2. tjedan

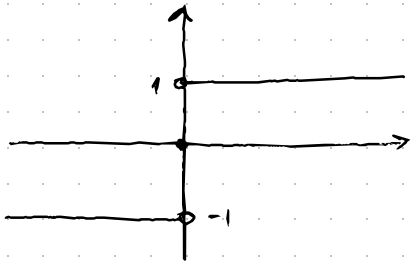
FUNKCIJE (nastavak)

Opća potencija

$$f(x) = x^c, \quad x > 0, \quad c \in \mathbb{R}$$

Funkcija signum (predznak)

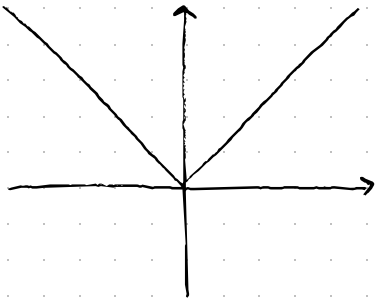
$$sgn: \mathbb{R} \to \mathbb{R}, \quad sgn(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$



Apsolutna vrijednost

$$f: \mathbb{R} \to [0, \infty\rangle$$

$$f(x) = |x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

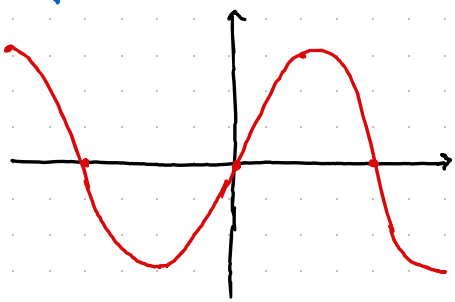


$|x - a|$ - udaljenost od x do a

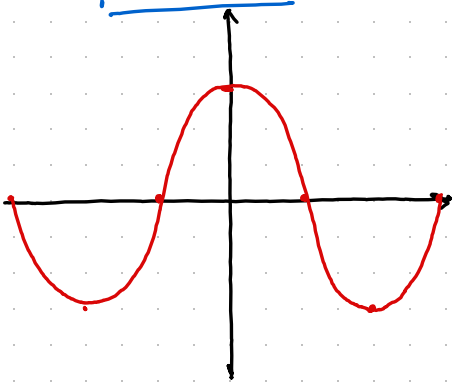
$|x - a| \leq b$

Trigonometrijske funkcije

- sinus



- kosinus



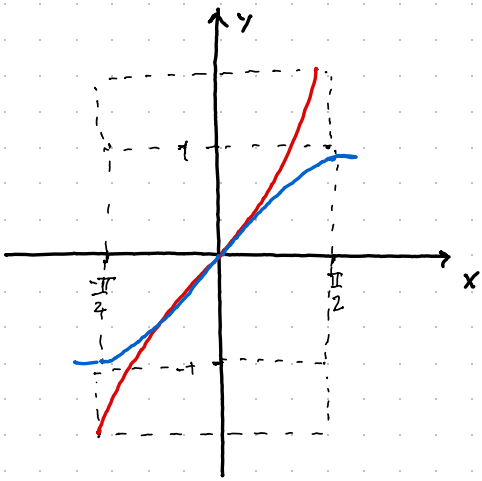
Ciklometrijske funkcije (arkus - fje)

.inverzne funkcije bijektivnih restrikcija trig. funkc.

ARKUS-SINUS

Restrikcija fja sin $\varphi: [\frac{-\pi}{2}, \frac{\pi}{2}] \to [-1,1]$, $g(x) = \sin x$ je bijekcija

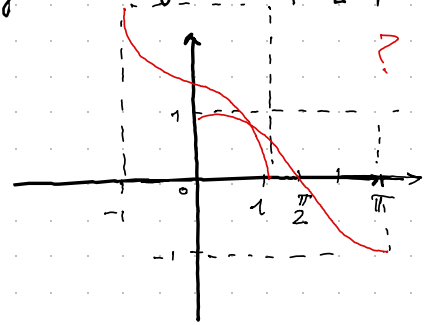
Njezin inverz je $f: [-1,1] \to [\frac{-\pi}{2}, \frac{\pi}{2}]$, $f(x) = \arcsin x$)



ARKUS KOSINUS

Restrikc. fje cos $g: [0,\pi] \to [-1,1]$, $g(x) = \cos x$ je bijekcija

Njen inverz je: $f: [-1,1] \to [0,\pi]$, $f(x) = \arccos x$

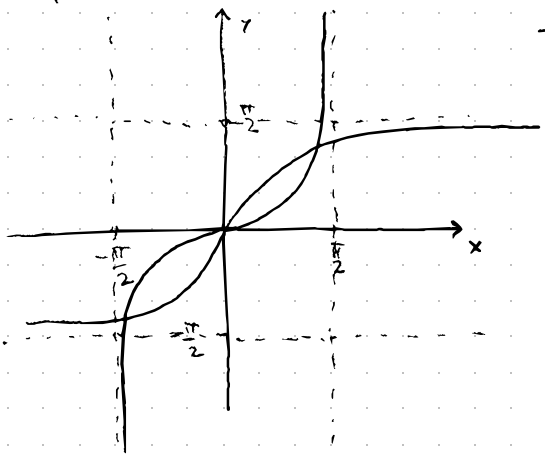


ARKUS TANGENS

Restrikc. fja tg $g: \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle \to \mathbb{R}$, $g(x) = \operatorname{tg} x$ je bijekcija

→ inverz. $f: \mathbb{R} \to \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$ $f(x) = \operatorname{arc}(\operatorname{tg} x)$

$D_f = \mathbb{R}$ $\quad H_f = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$



- neparna
- rastuća
- $\operatorname{tg}(\operatorname{arc}\operatorname{tg} x) = x$, $x \in \mathbb{R}$

$\operatorname{arc}\operatorname{tg}(\operatorname{tg} x) = y$ $\quad y = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

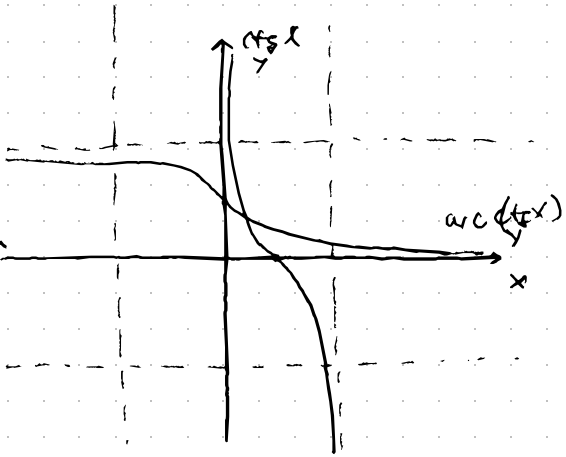
ARKUS KOTANGENS

Restrikc. f-je kotangens

$g: \langle 0, \pi \rangle \to \mathbb{R}$ je bijekcija

Inverz:

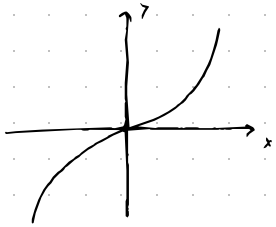
$f: \mathbb{R} \to \langle 0, \pi \rangle$, $f(x) = \operatorname{arc}(\operatorname{ctg} x)$



HIPERBOLNE (HIPERBOLIČKE) FUNKCIJE

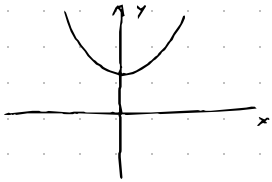
1. Sinus hiperbolni

$f: \mathbb{R} \to \mathbb{R} \qquad f(x) = \operatorname{sh} x = \frac{e^x - e^{-x}}{2}$



2. Kosinus hiperbolni

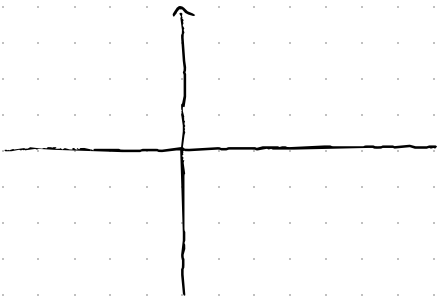
$f: \mathbb{R} \to [1, +\infty\rangle \qquad f(x) = \operatorname{ch} x = \frac{e^x + e^{-x}}{2}$



3. Tangens hiperbolni

$f: \mathbb{R} \to \langle -1, 1 \rangle$

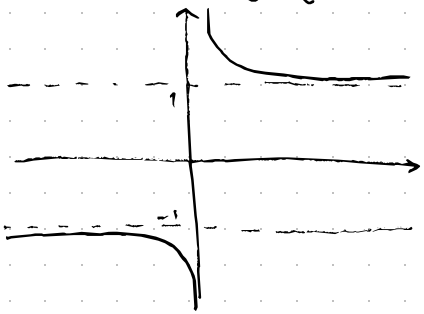
$f(x) = \operatorname{th} x = \frac{\operatorname{sh} x}{\operatorname{ch} x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$



4. Kotangens hip.

$f: \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus [-1, 1]$

$f(x) = \operatorname{cth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

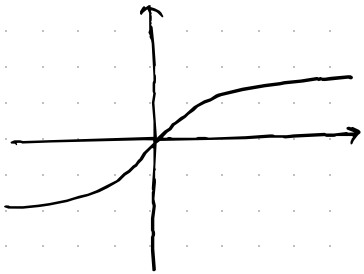


AREA FUNKCIJE

1. Area sinus

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = \operatorname{arsh} x = \ln\left(x + \sqrt{x^2+1}\right)$

Izvod formula



$y = \operatorname{sh} x$

$y = \frac{e^x - e^{-x}}{2} \quad / \cdot 2$

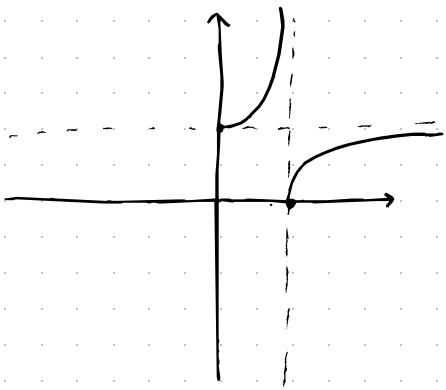
$2y = e^x - e^{-x} \quad / \cdot e^x$

$2y \cdot e^x = e^{2x} - 1$

2. Area kosinus

$f: [1, +\infty\rangle \longrightarrow [0, +\infty\rangle$

$f(x) = \operatorname{arch} x = \ln\left(x + \sqrt{x^2 - 1}\right)$



(To je inverz od restrikcije je
$g: [0, +\infty\rangle \to [1, +\infty\rangle$,
$g(x) = \operatorname{ch} x$)

IZVOD:

$y = \operatorname{ch} x$

$y = \frac{1}{2}\left(e^x + e^{-x}\right) \mid \cdot 2$

$2y = e^x + e^{-x} \mid \cdot e^x$

$2e^x \cdot y = e^{2x} + 1$ $\quad e^x = t$

$e^{2x} - 2y e^x + 1 = 0$

$t^2 - 2y \cdot t + 1 = 0$

$t_{1,2} = \frac{2y \pm \sqrt{4y^2 - 4}}{2}$

$t_{1,2} = \frac{2y \pm 2\sqrt{y^2 - 1}}{2}$

$t_{1,2} = y \pm \sqrt{y^2 - 1}$

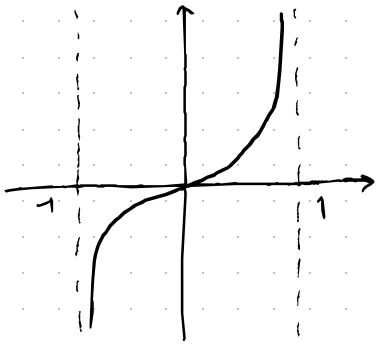
1.) $e^x = y + \sqrt{y^2 - 1}$

2.) $e^x = y - \sqrt{y^2 - 1}$

3. Area tangens

$f: \langle -1, 1 \rangle \to \mathbb{R}$

$f(x) = \operatorname{arth} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$



4. Area kotangens

$f: \underbrace{\langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle}_{\mathbb{R} \setminus [-1,1]} \Rightarrow \mathbb{R} \setminus \{0\}$

$f(x) = \operatorname{arth} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$