1. VEKTORSKE FUNKCIJE I FUNKCIJE VIŠE VARIJABLI

$f:\mathbb{R}\to\mathbb{R}$ (MATAN1)

$\underbrace{f:\mathbb{R}^n}_{\text{ulaz je n varijabla}}\underbrace{\to\mathbb{R}}_{\text{izlaz je jedna}}$ $f(x,y,z)=x^2\operatorname{arctg}(\sin z^2)$ [skalarna polja]

temp, masa ...

$f:\mathbb{R}\to\underbrace{\mathbb{R}^n}_{\text{izlaz je više funkcija}}$ $\vec f(t)=(\sin t, t^3, \operatorname{arch} t)$

više var. izlaz

zraće vektore

$f:\mathbb{R}^n\to\mathbb{R}^m$ $\vec f(u,v)=(uv^2, \sin^2 v, u^2)$ [vektorska polja]

el. /mag polje, brzina ..

1.1. VEKTORSKA FUNKCIJA

$f:\mathbb{R}\to\mathbb{R}^m$ $\vec f(t)=(x_1(t),\dots x_n(t))$

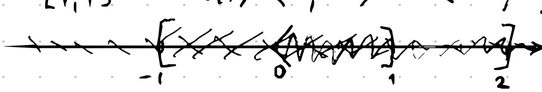
$x_i:\mathbb{R}\to\mathbb{R}$

Primjer:

$\vec f(t)=(\arcsin t, \ln(t), \sqrt{2-t})$ $\mathbb{R}\to\mathbb{R}^3$ trodimenzionalni prostor

DOMENA: presjek svih domena funkcija

$\arcsin t$ $[-1,1]$ $\ln(t)$ $\langle 0,\infty\rangle$ $\langle -\infty, 2]$ $\to$ $\underline{\underline{D_f=\langle 0,1]}}$



npr. uzmemo $1/2$ $\to$ dobit ćemo točku $\Rightarrow$ $\vec f\left(\frac{1}{2}\right)=\left(\frac{\pi}{6}, \ln\frac{1}{2}, \sqrt{\frac{3}{2}}\right)$

$x \longleftrightarrow$ prostorna krivulja

$\Rightarrow$ vekt. funkcija je parametrizacija krivulje, a parametar t opisuje točke po krivulji

▶ $\vec{r}(t) = (x(t), y(t), z(t)) = x(t)\vec{i}, y(t)\vec{j} + z(t)\vec{k}$

u primjeni: od radij vektora

! vektor je kodomena

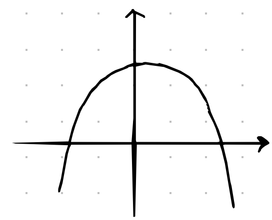
ZAD: Opišite i skicirajte sljedeće vekt. fje:

a) $\vec{r}(t) = (t, 2-t^2)$

$x = t$

$y = 2 - t^2$

$\Rightarrow y = 2 - x^2$

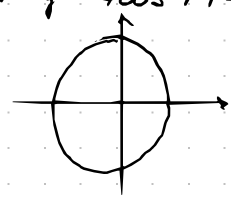


b) $\vec{r}(t) = (2\cos t, 2\sin t)$ $t = [0, 2\pi]$

$x = 2\cos t$

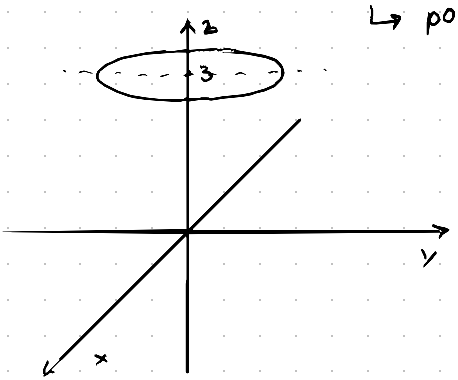
$y = 2\sin t$ $\Big/ ^2$ $\rightarrow$ $x^2 = 4\cos^2 t$

$y^2 = 4\sin^2 t$ $\Big/ +$ $\rightarrow$ $x^2 + y^2 = 4\cos^2 t + 4\sin^2 t$



c) $\vec{r}(t) = (2\cos t, 2\sin t, 3)$

↳ polegnuta na $z = 3$

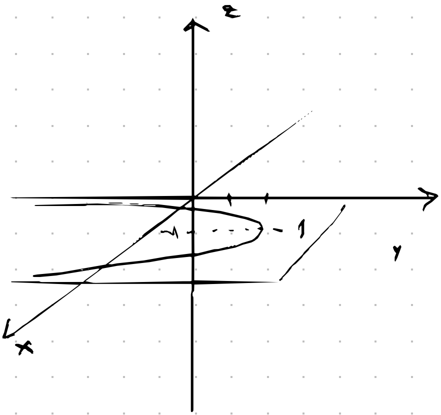


d) $\vec{r}(t) = (t, 2-t^2, -1)$ — $z = -1$

$x = t$

$y = 2 - t^2$

$y = 2 - x^2$



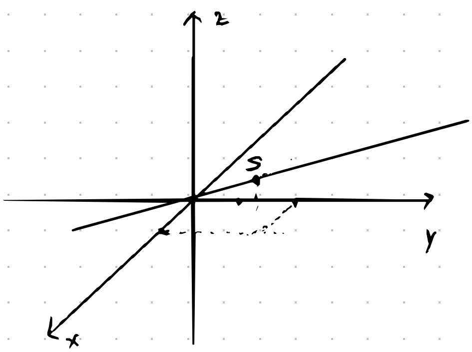
c) $\vec{r}(t) = (1-t, 1+2t, 2+t)$

$x = 1-t$

$y = 1+2t$

$z = 2+t$

parametarska jednadžba pravca



$$\frac{x-1}{-1} = \frac{y-1}{2} = \frac{z-2}{1}$$

$S(-1, 2, 1)$

1.2. DERIVACIJA VEKT. FIJE

DEF $\lim_{t \to t_0} \vec{r}(t) =$ za svaku komponentu gledamo di teži u tj vrijednosti

$$\lim_{t \to t_0} \vec{r}(t) = \lim_{t \to t_0} x(t)\,\vec{i} + \lim_{t \to t_0} y(t)\,\vec{j} + \lim_{t \to t_0} z(t)\,\vec{k}$$

DEF $$f'(t_0) = \lim_{\Delta t \to 0} \frac{f(t_0+\Delta t) - f(t_0)}{\Delta t}$$

TM Ako su komponente diferencijabilne, tada

$$\vec{r}\,'(t) = x'(t)\,\vec{i} + y'(t)\,\vec{j} + z'(t)\,\vec{k}$$

DOKAZ: (zadnji put 2015.)

$$\vec{r}\,'(t) = \lim_{\Delta t \to 0} \frac{f(t_0+\Delta t) - f(t_0)}{\Delta t} = \frac{x(t+\Delta t)\vec{i} + y(t+\Delta t)\vec{j} + z(t+\Delta t)\vec{k} - x(t)i - y(t)j - z(t)}{\Delta t}$$

$$\vec{r}\,'(t) = \lim_{\Delta t \to 0}$$

ZAD: $\vec{r}(t) = (\cos t,\ t^2 \sin t,\ -\cos^2 t)$, $\vec{r}\,'(\frac{\pi}{2}) = ?$, $\|\vec{r}\,'(\frac{\pi}{2})\| = ?$

$$\vec{r}\,'(t) = (-\sin t,\ 2t\sin t + t^2\cos t,\ -2\cos t \cdot (-\sin t))$$

$$\vec{r}\,'(t) = (-\sin t,\ 2t\sin t + t^2\cos t,\ 2\cos t \sin t)$$

$$r(\tfrac{\pi}{2}) = (-1, \pi, 0) \rightarrow |r(\tfrac{\pi}{2})| = \sqrt{1+\pi^2}$$

FIZIKALNA INTERPRETACIJA: $\vec{r}\,'(t) =$ brzina

duljina vektora je iznos brzine

GEOMETRIJSKA INTERPRETACIJA

→ derivacija je vektor!

- vektor smjera tangente na krivulju

$$\boxed{\frac{x-x_0}{x'(t)} = \frac{y-y_0}{y'(t)} = \frac{z-z_0}{z'(t)}}$$

jednadžba tangente: kanonski oblik

* tang. iz prošlog zad

$$\frac{x-0}{-1} = \frac{y-\frac{\pi}{4}}{\pi} = \frac{z-0}{0} \qquad \underline{T\left(0, \frac{\pi}{4}, 0\right)}$$

ZAD LJIR 2021/22

1) $T(-2, 0, 4)$

sve prikazati preko jednog parametra

$x^2 + y^2 = 4$

$y + z = 4$

$y = t \rightarrow$ $z = 4 - t$

$x^2 = 4 - t^2$

jer je $x < 0$ ← $x = \pm\sqrt{4-t^2}$

$\vec{r}(t) = (\sqrt{4-t^2}, t, 4-t)$

$\vec{r}'(t) = \left(\frac{t}{\sqrt{4-t^2}}, 1, -1\right)$

$t \ldots \frac{x+2}{0} = \frac{y-0}{-2} = \frac{z-4}{2}$

II. način) $\vec{r}(t) = (2\cos t, 2\sin t, 4 - 2\sin t)$

$\vec{r}'(t) = (-2\sin t, 2\cos t, -2\cos t)$

$T(-2, 0, 4)$ (uvrstimo u nederivirano)

$-2 = 2\cos t$

$-1 = \cos t \rightarrow t = \pi$ → $r'(\pi) = (0, -2, 2)$

$t \ldots \frac{x+2}{0} = \frac{y-0}{-2} = \frac{z-4}{2}$

DIR 2020/21

$$\begin{cases} x(t) = 3-t \\ y(t) = 8/t \\ z(t) = 6\ln(t) \end{cases} \qquad t \in [1,4]$$

$T_0 = ?$

$t \perp \pi$

atto sai $t \perp \pi \to t \parallel \vec{n}$

$\vec{t} \to \vec{c_t}$

$\downarrow$

$\vec{n} = \lambda \cdot c_T$

$\pi \equiv x+2y-3z-4=0 \longrightarrow \vec{n} = (1,2,-3) \longrightarrow \lambda \cdot \vec{c_t}$

$$\begin{cases} x'(t) = -1 \\ y'(t) = \dfrac{-8}{t^2} \\ z'(t) = \dfrac{6}{t} \end{cases} \qquad \vec{C_t} = \left(-1, \frac{-8}{t^2}, \frac{6}{t}\right) \Rightarrow (1,2,-3) = \left(-1, \frac{-8}{t^2}, \frac{6}{t}\right)\lambda$$

$1 = -1 \cdot \lambda$

$\lambda = -1$

$2 = \dfrac{-8}{t^2} \cdot (-1)$

$t^2 = \dfrac{8}{2}$ per $\notin [1,4]$

$t^2 = 4$ $\cancel{-2}$ 2

* rimanmo T_0, ali trebamo T_0

$x_0(t) = 1 \qquad y_0(t) = 4 \qquad z_0 = 6\ln(2)$

$T_0 = (1, 4, 6\ln(2))$