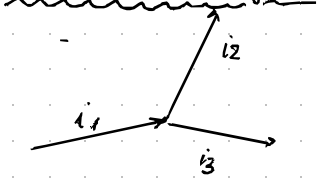
3. Kirchoffovi zakoni

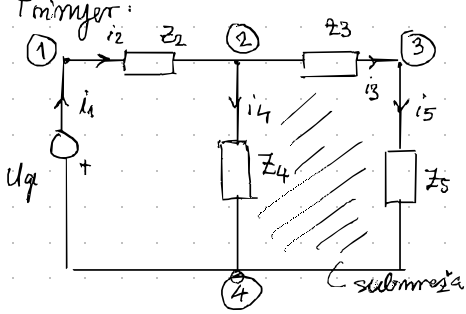
KZS (za struje)



- od čvorišta = +
- prema čvorištu = −

$i_1 = i_2 + i_3$

Primjer:



1) $i_1 = i_2$
2) $i_2 = i_3 + i_4$
3) $i_3 = i_5$
4) $i_1 = i_4 + i_5$

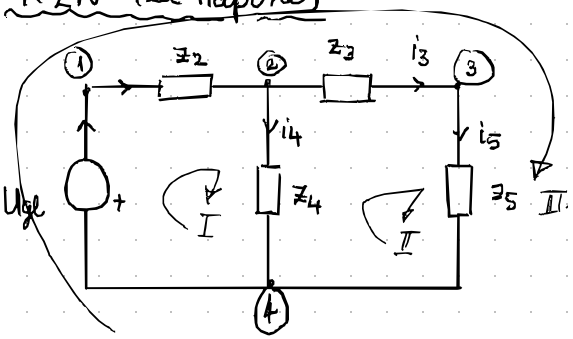
*ali to već znamo
→ najmanje jednu jed. moguće je izraziti kao linearnu kombinaciju ostalih
↳ ako nije moguće
↳ linearno neovisan je

$i_1 = i_3 + i_4$

→ za submrežu neke mreže → alg. suma svih struja koje ulaze u submrežu

▸ broj lin. nez. jed. KZS: $N_v - 1$ — v - vortex (čvorište)

KZN (za napone)



I) $-U_1 + U_2 + U_4 = 0$

II) $-U_4 + U_3 + U_5 = 0$

III) $-U_1 + U_2 + U_3 + U_5 = 0$

- ako su poznate dvije jednadžbe, poznata je također i treća → jedna je suvišna

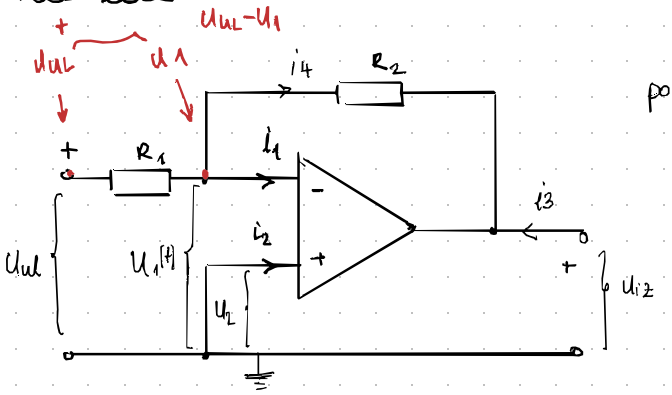
Općenito broj linearno neovisnih jednadžbi KZN je: $N_b - (N_v - 1) = N_b - N_v + 1$

b - branch (grana)

N_b (KZN + KZS), N_b - zakoni elemenata

PRIMJENA KIRCHOFFOVH ZAKONA

Primjer 1:



po def: $u_{iz} = A(u_2 - u_1)$ $(A \to \infty)$

$i_1 = i_2 = 0$

$u_2 = 0$ (uzemljeno)

KZS: $-i_3 + i_1 + i_4 = 0 \longrightarrow i_3 = i_4 = i$ $\qquad u_{iz} = -u_1 \cdot A \longrightarrow u_1 = \frac{-u_{iz}}{A}$

$$i = \frac{u_{ul} - u_1}{R_1} = \frac{u_1 - u_{iz}}{R_2} \qquad \frac{u_{ul} + \frac{u_{iz}}{A}}{R_1} = \frac{-\frac{u_{iz}}{A} - u_{iz}}{R_2}$$

$$\frac{u_{ul}}{R_1} + \frac{u_{iz}}{AR_1} = \frac{-u_{iz}}{AR_2} - \frac{u_{iz}}{R_2}$$

$$u_{iz}\left(\frac{1}{AR_1} + \frac{1}{AR_2} + \frac{1}{R_2}\right) = -\frac{u_{ul}}{R_1} \longrightarrow u_{iz} = \frac{-\frac{1}{R_1}u_{ul}}{\frac{1}{AR_1} + \frac{1}{AR_2} + \frac{1}{R_2}} \qquad A \to \infty$$

$\frac{1}{AR_1} \to 0$ $\qquad \frac{1}{AR_2} \to 0$

$$u_{iz} = \frac{-\frac{1}{R_1}u_{ul}}{\frac{1}{R_2}} \longrightarrow \boxed{u_{iz} = -\frac{R_2}{R_1}u_{ul}}$$

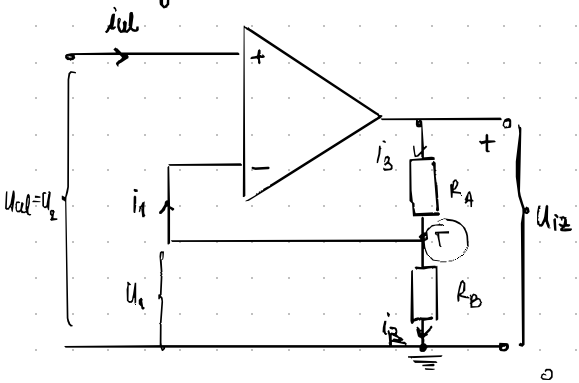
II. način

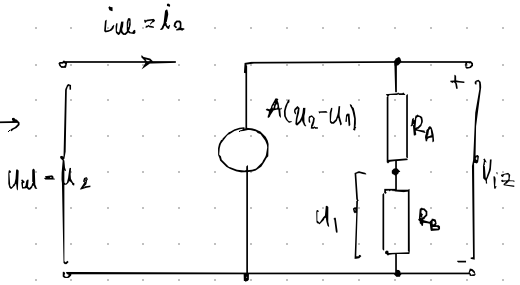
Def: $i_1 = i_2 = 0$ $\qquad u_2$ je spojen na $0 \to u_1 = u_2 = 0$

PVKS: $u_1 = u_2$

$$i = \frac{u_{ul} - 0}{R_1} = \frac{0 - u_{iz}}{R_2} \longrightarrow \frac{u_{ul}}{R_1} = \frac{-u_{iz}}{R_2} \Rightarrow \boxed{u_{iz} = \frac{-R_2}{R_1}u_{ul}}$$

Primjer 2)





Uizlazni = ?

$i_1 = i_2 = 0$

$U_+ = U_- \longrightarrow U_{ul} = U_1$

(T) $-i_1 + i_3 - i_R = 0 \rightarrow i_3 = i_R$

$$i = \frac{U_2 - U_1}{R_A} = \frac{U_{ul} - 0}{R_B} \Rightarrow \frac{U_2 - U_{ul}}{R_A} = \frac{U_{ul}}{R_B}$$

$$\boxed{U_{iz} = U_{ul}\left(1 + \frac{R_A}{R_B}\right)}$$

vs $U_{iz} = \frac{-R_2}{R_1} U_{ul}$ ovaj je invertirajući $R_1 > R_2 \rightarrow$ invertira

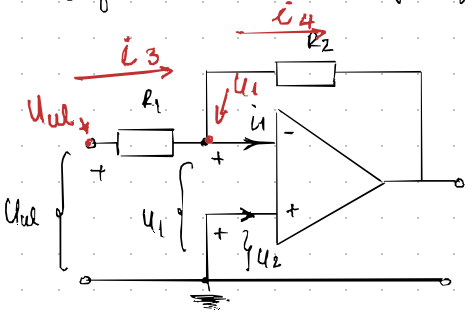
uvijek pojačava ili propušta točno

Operacijska pojačala (3. ppt)

Primjena Kirchhoffovih zakona:

Primjer 1) NONI $U_{iz} = ?$ (U_{ul} - poznato)

* uvijek krenuti od definicijskih jednadžbi



$U_{iz} = A \cdot (U_2 - U_1)$ $[A \to \infty]$

$i_2 = i_1 = 0$

$U_2 = \phi$

• moramo prevesti U_{ul} i U_{iz}

KZS: $i_3 = i_1 + i_4$ $\Big\}$ $\boxed{i_3 = i_4} = i$

$i_1 = \phi$

$$i = \frac{U_{ul} - U_1}{R_1} = \frac{U_1 - U_{iz}}{R_2}$$

$* \; U_{iz} = -U_1 \cdot A \longrightarrow U_1 = \frac{-U_{iz}}{A}$

$$\Rightarrow \frac{U_{ul}}{R_1} + \frac{U_{iz}}{AR_1} = \frac{-U_{iz}}{AR_2} - \frac{U_{iz}}{R_2}$$

$$U_{iz}\left(\frac{1}{R_2} + \frac{1}{AR_1} + \frac{1}{AR_2}\right) = \frac{-1}{R_1} U_{ul}$$

$$\longrightarrow U_{izl} = \frac{-1}{R_1}\left(\frac{1}{R_2} + \underbrace{\frac{1}{AR_1}}_{0} + \underbrace{\frac{1}{AR_2}}_{0}\right)^{-1} U_{ul} \quad / \lim_{A \to \infty}$$

$$\boxed{U_{izl} = -\frac{R_2}{R_1} U_{ul}}$$

$\longrightarrow$ naponom upravlja naponski izvor $\Rightarrow$ NONI

II. način $\to$ Princip virtualnog kratkog spoja (PVKS)

$i_1 = i_2 = 0$

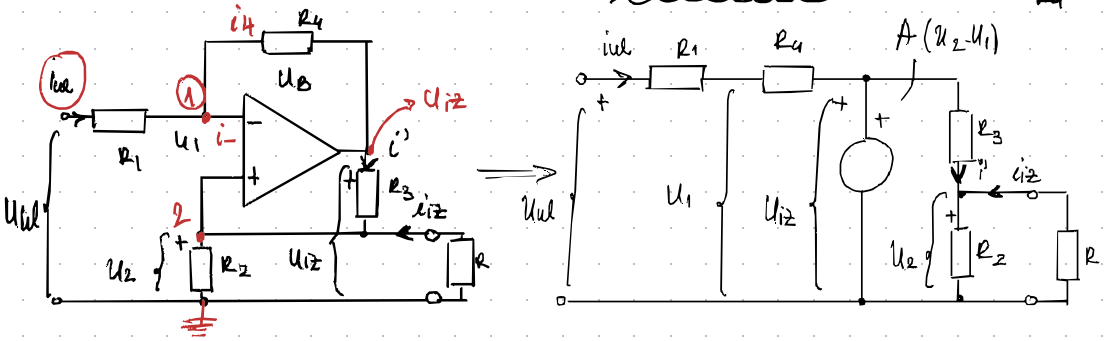
$U_1 = U_2$ $(U_+ = U_-)$

$U_2 = 0 \longrightarrow U_1 = U_2 = 0$

$$\longrightarrow i = \frac{U_{ul} - 0}{R_1} = \frac{0 - U_{izl}}{R_2}$$

$$\Rightarrow \boxed{U_{iz} = -\frac{R_2}{R_1} U_{ul}}$$ puno brže

Primjer 3) NOSI dokazati: $i_{iz} = \frac{1}{R_2} u_{ul}$ uz uvjet: $\frac{R_1}{R_4} = \frac{R_2}{R_3}$



PVKS:

$i_- = i_+ = 0$

$u_1 = u_2 \quad (u_+ = u_-)$

1) $i_{ul} = i_- + i_4$

$\rightarrow i_{ul} = i_4$

$\left(i_{ul} = \frac{u_{ul} - u_1}{R_1} \right)$

$\left(i_4 = \frac{u_1 - u_{iz}}{R_4} \right)$

$$\frac{u_{ul} - u_1}{R_1} = \frac{u_1 - u_{iz}}{R_4} \rightarrow -\frac{R_4}{R_1}(u_{ul} - u_1)$$

---

2) $\frac{u_1}{R_2} = i_{iz} + i'$

$\left(i' = \frac{u_{iz} - u_2}{R_3} = \frac{u_{iz} - u_1}{R_3} \right)$

$$i_{iz} = \frac{u_1}{R_2} - \frac{u_{iz} - u_1}{R_3} = \frac{u_1}{R_2} + \frac{R_4}{R_1 R_3}(u_{ul} - u_1)$$

$$i_{iz} = \frac{u_1}{R_2} + \frac{R_4}{R_1 R_3} u_{ul} - \frac{R_4}{R_1 R_3} u_1 = \frac{R_4}{R_1 R_3} u_{ul} + \left(\frac{1}{R_2} - \frac{R_4}{R_1 R_3} \right) u_1$$

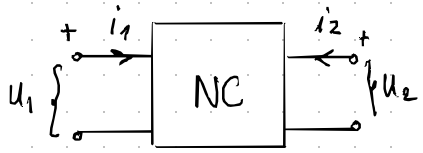
uvjet: $\left(\frac{R_4}{R_1 R_3} = \frac{1}{R_2} \right)$

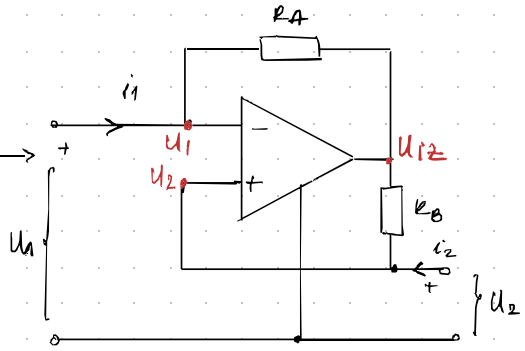
ako je ispunjen uvjet: $\frac{1}{R_2} = \frac{R_4}{R_1 R_3} \rightarrow i_{iz} = \frac{1}{R_2} u_{ul}$ NE OVISI O R

ako nije, u_1 možemo u_1 izraziti: $u_1 = -i_{iz} \cdot R$ OVISI O R

$$i_{iz} = \frac{R_4}{R_1 R_3} u_{ul} - \left(\frac{1}{R_2} + \frac{R_4}{R R_3} \right) i_{iz} \cdot R$$

Primjer 4.) Negativni konverter





$u_1(t) = k_1 u_2(t)$

$i_2(t) = k_2 i_1(t)$ $\quad k = k_1 \cdot k_2$

---

a) PVKS

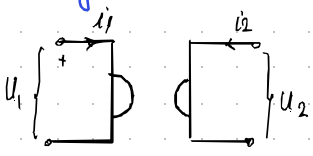
$u_1 = u_2 \rightarrow k_1 = 1$

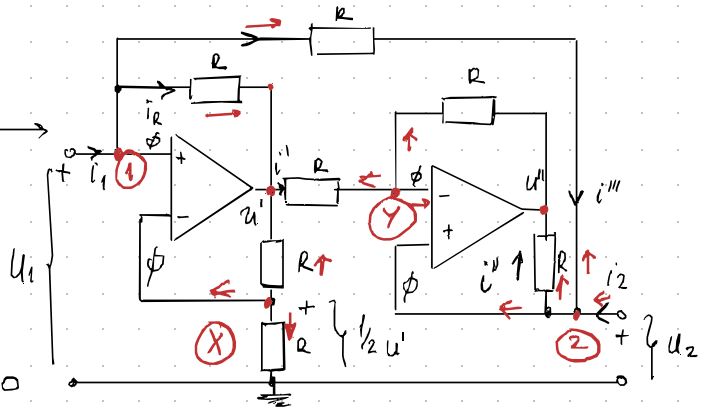
b) $i_1 = \frac{u_1 - u_{iz}}{R_A}$ $\quad i_2 = \frac{u_2 - u_{iz}}{R_B} = \frac{u_1 - u_{iz}}{R_B}$

$\Rightarrow R_A \cdot i_1 = R_B \cdot i_2$

$i_2 = \frac{R_A}{R_B} i_1 \rightarrow k_2 = \frac{R_A}{R_B}$

Primjer 5.) Girator





$u_1 = r\, i_2$

$u_2 = -r\, i_1$

---

1) $-i_1 + \frac{u_1 - u'}{R} + \frac{u_1 - u_2}{R} = 0$

X) $u_X = u_1$ (PVKS)

* Struja prema OP je ϕ

$\frac{u_1 - u'}{R} + \frac{u_1}{R} = 0 \Rightarrow 2u_1 = u'$

Y) $u_Y = u_2$ $\quad \frac{u_2 - u''}{R} + \frac{u_2 - u'}{R} = 0 \Rightarrow u'' = 2u_2 - 2u_1$ $\quad$ (def. jed. O.P. $\rightarrow u_3 = A(u_2 - u_1)$)

2) $-i_2 + \frac{u_2 - u_1}{R} + \frac{u_2 - u''}{R} = 0$

Girator. def. jed

① $i_1 = \frac{u_1 - 2u_1}{R} + \frac{u_1 - u_2}{R} = -\frac{u_1}{R} + \frac{u_1 - u_2}{R} = \frac{-u_2}{R} \rightarrow \boxed{u_2 = -R \cdot i_1}$

② $i_2 = \frac{u_2 - u_1}{R} + \frac{u_2 - 2u_2 + 2u_1}{R} = \frac{u_1}{R} \rightarrow \boxed{u_1 = R \cdot i_2}$

Dokazali smo da je ovo Girator

$r = R$

2. način

① $-i_1 + \frac{u_1 - u'}{R} + \frac{u_1}{R} = 0$

② $-i_2 - i_1 + \frac{u_2 - u''}{R} = 0 \rightarrow -i_2 - \left(\frac{u_1 - u_2}{R}\right) + \dots = 0$

$i_1 \quad -i_2 + \frac{u_2 - u_1}{R} + \dots = 0$ $\quad$ isto

---

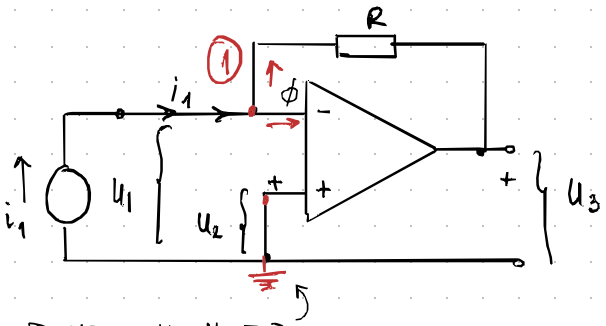
2a) $-i_2 - i''' + \dots$

2b) $-i_2 + i_g + \dots$

$\hookrightarrow -i'''$

Ovo je tu nešto svoje zalupao

Primjer 6.) SONI

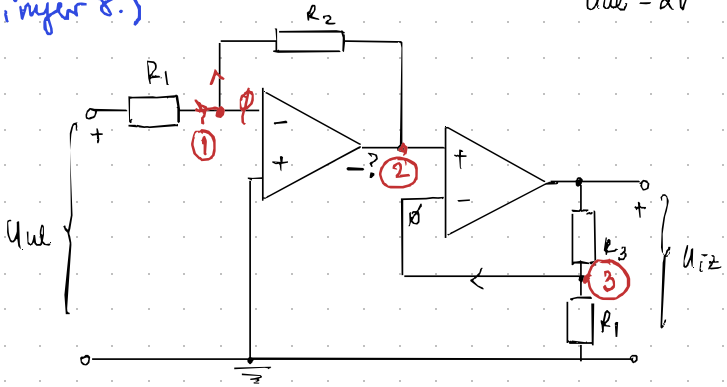


PVKS → $u_1 = u_2 = 0$

1) $-i_1 + \frac{0 - u_3}{R} = 0 \rightarrow u_3 = -R i_1$

Pr 7 nista specijalno

Primjer 8.)



$u_{ul} = 2V$ $R_1 = 2k\Omega$ $R_2 = 2k\Omega$ $R_3 = 2k\Omega$ $R_4 = 2k\Omega$ $u_{iz} = ?$

PVKS → $u_1 = u_2 = 0$

2) ne znamo što ulazi

1) $u_1 = 0$

$-\frac{u_{ul}}{R} + \frac{0 - u_2}{R} = 0 \rightarrow u_2 = -u_{ul} \rightarrow u_2 = -2V$

3) $u_3 = u_2$ zbog PVKS ??

$\frac{u_3 - u_{izl}}{R} + \frac{u_2 - 0}{R} = 0$

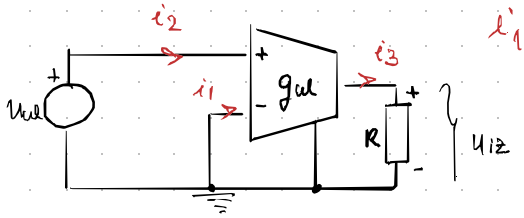
$\frac{u_2 - u_{izl}}{R} + \frac{u_2}{R} = 0 \rightarrow u_{iz} = 2u_2 = -4V$

Primjeri mreže sa OTA i CCII

Primjer 1: Strujno pojačalo (OTA) → $i_3 = g_m \cdot (u_2 - u_1)$, $i_2 = i_1 = 0$

Odredi pojačanje $k = \frac{u_{iz}}{u_{ul}}$

a)



$i_1 = i_2 = 0$

$i_3 = g_m (u_2 - u_1)$

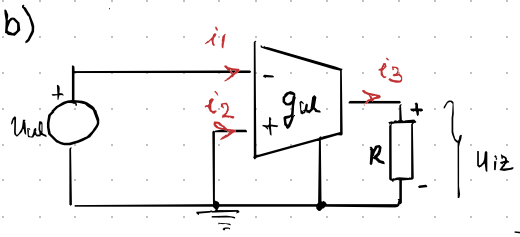
$u_2 = u_{ul}$ $u_1 = 0$

$\rightarrow i_3 = g_m \cdot u_{ul}$

$u_{iz} = i_3 \cdot R = g_m \cdot u_{ul} \cdot R \rightarrow K = \frac{u_{iz}}{u_{ul}} = \frac{g_m \cdot u_{ul} \cdot R}{u_{ul}}$

$K = g_m \cdot R$

b)

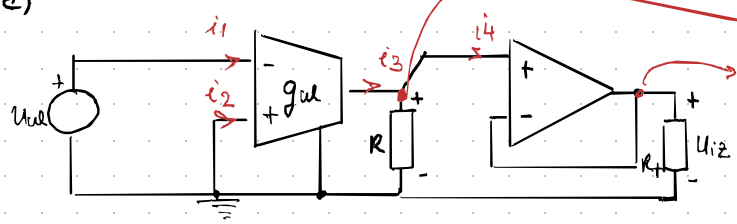


$i_2 = i_1 = 0$ $i_3 = g_m (0 - u_{ul}) = -g_m \cdot u_{ul}$

$u_2 = 0$ $u_1 = u_{ul}$

$\Rightarrow u_{iz} = i_3 \cdot R = -g_m u_{ul} \cdot R \rightarrow K = -g_m R$

c)



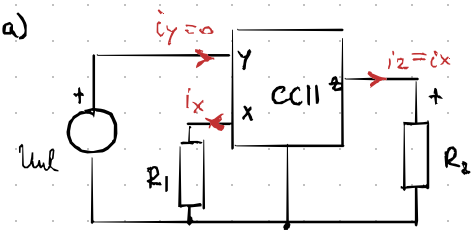
Zbog PVKS: $u_1 = u_2$

$u_2 = -g_m R \cdot u_{ul} = u_1$

$\rightarrow u_1 = u_{iz} = -g_m R \cdot u_{ul}$

Primjer 2: Strujni prijenosnik (CCII)

a)



$u_x = u_y$

$i_y = 0$

$i_z = i_x$

$u_{iz} = i_x \cdot R_2$

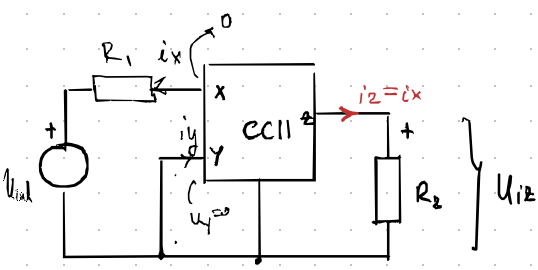
$u_x = i_x \cdot R_1$

$u_y = u_x = u_{ul} \rightarrow u_{ul} = i_x \cdot R_1$

$u_{iz} = \frac{u_{ul}}{R_1} \cdot R_2$

$\Rightarrow \frac{u_{iz}}{u_{ul}} = \frac{\frac{u_{ul}}{R_1} \cdot R_2}{u_{ul}} = \frac{R_2}{R_1} = K$

b)



$i_y = 0$

$u_x = u_y$

$u_{iz} = i_x \cdot R_2$

$u_y = 0$

$u_x = u_y = 0$

$i_x = \frac{0 - u_{ul}}{R_1}$

$i_x = \frac{-u_{ul}}{R_1}$

$\Rightarrow u_{iz} = \frac{-u_{ul}}{R_1} \cdot R_2 \rightarrow K = -\frac{R_2}{R_1}$

invertirajući spoj