Jednadžbe bridova

- Za analizu je potrebno
  - → br čvorišta N_V
  - → br grana N_B
  - → kako su one povezane

* broj nepoznanica je $2N_B$
(nepoznata I i nepoznata U za svaku granu)

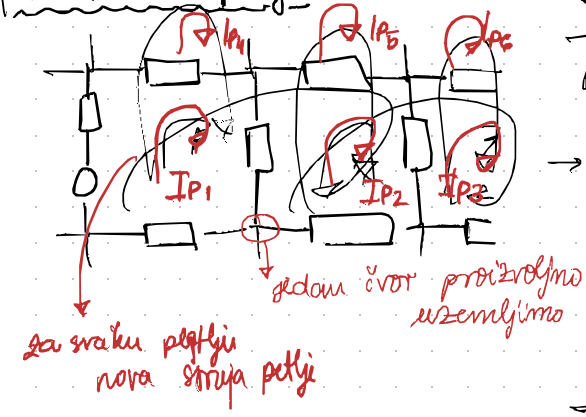
* $N_V - 1$ lin. nez jed. KZS

$N_B - N_V + 1$ lin. nez jed KZN

→ Želimo reducirati broj nepoznanica → uvodimo nove var.
- jednadžbe petlji i čvorišta

→ nove nepoznanice: struje petlji i naponi čvorišta

Jednadžbe petlji ili (konturnih struja)



→ odredimo potencijale ostalih čvorova pomoću log def v

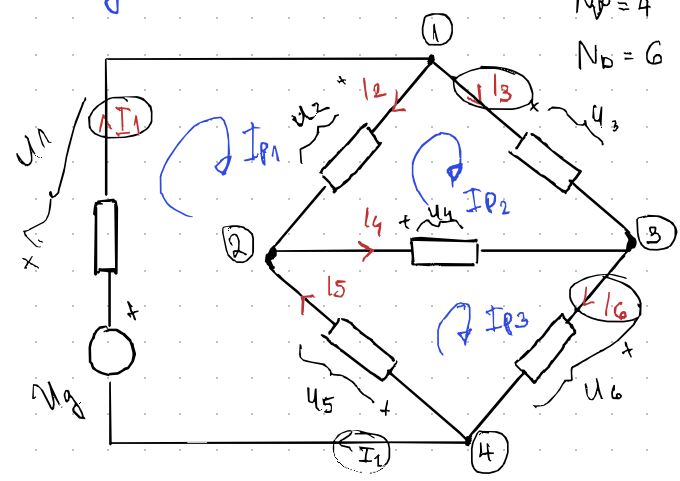
→ def KZN za petlje (njih $N_B - N_V + 1$)
(samo unutar okna)

→ uvodimo struje petlji
= struje grana izrazimo preko I_P

→ sve to u KZN ulaziti

* napon svake grane izrazimo kao $f(I_P)$

Primjer:



$N_V = 4$
$N_B = 6$

→ KZS = 3 $(N_V - 1)$
KZN = 3 $(N_B - N_V + 1)$

KZS: $-I_1 + I_2 + I_3 = 0$
$-I_2 + I_4 - I_5 = 0$
$-I_3 - I_4 + I_6 = 0$

KZN: $-U_1 + U_2 - U_5 = 0$
$-U_2 + U_3 - U_4 = 0$
$U_5 + U_4 + U_6 = 0$

uvodimo nove varijable: I_{P1}, I_{P2}, I_{P3}

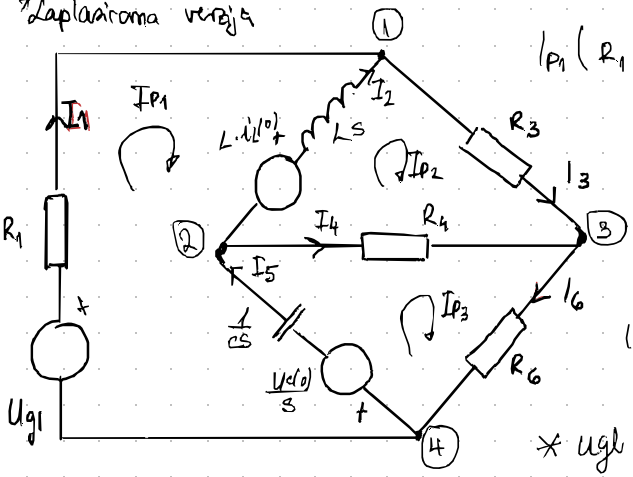
$I_{P1} = I_1$ $I_{P2} = I_3$ $I_{P3} = I_6$

→ $I_2 = I_{P1} - I_{P2}$ → $I_5 = I_{P3} - I_{P1}$

→ $I_4 = I_{P3} - I_{P2}$

• jednadžbe petlji je moguće odmah pisati u domeni kompleksne frekvencije s.

*Laplaceova verzija



$$I_{P1}\left(R_1 + Ls + \frac{1}{Cs}\right) - I_{P2}(Ls) - I_{P3}\frac{1}{Cs} = U_{g1} - Li_L(0) + \frac{U_c(0)}{s}$$

$$I_{P2}(Ls + R_3 + R_4) - I_{P1}(Ls) - I_{P3}(R_4) = Li_L(0)$$

$$I_{P3}\left(R_4 + R_6 + \frac{1}{Cs}\right) - I_{P1}\left(\frac{1}{Cs}\right) - I_{P2}(R_4) = -\frac{U_c(0)}{s}$$

* U_{g1} sve ono iz OE samo Laplace

⇒ jednadžbe konturnih struja

• u matričnoj formi: $\vec{U_g} = \vec{Z_p} \cdot \vec{I_p}$

$\vec{I_p}$ – vektor struja petlji $\quad \vec{I_p} = \begin{bmatrix} I_{P1} \\ I_{P2} \\ I_{P3} \end{bmatrix}$

$\vec{U_g}$ → vektor naponskih izvora $\quad \vec{U_g} = \begin{bmatrix} U_{g_1} - Li_L(0) + \frac{U_c(0)}{s} \\ Li_L(0) \\ -\frac{U_c(0)}{s} \end{bmatrix}$

$$\vec{Z_p} = \begin{bmatrix} R_1 + Ls + \frac{1}{Cs} & -Ls & -\frac{1}{Cs} \\ -Ls & Ls + R_3 + R_4 & -R_4 \\ -\frac{1}{Cs} & -R_4 & R_4 + R_6 + \frac{1}{Cs} \end{bmatrix}$$

– matrica impedancija petlji (3. grada)

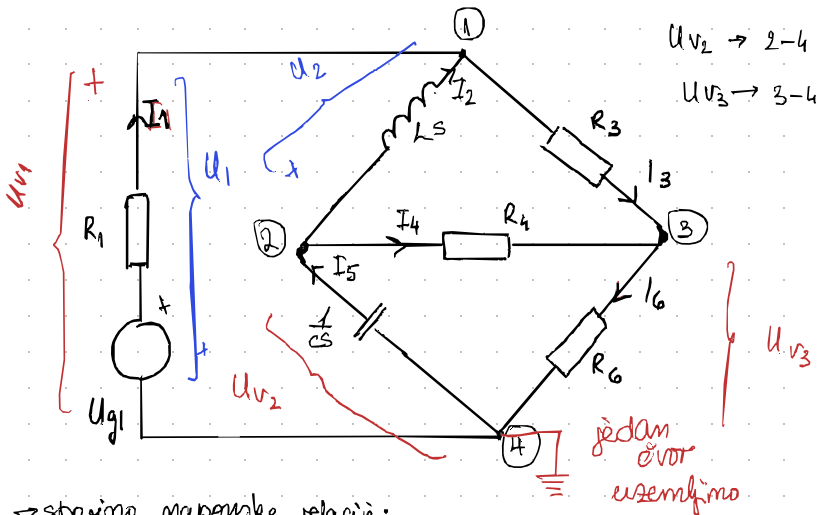
Jednodžba čvorišta *(MPČ)

$u_{v_1} \rightarrow$ 1-4

$u_{v_2} \rightarrow$ 2-4

$u_{v_3} \rightarrow$ 3-4

ako znamo njih, možemo dobiti napone i struje grana



$u_1 = -u_{v_1}$

$u_2 = u_{v_2} - u_{v_1}$

$u_3 = u_{v_1} - u_{v_3}$

$u_4 = u_{v_2} - u_{v_3}$

$u_5 = -u_{v_2}$

$u_6 = u_{v_3}$

→ strujno naponske relacije:

$$I_1 = \frac{u_{g_1} + u_1}{R_1} = \frac{u_{g_1} - u_{v_1}}{R_1}$$

$$I_2 = \frac{1}{L}\int_0^t u_2(\tau)\,d\tau + i_L(0) = \frac{1}{L}\int_0^t (u_{v_2} - u_{v_1})\,d\tau + i_L(0)$$

$$I_3 = \frac{u_3}{R_3} = \frac{u_{v_1} - u_{v_3}}{R_3} \qquad I_5 = C\frac{du_5}{dt} = -C\frac{du_{v_2}}{dt}$$

$$I_4 = \frac{u_4}{R_4} = \frac{u_{v_2} - u_{v_3}}{R_4} \qquad I_6 = \frac{u_6}{R_6} = \frac{u_{v_3}}{R_6}$$

$$-I_1 - I_2 + I_3 = 0$$

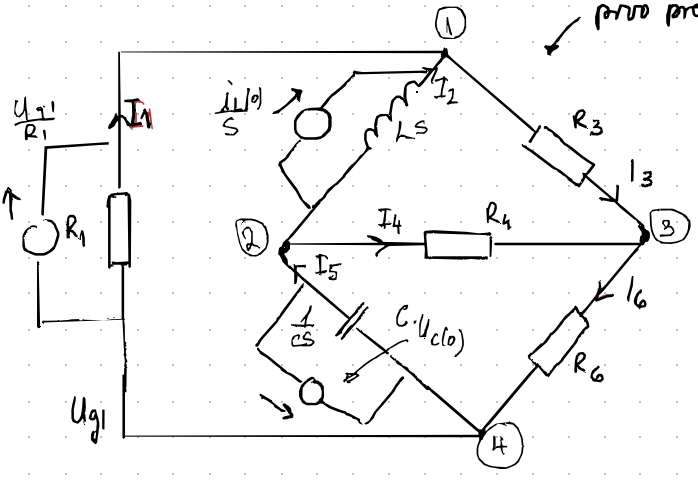
$$-I_5 + I_2 + I_4 = 0$$

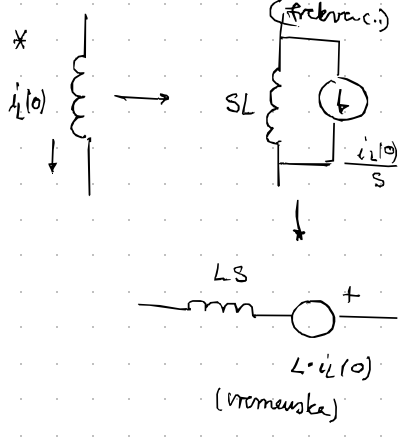
$$-I_3 - I_4 + I_6 = 0$$

supstituiramo i Laplasiramo

→ moguće direktno zapisati: teškoća da imamo sve strujne izvore!!

prvo pretvaramo u frekvencijsku domenu





po čvorištima (što je spojeno na njih)

$$\varphi_1\left(\frac{1}{Ls} + \frac{1}{R_3} + \frac{1}{R_1}\right) - \varphi_2\left(\frac{1}{Ls}\right) - \varphi_3\left(\frac{1}{R_3}\right) = \frac{u_{g_1}}{R_1} + \frac{i_L(0)}{s}$$

$$-\varphi_1\left(\frac{1}{Ls}\right) + \varphi_2\left(\frac{1}{Ls} + \frac{1}{R_4} + cs\right) - \varphi_3\left(\frac{1}{R_4}\right) = -\frac{i_L(0)}{s} - C \cdot u_c(0)$$

$$-\varphi_1\left(\frac{1}{R_3}\right) - \varphi_2\left(\frac{1}{R_4}\right) + \varphi_3\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6}\right) = 0$$

Maticni zapis: $\vec{I_g} = \vec{Y_V} \cdot \vec{U_V}$

$\vec{U_V}$ → vektor napona čvorišta $\vec{U_V} = \begin{bmatrix} U_{V1} \\ U_{V2} \\ U_{V3} \end{bmatrix} = * \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}$

$\vec{I_g}$ → vektor strujnih izvora $= \begin{bmatrix} \frac{U_{g1}(s)}{R} + \frac{i_L(0)}{s} \\ -\frac{i_L(0)}{s} - C\,U_c(0) \\ 0 \end{bmatrix}$

$$\vec{Y_V} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{LS} + \frac{1}{R_3} & -\frac{1}{LS} & -\frac{1}{R_3} \\ -\frac{1}{LS} & \frac{1}{LS} + \frac{1}{R_4} + CS & \frac{-1}{R_4} \\ \frac{-1}{R_3} & \frac{-1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \end{bmatrix}$$