1.1. Furierov red

$$f(x)=C_0+\sum_{n=1}^{\infty} C_n \sin(n\omega x+\varphi_n)$$

ideja: periodične funkcije u ovom zapisu!

Opća sinusoida

$$f(x)=C\sin(\omega x+\varphi)=C(\sin\omega x\cdot\cos\varphi+\cos\omega x\cdot\sin\varphi)$$

$$=\underbrace{C\sin(\omega x)\cdot\cos\varphi}_{A}+\underbrace{C\cos(\omega x)\cdot\sin\varphi}_{B}$$

$$=A\cos\varphi+B\sin\varphi$$

$* \; B=C\cos(\omega x)$

$A=C\sin(\omega x)$

$A^2+B^2=C^2$

$\operatorname{tg}\varphi=\frac{A}{B}$

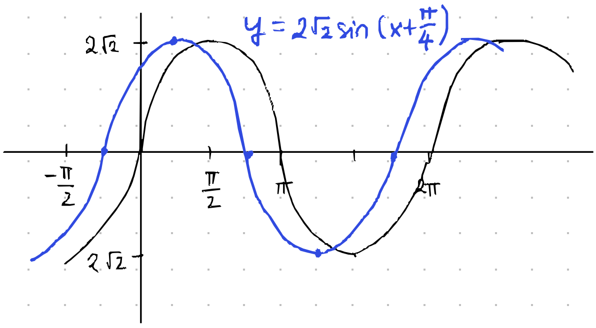
Primjer: Nacrtaj $f(x)=2\cos x+2\sin x$ u opću sinusoidu

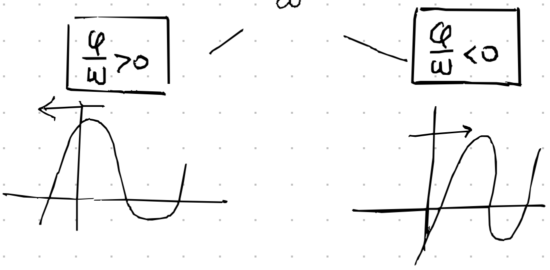
$C=\sqrt{A^2+B^2}\longrightarrow\sqrt{4+4}=2\sqrt{2}$

$\operatorname{tg}\varphi=\frac{2}{2}\longrightarrow\varphi=\frac{\pi}{4}$

$\Longrightarrow f(x)=2\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

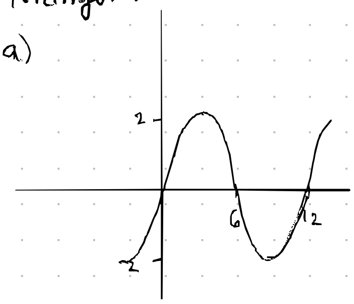
$\omega x+\varphi=0\longrightarrow x=\frac{-\varphi}{\omega}$





Primjer:

a)

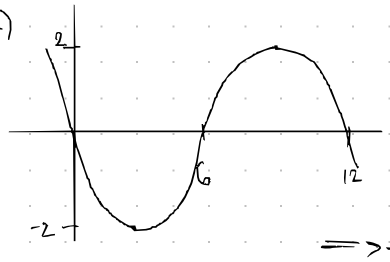


$T=12 \qquad T=\frac{2\pi}{\omega}\longrightarrow\omega=\frac{\pi}{6}$

$\varphi=0\longrightarrow$ nije pomaknuta

$\longrightarrow f(x)=2\sin\left(\frac{\pi}{6}x\right)$

a)



$f(x)=C\sin(\omega x+\varphi)=C\sin\omega\left(x+\frac{\varphi}{\omega}\right)$

$\frac{\varphi}{\omega}=6$ ili $\frac{\varphi}{\omega}=-6$

pomak je ili lijevo ili desno $\longrightarrow$ mi na slici vidimo $\frac{-\varphi}{\omega}$ koji može biti -6 ili 6, olage desno

$\Longrightarrow f(x)=2\sin\frac{\pi}{6}(x-6)$

ili $f(x)=2\sin\frac{\pi}{6}(x+6)$

$\frac{\varphi}{\omega}$

$f(x)=2\sin\left(\frac{\pi}{6}x-\pi\right)=$

$=2\sin\frac{\pi}{6}x\cos\pi-\underbrace{2\cos\frac{\pi}{6}\sin\pi}_{0}$

$=-2\sin\frac{\pi}{6}x$

Def - periodična funkcija

$f: D \to \mathbb{R}$, $D \subseteq \mathbb{R}$ je periodična ako $\exists T > 0$ t.d. $f(x+T) = f(x)$, $\forall x \in D$.

T nazivamo periodom f-je, a najmanji takav T, ako postoji je temeljni period

Primjer: Naći temeljni period

a) $f(x) = \sin^2 x$

$$f(x) = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{1}{2}\cos(2x) \quad \omega = 2$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$$

b) $f(x) = \sin(x^2)$

$$f(x+T) - f(x) = 0 \quad \forall x \in \mathbb{R}$$

$$\sin (x+T)^2 = \sin(x^2)$$

$$\sin(\underbrace{x^2 + 2Tx + T^2}) = \sin(x^2)$$

nije periodična f-ja

Operacije s periodičnim funkcijama

– suma f-ja različitih frekv –

$f(x) = A\cos\alpha x + B\sin\beta x\ ,\quad \alpha \neq \beta \quad \alpha, \beta > 0$

Je li periodična? Postoji li $T>0$ t.d $f(x+T) - f(x) = 0$

koristimo iz formule:

$^{*}\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\boxed{A\cos\alpha(x+T)} + \underline{B\sin\beta(x+T)} - \boxed{A\cos\alpha x} - \underline{B\sin\beta x} = 0$

$A(\cos\alpha(x+T) - \cos\alpha x) + B(\sin\beta(x+T) - \sin\beta x) = 0$

$A\left(-2\sin\alpha\frac{2x+T}{2}\cdot\sin\alpha\frac{T}{2}\right) + B\left(2\cos\beta\frac{2x+T}{2}\cdot\sin\frac{\beta T}{2}\right) = 0$

$-2A\sin\left(\alpha x + \frac{\alpha T}{2}\right)\cdot\underline{\sin\frac{\alpha T}{2}} + 2B\cos\left(\beta x + \frac{\beta T}{2}\right)\cdot\underline{\sin\frac{\beta T}{2}} = 0$

uočavamo:

$\sin\frac{\alpha T}{2} = 0 \qquad \frac{\alpha T}{2} = k_1\pi \longrightarrow T = \frac{2k_1\pi}{\alpha}$

$\sin\frac{\beta T}{2} = 0 \qquad \frac{\beta T}{2} = k_2\pi \longrightarrow T = \frac{2k_2\pi}{\beta}$

$\frac{k_1}{\alpha} = \frac{k_2}{\beta} \Rightarrow \boxed{\frac{\alpha}{\beta} = \frac{k_1}{k_2}}$

Funkcija f je <u>periodična</u> ako je <u>$\frac{\alpha}{\beta}$ racionalan broj</u>, kažemo da su α i β <u>sumjerljivi</u>

Primjer: Temeljni period?

$f(x) = \sin 2x + \cos 6x \qquad \frac{\alpha}{\beta} = \frac{6}{2} = 3$

$T = \pi \qquad T = \frac{\pi}{3}$

$\frac{\alpha}{\beta} = \frac{k_1}{k_2} \quad / \cdot k_2\beta$

$T = \frac{2k_1\pi}{\alpha} = \frac{2\cdot 1}{2}\pi = \underline{\pi}$

$k_2\alpha = k_1\beta$

$2k_2 = 6k_1$

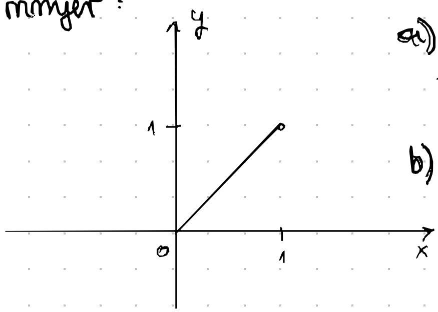
$3k_1 = k_2 \longrightarrow \begin{matrix} k_1 = 1 \\ k_2 = 3 \end{matrix}$

2.1.3 Periodična proširenja

$f: D \longrightarrow \mathbb{R}$

$f(-x) = -f(x) \quad \forall x \in D$ neparna $\qquad f(-x) = f(x) \quad \forall x \in D$ parna

Primjer:



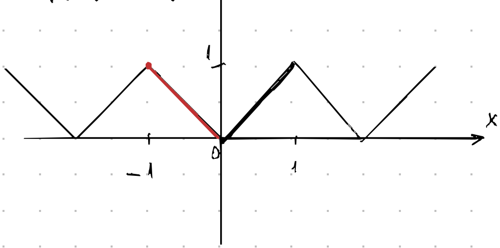
a) Nacrtaj graf periodičnog parnog proširenja funkcije f

b) –//– neparnog –//–

$T = 2$

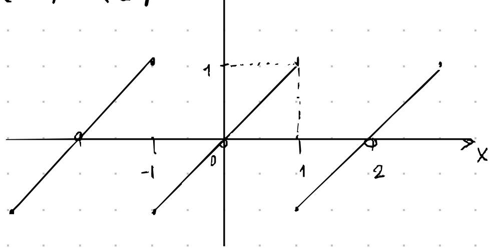
parna

a) $f(-x) = f(x)$



neparna

b) $f(-x) = -f(x)$



Primjer: $f(x) = \arcsin(\sin x)$

Rj:

$\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

$\arcsin : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$T = 2\pi \qquad f(x + 2\pi) = \arcsin(\sin(x + 2\pi))$

$= \arcsin(\sin x)$

$= f(x)$

$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \qquad \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(x) = \arcsin(\sin((x - \pi) + \pi))$

$= \arcsin(-\sin(x - \pi))$

$= -\arcsin(\sin(x - \pi)) \quad \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= -(x - \pi) = -x + \pi$

$$\longrightarrow f(x) = \arcsin(\sin x) = \begin{cases} x & x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ -x + \pi & \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \end{cases} \longrightarrow$$