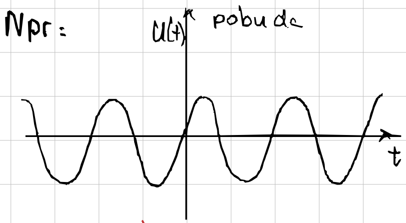
12.2. Funkcije mreže i stacionarno stanje sinusne pobude

· funkcija mreža $H(s)$ je omjer Laplaceovih trans odziva $Y(s)$ i pobude $X(s)$ uz početne uvjete=0

$$H(s)=\frac{Y(s)}{X(s)}$$

Npr:



$u(t)=\sin(\omega t) \quad -\infty < t < \infty$

$\boxed{\text{poč uvjeti} = 0}$

$H(s) = \frac{Y(s)}{X(s)}$ — odziv fazor / pobude fazor

i za fazore

oblik pobude → $X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$ → znači X i $Y(s)$ možemo prikazati kao fazor

$\boxed{s=j\omega}$

što je onda sa $H(s)$

$X(j\omega) = |X(j\omega)| \angle \varphi_x(\omega)$

ω → djeluje svagdje

$$\rightarrow \underline{H(j\omega)} = \frac{Y(j\omega)}{X(j\omega)} = \frac{|Y(j\omega)|e^{j\varphi_y(\omega)}}{|X(j\omega)|e^{j\varphi_x(\omega)}} = \frac{|Y(j\omega)|}{|X(j\omega)|}e^{j(\varphi_y-\varphi_x)}$$

općenito kompleksni broj

$$H(j\omega) = Re[H(j\omega)] + j\, Im[H(j\omega)]$$

sjeti se: $\varphi = \operatorname{arctg} \frac{Im}{Re}$

Polarne koord.

$$H(j\omega) = \underbrace{|H(j\omega)|}_{\text{a-f kar.}} \underbrace{e^{j\varphi(\omega)}}_{\text{f-f kar.}} \rightarrow \text{kompleksna frekv. karakter.}$$

ako je poznato $H(j\omega)$: * konjugiramo

$$H^2(j\omega) = \underbrace{H(j\omega)\cdot H(-j\omega)}_{|H^2(j\omega)|} \cdot \frac{H(j\omega)}{H(-j\omega)} = |H^2(j\omega)| \cdot e^{2j\varphi(\omega)}$$

$$|H(j\omega)| = \sqrt{H(j\omega)\cdot H(-j\omega)} \qquad \varphi(\omega) = \frac{1}{2j} \ln \left| \frac{H(j\omega)}{H(-j\omega)} \right|$$

$$H(s) = \frac{P(s)}{Q(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots a_0} = k\,\frac{(s-s_{o1})(s-s_{o2})\cdots(s-s_{om})}{(s-s_{p1})(s-s_{p2})\cdots(s-s_{pn})}$$

$$H(s) = k\,\frac{\prod_{i=1}^{m}(s-s_{oi})}{\prod_{j=1}^{n}(s-s_{pj})}$$

$s_{oi} = \sigma_{oi} + j\omega_{oi} \rightarrow$ nule funkcije mreže

$s_{pj} = \sigma_{pj} + j\omega_{pj} \rightarrow$ polovi funkcije mreže

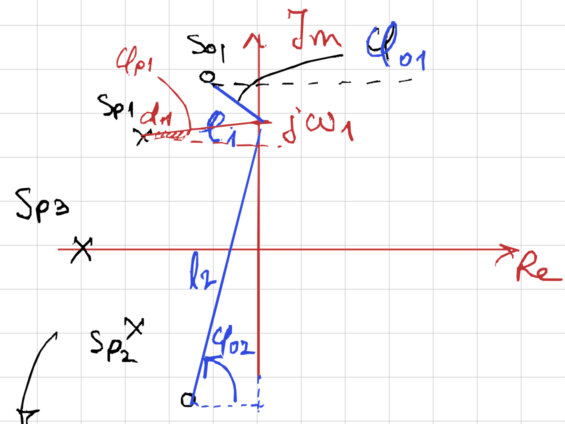
u uvjetima stacionarne sinusne pobude: $(s = j\omega)$

$$H(s) = k\,\frac{(j\omega - s_{o1})\cdots(j\omega - s_{om})}{(j\omega - s_{p1})\cdots(j\omega - s_{pn})}$$

novi kompleksni broj

$$\underbrace{|j\omega - s_{o1}|}_{l_1}\,\underbrace{e^{j\varphi_{o1}}}_{\varphi_{o1}}$$

$$(j\omega - s_{p1}) = \underbrace{|j\omega - s_{p1}|}_{d_1}\,\underbrace{e^{j\varphi_{p1}}}_{\varphi_{p1}}$$

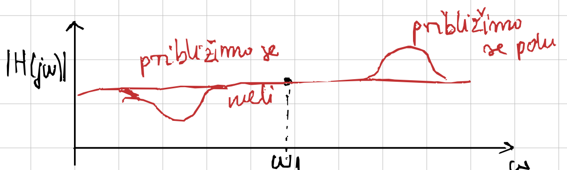


a-f karakteristika

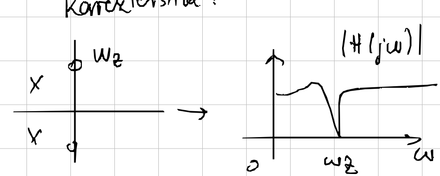
$$H(j\omega) = k \cdot \frac{|j\omega - s_{o1}| \cdots |j\omega - s_{om}|}{|j\omega - s_{p1}| \cdots |j\omega - s_{pn}|} \cdot e^{j(\varphi_{o1} + \varphi_{o2} + \dots \varphi_{om})} \cdot (\varphi_{p1} + \dots \varphi_{pn})$$

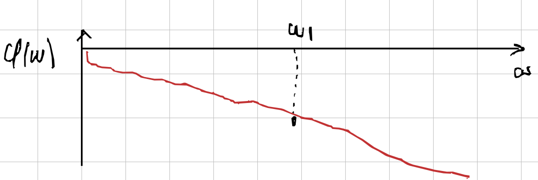
to je sve za jednu frekv. ω_1

$$|H(j\omega)| = \frac{l_1 l_2 \dots l_m}{d_1 d_2 \dots d_n} \qquad \varphi(\omega) = (\varphi_{o1} + \dots + \varphi_{om}) - (\varphi_{p1} + \dots \varphi_{pn})$$

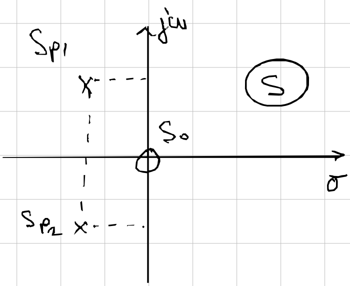


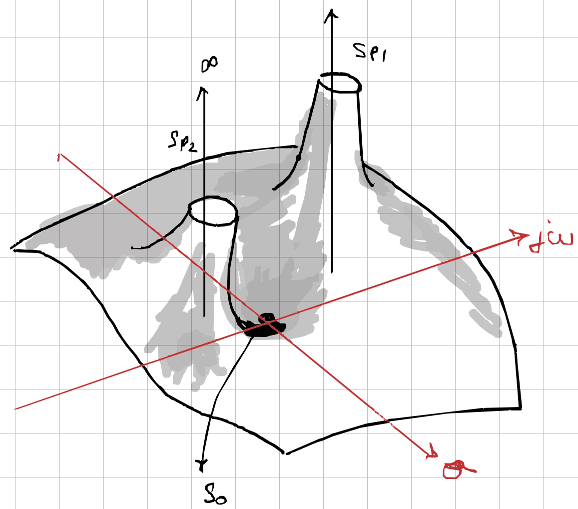
karakteristika:



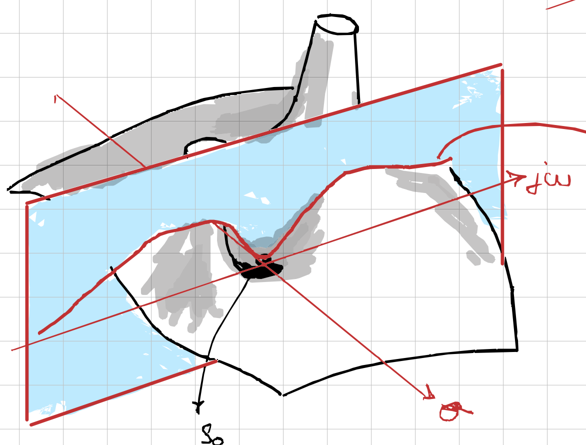


Primjer: $|H(s)|$ s dva pola i jednom nulom





ako kažemo $s = j\omega \rightarrow$ presjek



$|H(j\omega)|$
amplitudno frekvencijska karakteristika

Logaritamska mjera: često za prijenosne funkcije

$$\ln H(j\omega) = \underbrace{\ln}\left[\,|H(j\omega)| \cdot e^{j\varphi(\omega)}\right] = \ln|H(j\omega)| + j\varphi(\omega)$$

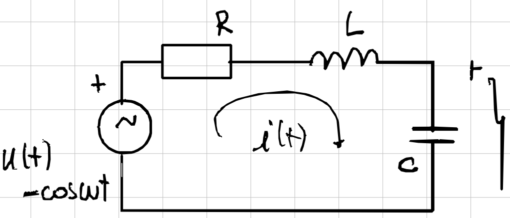
$\alpha_N(\omega) = \ln|H(j\omega)|$ – logaritamska mjera pojačanja u Neperima [N]

* često umjesto prirodnog → dekadski log

↳ $\alpha(\omega) = 20\log|H(j\omega)|$ → log. mjera pojačanja u dB

$\alpha(\omega)[dB] \cong 8.68\,\alpha_N(\omega)[N]$

Primjer : RLC



$\omega = 1$

$R = 2$

$C = 1$

$L = 2$

$\underline{U_c = ?}$

$u(t) = \cos \omega t$

$H(s) = \dfrac{U_c(s)}{U(s)}$

$U(s) = I(s) \cdot \left(R + Ls + \dfrac{1}{Cs} \right)$

$U_c(s) = I(s) \cdot \dfrac{1}{Cs}$

$$H(s) = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{sRC + s^2LC + 1}$$

$$H(j\omega) = \frac{U_c(j\omega)}{U(j\omega)} = \frac{1}{(j\omega)^2 LC + (j\omega)RC + 1} = \frac{1}{-\omega^2 LC + j\omega RC + 1} = \frac{1}{-2\omega^2 + 2j\omega + 1}$$

$U_c(j\omega) = ?$

$U_c(j\omega) = U(j\omega) \cdot H(j\omega)$ $\qquad U(j\omega) = 1\angle 0^\circ, \ \omega = 1$

$$U_c(j\omega) = \frac{U(j\omega)}{-2\omega^2 + 2j\omega + 1} = \frac{1}{-2 + 2j + 1} = \frac{1}{-1 + 2j}$$ – kalkulator

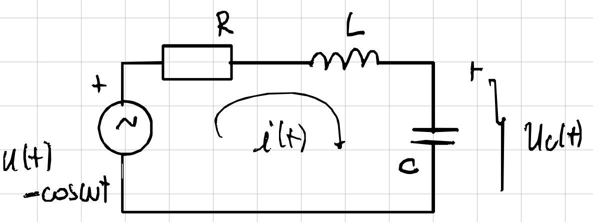
$\hookrightarrow = \underbrace{-\dfrac{\sqrt{5}}{5}}_{|U_c|} \underbrace{\angle \operatorname{arctg}(2)}_{\varphi_c}$

$U_c(t) = |U_c| \cos(\omega t + \varphi)$

$$\boxed{U_c = -\frac{\sqrt{5}}{5} \cos(t + \operatorname{arctg}(2))}$$

samo prepišemo

Primjer RLC: → Frekvencijski odziv ili karakteristika



$$H(s) = \frac{U_c(s)}{U_i(s)} = ?$$

$$H(s) = \frac{1}{s^2LC + sRC + 1}$$

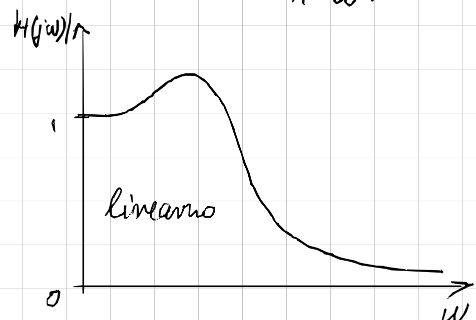
$$H(j\omega) \frac{1}{-\omega^2LC + j\omega RC + 1} = \frac{1}{(1-\omega^2LC) + j(\omega RC)}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2LC)^2 + (\omega RC)^2}}, \quad \varphi(\omega) = \underbrace{\varphi_{brojnik}}_{=0} - \underbrace{\varphi_{nazivnik}}_{\mathrm{arctg}\frac{\omega RC}{1-\omega^2LC}}$$

Za $R = L = C = 1$

$$\varphi(\omega) = -\,\mathrm{arctg}\frac{\omega RC}{1-\omega^2LC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}} = \frac{1}{\sqrt{1-\omega^2+\omega^4}}$$

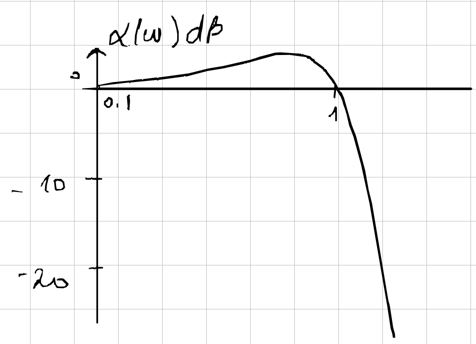


· logaritamska mjera pojačanja:

$$\alpha(\omega) = 20 \cdot \log|H(j\omega)|$$

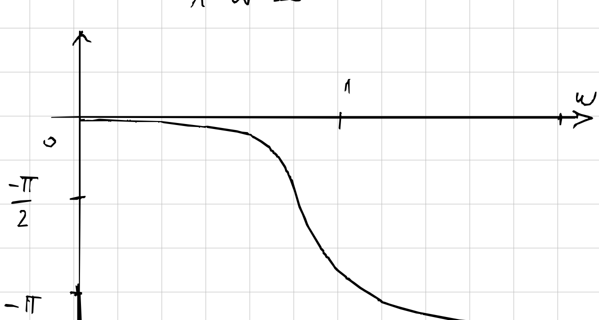
$$= 20 \cdot \log\left|\frac{1}{\sqrt{1-\omega^2+\omega^4}}\right|$$

$$= -10\log(1-\omega^2+\omega^4)$$



· fazna frekvencijska karakteristika

$$\varphi(\omega) = \mathrm{arctg}\frac{RC\omega}{1-\omega^2LC} \qquad \varphi(\omega) = -\,\mathrm{arctg}\frac{\omega}{1-\omega^2}$$



Kada je $H(s)$, odnosno $H(j\omega)$ višestupni polinom:

Primjer: $H(s) = \frac{s^5+3s^4+2s^3+s^2+s+1}{s^5+2s^4+s^3+7s^2+2s+2} = \frac{(3s^4+s^2+1)+(s^5+2s^3+s)}{(2s^4+7s^2+2)+(s^5+s^3+2s)}$

$H(s) = \frac{(3s^4+s^2+1)+s(s^4+2s^2+1)}{(2s^4+7s^2+2)+s(s^4+s^2+2)}$

$s=j\omega \rightarrow$

s^0	1
s^1	$j\omega$
s^2	$-\omega^2$
s^3	$-j\omega^3$ nestaje j
s^4	ω^4

$H(s) = \frac{(3\omega^4-\omega^2+1)+\overset{j\omega}{s}(\omega^4-2\omega^2+1)}{(2\omega^4-7\omega^2+2)+s(\omega^4-\omega^2+2)}$

$H(s) = \frac{(3\omega^4-\omega^2+1)\mp j\omega(\omega^4-2\omega^2+1)}{(2\omega^4-7\omega^2+2)\pm j\omega(\omega^4-\omega^2+2)}$ polinom od ω^2

$\rightarrow H(-j\omega) = H^*(j\omega)$ uvijek možemo s (odnosno $j\omega$) izlučiti

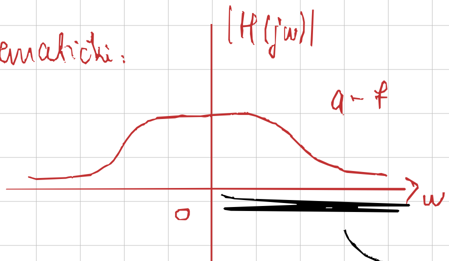
$\Rightarrow \mathrm{Re}[H(-j\omega)] = \mathrm{Re}[H^*(j\omega)]$

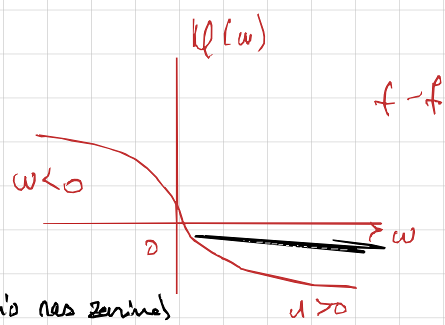
$\mathrm{Im}[H(-j\omega)] = \mathrm{Im}[H^*(j\omega)]$

Re dio od $H(j\omega) \rightarrow$ paran $y(-x) = y(x)$ — $|H(j\omega)|$

Im dio od $H(j\omega) \rightarrow$ neparan $y(-x) = -y(x)$ — $\varphi(\omega)$

matematički:





ovaj dio nas zanima