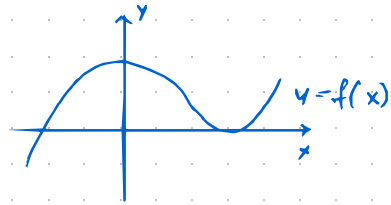


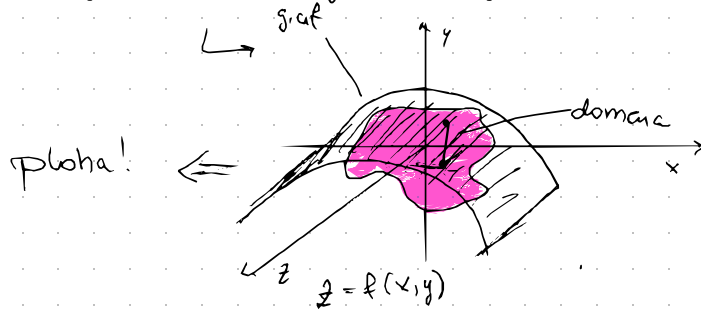
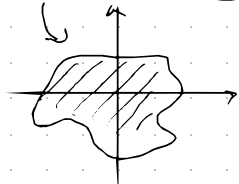
1.3. FIVE VIŠE VARIJABLI

$f: \mathbb{R} \rightarrow \mathbb{R}$
 |
 domena
 u 1-dimenz
 (neki interval) \rightarrow dospel



$D_f \rightarrow 1D$

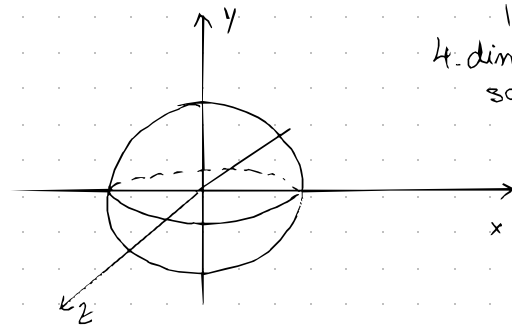
$f: \mathbb{R}^2 \rightarrow \mathbb{R}$
 |
 D_f dvodimenzionalna (2D) \rightarrow nije interval
 \rightarrow skup tačaka u ravlini (crtaemo u 2D)
 - nije uvijek kružnica
 \Rightarrow graf je 3D (jer 2 varijable + još jedna os $\rightarrow z \in \mathbb{R}$)



$f: \mathbb{R}^3 \rightarrow \mathbb{R}$ $D_f \Rightarrow 3D$

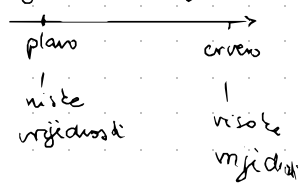
npr. domena je sfera

$(x, y, z) \in D_f$



graf \Rightarrow 4D

4. dimenz označavamo sa nijansom boje



Funkcije više (n) varijabli

↳ graf je (n+1)

↳ Df je $W(x_1, \dots, x_n)$

uvijek pišemo u
više dimenzija

Df crtamo!! nije više interval

Zad.) Odredite domenu slj. f-ja:

a) $f(x, y) = \ln(2 - x^2 - y^2) + \sqrt{x - y^2}$

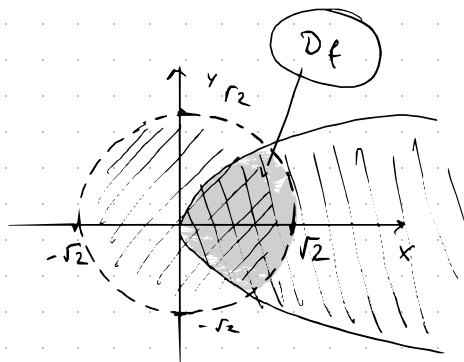
$2 - x^2 - y^2 > 0$

$x - y^2 \geq 0$

$x^2 + y^2 < 2$

↳ kružnica sa središtem u (0,0)

bez ruba



b) $f(x, y) = \ln(x \ln(y-x)) + \arctg(5x+2y)$

$x \ln(y-x) > 0$

$y-x > 0$

$y > x$

1. $x > 0$

$\ln(y-x) > 0 / e$

$y-x > 1$

$y > 1+x$

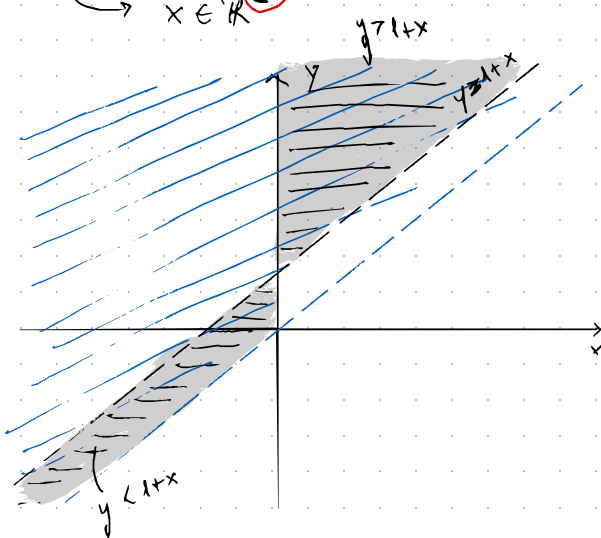
2. $x < 0$

$\ln(y-x) < 0 / e$

$y-x < 1$

$y < 1+x$

PAZITI U
koliko D
gledamo



c) $f(x,y) = \ln(\arcsin(\frac{x}{y}))$

$\arcsin(\frac{x}{y}) > 0 \quad (-1 \leq \frac{x}{y} \leq 1)$

$\Rightarrow 0 < \frac{x}{y} \leq 1$

$\frac{x}{y} > 0$ - prvi ili treći kvad ($\frac{+}{+}$ ili $\frac{-}{-}$)

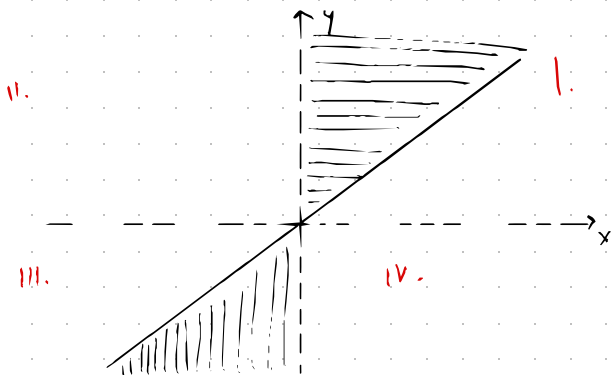
strahmo posas, ne moramo gledati kad je veći od -1 jer je $-1 \leq 0$

$\frac{x}{y} - 1 \leq 0 \rightarrow \frac{x-y}{y} \leq 0$

1. $x-y \geq 0 \rightarrow y \leq x$
 $y < 0$ manji od 0 manji od prava

2. $x-y \leq 0 \rightarrow y \geq x$
 $y > 0$ veći od prava
veći od 0

→ koord. osi su istaknute



d) $f(x,y,z) = \frac{\arccos x + \sqrt{1-y^2} + \arctan \sqrt{1-z^2}}{x^2+y^2+z^2}$

→ $\arccos x$

$-1 \leq x \leq 1$

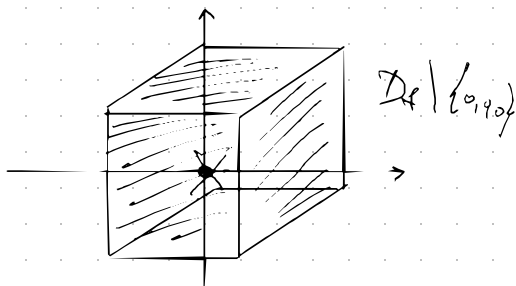
$x^2+y^2+z^2 \neq 0$ - moramo izbaviti točku (0,0,0) jer je zbog kvadrata uvijek > 0

→ $1-y^2 \geq 0$

$y^2 \leq 1 \rightarrow -1 \leq y \leq 1$

KOCKA

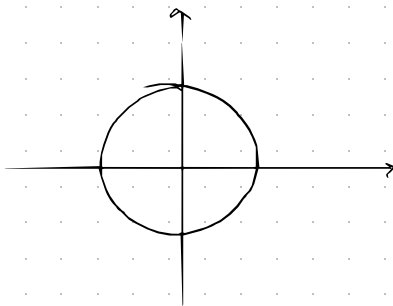
→ $z^2 \leq 1 \rightarrow -1 \leq z \leq 1$



e) $f(x,y,z) = \log_{\pi}(x^2+y^2+z^2-4)$

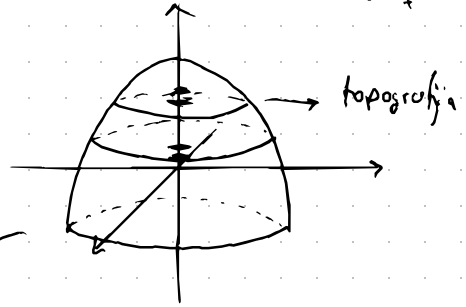
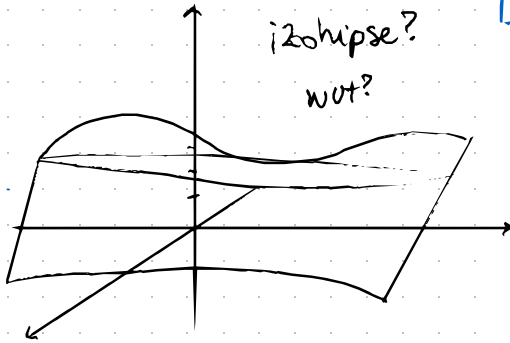
$x^2+y^2+z^2-4 > 0$

$x^2+y^2+z^2 > 4$ → sfera ali bez ruba sa deo uje

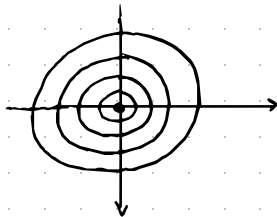


1.4. NIVO - SKUPOVI

DEF Skup točaka $(x,y) \in D_f$ takvih da je $f(x,y) = c$, $c \in \text{Im} f$



Plot

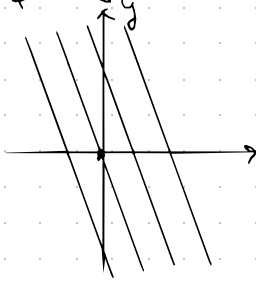


- višu mjesto
snažniji topliji
bogatije a niži
("udubine") hladniji

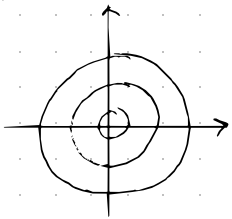
Zad. 1 Odredite i skicirajte nivo krivulje

a) $f(x,y) = 2x + y \Rightarrow 2x + y = c \rightarrow$ znači: to su pravci

$\hookrightarrow \text{Im} f = \mathbb{R}$



$y = c - 2x$ nagib -2



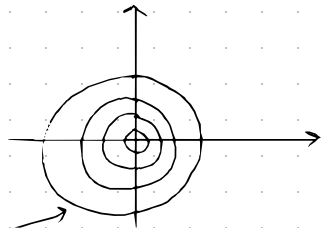
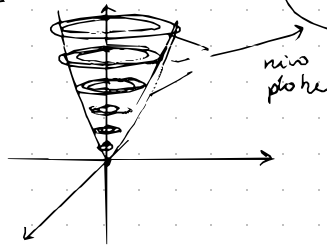
b) $f(x,y) = 2x^2 + 3y^2$

$c \in [0, +\infty)$ jer drugi kvadrat

$2x^2 + 3y^2 = c$

$\rightarrow z = 2x^2 + 3y^2$, graf u 3D \rightarrow

$\frac{x^2}{\frac{c}{2}} + \frac{y^2}{\frac{c}{3}} = 1$

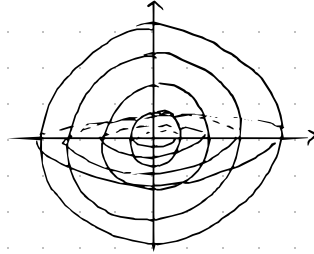


DEF Nivo plohe $f(x, y, z)$ je skup $(x, y, z) \in D_f$ t.dj. $f(x, y, z) = c$,
 $c \in \text{Im } f$.

P.r.) a) $f(x, y, z) = 2x + y \cdot z$

$$\underline{2x + y - z = c}$$

b) $f(x, y, z) = x^2 + y^2 + z^2$
nivo plohe = $x^2 + y^2 + z^2$ = sfere



15. PLOHE DRUGOG REDA

- Znamo: plohe prvog reda $Ax + By + Cz + D = 0$ (ravniina)

npr. $z = f(x, y) = 3x - 2y + 5$ je ravniina : to odmah prepoznamo

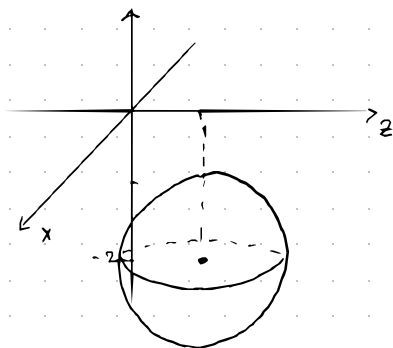
• krivulje drugog reda : kružnice, ellipse, hiperbole, parabole

• transformacije ravnog grafona (translacije, skaliranja, ...)

• geom. tijela : valjak, stožac, kugla ...

SFERA: $x^2 + y^2 + z^2 = r^2$

Pt.: $x^2 + (y-1)^2 + (z+2)^2 = 2$



ELIPSOID: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Pt.:)

$$x^2 + \frac{(y+1)^2}{9} + \frac{(z-2)^2}{4} = 1$$

$$a=1$$

$$b=3$$

$$c=2$$

