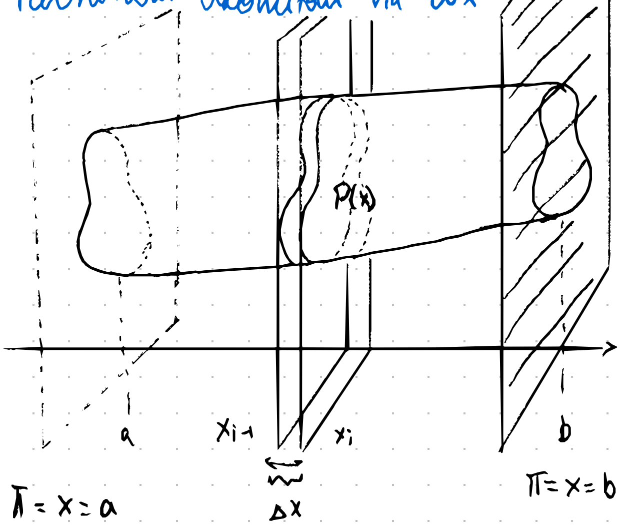
12.2. IZRAČUNAVANJE VOLUMENA TIJELA

- imamo $P = P(x)$ koja nastaje u presjeku promatranog tijela ravninom okomitom na Ox



$\Delta x = x_i - x_{i-1}$

- ako aproksimiramo volumen tako dobivenog dijela volumenom V_i valjka $(B \cdot h)$ sa $P(x_i) \cdot \Delta x_i$

baza, visina

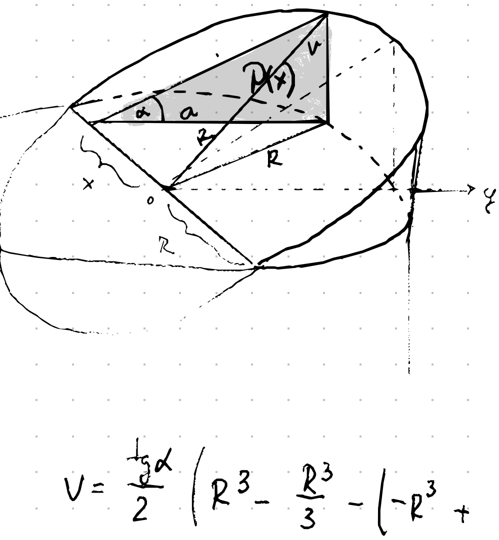
traženi volumen:

$$\sum_{i=1}^{n} V_i = \sum_{i=1}^{n} P(x_i) \cdot \Delta x_i$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} P(x_i) \Delta x_i$$

$$\boxed{V = \int_a^b P(x)\,dx}$$

Primjer 12.20.) Izračunaj volumen dijela valjka polumjera baze R presječenog ravninom koja prolazi kroz promjer baze i nagnuta je prema bazi za kut α.



$a = \sqrt{R^2 - x^2}$ $\quad v = \sqrt{R^2 - x^2} \cdot \operatorname{tg}\alpha$ $\quad \operatorname{tg}\alpha \to \frac{v}{a}$

$$V = \int_{-R}^{R} P(x)\,dx = \int_{-R}^{R} \frac{\sqrt{R^2 - x^2}\,\sqrt{R^2 - x^2}\,\operatorname{tg}\alpha}{2}\,dx$$

$$V = \int_{-R}^{R} \frac{R^2 - x^2}{2} \operatorname{tg}\alpha\,dx$$

$$V = \frac{\operatorname{tg}\alpha}{2}\left(\int_{-R}^{R} R^2\,dx - \int_{-R}^{R} x^2\,dx\right)$$

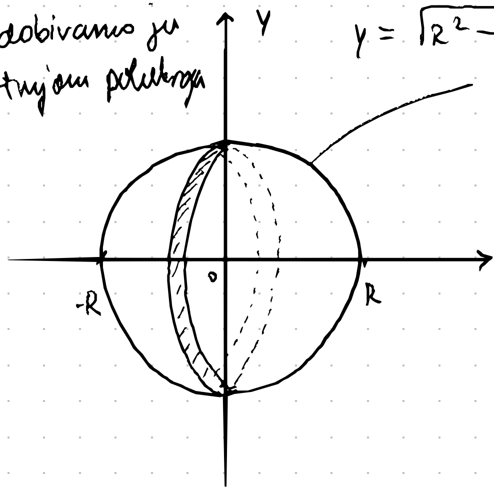
$$V = \frac{\operatorname{tg}\alpha}{2}\left(xR^2 - \frac{x^3}{3}\right)\Big|_{-R}^{R}$$

$$V = \frac{\operatorname{tg}\alpha}{2}\left(R^3 - \frac{R^3}{3} - \left(-R^3 + \frac{R^3}{3}\right)\right)$$

$$V = \frac{\operatorname{tg}\alpha}{\cancel{2}}\left(\cancel{2}R^3 - \frac{\cancel{2}R^3}{2}\right) = \boxed{\frac{2R^3}{2}\operatorname{tg}\alpha}$$

Primjer 22.) Izvod formule za volumen kugle polumjera R

- dobivamo ju vrtnjom polukruga



$y = \sqrt{R^2 - x^2}$ - polukrug

$V = \int_{-R}^{R} P_{valjka}$

$V_v = P \cdot v$

$P = r^2 \cdot \pi \to y^2\pi \quad \int \quad \sum_{i=1}^{n} y^2\pi \Delta x$

$V = \Delta x$

$$V = \int_{-R}^{R} \left(\sqrt{R^2 - x^2}\right)^2 \pi \, dx = 2\pi \int_{0}^{R} (R^2 - x^2) dx = 2\pi \left(xR^2 - \frac{1}{3}x^3\right)\Big|_0^R$$

$$V = 2\pi \left(R^3 - \frac{R^3}{3}\right) = 2\pi \cdot \frac{2R^3}{3} = \frac{4R^3\pi}{3}$$