

Forecasting Performance of Nonlinear Models for Intraday Stock Returns

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ABSTRACT

We studied the predictability of intraday stock market returns using both linear and nonlinear time series models. For the S&P 500 index we compared simple autoregressive and random walk linear models with a range of nonlinear models, including smooth transition, Markov switching, artificial neural network, non-parametric kernel regression and support vector machine models for horizons of 5, 10, 20, 30 and 60 minutes. The empirical results indicate that nonlinear models outperformed linear models on the basis of both statistical and economic criteria. Specifically, although return serial correlation receded by around 10 minutes, return predictability still persisted for up to 60 minutes according to nonlinear models, even though profitability decreases as time elapses. More flexible nonlinear models such as support vector machines and artificial neural network did not clearly outperform other nonlinear models. Copyright © 2011 John Wiley & Sons, Ltd.

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INTRODUCTION

The predictability of stock market returns has long attracted interest by financial researchers and practitioners alike, as it has profound theoretical and practical implications. The cornerstone of predictability is the idea of financial market efficiency that states that stock returns fully adjust to all relevant information and so cannot be forecast.

However, the process of adjustment to new and changing economic information is not, in practice, instantaneous. Traders require some time to ascertain whether new information is pertinent or not. Recently, Lo (2004) proposed an adaptive expectation hypothesis according to which important changes in information or economic circumstances lead to serial return correlation, which, in turn, gives rise to a kind of predictability that will be localized and limited in scope and duration until market participants learn to take advantage of it. Accordingly, tests of market efficiency

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examining the autocorrelation of returns find no evidence of statistically significant serial correlation at frequencies that are daily, weekly or monthly. Chordia *et al.* (2005), for example, show that daily returns for stocks listed in the NYSE are not serially correlated, so the efficiency-creation process—and thus predictability—must be looked for in intraday trading data. The evidence in Avramov *et al.* (2006) indicates that daily returns are not predictable enough to give rise to profit opportunities. Timmermann (2008), furthermore, shows, for a wide range of forecasting models, that monthly returns in the US market are poorly predicted.

This paper focuses on return predictability at the intraday level. Intraday trading data prediction is particularly important because it offers practitioners the opportunity to gain high annual returns from an intraday trading strategy. We should expect such predictability to weaken as trader actions to exploit profit opportunities expunge serial return correlation, with prices adjusting to new equilibrium levels quickly, given that the stock market nowadays is assumed to be highly efficient; hence predictability tends to recede over longer horizons. Intraday horizons are important, however, in the management of trading desk risk (see Chew, 1994). Despite its economic and financial importance, the analysis of stock return predictability for very short forecast horizons is under-represented in the academic literature, although worthy of mention is the study by Clements and Taylor (2003) describing an analysis of interval forecasts for high-frequency data.

We analysed in-sample and out-of-sample one-step-ahead point forecasts for the S&P 500 index for the period June 2003 to September 2003, for different within-day temporal horizons of 5, 10, 20, 30 and 60 minutes.

First we examined whether linear autoregressive (AR) models were able to predict stock returns at different time intervals as serial correlation persisted through time. Empirical evidence indicates that serial correlation recedes after 10 minutes and weak-form market efficiency rapidly becomes evident. This result is consistent with the rapid price adjustment found in Chordia *et al.* (2005) and Busse and Green (2002) for individual stocks.

We then examined whether nonlinear models were able to generate more accurate and longer time period predictions than linear models. Theoretically, there is no reason to suppose that stock prices must be intrinsically linear; in fact, prices result from a complex iteration between informed and uninformed traders in the market place. A nonlinear time series stock return model can have zero autocorrelation but a nonzero mean conditional on its past history, which implies predictable nonlinearity-in-mean (Hsiec, 1989, 1993). We explored intraday nonlinear return predictability using a set of nonlinear models that included a smooth transition autoregressive model, a smooth transition autoregressive model with generalized autoregressive conditional heteroscedasticity (GARCH) errors, a Markov switching model, an artificial neural network (ANN) model and a nonparametric time series kernel regression model. We also explored the forecasting ability with intraday data of support vector machines, a technique from statistical learning theory whose predictive ability for intraday stock return data—to the best of our knowledge—has not, as yet, been evaluated. Finally, we compared the forecasting performance of different models in terms of (a) statistical criteria, such as the mean squared error (MSE), the mean absolute error (MAE), the proportion of times the signs of returns were correctly forecasted (SIGN), the Pesaran and Timmermann (1992) directional accuracy test (DA test) and the popular Diebold–Mariano (1995) test (DM test) for the equality of accuracy of competing forecasts; and (b) economic criteria, using a simple trading strategy guided by forecasts, in order to test the relative payoffs generated by different forecasting models. Our main empirical findings verify the predictability of stock returns beyond when time serial correlation recedes; in other words, even if the market is weak-form efficient according to linear models—meaning that prices are not linearly predictable—predictable nonlinearity-in-mean is possible up to a

60-minute time interval. Surprisingly, the forecast performance of nonlinear models—mainly directionally predictable—decreases very slowly, which has important implications for market timing and active intraday asset allocation management.

The remainder of this paper is organized as follows: the next section provides a description of the different models and outlines the criteria for assessing forecasts; the third section describes the data and examines the predictive capacity of several models over different intraday time periods; and finally, the fourth section contains our conclusions.

EMPIRICAL METHODOLOGY

To forecast stock returns we used various models for $E[r_t/I_{t-1}]$, where r_t represents the first difference of the logarithmic stock price, and $I_{t-1} = \{r_{t-1}, r_{t-2}, \dots\}$ is the information set available at time $t-1$. The information set only included lagged returns, given that we only analysed the dynamic characteristics of returns and nonsynchronous trading effects may result in autocorrelation in returns. Below we briefly describe the different models considered for the conditional mean.

Forecasting models

We considered nine forecasting models, the simplest of which were the random walk model (RW) and an autoregressive (AR) specification:

$$r_{t+1} = \beta_0 + \sum_{j=1}^k \beta_j r_{t+1-j} + \varepsilon_{t+1} \quad (1)$$

where k was selected to minimize the Bayes information criterion. To account for the possibility that the conditional mean $E[r_t/I_{t-1}]$ could be time-varying in a nonlinear form we employed several nonlinear models in addition to the linear models used as a benchmark. We briefly discuss the set of nonlinear specifications below.

Smooth transition autoregressive models

Smooth transition autoregressive models (STAR) models account for the existence of different regimes with different dynamic properties and with smooth transition between regimes (Granger and Teräsvirta, 1993; Teräsvirta *et al.*, 1994). A first-order STAR model with two regimes takes the following form:

$$r_{t+1} = f_{10} + f_{11}r_t + [f_{20} + f_{21}r_t] F(z_{t+1}; \gamma, c) + \varepsilon_{t+1} \quad (2)$$

where $F(z_{t+1}; \gamma, c)$ is the smooth transition function that depends on the transition variable z_{t+1} and the parameters γ , which is the transition rate or smoothness parameter; and where c is the threshold value which represents the change from one regime to another. The transition variable can be defined as a linear combination of the lagged values of r_t , $z_{t+1} = \sum_{h=1}^H \alpha_h r_{t+1-h}$. The most widely used smooth transition functions are the logistic function and the exponential function: $F(z_{t+1}; \gamma, c) = 1/(1 + \exp(-\gamma(z_t - c)))$ and $F(z_{t+1}; \gamma, c) = 1 - \exp(-\gamma(z_t - c)^2)$, with $\gamma > 0$. This model is estimated by quasi-maximum likelihood using the logistic function.

To account for the possible effect of nonlinearities in variance on the mean, we also considered the model in equation (2) with GARCH(1,1) errors (STAR-GARCH). Thus $\varepsilon_{t+1} = \eta_{t+1} \sqrt{h_{t+1}}$, where

$$h_{t+1} = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_t \quad (3)$$

where $\alpha_0 > 0$, $\alpha_1, \alpha_2 \geq 0$, η_{t+1} is an i.i.d. process with zero mean and unit variance.

Markov switching models

The main feature of an autoregressive Markov switching (MS) model is the possibility for some of the parameters to switch across different regimes or states according to a Markov process governed by a state variable denoted by s_t (see Hamilton, 1989). A first-order autoregressive MS model has the following specification:

$$r_{t+1} = \alpha_{s_{t+1}} + \beta_{s_{t+1}} r_t + \varepsilon_{t+1} \quad (4)$$

where ε_{t+1} is i.i.d. $N(0, \sigma_{s_t}^2)$ and s_t is an unknown state variable that follows a first-order Markov chain, with a transition probability $\Pr(s_t = j | s_{t-1} = i) = p_{ij}$ that indicates the probability of switching from state i at time $t-1$ to state j at time t . For the sake of simplicity, we assume that there are only two states of the economy, denoted as state one and state two, as in Maheu and McCurdy (2000) and in Perez-Quiros and Timmerman (2000) (e.g. bull and bear markets; Chen and Shen, 2007) or low and high uncertainty in stock markets (Li, 2007). The ergodic probability of being in state one is given by $(1 - p_{22}) / (2 - p_{11} - p_{22})$. The MS model (4) generates nonlinearities in mean and also allows for nonlinearities in variance since volatility can change across different states of the economy.

The unknown parameters, $\theta = (\alpha_1, \alpha_2, \beta_1, \beta_2, \sigma_1, \sigma_2, p_{11}, p_{22})$, are estimated by maximum likelihood using the algorithm discussed in Hamilton (1989, 1994). For an observed value of r_t the likelihood can be written as

$$f(r_t | \Omega_{t-1}; \theta) = \sum_{i=1}^2 g(r_t | s_t = i, \Omega_{t-1}; \theta) \Pr(s_t = i | \Omega_{t-1}; \theta) \quad (5)$$

where $g(r_t | s_t = i, \Omega_{t-1}; \theta) = 1 / \sqrt{2\pi} \sigma_i \exp(r_t - \alpha_i - \beta_i r_{t-1} / 2\sigma_i)^2$ for $i = 1, 2$ is the likelihood value for an observed value of r_t in a given regime, and $\Pr(s_t = i | \Omega_{t-1}; \theta)$ is the conditional probability of being in state i at time t given all the information at time $t-1$, Ω_{t-1} . From the total probability theorem, the conditional state probabilities and densities can be obtained recursively:

$$P(s_t = i | \Omega_{t-1}; \theta) = \sum_{j=1}^2 P(s_t = i | s_t = j, \Omega_{t-1}; \theta) P(s_t = j | \Omega_{t-1}; \theta) \quad (6)$$

The updated regimen probabilities can be determined from the components of the conditional likelihood in (5):

$$\Pr(s_t = i | \Omega_t; \theta) = \frac{g(r_t | s_t = i, \Omega_{t-1}; \theta) \Pr(s_t = i | \Omega_{t-1}; \theta)}{\sum_{i=1}^2 g(r_t | s_t = i, \Omega_{t-1}; \theta) \Pr(s_t = i | \Omega_{t-1}; \theta)} \quad (7)$$

For given initial values of the vector of parameters, equations (6) and (7) can be iterated so as to recursively derive the state probabilities and obtain the parameters of the likelihood function:

$$\log L(r_1, r_2, \dots, r_T; \theta) = \sum_{t=1}^T \log(f(r_t | \Omega_{t-1}; \theta)) \quad (8)$$

The unconditional forecast of r_{t+1} based on information at t is related to the conditional forecasts:

$$E(r_{t+1} | r_t; \theta) = \sum_{j=1}^2 E(r_{t+1} | s_{t+1} = j; \theta) \Pr(s_{t+1} = j | \Omega_t; \theta) \quad (9)$$

Nonparametric kernel regression model

We can use nonparametric kernel regression (KR) when nonlinearity in the conditional mean cannot be characterized explicitly. In such cases the return conditional mean is specified in a general form as

$$E(r_{t+1} | I_t) = g(r_t, r_{t-1}, \dots, r_{t-p+1}) \quad (10)$$

where p is the number of lagged stock returns and where the function $g(\cdot)$ can be approximated locally at each point by a linear function.

Specifically, given a sample $\{(\mathbf{x}_i, y_i)\}_{i=1}^{T-p}$ with $\mathbf{x}_i = (r_i, r_{i+1}, \dots, r_{i+p-1})'$, $y_i = r_{i+p}$ the linear local polynomial regression at a point $\mathbf{x}_t = r_t$ results from the solution of the problem (see Fan and Gijbels, 1996):

$$\min_{\alpha_t, \beta_t} \sum_{i=1}^{T-p} [y_i - \alpha_t - \beta_t'(\mathbf{x}_t - \mathbf{x}_i)]^2 k_{\mathbf{H}}(\mathbf{x}_t - \mathbf{x}_i) \quad (11)$$

where $k: \mathbb{R}^d \rightarrow \mathbb{R}$ is a kernel function with $k_{\mathbf{H}}(\mathbf{u}) = |\mathbf{H}|^{-1} k(\mathbf{H}^{-1}\mathbf{u})$, and where \mathbf{H} is a window matrix $p \times p$, symmetrical and positive definite. The solution to the problem in (11) is

$$\hat{g}(\mathbf{x}_t) = \hat{\alpha}_t = \mathbf{e}_1' (\mathbf{X}_t' \mathbf{W}_t \mathbf{X}_t)^{-1} \mathbf{X}_t' \mathbf{W}_t \mathbf{y} \equiv \mathbf{s}_t' \mathbf{y}$$

where $\mathbf{e}_1 = (1, 0, \dots, 0)^T$, $\mathbf{y} = (y_1, \dots, y_{T-p})'$, $\mathbf{W}_t = \text{diag}\{k_{\mathbf{H}}(\mathbf{x}_t - \mathbf{x}_1), \dots, k_{\mathbf{H}}(\mathbf{x}_t - \mathbf{x}_n)\}$ and where \mathbf{X}_t is the design matrix with rows $(1, (\mathbf{x}_t - \mathbf{x}_i)')$, $i = 1, \dots, T-p$. It can be easily demonstrated that this estimator takes the following general form:

$$\hat{g}(\mathbf{x}_t) = \sum_{i=1}^{T-p} \omega_i(\mathbf{x}_t) k_{\mathbf{H}}(\mathbf{x}_t - \mathbf{x}_i)$$

where $\omega_i(\mathbf{x}_t)$ is a linear polynomial in $(\mathbf{x}_t - \mathbf{x}_i)$ (constant for the Nadaraya–Watson estimator used in this work). Hence the degree of smoothing of the estimator depends essentially on the degree of smoothing of the function $k_{\mathbf{H}}$, which in turn depends on the bandwidth parameter h controlling the size of the local neighbourhood used for estimating $g(\mathbf{x}_t)$.

Artificial neural network models

As an alternative to nonparametric nonlinear conditional mean, we considered ANNs, which have proven to be useful in capturing nonlinearity-in-mean for forecasting financial time series. ANNs are a universal approximator in a wide variety of nonlinear patterns (see Hornik *et al.*, 1990) and generate

good predictions (Swanson and White, 1995, 1997). The basic structure of neural networks combines many basic nonlinear functions via a multilayer structure, where there is at least one hidden layer between inputs and outputs. The idea is that explanatory variables simultaneously activate the units in the hidden layer through some function, and output is produced subsequently from the units in the hidden layer through another function. The specific type of ANN employed in this study is the multilayer perceptron (MLP) model, the most basic but perhaps most widely used neural network in economic and financial applications. Hence, MLP(p, q):

$$f(\mathbf{x}_t; \boldsymbol{\theta}) = \sum_{j=1}^q c_j \psi(\mathbf{w}'_j \mathbf{x}_t + w_{j0}) + c_0$$

where ψ is a sigmoid function (typically, a logistic or hyperbolic tangent function).

The universal approximation properties of MLP (see Leshno *et al.*, 1993) neural networks permit us to approximate any continuous or integrable function (the universal approximation property). In order to ensure consistency in a stochastic environment, in addition to considering the error associated with the approximation of the regression function via a finite number of parameters, it was necessary to consider the estimation error arising from the use of a limited quantity of data. The consistency of the MLP neural networks for different hypotheses was thus obtained (see Krzyzak *et al.*, 1996; Fine, 1999), and specifically for dependent observations (Chen and White, 1999). When the series responds to an autoregressive model, then

$$y_t = f(\mathbf{x}_t; \boldsymbol{\theta}) + \varepsilon_t$$

where $f(\mathbf{x}_t; \boldsymbol{\theta})$ is a neural network and ε_t is, for example, white noise. Trapletti *et al.* (2000) demonstrated the stationarity and strongly mixing nature of the series $\{y_t\}$, as also the consistency and asymptotic normality of the least squares estimator for a hypothesis that ensures the identifiability of the network parameters (Hwang and Ding, 1997).

MLP training, which usually uses the squared loss, is performed through nonlinear optimization algorithms. In this research we used the Bayesian algorithm proposed by Foresee and Hagan (1997) as it is less dependent on expert criteria.

Support vector machine for regression

Another alternative to the previous regression models is the support vector machine (SVM) (Vapnik, 1998; Schölkopf and Smola, 2002). The SVMs for regression are linear models obtained in a new feature space \mathcal{X} as a result of a transformation $\boldsymbol{\varphi} : \mathbb{R}^p \rightarrow \mathcal{X}$ of the input space, in which an inner product is defined through a positive definite function (kernel), $\langle \boldsymbol{\varphi}(\mathbf{x}_i), \boldsymbol{\varphi}(\mathbf{x}_t) \rangle = k(\mathbf{x}_i, \mathbf{x}_t)$. SVMs for regression have the following general formulation:

$$y_t = f(\mathbf{x}_t) = \langle \mathbf{w}, \boldsymbol{\varphi}(\mathbf{x}_t) \rangle + b$$

Given a sample, the parameters \mathbf{w} , b in the SVM are estimated as the solution to the following regularization problem:

$$\min_{\mathbf{w}, b} \left\{ \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^{T-p} \ell(y_i, f(\mathbf{x}_i)) \right\} \quad (12)$$

where C is a regularizing constant and ℓ is the ε -insensitive loss:

$$\ell(y, f(\mathbf{x})) = |y - f(\mathbf{x})|_{\varepsilon} = \max\{0, (|y - f(\mathbf{x})| - \varepsilon)\}$$

The only solution to the problem in equation (12) is a linear combination of key points of the sample (the support vectors), $\mathbf{w} = \sum_{i \in \text{s.v.}} \alpha_i \boldsymbol{\varphi}(\mathbf{x}_i)$, in such a way that the SVM results as

$$f(\mathbf{x}_t) = \sum_{i \in \text{s.v.}} \alpha_i \langle \boldsymbol{\varphi}(\mathbf{x}_i), \boldsymbol{\varphi}(\mathbf{x}_t) \rangle + b = \sum_{i \in \text{s.v.}} \alpha_i k(\mathbf{x}_i, \mathbf{x}_t) + b$$

SVMs share the general form of radial basis function neural networks, which are universal approximators to continuous or integrable functions (e.g. Park and Sandberg, 1993). Consistency results also exist (see Bousquet and Elisseeff, 2002; Steinwart, 2002).

Forecasting and evaluation method

We obtained in-sample and out-of-sample one-step-ahead forecasts that were composed of the estimated parameters of the model and lagged returns. Below we describe recursive parameter estimation and the corresponding predictions. To assess predictive ability, we evaluated different forecast models using different measures, given the difficulty in specifying a universally acceptable forecast evaluation criterion. First, forecast accuracy was evaluated by calculating the MSE and MAE. Second, as a measure of the market timing ability of the model we showed the proportion of times SIGNs were correctly predicted. Finally, we assessed directional prediction accuracy with the DA test devised by Pesaran and Timmermann (1992). The DA test is given by

$$\text{DA} = \frac{\text{SR} - \text{SRI}}{\sqrt{\text{var}(\text{SR}) - \text{var}(\text{SRI})}}$$

where SR is the fraction of return forecasts that have the same direction of change as the corresponding realizations, $\text{SRI} = P\hat{P} + (1 - P)(1 - \hat{P})$, with P the fraction of times that the realized return is positive and \hat{P} the proportion of return forecasts that are positive, $\text{var}(\text{SRI}) = n^{-1}(2P - 1)^2 \hat{P}(1 - \hat{P}) + n^{-1}(2\hat{P} - 1)^2 P(1 - P) + 4n^{-2}P\hat{P}(1 - P)(1 - \hat{P})$, and $\text{var}(\text{SR}) = n^{-1}\text{SRI}(1 - \text{SRI})$. The DA test is asymptotically distributed as standard normal under the null hypothesis of correct directional prediction. Finally, we employed the DM test to compare the predictive ability of pairs of competing models. Given the forecast accuracy for an arbitrary loss function (i.e. the MAE), the statistic to test the null of equal loss differential between two models was $\text{DM} = \bar{d} / \sqrt{\text{var}(\bar{d})}$, where $\bar{d} = 1/T \sum_{t=1}^T d_t$, d_t is the loss differential at time t , and $\text{var}(\bar{d})$ is the long-run variance estimated as $\text{var}(\bar{d}) = 1/T (\gamma_0 + 2 \sum_{j=1}^q \omega_j \gamma_j)$, $\gamma_j = \text{cov}(d_t, d_{t-j})$, $\omega_j = 1 - j/(q + 1)$. The DM test statistic has an asymptotic standard normal distribution.

The market timing ability of forecasting models was also compared with a simple buy-and-hold (B&H) intraday investment strategy for out-of-sample forecasting. This strategy implements a naïve allocation that consist of maintaining a 100% stock index or cash if the quantity predicted exceeds a threshold given by the transaction costs. Hence the forecast by each predictor determines the position to be taken for the following time period. Thus, if the share price is expected to fall below a threshold on the basis of a particular predictor, then shares are sold if the agent holds assets or they are not bought if the agent holds cash. In contrast, if the share price is expected to rise above a threshold on the basis of a particular predictor, then shares are bought if the agent holds cash or they are not sold if the agent holds assets. The threshold is determined by transaction costs, which are assumed to be low

for intraday transactions, as otherwise commissions would erode profits. The value of these costs were determined in our training set as the mean value of rises and falls divided by 1000 (around 3 bps).

DATA AND RESULTS

Data

Using intraday data for the S&P 500 index obtained from Bloomberg and covering the period 2 June 2003 to 30 September 2003, we constructed intervals of 5, 10, 20, 30 and 60 minutes. The value of the index was taken as the last index value closest to the end of the time interval (for example, for the 5-minute interval, the closest index value to 9:35 a.m., 9:40 a.m., etc.). Intraday stock returns were computed as the first difference of the logarithmic stock price index for each time interval; the first interval of each trading day was discarded, as it would have correlated with a lagged interval from the previous trading day. Similarly, taken as an explanatory variable was the lagged stock returns variable for the same trading day. Obviously, the size of the sample decreased as the time interval grew—from more than 6000 items for a 5-minute interval to less than 350 for a 60-minute interval.

Results

The in-sample and out-of-sample forecasting ability of the different linear and nonlinear models described above were assessed in terms of statistical accuracy and economic criteria. The analysis was based on one-step-ahead forecasting. Once each model had been estimated, we constructed the in-sample and out-of-sample forecasts without updating the parameters of the model that were obtained from the training set.

Table I shows the results of the goodness of forecasts for the different models and time periods and for the corresponding forecast horizons. The main conclusions are two.

First, the results were heterogeneous in terms of the log-likelihood (LogLik), MSE, MAE, SIGN and DA test measures. According to LogLik, the most useful model was the MS for all the time period returns and both in-sample and out-of-sample evaluations, with the exception of the out-of-sample 60-minute and in-sample 10-minute time horizons, where the MLP and KR models outperformed the MS model. In terms of MSE, the MLP tended to be the best model, even though it was occasionally bettered by other models, such as the STAR-GARCH in out-of-sample 30-minute and 20-minute horizons, the KR model in in-sample 5-minute and 10-minute horizons and the MS model for the out-of-sample 10-minute horizon. As for MAE, the results were more dependent on the time period, with the MLP and STAR-GARCH models appearing to be the best models for time periods longer than 20 minutes, and the SVM and KR models for shorter periods. Taking into account the proportion of times that SIGNs were correctly predicted, the MLP model generally performed far better than the other models for time periods longer than 20 minutes, whereas the MS and SVM models produced the best results for 5-minute returns and the STAR-GARCH model provided reasonable out-of-sample results for 20-minute and 30-minute returns. As for the DA test null hypothesis (in which forecast and realizations are independent at any significance level), the MS and MLP models rejected the null hypothesis for in-sample observations but not for out-of-sample returns. In general, of all the models only the RW, MLP and KR models for 5-minute returns rejected the null DA test hypothesis.

Second, the DM forecast accuracy test for the MAE loss function is shown as in-sample in the upper triangular matrix for each time period returns but as out-of-sample in the lower triangular matrix in Table II. Three comments are in order. First, comparing the RW model with all the other models, the DM test was positive and statistically significant at the 5% significance level for all the

Table I. Goodness of forecast in-sample and out-of-sample for all models and time periods

| Model/criteria | RW | AR(1) | AR(1) -GARCH(1,1) | MS (AR(1)) | ST (AR(1)) | ST(AR(1)) -GARCH(1,1) | MLP | SVM | KR |
|--|---|---------------------|-----------------------|----------------------------------|--------------------|--------------------------|-------------------------|-----------------------|--------------------------|
| <i>Panel A: 5-minute interval. # Obs. in-sample, 4527; out-of sample, 1940</i> | | | | | | | | | |
| LogLik. | Train 1702.75 Test 1125.93 | 3559.94 1897.40 | 4139.71 2171.31 | 4249.04 2214.11 | 3568.83 1901.39 | 4136.29 2169.98 | 3679.38 1925.78 | 3613.80 1914.84 | 3859.43 1847.88 |
| MSE ($\times 100$) | Train 4528 Test 1940 | 2.760 1.632 | 1.215 0.749 | 1.225 0.751 | 1.178 0.752 | 1.210 0.747 | 1.232 0.753 | 1.152 0.738 | 1.187 0.738 |
| MAE ($\times 100$) | Train 10.980 Test 8.847 | 7.358 5.904 | 7.344 5.893 | 7.255 5.897 | 7.363 5.890 | 7.350 5.898 | 7.273 5.890 | 7.314 5.882 | 7.069 6.117 |
| SIGN | Train 0.48 Test 0.48 | 0.50 0.50 | 0.49 0.50 | 0.53 0.51 | 0.50 0.50 | 0.48 0.48 | 0.48 0.48 | 0.51 1.78 | 0.52 3.13 |
| DA test | Train -2.40 Test -2.14 | -0.22 0.43 | -1.04 0.16 | 4.61 0.61 | 0.43 0.29 | -2.78 -1.51 | -2.95 -2.14 | 1.78 0.56 | 3.13 -2.86 |
| <i>Panel B: 10-minute interval. # Obs. in-sample, 2205; out-of sample, 945</i> | | | | | | | | | |
| LogLik. | Train 34.07 Test 262.58 | 1053.38 643.15 | 1261.69 733.89 | 1315.27 749.82 | 1156.78 667.55 | 1258.34 734.77 | 1166.93 665.70 | 1158.34 667.07 | 1319.34 642.93 |
| MSE ($\times 100$) | Train 5.677 Test 2.698 | 2.252 1.338 | 2.307 1.301 | 2.016 1.301 | 2.050 1.308 | 2.363 1.308 | 2.032 1.320 | 2.048 1.308 | 1.769 1.479 |
| MAE ($\times 100$) | Train 15.758 Test 12.252 | 10.576 8.335 | 10.449 8.243 | 10.076 8.241 | 10.287 8.267 | 10.495 8.302 | 10.216 8.268 | 9.642 8.756 | 9.832 8.758 |
| SIGN | Train 0.47 Test 0.47 | 0.51 0.52 | 0.50 0.51 | 0.55 0.51 | 0.50 0.48 | 0.49 0.47 | 0.51 0.52 | 0.56 0.47 | 0.56 0.47 |
| DA test | Train -2.43 Test -1.83 | 0.74 1.12 | 0.19 0.67 | 4.38 0.35 | -0.06 -1.25 | -1.16 -1.89 | 0.48 0.42 | 6.06 -1.57 | 6.06 -1.57 |
| <i>Panel C: 20-minute interval. # Obs. in-sample, 941; out-of sample, 403</i> | | | | | | | | | |
| LogLik. | Train -63.49 Test -3.94 | 264.08 160.60 | 282.25 164.54 | 309.91 189.87 | 264.17 161.09 | 279.52 167.62 | 269.95 156.89 | 262.94 161.73 | 263.61 161.96 |
| MSE ($\times 100$) | Train 6.701 Test 5.928 | 3.340 2.553 | 3.335 2.575 | 3.317 2.533 | 3.340 2.551 | 3.352 2.510 | 3.299 2.623 | 3.348 2.532 | 3.344 2.529 |
| MAE ($\times 100$) | Train 19.567 Test 17.945 | 13.737 11.530 | 13.755 11.587 | 13.701 11.489 | 13.735 11.534 | 13.719 11.441 | 13.640 11.621 | 13.715 11.464 | 13.724 11.473 |
| SIGN | Train 0.49 Test 0.46 | 0.49 0.49 | 0.49 0.46 | 0.51 0.47 | 0.49 0.48 | 0.49 0.52 | 0.52 0.51 | 0.48 0.50 | 0.49 0.49 |
| DA test | Train -0.36 Test -1.44 | -0.62 -0.54 | -0.82 -1.54 | 0.76 -1.14 | -0.69 -0.84 | -0.89 0.74 | 1.02 0.35 | -1.55 -0.05 | -0.69 -0.25 |

Table I. Continued

| Model/criteria | RW | AR(1) | AR(1) -GARCH(1,1) | MS (AR(1)) | ST (AR(1)) | ST(AR(1)) -GARCH(1,1) | MLP | SVM | KR |
|---|---------------|--------|----------------------|---------------|---------------|--------------------------|---------------|--------|-------------|
| <i>Panel D: 30-minute interval. # Obs. in-sample, 647; out-of sample, 277</i> | | | | | | | | | |
| LogLik. | Train -169.47 | 79.82 | 79.50 | 102.44 | 80.06 | 80.77 | 90.62 | 79.72 | 79.80 |
| | Test -22.20 | 84.91 | 87.13 | 93.79 | 85.05 | 88.06 | 83.08 | 84.58 | 84.96 |
| MSE ($\times 100$) | Train 9.886 | 4.575 | 4.575 | 4.498 | 4.571 | 4.583 | 4.424 | 4.576 | 4.575 |
| | Test 6.292 | 2.899 | 2.894 | 2.902 | 2.905 | 2.879 | 3.009 | 2.909 | 2.897 |
| MAE ($\times 100$) | Train 23.949 | 16.003 | 16.008 | 15.861 | 16.027 | 16.026 | 15.778 | 15.980 | 16.013 |
| | Test 19.521 | 13.151 | 13.145 | 13.154 | 13.176 | 13.125 | 13.431 | 13.166 | 13.150 |
| SIGN | Train 0.48 | 0.52 | 0.52 | 0.55 | 0.52 | 0.52 | 0.54 | 0.52 | 0.52 |
| | Test 0.49 | 0.52 | 0.53 | 0.51 | 0.52 | 0.53 | 0.53 | 0.49 | 0.53 |
| DA test | Train -1.02 | 1.02 | 1.02 | 2.70 | 0.79 | 1.02 | 2.16 | 0.87 | 1.02 |
| | Test -0.28 | 1.07 | 1.07 | 0.39 | 0.62 | 1.07 | 0.96 | -0.39 | 1.07 |
| <i>Panel E: 60-minute interval. # Obs. in-sample, 235; out-of sample, 101</i> | | | | | | | | | |
| LogLik. | Train -108.10 | -33.91 | -33.15 | -18.57 | -33.72 | -34.06 | -27.20 | -33.88 | -33.9 |
| | Test -33.27 | -6.73 | -8.12 | -5.68 | -6.10 | -6.15 | -4.46 | -6.05 | -6.21 |
| MSE ($\times 100$) | Train 14.692 | 7.729 | 7.735 | 7.550 | 7.801 | 7.818 | 7.380 | 7.812 | 7.813 |
| | Test 10.854 | 6.605 | 6.694 | 6.578 | 6.493 | 6.509 | 6.324 | 6.495 | 6.519 |
| MAE ($\times 100$) | Train 30.347 | 20.683 | 20.622 | 20.090 | 20.705 | 20.928 | 19.439 | 20.659 | 20.644 |
| | Test 24.471 | 18.962 | 19.141 | 18.793 | 18.798 | 18.889 | 18.396 | 18.684 | 18.710 |
| SIGN | Train 0.45 | 0.50 | 0.53 | 0.58 | 0.49 | 0.48 | 0.60 | 0.50 | 0.50 |
| | Test 0.46 | 0.41 | 0.42 | 0.47 | 0.43 | 0.46 | 0.50 | 0.43 | 0.44 |
| DA test | Train -1.68 | 0.07 | 1.00 | 2.64 | -0.33 | -0.74 | 3.35 | 0.07 | 0.07 |
| | Test -0.79 | -1.70 | -1.52 | -0.62 | -1.33 | -0.79 | 0.09 | -1.33 | -1.15 |

Note: The best model for all criteria is marked in bold, except for DA, where bold indicates rejection of the null hypothesis.

Table II. Diebold and Mariano test for MAE (upper triangle: in-sample; lower triangle: out-of-sample)

| | RW | AR(1) | AR(1)-GARCH(1,1) | MS(AR(1)) | ST(AR(1)) | ST(AR(1))-GARCH(1,1) | MLP | SVM | KR |
|------------------------------------|------|-------|------------------|-----------|-----------|----------------------|-------|-------|-------|
| <i>Panel A: 5-minute interval</i> | | | | | | | | | |
| RW | | 6.17 | 6.23 | 6.17 | 6.15 | 6.24 | 6.03 | 6.15 | 5.93 |
| AR(1) | 4.18 | | 0.83 | 4.50 | -0.64 | 0.45 | 1.35 | 1.66 | 2.64 |
| AR(1)-GARCH(1,1) | 4.19 | 0.82 | | 3.12 | -0.91 | -2.02 | 1.06 | 1.12 | 2.40 |
| MS(AR(1)) | 4.19 | 0.48 | -0.82 | | -4.69 | -3.13 | -0.32 | -2.01 | 1.87 |
| ST(AR(1)) | 4.17 | 1.21 | 0.19 | 0.34 | | 0.61 | 1.44 | 1.70 | 2.72 |
| ST(AR(1))-GARCH(1,1) | 4.19 | 0.39 | -1.84 | -0.49 | -0.48 | | 1.11 | 1.24 | 2.41 |
| MLP | 4.18 | 0.98 | 0.31 | 0.49 | 0.06 | 0.70 | | -0.99 | 3.54 |
| SVM | 4.18 | 1.59 | 1.14 | 1.37 | 0.50 | 1.40 | 0.67 | | 2.71 |
| KR | 4.26 | -2.11 | -2.26 | -2.26 | -2.17 | -2.25 | -2.25 | -2.27 | |
| <i>Panel B: 10-minute interval</i> | | | | | | | | | |
| RW | | 4.29 | 4.37 | 4.24 | 4.17 | 4.39 | 4.14 | 4.07 | 4.25 |
| AR(1) | 2.93 | | 2.10 | 3.36 | 1.63 | 1.02 | 1.70 | 2.47 | 3.45 |
| AR(1)-GARCH(1,1) | 2.97 | 1.61 | | 2.32 | 0.84 | -1.63 | 1.02 | 2.07 | 2.84 |
| MS(AR(1)) | 2.98 | 1.35 | -0.75 | | -2.10 | -2.30 | -1.16 | 1.71 | 2.47 |
| ST(AR(1)) | 2.96 | 1.23 | -2.01 | -1.27 | | -0.99 | 1.82 | 2.98 | 3.73 |
| ST(AR(1))-GARCH(1,1) | 2.99 | 0.40 | -1.89 | -2.19 | -0.89 | | 1.14 | 2.10 | 2.81 |
| MLP | 2.98 | 0.98 | -1.33 | -1.18 | -0.04 | 1.12 | | 3.17 | 3.25 |
| SVM | 2.98 | -2.12 | -2.64 | -2.66 | -2.53 | -2.69 | -2.65 | | -1.01 |
| KR | 2.98 | -2.13 | -2.64 | -2.66 | -2.53 | -2.70 | -2.65 | -1.47 | |
| <i>Panel C: 20-minute interval</i> | | | | | | | | | |
| RW | | 3.01 | 3.01 | 3.01 | 3.01 | 3.00 | 3.01 | 2.99 | 3.00 |
| AR(1) | 3.79 | | -0.99 | 1.79 | 0.09 | 0.38 | 1.64 | 0.54 | 0.59 |
| AR(1)-GARCH(1,1) | 3.80 | -1.94 | | 1.80 | 0.85 | 0.58 | 1.66 | 0.72 | 0.78 |
| MS(AR(1)) | 3.78 | 1.85 | 2.26 | | -1.47 | -0.40 | 1.12 | -0.36 | -0.85 |
| ST(AR(1)) | 3.79 | -0.36 | 1.76 | -1.86 | | 0.39 | 1.66 | 0.48 | 0.48 |
| ST(AR(1))-GARCH(1,1) | 3.76 | 1.62 | 1.79 | -1.52 | 1.71 | | 1.33 | 0.17 | -0.17 |
| MLP | 3.79 | -1.13 | -0.45 | -1.52 | -1.08 | -1.65 | | -1.14 | -1.54 |
| SVM | 3.78 | 1.54 | 2.01 | 0.96 | 1.54 | -0.56 | 1.62 | | -0.37 |
| KR | 3.77 | 1.93 | 1.96 | 0.70 | 1.89 | -1.16 | 1.57 | -0.24 | |
| <i>Panel D: 30-minute interval</i> | | | | | | | | | |
| RW | | 4.76 | 4.76 | 4.78 | 4.76 | 4.77 | 4.78 | 4.75 | 4.76 |
| AR(1) | 3.22 | | -0.66 | 3.04 | -1.03 | -0.97 | 1.78 | 1.17 | -1.66 |
| AR(1)-GARCH(1,1) | 3.23 | 0.63 | | 3.10 | -0.83 | -0.95 | 1.82 | 1.12 | -1.13 |
| MS(AR(1)) | 3.21 | -0.07 | -0.27 | | -3.25 | -3.04 | 0.69 | -2.43 | -3.19 |

Table II. Continued

| | RW | AR(1) | AR(1)-GARCH(1,1) | MS(AR(1)) | ST(AR(1)) | ST(AR(1))-GARCH(1,1) | MLP | SVM | KR |
|------------------------------------|------|-------|------------------|-----------|-----------|----------------------|-------|-------|-------|
| ST(AR(1)) | 3.22 | -1.27 | -1.66 | -0.64 | | 0.04 | 1.96 | 1.77 | 0.59 |
| ST(AR(1))-GARCH(1,1) | 3.23 | 1.09 | 0.97 | 0.59 | 1.41 | | 1.91 | 1.09 | 0.67 |
| MLP | 3.17 | -1.33 | -1.37 | -1.35 | -1.22 | -1.44 | | -1.60 | -1.85 |
| SVM | 3.21 | -0.67 | -0.77 | -0.91 | 0.33 | -0.93 | 1.28 | | -1.30 |
| KR | 3.23 | 0.29 | -0.49 | 0.11 | 1.36 | -1.25 | 1.33 | 0.59 | |
| <i>Panel E: 60-minute interval</i> | | | | | | | | | |
| RW | | 3.04 | 3.04 | 3.06 | 3.05 | 3.05 | 3.05 | 3.04 | 3.04 |
| AR(1) | 1.86 | | 1.22 | 2.28 | -0.19 | -1.60 | 2.26 | 0.24 | 0.38 |
| AR(1)-GARCH(1,1) | 1.83 | -1.71 | | 2.02 | -0.60 | -1.60 | 2.18 | -0.28 | -0.18 |
| MS(AR(1)) | 1.84 | 0.93 | 1.54 | | -2.43 | -2.68 | 1.65 | -2.53 | -2.50 |
| ST(AR(1)) | 1.88 | 1.07 | 1.46 | -0.03 | | -2.26 | 2.31 | 0.68 | 0.87 |
| ST(AR(1))-GARCH(1,1) | 1.89 | 0.41 | 0.99 | -0.39 | -0.91 | | 2.47 | 2.00 | 2.02 |
| MLP | 1.92 | 1.18 | 1.44 | 0.88 | 0.88 | 1.01 | | -2.30 | -2.29 |
| SVM | 1.88 | 1.47 | 1.65 | 0.73 | 1.38 | 1.33 | -0.93 | | 0.94 |
| KR | 1.88 | 1.44 | 1.66 | 0.62 | 1.16 | 1.16 | -0.69 | -1.00 | |

time periods and for in-sample and out-of-sample returns (10% significance level for 60 minute out-of-sample returns). Consequently, the linear AR model (any nonlinear model, in fact) improved on the RW model forecasts. Second, taking a closer look at the in-sample relative performance of the linear AR and nonlinear models, the MS model surpassed the linear AR model in all the time periods, the STAR and STAR-GARCH models were unable to improve on the AR model, the MLP model outperformed the AR model for long-horizon returns (60 minutes), and the SVM and KR models improved on the AR model for 5-minute and 10-minute returns but remained equal for longer time period returns. Of the nonlinear models, the MS model generally improved on the STAR and STAR-GARCH models, on the SVM model for 5-minute periods, but was indifferent for the MLP model and poorer than the KR and SVM models for longer time period returns. In general, the STAR and STAR-GARCH models did not outperform any other nonlinear model, the MLP model outperformed the KR model for longer time periods but performed less well for shorter time periods, and the KR model improved on the SVM model for shorter time periods, but was indifferent for longer time periods. Finally, if we examine out-of-sample relative performance, only the SVM and KR models surpassed the linear AR model for 5-minute and 10-minute time periods; for all the other nonlinear models and time periods the DM test did not reject the null hypothesis. Comparing the nonlinear models, for 5-minute periods the KR model dominated all other models, for 10-minute periods the SVM and KR models performed best, and there was little difference for longer periods.

To sum up, there is strong statistical evidence that nonlinear models outperform linear models. Even when serial correlation persisted and weak market efficiency did not hold for periods of 5 and 10 minutes, nonlinear models outperformed the linear autoregressive model. However, no linear model clearly outperformed the other models on the basis of all the criteria. For short-term period returns of 10 minutes or less, the MS, KR and SVM models seemed to perform well. For medium-term periods of around 30 minutes the STAR-GARCH model gave reasonable results, while for longer periods the MLP model behaved reasonably well.

Given that statistical and economic criteria can lead to different outcomes (Leitch and Tanner, 1991; Satchell and Timmerman, 1995), we also assessed relative forecast performance in a simple trading strategy using economic criteria and the one-step-ahead forecast for different models. For an initial endowment of 100 monetary units, an investor had to decide whether to invest or to keep this wealth in cash or assets on the basis of the return forecast for the out-of-sample period. Table III shows the final wealth the investor would achieve considering different time periods and linear AR and nonlinear models, and Figure 1 shows wealth time evolution through the investment period for 5 and 20 minutes. For 5-minute periods, the MS and SVM models gave better returns than any other model. Surprisingly, like the STAR-GARCH model, KR—a good model on statistical grounds—gave negative returns for the out-of-sample period. For short time periods, the MLP model did not give

Table III. Cumulative return over the out-of-sample period for different time periods

| | Obs. | AR(1) | AR(1)- GARCH(1,1) | MS (AR(1)) | ST (AR(1)) | ST(AR(1))- GARCH(1,1) | MLP | SVM | KR |
|--------|------|--------|----------------------|---------------|---------------|--------------------------|---------------|---------------|---------------|
| 5 min | 1940 | 102.78 | 102.85 | 104.49 | 103.72 | 98.83 | 100.89 | 104.60 | 95.27 |
| 10 min | 945 | 99.49 | 100.67 | 101.54 | 100.71 | 100.33 | 100.67 | 102.22 | 99.45 |
| 20 min | 403 | 100.60 | 100.14 | 100.68 | 99.76 | 101.85 | 100.30 | 100.79 | 100.01 |
| 30 min | 277 | 105.02 | 105.06 | 102.25 | 103.01 | 105.06 | 103.25 | 102.06 | 105.06 |
| 60 min | 101 | 99.75 | 99.86 | 99.98 | 100.03 | 101.77 | 101.75 | 99.55 | 100.00 |

Note: The two best models on the basis of profitability for each time period are indicated in bold.

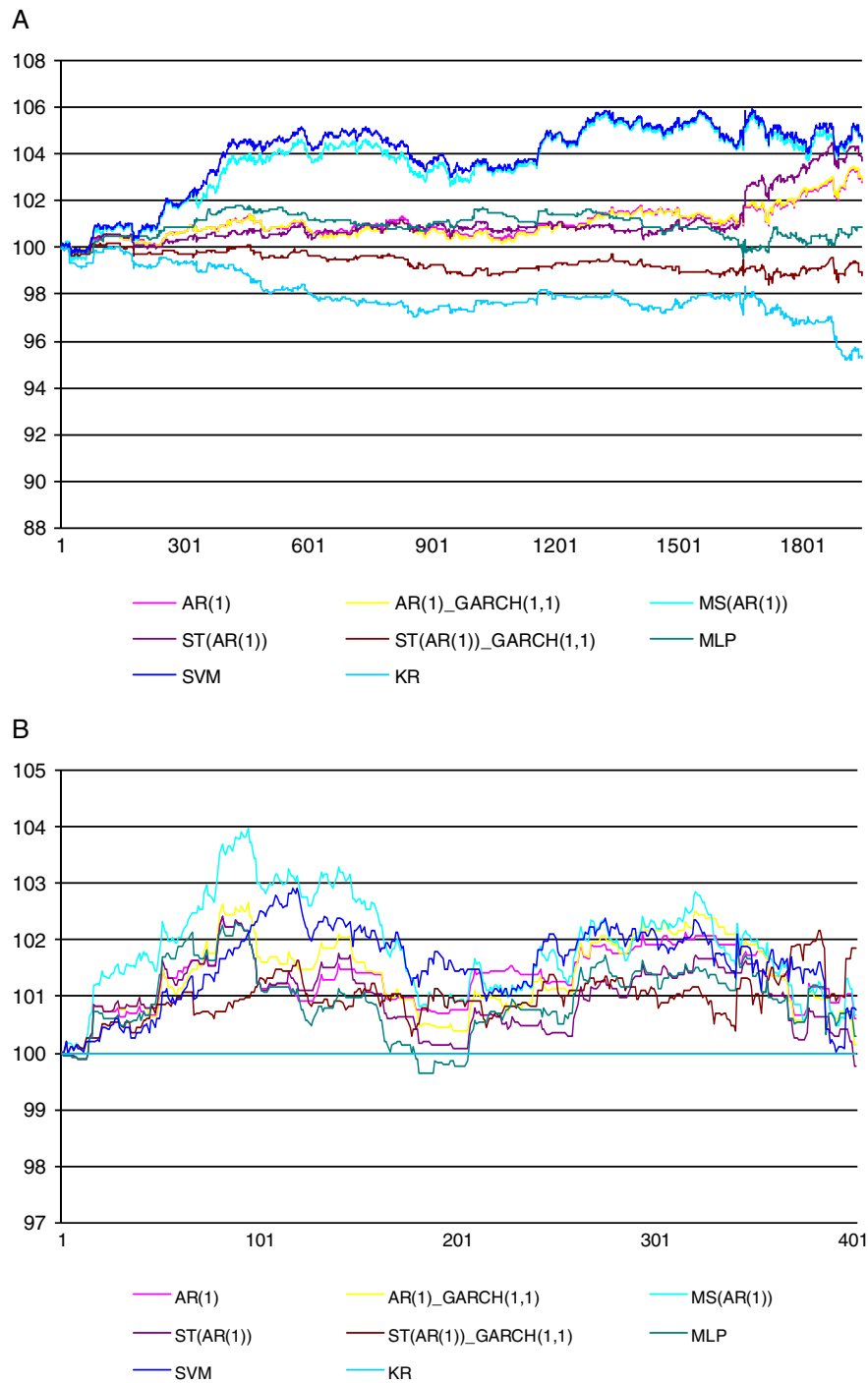


Figure 1. Cumulative return over the out-of-sample period for different forecasting models: (A) 5-minute interval; (B) 20-minute interval

good results. However, over longer time periods results changed. The SVM model remained profitable for returns of up to 20 minutes but decreased thereafter, and MS potency dissipated after 10 minutes. For time horizons greater than 20 minutes, the STAR-GARCH model performed best and the MLP model appeared to be useful for 60-minute returns. Worthy of note is the fact that, as periods lengthened, the profitability of the intraday investment strategies decreased, as a consequence of efficiency creation actions of traders expunging any predictability of returns over short periods of time. This conclusion is consistent with the empirical evidence, mentioned in the Introduction, showing that weak-form market efficiency is reached in a short period of time.

CONCLUSIONS

We performed an extensive evaluation of the in-sample and out-of-sample one-step-ahead forecasting performance of both linear and nonlinear models of stock market returns using high-frequency data. Using data for S&P 500 stock market returns for horizons that range from 5, 10, 20, 30 to 60 minutes, we showed that simple autoregressive and random walk linear models are surpassed by a wide range of nonlinear models, including smooth transition autoregressive, smooth transition autoregressive with GARCH errors, Markov switching, multilayer perceptron, nonparametric kernel regression and support vector machine models, which potentially capture nonlinearity-in-mean in intraday stock returns. Traditional statistical criteria suggest that nonlinearities-in-mean are relevant to forecasting intraday stock returns both in-sample and out-of-sample and for any intraday time return period. Of the nonlinear models, for short time periods of 5 minutes, the Markov switching model performed best for in-sample forecasting, whereas kernel regression was the best performer for out-of-sample forecasting. Despite return serial correlation receding and returns behaving as a random walk for more than 10 minutes, return predictability still persisted for up to 60 minutes according to nonlinear models. For the longer intraday time periods studied, smooth transition and neural network models appeared to be better performers, even though the Diebold–Mariano test was not conclusive on equal predictability among the models. We also evaluated linear and nonlinear models in terms of economic criteria, using a simple trading rule driven by model predictions and transaction cost. On economic grounds, trading rules based on Markov switching and support vector machine models were the most profitable for short time horizon returns, whereas smooth transition and neural networks behaved better for longer periods, even though profitability decreased as time elapsed.

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REFERENCES

- Avramov D, Chordia T, Goyal A. 2006. Liquidity and autocorrelation in individual stock returns. *Journal of Finance* **61**(5): 2365–2394.

- Bousquet G, Elisseeff A. 2002. Stability and generalization. *Journal of Machine Learning Research* **2**: 499–526.
- Busse JA, Green TC. 2002. Market efficiency in real time. *Journal of Financial Economics* **65**: 415–437.
- Chen S, Shen C. 2007. Evidence of the duration-dependence from the stock markets in the Pacific Rim economies. *Applied Economics* **39**: 1461–1474.
- Chen X, White H. 1999. Improved rates and asymptotic normality for nonparametric neural networks estimators. *IEEE Transactions on Information Theory* **45**: 682–691.
- Chew L. 1994. Shock treatment. *Risk* **7**: 63–70.
- Chordia T, Roll R, Subrahmanyam A. 2005. Evidence on the speed of convergence to market efficiency. *Journal of Financial Economics* **76**: 271–292.
- Clements MP, Taylor N. 2003. Evaluating interval forecasts of high-frequency financial data. *Journal of Applied Econometrics* **18**: 445–456.
- Diebold F, Mariano R. 1995. Comparing predictive accuracy. *Journal of Business and Economic Statistics* **13**(3): 253–263.
- Fan J, Gijbels I. 1996. *Local Polynomial Modeling and its Applications*. Chapman & Hall: London.
- Fine T. 1999. *Feedforward Neural Network Methodology*. Springer: Berlin.
- Foresee F, Hagan T. 1997. Gauss–Newton approximation to Bayesian regularization. In *Proceedings of the 1997 International Joint Conference on Neural Networks*; 1930–1935.
- Granger CWJ, Teräsvirta T. 1993. *Modelling Nonlinear Economic Relationships*. Oxford University Press: Oxford, UK.
- Hamilton JD. 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* **57**: 357–384.
- Hamilton JD. 1994. *Time Series Analysis*. Princeton University Press: Princeton, NJ.
- Hornik K, Stinchcombe M, White H. 1990. Universal approximation of an unknown mapping and its derivatives using multilayer feedforward networks. *Neural Networks* **3**: 551–560.
- Hsiec DA. 1989. Testing for nonlinear dependence in daily foreign exchange rates. *Journal of Business* **62**: 339–368.
- Hsiec DA. 1993. Implications of nonlinear dynamics for financial risk management. *Journal of Financial and Quantitative Analysis* **28**: 41–64.
- Hwang JTG, Ding AA. 1997. Prediction intervals for artificial neural networks. *Journal of the American Statistical Association* **92**: 748–757.
- Krzyzak A, Linder T, Lugosi G. 1996. Nonparametric estimation and classification using radial basis function nets and empirical risk minimization. *IEEE Transactions on Neural Networks* **7**: 475–487.
- Leitch G, Tanner JE. 1991. Economic forecast evaluation: profit versus the conventional error measures. *American Economic Review* **81**: 580–590.
- Leshno M, Lin VY, Pinkus A, Schocken S. 1993. Multilayer feedforward networks with a nonpolynomial activation function can approximate any function. *Neural Networks* **6**: 861–867.
- Li ML. 2007. Volatility states and international diversification of international stock markets. *Applied Economics* **39**: 1867–1876.
- Lo AW. 2004. The adaptive market hypothesis: market efficiency from an evolutionary perspective. *Journal of Portfolio Management* **30**: 15–29.
- Maheu JM, McCurdy TH. 2000. Identifying bull and bear markets in stock returns. *Journal of Business and Economic Statistics* **18**(1): 100–112.
- Park J, Sandberg IW. 1993. Approximation and radial-basis-function networks. *Neural Computation* **5**: 305–316.
- Perez-Quiros G, Timmerman A. 2000. Firm size and cyclical variations in stock returns. *Journal of Finance* **55**: 1229–1262.
- Pesaran MH, Timmermann A. 1992. A simple nonparametric test of predictive performance. *Journal of Business and Economic Statistics* **10**: 461–465.
- Satchell S, Timmerman A. 1995. An assessment of the economic value of non-linear foreign exchange rate forecasts. *Journal of Forecasting* **14**: 477–497.
- Schölkopf B, Smola AJ. 2002. *Learning with Kernels*. MIT Press: Cambridge, MA.
- Steinwart I. 2002. Support vector machines are universally consistent. *Journal of Complexity* **18**: 768–791.
- Swanson NR, White H. 1995. A model selection approach to assessing the information in the term structure using linear models and artificial neural networks. *Journal of Business and Economic Statistics* **13**(3): 265–275.
- Swanson NR, White H. 1997. Forecasting economic time series using flexible versus fixed specification and linear versus nonlinear econometric models. *International Journal of Forecasting* **13**: 439–461.

- Teräsvirta T, Tjøstheim D, Granger CWJ. 1994. Aspects of modeling nonlinear time series. In *Handbook of Econometrics*, Vol. IV, Engle RF, McFadden DL (eds). Elsevier: Amsterdam; 2919–2957.
- Timmermann A. 2008. Elusive return predictability. *International Journal of Forecasting* **24**: 1–18.
- Trapletti A, Leisch F, Hornik K. 2000. Stationarity and integrated autoregressive neural network processes. *Neural Computation* **12**: 2427–2450.
- Vapnik V. 1998. *Statistical Learning Theory*. Wiley: Chichester.

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