3.2. Zračenje crnog tijela
– Stefan Boltzmann & Wien –

Spektralna raspodjela zračenja CT

→ Stefan Boltzmanov ⇒ $\boxed{P = S\sigma T^4}$

Wienov zakon ⇒ $\boxed{\lambda_{max} T = 2{,}898 \times 10^{-3}\ \mathrm{Km}}$

Ukupni intenzitet $\xrightarrow{\text{zakon zračenja crnog tijela}}$ $\boxed{I = \int_0^\infty f(\lambda, T)\, d\lambda = \sigma T^4}$

→ valna duljina koja odgovara maksimumu zračene energije obrnuto je proporcionalna temperaturi

Rayleigh–Jeansova formula

· srednja en. svake slobode : $\bar{E} = kT$

* gledamo kao da brojimo stojne valove u volumenu

→ Br. stojnih valova u području frekvencije $(\nu, \nu + d\nu)$ ⇒ $dN = 8\pi \frac{V}{c^3} \nu^2 d\nu$

* izvod težak, neće nas tražiti

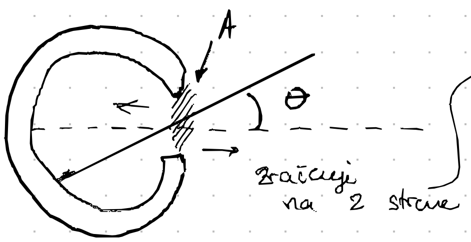
\+ Boltzmanova raspodjela po energijama : $P(E) = Ce^{-\frac{E}{kT}}$

Modul titranja: $dN = 8\pi \frac{V}{c^3} \nu^2 d\nu$

Srednja en : $dw = \bar{E} \cdot \frac{dN}{V} = kT \cdot \frac{8\pi}{} \frac{\nu^2}{c^3} d\nu$

„broj načina kroz volumen"

gustoća en.



$dA \cdot \cos\theta$

$2\, dE = w \cdot \underbrace{c \cdot \cos\theta \cdot \Delta t}_{\Delta x} \cdot \overbrace{dA'}$

P (snaga) → intenzitet

$w = \frac{2}{c} \cdot \frac{dE}{dt} \cdot \frac{1}{\langle \cos^2\theta \rangle} \cdot \frac{1}{A}$

$\langle \cos^2\theta \rangle$ ↓ $\underline{\text{usrednjeno}}$ $1/2$

I

$\Rightarrow w = \frac{2}{c} \left(\frac{P}{A}\right) \cdot \frac{1}{\frac{1}{2}}$

$w = \frac{4}{c} I \Rightarrow \underline{dI = \frac{c}{4} dw}$

$I = \int_0^\infty f(\nu, T)\, d\nu \longrightarrow dI = f(\nu, T)\, d\nu = \frac{c}{4} dw = \frac{c}{4} \cdot kT \frac{8\pi}{c^3} \nu^2$

$\Rightarrow f(\nu, T)\, d\nu = f(\lambda, T)\, d\lambda \,/ : d\lambda \Rightarrow f(\lambda, T) = \underline{f(\nu, T)\, \frac{d\nu}{d\lambda}}$ $\quad$ * $\lambda\nu = c$

$= \frac{2\pi kT}{c^2} \cdot \frac{c^2}{\lambda^2} \cdot \left(-\frac{c}{\lambda^2}\right)$

$\Rightarrow \boxed{f(\lambda, T) = 2\pi c \frac{kT}{\lambda^4}}$

Planckov zakon zračenja CT (crnog tijela)

- za razliku od klasičnog harmoničkog (1D) oscilatora čiju smo energiju opisali (računali) premda:

$$\bar{E} = \frac{\int_0^\infty E \cdot f(E) dE}{\int_0^\infty f(E) dE} = kT \quad \text{uz} \quad f(E) = [\text{konst}] \cdot \exp\left(\frac{-E}{kT}\right)$$

Planck je pretpostavio diskretnu varijablu: $\boxed{E = n h \nu}$ $n \in \mathbb{N}_0$ (Planck konstanta)

Zato integralni oblik prelazi u oblik suma

$$\bar{E} = \frac{[\text{konst}] \cdot \sum_{n=1} E \cdot \exp\left(\frac{-E}{kT}\right)}{[\text{konst}] \cdot \sum \exp\left(\frac{-E}{kT}\right)} = \left(\text{pokrata } x = \exp\left(\frac{-E}{kT}\right)\right)$$

$$= h x \nu \frac{1 + 2x + 3x^2 + \dots}{1 + x + x^2 + \dots} = h x \nu \frac{\frac{d}{dx}(1 + x + x^2 + \dots)}{\frac{1}{1-x}}$$

sumacija beskon. geom. reda: $S_\infty = \frac{a}{1-x}$

brojnik: $1 + 2x + 3x^2 + \dots =$

$$= \frac{d}{dx}(1 + x + x^2 + \dots) = \frac{d}{dx} \frac{1}{1-x}$$

$\longrightarrow$ brojnik $= \frac{1}{(1-x)^2}$ $\longrightarrow$ $\bar{E} = h x \nu \frac{\frac{1}{(1-x)^{\cancel{2}}}}{\frac{1}{\cancel{1-x}}} = h x \nu \frac{1}{1-x} = \left(\text{uvrstimo } x = \exp\left(\frac{-E}{kT}\right)\right.$ nazad

$\Longrightarrow$ srednja en. kvantnog oscilatora: $\bar{E} = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} = kT$ $\xrightarrow[\text{izraz umjesto } kT]{\text{uvrstimo u RJ}}$ $f(\lambda, T) = 2\pi c \frac{kT}{\lambda^4}$

$\Longrightarrow$ Planckov zakon zračenja crnog tijela: $\boxed{f(\lambda, T) = \frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}} = I_\lambda$

Povezanost RJ, SB i Wienovog zakona + Planckov

$\lambda \nu = c$

$\nu = \frac{c}{\lambda}$

$\lambda = \frac{c}{\nu}$

- povezanost RJ zakona i Planckovog zakona

$$\exp\left(\frac{h\nu}{kT}\right) \approx 1 + \frac{h\nu}{kT} \Rightarrow \bar{E} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \approx kT$$

- SB zakon će nam dati izraz za ukupni intenzitet (spektralnu raspodjelu valja integ.)

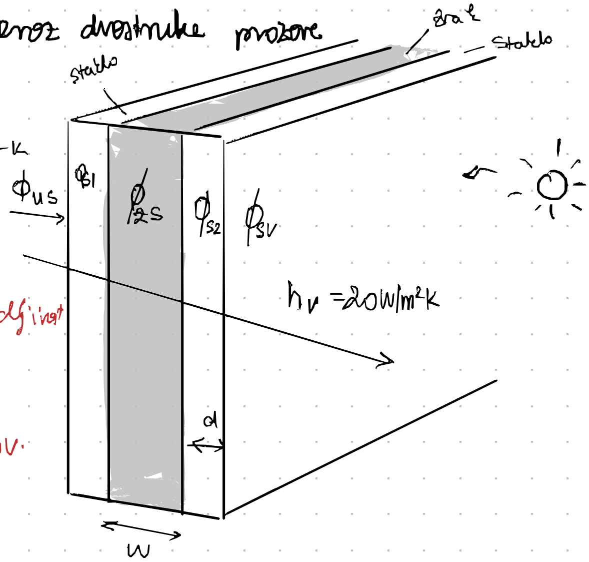
$$I = \int_0^\infty I_\lambda \, d\lambda = \int_0^\infty f(\nu, T) \, d\nu \quad \text{i} \quad x = \frac{h\nu}{kT} \longrightarrow I = \frac{2\pi k^4 T^4}{c^2 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx$$

- Weinov zakon pomaka: maksimum spektralne gustoće

$$\frac{d}{d\lambda} f(\lambda, T) = \frac{d}{d\lambda}\left(\frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}\right) = 0$$

PRIMJERI

Primjer 1.) Odrediti gubitke kroz dvostruke prozore



$\lambda_s = 0{,}8\,W/mK$ toplinska vodljivost

$d = 0{,}3\,cm$

$w = 2\,cm$

$A = 1\,m^2$

$\lambda_z = 0{,}025\,W/mK$ (zrak) toplinska vodljivost

$h_v = 20\,W/m^2K$ (vanjska) koef. konv.

$h_u = 6\,W/m^2K$ (unutarnja) koef. konv.

tok mora biti svuda isti:

„koliko unutra toliko van" $\rightarrow \phi_{conv} = \frac{\Phi}{t} = S \cdot q_v = \phi^*_{unutarnji\ na\ staklo}(\Delta T_{us}) = \phi^*_{staklo\ 1}(\Delta T_{s_1 z})$ (gustoća topl. toka)

$= \phi^*_{zrak\ staklo}(\Delta T_{zs}) = \phi^*_{staklo\ 2}(\Delta T_{zs_1}) = \phi^*_{staklo\ van}(\Delta T_{s_2 v})$

* temperature nisu jednake

$\Delta T_{us} = T_{unut} - T_{s1} \rightarrow q_{us_1} = (T_{unut} - T_{s1}) \cdot h_u$ konvekc

$\phi_{conv} = q \cdot S = \frac{\Delta T}{R} = \Delta T \cdot h_c \cdot S$

$= \lambda \frac{\Delta T}{\Delta x} \cdot S$

$\Delta T_{s_1 z} = T_{s1} - T_z \rightarrow q_{s_1 z} = \lambda_s \frac{T_{s1} - T_z}{d}$

$\Delta T_z = T_{s_1} - T_{s_2} \rightarrow q_z = \lambda_z \cdot \frac{T_{s_1} - T_{s_2}}{w}$ kondukcija

$\Delta T_{z s_2} = T_z - T_{s_2} \rightarrow q_{z s_2} = \lambda_s \frac{T_z - T_{s_2}}{d}$

ΔT možemo preko ϕ opisati

$\Delta T_{s_2 v} = T_{s_2} - T_v \rightarrow q_{s_2 v} = (T_{s_2} - T_v) h_v$ konveks

$\phi = \frac{\Delta T}{R} = \Delta T \cdot h_c S$

$\Rightarrow$ ukupna promjena $\rightarrow \Delta T = T_u - T_v = \sum \Delta T = [(T_u - T_{s_1}) + (T_{s_1} - T_z) + (T_{s_1} - T_{s_2}) + (T_z - T_{s_2}) + (T_{s_2} - T_v)]$

$R_{s_1 z} = R_{s_2 z} = \underbrace{R_s}$ $\leftarrow$ $R_k = \frac{1}{h_c S}$ $\quad R_c = \frac{\Delta x}{\lambda S}$

$\Delta T = \phi \cdot R_{uk} = \phi\,(R_{us} + 2R_s + R_z + R_{sv}) = \phi\left(\underbrace{\frac{1}{h_u \cdot S}} + \underbrace{\frac{2 \cdot d}{\lambda_s \cdot S}} + \underbrace{\frac{w}{\lambda_z \cdot S}} + \underbrace{\frac{1}{h_v \cdot S}}\right)$

$\left[\frac{K}{W}\right]$ $R_{us} = 0{,}17$ $\quad R_s = 3{,}8 \times 10^{-3}$ $\quad R_z = 0{,}8$ $\quad R_{sv} = 0{,}05$

$k = \frac{q}{\Delta T} = \frac{\phi}{S \cdot \Delta T} = \frac{1}{S \Delta T} \cdot \frac{\Delta T}{R_{uk}} = \frac{1}{S R_{uk}} = \boxed{0{,}97\,m^2K = k}$ $\quad \left(\phi \rightarrow \frac{\Delta T}{R}\right)$

to je valjda to?

WTF Gomzi

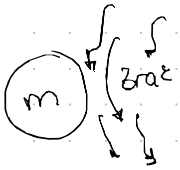
Primjer 2) #strujanje

$T_1 = 80°C$

$T_0 = 20°C$

$t_1 = 20s$ — $T = 60°C$

$t_2 = 50s \rightarrow T_3 = ?$



hlađenje tijela, ne tok!

$$\frac{dQ}{dt} = -h\,\Delta T$$

$$dQ = mc\,dT = C \cdot dT$$

$$\frac{C\,dT}{dt} = -h\,(T-T_0)$$

$$\frac{dT}{dt} = -h\,(T-T_0)\,C$$

$$\Rightarrow \frac{dT}{(T-T_0)} = -\frac{h}{C}\,dt \,/\int$$

$$\ln(T-T_0)\Big|_{T_1}^{T} = -\frac{h}{C}(t_1-0) \Rightarrow \ln\left(\frac{T-T_0}{T_1-T_0}\right) = \frac{-h}{C}t_1$$

$$\Rightarrow \ln\left(\frac{60-20}{80-20}\right) = \frac{-h}{C}\cdot 20s \Rightarrow \frac{-h}{C} = -0{,}0203/s$$

$$\Rightarrow \ln\left(\frac{T-20}{80-20}\right)\cdot\frac{1}{50s} = -0{,}0203/s$$

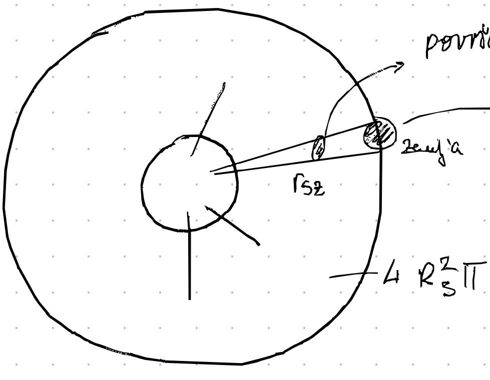
$$\ln\left(\frac{T-20}{60}\right) = -1{,}014 \,/e \rightarrow \frac{T-20}{60} = 0{,}363$$

$$\boxed{T = 41{,}77°C}$$

Primjer 3.) #zračenje

$R_{zemlja} = 6{,}37\times10^6 m$ $\quad R_{Sunce} = 6{,}95\times10^8 m$

$r_{Z-S} = 1{,}5\times10^{11} m$



površina koju zauzima Zemlja

$R_Z^2\pi$

a) Valna duljina za koju sunčeva svjetlost zrači max

⇓

Wienov zakon $\lambda_{max}\cdot T = 2{,}9\times10^{-3} mK$

$\rightarrow \boxed{\lambda_{max} = 0{,}5\mu m}$

b) Snagu koju zrači $1m^2$? (Sunca)

⇓

Stefan Boltzman $\Rightarrow P = S\sigma T^4$

σ - konstanta $\rightarrow \sigma = 5{,}67\times10^{-8} W/m^2K^4$

$S = 1m^2$

$\rightarrow \boxed{P = 64{,}16MW}$

→ Ukupna snaga zračenja Sunca

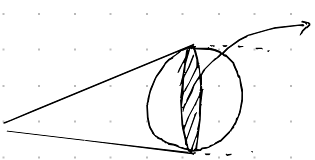
$P_{uk} = A\,\sigma\,T^4$

zračenje sunčeve površine

$$\Rightarrow P_{uk} = R_S^2\cdot4\pi\cdot(\sigma T^4 (1)) = R_S^2 4\pi\cdot 64{,}16MW$$

$$\boxed{P_{uk} = 3{,}89\times10^{26} W}$$

↳ koji dio prima zemlja?



$R_Z^2\pi$

?

$$P_Z = \frac{A_{zemlja}}{A_{zračenja\ Sunce}}\cdot P_S = \frac{R_Z^2\pi}{4R_S^2\pi}P_S$$

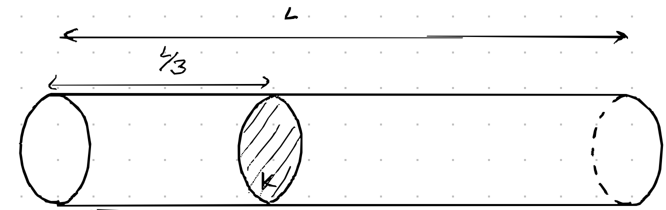
(P_S = P_{uk})

prostorni kut

$$P_Z = R_Z^2\pi\cdot\sigma T^4 = 8.17\times10^{21} W$$

Primjer (bio je na nekom ispitu)

$T = konst. \Rightarrow$ izotermni proces



$n_1 = 1\,mol$
$p_1 = 5\times10^5\,Pa$

$p_2 = 10^5\,Pa$

$$pV = nRT$$

$$p_1V_1 = n_1 \cdot \cancel{RT} \qquad p_2V_2 = n_2\cancel{RT}$$

$$\frac{p_1V_1}{n_1} = \frac{p_2V_2}{n_2}$$ nemamo

$\frac{2}{3}V$

$$n_2 = p_2 V_2 \cdot \frac{n_1}{p_1 V_1} \quad V_1 \to \frac{1}{3}V \Rightarrow n_2 = \frac{2}{\cancel{3}}\cancel{V}\cdot p_2 \cdot \frac{n_1 \cdot \cancel{3}}{p_1 \cdot \cancel{V}} = 2\,\frac{10^5}{5\times10^5}\cdot 1$$

$n_2 = 0{,}4\,mol$

$\Rightarrow$ u trenutku t_1

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u trenutku t_2 $\quad \frac{p_1'V_1'}{n_1} = \frac{p_2'V_2'}{n_2}$ $\quad$ zakon očuvanja mase n_1 i n_2 se ne mijenjaju

mičemo klip $\to F_R = 0$ jer $p_1'\cancel{S} = p_2'\cancel{S} \Rightarrow p_1' = p_2'$ (dolazi u ravnotežni položaj)

$$\frac{V_1'}{n_1} = \frac{V_2'}{n_2} \to \frac{V_1'}{V_2'} = \frac{n_1}{n_2} = \frac{1}{0{,}4} \quad \boxed{\frac{V_1'}{V_2'} = \frac{5}{2}} = \frac{V_1'}{V_2'} = \frac{L_1' \cdot S}{L_2' \cdot S} = \frac{5}{2}$$

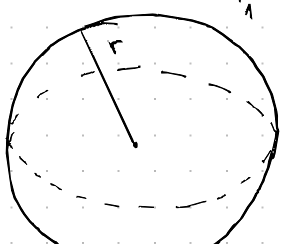
$$L_1 = \frac{5}{2}L_2$$

meaning
$\Downarrow$
L_1 je na $\frac{5}{7}$ od ruba

Primjer:

$r = 0{,}5\,m$

$T_1 \longrightarrow \lambda_{max} = 9{,}66\times10^{-7}\,m$



CT $\quad$ m?

$t = 2s \quad T_2 = 800K$

$T_{okol} = 0K$

$c = 155\,J/kgK \quad \left(c = \frac{C_t}{m}\right)$

Wienov zakon: $\lambda_{max}T \simeq 2{,}898\times10^{-3}$

$$T_1 = \frac{2{,}898\times10^{-3}\,mK}{9{,}66\times10^{-7}}$$

$T_1 = 3000K$

Stefan Boltzman: $P = S\sigma T^4$ $\quad \frac{dQ}{dt}$, $\quad S = 4r^2\pi$, $\quad \sigma$ konst.

$\Rightarrow -dQ = mc\,dT$ (hladi se)

$$\Rightarrow -mc\,dT = S\sigma T^4 \cdot dt$$

$$-mc\,\frac{dT}{T^4} = 4r^2\pi\,\sigma\,dt \,/\int$$

$$-mc\left(\frac{-1}{3}\right)\cdot\frac{1}{T^3}\Bigg|_{T_1}^{T_2} = 4r^2\pi\,(t - t_0)$$ $\quad t_0 = 0$

$$\frac{mc}{3}\left(\frac{1}{800} - \frac{1}{3000}\right) = 4r^2\pi\cdot 2\sigma \longrightarrow m = \frac{28\,8000}{11}\cdot\frac{r^2\pi}{c}\,\sigma$$

$\boxed{m = 7{,}5\,\mu kg}$ $\quad$ Gomzi je nekako dobio 11,2 kg?!?