MAXWELLOVE JEDNADŽBE → dobivanje valnih jednadžbi

I. $\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$ III. $\vec{\nabla}\times\vec{E} = \frac{-\partial B}{\partial t}$

II. $\vec{\nabla}\vec{B} = 0$ IV. $\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$$\boxed{\begin{aligned} &\vec{\nabla}\vec{E} = 0 &\quad& \vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} \\ &\vec{\nabla}\vec{B} = 0 && \vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\end{aligned}}$$

Maxwellove jed. u vakuumu imaju nesimetričan oblik

· ako brzine promjene el. i mag. toka nastaju samo zbog mijenjanja vektora $\vec{E}$ i $\vec{B}$ tada:

→ krećemo od rotacije el. i mag. polja

$$\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} \quad /\vec{\nabla}$$

$$\vec{\nabla}(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}\vec{\nabla}\times\vec{B} \longrightarrow \vec{\nabla}(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\times\vec{E}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \qquad * \ \vec{\nabla}(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\underbrace{\cancel{\vec{\nabla}\times\vec{E}}}_{0}) - \vec{\nabla}^2\vec{E}$$ linearna

$$\boxed{\vec{\nabla}^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$ Valna jednadžba el. polja

---

$$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad /\vec{\nabla}$$

$$\vec{\nabla}(\vec{\nabla}\times\vec{B}) = \mu_0\varepsilon_0\frac{\partial}{\partial t}\vec{\nabla}\times\vec{E} \longrightarrow \vec{\nabla}(\vec{\nabla}\times\vec{B}) = \mu_0\varepsilon_0\frac{\partial}{\partial t}\left(\frac{-\partial\vec{B}}{\partial t}\right)$$

$$\boxed{\vec{\nabla}^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t}}$$ Valna jednadžba mag. polja

$\mu_0\varepsilon_0$ je po definiciji $\frac{1}{c^2}$ → svugdje možemo zamijeniti

RAVNI LINEARNO POLARIZIRANI EMV

Valna jednadžba EMV

– val je ono što zadovoljava valnu jednadžbu → ne moramo imati sredstvo širenja vala!

· npr. jednadžba harmonijskog oscilatora (kao i u RLC) – ne tražimo opruge u RLC krugu

$$\left.\begin{aligned} &\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\ &\vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{B}=\frac{1}{c^2}\cdot\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\} \quad \begin{aligned}\nabla^2\vec{E}&=\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t}\\ \nabla^2\vec{B}&=\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t}\end{aligned}$$

jednadžba za el. polje:

$$\vec{\nabla}^2\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t}=0 \xrightarrow[\text{val } E(\vec{r},t)]{\text{njeno rj. je}} \vec{E}(\vec{r},t)=f(\omega t\pm\vec{k}\cdot\vec{r})$$

širi se brzinom c

↓ određuje smjer gibanja vala

valni vektor

$$\left.\begin{aligned}\vec{k}&=k_x\hat{x}+k_y\hat{y}+k_z\hat{z}\\ \vec{x}&=x\hat{x}+y\hat{y}+z\hat{z}\end{aligned}\right\}\quad \vec{k}\cdot\vec{r}=k_x x+k_y y+k_z z$$

→ određuje smjer gibanja vala

Ravni linearno polarizirani EMV

· poseban slučaj sinusoidnog vala za el. polje $\vec{E}=E_0\sin(\omega t-kx)\hat{E}$

kružna frekv. (ω), amplituda (E_0), $\vec{k}\vec{r}=k_x x=kx$

↳ pozitivan smjer x-osi $\boxed{k=+\hat{x}}$

· Zahtjevamo da EMV zadovoljava I. MAX

$$\vec{\nabla}=\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z} \longrightarrow \vec{\nabla}\vec{E}=\frac{\partial\vec{E}}{\partial x}\hat{x}+\underbrace{\cancel{\frac{\partial\vec{E}}{\partial y}}}_{=0}\hat{y}+\underbrace{\cancel{\frac{\partial\vec{E}}{\partial z}}}_{=0}\hat{z}=0$$

nemamo y i z komponentu

$$\vec{\nabla}\vec{E}=\frac{\partial}{\partial x}\left(E_0\sin(\omega t-kx)\right)\hat{E}\hat{x} \qquad \hat{E}\neq\hat{x}$$

$$\vec{\nabla}\vec{E}=-E_0\sin(\omega t-kx)\boxed{k_x\hat{x}\hat{E}}=0 \qquad \vec{k}\perp\vec{E}$$

→ E ne može sadržavati $\hat{x}$

→ smjer širenja vala uvijek mora biti okomit s obzirom na smjer širenja el. polja

$$\hat{E}\cdot\hat{k}=0$$

Magnetsko polje $\vec{B} = B_0 \sin(\omega t - kx)\hat{B}$

→ na sličan način dobivamo i za mag. polje $\vec{\nabla}\vec{B} = 0$

$$\vec{\nabla}\vec{B} = \frac{\partial}{\partial x}\left(B_0 \sin(\omega t - kx)\right)\hat{B}\hat{x}$$

$$\vec{\nabla}\vec{B} = -B_0 \cos(\omega t - kx)\boxed{k_x \hat{B}\hat{x}} = 0$$

opet $\vec{B} \perp \vec{k}$, B ne može sadržavati smjer x

⇒ $\vec{B}$ i $\vec{E}$ moraju biti okomiti na smjer širenja vala

Rotacija - odnos el. i mag. polja

$\vec{B} \perp \vec{k}$

$\vec{E} \perp \vec{k}$

$$\vec{\nabla}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

*ograničit ćemo na $\hat{E} = \hat{y}$ zbog jednostavnosti

$$\vec{E} = E_0 \sin(\omega t - kx)\hat{y}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \cancel{\hat{x}\cdot 0} + \cancel{\hat{y}\cdot 0} + \hat{z}\frac{\partial E_y}{\partial x}$$

$$\vec{\nabla}\times\vec{E} = \frac{\partial E_y}{\partial x}\hat{z} \rightarrow \boxed{\vec{\nabla}\times\vec{E} = -k E_0 \cos(\omega t - kx)\hat{z}}$$

budući da je

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

prema Maxwellu

onda mora biti jednako

$$-\frac{\partial \vec{B}}{\partial t} = -B_0 \omega \cancel{\cos(\omega t - kx)}\hat{B}$$

$$\vec{\nabla}\times\vec{E} = -k E_0 \cancel{\cos(\omega t - kx)}\hat{z}$$

$$\omega B_0 = k E_0$$

$$E_0 = \frac{\omega}{k} B_0$$

→ budući da je brzina širenja EMV c → $\boxed{E_0 = c B_0}$

iz jednadžbe vala

$$v = \lambda f = \frac{2\pi}{k}\cdot\frac{\omega}{2\pi}$$

$$v = \frac{\omega}{k}$$

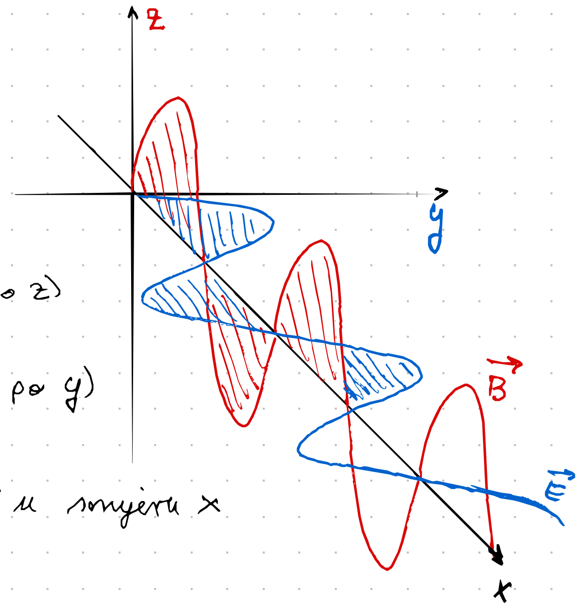
$E \perp k$, $B \perp k$, $E \perp B$

$$\boxed{\vec{E}\times\vec{B} = \vec{k}}$$

Kako se val širi?

- $\vec{B} = \frac{E_0}{c} \sin(\omega t - k_x x)\hat{z}$
- $\vec{E} = E_0 \sin(\omega t - k_x x)\hat{y}$

mora biti ista funkcija sinus



- mag polje u smjeru z (širi se po z)
- el. polje u smjeru y (širi se po y)

$\Rightarrow \hat{k} = +\hat{x} \longrightarrow$ val se širi u smjeru x

Zadatak (neki ne sa ppt)

$\vec{E} = E_0 \cos(\omega t + kz)(\hat{x} + \cancel{\hat{z}})$

ne može imati z komponentu jer je smjer vala z, a $E \perp k$!

$\hat{k} = -\hat{z}$

$\hat{E} = \hat{x}$

$E_0 = 1\ V/m$

$\omega = 10^{15}\ s$

$B = ?$

→ trebamo samo $\hat{B}$ i B_0 jer je trig. dio isti!

$c = \frac{\omega}{k} \longrightarrow k = \frac{\omega}{c}$

$\hat{k} = \hat{E} \times \hat{B}$

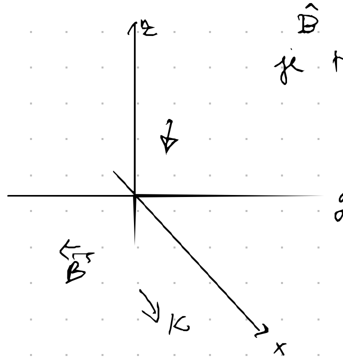
$\downarrow$

$-\hat{z} = \hat{x} \times \boxed{-\hat{y}}$

to jedino može jer je okomito na oboje

$E_0 = B_0 c$

$B_0 = \frac{1}{c}\ T$



$\vec{B} = \frac{1}{c} \cos(\omega t + kz)(-\hat{y})$

zap. minus nije ni bitan

Zadatak 2.)

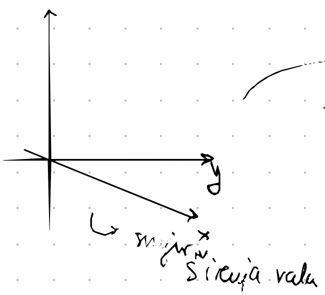
možemo pisati i cos

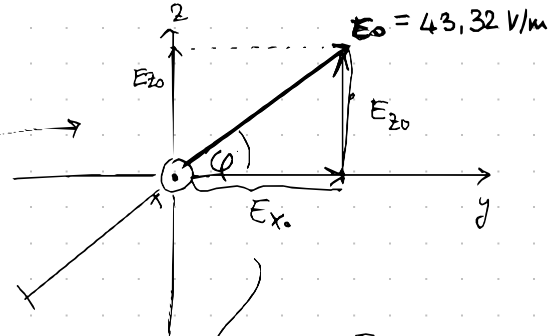
$f = 6 \cdot 10^{14}\,Hz$ $\qquad \hat{k} = \hat{x}$ $\qquad \vec{E} = E_0 \sin(\omega t - kx)\hat{E}$

$\omega = 2\pi f$ $\qquad E_0 = 42{,}32\,V/m$ $\qquad B = ?$

$k = \frac{\omega}{c}$ $\quad \rightarrow$ širi se u y-z ravnini pod kutom $45° \rightarrow \varphi = 45$

$\hat{E} \neq \hat{x}$





$\sin\varphi = \frac{E_{z0}}{E_0}$ $\qquad \cos\varphi = \frac{E_{y0}}{E_0}$

$\vec{E} = E_y \hat{y} + E_z \hat{z}$

$\underline{E_z = \frac{\sqrt{2}}{2} E_0}$ $\qquad \underline{E_{y0} = \frac{\sqrt{2}}{2}}$

$E = E_{y0} \sin(\omega t - kx)\hat{y} + E_{z0} \sin(\omega t - kx)\hat{z}$

$$\boxed{E = E_0 \frac{\sqrt{2}}{2} \cdot \sin(\omega t - kx)(\hat{y} + \hat{z})}$$

$$\vec{E} = E_0 \sin(\omega t - kx) \cdot \frac{\hat{y} + \hat{z}}{\sqrt{2}}$$

$\hat{E} \times \hat{B} = \hat{k}$

$\vec{B} = B_0 \sin(\omega t - kx)\hat{b}$

$$\frac{\hat{y} + \hat{z}}{\sqrt{2}} \times \boxed{\frac{\pm\hat{y} \pm \hat{z}}{\sqrt{2}}} = \hat{x}$$

$$\underbrace{\frac{\hat{y}}{\sqrt{2}} \times \left(\pm \frac{\hat{z}}{\sqrt{2}}\right)}_{\frac{\hat{x}}{2}} + \underbrace{\frac{\hat{z}}{\sqrt{2}} \times \left(\pm \frac{\hat{y}}{\sqrt{2}}\right)}_{\frac{\hat{x}}{2}} = \hat{x}$$

– želimo dobiti kombinaciju koja rezultira sa $\hat{x}$

$$\frac{\hat{x}}{2} + \frac{\hat{x}}{2} = \hat{x}$$

$\rightarrow \hat{b} = \frac{\hat{z}}{\sqrt{2}} - \frac{\hat{y}}{\sqrt{2}} \rightarrow \vec{B} = \frac{E_0 \sin(\omega t - kx)}{c} \frac{(\hat{z} - \hat{y})}{\sqrt{2}}$