5.4.1 NEODREĐENI OBLICI

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \Bigg\{$$

$$\lim_{x\to\pm} \frac{1}{x^r} = 0 \qquad r\in\mathbb{Q}^+$$

① $$\lim_{x\to+\infty} \frac{3x^4+x}{x^3-5} = +\infty$$

② $$\lim_{x\to+\infty} \frac{x+1}{x+\sqrt{4x^2+1}} = \left(\frac{\infty}{\infty}\right) = \lim \frac{x}{3x} = \frac{1}{3}$$

Nap: ako je $x<0$, onda se x napiše kao $-\sqrt{x^2}$

$$\lim_{x\to-\infty} \frac{2x+1 \;/:x}{\sqrt{4x^2+3}\;/:x} = \left(\frac{-\infty}{\infty}\right)$$

$$= \lim_{x\to-\infty} \frac{2+\frac{1}{x}}{-\sqrt{\frac{4x^2+3}{x^2}}} = -1$$

Zad.)

a) $$\lim_{x\to-\infty} \frac{x-1}{\sqrt{x^2+1}} = (x=-t) = \lim_{t\to+\infty} \frac{(-t-1)}{\sqrt{t^2+1}} = \lim_{t\to+\infty} \frac{-t}{t} = \underline{\underline{-1}}$$

b) $$\lim \left(\frac{x-2}{x+2}\right)^{2x+1} = (1^\infty) = \lim_{x\to+\infty} \left(\frac{x+2}{x-2}\right)^{-(2x+1)} = \lim_{x\to+\infty} \left(\frac{x-2+4}{x-2}\right)^{-(2x+1)}$$

$$= \lim \left(1+\frac{1}{x-2}\right)^{-(2x+1)} = \lim \left(\left(1+\frac{1}{\frac{x-2}{4}}\right)^{\frac{x-2}{4}}\right)^{\frac{4}{x-2}\cdot(-2x+1)} = \underline{\underline{e^{8}}}$$

$$\boxed{\lim_{x\to\pm\infty} \left(1+\frac{1}{x}\right)^x = e \iff \lim_{t\to 0} (1+t)^{\frac{1}{t}} = e}$$

c) $$\lim_{x\to 0} (1+5x)^{\frac{3}{x}} = (1^\infty) = \lim_{x\to 0} \left((1+5x)^{\frac{1}{5x}}\right)^{5x\cdot\frac{3}{x}} = \underline{\underline{e^{15}}}$$

d) $$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{1}{x}\cdot\ln(1+x) = \lim_{x\to 0} \ln(1+x)^{1/x} = \ln e = 1$$

e) $$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-\sqrt[3]{x}} = \left(\frac{0}{0}\right) = (x=t^6) = \lim_{t\to 1} \frac{1-t^3}{1-t^2} = \lim_{t\to 1} \frac{\cancel{(1-t)}(1+t+t^2)}{\cancel{(1-t)}(1+t)} = \frac{3}{2}$$

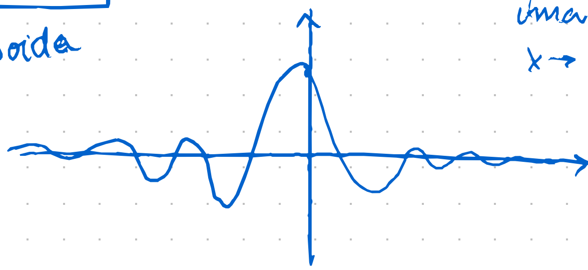
5.4.2. Sendvič tm

TM Neka je h t.d. $g(x) \leq h(x) \leq f(x)$ na nekoj okolini od $x=a$.
Ako je $\lim\limits_{x\to a}(g(x)) = \lim\limits_{x\to a}(f(x)) = L$, tada je i $\lim\limits_{x\to a} h(x) = L$.

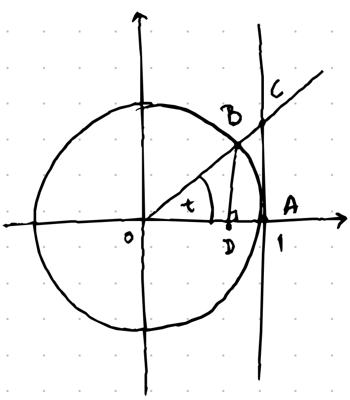
$\boxed{\text{Funkcija } f(x) = \frac{\sin x}{x}}$

prigušena sinusoida

Teorem Fja $f(x) = \frac{\sin x}{x}$ ima limes kad $x \to 0$ i $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$



Dokaz:



$P_{\triangle ODB} \leq P(\text{kr. isj. } BOA) \leq P_{\triangle OAC}$

$$\frac{1\cdot \sin t}{2} \leq \frac{1\cdot t}{2} \leq \frac{1\cdot \operatorname{tg} t}{2} \quad /\cdot 2$$

$$\sin t \leq t \leq \operatorname{tg} t \quad /:\sin t$$

$t \in [0, \frac{\pi}{2}]$ i to je bitno za predznake

$$1 \leq \frac{t}{\sin t} \leq \cos t$$

$$\boxed{\cos t \geq \frac{t}{\sin t} \geq 1}$$

tm. o sendviču $\Rightarrow$

$$\lim_{t\to 0^+}\frac{\sin t}{t} = 1$$

parna f

$f(x) = \frac{\sin t}{t}$ je parna (jer su i $\sin t$ i t neparne pa $\frac{-}{-} = +$)

$$\hookrightarrow \boxed{\lim_{t\to 0^-}\frac{\sin t}{t} = 1}$$

Zad. 1

1) $\lim\limits_{x\to 0}\frac{\sin(3x)}{x} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0}\frac{\sin(3x)}{3x}\cdot 3 = 3$

2) $\lim\limits_{x\to 0}\frac{\operatorname{tg} x}{x} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{1}{x} = 1$ //

3) $\lim\limits_{x\to 0}\frac{x}{\arcsin x}$ $\left(\frac{0}{0}\right) = \left(\begin{matrix} t = \arcsin x \\ x = \sin t\end{matrix}\right)$

5.4.3. Ekvivalentne neizmjerno male veličine

DEF Fje f i g su neizmjerno male veličine istog reda kad $x \to a$ ako je $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} (g(x)) = 0$ i $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = c \in \mathbb{R}^+$

Fje f i g su ekvivalentne neizmjerno male veličine ako je $c=1$,
tj $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = 1$ i pišemo $f(x) \sim g(x)$, $x \to a$.

Nap $\lim\limits_{x\to a} \frac{\overbrace{f(x)}^{\sim \ln(x),\ x\to a}}{g(x)} = \lim\limits_{x\to a} \frac{\ln(x)}{g(x)}$

$\lim\limits_{x\to a} f(x) - g(x) = \lim\limits_{x\to a}$