2.2 Trigonometrijski Furierov red

+ ideja da se periodična fj'a zapiše pomoću Furierovog reda

$$\frac{1}{2},\ \overbrace{\cos x,\ \sin x}^{2\pi},\ \overbrace{\cos 2x,\ \sin 2x}^{\pi},\ \dots,\ \overbrace{\cos nx,\ \sin nx}^{\frac{2\pi}{n}},\ \dots$$

$\Rightarrow$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ Furierov red funkcije f

- $a_0, a_1, a_2, \dots, a_n, b_1, \dots b_n$ - Furierovi koeficijenti
- pribrojnici u sumi – harmonici
- f je funkcija perioda 2π

Furierov polinom
$\hookrightarrow$ kada ima neki konačni član (ne ide u ∞)

Pitanja:

(1) Ako je f periodična 2π, kada će postojati njen Furier. red?

(2) Ako postoji Furierov red, kako računati koeficijente?

najlakše, ništa razmišljati samo računati, s tim krećemo

(3) U kojem smislu Furierov polinom aproksimira fj'u f?

Ortogonalnost funkcija

Funkcije $f, g : [a,b] \to \mathbb{R}$ su orto na $[a,b]$ ako vrijedi $\int_a^b f(x) g(x) dx = 0$

Lema: Trig sustav (*) je ortogonalan na $[-\pi, \pi]$ tj.

$$\int_{-\pi}^{\pi} \cos nx\, dx = \begin{cases} 0, & n>0 \\ 2\pi, & n=0 \end{cases}, \quad \int_{-\pi}^{\pi} \sin nx\, dx = 0$$

$$\int_{-\pi}^{\pi} \sin nx \cdot \sin mx\, dx = \begin{cases} \pi, & n=m \\ 0, & n \neq m \end{cases}, \quad \int_{-\pi}^{\pi} \sin nx \cdot \cos mx\, dx = 0$$

$$\int_{-\pi}^{\pi} \cos nx \cdot \cos mx\, dx = \begin{cases} \pi, & m=n \\ 0, & m \neq n \end{cases}$$

▶ Pretpostavimo $f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx) \Big/ \int_{-\pi}^{\pi}$

$$\int_{-\pi}^{\pi} f(x)dx = \underbrace{\frac{a_0}{2} \int_{-\pi}^{\pi} \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)dx}$$

općenito ne smije se zamijeniti $\int$ sa $\sum$, ali ovdje može jer Fourierov red trig jednoliko

$$\int_{-\pi}^{\pi} f(x)dx = \frac{a_0}{2}\cdot 2\pi + \sum_{m=1}^{\infty} \Big(a_m \underbrace{\int_{-\pi}^{\pi} \cos mx\,dx}_{=0} + b_m \underbrace{\int_{-\pi}^{\pi} \sin mx\,dx}_{=0}\Big)$$

$$\boxed{\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = a_0}$$

* ključno da je sustav orto!

$$\to \int_{-\pi}^{\pi} f(x)\cos mx\,dx = \frac{a_0}{2}\underbrace{\int_{-\pi}^{\pi} \cos mx\,dx}_{=0} + \sum_{m=1}^{\infty}\Big(a_m \int_{-\pi}^{\pi} \cos nx \cos mx\,dx + b_m \underbrace{\int_{-\pi}^{\pi} \cos nx \cdot \sin mx\,dx}_{=0}\Big)$$

$\int_{-\pi}^{\pi} \cos nx \cos mx\,dx$: $= 0$ za $m \neq n$, $= \pi$ za $m = n$

$$\Rightarrow \boxed{\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos mx\,dx = a_m}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos mx\,dx \quad n \geq 1$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx, \quad n \geq 1$$

* a_n ide uz cos
  b_n ide uz sin

Funkcija f ima Fourierov red $S(x)$

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

? Kada je $S(x) = f(x)$?

Postojanje i konvergencija Furierovog reda

Direkletovi uvjeti Kažemo da f zadovoljava D-uvjete na intervalu $[a,b]$ ako vrijedi:

① f je po dijelovima neprekinuta i ? vjerni? prekidi su prve vrste

② f je monotona ili nema najviše konačan broj strogih ekstrema

$\lim\limits_{x\to a^+} f(x) = f(a+0)$ limes zdesna

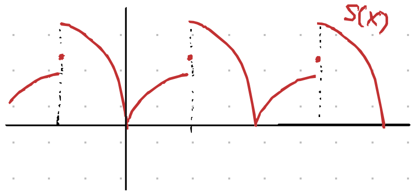
$\lim\limits_{x\to a^-} f(x) = f(a-0)$ limes slijeva

TM osnovni za Furierov red

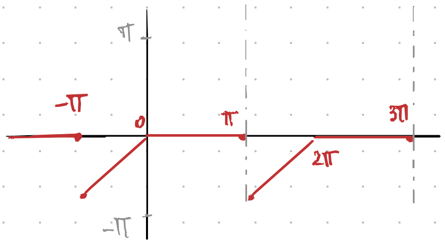
Neka je f po dijelovima glatka periodična funkcija perioda 2π koja zadovoljava Direkletove uvjete. Tada vrijedi F. red za $x\in[-\pi,\pi]$ i vrijedi:

1) $S(x) = f(x)$, ako je f neprekinuta u x

2) $S(x) = \frac{1}{2}\left(f(x-0)+f(x+0)\right)$, ako f ima prekid u x



Primjer: $T=2\pi$



$$f(x)=\begin{cases} x, & x\in[-\pi,0] \\ 0, & x\in[0,\pi] \end{cases}$$

$S(x) \overset{?}{=} f(x)$ određujemo po def ne po računu!

↳ odredimo u kojim točkama nisu jednaki

$S(x) \overset{?}{=} f(x) \quad \forall x, \in \mathbb{R}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx = \frac{1}{\pi}\frac{x^2}{2}\Big|_{-\pi}^{0} = \frac{-\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} x\cos nx\,dx = \frac{1}{\pi}\left(\frac{x}{n}\sin nx\Big|_{-\pi}^{0} - \frac{1}{n}\int_{-\pi}^{0}\sin nx\,dx\right) = \frac{-1}{n\pi}\int_{-\pi}^{0}\sin nx\,dx = \frac{1}{n^2\pi}\cos nx\Big|_{-\pi}^{0}$$

$$a_n = \frac{1}{n^2\pi}\left(1-\cos n\pi\right) \qquad n\geq 1$$

$a_{2n}=0, \quad n\geq 1 \qquad a_2=a_4=\dots=a_{2n}=0$

$$\boxed{a_{2n+1} = \frac{2}{(2n+1)^2\cdot\pi}} \qquad n\geq 0 \qquad \text{npr } a_1=\frac{2}{\pi},\ a_3=\frac{2}{9\pi},\dots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx = \frac{1}{\pi}\left(-\frac{x}{n}\cos nx\Big|_{-\pi}^{0} - \int_{-\pi}^{0}\frac{1}{n}\cos nx\,dx\right) = \frac{1}{\pi}\left(\frac{\pi}{n}\cdot\cos \pi n - \frac{1}{n^2}\sin nx\Big|_{-\pi}^{0}\right)$$

$u=x \to du=dx$

$dv=\sin nx \to \cos nx\cdot\frac{1}{n}$

$$= -\frac{\cos n\pi}{n} \quad \cos n\pi=(-1)^n \qquad = \frac{(-1)^{n+1}}{n} = b_n \qquad n\geq 1$$

$$f(x) = \frac{-\pi}{4} + \frac{2}{\pi}\left(\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x\right) + \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \dots$$

ne kužim koji je smisao?

▸ $T = 2L$ period funkcije f

$g(x) = f\left(\frac{T}{2\pi}x\right)$ Provjeri ima li g period 2π

$$g(x+2\pi) = f\left(\frac{T}{2\pi}(x+2\pi)\right) = f\left(\frac{T}{2\pi}x + T\right) = f\left(\frac{T}{2\pi}x\right) = g(x) \ \forall$$

$$g(x) = f\left(\frac{T}{2\pi}x\right) = f\left(\frac{L}{\pi}x\right)$$

$\cos\frac{2\pi x}{L}$ // $\sin\frac{2\pi x}{L}$ //

$$g\left(\frac{\pi x}{L}\right) = f(x) \longrightarrow f(x) = g\left(\frac{\pi x}{L}\right) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(n\frac{\pi x}{L}\right) + b_n \sin\left(n\frac{\pi x}{L}\right)$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$a_0, a_n, b_n = ?$

$$f(x) = g\left(\frac{\pi x}{L}\right) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$$

$T = 2L$

g 2π

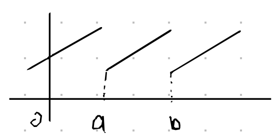
f $T = 2L$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\xi)\,d\xi = \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{L\xi}{\pi}\right)d\xi$$

$$\left(\begin{array}{l} x = \frac{L\xi}{\pi} \\ dx = \frac{L}{\pi}\,d\xi \end{array}\right) = \frac{1}{\pi}\int_{-L}^{L} f(x)\,\frac{\pi}{L}\,dx$$

T- formule

$T = b - a$



$$f(x) \sim \frac{a}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{T} + b_n \sin\frac{2n\pi x}{T}$$

$$a_0 = \frac{2}{T}\int_a^b f(x)\,dx$$

$$a_n = \frac{2}{T}\int_a^b f(x)\cos\frac{2n\pi x}{T}\,dx \qquad b_n = \frac{2}{T}\int_a^b f(x)\sin\frac{2n\pi x}{T}\,dx$$

f - parna ima samo kosinuse → a

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

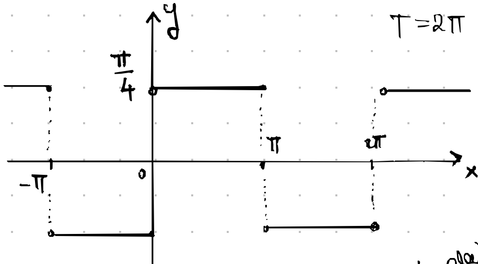
$$\to a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$\to a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

f - neparna $f(x) \sim \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L} \longrightarrow b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$

Primjer: $f(x)=\frac{\pi}{4}$ razvij u F red na $[0,\pi]$ po sinus funkcijama

suma $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots$?



$T=2\pi$

$f(x)=\sum_{n=1}^{\infty} b_n \sin(nx) \quad \forall x \neq k\pi$ $\qquad S(x)=f(x) \quad \forall x \neq k\pi$

$$b_n=\frac{2}{\pi}\int_0^{\pi}\frac{\pi}{4}\cdot\sin(nx)\,dx=\frac{-1}{2}\cdot\frac{1}{n}\cos(nx)\Big|_0^{\pi}$$

$$b_n=\frac{1}{2n}\left(1-(-1)^n\right)$$

propadaju za parne

$b_{2n}=0 \quad b_2=b_4=\dots=0$

$$b_{2n+1}=\frac{1}{2(2n+1)}\cdot 2=\frac{1}{2n+1}$$

$n=0 \quad b_n=1$

$n=1 \quad b_n=\frac{1}{3}$ $\qquad f(x)\sim$

$n=2 \quad b_n=\frac{1}{5}$

$\vdots$

$$f(x)\sim \sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+\dots$$

$\downarrow$

$$\frac{\pi}{4}=\sum_{n=0}^{\infty}\frac{\sin(2n+1)x}{2n+1}$$

$\cancel{x\neq k\pi}$

$x\in\left[0,\frac{\pi}{4}\right]$ bolji način

$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots$?

$x=\frac{\pi}{2}$

$$x=\frac{\pi}{2}\longrightarrow \sum_{n=0}^{\infty}\frac{\sin(2n+1)\cdot\frac{\pi}{2}}{2n+1}=\sin\frac{\pi}{2}+\frac{1}{3}\sin\frac{3\pi}{2}+\frac{1}{5}\sin\frac{5\pi}{2}+\frac{1}{7}\sin\frac{7\pi}{2}+\dots$$

2π $\quad f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \, dx$

? fja je jednaka Furierovom redu u točkama neprekinutosti

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx \qquad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad n \geq 1$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \quad n \geq 1$$

$2L$ $\quad f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \cos \frac{n\pi x}{L}$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\,dx \qquad a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} \, dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} \, dx$$

f - parna $\quad f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} \qquad a_0 = \frac{2}{L} \int_0^L f(x)\,dx \qquad a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} \, dx$

T $\quad T = 2L \qquad T = b - a$

$$f(x) \sim \frac{a_0}{2} + \sum_{n \geq 1}^{\infty} a_n \cos \frac{2n\pi x}{T} + b_n \sin \frac{2n\pi x}{T}$$

$$a_0 = \frac{2}{T} \int_a^b f(x)\,dx \qquad a_n = \frac{2}{T} \int_a^b f(x) \cos \frac{2n\pi x}{T} \qquad b_n = \frac{2}{T} \int_a^b f(x) \sin \frac{2n\pi x}{T}$$

Svojstva Furierovog reda

$f(x) = C\sin(\omega x + \varphi)$ $\qquad C = \sqrt{A^2+B^2}$ $\qquad tg\,\varphi = \frac{A}{B}$

$\omega_0 = \frac{2\pi}{T}$ $\qquad \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(n\omega_0 x) + b_n\sin(n\omega_0 x)$

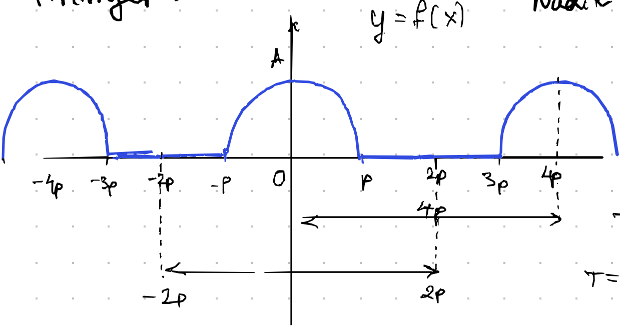
$= \pm\frac{C_0}{2} + \sum c_n \sin(n\omega_0 x + \varphi_n)$ $\quad c_0 = |a_0|$ $\quad c_n = \sqrt{a_n^2 + b_n}$ $\quad tg\,\varphi_n = \frac{a_n}{b_n}$

(a_n) kosinusni spektar $\qquad c_n$ - amplitudni spektar
(b_n) sinusni spektar $\qquad \varphi_n$ - fazni spektar
} diskretni spektri

$\boxed{TM}$ Ako periodične fje f i g zadovoljavaju i Diracove uvjete i imaju isti diskretni spektar, one se podudaraju osim možda u točkama prekida.

Primjer:

Nađite kosinusni spektar



$$f(x) = \begin{cases} A\cos\frac{\pi}{2p}x & |x| \leq p \\ 0, & p < |x| \leq 2p \end{cases}$$

$T = 4p \qquad L = 2p$

$T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T} \qquad \omega = \frac{2\pi}{4p} = \frac{\pi}{2p}$

$\cos\frac{\pi}{2p}$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{\cancel{2}}{2p}\int_0^{2p} A\cdot\cos\frac{\pi}{2p}x\,dx$$

$$a_0 = \frac{2A}{\pi}$$

$* \cos x\cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$

$$a_n = \frac{2}{2p}\int_0^p A\cos\left(\frac{\pi}{2p}x\right)\cos\left(\frac{n\pi x}{2p}\right)dx = \frac{A}{p}\int_0^p \cos\left(\frac{\pi x}{2p}\right)\cos\left(\frac{n\pi x}{2p}\right)dx =$$

$$a_n = \frac{A}{2p}\int_0^p \left(\cos\left(\frac{\pi x + n p x}{2p}\right) + \cos\frac{x(\pi - n\pi)}{2p}\right)dx =$$

$$a_n = \frac{A}{2p}\left(\frac{2p}{\pi(n+1)}\sin \qquad ??\right.$$

$$a_0 = \frac{2A}{\pi} \qquad a_n = \frac{A}{\pi}\left(\frac{\sin(n+1)\frac{\pi}{2}}{n+1} + \frac{\sin(n+1)\frac{\pi}{2}}{n-1}\right) = \frac{-2A}{\pi}\cdot\frac{\cos\frac{n\pi}{2}}{n^2-1} \qquad n \geq 2$$

$a_1 = ?$ $\quad$ 1. način: $a_1 = \frac{2}{2p}\int_0^p A\cos^2\frac{\pi x}{2p}\,dx$

2. način $\quad a_1 = \lim_{n\to 1} a_n$ $\quad$ u redu je, kod F reda smijemo zamijeniti poredak $\lim$ i $\int$.

$$a_n = \frac{-2A}{\pi}\frac{\cos\frac{n\pi}{2}}{n^2-1}, \quad n \neq 1$$

$$\lim_{n\to 1} a_n = \frac{-2A}{\pi}\lim_{n\to 1}\frac{\cos\frac{n\pi}{2}}{n^2-1} \overset{L'H}{=} \frac{-2A}{\pi}\lim_{n\to 1}\frac{-\frac{\pi}{2}\sin\frac{n\pi}{2}}{2n} = \frac{2A}{\pi}\cdot\frac{\pi}{2}\cdot\frac{1}{2} = \frac{A}{2}$$

Kosinusni spektar:

$$a_0 = \frac{2A}{\pi} \qquad a_1 = \frac{A}{2} \qquad a_n = \frac{-2A}{\pi}\frac{\cos\frac{n\pi}{2}}{n^2-1} \qquad n \geq 2$$

amplitudni spektar $\quad c_n = |a_n|$

$c_0 = \frac{2A}{\pi}, \quad c_1 = \frac{A}{2}$

$$c_{2n} = |a_{2n}| = \frac{2A}{\pi}\left|\frac{(-1)^{nešto}}{4n^2-1}\right|$$

$2n+1 \leftarrow \quad n = 2k+1$

$2n \leftarrow \quad n = 2k$

$c_{2n+1} = 0 \quad$ tj. $c_3 = c_5 = c_7 = \dots = 0$

$$c_{2n} = \frac{2A}{\pi}\cdot\frac{1}{4n^2-1} \qquad n \geq 1 \;? \quad \text{ne vidim}$$

Integriranje i deriviranje Furierovog reda

$$\int f(x)dx = \frac{a_0}{2}\int dx + \int \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)dx$$

$$\int f(x)dx = \dots + a_n \int \cos nx\, dx + \dots$$

$$= \dots + a_n \cdot \frac{1}{n} \sin n\, dx + \dots$$

Napomena: F red f kvg $\rightarrow \int f(x)dx =$ kvg

$\Downarrow$ f kvg ako f za D-uvjete ???

Parsevalova jednakost

$\omega_0 = \frac{2\pi}{T}$ $\quad \left\{ \frac{1}{2}, \sin(n\omega_0 x + \varphi_n) \right\}$ ortogonalni sustav na $[a,b]$ ako je $T = b-a$

tj $\longrightarrow$ $\int_a^b \sin(n\omega_0 x + \varphi_n)\cdot \sin(m\omega_0 x + \varphi_n) = 0$ za, $n \neq m$

$\rightarrow \int_a^b \frac{n}{2} \sin(n\omega_0 x + \varphi_n)\,dx = 0, \quad n \geq 1$

$$\int_a^b \sin^2(n\omega_0 x + \varphi_n)\,dx = \frac{T}{2} \longrightarrow \int_a^b \left(\frac{1}{2}\right)dx = \frac{1}{2}x\Big|_a^b = \left(\frac{1}{2}\right)(b-a) = \frac{T}{4}$$

$$\int_a^b |f(x)|^2\,dx = \int_a^b \left(\frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \sin(n\omega_0 x + \varphi_n)\right)^2 dx \quad \left[\text{koristi se } f(x) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \sin(n\omega_0 x + \varphi)\right]$$

$$= \int_a^b \frac{c_0^2}{4}\,dx + \int_a^b \sum_{n=1}^{\infty} c_n^2 \sin^2(n\omega_0 x + \varphi_n)\,dx = c_0^2 \cdot \frac{T}{4} + \sum_{n=1}^{\infty} c_n^2 \cdot \frac{T}{2}$$

$$\int_a^b |f(x)|^2\,dx = c_0^2 \cdot \frac{T}{4} + \sum_{n=1}^{\infty} c_n^2 \cdot \frac{T}{2}$$

$$(c_n^2 = a_n^2 + b_n^2)$$

$$\int_a^b |f(x)|^2\,dx = c_0^2 \cdot \frac{T}{4} + \frac{T}{2}\sum_{n=1}^{\infty} c_n^2 \longrightarrow \int_a^b |f(x)|^2\,dx = \frac{T}{4}a_0^2 + \frac{T}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

$$\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty} a_n^2 + \sum_{n=1}^{\infty} b_n^2 = \frac{2}{T}\int_a^b |f(x)|^2\,dx$$

Primjer:

Notebook 2 prije ovog

Kompleksni oblik kompleksnog reda

$f(t)$ fja perioda T, $T = b - a$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$
$$e^{-i\alpha} = \cos\alpha - i\sin\alpha \qquad /- \quad /+$$

$$e^{i\alpha} - e^{-i\alpha} = 2i\sin\alpha \; /:2i$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}$$

$$e^{i\alpha} + e^{-i\alpha} = 2\cos\alpha \; /:2$$
$$\cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

+ ideja, da članove F reda ne pišemo kao $\sin\alpha$ i $\cos\alpha$ nego u obliku $e^{i\alpha}$

$$a_n\cos n\alpha + b_n\sin n\alpha = c_n e^{in\alpha} + c_{-n} e^{-in\alpha}$$
$$= a_n \cdot \frac{e^{in\alpha} + e^{-in\alpha}}{2} + b_n \cdot \frac{e^{in\alpha} - e^{-in\alpha}}{2i}$$
$$= (\;\cdot\;)e^{in\alpha} + (\;\;)e^{-in\alpha} = \left(\frac{a_n}{2} + \frac{b_n}{2i}\cdot\frac{i}{i}\right)e^{in\alpha} + \left(\frac{a_n}{2} - \frac{b_n}{2i}\cdot\frac{i}{i}\right)e^{-in\alpha}$$
$$= \underbrace{\left(\frac{a_n}{2} - i\frac{b_n}{2}\right)}_{c_n} e^{in\alpha} + \underbrace{\left(\frac{a_n}{2} + i\frac{b_n}{2}\right)}_{c_{-n}} e^{-in\alpha}$$

$$\alpha = \frac{2\pi x}{T} \longrightarrow \text{F red (T formule)}: \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{T} + b_n\sin\frac{2n\pi x}{T}\right)$$

$$\Rightarrow c_0 + \sum_{n=1}^{\infty} c_n\left(e^{in\frac{2\pi x}{T}}\right) + c_{-n}\left(e^{-in\frac{2\pi x}{T}}\right) \Rightarrow \boxed{\sum_{n=1}^{\infty} c_n e^{\frac{2in\pi x}{T}}}$$ kompleksni oblik F reda

! ne uzimati pomoćnu varijablu x jer ju imamo u argumentu!

$$a_n = \frac{2}{T}\int_a^b f(\xi)\cos\frac{2n\pi\xi}{T}\,d\xi$$
$$b_n = \frac{2}{T}\int_a^b f(\xi)\sin\frac{2n\pi\xi}{T}\,d\xi$$
$$c_n = \frac{1}{2}(a_n - ib_n)$$

$$\to c_n = \frac{1}{T}\left(\int_a^b f(\xi)\cdot\cos\frac{2\pi\xi}{T}\,d\xi - i\int_a^b f(\xi)\sin\frac{2n\pi\xi}{T}\,d\xi\right)$$
$$= \frac{1}{T}\int_a^b f(\xi)\underbrace{\left[\cos\frac{2\pi n\xi}{T} - i\sin\frac{2\pi n\xi}{T}\right]}_{e^{-i\frac{2n\pi\xi}{T}}}d\xi \Rightarrow \boxed{\frac{1}{T}\int_a^b f(\xi)e^{-i\frac{2n\pi\xi}{T}}\,d\xi}$$