5.1.4. ZAMJENA VARIJABLI

Matan 1: $\int_a^b f(x)dx = \left| \begin{array}{l} x = \varphi(t) \\ dx = \varphi'(t)dt \end{array} \right| = \int_\alpha^\beta f(\varphi(t))\varphi'(t)\,dt$

Što je s 2D int?

$$\iint_D f(x,y)dxdy = \left| \begin{array}{l} x = x(u,v) \\ y = y(u,v) \end{array} \right| = \iint_D g(u,v) \cdot |\det J| \, du\,dv$$

↓ koristimo J

Prisjetimo se Jacobijeve matrice: $J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$

↳ $\det J$ = Jakobijan

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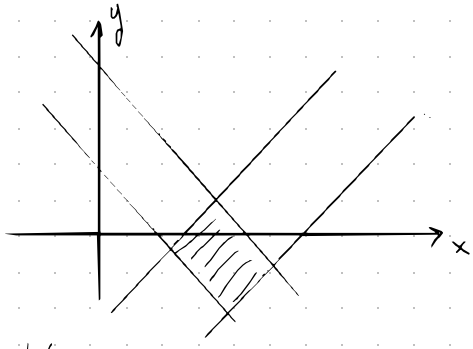
$$\iint_P \frac{\ln(3x+y)}{9x^2 - y^2}\,dxdy$$

$3x+y=3$
$3x+y=9$
$3x-y=3$
$3x-y=9$



$$\iint_P \frac{\ln(3x+y)}{(3x+y)(3x-y)}\,dxdy$$

u v

$\left| \begin{array}{l} u = 3x+y \\ v = 3x-y \end{array} \right| / +$ → $u+v = 6x$

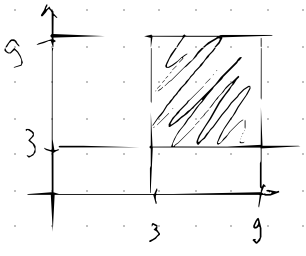
trebamo Jakobijan → $x = \frac{1}{6}u + \frac{1}{6}v$

$y = \frac{1}{2}u - \frac{1}{2}v$

$$\det J = \begin{vmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{12} - \frac{1}{12} = \left| \frac{-1}{6} \right| = \frac{1}{6}$$

→ $\iint_P \frac{\ln(u)}{u \cdot v}\,du\,dv$

$$\frac{1}{6}\int_3^9 du \int_3^9 \frac{\ln(u)}{v}\,dv \quad \ln v \Big|_3^9$$



$$= \frac{1}{6}\int_3^9 \frac{\ln(u)}{u}\left(\ln(v)\Big|_3^9\right)du = \left| \begin{array}{l} \ln(u) = t \\ \frac{1}{u} = dt \end{array} \right|$$