ELEKTRODINAMIKA

Magnetska i Lorenzova sila

→ $F_m \sim B \sim Q \sim v$ $\quad F_m = B \cdot q \cdot v$

→ $F_m \perp v \perp B$ ↳ Magnetska sila $\boxed{\vec{F}_m = q \cdot \vec{v} \times \vec{B}}$ — izvor mag sile je naboj

→ veza između magnetskih i električnih pojava je Lorenzova sila

Lorenzova sila = električna sila (Columbova) + magnetska

$$\boxed{F_L = q \cdot E + q \cdot \vec{v} \times \vec{B}}$$

Rad F_m: ↳ $\vec{v} \perp \vec{B}$ → skalarni produkt je 0! Rad je 0

↳ $W_m = \int \vec{F}_m \cdot d\vec{r} = \int q \overbrace{(\vec{v} \times \vec{B}) \cdot \vec{v}}^{0} \, dt = \underline{0}$

▸ mag sila ne obavlja rad mijenjanjem položaja

· znači $F_m \perp (v \perp B)$ ⟶ $F_m \cdot v = 0$ ⟹ $W = 0$

▸ ALI ujedno mijenja smjer → centripet

Kružno gibanje u mag. polju

· F_m — ne vrši rad, okomita na $\vec{v}$, mijenja smjer q

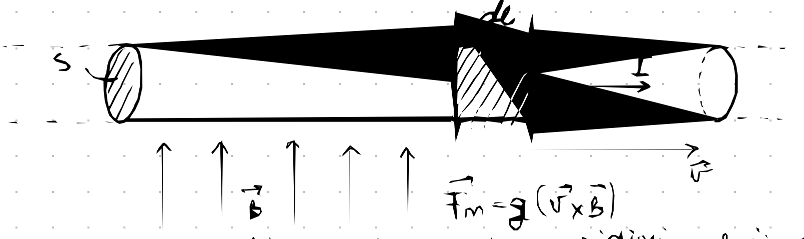
F_m je centripetalna sila

$$F_m = \frac{m v^2}{R} = Q v B \Rightarrow \boxed{R = \frac{m v}{q B}}$$

⟶ period (ako je $v = \omega \cdot R$)

$$\omega = \frac{v}{R} = \frac{qB}{m} = 2\pi f \longrightarrow \frac{qB}{2\pi m} = \frac{1}{T} \longrightarrow \boxed{T = 2\pi \frac{m}{qB}}$$

Sila na vodič u mag polju



$\vec{F}_m = q(\vec{v} \times \vec{B})$

↳ pojedini naboji će se gibati brzinom $\vec{v}$

$I = n q v S$ ↳ $nq = \frac{dQ}{dV}$

$n = \frac{N}{V}$ (koncentracija)

element sile po vodiču dl: $\vec{F}_m = q \vec{v} \times \vec{B} \, / d$

$d\vec{F}_m = dq (\vec{v} \times \vec{B})$ ⟵ $I = \frac{dq}{dt}$ $\quad v = \frac{dl}{dt}$

$d\vec{l} = dl \cdot \hat{v}$

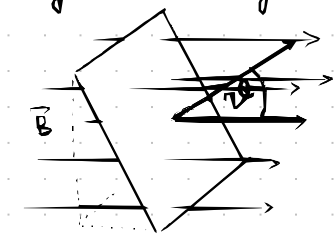
$\Rightarrow d\vec{F}_m = I \, dt \, \underbrace{\vec{v}}_{dl} \times \vec{B}$

$= I (d\vec{l}) \times \vec{B} = \underline{I (dl \cdot \hat{v}) \times \vec{B}} \, / \int$

$$\boxed{F_m = I \int^{B} d\vec{l} \times B}$$

Gaussov zakon i teorem za magnetsko polje

→ Gauss ⇒ iz gustoće silnica zaključujemo jakost mag polja



Tok mag. polja : $\Phi_B = B\,S\cos\alpha$

$(\sim \Phi_E)$

$\Phi_B = \vec{B}\,\vec{S}$ → podijelimo plohu na beskonačno male kvadratiće

$\Phi_B = \int B\, d\vec{S}$ [redacted]

⇒ nema izoliranih mag polova, mag. silnice se zatvaraju same u sebe

⟶ Mag. polje kroz bilokoju plohu je 0

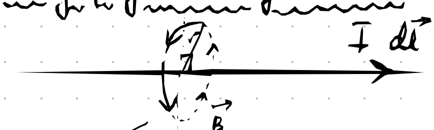
$$\oint_S \vec{B}\,d\vec{S} = 0$$

II. Maxwellova jed.

$$* \oint \vec{B}\,d\vec{S} \xrightarrow[\text{TM o diverg.}]{\text{Gaussov}} = \frac{Q_m}{\varepsilon_0}, \quad Q_m = 0 \rightarrow \boxed{\vec{\nabla}\vec{B} = 0}$$

posljedica nepostojanja mag monopola (nema izolacije pol.)

Mag. polje ravnog vodiča



B ⊥ na smjer gibanja struje

Pretpostavke Biot-Savartovog zakona

I) vekt mag polja $d\vec{B} \perp d\vec{l} \perp \vec{r}$

II) iznos $d\vec{B} \sim \frac{1}{r^2} \sim I\,dl$

III) $d\vec{B} \sim \sin(\measuredangle\, d\vec{l}, \vec{r})$

Biot-Savartov zakon

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,d\vec{l} \times \vec{r}}{r^2}$$

μ_0 – permitivnost u vakumu

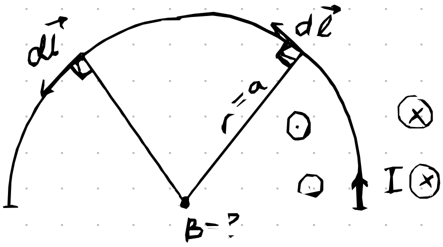
$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l} \times \vec{r}}{r^2}$$

$\left(\text{moći } \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2}\,\hat{r}\right)$

Primjer: Biot-Savartov zakon

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \hat{\vec{r}}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{I\, dl\, \overset{1}{r}\, \overset{1}{\sin 90}}{r^2}$$

$$B = \frac{\mu_0}{4\pi} I \int \frac{dl}{a^2}$$



→ budući da se radi o polukružnici

$l = [0, \frac{2\pi r}{2}] \rightarrow [0, \pi a]$

$$B = \frac{\mu_0}{4\pi} \cdot I \int_0^{\pi a} \frac{dl}{a^2} = \frac{\mu_0}{4\pi} \cdot I \cdot \frac{1}{a^{\cancel{2}}} \cdot \pi a$$ [redacted]

Amperov zakon

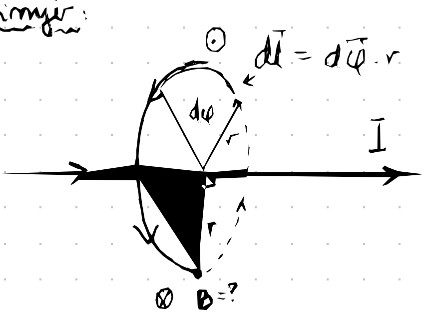
- na rubovima vodiča kroz koji prolazi struja I (rubovi površine s koja omeđuje površinu krivuljom l)

∟ stvara mag polje

[redacted]$d\vec{l} = \mu_0 I$ — cirkulacija polja $\vec{B}$

. koristimo u svrhu računanja mag polja oko vodiča

Primjer:



$d\vec{l} = d\vec{\varphi} \cdot r$ → polje ne ovisi o kutu → jednako svugdje na istom r

∟ cilindrična simetrija

$$\oint \vec{B} d\vec{l} = \oint \vec{B}\, r\, d\vec{\varphi} = \oint B\, r\, d\varphi \; \cancel{\cos 0}$$

$$= \int_0^{2\pi} B r\, d\varphi = \mu_0 I$$

$$B = \frac{\mu_0 I}{2 r \pi}$$

* uz pretpostavku da je žica $\langle -\infty, \infty \rangle$

Stokesov TM (TM o rotaciji)

* Gauss $\oint E\, d\vec{s} = \int \vec{\nabla} \vec{E}\, dV$ → veza plohe i volumena koji ona obuhvaća

→ ovdje: veza linije i plohe koju ona obuhvaća ⇒ $\oint \vec{F} d\vec{l} = \oint \vec{\nabla} \vec{F} \cdot d\vec{S}$

→ $\oint \vec{B} d\vec{s} = \oint_S \vec{\nabla} \times \vec{B} \cdot d\vec{s}$ ↳ gustoća struje: $I = \int \vec{J} d\vec{s}$

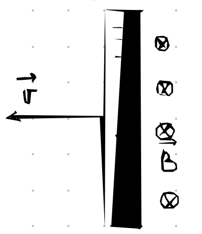
$\oint \vec{B} d\vec{s} = \mu_0 I$ → $\oint \vec{\nabla} \times \vec{B}\, d\vec{s} = \mu_0 \int \vec{J} d\vec{s}$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

* početak IV. MAX

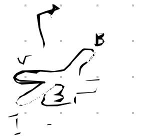
Faradayev zakon indukcije

▸ gibanje vodiča l u mag. polju $\vec{B}$ brzinom $\vec{v}$ $\Rightarrow$ $F_L = QE + q(\vec{v}\times\vec{B})$



$\vec{F}_L = q(\vec{E} + \vec{v}\times\vec{B})$

razdvajanje naboja

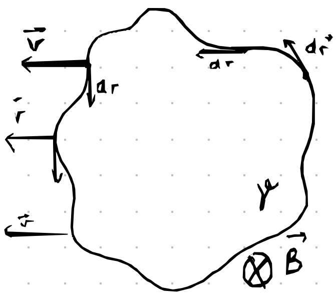


pravilo desne ruke

pojava mag. komponente F_L sile

· zbog premještanja naboja javlja se el. polje i time i napon (el. mot. sila)

$\hookrightarrow$ $E = \frac{\vec{F_L}}{q} = \vec{v}\times\vec{B}$



▸ E je jednako radu duž putanje $d\vec{r}$ po jedinicnom naboju q

$\llcorner$ integral el. mag. sile po jedinici naboja duž zatvorene krivulje γ

$\vec{v}\,dt \times d\vec{r} = d\vec{S}$ (površina)

$$\mathcal{E} = \oint_\gamma \frac{F_L}{q}\, d\vec{r}$$

$$\mathcal{E} = \oint_\gamma \frac{F_L}{q}\, d\vec{r} = \oint_\gamma (\vec{E} + \vec{v}\times\vec{B})\,d\vec{r} = 0 + \oint_\gamma \vec{v}\times\vec{B}\, d\vec{r}$$

potencijal je funkcija stanja / ne ovisi o putu ($\oint \vec{E}\, d\vec{l} = 0$)

$$\mathcal{E} = \frac{dt}{dt}\oint_\gamma \vec{v}\times\vec{B}\, d\vec{r} = \frac{d}{dt}\oint_\gamma d\vec{r}\,(\vec{v}\,dt \times \vec{B})$$

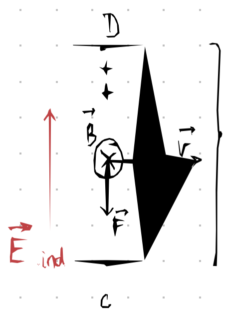
u formulama: $a(b\times c) = c(a\times b)$ $= -c(b\times a)$

$$\mathcal{E} = \frac{-d}{dt}\oint_\gamma \vec{B}\,\underbrace{(\vec{v}\,dt \times d\vec{r})}_{d\vec{S}} \Rightarrow \mathcal{E} = \frac{-d}{dt}\underbrace{\oint \vec{B}\cdot d\vec{S}}_{\phi_B} \Rightarrow \boxed{\mathcal{E} = \frac{-d}{dt}\phi_B}$$

promjena mag. polja u vremenu proporcionalna je nastanku napona ($\mathcal{E}$)

posljedica Lenzovog pravila („uvijek kontra")

$\hookrightarrow$ smjer $\mathcal{E}_{ind}$ je takav da stvorena struja (tj. napon) svojim učinkom želi poništiti uzrok koji ju je proizveo



prema pravilu desne ruke $\to$ $E_{ind} = \frac{\vec{F}}{q} = \vec{v}\times\vec{B}$

$\vec{F}$ bi trebao ići ↑

ALI $\vec{F}$ gibanje je uzrokovano induciranim nielektrostat. el. poljem

žica se električki polarizira (+ i −)

· $\mathcal{E}_{ind}$ je cirkulacija $\vec{E}_{ind}$

⇓

$\mathcal{E}_{ind} = \oint \vec{E}_{ind}\cdot d\vec{r}$

$\mathcal{E}_{ind} = \oint_0^l (\vec{v}\times\vec{B})\, d\vec{l} \Rightarrow (v\times B)l$ vektorski

$\longrightarrow$ kontra je pa je minus (−)

$\Rightarrow$ $\mathcal{E}_{ind} = vBl\sin\alpha$

▸ $\mathcal{E} = \oint_\gamma \vec{E}\,d\vec{r} = \frac{d}{dt}\phi_B = \frac{-d}{dt}\oint \vec{B}\,d\vec{S}$

III. MAXWELLOVA

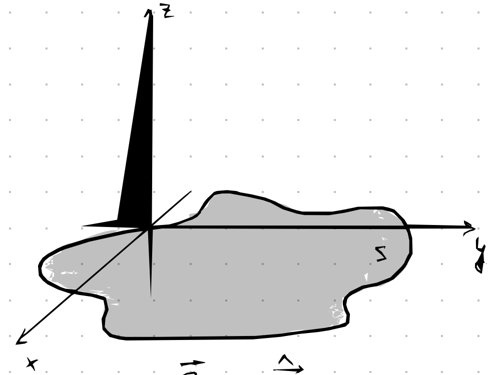
$\hookrightarrow$ $\oint \vec{E}\,d\vec{r} = \int \vec{\nabla}\times\vec{E}\, d\vec{S} \Rightarrow \int \vec{\nabla}\times\vec{E}\, d\vec{S} = \frac{-d}{dt}\int \vec{B}\,d\vec{S} \Rightarrow$ $\boxed{\vec{\nabla}\times\vec{E} = \frac{-\partial}{\partial t}\vec{B}}$

Primjer: $S = 0{,}65\,m^2$ u $z=0$

$$\vec{B} = B_0 \cos(\omega t)(\hat{y} + \hat{z}) \cdot \frac{1}{\sqrt{2}}$$

$B_0 = 0{,}05\,T$

$\omega = 10^3\,rad/s$ $\qquad \mathcal{E}_{ind} = ?$



$\vec{S} = S \cdot \hat{z}$

$$\mathcal{E}_{ind} = \frac{-d}{dt}\Phi = \oint \vec{E}\, d\vec{r} = \frac{-\partial}{\partial t}\int \vec{B}\, d\vec{S}$$

$$\Phi = \int \vec{B}\vec{S} = \vec{B} \cdot S \Rightarrow B_0 \cos(\omega t)(\hat{y} + \hat{z}) \cdot \frac{1}{\sqrt{2}} \cdot S \quad \hat{z}$$

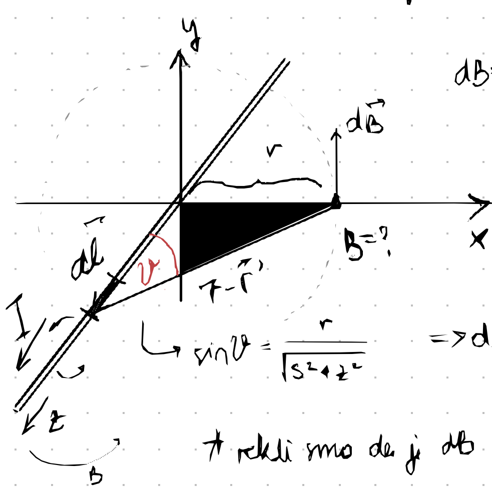
$$\Phi = \frac{B_0 \cdot S}{\sqrt{2}} \cos(\omega t) \longrightarrow \mathcal{E} = \frac{-\partial}{\partial t}\Phi = \frac{-B_0 S}{\sqrt{2}}(-\sin(\omega t)) \cdot \omega$$

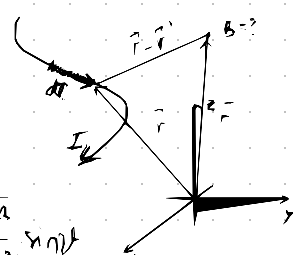
$\mathcal{E}_{max} \to$ onda kada je $\sin = 1 \longrightarrow$ $\boxed{\mathcal{E}_{max} = \frac{B_0 S}{\sqrt{2}}\omega}$

njihov vekt prod može biti samo na to Π

Primjer: Računanje $\vec{B}$ za stalne struje (Biot-Savart) $\qquad d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$

Beskonačna ravna žica, struja I





$$dB = \frac{\mu_0}{4\pi} I |dl| \frac{|r - r'|}{|\vec{r} - \vec{r}'|^3} \sin\vartheta$$

$$\Downarrow$$

$$dB = \frac{\mu_0}{4\pi} I\, dz \cdot \frac{\sqrt{z^2 + r^2}}{(\sqrt{z^2 + r^2})^{3/2}} \sin\vartheta$$

$\hookrightarrow \sin\vartheta = \frac{r}{\sqrt{s^2 + z^2}}$

$$\Rightarrow dB = \frac{\mu_0 I\, dz}{4\pi} \cdot \frac{1}{\sqrt{z^2 + s^2}} \cdot \frac{r}{\sqrt{z^2 + s^2}} = \frac{\mu_0 I r\, dz}{(z^2 + r^2)4\pi}$$

+ rekli smo da je dB u y osi $\rightarrow$ $B_y = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dz}{z^2 + r^2}$

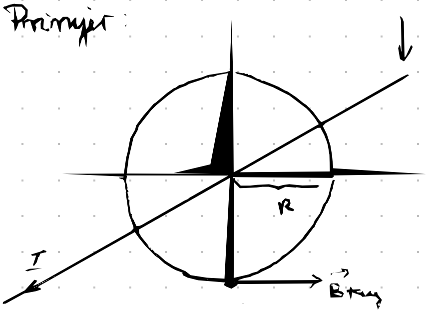
tablični int.
$\hookrightarrow$ kada je $[0, \infty)$, $\frac{1}{r^2}$
ali sada je duplo

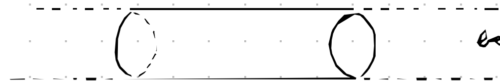
$$\Rightarrow B_y = \frac{\mu_0 I}{4\pi} \cdot r \cdot \frac{2}{r^2} \longrightarrow \boxed{B_y = \frac{\mu_0 I}{2\pi r}}$$

Amper - Maxwellov zakon

▶ **Ampèrov zakon**: $\oint_\gamma \vec{B}\,d\vec{r} = \mu_0 \underbrace{\int_S \vec{J}\cdot d\vec{S}}_{=I}$

Primjer:





beskonačno dugačak vodič

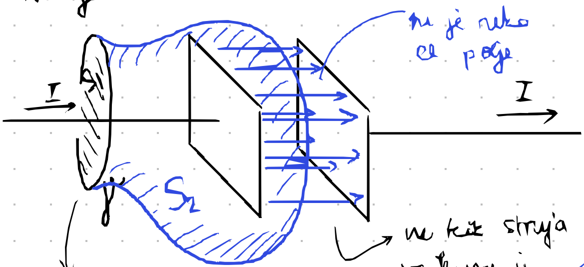
$$B = \frac{\mu_0 I}{2r\pi}$$ prema Biot-Savartu

isto je

$$\int \vec{B}\cdot d\vec{r} = \mu_0 \int \vec{J}\,d\vec{S} \longrightarrow B\cdot 2r\pi = \mu_0 I$$

* Ampere ne opisuje u potpunosti jer mu fali dio da je primjenjiv na sve

Primjer:



Amper - Maxwellov: IV. MAX jed.

$$\boxed{\oint \vec{B}\,d\vec{r} = \underbrace{\mu_0 \int \vec{J}\,d\vec{S}}_{\text{Amperov dio}} + \underbrace{\mu_0 \varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{S}}_{\text{MAXWELOV ČLAN}}}$$

① $\oint_\gamma \vec{B}\,d\vec{r} = \mu_0 I$ ✓

S_1: $\oint \vec{B}\,d\vec{r} = \mu_0 I + \underline{0}$ jer u području S_1 nema el. polja

S_2: $\oint \vec{B}\,d\vec{r} = 0$! prema Amperu

ali ovo nije 'shina'

↳ Ampèrov nije potpun

S_2: $\oint \vec{B}\,d\vec{r} = 0 + \text{MAXWELLOV ČLAN}$ zbog postojanja el. polja

$$E = \frac{\sigma}{\varepsilon_0} = \frac{\frac{q}{S}}{\varepsilon_0} = \underline{I\,t\,\frac{1}{\varepsilon_0 S}}$$

$$\Phi_E = E\,S = \frac{I t}{\varepsilon_0}$$

Max. član: $\mu_0 \varepsilon_0 \frac{d}{dt}\Phi_E = \mu_0 \cdot I$

▶ **Amper - Max drugi zapis**:

$$\underbrace{\oint_\gamma \vec{B}\,d\vec{r}}_{\text{Stokesov TM.}} = \mu_0 \int_S \vec{J}\,d\vec{S} + \mu_0 \varepsilon_0 \frac{d}{dt}\int_S \vec{E}\,d\vec{S}$$

$$\int_S (\vec{\nabla}\times\vec{B})\,d\vec{S} = \mu_0 \int_S \vec{J}\,d\vec{S} + \mu_0 \varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{S} \Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}}$$

IV. MAX. jed.