ELEKTROSTATIKA

Columbov zakon:

$$\boxed{\vec F = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r^2}\,\hat{r}}$$

svake dvije mirne čestice međudjeluju Columbovom silom

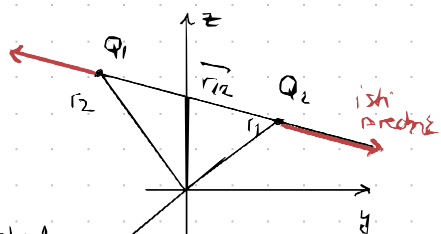
$r_{12} = |\vec r_{12}| = |\vec r_2 - \vec r_1|$

udaljenost između čestica $\rightarrow \hat r = \frac{\vec r_{12}}{r_{12}}$

$^{*}\ \varepsilon_0 = \frac{1}{(\mu_0 c^2)}$ u formulama

$q_{min} = e = 1{,}602\times 10^{-19}\,C$ (u formulama)

! naboj čestica je uvijek očuvan !

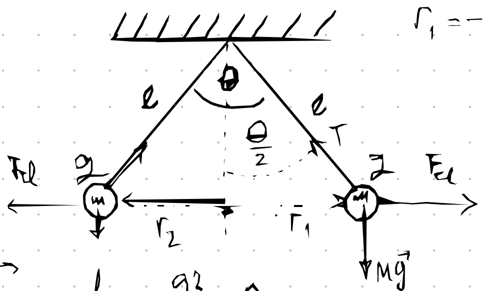


- odbojna kada čestice imaju isti predznak
- privlačna kada imaju suprotni predznak

Primjer: Elektroskop

koliko je naboja q potrebno da se rozmaknu za θ?

$\vec r_1 = -\vec r_2$



$\sin\frac{\theta}{2} = \frac{|\vec r_1|}{l}$

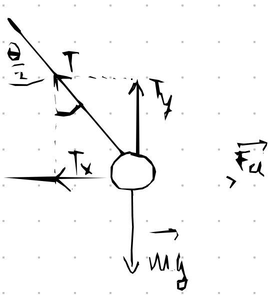
$|\vec r_1| = |\vec r_2| \longrightarrow |\vec r_{12}| = |\vec r_1 + \vec r_2| = r_{12}$

$r_{12} = l\sin\left(\frac{\theta}{2}\right) + l\sin\left(\frac{\theta}{2}\right)$

$r_{12} = 2l\sin\left(\frac{\theta}{2}\right)$

$$\vec F_{el} = \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{r^2}\,\hat{r}$$

$$F_{el} = \frac{q^2}{4\pi\varepsilon_0}\cdot\frac{1}{\left(2l\sin\left(\frac{\theta}{2}\right)\right)^2}$$



$\sin\left(\frac{\theta}{2}\right) = \frac{T_x}{T}$

$T_x = T\cdot\sin\left(\frac{\theta}{2}\right)$

$T_y = T\cos\left(\frac{\theta}{2}\right)$

$x:\ T_x = F_{el}$

$$T\sin\left(\frac{\theta}{2}\right) = \frac{q^2}{4\pi\varepsilon_0}\cdot\frac{1}{\left(2l\sin\left(\frac{\theta}{2}\right)\right)^2}$$

$y:\ T_y = m\cdot g$

$T\cos\left(\frac{\theta}{2}\right) = mg$

$\hookrightarrow T = \frac{mg}{\cos\left(\frac{\theta}{2}\right)}$

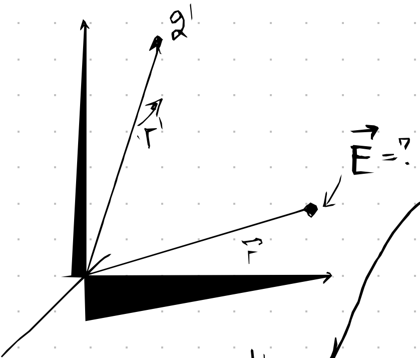
$$\rightarrow \frac{mg}{\cos\left(\frac{\theta}{2}\right)}\cdot\sin\left(\frac{\theta}{2}\right) = \frac{q^2}{4\pi\varepsilon_0}\cdot\frac{1}{\left(2l\sin\left(\frac{\theta}{2}\right)\right)^2}$$

$$\operatorname{tg}\left(\frac{\theta}{2}\right) = \frac{q^2}{4\pi\varepsilon_0\cdot 4\,l^2\sin^2\left(\frac{\theta}{2}\right)}\cdot\frac{1}{mg} \Rightarrow \boxed{q^2 = 16\pi\varepsilon_0\, l^2\cdot\frac{\theta^3}{8}}$$

* za $\theta \lll$ vrijedi $\theta \sim \sin\theta \sim \operatorname{tg}\theta$

Električno polje

$$\vec{E}[\vec{r}] = \frac{1}{4\pi\varepsilon_0} \frac{q'}{|\vec{r}-\vec{r}'|^2} \cdot \widehat{\vec{r}-\vec{r}'}$$



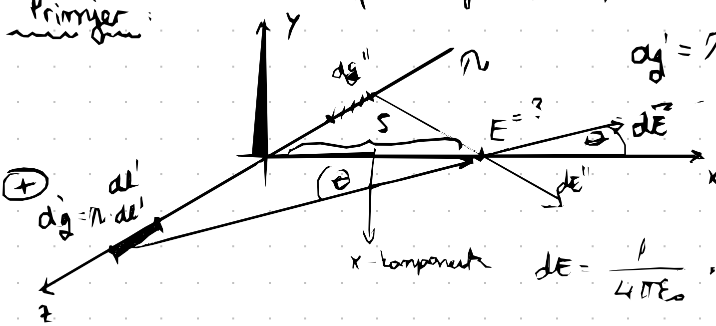
$$\widehat{\vec{r}-\vec{r}'} = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$

$$\vec{E}[\vec{r}] = \frac{q'}{4\pi\varepsilon_0} \cdot \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

ako to shvatimo kao diferencijal naboja u toj okolini točke: $d\vec{E} = \frac{\rho}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \cdot dq' \Big/ \int$

$\vec{E}[\vec{r}] = \int d\vec{E} = \frac{}{4\pi\varepsilon_0}\int$ [redacted] $\Rightarrow E_1 = \frac{F}{Q_2}$

Primjer: Beskonačni štap na kojem je raspoređen naboj



$dq' = \lambda \cdot dl'$

$dE_y = 0$

$dE = \frac{\rho}{4\pi\varepsilon_0} \cdot \frac{\lambda \, dl}{S^2+z^2} \cos\theta$

! ali zanima nas samo x komponenta

$\cos\theta = \frac{S}{S^2+z^2}$

$$dE_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda \, dz \cdot S}{(S^2+z^2)^{3/2}}$$

jer je na z-osi

$$E_x = \int dE_x = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cdot S}{(S^2+z^2)^{3/2}} dz = \frac{\lambda s}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{dz}{(s^2+z^2)^{3/2}}$$

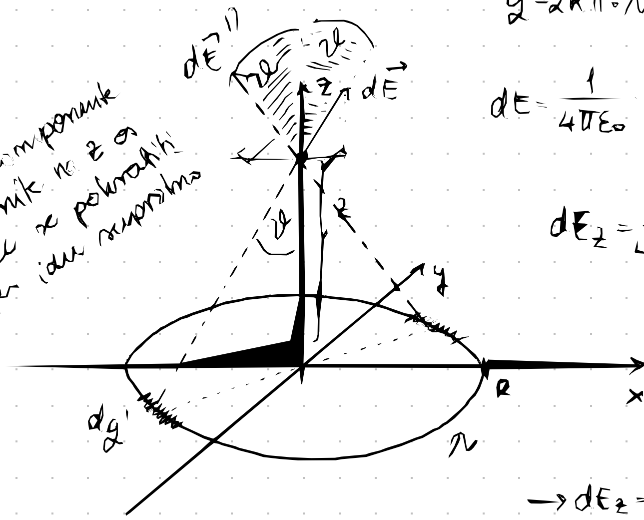
tablični integral

$$E_x = \frac{\lambda \cancel{S}}{4\pi\varepsilon} \cdot \frac{\cancel{2}}{S^{\cancel{2}}} \Rightarrow \boxed{E_x = \frac{\lambda}{2\pi\varepsilon_0 \cdot S}}$$

* linearna gustoća: $\frac{dq'}{dl} = \lambda \longrightarrow dq' = \lambda[r'] \cdot dl$
* površinska gustoća: $\frac{dq'}{dS} = \sigma \qquad dq' = \sigma[r'] \cdot dS$
* volumna gustoća: $\frac{dq'}{dV} = \rho \qquad dq = \rho[r'] \cdot dV$

Primjer: Prsten

$q = 2R\pi \cdot \lambda$



ove komponente
okomite na z os
će se pokratiti
jer idu suprotno

$$dE = \frac{1}{4\pi\varepsilon_0}\,\frac{dq'}{R^2+z^2}$$

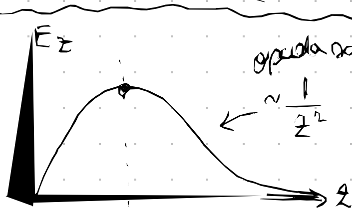
$$dE_z = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq'}{R^2+z^2}\cdot\cos\vartheta$$

$$\cos\vartheta = \frac{z}{\sqrt{R^2+z^2}}$$

$$\rightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\cdot\frac{z\cdot dq'}{(R^2+z^2)^{3/2}} \quad /\int$$

$$\rightarrow E_z = \frac{1}{4\pi\varepsilon_0}\cdot\frac{z}{(R^2+z^2)^{3/2}}\int dq'$$

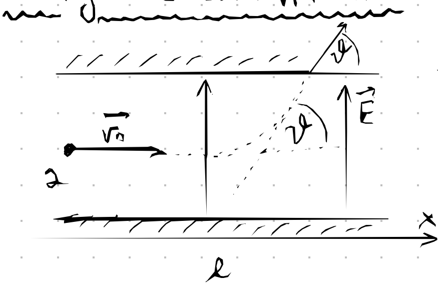
▸ ako se jako udaljimo od z osi → $R \to 0$
kao da sažmemo sav naboj u središte



opada sa $\sim \frac{1}{z^2}$

→ deriviramo po z i tražimo maksimum

Primjer: Sassich ppt #2



$$\vec{v_0} = v_0\cdot\hat{x} + \overset{0}{\vec{v_y}}$$

$$\vec{E} = E\cdot\hat{y}$$

x-os

$ma_x = 0$

y-os

$ma_y = F_y$

$ma_y = q\cdot E$

$$m\cdot\frac{dv_x}{dt} = 0 \;/\int$$

$$\int_{v_0}^{v_x(t)} dv_x = 0$$

$$v_x - v_0 = 0$$

$v_0 = v_x$ konst.

$$v_x = v_0 \Rightarrow \frac{l}{t}$$

$$\hookrightarrow t = \frac{l}{v_x}$$

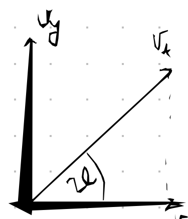
$$m\cdot\frac{dv_y}{dt} = q\cdot E$$

$$\int_0^{v(t)} dv_y = \int_0^{t} \frac{q\cdot E}{m}\,dt'$$

$$v_y(t) - 0 = \frac{qE}{m}\,t$$

to je taj neodređeni trenutak kad skrene

$$v_y(t) = \frac{qE}{m}\cdot\frac{l}{v_x}$$



$$tg\,\vartheta = \frac{v_y}{v_x}$$

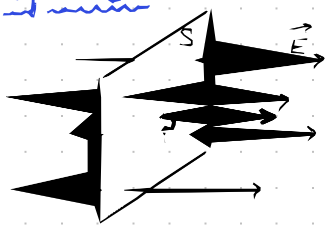
$$tg\,\varphi = \frac{v_y}{v_x} = \frac{1}{v_x}\cdot\frac{qE}{m}\cdot\frac{1}{v_x}$$

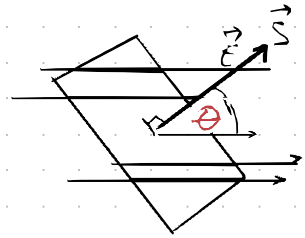
$$tg\,\varphi = \frac{qE}{mv_x^2} \longrightarrow \boxed{\varphi = arctg\left(\frac{qE}{mv_x^2}\right)}$$

Tok električnog polja = Φ

$$\boxed{\Phi_E = \oint_S \vec{E}\, d\vec{S}}$$

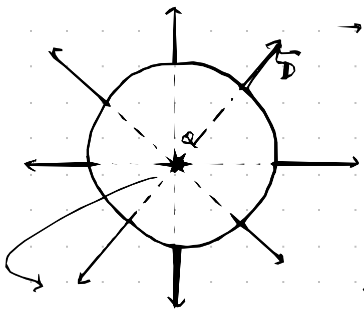
površina u el. polju





Skalarni umnožak: $\Phi_E = \oint E \cdot dS \cdot \cos\theta$

• u zatvorenoj plohi → točkasti naboj u sferi



→ tok el. polja je umnožak vektora $\vec{D}$ → silnice i ukupne površine kugle $\Phi = 4R^2\pi \cdot D = 4R^2\pi\, \varepsilon E$

→ TOK el. polja kroz neku sferu proporcionalan je naboju te kugle i neovisan je o R — Gauss

Jakost el. polja: $\oint \vec{E}\, d\vec{S} = 0$

$$E = \frac{Q}{4\pi\varepsilon_0 \cdot R^2}$$

$$D = \varepsilon \cdot E = \cancel{\varepsilon_0} \cdot \frac{Q}{4\pi\cancel{\varepsilon_0} R^2}$$

$$\Phi = D\,S = \frac{Q}{\cancel{4\pi R^2}} \cdot \cancel{4R^2\pi} \Rightarrow \boxed{\Phi = Q}$$

Gaussov zakon

=> el. tok u zatvorenoj plohi ~ ukupnom naboju koji smo zatvorili

$$\oint \vec{E}\, d\vec{S} = Q_{unutra}$$

vrijedi bez obzira na oblik

+ posljedica što F_{el} opada s kvadratom udaljenosti $F \sim \frac{1}{r^2}$

Gaussov teorem - tm o divergenciji

$$\boxed{\oint \vec{E}\, d\vec{S} = \int \nabla \cdot \vec{E}\, dV}$$

naboj raspodijeljen po V ⇒ $\frac{dq}{dV} = \rho$

$dq = \rho\, dV \,/\int$

$q = \int \rho\, dV$

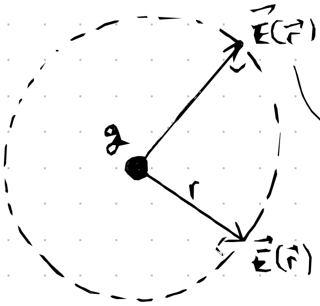
$$\oint \vec{E}\, d\vec{S} = \frac{q}{\varepsilon}$$

$$= \frac{1}{\varepsilon}\int \rho\, dV = \int \vec{\nabla}\vec{E}\, dV$$

$$\Rightarrow \boxed{\frac{\rho}{\varepsilon} = \vec{\nabla}\vec{E}}$$

1. MAX

Električno polje nabijene čestice



$$\oint \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon} = \Phi$$

*u formulama:

$dS = r^2 \sin\vartheta\, d\vartheta \cdot d\varphi \,/\int$

$$S = r^2 \int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{2\pi} d\varphi$$

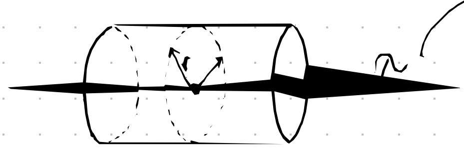
→ $E(r)$ je $\perp$ na sferu zbog prostorne simetrije

$$\oint_S \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0} = \Phi \longrightarrow ES = \frac{Q}{\varepsilon}$$

$$\Rightarrow E \cdot r^2 \int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{2\pi} d\varphi = \Phi \longrightarrow \boxed{\Phi = 4r^2 E = \frac{Q}{\varepsilon_0}}$$

El. polje jednoliko nabijene žice

$$\frac{dq}{dl} = \lambda \quad \left(\text{ili } \lambda = \frac{q}{l}\right)$$

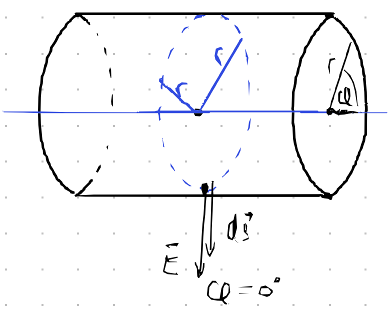


Gaussov zakon: $\oint \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0}$

cilindrična ploha $\Rightarrow dS = r\,d\varphi\,dz$

formule:

$$\oint \vec{E}\,d\vec{S} = \int E\,dS \cos\varphi$$



$$\Rightarrow \oint \vec{E}\,d\vec{S} = \int_{2\pi} E\,dS = \int \underbrace{E(r)}_{\text{konst}} \cdot r\,d\varphi\,dz$$

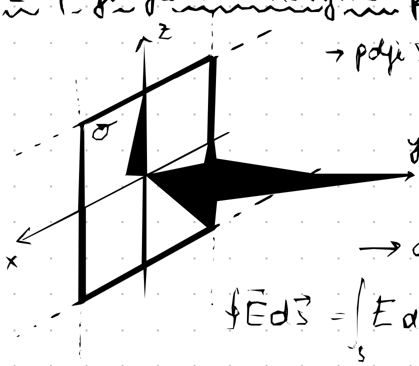
$$\oint \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0} = E r \int_0^{2\pi} d\varphi \int_{-z}^{z} dz = E \cdot r \int_0^{2\pi} 2z\,d\varphi$$

$$\frac{Q}{\varepsilon_0} = Er \cdot 2\pi \cdot 2z = \frac{l\lambda}{\varepsilon_0}$$

$-z$ do $z = l$

$$E r\, 2\pi \cdot \cancel{2z} = \frac{\cancel{2z}\,\lambda}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2r\pi\varepsilon_0}}$$

El. polje jednako nabijene plohe



→ polje se ne mijenja po x i y jer se udaljavamo/približavamo samo po y osi

$$\sigma = \frac{dq}{dS} \rightarrow \text{površinska gustoća}$$

$$\rightarrow dS = dx\,dz \longrightarrow S = \int_{-x}^{x} dx \int_{-z}^{z} dz$$

$$\oint \vec{E}\,d\vec{S} = \int_S E\,dS = 2\int E(y)\,dx\,dz$$

$$\frac{Q}{\varepsilon} = 2E(y) \int_{-x}^{x} dx \int_{-z}^{z} dz = 2E(y) \cdot \cancel{2x \cdot 2z} \Rightarrow \frac{\sigma \cdot S}{\varepsilon}$$

$S = 2x\,2z$

$$E(y) = \frac{\sigma}{2\varepsilon}$$

* zbog simetrije očekujemo da će polje biti okomito na plohu s obje strane (-y i y)

1. MAXWELLOVA JEDNADŽBA

$$\oint E d\vec{S} = \Phi = \frac{Q}{\varepsilon} \qquad \rho = \frac{dq}{dV} \implies \oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon} = \frac{1}{\varepsilon} \cdot \int \rho dV$$

$\hookrightarrow q = \int \rho dV$

Gaussov TM:

$$\int_S \vec{E} d\vec{S} = \oint \vec{\nabla}\vec{E} \cdot dV \longrightarrow \underline{\oint \vec{\nabla}\vec{E} dV = \frac{1}{\varepsilon} \int \rho dV}$$

Svaka zatvorena ploha okružuje neki volumen

$\Downarrow$

TM o divergenciji

$$\boxed{\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon}}$$

Konzervativnost i el. potencijal

Ako konz. polje F opisuje sila kojoj odgovara polje potencijalne energije U $\xrightarrow[\text{prema def}]{}$ $dU = -\vec{F} \cdot d\vec{r}$

$$dU = \vec{\nabla} U \cdot d\vec{r} \longrightarrow \vec{\nabla} U \cdot \cancel{d\vec{r}} = -\vec{F} \cdot \cancel{d\vec{r}}$$

$$\vec{\nabla} U = -\vec{F} \longrightarrow \underline{\vec{F} = -\vec{\nabla} U}$$

polje sile možemo dobit računanjem gradijenta polja potenc. en.

$\implies$ analogno: $\underline{\vec{E} = -\vec{\nabla} V}$

skal. polje = el. potencijal

$$\implies V = \int_A^B \vec{E} d\vec{l} = \frac{Q}{4\pi\varepsilon_0} \cdot \int_A^B \frac{dr}{r^2} \implies \underline{V = \frac{Q}{4\pi\varepsilon_0 \cdot r}}$$