Fundamental law of active management (FLAM) under fundamental factor models

Application in R based software application.

07 July 2023

Author: Sanele Makamo

Benguela Global Fund Managers

Abstract

In this paper we derive the fundamental law of active management (FLAM) based on cross-section fundamental factor model for residual returns and its application in R. Our focus is using the fundamental factor model to forecast asset expected returns from risk factors instead of predicting and controlling portfolio risk and then use those expected returns in active quadratic utility optimization with constraints to obtain optimal portfolio active weights which we use to calculate information coefficient and transfer coefficient for each time-period. We show that by setting a desired target risk and expected return one can obtain a desired optimal portfolio by maximizing expected return and minimizing forward-looking active risk. The results also show how multi-factor models can be extended to include the framework of the fundamental law of active management and R can be used as a software for the application of the law under factor risk models.

Introduction:

In their article on multi-factor models Grinold and Kahn (1994) show that beyond providing risk forecasts, multi-factor models can assist in providing forecasts of expected returns which in turn helps in the portfolio construction process by constructing optimal portfolios that implement bets based on the forecasted expected returns. The idea behind that lies in quadratic optimization by maximizing utility defined as a risk-adjusted expected return. This is known as the quadratic optimization problem which is solved to obtain optimal active weights.

In this analysis we use fundamental factor model for residual returns to derive the fundamental law of active management. The model is based on cross-section regression of asset returns against fundamental standardized exposures also know as risk factors. The cross-section fundamental model is based on five fundamentally derived variables, namely, financial risk which is constructed on financial variables based on firms' quick ratio, current ratio, interest cover etc. Business risk which is constructed on variables based on fluctuation of profit margins, revenue growth, operating cash flow, ROIC versus WACC, ROA, ROE etc. Market Risk, which is constructed on variables based on market price volatility, price-to-book variability, EV/EBIDTA etc. Lastly, the ESG factor which is constructed based on firms' sustainability, environmental and social scoring.

We use the cross-section factor model to compute the conditional mean alpha and their related conditional covariance matrix. To derive the fundamental law, we use residuals returns relative to the benchmark from the factor model as inputs to the portfolio optimization and the active management constraint is that the active weights must be dollar neutral.

The first part of the analysis we explore the fundamental law of active management, and the second part investigates the multi-factor models and the derivation of the law and lastly, we evaluate the quadratic utility optimization problem and how the inputs from the factor model become inputs to the derivation of the law.

1. Fundamental Law of Active Management

First described by Richard Grinold in 1988, the fundamental law of active management, the law expresses the information ratio in terms of three other statistics—the information coefficient, a measure of skill; breadth, a measure of diversification; and the transfer coefficient, a measure of efficiency of implementation:

$$\boldsymbol{IR} = \boldsymbol{IC} * \sqrt{\boldsymbol{BR}} * \boldsymbol{TC} \ \dots (\mathbf{1})$$

Information coefficient:

Information coefficient (IC) is the measure of skill, it measures the managers ability to process raw information into alphas. This is measured by taking the correlation between the forecasted active returns or alpha with the realized active returns of the portfolio.

Breadth:

Breadth or breadth of skill is the part of the fundamental law measures the number of independent bets the manager takes per year at an average skill level of IC. It measures diversification. We define breadth as bets per year because we define the information ratio as an annualized quantity. As the number of independent bets per year, it is a rate, not a number. It's not the number of assets in the portfolio. Breadth is hardest to understand since it is a rate and not the number of holdings in the portfolio since we expect twice as many bets over two years than one year (Grinold & Kahn, 2019).

To add more information on breadth, consider an investment process equilibrium where old information decays as new information arrives. In equilibrium, the two are in balance, and so the information turnover rate, γ, captures both the decay rate of old information and the arrival rate of new information (Grinold & Kahn, 2019). Assuming that these two processes are in balance, we can show breadth with the equation:

$$\boldsymbol{BR} = \boldsymbol{\gamma} * \boldsymbol{N} \dots (\mathbf{2})$$

Which shows how the breadth relates to both the number of assets under consideration and the information turnover rate.

Information turnover rate:

According to Grinold (2011) information turnover rate plays a significant role in modelling implementation efficiency. Consider a history of forecasted asset alphas, which are referred to as annualized forecasts of exception return, then Is it possible that there exists an implied rate of decay in the forecasting power of those alphas. Let us suppose there are N assets in the investment universe, and we receive new information every Δt. In our analysis we assume monthly updates $\Delta t = 1/12$. The information contained in each alpha forecast decays at the information turnover rate g and is replaced by new information. So, we expect,

$$\alpha_n(j) = \gamma \cdot \alpha_n(j-1) + \varepsilon_n(j) \dots (3)$$

Where $\alpha_n(j)$ alpha of asset n at time $j \cdot \Delta t$, *as* $\alpha_n(j)$ decays at time $(j-1)\Delta t$.
The term $\gamma \equiv e^{-g\Delta t}$ captures the information decay (the one-period-old forecast has less to say about this period's return), and $\varepsilon_n(j)$ represents the new information arriving over period Δt. The new information, by virtue of being new, is uncorrelated with $\alpha_n(j-1)$ and has an expected value of zero.

Since the regression coefficient, γ , depends on Δt . In any actual implementation, the estimated γ will vary from period to period; think, earnings season. On average, the coefficient gamma will be positive and less than one. We can then estimate the continuous rate of change in the alphas as

$$\boldsymbol{g} = \frac{-\boldsymbol{ln}\,(\boldsymbol{\gamma})}{\Delta \boldsymbol{t}} \ldots (\mathbf{4})$$

We can apply this technique—and estimate g —for any investment process with a history of return forecasts.

Transfer coefficient:

According to Grinold and Kahn (2019), It measures the correlation between the return of a paper portfolio that optimally implements the manager's views without regard to costs or constraints and the actual portfolio the manager is running. The information ratio of the paper portfolio is IC · BR. The information ratio of the actual portfolio—considering constraints, costs, and possibly even poor implementation—is typically much lower.

2. Quadratic Utility Optimization:

According to Grinold and Kahn (2000) on the derivation of the active management framework, the objective is to create a portfolio that maximises utility to achieve the highest information ratio. To do this, we make use of optimal active portfolio weights obtained using the expected quadratic utility variant of the Markowitz (1952) mean-variance portfolio optimization theory.

According to Ding and Martin (2016) quadratic utility decreases for sufficiently large values of wealth and has increasing absolute and relative risk aversion, all of which are inconsistent with typical investor behaviour. However, Levy and Markowitz (1979), provided reasonable justification for using the expected quadratic utility variant of mean-variance optimization where they showed that the mean-variance approach provides a good approximation to expected utility maximization for increasing concave utility functions.

Furthermore, quadratic utility is the approach is widely used by practitioners, mostly as part of portfolio construction process using a commercial portfolio optimization and risk management software product. Therefore, it is worthwhile to develop a fundamental law of active management in the expected quadratic utility framework.

In this short paper we apply the quadratic utility maximization approach first taken by Grinold and Kahn and by Clark et al. (2002, 2006) and later also used by Ding and Martin (2016) in calculating the conditional mean forecast of residual returns because we want to optimize the end of period portfolio performance based on the beginning of period information. We use the residual returns on the assumption that they are uncorrelated with the benchmark returns and do not contain any benchmark information.

The outcome is a benchmark neutral portfolio i.e., has zero beta on the benchmark. This construction of optimal active portfolio based on residual returns can allow portfolio managers to invest in a weighted combination of benchmark and the residual returns of the optimal portfolio to achieve a desired degree of exposure to the benchmark.

We use the alpha $\alpha_{A,t}$ less a penalty for active variance $\sigma^2_{A,t}$ while we are taking the active weights $\boldsymbol{w_{A,t}}$ as dollar-neutral, the quadratic utility active portfolio optimization problem is:

$$\underset{\Delta \boldsymbol{w_t}}{\boldsymbol{Max}} \ \boldsymbol{U_t} = \boldsymbol{\alpha_{A,t}} - \frac{\boldsymbol{\lambda}}{\mathbf{2}}\boldsymbol{\sigma^2_{A,t}} = \boldsymbol{w^T_{A,t}\alpha_{A,t}} - \frac{\lambda}{2}\boldsymbol{w^T_{A,t}}\sum\nolimits_{\boldsymbol{t}}\boldsymbol{w_{A,t}}, \dots (\mathbf{5})$$

$$s.t. \ \boldsymbol{w^T_{A,t}} \cdot \mathbf{1}'\boldsymbol{s} = 0$$

Were,

$\boldsymbol{\alpha_{A,t}} =$ portfolio active return conditional mean

$\boldsymbol{\sigma^2_{A,t}} =$ portfolio active return conditional variance

$\boldsymbol{\lambda} =$ a risk − aversion parameter

$\sum_t =$ conditional covariance matrix

1 = an *N*-vector of 1's

The solution for this optimization problem is easily obtained using the method of Lagrange. multipliers to handle the constraint, and the result is:

$$\boldsymbol{w_{A,t}} = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\lambda}}\sum\nolimits_{\boldsymbol{t}}^{-\mathbf{1}}\boldsymbol{\alpha_{A,t}} \dots (\mathbf{6})$$

Thus, we define the conditional information ratio as

$$\boldsymbol{IR} = \frac{\boldsymbol{\alpha_{A,t}}}{\sqrt{\boldsymbol{\sigma^2_{A,t}}}}, \dots (\mathbf{7})$$

The above general results hold under the quadratic utility optimization using mean forecast of residual return, $\boldsymbol{\alpha_{A,t}}$, and corresponding covariance matrix, $\sum_{\boldsymbol{t}}$. In what follows we extend the derive the fundamental law of active management (FLAM) under the cross-section factor models. This will provide more insights concerning the ex-ante information ratio.

3. Multi-Factor Models

Multi-factor models have been crucial in modelling asset expected returns and in the practice of portfolio optimization and risk management. Grinold and Kahn (1994) show that the goal of such risk-forecasting models is to help portfolio managers to analyse the sources of risk in their portfolios, that is, to determine (ex-post) if the risk were justified and in forecasting asset expected returns. Lastly, to construct portfolios that have desirable characteristics.

There are three types of factor models widely used in the industry: fundamental factor models, statistical factor models and time series models. The fundamental factor model is the most universally used model and more research exist for this model than previous two which includes written work by Fama-Macbeth (1973), Rosenberg (1974) and Fama-French (1992), among others.

The Fundamental factor models are typically cross-section regression models where the explanatory variables also referred to as 'exposures' to risk factors include a variety but limited continuous variables such book-to-market, earnings-to-price, size, momentum, value, growth, etc., and categorical variables reflecting sector, industry, and country exposures. Such models make use of a cross-sectional model fit which is computed at each time periods in an over a selected time periods. This model is mostly used for both risk analysis and asset return forecasting (Ding and Martin, 2016).

Using a similar approached by Ding and Martin (2016), the multi-factor model we use in the derivation of FLAM is part of the standard fundamental multi-factor model in which we assume the manager's alphas are residual to the risk model's factors. If we assume we have a covariance matrix $\sum_t$ based on a factor model so that:

$$\sum_t = \boldsymbol{\beta}_t \boldsymbol{F} \boldsymbol{\beta}_t{}^{\boldsymbol{T}} + \boldsymbol{\varepsilon}, \ldots (\mathbf{8})$$

Where $\boldsymbol{\beta}$ is the matrix of asset exposures to the model's factors, $\boldsymbol{F}$ is the factor covariance matrix and $\boldsymbol{\varepsilon}$ is the diagonal matrix of residual variances, $\boldsymbol{\sigma}_{i,t}^{2}$, for each asset i.e., specific variances. Thus, if the assumption holds, that the manager's alphas or forecast residual returns, $\boldsymbol{\alpha}_{i,t}$, are orthogonal to the risk model factors in a 'risk-adjusted' sense. i.e.,

$$\boldsymbol{\beta}^{\boldsymbol{T}} \cdot (\boldsymbol{\varepsilon}^{-1} \boldsymbol{\alpha}_{i,t}) = \mathbf{0}$$

And the active portfolio is factor neutral i.e.,

$$\boldsymbol{\beta}^{\boldsymbol{T}} \cdot \boldsymbol{\varepsilon}^{-1} \cdot \boldsymbol{w}_{A,t} = \mathbf{0}$$

The covariance between assets will be zero. Then information coefficient is defined as the cross-sectional correlation between the raw returns and standardized risk-adjusted forecasted residual returns or manager's alphas

$$\boldsymbol{IC}_t = \boldsymbol{corr}\left(\boldsymbol{r}_{i,t}, \frac{\boldsymbol{\alpha}_{A,t}}{\boldsymbol{\sigma}_{i,t}}\right), \ldots (9)$$

And here the transfer coefficient is defined as the cross-sectional correlation of the risk-adjusted holdings $\boldsymbol{\sigma}_{i,t} \cdot \boldsymbol{w}_{A,t}$ with the normalized alphas $\frac{\boldsymbol{\alpha}_{A,t}}{\boldsymbol{\sigma}_{i,t}}$

$$TC_t = corr\left(\frac{\alpha_{A,t}}{\sigma_{i,t}}, \sigma_{i,t} \cdot w_{A,t}\right), \dots (10)$$

Thus let $\boldsymbol{F} = (\boldsymbol{F_{1t}}, \boldsymbol{F_{2t}}, \dots \dots, \boldsymbol{F_{Kt}})$ be a $\boldsymbol{K} \times \boldsymbol{1}$ random vector of factor returns at time t whose values are not known at time $\boldsymbol{t-1}$. Then our multi-factor model is

$$r_{i,t} = \boldsymbol{\beta_{i,t-1} F_t} + \boldsymbol{\varepsilon_{i,t}}, \qquad \boldsymbol{i = 1, 2, \dots \dots, N}, \qquad \dots (\boldsymbol{11})$$

Were $\boldsymbol{\beta_{i,t-1}}$ is an $\boldsymbol{N} \times \boldsymbol{K}$ matrix of factor exposures and $\boldsymbol{\varepsilon_{i,t}}$ is the $\boldsymbol{K} \times \boldsymbol{1}$ error vector.

We make the following assumptions concerning the above cross-section multi-factor model:

I. The $\boldsymbol{\sigma_{i,t}}$ depend on the lagged returns, $r_{u,t}, u \leq t-1$ and these standard deviations have a lower bound $b_{lower} \geq 0$, and a finite upper bound, b_{upper}, uniformly across assets.

II. The exposures, $\boldsymbol{\beta_{i,t-1}}$ have unconditional mean zero, variance one and finite fourth moment, and for each $\boldsymbol{K}$ the exposures $\boldsymbol{\beta_{iK,t-1}}$ and $\boldsymbol{\beta_{jK,t-1}}$ are independent $\boldsymbol{i \neq j}$.

III.

IV. The exposures $\boldsymbol{\beta_{ik,t-1}}$ and $\boldsymbol{\beta_{im,t-1}}$ are uncorrelated for $\boldsymbol{k \neq m}$, $\boldsymbol{k, m = 1, 2, \dots, K}$, $\boldsymbol{i = 1, 2, \dots \dots, N}$

V. The random factor return $\boldsymbol{F_t}$ is a stationary random process with known mean $\boldsymbol{E[F_t] = F_t}$ and known covariance matrix $\sum_{\boldsymbol{t}}$.

VI. The error terms satisfy $E[\boldsymbol{\varepsilon_{i,t}}] = \boldsymbol{\sigma_{\varepsilon_{i,t}}}$ for all i.

4. Data & Methodology

For this study we consider a fundamental factor model using cross-sectional regression with FTSE/JSE asset returns for period of 5 years. The fundamental (cross-sectional) factor model uses ordinary least squares or robust regression. We use observable asset specific characteristics (or) fundamentals, like industry classification, market capitalization, style classification (value, growth) etc. to calculate the common risk factors.

For this analysis we consider fundamental constructed exposures which are typically chosen to be properly standardized firm specific quantities such as book-to-market, earnings-to-price, financial leverage, etc. These exposures are then grouped into five fundamental exposure variables namely, business risk, financial risk, market risk and ESG risk for a five-year period. We also include industry classification exposures and market capitalization to calculate the risk factors.

Concerning quadratic utility optimization, we use portfolio optimization with support for rebalancing periods for out-of-sample testing i.e., backtesting. Using the portfolio optimization with periodic rebalancing can help refine the constraints and objectives by evaluating the out of sample performance of the portfolio based on historical data. For this analysis we simulate 3000 random portfolios based on constraints and objectives, we then run portfolio optimization on the randomized portfolio with a monthly rebalancing time periods. The only constraints we consider is that the active weights must be dollar neutral and objective constraints on expected return and risk. There is no constrain on turnover limits, diversification target constraint etc.

We set an ex-ante tracking error target or active risk and expected return target on the both the risk and mean expected return objective constraints. The risk aversion objective target is also set.

We then use the optimal weights from the quadratic utility maximization with the residual returns from the factor model to derive the fundamental law of active management (flam) by calculating the transfer coefficient and information coefficient and use that as a standard comparison between the actual portfolio and optimal portfolio.

5. Results:

The use of factor models and optimisation in the derivation of the fundamental law of active management provides valuable insights concerning forecasting portfolio expected returns. Figure 1 confirms this as it shows the comparison between the actual portfolio and optimal portfolio, that is, by setting a correct target for active risk, expected return and risk aversion one can get an optimal portfolio with a high transfer coefficient, information coefficient and information ratio and this is achieved by maximization of expected return and minimisation of risk. Figure 2 further shows the distribution of the active weights along the efficient frontier for at least 300 random portfolios, the active weights for the optimal portfolio are bounded between 4% and -4% overweight and underweights position limits.

Figure 3 shows the risk-return scatter plot of the at least 1000 random portfolio generated from the optimization using inputs from the factor model. Figure 4 below shows the distribution of the active weights for the optimal portfolio while figure 5 shows the sector active weights for the optimal portfolio. Figure 6 shows the distribution of the factor characteristics exposures used in the factor model.

Figure 1: Shows the tables of flam values for Portfolio and Optimal Portfolio determined through the fundamental factor model.

Table 1: Portfolio

	Date	Portfolio TC (%)	Portfolio IC (%)	Breadth (BR)	IR=ICxBR (%)	IR=ICxBRxTC (%)
Portfolio	2023-05-31	65.4	-0.0111	42	-0.082	-0.0672

Table 2: Optimal Portfolio

	Date	Optimal TC (%)	Optimal IC (%)	Breadth (BR)	IR=ICxBR (%)	IR=ICxBRxTC (%)
Optimal Portfolio	2023-05-31	82	0.0106	55	0.0784	0.0643

Table 3: Optimization Summary

	Date	Target active risk (%)	Target expected return (%)	Alpha	Active risk (%)	Risk aversion (%)
Utility Maximisation	2023-05-31	3.5	0.06	-0.0018	3.17	6

Figure 2: shows the distribution of active weights along the efficient frontier.

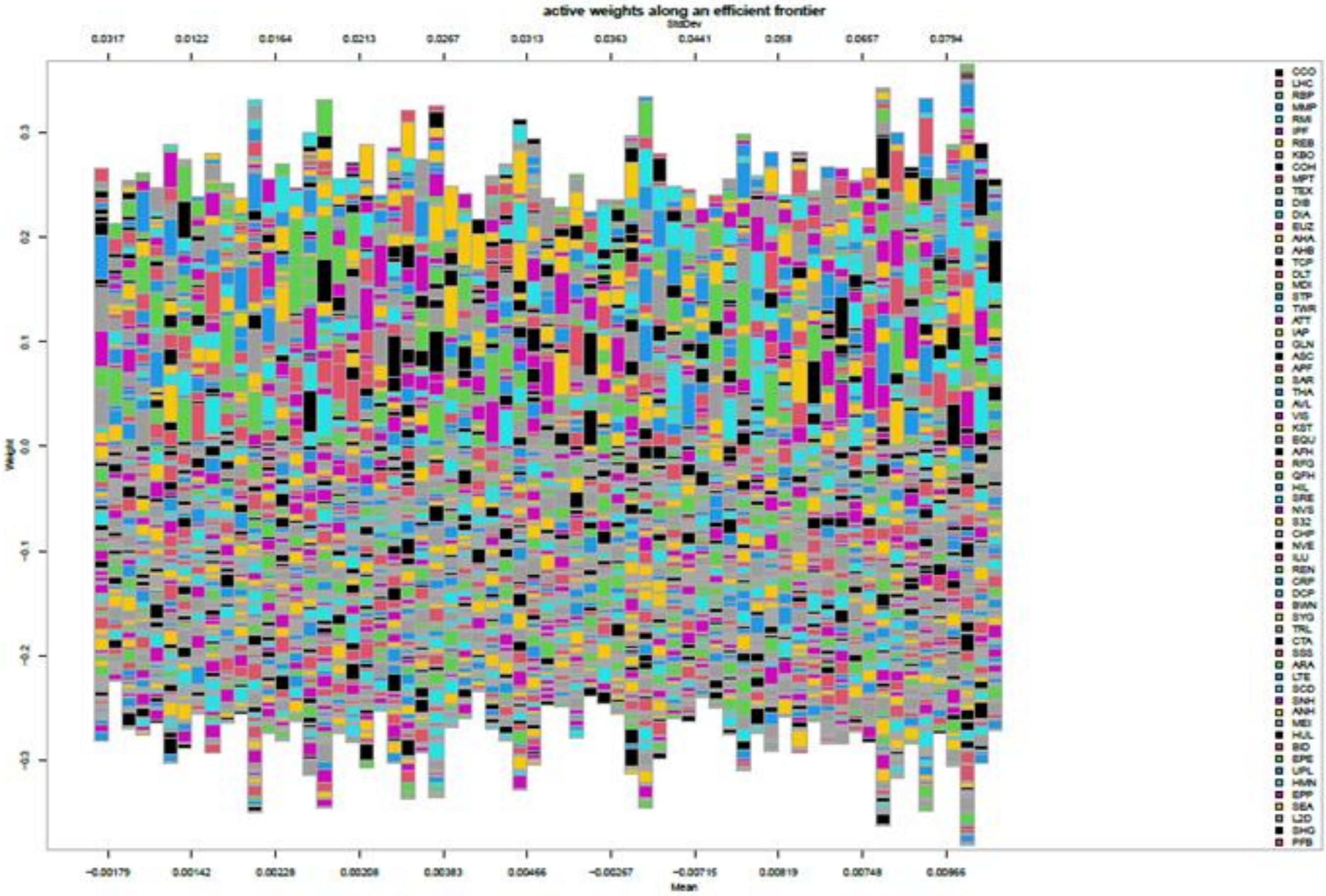


Figure 3: Shows the risk return scatter with the three distinguished paper portfolios from the optimization.

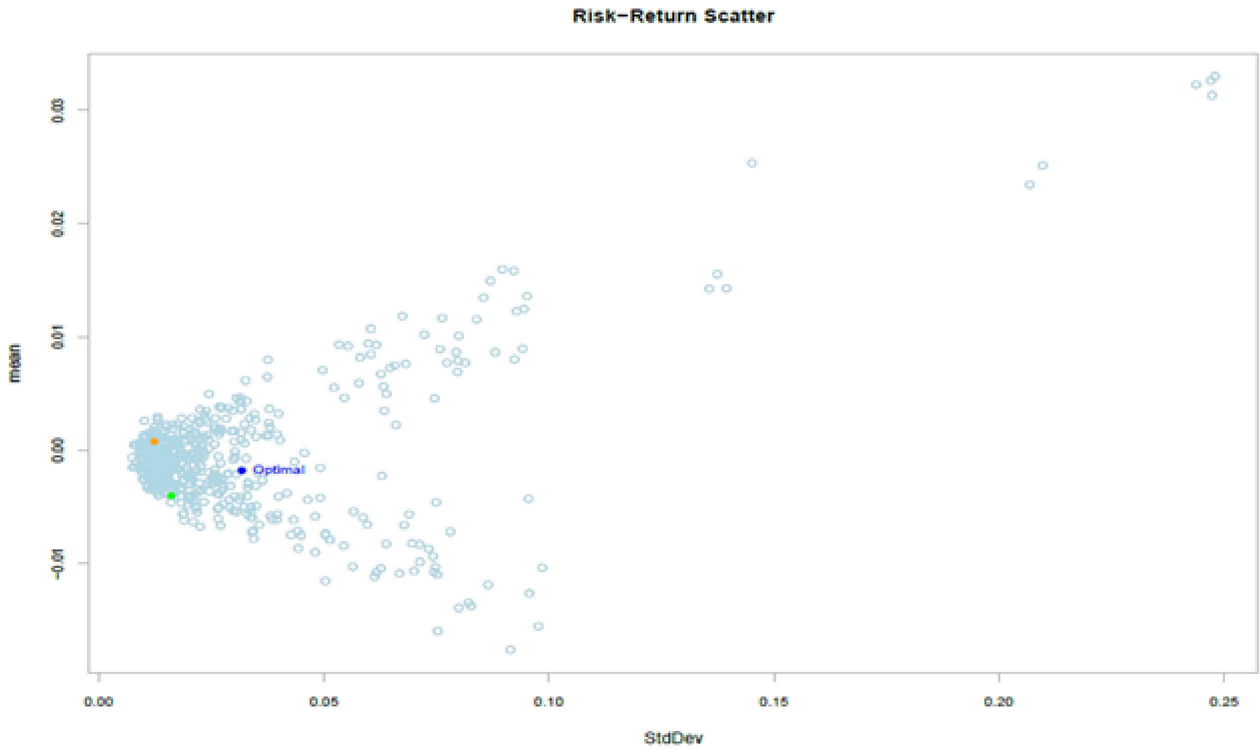


Figure 4: Shows the distribution of the optimal active weights.

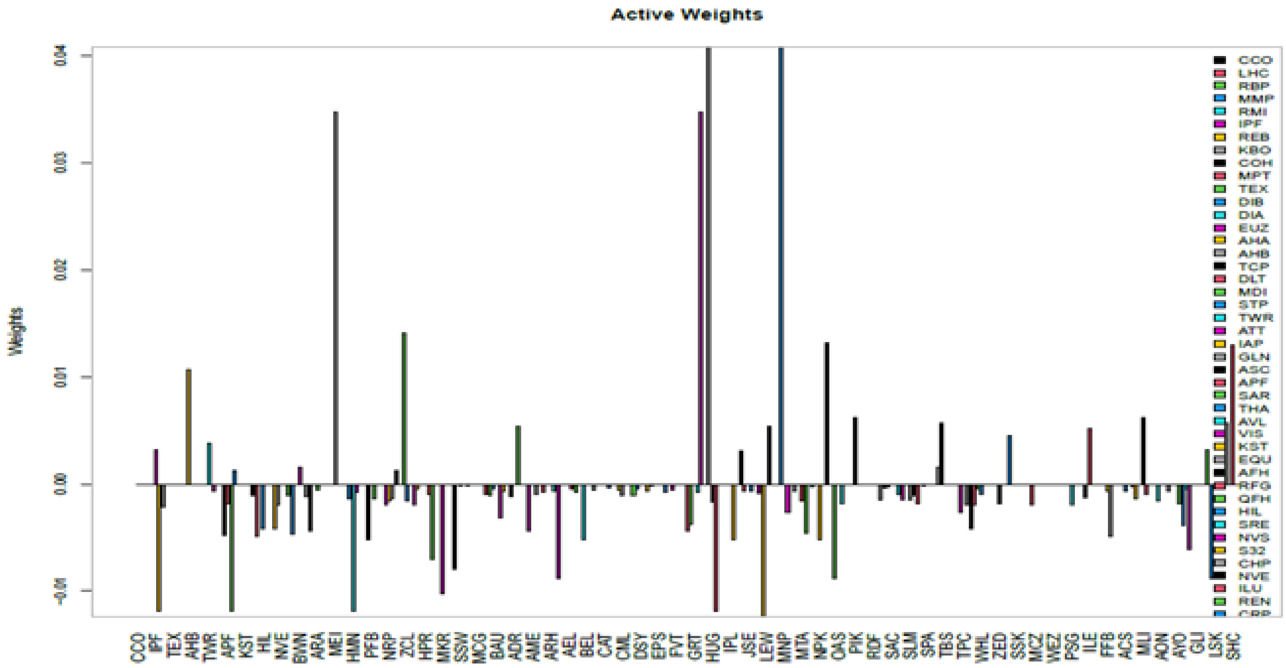


Figure 5: Shows the sector active weighs for the optimal portfolio.

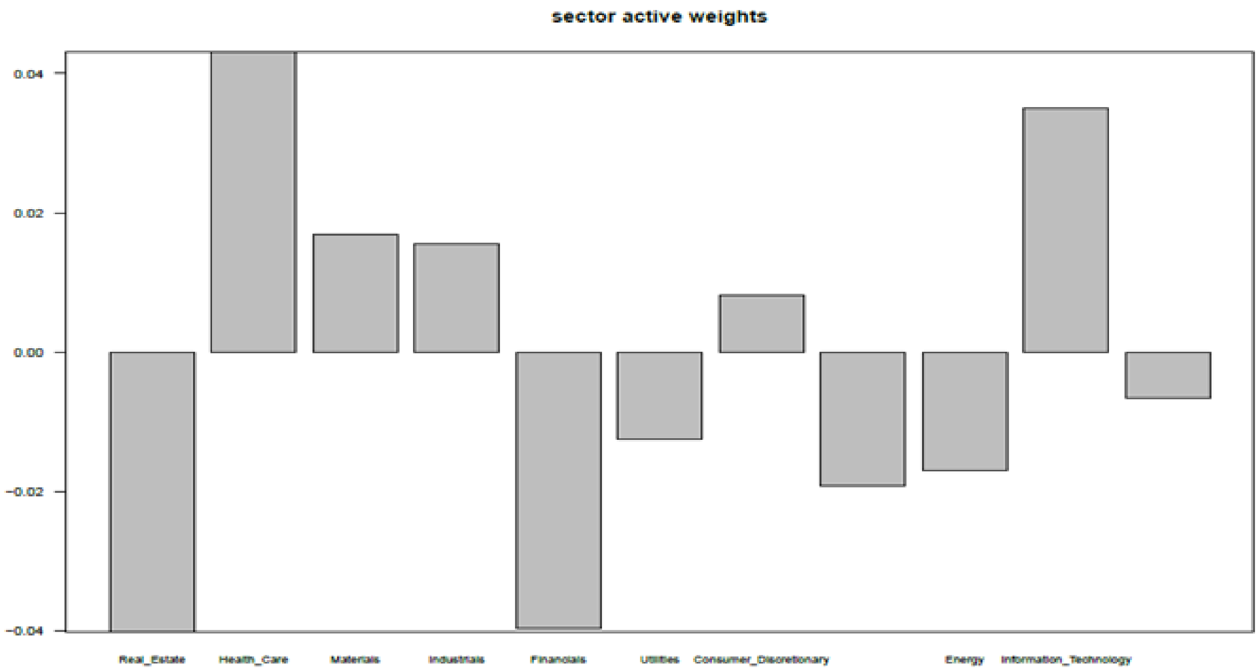
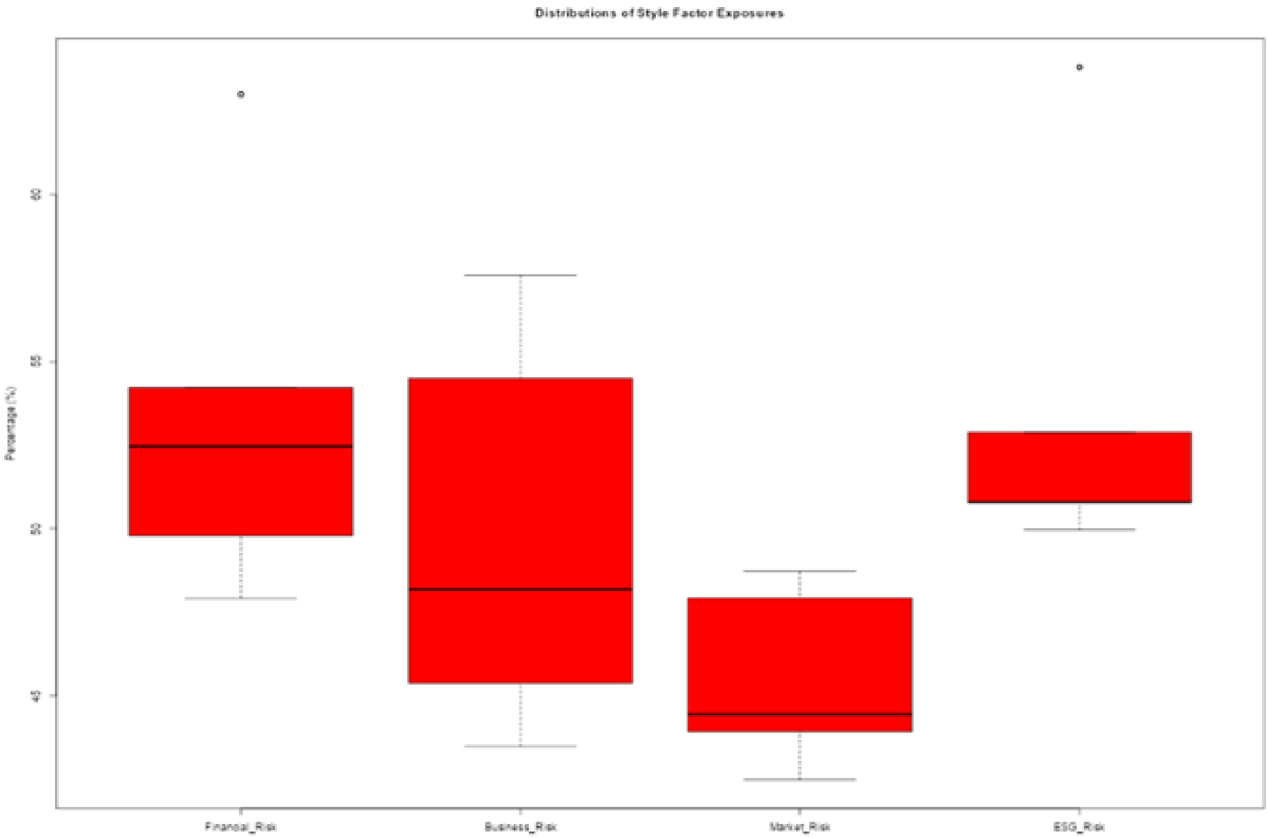


Figure 6: Shows the style factor exposure distribution from the factor model.



6. **Summary and conclusion:**

In this article we have shown how to use factor-based risk models and quadratic utility optimization to derive the fundamental law of active management. The results show that the use of factor models can assist managers forecasting expected returns and portfolio construction process who aim for a portfolio that maximises expected return and minimizes risk.

Style based factor models remain the most widely used models for the purpose of asset-pricing tests, attribution analysis, style evaluation and investment product development and management (Page D, McClelland D & Auret C, 2023). The results show the importance of these models in the derivation of the fundamental active management framework. This study further adds from the study done by Ding and Martin (2016), on incorporated the fundamental law of active management framework in the factor models to assist portfolio managers who wish to assess and evaluate their forecasting skill and implementation skill. The scope of the study only covered the random optimization and thus can be increased to include more recent and advanced optimisation algorithms such as Differential Evolution (DE) optimization algorithm, the ROI optimization, Particle Swarm Optimization (PSO) and Generalized Simulated Annealing (Gena) algorithm. The study was done using a Benguela Global risk and portfolio analytics software.

References

Buckle, D. "How to Calculate Breadth: An Evolution of the Fundamental Law of Active Management." Journal of Asset Management, Vol. 4, No. 6 (2004), pp. 393-405.

Clarke, Roger, Harindra de Silva, and Steven Thorley. 2002. "Portfolio Constraints and the Fundamental Law of Active Management." Financial Analysts Journal, September/October.

Ding, Z and Martin, R.D. 2016. The Fundamental Law of Active Management Redux. SSRN: https://ssrn.com/abstract=2730434 or http://dx.doi.org/10.2139/ssrn.2730434

Page D, McClelland D & Auret C. (2023) Pure quantile portfolios on the Johannesburg stock exchange, Cogent Economics & Finance, 11:2, 2231662.

Grinold, Richard C. 2006. "Attribution: Modelling asset characteristics as portfolios." Journal of Portfolio Management.

Grinold, Richard C. 2006. "A Dynamic Model of Portfolio Management." Journal of Investment Management 4 (2): 5–22.

Richard Grinold (2005) Implementation Efficiency, Financial Analysts Journal, 61:5, 52-64

Grinold, Richard C., and Ronald N. Kahn. 1995, 2000. Active Portfolio Management: Quantitative Theory and Applications. Probus Press. 2nd ed., McGraw Hill.

Grinold, Richard C., and Ronald N. Kahn. 2019. Advances in Active Portfolio Management: New Developments in Quantitative Investing. Probus Press. 1st ed., McGraw Hill.

Barra analytics. 2008. Measuring the Efficiency of Portfolio Construction., MSCI Inc.

Sid Browne, (2000) Risk-Constrained Dynamic Active Portfolio Management. Management Science 46(9):1188-1199. http://dx.doi.org/10.1287/mnsc.46.9.1188.12233

Grinold, R.C. "The Fundamental Law of Active Management." The Journal of Portfolio Management, Vol. 15, No. 3 (1989), pp. 30-37.