

# AKTIVNI EL. FILTRI

## Filtiri s aktivnim elementima

- filtri s pasivnim R, L, C elementima imaju nedostatke

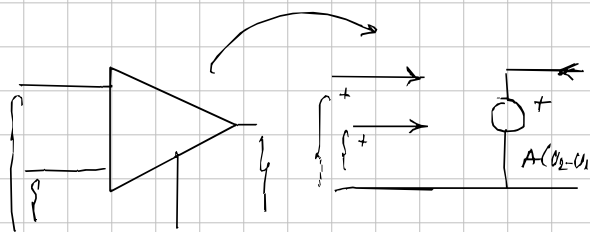
- karakt. filtra ovise o opterećenju

- prednost aktivnih filtera pred pasivnim RLC

- ne sadrže induktivite i njihove nedostatke
- moguće visoko odzivanje pojedinih stupnjeva filtra
- jednostavnije podešavanje karakteristika
- pojedini & niske frekv.

### PNEDOSTATCI

- nenačinost O.P.
- poteškoće pri visokim frekv
- potreba za izvorom napajanja



### Aktivni filtri 1. reda

#### NP filter s operacijskim pojačalom

opći oblik:  $H_{NP}(s) = \frac{k \cdot \omega_g}{s + \omega_g}$

$k = 1 + \frac{R_F}{R_0} \rightarrow$  pojačavajući pojačalo

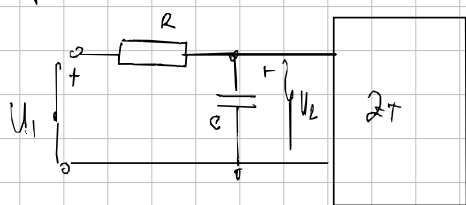
• očito da je  $k > 1$

$\omega_g$  - konstanta pasivne mreže

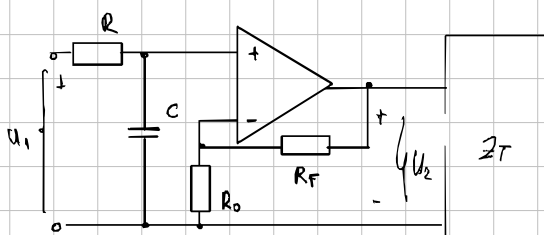
$$H(s) = \left(1 + \frac{R_F}{R_0}\right) \cdot \frac{\frac{1}{R_C}}{s + \frac{1}{R_C}}$$

O.P.  $\rightarrow$  vrlo niski izlazni otpor (idealno jednak nuli)

usporedba:



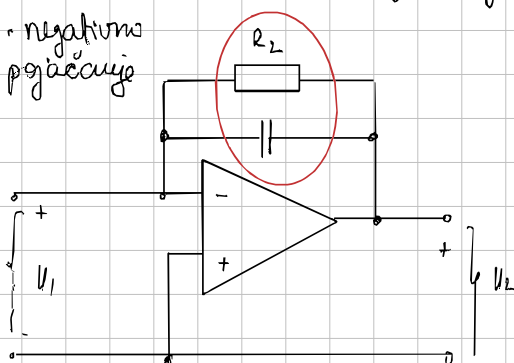
$$H(s) = \frac{\frac{1}{R_C}}{s + \frac{1}{R_C} + \frac{1}{C \cdot Z_T}} \rightarrow \text{direktno mjerenje } \omega_g$$



$$H(s) = \left(1 + \frac{R_F}{R_0}\right) \cdot \frac{\frac{1}{R_C}}{s + \frac{1}{R_C}} \rightarrow \text{ne mjerenja se}$$

### ALTERNATIVNO: neinvertirajuće pojačalo

• negativno pojačanje



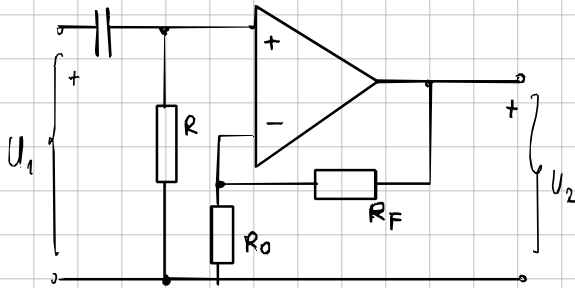
$$H(s) = -\frac{R_2}{R_1} \cdot \frac{\frac{1}{R_2 C}}{s + \frac{1}{R_2 C}}$$

$$k = -\frac{R_2}{R_1} \quad \omega_g = \frac{1}{R_2 C}$$

$$\omega_g = \frac{1}{R_2 C}$$

## VP filter 1. reda s operacijskim pojačalom

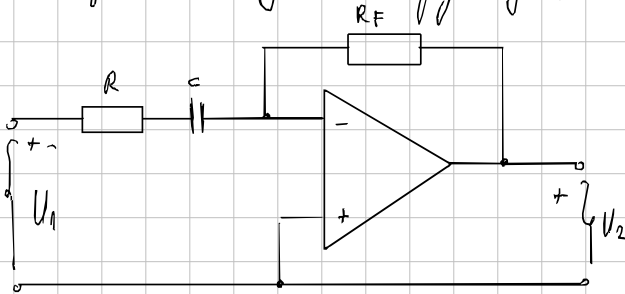
- neinvertirajući spoj O.P.



$$H(s) = \left(1 + \frac{R_F}{R_0}\right) \cdot \frac{s}{s + \frac{1}{RC}}$$

$$H_{VP}(s) = k \cdot \frac{s}{s + \omega_g} \rightarrow k = 1 + \frac{R_F}{R_0} > 1$$
$$\omega_g = \frac{1}{RC}$$

→ invertirajući (s negativnim pojačanjem)



$$H(s) = -\frac{R_F}{R} \cdot \frac{s}{s + \frac{1}{RC}}$$

$$k = -\frac{R_F}{R} \quad \omega_g = \frac{1}{RC}$$

• ponekad je dovoljno filter 1. reda realizirati kao pasivan RC filter  
s jediničnim pojačalom na izlazu

98. slajd, 17. ppt

## NP filter 2. reda

→ moguće ekstrahirati jedinice pojačalom

$$H_{NP}(s) = k \cdot \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

prjenosna funkcija

$$H(s) = \frac{\left(1 + \frac{G_0}{Q_F}\right) \frac{G_1 G_2}{C_1 C_2}}{s^2 + s \left[ \frac{G_1 + G_2}{C_1} + \frac{G_2}{C_2} \cdot \frac{G_0}{Q_F} \right] + \frac{G_1 G_2}{C_1 C_2} \omega_p^2}$$

⇒ usporedba s općom:  $\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$   $k = 1 + \frac{G_0}{Q_F}$   $Q_p = \frac{\sqrt{\frac{C_1 G_1}{C_2 G_2}}}{1 + \frac{G_1}{G_2} + (1-k) \cdot \frac{C_1}{C_2}}$

Primjer: Realizirati NP prjenosnu f.kju

$$H(s) = \frac{2}{s^2 + s + 1} = \frac{k \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$$k = 2$$

$$\omega_p = 1$$

$$Q_p = 1$$

3 uvjeta treba ispuniti

naša normirana vrijednost

čest 1. korak:  $C_1 = C_2 = C = 1$

$$\omega_p = 1 = \sqrt{\frac{G_1 \cdot G_2}{C=1}} = \sqrt{G_1 \cdot G_2}$$

$$G_1 = \frac{1}{G_2} \rightarrow G_2 = \frac{1}{G_1}$$

$$Q_p = \frac{G_1}{1 + G_1^2 + 1 - k} = \frac{G_1}{G_1^2 + 2 - k} = 2$$

$$Q_p = \frac{\sqrt{\frac{C}{C} \cdot \frac{G_1}{G_2}}}{1 + \frac{G_1}{G_2} + (1-k) \cdot 1} = \frac{\sqrt{G_1^2}}{1 + G_1^2 + (1-k)} = 2$$

$$k = 1 + \frac{G_0}{Q_F} = 2$$

$$Q_p = \frac{G_1}{G_1^2} = \frac{1}{G_1} = 1 \rightarrow G_2 = 1$$

$$\frac{G_0}{Q_F} = 1 \rightarrow G_0 = Q_F$$

$$Q_p = \sqrt{\frac{R_1}{R_2}}$$

$$\omega_p = \frac{1}{C \sqrt{R_1 R_2}}$$

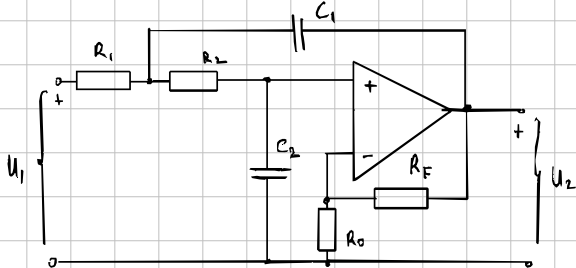
$$\frac{\omega_p}{Q_p} = \sqrt{\frac{R_2}{R_1}} \cdot \frac{C}{\sqrt{R_1 R_2}} = \frac{C}{R_1}$$

$$R_1 = \frac{Q_p C}{\omega_p} = 1$$

$$R_2 = \frac{k_1}{Q_p^2} = 1$$

$$\Rightarrow R_1 = R_2 = R_0 = R_F = 1$$

$$C_1 = C_2 = 1$$

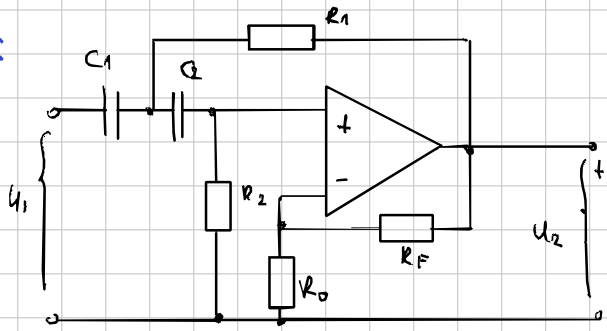


## VP Rhtar 2. reda

$$H_{VP}(s) = k \cdot \frac{s^2}{s^2 + \frac{\omega_p}{Q} s + \omega_p^2}$$

⇓

$$H(s) = \frac{\left(1 + \frac{G_0}{G_F}\right) s^2}{s^2 + s \left[ G_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right) - \frac{G_1 G_0}{C_1 G_F} \right] + \frac{G_1 G_2}{C_1 C_2}}$$



$$\begin{aligned} k &= 1 + \frac{G_0}{G_F} \\ \omega_p &= \sqrt{\frac{G_1 G_2}{C_1 C_2}} \quad Q = \frac{\sqrt{\frac{C_1 G_1}{C_2 G_2}}}{1 + \frac{C_1}{C_2} - \frac{G_1 G_0}{C_1 G_F}} \end{aligned}$$

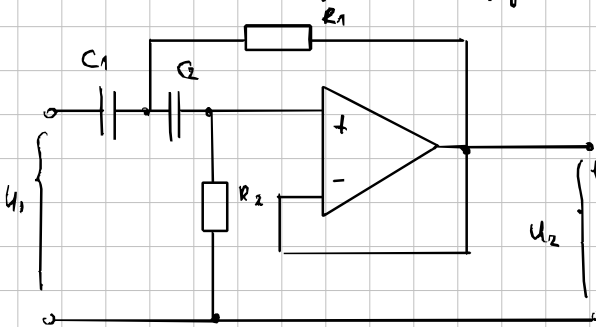
3 umjetka

6 parametara el, 3 umjetka

→ jednaki C ili jednaki R

→ ili  $k=1$

→ česta upotreba jediničnog pojačala (naponskog sledi)  $k=1$  →  $G_0 = \infty$  ili  $(R_F = \infty)$



$$k=1 \quad \omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$Q = \frac{\sqrt{\frac{C_1 G_1}{C_2 G_2}}}{1 + \frac{C_1}{C_2}}$$

Primer:  $Q=2$ ,  $\omega_p=10^3$  r/s odredite el. za konf  $R_1=R_2$  i  $C_1=C_2$

$$k = 1 + \frac{G_0}{G_F} \quad R_1=R_2 \rightarrow G_1=G_2 \Rightarrow R_1=R_2=R$$

$$\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}} = \sqrt{\frac{1}{R^2 \cdot C^2}} = \frac{1}{RC}$$

$$Q = \frac{\sqrt{\frac{C_1}{C_2} \frac{G_1}{G_2}}}{1 + \frac{C_1}{C_2} - \frac{C_1}{C_2} \frac{G_0}{G_F}} = \frac{\sqrt{1 \cdot 1}}{1 + 1 - 1 \cdot \frac{G_0}{G_F}}$$

$$Q = \frac{1}{2 - \frac{G_0}{G_F}} = 2 \rightarrow 1 = 4 - 2 \frac{G_0}{G_F} \rightarrow \frac{G_0}{G_F} = \frac{3}{2} \rightarrow \frac{R_F}{R_0} = \frac{3}{2}$$

$\hookrightarrow \boxed{K = \frac{5}{2}}$

pretpostavimo:  $R_0 = 10^3$

$$R_F = \frac{3}{2} R_0 = \frac{3}{2} \cdot 10^3 = \boxed{1.5 \text{ k}\Omega = R_F}$$

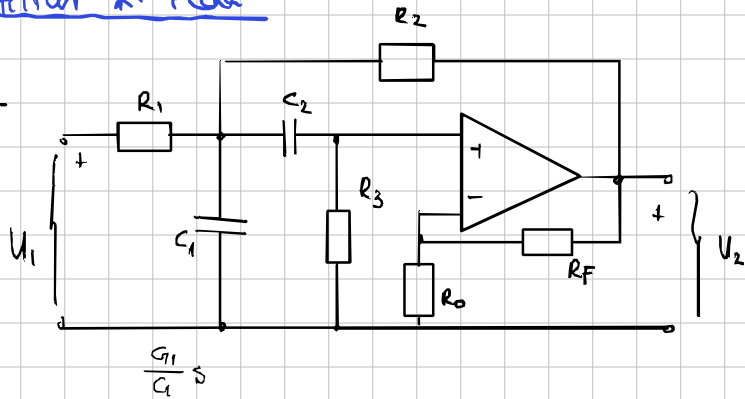
$$\omega_p = \frac{1}{RC} = 10^3 \rightarrow RC = \frac{1}{10^3}$$

$R = 10^3 \Omega \rightarrow$  otpor normalizacije

$$C = \frac{1}{10^3 \cdot 10^3} \rightarrow \boxed{C = 10^{-6} \text{ F}}$$

## P.P. filter 2. reda

$$H_{PP}(s) = k \frac{\omega_P}{Q_P s} \frac{1}{s^2 + \frac{\omega_P}{Q_P} s + \omega_P^2}$$



$$H(s) = \left(1 + \frac{R_F}{R_0}\right) \cdot \frac{\frac{G_1}{C_1} s}{s^2 + s \left[ \frac{G_3}{C_1} + \frac{G_3}{C_2} + \frac{G_1 + G_2}{C_1} - \left(1 + \frac{G_0}{G_F}\right) \frac{G_2}{C_1} \right] + \frac{(G_1 + G_2) G_3}{G_1 G_2}}$$

$$\omega_P = \sqrt{\frac{(G_1 + G_2) G_3}{C_1 C_2}}$$

$$Q_P = \frac{\sqrt{\frac{G_3}{G_1 + G_2} \cdot \frac{C_2}{C_1}}}{\frac{G_3}{G_1 + G_2} \cdot \left(1 + \frac{C_2}{C_1}\right) + \frac{C_2}{C_1} - \left(1 + \frac{G_0}{G_F}\right) \cdot \frac{G_2}{C_1 + G_2}}$$

## Prinzip: projektace

→ máme málo vstupů od elementů → můžeme prvky vzít d.  
ili dodat nové výstky

Dod výstky:  $G = C_2$  i  $R_F = R_0$

$$Q_P = \frac{\omega_P}{\frac{G_3}{C_1} + \frac{G_3}{C_2} + \frac{G_1 + C_2}{C_1} - \left(1 + \frac{G_0}{G_F}\right) \cdot \frac{G_2}{C_1}} = \frac{C \omega_P}{2G_3 + G_1 - G_2}$$

$$\omega_P = \sqrt{\frac{(G_1 + G_2) G_3}{C_1 C_2}} = \frac{\sqrt{(G_1 + G_2) G_3}}{C}$$

$$k = \frac{2 Q_P}{R_1 C \omega_P}$$

4 neznám.  
→ ale 1 prvky  
npr odlehneno C

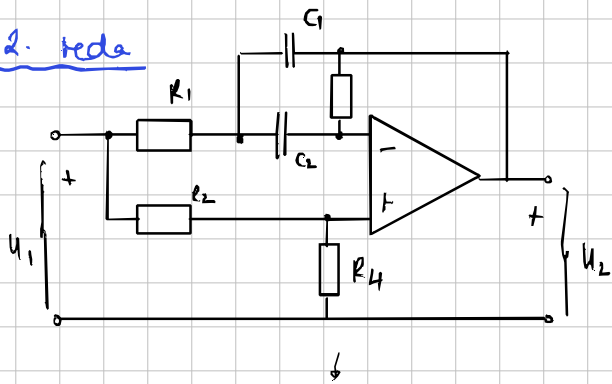
$$R_1 = \frac{2 Q_P}{k \omega_P C}$$

$$R_3 = \frac{k^2 R_1 (R_1 + R_2)}{4 Q_P^2 R_2}$$

$$R_2 = \frac{k R_1}{\sqrt{(k-1)^2 + 8 Q_P^2} - 1}$$

## P.B 2. reda

$$H_{PB}(s) = K \cdot \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$



Realizacija: nule na jw-osi

→ ako brojnik ne sadrži član sa  $s^n$

$$H_P(s) = K \cdot \frac{s^2 + \frac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

• Za pojačanu branu mora biti međuji član brojnika jednak nuli, tj  $Q_z \rightarrow \infty$

+ uočljivo da vrijedi  $\omega_z = \omega_p$

$$H(s) = \frac{G_3}{G_3 + G_4} \cdot \frac{s^2 + s \left[ G_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right) - \frac{G_1 G_4}{C_1 G_3} \right] + \frac{G_1 G_2}{C_1 C_2}}{s^2 + s \left[ G_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \right] + \frac{G_1 G_2}{C_1 C_2}}$$

×  $Q_z \rightarrow \infty$

$$\rightarrow G_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{G_1 G_4}{C_1 G_3} \Rightarrow H(s) = \frac{G_3}{G_3 + G_4} \cdot \frac{s^2 + \frac{G_1 G_2}{C_1 C_2}}{s^2 + s G_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right) + \frac{G_1 G_2}{C_1 C_2}}$$

$$\underbrace{\omega_p = \sqrt{\frac{G_1 G_2}{C_1 C_2}} \quad Q_p = \frac{\omega_p}{G_2} \frac{C_1 C_2}{C_1 + C_2} \quad K = \frac{G_3}{G_3 + G_4}}_{\text{uvijeti}}$$

Primjer: Realizacija PB za dimenziju signala 50Hz,  $Q_p = 2$ ,  $C_1 = C_2 = C$

$C_1 = C_2 = C = 10 \mu F \rightarrow$  proizvoljno

$$\omega_p = \frac{1}{C} \sqrt{G_1 G_2} = 2 \cdot \pi \cdot 50 \rightarrow 10 \cdot 10^{-6} \cdot 100 \cdot \pi = \sqrt{G_1 G_2}$$

$$\frac{1000}{\pi} = \sqrt{R_1 R_2} = 318.3 \Omega$$

$$Q_p = \frac{\omega_p}{G_2} \frac{C^2}{2C} = \frac{\omega_p \cdot C}{2G_2}$$

$$Q_p = \frac{\sqrt{G_1 G_2}}{2G_2} = \frac{1}{2} \sqrt{\frac{G_1}{G_2}} = 2 \rightarrow \sqrt{\frac{G_1}{G_2}} = 4 \rightarrow \sqrt{\frac{R_2}{R_1}} = 4 \rightarrow \begin{matrix} R_1 = 1.273 k\Omega \\ R_2 = 79.57 k\Omega \end{matrix}$$

× uvjet za pojačanu branu

$$G_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{G_1 G_4}{C_1 G_3} = \frac{2 G_2}{C} = \frac{G_1}{C} \frac{G_4}{G_3}$$

$$2 \frac{G_2}{G_1} = \frac{G_4}{G_3} \rightarrow \frac{1}{8} = \frac{G_4}{G_3} \rightarrow 8R_3 = R_4$$

→ uzmemo proizvoljno  
 $R_3 = 1 k\Omega$

$$\Rightarrow R_4 = 8 k\Omega$$