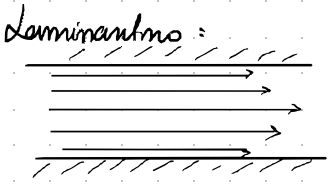
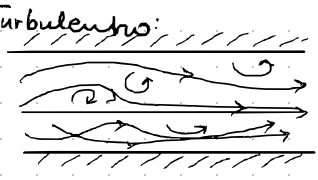
DINAMIKA FLUIDA

Idealni fluid ⟶ strujanje idealnog fluida

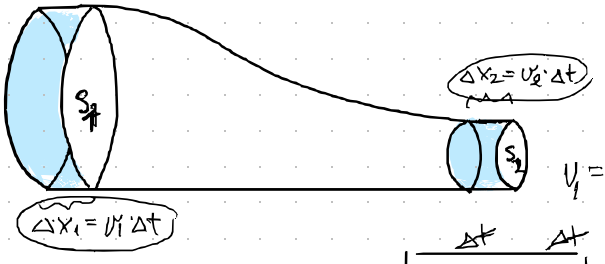
Pretpostavke za opis dinamike:

1) nestlačiv 2) konst. temp
3) tok fluida jednolik = brzina i tlak ne ovise o vremenu
4) tok fluida je laminaran - u slojevima
5) fluid nije viskozan





Jednadžba kontinuiteta



$$\Delta V_1 = S_1 \cdot \Delta x_1 = S_1 \cdot v_1 \cdot \Delta t$$
$$\Delta V_2 = S_2 \cdot \Delta x_2 = S_2 \cdot v_2 \cdot \Delta t \quad / \Delta t$$

$v_1 = v_2 \longrightarrow \frac{m_1}{\rho_1} = \frac{m_2}{\rho_2}$ $\qquad \rho = \frac{m}{V}$

nestlačivost: $\Delta V_1 = \Delta V_2 \longrightarrow \boxed{S_1 v_1 = S_2 v_2}$

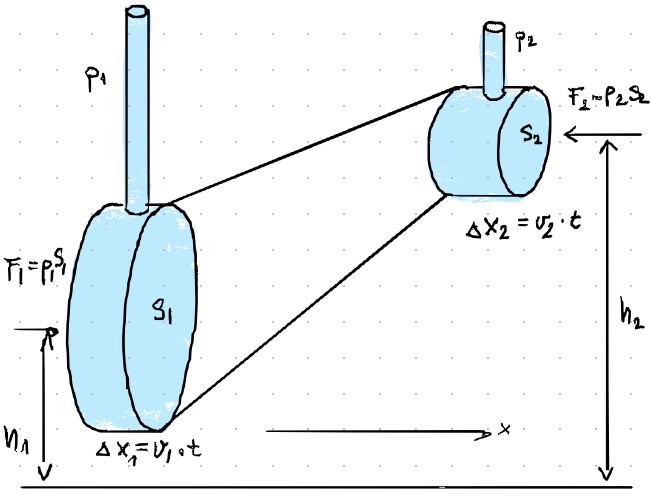
brzina na užem presjeku je iska ⟶ fluid nije viskozan

*ako ne mijenja nestlačivost

⇒ očuvanje mase: $\Delta m_1 = \Delta m_2$ $\qquad \rho_1 \Delta V_1 = \rho_2 \cdot \Delta V_2 \Rightarrow \boxed{\rho_1 S_1 v_1 = \rho_2 S_2 v_2}$

Bernoullijeva jednadžba

- kakve posljedice na tlak ima promjena brzine fluida?
  ↳ STATIČKI TLAK



$W_1 = F_1 \cdot \Delta x_1 = F_1 \cdot v_1 \cdot \Delta t$

$W_1 = p_1 S_1 v_1 \cdot \Delta t$ — fluid dobiva En

$W_2 = -F_2 \cdot \Delta x_2$ — ako jedan gura, drugi otpušta

$W_2 = -p_2 S_2 v_2 \Delta t$ — fluid gubi En

$\Delta W = W_1 + W_2$ $\qquad S_{1,2} v_{1,2} = \frac{\Delta m}{\rho}$

⟶ mijenja nestlačivost

$$\Delta W = (p_1 - p_2) \cdot \frac{\Delta m}{\rho} = \Delta E$$

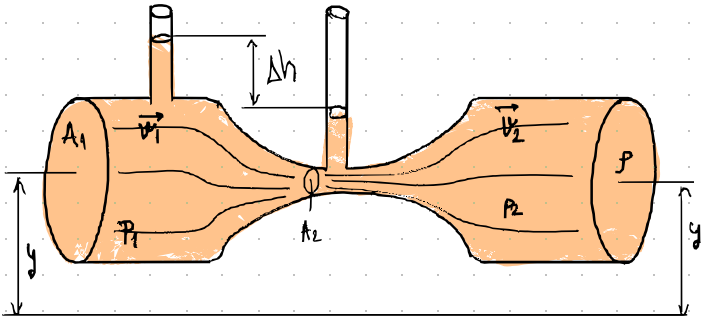
$$\Delta E = \Delta E_k + \Delta E_p = \left(\frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2\right) + (\Delta m g h_2 - \Delta m g h_1)$$

$$\Delta m \frac{1}{\rho}(p_1 - p_2) = \Delta m \left[\frac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1)\right] / \cdot \rho$$

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g (h_2 - h_1) \Rightarrow \boxed{p_1 + \frac{\rho}{2} v_1^2 + \rho h_1 = p_2 + \frac{\rho}{2} v_2^2 + \rho h_2}$$

Primjene Bernoullijeve jednadžbe

Venturijeva cijev



Zbog jednadžbe kontinuiteta, brzina je veća gdje je presjek cijevi manji ($A_1 v_1 = A_2 v_2$)

konstante

$$p_1 + \frac{\rho}{2} v_1^2 + \rho g y = p_2 + \frac{\rho}{2} v_2^2 + \rho g y$$

$$v_2^2 = \left(\frac{A_1}{A_2}\right)^2 v_1^2$$

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2) = \frac{\rho}{2} v_1^2 \left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) \rightarrow v_1^2 = \frac{2(p_1 - p_2)}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}$$

$\Delta p = \rho g \Delta h$ ako se tekućina kojom se mjeri tlak ista kao ona koja protječe

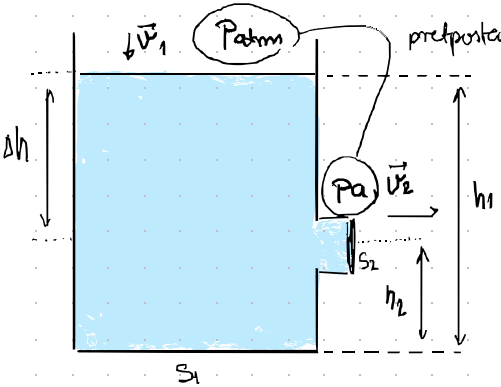
$$\Longrightarrow v_1^2 = \frac{2\rho g \Delta h}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)} = \frac{2\Delta h}{\left(\frac{A_1}{A_2}\right)^2 - 1}$$

$$\boxed{v_1 = \sqrt{\frac{2\Delta p}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}}$$

tok fluida: $\frac{\Delta V}{\Delta t} = A_1 v_1 \; (= A_2 v_2)$

- služi za mjerenje brzine i protoka fluida

Torricellijev zakon istjecanja



pretpostavlja se da je isti tlak zraka

$S_1 >>> S_2$

$v_1 \approx 0$ jer se toliko sporo mijenja

$$\cancel{p_{atm}} + \frac{\rho}{2} v_1^2 \overset{\approx 0}{} + \rho g h_1 = \cancel{p_{atm}} + \rho g h_2 + \frac{\rho}{2} v_2^2$$

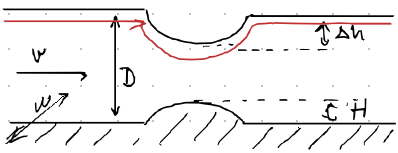
$$\rho g (h_1 - h_2) = \frac{\rho}{2} v_2^2$$

$$2g(h_1 - h_2) = v_2^2$$

$$\Downarrow$$

$$\boxed{v_2 = \sqrt{2g\Delta h}}$$

Primjeri – zadaci



$D = 3m$

$H = 20cm$

$\Delta h = 3cm$

$v = ?$

Kod mijenjanja presjeka

I. ↪ JEDNADŽBA KONTINUITETA

$$S_1 \cdot v_1 = S_2 v_2$$

$$D v_1 = (D - \Delta h - H) v_2$$

II. Bernoullijeva jed → strujnica na površini

$$\cancel{p_a} + \frac{1}{2}\rho v_1^2 + \cancel{\rho g D} = \cancel{p_a} + \frac{1}{2}\rho v_2^2 + \rho g (D - \Delta h)$$

$$\frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2 - \rho g \Delta h$$

$$\frac{1}{2} v_1^2 = \frac{1}{2} v_2^2 - g\Delta h \rightarrow \frac{1}{2}(v_2^2 - v_1^2) = g\Delta h$$

$$\frac{v_2^2 - v_1^2}{2g} = \Delta h$$

$$v_1 = \frac{(D - \Delta h - H) v_2}{D} \rightarrow v_2 = \frac{v_1 D}{(D - \Delta h - H)}$$

(tražiš se)

$$\Rightarrow \Delta h = \frac{1}{2g}\left(v_1^2 \frac{D^2}{(D - \Delta h - H)^2} - v_1^2 \right) = \frac{1}{2g} v_1^2 \left(\frac{D^2}{(D - \Delta h - H)^2} - 1 \right)$$

$$v_1^2 = \frac{2g\,\Delta h}{\left(\frac{D^2}{(D - \Delta h - H)^2} - 1 \right)} \rightarrow \boxed{v_1 = 1{,}84\ m/s}$$