

# An Adaptive Neuro-Fuzzy System for Stock Portfolio Analysis

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We propose an adaptive neuro-fuzzy inference system (ANFIS) for stock portfolio return prediction. Previous work has shown that portfolio optimization can be improved by using predicted stock earnings rather than historical earnings. We show that predicted portfolio returns can be improved by using ANFIS and taking as input a variety of technical and fundamental attributes about various indices of the stock market. To generate membership functions, we use a robust noise rejection-clustering algorithm. The neuro-fuzzy model is tested on portfolios constituted from the Tehran Stock Exchange. In our experiments, the proposed method performs better in predicting the portfolio return than the classical Markowitz portfolio optimization method, a multiple regression, a neural network, and the Sugeno–Yasukawa method. © 2010 Wiley Periodicals, Inc.

## 1. INTRODUCTION

Portfolio selection is concerned with trying to assign one's capital among different stocks subject to the investment goal.<sup>1</sup> The problem was addressed by Markowitz<sup>2</sup>, and his mean–variance approach has served as a basis for the development of modern financial portfolio theory. Markowitz combined probability theory and optimization to model the behavior of the economic agents under uncertainty. When investments are exposed to uncertainties, the investment selection framework must include a quantitative measure of the uncertainty of obtaining the expected return.

In the Markowitz model, the time series of returns of each stock follows a normal distribution and its mean is a prediction of the stock's future return, its variance is as a measure of the stock's risk, and the covariance of each pair of time

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series is a measure of joint risk of each pair of stocks. After the appearance of Markowitz's portfolio selection model, many other models that use its fundamental assumption have appeared. Despite the wide adoption of the classical models of portfolio selection, their fundamental assumption is inconsistent with the data. The distribution of the series of returns often departs from normality, exhibiting kurtosis and skewness.<sup>3</sup> The use of a mean return as a prediction of a stock's future return leads to imprecise estimates of a future return and is detrimental to the performance of the model.<sup>4</sup>

Since the introduction of the mean-variance portfolio selection model, many extensions have been explored.<sup>5–7</sup> The prediction of future returns in the context of portfolio selection tends to use the same prediction method employed in the mean-variance model, i.e., the mean of past returns. However, the historical mean returns are inadequate predictions of future returns.<sup>4</sup> The use of better prediction methods and associated risk measures can be used to improve portfolio selection models.

The forecasting of time series was traditionally tackled by the linear methods of time series analysis. Nevertheless, in recent years, soft computing techniques have been applied extensively to financial investing. In particular, artificial neural networks have been applied to financial investing.<sup>8–10</sup> Getting a neural network to handle the tremendous noise and complex dimensionality of stock price data has, however, often proven more of an art than a science.

The literature has shown that fuzzy systems may be used as an effective tool for many applications and have superiority to the classical techniques in many cases. Wang<sup>11</sup> constructed a data mart to reduce the size of stock data and combined fuzzification techniques with grey theory to develop a fuzzy grey prediction. Jilani and Burney<sup>12</sup> presented a simple time-variant fuzzy time series forecasting method. Chang and Liu<sup>13</sup> introduced a Takagi-Sugeno-Kang (TSK) type fuzzy rule based system for stock price prediction. Fazel Zarandi et al.<sup>14</sup> proposed an interval type-2 fuzzy rule based system for stock price prediction.

Freitas et al.<sup>15</sup> investigated the normality of the errors of weekly stock returns predicted by an autoregressive neural network. They found more evidence of normality on these errors of prediction than on the series of returns, and subsequently proposed a portfolio selection model that uses predicted returns as expected returns and portfolio expected return is the linear combination of the participation and predicted return of each stock in the portfolio.<sup>16</sup> Freitas et al.<sup>4</sup> use

- a neural network to predict each stock's return,
- errors in prediction of a stock's return as its variance or risk,
- covariance based on these error variances,
- the optimization technique of Markowitz to find the minimum risk portfolio for a given return, and
- the results of their system on the Brazilian Stock Exchange as a comparison to the results with the classic Markowitz method and with holding the Brazilian Stock Index.

Their experiments show that their method of portfolio optimization outperforms the traditional Markowitz method and the Brazilian Stock Market Index.

Boginski et al.<sup>17</sup> described *cliques* and *independent sets* of stocks. A clique is a set of stocks whose price fluctuations exhibit a similar behavior, i.e., a change of the price of any instrument in a clique is likely to affect all other instruments in this clique. On the other hand, an independent set of stocks consists of instruments that are negatively correlated and represent a diversified portfolio.

Since Freitas et al.<sup>4</sup> predict each stock's return individually and then calculate the portfolio's return by linear combination of each stock's return, they fail to identify cliques or independent sets. Hence, the correlation between different stocks' price time series is not considered. We extend the Freitas et al.<sup>4</sup> model by predicting portfolio returns based on data other than historical prices of individual stocks. We are given portfolios that a stockbroker working on the Tehran Stock Exchange generates each week for his clients. That broker has applied the Markowitz portfolio optimization method to predict earnings for his portfolio. We take his data as input and combine that with other information to predict the earnings of the stockbroker's portfolio. That other information includes time series about the price and volume of various indices in the Tehran stock market.

This paper contributes to the application of soft computing in finance in two ways. From the soft computing perspective, the use of a fuzzy rule based system to predict the stock portfolio return is new. Emphasis is usually placed on using neural networks and their hybrid systems. This study makes use of an adaptive neuro-fuzzy inference system (ANFIS) for stock portfolio return prediction. The initial rule base construction is a rational combination of existing methods. From the financial perspective, an approach to portfolio return prediction is proposed, which considers multiple markets and portfolio attributes and the natural ties (i.e., correlation) between stocks through fuzzy *if-then* rules.

The rest of the paper is organized as follows. In Section 2, we review portfolio selection theory and ANFIS. In Section 3, our neuro-fuzzy system for predicting portfolio return is presented. The experimental results of applying our system to the stock portfolio return prediction problem are presented in Section 4. Finally, Section 5 presents our conclusions.

## 2. BACKGROUND

For the reader unfamiliar with portfolio selection theory or ANFIS, the next two subsections give the relevant background.

### 2.1. Markowitz Portfolio Selection Model

In the Markowitz portfolio selection model, there are  $n$  securities denoted by  $S_j$  ( $j = 1, \dots, n$ ), the return of the security  $S_j$  is denoted by  $r_j$ , and the proportion of total investment funds devoted to this security is denoted as  $x_j$  ( $\sum_{i=1}^n x_i = 1$ ). Since  $r_j$  ( $j = 1, \dots, n$ ) vary across time, they are random variables and can be represented by the average vector and the covariance matrix. For instance, if returns on securities over  $m$  periods  $i$  ( $i = 1, \dots, m$ ) are given, then

- $n$  kinds of returns are denoted as a vector  $r_i = [r_{i1}, \dots, r_{in}]^t$ .
- The average vector of returns over  $m$  periods is denoted as  $r^0 = [r_1^0, \dots, r_n^0]^t$ , and
- the corresponding covariance matrix  $Q = [q_{ij}^2]$  can be written as

$$q_{ij}^2 = \sum_{k=1}^m (r_{ki} - r_i^0)(r_{kj} - r_j^0) / m(i, j = 1, \dots, n). \quad (1)$$

The return associated with the portfolio  $x$  is given by

$$z = x^t r. \quad (2)$$

The average and variance of  $z$  is given as

$$E(z) = E(x^t r) = x^t E r = x^t r^0, \quad (3)$$

$$V(z) = V(x^t r) = x^t Q x. \quad (4)$$

Since the variance is regarded as the risk of investment, the best investment is one with the minimum variance subject to a given average return  $r_s$ . This leads to the following quadratic programming problem:

$$\begin{aligned} \text{Min}_{x^t} \quad & x^t Q x \\ \text{S.t.} \quad & x^t r^0 \geq r_s, \quad \sum_{i=1}^n x_i = 1, \quad x_i \geq 0. \end{aligned} \quad (5)$$

## 2.2. Adaptive Neuro-Fuzzy Inference System

Takagi–Sugeno–Kang systems are widely used in the form of a neuro-fuzzy system called ANFIS<sup>18</sup> that is available from Mathworks in its Fuzzy Logic Toolbox. An ANFIS is a fuzzy inference system that can be trained to model some collection of input/output data. The training module allows the system to tune its parameters to learn the input/output relationships hidden in the data set. ANFIS incorporates two approaches: neural network and fuzzy.<sup>19</sup>

Our ANFIS architecture (type-3 ANFIS) has multiple layers. The first layer implements a fuzzification, the second layer executes the T-norm of the antecedent part of the fuzzy rules, the third layer normalizes the membership functions (MFs), the fourth layer calculates the consequent parameters, and the last layer computes the overall output as the summation of all incoming signals. The feedforward equations of this ANFIS are

$$w_i = \mu_{A_i}(x) \times \mu_{B_i}(y), \quad (6)$$

$$\bar{w}_i = \frac{w_i}{w_1 + w_2}, i = 1, 2. \quad (7)$$

$$\begin{cases} f_1 = p_1x + q_1y + r_1z \\ f_2 = p_2x + q_2y + r_2z \end{cases} \quad (8)$$

$$f = \frac{w_1f_1 + w_2f_2}{w_1 + w_2} = \bar{w}_1f_1 + \bar{w}_2f_2. \quad (9)$$

Note that the network's output is nonlinear in the weights  $w$ . The training of this multilayered neural network is thus a *nonlinear optimization* to which various methods can be applied.<sup>20</sup>

The neuro-fuzzy inference system is optimized by adapting the antecedent parameters and consequent parameters so that a specified objective function is minimized. A number of methods have been proposed for learning rules and for obtaining an optimal set of rules. For example, Mascioli et al.<sup>21</sup> proposed to merge Min–Max and ANFIS models to obtain a neuro-fuzzy network and determine an optimal set of fuzzy rules. Kumar and Garg<sup>22</sup> used Kohonen's map for training. Tang et al.<sup>23</sup> proposed a hybrid system combining a fuzzy inference system and genetic algorithms to tune the parameters in the TSK fuzzy neural network. Jang<sup>18</sup> proposed methods to update the ANFIS parameters involving gradient descent and least-square error. Several popular training algorithms for tuning parameters of ANFIS membership functions are compared by Chen.<sup>24</sup>

### 3. NEURO-FUZZY RULE-BASED MODELING

A mathematical definition of stock portfolio return prediction might use historical data vectors that have been made fuzzy. Suppose there are ND historical data vectors that are obtained from the subjects available and each data is associated with NV features, such as *total index*, *number of stocks in portfolio*, and *risk of portfolio*. Let  $X_1, X_2, \dots, X_{NV}$  be NV fuzzy linguistic variables in the universe of discourse  $U_1, U_2, \dots, U_{NV}$  and  $Y$  be a fuzzy linguistic variable in the universe of discourse  $V$ , which represents the output of the system. Here  $Y$  is the return of the portfolio. Let  $X_k = [x_{k,1}, \dots, x_{k,NV}]$  denote the input data vector of the  $k$ th sample, where  $k = 1, 2, \dots, ND$  and  $y_k$  is the output. Let  $R$  be a fuzzy relation in  $U_1 \times U_2 \times \dots \times U_{NV} \times V$ .

In general, a fuzzy if–then rule base has the following structure<sup>25</sup>:

$$R := \text{Also}_{i=1}^c \text{ IF antecedent}_i \text{ THEN consequent}_i$$

In the TSK fuzzy system modeling method, which is used in ANFIS, the consequent part of fuzzy rules are represented by a linear function of input variables. Hence, the TSK rule base can be represented as:

$$R := \text{Also}_{i=1}^c \text{ IF antecedent}_i \text{ THEN } y_i = a_i x^T + b_i$$

where  $X_k = [x_{k,1}, \dots, x_{k,NV}]$  is the input data vector,  $a_i = [a_{i,1}, \dots, a_{i,NV}]$  is the regression line coefficient vector associated with the  $i$ th fuzzy rule,  $a_{ij}$  is the

regression line coefficient in the  $i$ th fuzzy rule associated with the  $j$ th input variable, and  $b_i$  is the scalar offset of the regression line in the  $i$ th fuzzy rule.

The two basic steps in neuro-fuzzy modeling are *system identification* and *fuzzy reasoning*. In the system identification stage, the significant input variables are determined, the fuzzy if-then rules are generated, and the parameters of the model, such as the number of clusters, the level of fuzziness, and the operators to be used in the reasoning, are selected. The *fuzzy reasoning* is used to infer new knowledge from the identified rule base.<sup>25</sup> The basic steps are

1. input selection,
2. fuzzy clustering of the output,
3. input membership assignment, and
4. tuning the parameters of antecedent and consequent part of fuzzy rules.

### 3.1. Fuzzy Clustering of the Output

To determine the initial number of rules, we use a noise-rejection fuzzy clustering algorithm for clustering the output space. Melek et al.<sup>26</sup> find the number of clusters, select the weight exponent, choose the initial cluster centers, find the noisy data, and calculate the membership matrix. To find the number of clusters, we use a validity index as proposed by Kim et al.<sup>27</sup> and modified by Fazel Zarandi et al.<sup>14</sup> This minimizes

$$V_{\text{FNT}}(U, V; X) = \frac{2}{c(c-1)} \sum_{p \neq q}^c S_{\text{rel}}(A_p, A_q) \quad (10)$$

over the range of  $c$  values:  $2, \dots, c_{\text{max}}$ ; where  $S_{\text{rel}}(A_p, A_q)$  is the relative similarity between two fuzzy sets  $A_p$  and  $A_q$ .  $S_{\text{rel}}(A_p, A_q)$  is defined as

$$S_{\text{rel}}(A_p, A_q) = \sum_{i=1}^N S_{\text{rel}}(x_j : A_p, A_q) h(x_j) \quad (11)$$

where  $N$  is the number of data points and

$$h(x_j) = - \sum_{p=1}^c u_{A_p}(x_j) \log(u_{A_p}(x_j)) \quad (12)$$

where  $h(x_j)$  is the entropy of datum  $x_j$  and  $u_{A_p}(x_j)$  is the membership value with which  $x_j$  belongs to the cluster  $A_p$ .

For the choice of initial cluster centers, Melek et al.<sup>26</sup> suggested to use an agglomerative hierarchical clustering algorithm (AHC) as the initial clustering tool. Then, by defining a matrix of dissimilarities, the AHC merges two or more of these clusters. The process is repeated to form a sequence of nested clustering in which the number of clusters decreases gradually until the minimum required

number of clusters is obtained. To accommodate fuzzy logic concepts, we use a modified version of the Gustafson–Kessel clustering algorithm, which is introduced by Babuška et al.<sup>28</sup> to create initial cluster centers. Then methods developed by Melek et al.<sup>26</sup> are used to identify various parameters and from there we use the Possibilistic C-Means algorithm of Krishnapuram.<sup>29</sup>

### 3.2. Input Selection, Membership Assignment, and Rule Parameters

The performance of nonlinear identification techniques is often determined by the appropriateness of the selected input variables and the corresponding time lags.<sup>30</sup> For input variable selection, the variable selection algorithm proposed by Sugeno and Yasukawa<sup>31</sup> is used. All possible combinations of input candidates are considered. For each combination, two fuzzy models are built based on two separate sets of data, and a performance index called “regularity criterion” (RC) is calculated based on a method of group data handling in an attempt to cause data independence in model formation. After that, a combination of input variables is chosen, which has the minimum value of the performance index.

During the structure identification phase the fuzzy membership values can be identified on the basis of three different strategies with respect to how fuzzy clustering is utilized.<sup>25</sup> First, we can cluster the output space and obtain the fuzzy membership functions based on the projections of the output clusters onto the input space.<sup>31</sup> Second, we can first cluster the input space and project the input clusters to the output space.<sup>32</sup> Finally, we can cluster the input and output space and then project the multidimensional clusters to each one of the two spaces.<sup>33</sup> In this paper, we make use of the first approach and project the output clusters onto the input space and construct our rule base.

The ANFIS is used for neuro-fuzzy modeling. The ANFIS has two kinds of parameters that needed to be trained: the antecedent parameters and the consequent parameters. Gaussian membership functions are located in the antecedent part:

$$\mu_{A_i}(x) = \exp \left\{ - \left[ \left( \frac{x - c_i}{a_i} \right)^2 \right]^{b_i} \right\}. \quad (13)$$

The antecedent part has three types of parameters:  $\{a_i, b_i, c_i\}$ , where  $a_i$  is the variance,  $b_i$  is crossover slop, and  $c_i$  is the center of MFs. We apply the ANFIS Matlab toolbox default options to tuning the antecedent and conclusion parts of fuzzy rules.

### 3.3. Prediction-Based Portfolio Selection Model

After predicting the return of the portfolio by the proposed neuro-fuzzy rule-based system, one can use any risk minimization portfolio selection model. For example, if we consider the basic formulation of Markowitz, the prediction-based

portfolio optimization model can be formulated as

$$\begin{aligned} \text{Min}_x \quad & x^T Q x \\ \text{S.t.} \quad & r_p \geq r_s, \quad \sum_{i=1}^n x_i = 1, \quad x_i \geq 0 \end{aligned} \quad (14)$$

where  $r_p$  is the predicted return of the portfolio and the other parameters and variables are the same as in Equation 5.

## 4. EXPERIMENTAL RESULTS

This section presents the experiments we have used to evaluate the performance of our neuro-fuzzy rule-based predictors on our data set.

### 4.1. Data

Expert systems and evolutionary computing had become a popular choice for financial investing applications. Rada<sup>34</sup> conducted a comprehensive literature review of expert systems, which have been applied in this certain area. Among all types of expert systems, neuro-fuzzy systems, specially ANFIS, have been widely used for financial time series prediction problems.<sup>35–36</sup>

From the stocks that participated in the Tehran Stock Exchange between December 2005 and July 2008, a stockbroker selected regularly a portfolio of approximately five stocks and predicted the return of that portfolio based on the Markowitz model. We use the real historical data for these selected portfolios and their real returns. To predict the return of the given portfolio, the candidate variables of the system are shown in Table I. In this table, the portfolio return is the output variable. In this research, 280 data points have been selected where 200 data points are used for training and the rest for testing the model (Table II).

### 4.2. Prediction of Portfolio Return

The data of the portfolios are modeled into a multiple-input–single-output system. The steps and results of the development of the neuro-fuzzy rule-based model are as follows:

1. Using the Sugeno and Yasukawa method for variable selection, we begin with a fuzzy model with one input. We generate 26 models: one model for one particular input. Then the RC of each model is calculated and one model is selected to minimize RC from among the one-input models. Next, we fix that selected input and add another input to our fuzzy model among the remaining 25 candidates. At this stage, our fuzzy model has two inputs. The second input is also selected according to the value of RC. The above process is continued until the value of RC increases. The results of the obtained RC values are shown in Figure 1. As a result, L3MR is selected at the first step, FCWIC at the second step, CRC at the third step, ER at the fourth step, SN at the fifth step, and RE/R at the

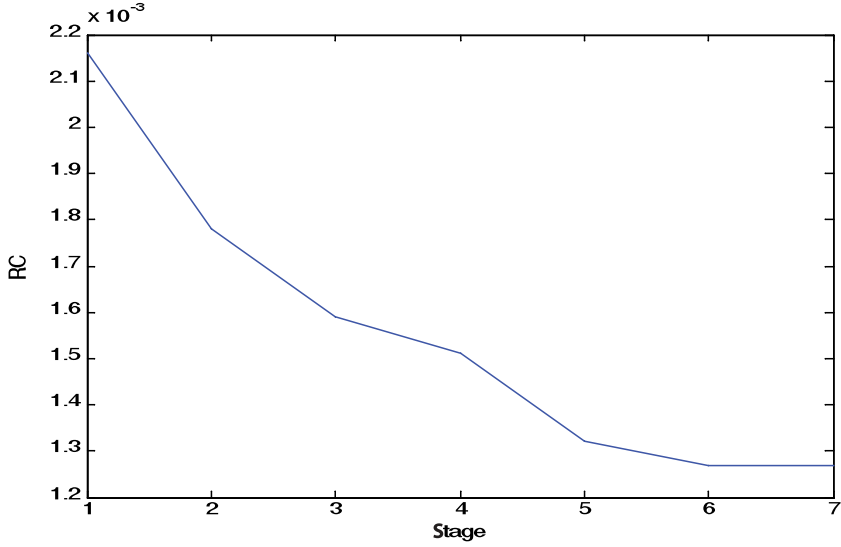


**Table I.** Variables of the system.

Variable name	Variable description	Variable
Trade date	Date of portfolio construction	D
Expected return	Expected return of portfolio calculated through Markowitz basic approach	ER
Risk	Risk of portfolio calculated through Markowitz basic approach	R
Number of stocks in portfolio	–	SN
Total index	Stock market total index	TI
Total index change	Stock market total index change	TIC
Fifty companies weighted index	Best 50 company weighted index	FCWI
Fifty companies weighted index change	Best 50 company weighted index change	FCWIC
Industry index	Weighted index of industrial companies	II
Industry index change	Weighted index of industrial companies change	IIC
Cash return and price index	Market profitability	PCR
Cash return and price index change	Market profitability change	PCRC
Cash return index cash return index change	Market net profitability market net profitability change	CR CRC
Fifty companies index	Best 50 companies' price index.	FCI
Fifty companies index change	Best 50 companies' price index change	FCIC
Last month return	Return of the given portfolio in the previous month	LMR
Last two month return	–	L2MR
Last three month return	–	L3MR
Return to risk ratio	–	RE/R
Trading value	Daily trading value	TV
Trading value change	Daily trading value change	TVC
Trading volume	Daily trading volume	TVO
Trading volume change	–	TVOC
Trading number	Popularity of the stock market	TN
Trading number change	–	TNC
Portfolio return	Output of the system	PR

**Table II.** Sample of data set.

L3MR	FCWIC	CRC	ER	SN	RE/R	RR
0.144	–0.43	1.17	0.047	6	5.44	0.013
0.120	–1.82	4.51	–0.007	7	–1.58	–0.013
0.261	0.54	0	–0.007	6	–0.52	–0.033
0.298	–0.97	0	0.006	6	0.84	–0.016
0.491	0.67	0	0.024	5	1.09	–0.022
0.382	0.63	0	0.017	5	1.07	–0.002
0.445	1.77	4.71	0.026	5	1.28	0.014
–0.141	0.8	3.41	–0.001	8	–0.28	0.002
–0.207	3.68	0	–0.014	8	–3.59	0.001
0.087	–0.75	0	0.014	6	13.31	–0.009
–0.082	1	0.07	–0.041	8	–10.73	0.042
0.233	0.4	21.22	0.035	7	7.32	0.003



**Figure 1.** Behavior of RC in the proposed model.

- sixth step. At the seventh step, all the values of RC for the seventh input are bigger than the minimal RC at the sixth step. So the search is terminated at this stage.
2. A robust noise rejection-clustering algorithm is implemented to cluster the output space. The suitable weight exponent is selected as  $m = 2.8$  (Figure 2). Then the cluster validity index based on similarity measure ( $V_{\text{FNT}}$ ) is implemented to determine the most suitable number of clusters or rules ( $c$ ). As shown in Figure 3, the best number of clusters based on this cluster validity index is eight clusters.
  3. The membership functions of the clustered output are projected onto the input spaces to generate the membership functions of inputs. It is assumed that inputs and output membership functions are Gaussian.
  4. For tuning the membership function parameters, the ANFIS toolbox of Matlab is used. We used its hybrid leaning algorithm option and ran the algorithm five times.

The Mamdani-style inference, min-max, sum-product operators, and some defuzzification methods such as centroid, bisector, middle of maximum, the smallest of maximum, and the largest of maximum are used. The best result of this system is obtained by min-max operators and the centroid defuzzification method.

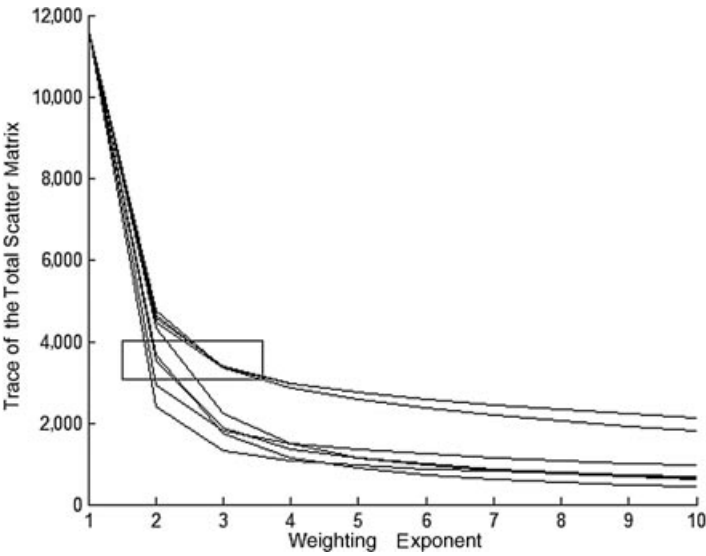
**4.3. Prediction Performance**

To show the superiority of the proposed neuro-fuzzy model rather than those approaches that individually predict the return of stocks and calculate the return of the portfolio by linear combination of them, we compare their prediction performance. Table III elaborates the prediction error of the proposed model and neural network in terms of root mean square error (RMSE).

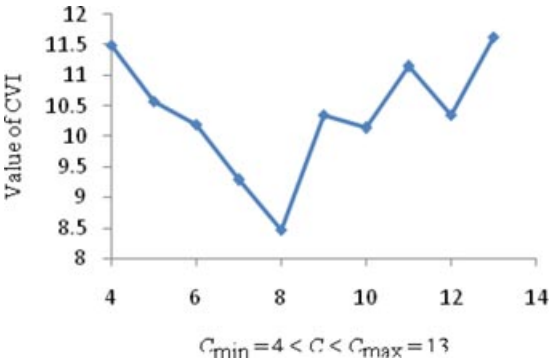
For validation of the proposed model, we compare the result of the neuro-fuzzy rule-based model with the results of the several other models.

**Table III.** Comparison between individually and integrated prediction approach.

	Average testing RMSE	
	Individually	Integrated
Neural network	0.0197	0.0145
Proposed model	0.0146	0.0108



**Figure 2.** Selection of level of fuzziness of the fuzzy-clustering algorithm for the proposed model.



**Figure 3.** Identification of the optimum number of clusters for the proposed model.

**Table IV.** Comparing between models for training data.

	Number of rules	Training RMSE		
		Minimum	Maximum	Average
Markowitz approach	–	–	–	0.1022
Multiple regression	–	–	–	0.1480
Neural network	–	0.0091	0.0219	0.0114
Sugeno–Yasukawa model	6	0.0552	0.0811	0.0664
Proposed model	8	0.0004	0.0287	0.0097

#### 4.3.1. Multiple Regressions

We have used the regression analysis of MATLAB. The regression equation is

$$y = 0.2121x_1 - 0.0019x_2 + 0.0114x_3 - 0.0001x_4 - 0.0173x_5 - 0.0003x_6 \quad (15)$$

After we found a linear relationship between these six parameters, we look at the residual plots. As shown in Figure 4, the linear formula for this relationship is a good estimation.

#### 4.3.2. Neural Network

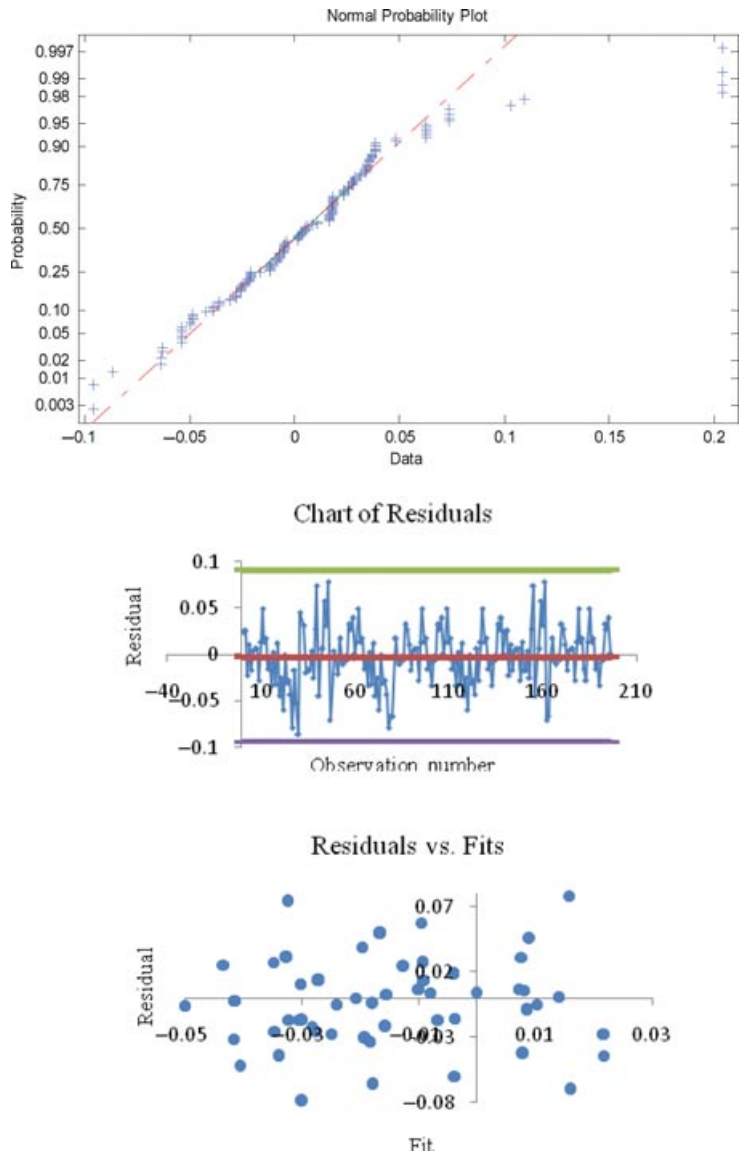
We use a  $10 \times 5 \times 1$  feed forward network to model our system. A tangent sigmoid activation functions was used in each node. The tests were accomplished for a maximum iteration number fixed in 500 epochs. The average results for five runs obtained with the training, and test data sets are depicted in Tables IV and V.

#### 4.3.3. Sugeno–Yasukawa Approach

We used the Sugeno–Yasukawa approach to obtain a fuzzy model with six rules, five inputs, and one output. The inputs are L3MR, FCWIC, CRC, ER, SN, and RE/R, and the output is portfolio return. We used Mamdani-style inference, min–max, sum–product operators, and centroid defuzzification.

**Table V.** Comparing between models for testing data.

	Number of rules	Testing RMSE		
		Minimum	Maximum	Average
Markowitz approach	–	–	–	0.1304
Multiple regression	–	–	–	0.1638
Neural network	–	0.0103	0.0281	0.0145
Sugeno–Yasukawa model	6	0.0816	0.0922	0.0898
Proposed model	8	0.0006	0.0411	0.0108



**Figure 4.** Residuals plot.

Tables IV and V show the average results for five runs obtained with the training and test data sets for the proposed neuro-fuzzy model with that of Sugeno and Yasukawa,<sup>31</sup> neural network, and Markowitz approach. The tests were performed for 500 epochs. To compare the performance of different models, we use the RMSE index.

For each modeling method, the RMSE error was obtained by contrasting the results on the trained data with the results on the test data. Table V shows that while

the RMSE of neural network is 0.0145 and that of Sugeno and Yasukawa's model is 0.0898, the RMSE of our model is a superior 0.0108.

## 5. CONCLUSION

In this paper, the stock portfolio return was predicted using a new approach named ANFIS. The obtained predicted return value can be used instead of expected return in the portfolio optimization models to develop the prediction-based portfolio optimization model.

An indirect approach is used to construct initial rule base by implementing a robust noise-rejection data-partitioning algorithm in a fuzzy-clustering approach. Then the Sugeno and Yasukawa method was used to select the most important variables for the rule-based fuzzy logic system. Next, the output membership values were projected onto the input spaces to generate the membership values of input variables, and the membership functions of inputs and output were tuned through ANFIS. The hybrid learning algorithm was used for tuning of the parameters.

Since previous work emphasized on individually prediction of the stocks' return and calculate portfolio return using linear combination of the stocks' return, we compared the performance of our method with the linear combination approach. The results showed that our integrated prediction approach outperforms the linear combination approach.

For the sake of validating the use of ANFIS for portfolio return prediction, the prediction performance of the ANFIS was compared with the Markowitz model, neural network, multiple regression, and the Sugeno–Yasukawa fuzzy systems modeling approach. The results showed the superiority of the ANFIS in the term of error minimization.

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