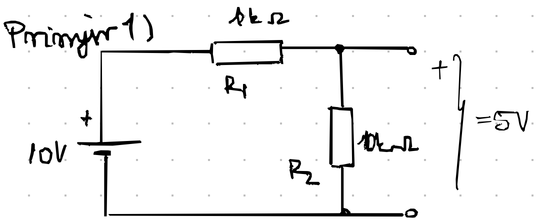
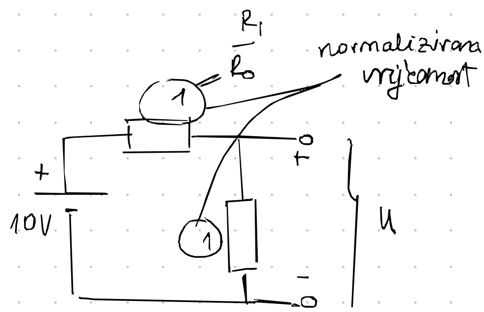
6.2. Normalizacija

1 Normiranje frekvencija

Primjer 1)





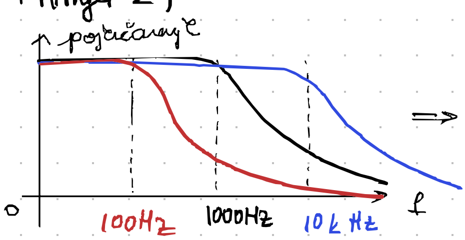
R_0 – normalizacijski otpor (odaberemo koliko očemo)

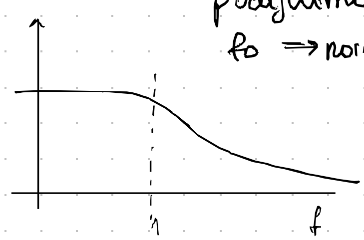
$R_0 = 100\,\Omega$

$$\frac{R_1}{R_0} = \frac{1k\Omega}{1000} = 1$$

$$\frac{R_2}{R_0} = \frac{1k\Omega}{1000} = 1$$

Primjer 2)





podijelimo f os sa nekim f_0 ⟹ normalizacija

→ umjesto kompleksne frekv. uvodi se normalizirana frekv: $S_n = \frac{S}{\omega_0}$ konstantna realna frekv.

→ X_C i X_L ostaju nepromijenjene, ali zato se c i L mijenjaju

2. Normiranje impedancije

→ impedancije mijenjaju vrijednost, ali zadržavaju oblik $Z_n(s) = \frac{Z(s)}{R_0}$

3. Normiranje Z i ω

→ u praksi se najčešće provodi istovremeno normiranje frekvencije i impedancije

$$Z_{Rn}(s) = \frac{R}{R_0} = R_n \rightarrow R_n = \frac{R}{R_0}$$ } otpor R = $R_n = \frac{R}{R_0}$

$$Z_{Ln}(s) = \frac{sL}{R_0}\frac{\omega_0}{\omega_0} = \frac{S}{\omega_0}\cdot\frac{\omega_0}{R}\cdot L = S_n \cdot \frac{\omega_0}{R}$$

$$\downarrow$$

$$Z_{Ln}(s) = S_n L_n \longrightarrow L_n S_n = \frac{\omega_0}{R} S_n \cdot L_0$$

induktivitet

$$L_n = \frac{\omega_0}{R} L_0$$

$$Z_{Cn}(s) = \frac{1}{R_0\,\omega_0 \frac{S}{\omega_0} C} = \frac{1}{C \cdot S_n \cdot R_0 \omega_0}$$

$$\downarrow$$

$$Z_{Cn}(s) = \frac{1}{sC_n} \longrightarrow \frac{1}{C_n \cdot S_n} = \frac{1}{C S_n R_0 W_0}$$

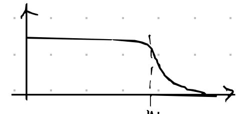
kapacitet

$$C_n = C R_0 W_0$$

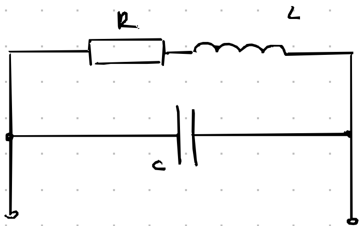
Denormalizacija - inverzan postupak

NORMALIZACIJA : zaključak

- za ω_0 se često uzima granična frekv.
- za R_0 se često uzima 600Ω



Primjer:



$R = 200\Omega$ $\quad \omega_0 = 10^8\ \mathrm{rad/s}$

$L = 2\mu H$ $\quad R_0 = 200\Omega$

$C = 50pF$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^8$$

$$R_0 = 200\Omega$$

$$Z_{ul} = \left(\frac{1}{Cs}\right) \parallel (R+Ls)$$

$$\left(Cs + \frac{1}{R+Ls}\right)^{-1} = \frac{R+Ls \quad /:CL}{1+RCs+s^2LC \quad /:CL}$$

$$Z_{ul} = \frac{\frac{R}{CL} + s\frac{1}{C}}{s^2 + s\frac{R}{L} + \frac{1}{CL}} = \frac{200 + 2\times10^{-6}s}{10^{-16}s^2 + 10^{-8}s + 1} = Z_{ul}$$

norm(ω) $\longrightarrow$ $R_n = 200\Omega$ $\quad L_n = 2\cdot10^{-6}\cdot10^{8} = 200$

$C_n = 50\cdot10^{-12}\cdot10^{8} = \frac{1}{200}$

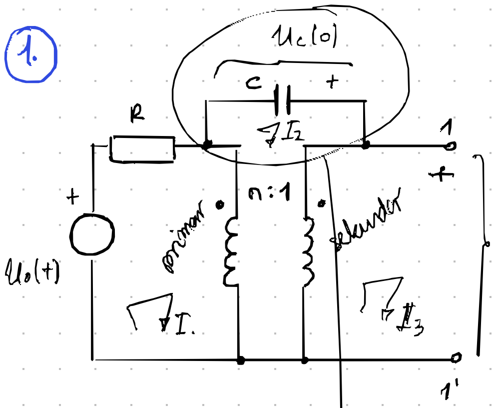
norm(Z) $\longrightarrow$ $R_n = 1$ $\quad L_n = 10^{-8}$ $\quad C_n = 10^{-8}$

norm(Z & ω) $\longrightarrow$ $R_n = 1$ $\quad L_n = 1$ $\quad C_n = 1$

↳ $$Z_{ul} = \frac{s+1}{s^2+s+1}$$ → normalizirana ulazna impedancija

Laplaceova transformacija – zadaci

1.



Odrediti parametre nadomjesne mreže po Thievenina $U_T(s)$ i $Z_T(s)$, s obzirom na priključnice 1-1'.

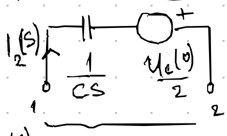
$R=1 \quad u_c(0)=1 \quad u_0(t)=S(t)$

$C=1 \quad n=2$

$u_c(0) \multimap \frac{1}{s}u_c(0)$

$u_T(t) \multimap U_T(s)$

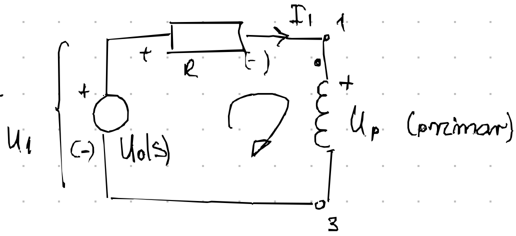
Korak 1



$U_2(s) = I_2(s)\cdot\frac{1}{Cs} - \frac{u_c(0)}{s}$

Korak 2

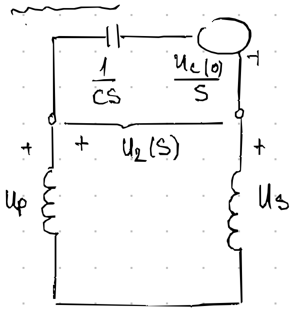
KZ 1.



$\boxed{N}$ $-U_0(s) + U_R(s) + U_p(s) = 0$

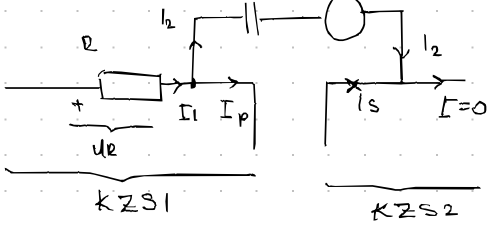
$\boxed{S}$ $-I_1 + I_2 + I_p = 0$

KZ 2.



$\boxed{N}$ $-U_p(s) + U_2(s) + U_s(s) = 0$

$\boxed{S}$ $-I_2 - I_s = 0$



Korak 3 strujno naponske relacije grana

$U_R(s) = I_1\cdot R$

$U_p(s) = n\cdot U_s(s)$

$U_2(s) = I_2(s)\cdot\frac{1}{Cs} - \frac{u_c(0)}{s}$

$I_p(s) = \frac{1}{n}I_s(s)$

Uvrstimo u Kirchoffa:

$-U_0(s) + I_1(s)\cdot R + n\cdot U_s(s) = 0$

$-n\cdot U_s(s) + I_2(s)\cdot\frac{1}{Cs} - \frac{u_c(0)}{s} + U_s(s) = 0$ KZN

$-I_1(s) + I_2(s) + \frac{1}{n}I_s(s) = 0$ $\quad I_1 = \frac{1-n}{n}I_s$

$-I_2(s) - I_s(s) = 0$ KZS

$\Rightarrow -U_0(s) + R\cdot\frac{1-n}{n}I_s + n\cdot U_s(s) = 0$

$-n\cdot U_s(s) - I_s(s)\cdot\frac{1}{Cs} - \frac{u_c(0)}{s} + U_s(s) = 0 \rightarrow I_s = \left[-nU_s(s) - \frac{u_c(0)}{s} + U_s(s)\right]\cdot Cs$

$I_s = Cs(1-n)U_s(s) - C\cdot u_c(0)$

$-U_0(s) + R\cdot\frac{1-n}{n}\cdot\left(Cs(1-n)U_s(s) - C\,u_c(0)\right) + nU_s(s) = 0$

$-U_0(s) + R\cdot\frac{(1-n)^2}{n}\cdot Cs\,U_s(s) - R\cdot\frac{1-n}{n}\cdot C\cdot u_c(0) + nU_s(s) = 0$

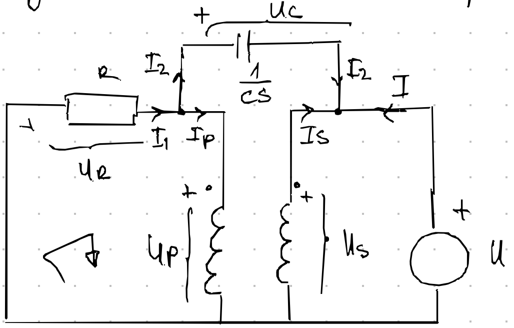
$U_s(s)\left[R(1-n)^2\cdot\frac{1}{n}\cdot Cs + n\right] = U_0(s) + RC\cdot\frac{1-n}{n}u_c(0)$

$U_s(s) = n\cdot\frac{U_0 + RC\frac{1-n}{n}u_c(0)}{RCs(1-n)^2 + n^2} = 2\cdot\frac{\frac{1}{s} + \frac{1-2}{2}\cdot 1}{s(1-2)^2 + 4} = 2\,\frac{\frac{1}{s} - \frac{1}{2}}{s+4} = \frac{2-s}{s(s+4)}$

$$\boxed{U_T(s) = U_s(s) = \frac{2-s}{s(s+4)}}$$

b) Impedancija $Z_T(s)$

→ ugasimo sve neovisne izvore i početne uvjete ali dodamo jedan pomoćni izvor



KZN: $U_R(s) + U_p(s) = 0$

$U_C(s) + U_S(s) - U_p = 0$

KZS: $-I_1(s) + I_2(s) + I_p(s) = 0$

$-I_2(s) - I_S(s) - I(s) = 0$

naponski strujne karakteristike ostaju iste

$U_R = R \cdot I_1$

$U_p = n \cdot U_S(s)$

$U_C(s) = I_2(s) \cdot \frac{1}{cs}$

$I_p(s) = I_S(s) \cdot \frac{1}{n}$

KZN

$R \cdot I_1(s) + n \cdot U_S(s) = 0$

$\frac{1}{cs} \cdot I_2(s) + U_S(s) - n \cdot U_S(s) = 0 \rightarrow I_2(s) = U_S(s)(n-1) \cdot cs$

KZS

$-I_1(s) + I_2(s) + \frac{1}{n} I_S(s) = 0$

$-I_2(s) - I_S(s) - I(s) = 0 \rightarrow I_2(s) = -I_S(s) - I(s)$

$-I_1(s) - I_S(s) - I(s) + \frac{1}{n} I_S(s) = 0$

$I_1(s) = \frac{1-n}{n} I_S(s) - I$

$\Rightarrow R \cdot \left(\frac{1-n}{n} I_S - I\right) + n U_S = 0$ KZN

$\frac{-1}{cs}(I_S + I) + U_S(1-n) = 0 \rightarrow$ $I_S = U_S(1-n) \cdot cs - I$

uvrsti

$R\left[\frac{1-n}{n}\left(U_S(1-n) \cdot cs - I\right) - I\right] + n U_S = 0$

$R\left[\frac{(1-n)^2}{n} U_S \cdot cs - \frac{1-n}{n} \cdot I - I\right] + n U_S = 0$

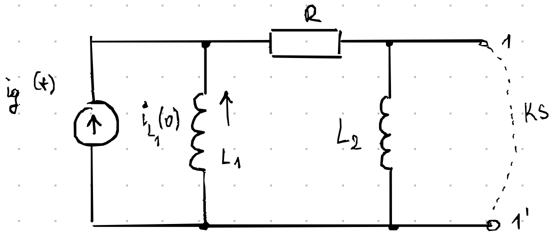
$R\left[\frac{(1-n)^2}{n} U_S \cdot cs - I\left(\frac{1\cancel{-n}+\cancel{n}}{n}\right)\right] + n U_S \Rightarrow \left[R \cdot \frac{(1-n)^2}{n} cs + n\right] U_S - \frac{1}{n} I \cdot R = 0$

$\hookrightarrow U_S = \frac{1}{n} I \cdot R \cdot \frac{1}{R \frac{(1-n)^2}{n} cs + n}$

$\rightarrow Z_S(s) = \frac{U_S(s)}{I} = \frac{R}{R(1-n)^2 cs + n^2}$

$$\boxed{Z_S(s) = \frac{1}{s+4}}$$

2. Nortonovi parametri $Y_n(s)$ i $I_N(s)$



$R = 1\,\Omega$

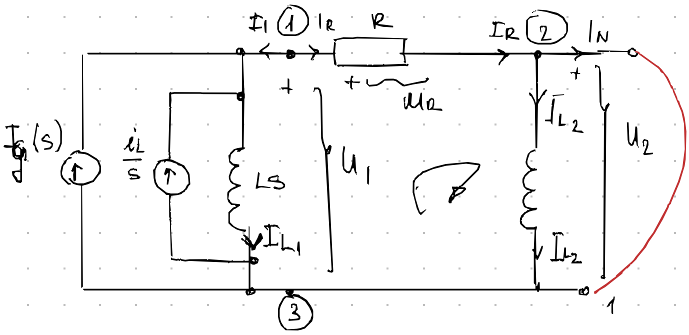
$L_1 = 1H$

$L_2 = 2H$

$i_g(t) = 2\delta(t)$

$i_{L_1}(0) = 1A$

⇓



KZN:

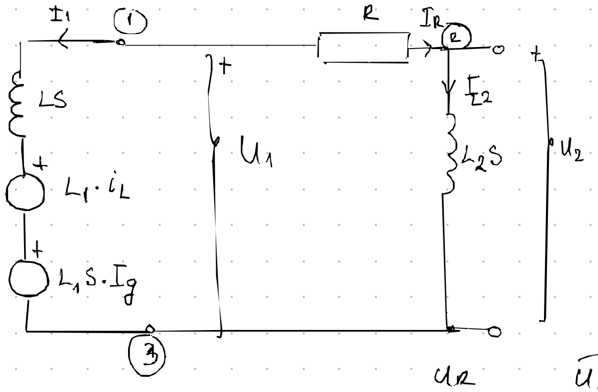
$-U_1 + U_R + U_2 = 0$

$U_2 = 0$ (KS)

KZS

$I_1 + I_R = 0$

$-I_R + I_N + I_{L_2} = 0$ (KS)



$\rightarrow U_1 = L_1 S \cdot I_g + L_1 \cdot i_L(0) + I_1 \cdot Ls$

$U_R = I_R \cdot R$

$U_2 = I_{L_2} \cdot L_2 S$

$\rightarrow U_1 = U_R + U_2$

$$-L_1 \cdot S I_g - L_1 \cdot i_L(0) - I_1 \cdot L_1 S + \underbrace{I_R \cdot R}_{U_R} + \underbrace{I_{L_2} \cdot L_2 S}_{U_2} \quad / : L_1 S$$

$$-I_g - \frac{i_L(0)}{s} - I_1 +$$

$$\dashrightarrow -I_g + \frac{i_{L1}}{s} + \frac{U_1}{L_1 S} + \frac{U_1 - U_2}{R} = 0$$

$$-\frac{U_1 - U_2}{R} + \frac{U_2}{L_2 S} + I_N = 0$$

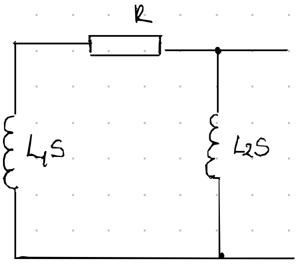
uvrstimo $U_2 = 0$

$$\Rightarrow -I_g + \frac{i_{L1}}{s} - \frac{U_1}{L_1 S} + \frac{U_1}{R} = 0 \rightarrow U_1 = \left[I_g + \frac{i_{L_1}(0)}{S} \right] \frac{S L_1 R}{R + S L_1}$$

$$-\frac{U_1}{R} + I_N = 0 \longrightarrow I_N = \frac{U_1}{R} = \left[I_g + \frac{i_L(0)}{S} \right] \frac{S L_1}{R + S L_1}$$

$$I_N(s) = \frac{U_1}{R} = \left(2 + \frac{1}{s}\right) \frac{s}{1+s} = \frac{2s+1}{s+1} = 2 - \frac{1}{s+1} \Rightarrow \boxed{i_N(t) = 2\delta(t) - e^{-t} S(t)}$$

b) $Y(s) = ?$ (isključimo izvore)



$$Y(s) = \frac{1}{Z(s)}$$

$$Z(s) = \left(\frac{1}{L_1 S + R} + \frac{1}{L_2 S} \right)^{-1} \rightarrow Y(s) = \frac{1}{L_1 S + R} + \frac{1}{L_2 S}$$

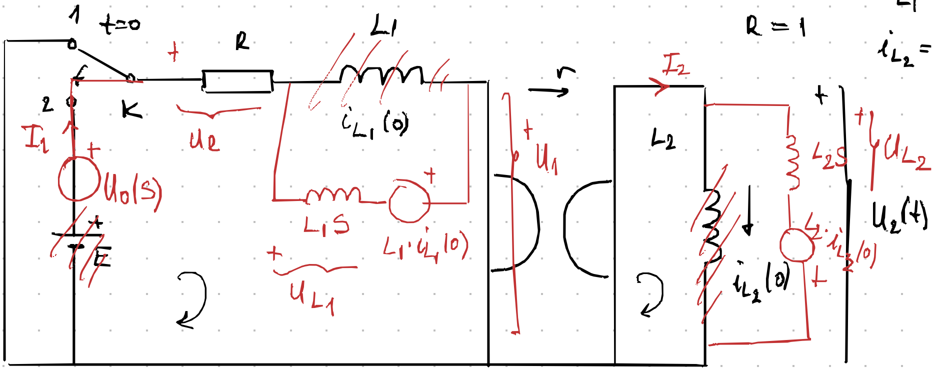
$$Y(s) = \frac{1}{s+1} + \frac{1}{2s} = \frac{2s + s + 1}{2s(s+1)} \Rightarrow \boxed{\frac{3s+1}{2s(s+1)} = Y(s)}$$

Zadatak 3.) $u_2(t)=?$ $t=0$ $1\to 2$

$E=1$ $r=\sqrt{2}$

$L_1=L_2=1$ $i_{L_1}=\frac{1}{2}$

$R=1$ $i_{L_2}=\frac{\sqrt{2}}{2}$



KZN:

$$-U_0(s)+U_R+U_{L_1}-L_1\cdot i_{L_1}(0)+U_1=0$$

$$U_{L_2}-L_2\cdot i_{L_2}(0)-U_2(s)=0$$

zašto minus??

$$-U_0+I_1\cdot R+I_1\cdot L_1 s-L_1\cdot i_{L_1}(0)-r\cdot I_2=0$$

$$I_2\cdot L_2 s-L_2\, i_{L_2}(0)+r\cdot I_1=0$$

$U_R=I_1\cdot R$ $U_1=-r\cdot I_2$

$U_{L_1}=I_1\cdot L_1\cdot s$ $U_2=-r\cdot I_1$

$U_{L_2}=I_2\cdot L_2 s$

$$I_1(R+L_1 s)-rI_2-U_0-L_1\cdot i_{L_1}(0)=0$$

$$I_1(R+L_1 s)-rI_2=U_0+L_1\cdot i_{L_1}(0)$$

$$I_2=\frac{L_2\cdot i_{L_2}(0)-rI_1}{L_2 s}$$

$$\to I_1(R+L_1 s)-r\cdot\frac{L_2\cdot i_{L_2}(0)-rI_1}{L_2 s}=U_0+L_1\cdot i_{L_1}(0)$$

$$I_1\left(R+L_1 s+\frac{r^2}{L_2 s}\right)=U_0+L_1\cdot i_{L_1}(0)+rL_2\, i_{L_2}(0)$$

$$I_1=\frac{U_0\cdot L_2 s+L_2 s\cdot L_1 i_{L_1}(0)+rL_2\cdot i_{L_2}(0)}{RL_2 s+L_1L_2 s^2+r^2}$$

$\to$ * $U_1=-rI_2$

$U_2=-r\cdot I_1$

$$U_2=-r\,\frac{U_0L_2 s+sL_1L_2\, i_{L_1}(0)+rL_2\, i_{L_2}(0)}{sL_2R+L_1L_2 s^2+r^2}$$

$$U_2=-\sqrt{2}\,\frac{s+s\cdot\frac{1}{2}+\sqrt{2}\cdot\frac{\sqrt{2}}{2}}{s+s^2+2}=-\sqrt{2}\cdot\frac{\frac{3}{2}s+1}{s+s^2+2}=-\frac{\sqrt{2}}{2}\cdot\frac{s+4}{s^2+s+2}$$

$$=-\frac{\sqrt{2}}{2}\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{7}{4}}+\frac{\frac{7}{2}}{(s+\frac{1}{2})^2+\frac{7}{4}}\right)$$

$2\cdot\frac{1}{2}s$ $\frac{1}{4}$ $\frac{8}{4}-\frac{1}{4}=\frac{7}{4}$

$$=\boxed{-\frac{\sqrt{2}}{2}\left(e^{-\frac{1}{2}t}\cdot\cos\left(\frac{\sqrt{7}}{2}t\right)+e^{-\frac{1}{2}t}\cdot\sin\left(\frac{\sqrt{7}}{2}t\right)\right)\cdot S(t)=u(t)}$$