7.7. RJEŠAVANJE DJ POMOĆU REDOVA

Primjer: $y''+y=0$, $y=\sum_{n=0}^{\infty} C_n x^n$ jer znamo da smijemo derivirati redove potencija → možemo ih koristiti

$$\sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$\downarrow$ $n \to n+2$

$$\sum_{n=0}^{\infty} (n+2)(n+1)C_{n+2}x^n + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} \Big[\underbrace{(n+2)(n+1)C_{n+2} + C_n}_{=0}\Big]x^n = 0$$

$\Longrightarrow (n+1)(n+2)C_{n+2} + C_n = 0$

$$C_{n+2} = -\frac{C_n}{(n+1)(n+2)}$$ rekurzivno zadan niz (MATAN 1)

$n \geq 0$

$n=0 \longrightarrow C_2 = \dfrac{-C_0}{1\cdot 2}$

$n=1 \longrightarrow C_3 = \dfrac{-C_1}{2\cdot 3}$

* C_0 i C_1 nemamo i ne možemo dobiti ali to su baš te konstante koje nam trebaju kao rješenja

$n=2 \longrightarrow C_4 = \dfrac{-C_2}{3\cdot 4} = \dfrac{C_0}{1\cdot 2\cdot 3\cdot 4} \to 4!$

$n=3 \longrightarrow C_5 = \dfrac{-C_3}{4\cdot 5} = \dfrac{C_1}{2\cdot 3\cdot 4\cdot 5} \to 5!$

$n=4 \to C_6 = \dfrac{-C_4}{5\cdot 6} = \dfrac{-C_0}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} \Rightarrow 6!$

$n=5 \to C_7 = \dfrac{-C_5}{6\cdot 7} = -\dfrac{C_1}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} \to 7!$

OPĆE RJEŠENJE:

$$y = \sum_{n=0}^{\infty} C_n x^n$$

$$y = C_0 + C_1 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 + \dots$$

$$= C_0 + C_1x + \frac{-C_0}{2!}x^2 - \frac{C_1}{3!}x^3 + \frac{C_0}{4!}x^4 + \frac{C_1}{5!}x^5 \dots$$

$$\Longrightarrow y = C_0\underbrace{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots\right)}_{\cos x} + C_1\underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots\right)}_{\sin x}$$

$$y = C_0 \cdot \cos x + C_1 \cdot \sin x \longrightarrow \boxed{y = M\cdot\cos x + N\cdot\sin x}$$

Zad.) $y'' - xy = \cos x$, $y(0) = y'(0) = 0$

2.r.

$\angle$ linearna drugog reda i imamo $\underline{\underline{x}}$

$\rightarrow$ ne može LDJ s k.k.

$$y = \sum_{n=0}^{\infty} c_n x^n \Longrightarrow \sum_{n=2}^{\infty} (n-1)(n)\, c_n x^{n-2} - x \sum_{n=0}^{\infty} c_n x^{n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

moramo imati x na istu potenciju $\rightarrow$ ne možemo pomicati za 3 jer je $n=2$

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 \ldots$$

$\boxed{y(0) = c_0 = 0}$ $\boxed{y'(0) = c_1 = 0}$

uz x^0: $1 \cdot 2 c_2 = 1 \longrightarrow c_2 = \frac{1}{2}$

uz x^1: $2 \cdot 3 \cdot c_3 - c_0 = 0 \longrightarrow c_3 = 0$

uz x^2: $3 \cdot 4 \cdot c_4 - c_1 = \frac{-1}{2} \rightarrow c_4 = \frac{-1}{24}$

$$\sum_{n=0} \left[(n+3)(n+2)c_{n+3} - c_n\right] x^{n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

LJIR-22-7)

a) $xy'' + y' = x^2$ — ne možemo MVK jer ne možemo dobiti homogenu j.

$y' = z$

$xz' + z = x^2$ → linearna prvog reda

$z' + \frac{1}{x}z = x$

$$z = e^{-\int \frac{1}{x}dx}\left[\int x e^{\int \frac{1}{x}dx}\,dx + C\right] = e^{-\ln x}\left[\int x^2\,dx + c\right] = \frac{1}{x}\left[\frac{x^3}{3} + c\right]$$

$$z = \frac{x^2}{3} + \frac{C}{x} = y' \quad / \int$$

$$y = \int \frac{x^2}{3}dx + c\int \frac{1}{x}dx = \boxed{\underbrace{\frac{1}{9}x^3}_{y_p} + \underbrace{c_1 \ln|x| + C_2}_{y_h}}$$

NE DA

opće rješenje

b)

- baza rješenja, Wronskijan, vrijedi za sve linearne

L baza rješenja homogenog dijela su $\ln|x|$ i 1 } homogeno ima konstante

$y_1 = \ln x$, $y_2 = 1$

$$W = \begin{vmatrix} \ln x & 1 \\ \frac{1}{x} & 0 \end{vmatrix}$$

$W = \frac{1}{x} \neq 0 \ \forall$ → baza rješenja $\boxed{\text{LIN. NEZ}}$

JIR-2019-8)

c) $y'' - 4y' + 4y = \frac{e^{2x}}{x^2}$ → ne možemo desnom stranom jer je x u nazivniku | ne možemo ~~$\frac{e^{2x}}{Ax^2 + Bx + C}$~~ ?

Bože sačuvaj

Zad.) $y'' + 4y' + 4y = sh(2x)$

pripadna homogena

1) $r^2 - 4r + 4 = 0$

$r_{1,2} = -2$ $\qquad y_h = C_1 e^{-2x} + C_2 x e^{-2x}$

2) MVK: $C_1'(x) e^{-2x} + C_2'(x) x e^{-2x} = 0$

$C_1'(x)(-2e^{-2x}) + C_2'(x)\left[e^{-2x} - 2xe^{-2x}\right] = sh(2x)$ $\Big\}\ +$

$C_2'(x) = sh(2x) e^{2x} = \frac{1}{2}(e^{2x} - e^{-2x}) e^{2x}$

$C_2'(x) = \frac{1}{2} e^{4x} - \frac{1}{2} \quad / \int dx$ ... dalje sami

2. način: $y'' + 4y' + 4y = \underbrace{\frac{1}{2} e^{2x}}_{f_1} \underbrace{- \frac{1}{2} e^{-2x}}_{f_2}$ $\quad *$

gledamo jel s homogenom isto!

$y_{p_1} = Ae^{2x}$

$y_{p_1}' = 2Ae^{2x}$

$y_{p_1}'' = 4Ae^{2x}$

$4Ae^{2x} + 4 \cdot 2Ae^{2x} + 4Ae^{2x} = \frac{1}{2} e^{2x}$

$4Ae^{2x} + 8Ae^{2x} + 4Ae^{2x} = \frac{1}{2} e^{2x}$

$16A\cancel{e^{2x}} = \frac{1}{2} \cancel{e^{2x}}$

$A = \frac{1}{32}$

$y_{p_2} = Be^{-2x} \cdot x^2$

$y_{p_2}' = -2Be^{-2x} \cdot x^2 + 2xBe^{-2x}$

$= e^{-2x}(2xB - 2x^2 B)$

$y_{p_2}'' = -2e^{-2x}(2xB - 2x^2 B) + e^{-2x}(-4Bx + 2B)$