9. AUD. VJEŽBE

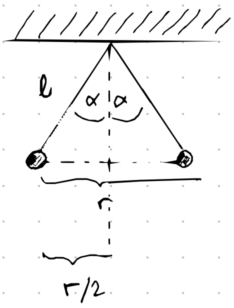
1.

$m = 30\,g$

$l = 0{,}15\,m$

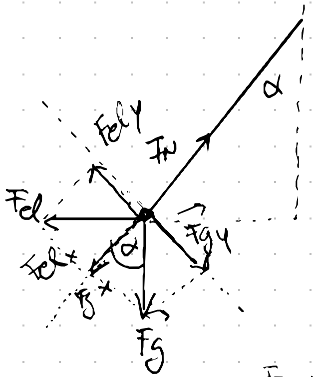
$\alpha = 5^\circ$

$q = ?$



$\frac{r}{2} = l \sin\alpha$

$r = 2l \sin\alpha$

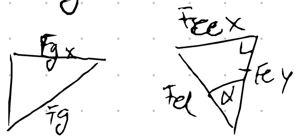


$$\vec{F}_N = \vec{F}_{gx} + \vec{F}_{el\,x}$$

$$F_{ely} = F_{gy}$$

općenito

$$F_{el} = \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{r^2}$$



$$F_{elx} = \sin\alpha \cdot F_{el}$$

$$F_{ely} = \cos\alpha\, F_{el}$$

$$F_{gy} = mg \sin\alpha$$

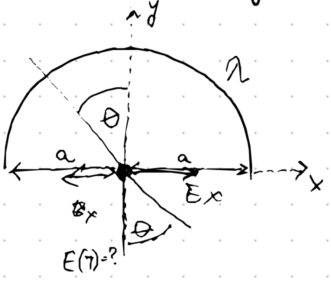
$$F_{el} \cdot \cos\alpha = F_g \cdot \sin\alpha$$

$$F_{el} = m \cdot g \cdot tg\alpha \Rightarrow \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{r^2} = m \cdot g \cdot tg\alpha$$

$$q = \sqrt{mg\, tg\alpha \cdot r^2 \cdot 4\pi\varepsilon_0} = r\sqrt{mg\, tg\alpha\, 4\pi\varepsilon_0} = 2l\sin\alpha\sqrt{mg\, tg\alpha\, 4\pi\varepsilon_0}$$

$$\boxed{q = 4{,}42 \times 10^{-8}\,C}$$

2. Izračunati el. polje u središtu poluprstena radijusa a i lin. gust. λ.



$$\lambda = \frac{dq}{ds} \rightarrow dq = \lambda \cdot ds$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, ds}{r^2}\,\hat{r}$$

$E_x = 0$ (poništite se doprinosi)

$E_y = E\cos\theta$

$E_y = ?$

$$E_y = \cos(\theta) \cdot \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda \hat{r}}{r^2}\, ds \xrightarrow{\text{na neku formu}} \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda}{r^2}\cos(\theta)\, ds$$

$a = r$ (konst)

Za prsten vrijedi

$s = a\theta$ (to se poklapa)

$ds = a\, d\theta$

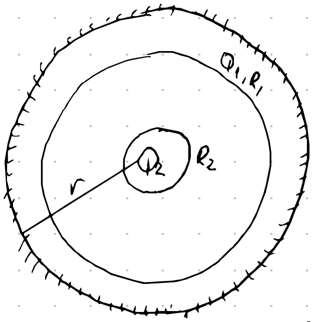
$$\Rightarrow E_y = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{1}{a^2} \cdot a \cos\theta\, d\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 a}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 a}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right)$$

$$\boxed{E_y = \frac{\lambda}{2\pi\varepsilon_0 a}}$$

③ Unutar sferne ljuske naboja Q_1 i R_1 nalazi se uniformno nabijena kuglica Q_2 i R_2 tako da im se središte poklapaju. Pomoću Gaussovog zakona izračunati el. polje:

a) izvan sferne ljuske



a) $r > R_1$

$$\frac{Q_{zat}}{\varepsilon_0} = \frac{Q_1 + Q_2}{\varepsilon_0}$$

$$\underbrace{\int \vec{E}\, d\vec{A}}_{LHS} = \underbrace{\frac{Q_{zat}}{\varepsilon_0}}_{RHS} = \int E\, dA$$

radijalno polje za r je konstantno → E = const

$$E \int dA =$$

u žutima

↳ $da = r^2 \sin\theta \, d\theta \, d\varphi$

$$= E r^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi$$

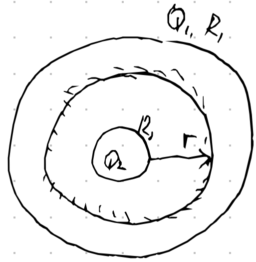
$$\Rightarrow E r^2 4\pi = \frac{Q_1 + Q_2}{\varepsilon_0}$$

$$\boxed{E = \frac{Q_1 + Q_2}{4\pi r^2 \varepsilon_0}}$$

b) u prostoru između ljuske i kuglice $R_2 < r < R_1$

$$\underbrace{\int \vec{E}\, d\vec{A}}_{LHS} = \underbrace{\frac{Q_{zat}}{\varepsilon_0}}_{RHS}$$

$$\frac{Q_{zat}}{\varepsilon_0} = \frac{Q_2}{\varepsilon_0}$$



$$E r^2 4\pi = \frac{Q_2}{\varepsilon_0} \rightarrow \boxed{E = \frac{Q_2}{r^2 4\pi \varepsilon_0}}$$

izvod pod (a)

c) $r < R_2$

$$\int \vec{E}\, dA = E r^2 4\pi$$

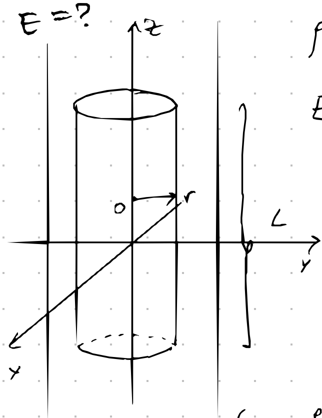
$Q_{zat} = ?$

$$\frac{Q_2}{\frac{4\pi}{3} R_2^3} = \frac{Q_{zat}}{\frac{4\pi}{3} r^3} \Rightarrow Q_{zat} = Q_2 \frac{r^3}{r_2^3}$$

$$\Rightarrow \boxed{E = \frac{Q_2\, r}{4\pi \varepsilon_0 r_2^3}}$$

4. Beskonačno dugi cilindar; $\rho = kr$, $k = konst.$

$E = ?$



$\rho = kr$

$E = ?$ (unutar)

$$\oint \vec{E}\,d\vec{A} = \frac{Q_{zat}}{\varepsilon_0}$$

$$\frac{dq}{dV} = \rho$$

$Q_{zat} = ?$

$$Q_{zat} = \int \rho\, dV$$

$$Q_{zat} = \int k r\, dV$$

u žetim koord. $dV = r\,dr\,d\Phi\,dz$, $r = konst.$

(pomaknemo cilindar da ide od 0)

$$Q_{zat} = \int_0^r kr^2 dr \int_0^{2\pi} d\varphi \int_0^L dz$$

$$Q_{zat} = \frac{2}{3}\pi k L r^3$$

$\oint \vec{E}\,d\vec{A} \Rightarrow E$ je radijalno za našu zatvorenu plohu je konst?

$E \oint dA =$ (u žetima) $\Rightarrow dA = r\,d\Phi\,dz$

$$= E r \int_0^{2\pi} d\varphi \int_0^L dz = E r 2\pi L$$

$$E r 2\cancel{\pi}\cancel{L} = \frac{2}{3}\cancel{\pi} k \cancel{L} r^3 \cdot \frac{1}{\varepsilon_0}$$

$$\boxed{E = \frac{2}{3\varepsilon_0} k r^2}$$