9.5. Integrali iracionalnih funkcija

Objasnimo na primjeru:

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$$

- najopćenitija metoda je odabir takve supstitucije koja bi eliminirala korijen
  $\hookrightarrow t^6 = x$

$t^3 = \sqrt{x}$ $\quad$ $t^2 = \sqrt[3]{x}$

OPĆENITO VRIJEDI:

ako imamo $\sqrt[p]{x}$, $\sqrt[q]{x}, \ldots$ uvodimo $x = t^{NZV(p,q\ldots)}$

$\hookrightarrow$ u ovom primjeru 3 i $2 \Rightarrow \underline{6}$

$$= \left| \begin{matrix} t^6 = x \\ dx = 6t^5 dt \end{matrix} \right| = \int \frac{6t^5 dt}{t^3+t^2} = \int \frac{\cancel{t^2}}{\cancel{t^2}} \cdot \frac{6t^3 dt}{t+1} = 6 \int \frac{t^3}{t+1} dt$$

$$\begin{array}{l} t^3 : (t+1) = t^2 - t + 1 \\ \underline{-t^3 - t^2} \\ 0 \quad -t^2 \\ \quad \underline{+t^2 + t} \\ \quad 0 \quad t \\ \qquad \underline{-t - 1} \\ \qquad 0 \quad -1 \end{array}$$

$$\rightarrow \frac{t^3}{t+1} = (t^2 - t + 1) + \frac{-1}{t+1}$$

$$\Rightarrow 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt = 6 \cdot \frac{1}{3} t^3 - 6 \cdot \frac{1}{2} t^2 + 6t - 6 \ln|t+1| + C$$

$$\boxed{= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln|\sqrt[6]{x}+1| + C}$$

Primjer 9.61) $\int \frac{dx}{\sqrt{x^2+x+2}} \longrightarrow x^2+2\cdot\frac{1}{2}\cdot 1x+\frac{1}{4}$

$$= \int \frac{dx}{\sqrt{(x+\frac{1}{2})^2+\frac{7}{4}}} = \left| \begin{array}{l} x+\frac{1}{2}=t \\ dx=dt \end{array} \right| = \int \frac{dt}{\sqrt{t^2+\frac{7}{4}}}$$

$$= \ln\left|t+\sqrt{t^2+\frac{7}{4}}\right| + c = \boxed{\ln\left|x+\frac{1}{2}+\sqrt{(x+\frac{1}{2})^2+\frac{7}{4}}\right|+C}$$

9.62.)

$$\int \frac{dx}{\sqrt{2x-x^2}} = \qquad 1+2x-x^2-1 \Rightarrow (1-x)^2-1$$

$$\int \frac{dx}{\sqrt{(1-x)^2-1}} = \left| \begin{array}{l} 1-x=t \\ dx=dt \end{array} \right| = \int \frac{dt}{\sqrt{1-t^2}} = \boxed{\arcsin(x-1)+c}$$