NUK

poredbeni
poredbeni - L
D'Alambert - L
Cauchy - L
Integralni

Aps. konverg.
Leibnitz

$\sum_{n=1}^{\infty} \frac{1}{n^r}$

ZI 2021.) ①

b) $\sum_{n=3} \frac{(-1)^n}{\ln(\ln n)}$

1) $\lim_{n\to\infty} \frac{1}{\underbrace{\ln(\ln n)}_{\infty}} = 0$

2) $a_n = \frac{1}{\ln(\ln n)}$ padajuća ILI $f(x) = \frac{1}{\ln(\ln x)}$ deriviramo

npr. matemat. indukcija
$a_{n+1} < a_n$

$f'(x) = \frac{-1}{\ln^2(\ln(x))} \cdot \frac{1}{\ln x} \cdot \frac{1}{x} < 0$

za $x \geq 3$
(jer ispod sume piše $n=3$)

ILI $\ln n \to$ rastuća,
kompozicija rastućih je rastuća
$\Rightarrow$ po Leibnitzu konv.

c) $\sum \left| \frac{(-1)^n}{\ln(\ln n)} \right| = \sum \frac{1}{\ln(\ln n)} \geq \sum \frac{1}{\ln n} \geq \sum \frac{1}{n}$ div

$\ln n \leq n \;/\ln$
$\ln(\ln n) \leq \ln n$

po poredbenom divergira

UBIT ću SE ♡

ZI 2022)

$\sum (-1)^n \frac{4^{n+1}}{3^{n-1}} \sin \frac{1}{3^n}$

$\rightarrow$ Leibnitz je pre shit za ovo
$\hookrightarrow$ „izgubiš volju za životom"

Leibniz
$12 \lim \left(\frac{4}{3}\right)^n \sin \frac{1}{3^n}$

Gledamo aps. $\rightarrow$ konvergira pa konvergira i početni red

$12 \sum \left| (-1)^n \left(\frac{4^n}{3^n}\right) \cdot \sin \frac{1}{3^n} \right| = \sum \left(\frac{4}{3}\right)^n \cdot \sin \frac{1}{3^n}$ jer $\sin x \sim x$ kada $x \to 0$

poredbeni
$\leq \sum \left(\frac{4}{3}\right)^n \cdot 1$
divergira
nema rezultata

poredbeni limes $\sum \left(\frac{4}{3}\right)^n \frac{1}{3^n}$

$\sum \left(\frac{4}{9}\right)^n$ $q < 1$
konvergira

Z1-23-1) c) ii)

$$\sum \frac{5^n}{\sqrt{n^3+1}} \ln\left(1+\frac{1}{3^n}\right) \qquad \frac{1}{3^n} \to 0$$

$\ln(1+x) \sim x$ kada $x \to 0$

div

$$\sim \sum \frac{5^n}{\sqrt{n^3}} \cdot \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \cdot \left(\frac{5}{3}\right)^n$$

diverzija

poredbeni limes

$$q = \lim \sqrt[n]{\frac{1}{n^{3/2}}\left(\frac{5}{3}\right)^n}$$

$q = \frac{5}{3} > 1$ div po Cauchyju

Z1-15) $\displaystyle\sum_{n=1}^{\infty} \underbrace{\sin^2\left(\frac{1}{n}\right)}_{\text{poz}} \underbrace{\cos(n)}_{\text{mijenja predznak zbog cos!}}$

poz jer je na kvad

konvergira 1 početni po tm. o aps. konv.

aps. konv: $\sum \sin^2\left(\frac{1}{n}\right) |\cos(n)|$ $\quad |\cos(n)| \to \in [0,1]$

poredbeni : $\leq \sum \sin^2\left(\frac{1}{n}\right)$ $\qquad \sin x \sim x$ kada $x \to 0$

poredbeni limes : $\sim \sum \frac{1}{n^2} \sim \int \frac{1}{x^2}\, dx$ limes