$\boxed{1.}$ $|z| = \frac{1}{2} \rightsquigarrow r = \frac{1}{2}$ $\qquad z = r(\cos\varphi + i\sin\varphi)$, $\quad r \geq 0$, $\varphi \in [0, 2\pi)$

$\operatorname{Im}(z^6) = \operatorname{Re}(z^3)$ $\qquad z^n = r^n(\cos(n\varphi) + i\sin(n\varphi))$

$\to r^6 \sin(6\varphi) = r^3 \cos(3\varphi)$

$\Rightarrow \frac{1}{2^6}\cdot 2\sin(3\varphi)\cdot\cos(3\varphi) = \frac{1}{2^3}\cos(3\varphi) \quad \Big| \cdot 2^5$

$\sin(3\varphi)\cos(3\varphi) = 4\cos(3\varphi)$

$\cos(3\varphi)\left[\sin(3\varphi) - 4\right] = 0$

$\cos(3\varphi) = 0 \Rightarrow 3\varphi = \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$

$\Rightarrow \varphi = \frac{\pi}{6} + k\frac{\pi}{3}, \ k \in \mathbb{Z}$

ili

$\cancel{\sin(3\varphi) = 4}$ Nemoguće!

Za $k = 0,1,2,3,4,5$ je $\varphi \in [0, 2\pi)$

O: $z = \frac{1}{2}\left[\cos\left(\frac{\pi}{6} + \frac{k\pi}{3}\right) + i\sin\left(\frac{\pi}{6} + \frac{k\pi}{3}\right)\right]$

gdje $k = 0, 1, 2, \ldots, 5$

4 velemajstora (L, Mk, Mn, R) i 12 ostalih

$\boxed{2.}$ (a)

$N_a = \binom{12}{3}\cdot\binom{9}{3}\cdot\binom{6}{3}\cdot\binom{3}{3}$ → preostala 3 su u skupini s R

→ od 6 biramo 3 uz Mn

biramo 3 od 12 šahista za skupinu u kojoj je L

od preostalih 9 biramo 3 uz Mk

(b) $\binom{10}{3}\cdot\binom{9}{3}\cdot\binom{6}{3}\cdot\binom{3}{3}$

biramo 3 od 12 bez A i B za skupinu s L

od preostalih 9 biramo 3 uz Mk

od 6 biramo 3 uz Mn

preostala 3 idu uz R

(drugi način)

A = {rasporedi gdje Ante i Luka igraju zajedno}

B = { — — — Brankica i Luka — — — }

$\# = |\bar{A} \cap \bar{B}| = N_a - |A \cup B| \overset{FUI}{=} N_a - |A| - |B| + |A \cap B|$

$|A| = |B| = \binom{11}{2}\cdot\binom{9}{3}\cdot\binom{6}{3}\cdot\binom{3}{3}$

$|A \cap B| = \binom{10}{1}\cdot\binom{9}{3}\cdot\binom{6}{3}\cdot\binom{3}{3}$

3. (a) T1 Točno

Neka je $f: D \to K$ neparna bijekcija i $f^{-1}: K \to D$ njen inverz.

Tvrdim da je f^{-1} neparna funkcija.

Neka je $y \in K$. Uzmimo $x = f^{-1}(y) \in D$. Tada $-x \in D$ i

$$f(-x) = -f(x) = -y \Rightarrow -y \in K \text{ i } f^{-1}(-y) = -x = -f^{-1}(y)$$

Dakle f^{-1} je neparna.

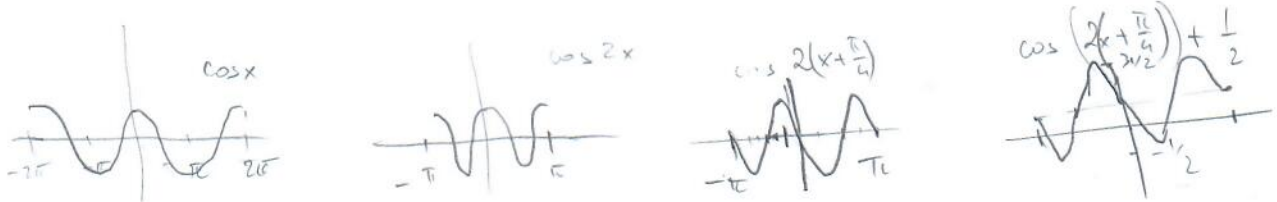
T2 Netočno

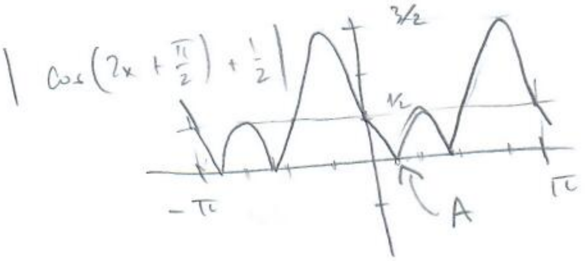
Periodička funkcija nikad nije injektivna pa ni bijektivna. Dakle inverz nikada ne postoji.

T3 Točno

po definiciji surjekcije.

(b)





$$\cos\left(2x+\frac{\pi}{2}\right) + \frac{1}{2} = 0$$

$$\Rightarrow \cos\left(2x+\frac{\pi}{2}\right) = -\frac{1}{2} \Rightarrow 2x+\frac{\pi}{2} = \frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$$

$$2x+\frac{\pi}{2} = \frac{4\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{12} + k\pi,\ k \in \mathbb{Z}$$

ili

$$x = \frac{5\pi}{12} + k\pi,\ k \in \mathbb{Z}$$

$\boxed{A = \frac{\pi}{12}}$ ← prva pozitivna nultočka

4

(a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h} = \lim_{h\to 0} \frac{x-2-(x+h-2)}{(x+h-2)(x-2)\cdot h} =$

$= \lim_{h\to 0} \frac{-h}{(x+h-2)(x-2)\cdot h} = \frac{-1}{(x-2)^2}$, za sve $x \in \mathbb{R}\setminus\{2\}$.

(b) Netočno! Funkcija $f(x)=|x|$ nije dif. u točki $x_0=0$ ali je neprekinuta u toj točki.

(c) $\left.\begin{array}{l} \lim_{x\to 3-} f(x) = \lim_{x\to 3-} x^2-8 = 1 \\ \lim_{x\to 3+} f(x) = \lim_{x\to 3+} \frac{1}{x-2} = 1 \end{array}\right] = f(3)$ pa je f-ja neprekinuta u $x_0=3$

$\left.\begin{array}{l} f'(3-) = (x^2-8)'\big|_{x=3} = (2x)\big|_{x=3} = 6 \\ f'(3+) = \left(\frac{1}{x-2}\right)'\Big|_{x=3} = \frac{-1}{(x-2)^2}\Big|_{x=3} = -1 \end{array}\right] \neq$ pa f-ja ne može biti difer. u $x_0=3$.

(d) $t_g \ldots \ y-g(3) = g'(3)(x-3)$

$y-1 = -1(x-3)$

$y = -x+4$

$t_h \ldots \ y-h(3) = h'(3)(x-3)$

$y-1 = 6(x-3)$

$y = 6x-17$

$y=-x+4 \longleftarrow$ NE PODUDARAJU SE! $\longrightarrow y=6x-17$

(Kada bi se ove tangente podudarale, onda bi i $f'(3-)=f'(3+)$ pa bi funkcija f bila diferencijabilna, a vidjeli smo da nije.)

$\boxed{5}$ (a) Fermatov teorem → vidi predavanja

(b) $f(x) = \dfrac{x+2}{\sqrt{x^2+2}}$, $D_f = \mathbb{R}$

≥ 2 ✓

$\lim\limits_{x \to +\infty} f(x) = 1$ ... $y = 1$ desna

$\lim\limits_{x \to -\infty} f(x) = -1$ ... $y = -1$ lijeva

} horizontalna asimptota

$$f'(x) = \frac{\sqrt{x^2+2} - (x+2)\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+2}}}{x^2+2} = \frac{\cancel{x^2}+2-\cancel{x^2}-2x}{(x^2+2)^{3/2}} = \frac{2(1-x)}{(x^2+2)^{3/2}}$$

$$f''(x) = \frac{-2(x^2+2)^{3/2} - 2(1-x)\cdot\frac{3}{\cancel{2}}(x^2+2)^{1/2}\cdot\cancel{2}x}{(x^2+2)^3} = \frac{\sqrt{x^2+2}\left[-2x^2-4-6x+6x^2\right]}{(x^2+2)^3} =$$

$$= \frac{4x^2-\overset{2x-6x}{6x}-4}{(x^2+2)^{5/2}} = \frac{2(2x+1)(x-2)}{(x^2+2)^{5/2}}$$

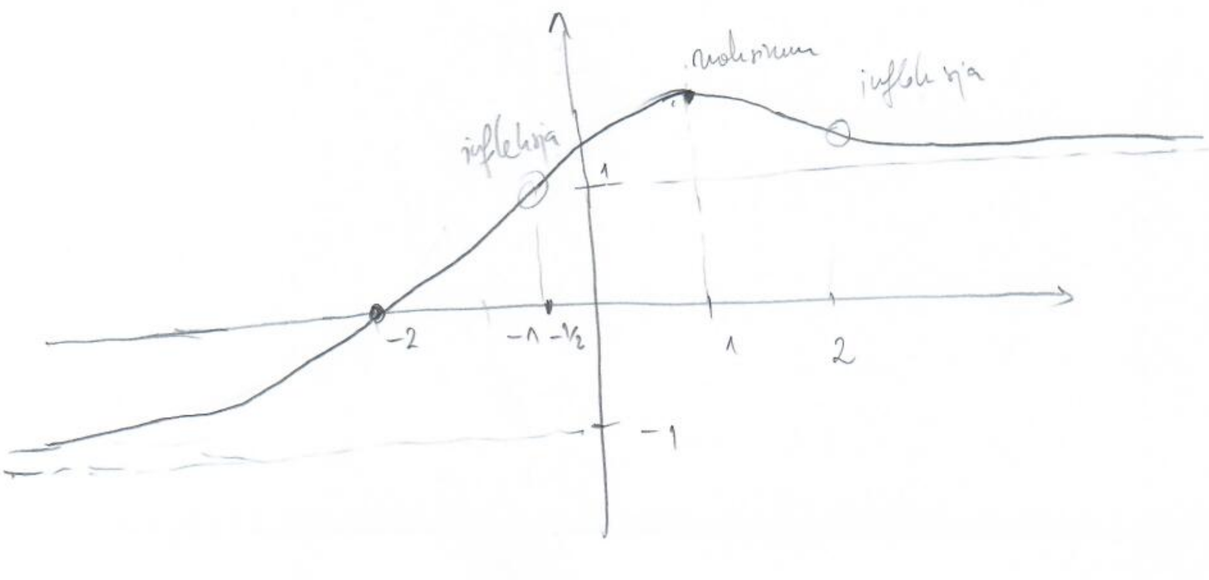
	$\langle -\infty, 1\rangle$	$\langle 1, +\infty\rangle$
f'	+	−
f	↗ max	↘

globalni maksimum u $x = 1$

$f(1) = \sqrt{3}$

	$\langle -\infty, -\frac{1}{2}\rangle$	$\langle -\frac{1}{2}, 2\rangle$	$\langle 2, +\infty\rangle$
f''	+	−	+
f	∪	∩	∩

točke infleksije u $x = -\frac{1}{2}$ i $x = 2$



6

(a) $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$ gdje $\Delta x = \frac{b-a}{n}$.

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3\cdot i}{n}}$

Uzmimo funkciju $f(x)=\sqrt{1+x}$ na intervalu $[0,3]$. Tada je n-ta integralna suma po ekvidistantnoj subdiviziji jednaka

$$\sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n} \sqrt{1+\frac{3}{n}i}\cdot\frac{3}{n},$$ gdje $x_i = i\cdot\frac{3}{n}$, za $i=0,1,2,\dots,n$; i $\Delta x = \frac{3}{n}$

Pa vrijedi da je $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}} = \int_0^3 \sqrt{1+x}\,dx = \left(\frac{2}{3}(1+x)^{3/2}\right)\Big|_0^3 =$

$$= \frac{2}{3}\left(4^{3/2} - 1^{3/2}\right) = \frac{14}{3}$$

(c) $\sigma_3 = \sum_{i=1}^{3} \frac{3}{3}\sqrt{1+\frac{3i}{3}} = \sqrt{2}+\sqrt{3}+\sqrt{4}$

Površina 3 osjenčana stupića.

$\boxed{7}$

(a) $$\int \frac{e^{3x}+e^{2x}}{(e^{2x}+1)(e^{x}-1)}\,dx = \left[\begin{matrix} t=e^{x} \\ dt=e^{x}dx \end{matrix}\right] = \int \frac{t^2+t}{(t^2+1)(t-1)}\,dt =$$

$$= \int \frac{1}{t^2+1}\,dt + \int \frac{1}{t-1}\,dt = \operatorname{arctg} t + \ln|t-1| + C$$

$$= \operatorname{arctg} e^{x} + \ln|e^{x}-1| + C$$

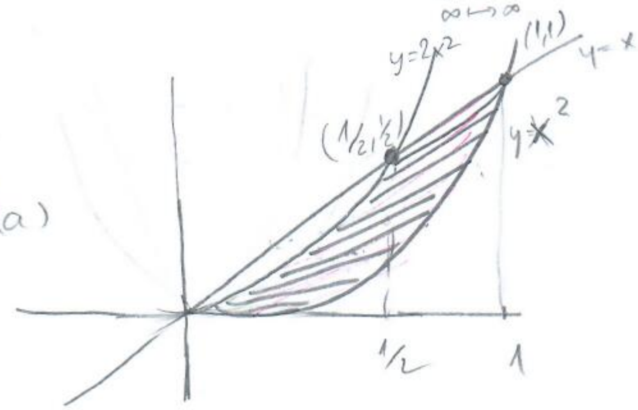
(b) $$\int_0^{\infty} \frac{dx}{25+x^2} = \left[\begin{matrix} t=x/5 \\ dt=\frac{dx}{5} \end{matrix}\right] = \int_0^{\infty} \frac{5\,dt}{25+25t^2} = \frac{1}{5}\int_0^{\infty}\frac{dt}{t^2+1} = \frac{1}{5}\operatorname{arctg}(t)\Big|_0^{\infty}$$

$0 \mapsto 0$
$\infty \mapsto \infty$

$$= \frac{1}{5}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{10}$$

$\boxed{8.}$

(a)

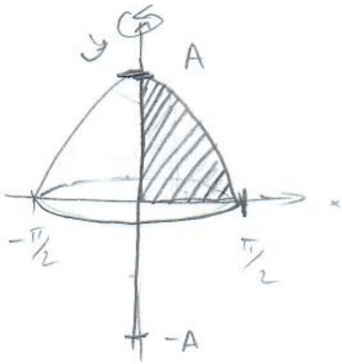


$$P_{ov} = \int_0^{1/2} (2x^2-x^2)\,dx + \int_{1/2}^{1} (x-x^2)\,dx$$

$$= \left(\frac{x^3}{3}\right)\Big|_0^{1/2} + \left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_{1/2}^{1}$$

$$= \frac{1}{24} + \underbrace{\left(\frac{1}{2}-\frac{1}{3}\right)}_{\frac{1}{6}} - \underbrace{\left(\frac{1}{8}-\frac{1}{24}\right)}_{\frac{1}{12}} = \frac{1+4-2}{24} = \frac{1}{8}$$

(b)



$$Vol = \int_0^{\pi/2} A\cos x \cdot 2\pi x\,dx = \left[\begin{matrix} u=2\pi A x & dv=\cos x\,dx \\ du=2\pi A\,dx & v=\sin x \end{matrix}\right]$$

$$= \left(2\pi A x \sin x\right)\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \cdot 2\pi A\,dx =$$

$$= 2\pi A\cdot\frac{\pi}{2} + \left(2\pi A\cdot\cos x\right)\Big|_0^{\pi/2} =$$

$$= A\pi^2 - 2\pi A = 2\pi^2 - 4\pi$$

$$\Rightarrow \boxed{A=2}$$