International Diversification Benefits with Foreign Exchange Investment Styles*

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Abstract. Style-based management of the foreign exchange (FX) component of international investments with carry trade, FX momentum, and FX value strategies provides economically large and significant diversification benefits. These speculative benefits go beyond the hedging benefits of FX risk documented in the earlier literature. Our results hold after transaction costs and are confirmed in an extensive out-of-sample experiment mimicking investor decisions in real time. Adding a composite FX style portfolio to diversified allocations of global bonds and stocks leads to a 64% increase in the out-of-sample Sharpe ratio from 0.64 to 1.05, without adverse impact on other portfolio characteristics such as skewness.

JEL Classification: F31, G12, G15

1. Introduction

Foreign exchange (FX) exposure is an inherent and important aspect of international investments. It is therefore quite surprising that relatively little attention has been paid to this key component of international portfolio decisions in the prior academic literature on international diversification. Work that does take the FX component seriously mostly focuses on how to eliminate the FX risk of international portfolios using various hedging strategies, or how to minimize the risk in global bond or equity portfolios using hedging positions in foreign currencies (e.g., Glen and Jorion, 1993; Campbell *et al.*, 2010). There is, however, an important speculative component of FX investing, and this aspect has so far almost entirely been neglected in studies of international diversification benefits.

Recent work on international asset pricing has demonstrated that there are significant and attractive returns to currency speculation. Carry trade, FX momentum, and FX value are multicurrency investment strategies that have shown to produce high Sharpe ratios (e.g., Lustig and Verdelhan, 2007; Ang and Chen, 2010; Burnside *et al.*, 2011b; Menkhoff *et al.*, 2012b; Asness *et al.*, 2013), thus illustrating that currency markets can possibly offer benefits to investors beyond hedging. Crucially, these multicurrency investment strategies are also among the most popular FX investment styles by professional currency fund managers (Pojarliev and Levich, 2008). A better understanding of their characteristics and the role such FX strategies can play in a well-diversified international asset portfolio is therefore of primary interest, not only from an academic but also from a practitioner's perspective.

Given the large Sharpe ratios of FX investment styles documented in the recent literature and their fairly low correlations to traditional asset classes, a straightforward question arises: Does adding FX investment strategies to the investment opportunity set—or, put differently, a style-based management of the FX component of international investments—improve the risk-return profile of well-diversified international portfolios?

We quantify the diversification benefits of FX styles on the basis of carry trade, FX momentum, and FX value strategies relative to a benchmark portfolio. As our benchmark allocation, we consider a broad portfolio of global bonds and stocks, comparable to earlier studies on international diversification (e.g., Britten-Jones, 1999; Errunza *et al.*, 1999; Eun *et al.*, 2010). To account for FX risk exposure, we carefully hedge our benchmark assets against exchange rate risk before evaluating whether there are any additional benefits from speculative FX style investing. The hedging strategies for the benchmark assets—a full hedge, an optimal hedge, and a conditional

optimal hedge—draw on recent work by Campbell *et al.* (2010). We then proceed by testing whether adding FX investment styles to the allocation provides significant diversification benefits relative to the benchmark. This procedure allows us to dissect the hedging benefits and the speculative benefits that can be derived from FX investing. Throughout the study, we adjust FX returns for transaction costs due to portfolio rebalancing based on quoted bid-ask spreads.

The empirical design of our study is in line with modern portfolio management practice, which often relies on a two-layer approach (see Olma and Siegel, 2004; Pojarliev and Levich, 2012). Typically, in a first step, a hedging strategy is implemented to remove "unintended" currency exposure in core assets such as bonds or stocks. In a second step, a return-seeking overlay is implemented and strategic positions in currency markets are established.[1]

We find strong evidence that FX investment styles provide considerable diversification benefits to portfolios consisting of global bonds and equities. For example, adding a composite FX portfolio based on the joint signals of all three investment styles raises the (annualized) Sharpe ratio from 1.25 (benchmark assets, conditional optimal hedge) to 1.60 (FX styles augmented portfolio), an increase of almost 30%. In the main part of our analysis, we conduct an extensive out-of-sample evaluation, which mimics investor decisions in real time. Naturally, the levels of Sharpe ratios that we obtain in the out-of-sample analysis are smaller compared to the in-sample optimized Sharpe ratios, and hence these out-of-sample tests can be regarded as the most realistic. Importantly, however, diversification benefits (measured by the increase of the Sharpe ratio) from augmenting the benchmark portfolio by FX style investments are economically large and statistically significant. Once portfolio decisions are taken in real time, the Sharpe ratio of the benchmark portfolio is 0.64, and the composite FX style portfolio elevates the Sharpe ratio by 64% to 1.05.

Our results also suggest to consider the information in the different conditioning variables of the FX styles jointly when constructing FX investment strategies. We find that a blended FX strategy improves portfolio allocations

[1] Several leading asset managers have recently added new forms of FX investment style-related currency management schemes to their product portfolios (which are similar to our set up), stressing the economic importance and relevance of the question we analyze. See, inter alia, UBS: "Currency Management in Equity Portfolios", 2006; Deutsche Bank: "Currency Indices in a Portfolio Context", 2007; J.P. Morgan: "Managing Currency", 2008; BNY Mellon: "New Approaches to Global Currency Management", 2010; PIMCO: "Asset Allocation Rx for Fx: A Risk Factor-Based Approach to Currencies", 2011. Russel Investments: "Conscious Currency – A new approach to understanding currency exposure", 2011. We thank Stephan Siegel who pointed this aspect out to us.

to a greater extent than relying on a single FX style in isolation. Correlations among the FX strategies are fairly low, which means that there can be benefits from diversifying within the space of FX investment strategies (a point which is also made by Jorda and Taylor, 2012). For this reason, we provide results for an FX style composite strategy where the instruments of the carry, FX momentum, and FX value strategies are equally weighted. In the robustness section, we also elaborate on (out-of-sample) optimally weighted FX style composite strategies, following the parametric approach of Brandt *et al.* (2009). These tests show that an equally weighted FX style portfolio comes very close to the feasible optimal portfolio approach in terms of performance.

It is well known that carry trade strategies are prone to occasional heavy losses, that is have negatively skewed return distributions (see, e.g., Gyntelberg and Remolona, 2007; Brunnermeier *et al.*, 2009). This is also what we find in our data. Other FX styles, however, including the FX composite strategy, are not subject to negative skewness of this magnitude or even have positively skewed returns. We also find that adding FX styles, even in case of the carry trade, does not in general lead to a deterioration in portfolio skewness, but instead often improves the downside risk characteristics of global bond and equity portfolios. Nevertheless, in our robustness tests, we consider two additional checks regarding the impact of higher moments. We start with an extreme scenario where the carry strategy (the only FX strategy with negative skewness in this sample) is hedged against downside risk using currency options (similar to Jurek, 2009 and Burnside *et al.*, 2011a). We find that the carry trade still provides some (albeit lower) improvements in the mean–variance space. As a second check, we rely on the stochastic dominance criterion, which is based on less restrictive assumptions compared to the traditional mean–variance framework. To that end, we apply second-order and third-order stochastic dominance tests proposed by Linton *et al.* (2005). Consistent with the research design in our main empirical analysis, these tests are applicable in an out-of-sample context. The third-order test explicitly allows to account for aversion against negative skewness (i.e., it takes into account the skewness which remains after FX styles become part of a well-diversified benchmark portfolio). The stochastic dominance tests corroborate our baseline results and imply that also an investor who dislikes negative skewness would strictly prefer FX style augmented portfolios over the benchmark allocation. These results on diversification benefits from FX style investments notwithstanding, our findings also highlight the importance of carefully accounting for the negative skewness of carry trade returns. While the baseline results that draw on the standard mean–variance framework suggest that the carry

trade provides highly significant diversification benefits, the statistical evidence for diversification benefits is somewhat weaker when accounting for higher moments by either using options-based hedges or stochastic dominance tests.

Our work relates to the extensive literature on international diversification benefits and earlier studies analyzing the role of currency risk in global asset portfolios. Since the work of Grubel (1968) and Solnik (1974), researchers have become aware of the potential benefits from international diversification. However, empirical studies often are used to find surprisingly small and insignificant diversification benefits for nonstyle-based international equity market investments (e.g., Britten-Jones, 1999; Errunza *et al.*, 1999; Kan and Zhou, 2012), but significant diversification benefits for style-based stock market investing (e.g., Eun *et al.*, 2008; Eun *et al.*, 2010). Most of these studies devote little attention, or no attention at all, to the role of the FX rate component, which is by construction an unavoidable element of international investments. Only few studies carefully consider the exchange rate component in foreign investments, in particular Glen and Jorion (1993), de Roon *et al.* (2003), and most recently Campbell *et al.* (2010). These studies are, first and foremost, interested in the role of individual currency positions in international portfolio allocations. Glen and Jorion (1993) and de Roon *et al.* (2003) do not find (significant) diversification benefits of simple currency positions that go beyond fully hedging the currency risk exposure of stock and bond portfolios. Campbell *et al.* (2010) find that positions in single currencies are attractive to minimize the risk of global equities. Interestingly, all three studies find further increased portfolio Sharpe ratios for a hedging strategy conditional on the interest rate differential of the domestic country to the foreign country (hence, mimicking some kind of simple carry trade strategy).

Our article contributes to these initial findings for simple conditional hedges based on carry in the older literature. We go beyond the existing literature by focusing on the diversification benefits due to the speculative component of FX investing as opposed to the gains stemming from the hedging component. Contrary to the prior literature, we employ investment strategies that condition on carry, momentum, and value signals. Such multicurrency strategies are commonly practiced among market participants nowadays and a key theme in the recent international asset pricing literature.

The article proceeds as follows. In Section 2, we provide a detailed description of the FX investment styles in a common framework. Section 3 describes our benchmark assets and the hedging schemes we apply to the benchmark assets before testing for mean–variance efficiency. Section 4 presents our major empirical results on the FX investment strategies and

illustrates the gains in international portfolio diversification that can be achieved by FX style investing. Section 5 provides robustness tests and looks at our baseline results from various additional angles. Finally, we conclude in Section 6.

2. FX Investment Styles

We study the diversification benefits of three FX investment styles, commonly known as the carry trade, FX momentum, and FX value. The end-of-month payoff on a long forward position (also denoted as the "FX excess return" in the following) for currency j is measured as

$$RX_{j;t+1} = \frac{S_{j;t+1} - F_{j;t}}{S_{j;t}}, \tag{1}$$

where $S_{j;t}$ is the spot US dollar (USD) price of one unit of foreign currency j at time $t = 0, ..., T$ and $F_{j;t}$ is the one period forward price. Computed this way, the FX return is an excess return since it is a zero net investment consisting of selling USD in the forward market for the foreign currency in t and buying USD at the future spot rate in $t + 1$.

All three FX investment strategies generally rely on long–short positions in foreign currencies conditional on a specific signal available one period before. In our empirical analysis, we use monthly observations and rebalance the style portfolios at the beginning of every month. Following Asness et al. (2013), we build currency portfolios by weighting them according to their time t specific cross-sectional rank:

$$w_{j;t}^{z(s)} = c_t \left(\text{rank}(z(s)_{j;t}) - \sum_{j=1}^{J_t} \text{rank}(z(s)_{j;t})/J_t \right), \tag{2}$$

$$RZ_{s;t+1} = \sum_{j=1}^{J_t} w_{j;t}^{z(s)} RX_{j;t+1} \quad s = \{CT, MO, VA\}, \tag{3}$$

where $RZ_{s;t+1}$ denotes the return on the style-based trading strategy and depends on the conditioning variable $z(s)_t$. J_t denotes the set of currencies that are available for investment in period t. The choice of the signal s determines the particular strategy, for example, the carry trade (CT), FX momentum (MO), or FX value (VA). The constant c_t is chosen such that the style portfolios are one USD long and one USD short. Using the rank instead of the particular signal mitigates outliers and reduces transaction costs due to portfolio rebalancing (see below).

2.1 CARRY TRADE

The carry trade exploits the well-established empirical failure of uncovered interest rate parity (UIP) known as the "forward premium puzzle" (Fama, 1984). Our carry trade strategy goes long (forward) in currencies with high nominal short-term interest rates (investment currencies) and short (forward) in low-interest rate currencies (funding currencies). Thus, our conditioning variable is the interest rate differential between the foreign and the US money market, which we infer from the FX forward premium/discount

$$z(\text{CT})_{j;t} = \frac{F_{j;t}}{S_{j;t}} - 1. \tag{4}$$

Carry trades are very profitable, typically have quite attractive risk-return characteristics, are widely used by practitioners and there is evidence that they leave their traces in FX turnover patterns (Galati et al., 2007). The recent academic debate has revolved around the question whether their returns can be explained by a risk premium or whether they should be attributed to the presence of frictions or behavioral biases.[2] The purpose of this article, in contrast, is to analyze if there is a (significant) demand for carry trade investments in an internationally diversified portfolio, or in other words, if they allow investors to improve their investment opportunity set.

2.2 FX MOMENTUM

Similar to the well-known momentum effect in stock markets (e.g., Jegadeesh and Titman, 1993), momentum profits have also been documented in FX markets (see, e.g., Okunev and White, 2003; Burnside et al., 2011b; Menkhoff et al., 2012b). Menkhoff and Taylor (2007) and Pojarliev and Levich (2008) report evidence for the popularity of trend-following FX strategies among currency fund managers. Our momentum portfolio goes long in a portfolio of currencies with the highest past cumulative returns (so-called "winners") and short in a portfolio of currencies with the lowest past returns (so-called "losers"). The conditioning variable of the momentum strategy is the cumulative return over the past 3 months

$$z(\text{MO})_{j;t} = \prod_{\tau=0}^{2} \left(1 + RX_{j;t-\tau}\right) - 1. \tag{5}$$

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[2] See, for example, Lustig and Verdelhan (2007), Brunnermeier et al. (2009), Bacchetta and van Wincoop (2010), Christiansen et al. (2010), Verdelhan (2010), Burnside et al. (2011a), Burnside et al. (2011c), Lustig et al. (2011), Menkhoff et al. (2012a).

Menkhoff *et al.* (2012b) show that the momentum signal in currency markets is stronger for more recent past returns. However, they also report that since 1-month past returns are more volatile than longer-horizon past returns, transaction costs due to portfolio rebalancing are larger for the former. To take this trade-off into account, we rely on 3-month past returns for our baseline FX momentum strategy. Further variations of the FX momentum strategy are covered in the Supplementary Appendix.

2.3 FX VALUE

The basic idea behind the value strategy is to go long in currencies considered to trade below a fundamental value and to short currencies which trade above a fundamental value. One may interpret this strategy as a contrarian or long-term reversal strategy. A widely used measure for fundamental value in currency markets is the real exchange rate defined as

$$Q_{j;t} = \frac{S_{j;t} P_{j;t}}{P_t^*}, \tag{6}$$

where $P_{j;t}$ is the price level of consumer goods in country j in the local currency, and P_t^* the corresponding US price level in USD. If purchasing power parity (PPP) holds between two countries, Equation (6) should be equal to one. Hence, currencies with real exchange rates below (above) unity may be regarded as "undervalued" ("overvalued"). PPP is a rather strong assumption, as an equilibrium real exchange rate can easily deviate from unity (Harrod–Balassa–Samuelson effects). Thus, to avoid the problem of defining an equilibrium real exchange rate, we use a measure of "value" defined as minus one times the cumulative 5-year change of the real exchange rate as our conditioning variable

$$z(\text{VA})_{j;t} = \left(\frac{Q_{j;t-3}}{\bar{Q}_{j;t-60}} - 1 \right) \times (-1), \tag{7}$$

where $\bar{Q}_{j;t-60}$ is the real exchange rate measured as the average over a period between 5.5 and 4.5 years in the past.[3] To avoid overlaps between the momentum and value conditioning variables, we skip changes of the real exchange rate of the past 3 months when constructing the FX value measure.

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[3] The construction of our FX value strategy is motivated by the measure studied by Asness *et al.* (2013). The correlation between our FX value portfolio return and theirs is as large as 0.86.

Further details on the construction of the value strategy are provided in the Supplementary Appendix.

2.4 FX COMPOSITE STRATEGY

We also construct an FX style composite strategy (CMP) combining all three signals. In our baseline results, we rely on a strategy which simply averages the ranks across the three signals (CT, MO, VA) for each currency and takes long–short positions according to Equation (2). As we show in Section 5, this approach performs well out-of-sample and compares favorably against an optimized construction of FX styles. For the optimized strategies, we combine the signals not just equally but in an optimal way using methods as in Burnside *et al.* (2006), and Brandt *et al.* (2009). We construct optimized portfolios in a realistic out-of-sample setting. This means that we lose several observations and thus can only compare the performance of optimized portfolios to the rank-based FX composite strategy for a shorter time period.

2.5 FX DATA

Bid and ask quotes for spot and 1-month forward exchange rates against the USD are from Barclays Bank International (BBI) and WM/Reuters (WMR) and are available via Thomson Reuters Datastream. The FX sample covers 30 currencies reflecting the lion's share of global market turnover.[4] We also perform tests based on a reduced set of developed market currencies, or "G10 currencies" (currencies of Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the UK against the USD). We use Consumer Price Index (CPI) data from the IMF's International Financial Statistics (IFS) to calculate real exchange rates for the value strategy.[5] The available currency data span the period from 02/1976 to 12/2011. For the value strategy, we need 5-year changes of the real exchange rate. Thus, the sample period for the FX style returns ranges from 02/1981 to 12/2011. The Supplementary Appendix provides further details on the data and on some

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[4] This sample includes currencies from Australia, Brazil, Canada, Denmark, Eurozone, France, Germany, Hungary, Iceland, India, Indonesia, Israel, Italy, Japan, Mexico, New Zealand, Norway, Philippines, Poland, Russia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, Ukraine, and the UK. According to BIS (2013), our set of currencies covers more than 95% of the global FX market turnover in April 2013.

[5] Since the CPI has an arbitrary base year unrelated to PPP, we use the PPP estimate of Heston *et al.* (2009) for the year 2000 to determine the level of the real exchange rate. The resulting conditioning variable for the value strategy is robust, as we use changes in the real exchange rate.

conservative data screens which we apply to ensure reliability of our data. Furthermore, we provide a detailed comparison of our FX investment style returns with similar portfolios studied by Burnside *et al*. (2011a), Lustig *et al*. (2011), and Asness *et al*. (2013).

2.6 TRANSACTION COSTS

Our FX strategies involve a reallocation of the positions in the individual currencies every month according to the signal given by the conditioning variable. Since this rebalancing involves transaction costs, we compute returns both with and without adjusting for bid-ask spreads. Our adjustment procedure is a conservative approach of accounting for transaction costs, given that the bid-ask spreads in the WMR/BBI database are based on indicative quotes and are thus likely to overstate the true transaction costs of an investor (Lyons, 2001). Moreover, in practice transaction costs may be substantially lower when currency positions are rolled via FX swaps as shown by Gilmore and Hayashi (2011). Details on the transaction cost adjustment are given in the Supplementary Appendix.

3. Benchmark Assets

3.1 GLOBAL BONDS AND GLOBAL STOCKS

We measure diversification benefits from style-based FX investing relative to a typical well-diversified international portfolio allocation consisting of global bonds and stocks. The countries are the same as those covered by the G10 currencies. We take the MSCI Total Return stock market indices for Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, the UK, and the USA from Thomson Reuters Datastream. These returns are measured in USD and are available for the period from 02/1981 to 12/2011. There are no official bond market indices covering the time period and the cross-section of countries required for our study. Following Campbell *et al*. (1997) and Campbell *et al*. (2010), we use a log yield-return approximation to derive bond market returns. Stock and bond market returns are monthly simple returns in excess of the 1-month US T-bill rate from Ibbotson (available on the web site of Kenneth R. French).

3.2 DISSECTING FX SPECULATION FROM FX HEDGING

In our analysis, we carefully account for the exchange rate risk exposure inherent in the benchmark assets in order to dissect the benefits of hedging

and strategic speculative currency positions. The international bond and stock market returns in our benchmark portfolio are exposed to exchange rate risk, that is these are unhedged returns $R_{j;t}^{uh}$. However, it is easy to counteract the exchange rate risk using a currency hedging strategy (e.g., Anderson and Danthine, 1981; Glen and Jorion, 1993; Jorion, 1994 and Campbell *et al.*, 2010). The currency hedged return $R_{j;t}^{h}$ can be written as

$$R_{j;t}^{h} = R_{j;t}^{uh} + \widetilde{\mathbf{\Psi}}'_{RM}\mathbf{H}_t, \tag{8}$$

where $\widetilde{\mathbf{\Psi}}'_{RM}$ is a vector of hedging positions, or risk management demands, and $\mathbf{H}_t$ is a vector with the returns of the hedges. For the US asset (bond or stock), we define $j = 0$ and for the foreign assets $j = 1, ..., 9$. In our baseline specifications, the hedges include the G10 currency excess returns ($\mathbf{H}_t = \mathbf{RX}_t$). The vector of G10 currencies follows the same order as the foreign assets, that is, $\mathbf{RX}_t = \begin{bmatrix} RX_{1,t}, ..., RX_{9,t} \end{bmatrix}'$.[6]

We consider three types of hedging strategies (full hedge, optimal hedge, and conditional optimal hedge). The most parsimonious strategy is the full hedge, where the investor hedges the full amount invested in a foreign asset with a counter position in the currency to which the investment is exposed. Accordingly, the vector of risk management demands for the j-th foreign asset is simply $\widetilde{\mathbf{\Psi}}_{RM} = -\mathbf{1}_j$, which denotes a position in currency j of -1 and zero for all other currencies. Campbell *et al.* (2010) also study optimal currency hedges, where risk management demands are calculated such that the variance of US and foreign bonds and stocks is minimized. Here, the vector of risk management demands is given by minus one times the beta coefficients of a regression of all j currency excess returns ($\mathbf{RX}_t$) on the fully hedged asset (country bond or stock market), which we denote $\widetilde{\mathbf{\Psi}}_{RM} = -\mathbf{B_{RX}}$. Furthermore, Campbell *et al.* (2010) propose a conditional optimal hedge, which allows for time-varying hedging positions conditional on a specific signal. The authors use foreign interest rate spreads, that is, this strategy also hedges against carry trade risk. To mimic a conditional optimal hedge, we include the FX investment styles as hedges, that is, $\mathbf{H}_t = \begin{bmatrix} \mathbf{RX}'_t, \mathbf{RZ}'_t \end{bmatrix}'$ where the vector $\mathbf{RZ}_t$ contains the FX investment styles we want to test; accordingly, we denote the risk management demands as $\widetilde{\mathbf{\Psi}}_{RM} = [-\mathbf{B_{RX}}, -\mathbf{B_{RZ}}]'$.[7]

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[6] For example, if $j = 1$ corresponds to the Australian bond (or stock) market return $R_{1;t}^{uh}$, then, $RX_{1,t}$ is the currency excess return of the Australian dollar against the USD.

[7] Campbell *et al.* (2010) propose two ways to implement a conditional hedge. First, they allow the single currency risk management demands ($-\mathbf{B_{RX}}$) to be time-varying conditional on foreign interest rate spreads. Second, in their Supplementary Appendix, they include the carry trade as a hedge together with the single currencies and consider constant risk management

We can exploit the results based on the optimally hedged benchmark assets to dissect the improvement of the slope of the mean–variance frontier into a speculative component and a hedging component. The intuition is simple. The hedging component is the part of the diversification benefits driven by the nonzero betas (correlations) between the test assets and the benchmark assets. The speculation component is the residual, and will be driven by the return and variance of the test assets. The optimal hedging positions simply orthogonalize the benchmark assets with respect to the hedges by construction. Thus, when the benchmark assets are orthogonalized with respect to the test assets, as is the case for the conditional optimal hedge, any improvement of the Sharpe ratio must be driven solely by the speculative component. This allows to make a clear cut between the currency hedging benefits documented by Campbell *et al*. (2010) and the benefits from speculative style investing we are interested in. The Supplementary Appendix covers further analytical details on the distinction between the speculation and the hedging component.

4. Empirical Results

4.1 RISK AND RETURN CHARACTERISTICS

Table I reports risk-return characteristics and descriptive statistics for the FX investment styles and the global benchmark assets.[8] The three FX investment styles show an attractive performance, with an annualized Sharpe ratio (before transaction costs) of 0.82, 0.70, and 0.62 for the carry trade, FX momentum, and FX value, respectively, in the baseline "all currencies" case. When adjusting the FX excess returns for transaction costs, the Sharpe ratios of the carry and the FX value strategy drop slightly to 0.75 and 0.57. More volatile signals lead to larger portfolio turnover and therefore higher transaction costs. This is particularly the case for the FX momentum strategy, where the Sharpe ratio is reduced to 0.49 after transaction costs.

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demands for the expanded set of hedges. Of course, the carry trade can be thought of as a managed portfolio exploiting information in foreign interest rate spreads (Cochrane, 2005, Chapter 8). Campbell *et al*. (2010) find that the second approach provides hedging results very similar to the first approach (see Table A.12 in their Supplementary Appendix).

[8] In most of our analysis, we use individual global bond and stock market returns in our benchmark. To conserve space, the descriptive statistics in Table I are based on portfolios of global bonds and stocks (GDP at PPP weights) rather than of individual country returns. Individual country return characteristics are reported in the Supplementary Appendix.

Table I. Risk and return characteristics of FX investment styles

The table reports the mean (in percentage points, p.a.), standard deviation (Std, p.a.), skewness (Skew), first-order autocorrelation (Ac1), the Sharpe ratio (p.a.), and the correlation of global bonds, global stocks, and FX investment styles. Global bond and stock market returns are in excess of the 1-month US T-bill rate. FX returns are zero-cost returns computed from FX forward positions. FX styles are conditional on time $t-1$ forward discounts (carry trade), last 3-month cumulative returns (FX momentum), the 5-year change of the real exchange rate (FX value), and a strategy combining all three signals (FX composite). Statistics for global bonds and stocks are based on a GDP weighted portfolio of ten international markets covering Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, the UK and the USA with fully hedged currency risk. The FX styles are based on a broad set of up to 30 currencies ("all currencies") or a smaller subset of major currencies ("G10"). The G10 currencies cover the same countries as the global bond and stock markets. Sharpe ratios for FX styles are reported before and after transaction costs. Correlations are for FX style returns after transaction costs. The sample period is from 02/1981 to 12/2011 (371 observations).

	Mean	Std	Skew	Ac1	Sharpe ratio			Correlation				
					All	All	G10	All currencies				
					Before	After	After	B	S	CT	MO	VA
		All currencies before transaction costs								All currencies			
Global bonds (B)	4.21	5.38	0.42	0.34	0.78			1.00				
Global stocks (S)	5.81	14.31	-1.03	0.12	0.41			0.06	1.00			
Carry trade (CT)	6.18	7.50	-0.63	0.10	0.82	0.75	0.54	-0.20	0.27	1.00		
FX momentum (MO)	5.34	7.68	0.25	-0.09	0.70	0.49	0.22	-0.01	0.00	-0.01	1.00	
FX value (VA)	4.18	6.69	-0.31	0.09	0.62	0.57	0.51	-0.02	0.01	0.05	0.09	1.00
FX composite (CMP)	8.23	7.22	-0.17	0.03	1.14	0.98	0.63	-0.13	0.12	0.51	0.64	0.54

Restricting the FX investment universe to the G10 currencies leads to less diversified style portfolios (by construction), manifesting itself in greater return volatility. In terms of Sharpe ratios, the carry trade and FX value portfolios constructed from the G10 currencies are fairly similar in magnitude. The investment restriction, however, has a large negative effect on FX momentum returns.[9] As shown in the right Panel of Table I, correlations between all three baseline FX styles are close to zero. This means that additional performance gains can be obtained by diversifying across FX strategies. In fact, the FX style composite portfolio, a strategy that equally combines all three signals, has a Sharpe ratio of roughly one (all currencies, after transaction costs). Cumulative returns on our benchmark and test assets are depicted in Figure 1.

Carry trade returns are known to be negatively skewed. This is also a feature of the carry strategy in our data, although the return distribution of global equities is even more negatively skewed. In contrast, the skewness of the FX value strategy is considerably smaller in absolute terms, and FX momentum returns are actually positively skewed. It therefore seems reasonable that combinations of FX styles could also improve portfolio characteristics in terms of higher moments. Indeed, the skewness of the FX style composite portfolio is only mildly negative, as shown in Table I. We will elaborate on this point in more depth in Section 5.

FX investment styles not only have attractive risk and return characteristics, their correlations with global bonds and stocks are also fairly low, as shown in the right panel of Table I, giving rise to potential benefits from diversification. In the following, we will quantify these diversification benefits from augmenting the benchmark portfolio by investments in FX investment styles.

4.2 DIVERSIFICATION BENEFITS OF FX STYLES: IN-SAMPLE ANALYSIS

We now formally test if the inclusion of the FX styles enhances the investment opportunity set spanned by the (hedged) global benchmark assets. Our primary question of interest is whether the Sharpe ratio of the tangency portfolio shifts up significantly once the FX investment styles are added to the benchmark allocation. We start with in-sample mean–variance efficiency

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[9] This is in line with Menkhoff *et al.* (2012b) who show that momentum returns based on advanced economy currencies have fallen over time and are no longer significant after transaction costs in recent data.
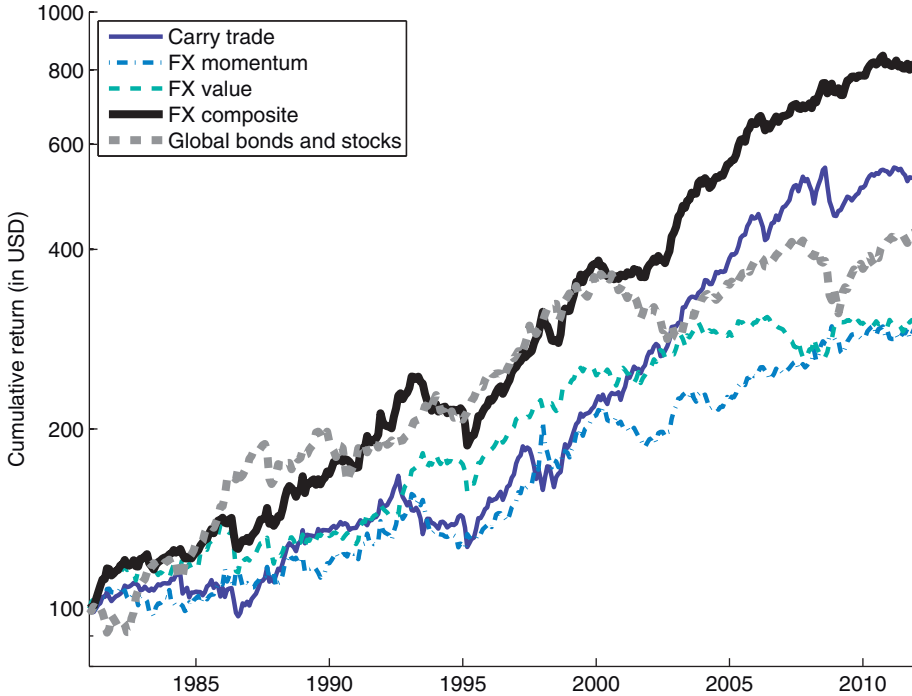
Figure 1. Cumulative returns. The figure shows cumulative returns of FX styles and our benchmark portfolio of global bonds and stocks. Returns of FX styles are reported after transaction costs due to portfolio rebalancing and based on up to 30 currencies. The benchmark portfolio shown in the figure is based on a balanced (50/50) allocation of GDP weighted global bonds and stocks. The sample period is from 02/1981 to 12/2011.

tests, which have been commonly employed in the analysis of diversification benefits in previous work. For statistical inference, we rely on standard regression-based (pv-*W*) and SDF-based (pv-*SDF*) tests, as outlined in the Appendix. The in-sample tests can be seen as a benchmark, as they imply optimal allocations based on full-sample estimates. However, an investor making portfolio decisions in real time would not have access to this kind of information. Thus, in our main analysis (Section 4.3), we conduct an extensive out-of-sample analysis of the diversification benefits of FX styles. In all tests reported and discussed in the following, we consider FX styles after accounting for transaction costs.

In Table II, we report in-sample tests of mean–variance efficiency and portfolio characteristics when FX styles are added to the benchmark allocation. Besides testing for the diversification benefits of individual FX styles, we also consider the case of including the FX style composite portfolio that

Table II. Mean–variance efficiency tests for FX styles

This table shows mean–variance frontier intersection tests for the tangency portfolio of 20 traditional assets (global bonds and stocks) when FX investment styles are added to the investment universe. The tested FX styles are the carry trade (CT), FX momentum (MO), FX value (VA), and a FX style composite portfolio (CMP). The FX styles are adjusted for transaction costs due to portfolio rebalancing. Three hedging schemes are applied to the benchmark before testing for mean–variance efficiency against the test assets: The conditional optimal hedge (CO) minimizes the standard deviation of the benchmark asset returns based on positions in the G10 currencies and in the specific FX style to be tested, the optimal hedge (OH) only includes positions in the G10 currencies, and the full hedge (FH) is a unitary hedge of country j specific currency risk. For the CO hedge (upper panel) the benchmark is test asset-specific; in the column "Benchmark", results based on hedge positions in all three FX styles are reported, which serves as the benchmark for the CMP strategy. The mean, standard deviation (Std), and the Sharpe ratio (SR) are annualized. The table reports the p-value of a regression-based test (pv-W) and the p-value of a stochastic discount factor based test (pv-SDF) for mean–variance efficiency. The bottom of the table provides a t-test for the difference in Sharpe ratio increases between two hedging schemes using the delta method (as described in the Appendix). All calculated test statistics are robust against heteroscedasticity and autocorrelation (Newey and West, 1987 kernel and automatic lag length according to Andrews, 1991). The sample period is from 02/1981 to 12/2011.

	Benchmark: global bonds and stocks				
		Added FX style (based on all currencies)			
	Benchmark	+ CT	+ MO	+ VA	+ CMP
	Conditional optimal hedge, $\widetilde{\Psi}_{RM} = [-\mathbf{B_{RX}}, -\mathbf{B_Z}]'$ (CO)				
Mean	6.06	5.96	5.33	4.98	6.33
Std	4.86	4.04	4.07	3.83	3.97
Skew	−0.08	−0.01	−0.01	0.05	−0.06
SR	1.25	1.48	1.31	1.30	1.60
pv-W		0.000	0.003	0.003	0.000
pv-SDF		0.002	0.005	0.008	0.000
	Optimal hedge, $\widetilde{\Psi}_{RM} = -\mathbf{B_{RX}}$ (OH)				
SR	1.20	1.46	1.31	1.31	1.58
pv-W		0.000	0.002	0.009	0.000
pv-SDF		0.001	0.003	0.017	0.000
	Full hedge, $\widetilde{\Psi}_{RM} = -\mathbf{1_j}$ (FH)				
SR	1.17	1.47	1.27	1.27	1.56
pv-W		0.000	0.002	0.009	0.000
pv-SDF		0.000	0.004	0.014	0.000
	Tests for difference in SR increases between two hedging schemes				
CO versus OH, t-stat		−1.98	−0.98	1.14	−0.90
OH versus FH, t-stat		−0.80	0.45	0.04	−0.41
CO versus FH, t-stat		−1.55	−0.44	0.81	−0.77

equally combines all three signals.[10] The results for different FX styles and the composite FX style portfolio are shown along the horizontal dimension of Table II. The benchmark assets are carefully hedged against FX risk to trace out the speculative component of FX style investments (as outlined in Section 3). The hedging schemes vary along the vertical dimension of Table II. In the upper panel, the benchmark assets are hedged using the conditional optimal hedge of Campbell *et al.* (2010), which serves as our baseline case. In the center of the table, the benchmark assets are hedged using the optimal hedge, whereas in the bottom panel the benchmark assets are fully hedged.

The annualized Sharpe ratio, that is the slope of the mean–variance frontier, of the benchmark portfolio is 1.25 when the currency risk of benchmark assets is conditionally optimally hedged. Now, what happens to the allocation if speculative FX style positions are added to the portfolio of these benchmark assets? Since the benchmark is hedged against FX style risk, the ensuing diversification benefits reflect the pure speculative component of FX investing.

Table II shows that the inclusion of the FX strategies leads to a marked improvement in the allocation. When we add the composite FX style portfolio to the benchmark assets (+ CMP), the Sharpe ratio increases to 1.60, a gain of 28% compared to the benchmark. This rise in the Sharpe ratio is highly significant, as indicated by both the regression-based test and the SDF-based test. Also the partial benefits from each individual FX style are economically and statistically significant. Adding individual positions in the carry trade, the FX momentum, or the FX value strategy produces Sharpe ratios of 1.48, 1.31, and 1.30, respectively, with p-values well below 1%.

Diversification benefits of FX style investments are more pronounced if we apply the optimal hedge or the full hedge to the benchmark assets, as shown by the center and bottom panels of Table II. While the Sharpe ratios of the benchmark are smaller compared to the conditional optimal hedge, the Sharpe ratios for the FX style augmented portfolios are of similar magnitude as before. We hence find a rise in the Sharpe ratio from inclusion of the composite FX style portfolio of 32% when benchmark assets are optimally hedged and 33% when they are fully hedged. In line with these

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[10] We also run tests where the three FX styles are added jointly as separate test assets (+CT, MO, VA). As the parsimonious composite FX style portfolio (+CMP) provides roughly similar diversification benefits as the three individual FX styles with ex post optimal weighting, we focus on these tests in the main article. Results for the case of benchmark portfolios augmented by all three FX styles are provided in the Supplementary Appendix.

observations, the p-values of the mean–variance efficiency tests indicate high statistical significance. Detailed results on the hedging demands and their significance are reported in the Supplementary Appendix.

One may ask whether the exact choice of the hedging scheme makes a significant quantitative difference when gauging diversification benefits of the FX styles. To investigate this formally from a statistical perspective, we perform tests based on the delta method and report the corresponding *t*-statistics at the bottom of the table (see the Appendix for computational details). We find that the differences in diversification benefits across hedging schemes are not statistically significant, except for the case when the carry trade is the single test asset. In this particular case, the difference in the Sharpe ratio is significantly changed when moving from the optimal hedge to the conditional optimal hedge. In all other cases, using a subopti-mal hedge does not (significantly) overstate the speculative diversification benefits of the FX styles.

We now conduct the same mean–variance tests as in Table II, but impose the restriction that the FX styles can only be constructed from the set of G10 currencies. Table III shows that the Sharpe ratio rises from 1.21 to 1.37 when the composite FX style portfolio is added, a gain of roughly 13%. This change in the Sharpe ratio is significant at conventional levels of significance. For the carry trade and the FX value strategy, the results are qualitatively similar to the all currencies case. Increases in Sharpe ratios are lower compared to the previous results, but still economically large and statistically significant. The most notable difference compared to the 30 currencies universe is FX momentum, which no longer provides significant diversifica-tion benefits if it is restricted to the G10 currencies. This reflects the fact, documented by Menkhoff *et al.* (2012b), that momentum strategies do not perform well for developed market currencies, and that transaction costs are large compared to the carry trade. In the Supplementary Appendix, we repeat the tests before applying our transaction costs adjustment. These results represent an upper bound of diversification benefits. Here, we find significant benefits also for FX momentum at the 5% level for all three hedging schemes.

4.3 OUT-OF-SAMPLE ANALYSIS

We now turn to the main results of the article, the results of our out-of-sample (OOS) analysis mimicking investor decisions in real time. The design of the OOS experiment closely follows DeMiguel *et al.* (2009) in that we compute (optimal) portfolio allocations using a rolling window approach. The window size is set to 120 months, which we roll forward with each

Table III. FX styles based on G10 currencies

This table shows mean–variance frontier intersection tests for the tangency portfolio of 20 traditional assets (global bonds and stocks) when FX investment styles are added to the investment universe. The tested FX styles are the carry trade (CT), FX momentum (MO), FX value (VA), and a FX style composite portfolio (CMP). The FX styles are restricted to the G10 currencies and are adjusted for transaction costs due to portfolio rebalancing. Three hedging schemes are applied to the benchmark before testing for mean–variance efficiency against the test assets: The conditional optimal (CO) hedge minimizes the standard deviation of the benchmark asset returns based on positions in the G10 currencies and in the specific FX style to be tested, the optimal hedge (OH) only includes positions in the G10 currencies, and the full hedge (FH) is a unitary hedge of country *j* specific currency risk. For the CO hedge (upper panel) the benchmark is test asset-specific; in the column "Benchmark", results based on hedge positions in all three FX styles are reported, which serves as the benchmark for the CMP strategy. The mean, standard deviation (Std), and the Sharpe ratio (SR) are annualized. The table reports the p-value of a regression-based test (pv-*W*) and the p-value of a stochastic discount factor based test (pv-*SDF*) for mean–variance efficiency. The bottom of the table provides a *t*-test for the difference in Sharpe ratio increases between two hedging schemes using the delta method (as described in the Appendix). All calculated test statistics are robust against heteroscedasticity and autocorrelation (Newey and West, 1987 kernel and automatic lag length according to Andrews, 1991). The sample period is from 02/1981 to 12/2011.

	Benchmark: global bonds and stocks				
		Added FX style (based on G10 currencies)			
	Benchmark	+ CT	+ MO	+ VA	+ CMP
	Conditional optimal hedge, $\widetilde{\mathbf{\Psi}}_{RM} = [-\mathbf{B_{RX}}, -\mathbf{B_Z}]'$ (CO)				
Mean	5.64	5.56	5.25	5.24	5.55
Std	4.65	4.11	4.29	4.08	4.07
Skew	−0.06	−0.05	−0.07	0.00	−0.02
SR	1.21	1.35	1.22	1.29	1.37
pv-*W*		0.003	0.199	0.012	0.001
pv-*SDF*		0.015	0.178	0.016	0.004
	Optimal hedge, $\widetilde{\mathbf{\Psi}}_{RM} = -\mathbf{B_{RX}}$ (OH)				
SR	1.20	1.33	1.22	1.29	1.35
pv-*W*		0.002	0.191	0.021	0.001
pv-*SDF*		0.011	0.168	0.026	0.004
	Full hedge, $\widetilde{\mathbf{\Psi}}_{RM} = -\mathbf{1_j}$ (FH)				
SR	1.17	1.34	1.19	1.26	1.34
pv-*W*		0.000	0.192	0.018	0.001
pv-*SDF*		0.002	0.182	0.021	0.002
	Tests for difference in SR increases between two hedging schemes				
CO versus OH, t-stat		−0.98	−0.05	0.71	0.15
OH versus FH, t-stat		−1.01	−0.11	−0.29	−0.79
CO versus FH, t-stat		−1.30	−0.12	0.39	−0.60

additional observation. There are various ways to compute the optimal allocations. In the main results (reported in Panel A of Table IV), we calculate optimal mean–variance allocations with a short sales constraint that maximize the ex ante Sharpe ratio. Furthermore, we study the performance of a naïve strategy in line with DeMiguel *et al.* (2009) (reported in Panel B of Table IV). In the Supplementary Appendix, we provide results for a host of additional popular portfolio optimization algorithms. Statistical inference on differences in Sharpe ratios is based on a test statistic using the delta method. Further details on the test methodology are described in the Appendix.

Panel A of Table IV reports the OOS Sharpe ratios and portfolio characteristics of the mean–variance optimized asset allocations. The setup of the table is the same as in Tables II and III. The different FX styles are tested along the horizontal dimension. Again, we consider three hedging schemes of the benchmark along the vertical dimension, to trace out the benefits from the speculative FX component.

The OOS portfolios exhibit a performance that can be considered more realistic compared to the in-sample results. For instance, the conditionally optimally hedged benchmark portfolio produces an OOS Sharpe ratio of 0.64, that is roughly one half of the Sharpe ratio based on full-sample estimates. Importantly, adding the composite FX portfolio based on the joint signals of the three strategies significantly increases the OOS Sharpe ratio to 1.05 (p-value below 1%), which is 64% higher than the one of the benchmark.[11] Individual positions in the three FX styles also provide significant diversification benefits in the out-of-sample context. Adding the three FX investment styles separately improves the OOS Sharpe ratios to 0.90 for the carry trade, 0.75 for FX momentum, and 0.81 for FX value. The increase in the OOS Sharpe ratio is significant at the 10% level for carry, and at the 5% and 1% level for FX momentum and FX value.

The benefits from adding FX styles are also evident for alternative definitions of the benchmark allocation. With a Sharpe ratio increase of 69%, the gains from adding the composite FX style portfolio to the optimally hedged benchmark allocation (OH) are roughly in the same ballpark as in the case of the conditionally optimally hedged benchmark (CO) discussed above. The out-of-sample Sharpe ratio of the fully hedged benchmark (FH) is higher than the one of the other two optimal hedges, unlike the in-sample case.

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[11] Likewise, adding all three FX styles generates an OOS Sharpe ratio of 1.09, an increase by 70% relative to the benchmark. The Supplementary Appendix to the paper reports results for the case when all three FX styles (+CT, MO, VA) are added jointly to the benchmark allocation.

Table IV. Out-of-sample performance

The table displays the characteristics of benchmark portfolios based on 20 global bonds and stocks and of FX-style augmented portfolios. The results are based on an out-of-sample analysis with a 120-month rolling window. The FX styles are the carry trade (CT), FX momentum (MO), FX value (VA), and a FX composite portfolio (CMP). The optimization portfolio formation rule in Panel A is a mean–variance tangency portfolio with short-selling constraints. For the naïve portfolio formation rule in Panel B, we allocate 1/2 to global stocks (GDP PPP weighted), and 1/2 to global bonds (GDP PPP weighted). For the augmented portfolios, we weight global bonds, global stocks, and FX styles 1/3. The table shows the p-value for the difference in Sharpe ratios between the benchmark allocation and the augmented portfolio using the delta method (pv-δ, Newey and West, 1987 kernel with four lags). The bottom of the table provides a *t*-test for the difference in Sharpe ratio increases between two hedging schemes using the delta method (as described in the Appendix). The mean, standard deviation (Std), and the Sharpe ratio (SR) are annualized. The sample period is from 02/1981 to 12/2011, and all portfolio performance results are based on the sample period 02/1991 to 12/2011.

	Benchmark: global bonds and stocks				
	Panel A: mean–variance optimized portfolio formation (without short-sales)				
		Added FX style (based on all currencies)			
	Benchmark	+ CT	+ MO	+ VA	+ CMP
	Conditional optimal hedge, $\widetilde{\Psi}_{RM} = [-\mathbf{B_{RX}}, -\mathbf{B_Z}]'$ (CO)				
Mean	3.11	3.76	3.14	3.20	4.30
Std	4.85	4.17	4.20	3.93	4.12
Skew	−0.45	−0.77	−0.27	−0.06	−0.20
SR	0.64	0.90	0.75	0.81	1.05
pv-δ		0.076	0.043	0.010	0.002
	Optimal hedge, $\widetilde{\Psi}_{RM} = -\mathbf{B_{RX}}$ (OH)				
SR	0.61	0.89	0.74	0.81	1.03
pv-δ		0.030	0.036	0.017	0.001
	Full hedge, $\widetilde{\Psi}_{RM} = -\mathbf{1_j}$ (FH)				
SR	0.76	1.09	0.87	0.96	1.21
pv-δ		0.008	0.090	0.023	0.000
	Tests for difference in SR increases between two hedging schemes				
CO versus OH, t-stat		−1.30	−0.17	0.46	−0.58
OH versus FH, t-stat		−0.73	0.94	0.33	−0.40
CO versus FH, t-stat		−1.16	0.70	0.61	−0.66

(continued)

Table IV. Continued

	Benchmark	Added FX style (based on all currencies)			
		+ CT	+ MO	+ VA	+ CMP
		Conditional optimal hedge, $\widetilde{\Psi}_{RM} = [-\mathbf{B_{RX}}, -\mathbf{B_Z}]'$ (CO)			
Mean	2.66	4.22	3.66	3.54	4.36
Std	6.65	5.08	5.19	5.16	5.04
Skew	0.43	−0.01	0.75	1.00	0.40
SR	0.40	0.83	0.71	0.69	0.86
pv-δ		0.004	0.021	0.011	0.000
		Optimal hedge, $\widetilde{\Psi}_{RM} = -\mathbf{B_{RX}}$ (OH)			
SR	0.50	0.84	0.71	0.72	0.92
pv-δ		0.006	0.025	0.012	0.000
		Full hedge, $\widetilde{\Psi}_{RM} = -\mathbf{1_j}$ (FH)			
SR	0.63	0.88	0.83	0.85	0.99
pv-δ		0.016	0.027	0.014	0.000
		Tests for difference in SR increases between two hedging schemes			
CO versus OH, t-stat		3.07	2.14	0.57	2.36
OH versus FH, t-stat		2.59	0.59	0.07	1.74
CO versus FH, t-stat		3.29	0.99	0.13	2.69

Benchmark: global bonds and stocks — *Panel B: naïve portfolio formation*

When it comes to the assessment of diversification benefits from the FX style investments, however, we still see economically sizeable and statistically significant benefits when taking the fully hedged assets as the benchmark. Performance gains when including the composite FX style portfolio are sizable, with an increase of 60% compared to the original OOS Sharpe ratio of the benchmark portfolio.

Panel B of Table IV compares the OOS Sharpe ratios of naïve asset allocations. DeMiguel *et al.* (2009) find that naïve portfolio diversification often performs better OOS than more sophisticated approaches, a result that we cannot confirm in our empirical setting. To construct the naïve benchmark portfolios, we allocate 50% to global bonds and 50% to global stocks. Within each asset class of the benchmark, the assets are weighted by GDP. The portfolios augmented with FX styles allocate 33% to the FX

style(s) to be tested, 33% to GDP-weighted global bonds, and 33% to GDP-weighted global stocks. Importantly, we again see significant improvements from the inclusion of FX styles into the investment set. For instance, adding a position in the FX style composite portfolio (+CMP) generates OOS Sharpe ratios in the range of 0.86–0.99 depending on the hedging scheme, a significant improvement in statistical and economic terms in each setting.

4.4 SUMMARY OF MAIN RESULTS

Overall, we find evidence of quantitatively large and significant diversification benefits for almost all of the three FX styles. The results generally hold when the FX styles are based on all currencies or based on G10 currencies, and when the benchmark assets are fully hedged, optimally hedged, or conditionally optimally hedged. The most notable exception is FX momentum, which only provides diversification benefits if investments in emerging market currencies are available to the investor.

The diversification benefits from FX styles show up in a realistic out-of-sample experiment. Not surprising, the levels of Sharpe ratios are lower out-of-sample, but also more realistic compared to the in-sample tests. Importantly, the differences in Sharpe ratios between the benchmark allocations and the allocations including FX styles are economically large and in most specifications statistically significant.

A blended FX strategy that combines the signals of the three FX styles performs very well in-sample and out-of-sample. Figure 2 summarizes our baseline results and documents how a (hedged) portfolio of global bonds and stocks can be improved (measured by the increase of the Sharpe ratio in %) by augmenting it with a parsimonious FX style composite strategy. The benefits from the speculative component of FX positions are sizeable. Based on these major findings, we explore our baseline results from several additional angles in the following section.

5. Additional Results and Robustness

5.1 OPTIMIZED FX STYLE COMPOSITE STRATEGY

Our baseline FX style composite portfolio weights all three instruments equally. In this subsection, we explore whether an *optimally* weighted FX style portfolio can further improve the performance. In line with our baseline empirical setup in Section 4.3, we construct optimized FX style composite portfolios in an out-of-sample setting. The first 120 observations are our initial values. Specifically, in a first step, we use a given window to calculate
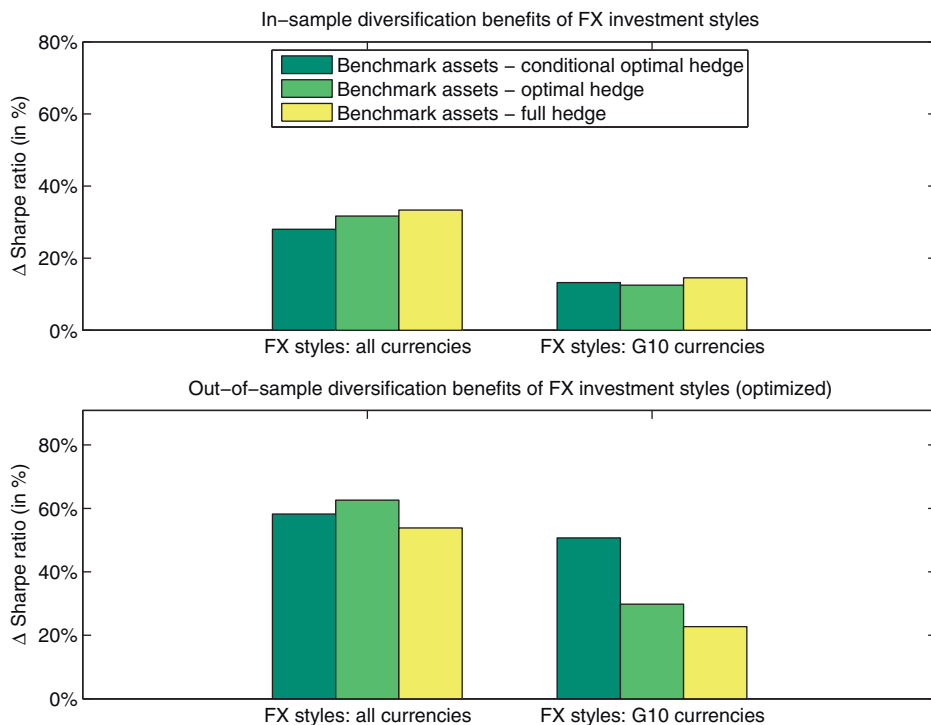
Figure 2. Overview of baseline results. The figure summarizes the increases of the Sharpe ratio (in %) when the FX style composite portfolio is added to a benchmark of global bonds and global stocks. Out-of-sample results are based on the mean–variance portfolio formation rule with short-sales constraints.

optimal weights. In a second step, we use these optimal weights to calculate portfolio returns of the next period. Finally, we expand the window by one period and repeat the previous steps. We consider two methods to obtain optimal weights for each individual currency when constructing the optimal FX style portfolios.

The first approach, which we label "traditional," picks optimal weights in the sense that the (ex ante) Sharpe ratio of the currency composite portfolio is maximized. This approach closely follows the optimized carry trade strategy of Burnside *et al*. (2006), but we expand this framework to include FX momentum and FX value instruments as well. To find the currency portfolio with the (ex ante) maximum Sharpe ratio, we need estimates of expected FX returns and a covariance matrix. We obtain expected returns as one-step ahead forecasts from linear regressions of the FX returns on the three instruments, and use the residuals' covariance matrix as the final input to solve for the optimal portfolio. In line with Brandt *et al*. (2009) and

a large literature cited therein, we find that unconstrained weights are extremely noisy and produce unstable results. Thus, we impose two constraints, namely (i) we do not allow for short sales (as in Burnside *et al.*, 2006), and (ii) we limit single currency positions to ±50%.

A second approach to construct optimized FX portfolios closely follows Brandt *et al.* (2009) and is labeled "parametric." The motivation for this method is to address the shortcomings of the traditional approach and to generate sensible portfolio weights with a robust performance. The parametric FX style composite portfolio obtains optimal weights from a simple linear function

$$w_{j;t}^{z(s)} = \bar{w}_{j;t} + \frac{1}{J_t} \boldsymbol{\theta}' \hat{\mathbf{z}}(s)_{j;t}, \tag{9}$$

where $\bar{w}_{j;t}$ is the weight of currency j in an equally weighted benchmark portfolio, $\hat{\mathbf{z}}(s)_{j;t}$ is a 3×1 vector of cross-sectionally standardized instruments (zero mean and unit standard deviation), and $\boldsymbol{\theta}$ is a 3×1 vector of coefficients to be estimated.[12] We also consider a parsimonious strategy that equally weights all three signals, that is $\boldsymbol{\theta}$ is a vector of ones. The difference of this strategy to our baseline composite portfolio is that it is based directly on the (standardized) signals of the instruments instead of ranks.

Table V shows performance characteristics and mean–variance efficiency tests for the optimized FX style composite portfolios. We find that only the parametric FX style composite portfolio is able to provide large Sharpe ratios comparable to our baseline rank-based portfolio. The poor OOS performance of the traditional optimization approach squares well with Brandt *et al.* (2009) and expands their results to currency markets. The FX style composite portfolio which simply equally weights the standardized instruments ($\boldsymbol{\theta} = \mathbf{1}$) leads to performance gains relative to the benchmark that are quantitatively in line with the ones of the fully optimized composite FX style portfolio (where $\boldsymbol{\theta}$ is estimated). The Supplementary Appendix provides further comparisons of the optimized FX style portfolios and our baseline FX style portfolios.

A key implication of these tests with optimal portfolios is that FX styles constructed via the rank-based approach come very close in terms of performance to the feasible optimal approach. Moreover, this method can be used for the entire sample period, whereas the optimal portfolio approach requires an initialization period to estimate parameters (in our case 10 years).

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[12] See Barroso and Santa-Clara (2013) for a comprehensive application of this method to FX markets.

Table V. Optimized FX style composite strategies

This table reports characteristics and mean–variance efficiency tests for optimally weighted FX style composite strategies. The optimized weights of the FX style composite strategies are determined out-of-sample with an expanding window and 120 starting observations. The first two strategies, labeled "traditional", solve a mean–variance problem to find optimal weights in each individual currency. In a first step, the three instruments carry, momentum, and value are used to estimate expected returns from linear regressions. In a second step, these estimates are applied to the (ex ante) optimal portfolio with the maximum Sharpe ratio. To reduce extreme portfolio weights, the weights for the first strategy are restricted to be nonnegative and for the second strategy to lie between −50% and +50%. The third and the fourth strategy, labeled "parametric", rely on the approach proposed by Brandt *et al.* (2009). The optimal weight in each currency is directly modeled as a linear function of the three instruments. The signals of the instruments are standardized and weighted by a coefficient θ, which must be estimated (out-of-sample). We also consider the case of applying equal weighting to the signal-based portfolios ($\theta = 1$). The sample period is from 02/1981 to 12/2011. The mean, standard deviation (Std), and the Sharpe ratio (SR) are annualized. Out-of-sample results on portfolio characteristics are based on the sample period 02/1991 to 12/2011.

	Out-of-sample optimized FX style composite strategies (exp. window using 120 starting obs., G10, after transaction costs)			
	"traditional" Mean–variance		"parametric" Brandt *et al.* (2009)	
	No short positions	Max/min ±50%	Theta: optimized	Theta: **1**
Panel A: descriptive statistics				
Mean	1.92	0.43	9.67	4.86
Std	10.18	15.82	13.16	6.11
Skew	−0.43	0.12	0.64	0.89
SR	0.19	0.03	0.73	0.80
Panel B: mean-var. efficiency tests, benchmark: global bonds and stocks				
SR	1.55	1.53	1.70	1.72
pv-*W*	0.279	0.910	0.004	0.003
pv-*SDF*	0.291	0.909	0.002	0.001

5.2 ACCOUNTING FOR SKEWED RETURNS

Some of the FX investment strategies, especially the carry trade, have negatively skewed return distributions. The standard mean–variance framework ignores this characteristic. In the following, we thus provide a detailed treatment of the impact on the higher moments of the portfolio returns that results from the inclusion of FX styles. We start by inspecting the

downside risk properties of FX style augmented portfolios of both the in-sample and out-of-sample analysis. Then, we consider two formal approaches to account for negative skewness. The first approach uses options to reduce the negative skewness of FX styles and benchmark assets. The second approach considers an alternative testing methodology, stochastic dominance, and explicitly accounts for skewed return distributions.

Before turning to the additional tests, it is interesting to note that the skewness at the portfolio level typically does not tend to worsen but rather in most cases improves slightly (i.e., becomes less negative) when adding FX investment styles to the investment set. The upper panel of Table II, for instance, reports the skewness of the optimal portfolio returns (with and without FX style investments, "all currency case") based on the in-sample optimized allocations. The downside-risk characteristics tend to improve slightly, even in the case when augmenting the benchmark allocation by an individual carry trade position. However, these differences in portfolio skewness are generally not statistically significant, when judged by (unreported) tests using the delta method. Also when inspecting the characteristics of the portfolio returns from the OOS analysis (reported in Table IV), we generally find an improvement in terms of skewness from the inclusion of the FX investments. The main exception is the carry trade, where the skewness of the augmented portfolio becomes slightly more negative compared to the benchmark. In all other cases (including the composite FX style portfolio), the skewness becomes less negative.

5.2.a. *Options*

We now proceed by studying a carry strategy where the downside risk is insured using FX options as in Jurek (2009) and Burnside *et al.* (2011a). For every long (short) position, the options-based hedging portfolio buys put (call) options such that the investor is compensated for large losses. This hedge for downside risk is rather extreme. By construction, given a specific threshold, the investor insures all of the downside risk of the carry trade. We obtain options data from J.P. Morgan for our set of G10 currencies for a limited sample period from 07/1997 to 12/2011. Further details on these data and the construction of the options-based hedge are provided in the Supplementary Appendix to this article.[13]

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[13] In these tests, we concentrate on the carry trade strategy. The FX value and FX composite strategy also exhibit some, albeit very mild, negative skewness over the full sample period (Table I). However, both strategies exhibit a positively skewed distribution during the sample period from 07/1997 to 12/2011 where we have options data available.

Note, however, that the return distributions of the stock market benchmark assets also exhibit a fair amount of negative skewness. Hence, to level the playing field these assets ought (in principle) be hedged for downside risk as well. Burnside *et al*. (2011a) show that a similar downside hedging strategy for US stocks reduces returns from 6.87% to −4.79% (p.a.). Downside risk protection for US stocks is far more costly than for the carry trade. We collect data on S&P 500 index options and replicate this result for a downside risk hedged US stock portfolio.[14]

Table VI reports the characteristics of the carry trade (based on G10 currencies) with and without downside risk insurance. As reported in the table, the other two FX styles as well as the FX style composite portfolio are not subject to negative skewness in this sample period. By construction, the skewness of carry returns is much reduced (in absolute terms) to −0.11 (+0.07) in case of the 10 delta (25 delta) downside risk hedge. Burnside *et al*. (2011a) report that the costs of these options-based hedges are low in terms of Sharpe ratios, in particular compared to a similar downside hedge in stock markets. In line with their results, we find that the Sharpe ratio of the 10 delta (25 delta) hedged carry trade is still as large as 0.47 (0.40) compared to 0.55 for the case without downside risk insurance.

The lower panel of Table VI provides tests for mean–variance efficiency for FX styles where negative skewness has been eliminated using the hedges. We test these FX styles against two benchmarks, in particular, (i) global bonds and unhedged global stocks (comparable to our baseline results), and (ii) global bonds and downside-hedged US stocks. To ensure that the benchmark assets are indeed hedged for downside risk in the latter case, we rely on regression-based mean–variance efficiency test with short-sales constraints by de Roon *et al*. (2001). Furthermore, all benchmark assets are hedged for currency risk based on the conditional optimal hedge.

Adding the baseline carry trade increases the Sharpe ratio of the global bond and equity portfolio that is unhedged for downside risk (but hedged for currency risk) from 1.62 to 1.75 for this shorter subsample. This increase is significant at the 10% level for both tests. The Sharpe ratio rises to 1.67 if the carry trade is downside-hedged using 10 delta ("far out-of-the-money") options, which is just significant at the 10% level in case of the SDF-based test, but not significant in the case of the Wald test. Finally, the increase in the Sharpe ratio is 1.66 if the carry trade is downside-hedged

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[14] Unfortunately, there are no such options data available covering the other nine international stock markets in our sample. However, we find that the US options-based hedge of a GDP weighted global stock portfolio works reasonably well in eliminating the downside risk (additional results are reported in the Supplementary Appendix).

Table VI. Using options to protect against downside risk

The table shows characteristics and mean–variance efficiency tests for FX styles where options are used to eliminate negative skewness. The carry trade (based on G10 currencies) is protected against downside risk using put and call options such that negative skewness cancels out by construction. Option data are based on implied volatility quotes and are from J.P. Morgan. Quotes are available for 25 delta ("out-of-the-money") and 10 delta ("far out-of-the-money") call and put options. The table also provides the characteristics of a US stock market portfolio protected in the same way against downside risk using S&P stock index options. Portfolio characteristics are reported in Panel A. The mean, standard deviation (Std), and the Sharpe ratio (SR) are annualized. The mean–variance tests in Panel B are conducted for two different benchmark assets. The first benchmark are global bonds and stocks without option-based protection against downside risk, as in the baseline results. The second benchmark relies on US stocks that are protected against downside risk using options. In the latter setting, to rule out short positions in the downside risk protected benchmark assets, the mean–variance efficiency test with short-sales constraints proposed by de Roon *et al.* (2001) and simulation-based p-values (pv-*DNW*) are applied. The currency risk of the benchmark assets is hedged using the conditional optimal hedge. The sample period is from 07/1997 to 12/2011.

	FX styles hedged for downside risk using options G10 currencies, 07/1997 to 12/2011						
	Carry trade			FX mom.	FX val.	FX comp.	US stocks
	Unhedged	Hedged (25 delta)	Hedged (10 delta)	Unhedged	Unhedged	Unhedged	Hedged (25 delta)
Panel A: descriptive statistics							
Mean	4.48	3.45	2.68	0.79	2.61	3.63	−4.78
Std	8.11	7.27	6.69	7.93	7.79	7.50	13.58
Skew	−0.43	−0.11	0.07	0.31	1.05	0.05	0.01
SR	0.55	0.47	0.40	0.10	0.34	0.48	−0.35
Panel B: mean–variance efficiency tests							
	Benchmark			Added FX style			
	Benchmark: global bonds and unhedged global stocks (country portfolios)						
SR	1.62	1.75	1.67	1.66	1.60	1.68	1.69
pv-*W*		0.050	0.103	0.151	0.754	0.335	0.131
pv-*SDF*		0.081	0.098	0.135	0.751	0.287	0.130
	Benchmark: global bonds and protected US stocks						
SR	1.19	1.33	1.28	1.26	1.20	1.24	1.28
pv-*DNW*		0.026	0.051	0.072	0.364	0.144	0.051

using 25 delta ("out-of-the-money") options and is no longer significant. However, when the skewness of the benchmark assets is removed as well via the options-based hedge, we find that even the 25 delta hedged carry portfolio produces a significant increase of the Sharpe ratio (10% level).

5.2.b. *Stochastic dominance tests*

As a second approach to account for skewness, we consider a stochastic dominance methodology which explicitly accounts for higher moments when testing for improvements in the asset allocation. This framework is based on less restrictive assumptions compared to the traditional mean–variance framework and allows to take skewness-loving preferences of investors into account. In particular, a portfolio is second-order stochastic dominance efficient (SSD) if it is optimal for a nonsatiable and risk-averse investor, and it is third-order stochastic dominance efficient (TSD) if it is optimal for a nonsatiable, risk-averse, and skewness-loving investor (Levy, 2006).

To be consistent with our baseline results, we perform stochastic dominance tests put forth by Linton *et al.* (2005) that are explicitly designed for an out-of-sample context. In these stochastic dominance tests, we compare the properties of the benchmark allocation and the portfolio augmented by FX styles as reported in Table IV. In technical terms, the SSD test statistic compares the integrated cumulative empirical distribution function of the benchmark and the augmented portfolio, whereas the TSD test compares the double integral of the cumulative empirical distribution function. P-values of the SSD and TSD test statistics are obtained via the subsample bootstrap method as suggested by Linton *et al.* (2005). Further details are provided in the Supplementary Appendix.

Table VII reports the results of the stochastic dominance tests. Based on these tests, we can reject that the allocations without FX styles stochastically dominate the allocations augmented by FX styles in the second-order sense. The largest p-value, 0.01, can be found when adding the FX value strategy to the benchmark allocation. We find similar results for the TSD tests. We always reject the null that allocations excluding FX styles stochastically dominate allocations with FX styles in the third-order sense. The rejection for the carry trade augmented portfolio, however, is only at the 10% level, whereas the lowest empirical significance level is 1% for the allocation that includes the parsimonious FX style composite portfolio.

Overall, these results speak a similar language as our baseline out-of-sample results in Table IV. Adding FX styles, in particular combinations of FX styles, to a well-diversified global bond and equity portfolio generally tends to improve (rather than worsen) the skewness properties of the

Table VII. Stochastic dominance tests for out-of-sample portfolios

The table shows p-values of second-order (SSD) and third-order (TSD) stochastic domin-ance tests, proposed by Linton *et al.* (2005), of a benchmark portfolio against a FX style augmented portfolio as reported in Panel A of Table IV (out-of-sample asset allocations, mean–variance optimized without short-sales). The benchmark portfolio consists of global bonds and stocks. The FX style augmented portfolio adds the carry trade (CT), FX momentum (MO), FX value (VA), or a FX style composite portfolio (CMP) to the invest-ment opportunity set. Low p-values indicate rejection of the hypothesis that the benchmark allocation stochastically dominates the FX style augmented portfolio. The reported p-values are calculated as in Linton *et al.* (2005) and are the medians of 20 subsample bootstraps with sizes from an equally spaced grid between the nearest integers of T/20 and T/4, where $T = 251$ is the number of observations. The sample period is from 02/1991 to 12/2011.

Stochastic dominance tests for out-of-sample optimized portfolios			
Added FX style:	CT	MO	VA	CMP
Conditional optimal hedge, $\widetilde{\Psi}_{RM} = [-\mathbf{B_{RX}}, -\mathbf{B_Z}]'$			
SSD	0.012	0.007	0.014	0.000
TSD	0.071	0.065	0.050	0.012

portfolio. The TSD tests are consistent with these earlier findings, as they suggest that also an investor who dislikes negative skewness would prefer a portfolio allocation with FX styles over an allocation that does not incorp-orate a speculative FX component. Our results also suggest that it is import-ant to carefully consider the negative skewness of carry trade returns when gauging diversification benefits. The baseline results based on the standard mean–variance framework suggest that the carry trade provides significant diversification benefits at the 5% level or even lower. However, the statistical evidence for diversification benefits from isolated carry investments is weaker when accounting for the impact on higher moments.

6. Conclusion

In this article, we go beyond currency hedging to shed more light on the speculative component of currency investments and their role in global asset portfolios. We study the implications of FX investment styles such as carry trades and widely practiced strategies known as FX momentum and FX value. These strategies are known to be profitable when considered in iso-lation (see, e.g., Lustig and Verdelhan, 2007; Ang and Chen, 2010; Burnside *et al.*, 2011b; Menkhoff *et al.*, 2012b; Asness *et al.*, 2013). But do they

provide diversification benefits when the investor already has access to a well-diversified global bond and equity portfolio?

To answer this question, we study whether investments in these strategies significantly shift the mean–variance frontier when a benchmark allocation consisting of global bonds and stocks is augmented by FX style investments. Importantly, the benchmark assets are thoroughly hedged against currency risk in order to dissect the diversification benefits into those deriving from hedging FX risk and those stemming from speculative FX positions.

Our results suggest that style-based FX investments generate significant improvements in the asset allocation. These findings hold after taking into account transaction costs and when controlling for the FX risk inherent in the benchmark assets. Importantly, these results are also confirmed in an extensive out-of-sample experiment with different portfolio formation rules mimicking investor decisions in real time. Augmenting the benchmark allocation by a composite FX style portfolio produces Sharpe ratios above one in the out-of-sample analysis (a gain by more 60% relative to the Sharpe ratio of the benchmark), without adverse impact on other portfolio characteristics such as skewness.

Supplementary Material

Supplementary data are available at *Review of Finance* online.

Appendix

MEAN–VARIANCE EFFICIENCY TESTS

Our major interest is whether adding FX styles to an internationally diversified portfolio improves the mean–variance frontier and may thus be beneficial from an investor's perspective. With the ability to borrow and invest in a risk-free asset, a test of mean–variance efficiency comes down to a test of a shift of the tangency portfolio, or in other words, to testing if the two mean–variance frontiers intersect at the point with the maximum Sharpe ratio. Following Glen and Jorion (1993), Harvey (1995), Eun *et al.* (2008), and Eun *et al.* (2010) among others, we use frontier intersection tests to analyze if FX investment styles significantly shift the investment opportunity set of a global bond and equity portfolio. We estimate the following pricing model:

$$RZ_{n;t} = \alpha_n + \mathbf{R}'_t \boldsymbol{\beta}_n + \varepsilon_{n;t}, \quad t = 1, \dots, T, \tag{10}$$

where $RZ_{n;t}$ is the return of $n = 1, ..., N$ test asset excess returns (FX styles), and $\mathbf{R}_t$ is a $K \times 1$ vector of benchmark asset excess returns (global bonds and global stocks). The null hypothesis of intersection is equivalent to the hypothesis that the N intercepts α_n are not significantly different from zero ($H_0 : \boldsymbol{\alpha} = \mathbf{0}_N$).[15] We report an asymptotic Wald test, $W \sim \chi_N^2$, which is robust against heteroscedasticity and autocorrelation (HAC). We use the Newey and West (1987) kernel and the automatic lag length selection procedure proposed by Andrews (1991).

Bekaert and Urias (1996) propose an alternative to the regression-based mean–variance efficiency tests, which exploits the duality between Hansen and Jagannathan (1991) bounds and mean–variance frontiers. We consider the general asset pricing restriction for the $N + K$ asset excess returns, $\widetilde{\mathbf{R}}_t = \left[\mathbf{RZ}_t', \mathbf{R}_t'\right]'$:

$$E\left(\widetilde{\mathbf{R}}_t m_t\right) = \mathbf{0}_{N+K}, \tag{11}$$

where m_t is the projection of a stochastic discount factor (SDF) with mean $v = E(m_t)$ onto the demeaned $N + K$ asset returns

$$m_t = v + \left[\widetilde{\mathbf{R}}_t - E\left(\widetilde{\mathbf{R}}_t\right)\right]' \mathbf{b}. \tag{12}$$

The SDF given by Equation (12) prices the $N + K$ asset returns correctly by construction. We can write the vector of SDF coefficients as $\mathbf{b} = [\mathbf{b}_N, \mathbf{b}_K]'$. Bekaert and Urias (1996) show that mean–variance efficiency of the K benchmark assets is implied by the N restrictions $\mathbf{b}_N = \mathbf{0}_N$. Put differently, only the benchmark assets are necessary to price the augmented set of $N + K$ assets. This estimation problem can be cast in a typical Generalized Methods of Moments (GMM) framework. We set $v = 1$, which corresponds to testing intersection at the tangency portfolio in the mean–variance space, and report an HAC-robust asymptotic GMM Wald test, SDF $\sim \chi_N^2$.[16]

DELTA METHOD

We apply the delta method and GMM framework for a test of the difference in Sharpe ratios between a benchmark portfolio (A) and a test asset portfolio

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[15] See Huberman and Kandel (1987), Gibbons et al. (1989), and Jobson and Korkie (1989).
[16] Kan and Zhou (2012) present some evidence on the power and size of the SDF-Wald test and perform a comparison to the regression-based approach. They find no important differences between the asymptotic test statistics when returns follow a multivariate normal distribution. However, their simulation study shows that the regression-based version is favorable to the SDF-based test when returns follow a multivariate Student-t distribution.

(*B*) that is HAC robust (Ledoit and Wolf, 2008). The difference in Sharpe ratios is a function of sample means

$$\Delta\text{SR} = g(\mathbf{m}) = \frac{E(R_{A;t})}{\sqrt{E(R_{A;t}R_{A;t}) - E(R_{A;t})^2}} - \frac{E(R_{B;t})}{\sqrt{E(R_{B;t}R_{B;t}) - E(R_{B;t})^2}}, \quad (13)$$

where

$$\mathbf{m} = \left[E(R_{A;t}); E(R_{A;t}R_{A;t}); E(R_{B;t}); E(R_{B;t}R_{B;t}) \right], \quad (14)$$

with covariance matrix $\Sigma_{\mathbf{m}}$. As shown by Hansen (1982) and Cochrane (2005), the asymptotic distribution of the estimate of ΔSR is

$$\sqrt{T}\left(\Delta\hat{\text{S}}\text{R} - \Delta\text{SR}\right) \to N\left(0, \frac{\partial g}{\partial \mathbf{m}} \Sigma_{\mathbf{m}} \frac{\partial g'}{\partial \mathbf{m}}\right). \quad (15)$$

We compute a HAC robust kernel estimate of $\Sigma_{\mathbf{m}}$ for a *t*-test

$$\Delta\hat{\text{S}}\text{R} \Bigg/ \left(\sqrt{T^{-1} \frac{\partial g}{\partial \mathbf{m}} \hat{\Sigma}_{\mathbf{m}} \frac{\partial g'}{\partial \mathbf{m}}}\right), \quad (16)$$

of the difference in Sharpe ratios in our out-of-sample analysis, and report the corresponding one-sided p-value (pv-δ). Ledoit and Wolf (2008) provide evidence on the size of this test statistic and find that asymptotic inference is already reliable for 120 observations.

   We also perform tests for difference in Sharpe ratios between two hedging schemes, in-sample and out-of-sample. For example, between the conditionally optimally hedged allocations (CO) and the optimally hedged allocations (OH), we want to test if the gain in the Sharpe ratio from augmenting the (hedged) benchmark by an FX-style augmented portfolio is the same for two different hedges. This allows us to assess if there is a significant error from relying on a suboptimal hedge when judging the diversification benefits from FX investment styles. Formally, we test the null hypothesis $\Delta\text{SR}^{\text{CO}} - \Delta\text{SR}^{\text{OH}} = 0$, where $\Delta\text{SR}^{\text{CO}} = \text{SR}_A^{\text{CO}} - \text{SR}_B^{\text{CO}}$ and $\Delta\text{SR}^{\text{OH}} = \text{SR}_A^{\text{OH}} - \text{SR}_B^{\text{OH}}$. Thus, the test statistic becomes

$$\Delta\text{SR}^{\text{CO}} - \Delta\text{SR}^{\text{OH}} \equiv g(\mathbf{m}) = \text{SR}_A^{\text{CO}} - \text{SR}_B^{\text{CO}} - \text{SR}_A^{\text{OH}} + \text{SR}_B^{\text{OH}}, \quad (17)$$

and the expressions for $\mathbf{m}$, $\frac{\partial g}{\partial \mathbf{m}}$, and $\Sigma_{\mathbf{m}}$ extend accordingly and provide us with all ingredients for the t-test as described above.

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