Fermi-Diracova raspodjela

$$W=\prod_i \frac{N_i!}{n_i!\,(N_i-n_i)!}\,, \quad \text{uz} \quad E=\sum n_i E_i \qquad N=\sum n_i$$

$$\delta f(n_i)=0 \qquad \text{Stirling} \quad N_i! \simeq \left(\frac{N_i}{e}\right)^{N_i}$$

$$f(n_i)=\ln \prod_i \frac{\left(\frac{N_i}{\cancel{e}}\right)^{N_i}}{\left(\frac{n_i}{\cancel{e}}\right)^{n_i}\left(\frac{N_i-n_i}{\cancel{e}}\right)^{(N_i-n_i)}}+a\left(N-\sum n_i\right)+b\left(E-\sum n_i E_i\right)$$

variranje, kao deriviranje

$$f(n_i)=\sum\Big[\underbrace{N_i \cdot \ln N_i}_{0 \text{ kada variramo } N_i}-n_i \ln n_i-(N_i-n_i)\ln(N_i-n_i)\Big]+a\left(N-\sum n_i\right)+b\left(E-\sum n_i E_i\right)\Big/\delta$$

$$\delta f(n_i)=\sum\Big[\varnothing-\delta n_i \ln n_i-\cancel{n_i}\cdot\frac{1}{\cancel{n_i}}\cdot\delta n_i+\delta n_i\cdot\ln(N_i-n_i)-\cancel{(N_i-n_i)}\frac{-\delta n_i}{\cancel{(N_i-n_i)}}\Big]-\left(a\sum\delta n_i\right)-\left(b\sum E_i\,\delta n_i\right)$$

$$\delta f(n_i)=\sum\Big[-\delta n_i\cdot\ln(n_i)-\cancel{\delta n_i}+\delta n_i\cdot\ln(N_i-n_i)+\cancel{\delta n_i}\Big]-a\sum\delta n_i-b\sum E_i\,\delta n_i=0$$

$$\delta f(n_i)\Rightarrow\sum\delta n_i\underbrace{\Big[-\ln n_i+\ln(N_i-n_i)-a-bE_i\Big]}_{=0}=0 \longrightarrow \text{tražimo ekstrem!}$$

$$\ln(N_i-n_i)-\ln n_i=a+bE_i$$

$$\ln\left(\frac{N_i-n_i}{n_i}\right)=a+bE_i\ \Big/\ e \longrightarrow \frac{N_i-n_i}{n_i}=e^{a+bE_i}$$

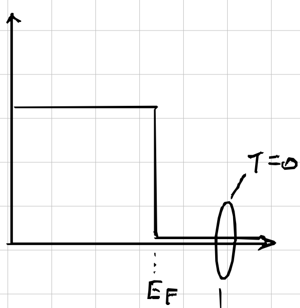
$$\Longrightarrow \frac{n_i}{N_i}=\frac{1}{e^{a+bE_i}+1}$$

→ za slučaj malih gustoća ($N_i \gg n_i$) nema izražaja Paulijevog principa
↳ raspodjela prelazi u Boltzmannovu

$$\Longrightarrow b=\frac{1}{kT}$$

$$e^{a+\frac{E_i}{kT}}$$

gledamo



↓ kakva je raspodjela na $T\to 0\,K$

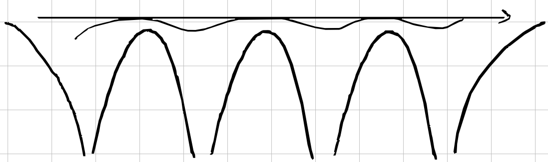
$$\frac{1}{e^{a+\frac{E_i}{kT}}+1}=\begin{cases}0\,, & a+\frac{E_i}{kT}>0\\ 1\,, & a+\frac{E_i}{kT}<0\end{cases} \longrightarrow E_i<-akT$$

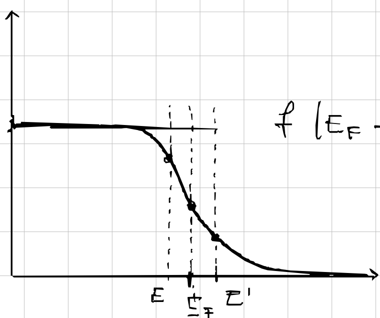
↳ $E_i < E_F$

$$E_F=-akT \qquad a=\frac{-E_F}{kT}$$

$$-\frac{E_F}{kT}+\frac{E_i}{kT}$$

$$\Longrightarrow \boxed{\frac{1}{e^{\frac{E_i-E_F}{kT}}+1}=\frac{n_i}{N_i}}$$





$$f(E_F - E') = 1 - f(E_F + E')$$

$$\frac{1}{e^{(E_F - E' - E_F)/kT} + 1} = \frac{1}{e^{-E/kT} + 1}$$

$$1 - f(E_F + E') = 1 - \frac{1}{e^{(E_F - E_F - E')/kT} + 1} = 1 - \frac{1}{e^{\frac{-E}{kT}} + 1}$$

Blochov teorem