DINAMIKA

I. & II. Newtonov zakon

"stanje ravnoteže"

→ čestica na koju ne djeluje sila giba se stalnom brzinom ili miruje

- u kojem ref. sustavu to vrijedi? NIJEDNOM!
  čim promatramo iz drugog sustava, to ne vrijedi (neinercijski ref. okvir)

ako pak sila djeluje onda je jednačba: $\vec{F} = m\vec{a}$ (samo inercijski ref. okviri)

* na čestici djeluje više sila, a I. N. Z. se odnosi na njihov (vektorski) zbroj ⇒ REZULTANTNA SILA

- beskonačno mnoštvo inercijskih referentnih okvira

→ količina gibanja: $\vec{p} = m\vec{v}$ [kgm/s]

$$\Rightarrow \vec{F} = \frac{d}{dt}\vec{p} = \frac{d}{dt}(m\vec{v}) = m\cdot\frac{d\vec{v}}{dt}\cdot\frac{ds}{dt} \Rightarrow \boxed{F = m\cdot\vec{a}}$$

Pokus: Kružno gibanje

kutna brzina: $\omega_z = \frac{d\Phi_z}{dt} \;/\cdot dt$

* jer je ⊥ i proteže se u z-osi

$\omega_z[t']\cdot dt' = d\Phi_z \;/\int_{t_0}^{t}$

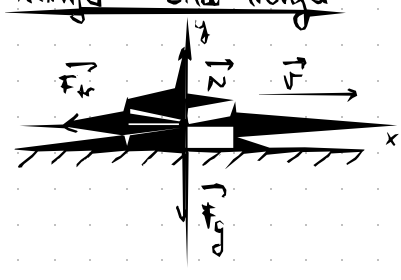
$\int_{t_0}^{t}\omega_z[t']\cdot dt' = \int_{t_0}^{t} d\Phi_z$

$\int_{t_0}^{t}\omega_z[t]\,dt = \underbrace{\Phi_z[t] - \Phi_z[t_0]}_{\Delta\Phi}$

→ integracijom ω_z možemo znati za koliko se vijak okrenuo

$$\Rightarrow \boxed{\int_{t_0}^{t}\omega_z[t']\cdot dt' + \Phi_z[t_0] = \Phi_z[t]}$$

Primjer: Sila trenja



jednadžbe gibanja:

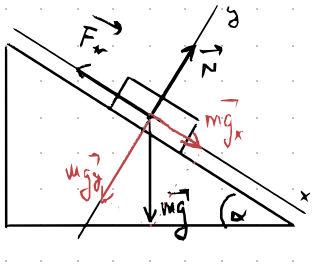
po y ↕: $ma_y = N - mg = 0 \longrightarrow N = mg$

po x ↔: $ma_x = -F_{tr} = -\mu N$

$ma_x = -\mu mg \Rightarrow a_x = -\mu g$

Pokus: kosina

- prema dolje



jednadžbe gibanja:

po y ↕: $m\vec{a}_y = N - mg_y = 0$ ($mg_y = mg\cos\alpha$)

$\Rightarrow N = mg\cdot\cos\alpha$

po x ↔: $m\vec{a}_x = mg_x - F_{tr}$ ($mg_x = mg\sin\alpha$, $F_{tr} = \mu N$)

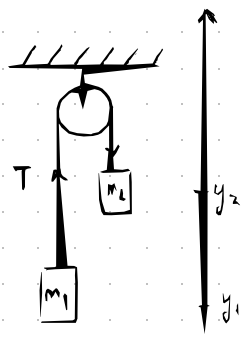
$m\vec{a}_x = mg\sin\alpha - mg\mu$

$\Rightarrow \vec{a}_x = g(\sin\alpha - \mu)$

uskoro ili se zbrajale →

treba znati izvod!

Primjer: kolotur (podosteg)



jednadžbe gibanja

* samo $y \Leftrightarrow \uparrow$ $\quad m_1 a_1 = T - m_1 g$ $\quad * a_1 = \ddot{y}_1$

$m_2 a_2 = T - m_2 g$

VEZA: $\Delta y_1 = -\Delta y_2$ (pomak jednog užeta = pomak drugog užeta)

$dy_1 = -dy_2 \;/\; \frac{1}{dt}$

$\frac{dy_1}{dt} = -\frac{dy_2}{dt} \;/\; \frac{d}{dt} \Rightarrow \ddot{y}_1 = -\ddot{y}_2$

$a_1 = -a_2$

$\Rightarrow m_1 a_1 = T - m_1 g$

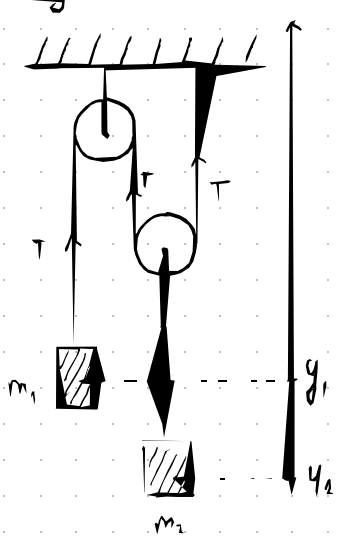
$-m_2 a_1 = T - m_2 g \;/\; -$

$a_1(m_1 + m_2) = -m_1 g + m_2 g$

$a_1(m_1 + m_2) = g(m_2 - m_1)$ $\longrightarrow$ $\ddot{y}_1 = g\frac{m_2 - m_1}{m_1 + m_2} = -\ddot{y}_2$

• znat izvod

Primjer: kolotur (2)



jednadžbe gibanja

$m_1 \ddot{y}_1 = T - m_1 g$

$m_2 \ddot{y}_2 = 2T - m_2 g$

$-m_1 2\ddot{y}_2 = T - m_1 g \;/\; (-2)$

$m_2 \ddot{y}_2 = 2T - m_2 g$

$4m_1 \ddot{y}_2 = -2T + 2m_1 g$

$m_2 \ddot{y}_2 = 2T - m_2 g \;/\; +$

$4m_1 \ddot{y}_2 + m_2 \ddot{y}_2 = 2m_1 g - m_2 g$

$\ddot{y}_2(4m_1 + m_2) = g(2m_1 - m_2)$

$\ddot{y}_2 = g\frac{2m_1 - m_2}{4m_1 + m_2} \Rightarrow \ddot{y}_1 = \frac{2(m_2 - 2m_1)}{4m_1 + m_2}$

VEZA:

$dy_1 = -2dy_2$

$\frac{dy_1}{dt} = -2\frac{dy_2}{dt}$

$\dot{y}_1 = -2\dot{y}_2 \;/\; \frac{d}{dt}$

$\ddot{y}_1 = -2\ddot{y}_2$

— sila ovisna o brzini —

$$m\frac{d\vec{v}}{dt} = \vec{F}[\vec{v}] \longrightarrow \text{slučaj gibanja u 1D (x-os)} \quad m\frac{dv_x}{dt} = F_x[v_x]$$

▸ separacija varijabli

$$m\frac{dv_x}{F_x[v_x]} = dt \quad / \int_{\text{početno stanje}}^{\text{konačno stanje}}$$

$$m\int_{v_0}^{v}\frac{dv_x'}{F_x} = \int_{t_0}^{t} dt' \longrightarrow m\int_{v_x[t_0]}^{v_x[t]}\frac{1}{F_x}\,dv_x' = t - t_0$$

▸ slučaj $F_x[v_x] = -bv_x$ — originira da brzina ima predznak koji nas „koči" (kontra smjer, ish. iznos)

* ono što je otpornik u elektrotehnici → WET FRICTION

→ trenje je DRY FRICTION

$$m\int_{v_x[t_0]}^{v_x[t]}\frac{1}{-bv_x}\,dv_x' \Rightarrow \frac{-m}{b}\int_{v_x[t_0]}^{v_x[t]}\frac{dv_x'}{v_x} = \frac{-m}{b}\cdot \ln|v_x'|\Big|_{v_x[t_0]}^{v_x[t]}$$

$$\Rightarrow \frac{-m}{b}\left(\ln|v_x[t]| - \ln|v_x[t_0]|\right) = \boxed{\frac{-m}{b}\cdot\ln\left|\frac{v_x[t]}{v_x[t_0]}\right|}$$

izjednačimo sa $\int_{t_0}^{t} dt'$

$$\frac{-m}{b}\ln\left|\frac{v_x[t]}{v_x[t_0]}\right| = t - t_0 \Rightarrow \ln\left|\frac{v_x[t]}{v_x[t_0]}\right| = \frac{-b}{m}(t-t_0) \quad / e$$

$$\frac{v_x[t]}{v_x[t_0]} = e^{-\frac{b}{m}(t-t_0)} \Rightarrow \boxed{v_x[t] = v_x[t_0]\cdot e^{-\frac{b}{m}(t-t_0)}}$$

Senzor sile: „load cell"