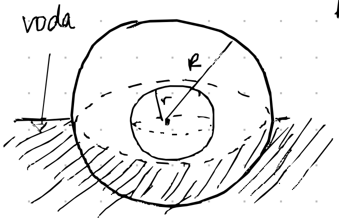
Auditorne – Statika i dinamika fluida

Zadatak 1.)



$R = 20\,cm$

$M = 20\,g$ (drvena)

$r = 10\,cm$ (metalna)

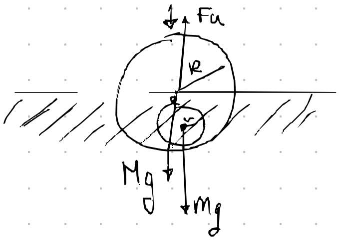
$\rho_m = ?$

$F_u = \rho_{tekućine} \cdot g \cdot V_{tijela}$

$Mg + mg = \rho_f \cdot g \cdot V_{tijela}$

Volumen kugle / uronjeni dio: $V = \frac{1}{2} \cdot \frac{4}{3} R^3 \pi$

$$V = \frac{2}{3}\pi R^3$$



$M + m = \rho_f \cdot V$

$m = \rho_m \cdot V_m$

$m = \rho_f \cdot \frac{2}{3}\pi R^3 - M$

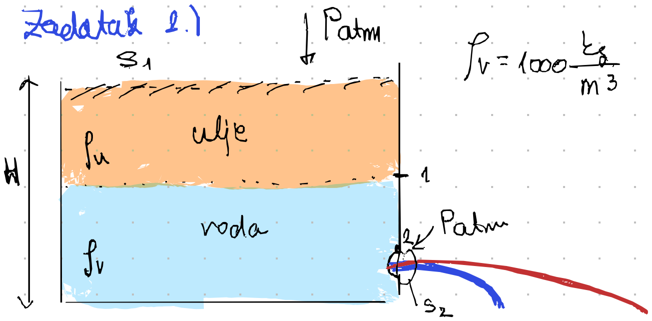
$\rho_m \cdot V_m = \rho_f \cdot \frac{2}{3}\pi R^3 - M \longrightarrow \rho_m = \dfrac{\rho_{voda} \cdot \frac{2}{3}\pi R^3 - M}{\frac{4}{3}\pi r^3}$

$\Rightarrow \boxed{\rho_{mala} = 3952{,}25\ kg/m^3}$

$R = 0{,}2\,m \quad r = 0{,}1\,m$

$M = 0{,}2\,kg$

Zadatak 2.)



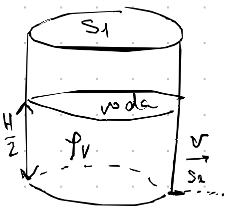
$\rho_v = 1000\,\frac{kg}{m^3} \quad h_1 = \frac{1}{2}H$

* Torricellijev zakon istjecanja

vanjski tlak

$\textcircled{P_1} + \rho g h_1 + \frac{\rho v_1^2}{2} = \textcircled{P_2} + \rho g h_2 + \frac{\rho v_2^2}{2}$

Samo voda:



$\cancel{P_1} + \rho g h_1 + \frac{\rho v_1^2}{2} = \cancel{P_2} + \cancel{\rho g 0} + \frac{\rho v_2^2}{2}$ — (v)

atmosferski; (dno)

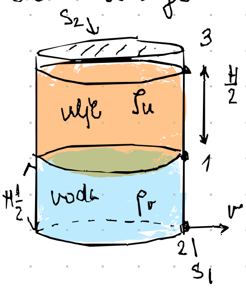
$S_1 v_1 = S_2 \cdot v \rightarrow S_1 >>> S_2 \Rightarrow v_1 \approx 0$

$\rho g \frac{H}{2} = \frac{\rho v_2^2}{2}$

$\longrightarrow v = \sqrt{gH}$

brzina „spuštanja" razine vode u posudi

dodа se ulje:



$v_1 \approx 0$

$v' = 1{,}1\, v$

voda istječe ali brže

$\cancel{P_1} + \rho_v g \frac{H}{2} + \rho_u g \frac{H}{2} + \cancel{0} = \cancel{P_2} + \rho_v \frac{v'^2}{2} \cdot \cancel{\rho g 0}$ (dno)

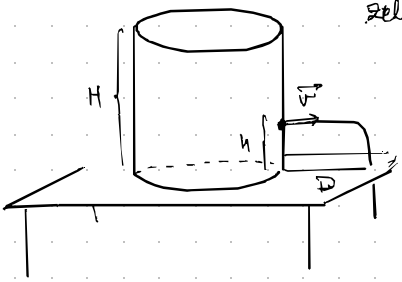
atmosf.; atmosf.

$g \frac{H}{2}(\rho_v + \rho_u) = \rho_v \cdot \frac{(1.1 v)^2}{2}$

$\cancel{g \frac{H}{2}}(\rho_v + \rho_u) = \rho_v \cdot (1.1)^2 \cdot \cancel{\frac{gH}{2}}$

$\rho_u = \rho_v \cdot (1{,}1)^2 - \rho_v = \rho_v((1.1)^2 - 1) = \boxed{210\,\frac{kg}{m^3}}$

Zadatak 3.)



Želimo izraziti domet kao funkciju visine

↪ $D(h) = ?$

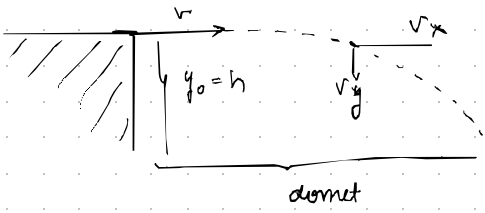
$\frac{dD}{dh} = 0 \rightarrow$ min / max zovet!

$\cancel{p_A} + \rho g H + \rho \frac{v_1^2}{2} = \cancel{p_A} + \rho g h + \rho \frac{v_2^2}{2}$ ($v_1 = 0$)

$\rho g H = \rho g h + \rho \frac{v_2^2}{2} \rightarrow v^2 = 2g(H-h)$

→ izraziti v kao fju od D

⟹ Horizontalni hitac



→ mijenjamo visinu: $y(t) = y_0 - \frac{gt^2}{2}$

$y(t) = h - \frac{gt^2}{2}$

udaljenost po x → $x(t) = vt$

Domet: $y = 0 = h - \frac{g t_D^2}{2}$

$x_D = D = v \cdot t_D$

$2h = g t_D^2$

$t_D = \sqrt{\frac{2h}{g}}$

$\rightarrow D = v \cdot \sqrt{\frac{2h}{g}} \rightarrow v^2 = D^2 \cdot \frac{g}{2h}$

$D^2 \cdot \frac{g}{2h} = 2g(H-h)$

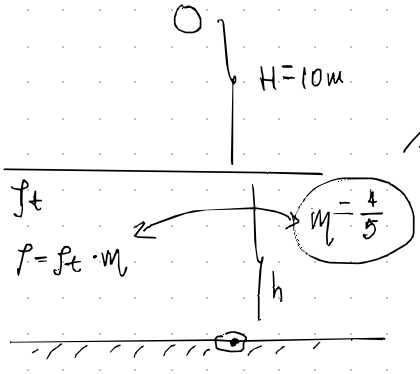
$\boxed{D = \sqrt{4h(H-h)}}$

max dometa dobijemo derivacijom

$\frac{dD}{dh} = \frac{1}{2\sqrt{4h(H-h)}} \cdot (4h(H-h))' = \frac{1}{2} \frac{1}{\sqrt{4h(H-h)}} \cdot (4(H-h) - 4h) = \frac{1}{4} \frac{1}{\sqrt{h(H-h)}} \cdot (4H - 8h)$

$\frac{dD}{dh} = \frac{H-2h}{\sqrt{Hh-h^2}} = 0 \qquad H - 2h = 0 \rightarrow \boxed{h = \frac{H}{2}}$

Zadatak 4.)



$F_u = \rho_t \cdot g \cdot V$

brzina gibanja do dna

ZOE: $mgH = m\frac{v^2}{2} \rightarrow v = \sqrt{2gH}$

$\frac{mv^2}{2} + mgh = W_u$

$W_u = \int_0^h f_u \cdot dh' = \int_0^h \rho_t \cdot gV\, dh' = \rho_t \cdot g \cdot V \cdot h$

$\Rightarrow \frac{mv^2}{2} + mgh = \rho_t \cdot g \cdot V \cdot h$

$\rho = \frac{m}{V} = \rho_t \eta$

$\rho V \frac{v^2}{2} + \rho V g h = \rho_t \cdot g \cdot V \cdot h$

$\rho \frac{2gH}{2} = gh(\rho_t - \rho) \longrightarrow \rho H = h(\rho_t - \rho) \;/ \cdot \rho$

$\rho_t = \frac{\rho}{\eta}$

$H = \left(\frac{\rho_t}{\rho} - 1\right) h = \left(\frac{1}{\eta} - 1\right) h$

$\longrightarrow h = \frac{H}{\frac{1}{\eta} - 1} = \frac{H}{\frac{5}{4} - 1} = 4H$

$\longrightarrow H = 10cm \rightarrow \boxed{h = 40cm}$