Review of Financial Economics 21 (2012) 93–101

Contents lists available at SciVerse ScienceDirect

Review of Financial Economics

journal homepage: www.elsevier.com/locate/rfe

True Markowitz or assumptions we break and why it matters ☆

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ARTICLE INFO

Available online 26 June 2012

Keywords:
Asset allocation
Markowitz optimization
Markowitz
Fund management
Portfolios
Risk management

ABSTRACT

Markowitz (1952, 1959) underlies modern corporate finance literature, from modern portfolio theory, option theory, to risk management (especially value at risk type methodologies). From it, *Diversify* has entered all languages, such is its power. Terms such as "the only free lunch" have become a way to give praise to Markowitz work. And, just as with all fundamental breakthroughs in the literature it has been extended many directions, sometimes not necessarily to the benefit of the original work, which often gets blamed when one rendition or another breaks down. With almost every MBA graduated believing they know what Markowitz optimization or portfolio theory means, it behooves us to step back and look at some of the basics, the assumptions that are made, the costs of breaking assumptions, and the potential disasters that can occur when those basics behind all of the theories dependent upon Markowitz' original work are ignored.

This paper lays out many of the basic underlying assumptions behind creation of Markowitz type portfolios, why they matter, and where those assumptions are ignored and/or broken. Breaking model assumptions is common in actual application of theory. Not understanding the implications of broken assumptions is almost a guarantee of failure for a money manager; it is just a matter of time. As one often hears, "Wall Street (or the City if in Europe) is littered with great ideas that do not work in practice." Some people throw Markowitz and portfolio optimization into that litter bin. We discuss several basic assumptions of modern portfolio theory, when and why they are commonly broken by the best of us in academia and in practice, and discuss the implications for breaking them under trying circumstances.

It has been sixty years since the finance literature was turned upside down by Harry Markowitz Noble Prize winning approach to portfolio creation and management. From his first path-breaking piece (Markowitz, 1952) and his extensive map of portfolio management theory and practice (Markowitz, 1959) we have all become modern portfolio theorists and practitioners. And as we have also benefited from multiple variations of Markowitz to the point almost every concept that is rolled out will have some relation, both good and bad, to basic Markowitz. Often modern portfolio theory (MPT) is appealed to when explaining everything from why SIVs (structured investment vehicles) were worthy of a AAA rating to explaining why those same SIV's blew up and were full of risk. It seems that all advisors appeal to "Portfolio Theory" at all levels no matter what is chucked into the portfolio. And, how could any investment committee of a foundation not have a computer generated asset allocation provided by their *Advisors*, which is a whole business in and of itself created by our enchantment with the concept of diversification. One could go on and on with one example after another. In fact MPT is ubiquitous to all financial theory and practice. By the same token, often the implementations of MPT break many of the basic assumptions behind MPT (and Markowitz) thereby making the conclusions derived from these actions extremely misleading, and in many cases completely incorrect.

Having experienced the inability of simple diversification to protect many managers from the crisis of 2008, some professors, spokesmen, and certainly journalists pronounced that Markowitz Theory is dead, misleading and just plain wrong. It was once overheard of a Dean of a major Business School stating after 2008 that "Mr. Markowitz had a lot of explaining to do and maybe should give back his Nobel Prize." One other quote (in print) that sums up this feeling suggested that Markowitz (along with others which followed on Markowitz such as Sharpe) should give back their Nobel prizes and the Nobel committee should be sued.[1] And many observers have attacked some of the underlying methodologies.[2] Railings of analysts against parts of MPT have

☆ The W. Frank Hipp Distinguished Professor of Business and Finance, The Citadel. This paper is a compilation of ideas resulting from several years of implementing and managing Markowitz based portfolios. As such it owes a great deal to others with whom Wilford worked over the years. Many are co-authors of papers noted. Particular thanks go to Bluford H. Putnam for his input to earlier drafts of this study. All errors are those of the author.

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[1] See http://www.bloomberg.com/news/2010-10-08/taleb-says-crisis-makes-nobel-panel-liable-for-legitimizing-economists.html for a discussion of some of these comments.

[2] Discussions of what is wrong with MPT and basic portfolio optimization have come from all perspectives. Some are very interesting such as discussions about Gaussian distributions versus more general ones (following on the problems with assuming normality of risk) and work discussions of chaos and complexity in systems. In particular see Mandelbrot (1963).

1058-3300/$ – see front matter © 2012 Published by Elsevier Inc.
doi:10.1016/j.rfe.2012.06.003

many forms, some justified and some simply conclusions reached because of misapplication of the theory. For example, value at risk (VaR) is often ascribed to MPT, and simplistic VaR calculations have proven anything but trustworthy. Did this mean MPT was wrong? Or, did the many mathematically sophisticated versions of VaR available just assume that because there was a distribution of risk apropos to Markowitz optimization, and diversification resulted, that VaR would provide appropriate answers to questions of risk. Was this the naivety of MPT (not sophisticated) or rather a result of the beauty of the math masking actual understanding of what assumptions underlay the math? Rather, did the GIGO (garbage in garbage out) optimized output simply reflect maximized garbage? An optimizer will maximize garbage if that is the input.

Many of these attacks on Markowitz and MPT result from the misapplication of basic concepts. To the extent these attacks are made by practitioners and portfolio managers, it likely reflects poor performance as a result of broken assumptions behind said MPT misapplication. In fact, it is difficult to find places where some assumption is not broken by the actual application of mean variance optimization. Many fixes that are applied to "solving" mean variance optimization issues further break the assumptions behind the basic theory, making it less useful and certainly less useful under stressful situations. Thus, from a practical application of MPT in creating efficient portfolios one must be absolutely cognizant of the basic assumptions of the theory, not just the mathematical models of optimization.

This paper considers several situations where accepted wisdom created a disconnect from the actual assumptions behind the theory. In doing so the discourse draws primarily upon the author's own mistakes, ones made over the years in utilizing basic Markowitz propositions to manage global asset allocation funds. As is often heard in the City or on Wall Street, "there is nothing like losing a great deal of money to focus one's attention." To avoid the problem of misreading the implied assumptions in a formula words will be used to highlight problems.

1. The basic model

Rather than quoting from Markowitz (1952, 1959) let us begin by seeing what is commonly referred to as Markowitz diversification. From the website Riskglossary.com (2012).[3]

> If we treat single-period returns for various securities as random variables, we can assign them expected values, standard deviations and correlations. Based on these, we can calculate the expected return and volatility of any portfolio constructed with those securities. We may treat volatility and expected return as proxy's for risk and reward. Out of the entire universe of possible portfolios, certain ones will optimally balance risk and reward. These comprise what Markowitz called an efficient frontier of portfolios. An investor should select a portfolio that lies on the efficient frontier.

From this definition or description it all seems very easy; no reason to read the original pieces of work. Simply calculate some expected values, make up a standard deviation for each asset to be considered in the portfolio, figure out some correlation matrix and *voila* one may find the efficient portfolio. An overhead download from a major university, which will go unnamed, states:

> "Markowitz (1959) mean-variance frontier:Investors select their portfolio in terms of expected return and the variance of returns."Optimally they will hold*A mean-variance efficient portfolio*, i.e. a portfolio with the highest expected return for a given level of variance."

Again, this is very straightforward. All one needs is to create a set of
Expected Returns
Variance
Correlations

And assuming a normal distribution of errors one can calculate an efficient portfolio.

Why then when most professionals, students, practicing portfolio managers, pension fund advisors, and so on actually do so they get portfolios that look silly or ones that just do not pass the common sense test?

To expedite our understanding go to a textbook on mean-variance optimization and see what it says. What is the favorite textbook of our MBA students at most universities? Wikipedia of course. From Wikipedia[4]:

> More technically, MPT models an asset's return as a normally distributed function (or more generally as an elliptically distributed random variable), defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets' returns. By combining different assets whose returns are not perfectly positively correlated, MPT seeks to reduce the total variance of the portfolio return. MPT also assumes that investors are rational and markets are efficient.
>
> MPT was developed in the 1950s through the early 1970s and was considered an important advance in the mathematical modeling of finance. Since then, many theoretical and practical criticisms have been leveled against it. These include the fact that financial returns do not follow a Gaussian distribution or indeed any symmetric distribution, and that correlations between asset classes are not fixed but can vary depending on external events (especially in crises). Further, there is growing evidence that investors are not rational and markets are not efficient.[3][4]

Perhaps Wikipedia has it right; this mean variance optimization is fraught with problems and as such it really does not matter much. This could be a first reading. (Perhaps the author's sarcasm is showing). Instead of finding fault with the fundamentals of the theory that was proposed by Markowitz, perhaps one should actually take the time to understand what those basic assumptions behind portfolio optimization are before attacking it. Doing so may reveal that much of the misunderstanding of MPT's implication and Markowitz mean variance optimization in particular, derives from our casual use of words such as expected returns, variance, correlations, (my favorite) "corner solutions" and so forth.

1.1. Revisiting Markowitz

First we must reread one of the most insightful set of quotes from his 1959 book[5]:

> 1. *Avoid "good" reading habits*. Some modern reading methods encourage the reader to grasp phrases in a glance, move steadily forward, never reread a passage, never mull over a detail. Although such practices may be excellent for quickly reading a novel, they are not suited to the comprehension of unfamiliar mathematical material! Rapid reading becomes increasingly out of the question as we introduce more compact notation. A few symbols can represent dozens of words of ordinary English. To attempt to swallow such a concentrated morsel in a single gulp is bound to lead to intellectual indigestion.

[3] http://www.riskglossary.com/link/portfolio_theory.htm (March, 2012).

[4] See http://en.wikipedia.org/wiki/Modern_portfolio_theory (March 9, 2012).

[5] Markowitz (1959 pages 37 and 38).

2. *Pay particular attention to definitions*. It is impossible for the reader to understand the significance of a theorem or follow a proof if he does not know the exact meaning of terms. Terms of special importance are in *italic type* when first introduced.

3. *Pay particular attention to theorems*. A theorem is a compact, formal statement of an important relationship between concepts. Most of our discussions are directly or indirectly related to some theorem. They explain theorems, prove theorems, illustrate the importance of theorems. If the theorems are understood, their applications to problems of portfolio selection follow as corollaries.

4. *Take time to understand proofs*. A proof shows that the relationships expressed in a theorem follow from definitions of terms and properties of numbers. A theorem learned by rote will soon loose meaning and slip from memory. Once the reasons for the validity of the theorem are seen, once its proof is understood, once the inevitability and logical necessity of the relationship are comprehended, the theorem becomes like an old friend not easily forgotten, quick to be recognized if met again.

In a sense the profession (the author included) is often guilty of breaking these commandments from his book. That is, we read quickly, assign what we think is meant by variance or risk for assets being forecasted as well as their correlations. It is easy to understand why one does so because sticking to the basic focus is difficult, nearly impossible sometimes. As such, second best solutions are often taken as received wisdom, thus forgetting the underlying assumptions.

Now consider a fairly straightforward representation of the mean-variance problem from basic Markowitz:

Harry Markowitz's work [including Markowitz (1959 and 1987) as he developed modern portfolio theory] indicated specifically the inputs needed to construct efficient portfolios. If one is in the forecasting business – that is, if one is seeking an efficient portfolio allocation not simply based on historical data – one needs to determine series for:

Expected returns,
A measure of forecast accuracy, as measured by the standard deviation of the forecasting errors, and
Correlation among the forecasting errors.

These are very different from using simple historical distributions. Markowitz recommended measuring the difference between expectations and outcomes rather than historical averages and outcomes using standard deviation as the measure of volatility. Under certain circumstances one could say that ex post measured variance covariance matrices are acceptable but those are only under those *specific* circumstances. [6]

An example is illustrative. Suppose one can forecast the return of a volatile asset with perfect accuracy. From the perspective of MPT or any approach to investment management, one should be willing to take a large position in a market where one's views have been very accurate. However, using a historical measure of risk, it could appear that, because the asset had been volatile historically, such a portfolio position was very risky. By the same token, consider the opposite example: imagine a model that made very extreme forecasts for a low volatility asset. In this case, a historical measure would tell one to take a large position because historically there has been little risk. In reality, our understanding of the volatility of forecase has not been accurate and thus we should take a more modest position.[7]

[6] Markowitz discusses these conditions in his chapter on variances and standard deviations in this 1959 book.

[7] This commentary and approach is from Norland and Wilford (2000) and Smithson and Wilford (2000).

From Markowitz (1987)
Expected return of an asset, i is: $\mu_i = E(r_i)$
Variance of an asset i is: $V(r_i) = \sigma_{ii} = E(r_i - \mu_i)^2$
Covariance of assets i and j is: $\sigma_{ij} = E\lfloor (r_i - \mu_i)(r_j - \mu_j) \rfloor$

What then is being said by these simple equations? In layman's terms it is exactly what was noted above. When forecasting a set of expected returns understanding not only what one's forecasts are, but also how proficient a forecaster one is in reality is essential. Furthermore, are the forecasting errors (not the underlying variables historical movements) related; I read this as, are your mistakes related to each other in a well defined or not so well defined relationship? As per Norland and Wilford (2000) this is <u>not</u> saying:

Variance of an asset *i* is: $V(r_i) = \sigma_{ii} = E(r_i - \bar{r}i)^2$
Covariance of assets *i* and *j* is: $\sigma_{ij} = E\left[(r_i - \bar{r}i)(r_j - \bar{r}j)\right]$
Where $\bar{r}$ is the average historical return of an asset i or j. [8]

This basic dichotomy sets the stage for our discussion of practical uses of mean-variance optimization, applications based upon MPT, and the common mistakes one finds in applications of the theory. Rather than calling these misapplications mistakes since in a sense they are not, we will discuss some of the ways the theory is utilized which may, under certain circumstances, break the fundamental assumptions embedded in the basic theory. Or, as Markowitz warned us, when we are not specific and speed-read we are likely to miss out on something critical to understanding and applying the theory.

1.2. Why does portfolio theory exist?

In providing a simple answer to this question one may better understand where there are embedded assumptions in the application of MPT that may easily be broken. Portfolios exist because no one is prescient. No forecaster will be perfect. Humans make errors in forecasting. Uncertainty in forecasting is a certainty, maybe the only one in financial markets. This does not mean that everyone should simply stop trying to forecast. That could be dangerous if crossing a busy street, motorway or even sometimes a country lane. By definition everyone forecasts often and intuitively place some type of measure of uncertainty about those forecasts. Still, no one is truly prescient about entering a motorway or arriving on the other side of the street. I do know, however that walking across Legare Street in Charleston SC is much less dangerous (success is very likely) than walking across I-95 in Jacksonville, Fl (success is much less likely). Simplistic? Yes it is, but what Markowitz saw was that in a world where no one is prescient it is not sufficient just to be a good forecaster, but rather one needed to have a sense of "how good", and that is true for each individual asset return being forecasted. Then if the errors are not perfectly related, overall portfolio risk can be less than the individual risks of the assets. Because no one is a perfect forecaster of the future, portfolio theory exists. With perfect foresight there is no need for a portfolio.

For our purposes let us rename these variables to be consistent with forecasts about walking across streets.

[8] When reading many thoughts by Markowitz it is not clear that he thought that measuring variance via historical returns differences was always incorrect. One could interpret under very specific circumstances where it is correct, but these are circumstances, which are of little use to the forecaster or professional money manager focused upon asset allocation; one using mean-variance for management purposes. It is often used under the conditions that if the future looks like the past, then historical returns, historical variances derived from actual returns and correlations may be utilized for illustrative purposes. It is one thing to "forecast the past and quite another to forecast the future." More importantly if one is prescient, and one should be about the past, why would one need to utilize MPT or mean variance or any tool for diversification in the first instance.

Expected Returns Making guesses about the future.
Variance in those expected returns How good a guesser am I?
Correlation in the errors in forecasting When I make one mistake does doing so provide information about other mistakes I may make?

Using terms such as mistakes and good guesses may help us remember that sophisticated mean variance optimization systems exist *because* one is making guesses about a future set of returns and will always make mistakes about those guesses, and may do so in a measurable way.

2. Practically applying mean-variance optimization

Optimally one would like to create a methodology which simultaneously created a set of forecasts for expected returns (our guesses), measured how good a forecaster one is (variance of the errors in our forecasts), and provided a quantitative measure of how errors in the forecasts are related (correlation of the errors in forecasting). If one had consistent measures of these series then well-defined portfolios which are well-behaved (do not look silly) would be a logical result. Obtaining this consistent set of measures is, however, easier said than done.

Indeed, it is the risk component that presents a significant difficulty for investors. Knowing how good a forecaster one actually is for each expected return is very difficult. People tend to be too confident or too pessimistic about their forecasts and misestimate errors in forecasting.[9] Further, if one makes more than one or two forecasts simultaneously, it is very difficult to keep all correlations of those errors in mind since the number of correlations increases in a quadratic manner.[10] Perhaps because of this problem managers find alternative ways of focusing upon their errors in forecasting. The first practical application many managers perform is to ignore actual forecast errors. And, as a result if one is utilizing something different than those errors in forecasting, the covariance matrix is also derived from something other than the theoretically correct set of estimates.

Many choices are made of how to handle this problem:

a. Using historical variances derived from the historical series of returns
b. Using some type of historical correlation matrix in combination
c. Using volatility measures derived from options markets with a correlation matrix derived from history
d. Separately forecasting, not simultaneously with the forecasts of expected returns, volatility and correlation matrices
e. Using proxies for forecasts of volatility such as a market measure of volatility
f. Calculating semi-variances
g. And the list goes on.

The literature is replete with different models for calculating the variance covariance matrix. And, under certain assumptions each individual methodology may have merit. The key is to understand when an approach will not work effectively. Each of these alternatives is theoretically incorrect. As such they break certain fundamental assumptions. Knowing when this is important or not is critical to determining if the portfolio created is logical and reflects reality.[11]

When practically applying MPT in a Mean Variance optimization form (MV) it is difficult to be pure, as noted. What, however, are the implications of not being so? Logically one should (and the key in practice is to) get as close as possible to the theoretically correct. When one deviates from the theory, at a minimum know the implications of deviations from theory.

Consider a sequence of events to illustrate where deviations may lead. Assume a sophisticated forecasting system, which does produce good and stable forecasts. Also assume that this methodology produces a measure of the errors in forecasting, but not a measure of the relationships of those errors to each other.[12] This is quite possible if a system exists which forecasts individual returns and variances for each of those individual returns independently. One may argue that such forecasting is the norm, not unusual and good, reliable forecasts are better than no forecasts at all or only ones generated in a random fashion. Using these data one might expect to build a consistent and well-behaved portfolio. To do so one has to make assumptions concerning the relationships between the errors in forecasting. One could assume that there are no relationships, implying correlations are zero. Doing so would not be dissimilar to what is done daily by bond traders who trade individual positions while not considering cross effects. With correlations assumed to be zero it is quite possible that one would achieve a portfolio that makes sense and believe that indeed it had the diversification that is promised by correctly implementing MV. What if, however, correlations are not zero; that is, the forecasting errors are not independent.

2.1. Non-zero correlations in forecasting errors: potential issues

Suppose that the correlations in the errors are rather high and our portfolio only allows for long only positions. In this case the estimates of risk will most likely be substantially below the actual risk, even without a crisis occurring. Now, if the portfolio is long-short (say a hedge fund), then the assumption of independence will likely generate a portfolio risk not dissimilar to that of the long only, even if there are long and short positions allowed. Zero correlation in forecasting errors eliminates the intended diversification associated with long-short portfolios.

If the forecasting errors are highly correlated then the long-short portfolio would tend to have a much lower estimated risk than the long only, one quite different from what is observed above, where this information was assumed away. More importantly, if the assumption of zero correlation in forecasting errors permeates the portfolio, not only is one likely to underperform in normal times, but also during periods of stress, such as a liquidity crisis, when the actual correlations in observed returns tend toward 1. In such an event the ex post risk of the whole portfolio (either long only or long-short) will be *significantly* greater than expected.

As a thought experiment assume zero correlation between two forecasts. Now allow the actual volatility in the asset prices to double from what was expected and observed correlations to move to one. In this two-asset model it is easy to deduce that the portfolio risk could more than double. This implies that the diversification one expected not only did not occur. Rather, the overall risk likely rose more than the risk of the individual return series themselves.[13]

Alternatively if (1) the two expected return forecasting errors were highly correlated, (2) the expected returns were different, (3) the long-short case MV was optimized with this information, and (4) the actual ex post correlation under the shock conditions (of a doubling of risk for each actual return) tended to 1, then the event may not create a significant increase in ex post portfolio risk. Indeed, it may be the case that the measured ex ante risk would be higher than the

[9] See Buede and Watson (1987) for an elaboration.
[10] Many of these difficulties are addressed by a series of articles by Putnam and Quintana et al. in a series of articles in *JASA*.
[11] See Silver and Wilford (2007) and Smithson and Wilford (2000) for examples and further discussion of the implications for both portfolio managers and those that use VaR based systems for measuring risk.

[12] Such a case is quite possible if the forecasts are human generated and someone has a methodology, which allows one to know how good each forecast is for each asset return being forecasted. Or, as noted each forecast is created in an independent format where there is some error calculated by a machine process.
[13] In the long only case the ex post risk of the portfolio will always be greater under these assumptions than the risks of the individual positions. It is not as clear in the long-short case, depending on the differences in expected returns the resulting risk will likely be much greater but may not more than double.

ex post risk resulting from the shock. No doubt this would be a pleasant surprise. These results can be expanded to multiple asset forecasts, forecasting errors and correlations. Similar logic can be used for long short portfolios where the shock risk is that correlations break toward negative one. [14]

The issue behind this discussion is the assumption of independence of return forecasts. This assumption hides many potential problems. Both underestimation of the risk in long only a portfolio occurs and overestimation of the risk in long-short portfolio occurs. "Intended" diversification can be created for highly correlated forecasting errors. Whereas underestimation of risk may not show up until a crisis, over estimation of risk will show up as underperformance of returns, since the actual positions taken would not be large enough to earn the benefits of "good" forecasts.

Of course a similar set of arguments can be made for the assumption of zero correlation when there are negative correlations in forecasting errors. Not to belabor this point, it is sufficient to say similar problems arise.

In all the cases outlined above the critical assumption in Markowitz MV optimization being broken is that we *know* the correlations in the errors of forecasts. Commonly broken, this assumption has multiple ramifications and the user must be cognizant of the implications of misapplying MV optimization under different circumstances.

2.2. Suppose one does not know the errors in forecasts

It is likely that the user of mean variance will not have measured the errors in forecasting effectively. In the example above it was assumed that this measure was correct and proceeded to show the implications of not knowing the actual correlation matrix and thus the correct variance covariance matrix to be utilized in Markowitz MV optimization. Thus, sorting out the potential errors resulting from multiple "breaks" in the assumptions behind MV optimization become even more difficult. Some people solve this problem by the KISS principle (keep it simple and stupid), or at least cover up the problem by doing so. That is, they say forecasts are derived from knowledge of the past, even though they are not the past, so the logical variance covariance matrix to be applied in the optimization process is that derived from history. It is simple, but wrong. Only a forecast of history will give one the historically consistent variance covariance matrix. The counter is that one knows what happened in the past. Why does one need a portfolio when one has perfect hindsight?

An alternative KISS principle, which may be apropos, however does exist. It is one espoused by Arnold Zellner (Zellner, 2000). Zellner's version of the KISS principal differs from the "keep it simple stupid" phrase. He points out that simple and stupid are sometimes one in the same. Zellner's version of KISS reads: "Keep it sophisticatedly simple." Models that are sophisticated and simple can work well in financial markets; models that are simple and stupid nearly always fail. Sophistication, however should not be confused with mathematically superior; these models often do quite the opposite in that they simplify the assumptions leading to very elegant (mathematically) KISS (and not the good one) portfolios. Although this will be covered later this problem arises clearly when models use constraints to cover up problems due to the original KISS principle being applied.[15]

This Zellner KISS principle is consistent with the dictums of Markowitz noted above. Practically applying MPT and MV optimization means that the systems utilized to do so must have inputs that are well understood and well defined in the context of the sophistication of the theory. Otherwise one is breaking assumptions without knowledge of the implications. Not having a good measure of the errors of ones forecast implies that the variance covariance matrix utilized will be incorrect leading to a portfolio that is not actually efficient. Dealing with this problem is not simple. Multiple assumptions could deal with the problem such as application of different time period variance covariance matrices, then using judgment as to the one best suited for the situation. This has obvious drawbacks but may be one KISS solution that somewhat corresponds to Zellner's definition.

2.3. The portfolio looks strange

Typically when a portfolio looks strange it is because one of the aforementioned assumptions was broken. Broken assumptions typically result in a portfolio with concentrated positions. In examining the output of a typical endowment or foundation fund advisor's asset allocation, one is often struck by the allocations symmetry. Large cap may have 20% allocation, high yield 10%, emerging markets 15%, etc. Although generated by their quantitative modeling these round numbers are usually the result of simple constrained optimizations and round to the point where the constraint is placed. Sitting in one of these meetings one is apt to hear "of course the model must be constrained otherwise it would put half of the allocation into emerging markets (or maybe it is high yield or maybe China or maybe commodities)." This is usually accepted as scientific. It is not scientific and typically reflects the user's dilemma of not knowing why the results look silly (*remember results that look silly are usually silly)* while knowing that the output is indeed silly. So the KISS (not Zellner's KISS) principal is to simply order up a mathematically elegant solution – the constrained optimization. Rather than asking what assumptions were broken in the original forecasting procedure, construction of the variance covariance matrix, the modeler constrains the portfolio such that it looks "diversified." Interestingly this is no different than simply "making up" the portfolio from the beginning, thus negating the need for the optimization process.[16]

If a portfolio looks silly it is usually because it is silly and one should try to analyse why. The answer is almost always found in the forecasts themselves (demonstrated in section V of this paper), or in the lack of consistency of the forecast with the measure of risk utilized, or the assumptions implicit in the correlation matrix. Often the problems are found to be in all three places.

Since theory would tell us that the forecasts of returns, the errors in forecasting (variance), and the correlations in those errors in forecasts should all be calculated simultaneously, then any deviation from the theoretically correct construct will lead to portfolios which may have inconsistencies and appear (because they are) silly. To solve this problem one should not compound the errors via constrained optimization, no matter how appealing. Most computer programs have the option built in so utilizing this crutch is easy. Rather, see where the misinformation lays and rework accordingly.

[14] Putnam and Wilford (1998) discuss these problems in more depth.

[15] Wilford, Norland and Quintana (2000) state:Sometimes, when practitioners apply modern portfolio theory incorrectly, their optimizers yield bizarre looking portfolios with extreme allocations called corner solutions. These managers typically then apply various constraints to limit position sizes. The need to apply such restrictions is indicative of problems with the inputs for the optimizer, not with the concept of applying mean-variance optimization itself. Such restrictions usually do not reduce risk. In fact, they sometimes have the perverse effect of increasing risk because managers have to take more aggressive positions in other areas. Thus, portfolio constraints nearly always have the effect of reducing risk-adjusted returns (see Putnam, 1997; Wilford & Quintana, 1998). For a more in depth discussion of the potential benefits and drawbacks of using quantitative systems to construct portfolio and to measure risk, please see Norland and Wilford (2000).

[16] Constraints come in many forms. Above we discussed the more common constraints used, but they may show up in many places. Use of constraints is widespread in the literature. See Pachamanova and Fabozzi (2010) Quantitative Equity Trading: Techniques and Strategies pages 327–333 for a discussion of constraints in MV optimization and Roudier (2007) for a constraint discussion dealing with size and turnover penalties, as well as other issues associated with MV analysis where forms of constraints may be employed.

2.4. Estimating excess returns, not total returns

Global asset allocation modeling uses exactly the same principles as domestic modeling. Doing so, however, highlights another of those assumptions that appear buried that is often broken. Portfolios should always be estimated in excess, not total, return space. What does this mean practically? To illustrate the problem assume that you have the choice in investing in the UK stock market or the Brazilian stock market or the U. S. stock markets or into a portfolio of the three. Now to make the problem even more simple, assume that the forecast made for the three markets will yield the same return of 8%. And, all three markets have the same volatility with cross correlations all the same. Also allow the portfolio to hold cash. Now optimize with a target return of 8%. Most likely on the back of the envelope one would find about a third invested in each market. Typically this is where the story would stop, but is it where it should stop? Implicit in the discussion was another assumption. That is, the numeraire that would be apropos is always the same. This is not the case. One can argue that the theoretically correct global portfolio (no matter the number the assets) should always be the same no matter what currency it is created in, but it will not be. Why? It is because we break another assumption. That is, we have implicitly assumed the same risk free rate or cash rate for each currency. They are different. Indeed, the 3 month rates for our example are very different:

3 month T-Bill yield March 2012	
Brazilian Real:	9.06
U. K. Sterling:	0.36
U. S. Dollar:	0.07

Thus, implicit in our forecasts were that the risky portion of the expected return in Brazil was actually a negative 1.06%, not a positive 8%. In this example there are different risk free rates for each currency. As such if one were considering a portfolio of investments, would one desire to invest into the Brazilian equity market with an expected return of 8% when a much higher return is available by avoiding the risk of the equity market, through a Real 3 month t-bill.

This simple illustration can be taken farther for purely domestic modeling as well. Suppose the expected total return of a bond is 5% and the estimated risk is 5% while the expected return on equity is 13% and the estimated risk is 13%. The risk/return ratio for both is 1. And if one is attempting to optimize in a manner to maximize the information ratio of the portfolio (allowing the model to simultaneously trade off risk and return) beginning with the information ratios for each asset would yield a portfolio which we will call A.

Now, perform the same exercise, but (instead of looking at the individual information ratios being one) separate the risky portion of the ratio from the part where there is minimal or no risk. This ratio is based upon excess return above libor or some risk free rate. We will define it as the expected return due to *actually* taking risk. If 3 month rates are 3% then

Total return	=	Excess return	+	Riskless return	Excess return ratio
Equity (TR)	=	10%	+	3%	10/13 or .77
Bond (TR)	=	2%	+	3%	2/5 or .40

Needless to say in excess return space the ratios are not equal. As such, would it not be more logical and safer to invest more into equities (given the first 3% is riskless) than in the case were we assumed away the excess return problem? This new portfolio, B, would look considerably different than A. Suppose the goal were to achieve a portfolio return of 7%; a portfolio made up largely of equities (assuming zero correlation of equities and bonds forecast errors, a big assumption) with a few bonds thrown into a mix with lots of cash should do the trick. Thus, for even a domestic portfolio with non-zero interest rates, misallocations will be made if one does not always ex ante create a series of expectations in excess return space. Although it may seem trite to highlight what appears to be a simple problem, this oversight shows up in most modeling systems. Few actually utilize excess return space although many papers have been written on the subject.[17] In an interconnected global economy making the adjustment to excess return space is necessary to create portfolios that truly reflect the risk being taken. And, of course one can change the numeraire simply by using a forward foreign exchange contract.

3. Estimation issues still arise

If one had a procedure to forecast expected excess returns, simultaneously calculate the errors in forecasting and the relationship among those errors, then one would be in position to create an optimal portfolio without breaking any of the aforementioned assumptions. In reality, however, there will still be issues that may affect the validity of an allocation.

Many of our simple models ignore the fact that there are transactions costs. For some allocation models these can be significant.[18] Where MV optimization is utilized to create portfolios that have reasonably high turnover then transactions costs are always of concern. Assume that the methodology to garner valid excess return forecasts and a consistent variance covariance matrix is available. Now, introduce the problem of transactions costs on trades. How can one handle this issue? Penalties for certain investments, lower expected returns (artificially), turnover penalties, and so forth. All of these are, in some manner, artificial to the system and may have effects not dissimilar to those of a constraint in optimization. A penalty function is in effect a constraint that is often used to solve this problem so a position that may have a large transaction cost associated with it is avoided. Entering constraints deals again with the symptom of the problem of cost and may thus simply be masking the fact that our assumptions (without transactions costs) are potentially wrong.

Often these problems show up in systems that trade illiquid securities. Excess returns appear high; the portfolio likes these illiquid securities since they promise higher returns if the liquidity issue is ignored. In fact liquidity may come and go for certain securities. Managers discovered this the hard way in 2008. As such assuming a constant penalty for lack of liquidity may create a significant deviant from an optimal portfolio that becomes very unstable during a crisis or on average undervalues the cost to the portfolio of transactions costs, thereby providing misleading transaction signals.

If constraints directly or in the form of penalties are not to be utilized then how does one manage the problem? There is no easy answer; however, addressing the fundamental issue head on appears to be the correct course of action. If one must impose said constraints it is likely because the forecasts of returns are too high or the risk associated with the particular asset too low. Here seems to be the correct place to address the problem, at its hart, not in masking the symptom of the problem via a constraint of one sort or another. [19]

Practically if one is to create a series of expected excess returns, the volatilities of the errors of those expectations and a consistent correlation matrix then most likely some type of quantitative system is being utilized to do so. Addressing these questions are essential to see where the disturbances that break the assumptions can creep in again. Markowitz, Xu and Putnam (1996) address many of these issues pointing the user to the spots where problems slip into quantitative estimation procedures.

Conclusions about methodology concerning transactions costs and illiquidity issues are simple; implementation is difficult however. Addressing the problem by covering up the symptoms through

[17] See Putnam, Bluford, "The Flaws in the US Asset Allocation Model" *Global Investor* March 1998. Norland and Wilford (2002a, 2002b), the JASA collection of articles cited below.

[18] See Al Janabi, in this issue on transactions costs.

[19] We refer the reader to Al Janabi, ibid. He addresses one aspect of the problem of correcting volatility to consider illiquidity in the measure of risk at the individual asset level. Also see Wilford and Quintana (1998) for examples of constraints effects on performance of a MV optimized strategy.

constraints during the optimization is not the best, although in many ways the common, solution. Addressing the correct risk for the expected return (understanding that risk in total for different periods of possible outcomes) being forecasted is the desired, even if difficult, solution. Further, understanding periodicity effects of misestimating transactions costs (overestimating expected returns) may offer a role for judgment, under the Zellner KISS principle.

4. Considering risk measurement and (mis)attributions to MV optimization

Risk measurement applications of Markowitz based concepts are often the most poorly applied. That is, they tend to be the ones that ignore what Markowitz type analytics can and cannot be used to deduce. With a mentality of diversify, diversify and diversify in the 1990s, the financial industry developed the concept of Value at Risk (VaR) which harkened to the simplicity of textbook Markowitz MV optimization. Rather than rehash VaR, consider some of the fundamental issues at work in applying VaR. First, VaR is an ex post concept that deals with risk estimation based up ex post volatility of assets and their relationship to each other. Second, instead of some measure of expected returns and the errors in those forecasts VaR is based on actual returns, a backward looking measure not a forward looking measure as is deemed essential in Markowitz.[20] Third, if Markowitz MV optimization underlies a portfolio based upon historical observations, then any VaR estimate of its risk has a built in downward bias. [21]

Under certain circumstances VaR can be a powerful tool and is, while under other circumstances it can be terribly misleading. The SIV crisis was a perfect example. VaR estimates of the risk embedded in SIVs were severely underestimated. Why? There are many reasons but the most simple is the fact that the assumptions behind its use (using historical data) were flawed. Using historical data on cash flows for the asset base would always tend to calculate a fairly low risk. Effectively the cash flows historically looked more like a set of coupons on a bond, even a high yield bond. Those cash flows have little variability, thus in a VaR context the volatility used in the measurements were low. Low volatilities imply a low volatility for the portfolio. As we all know writing a strip of options over time provides exactly the same type low volatility of returns but does that mean that the options have little risk (think hurricane insurance)? No, in fact quite the opposite. The risk may be large (thus the size of the cash flows from writing the options) because sometime the options may be exercised. SIV's were full of options embedded into its structure. Only with defaults (options being exercised) could one fully see from history the actual risk there. Belief in simple diversification based upon historical variance covariance matrices and their stability can be extremely misleading, especially when the structure of the instrument to be measured has a very different inherent embedded risk. Credit officers would not assume that just because a poor credit paid its last 3 years of coupons that it was ok for the next 27 years on a 30 year bond. But, in essence applying VaR models with three years of history to structures similar to SIV's is exactly the same thing. Stacking up models to justify "diversification" across all facets of a portfolio will likely embed a large number of broken assumptions.

Further, in simple applications of VaR based systems, even those that appeal to Monte Carlo, embedded distributional assumptions may make its use misleading. The classic one is assuming that the distribution of risk for each asset is normal, the future will look like the past, and correlations among those assets are stable on average. Suppose that in reality the world is better described by a bi-modal risk distribution. One may take a bi-modal distribution of risk and calculate from it a mean and a variance. Use those data (ignoring the fact that the distribution is not *fully* defined by those two moments) and underestimated tail risk (in particular the loss tail) will result.[22] Such a mis-measure is not necessarily critical, unless one is leveraged. For leveraged portfolios such mis-measurement can be serious.

From Norland and Wilford (2002a) we know that the probability of the capital being "wiped out" in a leveraged portfolio increases dramatically if the actual underlying distribution is bi-modal, given the same mean and variance, in comparison with a normally distributed portfolio. Using Monte Carlo simulations (perfect knowledge of the end result is imposed in both cases, so the end result is the same) they demonstrate that if the actual distribution is bi-model, the probability of default (for their experiment) at 10 times leverage is about 10 times MORE likely to default than the case where the portfolio distribution is normal and leverage is 10 times as well.

This probability of default ratio varies in their experiment with different leverage ratios, but the concepts of underestimation of actual risk (if the actual distribution is bimodal) holds. The *degree* of underestimation of risk is what is critical. Given that many investments and institutions are leveraged, such as banks, SIVs, leverage MBS funds, etc. underestimation of the risk, simply due to errors in judgment of the actual shape of the distribution of risk can have severe consequences. If senior management knew the actual tail risk of the SIV, would it have bought it with a leverage of 10 or 20 or an effective leverage above 50? Some did. A more correct measure of the VaR calculation, based on a bi-model distribution (maybe simply reflecting historical episodes of MBS hedge fund failures) would have indicated that even at 10 times risk the probability of default was large enough to make one fearful.

5. Some basic illustrations[23]

To illustrate some of the points raised above very simple optimizations are considered. The basic model does not suffer from constraints, misappropriate assumptions, a fixed risk or return target, but rather it maximizes information ratios seeking the best portfolio in excess return space. As such it is simple and fairly close to theoretically correct. It is not a model, however, to bet the ranch on! It is for illustrative purposes only. Historical data, with some of the flaws noted above, were used in creating the basic starting points for our illustrations. Further, adjustments to expected returns and/or the variance covariance matrix are made to illustrate the assumptions problems discussed. In this model, risk aversion can be adjusted as well. The same aversion factor is used throughout the exercise, however. In all cases the model is allowed to go long and short. Excess return estimates will be adjusted to force long only portfolios, thereby illustrating the issue of constrained portfolios (that is, the degree to which the adjustments would have to be made to correctly get a long only portfolio).

[20] See Smithson and Wilford (2000) to highlight the reasons that risk management issues need to be separated from implementation of Markowitz. The two concepts may be two sides of the same coin in some ways but heads are different from tails in this case.

[21] For example the LTCM crisis was continually explained by many of the LTCM management as a once in 10 thousand year event based upon their VaR calculations. Given that their speculative arbitrage portfolios were based upon historical data, they effectively optimized using history in creating a portfolio that reflected the most efficient allocations, if history simply repeated itself. Thus, when using the same history to estimate the risk of that portfolio it naturally underestimated the risk severely due to the bias that was introduced in the estimation procedure itself.

[22] See Putnam (2011), Putnam (2012) for a discussion of bi-modal distributional issues. Norland and Wilford (2002a) use a knock-out option concept to deal with the bi-modal problem.

[23] In this section complete data are not reported to conserve space; focus will be on the adjustments needed for each case to illustrate its point. These adjustments will be highlighted.

Six assets are chosen, plus cash. The numeraire is dollars in this case. They are:

US S&P500 (USD)
German MSCI Stock Index Hedged (USD)
Brazil MSCI Stock Index Hedged (USD)
EUR (USD per EUR)
BRL (USD per BRL)
US Treasury (10-Year Proxy)
USD Cash (Federal Funds)

Case 1. Excess return expectations based upon historical data for the period 2005–2011. Expected returns .5 times expected volatility except for Treasuries. Return assumption is zero for cash and treasuries.

Case 2. Increase expected returns of variables to force long only results.

Case 3. Adjust the long-short example imposing an assumption of zero correlation.

Case 4. Adjust Case 2 above for the assumption of zero correlations.

Case 5. Repeat Case 1 assuming volatility for 2008–2009 period and expected returns kept the same.

Case 6. Optimize in total return space assuming domestic interest rates are not taken out of the expectations for stocks and bonds. Zero rates are assumed in the US while the Brazilian interest is 10% and the Euro rate is assumed to be 2%. Exchange rates are already in excess return space by definition. The relevant comparison is to Case 2, which was long only since most portfolio models embed these assumptions.

Case 7. This is the same as Case 6 with assumption of zero correlations.

Beginning with Case 1. The set of expected returns are in excess return space, as are all of them with the exception of Cases 6 and 7. Expectations for excess returns are dampened with respect to the average volatility over the entire 2005–2011 period at a "normal" level of expected excess return, with exception of treasuries where a zero excess return is expected. The optimized model has no constraints and a reasonable risk aversion parameter. Finally the variance covariance matrix reflects the fact that there are correlations that are non-zero based upon estimates made from data for the period. Optimization allows for these facts. Results yield fairly large FX positions, both long and short, a short position in the Brazilian stock market with long positions in the S&P and German markets, a great deal of bonds and cash even though cash has no expected return. Bonds get an allocation even though return expectations are modest, however but in this model volatility of bonds is as well. The optimizer is not excited about the returns relative to risk with an information ratio not much greater than 0.5 (Table 1).

Case 2 allows for returns to increase while volatilities remain low. These sharp increases in expected returns "are necessary to get a long only portfolio" to occur. One could argue that the large across the board excess returns, especially for the equities' expected returns, is needed to compensate for the expected risks that remain the same as in Case 1. This illustrates the simple point that constraining a portfolio to be long only masks the fact that either the risk measure is incorrect for the inputs or the excess return estimates are too high, relative to the individual estimated risk for each expected return. These are the *symptoms* that are *masked* by a long only constraint.

By comparison with both Cases 1 and 2 respectively, Cases 3 and 4 illustrate the constraint of zero correlation. Case 3 uses the same parameter inputs as Case 1 but no short positions result in the optimization and the portfolio expected return is much higher. The information ratio is about 1. Imposition of zero correlations leads to underestimation of the actual risk in a portfolio relative to the positions taken (overestimating the expected portfolio return relative to the expected risk). It is interesting, however, that no short positions occur.

Where this assumption is particularly dangerous is when "actual" correlations spike toward 1 or negative 1 and actual volatility rises. In these cases this type of portfolio usually will have a greater rise in portfolio volatility than the rise in underlying volatilities. Short positions have dampening effect when correlations tend toward one and individual market volatilities rise sharply. Many traders implicitly, if not explicitly, assume a zero correlation in positions across a portfolio (perhaps not in pairs themselves) when putting on trades.

Case 4 is interesting relative to Case 2. In Case 2 we "over estimated expected return" relative to risk. Now, imposing the constraint of zero correlations simply provides an even greater sense of potential return relative to the risk taken in the portfolio. Here the information ratio is an unrealistic 1.7. And, of course, *if* there is a shock actual correlations will not be zero. Resulting volatility of the portfolio will be much higher than had been anticipated, since the same effect as noted would be exacerbated (positions are larger relative to cash since its weighting falls to 8% from 34%). Clearly the imposition of zero correlations implies a safety that is unrealistic and misleading, even too good to be true.

The GIGO principle comes fully into effect in Case 4, unrealistic expectations of returns relative to risk (similar to forcing a long-short portfolio to be long only if it is seeking a targeted return) combined with the assumption of independence. When experts estimate long-term capital market assumptions are they often not making both of these mistakes? Or, put another way, when one imposes certain constraints, here long only and zero correlation, is this not the same as overestimating returns and not recognizing the correlations in the errors of our estimates of returns?

Table 1
Asset Allocation for 7 Cases.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
ASSETS	Allocation	Allocation	Allocation	Allocation	Allocation	Allocation	Allocation
US S&P500 (USD)	17%	6%	11%	11%	2%	2%	11%
German MSCI Stock Index Hedged (USD)	12%	8%	11%	16%	0%	5%	18%
Brazil MSCI Stock Index Hedged (USD)	−1%	10%	10%	15%	2%	21%	23%
EUR (USD per EUR)	38%	16%	23%	23%	7%	11%	23%
BRL (USD per BRL)	−24%	4%	11%	26%	3%	9%	26%
US Treasury (10-Year Proxy)	33%	26%	0%	0%	4%	24%	0%
USD Cash (Federal Funds)	26%	30%	34%	8%	82%	28%	−2%
Portfolio summary							
Expected return	3.09%	5.66%	5.99%	13.71%	0.93%	9.73%	18.17%
Expected standard deviation	5.69%	5.06%	5.46%	8.28%	2.23%	6.51%	9.53%
Leverage	No leverage	No leverage	No leverage	No leverage	No leverage	No leverage	No leverage

Case 5 is an illustration of mis-estimation of the errors in forecasting problem. Its flipside is the implication of a VaR model, which utilizes history that is either unusually risky or unusually stable to evaluate underlying errors in forecasting. Case 5 assumes that risks hit the levels of the 2008 meltdown period for the selected markets. As a result the allocation basically goes to cash. Expected information ratio calculation (remember the optimization is trying to achieve an optimal information ratio given the risk aversion chosen) is paltry and only tiny positions are taken in risky assets. For comparison purposes this is similar to overestimation of risk. There exists excellent forecasting acumen, but the optimizer is not informed of this fact. It is because ex post measures of market volatility are used instead of actual measure of the errors in forecasts (the theoretically correct measure), which in this case would be small (excellent forecaster). Case 5 illustrates how under-invested one may be.

Case 6 considers the distortions caused by total return estimation. While the U. S. has essentially a zero cash rate of return the cash rate in Brazil is much higher at 10% and we assume 2% in Germany (perhaps we should have chosen Italy or Spain or Greece to show how big the distortion can truly be given the markets in May of 2012). The allocation thus reflects the “extra” return estimated relative to the excess return estimation utilized in Case 2. Focus is on the original long only case since most managers that create long only portfolios also commit this error. In comparison one observes that the information ratio is higher for Case 6 than 2, but not hugely. The difference primarily is in the sizable allocation increase to Brazilian equities relative to alternatives. More concentration of risk is taken, since the optimizer believes the expected return forecast (which is now seriously overestimated relative to the risk since the return inherent in the local interest rate is not separated out due to its (low or near zero) risk).

The final case chosen, Case 7, begins with Case 6 then imposes the zero correlations assumption. Larger positions are taken relative to cash, in general. The estimated information ratio is highest of all cases. Overestimation of returns for Brazilian and German equity markets plus the zero correlation assumption combines to give the optimizer a great deal of confidence. In this case it goes short cash so it can take risky positions across the board; again, note that the risk aversion parameter has been held constant across all cases.

One could have run many more cases to illustrate the implications of broken assumptions. These 7 cases just scratch the surface of mistakes the profession tends to make. They do, however, display how wildly different portfolios may appear, as some of the more common techniques are (mis) applied. Simply the dispersion of observed information ratio calculations should be a warning of the degree to which mis-estimations may occur. Careful consideration of any ignored assumption must be considered if Markowitz' brilliant insights into portfolio theory are to be fully understood and appreciated.

6. Concluding remarks

This paper highlighted the pitfalls of ignoring critical assumptions embedded in basic Markowitz. It argued that these assumptions are continuously broken, often resulting in portfolios that do not perform as intended. Underperformance has led to a rash of complaints about the basic theory. These complaints are often unfounded. Often they merely reflect a lack of knowledge about the essential assumptions behind any practical application of the basic Markowitz tenants. Markowitz provided us with a brilliant theory and a solid theoretical framework that has stood the test of time. Extensions and applications have not always adhered to the original rationale behind Markowitz, leading to disenchantment with the theory and poor performance. We have argued that much of this can be avoided by fully appreciating the errors that creep into application of basic theory and practical tools based upon MV analysis. If there is one lesson it is “do not throw out the baby with the bath water.” Markowitz portfolio theory is still the basis for the only free lunch in town. We just have to get the ingredients of the meal right if the cooking is to yield haute cuisine.

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