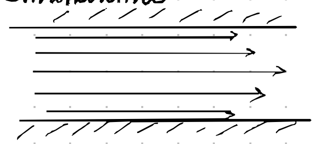
DINAMIKA FLUIDA

Idealni fluid ⟶ Strujanje idealnog fluida

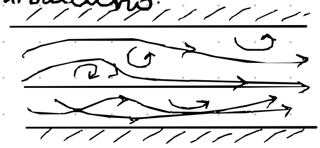
Pretpostavke za opis dinamike:

1) nestlačiv 2) konst. temp
3) tok fluida jednolik = brzina i tlak ne ovise o vremenu
4) tok fluida je laminaran – u slojevima
5) fluid nije viskozan

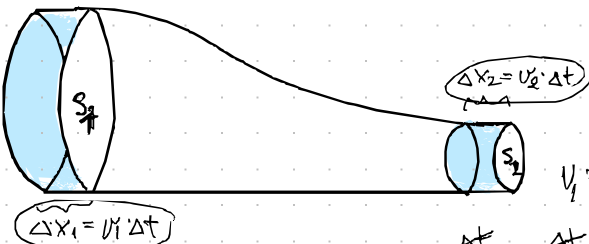
Laminarno:



Turbulentno:



Jednadžba kontinuiteta



$$\left.\begin{aligned} \Delta V_1 &= S_1 \cdot \Delta x_1 = S_1 \cdot v_1 \cdot \Delta t \\ \Delta V_2 &= S_2 \cdot \Delta X_2 = S_2 \cdot v_2 \cdot \Delta t \end{aligned}\right\} /\Delta t$$

$$V_1 = V_2 \longrightarrow \frac{m_1}{\rho_1} = \frac{m_2}{\rho_2} \qquad \rho = \frac{m}{V}$$

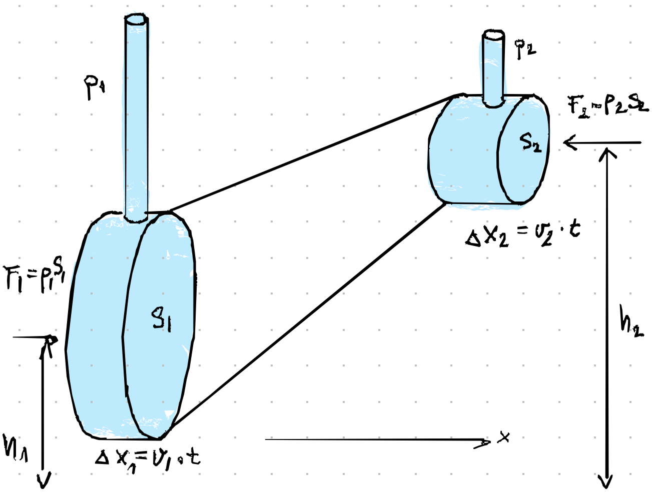
nestlačivost: $\Delta V_1 = \Delta V_2 \longrightarrow \boxed{S_1 v_1 = S_2 v_2}$

brzina na cijelom presjeku je ista
⟶ fluid nije viskozan

*ako ne vrijedi nestlačivost
⟹ očuvanje mase: $\Delta m_1 = \Delta m_2$ $\quad \rho_1 \Delta V_1 = \rho_2 \cdot \Delta V_2 \Rightarrow \boxed{\rho_1 S_1 v_1 = \rho_2 S_2 v_2}$

Bernoullijeva jednadžba

– kakve posljedice na tlak ima promjena brzine fluida?
↳ STATIČKI TLAK



$W_1 = F_1 \cdot \Delta x_1 = F_1 \cdot v_1 \cdot \Delta t$

$W_1 = p_1 S_1 v_1 \cdot \Delta t$ fluid dobiva En

ako jedan. gura, drugi otpušta

$W_2 = -F_2 \cdot \Delta x_2$

$W_2 = -p_2 S_2 v_2 \Delta t$ fluid gubi En

$\Delta W = W_1 + W_2 \qquad S_{1,2} v_{1,2} = \frac{\Delta m}{\rho}$

⟶ vrijedi nestlačivost

$$\Delta W = (p_1 - p_2) \cdot \frac{\Delta m}{\rho} = \Delta E$$

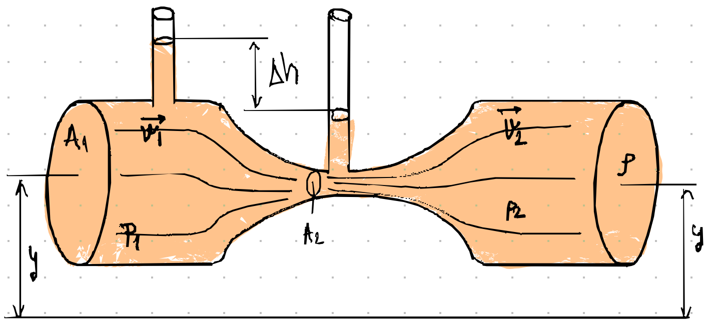
$$\Delta E = \Delta E_k + \Delta E_p = \left(\frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2\right) + (\Delta m g h_2 - \Delta m g h_1)$$

$$\cancel{\Delta m}\frac{1}{\rho}(p_1 - p_2) = \cancel{\Delta m}\left[\frac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1)\right] \Big/ \cdot \rho$$

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2) + \rho g (h_2 - h_1) \Rightarrow \boxed{p_1 + \frac{\rho}{2}v_1^2 + \rho h_1 = p_2 + \frac{\rho}{2}v_2^2 + \rho h_2}$$

Primjene Bernoullijeve jednadžbe

Venturijeva cijev



Zbog jednadžbe kontinuiteta, brzina je veća gdje je presjek cijevi manji ($A_1 v_1 = A_2 v_2$)

konstanta

$$p_1 + \frac{\rho}{2}v_1^2 + \rho g y = p_2 + \frac{\rho}{2}v_2^2 + \rho g y$$

$$v_2^2 = \left(\frac{A_1}{A_2}\right)^2 v_1^2$$

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2) = \frac{\rho}{2}v_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) \rightarrow v_1^2 = \frac{2(p_1 - p_2)}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}$$

$\rightarrow \Delta p = \rho g \Delta h$ ako se tekućina kojom se mjeri tlak ista kao ona koja protječe

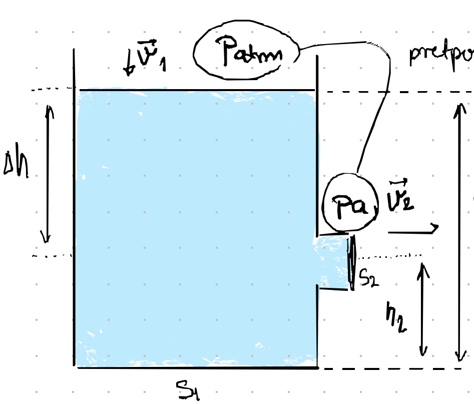
$$\Rightarrow v_1^2 = \frac{2\cancel{\rho} g \Delta h}{\cancel{\rho}\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)} = \frac{2\Delta h}{\left(\frac{A_1}{A_2}\right)^2 - 1}$$

$$\boxed{v_1 = \sqrt{\frac{2\Delta p}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}}$$

tok fluida: $\frac{\Delta V}{\Delta t} = A_1 v_1 \ (= A_2 v_2)$

– služi za mjerenje brzine i protoka fluida

Torricellijev zakon istjecanja



pretpostavlja se da je isti tlak izvana

$S_1 >>> S_2$

$v_1 \approx 0$ jer se toliko sporo mijenja

$$\cancel{p_{atm}} + \underbrace{\frac{\rho}{2}v_1^2}_{\approx 0} + \rho g h_1 = \cancel{p_{atm}} + \rho g h_2 + \frac{\rho}{2}v_2^2$$

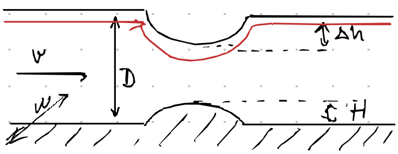
$$\cancel{\rho} g(h_1 - h_2) = \frac{\cancel{\rho}}{2}v_2^2$$

$$2g(h_1 - h_2) = v_2^2$$

$$\Downarrow$$

$$\boxed{v_2 = \sqrt{2g\Delta h}}$$

Primjeri – zadaci



$D = 3\,m$

$H = 20\,cm$

$\Delta h = 3\,cm$

$v = ?$

Kad mijenjamo presjeke

I. ↪ JEDNADŽBA KONTINUITETA

$$S_1 \cdot v_1 = S_2 v_2$$

$$D v_1 = (D - \Delta h - H) v_2$$

* na strujnicu djeluje samo promjena visine Δh, ne i H!

II. Bernoullijeva jed → strujnica na površini

$$\cancel{p_a} + \frac{1}{2}\rho v_1^2 + \cancel{\rho g D} = \cancel{p_a} + \frac{1}{2}\rho v_2^2 + \cancel{\rho g}(D - \Delta h)$$

$$\frac{1}{2}\cancel{\rho} v_1^2 = \frac{1}{2}\cancel{\rho} v_2^2 - \cancel{\rho} g \Delta h$$

$$\frac{1}{2} v_1^2 = \frac{1}{2} v_2^2 - g\Delta h \rightarrow \frac{1}{2}(v_2^2 - v_1^2) = g\Delta h$$

$$\frac{v_2^2 - v_1^2}{2g} = \Delta h$$

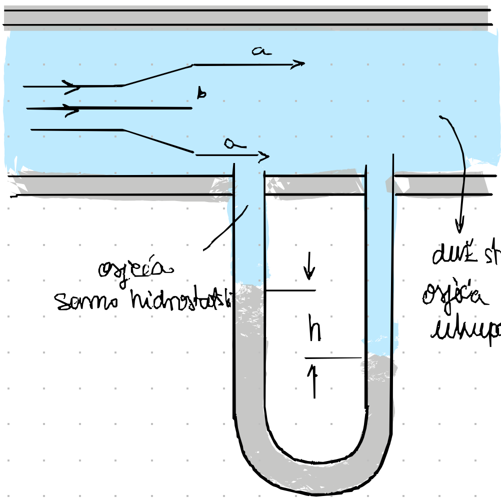
traži se

$$v_1 = \frac{(D - \Delta h - H) v_2}{D} \rightarrow v_2 = \frac{v_1 D}{(D - \Delta h - H)}$$

$$\Rightarrow \Delta h = \frac{1}{2g}\left(v_1^2 \frac{D^2}{(D - \Delta h - H)^2} - v_1^2 \right) = \frac{1}{2g} v_1^2 \left(\frac{D^2}{(D - \Delta h - H)^2} - 1 \right)$$

$$v_1^2 = \frac{2g\,\Delta h}{\left(\frac{D^2}{(D - \Delta h - H)^2} - 1 \right)} \longrightarrow \boxed{v_1 = 1{,}84\ m/s}$$

Pitot - Prandtlova cijev – služi za mjerenje brzine strujanja plina



$$p + \frac{1}{2}\rho v^2 + \rho g h = konst.$$

$$p_a + \Delta p = p_b$$

$$p_b = p_a + \frac{1}{2}\rho v^2$$

$$v = \sqrt{\frac{2\Delta p}{\rho}}$$

Primjer (D. Horvat, Fizika 1, Primjer 8.7)

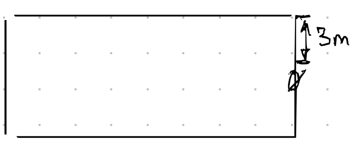
$m = 500\,kg$ $M = 4t$

$2r = 20cm \rightarrow r = 10cm$

$t = 1s$

$v_v = ?$ zanemari F_{tr}

Δm - izgubljena količina tekućine



$(m - \Delta m) v_{vagon} = \Delta m \cdot v_2$ Zoka

$q_v = \frac{\Delta V}{\Delta t} = S_2 v_2 \longrightarrow v_2 = \sqrt{2g\Delta h}$ $\Rightarrow (m - \Delta m) v_{va} = \sqrt{2gh} \cdot \Delta m \;/ : \rho$

$(V - \Delta V)\, v_{va} = \Delta V \sqrt{2g\Delta h}$

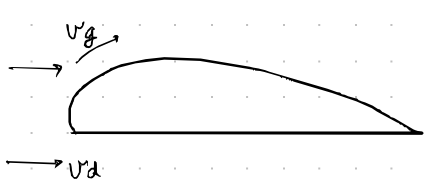
* raketa koja se giba isticanjem goriva!

$v(t) = v_0 + \underbrace{\sqrt{2g\Delta h}}_{v_{pl}} \cdot \ln \frac{m_0}{m}$

$\sum \frac{(x - v_i)^n}{n} \approx \frac{\Delta m}{m}$

* Aerodinamički uzgon ??

Primjer:



$h_1 \approx h_2$ zanemarimo razliku u visini

$v_g > v_d$

$$p_g + \frac{1}{2}\rho v_g^2 = p_d + \frac{1}{2}\rho v_d^2$$

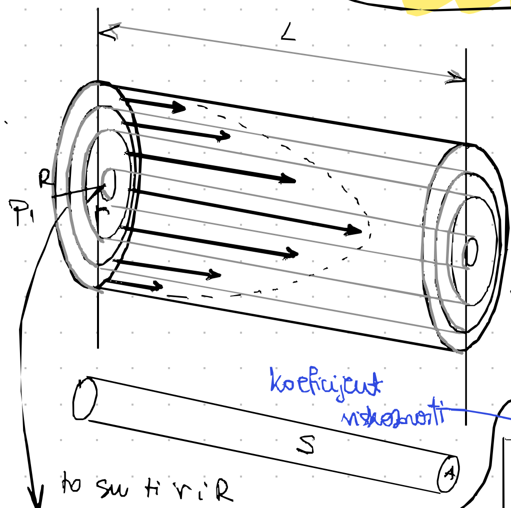
$$\underbrace{(p_d - p_g)}_{\Delta p} = \frac{1}{2}\rho (v_g^2 - v_d^2)$$

$$F_{uzg} = S \cdot \Delta p$$

* spoiler na autu → spoiler uništava drag zraka koji vuče i usporava auto (dio ide gore preko spoilera, a dio ispod spoilera)

Realni fluidi — Poiseuillov zakon

najvažnija idealizacija dosad: nepostojanje unutarnjeg trenja



to su ti r i R

$$\boxed{F = r^2\pi(p_1-p_2)}$$

Viskoznost: unutrašnje trenje između slojeva tekućine

(i dalje gledamo laminarno (nema miješanja) i stacionarno strujanje)

→ ukupna sila mora nestat jer teče $v = konst.$

$F = F_{tr} \rightarrow r^{\cancel{2}}\cancel{\pi}(p_1-p_2) = -\eta\,\underbrace{(L\cdot 2\cancel{r}\cancel{\pi})}_{S}\frac{dv}{dr} \Big/ dr$

koeficijent viskoznosti

$$\boxed{\begin{aligned} F_{visk} &= \eta\, S\,\frac{dv}{dr} \\ F_{tr} &= -\eta\, S\,\frac{dv}{dr} \end{aligned}}$$

* $F_{tr} = -F_{visk}$

$\Rightarrow \Delta p \cdot r\,dr = \eta\, 2L\, dv \;/ \int$

$F_{tr}\; dr = -\eta\, S\, dv$

$dr \uparrow \quad v \downarrow$

$$\frac{-\Delta p}{2\eta L}\cdot\frac{r^2}{2}\Big|_{r_1 \atop R}^{r_2\, r} = v\Big|_0^{v} \Rightarrow -\frac{\Delta p}{4\eta L}(r^2-R^2) = v$$

$$\boxed{v(r=0) = \frac{\Delta p}{4\eta L}\cdot R^2}$$

paraboliČna raspodjela brzina

$\frac{\Delta p}{4\eta L}(R^2-r^2) = v$

$dV = dA\cdot dl$

$dl = v\,dt$

$dA = 2r\pi\, dr$

$dV = 2r\pi\, v\, dr\, dt \;/ : dt$

$\frac{dV}{dt} = \text{volumni tok} = 2r\pi\, v(r)\, dr \;/ \int_0^R$

$$\frac{dV}{dt} = \frac{\pi\Delta p}{2\eta L}\int_0^R r(R^2-r^2)\,dr = \frac{\pi\Delta p}{2\eta L}\left[R^2\cdot\frac{r^2}{2}\Big|_0^R - \frac{r^4}{4}\Big|_0^R\right]$$

$$\frac{dV}{dt} = \frac{\pi\Delta p}{2\eta L}\left(\frac{R^4}{2}-\frac{R^4}{4}\right) = \frac{\pi\Delta p}{2\eta L}\cdot\frac{R^4}{4} \Rightarrow \boxed{\frac{dV}{dt} = \frac{\pi\Delta p}{8\eta L}R^4}$$

$$\rightarrow \boxed{q_V = \frac{\Delta V}{\Delta t} = \frac{\pi\Delta p}{8\eta L}R^4} \quad \text{tok}$$

Poiseuilleov zakon: srednja sila otpora

Prosječna sila koja djeluje po cijeloj površini A (presjek)

$F_{tr} = R^2\pi\Delta p = F \rightarrow$ to je jednako F koji djeluje zbog razlike tlakova

tok: $q_V = A\bar{v}$, $R^2\pi\bar{v}$

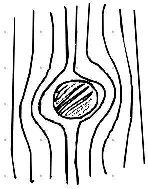
$$q_V = \frac{\pi\Delta p}{8\eta L}\cdot R^{4} \quad\Big\{\quad q_V = R^2\pi\Delta p\cdot\frac{R^2}{8\eta L} \Rightarrow F_{tr} = R^2\pi\Delta p = \frac{8\eta L}{R^2}q_V$$

$$F_{tr} = \frac{8\eta L}{\cancel{R^2}}\cdot\cancel{R^2}\pi\bar{v} = 8\eta L\pi\bar{v} \Rightarrow \boxed{F_{tr} = 8\eta L\pi\bar{v}}$$

Stokesov zakon - samo pamtimo formulu

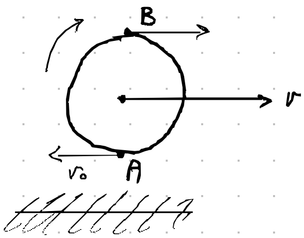
- tijelo radijusa R se nalazi u fluidu

$$\boxed{F_{otpor} = 6\pi\eta R v}$$



Magnusov učinak

relativna brzina (prema fluidu)

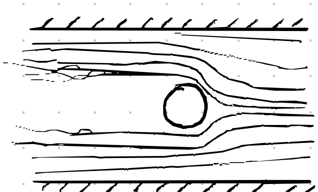


→ v_0 se giba u suprotnom smjeru pa se čini da se točka A giba brže.

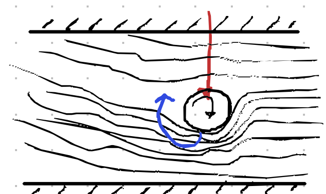
A: $v' = v + v_0$

B: $v' = v - v_0$

*gdje točka prolazi brže u odnosu na fluid, tlak na njoj će biti manji, zato je sila prema dolje



bez rotacije



s rotacijom

Turbulencije i Reynoldsov broj

- Kada će gibanje biti laminarno ili turbulentno → Reynoldsov broj

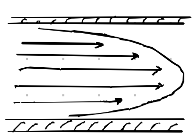
veličina kojom se određuje režim toka fluida

Reynoldsov broj: $\boxed{Re = \frac{\rho l v}{\eta}}$

* bezdimenzijonalna veličina

Re << viskoznost, laminarno

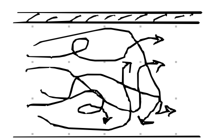
→ nema turbulencije, strujnice se povezuju oko tijela



uredno

Re >> inercijalni učinci, turbulencije

- nepredvidljivo



kaotično

Primjer :

$S = 0{,}01\,m^2$

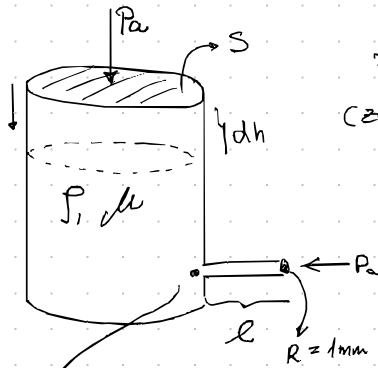
$\rho_{ulja} = 850\ kg/m^3$

$\mu = 0{,}25\ Pa\cdot s$ (viskoznost)

$l_{gevčica} = 2{,}5\,cm$

$2R = 2\,mm$

* ulje kroz gevčicu istječe u skladu s Poiseullovim zakonom (laminarno)



* dV je protekli volumen (znači onoliko koliko istekne van)

$t_{1/2} = ?$

$$q_V = \frac{dV}{dt} = \frac{\pi \Delta p}{8\eta l} R^4 \qquad \eta = \mu$$

razlika tlakova na rubovima cijevi

$$\Delta p = (\rho g h + p_a) - p_a \rightarrow \Delta p = \rho g h$$

povezati

$$\rightarrow \frac{dV}{dt} = \frac{\pi \rho g h}{8\mu L} R^4 \qquad * V = S\cdot h \,/\, d \longrightarrow dV = -S\,dh$$

smanjuje se razina vode jer istječe

najkompliciraniji korak jer je trebalo skužit poveznicu

$$-\frac{S\,dh}{dt} = \frac{\pi \rho g h}{8\mu L} R^4 \Big/ \cdot \frac{dt}{Sh}$$

$$-\frac{dh}{h} = \underbrace{\frac{\pi \rho g}{8\mu L}\cdot\frac{R^4}{S}}_{\text{konstanta}} dt \Big/ \int_0^{t_{1/2}} \Rightarrow \int_h^{\frac{h}{2}} \frac{dh}{h} = -Konst. \int_0^{t_{1/2}} dt$$

$$Pa = \frac{F}{S} = \frac{N}{m^2}$$

$$\ln\left(\frac{h}{2}\right) - \ln(h) = -Konst\cdot t_{1/2}$$

$$-\ln\frac{1}{2} = Konst\cdot t_{1/2} \longrightarrow t_{1/2} = \frac{-\ln\frac{1}{2}}{konst} = -\ln\left(\frac{1}{2}\right)\cdot\frac{8\mu L S}{\pi \rho g R^4}$$

$$t_{1/2} = \frac{8\cdot 0{,}25\ Pa\cdot s \cdot 2{,}5\times10^{-2}\ \cancel{m}\ \ 0{,}01\ \cancel{m^2}}{\pi \cdot 850\ kg\cdot \cancel{m^{-3}}\cdot 9{,}81\ \cancel{m}\cdot s^{-1}\cdot(10^{-3})^4\ \cancel{m^4}}\ m$$

$$t_{1/2} = \frac{8\cdot 0{,}25\cdot 2{,}5\cdot\times10^{-2}\cdot 0{,}01\cdot}{\pi\cdot 850\cdot 9{,}81\cdot 10^{-12}} = \boxed{13\,229{,}93\,s = 3{,}675\,h}$$