BINET-CAUCHYJEV TEOREM

$A, B \in M_n$, $\det(AB) = \det A \cdot \det B$

zad. 3.)

a) Nema za mat $A \in \mathbb{R}^{n \times m}$ vrijedi $AA^T = I$.
Koliko iznosi $\det(A)$.

$\det(AA^T) = \det(I) = 1$ $\Rightarrow \det(A) \cdot \det(A) = 1$

$\overset{\text{B.C.TM.}}{=}$ $\Rightarrow (\det(A))^2 = 1$

$\det(A) \cdot \det(A^T)$ $\Rightarrow \det A = \pm 1$

b) Ako je $\det(A) = 5$, Koliko je $\det(A^2)$?

$$\det(A^2) \overset{\text{B.C.TM.}}{=} (\det(A))^2 = 5^2 = 25$$

DZ - pitat Graciju

+ $\det(\lambda A) = \lambda^n \det(A)$

2.3. Računanje det. n-tog reda

T.B.

$$\begin{vmatrix} \alpha & \beta & 0 & \cdots & 0 & 0 \\ 0 & \alpha & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha & \beta \\ \beta & 0 & 0 & \cdots & 0 & \alpha \end{vmatrix} = \alpha \cdot (-1)^{1+1} \begin{vmatrix} \alpha & \beta & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & \alpha & \beta \\ 0 & 0 & \cdots & 0 & \alpha \end{vmatrix} + \beta(-1)^{n+1} \begin{vmatrix} \beta & 0 & \cdots & 0 & 0 \\ \alpha & \beta & & 0 & 0 \\ \vdots & \vdots & & & \\ 0 & 0 & & \alpha & \beta \end{vmatrix}$$

$$= \alpha \cdot \alpha^{n-1} + (-1)^{n+1} \cdot \beta \cdot \beta^{n-1} = \alpha^n + (-1)^{n+1} \cdot \beta^n$$

Primjer 8.)

c)

$$\begin{array}{r} \\ \\ \\ \\ n+1 \text{ red} \end{array} \begin{vmatrix} 1 & 3 & 5 & \cdots & 2n-1 \\ 1 & 2 & 5 & \cdots & 2n-1 \\ 1 & 3 & 4 & \cdots & 2n-1 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 3 & 5 & \cdots & 2n-2 \end{vmatrix} (-1) = \begin{vmatrix} 1 & 3 & 5 & \cdots & 2n+1 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{vmatrix} = (-1)^{n-1}$$

Zad. 6.)

A)

$$\begin{vmatrix} -1 & 2 & 2 & \cdots & 2 \\ 2 & -1 & 2 & \cdots & 2 \\ 2 & 2 & -1 & \cdots & 2 \\ \vdots & \vdots & \vdots & & \vdots \\ 2 & 2 & 2 & \cdots & -1 \end{vmatrix} +$$

Suma svakog stupca je $2(n-1)-1 = 2n-3$

$$= \begin{vmatrix} 2n-3 & 2n-3 & 2n-3 & \cdots & 2n-3 \\ 2 & -1 & 2 & \cdots & 2 \\ 2 & 2 & -1 & \cdots & 2 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 2 & 2 & \cdots & 1 \end{vmatrix} = (2n-3) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 2 & -1 & 2 & \cdots & 2 \\ 2 & 2 & -1 & \cdots & 2 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 2 & 2 & \cdots & 1 \end{vmatrix} (-2) = (2n-3) \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & -3 & 0 & \cdots & 0 \\ 0 & 0 & -3 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & -3 \end{vmatrix}$$

$$\boxed{= (2n-3)(-3)^{n-1}}$$

Zad. 7.)

A) $\begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ \boxed{-x \quad x} & & 0 & \cdots & 0 \\ 0 & \boxed{-x} & \boxed{x} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \boxed{-x \quad x} \end{vmatrix} = \begin{vmatrix} \sum_{i=1}^{n} a_i & a_2 & a_3 & \cdots & a_n \\ 0 & x & 0 & \cdots & 0 \\ 0 & -x & x & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & -x\,x \end{vmatrix}$

$= \left(\sum_{i=1}^{n} a_i\right) \cdot (-1)^{1+1} \begin{vmatrix} x & 0 & \cdots & 0 \\ -x & x & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & -x\,x \end{vmatrix} = x^{n-1} \left(\sum_{i=1}^{n} a_i \right)$