1. a) UZMIMO PROIZVOLJNE $x_1, x_2 \in I$, $x_1 \neq x_2$
PREMA TEOREMU SREDNJE VRIJEDNOSTI VRIJEDI DA POSTOJI $c \in \langle x_1, x_2 \rangle$ TAKAV DA

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

KAKO $c \in \langle x_1, x_2 \rangle \Rightarrow c \in I \Rightarrow f'(c) = 0$
IMAMO

$$0 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\Leftrightarrow \quad f(x_2) - f(x_1) = 0$$

$$\Leftrightarrow \quad f(x_2) = f(x_1)$$

PA JE FUNKCIJA KONSTANTNA.

1. b)

i. $f'(x_0) \neq 0 \Rightarrow f$ NEMA LOKALNI EKSTREM U x_0

OVA TVRDNJA JE ISTINITA JER JE OBRAT PO KONTRAPOZICIJI FERMATOVOG TEOREMA (SKRIPTA 9.2.1)

ii. $f'(x_0) = 0 \Rightarrow f$ IMA LOKALNI EKSTREM U x_0

OVA TVRDNJA JE NEISTINITA

NA PRIMJER $f(x) = x^3$

$f'(0) = 0$, ALI 0 NIJE LOKALNI EKSTREM

iii. f JE STROGO RASTUĆA FUNKCIJA NA I

$\Downarrow$

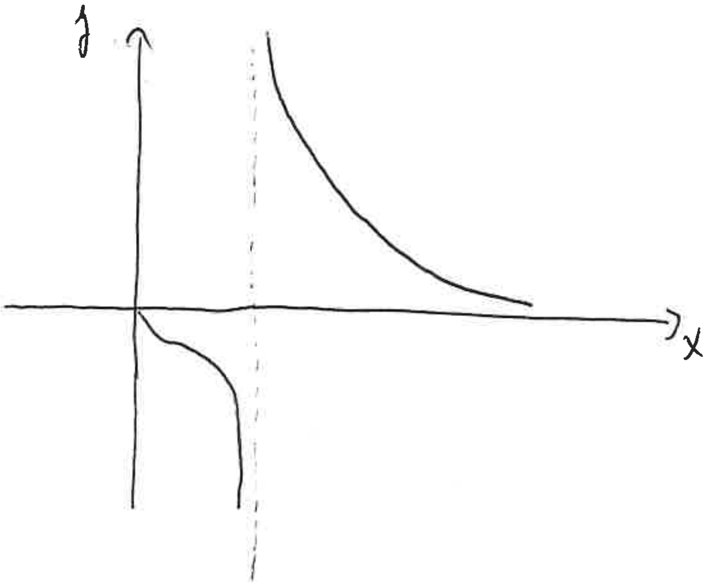
$f'(x) > 0 \quad \forall x \in I$

TVRDNJA NIJE ISTINITA

NA PRIMJER $f(x) = x^3$ JE STROGO RASTUĆA FUNKCIJA, ALI $f'(0) = 0$

2. a)

$$f(x) = \frac{1}{1+\ln x} \qquad D_f = \langle 0, \tfrac{1}{e} \rangle \cup \langle \tfrac{1}{e}, \infty \rangle$$



$$\lim_{x \to 0^+} \frac{1}{1+\ln x} = 0^-$$

$$\lim_{x \to \frac{1}{e}^-} \frac{1}{1+\ln x} = -\infty \qquad \lim_{x \to \frac{1}{e}^+} \frac{1}{1+\ln x} = \infty$$

$$\lim_{x \to \infty} \frac{1}{1+\ln x} = 0$$

$x = \frac{1}{e}$ JE VERTIKALNA ASIMPTOTA

$y = 0$ JE HORIZONTALNA ASIMPTOTA

FUNKCIJA NEMA NULTOČAKA

$$f'(x) = \frac{-\frac{1}{x}}{(1+\ln x)^2} \qquad D_{f'} = \mathbb{R} \setminus \{0, \tfrac{1}{e}\}$$

$f'(x) < 0 \quad \forall x \in D_f$

FUNKCIJA JE SVUDA PADAJUĆA

2. $f''(x) = \dfrac{\ln x + 3}{x^2(1+\ln x)^3}$

$f''(x) = 0 \Rightarrow x = \dfrac{1}{e^3}$

FUNKCIJA IMA TOČKU INFLEKSIJE U $\dfrac{1}{e^3}$

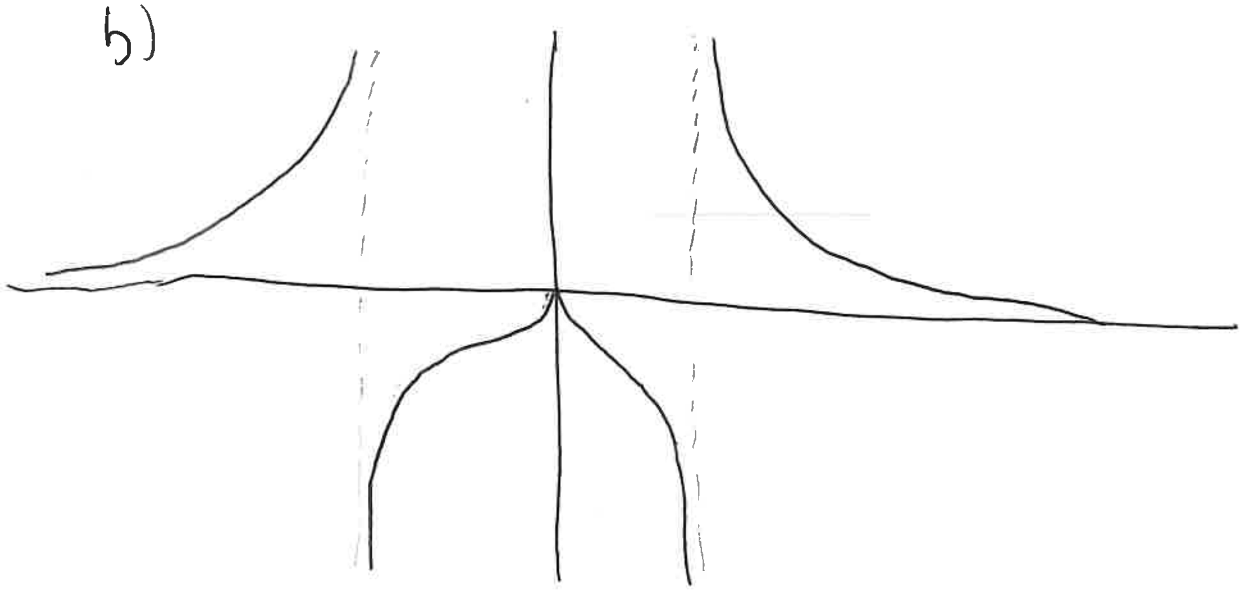
	0	$\frac{1}{e^3}$	$\frac{1}{e}$
$\ln x + 3$	$-$	$+$	$+$
$x^2(1+\ln x)^3$	$-$	$-$	$+$

$f''(x) > 0$ ZA $x \in \langle 0, \frac{1}{e^3}\rangle \cup \langle \frac{1}{e}, \infty\rangle$

$f''(x) < 0$ ZA $x \in \langle \frac{1}{e^3}, \frac{1}{e}\rangle$

FUNKCIJA JE KONVEKSNA NA INTERVALIMA $x \in \langle 0, \frac{1}{e^3}\rangle \cup \langle \frac{1}{e}, \infty\rangle$, A KONKAVNA NA $x \in \langle \frac{1}{e^3}, \frac{1}{e}\rangle$

b)



3. a) $\int_{-1}^{2} \frac{1}{x^2} = -\frac{1}{x}\Big|_{-1}^{2}$ JE POGREŠNO

KAKO JE $\frac{1}{x^2}$ NEDEFINIRAN ZA $x=0$, NEOMEĐEN U OKOLINI TE TOČKE, NE MOŽEMO ISKORISTITI NEWTON LEIBNITZOVU FORMULU

b) $\int_a^b x f''(x)\,dx$ $\left[\begin{array}{ll} u = x & du = dx \\ dv = f''(x)dx & v = f'(x) \end{array}\right]$

$= -\int_a^b f'(x)\,dx + x f'(x)\Big|_a^b$

$= b f'(b) - a f'(a) - f(x)\Big|_a^b$

$= b f'(b) - f(b) + f(a) - a f'(a)$

c) $\int_0^{\frac{\pi}{2}} x \sin x\,dx = \int_0^{\frac{\pi}{2}} x f''(x)\,dx$ ZA $f(x) = -\sin x$

$\int_0^{\frac{\pi}{2}} x \sin x\,dx = \frac{\pi}{2}\cdot\left(-\cos\frac{\pi}{2}\right) - \left(-\sin\frac{\pi}{2}\right) + (-\sin 0) - 0\cdot(-\sin 0)$

$= \frac{\pi}{2}\cdot 0 + 1 + 0 - 0$

$= 1$

4. a)

$$\int \frac{\sin x}{(\cos x+1)(\sin^2 x-2)}\,dx$$

$$= \int \frac{\sin x\,dx}{(\cos x+1)(1-\cos^2 x-2)}$$

$$= \int \frac{\sin x\,dx}{(\cos x+1)(-\cos^2 x-1)}$$

$$= \int \frac{-\sin x\,dx}{(\cos x+1)(\cos^2 x+1)}$$

$t = \cos x$

$dt = -\sin x\,dx$

$$= \int \frac{dt}{(t+1)(t^2+1)}$$

$$= \int \frac{At}{t^2+1} + \frac{B}{t^2+1} + \frac{C}{t+1}\,dt$$

$At^2 + At + Bt + B + Ct^2 + C = 1$

$A + C = 0$

$A + B = 0$

$B + C = 1$

$A = -\frac{1}{2} \quad B = \frac{1}{2} \quad C = \frac{1}{2}$

$$= -\frac{1}{2}\left[\int \frac{t\,dt}{t^2+1} - \int \frac{dt}{t^2+1} - \int \frac{dt}{t+1}\right]$$

4.a)

$$= \frac{1}{2}\left[\ln|t+1| + \operatorname{arctg} t - \int \frac{t\,dt}{t^2+1}\right]$$

$$u = t^2+1$$
$$du = 2t\,dt$$

$$= \frac{1}{2}\left[\ln|t+1| + \operatorname{arctg} t - \frac{1}{2}\int \frac{du}{u}\right]$$

$$= \frac{1}{2}\left[\ln|t+1| + \operatorname{arctg} t - \frac{1}{2}\ln|t^2+1|\right] + C$$

$$= \frac{1}{2}\ln|\cos x + 1| + \frac{1}{2}\operatorname{arctg}(\cos x) - \frac{1}{4}\ln|\cos^2 x + 1| + C$$

b) $\int_0^2 x^3 e^{x^2}\,dx$ $\quad t = x^2$, $dt = 2x\,dx$ $\quad x = 0 \to t = 0$, $x = 2 \to t = 4$

$$= \frac{1}{2}\int_0^4 t e^t\,dt$$

$u = t \quad du = dt$

$dv = e^t dt \quad v = e^t$

$$= \frac{1}{2}\left[t e^t\Big|_0^4 - \int_0^4 e^t\,dt\right]$$

$$= \frac{1}{2}\left[4e^4 - 0 - e^t\Big|_0^4\right]$$

$$= \frac{1}{2}\left[4e^4 - e^4 + 1\right] = \frac{3e^4+1}{2}$$

5. a)

i. $f(x) = \frac{1}{x-a}$

ii. $f(x) = \frac{1}{x-b}$

iii $f(x) = \frac{1}{(x-a)(x-b)}$

b) i. $\int_0^1 \frac{\ln(1+2x)}{x^3} dx$ INTEGRAL JE NEPRAV U LIJEVOM RUBU

PO USPOREDNOM KRITERIJU USPOREDIMO S FUNKCIJOM $\frac{1}{x^2}$

$$\lim_{x \to 0} \frac{\frac{\ln(1+2x)}{x^3}}{\frac{1}{x^2}} = \lim_{x \to 0} \frac{\ln(1+2x)}{x} = \lim_{x \to 0} 2 \frac{\ln(1+2x)}{2x}$$

$$= 2 \cdot 1 = 2 \neq 0$$

KONVERGENCIJA JE EKVIVALENTNA KONVERGENCIJI INTEGRALA $\int_0^1 \frac{1}{x^2}$ KOJI ZNAMO DA DIVERGIRA

5. b ii

$$\int_1^\infty \frac{\ln(1+2x)}{x^3}\,dx$$

ZA $x > 1$ VRIJEDI $\ln(1+2x) < 1+2x$

PA VRIJEDI $\frac{\ln(1+2x)}{x^3} < \frac{1+2x}{x^3}$

$$\int_1^\infty \frac{1+2x}{x^3}\,dx = \lim_{a\to\infty} \int_1^a \frac{1}{x^3}\,dx + \int_1^a \frac{2}{x^2}\,dx$$

$$= \lim_{a\to\infty} -\frac{1}{2}\frac{1}{x^2}\Big|_1^a - \frac{2}{x}\Big|_1^a$$

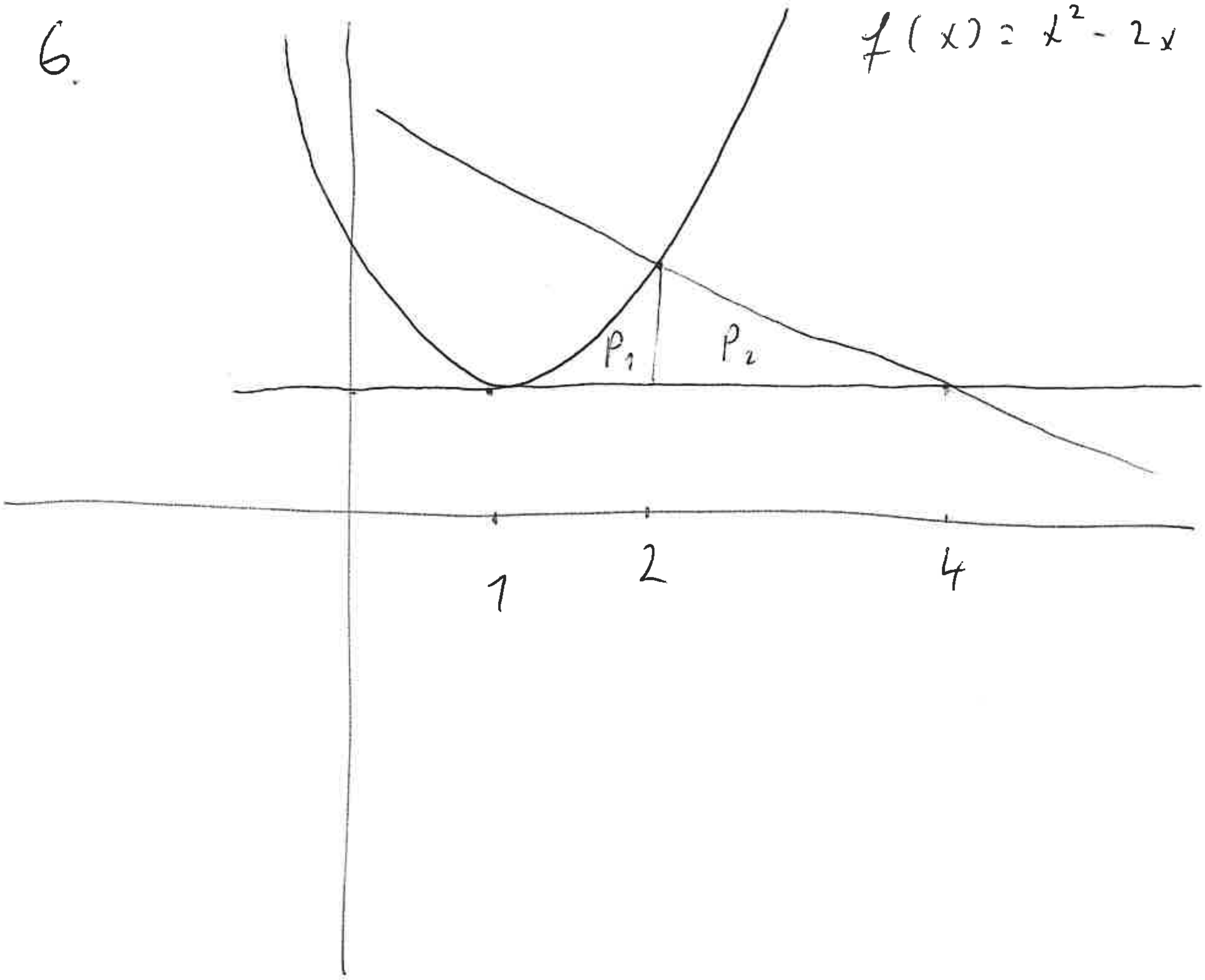
$$= \lim_{a\to\infty} -\frac{1}{2}\cdot\frac{1}{a^2} + \frac{1}{2}\cdot 1 - \frac{2}{a} + 2$$

$$= \frac{5}{2} < \infty$$

PA PO USPOREDNOM KRITERIJU KONVERGENCIJA $\int_1^\infty \frac{1+2x}{x^3}\,dx$ POVLAČI KONVERGENCIJU $\int_1^\infty \frac{\ln(1+2x)}{x^3}\,dx$, PA I TAJ INTEGRAL KONVERGIRA

6.

$f(x) = x^2 - 2x + 2 = (x-1)^2 + 1$



$f'(x) = 2x - 2$

$f'(1) = 0$ PA JE TANGNTA U $(1, 1)$ PARALELNA S x OSI

$f'(2) = 2$ NORMALA JE OKOMITA NA TANGENTU U OVOJ TOČKI PA JE NAGIB NORMALE $-\frac{1}{2}$

$-\frac{1}{2} \cdot 2 + b = 2$

$b = 3$

NORMALA U $(2, 2)$ ... $-\frac{1}{2}x + 3$

SJECIŠTE TANGENTE I NORMALE

$-\frac{1}{2}x + 3 = 1$

$x - 6 = -2$

$x = 4$

6.

PODIJELIMO UKUPNU POVRŠINU NA P_1 I P_2 KAO NA SKICI

$$P_1 = \int_1^2 f(x) - 1dx = \int_1^2 x^2 - 2x + 2 - 1dx = \int_1^2 x^2 - 2x + 1 dx$$

$$= \frac{1}{3}x^3\Big|_1^2 + (-x^2)\Big|_1^2 + x\Big|_1^2$$

$$= \frac{8}{3} - \frac{1}{3} + (-4 + 1) + 2 - 1$$

$$= 2 + \frac{1}{3} - 3 + 1$$

$$= \frac{1}{3}$$

$$P_2 = \int_2^4 -\frac{1}{2}x + 3 - 1 dx$$

$$= \int_2^4 -\frac{1}{2}x + 2 dx$$

$$= -\frac{1}{4}x^2\Big|_2^4 + 2x\Big|_2^4$$

$$= -\frac{1}{4}(16 - 4) + 2 \cdot 2$$

$$= -\frac{1}{4} \cdot 12 + 4$$

$$= -3 + 4 = 1$$

UKUPNA POVRŠINA JE $P_1 + P_2 = \frac{1}{3} + 1 = \frac{4}{3}$