Uvod u teoriju linija

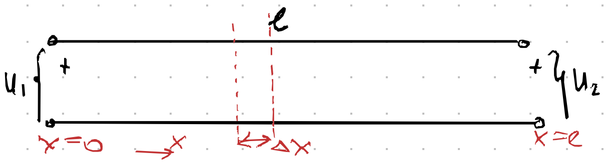
el. krugovi s koncentriranim elementima
↳ el. svojstva ne ovise o njihovim fiz dimenzijama
↳ trenutni odziv u svakoj drugoj točki mreže

Na vrlo visokim frekv $\rightarrow$ valna duljina signala može postati usporediva s fiz. dimenzijama
• nema trenutnog odziva na promjenu pobude

Električne prijenosne linije

* par paralelnih vodiča

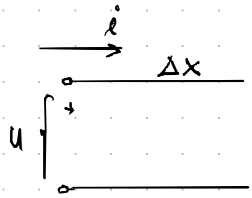
→ idealizirani model linije:



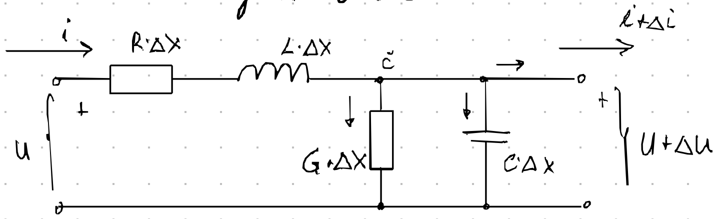
▷ liniju nije moguće prikazati koncentriranim elementima R, L, C, ako njena duljina nije puno kraća od najmanje valne duljine signala koji prenosi

$R \longrightarrow$ otpor linije po jedinici duljine $[\Omega/m]$
$L \longrightarrow$ induktivitet –||– $[H/m]$
$G \longrightarrow$ vodljivost –||– $[S/m]$
$C \longrightarrow$ kapacitet –||– $[F/m]$

zamislimo jedan segment linije Δx
↳ na njemu vrijede konc. parametri



$\Rightarrow$ to onda izgleda ovako:



Linije koje promatramo:
→ LINEARNE (ne ovise o $u(x,t)$ i $i(x,t)$)
→ VREMENSKI NEPROMJ. (neovisno o vremenu)
→ HOMOGENE (ne ovise o mjestu x)

jednadžbe:

$$\cancel{u} = (R\cdot\Delta x)\, i + (L\,\Delta x)\frac{di}{dt} + \cancel{u} + \Delta u$$

$$\cancel{i} = (G\cdot\Delta x)(u + \cancel{\Delta u}) + (C\cdot\Delta x)\frac{d}{dt}(u+\Delta u) + \cancel{i} + \Delta i \Rightarrow 0 = G\Delta x\, u + \overbrace{G\cdot\Delta x\cdot\Delta u}^{\approx 0} + C\Delta x\frac{du}{dt} + \underbrace{C\Delta x\cdot\frac{d\Delta u}{dt}}_{\approx 0} + \Delta i$$

↳ $0 = (R\Delta x)\, i + L\Delta x\frac{di}{dt} + \Delta u$

$0 = G\Delta x\cdot u + C\Delta x\frac{du}{dt} + \Delta i \quad / : \Delta x$

$$-\frac{\Delta u}{\Delta x} = Ri + L\frac{di}{dt}$$

$$-\frac{\Delta i}{\Delta x} = Gu + C\frac{du}{dt}$$

ako se Δx smanjuje t.d. $\Delta x \rightarrow \partial x$

$$\Rightarrow \boxed{\begin{aligned} -\frac{\partial u}{\partial x} &= Ri + L\frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial x} &= Gu + C\frac{\partial u}{\partial t} \end{aligned}} \Rightarrow \text{jednadžbe linija}$$

* $u = u(x,t)$
$i = i(x,t)$

u, i ↓ zavisne; (x, t) nezavisne var

Dif. jed moguće transform. u drugi oblik

$$\frac{\partial u(x,t)}{\partial x} + Ri + L\cdot\frac{\partial i(x,t)}{\partial t} = 0 \quad \Big/ \frac{\partial}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial x} + Gu + C\cdot\frac{\partial u(x,t)}{\partial t} = 0 \quad \Big/ \frac{\partial}{\partial x}$$

$$\Rightarrow \quad \boxed{-\frac{\partial^2 u}{\partial x \partial t}} = R\cdot\frac{\partial i}{\partial t} + L\frac{\partial^2 i}{\partial t^2}$$

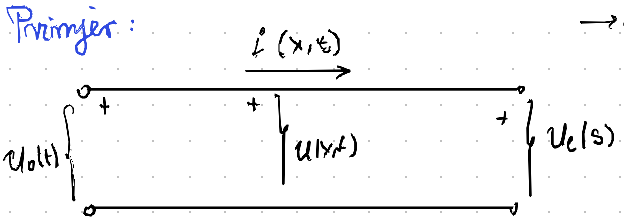
$$-\frac{\partial^2 i}{\partial x^2} = G\cdot\frac{\partial u}{\partial x} + C\cdot\frac{\partial^2 u}{\partial t \partial x}$$

$$\Rightarrow \frac{\partial^2 i(x,t)}{\partial x^2} = LC\frac{\partial^2 i(x,t)}{\partial t^2} + (LG+RC)\frac{\partial i(x,t)}{\partial t} + R\cdot G\cdot i(x,t) \to \text{dif jed za struju}$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2}(x,t) = LC\frac{\partial^2 u(x,t)}{\partial t^2} + (RC+LG)\frac{\partial u(x,t)}{\partial t} + R\cdot G\cdot u(x,t) \to \text{dif jed za napon}$$

L telegrafske jednadžbe linije

Primjer:

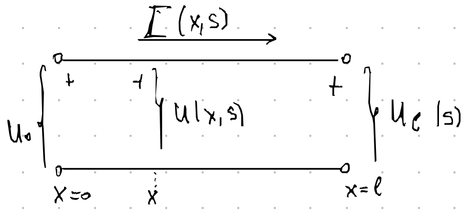


→ odrediti napon i struju na bilokojem mjestu x linije

• rješavanje dif jed → Laplaceove trans

$$-\frac{\partial u}{\partial x} = R\cdot i + L\cdot\frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = G\cdot u + C\cdot\frac{\partial u}{\partial t} \quad \xrightarrow{\mathcal{L}}$$



$$-\frac{\partial U}{\partial x} = R\cdot I(x,s) + L(s\cdot I(x,s) - i(x,0))$$ u vremenu t=0 — pretpostavka $u(x,0)$ i $i(x,0)=0$

$$-\frac{\partial I(x,s)}{\partial x} = G\cdot U(x,s) + C(s\cdot U(x,s) - u(x,0))$$

$$-\frac{\partial U}{\partial x} = (R+Ls)I(x,s) \quad \Big/\frac{\partial}{\partial x} \longrightarrow -\frac{\partial^2 U}{\partial x^2} = (R+Ls)\frac{\partial I(x,s)}{\partial x}$$

$$-\frac{\partial I}{\partial x} = (G+Cs)\,U(x,s)$$

$$\Rightarrow \frac{\partial^2 U}{\partial x^2} = \underbrace{(R+Ls)(G+Cs)}_{\gamma^2}U(x,s)$$

... isto tako i struju: $\frac{\partial^2 I}{\partial x^2} = (R+sL)(G+sC)\,I(x,s)$

$$\frac{\partial^2 U}{\partial x^2} - \gamma^2 U(x,s) = 0 \longrightarrow \text{HDJ 2. reda za raspodjelu napona duž linije}$$

$$\frac{\partial^2 I}{\partial x^2} - \gamma^2 I(x,s) = 0 \longrightarrow \text{-11- struje duž linije}$$

Opće rješenje za napon → eksp. oblika $U(x,s) = Ae^{px}$

$$\Rightarrow Ap^2e^{px} - \gamma^2\cdot Ae^{px} = 0 \quad \Big/ : Ae^{px}$$

$$p^2 - \gamma^2 = 0 \longrightarrow p_{1,2} = \pm\gamma \quad \text{jednadžba kar. jed.}$$

RJ. Dif J. za raspodjelu napona $\boxed{U(x,s) = A_1e^{-\gamma x} + A_2e^{\gamma x}}$

γ = faktor prijenosa $\gamma(s) = \sqrt{(R+sL)(G+sC)}$ ∗ ovisi o s (∗ faktor propagacije)

RJ. Dif jed za raspodjelu struje $\boxed{I(x,s) = B_1e^{-\gamma x} + B_2e^{\gamma x}}$

konstante se određuju iz rubnih uvjeta

r.u => $U(0,s)$ $I(0,s)$ $U(l,s)$ $I(l,s)$

Uvrštenjem rješenja za $U(x,s)$ i $I(x,s)$ dobiva se

$$A_1\gamma e^{-\gamma x} - A_2\gamma e^{\gamma x} = (R+sL)(B_1 e^{-\gamma x} + B_2 e^{\gamma x})$$

$$e^{-\gamma x}\left(A_1\gamma + (R+sL)B_1\right) - e^{\gamma x}\left(A_2\gamma + (R+sL)B_2\right) = 0 \quad / : \gamma$$

$$e^{-\gamma x}\left(A_1 + \boxed{\frac{R+sL}{\gamma}} B_1\right) - e^{\gamma x}\left(A_2 + \frac{R+sL}{\gamma} B_2\right) = 0$$

$$= \sqrt{\frac{R+sL}{G+sC}} = Z_0$$ valna ili karakt. impedancija hom. linije

$$\Rightarrow A_1 - Z_0 B_1 = 0 \rightarrow B_1 = \frac{A_1}{Z_0}$$

$$A_2 + Z_0 B_2 = 0 \rightarrow B_2 = -\frac{A_2}{Z_0}$$

$$\boxed{U(x,s) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}}$$

$$\boxed{I(x,s) = \frac{A_1}{Z_0} e^{-\gamma x} - \frac{A_2}{Z_0} e^{\gamma x}}$$

- Rubni uvjeti za $x=0 \longrightarrow U(0,s) = U(0)$
  $I(0,s) = I(0)$

$$U(0) = A_1 e^{-\gamma\cdot 0} + A_2 e^{\gamma\cdot 0} = A_1 + A_2$$

$$I(0) = \frac{A_1}{Z_0} - \frac{A_2}{Z_0}$$

$$\Longrightarrow \quad A_1 = \frac{U(0) + I(0)\cdot Z_0}{2}$$

$$A_2 = \frac{U(0) - I(0)\cdot Z_0}{2}$$

Rješenje za napon i struje na mjestu x linije:

$$\boxed{\begin{aligned} U(x,s) &= \frac{U(0) + I(0)\cdot Z_0}{2} e^{-\gamma x} + \frac{U(0) - I(0)\cdot Z_0}{2} e^{\gamma x} \\ I(x,s) &= \frac{\frac{U_0}{Z_0} + I(0)}{2} e^{-\gamma x} - \frac{\frac{U_0}{Z_0} - I(0)}{2} e^{\gamma x} \end{aligned}}$$

→ Drugi oblik: $ch(\gamma x)$ $sh(\gamma x)$

$$U(x) = U(0)\frac{e^{\gamma x} + e^{-\gamma x}}{2} - Z_0 I(0)\frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$I(x) = -\frac{U_0}{Z_0}\frac{e^{\gamma x} - e^{-\gamma x}}{2} + I(0)\frac{e^{\gamma x} + e^{-\gamma x}}{2}$$

$$U(x) = U(0)\,ch(\gamma x) - Z_0 I(0)\,sh(\gamma x) \qquad U(0) = U(x)\cdot ch(\gamma x) + Z_0 I(x)\,sh(\gamma x)$$

$$I(x) = -\frac{U(0)}{Z_0} sh(\gamma x) + I(0)\,ch(\gamma x) \qquad I(0) = \frac{U(x)}{Z_0} sh(\gamma x) + I(x)\cdot ch(\gamma x)$$

Prijenosne jednadžbe linije

- Linija je četveropol

$\longrightarrow$ prijenosni parametri

za $x = l$

$$U(0) = U(l)\,ch(\gamma l) + Z_0 I(l)\,sh(\gamma l)$$

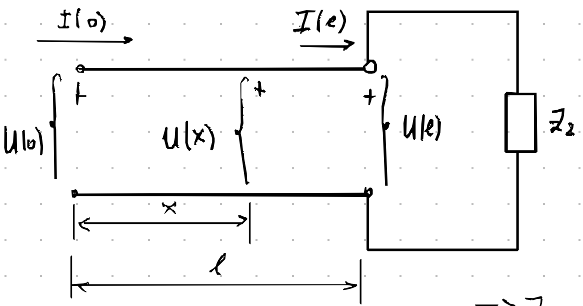
$$I(0) = \frac{U(l)}{Z_0} sh(\gamma l) + I(l)\,ch(\gamma l)$$

$\hookrightarrow$ prijenosne jed. linije

znači linija je simetrični četveropol

$$A = ch(\gamma l) = D \qquad B = Z_0\,sh(\gamma l) \qquad C = \frac{ch(\gamma l)}{Z_0}$$

Homogena linija zaključena impedancijom Z_2



za $x = l \rightarrow Z_2 = \frac{U(l)}{I(l)}$

$$U(l) = A_1 e^{-\gamma l} + A_2 e^{\gamma l}$$

$$I(l) = \frac{1}{Z_0}\left(A_1 e^{-\gamma l} - A_2 e^{\gamma l}\right)$$

$$\Rightarrow Z_2 = \frac{U(l)}{I(l)} = Z_0 \frac{A_1 e^{-\gamma l} + A_2 e^{\gamma l}}{A_1 e^{-\gamma l} - A_2 e^{\gamma l}}$$

$$U(l) = A_1 e^{-\gamma l} + A_2 e^{\gamma l}$$

$$Z_0 I(l) = A_1 e^{-\gamma l} - A_2 e^{\gamma l} \Rightarrow$$

$$A_1 = \frac{1}{2}\left(U(l) + Z_0 I(l)\right) e^{\gamma l} \rightarrow A_1 = \frac{I(l)}{2}(Z_2 + Z_0)\, e^{\gamma l}$$

$$A_2 = \frac{1}{2}\left(U(l) - Z_0 I(l)\right) e^{-\gamma l} \rightarrow A_2 = \frac{I(l)}{2}(Z_2 - Z_0)\, e^{-\gamma l}$$

$$\Rightarrow \boxed{\frac{A_2}{A_1} = \frac{Z_2 - Z_0}{Z_2 + Z_0} e^{-2\gamma l}}$$

koeficijent refleksije na izlazu

$$/\!/ \quad \Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} \Rightarrow \boxed{A_2 = A_1 \Gamma_2 e^{-2\gamma l}}$$

- napon na početku linije $x = 0$

$$U(0) = A_1 + A_2 = A_1\left(1 + \Gamma_2 e^{-2\gamma l}\right) \longrightarrow$$

$$A_1 = \frac{U(0)}{1 + \Gamma_2 e^{-2\gamma l}}$$

$$A_2 = \frac{\Gamma_2 e^{-2\gamma l}}{1 + \Gamma_2 e^{-2\gamma l}} U(0)$$

- ako je poznat napon na početku lin. onda napon i struja linije na bilo kojem mjestu x linije:

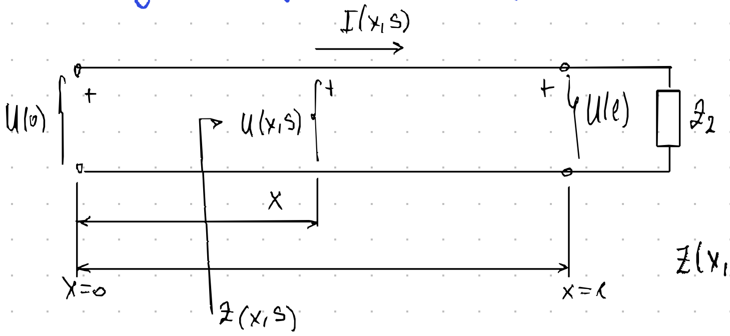
$$U(x,s) = \frac{U(0)}{1 + \Gamma_2 e^{-2\gamma l}}\left(e^{-\gamma x} + \Gamma_2 e^{-2\gamma l} e^{\gamma x}\right)$$

$$I(x,s) = \frac{U(0)}{Z_0\left(1 + \Gamma_2 e^{-2\gamma l}\right)}\left(e^{-\gamma x} - \Gamma_2 e^{-2\gamma l} e^{\gamma x}\right)$$

$$U(x,s) = U(0,s)\,\frac{e^{-\gamma(x-l)} + \Gamma_2 e^{\gamma(x-l)}}{e^{\gamma l} + \Gamma_2 e^{-\gamma l}}$$

$$I(x,s) = \frac{U(0,s)}{Z_0} \cdot \frac{e^{-\gamma(x-l)} - \Gamma_2 e^{\gamma(x-l)}}{e^{\gamma l} + \Gamma_2 e^{-\gamma l}}$$

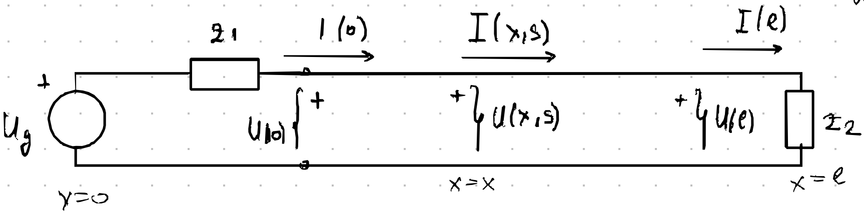
- Impedancija na mjestu x linije gledana prema izlazu



$$Z(x,s) = \frac{U(x,s)}{I(x,s)}$$

$$Z(x,s) = Z_0 \frac{e^{-\gamma(x-l)} + \Gamma_2 e^{\gamma(x-l)}}{e^{-\gamma(x-l)} - \Gamma_2 e^{\gamma(x-l)}}$$

Linija zaključena na oba kraja



$$U(0) = U_g - Z_1 I(0)$$

$$U(l) = I(l) \cdot Z_2$$

$$U(0) = A_1 + A_2 \qquad I(0) = \frac{A_1}{Z_0} - \frac{A_2}{Z_0}$$

$$A_1 + A_2 = U_g - Z_1 \left(\frac{A_1}{Z_0} - \frac{A_2}{Z_0} \right)$$

$$\Downarrow$$

$$A_1 \left(\frac{Z_0 + Z_1}{Z_0} \right) + A_2 \left(\frac{Z_0 - Z_1}{Z_0} \right) = U_g$$

$$\Downarrow$$

$$A_1 - A_2 \frac{Z_1 - Z_0}{Z_0 + Z_1} = U_g \frac{Z_0}{Z_0 + Z_1}$$

koeficijent refleksije na ulazu Γ_1

Vrijedi:

$$A_1 - \Gamma_1 A_2 = U_g \frac{Z_0}{Z_0 + Z_1} \qquad A_2 = A_1 \Gamma_2 e^{-2\gamma l}$$

$$A_1 - A_1 \Gamma_1 \Gamma_2 e^{-2\gamma l} = U_g \frac{Z_0}{Z_0 + Z_1}$$

$$\boxed{A_1 = U_g \frac{Z_0}{Z_0 + Z_1} \frac{1}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}}}$$

$$\boxed{A_2 = U_g \frac{Z_0}{Z_0 + Z_1} \cdot \frac{\Gamma_2 e^{-2\gamma l}}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}}}$$