KUTNA KOLIČINA GIBANJA

Količina gibanja u sustavu čestice

ZOKG:

$$\vec{P}_{uk} = \vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n = \sum_i \vec{p}_i \;\Big/\, \frac{d}{dt} \qquad \vec{F}_{ji} = -\vec{F}_{ij}$$

$$\frac{d}{dt}\vec{P}_{uk} = \sum_i \frac{d}{dt}\vec{p}_i = \sum_i \Big(\vec{F}_{i,ext} + \sum_{j\neq i}\vec{F}_{ji}\Big) = \sum_i \vec{F}_{i,ext}$$

• budući da je $\frac{d}{dt}\vec{P}_{uk} = \vec{F}_{i,ext}$ $\longrightarrow$ možemo zaključiti da je $\vec{P}_{uk}$ = konst.

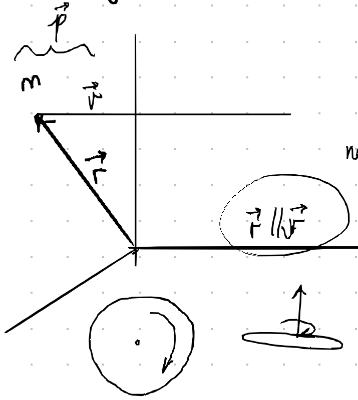
Kutna količina gibanja materijalne točke

linearno gibanje: $\vec{p} = m\vec{v}$

kod rotacije: kutna količina gibanja $\vec{L} = \vec{r}\times\vec{p}$

* u odnosu na neku os rotacije, tj. ovisi o izboru ishodišta

↳ ovisi o položaju tijela



nema kutne količine gibanja

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}}\times\vec{p} + \vec{r}\times\underbrace{\frac{d\vec{p}}{dt}}_{\vec{F}}$$

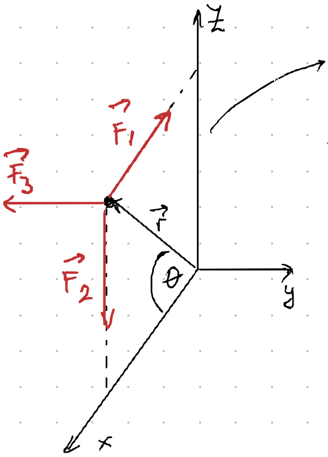
$$\frac{d}{dt}\vec{L} = \underbrace{\vec{v}\times\vec{p}}_{0} + \vec{r}\times\vec{F}$$

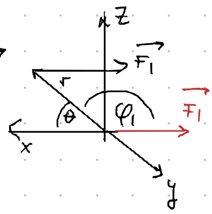
kada je paralelno

$\longrightarrow$ $\boxed{\vec{r}\times\vec{F} = \frac{d}{dt}\vec{L} = \vec{M} \quad \text{(moment sile)}}$

Primjer: računanje momenta sile

$r = 3\,m \qquad \theta = 30° \qquad F = 2\,N$

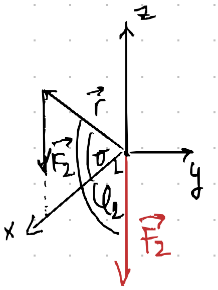




$\varphi_1 = 180° - \theta$

$|\vec{M}_1| = \vec{r}\times\vec{F}_1 = r\cdot F_1\cdot\sin\varphi_1$

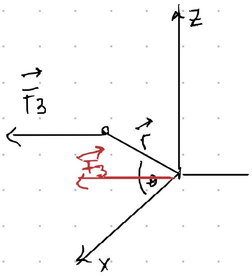
$|\vec{M}_1| = 3\,Nm$



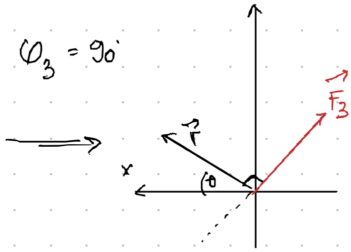
$\varphi_2 = 90° + \theta$

$|\vec{M}_2| = \vec{r}\times\vec{F}_2 = r\cdot F_2\cdot\sin\varphi_2$

$|\vec{M}_2| = 5{,}2\,Nm$



$\varphi_3 = 90°$



$|\vec{M}_3| = \vec{r}\times\vec{F}_3 = rF_3\sin 90°$

$|\vec{M}_3| = 6\,Nm$

Za točku gibanja po proizvoljnoj krivulji (ne nužno kružno)

$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times (m\vec{a}) = m\vec{r} \times (\vec{\alpha} \times \vec{r}) + m\vec{r} \times (-\hat{r}\, \omega^2 r)$

$M = m\vec{r} \times (\vec{\alpha} \times \vec{r}) = mr^2 \vec{\alpha} \longrightarrow \boxed{M = I \cdot \vec{\alpha}}$

$L = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) = m\vec{r} \, (\vec{\omega} \times \vec{r}) = mr^2 \vec{\omega} \Rightarrow \boxed{L = I\omega^2}$

$L = mr^2 \vec{\omega} \; / \frac{d}{dt} \longrightarrow M = \frac{d\vec{L}}{dt} = mr^2 \cdot \underbrace{\frac{d\vec{\omega}}{dt}}_{\vec{\alpha}} \Rightarrow \boxed{M = mr^2 \cdot \vec{\alpha}}$

Ukupna količina gibanja zatvorenog sustava je konstantna (ZOKKG)

$\Delta \vec{L}_{uk} = 0$ · vrijedi kada nema vanjskih momenata sile

promatramo sustav čestica: $\vec{L}_{uk} = \sum \vec{L}_i = \sum \vec{r}_i \times \vec{p}_i$

$\frac{d\vec{L}_{uk}}{dt} = \sum_i \frac{d}{dt}(\vec{r}_i \times \vec{p}_i) = \sum_i \vec{M}_i = \sum_i \left(\vec{M}_{i,ext} + \sum_{i \neq j} \vec{M}_{ji}\right) = \sum_i \vec{M}_{i,ext}$ poništ se

$\vec{F}_{ji} = -\vec{F}_{ij} \Rightarrow (\vec{r}_i - \vec{r}_j) \times \vec{F}_{ji} = 0$

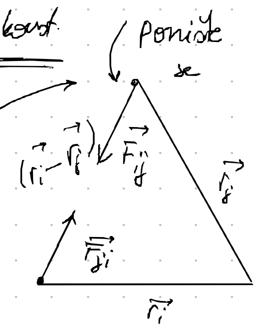
+kao i kod količine gibanja: $\vec{M}_{i,ext} - \frac{d\vec{L}_{uk}}{dt} = 0 \longrightarrow \underline{L_{uk} = konst.}$

$\measuredangle$ Momenti sile unutar objekta: $\sum_{j \neq i} \vec{M}_{ji}$

$\vec{M}_{ji} = \vec{r}_i \times \vec{F}_{ij}$

$\vec{M}_{ij} = \vec{r}_j \times \vec{F}_{ji} = -\vec{r}_j \times \vec{F}_{ij}$

$\Big\}\; (\vec{r}_i - \vec{r}_j) \times \vec{F}_{ij} = 0 \;\rightarrow$

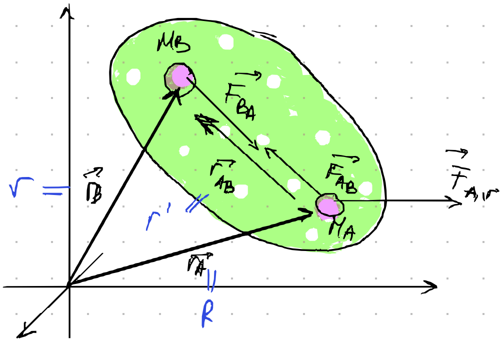


+gibanje planeta po elipsi oko Sunca $\rightarrow$ $r < \longrightarrow v >>$ $\Big\}$ Keppler

$L = \vec{r} \times m\vec{v}$

Sistem materijalnih točaka → centar mase

$\sum_i m_i \vec{r}_i = \vec{R}_{CM} \sum_i m_i$



$\vec{r}_B = \vec{r}_{AB} + \vec{r}_A \longrightarrow \vec{r}_i = \vec{R}_{CM} + \vec{r}_i\,' \; / \sum_i m_i$

kruto tijelo: $\vec{r}_{AB} = konst.$

$\vec{F}_{AB} = -\vec{F}_{BA}$

$\sum_i m_i r_i = \underbrace{\sum_i m_i}_{M} R_{CM} + \underbrace{\cancel{\sum_i m_i r_i'}}_{0}$ po definiciji

$\Rightarrow \vec{L} = \sum \vec{L}_i = \sum_i (\vec{r}_i + \vec{R}_{CM})$

$\hookrightarrow \sum_i m_i \vec{r}_i = R_{CM} \sum_i m_i = M \cdot R_{CM}$

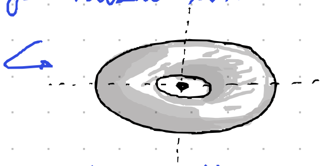
$\vec{L} = \sum_i \vec{L}_i = \sum_i \vec{r}_i \times \vec{p}_i = \sum_i \vec{r}_i \times (m_i \vec{v}_i) = \sum_i m_i \vec{r}_i \times \vec{v}_i = \sum_i m_i (\vec{R}_{CM} + \vec{r}_i\,') \times (\vec{V}_{CM} + \vec{v}_i\,')$

$\vec{L} = \underbrace{\sum \vec{R}_{CM} \times (m_i \vec{v}_{CM})} + \underbrace{\cancel{\sum_i \vec{R}_{CM} \times (m_i \vec{v}_i\,')}}_{0} + \underbrace{\cancel{\sum_i \vec{r}_i\,' \times (m_i \vec{v}_{CM})}}_{0} + \underbrace{\sum_i \vec{r}_i\,' \times (m_i \vec{v}_i\,')}$

$\boxed{\vec{L} = \vec{L}_{CM} + \vec{L}\,'}$

$R_{CM} = \dfrac{\sum m_i \vec{r}_i}{\sum m_i}$

* CM nije uvijek nužno unutar tijela



Fiz zakoni sustava čest. u lab. i sustavu CM

II. NZ

sustav centra mase (CM) $\quad \vec{r}_1 = \vec{r}_1\,' + \vec{R}_{CM} \quad \vec{v}_1 = \vec{v}_1\,' + \vec{V}_{CM} \quad \vec{V}_{CM} = konst$

$\hookrightarrow \vec{a} = \vec{a}\,'$

- ukupna kinetička energija:

$\sum_i E_{k,i} = \sum_i \frac{m_i}{2} \vec{v}_i^{\,2} = \sum_i \frac{m}{2} (\vec{v}_1\,' + \vec{V}_{CM})^2 = \frac{M}{2} \vec{V}_{CM}^{\,2} + \sum_i E'_{k,i}$

$\Rightarrow \boxed{E_{k,uk} = E'_{k,uk} + \vec{E}_{k,CM}}$

$\sum_i m_i \vec{v}_i = \sum_i m_i \vec{v}_i\,' + \sum_i m_i \vec{V}_{CM} \Rightarrow \vec{P}_{uk} = \vec{P}_{CM} \rightarrow$ tj. $\vec{P}_{uk}\,' = 0$

ukupna kol. gibanja

$\sum_i m_i \vec{r}_i\,' = 0 \;, \; \sum_i m_i \vec{v}_i\,' = 0 \quad$ iz definicije $\vec{R}_{CM}$

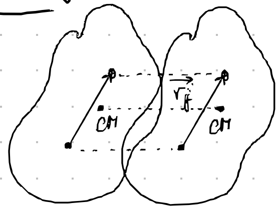
$\sum_i \vec{L}_i = \sum_i (\vec{r}_i\,' + \vec{R}_{CM}) \times m_i (\vec{v}_i\,' + \vec{V}_{CM}) \longrightarrow \boxed{\vec{L}_{uk} = \vec{L}_{uk}\,' + \vec{L}_{CM}}$

Ukupna kutna količina gibanja

STATIKA

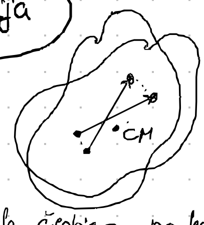
· ravnoteža i uvjet ravnoteže čestice – kruto tijelo ima dva NEZAVISNA načina gibanja:

translacija (centar mase se giba)



gibanje svih čestica istom brzinom + po istom pravcu

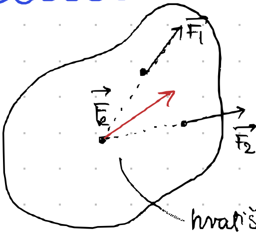
rotacija



– gibanje svih čestica po kružnim putanjama s centrom na osi rotacije

*ako je os rotacije CM, on ostaje nepomičan

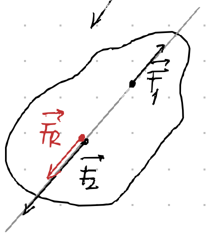
Konkurentne sile

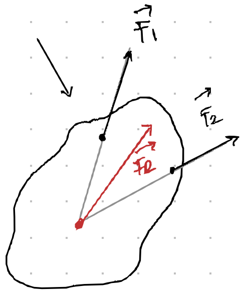


hvatište (može se pomicati duž pravca djelovanja sile bez utjecaja na gibanje)

konkurentne sile (imaju isto hvatište) → mogu se vekt zbrojiti u $\vec F_R$

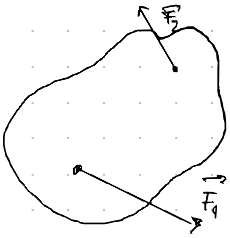
↳ $\vec F_R = \sum_i \vec F_i$





Ako je $\vec F_R = 0$ i na njega djeluju samo konkurentne sile => tijelo je u ravnoteži

Nekonkurentne sile



- tijelo nije ukočeno u centru mase

→ translacija – ako gledamo po rezultantnoj

rotacija – ako gledamo djelovanje rezultantnog momenta svih sila oko neke osi

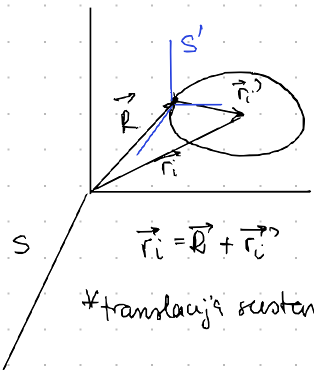
OPĆENITO UVJET RAVNOTEŽE : $\vec P = konst$, $\vec L = konst$

* nema translacije niti rotacije → $\sum_i \vec F_i = 0$ i $\sum_i \vec M_i = 0$

neovisno o izboru točke za račun momenta sile!

SAMO u slučaju ravnoteže

Zašto samo slučaj ravnoteže?



$\vec r_i = \vec R + \vec r_i\,'$

*translacija sustava

pretp: $\sum \vec F_i = 0$

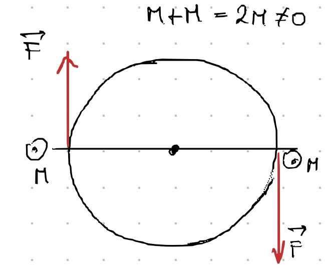
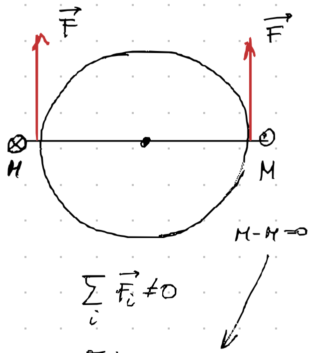
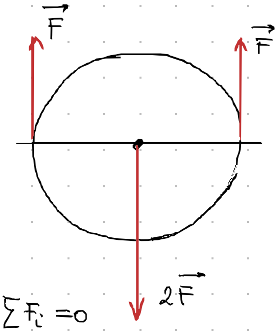
$\sum \vec M_i = \sum \vec r_i \times \vec F_i = 0$

$\vec M\,' = ? = \sum_i \vec r_i\,' \times \vec F_i = \sum_i (\vec r_i - \vec R) \times \vec F_i$

$= \underbrace{\sum_i \vec r_i \times \vec F_i}_{=0} - \underbrace{\vec R \sum_i \vec F_i}_{=0}$

Primjer: Ravnoteža oko krutog tijela

$M + M = 2M \neq 0$

$\sum_i \vec{F}_i = 0 \quad \sum_i \vec{M}_i \neq 0$

rotacija

$M - M = 0$

$\sum_i \vec{F}_i \neq 0$

$\sum_i M_i = 0$

translacija

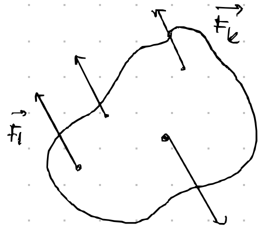
$\sum_i F_i = 0$

$\sum_i M_i = 0$

ravnoteža

Kruto tijelo je u ravnoteži ako mu je linearna akc. 0 i ako je kutna akceleracija oko svake točke jednaka 0

Nekonkurentne sile



Zbrajanje: $\vec{F_R} = \sum_i \vec{F_i} = \hat{n} \sum_i F_i$

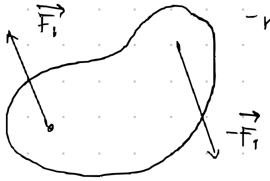
hvatište ~~rez~~ sile:

$$\sum_i \vec{M_i} = \sum_i \vec{r_i} \times \vec{F_i} = \left(\sum_i \vec{r} F_i\right) \times \hat{n}$$

$$\sum_i \vec{M_i} = \vec{R} \times \vec{F_R} = \left(\vec{R} \sum_i F_i\right) \times \vec{n}$$

$$R = \frac{\sum_i \vec{r_i} F_i}{\sum_i F_i}$$

• ako djeluju samo dvije sile za koje vrijedi $\sum F_i = 0$

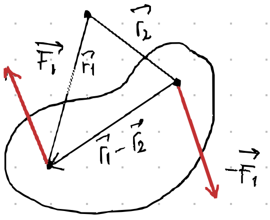
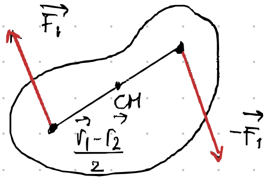


- ne možemo zamijeniti djelovanjem jedne (F_R) sile

→ nema translacije: $\vec{F_1} + \vec{F_2} = 0$ (nema pomaka CM)

→ ukupni moment $\vec{M_1} + \vec{M_2} = (\vec{r_1} - \vec{r_2}) \times \vec{F_1}$

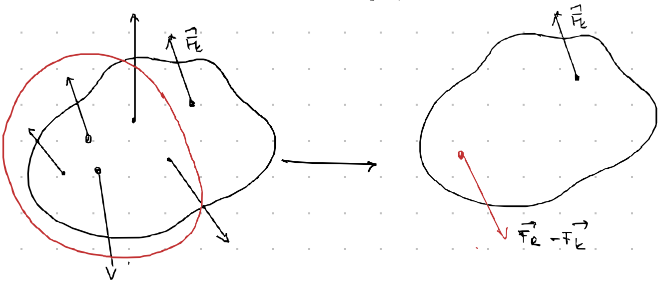
⇒ rotacija oko osi kroz CM → smjer određuje par sila

Na par sila moguće je svesti svaku raspodjelu sila za koju je $\sum_i \vec{F_i} = 0 \; ; \; \sum_i \vec{M_i} \neq 0$

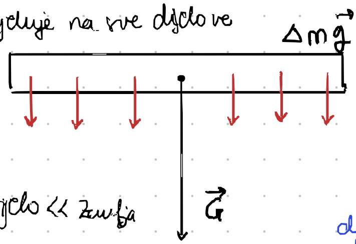
↳ par sila nije moguće uravnotežiti samo jednom silom već samo drugim parom sila

$$\vec{F_R} = \sum_{i=1}^{k} F_i = 0 = \sum_{i=1}^{k-1} \vec{F_i} + \vec{F_k}$$



Težište tijela

$\vec{G}$ djeluje na sve dijelove



hvatište sile teže: $\vec{r}_T = \dfrac{\sum_i \vec{r}_i m_i g}{\sum_i m_i g} = \vec{r}_{CM}$

*za tijelo << Zemlja $\vec{g}$ je svuda ista

moment $\vec{G}$ na tijelo jednak je momentu koji djeluje na ukupnu masu tijela smještenu na položaju CM

kontinuirana varijanta: $\Delta m = \rho \Delta V$

dif oblik za Kartezijeve koord: $dm = \rho dV = \rho\, dx\, dy\, dz$

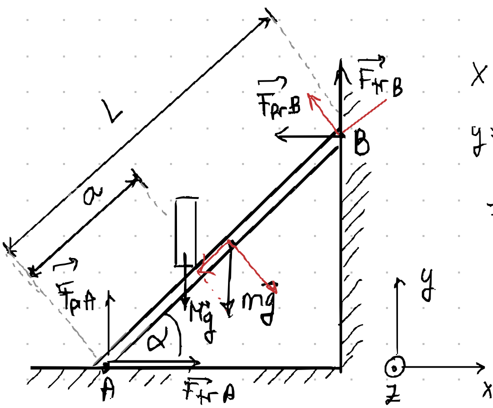
$$\vec{r}_T = \frac{\int \vec{r}\, dm}{\int dm} \quad / \cdot \hat{x} \qquad {}^{*}\vec{r}_T = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\hookrightarrow x_T = \frac{\int x\, dm}{\int dm} = \frac{\int x \rho\, dV}{\int \rho\, dV} \Longrightarrow x_T = \frac{\int x\, dV}{V}$$

homogeno tijelo

Primjer: Statika

F_{pr} – pritisak $\quad F_{tr}$ – trenje



$X: F_{trA} - F_{prB} = 0$

$y: F_{trB} + F_{prA} - Mg - mg = 0$

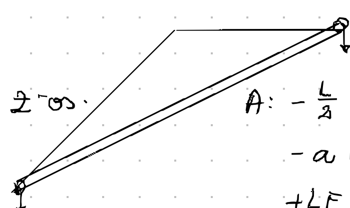
$\Sigma F_{x,y} = 0$

$F_{trA,B} = \mu_{AB} \cdot F_{prA,B}$

$x: F_{prB} = F_{trA}$

$y = F_{prA} = \Delta m g$

$F_{trA} = \mu_A F_{prA} = \mu_A \cdot \Delta m g$



z-os:

$A: -\frac{L}{2} mg \sin(90+\alpha)$
$- a\, Mg \sin(90+\alpha)$
$+ L F_{prB} \cdot \sin(180-\alpha)$
$+ L F_{trB} \sin(90-\alpha) = 0$

$A: + x \cdot 3mg \sin 75° + \frac{L}{2} mg \sin 45° = L \cdot \mu_A \Delta m g \cdot \sin 105°$

$\Longrightarrow 3 \sin 15° \frac{x}{\ldots} = \mu_A \sin 105° \ldots x \ldots$