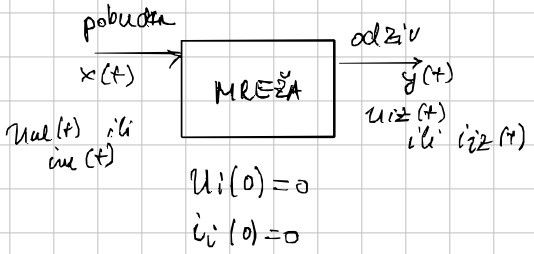
12. Funkcije mreža

- omjer L-transformacije odziva i pobude kada su svi poč uvjeti $\to 0$



$$H(s) = \frac{Y(s)}{X(s)}$$

$u_i(0) = 0$

$i_i(0) = 0$

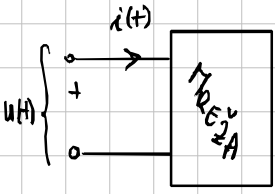
$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1} \cdot s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1} \cdot s^{n-1} + \dots + a_0} = \frac{P(s)}{Q(s)}$$

polinomi kompleksne var.

a_i, b_i - realni koef.

Funkcije mreža
- ulazne ili funkcije imitancije — dvopol
- prijenosne ili transfer funkcije — četveropol

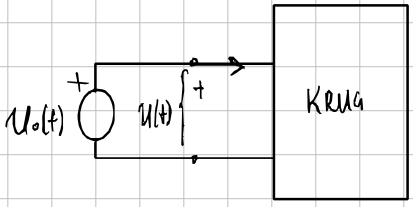
Funkcije imitancije → par priključnica ili 1 priključak



Slučaj 1)

pobuda → napon $u(t) \circ\!\!-\!\!\bullet U(s)$

odziv → struja $i(t) \circ\!\!-\!\!\bullet I(s)$

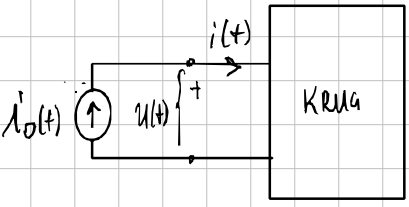


$$H(s) = \frac{I(s)}{U(s)} = Y(s) \to \text{Funkcija admitancije}$$

Slučaj 2)

pobuda → struja

odziv → napon



$$H(s) = \frac{U(s)}{I(s)} = Z(s)$$

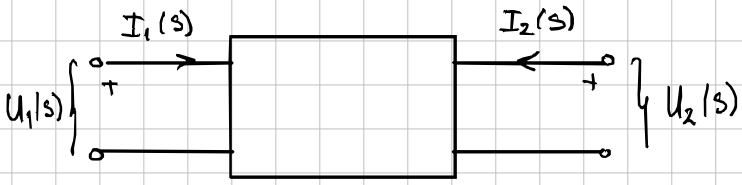
Funkcija impedancije

fije impedancije $Z(s)$

fije admitancije $Y(s)$

} imitancije

Prijenosne funkcije

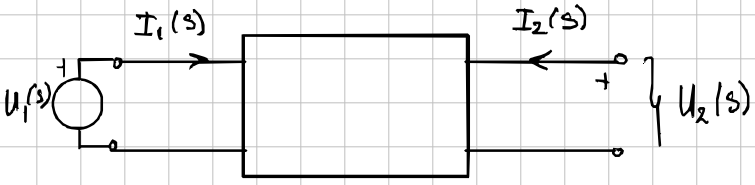


Pobuda (prilaz 1) — napon / struja

Odziv (prilaz 2) — napon / struja

$\Longrightarrow$ 4 tipa prijenosnih funkcija

1) Prijenosna funkcija (prijenosni omjer) napona

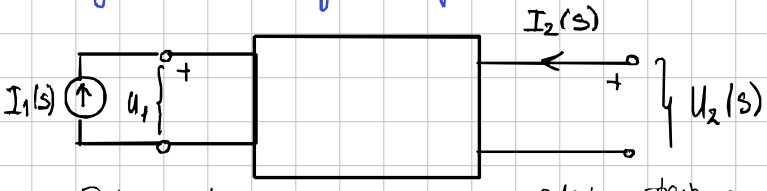


$$H(s) = \frac{U_2(s)}{U_1(s)} \quad \frac{(O)}{(P)}$$

Pobuda – napon 1

Odziv – napon 2

2) Prijenosna funkcija struja

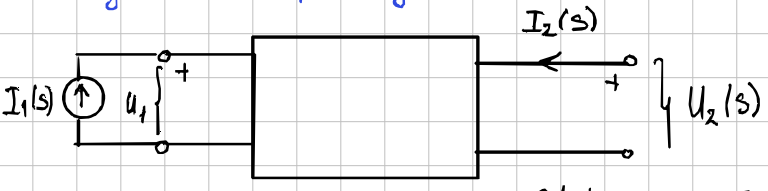


$$H(s) = \frac{I_2(s)}{I_1(s)} \quad \frac{(O)}{(P)}$$

Pobuda – struja 1

Odziv – struja 2

3) Prijenosna impedancija



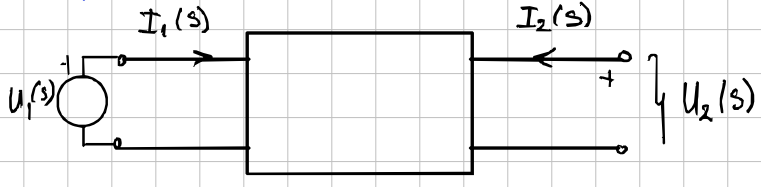
$$Z_{21}(s) = \frac{U_2(s)}{I_1(s)} \quad \frac{(O)}{(P)}$$

Pobuda – struja 1

Odziv napon 2

BITNO: $Z_{21} \neq \frac{1}{Y_{21}}$

4) Prijenosna admitancija

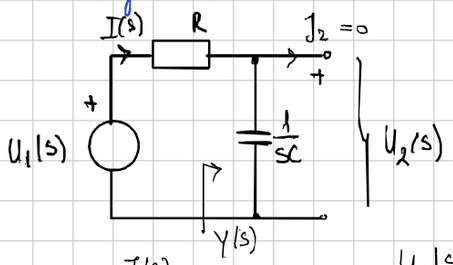


$$Y_{21}(s) = \frac{I_2(s)}{U_1(s)}$$

Pobuda = napon 1

Odziv = struja 2

Primjer 1) RC mreža



a) ulazna admitancija $Y(s)$

b) Prijenosna fja naponom $H(s)$

a)

$$Y(s) = \frac{J_1(s)}{U_1(s)}$$

$$U_1(s) = J_1(s) \cdot R + U_2(s) = J_1(s) \cdot R + \frac{1}{sC} \cdot J_1(s)$$

$$U_1(s) = J_1(s)\left(R + \frac{1}{Cs}\right)$$

$$Y(s) = \frac{1}{R + \frac{1}{Cs}} = \boxed{\frac{sC}{sRC + 1}}$$

*mala digresija

$$Y(s) = \frac{1}{R + \frac{1}{Cs}} = \frac{1}{R} \cdot \frac{1}{1 + \frac{1}{CsR}} = \frac{1}{\frac{R}{s}} \frac{1}{s + \frac{1}{CR}}$$

$$Y(s) = \frac{\frac{1}{R} \cdot s}{s + \frac{1}{CR}} \begin{matrix} \rightarrow P(s) \\ \rightarrow Q(s) \end{matrix}$$

$P(s) = a_1 \cdot s \Rightarrow a_1 = \frac{1}{R}$

$Q(s) = b_1 \cdot s + b_0 \Rightarrow b_1 = 1, \; b_0 = \frac{1}{RC}$

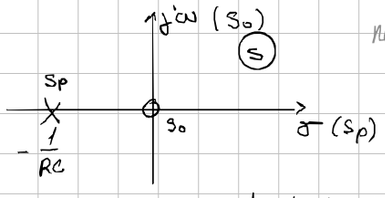
b) Prijenosna fja napona

$$Y(s) = \frac{1}{R} \frac{s}{s + \frac{1}{RC}}$$

Svaka fja mreže može se napisati u faktoriziranom obliku → nultočke

$P(s) = 0 \longrightarrow s_0 = 0$

$Q(s) = s + \frac{1}{RC} \Rightarrow s_p = -\frac{1}{RC}$



Ne kužim čemu ovo

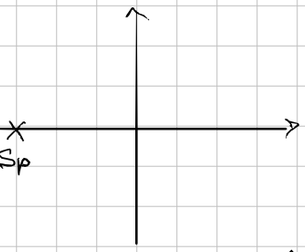
Isti brojnik, isti broj polova!

$$\frac{I}{U_1} = \frac{1}{R} \cdot \frac{s}{s + \frac{1}{RC}} \quad ; \quad U_2 = \frac{J}{sC} \rightarrow \frac{U_2}{J_1} = \frac{1}{sC}$$

$$H(s) = \frac{U_2(s)}{U_1(s)} = \underbrace{\frac{J_1}{U_1}}_{Y(s)} \cdot \frac{U_2}{J_1} = \frac{1}{R} \cdot \frac{s}{s + \frac{1}{RC}} \cdot \frac{1}{sC} = \frac{1}{RC} \cdot \frac{1}{s + \frac{1}{RC}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$$H(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}, \quad Y(s) = \frac{1}{R} \frac{s}{s + \frac{1}{RC}}$$

→ imamo isti pol

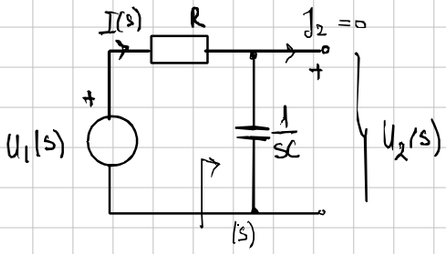


$$H(s) = \lim_{s \to \infty} \frac{1}{RC} \cdot \frac{1}{s + \frac{1}{RC}} = \left(\frac{1}{\infty}\right) = 0$$

↳ $s_0 = \infty$, drugi pol je u beskonačno

II. načina (b)

Mreža oblika je naponski djelitelj



↳ $U_2 = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \cdot U_1$

$U_1 = U_{Z1} + U_{Z2}$ $\quad Z_1 = R$

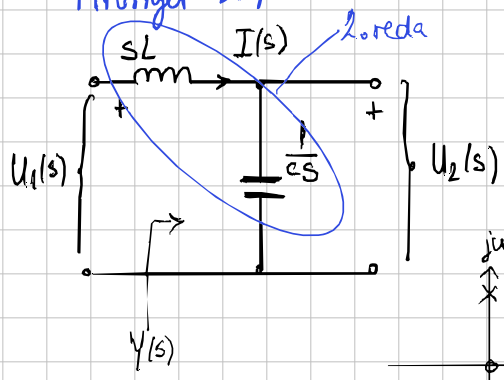
$U_1 = J \cdot Z_1 + J \cdot Z_2$ $\quad Z_2 = \frac{1}{sC}$

$U_2 = J \cdot \frac{1}{sC}$

$$\frac{U_2}{U_1} = \frac{J \cdot \frac{1}{sC}}{J \cdot R + J \cdot \frac{1}{sC}} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\frac{U_2}{U_1} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

Primjer 2.)

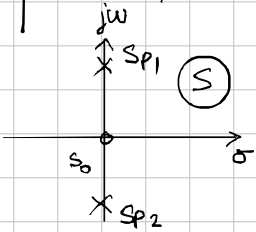


a) Ulazna admitancija

$$Y(s) = \frac{I(s)}{U_1(s)} = \frac{I(s)}{I(s)\left(SL + \frac{1}{Cs}\right)}$$

$$Y(s) = \frac{1}{SL + \frac{1}{Cs}} = \frac{s}{s^2L + \frac{1}{C}}$$

$$Y(s) = \frac{\frac{s}{L}}{s^2 + \frac{1}{LC}}$$



nule

$0 \rightarrow s_{o_1} = 0$

$\lim_{s\to\infty} \Rightarrow \frac{\frac{s}{L}}{s^2 + \frac{1}{LC}} \overset{L'H}{=} \left(\frac{\infty}{\infty}\right) = \frac{\frac{1}{L}}{2s} = \left(\frac{1}{\infty}\right) = 0 \rightarrow$ druga nula u ∞

$Q(s) = 0 \rightarrow s^2 = \frac{-1}{LC}$

$s_{p1,2} = \pm \frac{j}{\sqrt{LC}} \rightarrow$ polovi (2)

b) Prijenosna fja napona

$$H(s) = \frac{U_2}{U_1} = \underbrace{\frac{I}{U_1}}_{Y(s)} \cdot \frac{U_2}{I}$$

$$U_1 \cdot \frac{\frac{1}{Cs}}{SL + \frac{1}{Cs}} = U_2$$

$$H(s) = \frac{1}{\cancel{U_1}} \cdot \cancel{U_1} \cdot \frac{\frac{1}{Cs}}{SL + \frac{1}{Cs}}$$

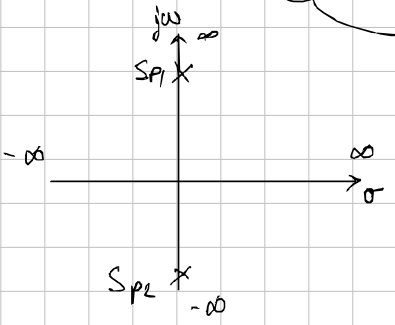
$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}}$$

polovi su isti kao kod Y(s)

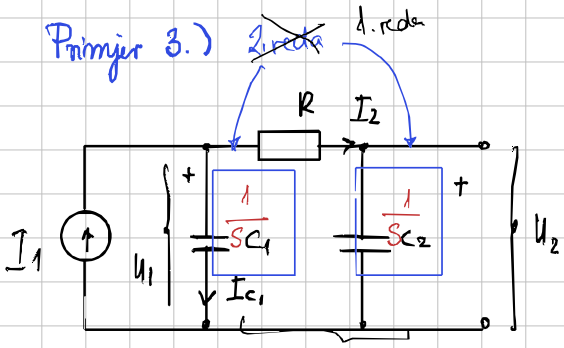
polovi $\rightarrow s_{p1,2} = \pm \frac{j}{\sqrt{LC}}$

nule $\rightarrow \lim_{s\to\infty} \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}} = \left(\frac{1}{\infty}\right) \rightarrow s_{o_1} = \infty$

$\rightarrow s_{o_2} = \infty$ (jer je kvadrat)



Primjer 3.) ~~2. reda~~ 1. reda



$H(s) = \frac{I_2}{I_1} = ?$

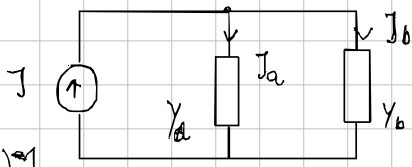
$Z_{21}(s) = \frac{U_2}{I_1} = ?$

$I_1 = I_{C_1} + I_2$

$Y_1 = SC_1 \qquad Y_2 = \left(R + \frac{1}{SC_2}\right)^{-1} = \left(\frac{SC_2 \cdot R + 1}{SC_2}\right)^{-1}$

$Y_2 = \frac{SC_2}{RSC_2 + 1}$

Strujni djelitelj



$J_a = \frac{Y_a \cdot J_1}{Y_a + Y_b} = \frac{Z_b}{Z_b + Z_a} \cdot J_1$

$I_2 = \frac{Y_2 \cdot I_1}{Y_1 + Y_2} \qquad I_1 \rightarrow H(s) = \frac{I_2}{I_1} = \frac{Y_2 \cdot \cancel{I_1} \cdot \frac{1}{\cancel{Y_1 + Y_2}}}{\cancel{Y_1 + Y_2}} = \frac{\frac{SC_2}{RSC_2 + 1}}{SC_1 + \frac{SC_2}{RSC_2 + 1}} = \frac{SC_2}{SC_1(RSC_2 + 1) + SC_2}$

$H(s) = \frac{SC_2}{RS^2C_1C_2 + SC_1 + SC_2} = \frac{1}{RSC_1 + \frac{C_1}{C_2} + 1} = \frac{1}{RC_1\left(S + \frac{1}{C_2R} + \frac{1}{RC_1}\right)}$

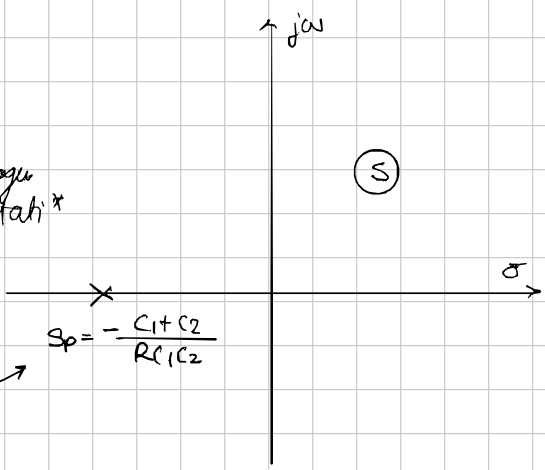
$H(s) = \frac{1}{RC_1} \cdot \frac{1}{S + \frac{(C_1 + C_2)}{RC_1C_2}}$

nule: da li $H(s) = 0$ → $s_0 = \infty$ / ne mogu ucrtati *

polovi: nazivnik = 0

$s_p + \frac{C_1 + C_2}{RC_1C_2} = 0$

$s_p = -\frac{C_1 + C_2}{RC_1C_2}$



* (1) R_1C → 1. red — C + U_{iz} = neovisna petlja => red(−1 stupanj)

(2) L_1C → 2. red — L u čvor & čine induktivni rez => rez (koji je neovisan) manje 1 stup.

(3) $R_1C_1C_2$ → ~~2. red~~ → 1. red — *ali je to uglavnom 2. red

Prijenosna impedancija

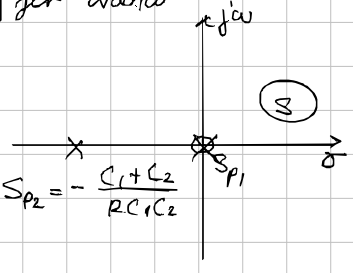
$Z(s) = \frac{U_2(s)}{I(s)} = \underbrace{\frac{I_2(s)}{I_1(s)}}_{H(s)} \cdot \frac{U_2(s)}{I_2(s)} \qquad U_2(s) = I_2(s) \cdot \frac{1}{SC_2}$

$Z(s) = \frac{1}{RC_1} \cdot \frac{1}{S + \frac{C_1 + C_2}{RC_1C_2}} \cdot \frac{1}{SC_2} = \frac{\frac{1}{RC_1C_2}}{S\left(S + \frac{C_1 + C_2}{RC_1C_2}\right)}$

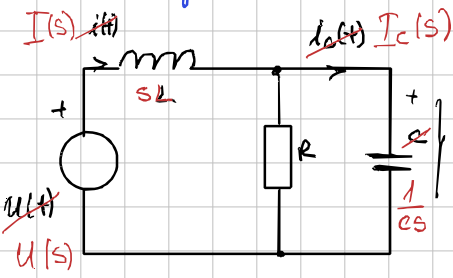
nule: $Z(s) = 0 \rightarrow \boxed{s_1 = \infty}$ i $\boxed{s_{0_2} = -\infty}$ jer kvadrat

polovi: $S\left(S + \frac{C_1 + C_2}{RC_1C_2}\right) = 0$

$\boxed{s_{p_1} = 0} \qquad \boxed{s_{p_2} = -\frac{C_1 + C_2}{RC_1C_2}}$



Primjer 4.) RLC krug



$R = 2$, $C = 1$, $L = 1$

Odziv → $I(s)$ → fija admit.

Odziv → $U_c(s)$ → prijenosna fja

Odziv → $I_c(s)$ → prijenosna fja

→ sigurno prijenosna fja drugog reda (R, L, C), poticaj pobude $u(t)$

a) $Y(s) = \frac{I(s)}{U(s)}$ funkcija admitancije

b) $H(s) = \frac{U_c(s)}{U(s)}$ prijenosna fja admitancije

c) $y_{21} = \frac{I_c(s)}{U(s)}$ prijenosna fja admitancije

$U(s) = I(s) \cdot sL + U_L(s)$

$U_c(s) = Z_{paralele} \cdot I(s) = I(s) \cdot \left(\frac{1}{R} + cs\right)^{-1}$

$U_c(s) = I(s) \cdot \frac{R}{1+RCs} = I(s) \cdot \frac{R/cs}{R + \frac{1}{cs}}$

$U(s) = I(s)\left(sL + \frac{\frac{R}{cs}}{R + \frac{1}{cs}}\right)$

$U(s) = I(s)\ \frac{sLR + \frac{sL}{cs} + \frac{R}{cs}}{R + \frac{1}{cs}} = \frac{s^2CLR + sL + R}{RCs + 1}\ I(s)$

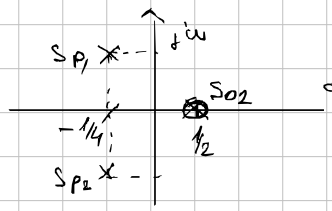
a)

$\Rightarrow Y(s) = \frac{SRC+1}{s^2CLR+SL+R} = \frac{1}{L} \cdot \frac{SRC+1}{s^2CR + s + \frac{R}{L}} = \frac{1}{L}\ \frac{s + \frac{1}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$

$$Y(s) = \frac{I(s)}{U(s)} = \frac{s + \frac{1}{2}}{s^2 + s\frac{1}{2} + 1}$$

$s_0 = \infty$, $s_{02} = -\frac{1}{2}$

$s_{p1,2} = \frac{-1 \pm \sqrt{15}j}{4}$



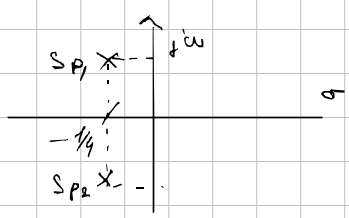
b) $H(s) = \frac{U_c(s)}{U(s)}$ $\quad U_c = I(s) \cdot \frac{R}{1+RCs} = \frac{U(s)}{Ls + \frac{R}{1+RCs}} \cdot \frac{R}{1+RCs}$

$U_c(s) = U(s) \cdot \frac{R}{Ls + RCLs^2 + R} \rightarrow H(s) = \frac{R}{sL + s^2RCL + R} = \frac{2}{s + 2s^2 + 2}$

$$H(s) = \frac{U_c(s)}{U(s)} = \frac{1}{s^2 + \frac{1}{2}s + 1}$$

nule → $s_{01,2} = \pm\infty$

polovi: $s_{p1} = \frac{-1 \pm \sqrt{15}j}{4}$

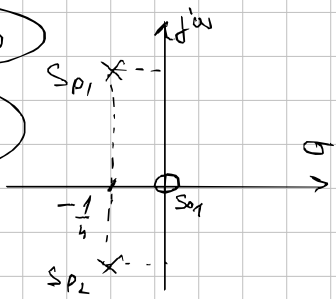


c) $H(s) = \frac{I_c(s)}{U(s)}$ $\quad I_c(s) = U_c(s) \cdot cs = U(s) \cdot \frac{RCs}{Ls + s^2RCL + R}$

$H(s) = \frac{I_c(s)}{U(s)} = \frac{2s}{2s^2 + s + 2}$

$s_{01} = 0$, $s_{02} = \infty$

$s_{p1,2} = \frac{-1 \pm \sqrt{15}j}{4}$



→ $$H(s) = \frac{s}{s^2 + \frac{1}{2}s + 1}$$

• Recipročna vrijednost funkcije imitancije također funkcija imitancije

dakle, funkcija mreže

Veza između Y i Z polova:

$Y: \circ\ x \mid x$ $\quad Z = \frac{1}{Y} \rightarrow$ $\ast\ \circ \mid \circ$ — zamjena polova i nula ✓ → je fja mreže

1 kci

! - nule mogu biti bilo gdje, ali polovi mogu samo lijevi!

• Recipročna vrijednost prijenosne fun je NIJE funkcija mreže

(nema jena svojstva)

npr: prijenosna admitancija

$y(s) =>$ $x \mid \circ$ / $x \mid \circ$ ✗

→ prijenosna

$Z = \frac{1}{y} \Rightarrow$ $\circ\ x \mid \circ\ x$ — ne može!

nestabilna funkcija

→ fja mreže

Za sve funkcije mreža vrijedi

- razlomljena realna funkcija f od „s"
- polovi samo u lijevoj poluravnini i na jw osi!
- polovi na jw osi jednostruki (✱, ne ✱✱)

▶ Za prijenosne funkcije → nule mogu biti i u desnoj poluravnini

▶ Za funkcije imitancija → nule ne smiju biti u desnoj poluravnini

→ Za njih vrijedi isto što i za polove