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A New Approach to International Arbitrage Pricing

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ABSTRACT

This paper uses a nonlinear arbitrage-pricing model, a conditional linear model, and an unconditional linear model to price international equities, bonds, and forward currency contracts. Unlike linear models, the nonlinear arbitrage-pricing model requires no restrictions on the payoff space, allowing it to price payoffs of options, forward contracts, and other derivative securities. Only the nonlinear arbitrage-pricing model does an adequate job of explaining the time series behavior of a cross section of international returns.

THE IDEA THAT A few relevant state variables explain expected returns is the main driving force of the seminal papers of Merton (1973) and Ross (1976). These seminal ideas have been extended to the pricing of international assets by Ikeda (1991), Ross and Walsh (1983), Solnik (1974, 1983), and Stulz (1981), among others.[1] The key implication of these arbitrage-pricing models is that only risks related to these factors (state variables) are relevant in determining asset prices.

Testable implications of these international asset-pricing models have been derived either by placing restrictions on the payoff structure (i.e., payoffs are linear in factors) as in Ikeda (1991), Ross and Walsh (1983), and Solnik (1983), or on the joint distributions of the payoffs and state variables as in Constantinides (1989). These restrictions lead to the testable implication that expected asset returns are linear in the conditional covariances with the factor payoffs. In what follows, we refer to these models as linear arbitrage models or linear models.

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[1] Also, there is a large literature on linear arbitrage pricing in a domestic context. Papers include Brock (1982), Bossaerts and Green (1989), Chamberlain (1983), Chamberlain and Rothschild (1983), Connor (1984), Connor and Korajczyk (1988, 1989), Dybvig (1983), and Grinblatt and Titman (1983), among others.

Previous empirical work using a linear one-factor model (e.g., capital asset-pricing model (CAPM) and consumption-based models) and multifactor extensions (using only equity-based factors) as developed in Korajczyk and Viallet (1989, 1992) are unable to simultaneously price international equity, bond, and in particular, forward currency returns (see Hodrick (1987), Dumas and Solnik (1992)). In this paper, we follow the nonlinear approach of Bansal and Viswanathan (1993), who observe that the pricing kernel from a linear model cannot price securities whose payoffs are nonlinear functions of the factors.[2] Such nonlinearities would arise if the primitive payoffs (e.g., equities) are themselves nonlinear functions of the factors. Even in the case when the primitive payoffs satisfy the assumptions of linear factor pricing, the presence of derivative securities (e.g., forward contracts and options) will lead to nonlinearities in the payoff set.[3] The ability of the pricing kernel from the nonlinear approach to price securities which are arbitrary nonlinear functions of the factors leads us to believe that we may have greater success in pricing simultaneously a rich collection of payoffs, including forward currency returns.

The nonlinear approach has other advantages over linear arbitrage-pricing models. The key restriction of linear arbitrage models is that there are only a few factors (relative to the number of traded securities) in the pricing kernel. Hansen and Jagannathan (1991) show that, for a given collection of payoffs, there always exists a unique pricing kernel (or stochastic discount factor) that is a linear combination of all the payoffs.[4] Linear arbitrage pricing further restricts this pricing kernel to be a linear combination of a few factor payoffs. If payoffs are arbitrary nonlinear functions of factors, the simple linear low-dimensional representation of the pricing kernel does not obtain (see Bansal and Viswanathan (1993)).[5] Our nonlinear approach delivers a pricing kernel which can accommodate this restriction of low dimensionality even when payoffs are complicated nonlinear functions of the factors.[6] Additionally, this nonlinear pricing kernel also prices dynamic trading strategies.

[2] The continuous time derivations of the linear models do not suffer from the inability to price nonlinear payoffs as all payoffs are locally linear. However, discrete time estimation of these models requires integration of the continuous time pricing kernel, leading to a discrete time nonlinear pricing kernel that depends on the specific stochastic processes used. Longstaff (1989) is an example of this approach.

[3] For example, Dybvig and Ingersoll (1982) show that even if stock returns satisfy the distributional restrictions required for the CAPM, the use of the CAPM (the one-factor model) to price options leads to violations of the no-arbitrage condition.

[4] Hansen and Jagannathan (1991) show that this linear combination of the payoffs is the minimum variance pricing kernel and is the unique projection of any pricing kernel on the space of payoffs.

[5] Bansal and Viswanathan (1993) show (see their Theorem 1) that the introduction of a new security will generically change the minimum variance pricing kernel.

[6] The critical distinction between a linear and nonlinear model can be illustrated in the following example suggested by René Stulz. Suppose the market proxy is a nonlinear function of the true underlying factor because of leverage. Any asset price correlated with leverage will help predict returns and might be identified as a factor in a linear model. However, if leverage is a function of total wealth, the market proxy will be a sufficient statistic but the linear beta relation will not hold. In this example, a linear model will typically yield more than one factor, while a nonlinear model will result in a single factor.

To construct the nonlinear pricing kernel, we first exploit the no-arbitrage restriction, which guarantees a nonnegative pricing kernel (see Kreps (1981), Harrison and Kreps (1979) and Ross (1978)). Then conditioning this nonlinear pricing kernel on the factors, we obtain a kernel which satisfies the restriction of low dimensionality. This construction does not require the nonlinear pricing kernel to belong to the set of traded securities,[7] so that nonnegativity can easily be imposed. In contrast, a linear arbitrage model restricts the pricing kernel to be a traded security which is a linear combination of factor payoffs. This linear combination (particularly when markets are incomplete) may take on negative values (see Hansen and Jagannathan (1991)), thus violating the no-arbitrage restriction.

As the exact functional form of the nonlinear pricing kernel is unknown, we nonparametrically approximate this function with a series expansion, as in Gallant and Tauchen (1989), Gallant and White (1989), and Bansal and Viswanathan (1993). In this paper, we use a polynomial series expansion. The advantage of the polynomial series is that its leading term is linear in the factors, which nests the unconditional versions of the international linear APT, the international CAPM (ICAPM), and the international discrete time ICAPM as special cases of the nonlinear model.

In addition to the nonlinear model and the associated nested unconditional linear model (as in Bansal and Viswanathan (1993)), here we also consider the conditional linear factor-pricing model, which restricts the minimum variance pricing kernel to be a conditional linear combination of a few factor payoffs. Since the conditional weights on this linear pricing kernel are unknown (and potentially complex) functions of variables in the information set, we estimate them using a nonparametric approach.[8]

Parameters of the polynomial series expansion are estimated using Hansen's (1982) generalized method of moments (GMM). Since the unconditional linear model is nested within the nonlinear model, a GMM-based likelihood ratio-type test allows a comparison between the two. However, as the nonlinear model and the conditional linear model are not nested, such a comparison is not possible. Instead, the three models can be directly compared using the distance measure suggested in Hansen and Jagannathan (1992). The Hansen-Jagannathan (HJ) distance measure is the distance between the pricing kernel under study (called the proxy) and the class of valid pricing kernels. A proxy that is a valid pricing kernel will have a zero HJ distance. Therefore, a proxy with a smaller HJ distance is closer to the class of valid pricing kernels and can be considered a better pricing kernel than one with a larger HJ distance. Consequently, we evaluate the three models using two different metrics: the GMM metric and the HJ distance.

[7] The pricing kernel that we present is not necessarily a payoff unless markets are complete. Thus it does not in general agree with the linear combination of payoffs identified by Hansen and Jagannathan (1991) as a pricing kernel. When markets are complete, there is a unique pricing kernel and the two pricing kernels must agree.

[8] Our nonparametric approach allows for time variation in a manner different from that in Gallant, Hansen, and Tauchen (1990).

The data we use are sampled on a weekly basis from January 1975 to December 1990. The raw data includes weekly observations on the world stock portfolio and equity indices from the United States, Japan, Germany, and the United Kingdom.[9] In addition, we use four-week forward contract returns on three foreign currencies (calculated using forward and spot exchange rates), the four-week U.S. Treasury bill return, and the seven-day Eurodollar deposit return.

The rest of the paper is organized in six sections. Section I discusses a theoretical derivation of the nonlinear APT and the restriction placed on security returns. It also presents a derivation of conditional linear models and contains details regarding the distance measure suggested by Hansen and Jagannathan (1992). Section II discusses the data and some of its properties. Section III presents the estimation strategy and Section IV discusses the factors, payoffs, and instruments used in estimation. Section V discusses the results and the cross-model comparisons and Section VI concludes.

I. A Theoretical Derivation

We present a simple derivation of the nonlinear APT in this section, which is similar to that in Bansal and Viswanathan (1993) for a one-country model.[10] In a world with N assets, the first-order condition that an investor in the United States willingly holds the ith asset for one period is (see Lucas (1982), Stulz (1981), Svensson (1987), and Bansal (1990) among others):

$$E[MRS_{t,t+1}x_i(t,t+1)|\Omega_t] = \pi(x_i(t,t+1)) \qquad \text{for } i = 1,\ldots,N, \quad (1)$$

where $x_i(t,t+1)$ is the payoff of the ith asset at time $t+1$ that has price $\pi(x_i(t,t+1))$ at time t, $MRS_{t,t+1}$ is the marginal rate of substitution of the investor from time t to time $t+1$, and Ω_t is the information set that the investor has at time t.

Then, the projection of the one-period marginal rate of substitution on the space of one-period payoffs, p^*_{t+1}, satisfies a similar condition that:

$$E[p^*_{t+1}x_i(t,t+1)|\Omega_t] = \pi(x_i(t,t+1)). \quad (2)$$

Hansen and Jagannathan (1991) show that this projection has the minimum variance in the class of all pricing kernels and is in general a linear combination of all the one-period payoffs under consideration.[11] In particular,

$$p^*_{t+1} = \sum_{j=1}^{N} \alpha_{jt}x_j(t,t+1), \quad (3)$$

[9] These indices were obtained from Morgan Stanley.

[10] See also Constantinides (1989).

[11] This projection is closely related to the maximally correlated portfolio in Breeden (1979). See also Shanken (1987).

where the conditional weighting vector $\alpha_t = [\alpha_{jt}]$ is given by

$$\alpha_t = [E[x_{t+1}x'_{t+1}\ \Omega_t]]^{-1}\pi_{t'} \tag{4}$$

where x_{t+1} is the payoff vector whose ith component is $x_i(t, t+1)$.

The linear factor-pricing model is derived by imposing the restriction that this projection, p^*_{t+1}, is a linear combination of only the factor payoffs. This restriction is justifiable when all the payoffs under consideration are linear in the factors or when the payoffs satisfy certain distributional restrictions. The existence of a payoff that is nonlinear in the factors or that does not satisfy the distributional restrictions would lead to the minimum variance pricing kernel involving this payoff in addition to the factors (see Bansal and Viswanathan (1993)). Thus the linear factor-pricing model holds only under restrictive assumptions on the payoff space.

Instead of imposing restrictions on the minimum variance pricing kernel, we follow an alternative approach which yields a pricing kernel that prices all securities and satisfies the nonnegativity restriction implied by no arbitrage. Our approach imposes a sufficient statistic restriction on the conditional expectation of the one-period marginal rate of substitution at time $t+1$. This approach leads to a low-dimensional, nonnegative pricing kernel that is typically different from the minimum variance pricing kernel unless markets are complete.

To do so, we use the law of iterated expectations to rewrite equation (1) for one-period-ahead payoffs as follows:

$$E[E[MRS_{t,t+1}|\Omega_{t+1}]x_i(t, t+1)|\Omega_t] = \pi(x_i(t, t+1)). \tag{5}$$

We impose the sufficient statistic restriction that the conditional expectation of the marginal rate of substitution between t and $t+1$ as of time $t+1$ is a function of p^b_{t+1}, a K-dimensional vector of basis variables:[12]

$$E[MRS_{t,t+1}|\Omega_{t+1}] = E\left[MRS_{t,t+1}|p^b_{t+1}\right] = G(p^b_{t+1}), \tag{6}$$

where K is a low number and $G(\bullet)$ is a well-behaved function. In the nonlinear arbitrage model, the information in the well-diversified basis variables p^b_{t+1} is sufficient for all the history in forming the conditional expectation of the marginal rate of substitution at time $t+1$. Equation (6) is the key dimensionality restriction that leads to a nonlinear arbitrage-pricing theory. As in linear arbitrage pricing, a low-dimensional pricing kernel that only depends on the factors exists. However, unlike linear pricing kernels, the above kernel is not necessarily linear in the factors. Also, it satisfies the nonnegativity restriction on the pricing kernel that is required by the absence of arbitrage opportunities in financial markets.

[12] The marginal rate of substitution between time t and $t+1$ is a random variable at time $t+1$ if we have durability of goods or non-time-separable preferences.

Equation (6) leads to the following restatement of the first-order condition for a marginal investor for one-period-ahead payoffs:

$$E\left[G(p_{t+1}^{b})x_i(t,t+1)\middle|\Omega_t\right] = \pi(x_i(t,t+1)). \tag{7}$$

This states that there exists a pricing kernel for one-period returns that is low dimensional. Recursive use of equation (7) along with the law of iterated expectations leads to the following restriction for s-period-ahead payoffs:[13]

$$E\left[\left(\prod_{r=1}^{s} G(p_{t+r}^{b})\right)x_i(t,t+s)\middle|\Omega_t\right] = \pi(x_i(t,t+s)). \tag{8}$$

This states that, for longer horizon returns, the product of the low-dimensional one-period pricing kernels is the appropriate pricing kernel. Equation (8) is the fundamental equation of interest for estimation purposes.

In summary, the nonlinear APT theory implies the existence of a low-dimensional pricing kernel that satisfies the following restrictions:

Restriction 1: The pricing kernel satisfies the condition that

$$E\left[\left(\prod_{r=1}^{s} G(p_{t+r}^{b})\right)x_i(t,t+s)\middle|\Omega_t\right] = \pi(x_i(t,t+s)). \tag{9}$$

Restriction 2: The pricing kernel is nonnegative; i.e. $G(\bullet) \geq 0$.

Restriction 3: The pricing kernel is low dimensional.

Dynamic theories of international asset pricing of the type tested in Bansal *et al.* (1992), Hodrick (1989), Wheatley (1988), and other related work test Restriction 1 (the functional forms for marginal utility in these models automatically ensure Restriction 2). Hansen and Jagannathan (1991) discuss Restriction 2 and the ways to implement it. Restriction 3 is implied by arbitrage-pricing theory and has been tested in the context of linear models where the pricing operator is a linear combination of a few factor portfolios. In our empirical work, we estimate the nonlinear model without imposing the no-arbitrage condition (Restriction 2) and test for Restrictions 1 and 3. Having done so, we proceed to check whether the estimated pricing kernel is positive. It turns out to be so in every case.

In addition to the nonlinear arbitrage model, we estimate the unconditional linear model (which is nested within the nonlinear model) and the conditional linear model. While the latter is not nested within the nonlinear model, we write its restrictions in terms of the basis variables of the nonlinear model.

Let the first asset be the riskless asset, so that $k = 2, \ldots, K$ denote the remaining $K - 1$ risky assets. From equation (3) and the fact that returns are unit cost payoffs, the sum of the conditional weights, $\alpha_t'\iota$ (where ι is a

[13] We have to consider returns with different period lengths as the forward contract returns are four week returns. In addition, we use the four-week return on a U.S. Treasury bill.

vector of 1's), is just the price of the minimum variance pricing kernel for one-period returns at time t, $\pi(p^*_{t+1})$. Dividing the conditional weights of the minimum variance pricing kernel by this price, $\pi(p^*_{t+1})$, we obtain a portfolio that is mean variance efficient among the class of one-period returns. Call this portfolio return $R^*(t, t+1)$.

Now, every conditional linear pricing model (e.g., the CAPM) implies that a portfolio of the $K-1$ risky factor returns is mean variance efficient. Call this portfolio return $R_M(t, t+1) = \Sigma_{k=2}^K \phi_{kt} p^b_{kt+1}$. In the case of the CAPM, this mean variance–efficient portfolio is just the market portfolio. Since every mean variance–efficient portfolio is a convex combination of the riskless asset and the mean variance–efficient portfolio $R_M(t, t+1)$ (see Hansen and Richard (1987)), we have

$$R^*(t, t+1) = \theta_t y(t, t+1) + (1-\theta_t) R_M(t, t+1), \tag{10}$$

where θ_t is the conditional weight and $y(t, t+1)$ is the return on the riskless asset. Multiplying by the price of the minimum variance pricing kernel, $\pi(p^*_{t+1})$, and expanding $R_M(t, t+1)$, we get the following expression for the minimum variance pricing kernel when the linear factor-pricing model holds:

$$\begin{aligned} p^*_{t+1} &= \pi(p^*_{t+1})[\theta_t y(t, t+1) + (1-\theta_t) R_M(t, t+1)] \\ &= \eta'_t p^b_{t+1}, \end{aligned} \tag{11}$$

where η_t is a vector whose first component is $\eta_{1t} = \pi(p^*_{t+1})\theta_t$ and whose remaining components are $\eta_{kt} = \pi(p^*_{t+1})(1-\theta_t)\phi_{kt}$ for $k = 2$ to K (all the factor payoffs other than the riskless payoff). The theoretical content of the conditional linear model is the restriction that the minimum variance pricing kernel is a conditional linear combination of the factor payoffs and the riskless asset. To price multiperiod returns using the one-period linear pricing kernel implied by the linear pricing models, we follow the strategy of using the law of iterated expectations to obtain

$$E\left[\left(\prod_{r=1}^{s} p^*_{t+r}\right) x_i(t, t+s) \middle| \Omega_t\right] = \pi(x_i(t, t+s)). \tag{12}$$

Equation (12) allows us to estimate and test the conditional linear model.

Finally, we compare the three models (unconditional linear, conditional linear, and nonlinear) using the Hansen and Jagannathan (1992) distance measure. Briefly, the Hansen Jagannathan (HJ) distance measure is a metric for determining how well a pricing kernel prices a given set of payoffs. Consider the one-period payoff set x_{t+1} with prices π_t. Hansen and Jagannathan (1992) prove the existence of a minimum variance pricing kernel p^*_{t+1} given in equations (3) and (4). Any other pricing kernel, m_{t+1}, can be rewritten as

$$m_{t+1} = p^*_{t+1} + \epsilon_m, \; E[\epsilon_m x_i(t, t+1)] = 0. \tag{13}$$

In particular, if m_{t+1} is a valid pricing kernel for the payoff set, its projection on the space of payoffs is the minimum variance pricing kernel p^*_{t+1}.

Hansen and Jagannathan (1992) suggest the following distance measure as a metric for comparing different pricing kernels for the payoff set x_{t+1}. Take any pricing kernel m_{t+1}. Project it on the space of payoffs and call its projection $\hat{m}_{t+1}$. If m_{t+1} is a valid pricing kernel for the payoff set, $\hat{m}_{t+1} = p^*_{t+1}$. If m_{t+1} is not a valid pricing kernel for the payoff set, $\hat{m}_{t+1} \neq p^*_{t+1}$. The distance between $\hat{m}_{t+1}$ and p^*_{t+1}, $E[(p^*_{t+1} - \hat{m}_{t+1})^2]$, is the minimal distance between the potential proxy and the class of valid pricing kernels for that payoff set. This distance is zero if m_{t+1} is a valid pricing kernel for the payoff set, and it is positive otherwise. Hansen and Jagannathan (1992) show how to estimate this distance measure.

II. Data Description

Our data are weekly return data on country capital market indices for the United States, Japan, Germany, and the United Kingdom collected by Morgan Stanley from January 1, 1975 to December 31, 1990. This gives us 832 observations for each time series of interest. The weekly indices are value-weighted indices that are not dividend adjusted. Monthly index data from Morgan Stanley have been used previously in work by Harvey (1991) (among others) and the daily data have been used by Chan, Karolyi, and Stulz (1992).[14] In addition to the four country stock market indices, we have a value-weighted weekly world index calculated by Morgan Stanley. The value-weighted world index does not adjust for the cross-corporate holdings in Japan and West Germany. This phenomenon has been documented for Japan by McDonald (1989) and French and Poterba (1990). Thus, the Morgan Stanley world index probably gives a higher weight to these countries than an index that adjusts for cross-corporate holdings.

All these indices are in dollar terms and are weekly returns calculated on each Wednesday 4:00 P.M. U.S. eastern standard time (EST). Since the foreign country stocks do not actually trade at this time, the prices used are the last closing prices before the Wednesday close on the New York Stock Exchange (4:00 P.M. EST). The returns are converted to dollar terms using the exchange rate prevailing at 4:00 P.M. EST.[15] The correlation between the Morgan Stanley U.S. Index and the S & P 500 Index (not dividend adjusted) over the period January 1975 to December 1989 is 0.9974. Similarly, the

[14] See Harvey (1991) for a discussion on the monthly data.

[15] Since the exchange rate is the 4:00 P.M. EST rate and the foreign index prices are those at the close that occurred some hours ago, there is a misalignment in the data. While this misalignment may create problems in daily returns, we do not think it is a problem in weekly returns. Our results are robust to lagging our instrument sets by one extra day, i.e., using Tuesday-to-Tuesday rather than Wednesday-to-Wednesday returns. Chan, Karolyi, and Stulz (1992) show that their statistical inference using daily returns is robust to nonsynchronicity, by making different adjustments to their data.

correlation between the Morgan Stanley Japan Index (the Japanese index not adjusted for the exchange rate) and the Nikkei 225 Index over the period October 1980 to September 1987 is 0.9085.[16] Thus, the Morgan Stanley weekly index data are closely correlated with the frequently used market indices. Furthermore, the Morgan Stanley U.S. Index has a correlation of 0.987 with the dividend-adjusted value-weighted index from the Center for Research in Security Prices, which indicates that the dividend adjustment does not make much difference in weekly returns.[17]

In addition to the stock market indices, we use the one-month Treasury bill rate for the United States and the seven-day Eurodollar rates obtained from the Board of Governors of the Federal Reserve System. We also use spot rates and four-week forward rates for the three foreign countries (Japan, Germany, and the United Kingdom) obtained from WEF Econometrics to construct an equally weighted portfolio of forward currency returns (see equation (23) below for a precise definition). All exchange rates are U.S. dollar prices of foreign currencies.

Table I, Panel A provides summary descriptions for the eight payoffs of interest. The four-week Treasury bill has the lowest return, while the Japanese stock market has the highest return. Also, the four-week Treasury bill has the lowest standard deviation, while the U.K. stock market has the highest standard deviation. The equally weighted forward portfolio return has a mean which is similar to that of Treasury bills, and a standard deviation which is smaller than that of stock indices but larger than that of Treasury bills.

Table I, Panel B gives the contemporaneous correlation across the eight payoffs. It is interesting to note that the two interest rates, Eurocurrency and Treasury bills, are highly correlated with each other. The stock returns are also positively correlated to each other; in particular, the U.S. and Japanese stocks are strongly correlated with the world index. However, there is no strong correlation between the equally weighted forward contract portfolio return and the other seven payoffs.

Table II shows the autocorrelation patterns of the eight payoffs. Panel A indicates that there is a slight amount of autocorrelation in the raw returns for stocks, and much stronger autocorrelation in interest rates and the equally weighted forward contracts. After performing a vector autoregression to correct for the conditional mean, we find strong evidence of autocorrelation of the squared residuals, as reported in Panel B. This is consistent with the presence of higher moment dependence in all these payoffs. This evidence is consistent with the evidence in Bansal, Gallant, Hussey, and Tauchen (1992), Engle and Gonzales-Riviera (1991) and Gallant, Hsieh, and Tauchen (1991).

[16] These numbers are comparable to the correlations reported by Harvey (1991) for monthly data.

[17] To evaluate the effect of any potential misspecification that may occur from this omission of dividends, we did some unconditional mean adjustments based on the monthly dividend yields. The adjustments made no substantitive difference to our results.

Table I

Descriptive Statistics

This table shows descriptive statistics for the international payoffs used in this paper. All payoffs except the Treasury bill return and the equally weighted forward contract return are one-period payoffs. The Treasury bill return and the equally weighted forward contract return are four-period payoffs. To allow proper comparison across payoffs, we divide each observation of the four-period payoffs by four to calculate the descriptive statistics. In the table, EDR is the one-week Eurodollar return and EWFC is the equally weighted portfolio of the forward contract return for Japan, the United Kingdom, and West Germany.

Panel A. Payoffs: Means and Standard Deviations

	Mean	Standard Deviation	Maximum	Minimum
EDR	1.001733	0.000641	1.004231	1.000781
World index	1.002243	0.018213	1.079840	0.864791
U.S. index	1.001915	0.021749	1.081693	0.848607
West German index	1.002301	0.025881	1.085662	0.865623
U.K. index	1.003331	0.031736	1.249334	0.837574
Japanese index	1.003647	0.027071	1.110665	0.885633
Treasury bill	1.001606	0.000566	1.003533	1.000612
EWFC	1.001921	0.006777	1.028097	0.982910

Panel B. Payoffs: Cross Correlations

	ECR	World	US	WG	UK	Japan	T-Bill	EWFC
EDR	1.000	−0.0664	−0.0398	−0.0628	−0.0521	−0.0524	0.9527	−0.1430
World		1.000	0.8413	0.5182	0.5752	0.6831	−0.0851	0.1491
U.S.			1.000	0.3087	0.3691	0.2875	−0.0503	−0.0386
West Germany				1.000	0.3485	0.3803	−0.0688	0.2349
U.K.					1.000	0.2996	−0.0762	0.1550
Japan						1.000	−0.0698	0.2655
Treasury bill							1.000	−0.1343
EWFC								1.000

III. Estimation Strategy

Call the realization of any payoff that we are interested in pricing, $x_i(t, t + s)$ (where the maturity of the ith payoff is s periods ahead). The orthogonality conditions that have to be satisfied by the payoffs in the nonlinear APT are

$$E\left[\left(\left(\prod_{r=1}^{s} G(p_{t+r}^{b})\right) x_i(t, t+s) - 1\right) Z_{1t}\right] = 0, \tag{14}$$

where Z_{1t} is any variable belonging to the information set Ω_t.

The function $G(\bullet)$ of the basis variables is an unknown function that we nonparametrically approximate using a multivariable polynomial series expansion, $G^q(\bullet)$, where q is the order of the expansion. Consistency of such nonparametric estimation requires that the order of the series expansion

Table II

Autocorrelation Patterns

The first panel of this table shows the autocorrelations up to order 8 for the Eurodollar returns (EDR), world index return, the four country indexes, the equally weighted forward contract portfolio (EWFC), and the Treasury bill returns.

The second panel of this table shows the autocorrelations in the squared residuals from a vector autoregression of the payoffs (up to lag length 8). These autocorrelations are also shown up to lag length 8.

	EDR	World Index	U.S. Index	German Index	U.K. Index	Japanese Index	EWFC	Treasury Bill
				Panel A. Autocorrelation of Raw Data				
Lag 1	0.958	0.092	0.005	0.125	0.073	0.085	0.798	0.980
2	0.954	0.038	−0.019	0.083	0.090	0.065	0.579	0.965
3	0.940	0.054	0.052	0.003	0.014	0.033	0.337	0.950
4	0.921	−0.014	−0.044	0.009	0.038	0.011	0.090	0.935
5	0.911	0.008	−0.014	−0.016	0.007	0.025	0.038	0.917
6	0.899	0.018	0.021	−0.041	−0.036	0.01	0.005	0.900
7	0.881	0.050	0.061	0.002	0.012	−0.001	0.003	0.883
8	0.873	−0.028	−0.025	−0.022	−0.034	−0.031	0.020	0.865
				Panel B. Autocorrelation of Squared VAR Residuals				
Lag 1	0.161	0.298	0.321	0.249	0.318	0.231	0.565	0.242
2	0.098	0.097	0.094	0.276	0.045	0.215	0.217	0.091
3	0.143	0.075	−0.006	0.188	0.020	0.232	0.032	0.116
4	0.122	0.044	−0.015	0.136	0.068	0.213	−0.005	0.100
5	0.054	0.046	0.015	0.116	0.058	0.155	−0.019	0.180
6	0.091	0.062	0.036	0.128	0.082	0.143	−0.004	0.037
7	0.084	0.041	0.056	0.087	−0.008	0.105	0.032	0.078
8	0.063	0.010	−0.004	0.069	0.028	0.081	0.014	0.117

increase with the sample size.[18] These and other technical issues are discussed in Gallant (1987), Gallant and White (1989), and Bansal and Viswanathan (1993).

The consistency of the nonparametric procedure allows us to replace $G(\bullet)$ by the polynomial series expansion, $G^q(\bullet)$, in the orthogonality condition

$$E\left[\left(\left(\prod_{r=1}^{s} G^q(p_{t+r}^b)\right) x_i(t,t+s) - 1\right) Z_{1t}\right] = 0, \tag{15}$$

where Z_{1t} is an instrument belonging to the information set Ω_t.
Write

$$u_{it+s} = \left(\prod_{r=1}^{s} G^q(p_{t+s}^b)\right) x_i(t,t+s) - 1. \tag{16}$$

[18] See Bansal and Viswanathan (1993) for a consistency proof.

The error u_{it+s} satisfies the restriction it is orthogonal to Z_{1t} which belongs to the information set Ω_t:

$$E[u_{it+s}Z_{1t}] = 0, \forall i, 1. \tag{17}$$

We construct sample versions of this restriction to estimate $G^q(\bullet)$. Define the $I \times J$ vector e_t whose components are $u_{it+s}Z_{1t}$ for $i = 1, \ldots, I$ and $l = 1, \ldots, L$. With I payoffs and L instruments, we have $IL = R$ orthogonality conditions that are used to estimate the pricing kernel. Call the sample mean of the vector e_t, se_t. $\hat{G}(p_{t+1}^b)$, the consistent estimator of $G^q(\bullet)$, is the function that minimizes the following GMM criterion function with respect to $G^q(\bullet)$:

$$N[se(G^q)'Wse(G^q)], \tag{18}$$

where W is a $R \times R$ optimal weighting matrix discussed in Hansen (1982) and Hansen and Singleton (1982). We minimize this criterion function with respect to the parameters of the series expansion $G^q(\bullet)$. Let $J(G^q, K)$ be the minimized value of the criterion function.

To test whether an additional factor is required in a K factor representation, we use a likelihood ratio-type test. To implement this test, we first estimate $G^q(\bullet)$ with K factors. Keeping the weighting matrix fixed, we reestimate $G^q(\bullet)$ with $K + 1$ factors. The specification with $K + 1$ factors nests the K factor specification. The difference in the objective value between the two specifications, $J(G^q, K + 1) - J(G^q, K)$, is distributed χ^2 with the degrees of freedom equal to the difference in the number of parameters between the two specifications. We use a similar approach to test the nested unconditional linear model against the nonlinear model.

In addition to estimating the nonlinear model, we estimate the conditional linear model which is not nested in the nonlinear model. In the conditional linear model, the conditional weights on the factor portfolios are an unknown (and potentially complicated) function of the history until that point in time. From equation (11), the pricing condition satisfied in the conditional model is

$$E\left[\left(\prod_{r=1}^{s} \eta_{t+r}' p_{t+r+1}^b\right) x_i(t, t+s) Z_{1t} - Z_{1t}\right] = 0. \tag{19}$$

We estimate the conditional weights in the above equation using a nonparametric method (see Gallant, Hansen, and Tauchen (1990) for another nonparametric approach). The conditional weights, η_{kt}, are nonparametrically estimated by

$$\eta_{kt} = \sum_{l=1}^{L} \lambda_{kl} Z_{1t'} \tag{20}$$

where we use exactly the same conditioning variables that are used as instruments; here, L is the number of instruments used in estimation. As the number of conditioning variables increases to infinity, we use all the relevant conditional information and this estimate of the conditional weight converges to the true conditional weight. Thus, this approach provides asymptotically consistent estimates without imposing the usual restrictive parametrization on the conditional mean process and the conditional covariance process of the (factor) payoffs.

Having estimated the pricing kernels, we compute their HJ distances. We do this for one payoff at a time. This particular strategy is followed for two reasons: first, because the minimum variance pricing kernel is different for payoffs of different holding periods, and second, because we want to keep the number of parameters to be estimated small.[19] For a one-period payoff, $x_i(t, t+1)$, we estimate the HJ distance of the pricing kernel m_{t+1} as

$$E[(m_{t+1} + \zeta'(x_i(t, t+1) \otimes Z_t))x_i(t, t+1)Z_{1t} - Z_{1t}] = 0, \qquad (21)$$

where Z_{1t} is the time t price of the "payoff" $x_i(t, t+1)Z_{1t}$.[20]

Let $Q_{it+1} = x_i(t, t+1) \otimes Z_t$ represent the L vector of "payoffs." The HJ distance measure for m_{t+1} can be expressed as

$$(E[Z_t] - E[Q_{it+1}m_{t+1}])'[E[Q_{it+1}Q'_{it+1}]]^{-1}(E[Z_t] - E[Q_{it+1}m_{t+1}]). \qquad (22)$$

This is akin to the GMM criterion function with the weighting matrix formed by the inverse of second moment matrix of the "payoffs" $Q_{it+1} = x_i(t, t+1) \otimes Z_t$. The HJ distance uses the same weighting matrix in calculating the distances for all pricing kernels under consideration. In contrast the optimal weighting matrix as used in the GMM criterion function is dependent on the proxy (pricing kernel) under consideration.

As previously mentioned, a valid pricing kernel must have a zero HJ distance, or equivalently, the vector of coefficients, ζ, must be zero. This hypothesis of equality of coefficients to zero is tested using a Wald test.[21,22]

IV. Estimation Details

The payoffs that we are interested in pricing are the weekly country stock market indexes, the seven-day Eurodollar return, the four-week holding period return on a U.S. Treasury bill, the four-week forward contract returns

[19] The minimum variance pricing kernel for one-period payoffs is the projection of the one-period pricing kernel on the space of one-period payoffs while the minimum variance pricing kernel for four-period payoffs is the projection of the four-period pricing kernel on the space of four-period payoffs.

[20] The estimation of the HJ distance using a subset of payoffs is legitimate provided these payoffs (the subset) were part of the original payoff set used to estimate the pricing kernels being compared.

[21] As in Hansen and Jagannathan (1992), we take the pricing kernel as given when we estimate the HJ distance and ignore any second-step estimation error issues.

[22] See Wong and Jagannathan (1993) for some recent work deriving the distribution of the Hansen-Jagannathan distance measure under the null and for an interesting application to the size effect.

for the United Kingdom, Japan, and Germany, and the seven-day return from holding the three foreign currencies (United Kingdom, Japan, and Germany).

We divide our payoffs into three sets (Sets 1, 2, and 3). In Set 1, we use only the weekly four-country index returns, $R_i(t, t+1)$, $i = US, WG, UK, JP$. We do this to compare our results with previous work that uses only stock index returns (see Harvey (1991)). In Set 2, we introduce interest rates into our payoffs, by adding to Set 1 the one-week Eurodollar return, $y_{US}(t, t+1)$, and the four-week U.S. Treasury bill return, $y_{US}(t, t+4)$. This yields a total of six payoffs. In Set 3, we introduce derivative securities by replacing the Japanese stock returns in Set 2 with an equally weighted portfolio of the three forward contracts (for the United Kingdom, Germany, and Japan), $f(t, t+4)$. The four-week equally weighted forward contract return, $f(t, t+4)$, is calculated as

$$\frac{1}{3} \sum_{k \in UK, WG, JP} \left[\frac{e_k(t+4) * y_{US}(t, t+4)}{F_k(t, t+4)} \right], \tag{23}$$

where $e_k(t+4)$ is the spot exchange rate four periods ahead and $F_k(t, t+4)$ is the one-month forward exchange rate today (exchange rates are U.S. dollar prices of a unit of foreign currency). For each currency k, this is the return on the trading strategy that invests $F_k(t, t+4)$ dollars in the four-week Treasury bill and buys $y_{US}(t, t+4)$ units of foreign currency forward contracts.[23] We use a portfolio instead of the individual forward contract returns to keep the number of nuisance parameters in estimation small. This payoff turns out to provide greater discrimination between the unconditional linear, conditional linear, and nonlinear models.

The holding period for the various payoff returns is one week except for the forward contract returns and the U.S. Treasury bill return where it is one month. However, the four-week Treasury bill and forward contract returns are sampled weekly. As in Hansen and Hodrick (1983), this leads to serial correlation in the errors. We adjust for this serial correlation by estimating our GMM weighting matrix using the procedure suggested by Newey and West (1987).

In estimation, we consider two basis variables: the known one-period Eurodollar interest rate and the value-weighted world index $R_W(t, t+1)$. The monthly value-weighted index has been used in previous research on the one-factor model (the CAPM) by Harvey (1991) and others. The use of the interest rate and the world index allows us to nest the one-factor unconditional linear model as a special case of the nonlinear model in our estimations.

[23] The trading strategy (for each currency k) has a price of $F_k(t, t+4)$ and a payoff of $F_k(t, t+4) y_{US}(t, t+4)$ on the investment in the Treasury bill and $(e_k(t+4) - F_k(t, t+4)) y_{US}(t, t+4)$ on the forward contract purchase. Thus, the overall payoff on the trading strategy is $e_k(t+4) y_{US}(t, t+4)$. Hodrick (1987) considers a similar trading strategy. In the event that covered interest arbitrage holds exactly, this is the uncovered return on holding an interest-bearing foreign currency deposit.

Given these basis variables, we approximate the unknown nonlinear pricing kernel as discussed previously using a multivariable polynomial series expansion.[24] To reduce the number of parameters to be estimated and to obtain a parsimonious representation, we use a fifth-order polynomial expansion with a number of terms in the series expansion suppressed. This strategy to reduce the number of parameters is also followed in Gallant, Rossi, and Tauchen (1992). We use the second and fifth orders of the world index return because the even and odd exponents seem to capture different kinds of function behavior and suppress the third and fourth orders of the market in the expansion.[25] Thus, for the one-factor nonlinear model, we use the series expansion[26]

$$G^q(p_{t+1}^b) = \beta_0 + \beta_{1y} y_0(t, t+1) + \beta_{1W} R_W(t, t+1) + \beta_{2W}[R_W(t, t+1)]^2 + \beta_{5W}[R_W(t, t+1)]^5. \qquad (24)$$

This particular formulation uses a linear combination of the factors as the leading term in the expansion. This linear leading term nests the linear pricing kernel (as implied by the restrictions of the unconditional linear factor-pricing models) in the nonlinear pricing kernel. For the one-factor (market) model considered above, the leading linear term is the CAPM pricing kernel. It should also be observed that the above formulation does not impose nonnegativity on the pricing kernel. The imposition of the nonnegativity restriction (as is done in Hansen and Jagannathan (1991) and Bansal and Viswanathan (1993)) is needed only if the unrestricted pricing kernel achieves any negative values, which is never the case in our estimation.

We also test whether an additional second factor is required in the unconditional linear model and the nonlinear model.[27,28] The second factor that we consider is the equally weighted foreign currency return, $(1/3) * \Sigma_{UK.WG.JP} [e_k(t+1)/e_k(t)]$, where $e_k(t)$ is the exchange rate for country k today. The equally weighted foreign currency return is the equally weighted average of the weekly price relative for the three foreign currencies, the Japanese yen, the German mark, and the British pound. This factor is chosen for the additional factor test because Dumas and Solnik (1992) present empirical work indicating that exchange rate factors have explanatory power in equity pricing (see also Roll (1992) and Bekaert and Hodrick (1992)).

[24] Good starting values for the parameters of the polynomial expansion can be easily obtained by two-stage least squares. Hence the polynomial expansion is numerically simpler to estimate than other series expansions.

[25] Using the fifth order instead of the third order was partly motivated by the need to reduce collinearity between the various powers of the expansion.

[26] For an application of nonparametric techniques to performance evaluation, see Glosten and Jagannathan (1992).

[27] We do not test for an additional factor in the conditional linear model as the number of parameters is very large (15 parameters) when we add a second factor.

[28] In the nonlinear model, the additional factor adds three parameters: a linear parameter, a second-order parameter, and a fifth-order parameter. Thus we treat the first factor and the additional factor symmetrically.

We consider five instrument sets in our estimation. These are labelled A, B, C, D, and E. Each set contains five instruments. The exact instrument sets used are listed in the Appendix. Each set of instruments is used with either six payoffs (payoff sets 2 and 3) or four payoffs (payoff set 1). This yields either 30 or 20 orthogonality conditions. To control for the number of parameters relative to the data size, we use the heuristic of saturation ratios (see Gallant and Tauchen (1989)). The saturation ratio is total number of observations (the number of orthogonality conditions times the length of the data) divided by the number of parameters to be estimated (which include the GMM weighting matrix parameters). The lowest saturation ratio is 52, which is obtained with payoff set 2 or 3 when estimating the conditional linear model. This is about three times the saturation ratios obtained in Gallant and Gauchen (1989). This heuristic of saturation ratios suggests that the number of parameters estimated is small relative to the number of observations.

Our choice of instruments is based on the following reasoning. Since lagged excess returns have been used in domestic studies, we use the difference between the world index return and the implied Japanese four-week interest rate in all five instrument sets.[29] To allow for nonlinear prediction we use the square of the difference between the U.S. index return and the one-week Eurodollar rate (instrument sets A, E) and the square of the difference between the world index return and the one-week Eurodollar rate (instrument sets B, C). We also use the difference between the one-week Japanese and U.S. interest rates (instrument sets A, C) and the U.S. interest rate (instrument set B, E). Finally, in all the instrument sets, we use the lagged equally weighted one-period exchange rate growth.

The instruments chosen strongly predict the payoffs we are interested in pricing. Table III contains results for projections of each payoff on the instruments.[30] The Wald test for the joint hypothesis that all the coefficients of the projection on the instruments are zero is strongly rejected. These rejections are especially striking for the world index, the typical p-values being less than 2 percent. Similar results hold for all the other payoffs. These results suggest that these chosen instruments allow a powerful test of the theory. While there is no unique way to decide on conditioning information, we feel that the results of the projections for the chosen instruments justify our conclusion that the instrument sets contain meaningful conditioning information. Hence, when estimating the conditional linear model, we use these instruments also as conditioning information to allow for time variation in the conditional weights.

[29] The variable we use is the Japanese four-week interest rate as implied by covered interest parity. Whether or not interest rate parity holds, it is in the information set and hence is a valid instrument.

[30] Since the four-week Treasury bill and seven-day Eurodollar returns are part of the instrument sets, we do not need to project these payoffs on the instruments in Table III.

Table III

Projection of Payoffs on Instruments

This table reports the results for the projection of the payoffs, i.e., the world index return, the four country indexes, and the equally weighted forward contract portfolio (EWFC) on the instruments. For each of the five instrument sets (Sets A, B, C, D, and E), we first estimate the projection

$$x_i(t, t+s) = \theta_0 + \nu' Z_t + v_t$$

where $x_i(t, t+1)$ is a payoff, Z_t is a set of instruments, θ_0 is a constant term, and ν is a four-dimensional coefficient vector (there are four variables in each instrument set). For the four period payoffs, the projection accounts for the MA structure. For each payoff and each instrument set, we report χ^2 value for the Wald test for the equality of vector ν to zero.

	World Index	U.S. Index	German Index	U.K. Index	Japanese Index	EWFC
Instrument set A						
Wald test $\chi^2(4)$	11.99	10.45	21.96	7.29	12.89	19.33
p-Value	0.017	0.033	0.00	0.12	0.012	0.00
Instrument set B						
Wald test $\chi^2(4)$	9.78	9.21	10.12	4.13	12.88	16.43
p-Value	0.044	0.056	0.038	0.38	0.012	0.002
Instrument set C						
Wald test $\chi^2(4)$	11.13	9.79	12.39	5.78	15.90	24.26
p-Value	0.025	0.044	0.015	0.22	0.003	0.00
Instrument set D						
Wald test $\chi^2(4)$	126.56	105.50	147.77	186.20	53.69	497.17
p-Value	0.00	0.00	0.00	0.00	0.00	0.00
Instrument set E						
Wald test $\chi^2(4)$	11.89	10.25	21.60	6.64	10.59	12.62
p-Value	0.018	0.036	0.00	0.15	0.032	0.013

For the parametrizations that we consider (one factor, five instruments), the conditional linear model has double the parameters of the nonlinear model (ten for conditional linear, five for nonlinear). This suggests that the nonlinear model leads to a more parsimonious specification. Note that any nonparametric estimation of the conditional weights will typically lead to even more parameters than the nonlinear model.

V. Results

Table IV shows the results for payoff set 1 that includes only the four country indexes. None of three models (linear, conditional linear, and nonlinear model) with one factor (the market returns) is rejected at the 5 percent level for either instrument set A or B. Consistent with previous research based on equity returns (Harvey (1991)), both the conditional and the unconditional linear model perform fairly well.[31] Finally, the nested model test for

[31] In addition, the test for an additional factor does not reject the one-factor model. This result is not reported in the tables.

Table IV

Payoff Set 1

This table lists results for payoff set 1, which includes the four country index returns. With five instruments, this yields 20 orthogonality conditions. For each instrument set, we report first, the GMM criterion function value (GMM F-value) for each of the three pricing kernels with the world index return as the only factor. Also reported with this χ^2 value are the degrees of freedom (dof) for the model under consideration and the associated p-value. Second, we present the mean and standard deviation (SD) of each of the three pricing kernels. Last, we report the results of the nested model test for the unconditional linear model against the nonlinear model (the nested test).

	Linear Model	Cond. Linear Model	Nonlinear Model
Instrument set A			
GMM F-value χ^2, dof, p-value	26.37, 17, 0.067	16.31, 10, 0.091	24.56, 15, 0.056
Pricing kernel mean, SD	0.999, 0.061	1.001, 0.111	0.998, 0.148
Nested test χ^2, dof, p-value			1.300, 2, 0.522
Instrument set B			
GMM F-value χ^2, dof, p-value	25.66, 17, 0.080	17.84, 10, 0.057	24.31, 15, 0.060
Pricing kernel mean, SD	0.997, 0.038	1.002, 0.162	0.995, 0.067
Nested test χ^2, dof, p-value			1.962, 2, 0.374

the nonlinear model versus the unconditional linear model does not reject the unconditional linear model.

Table V shows the HJ distance and the associated Wald tests using U.S. and Japan index returns. Consistent with the χ^2 tests based on the GMM criterion function, the hypothesis that these pricing proxies belong to the class of valid pricing kernels is not rejected by the HJ Wald test. More importantly, the equity indices do not seem to be able to discriminate between the three models.[32]

Table VI shows the results for payoff set 2. The introduction of interest rate payoffs results in rejections of the unconditional linear model for two instrument sets and a p-value marginally greater than 5 percent in the third instrument set. The conditional linear model and the nonlinear model have p-values for the GMM criterion function test that are in the 10 to 20 percent region. However, the p-values for the nonlinear model are much higher than those for the conditional linear model.

Not surprisingly, the test for a single factor in the unconditional model leads to a strong rejection (p-value of around 2 percent), indicating that a second factor is required. In the nonlinear model, a second factor is not required; the lowest p-value in the additional factor test is 70 percent. Finally, the nested linear model test rejects the unconditional linear model in favor of the nonlinear model.

[32] It is meaningful to compare the HJ distance only when the hypothesis that the HJ distance is zero is rejected. Here, the hypothesis that the HJ distance is zero cannot be rejected and we do not compare the HJ distance across models.

Table V

Hansen-Jagannathan Diagnostics for Payoff Set 1

This table represents the Hansen-Jagannathan distance measure tests for payoff set 1. For each of the two instrument sets (Set A and Set B), the HJ distance is estimated using the orthogonality condition

$$E[(m_{t+1} + \zeta'(x_i(t,t+1) \otimes Z_t))x_i(t,t+1)Z_{1t} - Z_{1t}] = 0$$

where Z_{1t} is the time t price of the "payoff" $x_i(t,t+1)Z_{1t}$. $x_i(t,t+1)$ is either the return on the U.S. index or the Japanese index and Z_t is the five-dimensional vector of instruments used in estimation. The estimated HJ distance and its associated standard deviation (SD) are first reported. For each of the two payoffs, a Wald test for equality of all the coefficients, ζ, to zero is reported next.

	Linear	Conditional Linear	Nonlinear
Instrument set A (U.S. index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.551, 3.332	6.742, 45.197	0.659, 2.091
Wald $\chi^2(5)$, p-value	2.59, 0.762	3.47, 0.627	0.842, 0.974
Instrument set A (Japanese index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.525, 2.311	6.833, 45.363	0.564, 3.3622
Wald $\chi^2(5)$, p-value	4.986, 0.417	4.035, 0.544	0.232, 0.998
Instrument set B (U.S. index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.551, 3.328	7.598, 47.386	0.632, 1.127
Wald $\chi^2(5)$, p-value	2.59, 0.762	3.404, 0.637	0.920, 0.968
Instrument set B (Japanese index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.525, 2.311	8.675, 61.469	0.313, 0.590
Wald $\chi^2(5)$, p-value	4.986, 0.418	4.216, 0.519	0.244, 0.998

For payoff set 2, the HJ distances and associated Wald tests are shown in Table VII. Generally, the nonlinear model has the lowest HJ distance measure for both the U.S. index and the Japanese index across all three instrument sets (sets A, B, and C).[33] This difference in the HJ distances between the nonlinear model and the conditional linear model is especially large. In addition, the p-values for the HJ Wald tests are always highest for the nonlinear model. While the nonlinear model is never rejected (the lowest p-values are around 20 percent), the unconditional linear model is rejected in three cases and the conditional linear model in two cases. Thus the GMM criterion function-based χ^2 test and the HJ Wald tests indicate that the nonlinear model is preferred to the two linear models.

Lastly, we turn to payoff set 3, which contains the equally weighted forward contract return and the interest rates along with the indices. The evidence in Table VIII shows rejections or marginal rejections for the unconditional linear model and marginal rejections for the conditional linear and nonlinear models. This indicates that the asset-pricing models

[33] The HJ distance comparison is valid in the cases where the hypothesis that the HJ distance is zero is rejected for the linear models and is not rejected for the nonlinear model. This occurs often with payoff set 2 and always with payoff set 3.

Table VI

Payoff Set 2

This table lists results for payoff set 2, which includes the four country index returns, the one-week Eurodollar return, and the four-week U.S. Treasury bill returns. With five instruments, this yields 30 orthogonality conditions. For each instrument set, we report first, the GMM criterion function value (estimated using a Newey-West MA (6) lag weighting matrix) for each of the three pricing kernels with the world index return as the only factor (GMM F-value). Also reported with this χ^2 value are the degrees of freedom for the model under consideration (dof) and the associated p-value. Second, the mean and standard deviation (SD) of each of the three pricing kernels. Third, for the unconditional linear model and the nonlinear model, we report the χ^2 value for the additional factor test (the Up test). The additional factor considered is the equally weighted average of the three foreign country one-period exchange rate growths. Last, we report the results of the nested model test for the unconditional linear model against the nonlinear model (the nested test).

	Linear Model	Cond. Linear Model	Nonlinear Model
Instrument set A			
GMM F-value χ^2, dof, p-value	41.78, 27, 0.034	27.84, 20, 0.113	32.51, 25, 0.144
Pricing kernel mean, SD	0.997, 0.043	0.998, 0.078	0.997, 0.046
Up test χ^2, dof, p-value	4.874, 1, 0.027		1.42, 3, 0.701
Nested test χ^2, dof, p-value			8.54, 2, 0.013
Instrument set B			
GMM F-value χ^2, dof, p-value	38.53, 27, 0.069	28.73, 20, 0.093	31.30, 25, 0.179
Pricing kernel mean, SD	0.997, 0.042	0.996, 0.061	0.997, 0.041
Up test χ^2, dof, p-value	6.07, 1, 0.013		0.732, 3, 0.865
Nested test χ^2, dof, p-value			11.39, 2, 0.003
Instrument set C			
GMM F-value χ^2, dof, p-value	41.12, 27, 0.040	25.49, 20, 0.183	32.93, 25, 0.137
Pricing kernel mean, SD	0.997, 0.041	0.996, 0.072	0.997, 0.043
Up test χ^2, dof, p-value	5.626, 1, 0.017		0.760, 3, 0.859
Nested test χ^2, dof, p-value			7.509, 2, 0.023

have the greatest difficulty in explaining forward contract returns. Again, in the unconditional linear model, a second factor is indicated in the additional factor test (p-values for four instrument sets are less than 4 percent). In contrast, in the nonlinear model, the second factor is not required as the lowest p-value obtained in the additional factor test is 23 percent. Finally, the nested model test always rejects the unconditional linear model in favor of the nonlinear model.

The HJ distances and associated Wald tests for payoff set 3 are shown in Tables IX and X. With the U.S. index return (Table IX), the nonlinear model has a much lower HJ distance measure than the conditional linear model. Also, the HJ Wald test yields p-values for the nonlinear model which are much higher than those for the linear models: the lowest p-value is 63 percent. The HJ Wald test actually rejects the unconditional linear model for two instruments and the conditional linear model for one instrument. Thus the HJ distance measure and the associated Wald tests favor the nonlinear model.

Table VII

Hansen-Jagannathan Diagnostics for Payoff Set 2

This table presents the Hansen-Jagannathan distance measure tests for payoff set 2. For each of the three instrument sets (Sets A, B, and C), the HJ distance is estimated using the orthogonality condition

$$E[(m_{t+1} + \zeta'(x_i(t,t+1) \otimes Z_t))x_i(t,t+1)Z_{1t} - Z_{1t}] = 0$$

where Z_{1t} is the time t price of the "payoff" $x_i(t,t+1)Z_{1t}$. $x_i(t,t+1)$ is either the return on the U.S. index or the Japanese index and Z_t is the five-dimensional vector of instruments used in estimation. The estimated HJ distance and its associated standard deviation (SD) are first reported. For each of the two payoffs, a Wald test for equality of all the coefficients, ζ, to zero is reported next.

	Linear	Conditional Linear	Nonlinear
Instrument set A (U.S. index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.255, 0.938	0.874, 7.213	0.1214, 0.263
Wald $\chi^2(5)$, p-value	13.008, 0.023	10.415, 0.063	2.686, 0.748
Instrument set A (Japanese index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.251, 1.453	1.153, 12.114	0.0608, 0.462
Wald $\chi^2(5)$, p-value	8.294, 0.141	9.348, 0.095	0.7213, 0.982
Instrument set B (U.S. index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.147, 0.392	1.294, 10.149	0.161, 0.297
Wald $\chi^2(5)$, p-value	13.531, 0.019	8.055, 0.153	2.644, 0.754
Instrument set B (Japanese index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.075, 0.879	1.545, 15.318	0.038, 0.084
Wald $\chi^2(5)$, p-value	2.817, 0.728	6.619, 0.251	0.734, 0.981
Instrument set C (U.S. index)			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.177, 0.367	0.364, 0.667	0.130, 0.249
Wald $\chi^2(5)$, p-value	12.109, 0.033	10.798, 0.055	7.255, 0.202
Instrument set C (Japanese index)			
H-J Dist($\times 10^4$, SD($\times 10^4$)	0.157, 0.797	0.436, 2.015	0.032, 0.162
Wald $\chi^2(5)$, p-value	6.816, 0.234	11.549, 0.042	2.219, 0.818

The results for the forward contract returns in Table X are also strongly supportive of the nonlinear model. The nonlinear model always has a much lower HJ distance measure than the conditional linear model. For two instrument sets, the unconditional linear model has a marginally lower HJ distance measure than the nonlinear model. In addition, the HJ Wald test of the hypothesis that the pricing kernel belongs to a class of valid pricing kernels virtually never rejects the nonlinear model. In contrast, the HJ Wald tests yield very strong rejections of the two linear models (p-values are 0.000).[34] These results are very strongly supportive of the nonlinear model as against the unconditional linear model and the conditional linear model.

[34] To check the sensitivity of our results on the conditional linear model to the number of instruments, we reestimated the conditional linear model with a lesser number of instruments. The GMM criterion function tests lead to greater rejections. The HJ distance was larger and the HJ Wald tests showed greater rejections. Thus the performance of the conditional linear model only deteriorates when we reduce the number of instruments.

Table VIII

Payoff Set 3

This table lists results for payoff set 3, which includes the three country index returns (United States, United Kingdom and West Germany), the one-week Eurodollar return, the four-week U.S. Tresury bill returns, and the equally weighted forward contract return. With five instruments, this yields 30 orthogonality conditions. For each instrument set, we report first, the GMM criterion function value (estimated using a Newey-West MA(6) lag weighting matrix) for each of the three pricing kernels with the world index return as the only factor (GMM F-value). Also reported with this χ^2 value are the degrees of freedom for the model under consideration (dof) and the associated p-value. Second, we present the mean and standard deviation (SD) of each of the three pricing kernels. Third, for the unconditional linear model and the nonlinear model, we report the χ^2 value for the additional factor test (Up test). The additional factor considered is the equally weighted average of the three foreign country one-period exchange rate growths. Last, we report the results of the nested model test for the unconditional linear model against the nonlinear model.

	Linear Model	Cond. Linear Model	Nonlinear Model
Instrument set A			
GMM F-value χ^2, dof, p-value	45.34, 27, 0.014	30.75, 20, 0.058	38.90, 25, 0.037
Pricing kernel mean, SD	0.998, 0.0320	0.996, 0.0461	0.997, 0.0288
Up test χ^2, dof, p-value	5.066, 1, 0.024		1.992, 3, 0.574
Nested test χ^2, dof, p-value			13.297, 2, 0.001
Instrument set B			
GMM F-value χ^2, dof, p-value	39.25, 27, 0.060	29.96, 20, 0.071	35.47, 25, 0.080
Pricing kernel mean, SD	0.998, 0.0253	0.996, 0.0367	0.997, 0.0211
Up test χ^2, dof, p-value	4.962, 1, 0.025		2.877, 3, 0.410
Nested test χ^2, dof, p-value			18.734, 2, 0.000
Instrument set C			
GMM F-value χ^2, dof, p-value	43.77, 27, 0.021	34.17, 20, 0.025	37.86, 25, 0.047
Pricing kernel mean, SD	0.997, 0.0284	0.997, 0.0343	0.997, 0.0238
Up test χ^2, dof, p-value	6.120, 1, 0.013		1.801, 3, 0.614
Nested test χ^2, dof, p-value			23.885, 2, 0.000
Instrument set D			
GMM F-value χ^2, dof, p-value	39.75, 27, 0.054	31.71, 20, 0.046	37.68, 25, 0.049
Pricing kernel mean, SD	0.997, 0.0239	0.996, 0.0701	0.996, 0.0394
Up test χ^2, dof, p-value	0.122, 1, 0.726		2.309, 3, 0.511
Nested test χ^2, dof, p-value			17.997, 2, 0.000
Instrument set E			
GMM F-value χ^2, dof, p-value	41.63, 27, 0.036	30.71, 20, 0.059	37.01, 25, 0.058
Pricing kernel mean, SD	0.998, 0.0284	0.996, 0.0329	0.997, 0.0257
Up test χ^2, dof, p-value	4.147, 1, 0.0417		4.259, 3, 0.234
Nested test χ^2, dof, p-value			13.027, 2, 0.002

Intuitively, the success of the nonlinear model can be explained by rewriting equation (1) in the unconditional form

$$E[m_t]E[x_i(t,t+1)] + \mathrm{Cov}(m_t, x_i(t,t+1)) = E[\pi(x_i(t,t+1))], \quad \text{for } i = 1,\ldots,N, \tag{25}$$

First, note that $E[\pi_i(t,t+1)]$ and $E[x_i(t,t+1)]$ are estimated at their sample means and are the same across the three models. Second, all three of

Table IX

Hansen-Jagannathan Diagnostics for Payoff Set 3 (U.S. Index)

This table presents the Hansen-Jagannathan distance measure tests for payoff set 3, where the HJ distance is estimated using the U.S. index return as the payoff. For each of the five instrument sets (Sets A, B, C, D, and E), the HJ distance is estimated using the orthogonality condition

$$E[(m_{t+1} + \zeta'(R_{US}(t,t+1) \otimes Z_t))R_{US}(t,t+1)Z_{1t} - Z_{1t}] = 0$$

where Z_{1t} is the time t price of the "payoff" $R_{US}(t,t+1)Z_{1t}$. $R_{US}(t,t+1)$ is the U.S. index return and Z_t is the five-dimensional vector of instruments. The estimated HJ distance and its associated standard deviation (SD) are first reported. A Wald test for equality of all the coefficients, ζ, to zero is reported next.

	Linear	Conditional Linear	Nonlinear
Instrument set A			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.0833, 0.4081	0.9502, 3.944	0.0623, 0.1489
Wald $\chi^2(5)$, p-value	13.196, 0.021	6.459, 0.264	1.089, 0.962
Instrument set B			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.0489, 0.1058	0.3212, 0.6858	0.0578, 0.2868
Wald $\chi^2(5)$, p-value	12.607, 0.027	8.393, 0.136	2.319, 0.803
Instrument set C			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.0588, 0.1323	0.4964, 1.0382	0.0931, 0.4777
Wald $\chi^2(5)$, p-value	11.705, 0.039	17.060, 0.004	3.445, 0.632
Instrument set D			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.0526, 0.1030	1.2508, 2.3276	0.1752, 0.3038
Wald $\chi^2(5)$, p-value	8.306, 0.139	7.393, 0.193	2.289, 0.807
Instrument set E			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.6551, 0.2962	0.3028, 0.5710	0.0422, 0.1027
Wald $\chi^2(5)$, p-value	14.414, 0.131	8.953, 0.111	1.973, 0.853

the pricing kernels have an unconditional mean equal to the average price of the one-period bond. Hence the difference in pricing errors across the three models is due to their ability to capture the covariances (risk premia) between the pricing kernel and the payoffs. From equation (22), we see that differences in the HJ distance measure are due to differences in the average pricing errors as the weighting matrix is held constant across the models. It immediately follows that the relative success of the nonlinear model can be attributed to its ability in capturing the risk premia on the different payoffs.

In our estimation, the estimated pricing kernel has a mean that is very close to 1 (it is 0.997 or 0.998 generally). From theory, the mean of the pricing kernel is the price of the unit riskless payoff the next period. Since the average Eurodollar deposit (gross) return over the week is around 1.001733, the estimated mean of the pricing kernel is consistent with theory. While all three pricing kernels have similar means, their standard deviation differs. The nonlinear pricing kernel tends to have the lowest standard deviation while the conditional linear pricing kernel has a much higher standard deviation. The lower standard deviation of the nonlinear pricing kernel is

Table X

Hansen-Jagannathan Diagnostics for Payoff Set 3 (Equally Weighted Forward Contract Portfolio)

This table presents the Hansen-Jagannathan distance measure tests for payoff set 3, where the HJ distance is estimated using the equally weighted forward contract return as the payoff. For each of the five instrument sets (Sets A, B, C, D, and E), the HJ distance is estimated using the orthogonality condition

$$E[(m_{t+4} + \zeta'(f(t,t+4) \otimes Z_t))f(t,t+4)Z_{1t} - Z_{1t}] = 0$$

where Z_{1t} is the time t price of the "payoff" $f(t,t+4)Z_{1t}$. $f(t,t+4)$ is the return on the forward contract portfolio and Z_t is the five-dimensional vector of instruments used in estimation. The estimated HJ distance and its associated standard deviation (SD) are first reported. A Wald test for equality of all the coefficients, ζ, to zero is reported next.

	Linear	Conditional Linear	Nonlinear
Instrument set A			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.2046, 0.9308	3.993, 6.393	0.1423, 0.8349
Wald $\chi^2(5)$, p-value	24.709, 0.000	34.252, 0.000	1.826, 0.872
Instrument set B			
H-J dist($\times 10^4$), SD($\times 10^4$)	0.1189, 0.6383	0.5824, 0.8499	0.1626, 0.4556
Wald $\chi^2(5)$, p-value	20.804, 0.001	34.382, 0.000	8.216, 0.145
Instrument set C			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.1469, 0.5690	2.0904, 3.0575	0.2836, 0.8448
Wald $\chi^2(5)$, p-value	20.233, 0.001	108.723, 0.000	14.783, 0.012
Instrument set D			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.4035, 0.7647	7.4205, 14.9733	0.3014, 0.5550
Wald $\chi^2(5)$, p-value	48.408, 0.000	52.921, 0.000	3.103, 0.684
Instrument set E			
H-J Dist($\times 10^4$), SD($\times 10^4$)	0.1356, 0.9508	0.3820, 0.4069	0.0948, 0.5043
Wald $\chi^2(5)$, p-value	22.414, 0.000	27.293, 0.000	5.264, 0.384

most striking in the third payoff set, which includes both bond and forward contract returns.

Our results indicate that the challenge in international asset pricing is in explaining forward contract and bond returns and not stock index returns. Stock index returns are explained by all the models that we consider and cannot discriminate between the three models. In contrast, the introduction of forward contracts leads to greater rejection of the models we consider and greater discriminatory power between the linear and nonlinear models in the cross-model tests.

All the above results were obtained without imposing the nonnegativity restriction on the pricing kernel. In a related context, Bansal and Viswanathan (1993) show how to impose this restriction in nonparametric estimation.[35] However, none of the estimated pricing kernels ever generate a negative value. Hence, the estimated pricing kernels always satisfy the nonnegativity restriction.

[35] See also Hansen and Jagannathan (1992).

VI. Conclusions

In this paper, we present an approach to arbitrage pricing that implies the existence of a low-dimensional, nonnegative, nonlinear pricing kernel. This pricing kernel prices all payoffs, including payoffs that are nonlinear in the factors and thus do not satisfy the usual linearity restrictions in linear arbitrage pricing. Allowing for nonlinear payoff structures is important because it makes possible the application of arbitrage-pricing theory to payoffs of derivative securities and fixed income securities. Moreover, there is no a priori reason to believe that primitive payoffs have linear factor representations.

The key object of interest in our approach is the low-dimensional, nonnegative, nonlinear pricing kernel. We present a nonparametric estimation method to estimate this pricing kernel and test the theory. The approach is implemented on weekly international data from January 1975 to December 1990. In addition to estimating and testing the unconditional linear factor and the nonlinear arbitrage-pricing theory, we also present a new approach to estimating the conditional linear factor-pricing model. The three models are then evaluated using the distance measure suggested in Hansen and Jagannathan (1992).

Our empirical results suggest that payoffs that include only equity index returns do not discriminate between the unconditional linear, conditional linear, and nonlinear one-factor models. None of the models are rejected in the GMM metric, and the HJ distance slightly prefers the nonlinear model to the linear models.

The addition of bond returns and especially forward contract returns to the payoff set sharply increases the power to discriminate among the three models in favor of the nonlinear model. In the GMM metric, the unconditional linear model is strongly rejected while the conditional linear model and the nonlinear model have greater success. In the HJ distance metric, the nonlinear model is strongly supported while the conditional and unconditional linear models are strongly rejected. The lowest HJ distances typically occur with the nonlinear model. The HJ distance test results clearly show that the one-factor (world index) unconditional linear and the conditional linear models have the greatest difficulty in pricing the forward contract. Only the nonlinear single factor model does an adequate job of pricing all these payoffs simultaneously. These results indicate that explaining forward contract and bond returns is the key challenge for an international asset-pricing model; all the single factor models considered here seem to do a reasonable job of explaining country index returns.

Finally, our results in this paper are strongly supportive of the approach to asset pricing that we present, emphasizing the restrictions of no arbitrage and low dimensionality. This approach yields a low-dimensional, nonnegative, nonlinear pricing kernel on which theory places restrictions. Our results using international data and those in Bansal and Viswanathan (1993) using U.S. data strongly support the nonlinear arbitrage-pricing model and suggest that this approach is a promising one.

Appendix: Instrument Sets

The Appendix lists the instrument sets that we use in our estimations. $R_{US}(t, t+1)$ is the one-week U.S. index return and $R_W(t, t+1)$ is the one-week world index return. $y_{US}(t, t+1)$ is the one-week Eurodollar return, while $y_{US}(t, t+4)$ is the four-week U.S. Treasury bill return. $y_{JP}(t, t+4)$ is a variable defined below as

$$y_{JP}(t, t+4) = \frac{e_{JP}(t)}{F_{JP}(t, t+4)} y_{US}(t, t+4), \tag{26}$$

where $F_{JP}(t, t+4)$ is the forward price today for yen four weeks ahead and $e_{JP}(t)$ is the spot exchange rate today. Finally, $e_{UK}(t)$ and $e_{WG}(t)$ are the spot exchange rates for the United Kingdom and West Germany.

Set A

Constant
$1 + 10^*[R_W(t-1, t) - y_{JP}(t-1, t+3)]$
$1 + (10*[R_{US}(t-1, t) - y_{US}(t-1, t+3)])^2$
$1 + [y_{JP}(t-1, t+3) - y_{US}(t-1, t+3)]$
$(1/3)*\Sigma_{UK, WG, JP}[e_k(t)/e_k(t-1)]$

Set B

Constant
$1 + 10*[R_W(t-1, t) - y_{JP}(t-1, t+3)]$
$1 + (10*[R_{US}(t-1, t) - y_{US}(t-1, t+3)])^2$
$1 + [100* y_{US}(t-1, t+3)]$
$(1/3)*\Sigma_{UK, WG, JP}[e_k(t)/e_k(t-1)]$

Set C

Constant
$1 + 10*[R_W(t-1, t) - y_{JP}(t-1, t\}3)]$
$1 + (10*[R_W(t-1, t) - y_{US}(t-1, t+3)])^2$
$1 + [y_{JP}(t-1, t+3) - y_{US}(t-1, t+3)]$
$(1/3)*\Sigma_{UK, WG, JP}[e_k(t)/e_k(t-1)]$

Set D

Constant
$1 + 10*[R_W(t-1, t) - y_{JP}(t-1, t+3)]$
$1 + R_W(t-1, t)$
$1 + y_{US}(t-1, t)$
$(1/3)*\Sigma_{UK, WG, JP}[e_k(t)/e_k(t-1)]$

Set E

Constant
$1 + 10*[R_W(t-1, t) - y_{JP}(t-1, t+3)]$
$1 + (10*[R_{US}(t-1, t+3) - y_{US}(t-1, t+3)])^2$
$1 + [100* y_{US}(t-1, t+3)]$
$(1/3)*\Sigma_{UK, WG, JP}[e_k(t)/e_k(t-1)]$

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