1 Stavimo $z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1)$

a) $z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + i\sin\varphi_1)}{r_2(\cos\varphi_2 + i\sin\varphi_2)} \cdot \frac{(\cos\varphi_2 - i\sin\varphi_2)}{\underbrace{(\cos\varphi_2 - i\sin\varphi_2)}_{1}} =$$

$$= \frac{r_1}{r_2}\Big(\cos\varphi_1\cos\varphi_2 + \sin\varphi_1\sin\varphi_2 + i\big(\sin\varphi_1\cos\varphi_2 - \cos\varphi_1\sin\varphi_2\big)\Big)$$

$$= \frac{r_1}{r_2}\big(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)\big)$$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \varphi_1 - \varphi_2$, ali i za svaki $k \in \mathbb{Z}$ $\arg\left(\frac{z_1}{z_2}\right) = \varphi_1 - \varphi_2 + 2k\pi$.

b) $\arg\left(\frac{z^2}{i \cdot \bar{z}}\right) = \frac{\pi}{2} + 2k\pi$, $Re(z^2) = 1$

$$z = r(\cos\varphi + i\sin\varphi)$$

$$\arg\left(\frac{z^2}{i \cdot \bar{z}}\right) = \arg\left(\frac{z^2}{i}\right) - \arg(\bar{z}) + 2k\pi$$

$$= \arg(z^2) - \arg(i) - \arg(\bar{z}) + 2k\pi$$

$$= 2\varphi - \frac{\pi}{2} - (-\varphi) + 2k\pi = 3\varphi - \frac{\pi}{2} + 2k\pi$$

$$3\varphi - \frac{\pi}{2} = \frac{\pi}{2} + 2k\pi$$

$$\Rightarrow \boxed{\varphi = \frac{(2k+1)\pi}{3}}$$

$k = 0 \Rightarrow \varphi_1 = \frac{\pi}{3}$

$k = 1 \quad \varphi_2 = \pi$

$k = 2 \Rightarrow \varphi_3 = \frac{5\pi}{3}$

$k = 3 = \frac{7\pi}{3} \simeq \frac{\pi}{3} + 2\pi$

$\vdots$

$Re(z^2) = 1$

$$r^2\cos 2\varphi = 1$$

$r^2 = \frac{1}{\cos 2\varphi}$. $\Leftarrow$ ovo je jedino moguće za $\varphi = \pi \Rightarrow r = 1$ $\Big\}$ $z = -1$ jedino rješenje.

(2)

5 studentica
7 studenata

6 osoba

a) $\binom{7}{5}\binom{5}{1}$ ← broj načina za odabrati 1 studenticu i 5 studenata

b) $\binom{7}{3}\binom{5}{3}$ ← broj načina za odabrati 3 studenta i 3 studentice

c) 4 ♂ 2 ♀ ili 5 ♂ 1 ♀ ili 6 ♂ 0 ♀ $= \binom{5}{0}\binom{7}{6} + \binom{5}{1}\binom{7}{5} + \binom{5}{2}\binom{7}{4}$.

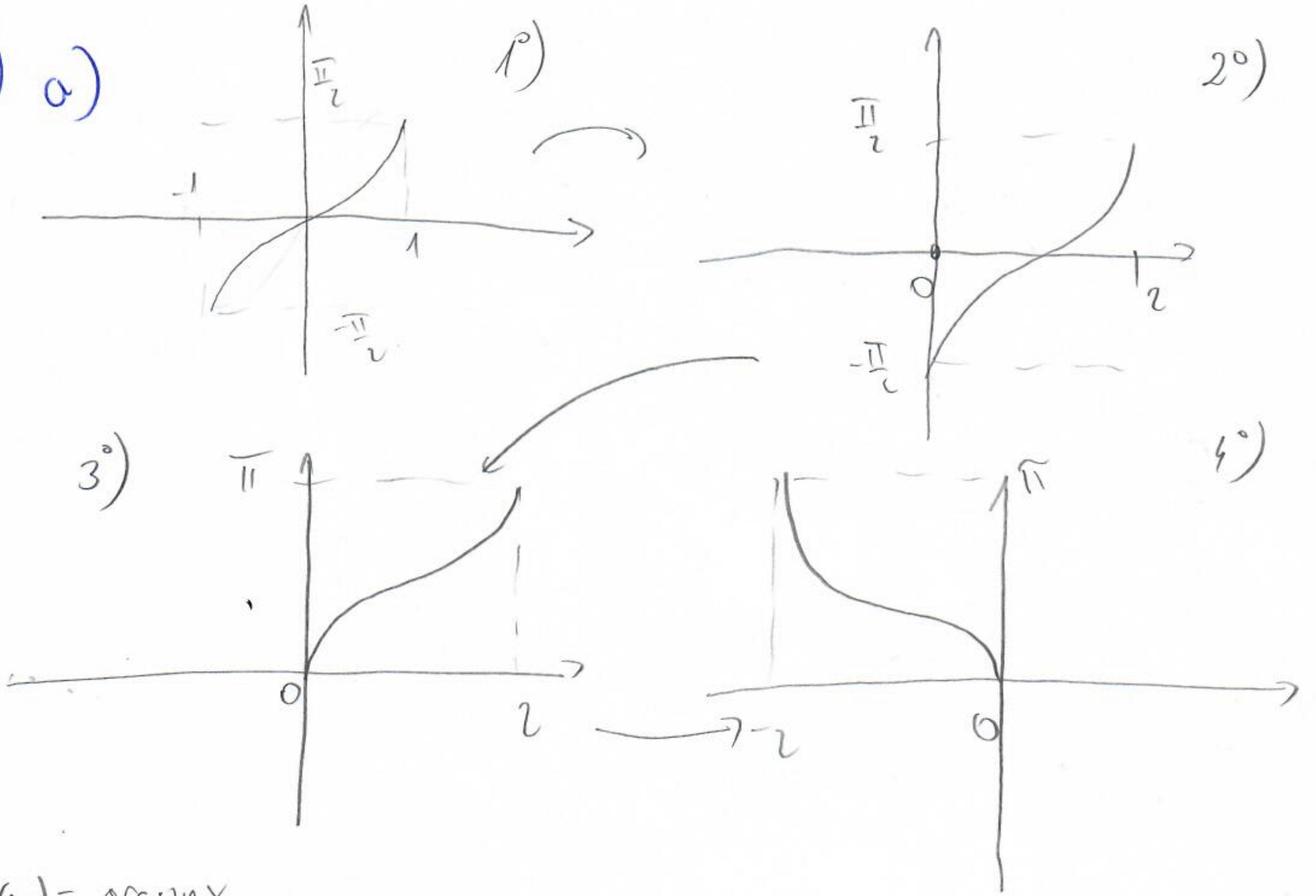
d) Prvu osobu odabiremo na n načina
Drugu na n − 1 n
⋮
k-tu na n − k + 1 načina

$$n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

Međutim nije nam bitan poredak:

$$\text{broj odabira} = \frac{n!}{(n-k)!\cdot k!} = \binom{n}{k}$$

3) a)



$f_1(x) = \arcsin x$

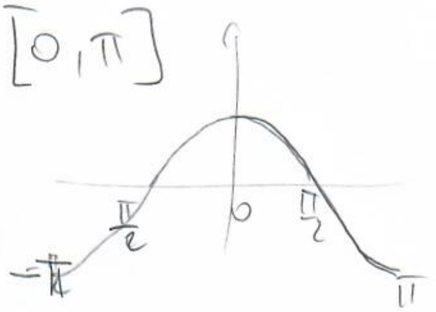
$f_2(x) = \arcsin(x-1)$

$f_3(x) = \arcsin(x-1) + \frac{\pi}{2}$

$f_4(x) = \arcsin(-x-1) + \frac{\pi}{2}$

$D_f = [-2, 0]$

$\text{Im} f = [0, \pi]$



b) $\cos(2\arccos(x)) = 2x$

$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

za $x \in [-1,1]$

$\arccos x \in [0, \pi]$

$\Rightarrow 2\cos^2(\arccos(x)) - 1 = 2x$

$\Rightarrow$ za $x \in [-1,1]$ je (inače nije definirano) $2x^2 - 1 = 2x$

$\Rightarrow x_{1,2} = \frac{1 \pm \sqrt{3}}{2}$

$\frac{1-\sqrt{3}}{2} \in [-1,1]$, $\cancel{\frac{1+\sqrt{3}}{2} \notin [-1,1]}$

4) $(f \circ g)'(x) = f'(g(x))\, g'(x)$

$(f \cdot g)'(x) = f'(x) g(x) + f(x)\, g'(x)$

$$\left(\frac{f}{g}\right)'(x) = \left(f \cdot \frac{1}{g}\right)'(x) = \left(f \cdot g^{-1}\right)'(x)$$

$$= f'(x) \cdot g^{-1}(x) - f(x) \cdot g^{-2}(x) \cdot g'(x)$$

$$= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)} .$$

b) $y = tg(x) = \dfrac{\sin x}{\cos x}$, $y' = \dfrac{\sin' \cdot \cos - \sin \cdot \cos'}{\cos^2}$

$$= \frac{\sin^2 + \cos^2}{\cos^2} = \frac{1}{\cos^2 x}$$

c) $x = f \circ g\,(x) \Rightarrow 1 = f'(g(x)) \cdot g'(x)$
$\Rightarrow f'(g(x)) = \dfrac{1}{g'(x)}$ $\Bigg\}$ $(f^{-1})' = \dfrac{1}{f'(f^{-1})}$

$f(y) = \operatorname{arctg}(y)$

$f'(y) = \cancel{\dfrac{1}{\operatorname{arctg}}}$ $(tg^{-1})'(y) = \dfrac{1}{tg'(\operatorname{arctg}(y))} = \dfrac{1}{\frac{1}{\cos^2(\operatorname{arctg}(y))}}$

$$= \cos^2(\operatorname{arctg}(y)) = \frac{1}{tg^2(\operatorname{arctg}(y)) + 1} = \frac{1}{y^2+1}$$

d) $y = \ln^3(\operatorname{arctg} x)$

$$y' = 3 \cdot \ln^2(\operatorname{arctg} x) \cdot \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1}$$

5. $f(x) = \sin(x)$

$$T_5(x) = \sum_{i=0}^{5} \frac{f^{(i)}(0)}{i!}(x-0)^i$$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 0$

$f'''(0) = -1$

$f^{(4)}(0) = 0$

$f^{(5)}(0) = 1$

$$T_5(x) = 0 + x + 0 - \frac{1}{6}x^3 + 0 + \frac{1}{120}x^5$$

$$= x - \frac{x^3}{6} + \frac{x^5}{120}$$

$$\sin(1) \approx T_5(1) = 0{,}841666 = \frac{101}{120}$$

$$R_n(x) = \frac{f^{(n+1)}(x_1)}{(n+1)!}(x-0)^{n+1}, \quad x_1 \in \langle 0, x \rangle$$

$$|R_5(x)| = \frac{|\sin(x_1)|}{6!}x^6 \leq \frac{1}{6!} = \frac{1}{720}$$

$$f(x) = T_5(x) + R_5(x)$$

(6) b)

$$\lim_{n\to\infty} \frac{3}{n} \sum_{k=1}^{n} \left(1+\sqrt{\frac{3k}{n}}\right)^2 = (*)$$

Stavimo:

$$\left.\begin{aligned} \Delta x_k &= \frac{3}{n} = \frac{b-a}{n} \\ x_k &= a + k\Delta x_k = \frac{3k}{n} \end{aligned}\right\} \begin{aligned} b&=3 \\ a&=0 \end{aligned} \qquad f(x) = (1+\sqrt{x})^2$$

$$(*) = \int_0^3 f(x)\,dx = \int_0^3 (1+\sqrt{x})^2\,dx = \int_0^3 1 + 2\sqrt{x} + x\,dx$$

$$= x + \frac{4}{3}x^{\frac{3}{2}} + \frac{x^2}{2}\Big/_0^3$$

$$= \frac{15}{2} + 4\sqrt{3}$$

a)

Iz kapsule.

7)

$$\int_{\frac{2}{\pi}}^{\frac{3}{\pi}} \frac{\sin(\frac{1}{x})}{x^2}\,dx = \begin{pmatrix} t=\frac{1}{x} & \frac{2}{\pi}\to\frac{\pi}{2} \\ dt=-\frac{dx}{x^2} & \frac{3}{\pi}\to\frac{\pi}{3}\end{pmatrix}$$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -\sin(t)\,dt = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(t)\,dt = -\cos(t)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= 0 + \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

b)

$$\int x^2 \operatorname{arctg}\left(\frac{1}{x}\right)dx = \begin{bmatrix} u=\operatorname{arctg}\left(\frac{1}{x}\right) & dv=x^2dx \\ du=-\frac{dx}{1+x^2} & v=\frac{x^3}{3}\end{bmatrix}$$

$$= \frac{x^3}{3}\operatorname{arctg}\left(\frac{1}{x}\right) - \int \frac{x^3}{3}\left(\frac{-dx}{1+x^2}\right)$$

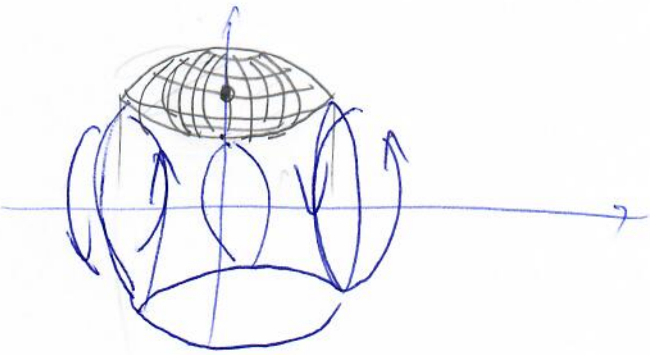
$$= \frac{x^3}{3}\operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx$$

$$= \frac{x^3}{3}\operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{3}\int x\,dx - \frac{1}{3}\int \frac{x}{x^2+1}\,dx$$

$$= \frac{x^3}{3}\operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{3}\frac{x^2}{2} - \frac{1}{3}\ln(x^2+1)\cdot\frac{1}{2} + C,$$

$C\in\mathbb{R}$

8) $|y-2|+x^2=1$



$|y-2| = 1-x^2 \geq 0 \Rightarrow x^2 \leq 1$

$x \in [-1, 1]$

gornji rob: $y-2 = 1-x^2$

$y = 3-x^2$

dolnji rob: $-y+2 = 1-x^2$

$y = 1+x^2$

- računanje volumna

$$V = \pi \int_{-1}^{1} y_1^2(x) - y_2^2(x)\, dx = \pi \int_{-1}^{1} \left((3-x^2)^2 - (1+x^2)^2 \right) dx = \frac{32\pi}{3}$$

- računanje oplošja:

$$O = O_1 + O_2 = 2\pi \int_{-1}^{1} y_1(x)\sqrt{1+(y_1'(x))^2}\, dx + 2\pi \int_{-1}^{1} y_2(x)\sqrt{1+(y_2'(x))^2}\, dx$$

$$= 2\pi \int_{-1}^{1} (3-x^2)\sqrt{1+4x^2}\, dx + 2\pi \int_{-1}^{1} (1+x^2)\sqrt{1+4x^2}\, dx =$$

$$= 2\pi \int_{-1}^{1} 4\sqrt{1+4x^2}\, dx = 16\pi \int_{0}^{1} \sqrt{1+4x^2}\, dx = \left| \begin{array}{l} x = \frac{1}{2}\operatorname{sh} t \\ dx = \frac{1}{2}\operatorname{ch} t\, dt \\ 0 \to 0 \\ 1 \to \operatorname{arsh} 2 \end{array} \right|$$

$$= 8\pi \int_{0}^{\operatorname{arsh} 2} \operatorname{ch}^2 t\, dt = 4\pi \int_{0}^{\operatorname{arsh} 2} (\operatorname{ch}(2t) + 1)\, dt =$$

$$= 4\pi \left(t + \frac{1}{2}\operatorname{sh}(2t) \right) \Big|_{0}^{\operatorname{arsh} 2} = 4\pi \left(\operatorname{arsh} 2 + 2\sqrt{5} \right)$$