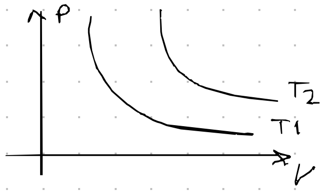
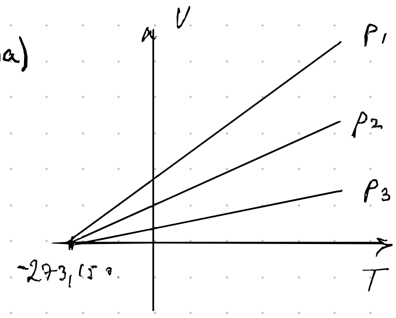
3.2 PLINSKI ZAKONI

- 3 eksperimentalna zakona



Boyle-Mariotteov zakon (izotermna promjena)

$p V = \underbrace{nR}_{konst}T$ → u izotermnim pojavama $T = konst.$ ⇒ $\boxed{pV = konst.}$



Guy-Lussacov zakon (izobarna promjena)

$p = konst$

$pV = nRT \to \frac{V}{T} = \frac{nR}{p} \to \boxed{\frac{V}{T} = konst}$

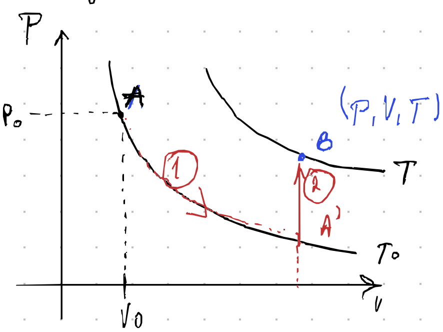
Charlesov zakon (izohorna)

$V = konst \to \frac{V}{nR} = \frac{T}{p} \to \boxed{\frac{p}{T} = konst.}$

→ Jednadžba stanja idealnog plina: $\boxed{\frac{pV}{T} = \frac{p_0 V_0}{T}}$

$p_0 = 101\,325\ Pa$ $\quad T_0 = 273,15\ K$

Primjer:



A → B

A → A'

$p_0 V_0 = p_{A'} \cdot V$

$\to p_{A'} = \frac{p_0 V_0}{V}$

A' → B

$\frac{p_{A'}}{T_0} = \frac{p}{T} \to p_{A'} = \frac{p T_0}{T}$

$\frac{p T_0}{T} = \frac{p_0 V_0}{V} \to \boxed{\frac{pV}{T} = \frac{p_0 V_0}{T}}$

Avogadrov zakon - jednaki volumeni svih plinova pri istom tlaku i temperaturi imaju jednak broj čestica

Br čestica - povezan s brojem molova ⇒ $N = \overset{[mol]}{n} \cdot N_A$

N_A - Avogadrova plinska konstanta $= 6,02 \times 10^{23}/mol$

V_{m0} - molarni volumen → s.u. $V = 22,4 \times 10^{-3}\ m^3/mol$

univerzalna plinska konst. $R = \frac{p_0 V_{0m}}{T_0} = 8,314 \frac{J}{mol\,K}$

$pV = \frac{p_0 V_0}{T_0} \cdot T$

$\frac{V}{N} = \frac{V_{m0}}{N_A} \to \frac{V}{n} = V_{m0}$

$\left.\right\} \frac{V}{n} = V_{m0} \Rightarrow pV = \underbrace{\frac{p_0 V_{m0}}{T_0}}_{R} n \cdot T$

$\boxed{pV = nRT}$

Boltzmanova konstanta $k_B = \frac{R}{N_A} = 1,38 \times 10^{-23}\ J/K \to \boxed{pV = N k_B T}$

Toplinski kapacitet i Kalorimetrija

Toplinski kapacitet : koliko topline dovesti da se temp promijeni za 1K? → $C_T = \frac{\Delta Q}{\Delta t}$

Specifični toplinski kapacitet $c = \frac{C_t}{m} = \frac{1}{m} \cdot \frac{\Delta Q}{\Delta t}$

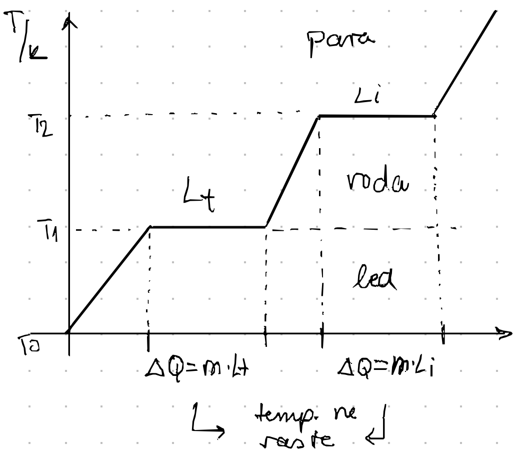
→ $\Delta Q = mc\Delta T$

$n = \frac{m}{M}$

$C_M = \frac{C_t}{n} = \frac{C_t}{m} \cdot M$

Molarni toplinski kapacitet : $C_m = \frac{C_t}{n} = M \cdot c$

Fazni prijelazi (promjene ag. stanja)



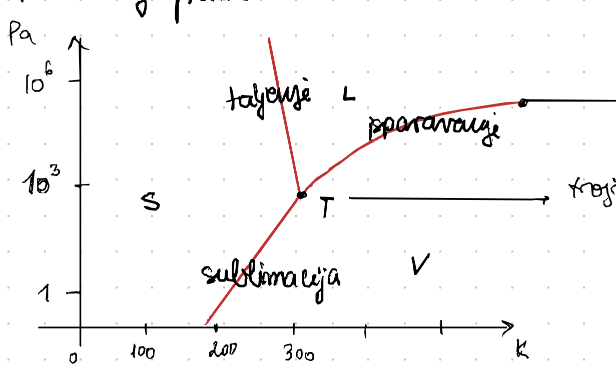
FAZA : tekuća, plinovita, no može biti više faza u istom ag. stanju

→ ako prilikom zagrijavanja dolazi do promjene faze, dovedena energija (toplina) se troši na promjenu kem. veza ili međudjelovanja

⇒ ne dolazi do promjene E_k !

→ pretvaranjem vode u led, energija se OSLOBAĐA → grije se okolni ~~zrak~~

fazni dijagrami :



→ trojna točka : $P = 610,620$ Pa

$T = 273,16$ K

Jednadžba stanja realnog plina

Idealni plinovi: $pV = nRT$

Za realne plinove uvode se dvije najvažnije popravke

1) $V \to V - nb_w$ (učinak smanjenja volumena zbog volumena molekula)

2) $p \to p + \frac{n^2 a_w}{V^2}$ (učinak porasta tlaka zbog privlačnih sila između mol. plina)

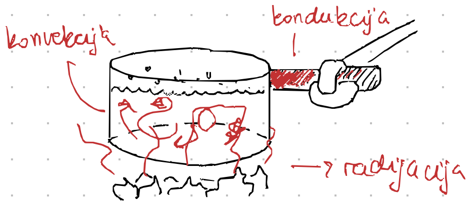
$\Rightarrow$ van der Waalsova jednadžba: $\left(p + \frac{n^2 a_w}{V^2}\right)(V - nb_w) = nRT$

PRIJENOS TOPLINE (kasnije će to u termodinamici biti opširnije)

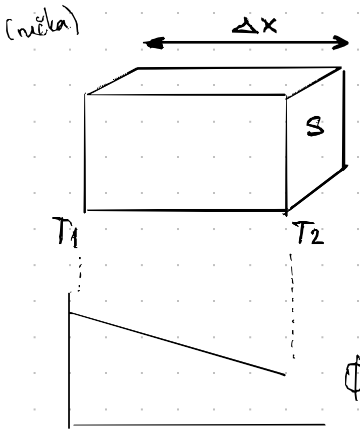
I. vođenje (kondukcija)

II. strujanje (konvekcija)

III. zračenje (radijacija)



→ vođenjem (kondukcijom)



$$Q = -\lambda \frac{\Delta T}{\Delta x} \cdot S \cdot t$$

promjena temp.

duljina

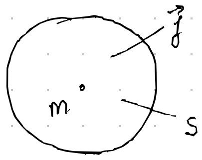
λ – koeficijent toplinske vodljivosti $\lambda \left[\frac{W}{mK}\right]$

$$\underbrace{\Phi_V = \frac{Q}{t}}_{\text{toplinski tok vođenja}} \rightarrow \boxed{\Phi_V = -\lambda \frac{\Delta T}{\Delta x} \cdot S}$$

$$\underbrace{q_V = \frac{\Phi_V}{S}}_{\text{gustoća toplinskog toka}}$$

$$\Phi_V = \frac{Q}{t} = -\lambda \frac{\Delta T}{\Delta x} \cdot S = \frac{\Delta T}{R} \rightarrow \boxed{R = \frac{\Delta x}{\lambda S}}$$

$$R_{tot} = \sum \frac{\Delta x_i}{\lambda_i S_i}$$



$$-\frac{dm}{dt} = \vec{S}\,\vec{j}$$

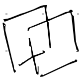
$$v = \frac{dx}{dt} \rightarrow dx = v \cdot dt$$

$$\rho = \frac{dm}{dV} \qquad m = \rho \cdot V = \rho \cdot S \cdot dx$$

$$\Rightarrow -\frac{\rho \cdot \cancel{S} \cdot v \cdot \cancel{dt}}{\cancel{dt}} = \cancel{\vec{S}} \cdot \vec{j} \Rightarrow \rho \cdot v = \vec{j}$$

* Sjeti se:

$$\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$$



$$\rightarrow E = \frac{\sigma}{\varepsilon_0} \rightarrow -\sigma = \rho \cdot v \cdot dt$$

$$E = -\underbrace{\rho v dt}_{dx} \cdot \frac{1}{\varepsilon_0} \rightarrow \nabla\left(-\rho v dt \frac{1}{\varepsilon_0}\right) = \frac{\rho}{\varepsilon_0}$$

površinska gustoća naboja

* $\rho = \frac{dq}{dV}$

$$\Rightarrow -\nabla \rho v dt = \rho \,/\, \frac{d}{dt} \longrightarrow \boxed{\nabla |\rho \vec{v}| = -\frac{\partial \rho}{dt}}$$

jednadžba kontinuiteta (dif. oblik)

$$\nabla \rho \vec{v} \cdot dt = -\frac{d^2 q}{ds} \Rightarrow \frac{dq}{dt} + \underbrace{\oint \rho \vec{v}\, d\vec{s}}_{\text{Gauss}} = 0$$

$$\frac{dq}{dt} + \int \nabla \rho v\, dV = 0 \,/\, :dV \longrightarrow \boxed{\frac{d\rho}{dt} + \int \nabla \rho v = 0}$$

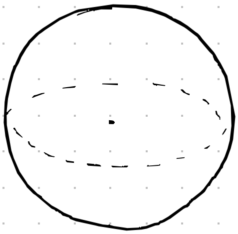
jednadžba kontinuiteta (integralno)

↳ standardno: $\frac{dq}{ds} + \oint \vec{j}\, d\vec{s} = 0$

općenito tok gledamo kao struju: $I = -\vec{S}\vec{j} = \frac{dQ}{dt} \qquad \frac{dm}{dt} = -\vec{S}\vec{j}$

TOK KROZ POVRŠINU KUGLE (Sferna simetrija)

$S = 4r^2\pi$



$$\Phi_{cond} = \overbrace{\frac{Q}{t}}^{\dot{Q}} = -\lambda \frac{\Delta T}{\Delta x} S = \lambda \cdot 4r^2\pi \frac{\Delta T}{\underbrace{\Delta x}_{dr}}$$

$$\frac{Q}{t} = -\lambda 4 r^2 \pi \frac{\partial T}{\partial r} \quad / \cdot \frac{dr}{r^2}$$

$$\dot{Q} \cdot \frac{dr}{r^2} = -\lambda 4 \pi \, dT \quad / \int$$

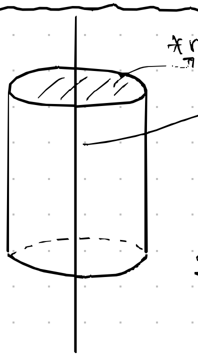
$$\dot{Q} \cdot \left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} = -4\pi\lambda \int dT \Rightarrow \dot{Q} = -4\pi\lambda \cdot \frac{T_2 - T_1}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$\dot{Q} = \frac{Q}{t} = -4\pi\lambda \overbrace{(T_2 - T_1)}^{\Delta T} \cdot \frac{1}{\frac{\overbrace{r_2 - r_1}^{\Delta r}}{r_2 r_1}} \Rightarrow \boxed{\frac{Q}{t} = -4\pi\lambda \, r_2 r_1 \cdot \frac{\Delta T}{\Delta r}}$$

→ ako je jako tanka fjiskica: $\Delta r << r_1 \simeq< r_2 \rightarrow r_1 r_2 = r^2$ $\quad \dot{Q} = -\lambda \cdot 4\pi r^2 \cdot \frac{\Delta T}{\Delta r}$

$$\boxed{\dot{Q} = -\lambda \cdot S \cdot \frac{\Delta T}{\Delta r}}$$

TOK KROZ POVRŠINU CILINDRA (cilindrična simetrija)



*nema toka kroz baze

npr ide grijač po sredini tako da samo kroz plašt zrači

$S = 2r\pi \cdot L$

$$\dot{Q} = -\lambda \frac{\Delta T}{\Delta x} \cdot S = -\lambda \cdot 2r\pi \cdot L \frac{\Delta T}{\underbrace{\Delta x}_{dr}}$$

$$\dot{Q} = -\lambda \cdot 2r\pi \cdot L \cdot \frac{\partial T}{\partial r}$$

$$\dot{Q} \cdot \frac{dr}{r} = -\lambda \cdot 2\pi L \cdot dT \quad / \int$$

$$\dot{Q} \cdot \ln(r)\Big|_{r_1}^{r_2} = -\lambda 2\pi L (T_2 - T_1)$$

$$\dot{Q} \cdot \ln\left(\frac{r_2}{r_1}\right) = -\lambda 2\pi L \Delta T$$

$$\dot{Q} = -2\pi\lambda \cdot L \cdot \Delta T \cdot \frac{1}{\ln\left(\frac{r_2}{r_1}\right)}$$

$r_2 \simeq r_1$ $\quad$ * $\ln(1+x) \simeq x$ za $|x| << 1$

$$\frac{r_2}{r_1} = 1 + x \Rightarrow \frac{r_2}{r_1} - 1 = x = \frac{\overbrace{r_2 - r_1}^{\Delta x}}{r_1}$$

$$\Rightarrow \dot{Q} = -2\pi\lambda L \cdot \Delta T \cdot \frac{1}{\frac{\Delta r}{r_1}} \longrightarrow \boxed{\dot{Q} = -\lambda 2\pi L \cdot \frac{\Delta T}{\Delta r} \cdot r_1}$$

→ prijenos topline strujanjem (konvekcija)

* difuzija topline

Newtnov zakon konvekcije: $q_c = h_c(T_p - T_f)$ površina, fluid

$\Phi = S \cdot q$, gustoća toplinskog toka → $\Phi = S \cdot h_{conv} \cdot \Delta T = \frac{\Delta T}{R_{conv}}$ → $\boxed{R_{conv} = \frac{1}{h_c S}}$

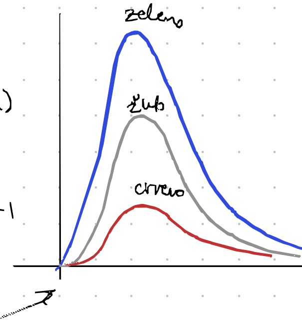
→ prijenos topline zračenjem (radijacija)

- EM zračenje * poseban dio

→ izvodi se iz QM (Planckov zakon, kvantizacija zračenja)

I - intenzitet, gustoća toka en. ($\frac{W}{m^2}$) ~> analogno q

$$I = \int_0^\infty f(\lambda,T)\,d\lambda \rightarrow f(\lambda,T) = \frac{2\pi h c^2}{\lambda^5}\left(\exp\left(\frac{hc}{\lambda kT}\right)-1\right)^{-1}$$



Stefan Boltzmannov zakon: $I = \sigma T^4$

$\sigma = 5{,}67 \times 10^{-8}\ W/m^2K^4$ (konstanta)

⇒ prijenos topline zračenjem: $P = \sigma S (T_1^4 - T_2^4)$

Toplinsko zračenje, termička ravnoteža, crno tijelo

! tijela nisu u kontaktu i u vakuumu je

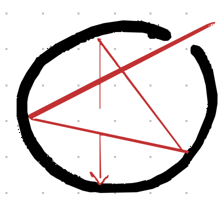
Termička ravnoteža → svako tijelo emitira koliko transmitira, apsorbira i reflektira

ukupni upadni tok: $\Phi_{upadno} = \Phi_{apsorbirano} + \Phi_{transmitirano} + \Phi_{reflektirano}$

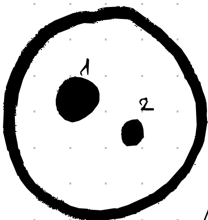
$$\rightarrow 1 = \alpha + \tau + \rho$$

$\alpha + \underset{\approx 0}{\tau} + \underset{\approx 0}{\rho} = 1$ (jer je apsolutno crno tijelo) ⇒ $\boxed{\alpha = 1}$ idealni apsorbens

(ne izlazi van)



Kirchoffov zakon – za dva tijela u term. ravnoteži, mogu imati različite faktore apsorpcije (α) (različiti materijali)



→ omjer emisije i apsorpcije mora biti isti za sva tijela, i može ovisiti samo o λ i T

*spektralna gustoća ovisi samo o λ i T

$$\frac{\epsilon(\lambda,T)_{mat}}{\alpha(\lambda,T)_{mat}} = f(\lambda,T)$$

faktor emisije, spektralna gustoća zračenja

* za crno tijelo $\alpha = 1$ $\Rightarrow$ $f(\lambda,T) = \epsilon(\lambda,T)_{mat}$

$$\rightarrow I = \int_0^\infty \epsilon(\lambda,T)_{mat}\, d\lambda$$

→ crno tijelo je idealno za zračenje za neku temp jer omjer $\frac{\epsilon(\lambda,T)_{mat}}{\alpha(\lambda,T)_{mat}}$ mora biti stalan; $\alpha\downarrow$ $\epsilon\downarrow$ i $\alpha\uparrow$ $\epsilon\uparrow$

* izvod nije trivijalan, nije jako bitan zakon (osim žutog), općenito nas to neće tražiti