# 4.5.2. ORTOGONALNE TRAJEKTORIJE

**Z1-19-3b** Pokažite da su familije krivulja

$$y_1' = -\frac{1}{y_2'}$$

krivulje:

$2x^2 + 5y^2 = C_1 \qquad C_1 > 0$

$y = C_2\sqrt{x^5} \qquad C_2 \in \mathbb{R}$

$$2x^2 + 5y^2 = C_1 \,\Big|'$$

$$4x + 10y \cdot y' = 0$$

$$y_1' = \frac{-4x}{10y} = \frac{-2x}{5y}$$

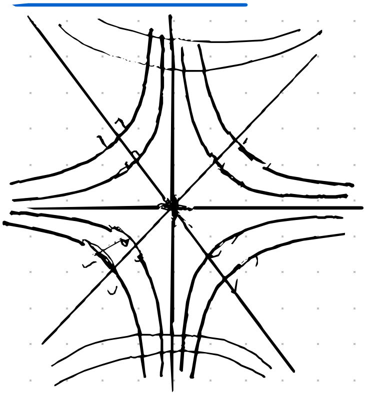
$$y = C_2\sqrt{x^5} \,\Big|' \longrightarrow \text{izrazimo } C_2$$

$$y' = C_2 \cdot \frac{5}{2} x^{3/2}$$

$$y' = \frac{y}{x^{\cancel{5/2}}} \cdot \frac{5}{2} \cdot \cancel{x^{3/2}}$$

$$y_2' = \frac{5y}{2x} = \frac{-1}{y_1'}$$

**WIR-20-6**



$$xy = a \,\Big|'$$

$$y + xy' = 0 \longrightarrow y' = -\frac{-y}{x}$$

za O.T. $\quad y' = \frac{x}{y}$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C \,\Big|\cdot 2$$

$$y^2 = x^2 + C$$

Zad.) Nađite $y$: $x^2y' - y\cdot y' = 2xy$ koje prolazi kroz $T(2,1)$

$y'(x^2-y) = 2xy$

$(x^2-y)\frac{dy}{dx} = 2xy$

$(x^2-y)dy = 2xy\,dx$

$y' = \frac{2xy}{x^2-y}$

$y' = \frac{1}{x'}$ ili eulerov multipl.

$y' = \frac{2xy}{x^2-y} = \frac{1}{x'} \to x'\cdot 2xy = x^2 - y \;/ :2xy$

$x' = \frac{x^2-y}{2xy}$

$x' - \frac{x^{\cancel{2}}}{2\cancel{x}y} = \frac{-\cancel{y}}{2x\cancel{y}}$

Bernoulli za kada gledamo $x(y)$ $\to$ $\underbrace{x' - \frac{x}{2y} = \frac{-1}{2x}}$

Bernoulli sa potencijom $-1$

supstitucija: $z = x^{1-\alpha} = x^2 \leftarrow$ $x' - \frac{1}{2y}x = \frac{-1}{2x}\cdot x^{-1} \;/ \cdot 2x$ $\quad \underline{\alpha = 1}$

$z' = 2x\cdot x'$

$2xx' - \frac{1}{y}x^2 = -1$

$z' - \frac{1}{y}z = -1$ LDJ

$$z = e^{-\int -\frac{1}{y}dy}\left[\int -1\, e^{\int -\frac{1}{y}dy}\, dy + c\right]$$

$$z = e^{\ln y}\left[\int -1\, e^{-\ln y}\, dy + c\right] = y\left[\int -1\cdot\frac{1}{y}\,dy + c\right] = \underline{y\left[-\ln y + c\right]} = x^2$$

$\underline{x^2 = -y\ln y + cy}$ $\quad T(2,1) \Rightarrow 4 = 0 + c \to c = 4$

$$\boxed{x^2 = -y\ln y + 4y}$$