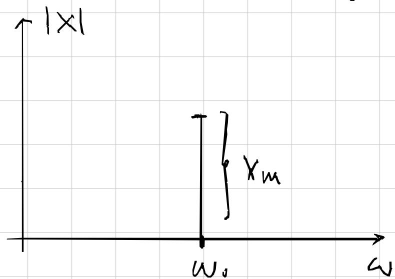
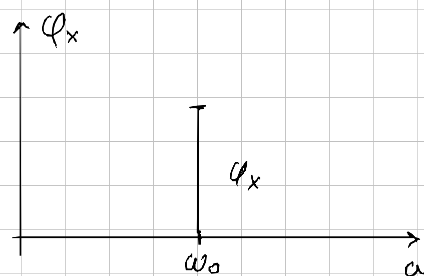
15. El filtri

- mijenja amplitudu i fazu frekvencijskih signala

Primjer: signal $x(t) = X_m \cdot \cos(\omega_0 t + \varphi_x) = Re[\underbrace{\dot{X}}_{\text{fazor sinusnog signala}} \cdot e^{j\omega_0 t}]$

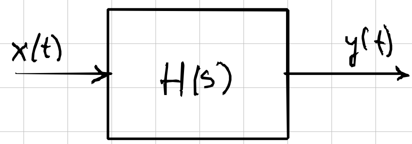
↳ amplitudu i fazu moguće prikazati kao funkcije frekvencije ω





Primjer: $x(t) \longrightarrow$ pobuda

$y(t) \longrightarrow$ odziv el. kruga



$$\left.\begin{array}{l} x(t) = X_m \cos(\omega_0 t + \varphi_x) \\ y(t) = Y_m \cdot \cos(\omega_0 t + \varphi_y) \end{array}\right\} \text{stacionarni sinusni signali} \rightarrow \left.\begin{array}{l} \dot{X} = X_m e^{j\varphi_x} \\ \dot{Y} = Y_m e^{j\varphi_y} \end{array}\right\} H = \frac{\dot{Y}}{\dot{X}}$$

$$H(s) = H(j\omega) = |H(j\omega)| \cdot e^{j\varphi(\omega)} \longrightarrow \dot{Y} = H(j\omega) \cdot \dot{X} \Rightarrow Y_m = |H(j\omega)| \cdot X_m$$

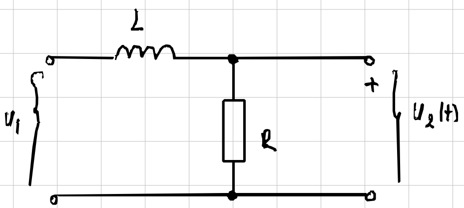
$$\varphi_y = \varphi(\omega) + \varphi_x$$

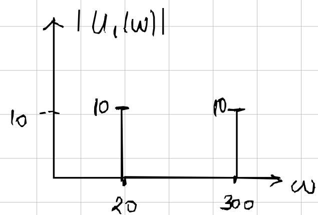
ako je $|H(j\omega)| \approx 1 \rightarrow Y_m \approx X_m$ propusni

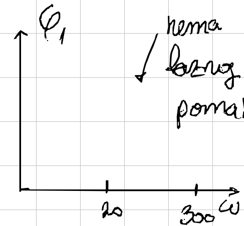
$|H(j\omega)| \approx 0 \rightarrow Y_m \approx 0$ gušenje

Primjer: Pobuda $u_1(t) = 10 \cdot \cos 20t + 10 \cos 300t$ (stacionarno

$R = 8\Omega \qquad L = 0{,}2\,H$







nema faznog pomaka

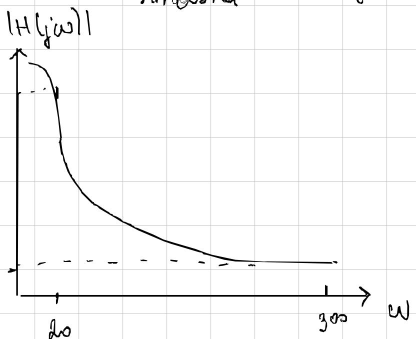
→ prijenosna f-ja:

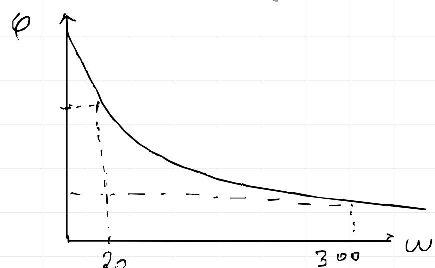
$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{R}{R+Ls} = \frac{8}{8+0{,}2s} = \frac{40}{40+s}$$

↳ Stacionarna sinusna: $H(j\omega) = \dfrac{40}{40+j\omega}$

$$|H(j\omega)| = \frac{40}{\sqrt{40^2+\omega^2}}$$

$$\varphi(\omega) = -\operatorname{arctg}\frac{\omega}{40}$$

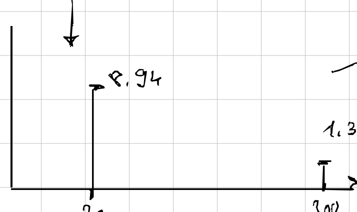




$|H(j20)| = 0.894 \qquad \varphi(20) = -26{,}6°$

$|H(j300)| = 0.132 \qquad \varphi(300) = -82{,}4°$

$u_2(t) = 8.94\cos(20t - 26.6°) + 1.32\cos(300t - 82{,}4°)$



→ nisko propusni filtar

→ RL filter

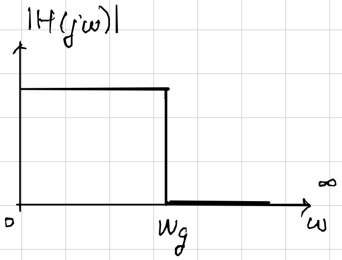
↳ propušta nisko frekv.

ne propušta visoke frekv.

Tipovi filtrara

NP – nisko propusni
VP – visoko propusni
PP – pojasno propusni
PB – pojasna brana

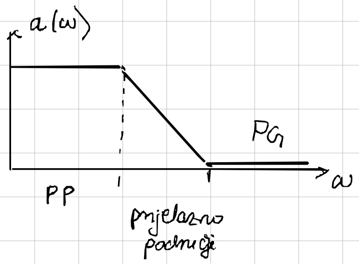
Nisko propusni filtar: $\boxed{NP}$



- propuštanje : $0 < \omega < \omega_g$
- gušenje : $\omega_g < \omega < \infty$

→ realne kar. to ne mogu ostvariti

→ u realnim filtrima nema oštre granice



Primjer: Opća prijenosna fja NP filtra 1. reda

$$H(s) = K \cdot \frac{\omega_g}{s + \omega_g}$$

→ pol: $s_p = -\omega_g$

nula: $s_0 \to \infty$

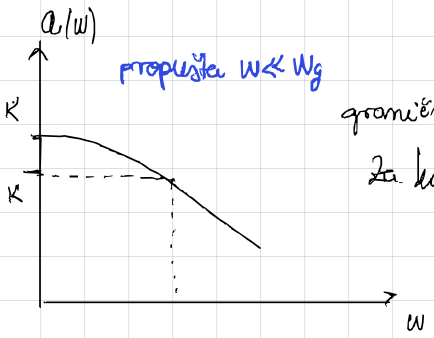
↓ frekv. kar.

$$H(j\omega) = K \cdot \frac{\omega_g}{j\omega + \omega_g} = K \cdot \frac{1}{j\frac{\omega}{\omega_g} + 1}$$

$$|H(j\omega)| = K \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_g}\right)^2}}$$

$$\varphi(j\omega) = -\operatorname{arctg}\left(\frac{\omega}{\omega_g}\right)$$

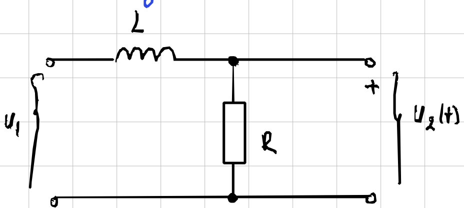
frekvencija pola i granična frekvencija



granična frekv. ona

za koju $a(\omega_g) = \frac{K}{\sqrt{2}} = 0.707K$

NP moguće realizirati RL četveropolom:



$$H(s) = \frac{U_2}{U_1} = \frac{R}{R + sL} = \frac{\frac{R}{L}}{s + \frac{R}{L}}$$

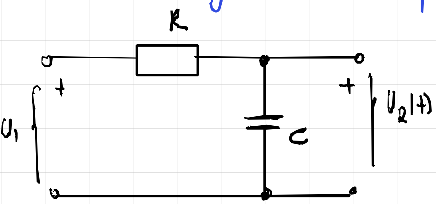
ω_g, $j\omega$, ω_g

→ $K \cdot \omega_g = \frac{R}{L}$

↓

$K = 1$

→ realizacija RC četveropolom:



$$H(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

ω_g

$K \cdot \omega_g = \frac{1}{RC}$ → $K = 1$

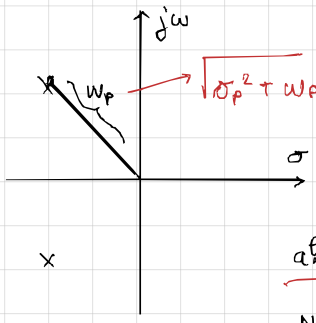
Opći oblik fje NP filtra 2. reda

$$H(s) = K \cdot \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

pojačanje

frekv. pola

faktor kvalitete pola



Polovi.

$$\sqrt{\sigma_p^2 + \omega_p}$$

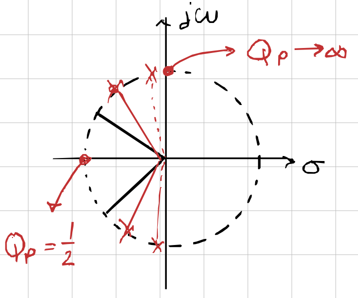
$$s_{p1,2} = \underbrace{\frac{-\omega_p}{2Q_p}}_{\sigma_p} \pm \boxed{j\omega_p\sqrt{1-\frac{1}{4Q_p^2}}}\ \tilde{\omega}_p$$

ako su kompleksni; $Q_p > 0{,}5$

Nule: $s_{o1,2} \to \infty$

Q_p - faktor kvalitete pola

$$Q_p = \frac{-\omega_p}{2\sigma_p} \longrightarrow \text{mjera udaljenosti pola od } j\omega \text{ osi}$$



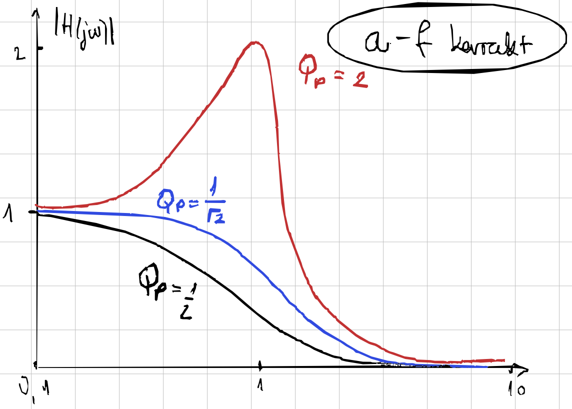
Q_p raste kada se pol približava $j\omega$ osi

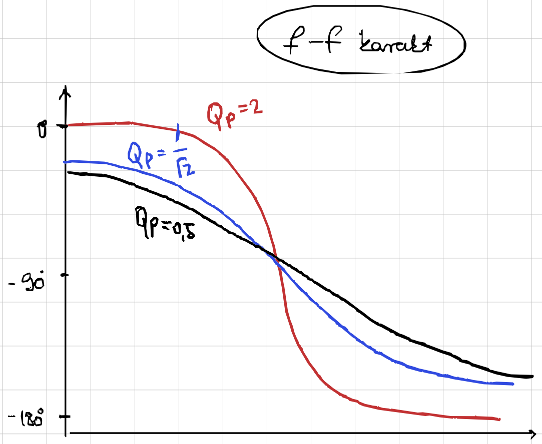
$$H(j\omega) = K \cdot \frac{\omega_p^2}{\omega_p^2 - \omega^2 + j\frac{\omega\omega_p}{Q_p}}$$

↓ a-f

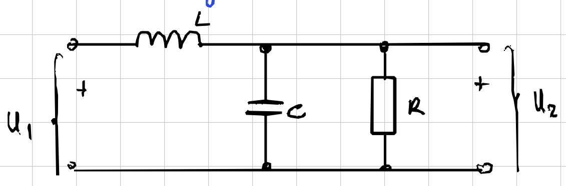
$$a(\omega) = |H(j\omega)| = K \frac{\omega_p^2}{\sqrt{(\omega_p^2-\omega^2)^2 + \left(\frac{\omega\omega_p}{Q_p}\right)^2}} = K \cdot \frac{\omega_p^2}{\sqrt{1-\left(\frac{\omega}{\omega_p}\right)^2 + \left(\frac{1}{Q_p}\cdot\frac{\omega}{\omega_p}\right)^2}}$$

→ više parametara → više krivulja





→ ostvarivanje RLC mrežom



$$H(s) = \frac{U_2}{U_1} \qquad U_2 = \frac{R \,\|\, \frac{1}{CS}}{R \,\|\, \frac{1}{CS} + LS} \cdot U_1$$

$$H(s) = \frac{\frac{R}{1+RSC}}{\frac{R}{1+RSC} + LS} = \frac{R}{R + LS + RCLS^2}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \qquad \omega_p^2 = \frac{1}{LC}$$

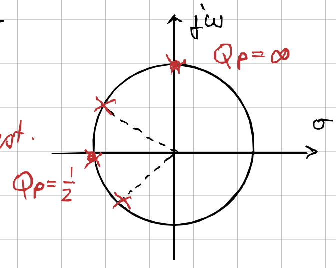
$$K \cdot \omega_p^2 = \frac{1}{LC}$$

$\hookrightarrow K = 1$

$$\frac{\omega_p}{Q_p} = \frac{1}{RC}$$

$$\to Q_p = \omega_p \cdot RC = R\sqrt{\frac{C}{L}}$$

$$\text{polovi: } s_{p1,2} = \frac{-1}{2RC} \pm j\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$$



promjenom R → promjena Q → ω_p = konst.

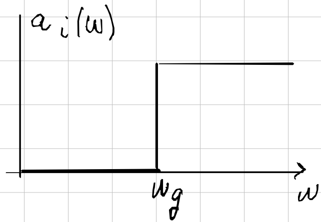
▶ polovi se pomiču po kružnici

3. red – kaskada I. i II. reda

4. red – kaskada II. + II. reda

Visokopropusni filtar VP

- propuštanje $\omega > \omega_g$
- gušenje $\omega < \omega_g$

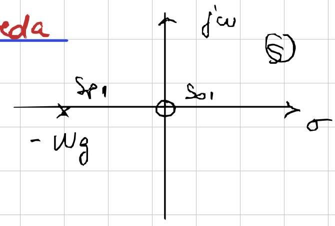


Najjednostavnija realizacija VP filtar 1. reda

$$H_{VP}(s) = K\cdot\frac{s}{s+\omega_g}$$

pol: $s_p = -\omega_g$

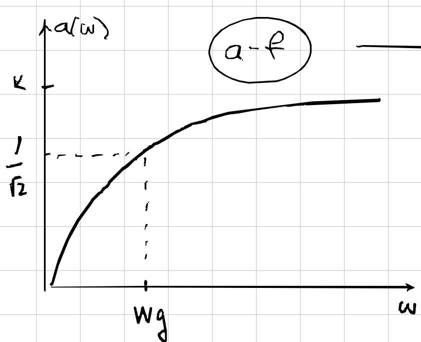
nula $s_0 = 0$



Frekv.: $H_{VP}(j\omega) = K\cdot\frac{j\omega}{j\omega+\omega_g}$

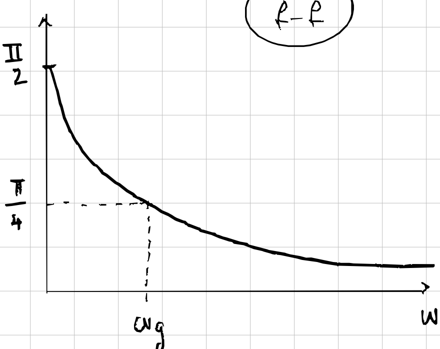
a-f

$$|H(j\omega)| = K\cdot\frac{|\omega|}{\sqrt{\omega^2+\omega_g^2}} = \frac{K\cdot\left|\frac{\omega}{\omega_g}\right|}{\sqrt{1+\left(\frac{\omega}{\omega_g}\right)^2}}$$

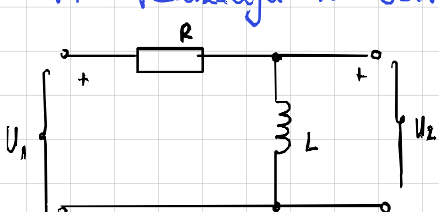


f-f

$$\varphi(\omega) = \frac{\pi}{2} - \mathrm{arctg}\left(\frac{\omega}{\omega_g}\right)$$



VP → realizacija RL četveropolom:



$$H(s) = \frac{U_2}{U_1} = \frac{s}{s+\frac{1}{RC}}$$

$\omega_g = \frac{1}{RC}$

$K = 1$

pol: $s_p = \frac{1}{RC}$

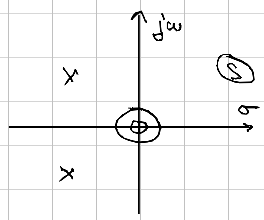
nula: $s_0 = 0$

VP filter 2. reda → daje bolju aprox idealnog filtra

↳ opći oblik : $H(s) = K \dfrac{s^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2}$

polovi : $s_{p1,2} = \dfrac{-\omega_p}{2Q_p} \pm j\omega_p \sqrt{1 - \dfrac{1}{4Q_p}}$

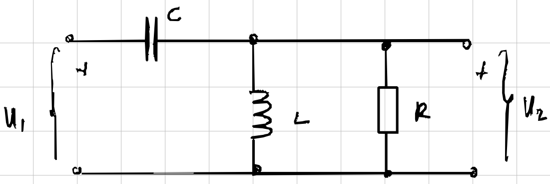
nule : $s_{0_{1,2}} = 0$



→ ako su kompleksni $[Q_p > 0{,}5]$!

ako je manji onda su na Re osi

Realizacija VP filtra 2. reda RLC četveropolom



$H(s) = \dfrac{U_2}{U_1} = \dfrac{\left(\frac{1}{R} + \frac{1}{Ls}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{Ls}\right)^{-1} + \frac{1}{Cs}}$

$H(s) = \dfrac{\frac{Ls+R}{RLs}}{\frac{Ls+R}{RLs} + \frac{1}{Cs}} = \dfrac{\frac{Ls+R}{\cancel{RLs}}}{\frac{Cs(Ls+R)+RLs}{Cs \cdot \cancel{RLs}}}$

$H(s) = \dfrac{(Ls+R)Cs}{CLs^2 + RCs + RLs} = \dfrac{Ls+R}{s^2 + \frac{Rs}{L} + \frac{Rs}{C}}$

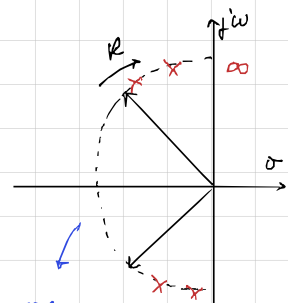
... $\rightarrow H(s) = \dfrac{s^2}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$

$\omega_p^2 = \dfrac{1}{LC}$

$\underline{\underline{K = 1}}$

$\dfrac{\omega_p}{Q_p} = \dfrac{1}{RC} \rightarrow \sqrt{\dfrac{1}{LC}} \cdot \dfrac{1}{Q_p} = \dfrac{1}{RC}$

↳ $Q_p = R\sqrt{\dfrac{C}{L}}$



promjenom R

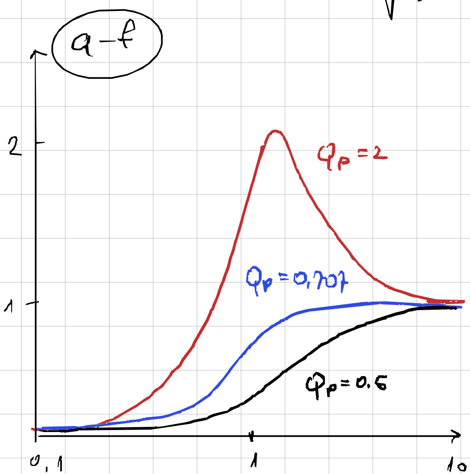
mijenja se Q_p po kružnici

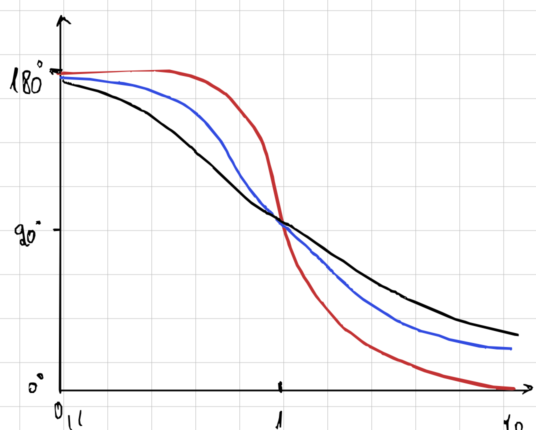
Frekv. kar:

$H(j\omega) = K \cdot \dfrac{-\omega^2}{-\omega^2 + j\omega \cdot \frac{\omega_p}{Q_p} + \omega_p^2}$

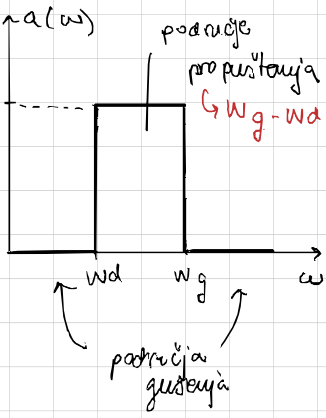
↳ $a(\omega) = |H(j\omega)| = K \cdot \dfrac{\omega^2}{\sqrt{(\omega_p^2 - \omega^2)^2 + \left(\frac{\omega\,\omega_p}{Q_p}\right)^2}}$, $\varphi(\omega) = \pi - \mathrm{arctg}\left(\dfrac{\frac{\omega_p\,\omega}{Q_p}}{\omega_p^2 - \omega^2}\right)$

iz bojnika



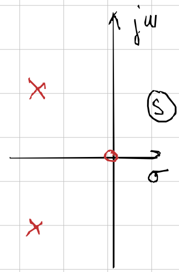


Pojasno propusni filter (PP) $\rightarrow$ ima donju i gornju ω_g



! moramo imati najmanje drugi red
$\downarrow$
PP filtar 2. reda

$$H_{PP}(s) = K \frac{s \cdot \frac{\omega_p}{Q_p}}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$



polovi: $-\frac{\omega_p}{2Q_p} \pm j\omega_p \sqrt{1 - \frac{1}{4Q_p}}$

nule: $s_{o_1} = 0$, $s_{o_2} \rightarrow \infty$

prijenosna fja $H_{pp}(j\omega)$

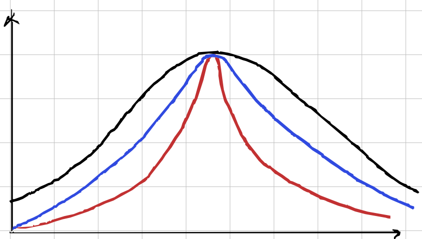
$$H_{pp}(j\omega) = K \frac{j \frac{\omega \omega_p}{Q_p}}{-\omega^2 + j \frac{\omega \omega_p}{Q_p} + \omega_p^2} = \frac{K}{1 + jQ_p\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)}$$

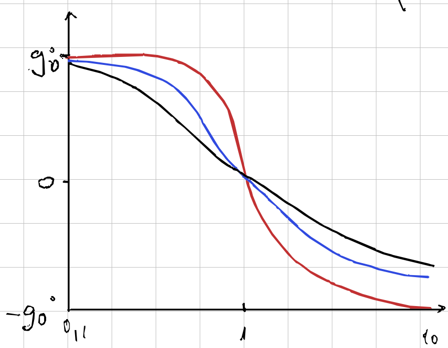
a-f

$$|H(j\omega)| = \frac{K}{\sqrt{1 + \left(Q_p\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)\right)^2}}$$

f-f

$$\varphi(\omega) = \frac{\pi}{2} - \mathrm{arctg}\left(Q_p\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)\right)$$





$\llcorner$ max $\rightarrow$ $\omega = \omega_p$ $\rightarrow$ postiže se K

– filtar ne propušta signale vrlo niskih i vrlo visokih frekv.

kako naći $\omega_{granično}$? $\longrightarrow$ $a_{pp}(\omega_d) = a_{pp}(\omega_g) = \frac{a_{pp}(\omega_p)}{\sqrt{2}} = \frac{K}{\sqrt{2}}$

$$|H(j\omega)| = \frac{K}{\sqrt{2}} \longrightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(Q_p\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)\right)^2}}\Bigg|_{\omega_{d,g}}$$

$\downarrow$
širina propusnog je $\omega_g - \omega_d = B$
$\downarrow$
$B = \frac{\omega_p}{Q_p}$

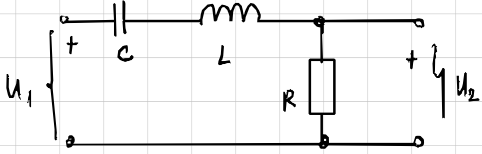
$\omega_p^2 = \omega_d \cdot \omega_g$ $\rightarrow$ geom. sredina od ω_d i ω_g

$\omega_p = \omega_c$ $\longrightarrow$ centralna frekv.

$\Rightarrow$ u praksi $B \ll \omega_c$ $\longrightarrow$ filteri za koje vrijedi $Q = \frac{\omega_c}{B} \geq 10$

tada je $\omega_{g,d} \cong \omega_p \pm \frac{\omega_p}{2Q_p} = \omega_p \pm \frac{1}{2} B$

Realizacija PP 2. reda serijskim RLC krugom



$$H(s) = \frac{R}{R + Ls + \frac{1}{Cs}} = \frac{s \cdot \frac{R}{L}}{s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}}$$

$\frac{\omega_p}{Q_p}$ $\qquad$ ω_p^2

$$\frac{1}{\sqrt{LC}} \cdot \frac{1}{Q_p} = \frac{R}{L}$$

$$\longrightarrow Q_p = \frac{1}{R}\sqrt{\frac{L}{C}}$$

a-f

$$H(j\omega) = \frac{j\omega \cdot \frac{R}{L}}{-\omega^2 + j\omega \cdot \frac{R}{L} + \frac{1}{LC}}$$

$$\hookrightarrow |H(j\omega)| = \frac{\omega \frac{R}{L}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega \cdot \frac{R}{L}\right)^2}}$$

rezonantna: $\omega = \omega_p = \frac{1}{\sqrt{LC}}$

$\longrightarrow$ za $R + L \| C$ je isto bla bla

Primjer: $20\,kHz \pm 250\,Hz$

$\downarrow \omega_p$ $\qquad$ $\downarrow + \omega_g$, $- \omega_d$

$L = 1\,mH$

$R, C = ?$

$$B = (250 - (-250))\,Hz \longrightarrow B = 500\,Hz$$

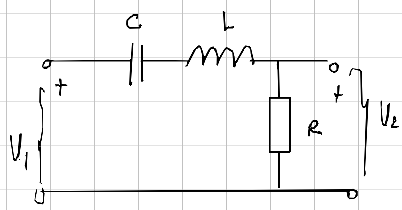
$$Q_p = \frac{\omega_p}{B} = \frac{20 \cdot 10^3}{500} = 40$$

$\hookrightarrow$ uskopojasni filter (10 je granica)

$f_g = 20.25\,kHz$

$f_d = 19,75\,kHz$

$f_e = \sqrt{f_g f_d} = 19,998\,kHz$



$$H(s) = \frac{s \cdot \frac{R}{L}}{s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}}$$

ω_p

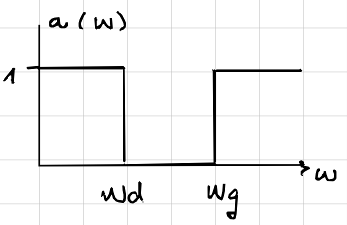
$$\omega_p = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_p^2 L}$$

$$C = \frac{1}{(2\pi \cdot 19,998)^2 \cdot 10^{-3}} = \boxed{63,3\,nF}$$

$$Q_p = \omega_p \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

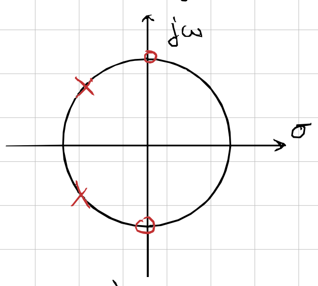
$$\hookrightarrow R = \frac{1}{Q_p} \cdot \sqrt{\frac{L}{C}} = \frac{1}{40}\sqrt{\frac{10^{-3}}{63,3 \cdot 10^{-9}}} \longrightarrow \boxed{R = 394,9\,\Omega}$$

Pojasna brana PB



→ dva područja propuštanja

$$H_{PB}(s) = k\frac{s^2+\omega_p^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

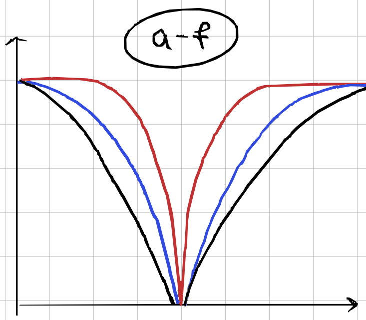


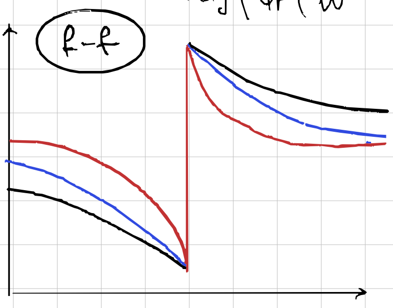
polovi: $s_{p1,2} = -\frac{\omega_p}{2Q_p} \pm j\omega_p\sqrt{1-\frac{1}{4Q^2}}$

nule: $s_{o1,2} = \pm j\omega_p$

$$\Rightarrow H_{PB}(j\omega) = \frac{K(\omega_p^2-\omega^2)}{\omega_p^2-\omega^2+j\frac{\omega\omega_p}{Q_p}} = K\frac{jQ_p\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)}{1+jQ_p\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)}$$

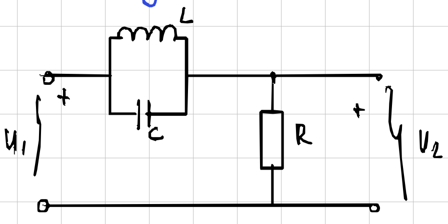
- $a_{PB}(\omega) = K\cdot\frac{Q_p\left|\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right|}{\sqrt{1+Q_p^2\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)^2}}$
- $\varphi_{PB}(\omega) = \mathrm{arctg}\left(KQ_p\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)\right) - \mathrm{arctg}\left(Q_p\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)\right)$





- propušta vrlo niske i vrlo visoke frekv.
- granične frekv. ω_d i ω_g dobivamo: $a_{PB}(\omega_d) = a_{PB}(\omega_g) = \frac{K}{\sqrt{2}}$

Realizacija PB sa RLC četveropolom



$$H(s) = \frac{s^2+\frac{1}{LC}}{s^2+s\cdot\frac{1}{RC}+\frac{1}{LC}}$$

$\omega_p = \frac{1}{\sqrt{LC}}$

$Q_p = RC\omega_p \rightarrow R\sqrt{\frac{C}{L}}$

$K = 1$

a-f → sve na isti način