EKSPONENC. ZAPIS KOMPL. BROJA

$e^{i\varphi} = \cos\varphi + i\sin\varphi$

$z = r(\cos\varphi + i\sin\varphi)$

$\boxed{z = r \cdot e^{i\varphi}}$ eksponencijalan zapis

$r = |z|$

$\varphi = \arg z$

Pr.)

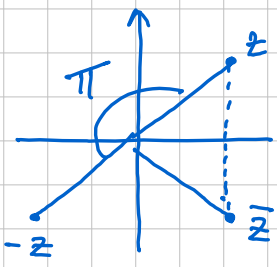
$z = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ $\quad r = |z| = \sqrt{\frac{2}{4} + \frac{2}{4}} = 1$

$\operatorname{tg}\varphi = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$ $\quad \varphi = \frac{7\pi}{4}$

$\boxed{z = 1 \cdot e^{\frac{7\pi}{4}}}$

VRIJEDI:

1. $\arg \bar{z} = -\arg z \ (+2k\pi)$
2. $\arg(-z) = \pi + \arg z \ (+2k\pi)$



3. $\arg(c \cdot z) = \arg c + \arg z$

$$= \begin{cases} 0 + \arg z, & c > 0 \\ \pi + \arg z, & c < 0 \end{cases}$$

4. $\arg(iz) = \arg i + \arg z = \frac{\pi}{2} + \arg z$

Zad.)

a) Skicirajte $\arg(z^3) = \frac{\pi}{2}$

b) Skicirajte sve $z \in \mathbb{C}$ td. $|z+2| = 1$

c) Nadite sve $z \in \mathbb{C}$ td $\arg(z^3) = \frac{\pi}{2}$; $|z+2| = 1$

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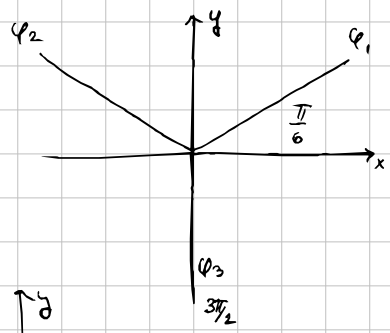
a) $\arg(z^3) = \frac{\pi}{2} + 2k\pi$ ili $3\varphi = \frac{\pi}{2} + 2k\pi$

$3\arg z = \frac{\pi}{2} + 2k\pi / : 3$ (umjesto arg pisati φ)

$\arg z = \frac{\pi}{6} + \frac{2}{3}k\pi$, $k = 0, 1, 2$

$\varphi_1 = \frac{\pi}{6}$ $\quad \varphi_2 = \frac{5\pi}{6}$ $\quad \varphi_3 = \frac{9\pi}{6} = \frac{3\pi}{2}$

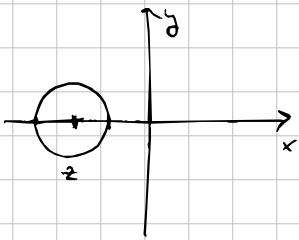
$\varphi = \frac{\pi}{6} \; , \; \frac{5\pi}{6} \; , \; \frac{3\pi}{2}$



b) $|z+2| = 1$

kružnica

$S(-2, 0)$, $r = 1$



c) Iz slika vidimo da je presjek moguć samo za

$\varphi_2 = \frac{5\pi}{6}$

$|z + 2| = 1$

$|r\cos\varphi + 2 + r \cdot i \sin\varphi| = 1$

$\sqrt{(r \cdot \cos\varphi + 2)^2 + (r \sin\varphi)^2} = 1 \; /^2$

$r^2 \cdot \cos^2\varphi + 4r\cos\varphi + 4 + r^2 \sin^2\varphi = 1$

$r^2(\cos^2\varphi + \sin^2\varphi) + 4r\cos\varphi + 3 = 0$

$r^2 + 4r\cos\varphi + 3 = 0$

$r^2 + 4 \cdot r\cos\left(\frac{5\pi}{6}\right) + 3 = 0$

$r_{1,2} = \frac{2\sqrt{3} \pm \sqrt{12 - 12}}{2} = \sqrt{3} > 0$ ✓

$z = \sqrt{3} \; cis \; \frac{5\pi}{6}$

Zad.) Odredite sve kompl. brojeve z koje je $z^2 + \frac{1}{z^2} = -1$ i $\mathbb{R} \in \mathbb{Z} \geq 0$

$z^2 + \frac{1}{z^2} = -1 \mid \cdot z^2$

$z^4 + 1 = -z^2$

$\boxed{z^4 + z^2 + 1 = 0}$

supst. $t = z^2$

$t^2 + t + 1 = 0$

$t_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

$t_1 = \frac{-1+\sqrt{3}i}{2} = \frac{-1}{2} + \frac{\sqrt{3}i}{2}$ $\quad r = 1$

$= \text{cis}\,\frac{2\pi}{3} \Rightarrow z^2 = \text{cis}\,\frac{2\pi}{3}$

$z_{1,2} = \sqrt{\text{cis}\,\frac{2\pi}{3}} = \text{cis}\,\frac{\frac{2\pi}{3} + 2k\pi}{2}$, $k = 0, 1$

$= \text{cis}\left(\frac{\pi}{3} + k\pi\right)$, $k = 0, 1$

I. kvadrant

$\boxed{z_1 = \text{cis}\,\frac{\pi}{3} \quad z_2 = \text{cis}\,\frac{4\pi}{3}}$

$t_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i = \text{cis}\,\frac{4\pi}{3}$

$z^2 = \text{cis}\,\frac{4\pi}{3} \mid \sqrt{\ } \longrightarrow z = \sqrt{\text{cis}\,\frac{4\pi}{3}} = \text{cis}\,\frac{\frac{4\pi}{3} + 2k\pi}{2}$

$z_3 =$

$\boxed{z_4 = \text{cis}\,\frac{5\pi}{3}}$

Zad.) Rijesite jednadzbu: $z^6 = \bar{z}(1-i)$

Jednakost dva kompleksna broja: $\begin{cases} |z_1| = |z_2| \\ \arg z_1 = \arg z_2 + 2k\pi \end{cases}$

$\boxed{z = r\,\mathrm{cis}\,\varphi}$

$z^6 = \bar{z}(1-i) \iff$

$|z^6| = |\bar{z}(1-i)|$

$\arg(z^6) = \arg(\bar{z}(1-i)) + 2k\pi$

(1.) $|z| = r$

$|z^6| = |\bar{z}| \cdot |1-i|$

$|z|^6 = |\bar{z}| \cdot |1-i|$

$r^6 = r \cdot \sqrt{2}$

$r^6 - \sqrt{2} \cdot r = 0$

$r(r^5 - \sqrt{2}) = 0$

$r_1 = 0$ $\qquad$ $r^5 = \sqrt{2}$

$\boxed{z_1 = 0}$ $\qquad$ $\boxed{r_2 = \sqrt[10]{2}}$

(2.) $\arg(z^6) = \arg(\bar{z}(1-i)) + 2k\pi$

$6\varphi = \arg(\bar{z}) + \arg(1-i) + 2k\pi$

$6\varphi = -\varphi + \frac{7\pi}{4} + 2k\pi$

$7\varphi = \frac{7\pi}{4} + 2k\pi \,/ : 7$

$\boxed{\varphi = \frac{\pi}{4} + \frac{2}{7}k\pi,\ k = 0, 1 \dots 6}$

$z_1 = 0$

$z_{1,2,3,4,5,6,7} = \sqrt[10]{2}\,\mathrm{cis}\left(\frac{\pi}{4} + \frac{2}{7}k\pi\right)$

$k = 0, \dots 6$

Zad.) Odredite se $z \in \mathbb{C}$ za koje vrijedi

$$\begin{cases} \arg(z^2 \cdot i) = \pi + 2k\pi \\ |z - 1 + i| = 2 \end{cases}$$

$z = \cos\varphi + i\sin\varphi$

$|r\cos\varphi - 1 + i\sin\varphi| = 2$

(1.) $\arg(z^2 \cdot i) = \pi + 2k\pi$

$\arg(z^2) + \arg i = \pi + 2k\pi$

$2\varphi + \frac{\pi}{2} = \pi + 2k\pi$

$2\varphi = \frac{\pi}{2} + 2k\pi \,/ :2$

$$\boxed{\varphi = \frac{\pi}{4} + k\pi, \; k = 0, 1}$$