MATEMATIČKI FORMALIZAM

(izdvojeno za EM)

Skalarna polja

· točkama iz prostora pridružuje vrijednost neke skalarne veličine

↳ $\phi[\vec{r},t] \rightarrow$ temperatura , $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \longrightarrow \phi(x,y,z,t)$

Gradijent skal. polja $\longrightarrow \vec{\nabla}\phi$ – smjer je smjer najvećeg pada

$\underline{d\phi = \vec{\nabla}\phi \cdot d\vec{r}}$ – promjena $d\phi$ iznosa polja ϕ pri pomaku $d\vec{r}$ mora biti dana skalarnim produktom $\vec{\nabla}\phi$ i $d\vec{r}$

$$\vec{\nabla}\phi = \frac{\partial\phi}{\partial x}\hat{x} + \frac{\partial\phi}{\partial y}\hat{y} + \frac{\partial\phi}{\partial z}\hat{z}$$

* ⇒ Ako konz polje F opisuje silu F kojoj odgovara polje potenc en $\underline{U}$ onda prema def potenc en ⇒ element prostornog pomaka ⇝ element potencijalne energije

$$\boxed{dU = -W = -\vec{F} \cdot d\vec{r}} \qquad dU \rightarrow \vec{\nabla} U \cdot d\vec{r}$$

$$-\vec{F}\,\cancel{d\vec{r}} = \vec{\nabla} U \cdot \cancel{d\vec{r}} \implies \underline{\vec{\nabla} U = -\vec{F}}$$

polje sile možemo dobiti računanjem gradijenta polja potenc en.

⇝ el. polje $\underline{E = -\nabla\varphi}$ ← u istom odnosu i el. polje

↳ el. potenc

Teorem o gradijentu

Integral skal. polja ϕ po bilokojoj krivulji od P do Q jednak je razlici vrijednosti :

$$\underline{\int_{\vec{r}_P}^{\vec{r}_Q} \vec{\nabla}\phi[\vec{r}] \cdot d\vec{r} = \phi[\vec{r}_Q] - \phi[\vec{r}_P]}$$

⇒ Gradijent svakog skal. polja je konzervativno vekt. polje

Vektorsko polje

• točkama -||- neke vektorske veličine

↳ vekt polje $\vec{A}$ točki $\vec{r}$ u trenutku t pridružuje vekt $\vec{A}[\vec{r},t]$

$$\rightarrow A[x,y,z,t] = A_x[x,y,z,t]\,\hat{x} + A_y[x,y,z,t]\,\hat{y} + A_z[x,y,z,t]\,\hat{z}$$

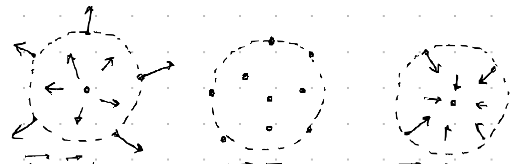
→ komponente vekt polja

Divergencija vekt. polja $\vec{\nabla}\cdot\vec{A}$

- vrijednost: tok polja kroz zatvorenu plohu koja omeđuje tu točku

↳ $d\Phi_A = \vec{\nabla}\cdot\vec{A}\cdot dV$ → tok polja $\vec{A}$ kroz zatvorenu plohu koja omeđuje dV

$$\Rightarrow \vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$



$\vec{\nabla}\cdot\vec{A} > 0$ $\quad$ $\vec{\nabla}\cdot\vec{A} = 0$ $\quad$ $\vec{\nabla}\cdot\vec{A} < 0$

Gaussov teorem o divergenciji vekt. p.

$$\int_V \vec{\nabla}\cdot\vec{A}[\vec{r}]\,dV = \oint_{\partial V} \vec{A}[\vec{r}]\cdot d\vec{S}$$

tok polja A kroz zatvorenu plohu ∂V koji omeđuje volumen V

Rotacija vekt polja $\vec{\nabla}\times\vec{A}$

↳ skalarna veličina $(\vec{\nabla}\times\vec{A})\,d\vec{S}$ odgovara integralu polja A po krivulji koja omeđuje element plohe $d\vec{S}$

• smjer: onaj u kojem treba usmjeriti el. plohe $d\vec{S}$ da bi $\int \vec{A}\,d\vec{s}$ bio MAX

$$\vec{\nabla}\times\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$$

Stokesov teorem

$$\int_S (\vec{\nabla}\times\vec{A})\,d\vec{S} = \oint_{\partial S} \vec{A}\cdot d\vec{r}$$

- int. rotacije vekt. polja A po otvorenoj plohi S

↳ integral polja A po zatvorenoj krivulji $\gamma = \partial S$ koja omeđuje plohu S

Očuvanje el. naboja i jednadžba kontinuiteta

$$\rho = \frac{dq}{dV} \longrightarrow q[t] = \int_V \rho[\vec{r}, t]\, dV$$ ↳ el. naboj u volumenu V

$$\vec{J} = \frac{I}{d\vec{S}} \longrightarrow I = \oint_{\partial V} \vec{J}[\vec{r}, t] \cdot d\vec{S} \overset{\text{Gauss}}{\rightsquigarrow} I = \int_V \vec{\nabla} \cdot \vec{J}[\vec{r}, t]\, dV$$ ↳ struja I el. naboja

<u>zakon očuvanja naboja</u>: naboj q sadržan u V se u vremenu dt može promijeniti samo za onoliko koliko je naboja proteklo u ∂V

$$\frac{dq}{dt} = -I$$ → pozitivna struja umanjuje naboj u volumenu

deriviramo prvi integral

$$\int_V \frac{dq}{\partial t}\, dV = -\int_V \vec{\nabla}\vec{J}\, dV \implies \int_V \frac{\partial \rho[\vec{r}, t]}{\partial t}\, dV = -\int_V \vec{\nabla}\vec{J}\, dV$$

$$\hookrightarrow \boxed{\frac{\partial \rho[\vec{r}, t]}{\partial t} = -\vec{\nabla} \cdot \vec{J}}$$

$\lambda = \frac{dq}{dl}$ — element krivulje

$\sigma = \frac{dq}{dS}$ — element plohe

$\rho = \frac{dq}{dV}$ — element volumena

Električna struja [A] ili [C/s]

- tok naboja kroz neku plohu ili kroz neki vodič

$$I = \frac{\Delta q}{\Delta t} \longrightarrow \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$$

↳ gustoća el. struje J $[A/m^2]$ $\longrightarrow dI = \vec{J} \cdot (d\vec{S} \cdot \hat{n})$ normala plohe

- smjer $\hat{J} \longrightarrow$ u smjeru $\hat{I}$ ako je $+q$
  - $\rightarrow$ u smjeru $-\hat{I}$ ako je $-q$

Magnetizacija materijala

M [A/m] → volumna gustoća mag. dipol. moment

$$\vec{m} = \int d\vec{m} = \int \vec{M}\, dV$$

ELEKTROMAGNETIZAM

Električno i magnetsko polje

polje u fizici: opisujemo fiz veličine koje su u svakom trenu prisutne u svim točkama fiz prostora

električno polje E [V/m] ili [N/C]   magnetsko polje B [T]

Lorenzova ili elektromagnetska sila $\vec{F}_L = \underbrace{q\vec{E}}_{\text{električni dio}} + \underbrace{q\vec{v}\times\vec{B}}_{\text{magnetski dio}}$

$$\vec{F}_L = \underbrace{q\vec{E}}_{\text{ima smjer el. polja (+), suprotan (-)}} + \underbrace{q\vec{v}\times\vec{B}}$$

- ako čestica miruje ili se giba u smjeru $\hat{B}$
  - inače: $\vec{F}_L \perp \vec{v}$ & $\vec{F}_L \perp (\vec{v}\times\vec{B})$
- okomitost sile $\Rightarrow$ rad $= 0$ $\rightarrow$ nema ΔK!

Rad F_m:

$$W_m = \int \vec{F}_m \cdot d\vec{r} = \left| \begin{array}{l} F_m = q(\vec{v}\times\vec{B}) \\ \vec{v} = \frac{d\vec{r}}{dt} \rightarrow dt\cdot\vec{v} \end{array} \right| = \int \overbrace{q(\vec{v}\times\vec{B})\cdot\vec{v}}^{F_m \perp v \rightarrow 0} \cdot dt = \int 0 \cdot dt = \underline{\underline{0}}$$

$\hookrightarrow$ centripetalna sila: isto ne vrši rad, okomita je, ne mijenja iznos, samo smjer

$$F_m = F_{cp} \longrightarrow R = \frac{mv}{qB}$$

$$qvB = \frac{mv^{\cancel{2}}}{R} \qquad \omega = \frac{v}{R} = 2\pi\cdot\frac{1}{T} \rightarrow v\cdot\frac{qB}{mv} = 2\pi\frac{1}{T} \longrightarrow T = 2\pi\cdot\frac{m}{qB}$$

Jednadžba gibanja nabijene čestice

$$\vec{F}_L = \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$v \approx c$ $\quad p = \gamma m v$

$v \ll c$ $\quad p = mv$

$$\rightsquigarrow m\vec{a} = q(\vec{E} + \vec{v} \times \vec{B})$$

Gibanje u hom. električnom polju

pretpostavimo da je $B = 0$, $v \ll c$

$r_0, v_0, t_0 \rightarrow$ početni trenutak & položaj

$$F_L = qE + 0 \longrightarrow m\cdot\vec{a} = \vec{F}_L$$

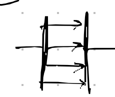
$$a = \frac{q\cdot E}{m}$$

$$\Downarrow$$

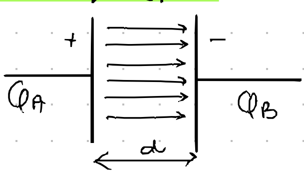
$$r[t] = r_0 + v_0[t - t_0] + \frac{a}{2}[t - t_0]^2$$

$$r[t] = r_0 + v_0[t - t_0] + \frac{qE}{2m}[t - t_0]^2$$

* u prirodi najsličnije hom polju je ono među pločama



Kondenzator:



$\longrightarrow U = \varphi_A - \varphi_B$

d = udaljenost ploča

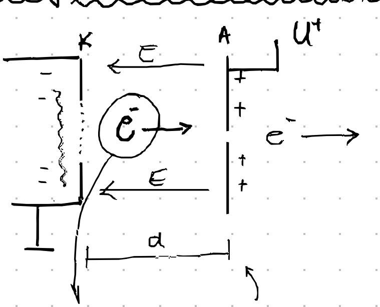
$$E = \frac{U}{d}$$

* kada počinjemo puniti kond. $U_{početni} = 0$ (nema naboja)

$\Rightarrow \uparrow q$ (dodajemo) $\sim U\uparrow$ + moramo uložit rad da bi se dodao naboj $\Rightarrow dW = U \cdot dq$, a $U = \frac{Q}{C}$ (osnove elektro)

$$\rightarrow dW = \frac{Q}{C}dq \;/\int \rightarrow \quad W = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C} \Rightarrow \boxed{W = \frac{1}{2}U^2\cdot C}$$

Primjer: Elektronski top



el. polje gaubrzava prema anodi

– stvara snop e^- željene E_k

m_e & e^- $\quad F_L = qE$

el. sila obavlja rad $\quad W = F\cdot d$

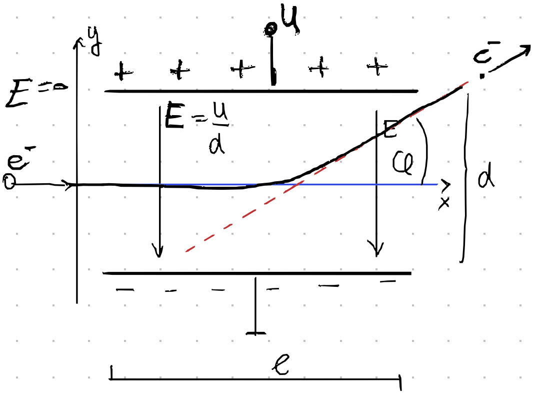
$$W = eEd$$

$$W = eU$$

$$\Delta K = \Delta W$$

$$\frac{m_e v^2}{2} = eU \longrightarrow \boxed{v = \sqrt{\frac{2eU}{m_e}}}$$

Otklonski sustav s el. poljem



Ulaz elektrona u $t=0$, $x=y=0$

– gledamo kao horizontalni hitac

$E=0$

$\Rightarrow x_0 = v_0 \cdot t$ $\quad y_0 = at^2$

$y_0 = \frac{qE}{2m} t^2$

$^{*}a = \frac{qE}{m} \longrightarrow$ $\quad y_0 = \frac{qU}{2m_e d} t^2$

$\ell = tg\,\alpha$

$t = \frac{x_0}{v_0} \longrightarrow y_0 = \frac{qU}{2m_e d} \cdot \frac{x_0^2}{v_0^2}$

nagib pravca: $tg\,\varphi = \frac{d}{dx} y[x]\Big|_{x=\ell} = \frac{eU}{\cancel{2}m_e d}\frac{\cancel{2}\ell}{v_0^2} \Rightarrow tg\,\varphi = \frac{eU\ell}{m_e d v_0^2}$

Za male kuteve $tg\,\varphi \simeq \varphi \rightarrow \varphi \approx \frac{eU\ell}{m_e d v_0}$

Sila koja djeluje na struju u mag. polju

$\odot\vec{B} \longrightarrow$ na element žice djeluje element sile

$dF = dq \cdot \vec{v} \times \vec{B} = dq \cdot \frac{d\vec{\ell}}{dt} \times \vec{B}$

$d\vec{F} = I\,d\vec{\ell} \times \vec{B} \rightarrow$ sila koja djeluje na element žice

$I = \frac{dq}{dt}$ $\quad d\ell = \frac{dq}{\lambda}$

Elektromagnetsko polje

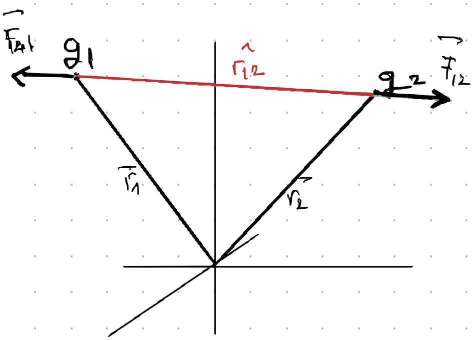
→ naboji miruju ili se gibaju na takav način da su električne struje stalne u vremenu

· E i B ne ovise o t i moguće ih je promatrati zasebno

! ako ovi uvjeti nisu ispunjeni → E i B ovise o t ~ Δq ili ΔI

Elektrostatika - EM - opis el. polja i raspodjele naboja u prostoru u situacijama u kojima $E = \text{const}, v_q = 0$.

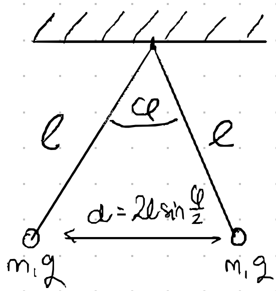
Columbov zakon



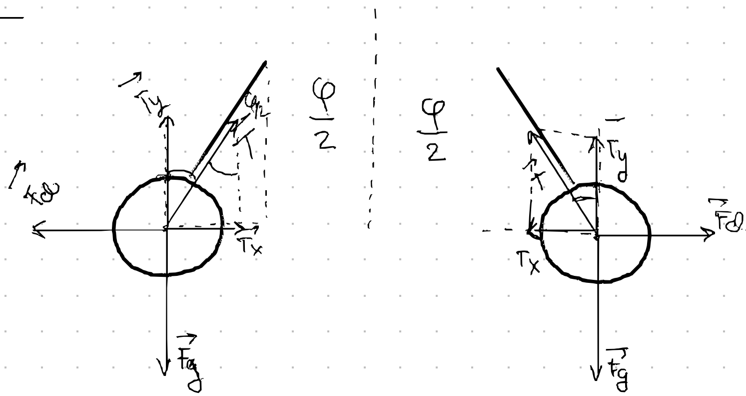
$$\longrightarrow \vec F_{21} = -\vec F_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|r_1-r_2|^3}\,\widehat{|r_1-r_2|}$$

$$\downarrow \; |r_{12}|$$

Primjer: Elektroskop



zbog istog naboja se odbijaju



$$F_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(2l\sin\left(\frac{\varphi}{2}\right)\right)^2}$$

$q = ? \rightarrow d$

$$F_{el} = T\cdot\sin\left(\frac{\varphi}{2}\right) \qquad F_g = T\cdot\cos\left(\frac{\varphi}{2}\right)$$

$$\hookrightarrow T = \frac{mg}{\cos\left(\frac{\varphi}{2}\right)}$$

$$T = \frac{F_{el}}{\sin\left(\frac{\varphi}{2}\right)}$$

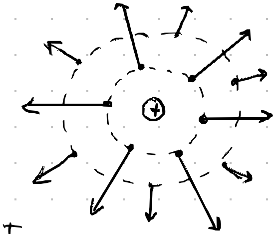
$$\Rightarrow \frac{F_{el}}{mg} = tg\left(\frac{\varphi}{2}\right)$$

$$\hookrightarrow tg\left(\frac{\varphi}{2}\right) = \frac{1}{mg}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{\left(2l\sin\left(\frac{\varphi}{2}\right)\right)^2} \longrightarrow q^2 = mg\,16\pi\varepsilon_0 l^2\sin^2\left(\frac{\varphi}{2}\right)\cdot tg\left(\frac{\varphi}{2}\right)$$

*za male kuteve $\sin\varphi \approx tg\varphi \approx \varphi$ $\longrightarrow q^2 = mg\,16\pi\varepsilon_0 l^2\cdot\frac{\varphi^3}{8}$

$$\boxed{q^2 = mg\,2\pi l^2\varepsilon_0\cdot\varphi^3}$$

Električno polje: *Coulombovo polje



$$\rightarrow \vec F_{12} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r_{12}^3}\cdot \vec r_{12}$$

↳ u obliku Lorenzove sile: $F_{el} = q\cdot E \longrightarrow E_1 = \frac{F_{12}}{q_2}$

$$\Rightarrow E_1[r] = \frac{q_1}{4\pi\varepsilon_0}\cdot\frac{\vec r-\vec r_1}{|\vec r-\vec r_1|^3}$$

zanima nas polje u točki r

čestica je u q_1

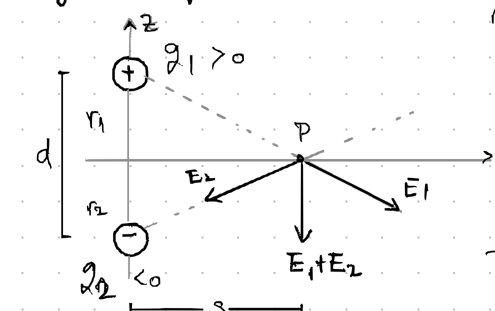
⇒ polje je moguće izračunati u svakoj točki osim u samoj točki izvora → r_1. $\longrightarrow r\neq r_1$

el. polje statične raspodjele naboja:

- ako je raspodjela zapisana volumno, površinski ili preko linijske gustoće onda q zamijenimo s dq $\longrightarrow E[r] = \frac{1}{4\pi\varepsilon_0}\int\frac{\hat r}{r^2}\,dq$

Polje el. dipola

- dipol: konfig. od naboja q i $-q$ na međusobnom razmaku d



$\rightarrow r_1 = +\frac{d}{2}\hat z$

$\longrightarrow r = s\hat x$

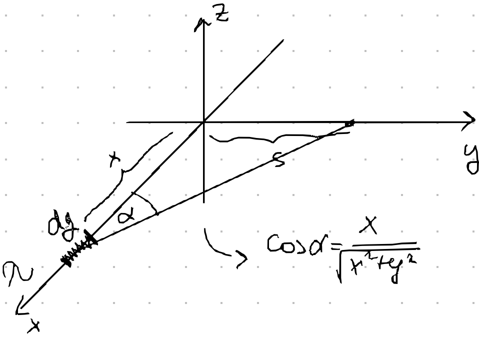
$\rightarrow r_2 = -\frac{d}{2}\hat z$

$E[r] = E_1 + E_2$

$$E[r] = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{\vec r-\vec r_1}{|r-r_1|^3}\underbrace{q_1}_{q} + \frac{\vec r-\vec r_2}{|r-r_2|^3}\cdot \underbrace{q_2}_{-q}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{s\hat x-\left(\frac{d}{2}\right)\hat z}{\left|s\hat x-\left(\frac{d}{2}\right)\hat z\right|^3}\,q_1 \;\; \frac{s\hat x+\left(\frac{d}{2}\right)\hat z}{\left|s\hat x+\left(\frac{d}{2}\right)\hat z\right|^{3}}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{\cancel{s}-\cancel{s}+d}{\left(s^2+\left(\frac{d}{2}\right)^2\right)^3}\right)\hat z = \frac{-q\,d}{4\pi\varepsilon_0\left(s^2+\left(\frac{d}{2}\right)^2\right)^3}$$

El. polje jednoliko nabijenog pravca



$$dE = \frac{1}{4\pi\varepsilon_0}\,\frac{dq\cdot\hat{r}}{r^2}$$

hipotenuza

jednoliko linijski nabijen štap

$\hookrightarrow \frac{dq}{dl} = \lambda \rightarrow dq = \lambda\cdot dl$

$\hookrightarrow \cos\alpha = \frac{x}{\sqrt{x^2+y^2}}$

$$r = \sqrt{x^2+y^2}$$

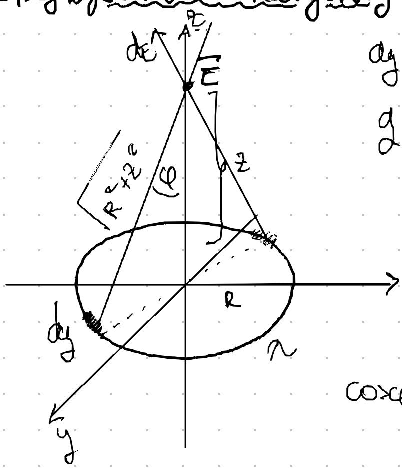
dy jer je po y-osi polje

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\cdot dl}{x^2+y^2}\cdot\cos\alpha = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dl}{x^2+y^2}\cdot\frac{x}{\sqrt{x^2+y^2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda}{(x^2+y^2)^{3/2}}\cdot x\,dy \;/\int$$

$$E = \frac{\lambda x}{4\pi\varepsilon_0}\underbrace{\int_{-\infty}^{\infty}\frac{dy}{(x^2+y^2)^{3/2}}}_{\text{tablični}} = \frac{\lambda \cancel{x}}{\underset{2}{\cancel{4}}\pi\varepsilon_0}\cdot\frac{\cancel{2}}{x^{\cancel{2}}} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0\cdot x}$$

El. polje jednoliko nabijenog prstena



$$dq = \lambda\cdot dl$$

$$q = \lambda\cdot 2r\pi$$

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{R^2+z^2}$$

ali budući da nas zanima po z osi $\rightarrow dE_z = dE\cdot\cos\varphi$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{R^2+z^2}\cdot\cos\varphi$$

$$\cos\varphi = \frac{z}{\sqrt{R^2+z^2}} \rightarrow \boxed{dE_z = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq\cdot z}{(R^2+z^2)^{3/2}}} \;/\int$$

$$\Rightarrow E_z = \frac{1}{4\pi\varepsilon_0}\,\frac{q\,z}{(R^2+z^2)^{3/2}}$$

El polje nabijene čestice

$$\oint E\,d\vec{S} = \frac{Q_{obuhvaćeni}}{\varepsilon}$$

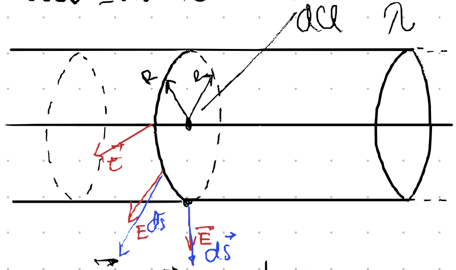
$\llcorner$ u formulama: $dS = r^2\sin\vartheta\,d\varphi$

$$E\cdot r^2\int_0^{\pi}\sin\vartheta\int_0^{2\pi}d\varphi = \frac{q}{\varepsilon}$$

$$E\cdot r^2(-\cos\vartheta)\Big|_0^{\pi}\cdot 2\pi = \frac{q}{\varepsilon} \rightarrow 2r^2\cdot 2\pi\cdot E = \frac{q}{\varepsilon}$$

$$E = \frac{q}{\varepsilon}\cdot\frac{1}{4r^2\pi}$$

El. polje jednoliko nabijene žice



linijski nabijena $\rightarrow \lambda = \frac{dq}{d\ell} \rightarrow \frac{q}{\ell}$

Gaussov zakon: $\oint \vec{E}\,d\vec{s} = \frac{Q_{obuhv}}{\varepsilon_0}$

ds: cilindrične: $r\,d\varphi\,dz$

$\Rightarrow \vec{E}\cdot d\vec{s}$ je skalarni $\Rightarrow E\cdot ds\cdot \cos\theta = E\cdot ds\cdot \cancel{\cos\theta}^{\,1}$

$$\Rightarrow E\cdot r\int_0^{2\pi} d\varphi \int_{-z}^{z} dz = \frac{q_{obuhv}}{\varepsilon_0}$$

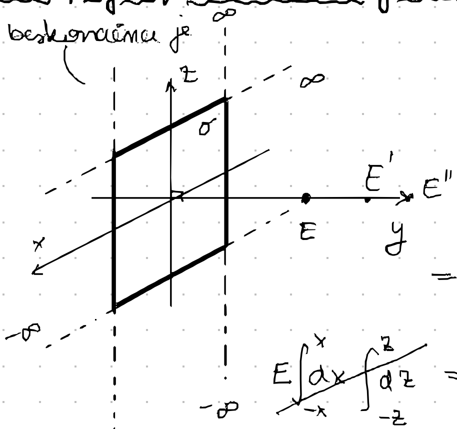
q obuhvaćeni $= \int \lambda\,d\ell = \lambda\cdot \ell$

$\ell = [-z, z] \rightarrow 2z$

$$E\cdot r\cdot 2\pi\cdot \cancel{2z} = \frac{\cancel{2z}\cdot\lambda}{\varepsilon_0}$$

$$E = \frac{\lambda}{2r\pi\cdot\varepsilon_0}$$

El. polje jednoliko nabijene plohe



- polje se ne mijenja po x i z, samo po y
- gustoća naboja je plošna $\rightarrow \sigma = \frac{dq}{ds}$
- obična pravokutna ploha: $ds = dx\,dz$

$\Rightarrow$ Gaussov zakon: $\oint \vec{E}\,d\vec{s} = \frac{q_{ob}}{\varepsilon_0}$

$$E\cancel{\int_{-x}^{x} dx \int_{-z}^{z} dz} = \frac{\sigma\int ds}{\varepsilon_0} = \frac{\sigma\cdot \cancel{\int dx \int dz}}{\varepsilon_0}$$

jer je simetrično po y i $-y$ $\quad 2E(y) = \frac{\sigma}{\varepsilon_0} \Rightarrow \boxed{E(y) = \frac{\sigma}{2\varepsilon_0}}$

Jednoliko nabijena ravnina #2

$E = \frac{\sigma}{2\varepsilon_0}$ $\Rightarrow$ E je okomit na ravninu i ne ovisi o udaljenosti od ravnine

poz nabijena $\sigma > 0$ od ravnine

neg nabijena $\sigma < 0$ prema ravnini

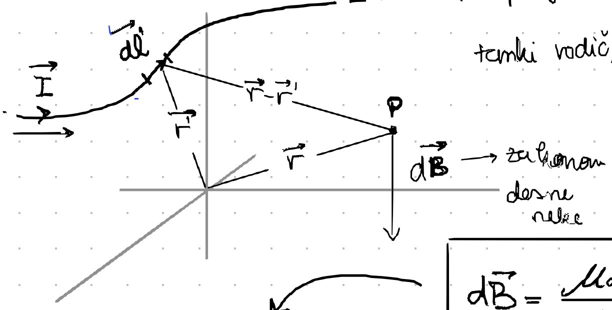
el. polje je s obje strane je jednoliko ali imaju međusobno suprotan smjer

Primjer: tanki nabijeni prsten

Biot-Savartov zakon & magnetostatika

magnetostatika – naboji se gibaju uz ograničenje da je gustoća el. struje svuda u prostoru stalna u vremenu

Biot-Savartovo pravilo: → računanje mag. polja u kojima je poznata raspodjela el. struje u prostoru



tanki vodič/žica

⇒ struja I koja teče elementom krivulje dl' stvara u točki P element mag. polja

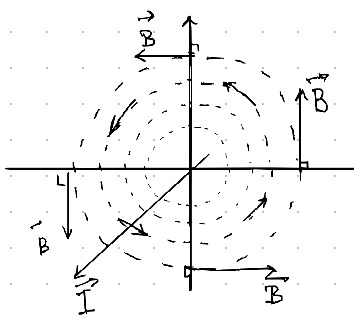
$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$

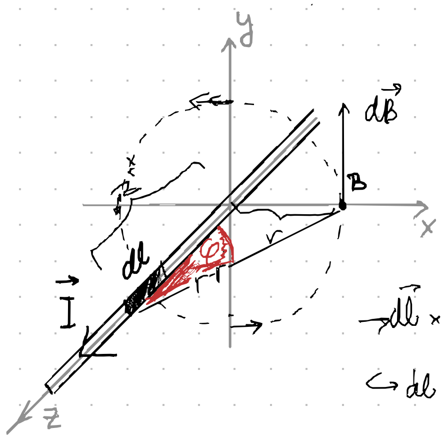
$$\vec{B}[r] = \frac{\mu_0}{4\pi} \int \frac{\vec{I}\, d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

* integracija se provodi duž čitave krivulje

Primjer: mag. polje beskonačno ravnog tankog vodiča

presjek žice:





$$dB = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times (r-r')}{|r-r'|^3}$$

$r' = z$

$r - r' = \sqrt{z^2+r^2}$

$\vec{dl} \times (r-r')$ = vekt umnožak

↳ $dl \cdot |r-r'| \cdot \sin\varphi$

$$\frac{r}{r^2+z^2} = \sin\varphi$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} I \cdot \frac{dl \cdot \sqrt{z^2+r^2} \cdot \sin\varphi}{(\sqrt{z^2+r^2})^{3/2}}$$

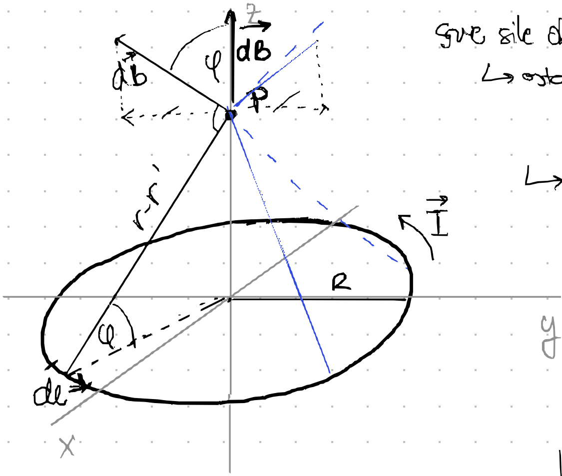
$dl = dz$ jer je na osi z

$$\Rightarrow dB = \frac{\mu_0}{4\pi} I \cdot dz \cdot \frac{\cancel{\sqrt{z^2+r^2}}}{(\sqrt{z^2+r^2})^{3/2}} \cdot \frac{r}{\cancel{\sqrt{z^2+r^2}}} = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot dz \cdot r}{(z^2+r^2)^{3/2}} = dB$$

· prema pravilu desne ruke $\hat{B}$ je $\hat{y}$

$$\hookrightarrow B_y = \frac{\mu_0}{4\pi} \cdot I r \int_{-\infty}^{\infty} \frac{dz}{(z^2+r^2)^{3/2}} = \frac{\mu_0}{\cancel{4}_{2}\pi} I \cdot \cancel{r} \frac{\cancel{2}}{r^{\cancel{2}}} \Rightarrow \boxed{B_y = \frac{\mu_0 \cdot I}{2\pi r}}$$

Magnetsko polje na osi kružne petlje



Sve sile okomite na z-os se pokrate
↳ ostaje samo dB u smjeru osi z ↑

↳ $\hat{B}=\hat{z}$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\cdot\left|\frac{I\cdot d\vec{l}\times(\vec{r}-\vec{r}')}{(r-r')^3}\right|$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\cdot I\cdot\frac{|d\vec{l}|\cdot\cancel{|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|^{\cancel{3}2}}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\cdot I\cdot\frac{dl}{|\vec{r}-\vec{r}'|^2}$$

$dB_z = |d\vec{B}|\cos\varphi$ $\qquad r-r'=\sqrt{R^2+z^2}$

↳ $\cos\varphi = \frac{R}{R^2+z^2}$

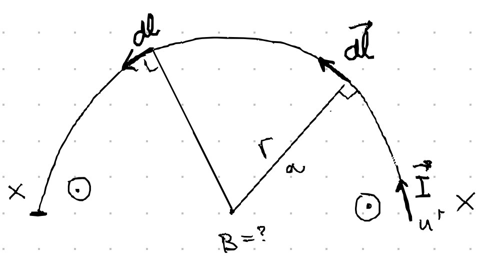
$$\Rightarrow dB_z = \frac{\mu_0}{4\pi}\cdot I\cdot\frac{dl}{\sqrt{R^2+z^2}}\cdot\frac{R}{\sqrt{R^2+z^2}}$$

$$\Rightarrow dB_z = \frac{\mu_0}{4\pi}\cdot\frac{I\cdot dl\cdot R}{(\sqrt{R^2+z^2})^{3/2}} \quad /\int$$

$$\Rightarrow B_z = \frac{\mu_0}{4\pi}\cdot\frac{IR}{(\sqrt{R^2+z^2})^{3/2}}\int_0^{2R\pi} dl$$

$$B_z = \frac{\mu_0}{\cancel{4\pi}}\cdot\frac{I\cdot R}{(\sqrt{R^2+z^2})^{3/2}}\cdot\cancel{2R\pi} \Rightarrow \boxed{B_z = \frac{1}{2}\,\frac{\mu_0\cdot I\cdot R^2}{(\sqrt{R^2+z^2})^{3/2}}}$$

Primjer Sossich



$$dB = \frac{\mu_0}{4\pi}\cdot I\cdot\frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\cdot I\cdot\frac{dl\cdot\cancel{1}\cdot\cancel{\sin 90^\circ}}{r^2}$$

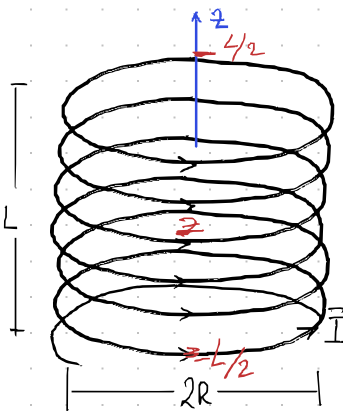
$$dB = \frac{\mu_0}{4\pi}\cdot I\cdot\frac{dl}{r^2} \quad /\int$$

$$B = \frac{\mu_0}{4\pi}\,\frac{I}{r^2}\int_0^{r\pi} dl = \frac{\mu_0}{4\cancel{\pi}}\,\frac{I}{r^{\cancel{2}}}\cdot\cancel{r\pi}$$

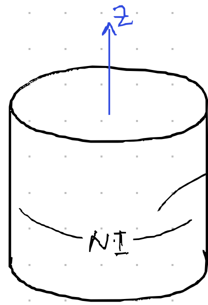
$l = \frac{\text{opseg}}{2} = r\pi$

$$\boxed{B = \frac{\mu_0 I}{4r}}$$

Magnetsko polje na osi zavojnice



N →



ravnomjerno raspoređena struja po plaštu

struju koja teče po namotajima zavojnice možemo shvatiti kao struju NI

$dz' \to$ visina „valjka"

$$z' \in \left[-\tfrac{L}{2}, \tfrac{L}{2}\right]$$

svakom takvom stazom širine dz' teče struja

$$dI' = N\cdot I\cdot \frac{dz'}{L}$$

→ jedna staza = kružna petlja → $B = \frac{\mu_0}{2}\,\frac{IR^2}{\left(\sqrt{R^2+z^2}\right)^{3/2}}$

$$\Rightarrow \quad dB_z = \frac{\mu_0}{2}\cdot\frac{R^2\cdot dI'}{\left(\sqrt{R^2+(z-z')^2}\right)^{3/2}} \longrightarrow dB_z = \frac{\mu_0}{2}\,\frac{R^2\cdot N\cdot I\cdot dz'/L}{\left(\sqrt{R^2+(z-z')^2}\right)^{3/2}} \Big/ \int$$

da ne pišemo stalno $z-z' \to \xi$

$$B_z = \frac{\mu_0}{2}\cdot\frac{R^2\cdot N\cdot I}{L}\cdot\underbrace{\int_{z-L/2}^{z+L/2}\frac{d\xi}{\left(\sqrt{R^2+\xi^2}\right)^{3/2}}}_{\text{tablični int}} \Rightarrow \frac{\mu_0}{2}\,\frac{\cancel{R^2}NI}{L}\cdot\frac{\xi}{\cancel{R^2}\sqrt{R^2+\xi^2}}\Bigg|_{z-\frac{L}{2}}^{z+\frac{L}{2}}$$

$$\Rightarrow \boxed{B_z = \frac{\mu_0}{2}\cdot\frac{NI}{L}\left(\frac{z+L/2}{\left(\sqrt{R^2+(z+L/2)^2}\right)^{3/2}} - \frac{z-\frac{L}{2}}{\left(\sqrt{R^2+(z-\frac{L}{2})^2}\right)^{3/2}}\right)}$$

MAXWELLOVE JEDNADŽBE

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\varepsilon_0 \frac{\partial E}{\partial t}$$

I. Maxwellova jednadžba : Gausov zakon za el. polje

tok el. polja kroz bilokoju zatvorenu plohu = naboj unutar plohe

$$\Phi_E = \frac{q_{obuhvaćeni}}{\varepsilon_0}$$ Gauss → integralni oblik Maxwellove jed.

$$\rho = \frac{dq}{dV} \rightarrow q = \int \rho dV \qquad \Phi_E = \oint \vec{E}\, d\vec{s} = \frac{q_{obuhv.}}{\varepsilon_0}$$

$$\rightarrow \boxed{\oint \vec{E}\, d\vec{s} = \frac{1}{\varepsilon_0}\int \rho\, dV}$$ * teorem o divergenciji $\oint \vec{E} d\vec{s} = \int \vec{\nabla}\vec{E}\, dV$

$$\Rightarrow \int \vec{\nabla}\vec{E}\, dV = \frac{1}{\varepsilon_0}\rho \int dV \rightarrow \vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$$

II. Maxwellova jed. : Gaussov zakon za mag. polje

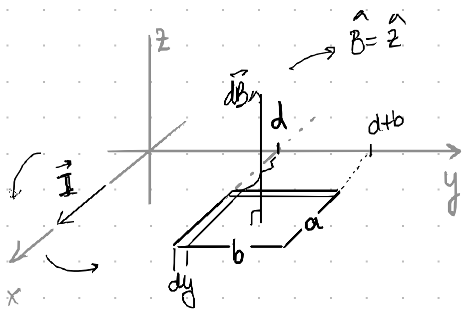
· nema izoliranih mag polova na plohi kojom $\vec{B}$ prolazi (mag silnice se zatvaraju same u sebe)

$$\Phi_B = \oint \vec{B} d\vec{s} \xrightarrow{Gauss} \frac{q_m}{\varepsilon_0} = 0$$ „nema naboja"

Integralni oblik: $\oint \vec{B} d\vec{s} = 0$

$$\vec{\nabla}\vec{B} = 0$$

Primjer: Tok mag polja kroz pravokutnik pored kojeg teče struja



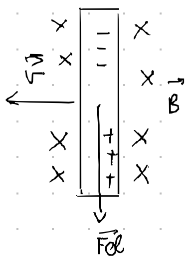
$$\vec{B}[y] = \frac{\mu_0 I}{2\pi y}\hat{z}$$

$$d\vec{S} = a\, dy\, \hat{z} \qquad \Phi_B = \int \vec{B}[y]\cdot d\vec{s}$$

$$\Phi_B = \int_d^{d+b} \frac{\mu_0 I}{2\pi y}\cdot a \cdot dy$$

$$\Rightarrow \Phi_B = a\frac{\mu_0 I}{2\pi}\int_d^{d+b}\frac{dy}{y} \longrightarrow \Phi_B = \frac{\mu_0}{2\pi} a I \ln\left|\frac{d+b}{d}\right|$$

III. Maxwell jednadžba: Faradayev zakon indukcije



$$\vec{F}_L = q\vec{E} + q(\vec{v}\times\vec{B})$$

$$\boxed{\mathcal{E} = \oint_\gamma \frac{\vec{F}_L}{q}\cdot d\vec{r}}$$

$\mathcal{E}$ - elektromotorna sila
↳ opisujemo jakost djelovanja meh koji pokreću slobodni naboj u gibanje u slob. žic

E ne ovisi o putu → = 0

$$\mathcal{E} = \oint_\gamma \left(\vec{E} + \vec{v}\times\vec{B}\right) d\vec{r} = \cancel{\oint_\gamma \vec{E}\, d\vec{r}} + \oint_\gamma \vec{v}\times\vec{B}\, d\vec{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\mathcal{E} = \frac{dt}{dt}\oint_\gamma \vec{v}\times\vec{B}\, d\vec{r} = \frac{d}{dt}\oint_\gamma \underset{a}{d\vec{r}}\left(\underset{b}{\vec{v}\cdot dt} \times \underset{c}{\vec{B}}\right)$$

površina: $\vec{v}dt\times d\vec{r}$

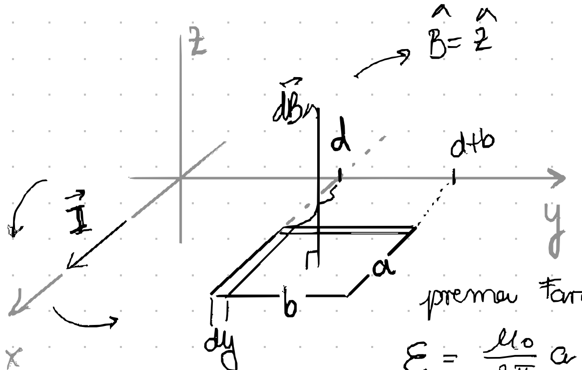
$$\mathcal{E} = \frac{d}{dt}\oint_\gamma \vec{B}\left(d\vec{r}\times\vec{v}\,dt\right) = -\frac{d}{dt}\oint_\gamma \vec{B}\underbrace{\left(\vec{v}dt\times d\vec{r}\right)}_{dS} = -\frac{d}{dt}\oint_\gamma \vec{B}\, d\vec{S}$$

integralni oblik

$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

Ako se mag. tok kroz neku površinu mijenja tokom vremena, bilo zbog promjene $\vec{B}$, površine S ili orijentacije površine → $\frac{\partial \Phi_B}{\partial t} \neq 0$!

Primjer: *prošli* EM indukcija u promjenjivom polju



$$I = I_0 \cos\omega t$$

$$\Phi_B = \frac{\mu_0 I}{2\pi} a \ln\left|\frac{d+b}{d}\right|$$

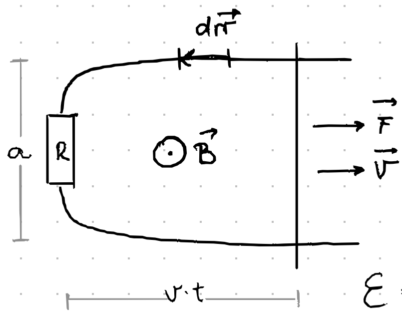
$$\Phi_B = \frac{\mu_0}{2\pi} a\cdot I_0 \cos(\omega t) \ln\left|\frac{d+b}{d}\right|$$

prema Faradayevom zakonu: $\mathcal{E} = -\frac{d}{dt}\Phi_B$

$$\mathcal{E} = \underbrace{\frac{\mu_0}{2\pi} a\cdot I_0\cdot \omega\cdot \ln\left|\frac{d+b}{d}\right|}_{\mathcal{E}_0} \sin(\omega t)$$

Primjer: Pretvorba meh rada u toplinu el. strujom

*! ovdje nešto fali!



· pomičemo štap djelujući silom $\vec{F}$, pomičemo stalnom $\vec{v}$
↳ EM indukcija
↳ el. struja
↳ zagrijavanje otpornika R

$\Phi_B = SB$

$S = a\cdot vt \Rightarrow \Phi_B = avt\, B$

$$\mathcal{E} = -\frac{d}{dt}\Phi_B = -avB$$

djeluje u suprotnom smjeru u odnosu na orijentaciju krivulje ($d\vec{r}$)

Ohmov zakon → $I = \frac{\mathcal{E}}{R} = \frac{-avB}{R}$

$$P = \mathcal{E}\cdot I = \frac{(avB)^2}{R}$$

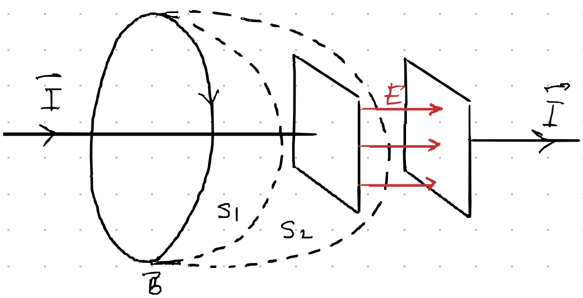
$$F = q(\vec{v}\times\vec{B}) = qvB = I\cdot \overset{a}{t\cdot v}\, B \Rightarrow F = IaB$$

IV. Maxwellova jednadžba : Ampère – Maxwellovo pravilo

* Ampère : došao je do prve polovice ali ona nije primjenjiva na sve slučajeve

$$\oint \vec{B} d\vec{r} = \mu_0 \int \vec{J} d\vec{s} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} d\vec{s} = \int_S (\vec{\nabla} \times \vec{B}) d\vec{s}$$

Primjer: Nepotpunost Ampèreovog zakona



S1 $\mu_0 \int J d\vec{s} \neq 0$ jer plohom S_1 prolazi struja I

$\mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} d\vec{s} = 0$ jer el. polje u području u kojem se nalazi S_1 smatramo zanemarivim

S1 $\Rightarrow \oint \vec{B} d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{s}$

S2 $\mu_0 \int \vec{J} d\vec{s} = 0$ jer el naboj ne prolazi

$\mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} d\vec{s} \neq 0$ jer je prisutno el. polje

↳ iznos tog polja kroz C se mijenja u vremenu

↳ $\Delta \Phi_E$

- površinska gustoća $\sigma = \frac{dq}{ds}$ $\quad I = \frac{q}{t} \Rightarrow \sigma = \frac{It}{S}$

$\int \vec{E} d\vec{s} = \frac{q}{\varepsilon_0} \longrightarrow EA = \frac{q}{\varepsilon_0} \longrightarrow E = \frac{It}{\varepsilon_0 S}$

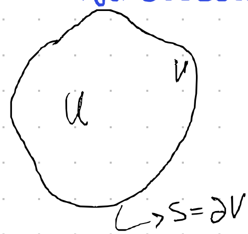
$\Phi_E = SE = \frac{It}{\varepsilon_0}$

$\mu_0 \varepsilon_0 \frac{d}{dt} \Phi_E = \mu_0 \varepsilon_0 \cdot \frac{I}{\varepsilon_0}$ S2 $\mu_0 \varepsilon_0 \frac{d}{dt} \Phi_E = \mu_0 I$ ↙ jednako je prvom članu

$\Rightarrow$ Ampère-Maxwellov zakon daje isti rez u oba slučaja

* Nepotpuni Ampèreov zakon dao bi dobar rez samo za S_1

Energija EM polja



Ukupna energija U sadržana u V

$\rightarrow$ $\underbrace{\text{meh. energija materije}}_{u_{meh}}$ + $\underbrace{\text{energija EM polja}}_{u_{em}}$ $\rightarrow$ volumne gustoće en.

$$\Rightarrow U = \int_V (u_{meh} + u_{em})\, dV$$

načelo očuvanja energije: očekujemo da se en. U sadržana unutar zatvorene plohe S može promijeniti ako dio energije EM polja napusti prostor ili uđe u prostor omeđen s S.

$\vec{S}$ – gustoća toka en. EM polja $\Rightarrow$ $\dfrac{d}{dt}U = -\oint_S \vec{S}\cdot d\vec{S}$

poz. tok en. kroz $S \sim \downarrow U$ sadržane unutar S

$$\Rightarrow \frac{d}{dt}\int_V (u_{meh} + u_{em})\, dV = -\oint_S \vec{S}\cdot d\vec{S}$$

$$\int_V \frac{d}{dt}(u_{meh} + u_{em})\, dV = -\int_V \vec{\nabla}\cdot\vec{S}\, dV$$

* jednadžba kontinuiteta

$$\Rightarrow \frac{d}{dt}(u_{meh} + u_{em}) = -\vec{\nabla}\cdot\vec{S}$$

Primjer: energija u kondenzatoru

$U = \frac{1}{2} C \cdot (\Delta\varphi)^2$ – napon

energija, kapacitet

$E = \dfrac{\Delta\phi}{d} \rightarrow u = \frac{1}{2} C \cdot E^2 \cdot d^2 \cdot dV$

$C = \varepsilon_0 \dfrac{S}{d} \rightarrow u = \frac{1}{2}\cdot\varepsilon_0\cdot\dfrac{S}{\cancel{d}}\cdot E^2 \cdot d^{\cancel{2}} \cdot dV \;/\int_V$

$$U = \int_V \frac{1}{2}\varepsilon_0 E^2\, dV = \frac{1}{2}\varepsilon_0 E^2 V = \frac{1}{2}\varepsilon_0 \cdot \frac{\Delta\varphi^2}{d^{\cancel{2}}}\cdot S \cdot \cancel{d} = \frac{1}{2} C (\Delta\varphi)^2$$

Razmatramo element snage dP kojom EM polje djeluje na naboje materije unutar volumena dV

$$dP = d\vec{F}_L \cdot \vec{v} = dq\,(\vec{E} + \underbrace{\vec{v}\times\vec{B})\cdot\vec{v}}_{0} = dq\,\vec{E}\cdot\vec{v} = \rho\cdot dV\cdot\vec{E}\cdot\vec{v}$$

$$\rho = \frac{dq}{dV} \qquad dV = dr\cdot dS \qquad v = \frac{dr}{dt} \qquad J = \frac{I}{dS} \qquad I = \frac{dq}{dt}$$

$$dP = \frac{dq}{dV}\cdot\frac{d\vec{r}}{dt}\,dV\cdot\vec{E} = I\cdot\underbrace{\frac{d\vec{r}}{dV}}_{\frac{1}{dS}}\cdot dV\cdot\vec{E} = \frac{I}{dS}\cdot dV\cdot\vec{E}$$

$$\Rightarrow \underline{dP = \vec{J}\,\vec{E}\,dV}$$

*obavljeni rad nad materijom mora biti jednak promjeni meh energije materije

$$\Rightarrow \frac{d}{dt}u_{meh} = \frac{dP}{dV} = \vec{E}\vec{J}$$

IV. maxwell: $\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\varepsilon_0\frac{\partial}{\partial t}E \;\Big/\cdot\frac{\vec{E}}{\mu_0}$

$$\frac{1}{\mu_0}\vec{E}(\vec{\nabla}\times\vec{B}) = \vec{E}\vec{J} + \varepsilon_0\vec{E}\frac{\partial}{\partial t}\vec{E} \Rightarrow \vec{E}\vec{J} = \frac{1}{\mu_0}\vec{E}(\vec{\nabla}\times\vec{B}) - \varepsilon_0\vec{E}\frac{\partial}{\partial t}\vec{E}$$

$$\vec{E}(\vec{\nabla}\times\vec{B}) = \vec{B}(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot\left(-\frac{\partial}{\partial t}\vec{B}\right) - \vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

$$\vec{E}\vec{J} = \frac{1}{\mu_0}\left(-\vec{B}\frac{\partial}{\partial t}\vec{B} - \vec{\nabla}(\vec{E}\times\vec{B})\right) - \varepsilon_0\vec{E}\frac{\partial}{\partial t}\vec{E}$$

$$= -\varepsilon_0\vec{E}\frac{\partial}{\partial t}\vec{E} - \frac{1}{\mu_0}\vec{B}\frac{\partial}{\partial t}\vec{B} - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B}) \qquad \vec{A}\frac{d}{dt}\vec{A} = \frac{1}{2}\frac{d}{dt}(\vec{A}\vec{A}) = \frac{1}{2}\frac{d}{dt}A^2$$

$$= -\varepsilon_0\cdot\underbrace{\frac{1}{2}\frac{\partial}{\partial t}}E^2 - \frac{1}{\mu_0}\underbrace{\frac{1}{2}\frac{\partial}{\partial t}}B^2 - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B})$$

$$= -\frac{1}{2}\frac{\partial}{\partial t}\left(+\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B})$$

$$\Rightarrow \frac{d}{dt}u_{meh} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B})$$

$$\boxed{u_{meh} = \frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right)}$$

Poyntingov teorem - meh rad izvršen od Lorenzove sile na naboj = − smanjenje en. u EM polju − en. koja je iscurila kroz S

$$\downarrow$$

$$\frac{\partial}{\partial t}u_{meh} = -\frac{\partial}{\partial t}u_{em} - \vec{\nabla}\cdot\vec{S}$$

$W = \int e_{meh}\,dV \rightarrow$ $\boxed{u_{em} = \frac{1}{2}\left(\varepsilon_0E^2 + \frac{1}{\mu_0}B^2\right)}$ $\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$

gustoća EM polja — Poyntingov vektor

EM valovi u vakumu

vakumu – nije prisutna materija

$\Downarrow$

ρ (volumna gustoća) $= 0 \rightarrow J = 0$

općenito Maxwellove	u vakumu
$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$	$\vec{\nabla}\cdot\vec{E} = 0$
$\vec{\nabla}\cdot\vec{B} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$
$\vec{\nabla}\times\vec{E} = \frac{-\partial}{\partial t}\vec{B}$	$\vec{\nabla}\times\vec{E} = \frac{-\partial}{\partial t}\vec{B}$
$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{E}$	$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{E}$

* za vakum znamo:

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\times10^8$$

– promjene brzine el. i mag. toka nastaju samo zbog mijenjanja vektora $\vec{E}$ i $\vec{B}$

$$\vec{\nabla}\times\vec{E} = \frac{-\partial}{\partial t}\vec{B} \,\Big/\, \vec{\nabla} \qquad \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{E}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \frac{-\partial}{\partial t}\vec{\nabla}\times\vec{B}$$

$$\vec{\nabla}(\underbrace{\cancel{\vec{\nabla}\cdot\vec{E}}}_{0}) - \nabla^2\vec{E} = \frac{-\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{E}\right) \Rightarrow \boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{E}}$$

$$\mu_0\varepsilon_0 = \frac{1}{c^2}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{\nabla}\times\vec{E}$$

$$\vec{\nabla}(\underbrace{\cancel{\vec{\nabla}\cdot\vec{B}}}_{0}) - \nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial}{\partial t}\left(\frac{-\partial}{\partial t}\vec{B}\right) \Rightarrow \boxed{\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{B}}$$

Rješenje EM vala: $\vec{E}(\vec{r},t) = \vec{f}(\omega t \pm \underbrace{\vec{k}\,\vec{r}}_{\text{smjer gibanja}})$ koji se širi brzinom c

$$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z} \longrightarrow \vec{k}\cdot\vec{r} = k_x\cdot x + k_y y + k_z z$$

Primjer: sinusoidni val za el. polje koje se giba u poz (+) x-osi

$$\vec{E} = E_0\sin(\omega t - kx)\hat{E}$$

* zahtjevamo da EMV zadovoljava prve Maxwellovu jednadžbu (u vakumu) $\vec{\nabla}\cdot\vec{E} = 0$

$$\vec{\nabla}\vec{E} = \frac{\partial E}{\partial x}\hat{x} + \underbrace{\cancel{\frac{\partial E}{\partial y}}}_{0}\hat{y} + \underbrace{\cancel{\frac{\partial E}{\partial z}}}_{0}\hat{z} = 0$$

$$\vec{\nabla}\vec{E} = \frac{\partial}{\partial x}\left(E_0\sin(\omega t - kx)\right)\hat{E}\cdot\hat{x} = 0 \longrightarrow \vec{\nabla}\cdot\vec{E} = \overbrace{E_0\cos(\omega t - kx)(-k_x)}^{\neq 0}\underbrace{\hat{E}\cdot\hat{x}}_{=0}$$

smjer širenja vala uvijek mora biti okomit s obzirom na smjer širenja polja $\longrightarrow k_x\cdot\hat{x}\cdot\hat{E} = 0 \hookrightarrow k_x\hat{x}\perp\hat{E}$!

Magnetsko polje : možemo na sličan način $\rightarrow \vec{B} = B_0 \sin(\omega t - kx)\,\hat{B}$

$\rightarrow \vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\cdot\vec{B} = \frac{\partial B}{\partial x}\hat{x} = 0 \rightarrow \vec{\nabla}\vec{B} = -B_0\cos(\omega t - kx)\,k_x\underbrace{\hat{x}\cdot\hat{B}}_{0\,!} = 0$

Odnos električnog i magnetskog polja = ?

i B mora biti okomit

Odnos električnog i magnetskog polja

$\vec{E}_y = E_0 \sin(\omega t - kx)\,\hat{y}$

- Rotiramo

uzmemo $\hat{E} = \hat{y}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \hat{x}\underbrace{\cancel{\frac{\partial}{\partial z}}E_y}_{0} + \cancel{\hat{y}\cdot 0} + \hat{z}\frac{\partial}{\partial x}E_y \Rightarrow \boxed{\vec{\nabla}\times\vec{E} = \frac{\partial E_y}{\partial x}\hat{z}}$$

$$\vec{\nabla}\times\vec{E} = \frac{-\partial B}{\partial t} = -k\cdot E_0\cos(\omega t - kx)\,\hat{z}$$

$$\vec{B} = B_0\sin(\omega t - kx)\,\hat{B} \longrightarrow \frac{-\partial B}{\partial t} = \omega B_0\cos(\omega t - kx)\,\hat{B}$$

$\vec{\nabla}\times\vec{E} = -kE_0\cancel{\cos(\omega t - kx)}\hat{z}$

$\frac{-\partial B}{\partial t} = \omega B_0\cancel{\cos(\omega t - kx)}\,\hat{B}$

$$\Rightarrow -kE_0\,\hat{z} = -\omega B_0\cdot\hat{B} \longrightarrow kE_0 = \omega B_0 \;/ :k$$

$E_0 = \frac{\omega}{k}B_0 \rightarrow$ iz jednadžbe vala

$v = \lambda f = \frac{2\pi}{k}\cdot\frac{\omega}{2\pi}$

$v = \frac{\omega}{k}$

zaključujemo iz konzistentnosti $\rightarrow$ ali budući da se EM šire brzinom svjetlosti

$\frac{\omega}{k} = c$

$\hat{B} = \hat{z} \rightarrow$ smjer širenja el. polja je $\perp$ na smjer mag.

$E \perp k$

$B \perp k$

$E \perp B$

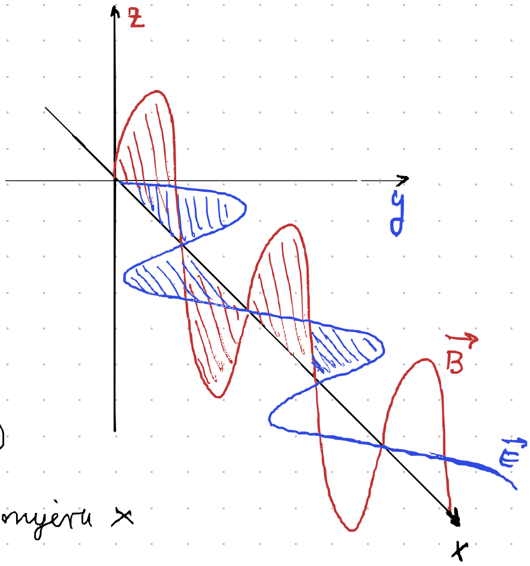
$$\boxed{E_0 = c\cdot B_0}$$

$$\boxed{\vec{E}\times\vec{B} = \vec{k}}$$

Kako se val širi?

mora biti ista funkcija sinus

- $\vec{B} = \frac{E_0}{c}\sin(\omega t - k_x x)\,\hat{z}$
- $\vec{E} = E_0 \sin(\omega t - k_x x)\,\hat{y}$



– mag polje u smjeru z (širi se po z)

– el. polje u smjeru y (širi se po y)

$\Rightarrow \hat{k} = +\hat{x} \longrightarrow$ val se širi u smjeru x

Intenzitet ravnog linearno polariziranog vala

Poyntingov vektor $\left[\frac{W}{m^2}\right]$

$\vec{E} \times \vec{B} = \vec{k}$

$\hat{x} \times [\bar{z}] = y$

· smjer uvijek u smjeru širenja vala

npr: $\vec{E} = E_0 \sin(\omega t - ky)\,\hat{x} \longrightarrow \vec{B} = -B_0 \cdot \sin(\omega t - ky) \cdot \hat{z}$

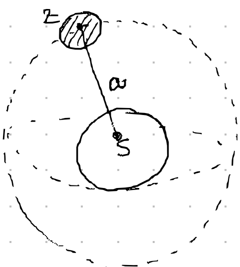
→ Poyntingov vektor: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{E_0^2}{\mu_0 c} \sin^2(\omega t - ky)\,\hat{y}$

srednja vrijednost PV: $\bar{S} = \frac{E_0^2}{2\mu_0 c}$

Primjer: Srednja vrijednost Poyntingovog vektora Sunčevog zračenja na Z.

luminozitet Sunca: $L_\odot = 3{,}828 \times 10^{26}$ W

1 AU = 149 597 870 700m = a



$I = \bar{S} = \frac{\text{omjer ukupne snage}}{\text{površine sfere}}$

srednja vr. Poyntingovog vektora u sferi

$I = \frac{P}{4\pi a^2} = \frac{L_\odot}{4\pi (1AU)^2} \simeq 1.361 \text{ kW/m}^2$

↳ Sunčeva konstanta

$\bar{S} = \frac{E_0^2}{2\mu_0 c} \rightarrow E_0 = \sqrt{\bar{S} \cdot 2\mu_0 \cdot c}$ → tako bismo izračunali amplitudu el. polja tog vala

Polarizacija: Malusov zakon

$I = I_0 \cdot \cos^2\varphi$ — polarizirano

$I = I_0 \cdot \frac{1}{2}$ → nepolarizirano