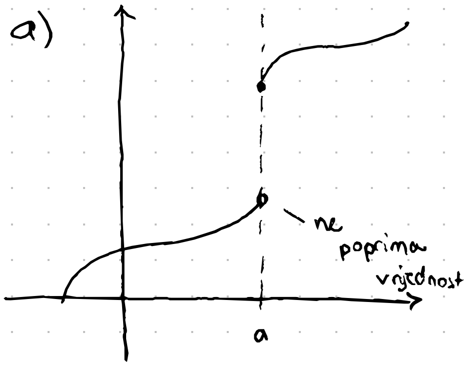
5.2. NEPREKINUTE Fje I LIMESI

Pr.) Postoji li $\lim_{x\to a} f(x)$? Vrijedi li $\lim_{x\to a} f(x) = f(a)$?

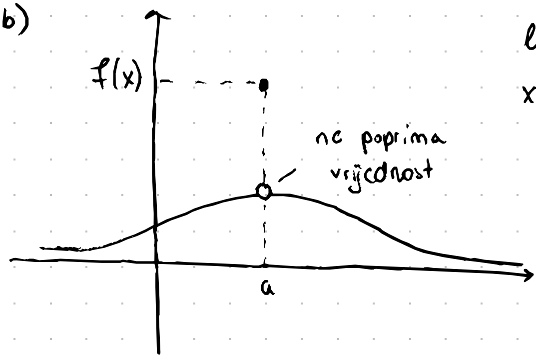
a)



$$\lim_{x\to a^+}(f(x)) \neq \lim_{x\to a^-}(f(x))$$

$\lim_{x\to a}(f(x))$ ne postoji

b)



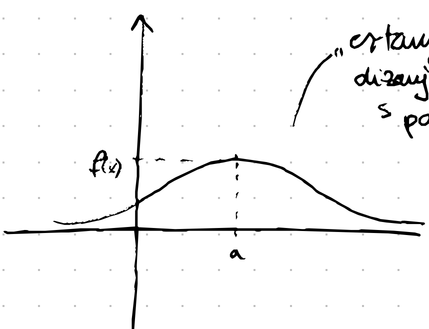
$$\lim_{x\to a^+}(f(x)) = \lim_{x\to a^-}(f(x))$$

$\lim_{x\to a} f(x)$ postoji

$\lim_{x\to a} f(x) \neq f(a)$

jer nije neprekinuta funkcija

c)



„crtanje bez dizanja olovke s papira"

$$\lim_{x\to a^+}(f(x)) = \lim_{x\to a^-}(f(x)) = \lim_{x\to a} f(x) = f(a)$$

fja je neprekinuta u a!

$\boxed{\text{DEF}}$ Neka je f definirana na otvorenom intervalu a. Kažemo da je f-ja f <u>NEPREKINUTA u točki a</u> ako vrijedi

$$\lim_{x\to a} f(x) = f(a)$$

Ako f nije neprekinuta u točki a, kažemo da je prekinuta ili ima prekid.

F-ja je neprekinuta na otvorenom intervalu $I \subseteq \mathbb{R}$ ako je neprekinuta u svakoj točki tog intervala.

<u>Nap:</u> f neprek. u točki $\Longleftrightarrow$ $\exists \lim\limits_{x\to a} (f(x))$ i $\lim\limits_{x\to a} (f(x)) = f(a)$

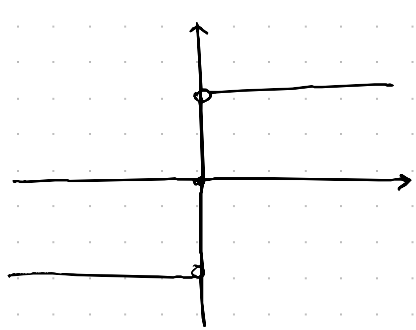
f ima prekid u $x=a$ $\Longleftrightarrow$ $\nexists \lim\limits_{x\to a} (f(x))$ ili $\lim\limits_{x\to a} \neq f(a)$

Pr.) Sve element. f-je su neprekinute na svojim domenama.

- polinomi na $\mathbb{R}$
- eksp, log, trig, arkus, area, hip, ...

Pr.)
$$\operatorname{sgn}(x) = \begin{cases} 1, & x>0 \\ \underline{0, \quad x=0} \\ -1, & x<0 \end{cases}$$

Ima prekid



$\boxed{\text{Teorem}}$ f, g nepr. u $x=a \Rightarrow f \pm g, f \cdot g, \frac{f}{g}$ $(g(a) \neq 0)$

su neprekinuti u $x=a$

$\boxed{\text{Tm}}$ Neka je $f \circ g$ dobro def.

(a) f nepr. $x=b$ i $\lim\limits_{x \to a} (g(x)) = b \Rightarrow \lim\limits_{x \to a} (f(g(x))) =$

b) g nepr. $x=a$ i f nepr. u $g(a)$ $\quad = f(\lim\limits_{x \to a} g(x)) = f(a)$

$\Rightarrow f \circ g$ npr u a.

Pr.) $\lim\limits_{x \to 1} \arcsin\left(\frac{1}{2-x}\right) = \arcsin\left(\lim\limits_{x \to 1} \frac{1}{2-x}\right) = \arcsin 1$

Pr.) $\lim\limits_{x \to 4} 2^{\sqrt{x^2-2x-4}} = 2^{\lim\limits_{x \to 4} \sqrt{x^2-2x-4}} = 2^2 = 4$

Pr.) Ispitajte neprekinutost fje $f(x) = \begin{cases} 1-x^2, & x \leq 1 \\ \ln(x), & x > 1 \end{cases}$?

Fja f je neprekinuta na

$\mathbb{R} \setminus \{1\}$ jer su kvad. polinom i $\ln$ nepr. fje ve

Je li f nepr u $x=1$?

uvjet nepr.:

$\boxed{\lim\limits_{x \to 1} (f(x)) = f(x)}$

$\lim\limits_{x \to 1^+} (f(x)) = \lim\limits_{x \to 1^+} (f(x)) = \ln(1) = 0$

$\lim\limits_{x \to 1^-} (f(x)) = \lim\limits_{x \to 1^-} (1-x^2) = 0$

$\lim\limits_{x \to 1} (f(x)) = 0 \quad = \quad f(1) = 0$

! Funkcija je neprekinuta u $x=1$.

Zad. | Odredite $a, b \in \mathbb{R}$ td. $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x<2 \\ ax^2-bx+3, & 2 \leq x < 3 \\ 2x-a+b, & x \geq 3 \end{cases}$

tako da bude neprekinuta.

F-ja je neprekinuta na $\mathbb{R} \setminus \{2, 3\}$ jer su rac. funkcija i polinomi neprekinute f-je.

f nepr. f-ja u $x=2$: $\lim_{x \to 2} (f(x)) = f(2)$

$$\lim_{x \to 2^+} (f(x)) = \lim_{x \to 2^+} (ax^2 - bx + 3) = 4a - 2b + 3 = f(2)$$

$$\lim_{x \to 2^-} \frac{x^2-4}{x-2} = \lim_{x \to 2^-} (x+2) = 4$$

f nepr. f u $x=3$: $\lim_{x \to 3} f(x) = f(3)$

$$\lim_{x \to 3^+} (2x - a + b) = 6 - a + b$$

$$\lim_{x \to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$$

1. uvjet: $4a - 2b + 3 = 4$

$\boxed{4a - 2b = 1}$

2. uvjet: $6 - a + b = 9a - 3b + 3$

$\boxed{10a - 4b = 2}$

$4a = 1 - 2b \, / : 4$

$a = \frac{1-2b}{4}$

$\cancel{10}^{5} \cdot \frac{1-2b}{\cancel{4}_2} - 4b = 2$

$5\left(\frac{1}{2} - b\right) - 4b = 2$

$\frac{5}{2} - 5b - 4b = 2$

$-b = 2 - \frac{5}{2}$

$b = \frac{5}{2} - 2 \longrightarrow b = \frac{1}{2}$

$a = \frac{1 - \cancel{2} \cdot \frac{1}{\cancel{2}}}{4}$

$a = \frac{1}{4}$

Pr.) Je li $f(x) = 1 - \sqrt{1-x^2}$ nepr. na $[-1, 1]$

$\lim\limits_{x \to -1^+} f(x) = 1$ $\lim\limits_{x \to 1^-} f(x) = 1$

$\boxed{\text{DEF}}$ $a \in D(f) \subset \mathbb{R}$

- f je neprekinuta s desna u $x=a$ $\lim\limits_{x \to a^+} f(x) = f(a)$
- f je neprekinuta s lijeva u $x=a$. $\lim\limits_{x \to a^-} (f(x) - f(a)$
- f je neprekinuta na $[c, d]$.

f je nepr. na $\langle c, d \rangle$ + f nepr. s desna u $x=c$, i s lijeva $x=d$.

$\boxed{\text{TM}}$ f nepr. f-je na $[a, b]$

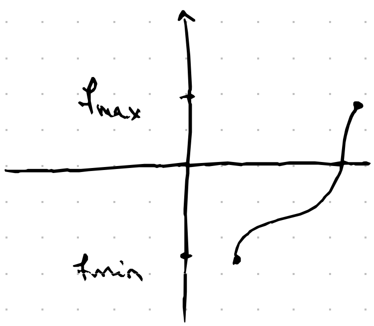
(a) F-ja f na $[a,b]$ minimum i maximum tj:

$$\exists x_m, x_n \in [a,b] \text{ t.d. } f(x_m) \le f(x) \le f(x_n), \forall x \in [a,b]$$

b) Za $\forall y^* \in \langle f(x_m), f(x_M) \rangle$ $\exists x^* \in [a,b]$ t.d. $f(x^*) = y^*$

Nepr. f-ja poprima sve vrijednosti između f_{min} i f_{max}

Nap. $f([a,b]) = [f(x_m), f(x_M)]$, f nepr.



$f \nearrow$ + nepr.: $f([a,b] = [f(a), f(b)])$

$f \searrow$ + nepr.: $f([a,b] = [f(b), f(a)])$

$\boxed{\text{KOROLAR}}$ $f : I \subset \mathbb{R} \to \mathbb{R}$ nepr. Ako f na I mijenja predznak tada