3.1. INTEGRALI OVISNI O PARAMETRU

$$\int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx$$

možemo ga shvatiti kao funkciju $I(\alpha)$ jedne varijable α:

$$\alpha \longmapsto I(\alpha) = \int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx$$

računanje derivacije po parametru α:

$$I'(\alpha) = \frac{d}{d\alpha}\int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx$$

$\Rightarrow$ motivacija: Laplaceova transf., Fourierova transf.

TM Derivacija integrala ovisnog o parametru (Leibnizovo pravilo)

Neka je

$$I(\alpha) = \int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx$$

neprekinuto diferenc. fije

fija 2 var, diferenc i parc. deriv. su nepr. fije

određeni integral ovisan o parametru $\alpha \in \mathbb{R}$, i neka su φ i ψ neprekinuto diferencijabilne funkcije u varijabli α,
a f je funkcija dviju var. x i α, klase C^1

* $\hookrightarrow$ diferencijabilna je i parcijalne derivacije su neprekinute funkcije.

Tada je $I(\alpha)$ diferencijabilna fija i vrijedi:

$$I'(\alpha) = \frac{d}{d\alpha}\int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx$$

$$\boxed{I'(\alpha) = f[\psi(\alpha),\alpha]\cdot\psi'(\alpha) - f[\varphi(\alpha),\alpha]\cdot\varphi'(\alpha) + \int_{\varphi(\alpha)}^{\psi(\alpha)} \frac{\partial f(x,\alpha)}{\partial\alpha}\,dx}$$

DOKAZ ne radimo *

KADA SE KORISTI KOJA FORMULA:

▶ Ako granice ne ovise o α:

$$\frac{d}{d\alpha}\int_a^b f(x,\alpha)\,dx = \int_a^b \frac{\partial f(x,\alpha)}{\partial \alpha}\,dx$$

$$\Downarrow$$

$$\frac{d}{d\alpha}\int_a^\infty f(x,\alpha)\,dx = \int_a^\infty \frac{\partial f(x,\alpha)}{\partial \alpha}\,dx$$

— vrijedi i kada su jedna ili obje granice integracije u beskonačnosti

* ukoliko nepravi integ jednoliko konvergira

- postupamo na isti način kad su obje granice integracije konačne

▶ Ako $f(x,\alpha)$ ne ovisi o α i kad je $\varphi(\alpha) = a$ neka konstanta, a $\psi(\alpha) = \alpha$, onda se formula:

$$\frac{d}{d\alpha}\int_{\varphi(\alpha)}^{\psi(\alpha)} f(x,\alpha)\,dx = f[\psi(\alpha),\alpha]\cdot\psi'(\alpha) - f[\varphi(\alpha),\alpha]\cdot\varphi'(\alpha) + \int_{\varphi(\alpha)}^{\psi(\alpha)} \frac{\partial f(x,\alpha)}{\partial\alpha}\,dx$$

$\Longrightarrow$ svodi se na formulu poznatu iz MAT1:

$$\frac{d}{d\alpha}\int_a^\alpha f(x)\,dx = f(\alpha) \quad \Longrightarrow \quad \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Primjer: ako je $F(\alpha) = \int_2^{\sin\alpha} \frac{tg(\alpha x)}{x}\,dx$, izračunajte $F'(\alpha)$

Podintegralna fja oblika $f(x,\alpha) = \frac{tg(\alpha, x)}{x}$, uvrstimo u formulu:

$$F'(\alpha) = f(\sin\alpha, 2)\cdot \sin\alpha' - f(2,\alpha)\cdot(2)' + \int_2^{\sin\alpha} \frac{\partial}{\partial\alpha}\left(\frac{tg(\alpha x)}{x}\right)dx$$

$$F'(\alpha) = \frac{tg(\alpha\cdot\sin\alpha)}{\sin\alpha}\cdot\cos\alpha - \frac{tg(2\alpha)}{2}\cdot 0 + \int_2^{\sin\alpha}\left(\frac{1}{\cancel{x}}\cdot\frac{1}{\cos^2(\alpha x)}\cdot\cancel{x}\right)dx$$

$$= \frac{tg(\alpha\cdot\sin\alpha)}{\sin\alpha}\cdot\cos\alpha + \int_2^{\sin\alpha}\frac{1}{\cos^2(\alpha x)}\,dx$$

$$= ctg\alpha\cdot tg(\alpha\cdot\sin\alpha) + \frac{1}{\alpha}\cdot tg(\alpha x)\Big|_2^{\sin\alpha}$$

$$= ctg\alpha\cdot tg(\alpha\cdot\sin\alpha) + \frac{1}{\alpha}\left(tg(\alpha\cdot\sin\alpha) - tg(2\alpha)\right)$$

$$\rightarrow \boxed{F'(\alpha) = ctg\alpha\cdot tg(\alpha\cdot\sin\alpha) + \frac{1}{\alpha}\cdot tg(\alpha\sin\alpha) - \frac{1}{\alpha}tg(2\alpha)}$$

ovo je muka Isusova