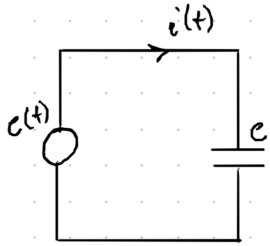
1.7. DIRACOVA DELTA FUNKCIJA



$i(t) = ?$

$$i(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

$e(t) = u(t)$

$e(t) = \frac{q(t)}{C}$

$$i(t) = \frac{dq(t)}{dt} = \frac{d(C\cdot e(t))}{dt} = C \cdot \frac{du(t)}{dt} \Big/ \int_{-\infty}^{\infty} \longrightarrow \int_{-\infty}^{\infty} i(t)\,dt = C(1-0) = \underline{\underline{C}}$$

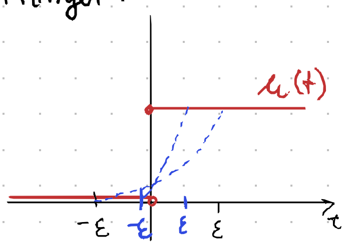
→ ovo je intuitivno računanje jer ova „funkcija" ima prekid pa ne bi trebala biti derivabilna → to se zovu DISTRIBUCIJE

$$\int_{-\infty}^{\infty} i(t)\,dt = 1$$

Tražimo f-ju koja zadane uvjete 1, 2.

→ Objekt koji će zadovoljavati te uvjete neće biti funkcija, ali ćemo ga zvati Diracova funkcija.

Primjer:



$$u_\varepsilon(t) = \begin{cases} 0, & t < -\varepsilon \\ \neq 0, & t \in [-\varepsilon, \varepsilon] \\ 1, & t > \varepsilon \end{cases}$$

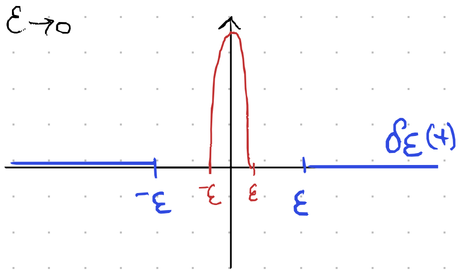
$\lim_{\varepsilon \to 0} u_\varepsilon(t) = u(t)$ $\exists\, u_\varepsilon'(t)$ $\forall t$

kada $\varepsilon \to 0$ oni su u step funkcija

što više smanjujemo $|\varepsilon|$ onda je što strmije uspinjanje

⟵ njihove $(u_\varepsilon(t))$ derivacije su $\delta_\varepsilon(t)$!!

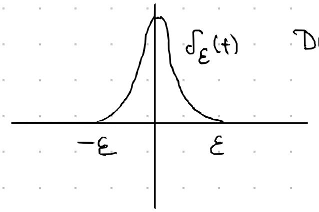
Diracovu $\delta_\varepsilon(t)$ definiramo kao $\delta_\varepsilon(t) = u_\varepsilon'(t)$ kada $\varepsilon \to 0$



Def: $\lim_{\varepsilon \to 0} \delta_\varepsilon(t) = \delta(t)$

$$\delta_\varepsilon(t) = u'_\varepsilon(t)$$

$$\delta(t) \overset{def}{=} \lim_{\varepsilon \to 0} \delta_\varepsilon(t)$$



Diracova δ- funkcija

uvjeti: $i(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$

$$\int_{-\infty}^{\infty} i(t)\,dt = 1$$

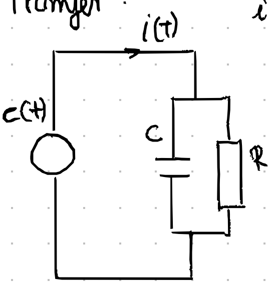
Provjera uvjeta:

$$\int_{-\infty}^{\infty} \delta_\varepsilon(t)\,dt = \int_{-\varepsilon}^{\varepsilon} u'_\varepsilon(t) = u_\varepsilon(t)\Big|_{-\varepsilon}^{\varepsilon} = 1 - 0 = 1$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = \int_{-\infty}^{\infty} \lim_{\varepsilon \to 0} \delta_\varepsilon(t)\,dt \overset{?}{=} \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} \delta_\varepsilon(t)\,dt = 1$$

Nap: Općenito ova jednakost ne vrijedi, ali za δ-fiju vrijedi.

Primjer:



$i(t) = ?$

$e(t) = u(t)$

$E(s) = \frac{1}{s}$

$$\cancel{Z(s) = \left(\frac{1}{Cs} + \frac{1}{R}\right)^{-1}}$$

$$\cancel{Z(s) = \frac{RCs}{R + Cs}} \quad ?$$

$$Z(s) = \frac{R}{RCs + 1}$$

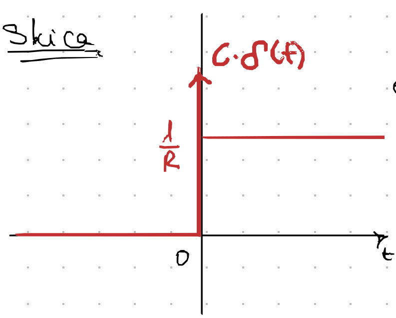
$$I(s) = \frac{1}{s} \cdot \frac{RCs + 1}{R}$$

$$I(s) = \frac{1}{s}\left(Cs + \frac{1}{R}\right)$$

$$I(s) = C + \frac{1}{Rs}$$

$$i(t) = C \cdot \delta(t) + \frac{1}{R} u(t)$$

Skica



električni udar