1\. a)

REFLEKSIVNOST

SIMETRIČNOST

ANTISIMETRIČNOST

TRANZITIVNOST

b)

ρ NIJE REFLEKSIVNA JER $(3,3) \notin \rho$

ρ NIJE SIMETRIČNA JER $(2,1) \notin \rho$ A $(1,2) \in \rho$

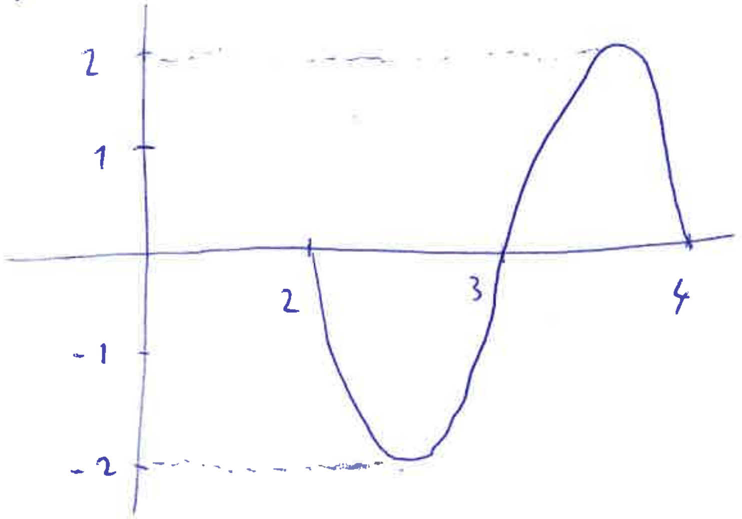
ρ NIJE ANTISIMETRIČNA JER $(1,4) \in \rho$ I $(4,1) \in \rho$

ρ NIJE TRANZITIVNA JER SU $(2,4) \in \rho$ I $(4,1) \in \rho$, ALI $(2,1) \notin \rho$

MORAMO DODATI : $(3,3)$ I $(2,1)$ DA BI ρ BILA RELACIJA EKVIVALENCIJE

KLASE : $\begin{cases} [1] = \{1,2,4\} \\ [3] = \{3\} \end{cases}$

2. d)



$$f(x) = A \sin(wx + \varphi)$$

AMPLITUDA JE 2 $\Rightarrow$ $A = 2$

NULTOČKE SU NA RAZMAKU 1

$$\Rightarrow \quad 3w + \varphi = k\pi$$

$$4w + \varphi = (k+1)\pi$$

$$4w + \varphi = 3w + \varphi + \pi$$

$$w = \pi$$

FUNKCIJA JE SINUS POMAKNUT ZA 1, PA

$$w(x-1) = wx + \varphi$$

$$\varphi = -\pi$$

JEDNO RJEŠENJE:

$A = 2$
$w = \pi$
$\varphi = -\pi$

DRUGA MOGUĆA RJEŠENJA

$A = \pm 2$
$w = \pm\pi$
$\varphi = (2k+1)\pi$

2. b) FUNKCIJA g MORA BITI INJEKTIVNA i SURJEKTIVNA

$g^{-1}: K \to D$

$$g^{-1}(g(x)) = x \quad \forall x \in \mathbb{R}$$

$$g(g^{-1}(y)) = y \quad \forall y \in K$$

c) INTERVAL I JE OD MINIMUMA DO MAKSIMUMA FUNKCIJE, TO JEST

$$I = [2.5, 3.5]$$

NA TOM JE INTERVALU FUNKCIJA INJEKTIVNA I POSTIŽE SVE VRIJEDNOSTI OD -2 DO 2

$$h : [2.5, 3.5] \to [-2, 2], \quad h(x) := 2 \sin(\pi x - \pi)$$

h JE BIJEKCIJA PA IMA INVERZ

$$D_h = [2.5, 3.5]$$

$$Im(h) = [-2, 2]$$

POMAK ZBOG arcsin

$$h^{-1}(x) = \frac{\arcsin\left(\frac{x}{A}\right) - \varphi}{\omega} = 2 + \frac{\arcsin\left(\frac{x}{2}\right) + \pi}{\pi}$$

$$D_{h^{-1}} = [-2, 2]$$

$$Im(h^{-1}) = [2.5, 3.5]$$

3. a)

i) $$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

ii)

$$\lim_{h \to 0} \frac{e^{2(x+h)} - e^{2x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{2x} e^{2h} - e^{2x}}{h}$$

$$= \lim_{h \to 0} e^{2x} \frac{e^{2h} - 1}{h}$$

$$t := 2h$$

$$= \lim_{t \to 0} e^{2x} \frac{e^t - 1}{\frac{t}{2}}$$

$$= \lim_{t \to 0} 2 e^{2x} \frac{e^t - 1}{t}$$

$$= 2 e^{2x}$$

b)

$$f(x) = \ln^2(1 + tg(x))$$

$$f'(x) = 2\ln(1+tg(x)) \frac{1}{1+tg(x)} \cdot \frac{1}{\cos^2(x)}$$

c) TANGENTA $t \ldots y = ax + b$

$$a = f'(x_0)$$

$$= 2\ln(1+1) \cdot \frac{1}{1+1} \cdot \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= 2\ln(2) \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{\ln 2}{2}$$

$$f(x_0) = a x_0 + b$$

$$\ln^2(1+1) = \frac{\ln 2}{2} \frac{\pi}{4} + b$$

$$b = \ln^2(2) - \frac{\pi \ln 2}{8}$$

$$t \ldots \quad y = \frac{\ln 2}{2} x + \ln^2(2) - \frac{\pi \ln 2}{8}$$

4. NEKA JE $\lim_{n\to\infty} a_n = L_1$ I $\lim_{n\to\infty} b_n = L_2$

$L := L_1 + L_2$

$\forall \varepsilon > 0 \ \exists n_1, n_2 : \forall n > n_1 \ |L_1 - a_n| < \frac{\varepsilon}{2}$

$\forall n > n_2 \ |L_2 - b_n| < \frac{\varepsilon}{2}$

DAKLE ZA $n_0 = \max\{n_1, n_2\}$ VRIJEDI

$\forall n > n_0 \ |L_1 - a_n| < \frac{\varepsilon}{2}$

$\forall n > n_0 \ |L_2 - b_n| < \frac{\varepsilon}{2}$

SADA, ZA $n > n_0$

$$|L - (a_n + b_n)| = |L_1 - a_n + L_2 - b_n|$$
$$\leq |L_1 - a_n| + |L_2 - b_n|$$
$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$
$$= \varepsilon$$

PA $|L - (a_n + b_n)| < \varepsilon \ \forall n > n_0$

KAKO JE ε BIO ODABRAN PROIZVOLJNO, TO VRIJEDI ZA SVAKI ε, PA ZATO

$$\lim_{n\to\infty} (a_n + b_n) = L_1 + L_2 = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$$

4. b)

$$\lim_{n\to\infty}\left(2n-\sqrt[3]{8n^3+n^2}\right)n^a$$

$$=\lim_{n\to\infty}\left(2n-\sqrt[3]{8n^3+n^2}\right)n^a\cdot\frac{4n^2+2n\sqrt[3]{8n^3+n^2}+\sqrt[3]{(8n^3+n^2)^2}}{4n^2+2n\sqrt[3]{8n^3+n^2}+\sqrt[3]{(8n^3+n^2)^2}}$$

$$=\lim_{n\to\infty}n^a\cdot\frac{8n^3-8n^3-n^2}{4n^2+2n\sqrt[3]{8n^3+n^2}+\sqrt[3]{(8n^3+n^2)^2}}$$

$$=\lim_{n\to\infty}n^a\frac{-n^2}{4n^2+2n\sqrt[3]{8n^3+n^2}+\sqrt[3]{(8n^3+n^2)^2}}$$

$$=\lim_{n\to\infty}n^a\frac{-1}{4+2\sqrt[3]{8+\frac{1}{n}}+\sqrt[3]{(8+\frac{1}{n})^2}}$$

1° $a=0$ $\quad\lim_{n\to\infty}\dfrac{-1}{4+2\sqrt[3]{8+\frac{1}{n}}+\sqrt[3]{(8+\frac{1}{n})^2}}=\dfrac{-1}{12}$

2° $a>0$ $\quad\infty$

3° $a<0$ $\quad 0$

5.

a) DEFINICIJA 9.2.1

b) TEOREM 9.2.1

c) TEOREM 9.2.1

6.

$$D_f = \langle -\infty, -2\rangle \cup \langle -1, 0\rangle \cup \langle 1, \infty\rangle$$

$$\lim_{x\to -2^-} f(x) = -\infty \qquad \lim_{x\to -1^+} f(x) = +\infty$$

$$\lim_{x\to 0^-} f(x) = +\infty \qquad \lim_{x\to 1^+} f(x) = -\infty$$

VERTIKALNE ASIMPTOTE SU

$x = -2$
$x = -1$
$x = 0$
$x = 1$

HORIZONTALNA ASIMPTOTA JE

$y = 0$

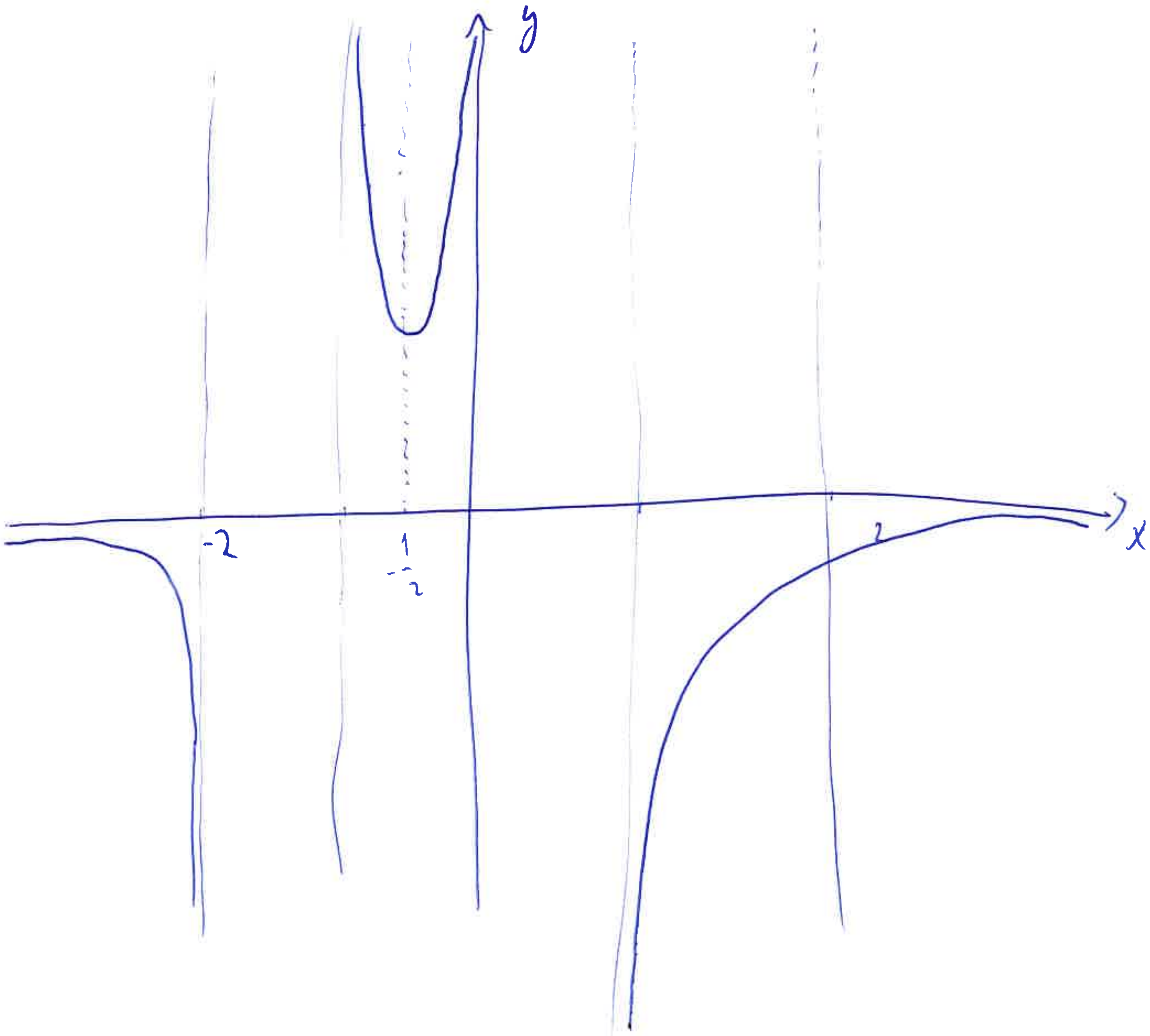
KOSIH ASIMPTOTA NEMA

$$f'(x) = \frac{1}{1-\frac{2}{x^2+x}} \cdot \frac{2(2x+1)}{(x^2+x)^2}$$

$$f'(x) = 0 \Rightarrow 2x+1 = 0 \Rightarrow x = -\frac{1}{2}$$

x	$\langle -\infty, -2\rangle$	$\langle -2, -1\rangle$	$\langle -1, -\frac{1}{2}\rangle$	$\langle -\frac{1}{2}, 0\rangle$	$\langle 0, 1\rangle$	$\langle 1, \infty\rangle$
$f'(x)$	$-$	NIJE DEF	$-$	$+$	NIJE DEF	$+$
$f(x)$	$\searrow$	NIJE DEF	$\searrow$	$\nearrow$	NIJE DEF	$\nearrow$

6. GRAF



7. a) TEOREM 12.2.1

b)

$$\int x e^{-2x} dx = \begin{bmatrix} u = x & du = dx \\ dv = e^{-2x} dx & v = \frac{e^{-2x}}{2} \end{bmatrix}$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \frac{e^{-2x}}{-2} + C$$

$$= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C \qquad C \in \mathbb{R}$$

c)

$$\int \frac{tg(\ln x)}{x} dx = \begin{bmatrix} \ln x = t \\ \frac{1}{x} dx = dt \end{bmatrix}$$

$$= \int tg\, t \, dt = \int \frac{\sin t}{\cos t} dt = \begin{bmatrix} \cos t = u \\ -\sin t \, dt = du \end{bmatrix}$$

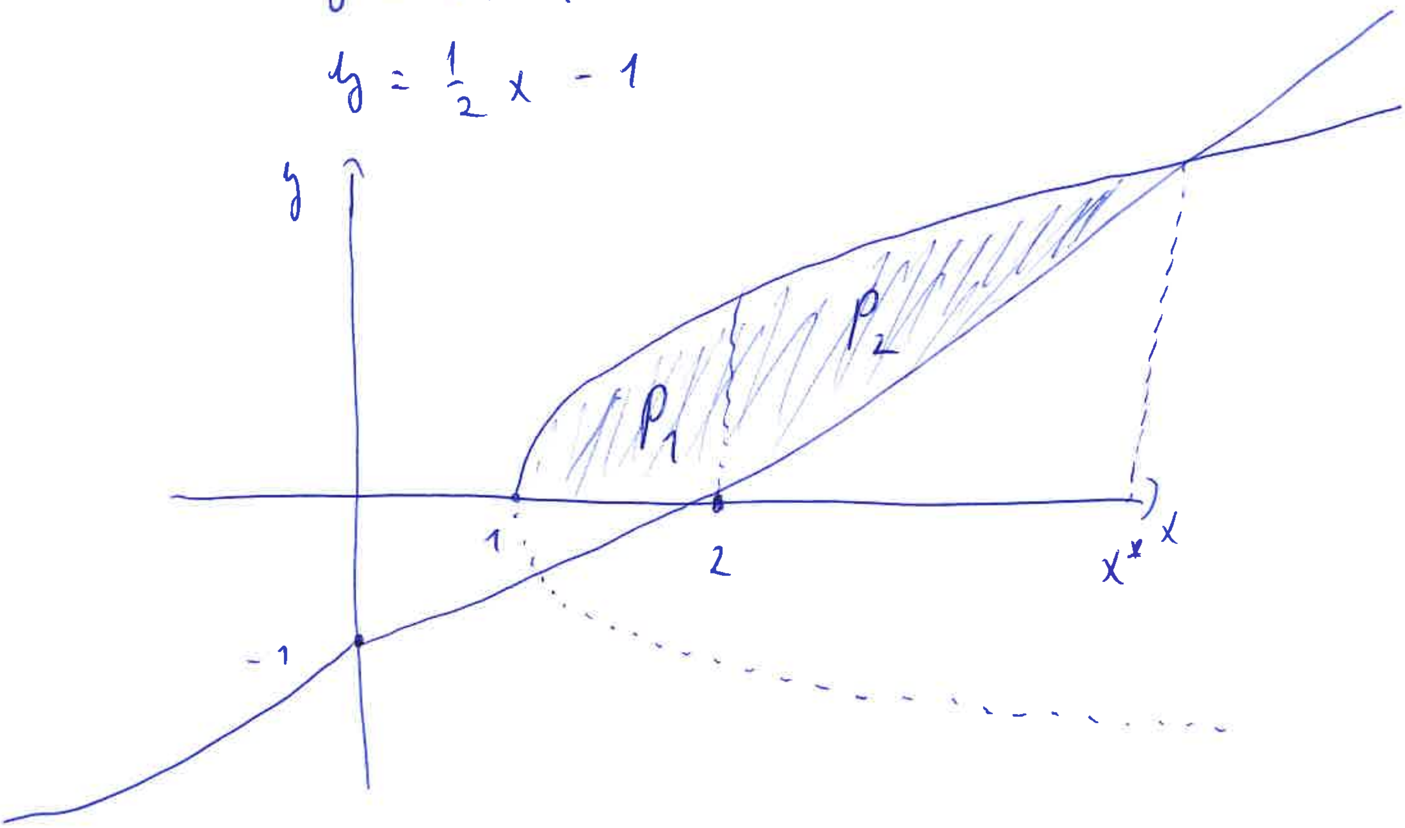
$$= - \int \frac{du}{u} = \ln |u| + C = \ln |\cos(\ln x)| + C$$

$$C \in \mathbb{R}$$

8.

a) $y = \sqrt{x-1}$

$y = \frac{1}{2}x - 1$



$$\sqrt{x-1} = \frac{1}{2}x - 1 \quad / \cdot 2$$

$$2\sqrt{x-1} = x - 2 \quad / ()^2$$

$$4(x-1) = (x-2)^2$$

$$4x - 4 = x^2 - 4x + 4$$

$$x^2 - 8x + 8 = 0$$

$$x_{1,2} = \frac{8 \pm \sqrt{64-32}}{2}$$

$$x_{1,2} = \frac{8 \pm \sqrt{32}}{2} = \frac{8 \pm 4\sqrt{2}}{2} = 4 \pm 2\sqrt{2}$$

$$x^* = 4 + 2\sqrt{2}$$

$$P = P_1 + P_2$$

$$P_1 = \int_1^2 \sqrt{x-1}\, dx$$

$$P_1 = \frac{2}{3}(x-1)^{\frac{3}{2}}\Big|_1^2 = \frac{2}{3}$$

$$P_2 = \int_2^{x^*} \sqrt{x-1} - \frac{1}{2}x + 1\, dx$$

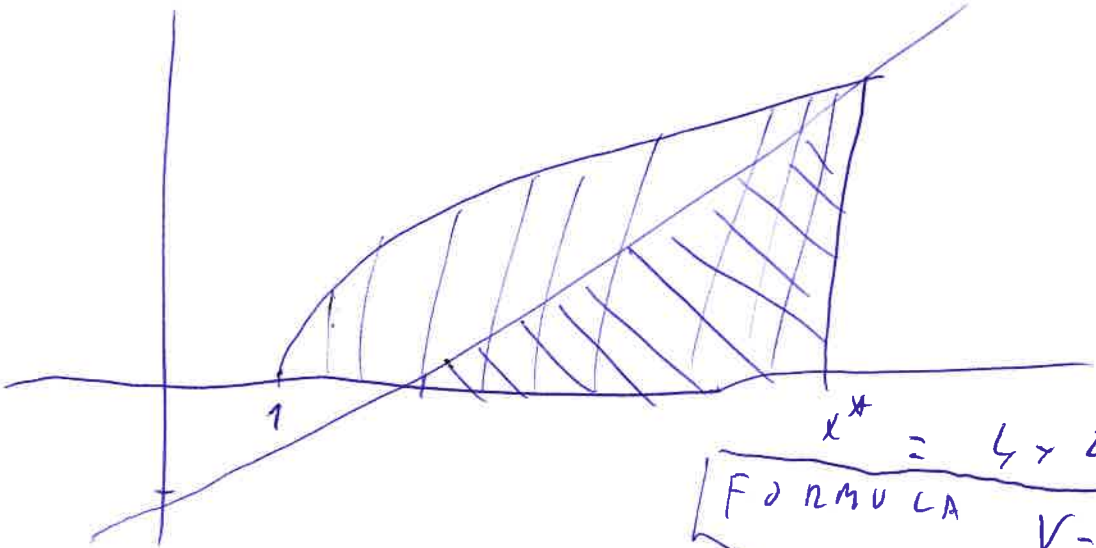
$$P_2 = \int_2^{x^*} \sqrt{x-1}\, dx - \frac{1}{2}\int_2^{x^*} x\, dx + \int_2^{x^*} dx$$

$$P_2 = \frac{2}{3}(x-1)^{\frac{3}{2}}\Big|_2^{4+2\sqrt{2}} - \frac{x^2}{4}\Big|_2^{4+2\sqrt{2}} + x\Big|_2^{4+2\sqrt{2}}$$

$$P_2 = 1 + \frac{4\sqrt{2}}{3}$$

$$P = \frac{5 + 4\sqrt{2}}{3}$$

8. b) VOLUMEN DOBIVAMO ODUZIMANJEM VOLUMENA TIJELA OMEĐENOG PRAVCEM $y = \frac{1}{2}x - 1$ I OSI x OD VOLUMENA ROTACIJSKOG TIJELA OMEĐENOG S $y = \sqrt{x-1}$ I x OSI



$x^* = 4 + 2\sqrt{2}$

$\boxed{\text{FORMULA } V = \int_a^b y^2(x)\,dx}$

$$V = V_1 - V_2$$

$$V_1 = \pi \int_1^{x^*} \sqrt{x-1}^{\,2}\,dx$$

$$V_1 = \pi \int_1^{x^*} x - 1\,dx = \pi\left[\frac{x^2}{2}\Big|_1^{4+2\sqrt{2}} - x\Big|_1^{4+2\sqrt{2}}\right]$$

$$V_1 = \frac{17}{2} + 6\sqrt{2}$$

V_2 = VOLUMEN STOŠCA VISINE $h = 3 + 2\sqrt{2}$ I RADIJUSA BAZE $r = 1 + \sqrt{2}$

$$V_2 = \pi r^2 \frac{h}{3} = \frac{\pi}{3}(9\sqrt{2} + 13)$$