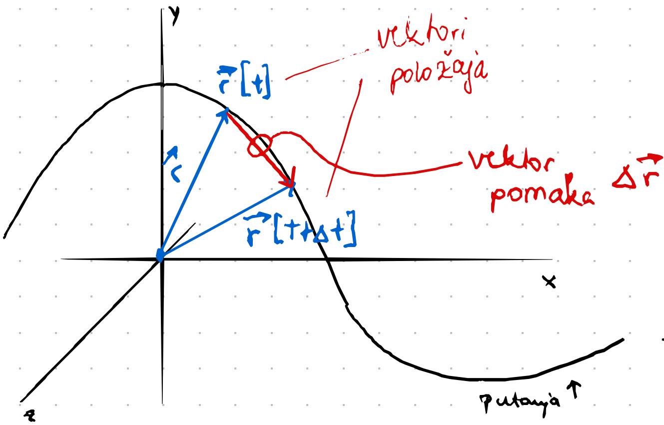
KINEMATIKA ČESTICA U 3D PROSTORU

PUTANJA Č. i POLOŽAJ Č.



$\vec{r}[t+\Delta t] = \vec{r}[t] + \Delta\vec{r}$

$\Downarrow$

Pomak:

$$\Delta\vec{r} = \vec{r}[t+\Delta t] - \vec{r}[t]$$

- koliko god smanjimo Δt, smanjit ćemo i duljinu $\Delta\vec{r}$ (modul)

avg. velocity

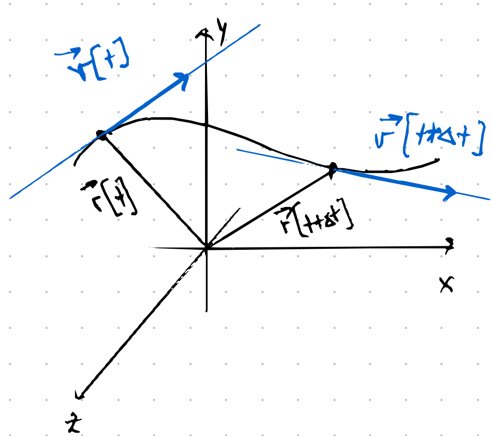
brzina: (vektor) $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$ → vektor smijemo pomnožiti skalarom $\left(\frac{1}{\Delta t}\right)$ ali nije očito pa možemo napisati ovako

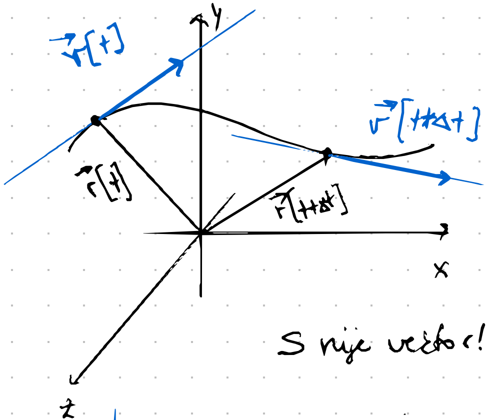
→ shvatimo $r[t]$ kao funkciju vremena

$$\Rightarrow \vec{v} = \lim_{\Delta t \to 0} \frac{\vec{r}[t+\Delta t] - \vec{r}[t]}{\Delta t}$$ derivacija!

$$\vec{v} = \frac{d}{dt}\vec{r}[t]$$

iznos brzine: $v = |\vec{v}|$ avg. speed!





s nije vektor!

elemenat duljine prevaljenog puta:

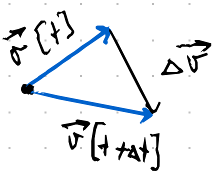
→ matematički → diferencijal

$$ds = |\vec{v}|\,dt \Rightarrow ds = v\,dt$$

iznos brzine: $v = |\vec{v}| = \dfrac{ds}{dt}$

put funkcija po vremenu → $v = \dfrac{d}{dt} s(t)$

vektore brzine možemo translatirati u istu točku da ih usporedimo



$\Delta\vec{v}$ je razlika brzina koja je nastala u intervalu vremena Δt

v je f-ja vremena

→ kada taj Δt teži u 0:

akceleracija: $\lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{d}{dt} v(t)$

budući da znamo da je $\vec{v}(t) = \dfrac{d}{dt}\vec{r}(t)$

onda znamo, tj. možemo napisati $\boxed{\vec{a} = \dfrac{d^2}{dt^2}\vec{r}(t)}$

Položaj brzina i akceleracija u pravok. koord. sust.

položaj $\vec{r}(t) = x(t) \cdot \hat{x} + y(t) \cdot \hat{y} + z(t) \cdot \hat{z}$

→ x koord. položaja č. u t.

konstanta (ne deriviramo)

brzina: $\vec{v}(t) = \dfrac{d}{dt}\vec{r}(t) = \left(\dfrac{d}{dt}x(t)\right)\hat{x} + \left(\dfrac{d}{dt}y(t)\right)\hat{y} + \left(\dfrac{d}{dt}z(t)\right)\cdot\hat{z}$

$$= v_x(t)\,\hat{x} + v_y(t)\,\hat{y} + v_z(t)\,\hat{z}$$

akceleracija: $\vec{a}(t) = \dfrac{d}{dt}\vec{v}(t) = \left(\dfrac{d}{dt}v_x(t)\right)\hat{x} + \ldots = a_x(t)\,\hat{x} \ldots$

$$a_x(t) = \frac{d}{dt}v_x(t) = \frac{dv_x}{dt} = \dot{v}_x \Rightarrow \frac{d^2x}{dt^2} = \ddot{x}$$

Inverzne relacije za $\vec{v}$ i $\vec{r}$

BRZINA

* akceleracija $= \frac{d\vec{v}}{dt} \Rightarrow \boxed{d\vec{v} = \vec{a} \cdot dt}$

$$d\vec{v} = \vec{a}[t']dt' \quad / \int_{t'=t_0}^{t'=t}$$

$$\int_{t'=t_0}^{t'=t} d\vec{v} = \int_{t'=t_0}^{t'=t} \vec{a}[t']dt'$$

$$\vec{v}[t] - \vec{v}[t_0] = \int_{t'=t_0}^{t'=t} \vec{a}[t']dt'$$

$$\vec{v}[t] = \vec{v}[t_0] + \int_{t'=t_0}^{t'=t} \vec{a}[t']dt'$$

* brzina $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow d\vec{r} = \vec{v} \cdot dt \quad / \int_{t_0}^{t'=t}$

$$\vec{r}[t] - r[t_0] = \int_{t'=t_0}^{t'=t} \vec{v}[t']dt'$$

$$\boxed{\Rightarrow \vec{r}[t] = \vec{r}[t_0] + \int_{t'=t_0}^{t'=t} \vec{v}[t']dt'}$$

GIBANJE STALNOM BRZINOM

$\vec{a} = 0 \qquad \vec{v} = \vec{v_0} = \overrightarrow{konst.}$

ako je brzina konstantna, jedino se mijenja položaj

$$\vec{r}[t] = \vec{r}[t_0] + \int_{t_0}^{t} \underbrace{\vec{v}[t']}_{\vec{v}_0} dt' = \vec{r}[t_0] + \int_{t_0}^{t} \vec{v}_0 dt'$$

$$= \vec{r}[t_0] + v_0 \int_{t_0}^{t} dt' = \underline{\vec{r}[t_0] + v_0(t - t_0)}$$

GIBANJE STALNOM AKCELERACIJOM

$\vec{a} = \vec{a_0} = \overrightarrow{konst.}$

$$\vec{v}(t) = \vec{v}[t_0] + \int_{t_0}^{t} \underbrace{\vec{a}[t']}_{\vec{a}_0} dt = \vec{v}[t_0] + a_0 \int_{t_0}^{t} dt = \underline{\vec{v}[t_0] + \vec{a_0}(t - t_0)}$$

$$\vec{r}[t] = \vec{r}[t_0] + \int_{t_0}^{t} \vec{r}[t'] dt'$$

$$\vec{r}[t] - \vec{r}(t_0] = \int_{t_0}^{t} \left(\vec{v}[t_0] + \vec{a_0}(t' - t_0)\right) dt'$$

$$= \int_{t_0}^{t} \vec{v}[t_0] dt' + \int_{t_0}^{t} \vec{a_0} (t' - t_0) dt' = \vec{v_0}[t_0] \int_{t_0}^{t} dt + \vec{a_0} \int_{t_0}^{t} \underbrace{(t' - t_0)}_{=\tau} dt'$$

$$= \vec{v_0}[t_0](t - t_0) + \vec{a_0} \int_{0}^{t-t_0} \tau \, d\tau = \vec{v_0}[t_0](t - t_0) + \vec{a_0} \frac{\tau^2}{2} \Big|_{0}^{t-t_0}$$

$$\Rightarrow \boxed{\vec{r}[t] - \vec{r}[t_0] = \vec{v_0}[t_0](t - t_0) + \frac{\vec{a_0}}{2}(t - t_0)^2}$$