1 (a) $f(a) = f(b) \iff sh^2(a) = sh^2(b)$

Npr. $a = -1$, $b = 1$ zadovoljavaju uvjet jer je $f(x)$ parna

Prema Rolle-ovom teoremu $\exists c \in \langle a,b \rangle$ t.d. $f'(c) = 0$

$$f'(c) = 0 \iff f'(x) = 2\,sh(x)\,ch(x) = 0$$

$$\Rightarrow \quad c = 0 \in \langle -1, 1]$$

(b) Skripta, Poglavlje 9, Teorem 9.2.3

(c)
$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

Neka je $a = 0$, $b = 1$ $\Rightarrow$ $f(0) = 5$, $f(1) = 3$

$$f(x) = x^2 - 3x + 5 \quad \Rightarrow \quad f'(x) = 2x - 3$$

$$\Rightarrow \quad 2c - 3 = \frac{3-5}{1-0} \quad \Rightarrow \quad 2c - 3 = -2$$

$$2c = 1$$

$$\boxed{c = \frac{1}{2}}$$

② Područje definicije $\mathcal{D}_f = \{x \in \mathbb{R} : x-2 \neq 0\} = \mathbb{R} \setminus \{2\} = \langle -\infty, 2\rangle \cup \langle 2, +\infty\rangle$

V.A. $\lim\limits_{x \to 2^{\pm}} f(x) = \lim\limits_{x \to 2^{\pm}} \frac{x^3}{(x-2)^2} = \frac{2^3}{0^+} = +\infty \Rightarrow x=2$ je obostrana vertikalna asimptota

K.A. $\lim\limits_{x \to \pm\infty} \frac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \frac{x^2}{(x-2)^2} = \lim\limits_{x \to \pm\infty} \left(\frac{x}{x-2}\right)^2 = \lim\limits_{x \to \pm\infty} \left(\frac{1}{1-\frac{2}{x}}\right)^2 = \boxed{1}$

$$\lim_{x \to \pm\infty} f(x) - x = \lim_{x \to \pm\infty} \frac{x^3 - x(x-2)^2}{(x-2)^2} = \lim_{x \to \pm\infty} \frac{4x^2 - 4x \;|:x^2}{x^2-4x+4 \;|:x^2} = \lim_{x \to \pm\infty} \frac{4-\frac{4}{x}}{1-\frac{4}{x}+\frac{4}{x^2}} = \boxed{4}$$

$\boxed{y = x+4}$ je i desna i lijeva kosa asimptota

$$f'(x) = \frac{3x^2(x-2)^{\cancel{2}} - x^3 \cdot 2\cancel{(x-2)}}{(x-2)^{\cancel{4}}} = \frac{3x^3 - 6x^2 - 2x^3}{(x-2)^3} = \boxed{\frac{x^2(x-6)}{(x-2)^3}}$$

$$f''(x) = \frac{(3x^2-12x)(x-2)^{\cancel{3}} - x^2(x-6)\cdot 3\cancel{(x-2)^2}}{(x-2)^{\cancel{6}\,4}} = \frac{3x^2-12x^2-6x^2+24x-3x^3+18x^2}{(x-2)^4} = \frac{24x}{(x-2)^4}$$

Stacionarne točke

$$f'(x) = \frac{x^2(x-6)}{(x-2)^3} = 0$$

$\Leftrightarrow$ $\boxed{x=0}$ ili $\boxed{x=6}$

Kandidati za točke infleksije

$$f''(x) = \frac{24x}{(x-2)^4} = 0$$

$\Leftrightarrow$ $\boxed{x=0}$

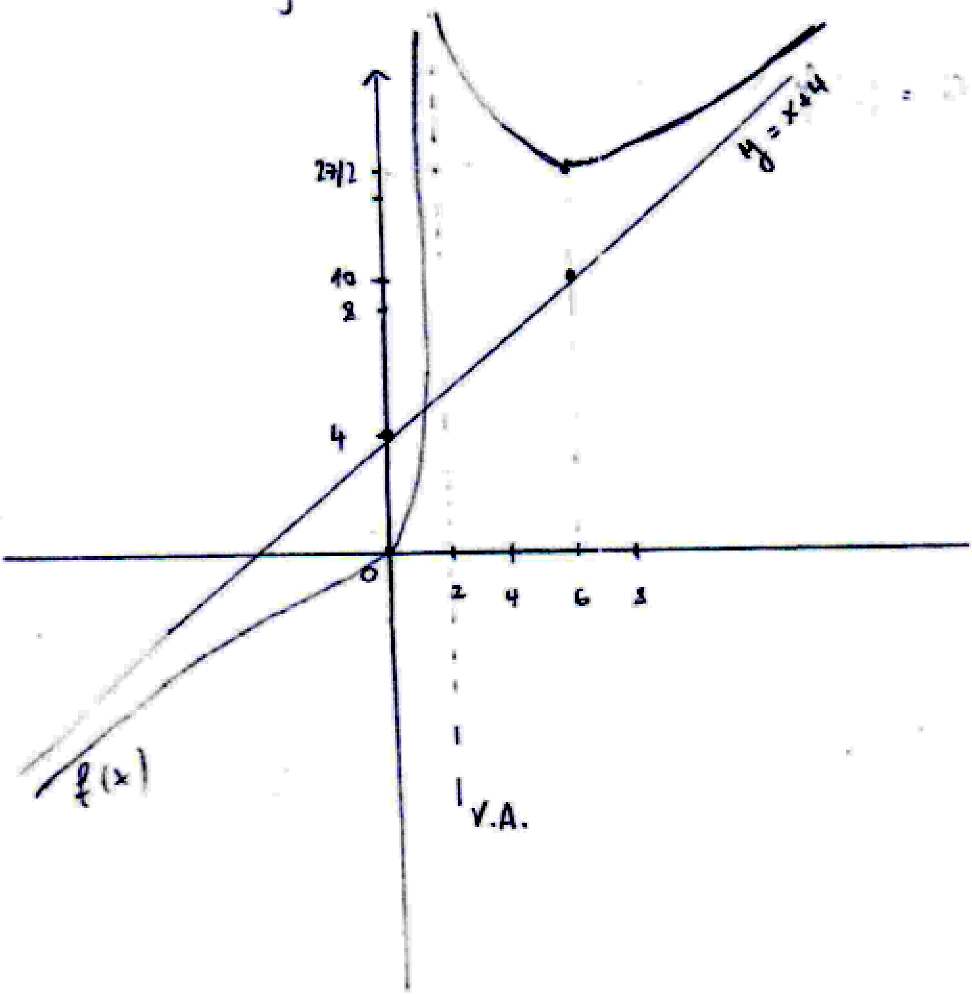
	$\langle -\infty, 0\rangle$	0	$\langle 0, 2\rangle$	2	$\langle 2, 6\rangle$	6	$\langle 6, +\infty\rangle$
f'	+	0	+	×	−	0	+
f	↗		↗	×	↘		↗
f''	−		+	×	+		+
f	$\cap$ konkavna		$\cup$ konveksna		$\cup$ konveksna		$\cup$ konveksna

← intervali monotonosti

← intervali konveksnosti/konkavnosti

U točki $\underline{x = 0}$ nije lokalni ekstrem već točka infleksije

U točki $\underline{x = 6}$ je lokalni minimum $f(6) = 13.5$, $f(0) = 0$



③ (a) Skripta, Poglavlje 11, Teorem 11.3.2

(b) $$\lim_{x \to 3} \frac{\int_3^x \sqrt{t^2+t+4}\,dt}{x^2-9} \overset{L'H}{=} \lim_{x \to 3} \frac{\sqrt{x^2+x+4}}{2x} = \frac{4}{6} = \frac{2}{3}$$

④ (a) $$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

<u>DOKAZ</u>: Deriviranjem po x

$$f(x)g'(x) = \cancel{f'(x)g(x)} + f(x)g'(x) - \cancel{f'(x)g(x)}$$

dobivamo jednakost

(b)

$$\int \operatorname{arctg}(\sqrt{x})\,dx = \left| \begin{matrix} t = \sqrt{x} \\ dt = \frac{dx}{2\sqrt{x}} = \frac{dx}{2t} \end{matrix} \right| = 2\int t \operatorname{arctg} t\, dt$$

$$= \left| \begin{matrix} u = \operatorname{arctg} t \to du = \frac{dt}{1+t^2} \\ dv = t\,dt \to v = \frac{t^2}{2} \end{matrix} \right| = 2\left(\frac{t^2}{2}\operatorname{arctg} t - \frac{1}{2}\int \frac{t^2\,dt}{1+t^2} \right)$$

$$= t^2 \operatorname{arctg} t - \int \left(1 - \frac{1}{1+t^2}\right) dt =$$

$$= t^2 \operatorname{arctg} t - t + \operatorname{arctg} t + C$$

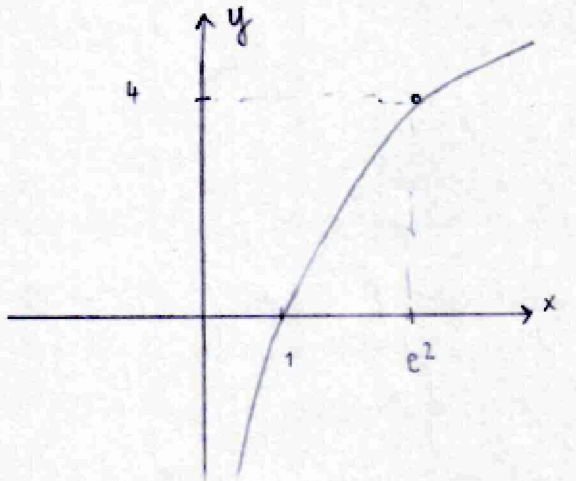
$$= (x+1)\operatorname{arctg}(\sqrt{x}) - \sqrt{x} + C$$

⑤ (a) $I = \int_{1}^{+\infty} \underbrace{\frac{\sin^2(x)}{x^2}} \, dx$

$$0 \leq \frac{\sin^2(x)}{x^2} \leq \frac{1}{x^2} \quad \text{za} \quad x \geq 1$$

Integral $\int_{1}^{+\infty} \frac{dx}{x^2}$ konvergira $\Rightarrow$ I konvergira

(b)



$$I = \lim_{\delta \to 0^+} \int_{\delta}^{e^2} \ln(x^2)\, dx = \left| \begin{array}{ll} u = \ln(x^2), & dv = dx \\ du = \frac{1}{x^2} \cdot 2x\, dx, & v = x \end{array} \right|$$

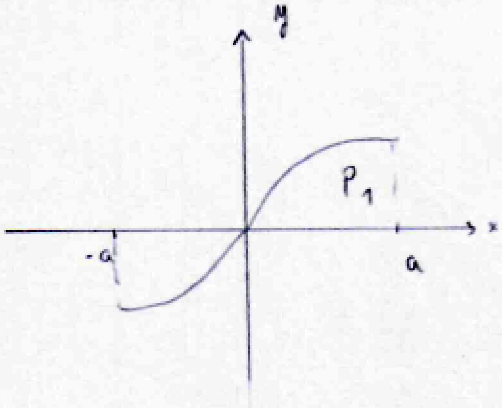
$$= \lim_{\delta \to 0^+} \left(x \ln(x^2) - \int_{\delta}^{e^2} x \cdot \frac{2}{x}\, dx \right) = \lim_{\delta \to 0^+} \left(x \ln(x^2) - 2x \right) \Big|_{\delta}^{e^2} =$$

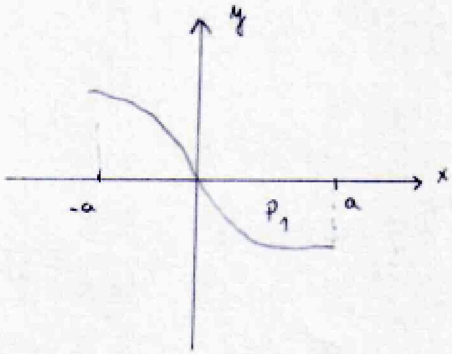
$$= \lim_{\delta \to 0^+} \left(e^2 \ln(e^4) - 2e^2 - \delta \ln(\delta^2) + 2\delta \right) =$$

$$= 4e^2 - 2e^2 - \underbrace{\lim_{\delta \to 0^+} (2\delta \ln \delta)} = \boxed{2e^2}$$

$$\lim_{\delta \to 0^+} \delta \cdot \ln \delta \underset{[0 \cdot (-\infty)]}{\overset{\uparrow}{=}} \lim_{\delta \to 0^+} \frac{\ln \delta}{\frac{1}{\delta}} = \lim_{\delta \to 0^+} \frac{\frac{1}{\delta}}{-\frac{1}{\delta^2}} = 0$$

⑥ (a)





<u>1. slučaj</u>

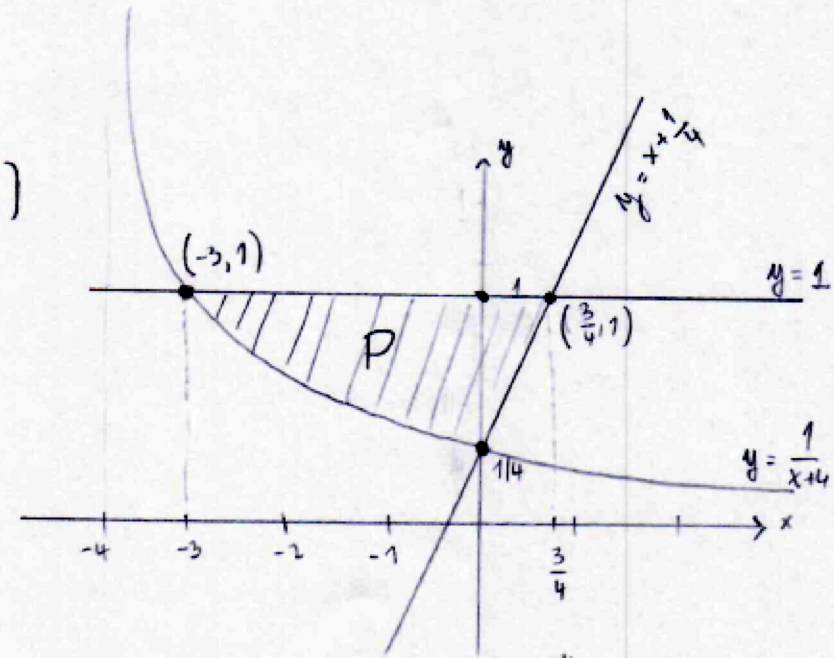
$$P = 2P_1 = 2 \cdot \int_0^a f(x)\,dx = 2\int_0^a |f(x)|\,dx$$

<u>2. slučaj</u>

$$P = 2P_1 = 2\int_0^a (0 - f(x))\,dx$$

$$= 2\int_0^a |f(x)|\,dx$$

(b)



$$P = \int_{-3}^{0} \left[1 - \frac{1}{x+4}\right] dx + \int_0^{3/4} \left[1 - \left(x + \frac{1}{4}\right)\right] dx$$

$$= \left(x - \ln(x+4)\right)\Big|_{-3}^{0} + \left(x - \frac{x^2}{2} + \frac{1}{4}x\right)\Big|_0^{3/4}$$

$$= \left[0 - \ln(4) - \left(-3 - \ln(1)\right)\right] + \left(\frac{3}{4} - \frac{1}{2}\cdot\frac{9}{16} + \frac{1}{4}\cdot\frac{3}{4}\right) =$$

$$= 1.894$$