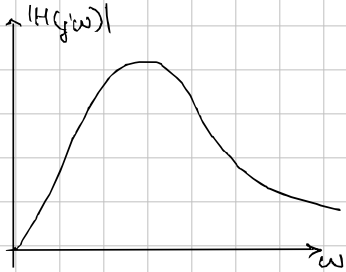
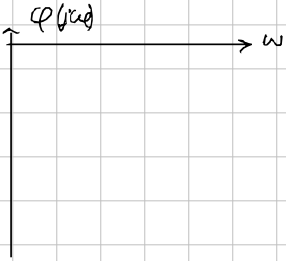
13. Bodeovi dijagrami

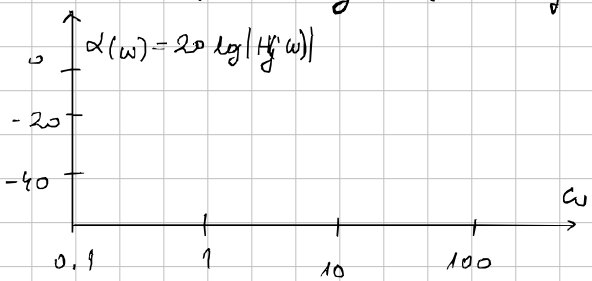
- dosada (kada su normirane vrijednosti el. u brojevima)





Bodeov prikaz → logaritamska mjera pojačanja filtra u decibelima

$$\alpha(\omega) = 20 \log \alpha(\omega) = 20 \log |H(j\omega)|$$



→ prikaz u širem području

! ovo ide bez računanja, lakše

▷ filtri, pojačala, regul. sustavi...

→ Faktorizirane prijenosne funkcije $H(s) = k \cdot H_1(s) \cdot H_2(s)$

realna konstanta

- nakon fazit umjetavamo $s = j\omega$

$$\rightarrow H(j\omega) = |H(j\omega)| e^{j\varphi(\omega)} = |H_1(j\omega)| e^{j\varphi_1(\omega)} \cdot |H_2(j\omega)| e^{j\varphi_2(\omega)} \dots$$

$$a(\omega) = |H(j\omega)| = |k| \cdot a_1(\omega) \cdot a_2(\omega)$$

$$\hookrightarrow a_1(\omega) = |H_1(j\omega)|$$

$$\varphi(\omega) = \angle H(j\omega) = \angle k + \varphi_1(\omega) + \varphi_2(\omega)$$

$$\alpha(\omega) = 20 \log \alpha(\omega) = 20 \log |H(j\omega)|$$

[dB] „decibel pojačanje"

deci - bel

Doprinos svakog faktora funkcije $H(s)$

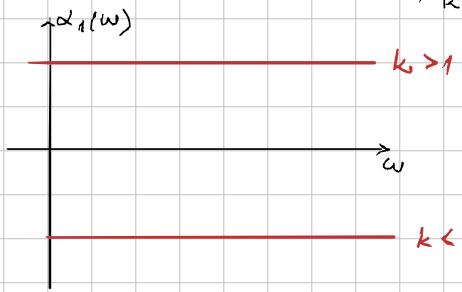
Pribrojnici koji vodi: faktor brojnika – pozitivan predznak

faktor nazivnika – negativan predznak

a) konstanta k

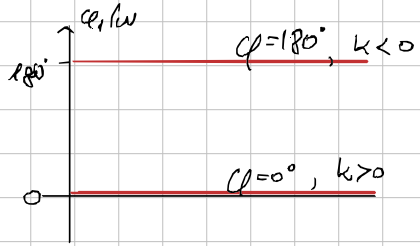
$\alpha_1(\omega) = 20 \log|k| = k_{dB}$

- $k_{dB} > 0$ ako $|k| > 1$
- $k_{dB} < 0$ ako $|k| < 1$



pripadajuća fazna karakt.: $0° \rightarrow k > 0$

$180° \rightarrow k < 0$

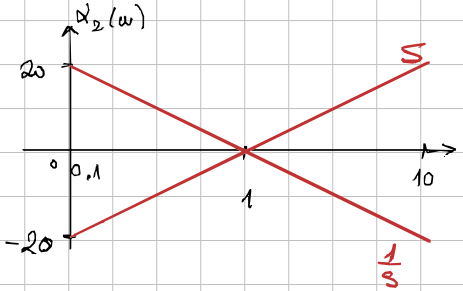


b) faktor s korijenom u nuli $H_2(s) = s$

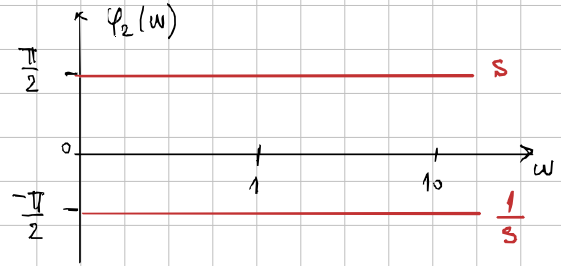
$\alpha_2(\omega) = 20 \log|j\omega| = 20 \log(\omega)$

u brojniku: $\alpha_2 = 20 \log\left|\frac{1}{j\omega}\right| = -20 \log(\omega)$

u nazivniku: $\alpha_2 = 20 \log(j\omega) = 20 \log(\omega)$



▷ fazna kar.:



konstantna fazna karakteristika

c) faktor s realnim korjenom $\neq 0 \rightarrow H_3(s) = s + \sigma$

$$s + \sigma \rightarrow \sigma \left(1 + \frac{s}{\sigma}\right)$$

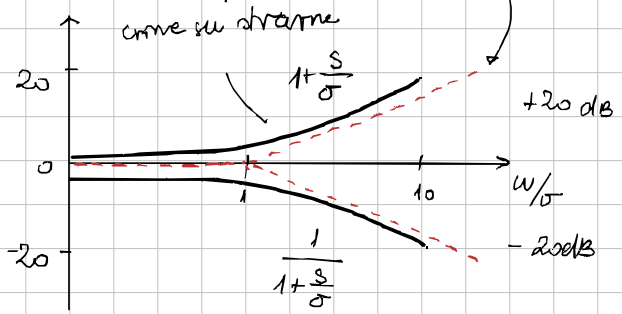
$$\Rightarrow \alpha_3(\omega) = 20 \log \left| 1 + \frac{j\omega}{\sigma} \right| = \dots = 10 \log \left[1 + \left(\frac{\omega}{\sigma} \right)^2 \right]$$

brojnik: $\alpha_3(\omega) = -20 \log \left| 1 + \frac{j\omega}{\sigma} \right| = -10 \log \left[1 + \left(\frac{\omega}{\sigma} \right)^2 \right]$

nazivnik $\alpha_3(\omega) = 20 \log \left| 1 + \frac{j\omega}{\sigma} \right| = 10 \log \left[1 + \left(\frac{\omega}{\sigma} \right)^2 \right]$

- ako vrijedi $\omega << \sigma$: $\alpha_3(\omega) \cong \pm 20 \log 1 = 0\,dB$
- ako vrijedi $\omega >> \sigma$: $\alpha_3(\omega) \cong 20 \log \left(\frac{\omega}{\sigma} \right)$

spomenuti pravci = asimptote

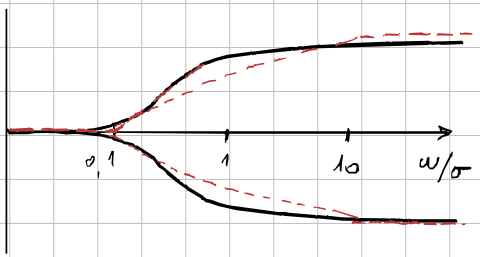


FAZNO-FREKV $\varphi_3 = \pm \operatorname{arctg} \left(\frac{\omega}{\sigma} \right)$ je li faktor u brojniku ili nazivniku

$\omega << \sigma \Rightarrow \varphi_3 \cong 0°$

$\omega >> \sigma \Rightarrow \varphi_3 \cong 90°$

$\omega = \sigma \Rightarrow \varphi_3 = \pm 45°$



d) faktor koji sadrži konjugirano kompleksni par korijena

$$H_4(s) = \underbrace{\left(s^2 + 2\xi s\omega_0 + \omega_0^2\right)}$$

ω_0 – modul korijena (neprigušena kružna frekvencija)

$$\omega_0^2\left(\left(\frac{s}{\omega_0}\right)^2 + 2\xi\frac{s}{\omega_0} + 1\right)$$

ξ – koeficijent prigušenja

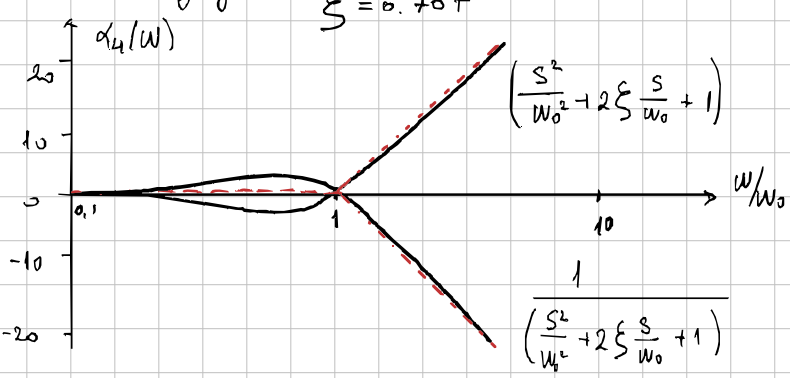
$$\alpha_4 = \pm 20\log\left|1 - \frac{\omega^2}{\omega_0^2} + \frac{2j\xi\omega}{\omega_0}\right|$$

$$= \pm 10\log\left[\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\xi\omega}{\omega_0}\right)^2\right]$$

$\omega \ll \omega_0 \quad \alpha_4 = \pm 20\log(1) = 0\ dB$

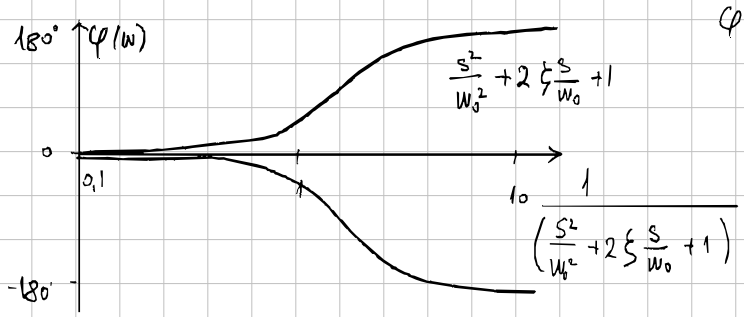
$\omega \gg \omega_0 \quad \alpha_4 = \pm 20\log\left|\frac{\omega^2}{\omega_0^2}\right| = \pm 40\log\left(\frac{\omega}{\omega_0}\right)$

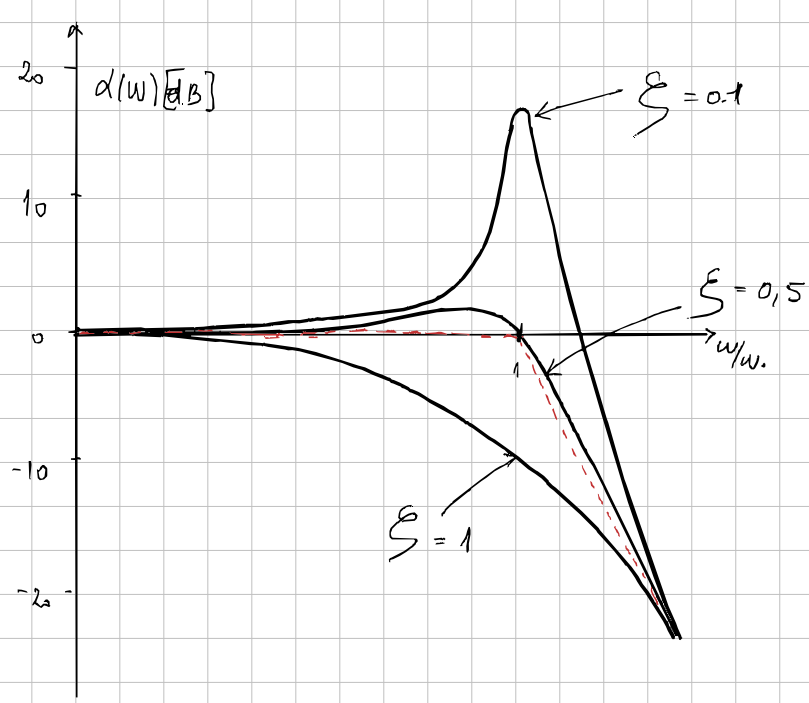
Bodeov dijagram: $\xi = 0.707$



Iznos faze:

$$\varphi_4 = \pm\arctg\left(\frac{2\xi\omega\omega_0}{\omega_0^2 - \omega^2}\right)$$





Primjer $H(s) = \frac{10^4 (s+2)}{(s+10)(s+100)} = 20 \frac{\left(1+\frac{s}{2}\right)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$

$\omega_1 = 2$

$\omega_2 = 10$

$\omega_3 = 100$