

# TRANSFORMACIJE NAD GRAFOVIMA FUNKCIJA

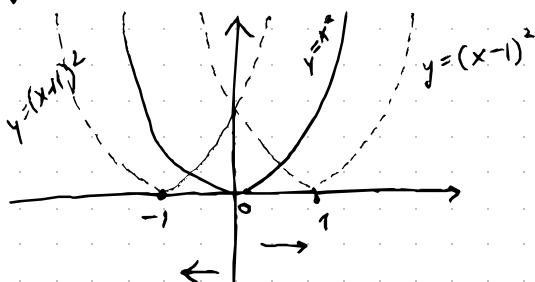
1. TRANSLACIJA
2. SKALIRANJE
3. ZRCALJENJE

## 1.1. Horizontalna translacija (u smjeru osi x)

$$y = f(x) \Rightarrow g(x) = f(x - x_0)$$

$x_0 > 0$  transl. udesno  $\rightarrow$

$x_0 < 0$  transl. uljevo  $\leftarrow$

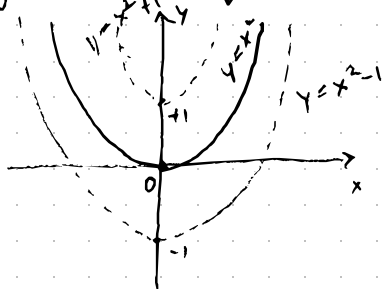


## 1.2. Vertikalna translacija (y os)

$$y = f(x) \Rightarrow g(x) = f(x) + y_0$$

$y_0 > 0$  gore  $\uparrow$

$y_0 < 0$  dolje  $\downarrow$



Pr:  $g_1(x) = e^x + 1 \uparrow y_0 = 1$

$g_2(x) = c^x - 2 \downarrow y_0 = -2$

## 2.1. Skaliranje u smjeru x osi

$$y = f(x) \rightarrow g(x) = f(kx)$$

$k > 1$  kontrakcija (sukracenje)

$0 < k < 1$  dilatacija (rastecenje)

$k = -1$  zrcaljenje u odnosu na y os

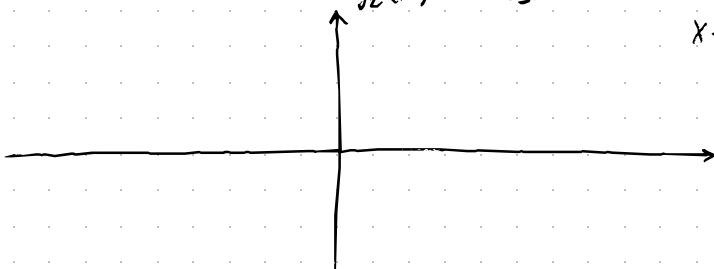
Primer:  $f(x) = \sin x$   $g_1(x) = \sin(2x)$

$g_2(x) = \sin\left(\frac{x}{3}\right)$

$$2x = k\pi / 2$$

$$x = \frac{k\pi}{2}$$

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## 2.2 SKALIRANJE U SMJERU OSI Y

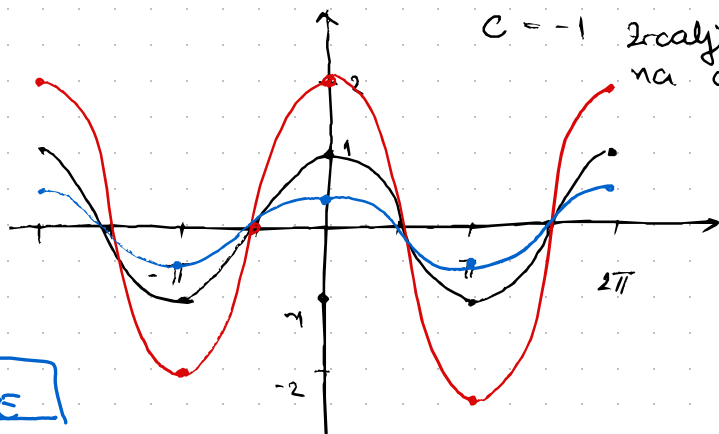
$$y = f(x) \Rightarrow g(x) = C \cdot f(x)$$

$C > 1$  "amplituda" je veća  
 $0 < C < 1$  — manja  
 $C = -1$  zrcaljenje s dr. na  $\cos x$

$$f(x) = \cos x$$

$$f_1(x) = 2\cos x$$

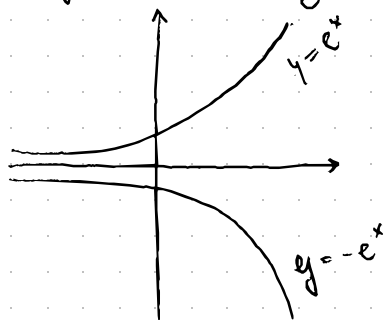
$$f_2(x) = \frac{1}{2}\cos x$$



## 3 ZRCALJENJE

### 3.1 Skaliranje s drzom na os x

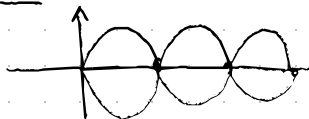
$$y = f(x) \Rightarrow g(x) = -f(x)$$



Napomena:

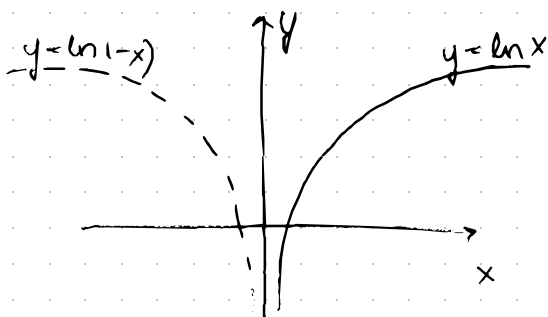
$$y = f(x) \Rightarrow g(x) = |f(x)| = \begin{cases} f(x) & f(x) \geq 0 \\ -f(x) & f(x) < 0 \end{cases}$$

Pr.  $f(x) = |\sin x|$



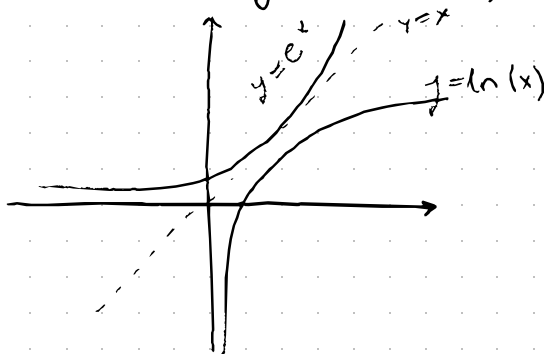
### 3.2 Zrcaljenje s drzom na os y

$$y = f(x) \Rightarrow g(x) = f(-x)$$



### 3.3. Zrcaljenje s obzirom na pravac $y=x$

$$y = f(x) \Rightarrow g(x) = f^{-1}(x)$$



**2ad.** Ako graf fije  $f(x) = 2x^2 + 1$  translacijom 2 1 udesno i 2a 2 prema dolje te dobiveni graf zrcalimo s obzirom na os  $x$ . Kako glasi formula fije  $g(x)$ , čiji je graf?

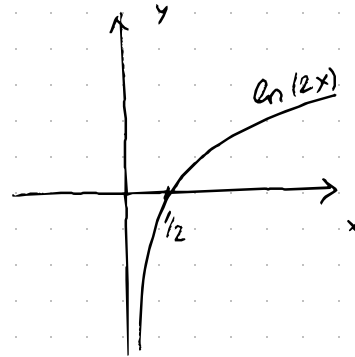
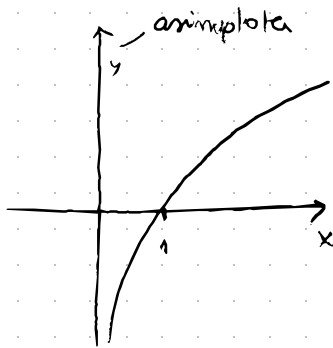
$$f(x) = 2x^2 + 1 \xrightarrow{(-1)} f(x)_1 = 2(x-1)^2 + 1 \xrightarrow{(-2)} f(x)_2 = 2(x-1)^2 - 1$$

$$f(x) = 2(x-1)^2 - 1 \xrightarrow[\text{os } x]{\text{zrcaljenje}} f(x)_3 = -f(x)_2 = -[2(x-1)^2 - 1] = -2(x-1)^2 + 1$$

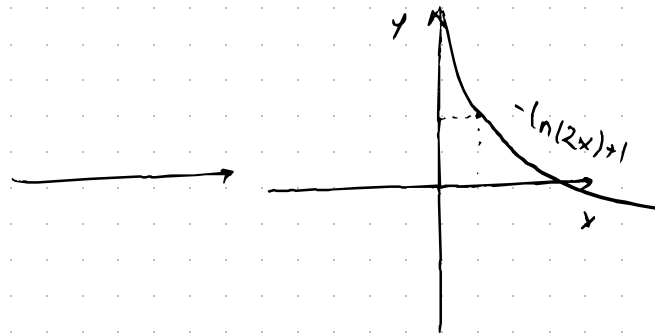
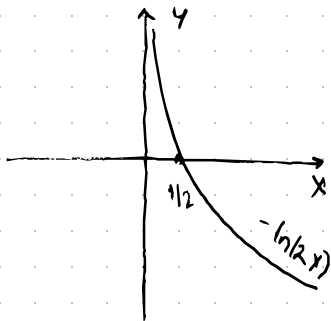
$$\boxed{g(x) = -2(x-1)^2 + 1}$$

Zad. 1 Skicirajte graf. fje  $f(x) = -\ln(2x) + 1$

$$y = \ln(x) \xrightarrow[\text{u skali } x]{\text{skaliranje}} y = \ln(2x)$$



$$y = \ln(2x) \xrightarrow[\text{na os } x]{\text{zrc. o osi}} y = -\ln(2x) \xrightarrow{x+1} f(x) = -\ln(2x) + 1$$



## Opcenita sinusoida

$$y = A \sin(\omega(x - x_0)) + B$$

vert. pomak

└─ horizont. pomak

ZAD. Skicirajte graf sinusoida  $f(x) = 3 \sin(2x - \frac{\pi}{3})$

$$f(x) = 3 \sin 2(x - \frac{\pi}{6})$$

$$A = 3$$

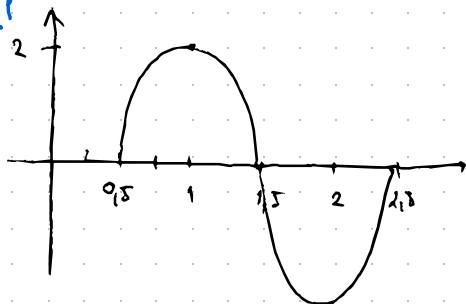
$$\omega = 2$$

$$x_0 = \frac{\pi}{6}$$

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$$T = \frac{2\pi}{\omega} = \underline{\underline{\pi}}$$

ZAD.



$$T = 2 \Rightarrow \omega = \frac{2\pi}{2} = \pi$$

$$A = 2$$

$$x_0 = \frac{1}{2}$$

$$f(x) = 2 \sin \pi(x - \frac{1}{2})$$

ZAD. Zadane su fije  $f(x) = 2 \arcsin(3x)$  i  $g(x) = 1 - e^x$

Skicirajte grafove te odredite domene i slike tih fija.

$$D_f = ?$$

$$3x \in [-1, 1] : 3$$

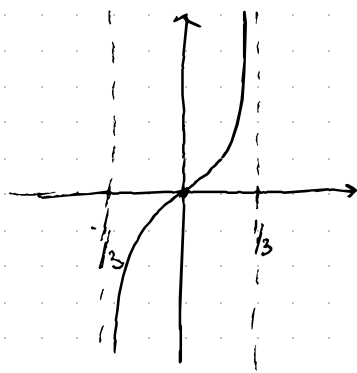
$$x \in \left[-\frac{1}{3}, \frac{1}{3}\right] = D_f$$

$$S_f = ?$$

$$3x \in [-1, 1] / \arcsin$$

$$\arcsin(3x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] / \cdot 2$$

$$2 \arcsin(3x) \in [-\pi, \pi]$$



Zad.  $y = e^x \rightarrow y = e^{-x} \rightarrow y = -e^{-x}$

D2.

1. Zadatak je fija  $f: [0, \pi) \rightarrow \mathbb{R}$ ,  $f(x) = |\sin(2x) + \frac{1}{2}|$
- Skicirajte graf od  $f$
  - Je li  $F$  injektivna?
  - Odredite skup  $y$  t.d.  $f: [0, \pi) \rightarrow y$  bude surjektivna

2. Funkcija  $f$  je zadana  $\rightarrow f(x) = A \cos(3x + b) + c$
- Odredite  $A, b, c$  ( $A, b > 0$ ) t.d.  $f: [-\frac{\pi}{6}, \frac{\pi}{6}] \rightarrow [0, 4]$  injektivna.
  - Skicirajte graf.
  - Odredite  $f^{-1}(x)$  i skicirajte graf