COULOMBOV ZAKON I EL. POLJE

Coulombov zakon – sila kojom djeluju dva naboja

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

Sila kojom međudjeluju DVA naboja

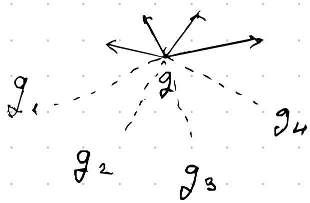
Puno veći od onog G iz gravitacijske sile

ε_0 - permitivnost vakuuma

$q_{min} = e = 1,602 \times 10^{-19}$ C $\longrightarrow$ [C] = [Sek · Amp]

▸ naboj je očuvan UVIJEK! Q_{uk} (prije) = Q_{uk} (poslije)

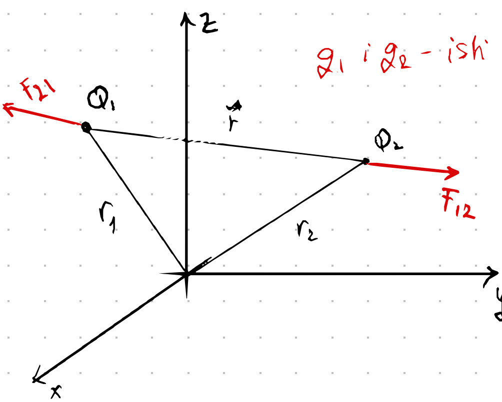
$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i$$



- Coulombova sila je odbojna kada naboji imaju isti predznak

–||– privlačna –||– suprotni (+ i −)

q_1 i q_2 - isti predznak

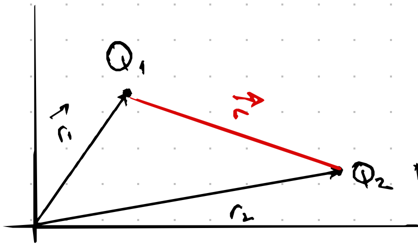


- neovisno jesu li naboji čestica istog ili suprotnog predznaka, uvijek vrijedi:

$$\underbrace{F_{12} = -F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

ELEKTRIČNO POLJE

- prostor u kojem postoje zamišljene el. silnice iz izvora Q_1



$$E_1 = \frac{F}{Q_2}$$

*za el polja vrijedi načelo superpozicije

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{Q_i}{r^2}\hat{r} \Rightarrow \vec{E_i} = \sum_{i\neq j}\vec{E_j} = \vec{E}_{i1} + \vec{E}_{i2} + \dots$$

$$\vec{F_i} = \sum_{i\neq j}\vec{F_j} = \vec{F}_{i1} + \vec{F}_{i2}$$

Coulombova sila koja ovdje djeluje:

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1\cdot q_2}{|r_2 - r_1|^{3/2}}\cdot\cancel{(\vec{r_2}-\vec{r_1})} \Rightarrow F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r_2-r_1|^2}\cdot\hat{r}$$

ostane smjer

Ako silu F_{12} napišemo u obliku el. djela Lorentzove sile: $F_{12} = q_2 E_1[r_2]$

$$\rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_1\cancel{q_2}}{|r_2-r_1|^2}\vec{r} = \cancel{q_2}E_1$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|r_2-r_1|^2}\vec{r}$$

- Ako je raspodjela el. naboja u prostoru opisana volumnom, površinskom ili linijskom gustoćom naboja

$\Rightarrow$ naboj $q \rightarrow dq'$ (nalazi se u okolini točke r')

$\Rightarrow$ sumu zamijenjujemo integralom

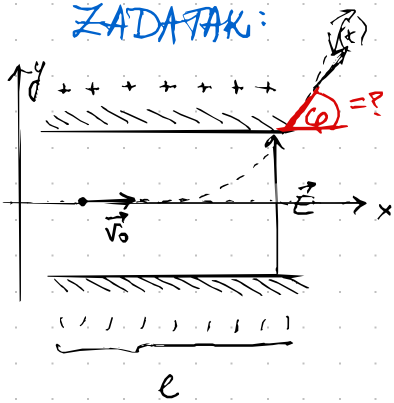
$$E[r] = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{|r-r'|^3}(r-r')$$

linearna gustoća $\rightarrow \frac{dq'}{dl} = \lambda \rightarrow dq' = \lambda[r']\cdot dl$ — linijska gustoća

površinska gustoća $\Rightarrow \frac{dq'}{dS} = \sigma \rightarrow dq' = \sigma[r']dS$ — površinska

volumna gustoća $\Rightarrow \frac{dq'}{dV} = \rho \rightarrow dq' = \rho[r']dV$ — volumna

ZADATAK:



$\vec{v}_0 = v_0\hat{x}$

$\vec{E} = E\hat{y}$

$F = m\cdot a = qE$

x

$ma_x = 0$

$$\frac{dv_x}{dt} = 0$$

$$\int_{v_{x_0}}^{v_x(t)} dv_x = 0$$

$v_x - v_0 = 0$

$v_x = v_0$ konst.

$\frac{dx}{dt} = v_0$

$x(t) = v_0 t$

y

$ma_y = qE$

$$\frac{dv_y}{dt} = \frac{qE}{m} \quad / \int dt$$

$$\int_0^{v_y(t)} dv_y = \frac{qE}{m}\int_0^t dt$$

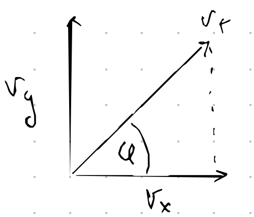
$$v_y(t) = \frac{qE}{m}\cdot t$$

→ neodređeni trenutak

određuje l (put)

$$v_y(t) = \frac{qE}{m}\cdot\frac{l}{v_0}$$

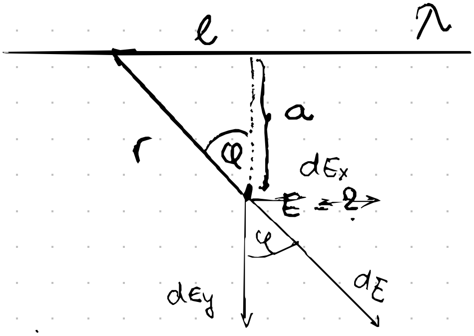
$v_0 = \frac{l}{t} \rightarrow t = \frac{l}{v_0}$



$$tg\varphi = \frac{v_y}{v_x}$$

$$\varphi = arctg\left(\frac{qE}{mv_0}\cdot\frac{1}{v_x}\right) = arctg\left(\frac{qE}{mv_0^2}\right)$$

ZADATAK 3:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{\vec{r}}$$

$$\frac{dq}{dl} = \lambda \rightarrow dq = \lambda \cdot dl$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dl}{r^2}\,\hat{\vec{r}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\, dl}{r^2}\,\hat{\vec{r}}$$

$$dE_x = 0$$

$$dE_y = dE\cos\varphi$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dl}{r^2}\cdot\cos\varphi$$

r želimo izraziti preko cosφ (a)
jer nam je to konstanta,
a sinus bi bio sa l koji je nedefiniran

$$\cos\varphi = \frac{a}{r} \rightarrow r = \frac{a}{\cos\varphi}$$

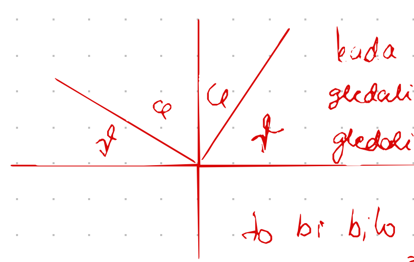
$$r^2 = \frac{a^2}{\cos^2\varphi}$$

$$\operatorname{tg}\varphi = \frac{l}{a} \rightarrow l = \operatorname{tg}\varphi\cdot a \qquad dl = d\varphi\, a \qquad (\operatorname{tg}\varphi)' = \frac{1}{\cos^2\varphi}$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int \frac{\cos^{\cancel{2}}\varphi}{a^{\cancel{2}}}\cdot\lambda\cdot\frac{\cancel{a}}{\cancel{\cos^2\varphi}}\,d\varphi$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\cos\varphi}{a}\,d\varphi$$

koje kutove za granice uvrstiti?



kada bismo gledali thetu ϑ gledali bi 0−π

to bi bilo E_x

$$E_x = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda}{a}\int_0^{\pi}\cos\varphi\, d\varphi$$

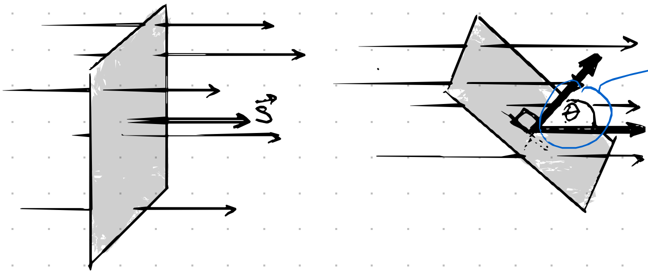
$$E_y = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda}{a}\int_{-\pi/2}^{\pi/2}\cos\varphi\, d\varphi$$

TOK električnog polja = Φ

$$\boxed{\Phi_E = \oint_S \vec{E}\, d\vec{S}}$$

zatvorena plohe

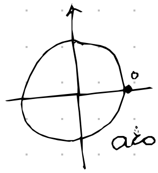
- promatramo površinu S kroz el. polje → kroz nju prolazi više ili manje silnica



kut između silnice el. polja i smjera okomice

skalarni umnožak $|E|\,|dS|\cdot\cos\theta$

* vektor $\vec{S}$ ima
  - iznos vektora površine
  - smjer vektora normale

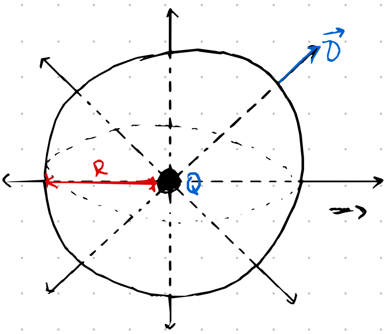


ako je kut neki θ:

$$\Phi_E = \oint_S E\, dS \cos\theta$$

TOK EL POLJA TOČKASTOG NABOJA Q (zatvorena ploha S)

→ zamišljena ploha S je kugla (sfera)



- silnice okomito probadaju kuglinu plohu
- iznos vektora D jednak je u svim točkama kugline plohe

→ tok el. polja jednak umnošku iznosa vektora $\vec{D}$ i ukupne površine kugle: $\Phi = 4R^2\pi D = 4R^2\pi\varepsilon E$

u središtu naboj Q, polje na površini: $E = \dfrac{Q}{4\pi\varepsilon R^2}$

TOK električnog polja kroz neku kuglinu plohu proporcionalan je naboju unutar te kugle (i neovisan je o R)

↓

Gauss

→ $D = \varepsilon E = \dfrac{Q}{4\pi R^2}$

→ $\Phi = D\cdot S = \dfrac{Q}{\cancel{4\pi R^2}}\cdot \cancel{4R^2\pi} = \underline{\underline{Q}}$

JAKOST EL. POLJA:

$$\oint \vec{E}\, d\vec{s} = 0$$

Gaussov zakon

→ el. tok po zatvorenoj plohi ~ ukupnom nabolu koji smo obuhvatili tom plohom

$$\boxed{\oint_S \vec{E}\, d\vec{s} = \frac{Q_{unutra}}{\varepsilon_0}}$$

→ fizikalna posljedica toga što elektrostatska sila opada s kvadratom udaljenosti $F \sim \frac{1}{r^2}$

Gaussov teorem - teorem o divergenciji

$$\oint \vec{E}\, d\vec{s} = \oint \vec{\nabla}\vec{E}\, dV$$

- zatvorena ploha S obuhvaća neki volumen V

volumna gustoća: $dq = \rho \cdot [r']\, dV$

$q = \int \rho \, dV$

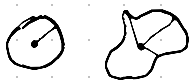
$\oint \vec{E}\, d\vec{s} = \frac{q}{\varepsilon} \Rightarrow \frac{1}{\varepsilon}\int \rho\, dV = \int \vec{\nabla}\vec{E}\, dV \Rightarrow$

PRVA MAXWELLOVA

$$\boxed{\frac{\rho}{\varepsilon_0} = \vec{\nabla}\vec{E}}$$

JEDNADŽBA!

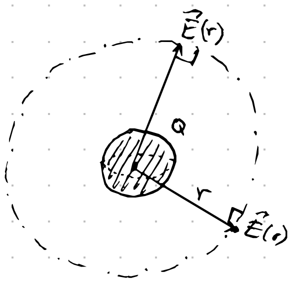
→ Gaussov zakon vrijedi bez obzira na oblik



El. polje nabijene čestice

polje točkastog naboja

$$\oint_S \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0} = \Phi_E$$



$dV = dx\, dy\, dz$

$ds = dx\, dy$

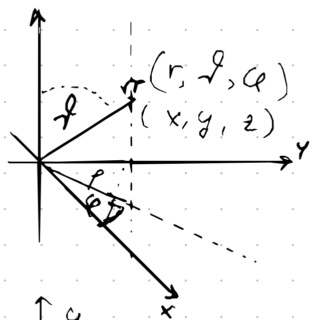
$V = \frac{4}{3} r^3 \pi$

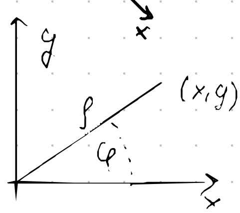
$ds = r^2 \sin\theta\, d\theta\, d\varphi$

$S = r^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi$

ploha koja obuhvaća naboj Q

SFERNI KOORD.:

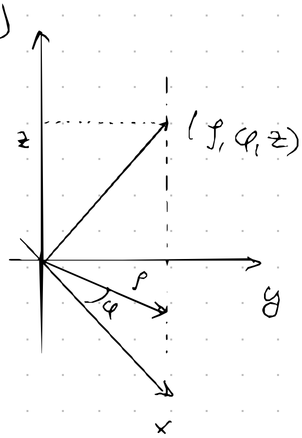




• E(r) je ⊥ na sferu zbog prostorne simetrije

- polje ima isključivo komponentu koja je okomita na q jer tangencijalna ne može biti izražena (fizički sustav ne sadrži niti jedno svojstvo pomoću kojeg bismo odredili smjer)

CILINDRIČNI:

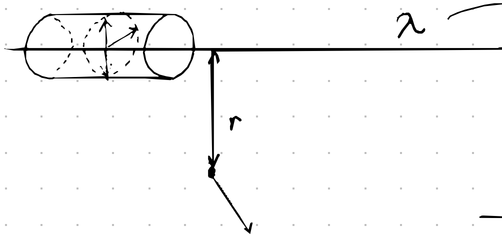


$$\oint_S \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0} = \Phi_E$$

$$E s = \frac{Q}{\varepsilon_0} \Rightarrow E r^2 \overbrace{\int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi}^{\text{ubacimo u kalkulator broj.}}$$

$$\boxed{\Phi_E = 4 r^2 E = \frac{Q}{\varepsilon_0}}$$

POLJE jednoliko nabijene žice



linearna gustoća $\lambda = \frac{Q}{l}$

$\hookrightarrow Q = \lambda l$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$ (dolje postupak)

- rješavamo pomoću cilindričnih ploha

$d\vec{S} = dx\,dy\,\hat{z}$

$dS = r^2 \sin\vartheta \, d\vartheta \, d\varphi$?

$\boxed{dS = r\,d\varphi\,dz}$

$dV = \underline{dx\,dy}\,dz$

$dV = r^2 dr \sin\vartheta \, d\varphi$

$dV = r\,dr\,d\varphi\,dz$

$\vec{E} \downarrow\downarrow d\vec{S}$

vekt. umnožak

kut je 0, cos = 1

$$\oint \vec{E}\,d\vec{S} = \int E\, \overbrace{dS \cos(\underline{E, dS})}^{1}$$

$$\oint \vec{E}\,d\vec{S} = \oint E\,dS = \oint \underbrace{E(r)\,r}_{\text{konst}}\,d\varphi\,dz$$

$$E(r)\,r \oint_0^{2\pi} d\varphi \int_{-z}^{z} = \underline{E(r)\,r\,2\pi\,2z}$$

Gaussov zakon: $\oint \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0}$

$E(r)\,r\,2\pi 2z = \frac{Q}{\varepsilon_0}$ $\qquad Q = \lambda l, \quad l = 2z \rightarrow Q = 2z \cdot \lambda$

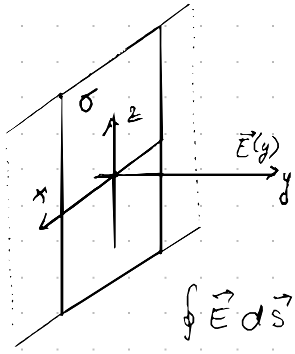
$E(r)\,r\,2\pi \cancel{2z} = \frac{\lambda \cancel{2z}}{\varepsilon_0}$

$E \qquad 2\pi r$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

*polje unutar i izvan nabijene kugle => AUDITORNE

POLJE jednako nabijene plohe



- polje se ne mijenja po x i z, samo po y jer se pomicanjem po y približavamo ili udaljavamo od plohe

$\sigma = \frac{Q}{S}$ - plošna gustoća naboja

↳ okomita je na ravninu

$S = \int_{-x}^{x} dx \int_{-z}^{z} dz$

$\oint \vec{E} d\vec{s}$ $\quad \sigma = \frac{dq}{dS}$ $\quad dS = dx \cdot dz$ (površina) $\quad S = 2x2z$

$$\oint E(y)\, dx dz \hat{y} = \oint E(y)\, dx dz = \oint 2E(y)\, dx dz = 2E(y) \int_{-x}^{x} dx \int_{-z}^{z} dz$$

$$\Rightarrow 2E(y) \cdot 2x \cdot 2z = \frac{Q}{\varepsilon_0} \rightarrow \sigma \cdot S = \frac{\sigma S^{\rightarrow 2x2z}}{\varepsilon_0}$$

otkud taj 2? zbog zatvorenog $\int$?

$$2E(y) = \frac{\sigma}{\varepsilon_0} \rightarrow \boxed{E_y = \frac{\sigma}{2\varepsilon_0}}$$

I. MAXWELLOVA JEDNADŽBA

▶ POLJE unutar i izvan jednoliko nabijene kugle

$\rho = \frac{dQ}{dV}$ → gustoća naboja

↳ $Q = \int \rho dV$

- svaka zatvorena ploha obuhvaća neki volumen

⇓

otud slijedi Gaussov TM (TM o divergenciji)

$$\Phi_E = \frac{Q}{\varepsilon_0} \qquad \oint_S \vec{E} d\vec{s} = \frac{Q}{\varepsilon_0} = \frac{\int \rho dV}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho dV$$

ta ploha obuhvaća neki volumen (zatvoreni $\int$)

Gaussov TM.

$$\rightarrow \oint_S F \cdot d\vec{s} = \int \vec{\nabla} \vec{F} dV$$

Ako imamo zatvorenu plohu i ako tražimo $\int$ možemo ga naći pomoću divergencije

$$\Rightarrow \oint \vec{E} d\vec{s} = \frac{q}{\varepsilon_0} \qquad \oint \vec{E} d\vec{s} = \int \vec{\nabla} \vec{E} dV$$

$$\Rightarrow \frac{1}{\varepsilon_0} \int \rho dV = \int \vec{\nabla} \vec{E} dV$$

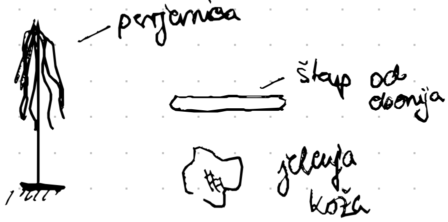
$$\rightsquigarrow \int \underbrace{\frac{\rho}{\varepsilon_0}} dV = \int \underbrace{\vec{\nabla} \vec{E}} dV \Rightarrow \boxed{\vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0}}$$

analogno će vrijediti za mag polje, ali o tome kasnije

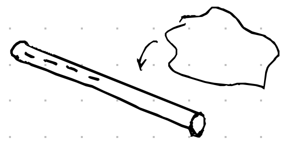
$\nabla \; (\vec{\nabla} \vec{B} = 0)$

Razni pokusi: Elektrostatika

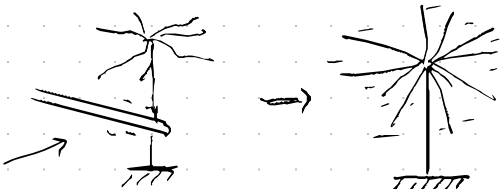
I.) nabijeni štap i perjanice



1.) elektriziramo štap trljanjem u krpu

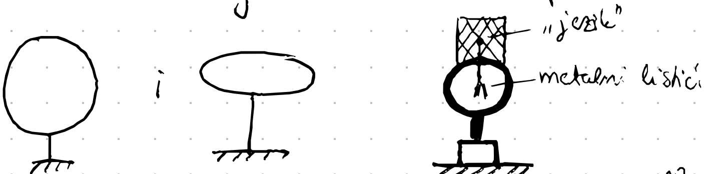


2.) štapom prođemo po metalnoj kvasi perjanice i prenesemo naboj

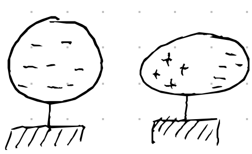


višak istog naboja se odbija pa se perjanica „napuhne"

II.) Imamo dva tijela: i elektroskop



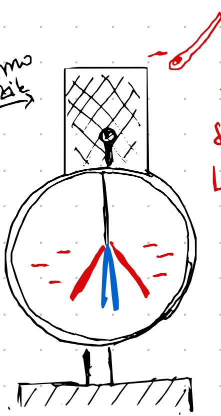
1) metalnu kuglu nabijemo npr. neg. nabojem i stavimo pored elipsoida:



→ na elipsoidu dolazi do separacije naboja

2) sa metalnom „žličicom" uzmemo malo naboja sa daljeg (−) dijela elipsoida

i stavimo na jezik

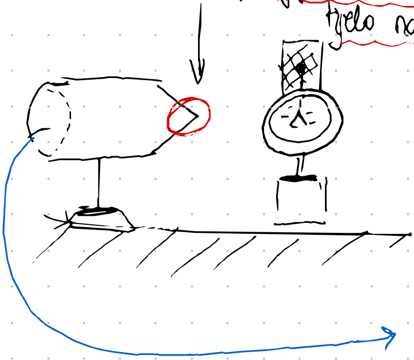


- listići su se odmakli
↳ dokaz da je tijelo ELEKTRIZIRANO

3) ako ponovimo to isto, ali ovaj put pokupimo (+) sa strane bliže kugli → SPOJIT ĆE SE (neutralizacija)

III.)

naboj je najgušći tamo gdje je tijelo najšpičastije



ali potencijal je jednak svugdje

u unutrašnjosti šupljeg tijela gustoća naboja je 0

1. MAXWELLOVA jednadžba

$$\vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} = \frac{d}{dx}\hat{x} + \frac{d}{dy}\hat{y} + \frac{d}{dz}\hat{z}$$

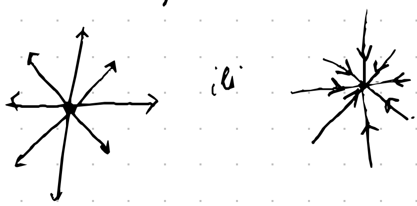
$\vec{\nabla} \cdot U$ - gradijent !

$\vec{\nabla} \cdot \vec{E}$ - divergencija

$\vec{\nabla} \times \vec{E}$ - rotacija

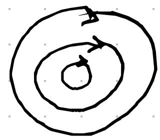
divergencija $\neq 0$

$(\vec{\nabla}\vec{E} \neq 0)$



divergencija $= 0$

$(\vec{\nabla}\vec{E} = 0)$



Neko Markovo računanje gradijenata i ostalih magija:

$$U(r) = \int \frac{\partial U}{\partial \vec{r}} \, d\vec{r}\,' \;\rightarrow$$ integracija gradijenta koju znamo otprije

↳ isto možemo zapisati: $\int U dr' = \int \frac{\partial U}{\partial \vec{r}} dr'$

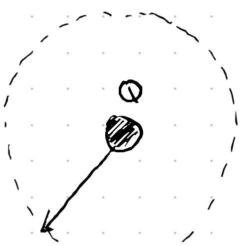
vekt

$$\int E \, dr' = \int (\vec{\nabla} \times \vec{E}) \, d^2 r$$

$$\rightarrow \int \vec{E} \, d^2 r = \int \underbrace{(\vec{\nabla} \cdot \vec{E})}_{\text{skal}} \, d^3 r$$

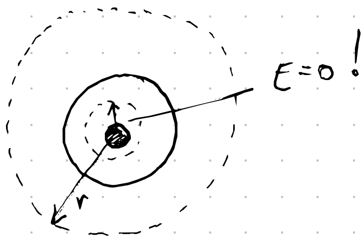
HUH?

Unutar kugle



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

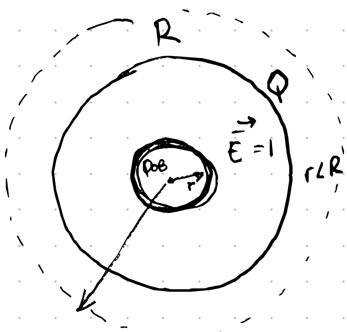
* sa ljuskom



$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

HOMOGENA KUGLA:

$$\rho = \frac{Q}{V} \rightarrow \rho = \frac{dQ}{dV} \rightarrow \rho(\cancel{r, \theta, \phi}) = \rho$$



$$\vec{E} = \frac{Q}{4\pi r^2 \varepsilon_0}\hat{r}$$

KONZERVATIVNOST COULOMBOVE SILE & EL. POTENCIJAL

Prva Maxwellova jednadžba: $\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$

Coulombova sila je konzervativna jer vrijedi: $\vec{\nabla}\times\vec{F} = 0$
└ rotacija je ∅

↳ to vrijedi samo u elektrostatici, ne i u magnetizmu *

kasnije smo nazvali potencijalnom energijom

⟹ slijedi: postoji skalarno polje φ za koje vrijedi $F = -\vec{\nabla}\varphi$

analogno vrijedi: $\vec{\nabla}\times\vec{E} = 0$ (el. polje je konzervativno)

⟹ -||- : postoji negativni gradijent $E = -\vec{\nabla}V$ — *potencijal (skal. polje $V(x,y,z)$)

$F = -\vec{\nabla}\varphi = -\frac{\partial\varphi}{\partial l}$ — po duljini vodiča

$E = -\vec{\nabla}V = -\frac{\partial V}{\partial l}$

$E = \frac{F}{q}$

⟹ $V = -\int\vec{E}\,d\vec{l}$

$\varphi = -\int\vec{F}\,d\vec{l}$ električni potencijal

⇝ $V = -\int\frac{\vec{F}}{q}\,d\vec{l} = \frac{1}{q}\varphi$ ⟹ $\varphi = qV \rightarrow \boxed{U = qV}$

- el. potencijal po jednakoj analogiji s potencijalnom energijom možemo prikazati kao:

$V = -\int\vec{E}\,d\vec{r}$ $d\vec{r} \rightarrow dr\cdot\hat{r}$

$* \; E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\cdot\hat{r}$ jed. vektor smjera

$V = -\int\frac{q}{4\pi\varepsilon_0 r^2}\,\cancel{\hat{r}}\cdot dr\,\cancel{\hat{r}}$

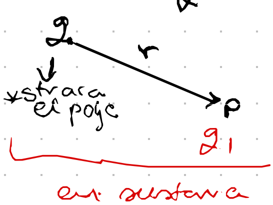
LINALG: $\hat{r}\cdot\hat{r} = 1$

$V = -\int\frac{q}{4\pi\varepsilon_0 r^2}\,dr \longrightarrow \boxed{V = \frac{q}{4\pi\varepsilon_0 r} + \cancel{C}^{\,0}}$

*isto kao potencijalna energija

- potencijal koji stvara neki q u udaljenosti r

↳ $U = q_1 V$



- dovodimo q_1 iz beskonačnosti u točku potencijala P

⟹ možemo izračunati energiju sustava

$U = q_1 V = \frac{q_1 q}{4\pi\varepsilon_0 r}$

- veza el. polja i el. potencijala: $\vec{E} = -\vec{\nabla}V$ (gradijent potencijala)

21.5.2024.

KONZERVATIVNOST COULOMBOVE SILE & ELEKTRIČNI POTENCIJAL

Prva Max. jed.: $\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$

$\vec{\nabla}\vec{B} = ?$

$\vec{\nabla}E = ?$

$\vec{\nabla}\times\vec{B} = ?$

- Kulonova sila je konzervativna
  - ↳ rotacija je 0 (u elektrostatici) *ne i u magnetizmu

$$\underbrace{\vec{\nabla}\times\vec{F} = 0}_{\text{sila je konz}} \qquad \underbrace{\vec{\nabla}\times\vec{E} = 0}_{\text{El polje je konz.}}$$

* sila je negativni gradijent potencijalne energije

$$\vec{E} = -\vec{\nabla}U$$

iz konzervativnosti polja slijedi da postoji negativni gradijent

$\vec{E} = -\vec{\nabla}V$ — potencijal

$$\longrightarrow \left.\begin{aligned} V &= -\int \vec{E}\,d\vec{l} \\ U &= -\int \vec{F}\,dl \end{aligned}\right\}$$

el. potencijal

$U = qV$

- el. potencijal po jednakoj analogiji s potencijalnom energijom možemo prikazati kao:

$V = -\int \vec{E}\,d\vec{r} \rightarrow dr\cdot\hat{r}$

$* \; E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\cdot\hat{r}$ jed. vektor smjera

$V = -\int \frac{q}{4\pi\varepsilon_0 r^2}\,\cancel{\hat{r}}\cdot dr\,\cancel{\hat{r}}$

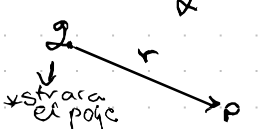
LINALG: $\hat{r}\cdot\hat{r} = 1$

$V = -\int \frac{q}{4\pi\varepsilon_0 r^2}\,dr \longrightarrow \boxed{V = \frac{q}{4\pi\varepsilon_0 r} + \cancelto{0}{C}}$

*isto kao potencijalna energija

- potencijal koji stvara neki q u udaljenosti r

↳ $U = q_1 V$



$\underbrace{\qquad q_1}_{\text{en. sustava}}$

- dovodimo q_1 iz beskonačnosti u točku potencijala P

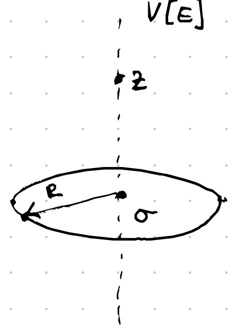
⟹ možemo izračunati energiju sustava

$$U = q_1 V = \frac{q_1 q}{4\pi\varepsilon_0 r}$$

- veza el. polja i el. potencijala: $\vec{E} = -\vec{\nabla}V$ (gradijent potencijala)

Zad. El. potencijal na bilo kojoj točki sred. osi homogeno nabijenog diska je dan izrazom: $V(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$.

Izvedite izraz za el. polje:



$$V(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

$$\sigma = \frac{Q}{R^2\pi} \quad \text{(plošna gustoća)}$$

$$\vec{E} = -\left(\cancel{\frac{\partial V}{\partial x}\hat{x}} + \cancel{\frac{\partial V}{\partial y}\hat{y}} + \frac{\partial V}{\partial z}\hat{z}\right)$$

$$\vec{E} = -\frac{\partial V}{\partial z}\hat{z}$$

$$E = \frac{-\sigma}{2\varepsilon_0}\left(\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{z^2+R^2}}\cdot\cancel{2}z - 1\right) = -\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2+R^2}} - 1\right)$$

Ako je $\Phi_E = \oint \vec{E}\, d\vec{S}$ (gaussov zakon)

$$\Phi_E = \oint \vec{E}\, d\vec{S} = \frac{Q_{OBUHVAĆENI}}{\varepsilon_0} \qquad \Phi \sim \int \rho\, dV$$

$\rightarrow$ $\oint \vec{E}\, d\vec{S} = \int \vec{\nabla}\vec{E}\, dV$

nešto ???