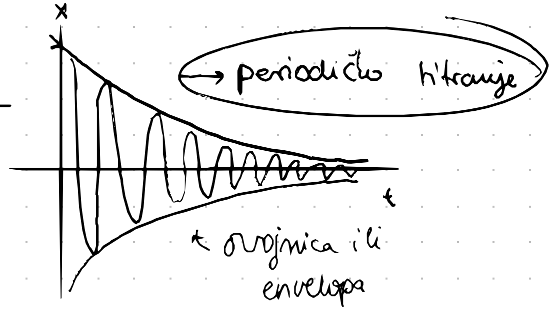
Prigušeno titranje

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0$$

ω_0 – kojom bi oscilacijom titralo da prigušenja nema

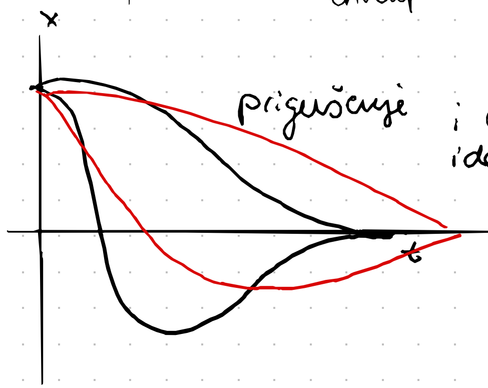
$$x = e^{\lambda} \Rightarrow \lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$



Slučaj podkritičnog prigušenja $\delta < \omega_0$

$$\omega^2 = \omega_0^2 - \delta^2$$

$$x(t) = A e^{-\delta t} \cos(\omega t + \phi)$$



prigušenje i oscilator ide u 0

Kritično prigušenje $\delta = \omega_0$

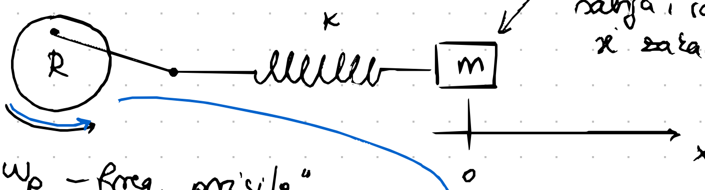
$$x(t) = (a_1 + a_2 t)\, e^{-\delta t}$$

Nadkritično prigušenje $\delta > \omega_0$

$$\Omega^2 = \delta^2 - \omega_0^2$$

$$x(t) = (a_1 e^{\Omega t} + a_2 e^{-\Omega t})\, e^{-\delta t}$$

Prisilno titranje



kako se R kreće vrti tako sabija i rasteže oprugu na koju je zakačeno tijelo mase m

ω_p – freq. „prisile"

$F_p = k \cdot R$

* u žutim formulama piše ω umjesto ω_p

$$m\ddot{x} = -kx - b\dot{x} + F_p \cos \omega_p t$$

$$\boxed{\ddot{x} + \underbrace{2\delta}_{\frac{b}{m}} \dot{x} + \underbrace{\omega_0^2}_{\frac{k}{m}} x = \underbrace{f_p}_{\frac{F_p}{m}} \cos \omega t}$$

$f_p = \frac{kR}{m}$

Probno rješenje:

$x[t] = A\cos[\omega t - \Phi]$ – kašnjenje

jednadžba gibanja $\quad \ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f_p e^{i\omega_p t}$

probno rješenje $\quad x = A e^{i(\omega_p t - \Phi)}$ $\quad \Rightarrow$

$$\Rightarrow (-\omega^2 + i\,2\delta\omega_p + \omega_0^2)\boxed{A}\, e^{-i\Phi} = f_p$$

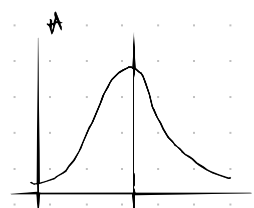
$$A = \frac{f_p = \frac{kR}{m} = \omega_0^2 R}{\sqrt{(\omega_0^2 - \omega_p^2)^2 + (2\delta\omega_p)^2}}$$

* trebamo modul

— ali ako $\omega_p \to \infty \Rightarrow A = 0$

„(jer nazivnik ide u ∞)

↳ $A = \frac{\omega_0^2 R}{\sqrt{\omega_0^4 + 0}} = \frac{\cancel{\omega_0^2}}{\cancel{\omega_0^2}} R \Rightarrow A = R$



=> Maksimum pri rezonantnoj freq

-dobijemo deriviranjem po freq. vanjske sile

$$\omega_{rez.} = \sqrt{\omega_0^2 - 2\delta^2}$$

$$A_{rez} = \frac{R\omega_0^2}{2\delta\sqrt{\omega_0^2 - \delta^2}}$$

Slaganje titranja na pravcu (u 1D)

$$x[t] = A_1 \cos[\omega_1 t + \phi_1] + A_2 \cos[\omega_2 t + \phi_2] + \dots$$

Primjer: $A_1 = A_2 = A$, $\omega_1 \simeq \omega_2$, $\phi_1 = \phi_2 = 0$ mičemo fazni pomak

$$\delta = \frac{\omega_1 - \omega_2}{2} \qquad \bar{\omega} = \frac{\omega_1 + \omega_2}{2}$$

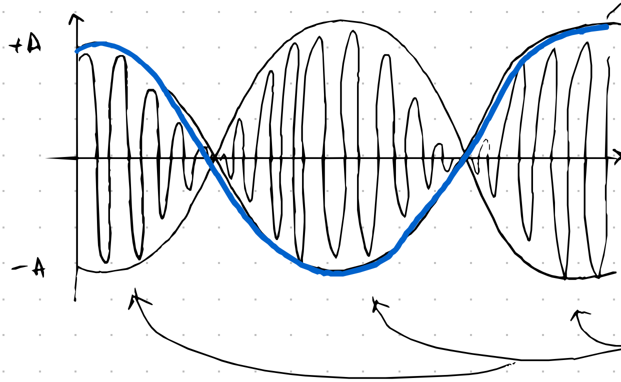
$$x[t] = A\cos[(\bar{\omega}+\delta)t] + A\cos[(\bar{\omega}-\delta)t]$$

$$= A \cdot \mathrm{Re}\left[e^{i(\bar{\omega}+\delta)t} + e^{i(\bar{\omega}-\delta)t}\right]$$

$$= A \cdot \mathrm{Re}\left[e^{i\bar{\omega}t}\underbrace{\left(e^{i\delta t} + e^{-i\delta t}\right)}_{=2\cos\delta t}\right]$$

zar ne bi trebalo $e^{\delta} + e^{-\delta}$?

$$= 2A\cos\bar{\omega}t\cos\delta t$$



envelopa ide frekvencijom delta (δ)

→ freq. udara duplo je veća od f envelope jer je to svaki ovaj brijeg (čujemo freq udara u stvari)

Primjer: $\omega_1 = \omega_2 = \dots = \omega$

$$x[t] = A_0 e^{i(\omega_1 + \phi_1)} + A_2 e^{i(\omega + \phi_2)} + \dots$$

$$x[t] = (A_p e^{i\phi_1} - A_2 e^{i\phi_2}) + \dots) e^{i\omega t}$$

otkud t? jel zaboravljen, opet sno izlučivanje