SPECIJALNA TEORIJA RELATIVNOSTI

Lorentzove transformacije

I. → nove transformacije moraju prijeći u Galilejeve transformacije kada uzmimo da je relativna brzina $\boxed{v \ll c}$

· brzina svjetlosti je u svim sustavima c

II. $v' = -v$ moraju biti simetrične

III. Za pretpostavku uzimamo da su transformacije linearne $\boxed{x = \alpha x' + \beta t'}$

III

$\Rightarrow \quad x = \alpha x' + \beta t'$

$$x = \gamma (x' + vt')$$

$$\left.\begin{aligned} x &= ct \\ x' &= ct' \end{aligned}\right\} \text{ u svim sustavima } c = c \quad \text{I}$$

$$\hookrightarrow x' = \gamma (x' - vt) \rightarrow \text{II} \quad v = -v$$

$$\Rightarrow x = ct = \gamma (x' + vt')$$

$$= \gamma (ct' + vt') \Rightarrow \gamma t' (c + v)$$

$$\Rightarrow \frac{x}{\gamma} = t'(c+v) \qquad \frac{x'}{\gamma} = t'(c-v)$$

$$\Rightarrow x' = ct' = \gamma (x - vt) = \gamma (ct - vt)$$

$$x' = \gamma t (c - v) \longrightarrow t = \frac{x'}{\gamma (c - v)}$$

uvrstimo: $x = ct$

$$t' = \frac{x}{c}$$

$$x = c \frac{x'}{\gamma (c - v)} = \gamma (x' + vt) \longrightarrow \gamma \left(x' + v \frac{x'}{c}\right)$$

$$\frac{c \cancel{x'}}{\gamma (c - v)} = \gamma \cancel{x'} \left(1 + \frac{v}{c}\right) \rightarrow \frac{c}{\gamma (c - v)} = \gamma \left(1 + \frac{v}{c}\right) \Rightarrow c = \gamma^2 (c - v) \left(1 + \frac{v}{c}\right)$$

$$c = \gamma^2 (c - v) \left(1 + \frac{v}{c}\right) = \left(c + \cancel{v} - \cancel{v} - \frac{v^2}{c}\right)$$

$$= \gamma c \left(1 - \frac{v^2}{c^2}\right) \rightarrow 1 = \gamma \left(1 - \frac{v^2}{c^2}\right)$$

$$\boxed{\gamma = \frac{1}{\sqrt{1 - v^2}}}$$

transformacija

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Dilatacija vremena t_1', t_2' $x_2' = x_1$

$$t_2 - t_1 = \gamma\left((t_2' - t_1') + \frac{v}{c^2}(x_2' - x_1')\right)$$

$$\to \Delta t = \gamma \cdot \Delta t'$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c}}}$$

Kontrakcija duljine x_2', x_1' $t_1 = t_2$

$$x_2' - x_1' = \gamma\left(x_2 - x_1 - v(t_2 - t_1)\right)$$

$$\ell = \gamma' \ell$$

Brzina čestica

u_x – brzina u sustavu S

u_x' – -||- u sustavu S'

$$u_x = \frac{dx}{dt} = \frac{d}{dt}\gamma(x' + vt') = \frac{dt'}{dt} \cdot \frac{d}{dt'}\left[\gamma(x' + vt')\right]$$

$$u_x = \frac{dt'}{dt}\left[\gamma\left(\overbrace{\frac{dx'}{dt'}}^{u_x'} + v\right)\right] \longrightarrow u_x = \frac{dt'}{dt}\left[\gamma(u_x' + v)\right]$$

$z = z'$ $y = y'$
nema promjene

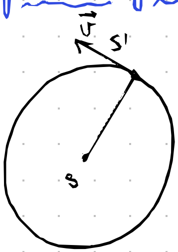
$$t = \gamma \cdot t' = \gamma\left(t' + \frac{v}{c^2}x'\right) \Big/ \frac{d}{dt'}$$

$$\frac{dt}{dt'} = \gamma\left(1 + \frac{v}{c^2} \cdot u_x'\right)$$

$$\longrightarrow u_x = \frac{\cancel{\gamma}(u_x' + v)}{\cancel{\gamma}\left(1 + \frac{v}{c^2}u_x'\right)} = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'} = u_x$$

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}$$

Gravitacijska konja



- nije inercijski sustav pa se zakonitosti ne poklapaju

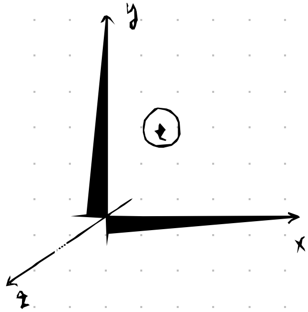
R se ne transformira jer je S okomit na S

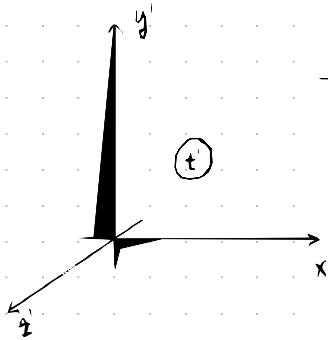
$\Rightarrow$ S' < S, S'

Lorentzove transformacije i njihova svojstva

+ Saša

- dva inercijska sustava : S i S'





$$\Delta x' = \gamma(x - v\Delta t)$$

$$\Delta y' = \Delta y \qquad \Delta z' = \Delta z$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{c}{v} < 1$$

$$u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'} \qquad u_y = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)} \qquad u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$$

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \qquad u_y' = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} \qquad u_z' = \frac{u_z}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

KORISNO: (izvodi)

$$\frac{dt'}{dt} = \frac{\Delta t'}{\Delta t} = \frac{\gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)}{\Delta t} = \gamma\left(1 - \frac{v}{c^2}\overbrace{\left(\frac{\Delta x}{\Delta t}\right)}^{u_x}\right) \rightarrow \boxed{\gamma\left(1 - \frac{v}{c^2}\cdot u_x\right)}$$

$$u_x' = \frac{dx'}{dt'} = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x'}{\Delta t}\cdot\frac{\Delta t}{\Delta t'} = \frac{\gamma(\Delta x - v\Delta t)}{\Delta t}\cdot\frac{1}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

$$u_x' = \frac{(u_x - v)}{\left(1 - \frac{v}{c^2}u_x\right)}$$

$$u_y' = \frac{\Delta y'}{\Delta t'} = \frac{\Delta y'}{\Delta t}\cdot\frac{\Delta t}{\Delta t'} = \overbrace{\frac{\Delta y}{\Delta t}}^{u_y}\cdot\frac{1}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

RELATIVISTIČKA KOLIČINA GIBANJA & ENERGIJA ČESTICE:

Količina gibanja: ako se čestica mase m giba brzinom kojoj odgovaraju:

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \qquad \vec{p} = \gamma m \vec{u}$$

→ Newtnova jednadžba gibanja mora odgovarati i za relativističku i ne relativističku fiziku: $F = \frac{dp}{dt}$

- $u \to c$ $\quad \gamma \to \infty$ $\quad \vec{p}$ može biti proizvoljno velika bez $u \geq c$
- $u \to 0$ $\quad \gamma \to 1$ $\quad \vec{p}$ je jednaka nerelativističkoj (običnoj)

Relativistički izraz za silu u čestice mase m pod djelovanjem F

$$F = \frac{d\vec{p}}{dt} \qquad p[t] = Ft \longrightarrow p(t) = \frac{m \cdot u[t]}{\sqrt{1-\frac{u^2[t]}{c^2}}} \text{ jer } p[t] = \gamma m \vec{u}$$

$$\vec{u} = \frac{p[t]}{\gamma m} = \frac{Ft}{\gamma m} \to \vec{u}[t] = \frac{Ft}{\sqrt{1-\frac{u^2}{c^2}} \cdot m}$$

Kinetička energija $K = (\gamma - 1)mc^2$

- $u \to c$ $\quad \gamma \to \infty$ $\quad$ → K može biti >> bez da $u \geq c$
- $u/c \ll 1$ $\quad u/c \to 0$ $\quad K = \frac{1}{2}mu^2$ $\quad$ odgovara nerelativističkoj (običnoj) jednadžbi

Rad i energija u specijalnoj teoriji relativnosti

- $\vec{p} = \gamma m \vec{u}$
- $\vec{F} = \frac{d\vec{p}}{dt}$

izvršeni rad pod djelovanjem sile: $W = \int_1^2 \vec{F}\, d\vec{s}$

→ u 1D: $\int_1^2 F dx = \int_1^2 \frac{dp}{dt} dx = \int_1^2 v\, dp$

* podintegralni izraz možemo zapisati: $d(vp) = p\,dv + v\,dp$

→ $v\,dp = d(vp) - p\,dv$

& $1 \to v_1 = 0$

$2 \to v_2 = v$

$$W = \int_0^v v\,dp = \int_0^v d(vp) - \int_0^v p\,dv$$

$$W = vp\Big|_0^v - \int_0^v \gamma m v\,dv$$

$$W = vp\Big|_0^v - m\int_0^v \gamma[v]\, v\,dv = v\gamma m v - m\int_0^v \frac{v\,dv}{\sqrt{1-\frac{v^2}{c^2}}} = \dots$$

$$\Rightarrow \gamma mc^2 - mc^2 = E_k \longrightarrow \boxed{mc^2(\gamma - 1) = E_k} = W$$

Teorem o radu i kinetičkoj energiji

$E = \gamma mc^2$ – ukupna en.

$E_0 = mc^2$ – en. mirovanja

$E = E_0 + E_k$

$\gamma mc^2 = mc^2 + E_k$

Relativistička energija čestice $E = \underline{mc^2} + K = \gamma mc^2$

mc^2 – energija mirovanja

vrijedi: $E^2 = (mc^2)^2 + (pc)^2$

Relativistički savršeno neelastičan sudar:

O.K.G: $\gamma mu = \Gamma M u$ $\quad \gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ (samo veliko slovo)

O.Rel.E: $\gamma mc^2 + mc^2 = \Gamma M c^2$

$\Rightarrow \quad u = \frac{\gamma u}{\gamma + 1}$

- $u \ll c, \ \gamma \simeq 1 \qquad u_{sm} = \frac{u}{2} \qquad M = 2m\sqrt{(1+\gamma)/2}$
- $u \to c, \ \gamma \to \infty \qquad u \to u$

Bezmasene čestice $E = pc$ za $m = 0$; npr. fotoni