MAGNETSKA I LORENZOVA SILA

pokusi:

→ $F_M \sim B \sim q \sim v$

→ $F_M \perp v \perp \vec{B}$

$$\boxed{\text{Magnetska sila} \Rightarrow F_M = q\vec{v}\times\vec{B}}$$

izvor mag. sile je el. naboj

— veza između magnetnih i električnih pojava: Lorenzova sila

(djeluju na el. naboj q)

$$\boxed{F_L = q\cdot\vec{E} + q\vec{v}\times\vec{B}}$$

Može li mag. polje vršiti rad? NE

· $\vec{v} \perp \vec{B}$ => skalarni produkt je 0! ⟶ $\vec{a}\times\vec{b} = |a||b|\overset{0}{\cos\varphi}$

↳ $W = \int \vec{F}_M \cdot d\vec{r} = \int q(\vec{v}\times\vec{B})^{\to 0} d\vec{r}$ → $\vec{v} = \frac{d\vec{r}}{dt}$ → $\vec{v}\,dt = d\vec{r}$

$$W = \int q(\vec{v}\times\vec{B})\vec{v}\,dt = \underline{0} \;!$$

* mag. sila ne obavlja rad mijenjanjem položaja naboja

ALI cijelo vrijeme mijenja smjer naboja (centripetalna sila)

- znači $F_M \perp (\vec{v},\vec{B})$ ⟶ $F_M \perp \vec{v} \perp \vec{B}$

$F_M \cdot v = 0$ ⟶ $\boxed{W=0}$

Primjer: $F_M = q\vec{v}\times\vec{B}$

Zadano:

$v = \frac{v_0}{\sqrt{2}}(\hat{x} - \hat{y})$

$B = \frac{B_0}{\sqrt{2}}(\hat{y} - \hat{z})$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{v_0}{\sqrt{2}} & -\frac{v_0}{\sqrt{2}} & 0 \\ 0 & \frac{B_0}{\sqrt{2}} & -\frac{B_0}{\sqrt{2}} \end{vmatrix} = \frac{B_0 v_0}{2}\cdot\hat{x} + \frac{B_0 v_0}{2}\hat{y} + \frac{v_0 B_0}{2}\hat{z}$$

⟶ $\vec{v}\times\vec{B} = \frac{B_0 v_0}{2}(\hat{x} + \hat{y} + \hat{z})$

↳ $$\boxed{F_M = q\,v\times B = -|q|\frac{B_0 v_0}{2}(\hat{x} + \hat{y} + \hat{z})}$$

GIBANJE U MAG. POLJU.

→ matemat. oblik: $F_M = q\vec{v}\times\vec{B}$; ne vrši rad ; $\perp$ na $\vec{v}$

↳ sila koja $W=0$, $F \perp v$ => centripetalna sila

→ $F_M = F_{cp}$

$$F_m = qvB = \frac{mv^{\cancel{2}}}{R} = q\cancel{v}B\underbrace{\sin(\overset{90^\circ}{v,B})}_{1}$$

$$F_m = \frac{mv}{R} = qB \implies \boxed{R = \frac{mv}{qB}}$$ polumjer kruženja naboja

• možemo izračunati period ako je $v = \omega R$

$$\omega = \frac{v}{R} = \frac{qB}{m}$$, a za ω općenito vrijedi $\omega = \frac{2\pi}{T}$

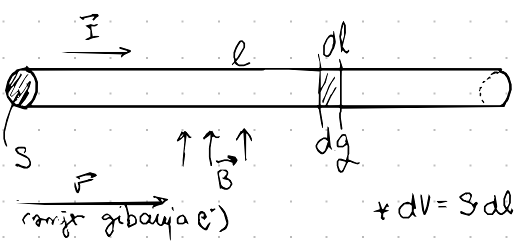
$$\implies T = \frac{2\pi}{\frac{qB}{m}} \implies T = 2\pi\frac{m}{qB} \qquad T = \frac{2\pi}{\omega}$$

odnosno, gibanje je kružno

SILA NA VODIČ U MAG. POLJU

• vodič duljine (dl) površine pop. presjeka (S) kojim teče struja (I) → struju definiramo: $I = nq\,v\cdot S$

$nq = \frac{dq}{dV}$ → $n = \frac{N}{V}$ broj naboja u volumenu



$* \; dV = S\,dl$

↳ stoga se u mag polju $\vec{B}$ pojedini naboji gibaju brzinom v zbog sile $F_M = q\vec{v}\times\vec{B}$

↓ slijedi da je sila na vodiču:

$I = n\cdot q\cdot v\cdot S$

$d\vec{F} = q(v\,\hat{v}\times\vec{B})\,n\,S\,dl$

$dF = I(\hat{v}\times\vec{B})\,dl$

$d\vec{l} = \hat{v}\cdot dl$ (dodali smo smjer promjeni puta našeg vodiča u smjeru kretanja naboja)

$F_{L,M} = q\vec{v}\times\vec{B}$ / $\frac{d}{dq}$ $\qquad I = \frac{dq}{dt} \qquad v = \frac{d\vec{l}}{dt}$

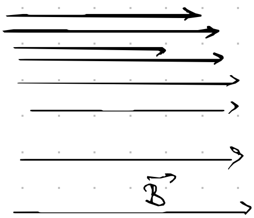
$$dF_M = dq\,\vec{v}\times\vec{B} = I\cdot dt\cdot\vec{v}\times\vec{B} = I(dl\cdot\hat{v})\times\vec{B}$$

$d\vec{l} = \hat{v}\cdot dl$

$$\boxed{F = I\int_A^B d\vec{l}\times\vec{B}}$$

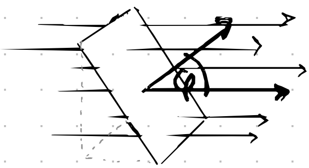
Gaussov zakon i teorem za magnetsko polje

Gaussov zakon: – iz gustoće silnica zaključujemo je li polje jako ili slabo



· ondje gdje su silnice gušće → mag. ind (B_{ind}) & jakost mag. polja (B) su veće

homogeno mag. polje



$\Phi_B = BS\cos\varphi$ => skalarni umnožak vektora

$\Phi_B = \vec{B} \cdot \vec{S}$ – plohu moramo podijeliti na beskonačno male komadiće dS

$$\boxed{\Phi_B = \int \vec{B}\, d\vec{S}} \quad [Wb] = [T\, m^2] = [V\, s]$$

(weber)

$* = 0$

→ tok el. polja E (Φ_E) kroz neku plohu je obuhvaćen tom plohom

↳ isto vrijedi i za tok mag polja => nema izoliranih mag polova, mag. silnice se zatvaraju u sebe

=> MAG. TOK. (Φ_B) kroz bilokoju zatvorenu plohu je 0

$$\boxed{\oint_S \vec{B}\, d\vec{S} = 0}$$

II. MAXWELLOVA JEDNADŽBA u integralnom obliku

$$\oint \vec{B}\, d\vec{S} = \frac{Q_M}{\varepsilon_0} = 0 \longrightarrow \boxed{\vec{\nabla}\vec{B} = 0}$$ (običan oblik)

(primjenom Gaussovog TM o divergenciji

Druga Maxwellova jednadžba je posljedica nepostojanja mag. monopola

Magnetski monopol: postojanje električno nabijenja strana (pol)

=> hence nema polova

GIBANJE U MAG. POLJU :

- iz matemat. oblika za mag. silu → $F_m = q\vec{v}\times\vec{B}$ (ona ne vrši rad)

→ sila koja također ne vrši rad je centripetalna sila

↳ $\vec{F}_m = \vec{F}_{cp}$

$$F_m = qvB = \frac{mv^{\cancel{2}}}{R} = q\cancel{v}B\underbrace{\sin(\overset{90°}{v,B})}_{1}$$

$$F_m = \frac{mv}{R} = qB \longrightarrow \boxed{R = \frac{mv}{qB}}$$ polumjer kružnog gibanja

- možemo izračunati period gibanja ako je $v = \omega R$

→ $\omega = \frac{v}{R} = \frac{qB}{m}$, a za ω općenito vrijedi $\omega = \frac{2\pi}{T}$

$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow T = \frac{2\pi}{\frac{qB}{m}} = 2\pi\frac{m}{qB}$$

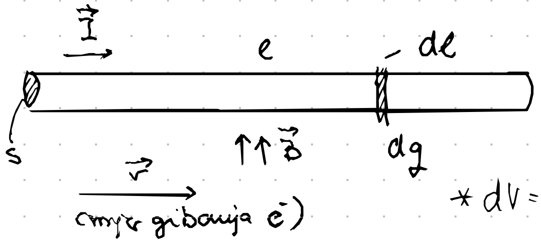
odnosno, gibanje je kružno

SILA NA VODIČ U MAG. POLJU

- vodič duljine $d\vec{l}$, površina poprečnog presjeka S, struja kroz njega je I

⟹ struja: $I = nqvS$

$n_q = \frac{dQ}{dV}\left[\frac{N}{V}\right]$ broj naboja u volumenu (koncentracija)



* $dV = S\cdot dl$

zbog toga u mag polju $\vec{B}$ se naboji gibaju brzinom $\vec{v}$ zbog sile $F_m = q\vec{v}\times\vec{B}$

slijedi da je sila na vodiču:

$$I = n\cdot q\cdot v\cdot S$$

$$d\vec{F} = q(v\,\hat{v}\times\vec{B})\,nS\,dl$$

$$dF = I(\hat{v}\times\vec{B})\,dl$$

$$\boxed{dl = \hat{v}\,dl}$$

$d\vec{l} = \hat{v}\cdot dl$ (dali smo smjer našem vodiču dl)

$$d\vec{F} = I\,d\vec{l}\times\vec{B}$$

$$\vec{F} = \int_A I(d\vec{l}\times\vec{B})$$

$$\boxed{\vec{F} = I\int_A^B d\vec{l}\times\vec{B}}$$

Zadatak za promišljanje:

$$\vec{B} = B_0 \hat{x} \qquad \vec{v} = \frac{v_0}{\sqrt{2}}(\hat{x} + \hat{y}) \qquad q \;\Big\}\; \text{zadano} \qquad F_M = ?$$

$$F = |q| \cdot \vec{v} \times \vec{B} \longrightarrow \vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{v_0}{\sqrt{2}} & \frac{v_0}{\sqrt{2}} & 0 \\ B_0 & 0 & 0 \end{vmatrix} = q\left(0 + 0 - \frac{B_0 v_0}{\sqrt{2}} \hat{z}\right)$$

$$F_M = -q \frac{B_0 v_0}{\sqrt{2}} \hat{z}$$

↳ možemo izračunati v

$$m a_x = 0 \qquad m a_y = 0 \qquad m a_z = -\frac{q v_0 B_0}{\sqrt{2}}$$

$$v_x(t) = \frac{v_0}{\sqrt{2}} \qquad v_y(t) = \frac{v_0}{\sqrt{2}} \qquad v_z(x) =$$

← ovo nije dobro jer ako smo dobili da postoji neka v_z kako smo u matrici imali 0?

=> treba biti

$$F = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_0 & 0 & 0 \end{vmatrix} = [\cancel{\hat{x}\, 0} + \hat{y} \cdot B_0 v_z + \hat{z} B_0 v_y]$$

$$F = q B_0 (v_z \hat{y} - v_y \hat{z})$$

↳ specifičnost mag. polja B je da dodaje nove komponente

$$m a_x = 0 \qquad m a_y = q v_z B_0 \qquad m a_z = -q v_y B_0$$

$$v_x = \frac{v_0}{\sqrt{2}} \qquad a_y = \frac{q v_z B_0}{m} \qquad \ldots \text{isto}$$

$$\frac{dv_y}{dt} = \frac{q v_z B_0}{m} \Big/ \cdot \frac{d}{dt} \qquad \frac{dv_z}{dt} = -\frac{q v_y B_0}{m}$$

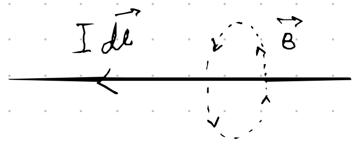
$$\frac{d^2 v_y}{dt^2} = \frac{d}{dt} \cdot \frac{q v_z B_0}{m} = \frac{dv_z}{dt} \frac{q B_0}{m} = \frac{q B_0}{m} \cdot \left(-\frac{q v_y B_0}{m}\right)$$

$$\rightarrow \frac{d^2 v_y}{dt^2} = -\left(\frac{q B_0}{m}\right)^2 v_y$$

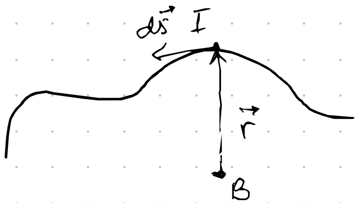
$$\ddot{v}_y = -\left(\frac{q B_0}{m}\right)^2 v_y$$

$$\ddot{v}_y + v_y \left(\frac{q B_0}{m}\right)^2 = 0 \rightarrow \omega = \frac{q B_0}{m}$$

Magnetsko polje ravnog vodiča



mag polje B je uvijek okomito na smjer gibanja struje $\vec{dl}$



Pretpostavke Biot-Savartovog zakona

I) vekt. mag polja $\vec{dB} \perp \vec{ds} \perp \vec{r}$

II) iznos $\vec{dB} \sim \frac{1}{r^2} \sim I\,ds$

III) iznos $\vec{dB}$ je ovisan o kutu φ između $\vec{ds}$ i $\vec{r}$ (mijenja se sa sinusom)

$\vec{dB} \sim \sin(\measuredangle \vec{ds}, \vec{r})$

Biot-Savartov zakon

- iz gore navedenih pretpostavki:

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{ds} \times \vec{r}}{r^2} \quad / \int$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{ds} \times \vec{r}}{r^2}$$

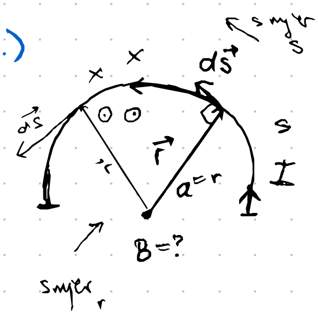
konst

→ lakše je to nego

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \vec{r}$$

el polje

Pr.)



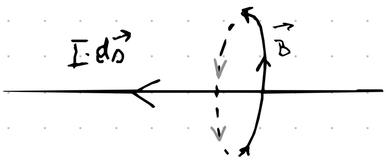
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{ds} \times \vec{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{ds \overbrace{\sin 90}^{1}}{r^2}$$

- $\vec{ds} \perp \vec{r}$ u svakoj točki
- radi se o polukružnici u ovom zad
  - ↳ put je $0 - 2\pi r/2 \to [0, \pi r]$ $r = a$

$$\rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi a} \frac{ds}{a^2} = \frac{\mu_0 I}{4\pi} \frac{1}{a^2} \int_0^{\pi a} ds$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{1}{a^2} \cdot \pi a = \frac{\mu_0 I}{4a}$$

Magnetsko polje ravnog vodiča

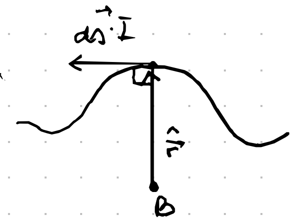


· mag. polje B je uvijek okomito na smjer gibanja struje $\underline{\underline{d\vec{s}}}$ ($d\vec{s}$ je za površinu, a $d\vec{l}$ je kasnije)

Pretpostavke BIOT - SARVARTOVOG zakona:

I.) $d\vec{B} \perp d\vec{s} \perp \hat{r}$

položaje od djelića $d\vec{l}$ do točke izvan vodiča u kojoj promatramo i određujemo mag. polje $d\vec{B}$



II.) $d\vec{B} \sim \frac{1}{\vec{r}^2} \sim I d\vec{s}$

III.) $d\vec{B} \sim \sin(\angle d\vec{s}, \hat{r})$ iznos $d\vec{B}$ ovisan je o kutu θ između $d\vec{l}$ i $\hat{r}$

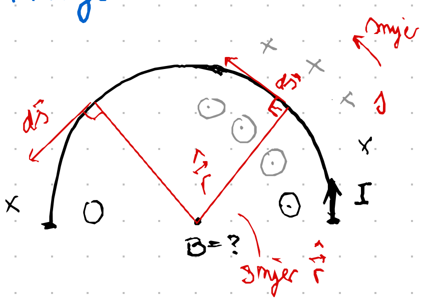
=> **Biot-Sarvartov zakon** $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{s} \times \hat{r}}{r^2}$

$$\vec{B} = \frac{\mu_0}{4\pi} I \cdot \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

* μ_0 - permitivnost vakuuma : 10^{-7} Wb/Am

* lakše nego el. polje $\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \hat{r}$

Primjer:



$= d\vec{s}$
$(\hat{r} \; ds = d\vec{s})$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{l_0}^{l_1} \frac{ds \times \hat{r}}{r^2} =$$

jedinični vektor je 1

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{l_0}^{l_1} \frac{ds \cdot 1 \cdot \overbrace{\sin\theta}^{1}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{ds}{r^2}$$

↓

(budući da je radi o polukružnici, $\in [0, \pi r]$ → $\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi r} \frac{ds}{r^2}$

(pola kruga je $\frac{2\pi r}{2} = \pi r$)

→ zadano je da je $r = a =$ konst.

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{1}{a^2} \int_0^{\pi r} ds$$

$$\boxed{B = \frac{\mu_0 I}{4a}}$$