1 a) $(z+\sqrt{2})^4 = -16\,\text{cis}(\pi)$

$$z+\sqrt{2} = \sqrt[4]{16\,\text{cis}\,\pi} = \sqrt[4]{2^4}\ \text{cis}\ \frac{\pi+2k\pi}{4}, \quad k=0,1,2,3$$

$$z+\sqrt{2} = 2\,\text{cis}\left(\frac{\pi}{4}+\frac{k\pi}{2}\right), \quad k=0,1,2,3$$

$$z_1+\sqrt{2} = 2\,\text{cis}\,\frac{\pi}{4} = 2\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = \sqrt{2}+\sqrt{2}\,i$$

$$\boxed{z_1 = \sqrt{2}\,i}$$

$$z_2+\sqrt{2} = 2\,\text{cis}\,\frac{3\pi}{4} = 2\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = -\sqrt{2}+\sqrt{2}\,i$$

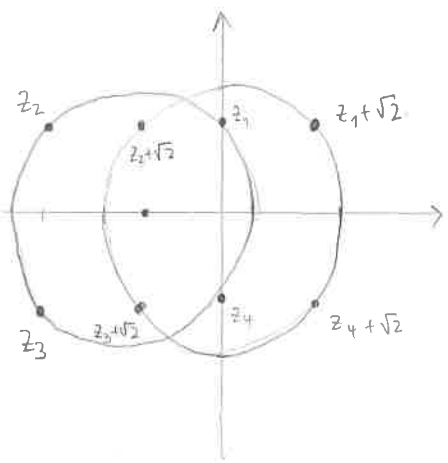
$$\boxed{z_2 = -2\sqrt{2}+\sqrt{2}\,i}$$

$$z_3+\sqrt{2} = 2\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = -\sqrt{2}-\sqrt{2}\,i$$

$$\boxed{z_3 = -2\sqrt{2}-\sqrt{2}\,i}$$

$$z_4+\sqrt{2} = 2\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = \sqrt{2}-\sqrt{2}\,i$$

$$\boxed{z_4 = -\sqrt{2}\,i}$$



b) KRUŽNICA

$$|z+\sqrt{2}| = 2$$

2 (a) $x_1 + x_2 + \cdot + x_n = k$ iznosi $\binom{n+k+1}{k}$

2 (b_1) Kombinacije s ponavljanjem ili broj rj. jdn. $x_1+x_2+x_3+x_4=10$

$n=4,\ k=10$

$$\binom{10+4-1}{10} = \binom{13}{10}$$

2 (b_2) A = Barem jedna osoba ne dobije niti jedan predmet.

A je komplement od:

$\bar{A}$ = svaka osoba dobije barem jedan predmet.

$$\underbrace{|\bar{A}| = \binom{6+4-1}{6} = \binom{9}{6}}$$

- podijelimo svima po 1 predmet
- kombinacije s ponavljanjem za preostalih 6 predmeta

$$\Rightarrow \quad |A| = |X| - |\bar{A}| = \binom{13}{10} - \binom{9}{6}$$

2 (b3) 1. način: produktno pravilo $\binom{8}{2} \cdot \binom{6}{2} \cdot \binom{4}{2} \cdot \binom{2}{2}$

2 način: permutacije s ponavljanjem (predmetu pridružujemo osobu)

$$\frac{8!}{2!\,2!\,2!\,2!}$$

3 (a) $a_n = n$

$$\frac{1}{a_n^3} = \frac{1}{n^3} \longrightarrow 0 \text{ kada } n \to +\infty$$

3 (b) JEDINO GOMILIŠTE JE $\frac{7}{6}$. NIZ JE KONVERGENTAN S LIMESOM $\frac{7}{6}$.

3 (c) $$\lim_{n\to\infty} \frac{\cos(3^n) + 2\cdot 3^n}{2^n + 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n\to\infty} \frac{\frac{\cos(3^n)}{3^n} + 2}{\left(\frac{2}{3}\right)^n + 1} = 2$$

④ (a) Vrijedi:

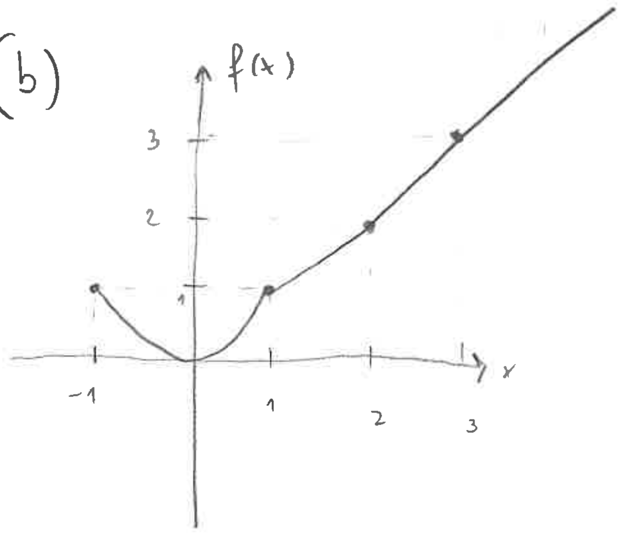
$$\lim_{x\to 1^-} f(x) = 1 \qquad \lim_{x\to 1^+} f(x) = a$$

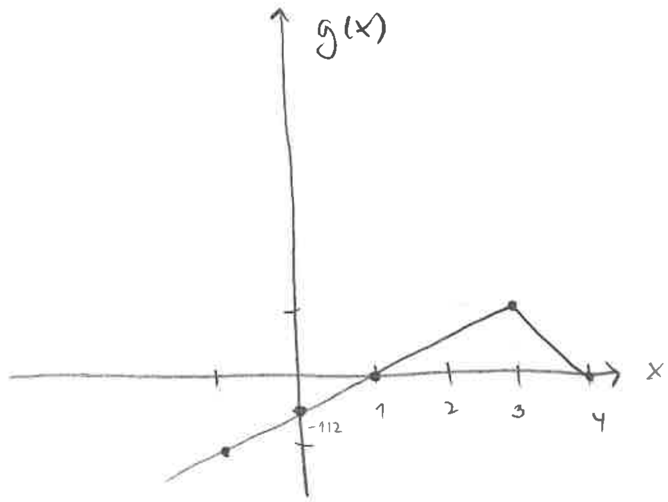
pa je $a = 1$. Također

$$\lim_{x\to 3^-} g(x) = 1 \qquad \lim_{x\to 3^+} g(x) = b-3$$

pa je $b = 4$.

④ (b)





④ (c)

$$h_1'(2) = f'(g(2)) \cdot g'(2) = f'\left(\tfrac{1}{2}\right) \cdot \tfrac{1}{2} = \tfrac{1}{2}$$

$$h_2'(1) = g'(f(1)) \cdot f'(1) = g'(1) \cdot 2 = 1$$

(5) $\mathcal{D}(f)=\mathbb{R}$

- NEMA V.A

$\in[-1,1]$

- K.A. $\quad k=\lim\limits_{x\to\pm\infty}\dfrac{3\cos\left(\frac{1}{x^2+1}\right)}{x}=0$

$$l=\lim_{x\to\pm\infty} 3\cos\left(\frac{1}{x^2+1}\right)=3\cos\left(\frac{1}{+\infty}\right)=3\cos(0)=3$$

$\boxed{y=3 \text{ JE D. i L. H.A}}$

$$f'(x)=\frac{6x\sin\left(\frac{1}{x^2+1}\right)}{(x^2+1)^2}=0$$

$$\Leftrightarrow\ 6x\,\sin\left(\frac{1}{x^2+1}\right)=0$$

$\frac{1}{x^2+1}\in\langle 0,1]$

$\neq 0$

$\Leftrightarrow\ \boxed{x=0}$ STACIONARNA TOČKA

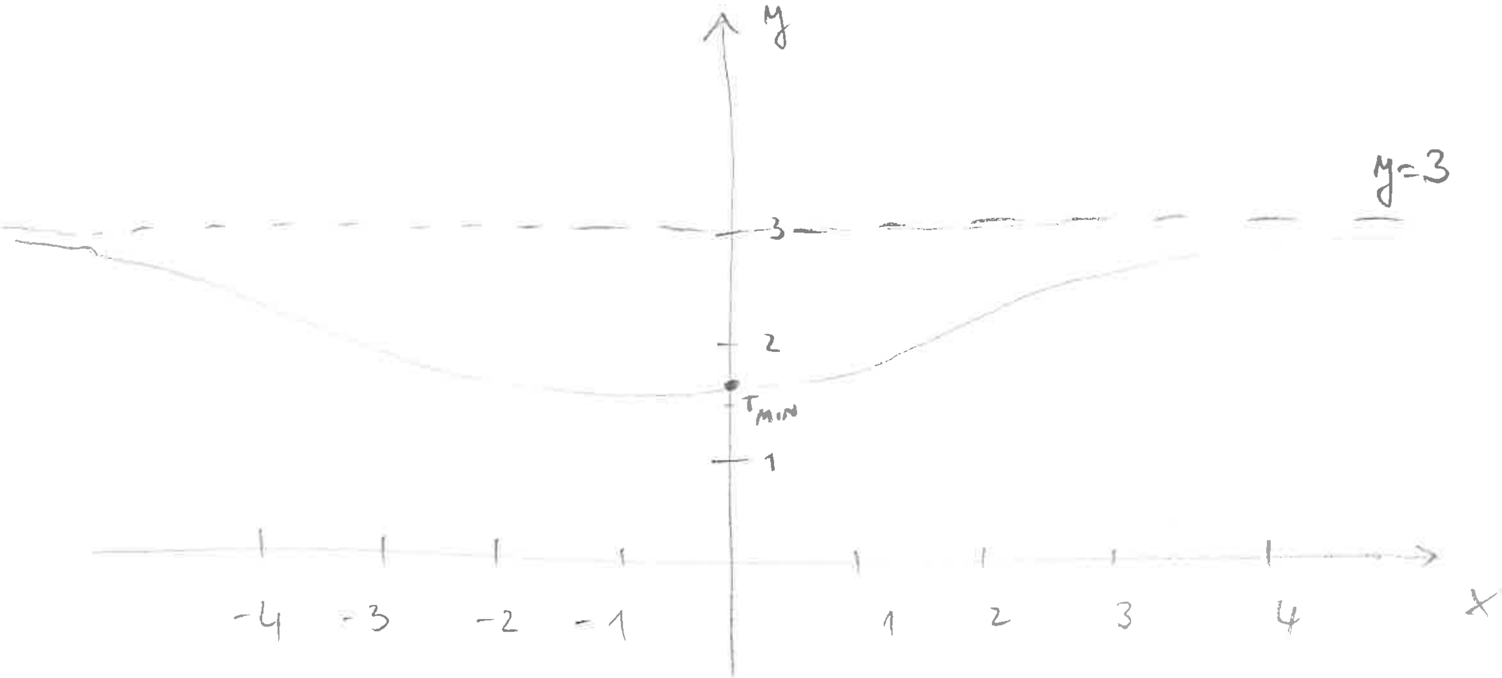
	$-\infty$		0		∞
f'		$-$		$+$	
f		↘	LOK. MIN.	↗	

$T_{MIN}\left(0,\ 3\cos(1)\right)$

$$f(x) = 0 \iff 3\cos\left(\frac{1}{x^2+1}\right) = 0$$

$$\frac{1}{x^2+1} \in \langle 0, 1] \quad \forall x \in \mathbb{R}$$

$\Rightarrow$ NEMA NULTOČAKA



(6) (a) $$\left.\begin{aligned}\Phi(x) &= \int_a^x f(t)\,dt \\ F(x) &= \text{prim. fja}\end{aligned}\right\} \Rightarrow \Phi(x) = F(x) + C$$

$$\boxed{x=a} \quad \left.\begin{aligned}\underset{\underset{0}{\parallel}}{\Phi(a)} = F(a) + C\end{aligned}\right\} \Rightarrow F(a) = C \Rightarrow \Phi(x) = F(x) - F(a)$$

$$\boxed{x=b} \quad \Phi(b) = F(b) - F(a)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(6)

(b) $$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2(x)\cos(x)\ln(\sin x)\,dx = \left(\begin{array}{lll} \underline{\text{Supst}} & & \\ \sin x = t & x=\frac{\pi}{4} & t=\frac{\sqrt{2}}{2} \\ \cos x\,dx = dt & x = \frac{\pi}{2} & t=1 \end{array}\right)$$

$$= \int_{\sqrt{2}/2}^{1} t^2 \ln t\,dt = \left(\begin{array}{ll} \text{P. I} & \\ \ln t = u & t^2 dt = dv \\ \frac{1}{t}dt = du & v = \frac{1}{3}t^3 \end{array}\right)$$

$$= \frac{1}{3}t^3\ln t\Big|_{\sqrt{2}/2}^{1} - \frac{1}{3}\int_{\sqrt{2}/2}^{1} t^2\,dt = \frac{1}{3}\left(1\cdot\underbrace{\ln(1)}_{=0} - \left(\frac{\sqrt{2}}{2}\right)^3 \ln\frac{\sqrt{2}}{2}\right) - \frac{1}{3}\frac{t^3}{3}\Big|_{\sqrt{2}/2}^{1}$$

$$= -\frac{\cancel{2}\sqrt{2}}{\cancel{24}_{12}}\ln\frac{1}{\sqrt{2}} - \frac{1}{9}\left(1 - \left(\frac{\sqrt{2}}{2}\right)^3\right) = \frac{\sqrt{2}}{24}\ln 2 + \frac{\sqrt{2}}{36} - \frac{1}{9}$$

7 (a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{\dfrac{x}{\sqrt[4]{x^5+x+1}}}{\dfrac{1}{\sqrt[4]{x}}} = \lim\limits_{x\to\infty} \sqrt[4]{\dfrac{x^5}{x^5+x+1}} = 1$

tj $\dfrac{x}{\sqrt[4]{x^5+x+1}} \sim \dfrac{1}{\sqrt[4]{x}}$ za $x \to \infty$

Po teoremu usporedbe nepravih integrala je:

$$I = \int_1^\infty \frac{x\,dx}{\sqrt[4]{x^5+x+1}} \sim \int_1^\infty \frac{dx}{\sqrt[4]{x}} = \frac{4}{3}\sqrt[4]{x^3}\Big|_1^\infty \longrightarrow \infty$$

Dakle, zadani integral DIVERGIRA

7 (b) $p = 1 \Rightarrow \int_0^a \frac{dx}{x}$ DIVERGIRA

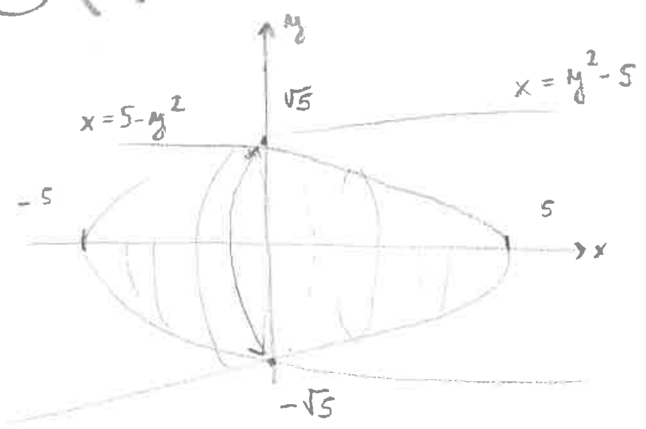
$p \neq 1 \Rightarrow \int_0^a \frac{dx}{x^p} = \lim\limits_{\delta\to 0^+} \int_\delta^a \frac{dx}{x^p} = \lim\limits_{\delta\to 0^+} \left(\frac{1}{-p+1} x^{-p+1}\right)\Big|_\delta^a$

$$= \frac{1}{-p+1} a^{-p+1} - \lim_{\delta\to 0^+}\left(\frac{1}{-p+1}\delta^{-p+1}\right)$$

$$= \begin{cases} \text{konvergira za } p<1 \\ \text{divergira za } p>1 \end{cases}$$

7 (c) $\lim\limits_{x\to 0^+} \dfrac{\dfrac{e^x-1}{\sqrt{x^5}}}{\dfrac{x}{\sqrt{x^5}}} = 1 \Rightarrow \displaystyle\int_0^1 \frac{e^x-1}{\sqrt{x^5}} \sim \underbrace{\int_0^1 \frac{x}{\sqrt{x^5}}}_{\text{DIVERGIRA}}$

8 (a)



$$P = \int_{-\sqrt{5}}^{\sqrt{5}} \Big((5 - y^2) - (y^2 - 5) \Big)\, dy$$

$$= \int_{-\sqrt{5}}^{\sqrt{5}} (10 - 2y^2)\, dy$$

$$= \left(10y - \frac{2}{3} y^3\right) \Bigg|_{-\sqrt{5}}^{\sqrt{5}}$$

$$= 20\sqrt{5} - \frac{20}{3}\sqrt{5} = \frac{40}{3}\sqrt{5}$$

8 (b) $$V = \int_{-5}^{5} P(x)\, dx = 2 \int_{0}^{5} y^2 \pi\, dx$$

$$= 2\pi \int_{0}^{5} (5 - x)\, dx =$$

$$= 2\pi \left(5x - \frac{x^2}{2}\right) \Bigg|_{0}^{5}$$

$$= 2\pi \left(25 - \frac{25}{2}\right)$$

$$= 25\pi$$