ELEKTRIČNE PRIJENOSNE LINIJE (dio II.)

Valni raspored napona i struja duž linije

Napon i struja na liniji su

$$U(x,s) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}$$

$$I(x,s) = \frac{A_1}{A_0} e^{-\gamma x} - \frac{A_2}{Z_0} e^{\gamma x}$$

Stacionarna sinusna pobuda

$$u(x,t) = |U| e^{j(\omega t + \varphi)}$$

$$i(x,t) = |I| e^{j(\omega t + \varphi)}$$

tada su i $A_1 = |A_1| e^{j(\omega t + \varphi)}$

$A_2 = |A_2| e^{j(\omega t + \varphi)}$

$$u(x,t) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}$$

$$i(x,t) = B_1 e^{-\gamma x} + B_2 e^{\gamma x}$$

$\Rightarrow$ Z_0 i γ = funkcije od $j\omega$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \rightarrow \text{kompleksni}$$

$\gamma = \alpha + j\beta$

$$\alpha = \sqrt{\frac{1}{2}\left(RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right)}$$

karakt. ili zrcalni fakt. gušenja

$$\beta = \sqrt{\frac{1}{2}\left(\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right)}$$

karakt. ili zrcalni faktor faze

* konstante

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sve to uvrstimo (sljedeća stranica)

$$u(x,t) = \underline{A_1} e^{-\gamma x} + \underline{A_2} e^{\gamma x} = A_1 e^{-(\alpha + j\beta)x} + A_2 e^{(\alpha + j\beta)x}$$

$$A_1 = |A_1| e^{j(\omega t + \varphi_1)} \qquad A_2 = |A_2| e^{j(\omega t + \varphi_2)}$$

$$\Rightarrow u(x,t) = |A_1| e^{j(\omega t + \varphi_1)} e^{-(\alpha + j\beta)x} + |A_2| e^{j(\omega t + \varphi_2)} \cdot e^{(\alpha + j\beta)x}$$

$$\Rightarrow i(x,t) = \frac{|A_1|}{Z_0} e^{j(\omega t + \varphi_1)} e^{-(\alpha + j\beta)x} - \frac{|A_2|}{Z_0} e^{j(\omega t + \varphi_1)} e^{-(\alpha + j\beta)x}$$

utječe samo na fazu

$$u(x,t) = \underbrace{|A_1| e^{-\alpha x} \cdot e^{j(\omega t - \beta x + \varphi_1)}}_{\text{val koji putuje od početka do kraja linije}} + \underbrace{|A_2| e^{\alpha x} \cdot e^{j(\omega t + \beta x + \varphi_2)}}_{\text{val koji ide s kraja do početka linije}}$$

dolje spotrenut

gušenje

Zanima nas koliko brzo to sve putuje (brzina propagacije)

$f(\omega t_1 - \beta x_1) = z_1 = f(\gamma_1) \rightarrow$ stanje na liniji na mjestu x_1 u času t_1

u času $t_2 = t_1 + \Delta t \longrightarrow$ isto stanje z_1 na mjestu $x_2 = x_1 + \Delta x$

$$\omega t_1 - \beta x_1 = \gamma_1$$

$$\cancel{\omega t_1} + \Delta t \cdot \omega - \beta(\cancel{x_1} + \Delta x) = \gamma_1 = \cancel{\omega t_1 - \beta x_1}$$

$$\omega \cdot \Delta t - \beta \cdot \Delta x = 0 \longrightarrow \frac{\Delta x}{\Delta t} = \text{brzina} = \frac{\omega}{\beta} \longrightarrow \boxed{\frac{dx}{dt} = \frac{\omega}{\beta} = v}$$

$f(\omega - \beta t) \longrightarrow$ poč - kraj

$f(\omega + \beta t) \Rightarrow$ kraj - poč.

obje brzine su oblika $v = \frac{\omega}{\beta}$

Prema tome: napon i struja na liniji:

$$u(x,t) = u_p(x,t) + u_r(x,t)$$

$$i(x,t) = i_p(x,t) - i_r(x,t)$$

polazni val $\longrightarrow$ reflektirani val $\longleftarrow$

Periodične fje:

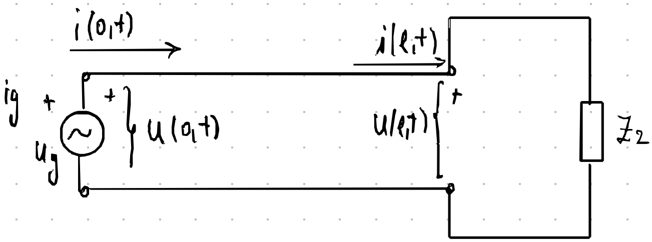
$x_1 - x_2 = \frac{n \cdot P}{\beta}$ (matematički izvod)

$\hookrightarrow$ temeljni period

$x_1 - x_2 = \frac{P}{\beta} = \lambda \rightarrow$ valna duljina, $P = 2\pi$

period jednog titraja: $\frac{2\pi}{\omega}$

Polazni i reflektirani val kod sinusoidnog signala



$$u_g(t) = U_g \cos(\omega t + \varphi_g)$$

fazorski $\dot{U}_g = U_g \cdot e^{j\varphi_g}$

$$\hookrightarrow u_g(t) = Re\left[\dot{U}_g \cdot e^{j\omega t}\right]$$

$$\hookrightarrow i_g(t) = Re\left[\frac{\dot{U}_g}{Z_{ul}} \cdot e^{j\omega t}\right] = I_g \cos(\omega t + \psi_g)$$

Rubni uvjeti: $u_g(t) = u(0,t)$ $\qquad u_2(t) = u(l,t)$

$i_g(t) = i(0,t)$ $\qquad i_2(t) = i(l,t)$

$$\Rightarrow \quad A_1 = \frac{\dot{U}_g + \dot{I}_g Z_0}{2} e^{j\omega t} = |A_1| e^{j(\omega t + \varphi)}$$

$$A_2 = \frac{\dot{U}_g - \dot{I}_g \cdot Z_0}{2} e^{j\omega t} = |A_2| e^{j(\omega t + \varphi)}$$

$$Z_0 = |Z_0| e^{j\xi_0}$$

izrazi za napon i struju

$$u(x,t) = Re\left[|A_1| e^{j\varphi_1} e^{j\omega t} e^{-(\alpha x + j\beta x)}\right] + Re\left[|A_2| e^{j\varphi_2} \cdot e^{j\omega t} e^{(\alpha + j\beta)x}\right]$$

$$i(x,t) = Re\left[\frac{|A_1|}{|Z_0|} e^{j(\varphi_1 - \xi_0)} e^{j\omega t} e^{-(\alpha x + j\beta x)}\right] - Re\left[\left|\frac{A_2}{Z_0}\right| e^{j(\varphi_2 - \xi_0)} e^{j\omega t} e^{(\alpha + j\beta)x}\right]$$

Ako se riješimo Re

$$u(x,t) = |A_1| e^{-\alpha x} \cdot \cos(\omega t - \beta x + \varphi_1) + |A_2| e^{\alpha x} \cdot \cos(\omega t + \beta x + \varphi_2)$$

$$i(x,t) = \left|\frac{A_1}{Z_0}\right| e^{-\alpha x} \cos(\omega t - \beta x + \varphi_1 - \xi_0) + \left|\frac{A_2}{Z_0}\right| e^{\alpha x} \cdot \cos(\omega t + \beta x + \varphi_2 - \xi_0)$$

polazni valovi

reflektirani valovi

Faktor refleksije

polazni : $u_p(x,t) = |A_1| e^{-\alpha x} \cdot \cos(\omega t - \beta x + \varphi_1)$

$$A_1 = |A_1| e^{j(\omega t + \varphi_1)} \longrightarrow A_2 = \Gamma_2 \cdot A_1 e^{-2\gamma l} = \Gamma_2 \cdot A_1 e^{-2(\alpha + j\beta) l}$$

reflektirani

$$\Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = |\Gamma_2| e^{j\theta_2} = \left| \frac{Z_2 - Z_0}{Z_2 + Z_0} \right| e^{j\theta_2}$$

$$\Rightarrow A_2 = |A_2| e^{j(\omega t + \varphi_2)} = |A_1| \cdot |\Gamma_2| \cdot e^{-2(\alpha + j\beta) l} \cdot e^{j(\omega t + \varphi_1)} \cdot e^{j\theta_2}$$

$$A_2 = |A_1| |\Gamma_2| e^{-2\alpha l} \cdot e^{j(\omega t - 2\beta l + \varphi_1 + \theta_2)} = |A_2| e^{j(\omega t + \varphi_2)}$$

$$\boxed{|A_2| = |A_1| |\Gamma_2| e^{-2\alpha l} \quad , \quad \varphi_2 = \varphi_1 + \theta_2 - 2\beta l}$$

omjeri amplituda i faza

$$u_p(x,t) = |A_1| e^{-\alpha x} \cos(\omega t - \beta x + \varphi_1)$$

$$u_r(x,t) = |A_2| e^{\alpha x} \cos(\omega t + \beta x + \varphi_2) = |A_1| |\Gamma_2| e^{-2\alpha l} e^{\alpha x} \cos(\omega t + \beta x + \varphi_1 + \theta_2 - 2\beta l)$$

amplituda na mjestu $x = l$

$$u_r(x,t) = |A_1| |\Gamma_2| e^{-\alpha l}$$

$$u_p(x,t) = |A_1| e^{-\alpha l}$$

$$\boxed{\frac{\text{ampl } u_r(l,t)}{\text{ampl } u_p(l,t)} = |\Gamma_2|}$$

$$\text{faza}\left[\frac{u_r(l,t)}{u_p(l,t)}\right] = \cancel{\beta l} + \cancel{\varphi_1} + \theta_2 - \cancel{2\beta l} + \cancel{\beta l} - \cancel{\varphi_1} = \theta_2 = \arg(\Gamma_2)$$

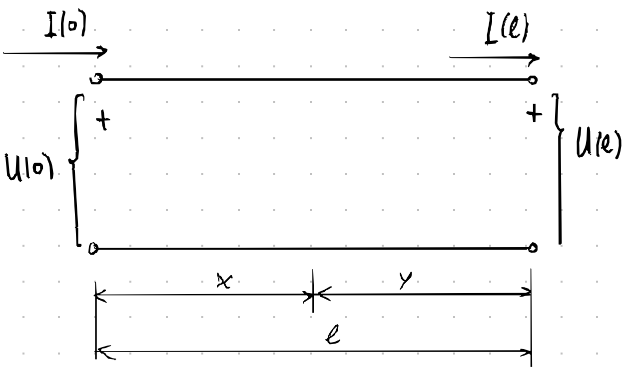
1. LINIJA BEZ GUBITAKA

$$R=0\ ,\ G=0 \longrightarrow Z_0=\sqrt{\frac{R+sL}{G+sC}}=\sqrt{\frac{L}{C}}=konst=R_0$$

$$\hookrightarrow \gamma=\sqrt{(R+sL)(G+sC)}=s\sqrt{LC}$$ — faktor propagacije

Za $s=j\omega \longrightarrow$ sinusna pobuda

$$\gamma=j\omega\sqrt{LC}=\underline{\underline{j\beta}} \qquad \underline{\underline{\alpha=0}} \qquad v=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}$$



$$y=l-x$$

$$U(x)=U(l)\,ch(\gamma y)+I(l)\cdot Z_0\,Sh(\gamma y)$$

$$I(x)=\frac{U(l)}{Z_0}\,sh(\gamma y)+I(l)\,ch(\gamma y)$$

$$\gamma=j\omega\sqrt{\frac{L}{C}}=j\beta$$

$$U(y)=U(l)\cos\beta y+j\,I(l)Z_0\cdot\sin\beta y \qquad U(l)=|U(l)|e^{j\varphi}e^{j\omega t}$$

$$I(y)=j\frac{U(l)}{Z_0}\sin\beta y+I(l)\cos\beta y \qquad I(l)=|I(l)|e^{j\psi}e^{j\omega t}$$

<u>Možemo napisati izraze za napon i struju</u>

$$u(y,t)=Re[U(y)]=Re\left[|U(l)|e^{j(\omega t+\varphi)}\cos\beta y+j\,|I(l)|Z_0\,e^{j(\omega t+\psi)}\sin\beta y\right]$$

$$=\left[|U(l)|\cos(\omega t+\varphi)\cos\beta y-|I(l)|Z_0\sin\beta y\cdot\sin(\omega t+\psi)\right]$$

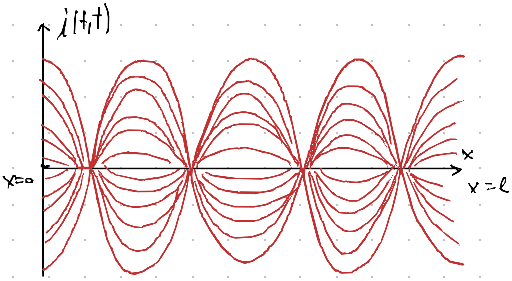
$$i(y,t)=Re[I(y)]=Re\left[j\frac{|U(l)|}{Z_0}e^{j(\omega t+\varphi)}\sin\beta y+|I(l)|e^{j(\omega t+\psi)}\cos\beta y\right]=$$

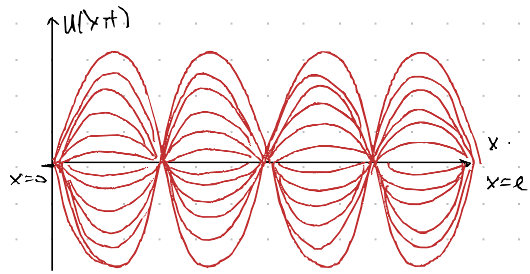
$$=-\frac{|U(l)|}{Z_0}\sin\beta y\cdot\sin(\omega t+\varphi)+|I(l)|\cos\beta y\cdot\cos(\omega t+\psi)$$

• Postoje 2 slučaja: $I=0$ i $U=0$

$\boxed{I_2 = 0} \longrightarrow \Gamma_2 = 1 \quad Z_2 = \infty \rightarrow$ kratko spojimo 2–2'

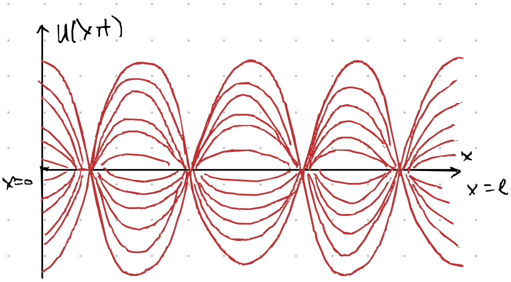
$$u(y,t) = |U(l)| \cos \beta y \cdot \cos(\omega t + \varphi)$$
$$i(y,t) = -\frac{|U(l)|}{Z_0} \cdot \sin \beta y \, \sin(\omega t + \varphi)$$
} stojni val

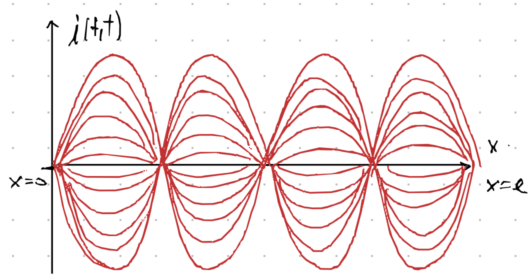




$\boxed{U_l = 0} \longrightarrow Z_2 = 0 \quad \Gamma_2 = -1$

$$u(y,t) = -|I(l)| Z_0 \sin \beta y \, \sin(\omega t + \varphi)$$
$$i(y,t) = |I(l)| \cos \beta y \cos(\omega t + \psi)$$





2. LINIJA BEZ DISTORZIJE

$$\boxed{\frac{R}{L} = \frac{G}{C} \rightarrow \frac{R}{G} = \frac{L}{C} \Rightarrow RC = GL}$$

ne mijenja valni oblik signala

$$Z_0 = \sqrt{\frac{R+sL}{G+sC}} = \sqrt{\frac{\frac{R}{L}+s}{\frac{G}{C}+s}\cdot\frac{L}{C}} = \sqrt{\frac{L}{C}} = Z_0$$

$$\gamma = \sqrt{(R+sL)(G+sC)} = \sqrt{LC\left(\frac{R}{L}+s\right)\left(\frac{G}{C}+s\right)} = \sqrt{LC}\left(\frac{R}{L}+s\right) = \sqrt{RG} + s\sqrt{LC} = \gamma$$

za slučaj sinusne pobude $s = j\omega \rightarrow \gamma = \sqrt{RG} + j\omega\sqrt{LC}$

$$\alpha = R\sqrt{\frac{C}{L}} = \sqrt{RG} \qquad \beta = \omega\sqrt{LC} \qquad v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

3. RC - LINIJA

$$\boxed{G = 0 \quad L = 0}$$

$$Z_0 = \sqrt{\frac{R+sL}{G+sC}} = \sqrt{\frac{R}{sC}} = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}}\cdot e^{-j45^\circ} = Z_0$$

$$\gamma = \sqrt{R\cdot j\omega C} = \sqrt{\omega RC}\,e^{j45^\circ} = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}} \hookrightarrow \alpha = \beta = \sqrt{\frac{\omega RC}{2}}$$

4. LINIJA S MALIM GUBICIMA

$$\boxed{\omega L \gg R \qquad \omega C \gg G}$$

$$\gamma = j\omega\sqrt{LC}\sqrt{\left(1-j\frac{R}{\omega L}\right)\left(1-j\frac{G}{\omega C}\right)}$$

$$\simeq j\omega\sqrt{LC}\left(1-j\frac{R}{2\omega L}\right)\left(1-j\frac{G}{2\omega C}\right)$$

$$Z_0 = \sqrt{\frac{L}{C}}\,e^{-j\left(\frac{R}{2\omega L}-\frac{G}{2\omega C}\right)}$$

$$\gamma \simeq \left[\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right] + j\omega\sqrt{LC}$$

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \qquad \beta = \omega\sqrt{LC}$$

Ulazna impedancija linije

$$Z_{ul} = \frac{U(0)}{I(0)} \qquad Z_2 = \frac{U(l)}{I(l)} \; ?$$

$$Z_{ul} = \frac{U(l)\cdot ch(\gamma l) + Z_0 I(l)\cdot sh(\gamma l)}{\frac{U(l)}{Z_0} sh(\gamma l) + I(l) ch(\gamma x)} = \frac{Z_2 ch\gamma l + Z_0 sh\gamma l}{\frac{Z_2}{Z_0} sh\gamma l + ch\gamma l}$$

$$Z_{ul} = Z_0 \frac{Z_2 ch\gamma l + Z_0 sh\gamma l}{Z_2 sh\gamma l + Z_0 ch\gamma l}$$

$\xrightarrow{Z_2 = 0}$ $Z_{ul} = Z_0 th\gamma l = \frac{1}{y_{11}} = Z_k$

$\xrightarrow{Z_2 = \infty}$ $Z_{ul} = Z_0 cth\gamma l = Z_p = Z_{11}$

$^* Z_0 = \sqrt{Z_p \cdot Z_k}$

$$Z_{ul} = Z_0 \frac{e^{2\gamma l} + \Gamma_2}{e^{2\gamma l} - \Gamma_2}$$

$\xrightarrow{Z_2 = Z_0}$ $Z_{ul} = Z_0 \Rightarrow$ prilagođenje

Beskonačno duga linija

nikada nema povratnog vala → nema kraja pa se nema što vratiti

$l \to \infty$

Samo polazni val $\quad U(0) = A_1 \quad I(0) = \frac{A_1}{Z_0}$

$$\left.\begin{aligned} U(x) &= U(0)e^{-\gamma x} \\ I(x) &= \frac{U(0)}{Z_0} e^{-\gamma x} \end{aligned}\right\} Z_{ul} = \frac{U(0)}{I(0)} = Z_0$$

Linija zaključena sa Z_0 → kao beskonačno duga linija

$Z_2 = Z_0 \rightarrow \Gamma_2 = 0$ nema refleksije

$A_2 = \Gamma_2 A_1 e^{-2\gamma l} \Rightarrow 0$

$$U(x) = A_1 e^{-\gamma x} = U(0)e^{-\gamma x}$$

$$I(x) = \frac{U(0)}{Z_0} e^{-\gamma x}$$