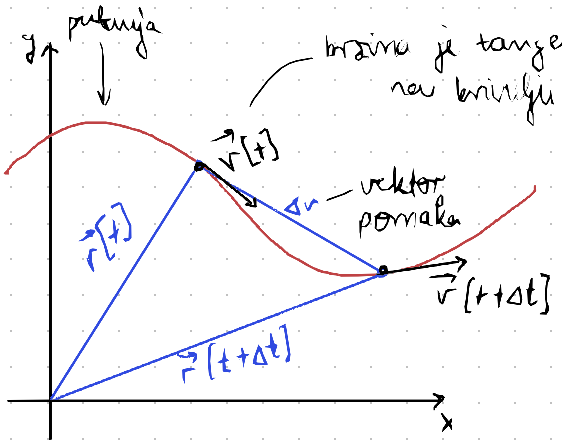
KINEMATIKA



brzina je tangenta (derivacija) na krivulju putanje

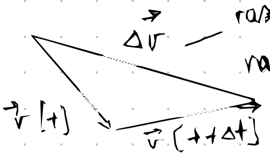
BRZINA: $v[t] = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$

$$v[t] = \lim_{\Delta t \to 0} \underbrace{\frac{r(t+\Delta t) - r[t]}{\Delta t}}_{\text{definicija derivacije}}$$

$$v[t] = \frac{d}{dt}\vec{r}[t]$$

iznos brzine – skalar: $v = |\vec{v}|$

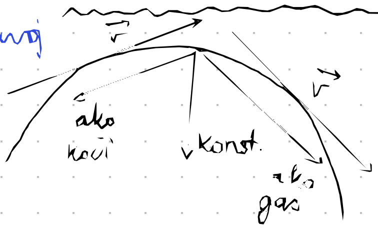
element duljine prevaljenog puta: $ds = |\vec{v}| \cdot dt$



AKCELERACIJA: $\vec{a}[t] = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \dots$

$$\vec{a}[t] = \frac{d}{dt}\vec{v}[t] = \frac{d^2}{dt^2}\vec{r}(t)$$

* vektor ne leži na tangenti, već gleda u zavoj



$$\vec{r}[t] = x[t]\,\hat{\vec{x}} + y[t] \cdot \hat{\vec{y}} + z[t] \cdot \hat{\vec{z}}$$

$$\vec{v}[t] = \frac{d}{dt}\vec{r}[t] = \frac{d}{dt}x[t]\,\hat{\vec{x}} + \frac{d}{dt}y[t] \cdot \hat{\vec{y}} + \frac{d}{dt}z[t] \cdot \hat{\vec{z}} = v_x[t]\,\hat{\vec{x}} + v_y[t] \cdot \hat{\vec{y}} + v_z[t] \cdot \hat{\vec{z}}$$

$$\vec{a}[t] = \frac{d}{dt}\vec{v}[t] \quad \dots = a_x(t) \cdot \hat{\vec{x}} + a_y[t] \cdot \hat{\vec{y}} + a_z[t] \cdot \hat{\vec{z}} = \ddot{x}\hat{\vec{x}} + \ddot{y}\hat{\vec{y}} + \ddot{z}\hat{\vec{z}}$$

Inverzne relacije za $\vec{v}$ i $\vec{r}$

- brzina: $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}[t'] \cdot dt' \,/\int$

$$\int_{t_0}^{t} dv = \int_{t_0}^{t} a[t'] \cdot dt'$$

$$\vec{v}[t] - \vec{v}[t_0] = \int_{t'=t_0}^{t'=t} a[t'] \cdot dt' \qquad \Rightarrow \vec{v}[t] = \vec{v}[t_0] + \int_{t_0}^{t} a[t']\,dt'$$

- položaj: $\vec{v} = \frac{d\vec{r}}{dt} \to d\vec{r} = \vec{v}\,dt \,/\int$

$$\int_{t_0}^{t} d\vec{r} = \int_{t_0}^{t} v[t']\,dt' \Rightarrow$$

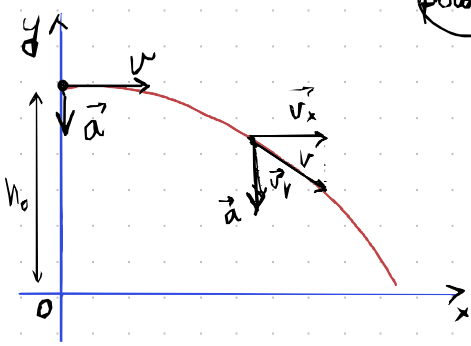
$$\vec{r}[t] - \vec{r}[t_0] = \int_{t_0}^{t} v[t']\,dt' \qquad \Rightarrow \vec{r}[t] = \vec{r}[t_0] + \int_{t_0}^{t} v[t']\,dt'$$

- duljina puta: $ds = |d\vec{r}|$

$$ds = |d\vec{r}| = |\vec{v}\,dt| = |\vec{v}|\,dt = \overset{\text{iznos}}{v}\,dt \quad /\int$$

$$s = \int_{t_1}^{t_2} \sqrt{\underbrace{v_x^2 + v_y^2 + v_z^2}_{\text{trebam modul jer gledam iznos}}} \cdot dt$$

Vodoravni hitac



Položaj $\quad \vec r[t] = v_0 t\,\hat x + \underbrace{\left(h_0 - \frac{g}{2}t^2\right)}\hat y$

jer se smanjuje visina na kojoj se nalazi loptica

$\longrightarrow \quad x = v_0 t \qquad y = h_0 - \frac{g}{2}t^2$

$t = \frac{x}{v_0} \longrightarrow y = h_0 - \frac{g}{2}\cdot\frac{x^2}{v_0^2}$

↳ parabola s tjemenom u $x=0$ i $y=0$

Brzina

$\vec v[t] = v_0\hat x - gt\,\hat y$ $\longrightarrow$ $v_y = -gt$

↳ usmjerena prema dolje i s vremenom se povećava

$v_x = v_0$ → x-komp. je stalna u vremenu

Akceleracija $\quad \vec a[t] = 0\hat x - g\hat y \Rightarrow$ akceleracija slobodnog pada

Gibanje stalnom brzinom:

$\vec a = 0$

$\vec v = \vec v_0 = \text{konst.}$

$\vec r[t] = \vec r[t_0] + v_0(t - t_0)$

$* \quad \vec r[t] = \vec r[t_0] + \int_{t_0}^{t} \vec v[t']\,dt'$

$\vec r[t] = \vec r[t_0] + \vec v(t - t_0)$

Gibanje stalnom akceleracijom

$\vec a = \text{konst.} = \vec a_0$

$\vec v[t] = \vec v[t_0] + \int_{t_0}^{t} \vec a[t']\,dt$

$\vec v[t] = \vec v[t_0] + \vec a_0(t - t_0) \qquad \vec r[t] = \vec r[t_0] + \int_{t_0}^{t} \vec v[t']\,dt'$

$\vec r[t] = \vec r[t_0] + \int_{t_0}^{t}\left(\vec v[t_0] + \vec a(t - t_0)\right)dt'$

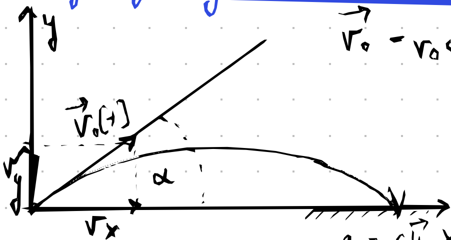
$\vec r[t] - \vec r[t_0] = \int_{t_0}^{t} \vec v[t_0]\,dt' + \int_{t_0}^{t} \vec a(t - t_0)\,dt' = \vec v[t_0]\int_{t_0}^{t} dt' + \vec a_0\int_{0}^{t-t_0} \tau\,d\tau$ $\quad (=\tau)$

$= \vec v[t_0](t - t_0) + \vec a_0 \cdot \frac{\tau^2}{2}\Big|_0^{t-t_0}$

$\vec r[t] - \vec r[t_0] = \vec v[t_0](t - t_0) + \vec a_0$ [redacted]

Primjer gibanja stalnom akceleracijom - kosi hitac

$\downarrow \vec{g}$



$a = -g\hat{y}$

$\vec{v}_0 = v_0\cos\alpha\,\hat{x} + v_0\sin\alpha\,\hat{y}$

položaj $\vec{r}(t_0) = 0$

$$\vec{r}(t) = v_0(\cos\alpha\,\hat{x} + \sin\alpha\,\hat{y})t + \frac{(-g\hat{y})}{2}t^2$$

$$\vec{r}(t) = \underbrace{v_0\cos\alpha\,\hat{x}t}_{x\text{-koord }(r_x)} + \underbrace{v_0\sin\alpha\,\hat{y}t - \frac{gt^2}{2}\hat{y}}_{y\text{-koord }(r_y)}$$

putanja:

$x(t) = v_0\cos\alpha \cdot t$

$y(t) = v_0\sin\alpha t - \frac{gt^2}{2}$

$t = \frac{x(t)}{v_0\cos\alpha}$

$$\Rightarrow \cancel{v_0}\sin\alpha \frac{x}{\cancel{v_0}\cos\alpha} - \frac{g}{2}\frac{x^2}{v_0^2\cos^2\alpha} \Rightarrow y(t) = x\cdot tg\alpha - \frac{x^2 g}{2v_0^2\cos^2\alpha}$$

brzina $\vec{v} = \frac{d\vec{r}}{dt}$

$v_x = \frac{d}{dt}x(t) = v_0\cos\alpha$

$v_y = \frac{d}{dt}y(t) = v_0\sin\alpha - gt$

akceleracija $\vec{a} = \frac{d\vec{v}}{dt}$

$a_x = 0$

$a_y = -g$

najveća visina · kada je y-komponenta brzine $= 0$ (leti vodoravno)

uvjet $\rightarrow v_y(t') = 0$

$v_0\sin\alpha - gt' = 0 \rightarrow t' = \frac{v_0\sin\alpha}{g}$

najveća visina: $y(t') = v_0\sin\alpha\, t - \frac{g}{2}t^2$

$= v_0\sin\alpha \cdot \frac{v_0\sin\alpha}{g} - \frac{\cancel{g}}{2}\cdot\frac{v_0^2\sin\alpha^2}{g^{\cancel{2}}}$

$$y(t') = \frac{1}{2}\frac{v_0^2\sin^2\alpha}{g}$$ y - koord

$$x(t') = v_0\cos\alpha\frac{v_0\sin\alpha}{g} = \frac{v_0^2}{g}\sin\alpha\cos\alpha = \frac{v_0^2}{g}\cdot\frac{\sin 2\alpha}{2}$$ x- koord

domet na vodoravnom tlu

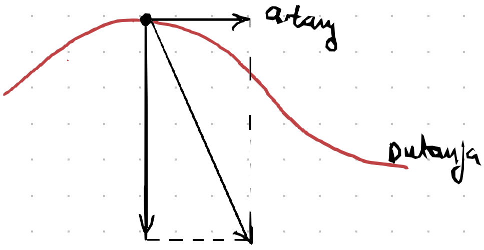
uvjet $y(t') = 0 \Rightarrow \frac{gt^{\cancel{2}}}{2} = v_0\sin\alpha\,\cancel{t}$

$t = \frac{2}{g}v_0\sin\alpha$

$x(t') = v_0\cos\alpha\cdot\frac{2}{g}v_0\sin\alpha$

$$x(t') = \frac{v_0^2}{g}\sin 2\alpha$$

Centripetalna akceleracija



Brzina: $\vec v = |\vec v| \cdot \hat v$

velocity

iznos (speed)

$\hat v$ ↳ smjer (direction of motion)

Akceleracija: $\vec a = \frac{d\vec v}{dt}$

$\vec a = \frac{d}{dt}\vec v$ = (derivacija umnoška) $= \frac{d}{dt}(|\vec v| \cdot \hat v)$

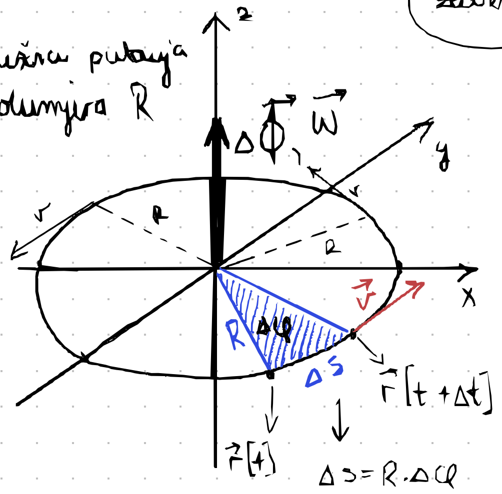
$\vec a = \frac{d|\vec v|}{dt} \cdot \hat v + \boxed{\frac{d\hat v}{dt}|\vec v|}$ → a_{cp}

a_{tang}
- isti smjer (|| sa $\vec v$)
- tangenta putanje
- naslijeđuje se smjer, mijenja se iznos brzine od originalne

a_{cp}
- okomit na jed. vektor
- naslijeđuje iznos, mijenja se smjer brzine od originalne

Kružno gibanje

kružna putanja polumjera R



Vektor zakreta: $|\Delta\vec\Phi| = \Delta\varphi$ [rad]

smjer - pravilo desne ruke

Vektor kutne brzine: $\vec\omega = \lim_{\Delta t \to 0} \frac{\Delta\Phi}{\Delta t} = \frac{d\Phi}{dt}$

- okomit na ravninu gibanja

Brzina: S obzirom da pri zakretu vektora r ($\vec r[t] \to \vec r[t+\Delta t]$) za kut $\Delta\varphi$ čestica prevali put duljine $\Delta S = R\,\Delta\varphi$,

iznos brzine čestice: $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

$\Rightarrow \vec v = \lim_{\Delta t \to 0} \frac{\Delta\vec\varphi \cdot \vec r}{\Delta t} = \frac{d\vec\varphi}{dt}\vec r \to \vec v = \vec\omega \times \vec r$

$\to \vec v = \vec\omega \times \vec r$

$v = |\vec\omega| \cdot |\vec r| \cdot \sin\varphi$ $\quad v = \omega \cdot R$ $\quad$ > okomit → $\sin\frac{\pi}{2} = 1$

Akceleracija: $a = \frac{dv}{dt} = \frac{d}{dt}(\vec\omega \times \vec r) = \underbrace{\frac{d\vec\omega}{dt} \times \vec r}_{\text{na tangenti}} + \vec\omega \times \frac{d\vec r}{dt}$ ($\frac{d\vec r}{dt} = \vec v$)

$a = a_t + a_{cp}$

$\vec a_{cp} = -\omega^2 \vec r$ — gleda prema središtu

↳ $a_{cp} = \omega^2 R = \frac{v^2}{R}$

vektorski umnožak

$\vec\omega \times \vec v = \vec\omega \times (\vec\omega \times \vec r)$

$\vec\omega \times \vec v = (\vec\omega \cdot \vec r)\vec\omega - (\vec\omega \cdot \vec\omega)\vec r$

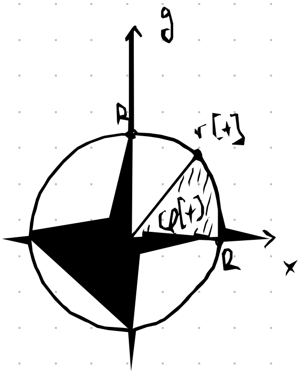
0 (jer su okomiti)

Tangencijalna akceleracija - vektor tang. akc. pri kružnom gibanju može se izraziti kao $\vec a_{tang} = \vec\alpha \times \vec r$

Kutna akceleracija - vektorska veličina

$\vec\alpha = \frac{d\vec\omega}{dt} = \frac{d^2\varphi}{dt} \cdot \vec r$

Kružno gibanje u pravokutnom koord sustavu



položaj $\vec{r}[t] = R\cdot\hat{\vec{r}}[t] = R\left(\cos(\varphi[t])\hat{\vec{x}} + \sin(\varphi[t])\hat{\vec{y}}\right)$

vektor kutne brzine - okomit na ravninu u kojoj leži kružnica

$$\vec{\omega}[t] = \omega_z[t]\cdot\hat{\vec{z}}$$

$$\downarrow$$

$$\omega_z[t] = \frac{d}{dt}\varphi[t]$$

- kad se kutna koord povećava u vremenu z-komponenta kutne brzine ω_z je pozitivna
↪ $\vec{\omega}_z$ gleda u poz smjeru (pravilo desnog vijka)

brzina čestice $\vec{v} = \frac{d}{dt}\vec{r}[t]$: 1. način

$$\vec{v} = R\cdot\frac{d}{dt}\left(\cos(\varphi[t])\cdot\hat{\vec{x}} + \sin(\varphi[t])\cdot\hat{\vec{y}}\right)$$

$$\vec{v} = R\cdot\left(-\sin\varphi\,\hat{\vec{x}} + \cos\varphi\,\hat{\vec{y}}\right)\varphi[t]' = \frac{d\varphi}{dt} = \omega_z$$

$$\vec{v} = R\left(-\sin\varphi\,\hat{\vec{x}} + \cos\varphi\,\hat{\vec{y}}\right)\cdot\omega_z$$

dobije se isto

2. način

$$v = \omega\times r = \begin{vmatrix} \hat{\vec{x}} & \hat{\vec{y}} & \hat{\vec{z}} \\ 0 & 0 & \omega_z \\ R\cos\varphi & R\sin\varphi & 0 \end{vmatrix} = \hat{\vec{x}}\left(-\omega_z R\sin\varphi\right) + \hat{\vec{y}}\left(\omega_z R\cos\varphi\right)$$

centripetalna akceleracija $\vec{a}_{cp} = -\omega^2\vec{r} = -\omega^2 R\left(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}\right)$

kutna akceleracija $\vec{\alpha} = \alpha_z\hat{z} \longrightarrow \alpha_z = \frac{d}{dt}\omega_z$

tangencijalna akc. $\vec{a}_t = \vec{a} - \vec{a}_{cp} = \frac{d\vec{v}}{dt} - (-\omega^2\vec{r}) = R\alpha_z\left(-\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}\right)$

ili drugi način

$$\vec{a}_{tang} = \vec{\alpha}\times\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \alpha_z \\ R\cos\varphi & R\sin\varphi & 0 \end{vmatrix} = \hat{x}\left(-\alpha_z R\sin\varphi\right) + \hat{y}\left(\alpha_z R\cos\varphi\right)$$