters from panel A of table 1, and $UR_{iq}$ and $UE_{iq}$ are defined in table 1.

The Wilcoxon signed-rank test in the last column is computed from the ranks of the following differences in squared residuals:

$$(UR_{iq} - \hat{\gamma}_{0q} - \hat{\gamma}_{1q}\, UE_{iq})^2 - [UR_{iq} - \hat{a}_{0q} - \hat{a}_{1q} \arctan(\hat{a}_{2q} UE_{iq})]^2.$$

Positive $t$ statistics indicate that the nonlinear model is more accurate than the linear model. For two-tailed tests, $t$ statistics greater than 2.58 (1.96) are significant at the .01 (.05) level.

model exceed those from the nonlinear model and that this difference is significant at the .01 level.[11]

Each of the first six rows of table 3 compares pooled explanatory power over a selected range of $|UE|$ magnitude for cross-sectional regressions estimated on untrimmed samples. In all but the most extreme category (about 3% of the sample), the explanatory power of the nonlinear model is significantly greater than the explanatory power of the linear model. On an untrimmed sample, the linear model provides a "good fit" for extreme observations only. Since the weights in the linear model's slope coefficient increase with $|UE|$, the nonlinear model's explanatory advantage over the linear model declines as $|UE|$ increases.

### 3.3 COMPARISON OF MARGINAL RESPONSES AND PRICE–EARNINGS RATIOS

In theory, *ERC*s measure the change in the present value of expected future dividends per dollar of *unexpected* earnings. This definition suggests a link between *ERC*s and price–earnings (*PE*) ratios, which are the present value of future expected dividends per dollar of *current* earnings. Both cross-sectional and time-series studies consistently report *ERC*s well below *PE*s, but reasons for this difference have not been explored from the perspective of a nonlinear earnings–return relation.[12]

In certain well-defined settings, the earnings–return relation can be derived analytically. For example, Ohlson [1991] describes conditions sufficient for a linear relation if all earnings have equal persistence. In Ohlson's setting, permanent earnings surprises have an *ERC* of one plus the inverse of the risk-free rate. If transitory and price-irrelevant earnings are uncorrelated across firms at a point in time, cross-sectional "composite *PE*" is a ratio of price to permanent earnings.[13] This ratio is also equal to one plus the inverse of the risk-free rate in Ohlson's model; and a sample of permanent earnings surprises would have a cross-sectional *ERC* equal to composite *PE*. In contrast, a sample of transitory earnings would have an *ERC* of one, and a sample of price-

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[11] Parametric $t$ tests on the difference in squared residuals provide similar results. Note that the Wilcoxon statistic is positive (but insignificant) for the 346 observations with $|UE| > .10$, even though the linear model has greater squared residuals than the nonlinear model. The parametric test for this subsample of extreme observations is negative and insignificant. The tests have different signs because the nonparametric test is based on ranks of differences and the parametric test is based on the raw differences. Parametric and nonparametric tests on matched pairs of absolute residuals provide results similar to tests on squared residuals. We also repeated the analysis in table 3 on residuals from 13 pairs of cross-sectional regressions estimated on the 12,035 observations with $|UE| \leq .10$, with no qualitative differences in results.

[12] Studies that have reported *ERC*s well below average *PE*s include Beaver, Lambert, and Morse [1980], Brown et al. [1987], and Beaver, Lambert, and Ryan [1987]. Those studies attribute low *ERC*s to measurement error in the researcher's proxy for unexpected earnings. As noted earlier, positive correlation between measurement error and $|UE|$ may contribute to the *S*-curve.

[13] Composite *PE* in quarter $q$ is defined as $\Sigma_i P_{iq}/(4 \times \Sigma_i E_{iq})$.

**TABLE 4**

*Summary of Price–Earnings Ratios (PEs) and Marginal Price Responses from Cross-Sectional Regressions**

| Cross-Sectional Estimate for Quarter $q$ = 1984:3–1987:3 | Summary of Cross-Sectional Statistics from 13 Quarters | | | |
|---|---|---|---|---|
| | Minimum | Maximum | Mean | Median |
| Average $PE_q$ | | | | |
| Mean$_i$ $[P_{iq} / (4 \times E_{iq})]$ | 9.03 | 18.45 | 14.25 | 14.11 |
| Median$_i$$[P_{iq} / (4 \times E_{iq})]$ | 9.25 | 15.23 | 12.45 | 12.23 |
| Composite $PE_q$ | | | | |
| $\Sigma_i P_{iq} / (4 \times \Sigma_i E_{iq})$ | 10.79 | 22.79 | 16.72 | 16.77 |
| Nonlinear Model $\partial UR/\partial UE_q$ | | | | |
| at $UE = 0$ | 8.05 | 34.20 | 13.96 | 12.90 |
| at $UE = .001$ | 7.24 | 15.17 | 11.32 | 12.12 |
| at $UE = .005$ | .77 | 7.50 | 3.78 | 3.26 |
| at $UE = .01$ | .20 | 3.00 | 1.31 | 1.16 |
| at $UE = .05$ | .01 | .15 | .06 | .05 |
| at $UE = .10$ | .00 | .04 | .02 | .01 |
| Linear Model $ERC_q$ | –.35 | .83 | .28 | .21 |

*This table summarizes cross-sectional *PE* ratios and cross-sectional marginal price responses for the 13 quarters listed in panel A of table 1. Within each quarter, three *PE* statistics are calculated: two "average" *PE* ratios (the mean *PE* and the median *PE*) and a "composite" *PE* ratio (the sum of per-share price divided by four times the sum of per-share earnings). Regression models and variables used in estimating marginal responses are described in table 1.

irrelevant earnings innovations would have an *ERC* of zero. In general, *ERC*s should be approximately equal to *PE*s if unexpected earnings have about the same degree of permanence as current earnings.

Table 4 compares the distribution of marginal price responses from panel A of table 1 with composite *PE*s. The first column of the table describes summary *PE* calculations and marginal responses for 1984:3 through 1987:3. The next four columns report the minimum, maximum, mean, and median of each quarterly statistic. For the 13 quarterly ratios in our sample, the median composite *PE* is 16.77. When $|UE| \leq .001$ (about 22% of the sample), the median $\partial UR/\partial UE \geq 12.12$. That is, average marginal responses are at least 72% of composite *PE* when the difference between announced earnings and expected earnings is within .1% of firm value. However, once unexpected earnings reach .5% of firm value, an additional dollar of earnings is capitalized at $3.26, on average. Further earnings increases have even less value. For unexpected earnings of 1% of firm value, the next dollar of earnings is capitalized at $1.16. Since 76% of our sample has unexpected earnings magnitudes of 1% of firm value or less, we conclude that most earnings announcements result in the capitalization of unexpected earnings at a ratio greater than 1:1. In contrast, *ERC*s from linear regressions on untrimmed samples imply that, on average, unexpected earnings are capitalized at less than 1:1—e.g., the median (mean) *ERC* from the 13 cross-sectional regressions in panel A of table 1 is .213 (.283). For drastically trimmed samples, linear model *ERC*s are approximately equal to composite *PE*s. For example, the *ERC* for the sample with $|UE|$ trimmed to .1% of firm value is 17.22, as reported in panel B of table 1.

### 3.4 ASYMMETRIC RESPONSES TO ANALYSTS' FORECAST ERRORS

Philbrick and Ricks [1991], among others, report that analysts' forecasts are optimistic, on average.[14] In our sample, the mean forecast error is –.65% of firm value. Results in table 1 suggest that investors are aware of the bias in analysts' forecasts—i.e., $\hat{a}_0 > 0$ indicates a positive price response for zero forecast errors. Since equation (3) describes a symmetric $UR$–$UE$ relation around $(UR, UE) = (a_0, 0)$, there appear to be greater absolute price responses to positive forecast errors than to negative forecast errors of equal absolute magnitude. In the pooled regression with all 12,381 observations (row 1, panel B, table 1), the absolute value of predicted returns for equal levels of positive and negative $UE$ differs by 1.6% (or twice the intercept of .008).

A positive intercept adjusts only partially for analysts' bias because equation (3) requires a maximum marginal response at $UE = 0$. If analysts' forecasts are biased and if investors recognize this, the maximum marginal response could occur at $UE \neq 0$. Equation (4) allows for this possibility by adding the parameter $a_3$ to the arctan argument:

$$UR_i = a_0 + a_1 \arctan(a_2\ UE_i + a_3) + error_i, \tag{4}$$

so that the maximum $\partial UR/\partial UE$ occurs at $UE = -a_3/a_2$.[15] If the market recognizes the optimism in analysts' forecasts and if the smallest "true" earnings surprises have the greatest marginal price response, then the maximum marginal response should occur when $UE$ is negative—i.e., $\hat{a}_2$ and $\hat{a}_3$ should have the same sign.

The nonlinear regressions in table 1 were replicated with equation (4). The pooled regression with all 12,381 observations yields the following parameter estimates: $\hat{a}_0 = .001$, $\hat{a}_1 = .039$, $\hat{a}_2 = 340.6$, $\hat{a}_3 = .364$. The latter three coefficients are significantly different from zero at the .01 level.[16] The results for equation (4) are consistent with a market that is aware of the optimism in analysts' forecasts (predicted returns at $UE = 0$ are 1.5%) and that assigns the highest marginal price response to the smallest unbiased earnings surprises (the maximum marginal response of 13.3 occurs at $UE = -.1\%$ of firm value).[17]

Equation (4) also provides evidence of symmetric responses to unbiased forecast errors. Since equation (4) is symmetric about $UR = a_0$

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[14] Schipper [1991] summarizes evidence of forecast bias from several empirical studies.

[15] For equation (4), $\partial UR/\partial UE = a_1 a_2 \, / \, [1 + (a_2 UE + a_3)^2]$.

[16] All three coefficients are significantly positive for the pooled samples with $|UE| \leq$ .10, .05, and .01. For the 13 untrimmed cross-sectional samples, $\hat{a}_3$ is positive in ten quarters but is significantly greater than zero at the .05 level only once. In the other three quarters, $\hat{a}_3$ is negative but is not significantly different from zero.

[17] Maximum $\partial UR/\partial UE = \hat{a}_1 \hat{a}_2 = .039 \times 340.6 \approx 13.3$ when $UE = -\hat{a}_2/\hat{a}_3 = -.364/340.6 \approx -.001$.

and $UE = -a_3/a_2$, a negative estimate of $a_0$ indicates stronger market responses to bad news than to equal amounts of good news; $\hat{a}_0 > 0$ would indicate the converse. Since $\hat{a}_0$ is not significantly different from zero, results from equation (4) suggest that price responses to unbiased unexpected earnings are approximately symmetric.

## 4. *Other Determinants of ERCs*

The empirical methods in section 3 do not control for four potentially confounding variables investigated in earlier studies: average persistence of the firm's earnings series (Kormendi and Lipe [1987]), average predictability of the earnings series (Lipe [1990]), average time-series systematic risk (Collins and Kothari [1989] and Easton and Zmijewski [1989]), and the ratio of market value to book value of equity (Collins and Kothari [1989]). Those studies find that *ERC*s are positively correlated with time-series persistence and market-to-book values, and are negatively correlated with systematic risk and the variance of earnings forecast errors.[18] To gauge the strength of the nonlinear relation versus these other factors, this section investigates the incremental role of $|UE|$ magnitude while controlling for these four factors.

Our proxies for each of the four potentially confounding effects are similar to those used in earlier studies. Average time-series persistence for firm $i$ is the autocorrelation parameter ($\varphi_i$) from Foster's [1977] first-order autoregressive model in seasonally differenced earnings (hereafter, an *AR1* model) estimated on a minimum of 16 observations from 1980:1 through 1987:3. Firm $i$'s earnings predictability ($\rho_{iq}$) is the variance of the *AR1* errors deflated by the price per share at the beginning of the quarter. Systematic risk is the Scholes-Williams market model beta ($\beta_{iq}$) estimated over the calendar year preceding the current fiscal quarter. Finally, the market-to-book ratio for firm $i$ in quarter $q$ ($\mu_{iq}$) is the ratio of market price to book value of outstanding common stock at the beginning of the quarter.[19]

### 4.1 MULTIVARIATE REGRESSION MODELS

The models we use to examine the sensitivity of *ERC*s to $|UE|$ and to each of the characteristics investigated in previous research impose different ad hoc relations between *ERC*s and each characteristic, but all share the following form:

$$UR = \gamma_0 + \gamma_1\, UE_i + \sum_{k=1}^{5} c_k X_{ki}\, UE_i + error_i. \tag{5}$$

[18] In Lipe's [1990] model, greater variance of earnings surprises implies less predictable earnings series and lower *ERC*s.

[19] We obtain similar results with undeflated *AR1* variance and with a squared price deflator, $P^2_{iq-1}$. Results for systematic risk are very similar across several alternative measures of beta. The market-to-book ratio is based on price and book value from *Compustat.*

Coefficients ($\gamma_0$, $\gamma_1$, and $c_k$) are estimated by cross-sectional regressions in each of the 13 sample quarters.

In the first model (model 1), $X_{ki}$ is an indicator variable with a value of one if firm $i$'s characteristic $k$ is above the sample median in quarter $q$ and a value of zero otherwise. The five characteristics (variables) are: the absolute magnitude of unexpected earnings ($X_1 = 1$ if $|UE_i|$ is high), *AR1* persistence ($X_2 = 1$ if $\varphi_i$ is high), *AR1* predictability ($X_3 = 1$ if $\rho_i$ is high), systematic risk ($X_4 = 1$ if $\beta_i$ is high), and market-to-book of common equity ($X_5 = 1$ if $\mu_i$ is high). The expected signs of the related coefficients are: $c_1 < 0$, $c_2 > 0$, $c_3 < 0$, $c_4 < 0$, and $c_5 > 0$. Each $c_k$ coefficient represents the average difference in *ERC*s between firms with below- and above-median levels of a characteristic. Model 1 provides a rough indication of the relative economic importance of each characteristic. For example, if $|c_2| < |c_1|$, the average *ERC* difference between firms with high versus low levels of time-series persistence is less than the *ERC* difference between high versus low $|UE|$ magnitude.

The theoretical arguments linking *ERC*s to each factor imply that $ERC_i > ERC_j$ when $|UE_i| < |UE_j|$, $\varphi_i > \varphi_j$, $\rho_i < \rho_j$, $\beta_i < \beta_j$, or $\mu_i > \mu_j$. Model 1 does not impose additional conditions on the relation between *ERC*s and these characteristics, but it may employ inefficient significance tests because it allows only two classes of each characteristic. In a search for more efficient tests, we estimate a model with an ordinal measure (model 2) and two models (models 3 and 4) with cardinal measures, for each variable. In model 2, $X_{ki}$ is the standardized rank of firm $i$'s characteristic $k$; in model 3, $X_{ki}$ is the percentage of characteristic $k$'s range; and in model 4, $X_{ki}$ is the estimated value of firm $i$'s characteristic $k$. The predicted signs of coefficients do not differ across models.

Model 4 is included for comparability with prior research; it is the direct regression analogue of Collins and Kothari's [1989] reverse regressions. The five $X_k$ variables in model 4 are $|UE|$, $\varphi$, $\rho$, $\beta$, and $\mu$. As discussed by Collins and Kothari [1989], any comparison of coefficient magnitudes in model 4 is not meaningful because each coefficient is affected by the scale of characteristic $k$ and its deflator. Models 2 and 3 overcome this limitation by scaling each $X_k$ so that it is distributed on the [0,1] interval in each quarter. In models 2 and 3, the $c_k$ coefficients can be interpreted as the *ERC* difference between firms with the highest and lowest values of characteristic $k$. Model 3 standardizes each variable by expressing it as a percentage of the variable's range—e.g.:

$$X_{4iq} = [\beta_{iq} - \mathrm{Min}_i(\beta_{iq})] \;/\; [\mathrm{Max}_i(\beta_{iq}) - \mathrm{Min}_i(\beta_{iq})].$$

Since the $c_k$ coefficient estimates in models 3 and 4 may be influenced by outliers in the cardinal $X_k$ measures, we include model 2, which places each $X_k$ variable on the [0,1] interval as a percentage of the variable's rank—e.g.:

$$X_{4iq} = [\mathrm{Rank}_i(\beta_{iq}) - 1] \;/\; [(\text{Number of firms in quarter } q) - 1].$$

### 4.2 RESULTS FROM MULTIVARIATE REGRESSIONS

To enhance comparability with Collins and Kothari [1989], we estimate each model by cross-sectional regressions in each quarter and base statistical tests on coefficient means. We are able to calculate all four additional characteristics ($\varphi$, $\rho$, $\beta$, and $\mu$) for almost two-thirds of the sample (8,229 observations). Mean coefficients from the cross-sectional regressions are reported in panel A of table 5; all have the predicted signs except the mean predictability coefficient ($c_3$) in model 2. In that limited sense, our results for quarterly data and analysts' forecast errors are consistent with earlier results based on annual data and earnings innovations from various time-series models. However, only $|UE|$ magnitude and predictability are statistically significant in model 1, and $|UE|$ magnitude has the largest *ERC* effect in that model.[20] The average difference in *ERC*s for low- and high-$|UE|$ magnitude is 11.12, but the average difference for low- versus high-predictability is only 1.29. In models 2, 3, and 4, the coefficients for $|UE|$ magnitude and time-series persistence have the predicted signs in at least 10 of the 13 quarters.

To assess the influence of $|UE|$ outliers on the results in panel A of table 5, we repeat the multivariate regressions on 8,069 observations with data available for all five characteristics and with $|UE|$ no more than .10 of firm value. Mean coefficients for this restricted sample appear in panel B of table 5. Only the $|UE|$ magnitude coefficient ($c_1$) and the market-to-book coefficient ($c_5$) have the predicted signs in all four models, and the significance tests consistently indicate that $|UE|$ magnitude provides more explanatory power than *AR1* persistence, *AR1* predictability, systematic risk, or market-to-book of common equity.

## 5. *Summary and Conclusions*

This study reports cross-sectional differences in security price responses per unit of unexpected earnings. We argue that the permanent component of earnings surprises (as a percentage of total earnings surprises) increases as unexpected earnings approach zero because analysts and investors forecast high-value permanent earnings more accurately than low-value transitory earnings. This possibility allows us to predict that the marginal price response to earnings surprises should approach a composite price–earnings ratio as the earnings surprise approaches zero. Prior research hypothesizes that low *ERC*s from linear regressions are due to measurement errors in the researcher's unexpected

[20] Insignificant statistical results could also be due to poor proxies for average persistence, risk, and growth. For example, Lipe and Kormendi [1991] report that the largest correlations between average persistence and *ERC*s occur for persistence measures based on the higher-order properties of annual earnings. Since we examine responses to quarterly earnings, Lipe and Kormendi's results are not directly applicable here. Our risk and growth proxies are consistent with variables used in prior work.

**TABLE 5**

*Incremental Sensitivity of ERCs to |UE| Magnitude and Firm Characteristics Examined in Prior Studies*

$$UR_i = \gamma_0 + \gamma_1\, UE_i + \sum_{k=1}^{5} c_k\, X_{ki}\, UE_i + error_i{}^*$$

| Model | $\gamma_0$ | $\gamma_1 > 0$ | $\lvert UE \rvert$ Magnitude: $c_1 < 0$ | *AR1* Persistence: $c_2 > 0$ | *AR1* Predictability: $c_3 < 0$ | Systematic Risk: $c_4 < 0$ | Market-to-Book of Equity: $c_5 > 0$ |
|---|---|---|---|---|---|---|---|
| | Mean Coefficients and Predicted Signs for 13 Cross-Sectional Regressions (*t* statistic, number of coefficients with predicted sign) | | | | | | |
| **Panel A: All Observations Included** | | | | | | | |
| 1 | .008 | 12.95[a,c] | −11.12[a,c] | .26 | −1.29[a,c] | −.16 | .03 |
| | *(2.78)* | *(7.93, 13)* | *(−7.15, 13)* | *(1.24, 9)* | *(−4.07, 13)* | *(−.57, 8)* | *(.16, 6)* |
| 2 | .012 | 23.23[a,c] | −24.07[a,c] | .49[b,d] | .91 | −.28 | .26 |
| | *(3.76)* | *(8.28, 13)* | *(−7.55, 13)* | *(2.16, 10)* | *(1.39, 9)* | *(−1.12, 9)* | *(1.28, 8)* |
| 3 | .009 | .48 | −1.23[b,d] | 3.29[b,d] | −.42 | −.48 | .20 |
| | (2.92) | (.88, 8) | *(−1.85, 11)* | *(2.23, 11)* | *(−.36, 7)* | *(−.84, 6)* | *(.51, 6)* |
| 4 | .009 | .80[b] | −.19[d] | .68[b,d] | −.33 | −.09 | .04 |
| | *(2.92)* | *(2.11, 9)* | *(−.10, 11)* | *(2.23, 11)* | *(−.30, 7)* | *(−.56, 6)* | *(.53, 6)* |
| **Panel B: Observations with $\lvert UE \rvert$ > .10 Excluded** | | | | | | | |
| 1 | .010 | 12.41[a,c] | −10.35[a,c] | .03 | −.71[b] | .20 | .49[b] |
| | *(3.20)* | *(8.10, 13)* | *(−7.70, 13)* | *(.06, 8)* | *(−1.79, 8)* | *(.84, 5)* | *(2.45, 9)* |
| 2 | .012 | 21.42[a,c] | −22.31[a,c] | .46 | .99 | .86 | .65[a,c] |
| | *(3.55)* | *(9.34, 13)* | *(−8.81, 13)* | *(.80, 8)* | *(.92, 4)* | *(1.86, 3)* | *(4.26, 12)* |
| 3 | .012 | 3.66[a,d] | −5.27[a,c] | .32 | −.45 | 1.94 | .39 |
| | *(3.71)* | *(5.43, 11)* | *(−12.12, 13)* | *(.23, 6)* | *(−.16, 7)* | *(2.33, 3)* | *(1.41, 7)* |
| 4 | .012 | 3.88[a,c] | −56.32[a,c] | −.00 | .05 | .59 | .08 |
| | *(3.71)* | *(8.19, 13)* | *(−12.90, 13)* | *(−.01, 6)* | *(.02, 7)* | *(1.93, 3)* | *(1.40, 7)* |

*This table reports regression results for 13 quarterly cross-sectional samples. Panel A's results are based on a total of 8,229 observations with no restrictions on $|UE|$ magnitude. Panel B's results are based on 8,069 observations with $|UE| \leq .10$.

Variable definitions:

$UR_{iq}$ and $UE_{iq}$: See table 1.

$X_{kiq}$ in Model 1: $X_{kiq} = 1$ if firm $i$'s characteristic $k$ is greater than the median characteristic $k$ in quarter $q$; otherwise, $X_{kiq} = 0$.

$X_{kiq}$ in Model 2: $X_{kiq} = (\text{Rank}_{kiq} - 1) / [(\text{Number of firms in quarter } q) - 1]$, where $\text{Rank}_{kiq}$ is the rank of firm $i$'s characteristic $k$ in quarter $q$. In this model, $X_{kiq}$ is an ordinal measure on the [0,1] interval.

$X_{kiq}$ in Model 3: $X_{kiq} = [A_{kiq} - \text{Min}_i(A_{kiq})] / [\text{Max}_i(A_{kiq}) - \text{Min}_i(A_{kiq})]$, where $A_{kiq}$ is the raw measure of firm $i$'s characteristic $k$ in quarter $q$, $\text{Min}_i(A_{kiq})$ is the minimum value of characteristic $k$ for all firms in quarter $q$, and $\text{Max}_i(A_{kiq})$ is the maximum value of characteristic $k$ for all firms in quarter $q$. In this model, $X_{kiq}$ is a cardinal measure on the [0,1] interval.

$X_{kiq}$ in Model 4: $X_{kiq} = A_{kiq}$, which is the value of firm $i$'s characteristic $k$ in quarter $q$.

$k = 1$: The absolute value of price deflated unexpected earnings, $|UE|$.

$k = 2$: The estimate of the *AR1* correlation parameter ($\varphi$) for at least 16 pairs of earnings observations (after differencing).

$k = 3$: The estimated variance of the *AR1* time-series model ($\sigma^2$), deflated by price per share of common stock at the beginning of the quarter ($\rho = \sigma^2/P$) and estimated over the same period as $\varphi$.

$k = 4$: Market model systematic risk ($\beta$) estimated over the calendar year preceding the current quarter.

$k = 5$: The ratio ($\mu$) of market price per share of common equity divided by the book value per share of common equity both measured at the beginning of the quarter.

[a,b]Significant $t$-statistic (at the $a = .01$ or $b = .05$ level for one-tailed test) for the mean of 13 cross-sectional coefficients.

[c,d]Significant binomial test (at the $c = .01$ or $d = .05$ level) for the number of coefficients with the predicted sign.

earnings proxy; such errors could keep estimated marginal responses below composite *PE* even for very small unexpected earnings; but after eliminating large positive and negative earnings innovations, we find that estimated marginal responses are in the neighborhood of composite *PE*. As the absolute magnitudes of the earnings innovations increase, marginal price responses decline to levels near zero. Accordingly, our results suggest that studies which use linear regression models have reported low *ERC*s due to the nonlinearities in the returns–earnings relation. However, the linear model appears to be well specified when the earnings surprise is no greater than .5% of firm value, which includes approximately 60% of our sample.

Several previous studies document cross-sectional and time-series differences in linear-model *ERC*s. One implication of our work is that the underlying cause of such *ERC* differences may be forecast error magnitude—e.g., firms with low time-series persistence may simply have many large-magnitude forecast errors during the sample period. To illustrate the economic significance and statistical robustness of the nonlinear earnings response relation, we examine the sensitivity of *ERC*s to forecast error magnitude and to four firm-specific characteristics examined in earlier empirical studies. After controlling for the magnitude of unexpected earnings, we find that time-series estimates of average earnings persistence, average earnings predictability, and systematic risk have very weak associations with earnings response coefficients. We also find a weak positive relation between *ERC*s and market-to-book of common equity (a proxy for expected economic growth). In contrast, marginal price responses remain highly sensitive to forecast error magnitude after controlling for other potential determinants of *ERC*s.

Research which has documented additional cross-sectional *ERC* differences and event-related *ERC* changes over time may be comparing samples with different forecast error magnitudes. If so, an investigation of the relation between forecast error magnitude and the characteristics of interest might enhance our understanding of earnings time-series properties and of earnings management. In general, our results suggest that linear regressions are likely to indicate statistical significance for any nonearnings variable which is correlated with the magnitude of unexpected earnings.

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