1.1. Fourierov red

$f(x) = c_0 + \sum_{n=1}^{\infty} c_n \sin(n\omega x + \varphi_n)$ — idеja: periodične funkcije u ovom zapisu!

Opća sinusoida

$$f(x) = c\sin(\omega x + \varphi) = c(\sin\omega x \cdot \cos\varphi + \cos\omega x \cdot \sin\varphi)$$

$$= \underbrace{c\sin(\omega x)}_{A}\cdot\cos\varphi + \underbrace{c\cos(\omega x)}_{B}\cdot\sin\varphi$$

$$= A\cos\varphi + B\sin\varphi$$

* $B = c\cos(\omega x)$

$A = c\sin(\omega x)$

$A^2 + B^2 = c^2$

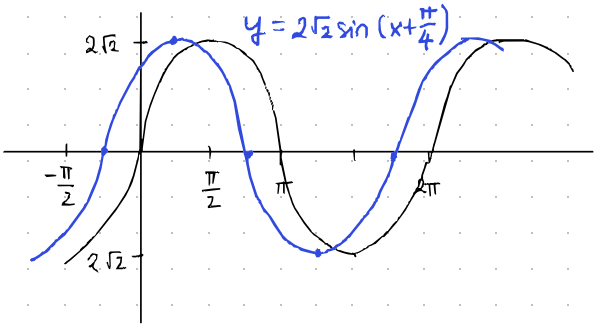
$\operatorname{tg}\varphi = \frac{A}{B}$

Primjer: Nacrtaj $f(x) = 2\cos x + 2\sin x$ u općem sinusoidu

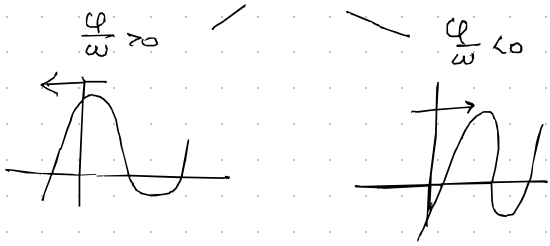
$c = \sqrt{A^2+B^2} \longrightarrow \sqrt{4+4} = 2\sqrt{2}$

$\operatorname{tg}\varphi = \frac{2}{2} \longrightarrow \varphi = \frac{\pi}{4}$

$\Longrightarrow f(x) = 2\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

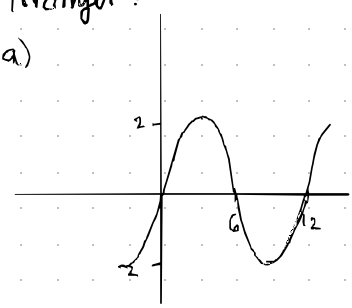


$\omega x + \varphi = 0 \longrightarrow x = \frac{-\varphi}{\omega}$



Primjer:

a)

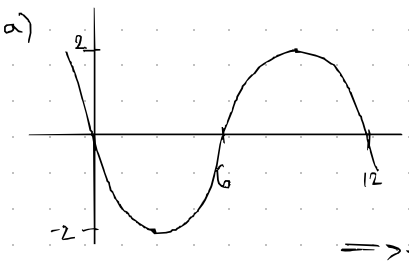


$T = 12 \qquad T = \frac{2\pi}{\omega} \longrightarrow \omega = \frac{\pi}{6}$

$\varphi = 0 \longrightarrow$ nije pomaknuta

$\longrightarrow f(x) = 2\sin\left(\frac{\pi}{6}x\right)$

a)



$f(x) = c\sin(\omega x + \varphi) = c\sin\omega\left(x + \frac{\varphi}{\omega}\right)$

$\frac{\varphi}{\omega} = 6$ ili $\frac{\varphi}{\omega} = -6$

pomak je ili lijevo ili desno $\longrightarrow$ mi na slici vidimo $\frac{-\varphi}{\omega}$ koji može biti -6 ili 6, ovisi o lijevo/desno

$\Longrightarrow f(x) = 2\sin\frac{\pi}{6}(x-6)$

ili $f(x) = 2\sin\frac{\pi}{6}(x+6)$ $\quad \frac{\varphi}{\omega}$

$f(x) = 2\sin\left(\frac{\pi}{6}x - \pi\right) =$

$= 2\sin\frac{\pi}{6}x\cos\pi - 2\cos\frac{\pi}{6}\sin\pi$ ($\sin\pi = 0$)

$= -2\sin\frac{\pi}{6}x$

Def - periodična funkcija

$f: D \to \mathbb{R}$, $D \subseteq \mathbb{R}$ je <u>periodična</u> ako $\exists T > 0$ t.d. $f(x+T) = f(x)$, $\forall x \in D$.

T nazivamo periodom fje, a najmanji takav T, ako postoji je (temeljni period)

Primjer: Nađi temeljni period

a) $f(x) = \sin^2 x$

$$f(x) = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{1}{2}\cos(2x) \qquad \omega = 2$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$$

b) $f(x) = \sin(x^2)$

$f(x+T) - f(x) = 0 \quad \forall x \in \mathbb{R}$

$\sin(x+T)^2 = \sin(x^2)$

$\sin(x^2 + 2Tx + T^2) = \sin(x^2)$

nije periodična fja

Pojer - suma fja različitih frekvencija

$f(x) = A\cos\alpha x + B\sin\beta x$, $\alpha \neq \beta$ $\alpha, \beta > 0$

Je li periodična? Postoji li $T>0$ t.d $f(x+T) - f(x) = 0$

koristimo iz formule:
$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$
$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$$A\cos\alpha(x+T) + B\sin\beta(x+T) - A\cos\alpha x - B\sin\beta x = 0$$

$$A(\cos\alpha(x+T) - \cos\alpha x) + B(\sin\beta(x+T) - \sin\beta x) = 0$$

$$A\left(-2\sin\alpha\frac{2x+T}{2}\cdot\sin\alpha\frac{T}{2}\right) + B\left(2\cos\beta\frac{2x+T}{2}\cdot\sin\frac{\beta T}{2}\right) = 0$$

$$-2A\sin\left(\alpha x + \frac{\alpha T}{2}\right)\cdot\sin\frac{\alpha T}{2} + 2B\cos\left(\beta x + \frac{\beta T}{2}\right)\cdot\sin\frac{\beta T}{2} = 0$$

množavamo:

$\sin\frac{\alpha T}{2} = 0$ $\quad \frac{\alpha T}{2} = k_1\pi \longrightarrow T = \frac{2k_1\pi}{\alpha}$

$\sin\frac{\beta T}{2} = 0$ $\quad \frac{\beta T}{2} = k_2\pi \longrightarrow T = \frac{2k_2\pi}{\beta}$

$\frac{k_1}{\alpha} = \frac{k_2}{\beta} \Rightarrow \boxed{\frac{\alpha}{\beta} = \frac{k_1}{k_2}}$

Funkcija f je periodična ako je $\frac{\alpha}{\beta}$ racionalan broj, kažemo da su α i β **sumjerljivi**

Primjer: Temeljni period?

$f(x) = \sin 2x + \cos 6x$

$\frac{\alpha}{\beta} = \frac{6}{2} = 3$

$T = \pi$ $\quad T = \frac{\pi}{3}$

$\frac{\alpha}{\beta} = \frac{k_1}{k_2} \quad / \cdot k_2/\beta$

$T = \frac{2k_1\pi}{\alpha} = \frac{2\cdot 1}{2}\pi = \pi$

$k_2\alpha = k_1\beta$

$2k_2 = 6k_1$

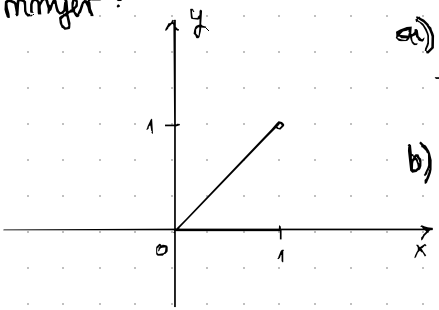
$3k_1 = k_2 \longrightarrow$ $k_1 = 1$, $k_2 = 3$

Periodička proširenja

$f: D \to \mathbb{R}$

$f(-x) = -f(x)$ $\forall x \in D$ neparna $\quad f(-x) = f(x)$ $\forall x \in D$ parna

Primjer:

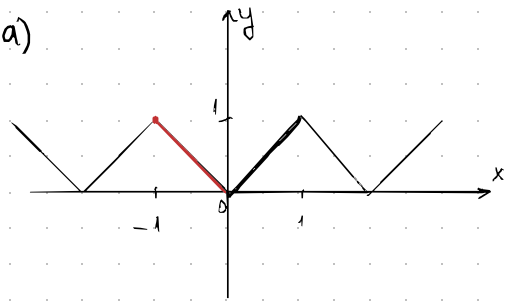


a) Nacrtaj graf periodičnog parnog proširenja funkcije f

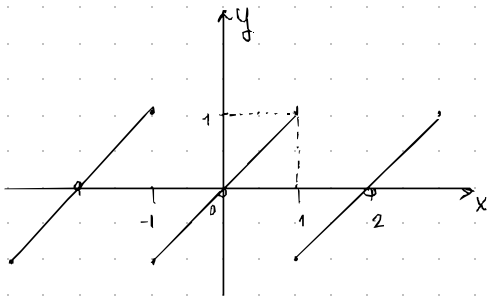
b) -//- neparnog -//-

$T = 2$

a)



b)



Primjer: $f(x) = \arcsin(\sin x)$

Rj:

$\sin: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$

$\arcsin: [-1, 1] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$T = 2\pi \qquad f(x+2\pi) = \arcsin(\sin(x+2\pi))$

$= \arcsin(\sin x)$

$= f(x)$

$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$f(x) = \arcsin\left(\sin\left(\underbrace{x-\pi}_{\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} + \pi\right)\right)$

$= \arcsin(-\sin(x-\pi))$

$= -\arcsin(\sin(x-\pi)) \quad \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= -(x-\pi) = -x+\pi$

$\longrightarrow f(x) = \arcsin(\sin x) = \begin{cases} x & x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \\ -x+\pi & \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \end{cases} \longrightarrow$