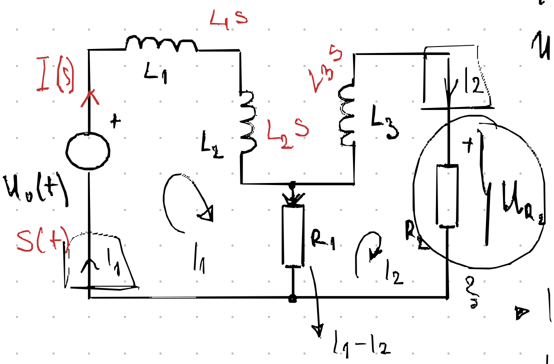
# 7.2. Primjeri za pripremu međuispita

## Zadatak 1.)




$R_1=1 \quad R_2=1 \quad L_1=1 \quad L_2=2 \quad L_3=4$

$u_0(t)=S(t)$

1.) $I_1(L_1 s + L_2 s + R_1) - I_2 \cdot R_1 = \frac{1}{s}$

2.) $-I_1 \cdot R_1 + I_2(L_3 s + R_1 + R_2) = 0$

$I_1(s+2s+1) - I_2 = \frac{1}{s}$

$I_1(1+3s) - I_2 = \frac{1}{s}$

$-I_1 + I_2(4s+2) = 0$

$I_1 = I_2(4s+2)$

$I_2(4s+2)(1+3s) - I_2 = \frac{1}{s}$

$I_2((4s+2)(1+3s)-1) = \frac{1}{s} \Rightarrow I_2(4s+12s^2+2+6s-1) = \frac{1}{s}$

$u_{R_2} = R_2 \cdot I_2$

$U_{R_2} = 1 \cdot I_2$

$$I_2 = \frac{1}{s(12s^2+10s+1)} = \frac{1}{12s(s+0.7173)(s+0.1162)}$$

$$12s(s+0.7173)(s+0,1162) = \frac{A}{12s} + \frac{B}{s+0,7173} + \frac{C}{s+0,1162}$$

$A(s+0.7173)(s+0.1162) + 12s(s+0.1162)B + 12s(s+0.7173)C = 1$

$As^2 + 0,1162As + 0.7173As + 0,0832A + 12s^2B + s \cdot B \cdot 1,3944$

$12s^2 \cdot C + 8.6076s \cdot C = 1$

$+12s^2 \cdot C$

$As^2 + 12s^2 \cdot B + 0.8335As + 1,3944Bs + 8.6076 \cdot Cs + 0.0832A = 1$

$A = 12$

$A + 12B + 12C = 0$

$B + C = -1$

$B = -1 - C$

$B = -1.193$

$1,3944B + 8.6076 \cdot C = 0$

$-1.3944(1+C) + 8.6076C = 0$

$-1.3944 - 1.3944C + 8.6076 = 0$

$7.2132C = 1.3944$

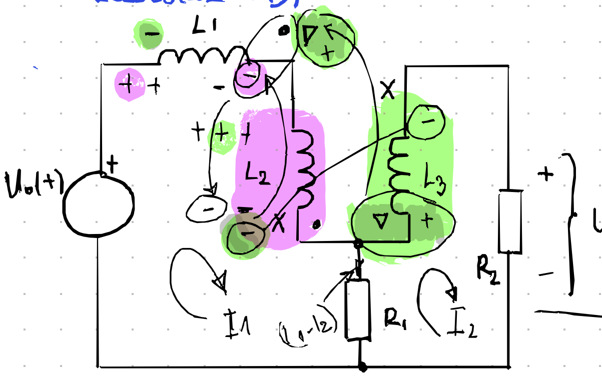
$C = 0.193$

$$U_R(s) = \frac{1}{s} - \frac{1.193}{s+0,7173} + \frac{0.193}{s+0.1162}$$

$e^{-at} \circ\!\!-\!\!\bullet \frac{1}{s+a}$

$$U_R(t) = \left[1 - 1.193 \cdot e^{-0,7173} + 0.193e^{-0.1162}\right] S(t)$$

Zadatak 1B)



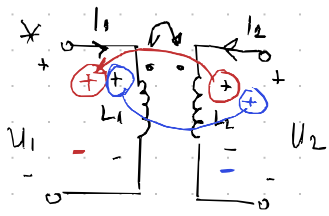

$R_1 = 1$ $\quad L_1 = 1$ $\quad M_{12} = 1/2$ •

$R_2 = 1$ $\quad L_2 = 2$ $\quad M_{13} = 2$ ▽

$L_3 = 4$ $\quad M_{23} = 3$ ×

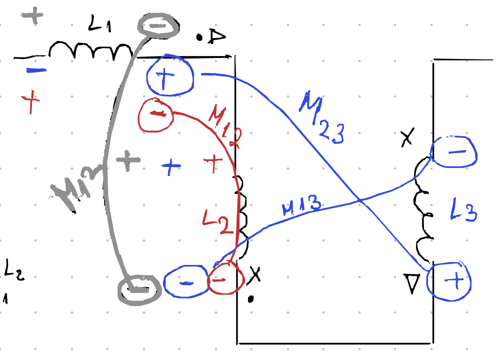
$u_0(t) = S(t)$

ovako se prenose predznaci




$U_1 = I_1 \cdot L_1 s + I_2 M_{12} s$

$U_2 = I_1 \cdot M_{12} s + I_2 \cdot L_2 s$



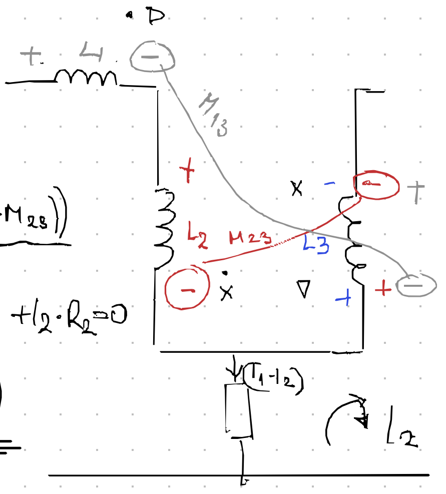

jednom $L_1$ utječe na $L_2$, drugi put $L_2$ na $L_1$

Petlje

1.) $I_1 L_1 s + I_1 \cdot L_2 s + I_1 \cdot M_{12} s + I_1 \cdot M_{12} \cdot s - I_2 \cdot M_{13} \cdot s + I_2 \cdot M_{23} \cdot s + (I_1 - I_2) R_1 = u_0$

to je ono što se inducira

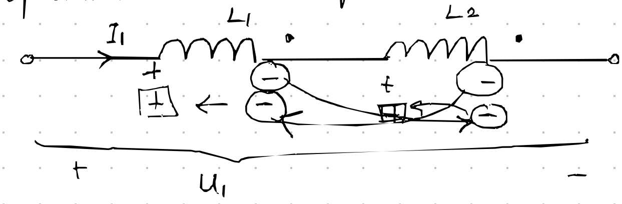
$\rightarrow U_0 = I_1(s(L_1 + L_2 + 2M_{12}) + R_1) - I_2(R_1 + s(M_{13} - M_{23}))$




2) $-(I_1 - I_2) \cdot R_1 + I_2 L_3 \cdot s + I_1 \cdot M_{23} s - I_1 M_{13} s + I_2 \cdot R_2 = 0$

$\rightarrow 0 = -I_1(R_1 + s(M_{13} - M_{23})) + I_2(sL_3 + R_1 + R_2)$

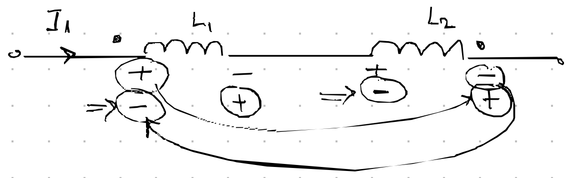
općenito za vezane zavojnice:




$\rightarrow U_1 = I_1 \cdot L_1 \cdot s + I_1 \cdot L_2 \cdot s + L_1 \cdot M_{12} s + L_1 \cdot M_{12} s$

$U_1 \rightarrow I_1 L_1 s + I_1 L_2 s + 2 I_1 \cdot M_{12} \cdot s$

$\rightarrow L_{ekvival} = L_1 + L_2 + 2M_{12}$




$\Rightarrow U_1 = I_1 \cdot L_1 \cdot s + I_1 L_2 s - I_1 \cdot M_{12} s - I_1 M_{12} s$

$\Rightarrow U_1 = I_1 L_1 s + L_1 L_2 s - 2M_{12} s$

$\Rightarrow J(\text{kontura}) \Rightarrow \frac{1}{s} = I_1(4s+1) - I_2(1-s)$

$0 = -I_1(1-s) + I_2(4s+2) \rightarrow I_1 = I_2 \frac{4s+2}{1-s}$

$\frac{1}{s} = I_2 \frac{(4s+1)(4s+2)}{1-s} - I_2(1-s) = \frac{I_2}{1-s}(16s^2 + 8s + 4s + 2 - 1 + 2s - s^2)$

$\frac{1}{s} = \frac{I_2}{1-s}(15s^2 + 14s + 1) \rightarrow I_2 = \frac{1-s}{s(15s^2 + 14s + 1)}$ $\quad s_1 = -0{,}07794$, $s_2 = -0{,}8554$

$I_2 = \frac{1-s}{s(s+0{,}07794)(s+0{,}8554)} \cdot \frac{1}{15}$

$1 - s = A(s+0{,}07794)(s+0{,}8554) + Bs(s+0{,}8554) + Cs(s+0{,}07794)$

$1 - s = As^2 + As \cdot 0{,}8554 + As \cdot 0{,}07794 + A \cdot 0{,}0667$
$+ Bs^2 + Bs \cdot 0{,}8554$
$+ Cs^2 + Cs \cdot 0{,}07794$

$A + B + C = 0$

$0.933A + 0{,}855B + 0{,}078C = -1$

$A \cdot 0{,}0667 = 1 \rightarrow \boxed{A = 14.9925}$

$B = -C - 14.993$

$B \cdot 0{,}855 + 0{,}078C = -1 - 14.888$

$(-C - 14.993) \cdot 0{,}855 + 0{,}078 \cdot C = -15.888$

$-C \cdot 0{,}77 = -3.069$

$\boxed{C = 3{,}95} \rightarrow \boxed{B = -18.943}$

Video: 1.186

0.186 ??

$\Rightarrow I_2 = \frac{1}{s} - \frac{1.263}{s+0.078} + \frac{0.263}{s+0.855}$

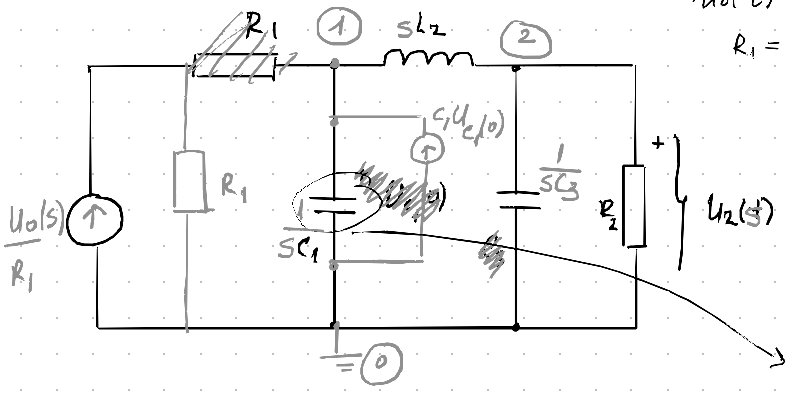
$U_{R_2}(s) = R_2 \cdot I_2 \Rightarrow I_2(s)$ $\quad$ ($R_2$ "$=1$")

$\boxed{U_R(t) = [1 - 1.263 \cdot e^{-0{,}078t} + 0{,}263 \cdot e^{-0{,}855t}] S(t)}$

## Zadatak 2.) Laplace

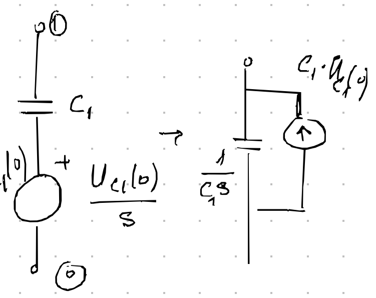
$u_0(t)=\delta(t)$ $\quad u_{c_1}(0)=1$

$R_1=R_2=1$ $\quad C_1=C_3=1$ $\quad L_2=2$




! MČ

$\varphi$ · sve admitancije




1) $\varphi_1\left(\frac{1}{R_1}+sC_1+\frac{1}{sL_2}\right)-\varphi_2\left(\frac{1}{sL_2}\right)=\frac{U_0(s)}{R_1}+C_1 U_{c_1}(0)$

2) $-\varphi_1\left(\frac{1}{sL_2}\right)+\varphi_2\left(\frac{1}{sL_2}+sC_3+\frac{1}{R_2}\right)=0$

$\hookrightarrow \varphi_1=\varphi_2\left(\frac{1}{sL_2}+sC_3+\frac{1}{R_2}\right)\cdot sL_2=\varphi_2\left(1+L_2C_3s^2+s\frac{L_2}{R_2}\right)$

$\left(\left(1+L_2C_3s^2+s\frac{L_2}{R_2}\right)\left(\frac{1}{R_1}+sC_1+\frac{1}{sL_2}\right)-\frac{1}{sL_2}\right)\varphi_2=\frac{U_0(s)}{R_1}+C_1U_{c_1}(0)$

$\left((1+2s^2+2s)\left(1+s+\frac{1}{2s}\right)-\frac{1}{2s}\right)\cdot\varphi_2=U_0(0)+U_{c_1}(0)$

$\left((1+2s^2+2s)\left(\frac{2s+2s^2+1}{2s}\right)-\frac{1}{2s}\right)\cdot\varphi_2=1+1 \quad /\cdot 2s$

$\varphi_1=U_1$

$\varphi_2=U_2$

$\left((1+2s^2+2s)^2-1\right)\varphi_2=4s$

$(1+2s^2+2s)(1+2s^2+2s)=1+2s^2+2s+2s^2+4s^4+4s^3+2s+4s^3+4s^2=$

$=4s^4+8s^3+8s^2+4s+1$

$(4s^4+8s^3+8s^2+4s)\varphi_2=4s \quad /:4s$

$(s^3+2s^2+2s+1)=\frac{1}{\varphi_2} \rightarrow U_2=\frac{1}{(s+1)(s^2+s+1)}=$ parcijalni

$(s^3+1)+2s(s+1) \rightarrow (s+1)(s^2-s+1)+2s(s+1)$

$\Rightarrow 1=A(s^2+s+1)+(Bs+C)(s+1)=As^2+As+A$
$+Bs^2+Bs+Cs+C$

$\rightarrow$ $A+B=0$ $\quad B=-1$

$A+B+C=0 \rightarrow \boxed{C=0}$

$A+C=1 \rightarrow A=1$

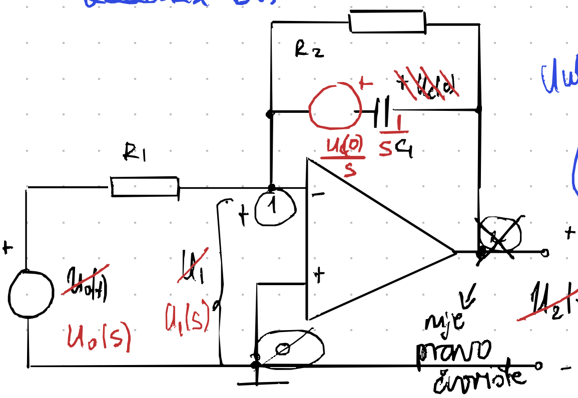
$u_2=\frac{1}{s+1}-\frac{s}{s^2+s+1}=\frac{1}{s+1}-\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}+\frac{\frac{1}{2}}{(s+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)$

$U_2=\frac{1}{s+1}-\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{1}{\sqrt{3}}\cdot\frac{\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

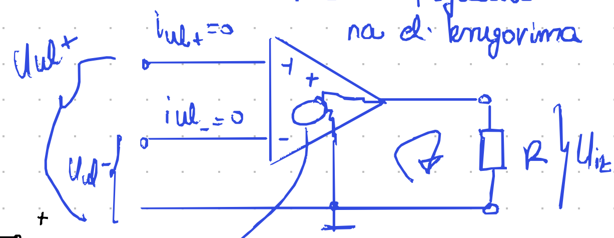
$U_2(t)=\left[e^{-t}-e^{-\frac{1}{2}t}\cdot\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{1}{\sqrt{3}}\cdot e^{-\frac{1}{2}t}\cdot\sin\left(\frac{\sqrt{3}}{2}t\right)\right]\cdot S(t)$

$\rightarrow \boxed{U_2(t)=\left[e^{-t}-e^{-\frac{1}{2}t}\left(\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{1}{\sqrt{3}}\cdot\sin\left(\frac{\sqrt{3}}{2}t\right)\right)\right]S(t)}$

# Zadatak 3.)




idealno pojačalo na el. krugovima




$$U_{iz} = A\cdot(U_{ul+} - U_{ul-})$$

$$i_{ul+} = i_{ul-} = 0$$

$$A \to \infty$$

naponsko pojačalo ima naponski ovisni naponski izvor $\to U_{iz}$

$\to$ $Z_{ul}$ ili $Z_{iz} = 0$ (pravi naponski izvor)

∀ ne moramo u strujne izvore

$U_2 \to$ naponski izvor !

$$\varphi_1\left(\cancel{\frac{1}{R_1} + \frac{1}{R_2} + sC_1}\right) - \varphi_2\left(\frac{1}{R_2} + sC_1\right) = \frac{U_0(s)}{R_1} - C_1 \cdot U_c(0)$$

$\hookrightarrow$ $U_1 = \varphi_1$ ne teče struja, $U_1 = \varphi_1 = 0$ zbog prividnog kratkog spoja

$\to$ u to pojačalo ne teče unutar nikakva struja $\to$ posljedica beskonačnog pojačanja $A \to \infty$

$$U_0(t) = S(t) \circ\!\!-\!\!\bullet \frac{1}{s}$$

$$U_c(0) = 1$$

$$R_1 = 1 \quad R_2 = 1 \quad C_1 = 1$$

$$\to \quad -\varphi_2\left(\frac{1}{R_2 \cdot C_1} + s\right) = \frac{U_0(s)}{R_1 \cdot C_1} - U_c(0)$$

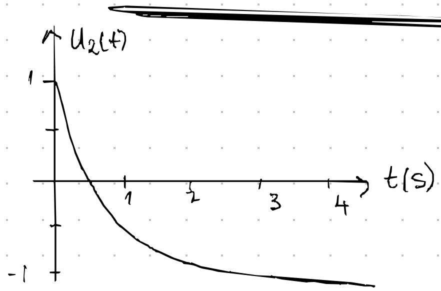
$$\varphi_2 = \frac{U_c(0)}{\frac{1}{R_2C_1} + s} - \frac{U_0(s)}{R_1C_1} \cdot \frac{1}{\frac{1}{R_2C_1} + s} \overset{\text{uvrsti vr.}}{=} \frac{1}{s+1} - \frac{1}{s} \cdot \frac{1}{s+1}$$

$$\frac{1}{s} \cdot \frac{1}{s+1} = \frac{A}{s} + \frac{B}{s+1}$$

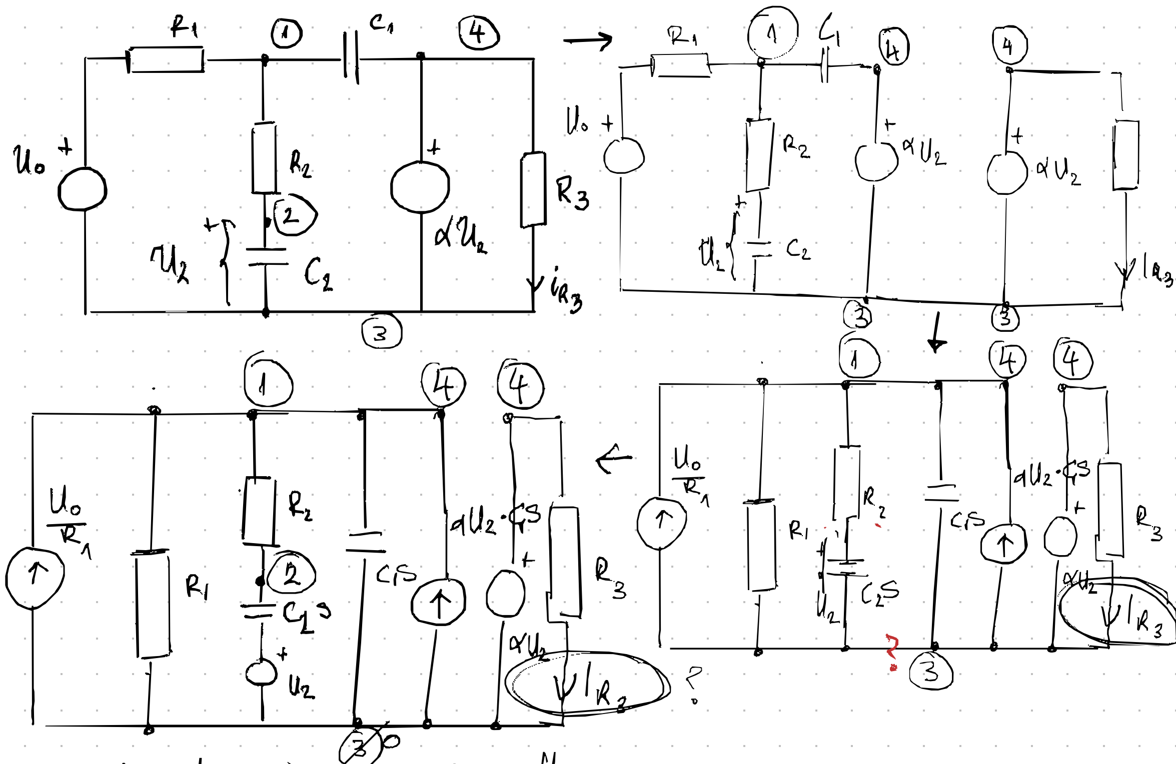
$$1 = As + A + Bs \to A = 1 \quad B = -1$$

$$\to U_2 = \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s+1} = \frac{2}{s+1} - \frac{1}{s}$$

$$\boxed{U_2(t) = \left[2e^{-t} - 1\right] S(t)}$$



Zadatak 4.) $u_0(t)=\delta(t)$ $R_1=R_2=1$, $C_1=C_2=2$ $\alpha=2$, $R_3=2$




1) $U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+C_1 s\right) - U_2\left(\frac{1}{R_2}\right) = \frac{U_0}{R_1} + \alpha C_1 s\cdot U_2$

2) $-U_1\left(\frac{1}{R_2}\right) + U_2\left(\frac{1}{R_2}+C_2 s\right) = 0$

$\hookrightarrow U_2(1+R_2C_2 s) = U_1 \rightarrow U_2\left((1+R_2C_2s)\left(\frac{1}{R_1}+\frac{1}{R_2}+C_1 s\right)-\left(\frac{1}{R_2}\right)\right)=\frac{U_0}{R_1}+\alpha C_1 s U_2$

$U_2\left[\frac{1}{R_1}+\cancel{\frac{1}{R_2}}+C_1S+\frac{R_2C_2}{R_1}s+C_2s+R_2C_1C_2s^2-\cancel{\frac{1}{R_2}}\right]=\frac{U_0}{R_1}+\alpha C_1 s\cdot U_2 \,/\cdot$

$U_2\left[1+R_1C_1S+R_2C_2s+R_1C_2S+R_1R_2C_1C_2s^2-\alpha R_1C_1S\right]=U_0$

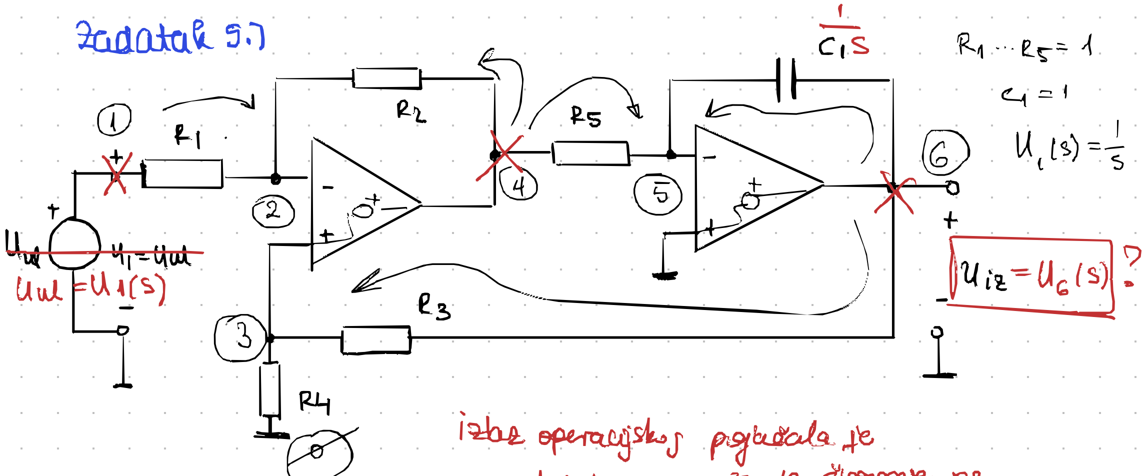
$U_2=\frac{U_0}{1+R_1C_1S(1-\alpha)+R_1C_2S+R_2C_2S+R_1R_2C_1C_2S^2}=\frac{U_0}{1\cancel{-2s}\cancel{+2s}+2s+4s^2}$

$U_2=\frac{U_0}{4s^2+2s+1}=\frac{1}{4}\cdot\frac{U_0}{s^2+\frac{1}{2}s+\frac{1}{4}}=\frac{1}{4}\cdot\frac{U_0}{(s+\frac{1}{4})^2+\frac{3}{16}}$ $U_0=1$

$I_{R_3}=\frac{\alpha U_2}{R_3}=\frac{2}{2}U_2=\frac{1}{\cancel{4}}\cdot\frac{\frac{\sqrt{3}}{4}}{(s+\frac{1}{4})^2+\left(\frac{\sqrt{3}}{4}\right)^2}\cdot\frac{\cancel{4}}{\sqrt{3}}$

$$\boxed{\rightarrow I_{R_3}=e^{-\frac{1}{4}t}\cdot\cos\left(\frac{\sqrt{3}}{4}\right)\cdot\frac{1}{\sqrt{3}}\cdot S(t)}$$

## Zadatak 5.7




izbor operacijskog pojačala je naponski izvor → za te čvorove ne pišemo jednadžbe (1, 4, 6)

MČ je najbolja metoda ~~za~~ op. pojačala

$$U_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{U_1}{R_1}+\frac{U_4}{R_2}$$

$$U_3\left(\frac{1}{R_4}+\frac{1}{R_3}\right)=\frac{U_6(s)}{R_3}$$

$$U_5\left(\frac{1}{R_5}+C_1 s\right)=\frac{U_4(s)}{R_5}+U_6 C_1 s$$

jednadžbe koje opisuju operacijska pojačala

$$U_4(s) = A(U_3-U_2) \rightarrow \frac{U_4}{A}=U_3-U_2$$

$$U_6(s) = A(0-U_5) \rightarrow -\frac{U_6}{A}=U_5$$

kada je $A \to \infty$

$$\rightarrow U_3 = U_2 \qquad U_5 = 0$$

$$U_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{U_1}{R_1}$$

$$U_2\left(\frac{1}{R_4}+\frac{1}{R_3}\right)=\frac{U_6}{R_3}$$

$$0=\frac{U_4(s)}{R_5}+U_6\cdot C_1 s$$

$$U_4(s) = -U_6 R_5 \cdot C_1 s$$

$$U_2 = U_6\cdot\frac{1}{R_3}\cdot\left(\frac{1}{R_4}+\frac{1}{R_3}\right)^{-1} = U_6\cdot\frac{1}{\cancel{R_3}}\cdot\frac{R_4\cancel{R_3}}{R_4+R_3} \rightarrow U_2 = U_3 = U_6\cdot\frac{R_4}{R_4+R_3}$$

$$U_2\left(\frac{R_1+R_2}{R_1R_2}\right)=\frac{U_1}{R_1}-U_6\frac{R_5C_1}{R_2}s$$

$$U_6\frac{R_4(R_1+R_2)}{R_1R_2(R_4+R_3)}+U_6\frac{R_5C_1}{R_2}s=\frac{U_1}{R_1} \quad \Big/\cdot R_1$$

$$U_6\left(\frac{R_4(R_1+R_2)}{R_2(R_4+R_3)}+\frac{R_1R_5C_1(R_4+R_3)s}{R_2(R_4+R_3)}\right)=U_1$$

$$A+AS+BS=1$$

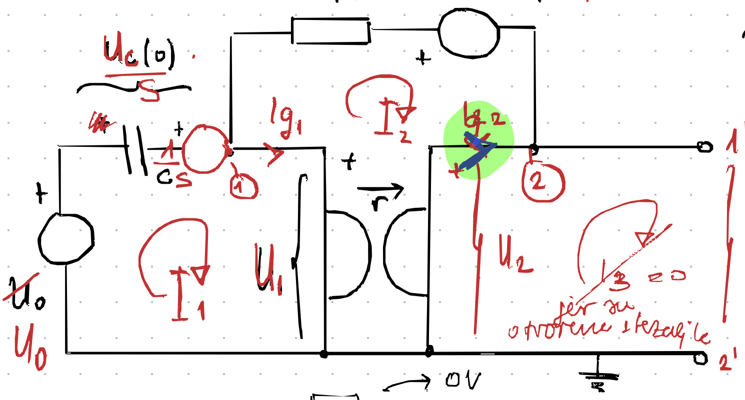
$$A=1$$

$$B=-1$$

$$U_6 = U_1\frac{R_2(R_4+R_3)}{R_4(R_1+R_2)+R_1R_5C_1(R_4+R_3)s}=\frac{1}{S}\cdot\frac{2}{2+2S}=\frac{1}{S}\cdot\frac{1}{1+S}$$

$$\boxed{U_6=\frac{1}{S}-\frac{1}{1+S}=U_{iz}(s)}$$

Zadatak 6.)




$R=0{,}5 \quad r=0{,}5 \quad \mu=0{,}5 \quad c=1$

$u_c(0)=2 \quad U_0(t)=S(t)$

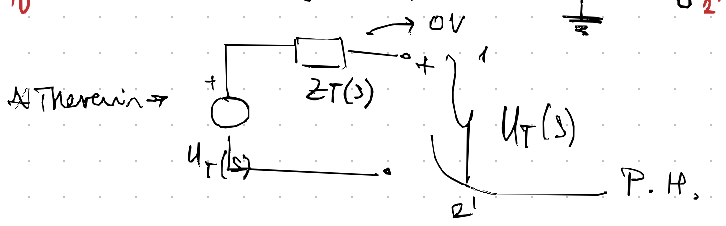
⟶ primjenom jed. petlji

Thevenin prazni hod

Girator:

$U_1(s)=r\cdot I_{g_2}(s)$

$U_2(s)=-rI_{g_2}(s)$




1.) $I_1\left(\frac{1}{cs}\right)=U_0-\frac{U_c(0)}{s}-U_1$

2) $I_2\cdot R=-\mu\cdot U_1-U_2+U_1$ ? ⟶ $I_{g_1}=I_1-I_2$ ↳ dodirna točka

$I_{g_1}=I_1-I_2 \qquad U_1=-r\cdot I_{g_2}=I_2\cdot r$

$I_{g_2}=-I_2 \qquad \boxed{U_2=-r\cdot I_{g_1}=I_2\cdot r-I_1\cdot r}$

$U_T=U_2-\cancel{U_1}^{\,0} \rightarrow U_T=U_2$

$U_0-\frac{U_c(0)}{s}=U_1+I_1\cdot\frac{1}{cs} \Rightarrow I_2\cdot r+I_1\cdot\frac{1}{cs}=U_0-\frac{U_c(0)}{s}$

$I_2\cdot R=-\mu\cdot I_2\cdot r-\cancel{I_2\cdot r}+I_1\cdot r+\cancel{I_2 r} \Rightarrow I_2(R+\mu r)-I_1\cdot r=0$

$\Rightarrow I_2\cdot r+I_2\cdot\frac{R+\mu r}{r}\cdot\frac{1}{cs}=U_0-\frac{U_c(0)}{s} \qquad I_1=I_2\frac{R+\mu r}{r}$

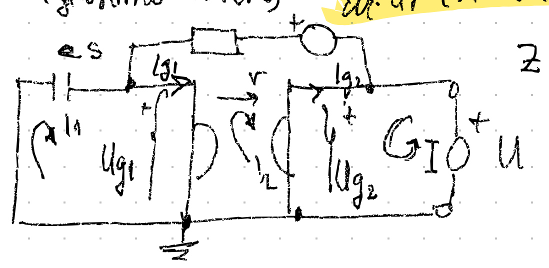
$I_2\left(r+\frac{R+\mu r}{r}\cdot\frac{1}{cs}\right)=U_0-\frac{U_c(0)}{s} \rightarrow I_2=\frac{\frac{sU_0-U_c(0)}{s}}{\frac{r^2cs+R+\mu r}{rcs}}$

$I_2=r\frac{csU_0-cU_c(0)}{r^2cs+R+\mu r}=\frac{1-2}{\frac{1}{4}s+\frac{3}{4}}\cdot 0{,}5=\frac{-2}{s+3}=I_2$

$I_1=I_2\cdot\frac{R+\mu r}{r}=\frac{-2}{s+3}\cdot 2\cdot\left(\frac{1}{2}+\frac{1}{4}\right)=\frac{-4}{s+3}\cdot\frac{3}{4}\Rightarrow I_1=\frac{-3}{s+3}$

$U_T=U_2=\frac{1}{2}\cdot I_2-\frac{1}{2}\cdot I_1=\frac{1}{2}\left(\frac{-2+3}{s+3}\right)=\boxed{\frac{1}{2}\cdot\frac{1}{s+3}=U_T}$

b) Theveninova impedancija $Z_T(s)$
(gasimo izvore) $\mu\cdot U_1$ ovisni izvori ostaju




$Z_T=\frac{U}{I}$

$I_1\cdot\frac{1}{cs}+U_1=0 \qquad U_1(1-\mu)$

$I_2\cdot R=+U_1-\mu U_1-U_2$

$U_2=U$

$U_1=-r\cdot I_{g_2}=r(I_2+I)$

$U_2=-r\cdot I_{g_1}=r(I_2-I_1)$

$\frac{1}{sc}I_1+rI_2+rI=0$

$R\cdot I_2-r(I_2+I)(1-\mu)+r(I_2-I_1)=0$

$r(I_2-I_1)=U$

$\frac{1}{sc}I_1+rI_2+rI_1=0$

~~$R\cdot I_2-r_2I_2+r\mu I_2-rI+r\mu I+rI_2-rI_1=0$~~

$-rI_1+I_2(R+r\mu)+rI(\mu-1)=0 \qquad -rI_1+rI_2=U$

$$\begin{bmatrix}\frac{1}{sc} & r & r\\ -r & R+r\mu & r(\mu-1)\\ -r & r & 0\end{bmatrix}\begin{bmatrix}I_1\\ I_2\\ I\end{bmatrix}=\begin{bmatrix}0\\0\\U\end{bmatrix}$$

→ nije simetrična
↳ ova mreža nije recipročna

matrica impedancija petlji

zanima nas

$\Delta=\begin{vmatrix}\frac{1}{sc} & r & r\\ -r & R+r\mu & r(\mu-1)\\ -r & r & 0\end{vmatrix}=r^2\left(R+\frac{1-\mu}{sc}\right)$

$\Delta_3\rightarrow(I)$

$\Rightarrow \Delta_3=\begin{vmatrix}\frac{1}{sc} & r & 0\\ -r & R+r\mu & 0\\ r & r & U\end{vmatrix}=U\left(\frac{\mu r+R}{sc}+r^2\right)$

$\left(\frac{U}{I}=Z_T\right)$

$\rightarrow I=\frac{\Delta_3}{\Delta}=U\cdot\frac{\mu r+R+r^2sc}{r^2(Rsc+1-\mu)}=U\cdot 2\frac{s+3}{s+1}\ /\cdot^{-1}$

$\boxed{\frac{1}{2}\cdot\frac{s+1}{s+3}=Z_T(s)}$

zaključak, ne isplati se preko petlji ovake

And. !