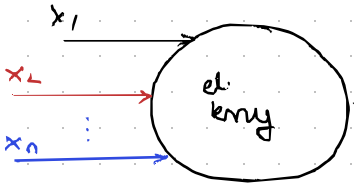
4.2. TEOREMI MREŽA

1. TEOREM SUPERPOZICIJE

- ako imamo više izvora → ukupni odziv je suma svih odziva

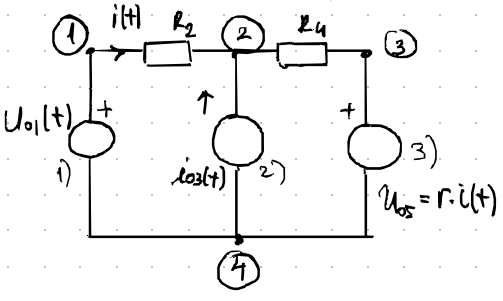


$$y = y_1 + y_2 + \dots + y_n$$

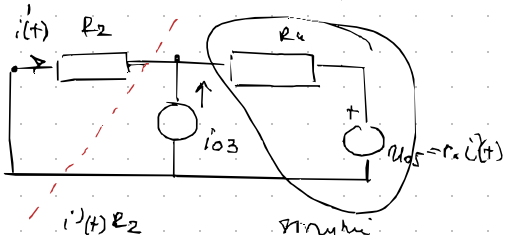
$$y(t) = \sum y_k(t)$$

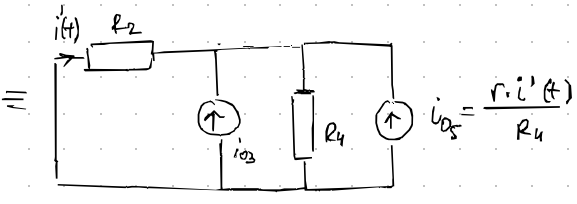
→ promatramo pojedini izvor isključivanjem svih ostalih izvora

Primjer: $i(t)$ primjenom TM superpozicije $R_2 = 3\,\Omega$ $R_4 = 2\,\Omega$ $r = 3\,\Omega$ $u_{01} = 24\,V$ $i_{03} = 7\,A$



1) isključimo 1 izvor, npr lijevi





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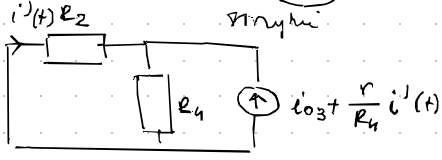

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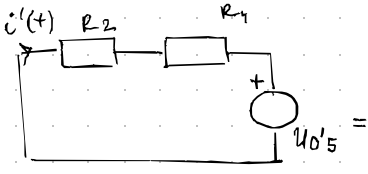
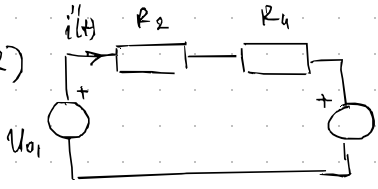


$$u_{05}' = R_4 \cdot i_{03} + r \cdot i'(t) \Rightarrow i'(t) = \frac{u_{05}'}{R_2 + R_4} \dots = \frac{-7}{4}\,A$$

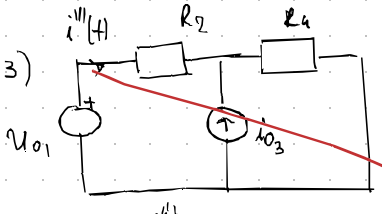
2)



$$\rightarrow i''(t) = \frac{u_{01} - u_{05}}{R_2 + R_4} = \dots = \frac{12}{4}\,A$$

$$\Rightarrow i = \frac{5}{4}\,A$$

3)



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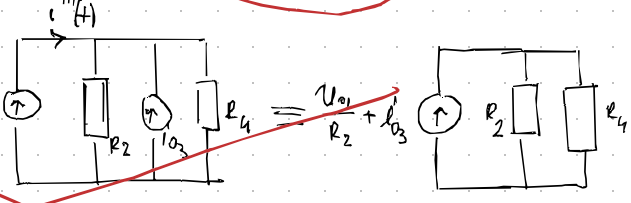

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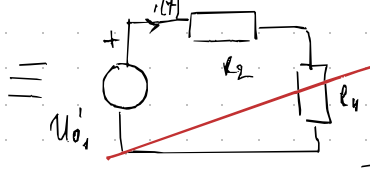


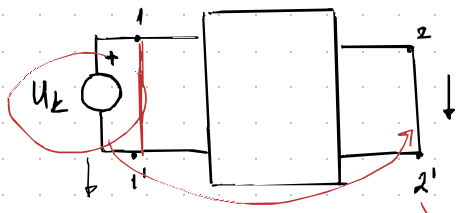
$$u_{01}' = u_{01} + i_{03} \cdot R_2$$

$$\rightarrow i''(t) = \frac{u_{01} + i_{03} \cdot R_2}{R_2 + R_4} = \dots =$$

NE JER JE TO OVISAN IZVOR

2. TEOREM RECIPROČNOSTI

→ odnosi za pasivne krugove sastavljene od R, L, C, M idealnih transformatora jednog onimog naponskog / strujnog izvora



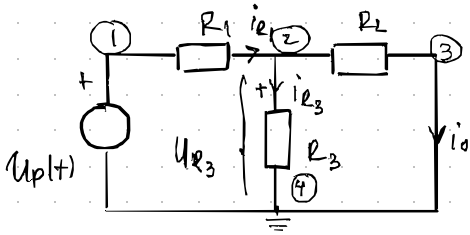
napon kao pobuda u k-toj grani

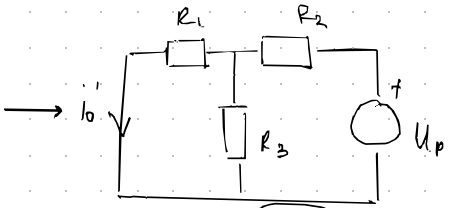
struja kao odziv u j-toj grani

Zamjena mjesta pobude i odziva

$U_j = U_k$, ako je $I_k = I_j$ → krug je reciprocan

Primjer: Pokazati da je krug reciprocan





→ bez obzira na R_1 i R_2, ovaj spoj je uvijek reciprocan

ako je $I_0 = I_0'$? → reciprocan je

1. korak

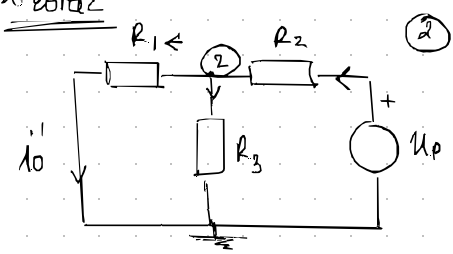
(2): $-i_{R_1} + i_{R_3} + i_0 = 0$

$$i_{R_1} = \frac{U_{p(t)} - U_2}{R_1} \qquad i_{R_3} = \frac{U_2}{R_3} \qquad i_0 = \frac{U_2}{R_2} \Rightarrow \frac{U_2 - U_p}{R_1} + \frac{U_2}{R_3} + \frac{U_2}{R_2} = 0$$

$$U_2\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2}\right) = \frac{U_p}{R_1} \longrightarrow I_0\left(\frac{R_2}{R_1} + \frac{R_2}{R_3} + 1\right) = \frac{U_p}{R_1}$$

$$I_0 = U_p \cdot \frac{1}{R_1}\left(\frac{R_1}{R_2} + \frac{R_3}{R_2} + 1\right) \Rightarrow i_0 = U_p\left(\frac{1}{R_2} + \frac{R_3}{R_1 R_2} + \frac{1}{R_1}\right)$$

2. korak



(2): $\frac{U_2}{R_1} - \frac{U_2 - U_p}{R_2} + \frac{U_2}{R_3} = 0$

$$U_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{U_p}{R_2}$$

$$i_0'\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = \frac{U_p}{R_2}$$

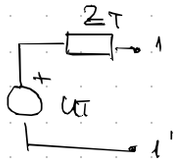
$$i_0' = \frac{U_p}{R_2}\left(\frac{R_3}{R_1} + \frac{R_2}{R_1} + 1\right) = U_p\left(\frac{R_3}{R_1 R_2} + \frac{1}{R_1} + \frac{1}{R_2}\right)$$

isto

$i_0 = i_0'$ ✓

3. THEVENINOV TEOREM

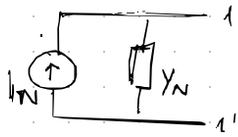
- odnosi na aktivne mreže s 1 priključkom
- moguće odrediti ekvivalentan izvor → Z_T, U_T



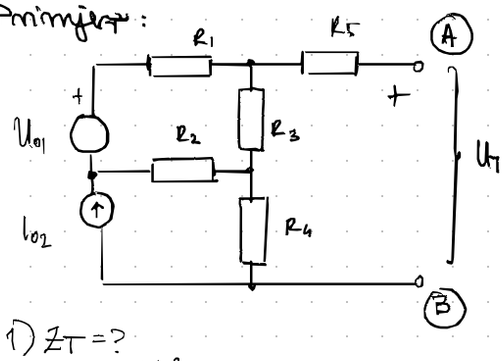
odgovarajuće međusobne transformacije izvora

4. NORTONOV TEOREM

→ strujni izvor i serijski spojena admitancija →



Primjer:



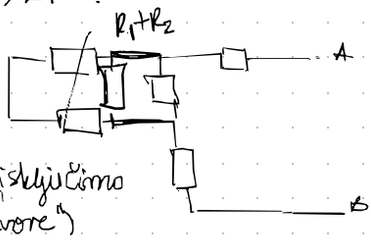
Thevenina i Nortona?

$U_{01} = 18V$ $R_1 = 4k\Omega$ $R_3 = 3k\Omega$

$I_{02} = 12mA$ $R_2 = 2k\Omega$ $R_4 = 1k\Omega$

$R_5 = 4k\Omega$

1) $Z_T = ?$



(isključimo izvore)

$$Z_T = R_5 + R_4 + \left(\frac{1}{R_1+R_2} + \frac{1}{R_3}\right)^{-1}$$

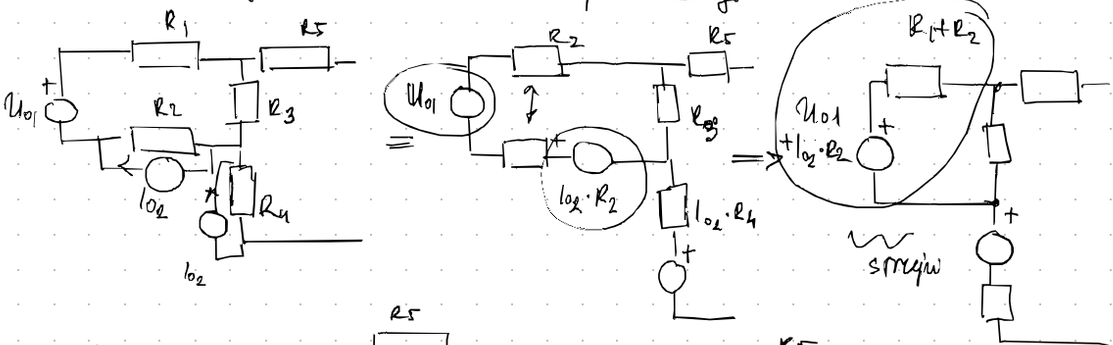
$$Z_T = 7k\Omega$$

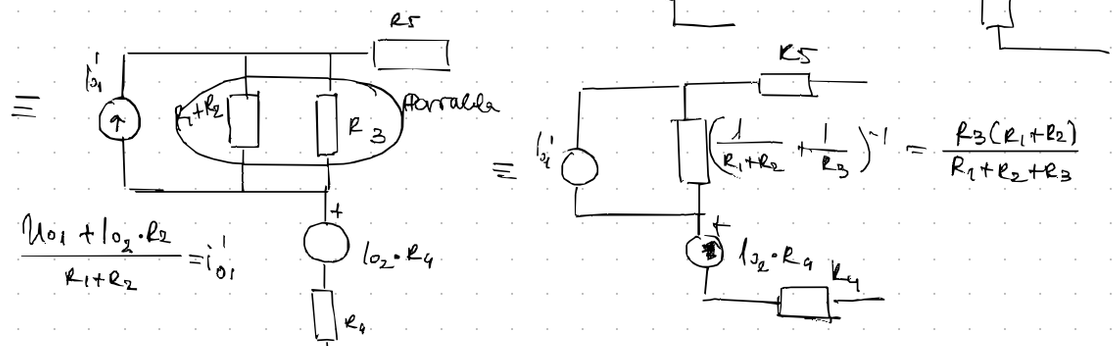
2) $U_T = ?$ *Superpozicija ili *pomicanjem *trauf

$U \rightarrow$ strujni prekid R_1

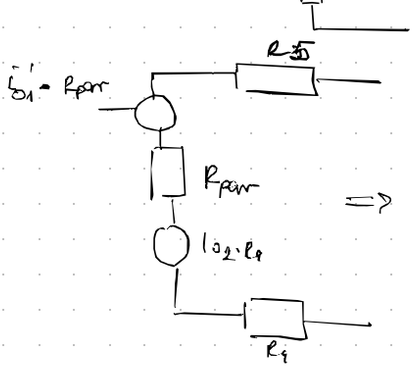
$I_{02} \rightarrow$ u naponski ne možemo transf. (direktno) → pomicanje

* malo za promjenu ; pomicanje

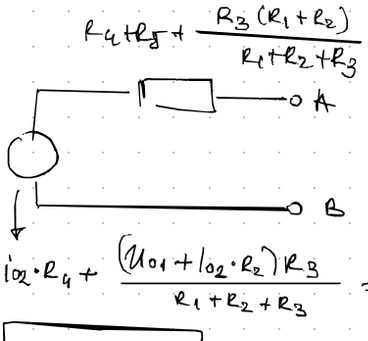




$$\frac{U_{01}+I_{02}\cdot R_2}{R_1+R_2} = I_{01}'$$



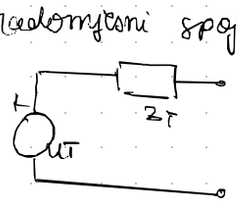
$$\Rightarrow$$



$$I_{02}\cdot R_4 + \frac{(U_{01}+I_{02}\cdot R_2)R_3}{R_1+R_2+R_3} = U_T$$

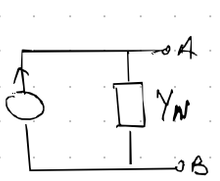
$$U_T = 26V$$

$\Rightarrow$ neopterećeni spoj:



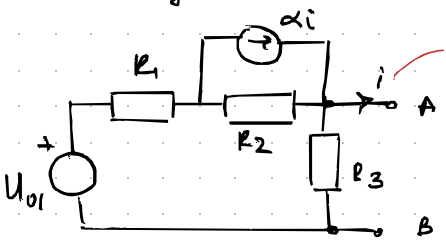
$$I_N = \frac{U_T}{Z_T} = 3.71mA$$

$\Longleftrightarrow$



$$Y_N = \frac{1}{Z_T} = 1.43 \text{ siemens?}$$

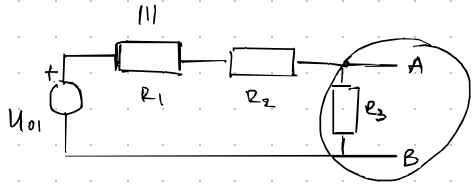
Primjer 6.) * ovisni izvor



struja ide točno u čvorište A → $i(t) = \alpha i = 0$
samo ga maknemo

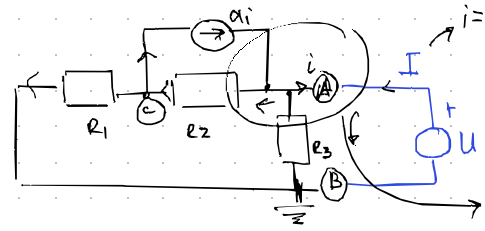
"u zraku" su
- jer su stezaljke otvorene,
- te struja izađe i nikad se ne vrati

III

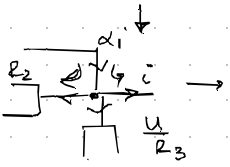


napon na R_3 $\quad U_3 = U_T = \dfrac{R_3}{R_1 + R_2 + R_3} = U_T$

$Z_T = ?$



Ulazi = minus



$U = U_A$

Ⓐ $\quad -i - \alpha i + \dfrac{U_A - U_C}{R_2} + \dfrac{U_A}{R_3}$

$-I + \alpha I + \dfrac{U}{R_3} + \dfrac{U - U_C}{R_2} = 0$

Ⓒ $\quad \dfrac{U_C}{R_1} + \alpha i - \dfrac{U_A - U_C}{R_2} = 0$

$\dfrac{U_C}{R_1} - \alpha I - \dfrac{U - U_C}{R_2} = 0 \longrightarrow \alpha I = \dfrac{U_C}{R_1} - \dfrac{U - U_C}{R_2}$

$I(-1 + \alpha) = -\dfrac{U}{R_3} - \dfrac{U - U_C}{R_2}$

$\alpha I = \dfrac{U_C}{R_1} - \dfrac{U}{R_2} + \dfrac{U_C}{R_2} = U_C\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) - \dfrac{U}{R_2} \quad / \cdot \dfrac{1}{R_2}$

$I(\alpha - 1) = \dfrac{U}{R_3} + \dfrac{U}{R_2} - \dfrac{U_C}{R_2} = U\left(\dfrac{1}{R_2} + \dfrac{1}{R_3}\right) - U_C \quad / \cdot \dfrac{1}{R_1 + R_2}$

$I\left[\alpha\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) - \alpha\dfrac{1}{R_2} - \dfrac{1}{R_1} - \dfrac{1}{R_2}\right] + U\left[\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\left(\dfrac{1}{R_2} + \dfrac{1}{R_3}\right) - \dfrac{1}{R_2^2}\right] = 0$

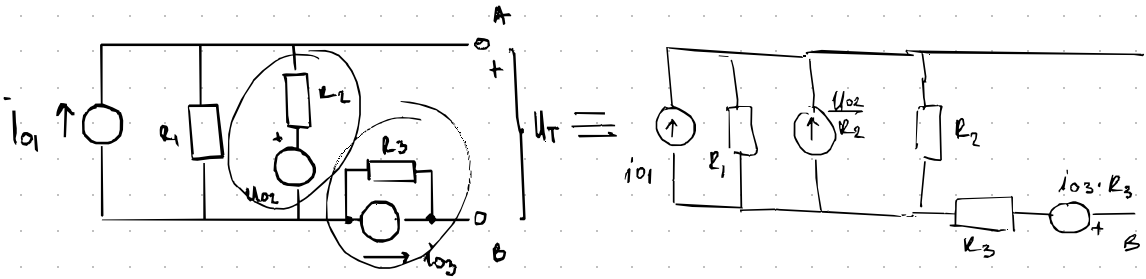
$I\left[\alpha\dfrac{1}{R_1} - \dfrac{1}{R_1} - \dfrac{1}{R_2}\right] + U\left[\dfrac{1}{R_1 R_3} + \dfrac{1}{R_1 R_2} + \dfrac{1}{R_2 R_3}\right] = 0$

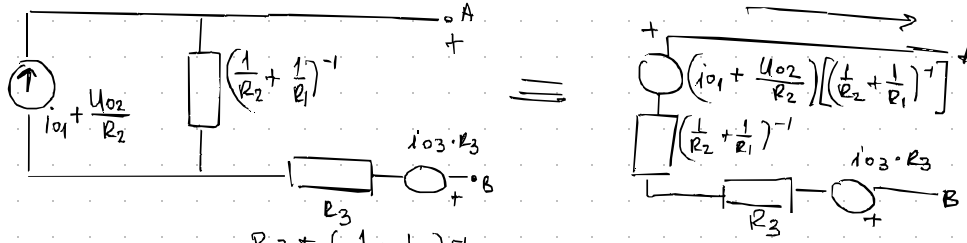
$I = \dfrac{\dfrac{1}{R_1 R_2} + \dfrac{1}{R_1 R_3} + \dfrac{1}{R_2 R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} - \alpha\dfrac{1}{R_1}} U \longrightarrow \boxed{Z = \dfrac{U}{I} = \dfrac{R_1 + R_2(1-\alpha)}{R_1 + R_2 + R_3} R_3}$

$G_N = Z^{-1}$

Nortonova struja $= i_N = \dfrac{U_T}{Z_T} = \dfrac{U_{01}}{R_1 + R_2(1-\alpha)}$

Primjer 7.) $R_1 = 5\,k\Omega$ $R_2 = 10\,k\Omega$ $R_3 = 10\,k\Omega$





$$R_3 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = Z_T$$

$$\left(i_{01} + \frac{U_{02}}{R_2}\right)\left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}\right] - i_{03} \cdot R_3 = U_T = \left(i_{01} + \frac{U_{02}}{R_2}\right) \cdot \frac{R_1 \cdot R_2}{R_1 + R_2} - i_{03} \cdot R_3$$

$$= \frac{i_{01} \cdot R_2 + U_{02}}{R_1 + R_2} \cdot R_1 - i_{03} \cdot R_3$$

$$= i_{01} \cdot 3{,}333 \times 10^3 + U_{02} \cdot 0{,}333 - i_{03} \cdot 10^4$$

$$R_T = R_3 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = 13{,}33\,k\Omega$$

$$G_N = \frac{1}{R_T} \qquad I_N = \frac{U_T}{R_T}$$

(5) TELLEGENOV TEOREM

$$\sum_{k=1}^{N_b} u_k(t)\, i_k(t) = 0$$

→ za mrežu sa N_b grana sa strujama grana $i_1, i_2, i_3 \dots i_{N_b}$

+ odg. naponom $u_1, u_2 \dots u_{N_b}$

⟹ ukupna suma snaga na svim granama u svakom trenu je $\emptyset$!

· indirektno kaže: koliko P trošila troše, toliko P izvor daje

* dokaz niže bitan