7. DZ Rješenja jednadžbi mreža

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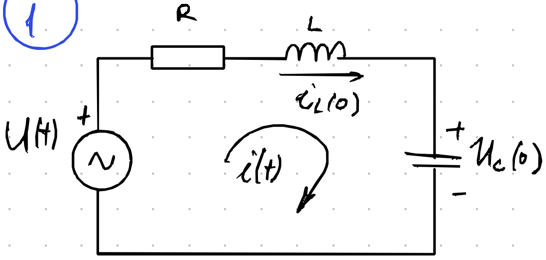


odrediti polove prijenosne funkcije

$$H(s)=\frac{U_c(s)}{U(s)} \qquad R=1,\ L=1,\ C=1$$

$$U(t)=i(t)\cdot R+L\cdot\frac{di(t)}{dt}+\underbrace{U_c(0)+\frac{1}{C}\int_0^t i(\tau)\,d\tau}_{U_c}$$

$$U(s)=I(s)\cdot R+L\cdot\left(sI(s)-i_L(0)\right)+\underbrace{U_c(0)\frac{1}{s}+\frac{1}{Cs}\cdot I(s)}$$

$$i_L(0)=0 \qquad U_c(0)=0 \qquad U_c(s)=U_c(0)\cdot\frac{1}{s}+\frac{1}{Cs}I(s)$$

$$U(s)=I(s)+sI(s)+\frac{I(s)}{s} \qquad U_c(s)=\frac{I(s)}{s}$$

$$H(s)=\frac{U_c(s)}{U(s)}=\frac{\frac{\cancel{I(s)}}{s}}{\cancel{I(s)}\left(1+s+\frac{1}{s}\right)}=\frac{1+s+\frac{1}{s}}{s}=\frac{1}{s}+1+\frac{1}{s^2}=\frac{s+s^2+1}{s^2}$$

$$s_{1,2}=\frac{1\pm\sqrt{1-4\cdot1\cdot1}}{2}=\frac{1}{2}\pm\frac{\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}$$

2 Termin „slobodne mreže" znači da:

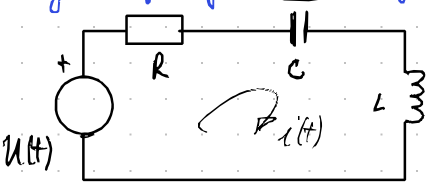
$\longrightarrow$ Poticaj mreže jednak je nuli

3 Kombinacija od dva poticaja $X_1=x(t)-x(t-\Delta t)$ daje odziv $y_1=y(t)-y(t-\Delta t)$

4 Kako glasi homogeno rješenja $i_h(t)$ struje $i(t)$ ako je zadano:

$R=3$
$L=2{,}5$
$C=2$

početni uvjeti = 0



$$U(t)=i(t)\cdot R+\frac{1}{C}\int_{-\infty}^{t}i(\tau)\,d\tau+L\cdot\frac{di(t)}{dt}\quad \Big/\ \frac{d}{dt}$$

$$\frac{U(t)}{dt}=R\,\frac{di}{dt}+\frac{1}{C}\cdot i+L\frac{d^2i}{dt^2}$$

1) $\frac{1}{C}i+R\frac{di}{dt}+L\frac{d^2i}{dt^2}=0$ $\qquad$ pretpostavka: $i(t)=Ae^{st}$

$$\frac{1}{C}\cdot Ae^{st}+R\cdot A\cdot se^{st}+L\cdot As^2e^{st}=0\quad\Big/ : Ae^{st}$$

$$\frac{1}{C}+R\cdot s+Ls^2=0\quad\Big/ : L$$

$$\underbrace{s^2+\frac{R}{L}s}_{a_1}+\underbrace{\frac{1}{LC}}_{a_0}=0 \longrightarrow s_{1,2}=\frac{-R}{2L}\mp\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}=\frac{-3}{5}\pm\sqrt{\frac{9}{4\cdot6{,}25}-\frac{1}{5}}$$

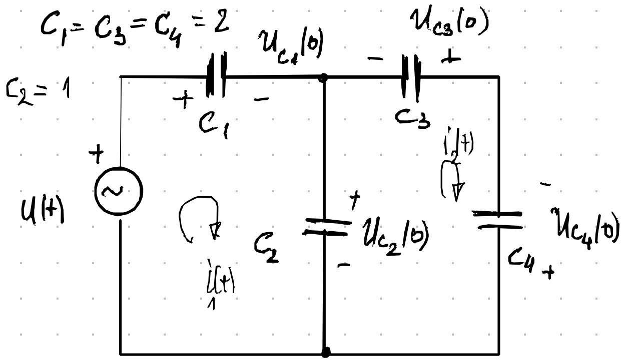
$$s_{1,2}=\frac{-3}{5}\pm\frac{2}{5} \quad s_1=\frac{-1}{5} \quad s_2=-1$$

$$i(t)=A_1e^{s_1t}+A_2e^{s_2t}=A_1\cdot e^{-\frac{1}{5}t}+A_2e^{-t}$$

5 Odrediti nule prijenosne funkcije $H(s) = \frac{U_{c_4}(s)}{U(s)}$ za mrežu na slici za slučaj da su svi početni uvjeti jednaki nuli:

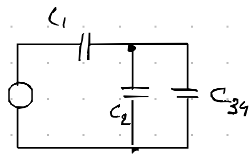
$C_1 = C_3 = C_4 = 2$

$C_2 = 1$



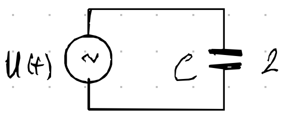
$\rightarrow C_{34} = \left(\frac{1}{C_3} + \frac{1}{C_4}\right)^{-1}$ (serija)

$C_{34} = \left(\frac{1}{2} + \frac{1}{2}\right)^{-1} = \underline{\underline{1}}$



$C_{234} = C_2 + C_{34} = 1 + 1 = \underline{\underline{2}}$

$C_{uk} = \left(\frac{1}{C_1} + \frac{1}{C_{ost}}\right)^{-1} = \underline{\underline{1}}$



$U_{c_{34}}(t) = \frac{1}{C_{34}} \cdot \frac{1}{s} I(s)$

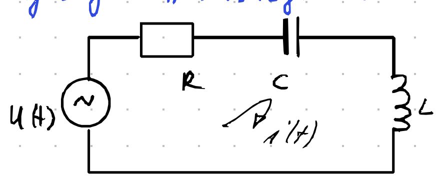
$u(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau)\, d\tau$

$H(s) = \frac{1 \cdot \cancel{\frac{1}{s} I(s)}}{1 \cdot \cancel{\frac{1}{s}(I(s))}} = 1$ nula je u nuli

$\underline{\underline{U(s) = \frac{1}{C_{uk}} \cdot \frac{1}{s} I(s)}}$

6. Kako glasi homogeno rješenje $i_h(t)$ struje $i(t)$ ako je zadano:

$R = 3 \quad L = 2{,}5 \quad C = 2$



$u(t) = i(t) \cdot R + \frac{1}{C} \int_{-\infty}^{t} i(\tau)\, d\tau + L \frac{di}{dt} \Big/ \cdot \frac{d}{dt}$

$\frac{du(t)}{dt} = R \frac{di}{dt} + \frac{1}{C} \cdot i(t) + L \frac{d^2 i}{dt}$

1) $\frac{1}{C} i(t) + R \frac{di}{dt} + L \frac{d^2 i}{dt} = 0$

pretpostavka: $i(t) = Ae^{st}$

$\frac{1}{C} Ae^{st} + R \cdot Ase^{st} + L \cdot As^2 e^{st} = 0 \Big/ : Ae^{st}$

$\frac{1}{C} + Rs + Ls^2 = 0 \Big/ : L$

$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$

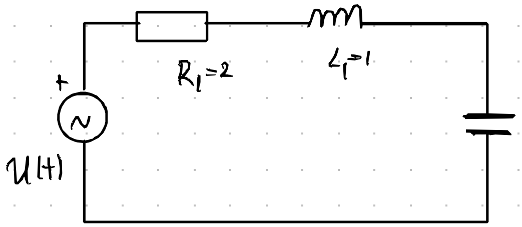
$S_{1,2} = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$

[redacted]

$S_{1,2} = \frac{-3}{5} \pm \frac{2}{5}$

$S_1 = \frac{-1}{5}$ $\quad S_2 = -1$

7. Odrediti vrstu prigušenja odziva



$$u(t) = i(t)\cdot R + L\cdot\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

$$U(s) = R\cdot I(s) + L(sI(s) - i(0)) + \frac{1}{C}\cdot\frac{1}{s}\cdot I(s)$$

$$U(s) = I(s)\left(R + Ls + \frac{1}{Cs}\right) \cancel{- L\,i(0)}$$

$$I(s) = \frac{U(s)}{L\left(s + \frac{R}{L} + \frac{1}{LCs}\right)}\cdot\frac{s}{s} = \frac{s\cdot U(s)}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

polovi:

$$s_{1,2} = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\frac{R}{2L} = \alpha \qquad \frac{1}{LC} = \omega_0^2$$

a) Nadkritično gušenje: $\alpha > \omega_0 \Rightarrow R > 2\sqrt{\frac{L}{C}}$

b) Kritično: $\alpha = \omega_0 \Rightarrow R = 2\sqrt{\frac{L}{C}}$

c) Podkritično $\alpha < \omega_0 \Rightarrow R < 2\sqrt{\frac{L}{C}}$

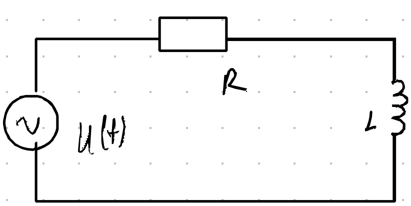
d) Neprigušeni $\alpha = 0 \rightarrow R = 0$

$$\Rightarrow \text{polovi: } s_{1,2} = \frac{-2}{2\cdot 1} \pm \sqrt{\frac{2^2}{4\cdot 1} - \frac{1}{1\cdot 1}}$$

$$\frac{R}{2L} = \alpha \Rightarrow \alpha = 1 \quad \text{KRITIČNO!}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = 1$$

8. Odrediti i(t).



$U(s) = 1$

$R = L \quad i_L(0) = 0$

$$u(t) = R\cdot i(t) + L\cdot\frac{di}{dt} \quad \Big/ \frac{d}{dt}$$

$$\frac{du(t)}{dt} = R\frac{di}{dt} + L\cdot\frac{d^2i}{dt^2}$$

1) $R\cdot\frac{di}{dt} + L\cdot\frac{d^2i}{dt^2} = 0$

prep: Ae^{st}

$$R\cdot Ase^{st} + L\cdot As^2e^{st} = 0$$

$$Rs + Ls^2 = 0 \qquad s(R + Ls) = 0 \qquad s_1 = 0$$

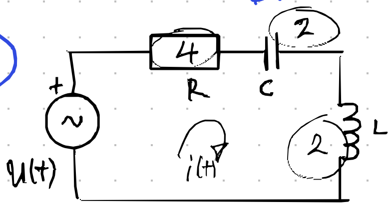
$$s_2 = \frac{-R}{L} = -1$$

$$i(t) = A_1e^{0} + A_2e^{-t}$$

$$i'(t) = A_1 + A_2e^{-t}$$

MOJA DZ:

(1)



$$u(t) = i(t)\cdot R + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau + L\frac{di}{dt}$$

$$U(s) = I(s)\cdot R + \frac{1}{C}\cdot\frac{I(s)}{s} + L\left(sI(s) \cancel{- i(0)}\right)$$

$$U(s) = I(s)\cdot R + \frac{1}{C}\,\frac{I(s)}{s} + Ls\cdot I(s) = I(s)\left(R + \frac{1}{Cs} + Ls\right)$$

$$I(s) = \frac{s\cdot U(s)}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \longrightarrow s_{1,2} = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0^2 = \frac{1}{LC}$$

$$\alpha = \frac{4}{2\cdot 2} = 1 \qquad \omega_0 = \sqrt{\frac{1}{2\cdot 2}} = \frac{1}{2}$$

$\alpha > \omega_0 \longrightarrow$ Nadkritično

(2) Za zadanu funkciju odrediti prirodne frekv.

$$F(s) = \frac{s^2 - 4s + 1}{s^3 + s^2} \cancel{= \frac{1}{s^2}\cdot\frac{s^2 - 4s + 4 - 3}{s+1}}$$

$$\cancel{\frac{1}{s^2}\left(\frac{(s-2)^2}{s+1} - \frac{3}{s+1}\right)}$$

polovi

$$s^2(s+1) = 0$$

$s_{1,2} = 0$ dvostruki pol u ishodištu

$s = -1$ stvarni pol

Nule sustava:

$$s^2 - 4s + 1 = 0 \longrightarrow s_{1,2} = \frac{4 \pm \sqrt{16 - 4\cdot 1}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$s_{1,2} = 2 \pm \sqrt{3}$$

$$s_1 = 2 + \sqrt{3} \qquad s_2 = 2 - \sqrt{3}$$

nule su u desnoj poluravnini

$\Longrightarrow$ nestabilno