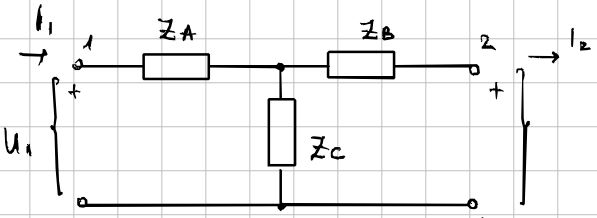
Četveropoli II. dio

- dva četveropola su ekvivalentna ako njihovom međusobnom zamjenom STRUJE I NAPONI NA PRIKLJUČNICAMA ostaju nepromijenjeni

Ekvivalentni četveropoli u T-spoju

- svakom recipročnom četveropolu zadanom Z-parametrima moguće je odrediti ekvivalentni četveropol u T-spoju



$$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12}$$

$$U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22}$$

$$Z_A = Z_{11} - Z_{12} \qquad Z_{11} = Z_A + Z_C \qquad Z_{12} = Z_C$$

$$Z_B = Z_{22} - Z_{12} \qquad Z_{21} = Z_C \qquad Z_{22} = Z_B + Z_C$$

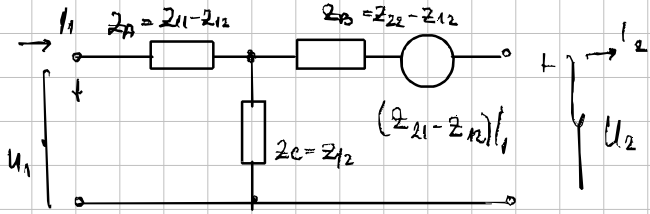
$$Z_C = Z_{12} = Z_{21}$$

→ ako četveropol nije recipročan $Z_{12} \neq Z_{21}$

$$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12}$$

$$U_2 = I_1 \cdot Z_{21} - I_2 \cdot Z_{22} \qquad U_2 = I_1 \cdot Z_{12} - I_2 \cdot Z_{22} + I_1 (Z_{21} - Z_{12})$$

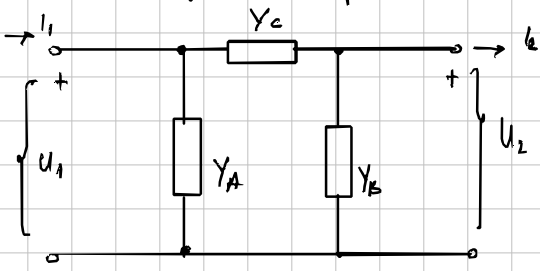
* nismo ništa zeznuli



interpretiramo kao naponski izvor

Ekvivalentan četveropol u Π-spoju

- kao što je za Z parametre bio T, za y parametre je Π



$$I_1 = U_1 \cdot y_{11} - U_2 y_{12}$$

$$I_2 = U_1 \cdot y_{21} - U_2 \cdot y_{22}$$

$$y_{11} = Y_A + Y_C \qquad y_{12} = Y_C$$

$$y_{21} = Y_C \qquad Y_B + Y_C$$

$$Y_A = y_{11} - y_{12}$$

$$Y_B = y_{22} - y_{12}$$

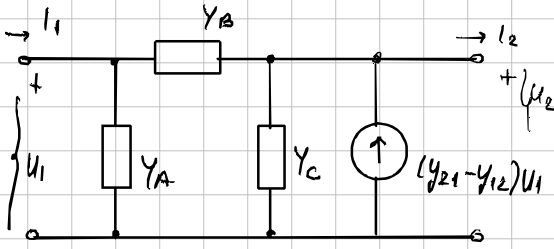
$$Y_C = y_{12} = y_{21}$$

→ za nerecipročan vrijedi opet $y_{12} \neq y_{21}$

$$I_1 = U_1 \cdot y_{11} - U_2 \cdot y_{12}$$

$$I_2 = U_1 \cdot y_{12} - U_2 \cdot y_{22} + U_1 (y_{21} - y_{12})$$

Strujni izvor

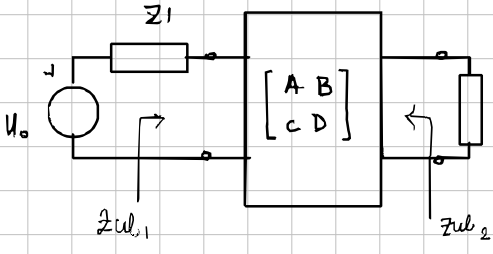


Zrcalni ili valni parametri četveropola

* sjest će bolje na kon predavanja s limjarna

- zrcalne impedancije Z_{c1} i Z_{c2}
- zrcalna konstanta prijenosa g

definiranje:



Ulazne impedancije:

$$Z_{ul_1} = \frac{AZ_2 + B}{CZ_2 + D} \qquad Z_{ul_2} = \frac{DZ_1 + B}{CZ_1 + A}$$

uvjet: $Z_{ul_1} = Z_1 \qquad Z_{ul_2} = Z_2$

$$\Rightarrow Z_1 = \frac{AZ_2 + B}{CZ_2 + D} \Big/ \cdot (CZ_2 + D) \qquad Z_2 = \frac{DZ_1 + B}{CZ_1 + A} \Big/ \cdot (CZ_1 + A)$$

$$Z_1 (CZ_2 + D) = AZ_2 + D \qquad Z_2 (CZ_1 + A) = DZ_1 + B$$

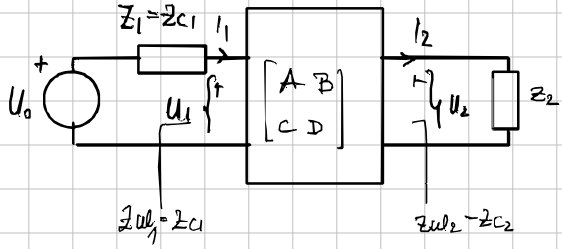
$$CZ_1 Z_2 - B = 0 \Rightarrow \boxed{Z_1 \cdot Z_2 = \frac{B}{C}}$$

$$\cdots DZ_1 - AZ_2 = 0 \Rightarrow \boxed{\frac{Z_1}{Z_2} = \frac{A}{D}}$$

$$\Rightarrow \boxed{Z_1 = Z_{c1} = \sqrt{\frac{AB}{CD}} \qquad Z_2 = Z_{c2} = \sqrt{\frac{BD}{AC}}}$$

zrcalne ili valne impedancije

Prijenosne jednadžbe i zrcalni parametri



$$U_1 = A \cdot U_2 + B \cdot I_2$$

$$I_1 = C \cdot U_2 + D \cdot I_2$$

$$* \; Z_{c2} = \sqrt{\frac{BD}{AC}}$$

$$U_1 = A U_2 + \frac{B}{Z_{c2}} \cdot U_2 \Rightarrow \frac{U_1}{U_2} = A + \frac{B}{Z_{c2}}$$

$$I_1 = C I_2 \cdot Z_{c2} + D \cdot I_2 \Rightarrow \frac{I_1}{I_2} = C Z_{c2} + D$$

$$\frac{U_1}{U_2} = A + \frac{B}{\sqrt{\frac{BD}{AC}}} = \sqrt{\frac{A}{D}}\left(\sqrt{AD} + \sqrt{BC}\right)$$ omjer prijenosa napona

$$\frac{I_1}{I_2} = C\sqrt{\frac{BD}{AC}} + D = \sqrt{\frac{D}{A}}\left(\sqrt{AD} + \sqrt{BC}\right)$$ omjer prijenosa struje

$$\frac{A}{D} = \frac{Z_{c1}}{Z_{c2}} \qquad \sqrt{\frac{A}{D}} = \sqrt{\frac{Z_{c1}}{Z_{c2}}} = n$$ → omjer transformacije

$$\frac{D}{A} = \frac{Z_{c2}}{Z_{c1}} \qquad \sqrt{\frac{D}{A}} = \sqrt{\frac{Z_{c2}}{Z_{c1}}} = \frac{1}{n}$$

$$\sqrt{AD} + \sqrt{BC} = e^{g} \Rightarrow g = \ln\left(\sqrt{AD} + \sqrt{BC}\right)$$

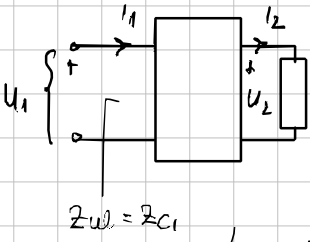
g → zrcalni koeficijent prijenosa

- u uvjetima stacionarne sinusne pobude

$$g = g(j\omega) = a(\omega) + jb(\omega)$$

a → zrcalni koeficijent gušenja

b → zrcalni koeficijent faze



$$\frac{U_1}{U_2} = n \cdot e^{g} \quad \text{uz } Z_2 = Z_{c2}$$

$$\frac{I_1}{I_2} = \frac{1}{n} \cdot e^{g} \quad \text{uz } Z_2 = Z_{c2}$$

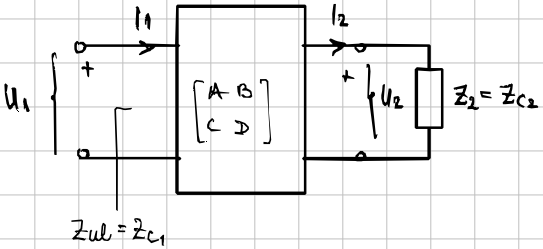
Ako je zrcalni koeficijent prijenosa $e^{g} = \sqrt{AD} + \sqrt{BC}$ pomnoži s $\left(\sqrt{AD} - \sqrt{BC}\right)$

$$\Rightarrow e^{g}\left(\sqrt{AD} - \sqrt{BC}\right) = AD - BC \xrightarrow{\text{s obzirom da je recipročan}} AD - BC = 1$$

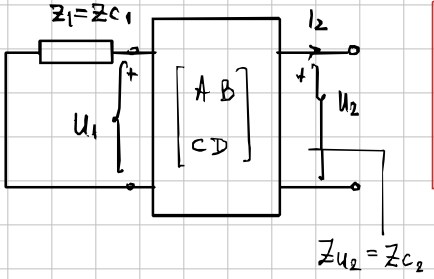
$$e^{g}\left(\sqrt{AD} - \sqrt{BC}\right) = 1 \rightarrow \left(\sqrt{AD} - \sqrt{BC}\right) = e^{-g}$$

bla bla bla i formulama

Prijenosne jednadžbe



$$U_1 = n\left(U_2 \cdot ch g + I_2 \cdot Z_{c_2} \cdot sh g\right)$$
$$I_1 = \frac{1}{n}\left(U_2 \cdot \frac{sh g}{Z_{c_2}} + I_2 \cdot ch g\right)$$



$$U_2 = \frac{1}{n}\left(U_1 \cdot ch g - I_1 \cdot Z_{c_1} \cdot sh g\right)$$
$$-I_2 = n\left(\frac{U_1}{Z_{c_1}} \cdot sh g - I_1 \cdot ch g\right)$$

Impedancije na kratko i prazno:

$U_1 = A U_2 + B I_2 \qquad I_2 = 0 \Rightarrow Z_{p_1} = Z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = \frac{A}{C}$

$I_1 = C U_2 + D I_2 \qquad U_2 = 0 \Rightarrow Z_{k_1} = \frac{1}{Y_1} = \left.\frac{U_1}{I_1}\right|_{U_2=0} = \frac{B}{D}$

$Z_{c_1} = \sqrt{\frac{AB}{CD}} = \sqrt{Z_{p_1} Z_{k_1}} = \sqrt{\frac{Z_{11}}{Y_{11}}}$ — puno brže nego da sve posebno!

$I_1 = 0 \Rightarrow Z_{p_2} = Z_{22} = \left.\frac{-U_2}{I_2}\right|_{I_1=0} = \frac{D}{C}$

$U_2 = 0 \Rightarrow Z_{k_2} = \frac{1}{Y_{22}} = \left.\frac{-U_2}{I_2}\right|_{U_2=0} = \frac{B}{A}$

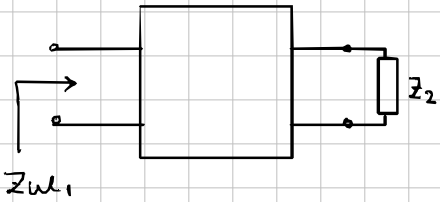
$Z_{c_2} = \sqrt{\frac{DB}{AC}} = \sqrt{Z_{p_2} \cdot Z_{k_2}} = \sqrt{\frac{Z_{22}}{Y_{22}}}$

također vrijedi: $\frac{Z_{p_1}}{Z_{p_2}} = \frac{A}{D} \qquad \frac{Z_{k_1}}{Z_{k_2}} = \frac{A}{D} \Rightarrow \frac{Z_{p_1}}{Z_{k_1}} = \frac{Z_{p_2}}{Z_{k_2}}$

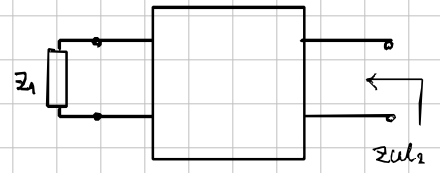
$th(g) = \frac{sh(g)}{ch(g)} = \sqrt{\frac{BC}{AD}} = \sqrt{\frac{Z_{k_1}}{Z_{p_1}}} = \sqrt{\frac{Z_{k_2}}{Z_{p_2}}}$

$\Rightarrow$ $Z_{c_1} = \sqrt{Z_{p_1} \cdot Z_{k_1}}$ $\qquad$ $Z_{c_2} = \sqrt{Z_{p_2} \cdot Z_{k_2}}$ $\qquad$ $th(g) = \sqrt{\frac{Z_{k_1}}{Z_{p_1}}} = \sqrt{\frac{Z_{k_2}}{Z_{p_2}}}$

Ako četveropol nije prilagođen $\rightarrow Z_2 \neq Z_{c_2} \Rightarrow Z_{ul} \neq Z_{c_1}$



$$Z_{ul_1} = \frac{U_1}{I_1} = \frac{sh\gamma + \frac{Z_2}{Z_{c_2}} ch\gamma}{ch\gamma + \frac{Z_2}{Z_{c_2}} sh\gamma} Z_{c_1}$$



$$Z_{ul_2} = \frac{-U_2}{I_2} = \frac{sh\gamma + \frac{Z_1}{Z_{c_1}} ch\gamma}{ch\gamma + \frac{Z_1}{Z_{c_1}} sh\gamma} Z_{c_2}$$

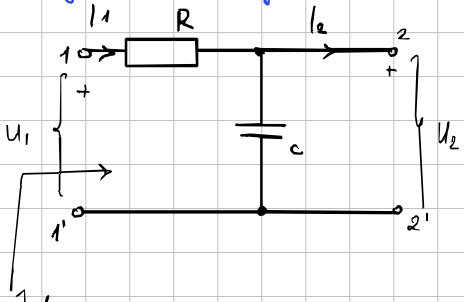
$Z_1 \neq Z_{c_1} \Rightarrow Z_2 \neq Z_{c_2}$

Mjera odstupanja od prilagođenja

$$p_1 = \frac{Z_1 - Z_{c_1}}{Z_1 + Z_{c_1}}$$ } pogreška prilagođenja ili koeficijent refleksije na ulazu

$$p_2 = \frac{Z_2 - Z_{c_2}}{Z_2 + Z_{c_2}}$$ } pogreška pr. ili koeficijent refleksije na izlazu

Primjer: Izračunaj zrcalne parametre



Z_{k_1} – impedancija sa 1-1' na KS na 2-2'

Z_{p_1} – impedancija na prazno na 2-2'

$$Z_{k_1} = R \qquad Z_{k_2} = \left(R \parallel \frac{1}{sC}\right) = \left(\frac{1}{R} + sC\right)^{-1} = \frac{R}{RSC+1}$$

$$Z_{p_1} = R + \frac{1}{sC} \qquad Z_{p_2} = \frac{1}{sC}$$

$$\Rightarrow Z_{c_1} = \sqrt{Z_{p_1} \cdot Z_{k_1}} = \sqrt{R\left(R + \frac{1}{sC}\right)} = \sqrt{\frac{R}{sC}\left(RSC+1\right)}$$

$$Z_{c_2} = \sqrt{Z_{p_2} \cdot Z_{k_2}} = \sqrt{\frac{1}{sC}\left(\frac{R}{RSC+1}\right)} = \sqrt{\frac{R}{sC}\left(\frac{1}{RSC+1}\right)}$$

$$th\gamma = \sqrt{\frac{Z_{k_1}}{Z_{p_1}}} = \sqrt{\frac{Z_{k_2}}{Z_{p_2}}} \rightarrow th\gamma = \sqrt{\frac{R \cdot sC}{RSC+1}}$$

5 načina spajanja četveropola

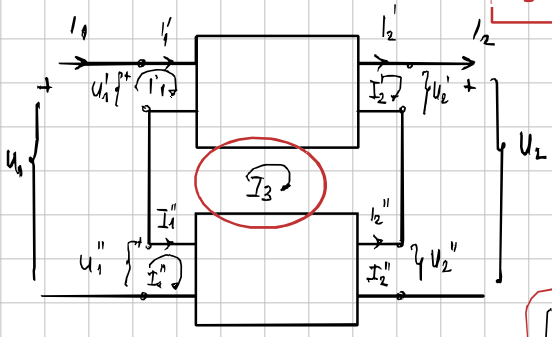
→ serijski → serijsko paralelni

→ paralelni → paralelno serijski

→ spoj u lanac ili kaskadu

Serijski spoj četveropola

$I_3 = 0$!



$U_1 = U_1' + U_1''$ $\quad I_1 = I_1' = I_1''$

$U_2 = U_2' + U_2''$ $\quad I_2 = I_2' = I_2''$

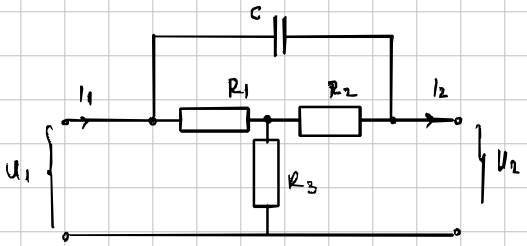
... matrični oblik ...

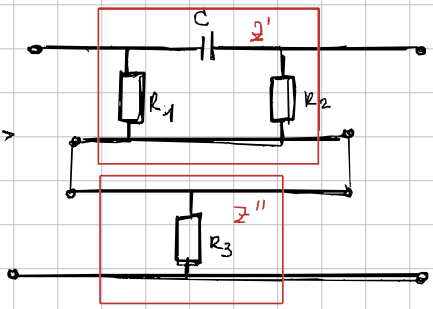
$$[Z] = [Z'] + [Z'']$$

$Z_{11} = Z_{11}' + Z_{11}''$ ---

---- $Z_{22} = Z_{22}' + Z_{22}''$

Primjer: Premošteni T-četveropol





$$[Z] = [Z'] + [Z''] \Rightarrow Z' = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}'' \end{bmatrix}$$

$$Z_{11}' = R_1 \| \left(\frac{1}{Cs} + R_2\right) = \left(\frac{1}{R_1} + \frac{1}{\frac{1}{Cs} + R_2}\right)^{-1}$$

$$Z_{11}' = \frac{R_1\left(\frac{1}{Cs} + R_2\right)}{\frac{1}{Cs} + R_2 + R_1} = \frac{R_1(1+SCR_2)}{1+SC(R_2+R_1)}$$

$U_1' = Z_{11}' \cdot I_1' \oplus Z_{12}' \cdot I_2'$

$U_2' = Z_{21}' \cdot I_1' \oplus Z_{22}' \cdot I_2'$

$$Z_{12} = Z_{21} = \frac{R_1 R_2 SC}{SC(R_1+R_2)+1}$$

$$Z_2 = \frac{(SCR_1+1)R_2}{SC(R_1+R_2)+1}$$

$Z_{11}'' = Z_{12}'' = Z_{21}'' = Z_{22}'' = R_3$

$$Z = \begin{bmatrix} \frac{R_1(1+SCR_2)}{1+SC(R_2+R_1)} + R_3 \oplus \left(\frac{R_1R_2SC}{SC(R_1+R_2)+1} + R_3\right) & -\left(\frac{R_1R_2SC}{SC(R_1+R_2)+1} + R_3\right) \oplus \left(\frac{(SCR_1+1)R_2}{SC(R_1+R_2)+1} + R_3\right) \end{bmatrix}$$

*ostali spojevi su u formulama

Simetrični četveropoli

$Z_{11} = Z_{22} \longrightarrow Z_{P1} = Z_{P2}$

$\rightarrow [y] = [z]^{-1} \Rightarrow y_{11} = y_{22}$ tj. $z_{k_1} = z_{k_2}$

Prijenosni parametri: $A = D$

Hibridni: $h \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = -1$

hibridni g: $\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$

Z-calni parametri

$z_{c_1} = z_{c_2} = z_c = \sqrt{\frac{B}{C}}$

$g = \ln\left(\sqrt{AD} + \sqrt{BC}\right) = \ln\left(A + \sqrt{BC}\right)$

$AD - BC = 1 \rightarrow BC = AD - 1$

$\hookrightarrow BC = A^2 - 1$

$\Rightarrow g \ln\left(A + \sqrt{A^2+1}\right)$

Prijenosne jed

$U_1 = n\left(U_2 \operatorname{ch}(g) + I_2 z_2 \operatorname{sh}(g)\right)$

$I_1 = \frac{1}{n}\left(\frac{U_2 \operatorname{sh}(g)}{z_{c_2}} + I_2 \operatorname{ch}(g)\right)$

$U_1 = n\left(U_2 \operatorname{ch}(g) + I_2 \cdot z_c \cdot \operatorname{sh}(g)\right)$

$I_1 = \frac{U_2}{z_c} \operatorname{sh}(g) + I_2 \cdot \operatorname{ch}(g)$

pošto je $n = \sqrt{\frac{z_{c_1}}{z_{c_2}}} = 1 \rightarrow z_{c_2} = z_c$