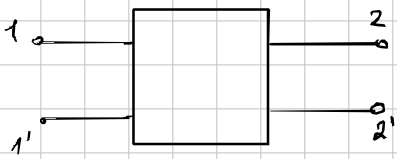
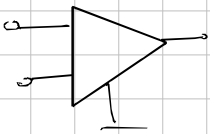
14.1 UVOD U TEORIJU ČETVEROPOLA

two-port networks

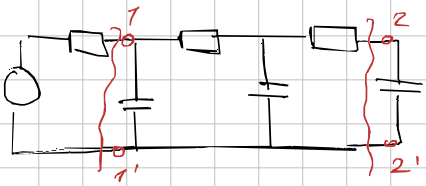


→ „dvo-pristupne" mreže (ne uvijek tho)

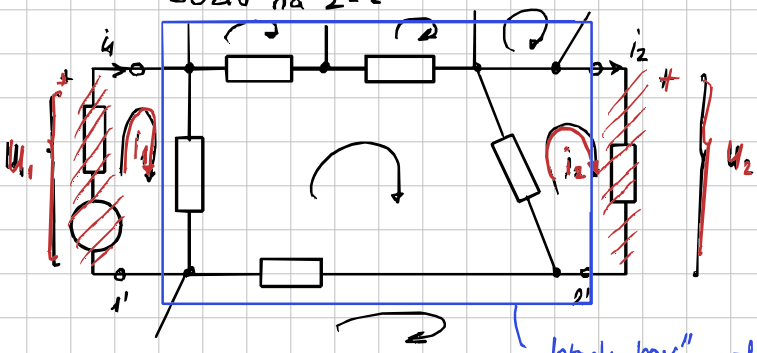


- element mreže ili submreže

* četveropol može biti dio mreže!



Primjer: mreža s pobudom na 1-1' odziv na 2-2'



napon i struja na priključnicama
↳ varijable četveropola

„black box" - eliminiramo sve nepoznate struje od I_3 nadalje

Jednadžbe petlji

$$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12} - I_3 \cdot Z_{13} - \dots - I_n \cdot Z_{1n}$$

$$U_2 = I_2 \cdot Z_{21} - I_2 \cdot Z_{22} - I_2 \cdot Z_{23} - \dots - I_n \cdot Z_{2n}$$

$$0 = -I_1 \cdot Z_{31} - I_2 Z_{32} + I_3 \cdot Z_{33} - \dots - I_n Z_{3n}$$

$$0 = -I_1 \cdot Z_{n1} - I_2 \cdot Z_{n2} - I_3 \cdot Z_{n3} - \dots + I_n Z_{nn}$$

$$I_1 = \left(\frac{\Delta_{11}}{\Delta}\right)_Z U_1 - \left(\frac{\Delta_{21}}{\Delta}\right)_Z U_2 \quad (= y_{11}, \; = y_{12})$$

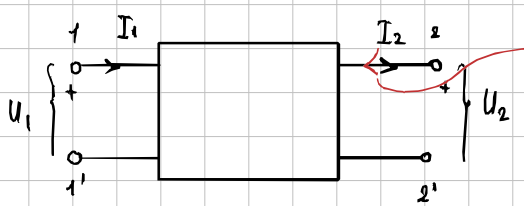
$$I_2 = \left(\frac{\Delta_{12}}{\Delta}\right)_Z \cdot U_1 - \left(\frac{\Delta_{22}}{\Delta}\right)_{22} \cdot U_2 \quad (= y_{21}, \; = y_{22})$$

$\Delta \rightarrow$ det sustava

Δ_{ij} - kofaktori det sustava

$\frac{\Delta_{ij}}{\Delta}$ – ima dimenziju $\frac{1}{\Omega}$

- ajelu mreže moguće je promatrati kao:



$$I_1 = y_{11} \cdot U_1 + y_{12} \cdot U_2$$

$$I_2 = y_{21} \cdot U_1 + y_{22} \cdot U_2$$

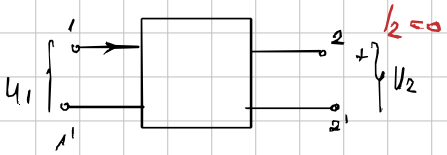
Struja ne jed. četv.

$y_{11}, y_{12}, y_{21}, y_{22}$ → y-parametri četveropola

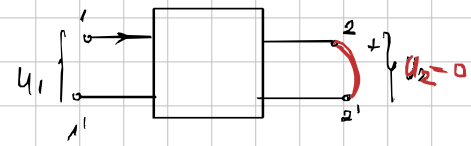
Određivanje parametara četveropola

→ iz uvjeta na prazno i kratko

⇒ četveropol na (prazno) → prazni hod
na priključku 1-1' i/i 2-2'



⇒ četveropol na (kratko) → kratki spoj
na priključku 1-1' i/i 2-2'



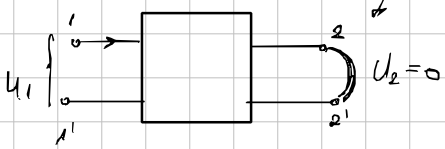
Strujne jednadžbe → y-parametri četveropola

y-parametri → iz četveropola na kratko

$U_2 = 0$ → KS

$I_1 = (y_{11}) U_1 - y_{12} \cdot U_2$

$I_2 = y_{21} \cdot U_1 - y_{22} \cdot U_2$



$I_1 = U_1 \cdot y_{11}$

$$\rightarrow y_{11} = \frac{I_1}{U_1}\Big|_{U_2=0} \qquad y_{21} = \frac{I_2}{U_1}\Big|_{U_2=0}$$

ulazna admitancija na kratko $U_2 = 0$

prijenosna admitancija na kratko $U_2 = 0$

▶ ako je četveropol recipročan $\Delta_{12} = \Delta_{21}$ pa je $y_{12} = y_{21}$

* možemo org. matrično

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}}_{[y]} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = [y] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$[y]$ matrica y-parametara

Naponske jed. → Z-parametri četveropola

$I_1 = y_{11} \cdot U_1 - y_{12} \cdot U_2$

$I_2 = y_{21} \cdot U_1 - y_{22} \cdot U_2$

$$\rightarrow \quad U_1 = \frac{y_{22}}{D_y} I_1 - \frac{y_{12}}{D_y} I_2$$

$$U_2 = \frac{y_{21}}{D_y} I_1 - \frac{y_{11}}{D_y} I_2$$

$D_y = y_{11} \cdot y_{22} - y_{12} \cdot y_{21}$

$$D_y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$$

$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12}$

$U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22}$

} naponske jed. četveropola

recipročni:

$Z_{21} = Z_{12}$

Odredivanje: $I_2 = 0$

$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12}$ ($I_2 = 0$) $\rightarrow$ $Z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0}$ ulazna impedancija

$U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22}$

$Z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0}$ prijenosna impedancija

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z_1 & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad [Z] = [y]^{-1}$$

* matrica Z parametara je inverz matrice Y parametara

Prijenosne jednadžbe → a-parametri četveropola

- prijenos signala s 1-1' ulaza na izlaz 2-2'
- ovisnost U_1 i I_1 o U_2 i I_2

$I_1 = U_1 \cdot y_{11} - U_2 \cdot y_{12}$

$I_2 = U_1 \cdot y_{21} - U_2 \cdot y_{22}$

$U_1 = U_2 \cdot \underbrace{\frac{y_{22}}{y_{21}}}_{A} + \underbrace{\frac{1}{y_{21}}}_{B} I_2$

$I_1 = \underbrace{\frac{\Delta y}{y_{21}}}_{C} U_2 + \underbrace{\frac{y_{11}}{y_{21}}}_{D} \cdot I_2$

$U_1 = AU_2 + BI_2$

$I_1 = CU_2 + DI_2$

} prijenosne jed. četveropola

→ odredujemo preko kratkog i praznog

$I_2 = 0$

$U_1 = AU_2$

$I_1 = CU_2$

$A = \left.\frac{U_1}{U_2}\right|_{I_2=0}$ omjer napona

$C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$ recipročna vrijednost admit na praznom

$U_2 = 0$

$U_1 = BI_2$ $\rightarrow$ $B = \left.\frac{U_1}{I_2}\right|_{U_2=0}$ recipročna vrijednost prijenosne admit

$I_1 = DI_2$ $\rightarrow$ $D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$ omjer struja na kratko pri $U_2 = 0$

matrica prijenosnih parametara $[a] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

UVJET RECIPROČNOSTI: $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC = 1$

Prijenos signala od izlaza prema ulazu

$$U_1 = AU_2 + BI_2$$
$$I_1 = CU_2 + DI_2$$

$$\Rightarrow \begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \quad / \cdot \text{ lijeva } \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}$$

$$\rightarrow \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$$

$$U_2 = \frac{D}{\Delta_A} U_1 - \frac{B}{\Delta_A} I_1 \qquad (D, B)$$

$$-I_2 = \frac{C}{\Delta_A} U_1 - \frac{A}{\Delta_A} I_1 \qquad (C, A)$$

$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

Za recipročne četveropole vrijedi

$$\Delta_A = AD - BC = 1$$

samo za recipročne!

$$\boxed{\begin{aligned} U_2 &= DU_1 - BI_1 \\ -I_2 &= CU_1 - AI_1 \end{aligned}}$$

inverzne prijenosne jednadžbe

→ parametre određujemo preko pokusa na kratko i prazno

Hibridne jednadžbe → h - parametri četveropola

U_1 I_2 — napon na ulazu + struja na izlazu ← preko struje na ulazu I_1 i napona na izlazu U_2

$$I_1 = U_1 y_{11} - U_2 y_{12}$$
$$I_2 = U_1 y_{21} - U_2 y_{22}$$

$$\Rightarrow \quad U_1 = \underbrace{\frac{1}{y_{11}}}_{h_{11}} I_1 + U_2 \underbrace{\frac{y_{12}}{y_{11}}}_{h_{12}}$$

$$I_2 = \underbrace{\frac{y_{21}}{y_{11}}}_{h_{21}} I_1 + \underbrace{\frac{y_{12} \cdot y_{21} - y_{11} \cdot y_{22}}{y_{11}}}_{h_{22}} \cdot U_2$$

$$U_1 = h_{11} \cdot I_1 + h_{12} U_2$$
$$I_2 = h_{21} I_1 + h_{22} U_2$$

→ uvrštavamo u matrične oblike i gledamo $U_2 = 0$ i $I_1 = 0$ (za I_2) (za U_1)

Hibridne jed → g - parametri četveropola

→ invertiramo prošlu matricu

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} \longrightarrow \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

} matrica g parametara

više na 36. slajdu

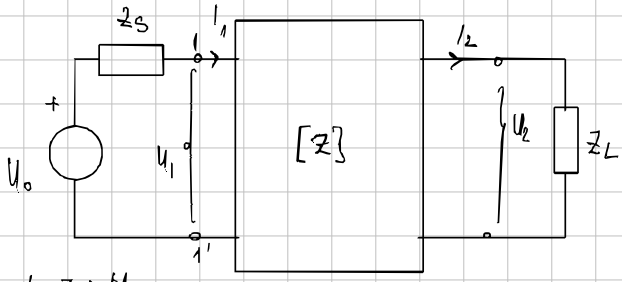
Prijenosne i ulazne funkcije četveropola

Cilj = analizirati mrežu koristeći parametre četveropola
= dobiti rezultate koji vrijede za bilokoji četveropol

Posebno odrediti fje vezane za priključke četveropola

- ▷ prijenosnu fju napona $H_u = \frac{U_2}{U_1}(s)$
- ▷ -||- struje $H_i = \frac{I_2}{I_1}(s)$
- ▷ ekv. ul. imped. $Z_u = \frac{U_1}{I_1}$
- ▷ ekv. izl. imped $Z_i = \frac{U_2}{I_2}$

Primjer: zadani Z parametri



$U_1 + I_1 \cdot Z_S = U_0$

Y - u ppt

$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12}$

$U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22}$

↓

za Y koristimo strujne jednadžbe

$U_2 = I_2 \cdot Z_L \Longrightarrow U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22} \Longrightarrow I_2 Z_L = I_1 \cdot Z_{21} - I_2 \cdot Z_{22}$

prijenosna funkcija struje: $H_i(s) = \frac{I_2}{I_1} = \frac{Z_2}{Z_L + Z_{22}}$

$H_i = \frac{Y_L \, Y_{21}}{Y_{11}(Y_{22} + Y_L) - Y_{12} Y_{21}}$

- naponska prijenosna funkcija → iz prve jednadžbe

$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12} = I_1 \cdot Z_{11} - H_i(s) \cdot I_1 \cdot Z_{12}$

$U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22} = I_1 \cdot Z_{21} - H_i(s) \cdot I_1 \cdot Z_{22}$

$H_u(s) = \frac{U_2}{U_1} = \frac{Y_{21}}{Y_L + Y_{22}}$

↓

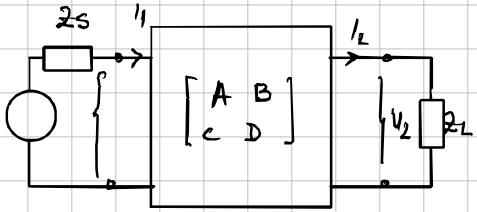
$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L \cdot Z_{21}}{Z_{11}(Z_{22} + Z_L) - Z_{12} \cdot Z_{21}} = \frac{Z_L \cdot Z_{21}}{\Delta Z + Z_{11} \cdot Z_L}$

$\Delta Z = Z_{11} Z_{22} - Z_{12} \cdot Z_{21}$

Prijenosna fja $\frac{U_2}{U_0}$ $\quad U_0(s) = U_1(s) + I_1(s) \cdot Z_S$

$H_u(s) = \frac{U_2}{U_0} = \frac{Z_L \cdot Z_{21}}{(Z_{11} + Z_S)(Z_{22} + Z_L) - Z_{12} \cdot Z_{21}}$

Prijenosne fje izražene prijenosnim parametrima



$$U_1 = A U_2 + B I_2$$

$$I_1 = C U_2 + D I_2$$

$$U_2 = I_2 \cdot Z_L \Rightarrow U_1 = A U_2 + B I_2 \Rightarrow U_1 = A U_2 + B \frac{U_2}{Z_L}$$

$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L}{A Z_L + B}$$ ↳ prijenosna fja napona

Strujna prijenosna fja iz druge jed:

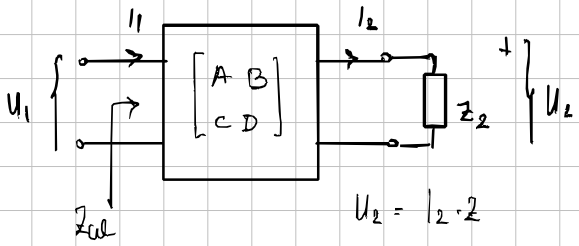
$$I_1 = C U_2 + D I_2 = C I_2 Z_L + D I_2$$

$$H_i(s) = \frac{I_2}{I_1} = \frac{1}{C Z_L + D}$$

Naponska prijenosna funkcija $\frac{U_2}{U_0}$:

$$U_0 = U_1 + I_1 \cdot Z_S = A U_2 + B \frac{U_2}{Z_L} + C U_2 \cdot Z_S + D \frac{U_2}{Z_L} Z_S$$ ↳ $$H(s) = \frac{Z_L}{A Z_L + B + Z_S (C Z_L + D)}$$

Ulazne funkcije četveropola → Ulazne impedancije



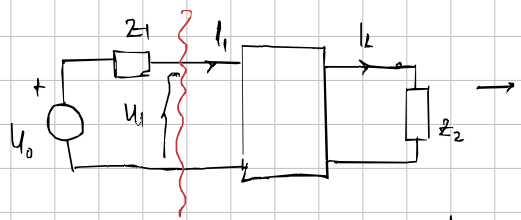
$$U_2 = I_2 \cdot Z_2$$

$$U_1 = A U_2 + B I_2 = (A Z_2 + B) I_2$$

$$U_1 = C U_2 + D I_2 = (C Z_2 + D) I_2$$

$$Z_{ul} = \frac{U_1}{I_1} = \frac{A Z_2 + B}{C Z_2 + D}, \quad Y_{ul} = \frac{1}{Z_{ul}}$$

možemo shvatiti kao



$$I_1 = \frac{U_0}{}$$

$$U_1 = I_1 \cdot Z_{ul} = U_0 \frac{Z_{ul}}{}$$

$Z_1 + Z_{ul}$