Dinamika krutog tijela
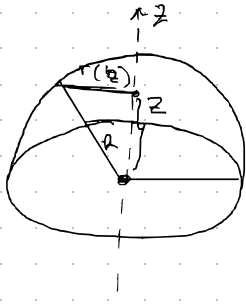
Zadatak 1.)
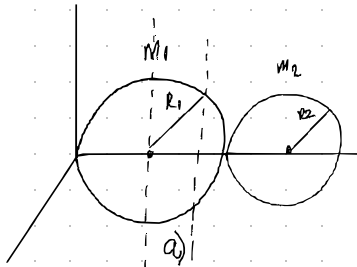

$$I_z = \int (x^2+y^2)\, dm = \int \rho\, r^2\, dV = \rho \int_0^{2\pi} d\varphi \int_0^{r(z)} r^3 dr \int_0^{R} dz$$

$$I_z = \rho \int_0^{2\pi} d\varphi \int_0^{R} dz \int_0^{\sqrt{R^2-z^2}} r^3 dr$$

$$I_z = 2\pi \rho \cdot \int_0^{R} \frac{(R^2-z^2)^2}{4} dz = \frac{\pi \rho}{2} \int_0^{R} R^4 - 2R^2 z^2 + z^4\, dz$$

$$I_z = \frac{\pi \rho}{2} \cdot R^5 \left[1 - \frac{2}{3} + \frac{1}{5} \right] = \boxed{\left(\frac{1}{2} \cdot \frac{4}{3} R^3 \pi \cdot \rho \cdot\right)}_{M} \frac{2}{5} R^2$$

Zadatak 2)

a) $I = ?$ oko osi koja prolazi kroz središte kugle 1

$$I = \underbrace{I_{1CM}}_{\frac{2}{5} m_1 R_1^2} + \underbrace{I_{steiner_2}}_{I_{CM} + m_2(R_1+R_2)^2} = \frac{2}{5}(m_1 R_1^2 + m_2 R_2^2) + m_2(R_1+R_2)^2$$

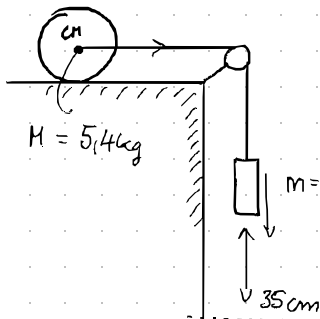$$\underbrace{}_{\frac{2}{5}m_2 R_2^2}$$

$$\boxed{= 23,6\ kg\, m^2}$$

b) Aditivnost:

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 R_1 + m_2(2R_1 + R_2)}{m_1 + m_2} = \frac{6+10}{5} m = \frac{16}{5} m$$

$$I = I_{1CM} + I_{1St} + I_{2CM} + I_{2St} = \frac{2}{5} m_1 R_1^2 + \frac{2}{5} m_2 R_2^2 + m_1\left(\frac{16}{5} - 2\right)^2 + m_2\left(5 - \frac{16}{5}\right)^2$$

$$\boxed{I = 16,4\ kg\, m^2}$$

Zadatak 3.)

$M = 5,4\ kg$

$m = 4,3\ kg$

$35\ cm$

$$I_{valjak} = \frac{MR^2}{2}$$

$$E_p = mgh$$

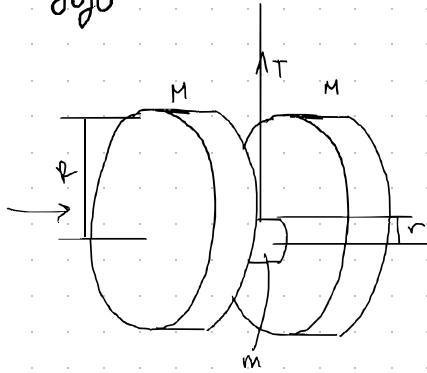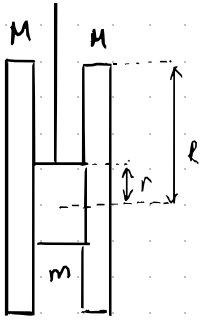$$E_k = \frac{mv^2}{2} + \frac{Mv^2}{2} + \frac{1}{2}\omega^2 \qquad \left(\frac{v}{R}\right)^2$$
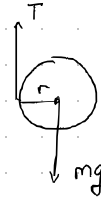
$$mgh = \left(\frac{m}{2} + \frac{3}{4}M\right)v^2$$

$$mgh = \left[\frac{m}{2} + \frac{M}{2} + \frac{MR^2}{2} \cdot \frac{1}{2R^2} \right]\cdot v^2$$

$$v = \sqrt{\frac{mgh}{\left(\frac{m}{2} + \frac{3}{4}M\right)}} \implies \boxed{v = 1,543\ m/s}$$

Zadatač 4.) Jojo

M   M

ℓ

r

m

$\rightarrow$ P

M

M

T

r

m

$a = \alpha r \qquad v = r\omega$

T

r

mg

$T = \dfrac{I\alpha}{r} = \dfrac{Ia}{r^2}$

$a(2M+m) =$