1. Vektorska funkcija i funkcije vise varijabli

zadaci sa ispita

JIR201

1. **(7 bodova)**

   (a) **(3b)** Odredite prirodnu domenu, sliku i nivo-plohe funkcije

   $$f(x, y, z) = 5 - \sqrt{x^2 + 4y^2 + 9z^2}.$$

   (b) **(4b)** Skicirajte i imenujte nivo-plohu koja prolazi točkom $T(\sqrt{3}, 1, 1)$, te odredite tangencijalnu ravninu na tu plohu u zadanoj točki T.

① (a)

$D_f = \mathbb{R}^3 \quad (x^2 + 4y^2 + 9z^2 \geq 0 \quad \forall (x,y,z) \in \mathbb{R}^3)$

$\mathrm{Im} f = \langle -\infty, 5]$ $\qquad f(x,y,z) = 5 - \underbrace{\sqrt{x^2+4y^2+9z^2}}_{\geq 0} \leq 5 \quad \forall (x,y,z) \in \mathbb{R}^3$

$\Rightarrow \forall\, C \in \langle -\infty, 5]$ postoji $(x,y,z) = (5-C,0,0)$ t.d.

$$f(5-C,0,0) = 5 - \underbrace{\sqrt{(5-C)^2}}_{\geq 0} = 5 - (5-C) = C \quad \checkmark$$

Nivo-plohe za $C \in \langle -\infty, 5]$

$$5 - \sqrt{x^2+4y^2+9z^2} = C$$

$$\underbrace{5-C}_{\geq 0} = \sqrt{x^2+4y^2+9z^2} \iff \underbrace{(5-C)}_{\text{nova konstanta}}^2 = x^2+4y^2+9y^2$$

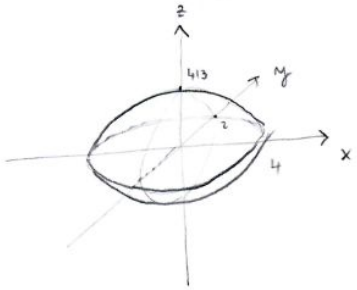
$\Rightarrow$ to su elipsoidi

① (b) nivo-ploha skozi $T(\sqrt{3}, 1, 1)$ je

$$5 - \sqrt{x^2 + 4y^2 + 9z^2} = f(\sqrt{3}, 1, 1) = 5 - \sqrt{16} = 1$$

$$4 = \sqrt{x^2 + 4y^2 + 9z^2}$$

$$\boxed{16 = \underbrace{x^2 + 4y^2 + 9z^2}_{g(x,y,z)}} \quad \text{elipsoid}$$



tangencijalna ravnina ... $\nabla g = (2x, 2y, 18z)$

$$\nabla g(\sqrt{3}, 1, 1) = (2\sqrt{3}, 2, 18) = \vec{m}$$

Tangencijalna ravnina gleši ... $(x - \sqrt{3})\,2\sqrt{3} + (y-1)\,2 + (z-1)\,18 = 0$

$$2\sqrt{3}x + 2y + 18z = 32$$