# SPECIJALNA TEORIJA RELATIVNOSTI

- relativnost gibanja inercijalnih sustava

## ETER i Michelson-Morleyev pokus

c - brzina svjetlosti, ne ovisi o brzini sustava

- nedokazano su se preseravali jer nisu htjeli priznat da je c konstantna

▸ Galilejeve transformacije : $\boxed{\vec{r} = \vec{r}\,' + \vec{v}t}$

• ako je c konst :

$ct = r \;/^2$

$c^2t^2 = r^2$

$\boxed{c^2t^2 = x^2+y^2+z^2}$ kugla!

$c^2t'^2 = x'^2+y'^2+z'^2$

$c = \frac{r}{t}$

— ali ako idemo koristiti Galilejeve transf. → $c^2t'^2 = (x'+vt')^2+y'^2+z'^2$

▸ Lorentzove transformacije *ovo bi moglo biti na usmenom

I. - nove transformacije moraju prijeći u Galilejeve kada uzmemo da je relativna brzina $v \ll c$ brzina svjetlosti je u svim sustavima c

II. - moraju biti simetrične $v = -v$ (s obzirom na v transformacije)

III. - kao pretpostavku : transf. su linearne $E = h\cdot\nu$ , $x = \alpha x' + \beta t'$

$x = \alpha x' + \beta t'$

$x = \gamma(x' + vt')$

$x' = ct'$

$x = ct$

u svim sustavima je brzina svjetlosti c

$x' = \gamma(x - vt)$ (jer $v = -v$ → II. pretpostavka)

$\Rightarrow x = ct = \gamma(x'+vt') = \gamma(ct'+vt') = \gamma t'(c+v)$

$\hookrightarrow \frac{x}{\gamma} = t'(c+v)$

$\frac{x'}{\gamma} = t(c-v)$

znači :

C = konst.

(I.) $x' = ct'$, $x = ct$

$v \ll c$

simetričnost

(II.) $x = \gamma(x'+vt')$, $x' = \gamma(x-vt)$

$v = -v$

linearnost

(III.) $x = \alpha x' + \beta t'$

$E = h\cdot\nu$?

$\Rightarrow x = ct = \gamma(x' + vt') = \gamma(ct' + vt') = \gamma t'(c+v)$

$\frac{x}{\gamma} = t'(c+v)$

$\Rightarrow x' = ct' = \gamma(x - vt) = \gamma t(c-v) \longrightarrow \frac{x'}{\gamma} = t(c-v)$

$\frac{x'}{\gamma(c-v)} = t$

uvrstimo:

$x' = ct' \to t' = \frac{x'}{c}$

$x = c \cdot \frac{x'}{\gamma(c-v)} = \gamma(x' + vt')$

$\frac{c\cancel{x'}}{\gamma(c-v)} = \gamma\left(\cancel{x'} + v\frac{\cancel{x'}}{c}\right)$

$\frac{c}{\gamma(c-v)} = \gamma\left(1 + \frac{v}{c}\right)$

$c = \gamma^2(c-v)\left(1+\frac{v}{c}\right) = \gamma^2\left(c + \cancel{v} - \cancel{v} - \frac{v^2}{c}\right) = \gamma^2 \cdot c\left(1 - \frac{v^2}{c^2}\right)$

$1 = \gamma^2\left(1 - \frac{v^2}{c^2}\right) \longrightarrow \boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$

## Dilatacija vremena

$t_1', \; t_2'$ $\qquad x_2' = x_1'$

$t_2 - t_1 = \gamma\left((t_2' - t_1') + \frac{v}{c^2}(x_2' - x_1')\right)$

$\boxed{\Delta t = \gamma \Delta t'}$

## Kontrakcija duljine

$t_2 = t_1$

$x_2' - x_1' = \gamma(x_2 - x_1 - v(t_2 - t_1))$

$l_0 = \gamma l \;\; — \;\; l = \frac{l_0}{\gamma}$

$l = \underbrace{\sqrt{1-\frac{v^2}{c^2}}}_{<1} \, l_0$

## Zad. 2.)


$- \Delta t'$ – manji od $\Delta t$

$\Delta t' = 5$ godina

$v = ?$

$\Delta t = \dfrac{D}{v}$

$v = \dfrac{s}{t}$

$D$ ↳ 10 svj god

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} > 1, \quad \frac{v^2}{c^2} < 1$$

↳ budući da je $\Delta t$ veći od $\Delta t'$

$$\Delta t = \gamma \Delta t'$$

$$\longrightarrow \frac{D}{v} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \Delta t' \quad /^2$$

$$\frac{D^2}{v^2} = \frac{(\Delta t')^2}{1-\frac{v^2}{c^2}}$$

$$D^2 - D^2 \cdot \frac{v^2}{c^2} = v^2 \cdot (\Delta t')^2$$

$$D^2 = v^2 (\Delta t')^2 + D^2 \cdot \frac{v^2}{c^2}$$

$$D^2 = v^2\left((\Delta t')^2 + \frac{D^2}{c^2}\right)$$

$$v = \frac{D}{\sqrt{(\Delta t')^2 + \frac{D^2}{c^2}}} \approx \underline{\underline{0{,}894c}}$$

# BRZINA ČESTICA I ZBRAJANJE BRZINA

## Brzina čestica:

$u_x$ - brzina čestice u sustavu S

$u_x'$ - -||- u S'

$$x = \gamma(x' + vt')$$

$$u_x = \frac{dx}{dt} = \frac{d}{dt}\gamma(x' + vt')$$

$$u_x = \frac{dt'}{dt}\cdot\frac{d}{dt'}[\gamma(x'+vt')]$$

$$u_x' = \frac{dx'}{dt}$$

$$u_x = \frac{dt'}{dt}\left[\gamma\left(\frac{dx'}{dt'}+v\right)\right]$$

$$u_x = \frac{dt'}{dt}\gamma(u_x'+v)$$

novi način

$$\cancel{t = \gamma\left(t' + \frac{v}{c^2}x'\right)}$$

$$\frac{dt}{dt'} = \gamma\left(1 + \frac{v}{c^2}u_x'\right)$$

$$\frac{dt'}{dt} = \frac{1}{\gamma\left(1+\frac{v}{c^2}u_x'\right)}$$

$$u_x = \frac{\cancel{\gamma}(u_x'+v)}{\cancel{\gamma}\left(1+\frac{v}{c^2}u_x'\right)}$$

$$u_x = \frac{u_x'+v}{1+\frac{v}{c^2}u_x'}$$

$z = z'$
$y = y'$ } nema promjene $\Rightarrow$ $u_y = \frac{dy}{dt} = \frac{dy'}{dt} \Rightarrow \frac{dt'}{dt}\frac{dy'}{dt'}$ ($u_y'$)

$$u_y = \frac{u_y'}{\gamma\left(1+\frac{v}{c^2}u_x'\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1+\frac{v}{c^2}u_x'\right)}$$

$$u_y' = \frac{u_y}{\gamma\left(1-\frac{v}{c^2}u_x\right)}$$

$$u_x = \frac{u_x'+v}{1+\frac{v}{c^2}u_x'} \qquad u_x' = \frac{u_x - v}{1-\frac{v}{c^2}u_x}$$

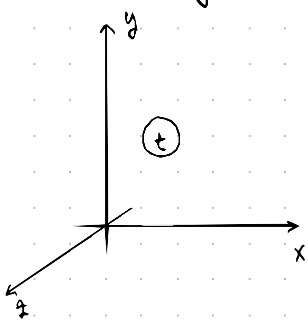
$$u_y = \frac{u_y'}{\gamma\left(1+\frac{v}{c^2}u_x'\right)} \qquad u_y' = \frac{u_y}{\gamma\left(1-\frac{v}{c^2}u_x\right)}$$

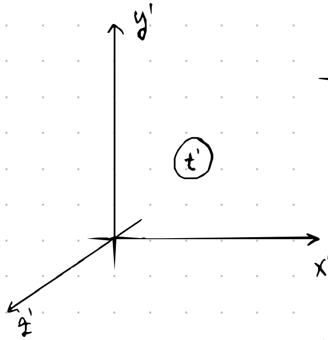
$$u_z = \frac{u_z'}{\gamma\left(1+\frac{v}{c^2}u_x'\right)} \qquad u_z' = \frac{u_z}{\gamma\left(1-\frac{v}{c^2}u_x\right)}$$

# Lorentzove transformacije i njihova svojstva

Saša

- dva inercijska sustava : S i S'




$$\Delta x' = \gamma(x - v\Delta t)$$

$$\Delta y' = \Delta y \qquad \Delta z' = \Delta z$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{c}{v} < 1$$

$$u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'} \qquad u_y = \frac{u_y'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)} \qquad u_z = \frac{u_z'}{\gamma\left(1 + \frac{v}{c^2}u_x'\right)}$$

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \qquad u_y' = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} \qquad u_z' = \frac{u_z}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

KORISNO: (izvodi)

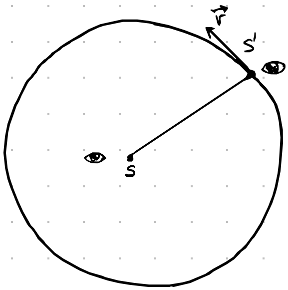
$$\frac{dt'}{dt} = \frac{\Delta t'}{\Delta t} = \frac{\gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)}{\Delta t} = \gamma\left(1 - \frac{v}{c^2}\overset{u_x}{\left(\frac{\Delta x}{\Delta t}\right)}\right) \rightarrow \boxed{\gamma\left(1 - \frac{v}{c^2}\cdot u_x\right)}$$

$$u_x' = \frac{dx'}{dt'} = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x'}{\Delta t}\cdot\frac{\Delta t}{\Delta t'} = \frac{\gamma(\overset{u_x}{X} - v\Delta t)}{\Delta t}\cdot\frac{1}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

$$u_x' = \frac{(u_x - v)}{\left(1 - \frac{v}{c^2}u_x\right)}$$

$$u_y' = \frac{\Delta y'}{\Delta t'} = \frac{\Delta y'}{\Delta t}\,\frac{\Delta t}{\Delta t'} = \overset{u_y}{\frac{\Delta y}{\Delta t}}\cdot\frac{1}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}$$

## Gravitacijska teorija




nije inercijski sustav pa se zakonitosti ne poklapaju

$s = 2\pi$

R se ne transformira jer je s' okomit na s

$\Rightarrow s' < s \; , \; s'$

# RELATIVISTIČKA KOLIČINA GIBANJA & ENERGIJA ČESTICE:

**Količina gibanja:** ako se čestica mase m giba brzinom kojoj odgovaraju:

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \qquad \vec{p} = \gamma m \vec{u}$$

→ Newtnova jednadžba gibanja mora odgovarati i za relativističku i ne relativističku fiziku: $F = \frac{d\vec{p}}{dt}$

- $u \to c$ $\quad \gamma \to \infty$ $\quad \vec{p}$ može biti proizvoljno velika bez $u \geq c$
- $u \to 0$ $\quad \gamma \to 1$ $\quad \vec{p}$ je jednaka nerelativističkoj (običnoj)

Relativistički izraz za silu u čestice mase m pod djelovanjem F

$$F = \frac{d\vec{p}}{dt} \Big/ \int \qquad p[t] = Ft \longrightarrow p[t] = \frac{m \cdot u[t]}{\sqrt{1 - \frac{u^2[t]}{c^2}}} \text{ jer } p[t] = \gamma m \vec{u}$$

$$\vec{u} = \frac{p[t]}{\gamma m} = \frac{Ft}{\gamma m} \rightarrow \vec{u}[t] = \frac{Ft}{\sqrt{1 - \frac{u^2}{c^2}} \cdot m}$$

**Kinetička energija** $\quad K = (\gamma - 1) mc^2$

- $u \to c$ $\quad \gamma \to \infty$ $\quad$ → K može biti >> bez da $u \geq c$
- $u/c \ll 1$ $\quad u/c \to 0$ $\quad K = \frac{1}{2} m u^2$ $\quad$ odgovara nerelativističkoj (običnoj) jednadžbi

# Rad i energija u specijalnoj teoriji relativnosti

- $\vec{p} = \gamma m \vec{u}$
- $\vec{F} = \frac{d\vec{p}}{dt}$

izvršeni rad pod djelovanjem sile: $W = \int_1^2 \vec{F} \cdot d\vec{s}$

$\rightarrow$ u 1D: $\int_1^2 F dx = \int_1^2 \frac{dp}{dt} dx = \int_1^2 v\, dp$

* podintegralni izraz možemo zapisati: $d(vp) = p\,dv + v\,dp$

$\rightarrow v\,dp = d(vp) - p\,dv$

& $1 \rightarrow v_1 = 0$

$2 \rightarrow v_2 = v$

$$W = \int_0^v v\,dp = \int_0^v d(vp) - \int_0^v p\,dv$$

$$W = vp\Big|_0^v - \int_0^v \gamma m v\,dv$$

$$W = vp\Big|_0^v - m\int_0^v \gamma[v]\, v\,dv = v \cdot \gamma m v - m \int_0^v \frac{v\,dv}{\sqrt{1-\frac{v^2}{c^2}}} = \ldots$$

$\Rightarrow \gamma mc^2 - mc^2 = E_k \longrightarrow \boxed{mc^2(\gamma - 1) = E_k} = W$

## Teorem o radu i kinetičkoj energiji

$E = \gamma mc^2$ – ukupna en.

$E_0 = mc^2$ – en. mirovanja

$E = E_0 + E_k$

$\gamma mc^2 = mc^2 + E_k$

## Relativistička energija čestice

$E = \underline{mc^2} + K = \gamma mc^2$

energija mirovanja

vrijedi: $E^2 = (mc^2)^2 + (pc)^2$

Relativistički savršeno neelastičan sudar:

O.K.G: $\gamma m u = \Gamma M u$ $\quad \gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ (samo veličina bitna)

O. Rel. E: $\gamma^2 mc^2 + mc^2 = \Gamma M c^2$

$$\Rightarrow \quad u = \frac{\gamma u}{\gamma + 1}$$

- $u \ll c$, $\gamma \simeq 1$ $\quad u_{sm} = \frac{u}{2}$ $\quad M = 2m\sqrt{(1+\gamma)/2}$
- $u \to c$, $\gamma \to \infty$ $\quad u \to u$

Bezmasene čestice $E = pc$ za $m=0$; npr. fotoni

Primjer: UDŽ.

Elektron se giba brzinom $v = 0{,}85c$. Kolika je kinetička energija $e^-$ i njegova ukupna energija? Izrazite rezultat u eV.

Ukupna energija: $E = \gamma mc^2$

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{9{,}10938 \times 10^{-31} \cdot 3\times 10^{8}}{\sqrt{1-\frac{0{,}85^2 \cdot \cancel{c^2}}{\cancel{c^2}}}} \cdot \frac{1}{e} = \frac{511\,keV}{0.527}$$

$$\boxed{E = 969{,}64\,keV}$$

$$E = E_0 + E_k$$

$$E = mc^2 + E_k \longrightarrow E_k = mc^2(\gamma - 1)$$

$$\boxed{E_k = 458{,}64\,keV}$$

Zadatak: Ukupna energija protona trostruko je veća od njegove energije mirovanja. Odredite (a) brzinu protona, (b) kinetičku en. (c) količinu gib.

a)

$$\gamma \cancel{mc^2} = 3 \cdot \cancel{mc_0^2}$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 3 \quad /^2$$

$$9\left(1-\left(\frac{v}{c}\right)^2\right) = 1$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{v^2}{c^2} = \frac{8}{9}$$

$$v = \sqrt{\frac{8}{9}} \cdot c$$

$$\boxed{v = \frac{\sqrt{8}}{3}c}$$

b) $E_k = E - E_0$

$$E_k = 3E_0 - E_0 = 2E_0$$

$$E_k = 2mc^2 \quad (:e \text{ jer eV})$$

$$\boxed{E_k = 1{,}8765\,GeV}$$

c) $(pc)^2 = E^2 - (mc^2)^2$

$$p = \sqrt{\frac{E^2 - (mc^2)^2}{c^2}}$$

$$p = \gamma m v \longrightarrow p = \frac{m \cdot \frac{\sqrt{8}}{3}c}{\sqrt{1-\frac{\left(\frac{\sqrt{8}}{3}\right)^2 \cancel{c^2}}{\cancel{c^2}}}}$$

$$p = \frac{m_p \frac{\sqrt{8}}{3}c}{\sqrt{1-\left(\frac{\sqrt{8}}{3}\right)^2}} \cdot \frac{c}{e}$$

$$\boxed{p = 2{,}6538\,GeV/c}$$

zbog mj. jed.