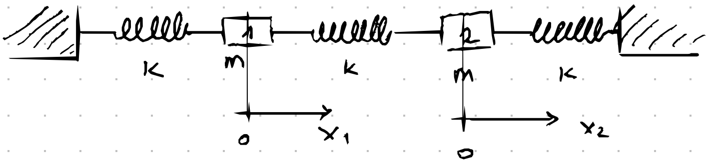
6. MEHANIČKI VALOVI

Primjer: Titranje u fazi

Formula: $-kx$
- uvrstiti x zajedno sa predznakom → sila nas uvijek „vraća nazad"



$x_1 = x_2 \longrightarrow 2m(\ddot{x}_{1,2}) = -2k\,x_{1,2}$ (u istom smjeru se miču m_1 i m_2)

$$\boxed{\Rightarrow \omega_\Delta^2 = \frac{k}{m}}$$

Titranje u protufazi $x_1 = -x_2$ (pomak je u suprotnom smjeru)

$m\ddot{x}_1 = -kx_1 - k'(x_1 - x_2) \longrightarrow 2kx_1$

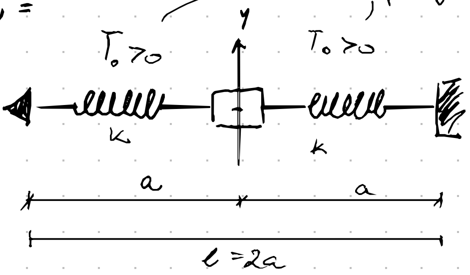
$m\ddot{x}_1 = -kx_1 - 2k'x_1$

$m\ddot{x}_1 = -x_1(k + 2k')$

$$\boxed{\Rightarrow \omega^2 = \frac{k + 2k'}{m}}$$

vlastiti mod = opruge su napete u stanju mirovanja

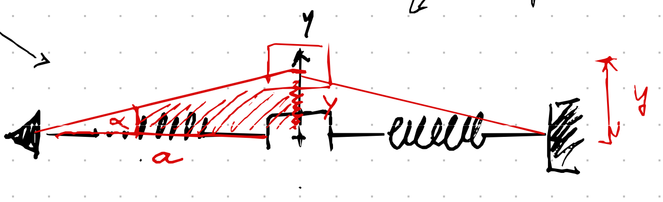
Primjer:



Napetost opruge pri otklonu u y

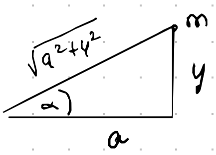
$T[y] = T_0 + k\left(\sqrt{a^2 + y^2} - a\right)$ ali ovo nam neće trebati

jednadžba gibanja



$m\ddot{y} = -2T[y] \cdot \sin\alpha$

dvije opruge koje pokušavaju vratiti u ravnotežni položaj



$\rightarrow \sin\alpha = \frac{y}{\sqrt{a^2 + y^2}}$

$m\ddot{y} = -2\left[T_0 + k\left(\sqrt{a^2 + y^2} - a\right)\right] \cdot \frac{y}{\sqrt{a^2 + y^2}} \quad / : m$

$\ddot{y} + \boxed{\frac{2}{m}\left(T_0 + k\sqrt{a^2+y^2} - ka\right) \cdot \frac{1}{\sqrt{a^2+y^2}}} \cdot y = 0$

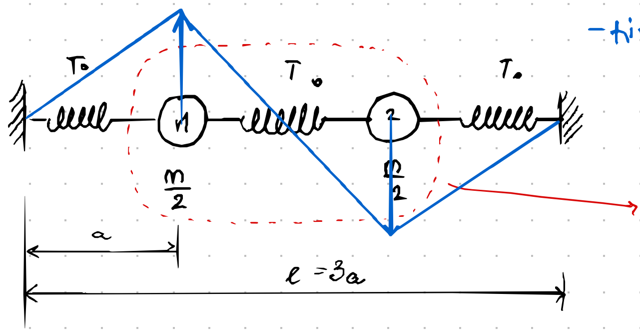
$\hookrightarrow \omega^2[y]$

$\omega^2 = \lim_{y \to 0} \omega_0^2(y) = \frac{2T_0}{m} \cdot \frac{1}{\sqrt{a^2}} = \boxed{\frac{2T_0}{ma}}$

$\omega^2 = \frac{4T_0}{m \cdot l} \longleftarrow l = 2a$

Primjer

- titranje u protufazi



u fazi: $y_1 = y_2$

$$m\ddot{y}_{1,2} = -2T_0 \cdot \frac{y_{1,2}}{a}$$

$$\ddot{y}_{1,2} + \underbrace{\frac{2T_0}{ma}}_{\omega_0^2} \cdot y_{1,2} = 0 \quad \rightarrow \quad \omega_A^2 = \frac{6T_0}{m \cdot \ell}$$

u protufazi:

- dvije čestice i svaka ima polovicu mase

$$\frac{m}{2}\ddot{y}_1 = -T_0 \frac{y_1}{a} - T_0 \frac{2y_1}{a} = -\frac{3T_0}{a} y_1$$

$$\ddot{y}_1 + \boxed{\frac{6T_0}{ma}} y_1 = 0$$

$\hookrightarrow = \omega^2$

Harmonjski val : prijamnik opaža titranje uz kašnjenje $= \frac{r}{v}$

$$\frac{\partial^2}{\partial t^2} y[x,t] - \overbrace{\frac{T}{\mu}}^{=v^2} \cdot \frac{\partial^2}{\partial x^2} y[x,t] = 0$$

$$y[x,t] = f[x \pm vt]$$

$$\frac{\partial}{\partial t} f[x \pm vt] = f'[x \pm vt] \cdot \frac{d}{dt}(x \pm vt)$$

$$= f'[x \pm vt] \cdot (\pm v) \Rightarrow \boxed{\pm v \cdot f'}$$

$$\frac{\partial^2}{\partial t^2} y[x,t] + \frac{T}{\mu} \frac{\partial^2}{\partial x^2} y[x,t] = 0 \Rightarrow f''$$

$$v^2 f \qquad v^2 \rightarrow v = \sqrt{\frac{T}{\mu}}$$

$$\frac{\partial^2}{\partial x^2} f[x \pm vt] = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f[x \pm vt]\right)$$

$$= \frac{\partial}{\partial x}\left(f'[x \pm vt] \underbrace{\frac{d}{dx}(x \pm vt)}_{1}\right)$$

$$= \frac{\partial}{\partial x} f'[x \pm vt]$$

$$= f''[x \pm vt) \underbrace{\frac{d}{dx}(x \pm vt)}_{1}$$

$$\frac{\partial^2}{\partial x^2} f = f''$$