1. a) TVRDNJA NIJE ISTINITA. NA PRIMJER

$$z_1 = 1 \quad , \quad z_2 = 2$$

$$z_1 \neq z_2 \ , \ \arg z_1 = \arg z_2$$

b)

$$\arg z = \frac{5}{4}\pi$$

$$|z| = \sqrt{2} \qquad \Rightarrow \quad z = -1 - i$$

$$\frac{1}{z+2i} = \frac{1}{-1-i+2i} = \frac{1}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{-1-i}{\sqrt{2}}$$

$$|-1-i| = \sqrt{2}$$

$$\arg(-1-i) = \frac{5}{4}\pi$$

$$\frac{-1-i}{\sqrt{2}} = \text{cis}\,\frac{5\pi}{4}$$

c)

$$z^5 = \bar{z}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$z = r\,\text{cis}\,\varphi$$

$$r^5 = r\sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$r^5 = r$$

$$r \in \mathbb{R} \Rightarrow \quad r = 0 \text{ ili } r = 1 \qquad \Rightarrow \ z_0 = 0$$

$$5\varphi = -\varphi + \frac{\pi}{3} + 2k\pi \qquad z_1 = \text{cis}\,\frac{\pi}{18}$$

$$6\varphi = \frac{\pi}{3} + 2k\pi \qquad z_2 = \text{cis}\,\frac{7\pi}{18}$$

$$\varphi = \frac{\frac{\pi}{3} + 2k\pi}{6} \Rightarrow \qquad z_3 = \text{cis}\,\frac{13\pi}{18}$$

$$k \in \{0,1,2,3,4,5\} \qquad z_4 = \text{cis}\,\frac{19\pi}{18}$$

$$z_5 = \text{cis}\,\frac{25\pi}{18}$$

$$z_6 = \text{cis}\,\frac{31\pi}{}$$

2. $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)(n-2)\cdot\ldots\cdot(n-k+1)}{k!}$

a)

$$(x+y)^n = \sum_{k=0}^{n} x^k y^{n-k} \binom{n}{k}$$

$$x=1 \quad y=1 \Rightarrow (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} \Rightarrow 2^n = \sum_{k=0}^{n} \binom{n}{k}$$

KOMBINATORNA INTERPRETACIJA

2^n - BROJ SVIH PODSKUPOVA n-ČLANOG SKUPA

$\binom{n}{k}$ - BROJ k-ČLANIH PODSKUPOVA n-ČLANOG SKUPA

b)

A = {NISU ZASTUPLJENI PROGRAMERI}

B = {NISU ZASTUPLJENI WEB DIZAJNERI}

C = {NISU ZASTUPLJENI MENADŽERI}

S - {SVE KOMBINACIJE}

$|S| = \binom{50}{10}$

$$|S| - |A \cup B \cup C| = |S| - |A| - |B| - |C| + |A \cap B| + |A \cap C| + |B \cap C$$

$$- |A \cap B \cap C|$$

$$= \binom{50}{10} - \binom{29}{10} - \binom{35}{10} - \binom{36}{10} + \binom{14}{10} + \binom{15}{10} + \binom{21}{10} -$$

3.

a) $$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$\sin'(3\tfrac{\pi}{4}) = \lim_{h \to 0} \frac{\sin(3\cdot(\frac{\pi}{4}+h)) - \sin(3\cdot\frac{\pi}{4})}{h}$$

$$= \lim_{h \to 0} \frac{\sin[3\cdot\frac{\pi}{4} + 3h] - \sin(3\cdot\frac{\pi}{4})}{h}$$

$$= \lim_{h \to 0} \frac{\sin(3\frac{\pi}{4})\cos(3h) + \cos(3\frac{\pi}{4})\sin 3h - \sin(3\frac{\pi}{4})}{h}$$

$$= \lim_{h \to 0} \frac{\sin(3\frac{\pi}{4})[\cos(3h) - 1] + \cos 3\frac{\pi}{4}\sin 3h}{h}$$

$$\lim_{h \to 0} \frac{\cos(3h) - 1}{h} = 0$$

$$\lim_{h \to 0} \frac{\sin 3h}{h} = 3$$

$$= 3\cos 3\frac{\pi}{4} = -\frac{3\sqrt{2}}{2}$$

b) $$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\sqrt[3]{h} - \sqrt[3]{0}}{h} = \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} = \lim_{h \to 0} \frac{1}{\sqrt[3]{h^2}} = +\infty$$

PA FUNKCIJA NIJE DIFERENCIJABILNA U a.
IMA VERTIKALNU TANGENTU OBLIKA $x = a$

c)
$$h(x) = \ln^2(\operatorname{arctg}(5x))$$

$$h'(x) = 2\ln(\operatorname{arctg}(5x)) \cdot \frac{1}{\operatorname{arctg}(5x)} \cdot \frac{1}{1+25x^2} \cdot 5$$

$$= 10\,\frac{\ln(\operatorname{arctg}(5x))}{\operatorname{arctg}(5x)(1+25x^2)}$$

4. c)

$$\lim_{n\to\infty}\left(\frac{2n}{2n+3}\right)^n = \lim_{n\to\infty}\left(1+\frac{-3}{2n+3}\right)^n$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{-3}{2n+3}\right)^{2n}\right]^{\frac{1}{2}}$$

$$= \left[\lim_{n\to\infty}\left(1+\frac{-3}{2n+3}\right)^{2n+3}\cdot\left(1+\frac{-3}{2n+3}\right)^{-3}\right]^{\frac{1}{2}}$$

$$= \left[e^{-3}\cdot 1\right]^{\frac{1}{2}}$$

$$= e^{-\frac{3}{2}}$$

a) Broj $L\in\mathbb{R}$ zovemo limes niza (a_n) ako za svaki $\varepsilon>0$ postoji $n_0\in\mathbb{N}$ takav da za svaki $n\geq n_0$ vrijedi $|a_n-L|<\varepsilon$

b) Ne, jer ima gomilište $\frac{3}{2}$, $-\frac{3}{2}$

$$\lim_{n\to\infty} a_{2n}=\frac{3}{2}, \quad \lim_{n\to\infty} a_{2n+1}=-\frac{3}{2}$$

5. a) $x_1, x_2 \in I$ TAKVI DA $x_1 < x_2$

PO TEOREMU SREDNJE VRIJEDNOSTI

$\exists\, c \in \langle x_1, x_2 \rangle$ T.D. $f(x_1) - f(x_2) = f'(c)(x_2 - x_1)$

$x_2 - x_1 > 0$

$c \in \langle x_1, x_2 \rangle \Rightarrow c \in I \Rightarrow f'(c) < 0$

$f(x_1) - f(x_2) = f'(c) \cdot (x_2 - x_1) < 0$

6.5. b) NEKA JE ~~TOČKA~~ VRHA KUT PRAVOKUTNIKA U PRVOM KVADRANTU, DANA KOORDINATAMA

$$\left(t, \sqrt{1-\frac{t^2}{16}} \right)$$

POVRŠINA P KAO FUNKCIJA t IZGLEDA KAO

$$P(t) = 4t\sqrt{1-\frac{t^2}{16}} = t\sqrt{16-t^2} \qquad t \in [0,4]$$

$P'(t)=0$ JE GLOBALNI MAKSIMUM!

$$P'(t) = \sqrt{16-t^2} + t\frac{1}{2\sqrt{16-t^2}}\cdot(-2t)$$

$$= \sqrt{16-t^2} - \frac{t^2}{\sqrt{16-t^2}}$$

$$= \frac{16-2t^2}{\sqrt{16-t^2}}$$

$$P'(t_0) = 0$$

$$\Updownarrow$$

$$16 - 2t_0^2 = 0$$

$$t_0 = 2\sqrt{2} \qquad \text{JER } t \in \langle 0, 4]$$

$$P_{MAX} = P(2\sqrt{2}) = 2\sqrt{2}\cdot\sqrt{16-8} = 8$$

4.6 a) $\int x^2 e^x \, dx$

$$= \begin{bmatrix} u = x^2 & dv = e^x dx \\ du = 2x\,dx & v = e^x \end{bmatrix}$$

$$= x^2 e^x - \int 2x e^x \, dx$$

$$= \begin{bmatrix} u = 2x & dv = e^x dx \\ du = 2dx & v = e^x \end{bmatrix}$$

$$= x^2 e^x - \left[2x e^x - \int 2 e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2 e^x + C$$

b)

$$\frac{x^2 + 2x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx + C}{x^2+x+1}$$

$$x^2 + 2x = A(x^2+x+1) + (Bx + C)(x-1)$$

$$x = 1 \Rightarrow 3 = 3A \Rightarrow A = 1$$

$$x = 0 \Rightarrow 0 = A - C \Rightarrow C = 1$$

$$x = -1 \Rightarrow -1 = A - 2(C - B) \Rightarrow B = 0$$

$$\int \frac{x^2+2x}{x^3-1} = \int \frac{1}{x-1} + \frac{1}{x^2+x+1} \, dx$$

$$= \int \frac{dx}{x-1} + \int \frac{dx}{(x+\frac{1}{2})^2 + \frac{3}{4}}$$

$$= \int \frac{dx}{x-1} + \frac{4}{3} \int \frac{dx}{\frac{4}{3}(x+\frac{1}{2})^2 + 1}$$

$$= \ln|x-1| + \frac{2}{\sqrt{3}} \operatorname{arctg}\left(\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right) + C$$

7\. a) SKRIPTA

b) $\int_0^1 e^x \, dx = e^c \cdot (1-0)$

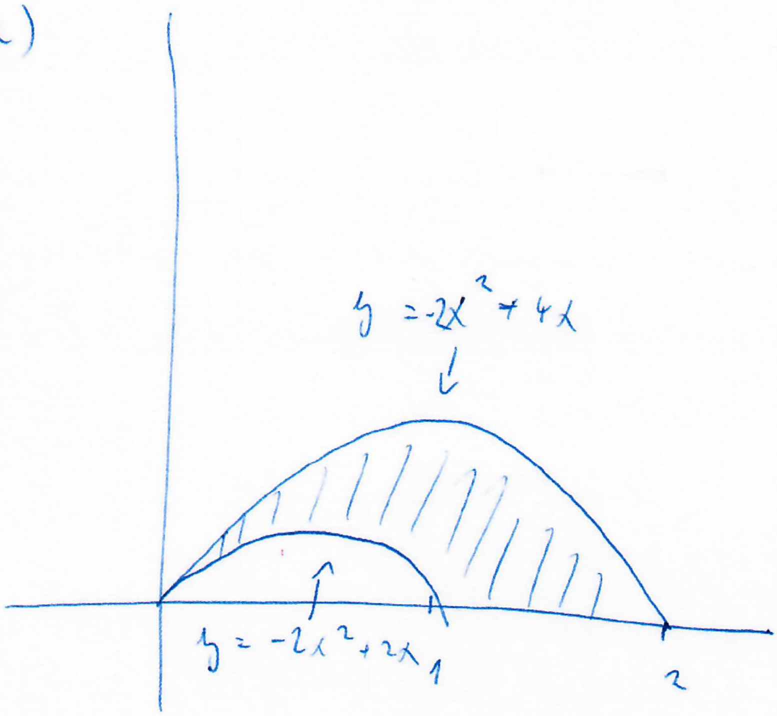
$e^x \Big|_0^1 = e^c$

$e - 1 = e^c$

$c = \ln(e-1)$

8.

a)



$$P = \int_0^2 -2x^2 + 4x \, dx - \int_0^1 -2x^2 + 2x \, dx$$

$$= \left(-\frac{2}{3}x^3 + 2x^2\right)\Big|_0^2 - \left(-\frac{2}{3}x^3 + x^2\right)\Big|_0^1$$

$$= -\frac{16}{3} + 8 + \frac{2}{3} - 1$$

$$= \frac{7}{3}$$

b)

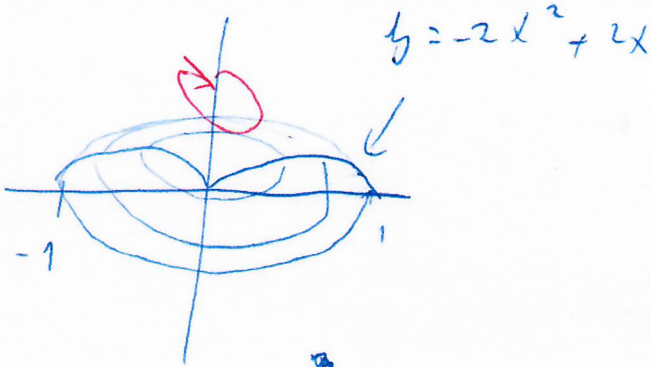
$$\cancel{V = \pi \int_0^2 (-2x^2 + 4x)^2 \, dx}$$

$$\cancel{V = \pi \int_0^2 4x^4 - 16x^3 + 16x^2 \, dx}$$

$$\cancel{= \pi \left[\frac{4}{5}x^5 - 4x^4 + \frac{16}{3}x^3\right]\Big|_0^2}$$

$$\cancel{= \pi \left[\frac{128}{5} - 64 + \frac{128}{3}\right]}$$

8. b)



$$V = 2\pi \int_0^1 x(-2x^2 + 2x)\,dx$$

$$= 2\pi \int_0^1 -2x^3 + 2x^2\,dx$$

$$= 2\pi \int_0^1 -2x^3 + 2x^2\,dx$$

$$= 2\pi \left[-\frac{x^4}{2} + \frac{2}{3}x^3 \right]\Big|_0^1$$

$$= 2\pi \left[-\frac{1}{2} + \frac{2}{3} \right]$$