7.4.2. Metoda oblika desne strane

$\underset{\sim\sim}{Ly = f(x)} \rightarrow$ ako je $f(x) = e^{\alpha x}\left[Q_1(x)\cos\beta x + Q_2(x)\sin\beta x\right]$, tada:

Rj: $y = y_h + y_p \longrightarrow y_p = e^{\alpha x}\left[R_1(x)\cos\beta x + R_2(x)\sin\beta x\right]\cdot x^k$

k je kratnost nultočke $\alpha \pm i\beta$ u karakt. polinomu

gdje su $R_1(x)$ i $R_2(x)$ polinomi stupnja $\max\{st(Q_1), st(Q_2)\}$ čije koeficijente dobijemo uvrštavanjem u početnu jednadžbu

Npr. : $y'' + y = x^2$

1) $r^2 + 1 = 0$

$r = \pm i$

$y_h = C_1\cos x + C_2\sin x$

isto je i y_p ali s nekim neodređenim koeficijentima

2) $y_p = Ax^2 + Bx + C \longrightarrow$ mora biti istog stupnja kao g i

moramo provjeriti je li lin. nezavisno sa y_h i u odnosu na to množimo sa x^k

• ne moramo množiti s x jer je $\alpha = 0$, $\beta = 0$

$y' = 2Ax + B \qquad y'' = 2A$

$2A + Ax^2 + Bx + C = x^2$

$A = 1 \qquad B = 0 \qquad 2A + C = 0$

$C = -2$

$$\Rightarrow \boxed{y_p = C_1\cos x + C_2\sin x + x^2 - 2}$$

Pr.) $y'' - y = 2\sin x - \cos x$

1) $\left.\begin{matrix} r^2 - 1 = 0 \\ r = \pm 1 \end{matrix}\right\}$ $y_h = C_1 e^x + C_2 e^{-x}$

ovo nisu isti α i β!

$\alpha = 0$, $\beta = 1$

2) $y_p = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

$\Rightarrow -A\sin x - B\cos x - A\sin x - B\cos x = 2\sin x - \cos x$

$-2B = -1$

$B = \frac{1}{2}$

$$\boxed{y = C_1 e^x + C_2 e^{-x} - \sin x + \frac{1}{2}\cos x}$$

Pr.)

a) $f(x) = x^3 e^{-x}$ $\qquad \alpha = -1$

$y_p = (Ax^3 + Bx^2 + Cx + 1)e^{-x}$

b) $f(x) = \sin 3x - x\cos 3x$

$y_p = (Ax+B)\sin 3x + (Cx+D)\cos 3x$

Zadatak LJIR - 2022)

(8) $y^{(4)} = y + 8xe^{x} \longrightarrow y^{(4)} - y = \overbrace{8xe^{x}}^{f(x)}$

1) $r^4 - 1 = 0$

$(r^2-1)(r^2+1) = 0$

$(r-1)(r+1)(r^2+1) = 0$

$r_{1,2} = \pm 1$

$r_{3,4} = \pm i$

$\Rightarrow y_h = C_1 e^{x} + C_2 e^{-x} + C_3 \sin x + C_4 \cos x$

2) $y_p = (Ax+B)e^{x}$ – ovo nije lin nezavisno jer Be^{x} je konstanta $\cdot e^{x}$

< moramo množiti s x

$y_p = (Ax+B)e^{x} \cdot x^{\alpha=1} = (Ax^2+Bx)e^{x}$

$y_p' = (2Ax+B)e^{x} + (Ax^2+Bx)e^{x}$

$y'', y''', y^{(4)} \ldots$

$(Ax^2 + 8A + B)x + 12A + 4B)e^{x} - (Ax^2+Bx)e^{x} = 8xe^{-x}$

$8A = 8 \rightarrow A = 1$ $\qquad 12A + 4B = 0 \rightarrow B = -3$

LJIR - 21 - 8) -> DZ

$y''' - y' = \frac{x}{e^x}$

1) $r^3 - r = 0$

$y_h = C_1 + C_2 e^{x} + C_3 e^{-x}$ $\quad$ isto η ?

2) MVK 3×3, ... $\qquad \alpha = -1$

ili MODS: $y_p = (Ax+B)e^{-x} \cdot x$

$y''' = \ldots$ uvrstit

Pr.) a) $f(x) = x^3 + e^{-x}$

$x^3 \to y_{P_1}$, $e^{-x} \to y_{P_2}$

DOKAZIĆ KOJI JE BIO PROŠLE GODINE:

$$L(y_{P_1} + y_{P_2}) = Ly_{P_1} + Ly_{P_2}$$
$$= f_1(x) + f_2(x)$$

Z1-20-8) $y'' - 5y' + 6y = \underbrace{3x}_{f_1} + \underbrace{e^{2x}}_{f_2}$

1) $r^2 - 5r + 6 = 0$

$r_1 = 2 \quad r_2 = 3$

$y_h = C_1 e^{2x} + C_2 e^{3x}$

jer ovaj ce^{2x} imamo!

2) $y_{P_1} = Ax + B \quad (\alpha = 0)$ nije rj.

$y' = A, \quad y'' = 0$

$0 - 5A + 6Ax + 6B = 3x$

$6A = 3$

$A = \frac{1}{2}$

$-5A + 6B = 0$

$B = \frac{5}{2} \cdot \frac{1}{6} = \frac{5}{12}$

prvi dio

3) $y_{P_2} = Ce^{2x} \cdot x \quad \alpha = 2$ ✓

$y' = 2Ce^{2x} \cdot x + Ce^{2x}$

$y'' = 4Ce^{2x} \cdot x + 2Ce^{2x} + 2Ce^{2x}$

drugi dio

$\Rightarrow 4Ce^{2x} \cdot x + 4Ce^{2x} - 5(2Ce^{2x} \cdot x + Ce^{2x}) + 6(Ce^{2x} \cdot x)$

$\Rightarrow \cancel{4Ce^{2x}x} + 4Ce^{2x} - \cancel{10Ce^{2x} \cdot x} + Ce^{2x} + \cancel{6Ce^{2x} \cdot x} = e^{2x}$

$-C = 1 \longrightarrow \boxed{C = -1}$

OPĆE: $y = y_h + y_{P_1} + y_{P_2} = C_1 e^{2x} + C_2 e^{3x} + \underbrace{\tfrac{1}{2}x + \tfrac{5}{12}}_{P_1} \underbrace{- xe^{2x}}_{P_2}$

DZ

4) $y' = 2C_1 e^{2x} + 3C_2 e^{3x} + \frac{1}{2} - e^{2x} - 2xe^{2x}$

$y'(0) = 1 \Rightarrow y'(0) = 2C_1 + 3C_2 + \frac{1}{2} - 1 = 1$

$y(0) = 0 \Rightarrow y(0) = C_1 + C_2 + \frac{5}{12} = 0$

dovršiti

$C_1 = \frac{-11}{4}, \quad C_2 = \frac{7}{3}$

Z1-21-6) $y'' + 4y + 2 = \sin 2x \Rightarrow y'' + 4y = \sin 2x - 2$

↳ u partikularnom su i SINUS i COSINUS
dolaze u paru jer konjugirano kompleksne
nultočke su uvijek u paru!

1) $r^2 + 4 = 0$

$r = \pm 2i$

$y_h = C_1 \cos 2x + C_2 \sin 2x$

$\alpha = 0 \quad \beta = 2$

2)

$y_{p_1} = (A \sin 2x + B \cos 2x) \cdot x$

$y' = A \sin 2x + B \cos 2x + (2A \cos 2x - 2B \sin 2x)x$

$y'' = 2A \cos 2x - 2B \sin 2x + 2A \cos 2x - 2B \sin 2x + (-4A \sin 2x - 4B \cos 2x)x$

$4A \cos 2x - 4B \sin 2x + \cancel{(-4A \sin 2x - 4B \cos 2x)x} + \cancel{4x(A \sin 2x + B \cos 2x)} = \sin 2x$

$-4B = 1 \qquad 4A = 0$

$B = -\frac{1}{4} \qquad A = 0$

$y_{p_1} = (\cancel{0 \sin 2x} - \frac{1}{4} \cos 2x)x$

3) $y_{p_2} = C \qquad y' = 0, \quad y'' = 0$

$0 + 4C = -2$

$C = -\frac{1}{2}$

$\Rightarrow$ OPĆE:

$$\boxed{y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{4} x \cos 2x - \frac{1}{2}}$$

DIR-21-8) $y'' + 4y = \underbrace{\cos 2x}_{y_{p1}} + \underbrace{x \sin x}_{p2}$

1) $r^2 + 4 = 0$

$r_{1,2} = \pm 2i$

$y_h = C_1 \sin 2x + C_2 \cos 2x$

množimo s x

$y_{p_1} = (A \cos 2x + B \sin 2x)x$

$y_{p_2} = (Cx + D) \sin x + (Ex + F) \cos x$