6. Laplaceova transformacija

Laplace u primjeni: $\int_0^{\infty} e^{-st} f(t)\,dt$ $\quad \boxed{s = \sigma + j\omega}$

Lapl. trans. postoji ako je: $\int_0^{+\infty} |f(t)|\, e^{-\sigma_0 t}\, dt < \infty$

$\sigma_0 \rightarrow$ apscisa apsolutne konv

Primjer: $f(t) = S(t)$ (step)

$$\int_0^{\infty} e^{-st} f(t)\,dt = \int_0^{\infty} e^{-st} \cdot \underbrace{S(t)}_{=1}\, dt = \int_0^{\infty} e^{-st}\,dt \qquad s = \sigma + j\omega$$

$$= \frac{-1}{s} e^{-st}\Big|_0^{\infty} = -\frac{1}{s}\left(e^{-s\cdot\infty} - e^{0}\right) = \frac{-1}{s}\left(e^{-(\sigma+j\omega)\infty} - 1\right)$$

?

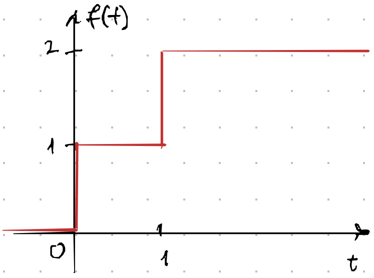
$$\Longrightarrow e^{-(\sigma+j\omega)\infty} = \underbrace{e^{-\sigma\infty}} \cdot e^{-j\omega\infty}$$ ne znamo što je to ali je sigurno na kompleksnoj ravnini kružnica

$\sigma > 0$: $= 0$ $\qquad \sigma < 0$: $= \infty$

* sva svojstva iz MATANA 3

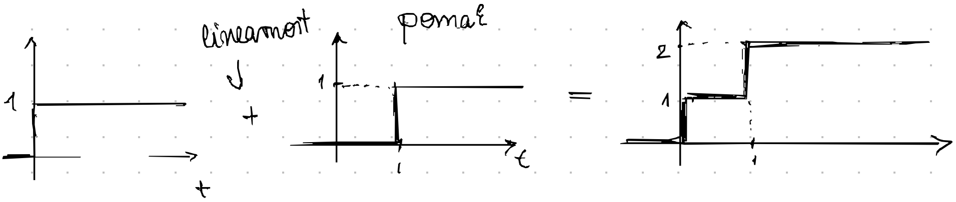
Primjena na signale

Primjer: Odrediti Laplaceovu transf. signala prikazanog slikom

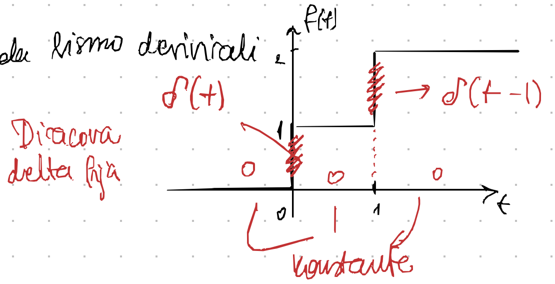


$$f(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 1 \\ 2, & 1 < t \end{cases}$$

$\rightarrow$ step funkcija



* kada bismo derivirali



$\Longrightarrow f'(t) = \delta(t) + \delta(t-1)$

$\hookrightarrow \mathcal{L}(f'(t)) = \mathcal{L}(\delta(t)) + \mathcal{L}(\delta(t-1))$

$\Longrightarrow \mathcal{L}[f'(t)] = 1 + 1\cdot e^{-s}$

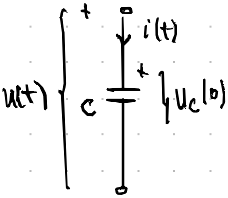
$\mathcal{L}(f'(t)) = 1 + e^{-s}$

$\longrightarrow \quad \mathcal{L}(f'(t)) = sF(s) - \underbrace{f(0)}_{=0} \Longrightarrow sF(s) = 1 + e^{-s} \;/ : s$

$$\boxed{F(s) = \frac{1 + e^{-s}}{s}}$$

Primjena na elemente el. krugova

KAPACITET



početni napon na kondenzatoru

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = u_c(0) + \frac{1}{C}\int_{0}^{t} i(\tau)\,d\tau$$

$\Rightarrow$ Laplace $\Rightarrow$ $\mathcal{L}(5) := 5 \circ\!\!-\!\!\bullet \frac{1}{s}\cdot 5 \Rightarrow \mathcal{L}[5] = \frac{5}{s}$

$$\mathcal{L}(u_c(0)) = \frac{u_c(0)}{s}$$

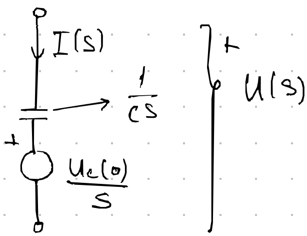
$$\mathcal{L}\left(\frac{1}{C}\cdot\int_0^t i(\tau)\,d\tau\right) = \frac{1}{C}\cdot\frac{I(s)}{s} = \frac{I(s)}{Cs}$$

„naponski izvor"

$$\boxed{U(s) = \frac{u_c(0)}{s} + \frac{1}{Cs} I(s)}$$

$$U(s) = R\cdot I(s)$$

Laplace shema



Laplace nam pomaže da rješavamo mreže kao da se radi o samim otpornicima

Drugi oblik Laplace transformacije:

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau \;\Big/\; \frac{d}{dt} \rightarrow i(t) = C\,\frac{du(t)}{dt} \;\Big/\mathcal{L}$$

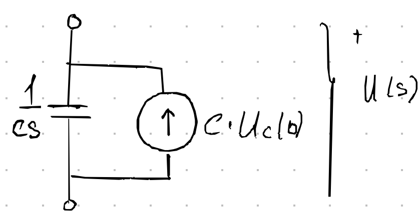
$C\cdot u'(t)$

$$\mathcal{L}(i(t)) = C\cdot(s\cdot U(s) - u(0)) = Cs\cdot U(s) - U(0)\cdot C = I(s)$$

$$I(s) + C\cdot U(0) = Cs\cdot U(s) \;/:Cs \longrightarrow \boxed{U(s) = \frac{1}{Cs}I(s) + \frac{1}{s}\cdot u(0)}$$

nova shema (ekvivalentna mreža)

kao da smo naponski pretvorili u strujni



Impedancija kapaciteta: $Z(s) = \frac{U(s)}{I(s)} = \frac{1}{Cs}$

Admitancija kapaciteta: $Y(s) = \frac{I(s)}{U(s)} = Cs$

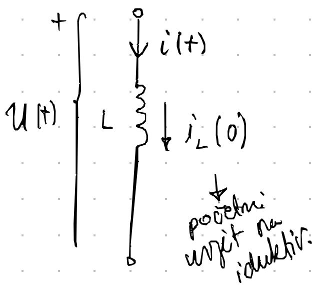
Induktivitet

* Kapacitet: $i(t) = C\,\dfrac{du(t)}{dt}$

Induktivitet: $u(t) = L \cdot \dfrac{di(t)}{dt}$

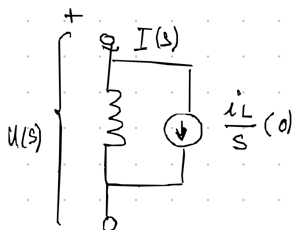
$i(t) \longleftrightarrow u(t) \Rightarrow$ Dualnost

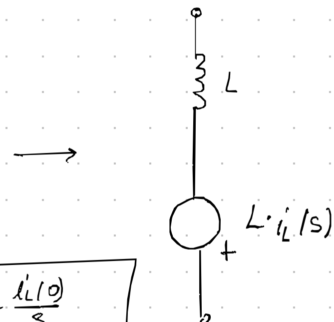
$L \longleftrightarrow C$



vremenska domena

Frekvencijska domena

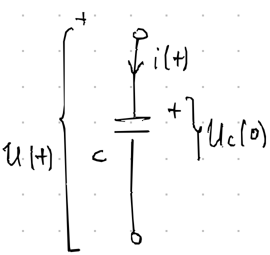


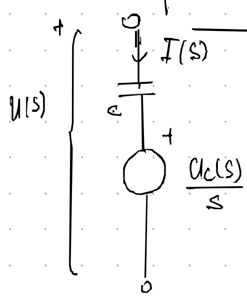


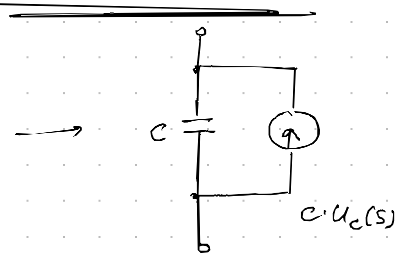
$$I(s) = \frac{1}{sL}\,U(s) + \frac{i_L(0)}{s}$$

$$U(s) = sL\,I(s) - L\,i_L(0)$$

* kapacitet







Impedancija $Z(s) = \dfrac{U(s)}{I(s)} = sL$

Admitancija $Y(s) = \dfrac{I(s)}{U(s)} = \dfrac{1}{sL}$

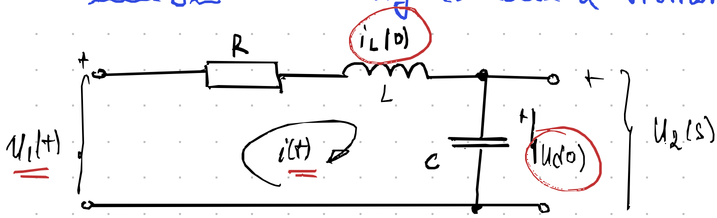
Znači:

$u(t) = R \cdot i(t) \longrightarrow U(s) = R \cdot I(s)$

$Z = \dfrac{U(s)}{I(s)}$ otpori: R, $\dfrac{1}{Cs}$, Ls

admitancije: $\dfrac{1}{R}$, Cs, $\dfrac{1}{Ls}$

Primjer: RLC krug (zadan u vremenskoj domeni)



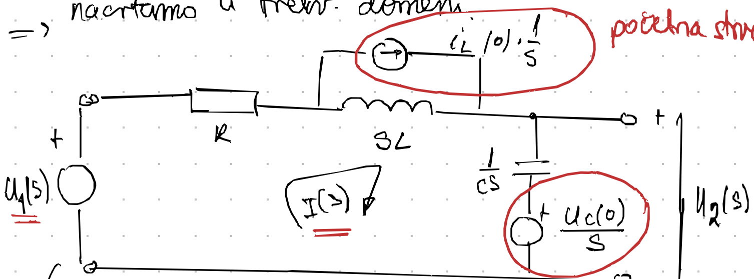
$u_c(0) = 1V$

$i_L(0) = i(0) = 1A$

$u(t) = \delta(t) \longrightarrow$

$R = 1\Omega \quad L = 1H \quad C = 1F$

=> nacrtamo u frekv. domeni

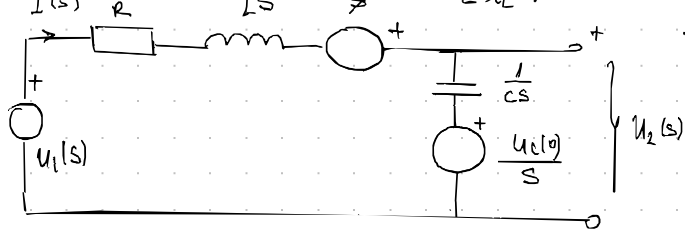


početna struja kroz induktivitet

ne moramo ali je jako pametno

početni napon kroz kapacitet

$\frac{i_L}{\cancel{s}}\cdot L\cancel{s} = L\cdot i_L(0)$



$\longrightarrow U_1(s) = R\cdot I(s) + Ls\cdot I(s) - L\cdot i_L(0) + \frac{1}{Cs} I(s) + \frac{U_c(0)}{s}$

$$U_1(s) + Li_L(0) - \frac{U_c(0)}{s} = \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

$$I(s) = \frac{U_1(s) + Li_L(0) - \frac{1}{s}U_c(0)}{R + Ls + \frac{1}{Cs}} = \frac{\cancel{\frac{1}{s}} + 1 - \cancel{\frac{1}{s}\cdot 1}}{1 + s + \frac{1}{s}} = \frac{1}{\frac{s+s^2+1}{s}} = \frac{s}{s+s^2+1}$$

$$i(t) = \mathcal{L}^{-1}(I(s)) = \frac{s}{(s+\frac{1}{2})^2 + \frac{3}{4}} = \underbrace{\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}}_{\cos} - \frac{1}{\sqrt{3}}\underbrace{\frac{\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}}_{\sin}$$

$2\cdot\frac{1}{2}s$

$\cos(t) \circ\!\!-\!\!\bullet \frac{s}{s^2+w^2}$

prigušenje $e^{-at}f(t) \circ\!\!-\!\!\bullet F(s+a)$

$$i(t) = \left[e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right] S(t)$$