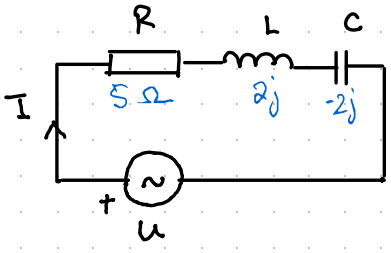
9. REZONANCIJA



ICE → u kondenzatoru struja prethodi naponu za 90°

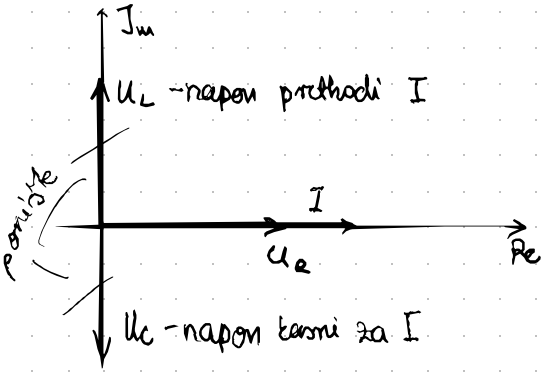
i

ELI → u zavojnici napon prethodi struji za 90° (struja kasni)

- budući da se radi o serji, struja je na svim elementima ista i nju gledamo da je na 0°

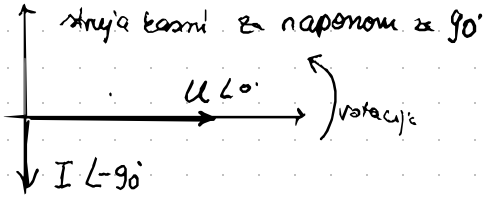
$$\frac{U}{I} = X_L = j\omega L = \omega L \angle 90°$$

↳ $U = I \cdot \omega L \angle 90°$



zaključimo: $I \angle -90°$ da bi se moglo pomnožiti sa $\angle 90°$ i reći da je $U \angle 0°$

$$U\angle 0° = I\angle -90° \cdot \omega L \angle 90°$$

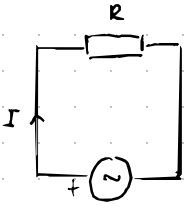


→ R - nema j → struja i napon u fazi

REZONANCIJA = samo kada je $X_C = X_L$

- struja je konstantna
- $U_C = U_L$

} u serijskoj rezonanciji se ponište pa je spoj zapravo



$U = \cancel{U_C} + U_R + \cancel{U_L}$ → $U = U_R$

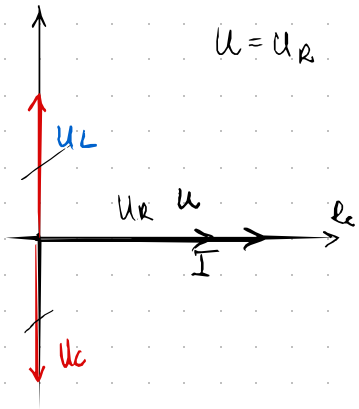
→ iz slike: I i U su u fazi

- imaginarni dio od Z mora biti 0 (poništi se)

$Z = R + j(\cancel{X_L - X_C})$ → U Rezonanciji $Z = R$ → zato je $\varphi = 0°$

- impedancija je min, admitacija min
- struja je max

(serijska ili naponska rezonancija jer je serijski spoj i jer su naponi na R i C su isti)



$U = U_R$

- Struja i napon izvora su u fazi (pod istim kutem) — u fazi I i U

↳ znači da imaginarni dio mora biti 0 da bi to bilo ostvareno

- $Z = R + j(X_L - X_C)$ — $\mathrm{Im}\{Z\} = 0$

- Z je minimalan — $Z = \min$

↳ $I = \frac{U}{Z}$ $\Rightarrow$ $I = \max$

↳ $Z \rightsquigarrow 0$

→ budući da admitaciju Y gledamo u paraleli, onda tada želimo da je Y min

(Z za seriju, Y za paralelu)

$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$
$$X_C = X_L$$

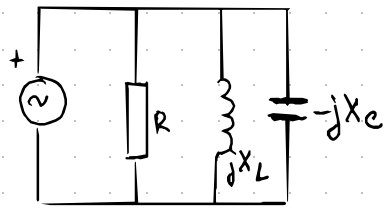
$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

! struja dolazi do tog nekog peaka kada pogodimo rezonantnu frekv

(gledamo osciloskop npr.)

Primjer



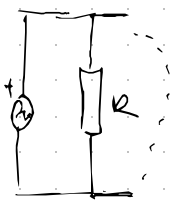
- pojednostavimo sklop
  - tretiramo L i C kao jedan zajednički element koji izračunamo kao paralelu

$$X_L = X_C$$

$$R_{LC} = \left(\frac{1}{jX_L} + \frac{1}{-jX_C}\right)^{-1} = \left(\frac{1}{2j} + \frac{1}{-2j}\right)^{-1} = (0)^{-1}$$

*u elektrotehnici možemo dobiti ∞ i to nam kaže da tamo nema protoka struje →

! $R_{LC} = \infty$ → beskonačni otpornik



$$I = \frac{U}{R}$$

$\mathrm{Im}\{Z\} = 0$, imamo samo realan dio

→ zaključak:

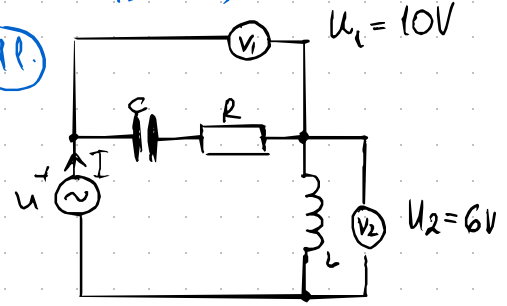
kada gledamo da su X_L i X_C jednaki u paraleli, njihov „element" gledamo kao beskonačni otpor, odnosno kao da ničeg nema jer struja ne prolazi

ZADACI

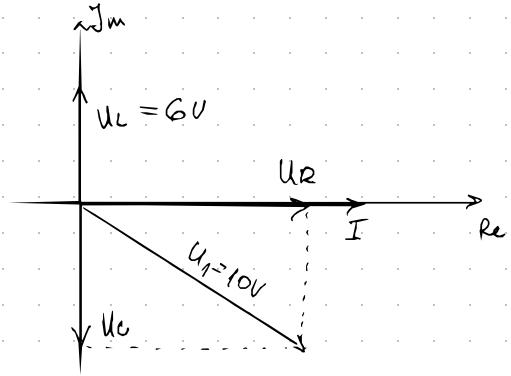
ICE ELI

ZI 19/20.)

11.)



$U_1 = 10V$



- rezonancija! $\rightarrow Im\{\underline{Z}\} = 0$

$U = ?$

$X_C = X_L$

$\Downarrow$

$U_C = U_L$

$U_1 = \sqrt{U_C^2 + U_R^2}$

$U_R = \sqrt{U_{V_1}^2 - U_C^2}$

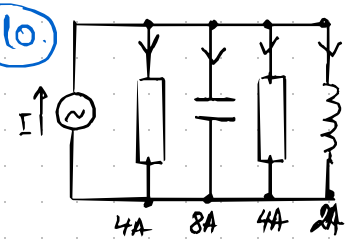
$U_R = 8$

$\dot{u} = \dot{u}_R + \dot{u}_L + \dot{u}_C$

$U = U_R + 6V - 6V \rightarrow \dot{u} = \dot{u}_R$

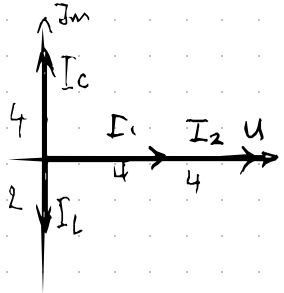
MI 2015)

10.)

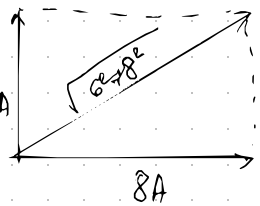


→ napon je srugdje isti (paralela)

$\underline{I} = 4\dot{A} + 8\dot{A} + 4\dot{A} + 2\dot{A}$



ICE ELI

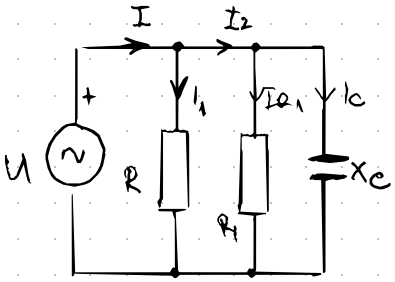


$I = 10A$

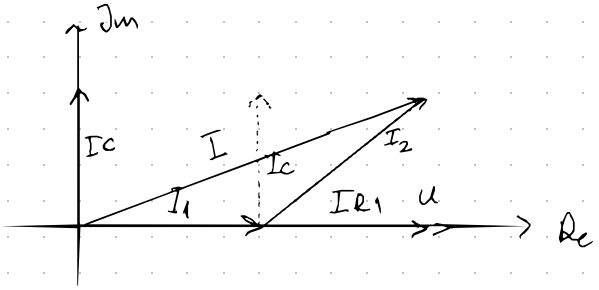
Zadatak Pavić - Feljo

$I = 3A \quad I_1 = 2A \quad I_2 = 2A$

ICE



$R_1 = 4\Omega$



$I = I_1 + I_2$

$I_2^2 = I_c^2 + I_{R_1}^2$

$I^2 = (I_c)^2 + (I_1 + I_{R_1})^2$

$I^2 = I_c^2 + I_1^2 + 2 I_1 I_{R_1} + I_{R_1}^2$

$U = R_1 \cdot I_{R_1}$

ako nađemo struju, imamo napon

$I^2 = I_2^2 + I_1^2 + 2 I_1 I_{R_1}$

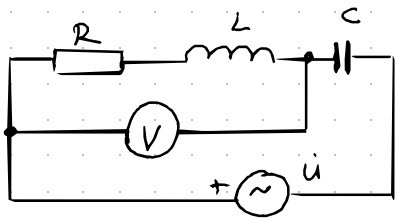
$$I_{R_1} = \frac{I^2 - I_2^2 - I_1^2}{2 I_1} = \frac{9 - 4 - 4}{4}$$

$I_{R_1} = \frac{1}{4} A$

$U = 4\Omega \cdot \frac{1}{4} A = 1V$

DEK 20./21.)

2.)



$Q_s = 5$

$U = 2V$

Rezonancija

$U_V = ?$

uvjeti rezonancije

$X_L = X_C$

$I = \frac{U}{R}$

*Im(Z) = 0

formule

$Q_s = \frac{f}{R}$

$f = \frac{1}{\sqrt{}} = \sqrt{\frac{L}{C}}$

$Q_s = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$

$X_L = X_C$

$U_V = I \cdot (R + j X_L)$

$U_V = \frac{U}{R}(R + j\omega L)$

$\omega^2 = \frac{1}{CL} \rightarrow C = \frac{1}{\omega^2 L} \rightarrow Q_s = \frac{1}{R}\sqrt{\omega^2 L^2}$

$U_V = U + \frac{U}{R} j(\omega L) = U + jU \cdot Q_s$

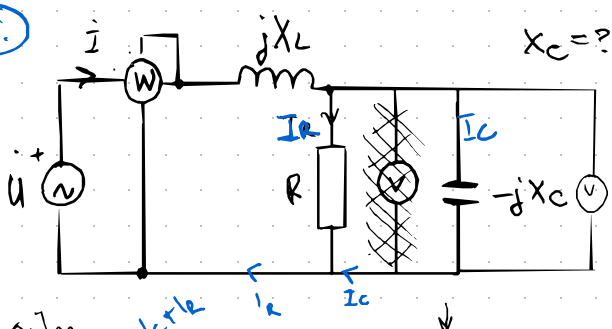
$Q_s = \frac{\omega L}{R}$

$U_V = |2 + j \cdot 2 \cdot 5| = \boxed{10{,}2\ V}$ (modul)

JES 18./19.)

u i I su u fazi

5.



$X_C = ?$

ELI ; ICE

$U = 40V$

$P = 333,3\ W$

$U_V = 50V$

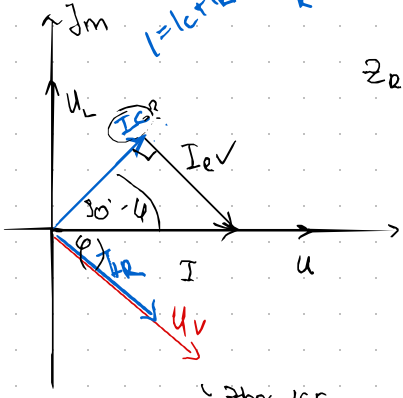
$I = I_C + I_R$

I_R

I_C

$$Z_{RC} = \frac{-jX_C \cdot R}{R - jX_C}$$

(P) Snaga se stvara samo na Realnim elementima



I tu ne vrijedi $X_L = X_C$

for to vrijedi samo za paralelnu i nerijeku R.R. (ona je miješovita)

$$P = \frac{U_V^2}{R} = \frac{50^2}{R}$$

$R = 7,5\ \Omega$

$\hookrightarrow I_R = \frac{U_V}{R} \Rightarrow I_R = 6,67A$

Zbog ICE, struja od C mora biti 90° prije od U_V

$$Z_{RC} = \frac{-jX_C \cdot R}{R - jX_C} \rightarrow (a) + (-b)j \quad \text{(kondenzatorski)}$$

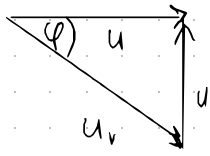
$a \neq R$

$-b \neq X_C$

ICE - za ČISTI C vrijedi da kasni za 90°, ali kombinacija samo poprima ponašanje na nekom kutu

$\hookrightarrow U_V = U_C = U_R$ (paralelni spoj)

translacijom konstruiramo:



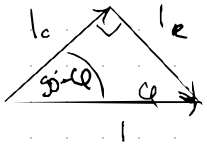
$\rightarrow U_L = \sqrt{U_V^2 - U^2}$

$U_L = 30V$

$\cos\varphi = \frac{U}{U_V}$

$\varphi = 36,86°$

- trebam I_C da bih izračunala X_C



$\operatorname{tg}\varphi = \frac{I_C}{I_R}$

$\hookrightarrow I_C = I_R \cdot \operatorname{tg}\varphi$

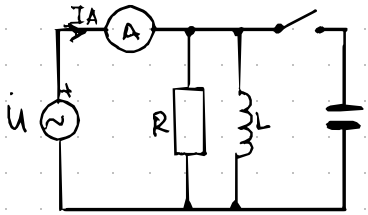
$I_C = 5A$

$X_C = \frac{U_V}{I_C} \Leftrightarrow X_C = 10\ \Omega$

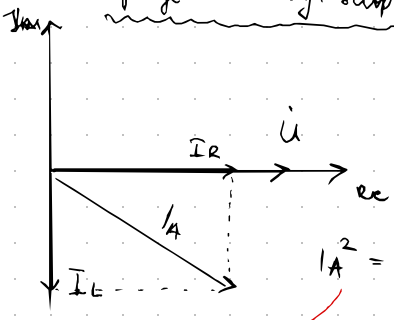
DEK 20./21.)

⑥ Pri otvorenoj i zatvorenoj sklopci ampermetar pokazuje $I = 4A$.

$X_L = 40\,\Omega$, $X_C = ?$



prije zatvaranja sklopke



ELI

$I_A^2 = I_R^2 + I_L^2$

$I_A^2 = I_R^2 + (I_C - I_L)^2$

$I_A^2 = I_R^2 + I_C^2 - 2 I_C I_L + I_L^2$

$4^2 = 4^2 - 2 I_C I_L + I_C^2$

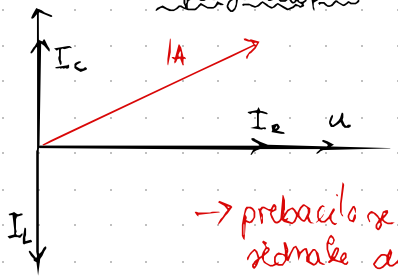
$I_C^2 = 2 I_C I_L$

$I_C = 2 I_L$

$\frac{U}{X_C} = 2 \cdot \frac{U}{X_L}$

$X_C = \frac{X_L}{2} = 20\,\Omega$

poslje sklopke (dodamo C) (39:40)



ICE

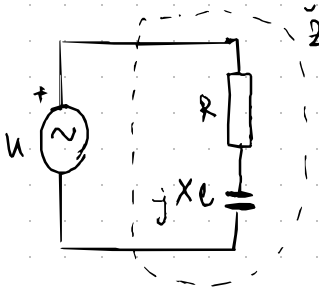
→ prebacila se gore ali je iznosa duljine

↳ prebacila se gore jer I_L i I_R nisu mijenjane, a budući da je dodan I_C, jedini način da se očuva $I_A = 4A$ je da se prebaci gore

MI-2016)

7. $U_n = 110V$ $U = 220\,V$ $C = ?$

$P_n = 100W$ $f = 50Hz$ $\hookrightarrow P = P_n$



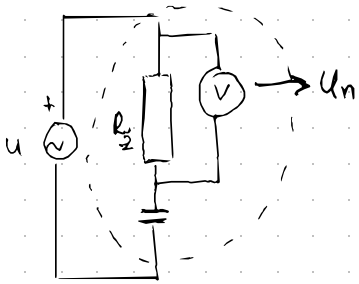
$P = $ na realnim el. $= I^2 \cdot R_{\check{z}}$

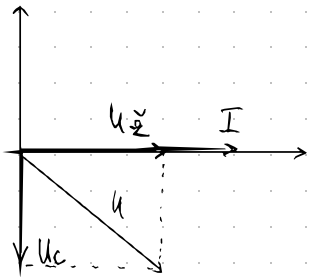
$\omega = 2\pi f \rightarrow X_C = \frac{1}{\omega C} \rightarrow C = \frac{1}{X_C \omega} = \frac{1}{X_C 2\pi f}$

$P_n = \frac{U_n^2}{R_{\check{z}}} \rightarrow R_{\check{z}} = \frac{U_n^2}{P_n} = \frac{110^2}{100} = 121\,\Omega$

$P_{220} = \frac{U}{R_{\check{z}}} = 400\,W$ ↯ ne valja da žarulja predviđena za 100W svijetli na 400!

– zato dodajemo kondenzator C (da pokupi dio napona)





ICE

$U^2 = U_{\check{z}}^2 + U_C^2$

$U_C = \sqrt{U^2 - U_{\check{z}}^2}$

$U_C = 190{,}53\,V$

$X_C = \frac{U_C}{I}$

$I = \sqrt{\frac{P_n}{R_{\check{z}}}} = \sqrt{\frac{100}{121}}$

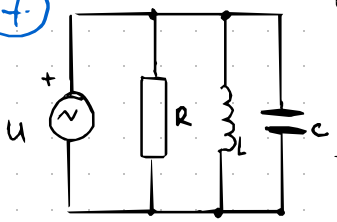
$I = 0{,}9A$

$\hookrightarrow X_C = 211{,}1\,\Omega$

$C = \frac{1}{2\pi f \cdot X_C} = 15{,}07\,\mu F$

M 2018.)

(7.)



$\dot{u} = 12\angle 0^\circ$

$R = 15\,\Omega$

$L = 20\,mH$

$C = 24\,\mu F$

$\dot{I} = ?$

EL ou the LCE.

admitacija je min $y = \frac{1}{z}$

↳ rezonancija

$X_C = X_L$

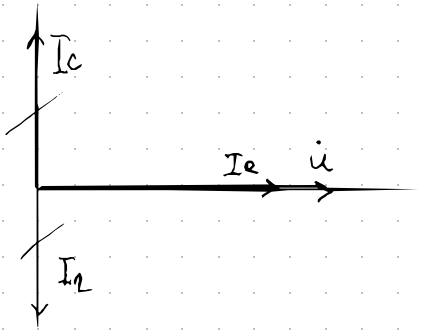
↓

ponište se

↓

* lakše za računati paralele

$\frac{1}{z_{uk}} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$

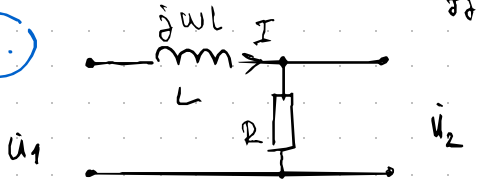


$\Rightarrow \dot{I} = \frac{U}{R} = 0{,}8A$

$y = G + \frac{1}{jX_L} + jX_C$

LJIR 2018/19.)

(9.)



$f_z = ?$

$L = 40\,mH$

$R = 100\,\Omega$

$\omega = 0$ + ⎓ (istosmjerni)

$X_L = 0$ namot žica bez otpora

$U_2 = U_1$

Granični slučaj: $|U_2| = \left|\frac{U_1}{\sqrt{2}}\right|$

$U_2 = I \cdot R$

$I = \frac{U_1}{Z}$ $\quad Z = R + jX_L$

$\Rightarrow U_2 = \frac{U_1 R}{R + jX_L} \rightarrow \frac{U_1}{\sqrt{2}} = \frac{U_1 R}{R + jX_L}$

$\sqrt{2}R = R + jX_L$ → ali moramo gledati module (nemamo Im ni Re)

$|\sqrt{2}R| = |R + jX_L|$

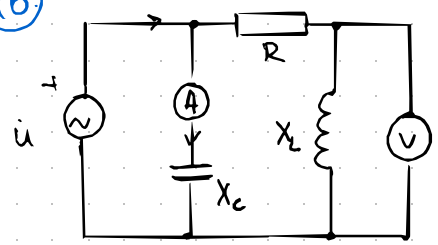
$\sqrt{2}R = \sqrt{R^2 + \omega^2 L^2}\ /^2$

$2R^2 = R^2 + \omega^2 L^2 \rightarrow \frac{R}{L} = \omega \rightarrow \frac{R}{2\pi L} = f$

$f = 397{,}88\,Hz$

MI 2016)

10.



$U_L = 6V$

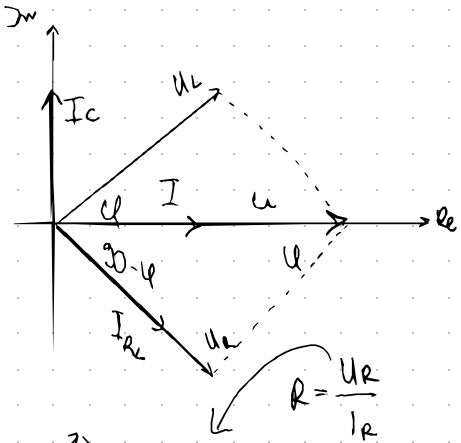
$I_C = 1A$

$U = 10V$

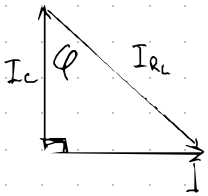
$R = ?$

ELI on the ICE

I u fasi sa U



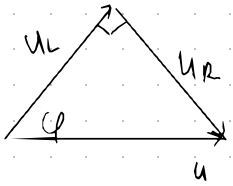
$\underline{I} = \underline{I}_C + \underline{I}_{RL}$



$\rightarrow I = \sqrt{I_{RL}^2 - I_C^2}$

$\underline{I}_{RL} = \frac{\underline{U}_L}{jX_L}$

$R = \frac{U_R}{I_R}$



$\rightarrow tg\varphi = \frac{U_R}{U_L}$

$U_R = \sqrt{U^2 - U_L^2}$

$U_R = 8V$

$\varphi = 53,13°$

$\cos\varphi = \frac{I_C}{I_{RL}}$

$I_{RL} = \frac{I_C}{\cos\varphi}$

$I_{RL} = 1,67A$

$R = 4,8\Omega$