2.7. IMPLICITNO DERIVIRANJE

Matan 1: $x^2+y^2=4 \quad / \frac{d}{dx}$ ($F(x,y)$)

! paziti da ne primjenjujemo parcijalno ako nam se pomiješa!

$2x + 2y \cdot y' = 0$

$$y' = \frac{-2x}{2y} = \frac{-x}{y}$$

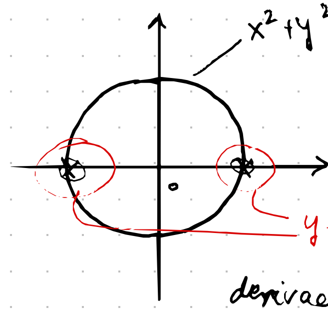
$\frac{\partial f}{\partial x}$ $\quad$ $\frac{\partial f}{\partial y}$

$$y' = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

vrijedi samo ako je $y \neq 0$

→ parcijalna deriv. mora biti $\neq 0$

$x^2+y^2=4$



dvije različite f-je

$y = \pm\sqrt{4-x^2}$

derivacija u tim točkama ne postoji (y')

* postoje parc. deriv.

TM Neka je $\frac{\partial f}{\partial y}(x_0,y_0) \neq 0. \Rightarrow$ jed. f-ja $y=y(x)$

ako je parc. deriv. $\neq 0$, tada postoji jedinstvena implicitno zadana f-ja

↳ zadovoljava $f(x,y)=0$, a njena deriv se računa po formuli

$$y' = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

Ako je krivulja $y=y(x)$ zadana implicitno s $f(x,y)=0$, onda je tangenta na tu krivulju u točki $T_0(x_0,y_0)$

$$y-y_0 = y'(x_0)(x-x_0)$$

$$y-y_0 = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}(x-x_0) \Longrightarrow \left.\frac{\partial f}{\partial x}\right|_{x_0,y_0}(x-x_0) + \left.\frac{\partial f}{\partial y}\right|_{x_0,y_0}(y-y_0) = 0$$

$\boxed{\text{TM}}$ $f(x,y,z)=0$. Ako je $\frac{\partial f}{\partial z}(T_0)\neq 0$. Tada postoji jedinstvena implicitno zadana $z(x,y)$ u T_0, t.d.

$$\frac{\partial z}{\partial x}(T_0) = -\frac{\frac{\partial f}{\partial x}(T_0)}{\frac{\partial f}{\partial z}(T_0)}$$

Zad: $x^3y + xz + yz^3 = 2$, $T(1,0,2)$

$$\frac{\partial z}{\partial x} = -\frac{3x^2y + z + 0}{x + 3z^2y}, \quad \frac{\partial f}{\partial x}(T) = -2$$

$$\frac{\partial z}{\partial y} = -\frac{x^3 + z^3}{x + 3yz^2}, \quad \frac{\partial f}{\partial y}(T) = -9$$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{\left(6xy + \frac{\partial z}{\partial x}\right)(x + 3z^2y) - (3yx^2 + z)\left(1 + 6yz\frac{\partial z}{\partial x}\right)}{(x+3yz^2)^2} = 4$$

deriv. umnoška jer ima i z i y!

$$\frac{\partial^2 z}{\partial y^2} = -\frac{\left(0 + 3z^2\frac{\partial z}{\partial y}\right)(x + 3yz^2) - (x^3+z^3)\left(0 + 3z^2 + 6yz\frac{\partial z}{\partial y}\right)}{(x+3yz^2)^2} = 216$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{\left(3x^2 + 3z^2\cdot\frac{\partial z}{\partial x}\right)(x + 3yz^2) - (x^3+z^3)\left(1 + 6yz\frac{\partial z}{\partial x}\right)}{(x+3yz^2)^2} = 30$$

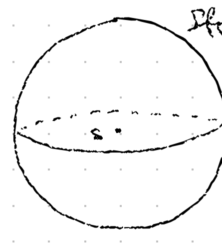
2. 1.

Tangencijalna ravnina:

$$\frac{\partial f}{\partial x}(T_0)(x - x_1) + \frac{\partial f}{\partial y}(T_0)(y - y_1) + \frac{\partial f}{\partial z}(T_0)(z - z_0) = 0$$

$\boxed{\text{M1-20-2}}$ $x^2 + y^2 + z^2 - 2y + 2z = 0$ $\longrightarrow x^2 + (y-1)^2 + (z-1)^2 = 2$, $T(0,2,0)$

Sfera



$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2y - 2 \quad \frac{\partial f}{\partial z} = 2z + 2$$

$$0(x - 0) + 2(y - 2) + 2(z - 0) = 0$$

$2y + 2z = 4$ tang. ravnina