6.2.3. DERIVIRANJE I INTEGRIRANJE REDOVA

Pr.)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots$$

deriv.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots$$

deriviramo u beskonačnost ovu jednakost

budući da možemo derivirati, možemo i integrirati

- deriviramo član po član

TM Neka je $f(x) = \sum_{n=0}^{\infty} a_n (x-x_0)^n$ s polumjerom R.
Tada i $f'(x) = \sum_{n=0}^{\infty} a_n \cdot n (x-x_0)^{n-1}$ & $\int f(x) = \sum_{n=0}^{\infty} a_n \frac{(x-x_0)^{n+1}}{n+1} + C$

→ kod deriviranja se gubi prvi član, ali kod integriranja ne

Pritom se R ne mijenja! Deriv. i integ. ne mijenjaju radijus konvergencije, ALI konvergencija na rubu se može promijeniti.

21-21-2) $\sum_{n=1}^{\infty} \frac{n}{n+1} \cdot \frac{x^n}{3^{n+1}}$

a) polumjer i rubovi

$$R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{\cancel{\frac{n}{n+1}} \cdot \frac{1}{3^{n+1}}}} = \frac{1}{\frac{1}{3}} = \underline{\underline{3}}$$

$x_0 = 0$

$x \in \langle -3, 3 \rangle$

ponašanje na rubu:

$x = 3$

$\sum \frac{n}{(n+1)3}$ — NUK $\lim_{n\to\infty} \frac{n}{(n+1)3} = \frac{1}{3} \neq 0$

na oba ruba divergira

— II — za -3

b) Derivacija reda i R

paziti ide li od prvog člana (ostaje li n)

$$\sum_{n=1}^{\infty} \frac{n^2}{n+1} \cdot \frac{x^{n-1}}{3^{n+1}}, \quad R = 3 \text{ po teoremu!}$$

c) Integracija i odrediti područje

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)^2} \cdot \frac{x^{n+1}}{3^{n+1}}, \quad R = 3, \quad x \in \langle -3, 3 \rangle$$

za $x = -3$: $\sum \frac{n}{(n+1)^2} \frac{(-3)^{n+1}}{3^{n+1}} + C = \sum (-1)^{n+1} \cdot \frac{n}{(n+1)^2}$ po Leibnizu konvergira

(alternativa i padajući je)

za $x = 3$: $\sum \frac{n}{(n+1)^2} \sim \sum \frac{1}{n}$ (po aps. $\sim \frac{1}{n}$ div. nema zagrade)

poredbeni limes

21-19.-2) $f(x)=\operatorname{arctg} x \;/'$ oko $x=0$

a)

$$f'(x)=\frac{1}{1+x^2}=\sum_{n=0}^{\infty}(-1)^n\cdot(x^2)^n=\sum_{n=0}^{\infty}x^{2n} \quad \Big/\!\!\Big/ \int dx$$

$$\operatorname{arctg} x=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{2n+1}}{2n+1}=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\dots+c$$

$\downarrow$ razvijamo oko 0

$x=0:\quad \operatorname{arctg}(0)=0-0+\dots c=0$

$x=1\quad \frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots \quad /\cdot 4$

$\pi=$

b) $R=1$ (ne mijenja se) $x\in\langle -1,1\rangle$ $\frac{1}{1+x}=\sum(-1)^n x^n$

$R=1$

$x=1:\quad \sum(-1)^n\frac{1}{2n+1}$ Leibniz

$x=-1:\quad \sum(-1)^n\cdot\left(\frac{-1}{2n+1}\right)$ Leibniz $\qquad x\in[-1,1]$

c)

$$\sum\frac{(-1)^n}{3^n}\cdot\frac{1}{2n+1}=\sqrt{3}\sum\frac{(-1)^n}{2n+1}\left(\frac{1}{\sqrt{3}}\right)^{2n+1}=\sqrt{3}\operatorname{arctg}\frac{1}{\sqrt{3}}=\frac{\pi\sqrt{3}}{6}$$

VIR-23-4) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{2n+1}{2^{2n-1}} \cdot \frac{\pi^{2n}}{(2n)!}$ izračunati sumu reda

$\cos x = \sum (-1)^n \frac{x^{2n}}{(2n)!}$ — mo ramo dobiti $2n+1$ - zato množimo s x da kad deriviramo spustimo $2n+1$

$x\cos x = \sum (-1)^n \frac{x^{2n+1}}{(2n)!} \Big/ '$

$1\cos x - x\sin x = \sum (-1)^n (2n+1) \frac{x^{2n}}{(2n)!}$, $x = \frac{\pi}{2}$

$\underbrace{\cos\frac{\pi}{2}}_{0} - \frac{\pi}{2}\underbrace{\sin\frac{\pi}{2}}_{1} = \sum (-1)^n (2n+1) \frac{\pi^{2n}}{2^{2n}(2n)!} \Big/ \cdot \left(\frac{-1}{2^{-1}}\right) = -2$

$\Rightarrow S = -2\left(0 - \frac{\pi}{2} \cdot 1\right) = \underline{\underline{\pi}}$

VIR-22-5) suma reda $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n}$ — geometrijski

c) $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \Big/ \int$ $\quad \hookrightarrow = \sum_{n=1}^{\infty} \frac{1}{n+1}\left(\frac{1}{2}\right)^n \quad x = \frac{1}{2}$

$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln(1-x) \Big/ :x$

$\sum \frac{x^n}{n+1} = \frac{-\ln(1-x)}{x} \Big/ '$

$\sum \frac{n}{n+1} x^{n-1} = \frac{\frac{1}{1-x}(x) + \ln(1-x) \cdot 1}{x^2} \Big/ \cdot x$

$\sum \frac{n}{n+1} x^n = \frac{\frac{x}{1-x} + \ln(1-x)}{x} \longrightarrow$ uvrstimo $x = \frac{1}{2}$

$\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n} = \frac{\cancel{\frac{1/2}{1/2}} + \ln\frac{1}{2}}{\frac{1}{2}} = \boxed{2\left(1 + \ln\frac{1}{2}\right)}$

VIR-21-5) c) Suma reda $\sum_{n=1}^{\infty} n(n+1)\frac{2^{n-1}}{7^{n+1}}$

$$\sum_{n=1}^{\infty} n(n+1)\left(\frac{2}{7}\right)^n \cdot \frac{1}{14} = \frac{1}{14}\sum_{n=1}^{\infty} n(n+1)\left(\frac{2}{7}\right)^n \longrightarrow x=\frac{2}{7}$$

$$\sum_{n=1}^{\infty} x^n = \frac{1}{1-x} \quad /'$$

$$\sum_{n=1}^{\infty} n\cdot x^{n-1} = \frac{1}{(1-x)^2} \quad /\cdot x^2$$

$$\sum_{n=1}^{\infty} n\cdot x^{n+1} = \frac{x^2}{(1-x)^2} \quad /'$$

$x=\frac{2}{7}$

$$\sum_{n=1}^{\infty} n(n+1)\,x^n = \frac{2x(1-x)^2+2x^2(1-x)}{(1-x)^4} = \frac{2x(1-x)+2x^2}{(1-x)^3}$$

$$\sum_{n=1}^{\infty}(n+1)n\,x^n = \frac{2x-\cancel{2x^2}+\cancel{2x^2}}{(1-x)^3} = \frac{2x}{1-x^3} = \frac{\frac{4}{7}}{\left(\frac{5}{7}\right)^3} = \frac{4}{\cancel{7}}\cdot\frac{\cancel{7^3}\,7^2}{5} = \cancel{16}$$

resto

MIZI-20-5) $\sum_{n=0}^{\infty}\frac{n+1}{2^n n!} = e^{1/2}+\frac{1}{2}e^{1/2} = \sqrt{e}+\frac{1}{2}\sqrt{e}$