5.4. PRIKAZ SINUSNIH VELIČINA U KOMPLEKSNOJ DOMENI

Eulerova formula: $e^{j\alpha} = \cos\alpha + j\sin\alpha$

Dva parametra kompleks. broja i dva načina prikazivanja

a) REALNI I IMAGINARNI DIO $\underline{Z} = x + jy$

$Re_z = x \qquad Im_z = y$

b) MODUL I ARGUMENT $\underline{Z} = x + jy = A \angle \alpha$

A - modul $\longrightarrow$ $A = \sqrt{x^2 + y^2}$, $\alpha = \operatorname{arctg}\left(\frac{y}{x}\right)$

sinusna veličina $A_m \sin(\omega t + \alpha_0)$ ima 4 parametra (trenutna vrijednost)

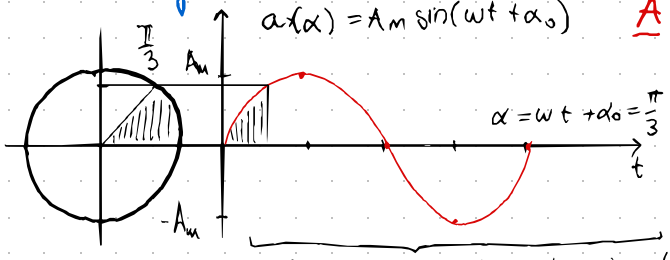
- A_m – amplituda
- ω – kružna frekvencija
- α_0 – fazni kut

↳ da bismo prešli u $\underline{C}$, trebamo 4 parametra => svesti na 2

=> a) A => amplituda ili ef (Jef, Uef ...)

b) α_0 => argument α

Sustav rotirajućih vektora



projekcija vektora (priključan u $\underline{C}$ br.)

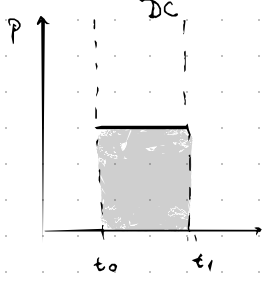
sinusna vel. koja prikazuje $a(t)$ u vrem. području

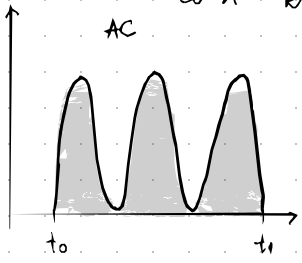
$\underline{A} = A_M e^{j\alpha_0} \cdot e^{j\omega t}$

$= A_M \cos(\omega t + \alpha_0) + j A_M \sin(\omega t + \alpha_0)$

a_t

Efektivne vrijednosti $\longrightarrow$ pokazuje mjerni instrument – u AC krugu su vrijednosti AC struje





$\Delta W = P \Delta t = R I^2 \cdot \Delta t$

$\Delta W = \int_{t_0}^{t_1} P\,t\,dt = \int_0^{t_1} R\,i(t)\,dt$

FAZORI, TRANSFORMACIJA SINUSNIH FUNKCIJA U KOMPLEKSNU DOMENU (i obratno)

Podrazumijevaju se 3 parametra pili d
- am u a
- fazni kut
- freq

$\Rightarrow$ treba svesti na 2

$$A_m e^{j(\omega t+\alpha_0)} = \overbrace{A_m \cos(\omega t+\alpha_0)}^{Re} + j\overbrace{A_m \sin(\omega t+\alpha_0)}^{Im}$$

$f = \frac{1}{T}$ $\quad \cos(t) = \sin(t+\frac{\pi}{2})$

$\omega = 2\pi f$ $\quad i = j = \sqrt{-1}$

$\dot{I} = I_{ef}$, $\dot{U} = U_{ef}$

$j = 1 \angle 90°$

* umjesto amplitude treba efektivnu vrijednost staviti

$A_{ef} = \frac{A_m}{\sqrt{2}} = |\dot{A}| \Rightarrow \dot{A} = A \cdot e^{j(\omega t+\alpha_0)}$

FAZOR: $\dot{A} = Ae^{j\alpha_0} = |\dot{A}| \angle \alpha_0 = A \angle \alpha_0$

↳ kompleks br. / vektor u $\mathbb{C}$

$$i(t) = \boxed{I_0} \sin(\omega t + \alpha_i) \rightarrow \operatorname{tg}\varphi = \frac{\Delta y}{\Delta x}$$

$I_0 = I_{ef} \cdot \sqrt{2}$

- ako su poznati A_m, α_0 i ω tada je fazor $\dot{A}$:

$$a(t) = A_m \sin(\omega t+\alpha_0) \xrightarrow{A = \frac{A_m}{\sqrt{2}}} \dot{A} = A \angle \alpha_0$$

u transform. ne ulazi frekvencija

→ frekvencija je ista i poznata za cijelu mrežu

- ako nije zadana → $f = 50Hz$ (gradska mreža)

obrnuti smjer: $\dot{A} = A \angle \alpha_0 \xrightarrow{A_m = \sqrt{2} \cdot A} A_m \sin(\omega t + \alpha_0)$

Mjerenje struje i napona kod sinusnih izvora

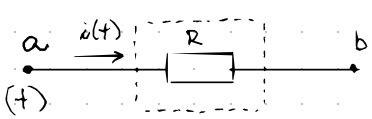
→ Idealni mjerni instrumenti prikazuju efektivnu vrijednost

VOLTMETAR: ako je fazor napona $\dot{U}_{12} = 3 - j4\,V$, voltmetar:

$$U_V = |\dot{U}_{12}| = \sqrt{3^2+4^2} = 5V$$

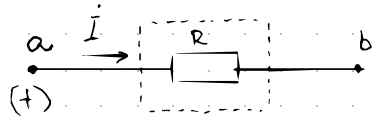
AMPERMETAR: fazor struje $\dot{I} = 12 \angle 60°\,A$, ampermetar:

$$I_A = |\dot{I}| = \sqrt{12^2} = 12A$$



$u_R(t) = u_{ab}(t) = R \cdot i(t)$

→ shema otpora u vremenskoj domeni



$\dot{U}_R = \dot{U}_{ab} = R \cdot \dot{I}$

→ shema otpora u kompleksnoj domeni

Otpor (R)

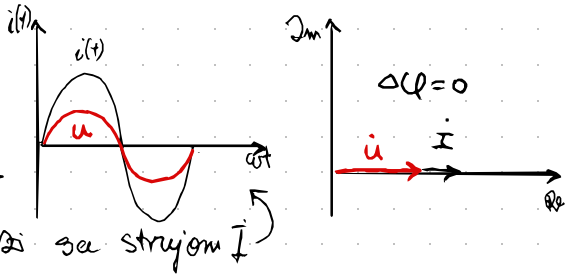
- otpor R priključen na napon $u_R(t) = u(t) = U_M \sin(\omega t + \alpha_u)$
  - $\alpha_u = 0$ → trebamo odrediti fazni pomak $\varphi = \alpha_u - \alpha_i$ → φ mora biti $0 \rightarrow \alpha_u = \alpha_i = 0$

$$\dot{U}_{ef} = \dot{U} = \frac{U_M}{\sqrt{2}} \angle \alpha_u = |\dot{U}| \angle \alpha_u = U \angle \alpha_u = \boxed{U \angle 0^\circ}$$

▶ a struja kroz otpor R: $i(t) = \frac{u_R(t)}{R} = \frac{1}{R} \underbrace{U_M \sin(\omega t)}_{\text{vremenski promjenjiv član}}$ / konstantan član

$$\Rightarrow \dot{I} = \frac{1}{R} \cdot \frac{U_M}{\sqrt{2}} \angle 0^\circ = \boxed{\frac{\dot{U}}{R}}$$

$$\Rightarrow \boxed{\begin{aligned} \dot{U}_R &= U_{ef} \cdot \angle 0^\circ \\ \dot{I}_R &= I_{ef} \cdot \angle 0^\circ \end{aligned} \qquad R = \frac{\dot{U}}{\dot{I}}}$$



napon na otporu u_R je u fazi sa strujom $\dot{I}$

$$\Rightarrow \dot{I}_R = \frac{\dot{U}}{R}$$

$$\Rightarrow \text{napon i struja su u fazi} \rightarrow \varphi = 0^\circ \quad (\text{fazni pomak } \varphi = \alpha_u - \alpha_i = 0^\circ)$$

Induktivitet (L)

$u_L(t) = U_M \sin(\omega t + \alpha_u)$

$\varphi = 0^\circ \qquad \alpha_u = 0^\circ$

induktivna reaktancija

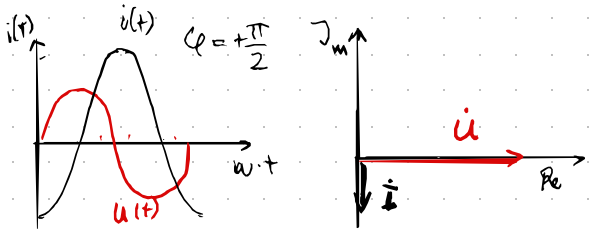
$$\dot{U}_L = \dot{U}_{ef} \angle 0^\circ$$
$$\dot{I} = \frac{\dot{U}_R}{R} = \frac{U_{ef} \angle 0^\circ}{j X_L}$$
$$\dot{I} = I_{ef} \angle -90^\circ$$

$$\dot{U}_L = \dot{U} = \frac{U_M}{\sqrt{2}} \angle \alpha_u = |\dot{U}| \angle \alpha_u = U \angle \alpha_u = \underline{U \angle 0^\circ}$$

$$i(t) = \frac{1}{L} \int u_L(t)\,dt = \frac{1}{L} \int U_M \cdot \sin(\omega t)\,dt = \frac{-1}{\omega L} \cdot U_M \cos(\omega t) = \underline{\frac{-1}{\omega L} U_M \sin\left(\omega t - \frac{\pi}{2}\right)}$$

pretvaranje u fazor:

$$\dot{I} = \frac{1}{\omega L} \cdot \frac{U_M}{\sqrt{2}} \cdot \angle -90^\circ = \frac{1}{\omega L} |\dot{U}| \angle -90^\circ$$
$$= \frac{1}{\omega L} \cdot |\dot{U}| \angle 0^\circ \cdot 1 \angle -90^\circ =$$

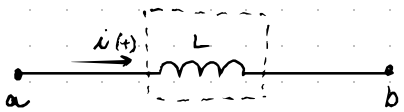


$$\dot{I} \rightarrow \frac{1}{j\omega L} \cdot \dot{u} = \frac{\dot{U} \angle 0^\circ}{j X_L}$$

IND otpor $\hookrightarrow X_L = \omega \cdot L$

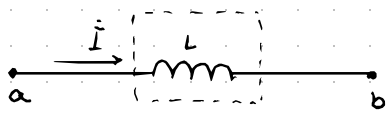
napon >> struja (prethodi)

$\varphi = \alpha_u - \alpha_i = +90^\circ$



(t) $u_L(t) = u_{ab}(t)$

vremenska domena

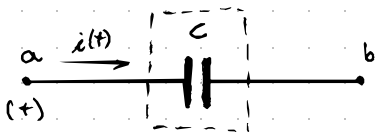


(f) $\dot{U}_L = \dot{U}_{ab} = j X_L \cdot \dot{I}$

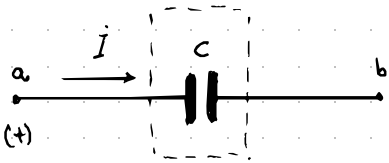
kompleksna domena

Kapacitet C

$u_c(t) = U_M \sin(\omega t + \alpha_0)$



$u_c(t) = u_{ab}(t)$



$\dot{u}_c = \dot{u}_{ab} = -j X_C \cdot \dot{I}$

$\alpha = 0$

$i(t) = C \frac{d}{dt} u(t) = C \frac{d}{dt} \cdot U_M \sin(\omega t) = C \cdot U_M \cdot \cos(\omega t)$

$i(t) = \omega C \cdot U_M \sin(\omega t + \frac{\pi}{2})$

→ pretvaranje u fazor:

$i(t) = \omega \cdot C \cdot U_m \sin(\omega t + \frac{\pi}{2})$

$\dot{I} = \omega \cdot C \cdot \frac{U_{ef}}{\sqrt{2}} \cdot \angle 90^\circ$

$= \omega \cdot C \cdot |\dot{u}| \angle 0^\circ \cdot 1 \angle +90^\circ$

$= j\omega C \dot{u} = \frac{\dot{u}}{-jX_C}$

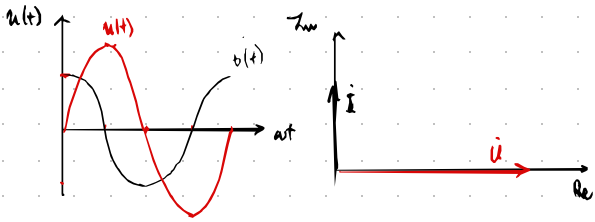
$\dot{U}_c = U_{ef} \angle 0^\circ$

$\dot{I} = \frac{U_{ef} \cdot \angle 0^\circ}{-j X_C}$

$\dot{I} = I_{ef} \angle 90^\circ$

kapacitivna reaktancija

kapacitativni otpor $\frac{1}{\omega C}$



α_u - početak napona $2\alpha + \frac{\pi}{2}$

napon << struja

↓

kasni

$\varphi = \alpha_u - \alpha_i = -90^\circ$