ZADACI

1. Odredite prirodne domene:

a) $f(x)=\sqrt{x^3-x^2-4x-6}$

b) $f(x)=\sqrt{1-|x^2-3|}$

c) $f(x)=\sqrt{2-\log_2(x^2-1)}$

d) $f(x)=\dfrac{2}{ch\,\ln(x)-x}$

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a) $D_f=?$

tražimo nultočku: djelitelji od -6

$\pm1,\ \pm2,\ \pm3,\ \pm6$

↳ uvrštavamo umjesto x

Uvjet: $x^3-x^2-4x-6\geq 0$

$(+3)^3-(+3)^2-(+3)\cdot4-6\geq0$

$\longrightarrow x=3$ je nultočka

$(x^3-x^2-4x-6):(x-3)=x^2+2x+2$

$-\ x^3+3x^2$

$0\ +2x^2$

$-\ 2x^2+6x$

$0\ +2x$

$-\ 2x+6$

0

$\underbrace{(x^2+2x+2)}_{\geq 0\ \forall x\in\mathbb{R}}(x-3)\geq0$

↳ Pozitivno kada je $\boxed{x\geq3}$

b) $D(f)=?$ $\quad f(x)=\sqrt{1-|x^2-3|}$

$1-|x^2-3|\geq0$

$|x^2-3|\leq1$

$-1\leq x^2-3\leq1$

ove dvije nejednakosti vrijede istovremeno

$\Downarrow$

PRESJEK

$-1\leq x^2-3$

$x^2-2\geq0$

$x_{1,2}=\pm\sqrt2$

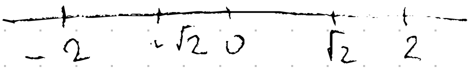
$x^2-3\leq1$

$x^2-4\leq0$

$(x-2)(x+2)\leq0$

$x_{1,2}=\pm2$

$\boxed{D_f=[-2,-\sqrt2]\cup[\sqrt2,2]}$



c) $f(x)=\sqrt{2-\log_2(x^2-1)}$

uvjet: (log) $\quad x^2-1>0$

$x^2>1$

$x\in\langle-\infty,-1\rangle\cup\langle1,\infty\rangle$

($\sqrt{\ }$) $\quad 2-\log_2(x^2-1)\geq0$

$\log_2(x^2-1)\leq2$

$x^2-1\leq4$

$x^2-5\leq0$

$x\in[-5,5]$

$\boxed{[-\sqrt{5}-1\rangle\cup\langle1,5]}$

d) $f(x)=\dfrac{2}{ch(\ln x)-x}$

1. uvjet (ln): $x>0$

2. uvjet (nazivnik): $ch(\ln x)-x\neq0$

$ch(\ln x)\neq x$

$\frac{1}{2}(e^{\ln x}+e^{-\ln x})\neq x$

$\frac{1}{2}\left(x+\frac{1}{x}\right)\neq x \quad /\cdot2$

$x+\frac{1}{x}\neq2x$

$\frac{1}{x}\neq x \quad /\cdot x$

$1\neq x^2$

$x\neq\pm1$

$\boxed{D_f=\langle0,+\infty\rangle\setminus\{1\}}$

2. Odredite $D(f)$, f^{-1} i $D(f^{-1})$ ako je f zadana s $f(x)=\ln\left(\dfrac{3x+1}{x-3}\right)$

1. uvjet: (naz) $\quad x-3\neq0$

$x\neq3$

2. uvjet (ln) = $\dfrac{3x+1}{x-3}>0$

	$-\infty$	$-1/3$	3	$+\infty$
$3x+1$	$-$ 0	$+$	$+$	
$x-3$	$-$	$-$	✕ $+$	
	$+$	$-$		

$\boxed{D(f)\in\langle-\infty,-1/3\rangle\cup\langle3,+\infty\rangle}$

* $Im(f)$ raspisati tako da se riješimo x u brojniku

inverz f^{-1}

$y=\ln\left(\dfrac{3x+1}{x-3}\right)$

$\dfrac{3x+1}{x-3}=e^y \quad /\cdot(x-3)$

$3x+1=e^y\cdot(x-3)$

$3x+1=e^y\cdot x-3e^y$

$x\cdot e^y-3x=3e^y+1$

$x(e^y-3)=3e^y+1$

$x=\dfrac{3e^y+1}{e^y-3}$

* $e^y-3\neq0$

$e^y\neq3$

$y\neq\ln3$

$\boxed{f^{-1}(y)=\dfrac{3e^y+1}{e^y-3}}$

$\boxed{D(f^{-1})=\mathbb{R}\setminus\{\ln3\}}$

$\dfrac{3x+1}{x-3}=\dfrac{3(x-3)+10}{x-3}=3+\dfrac{10}{x-3}$, $x\neq3$

s obzirom da je ln onda $\ln3\neq0$ je slika (isto kao $D(f^{-1})$)

③ $f(x) = \sqrt{3\arccos x - \pi}$

a) D_f i Im_f =?

b) Pokažite da je f stoga padajuća

c) Ako postoji, odredite f^{-1} za f-ja $f: D(f) \to Im(f)$

a) $D(f)$ = ?

1. uvjet (arc cos) = $x = [-1, 1]$

2. uvjet ($\sqrt{\ }$) = $3\arccos x - \pi \geq 0$

djelujemo s inverznom funkcijom

$\arccos x \geq \frac{\pi}{3}$ | cos | $[0, \pi]$ ↓ padajuća

$\cos|_{[0,\pi]}(\arccos x) \leq \cos \frac{\pi}{3}$

$\boxed{x \leq \frac{1}{2}}$ $D_f = [-1, \frac{1}{2}]$

Im_f = ?

$\frac{1}{2} \geq x \geq -1$ | arc cos ↓

$\arccos \frac{1}{2} \leq \arccos x \leq \arccos(-1)$

$\frac{\pi}{3} \leq \arccos x \leq \pi$ / · 3

$\pi \leq 3\arccos x \leq 3\pi$ / $-\pi$

$0 \leq 3\arccos x - \pi \leq 2\pi$ / $\sqrt{\ }$

$0 \leq \sqrt{3\arccos x - \pi} \leq \sqrt{2\pi}$

dobili smo to isto

$\boxed{Im_f = [0, \sqrt{2\pi}]}$

$f(x)=\sqrt{3\arccos x-\pi}$

b) $f=f_3\circ f_2\circ f_1$

$f_1(x)=\arccos x \searrow$

$f_2(x)=3x-\pi \nearrow$

$f_3(x)=\sqrt{x} \nearrow$

$f_2\circ f_1 \searrow$

$f_3\circ f_2\circ f_1 \searrow$

c) $f: D_f \longrightarrow Im(f)$ — ovako zapisano znači da je cijela kodomena slika

f bijekcija

surj: je jer je cijela kodomena slika

injekt. je jer je kompozicija $f_3\circ f_2\circ f_1$ + svo su f-je bijekcije (f_3, f_2, f_1)

inverz (f^{-1}):

$$y=\sqrt{\arccos x-\pi} \quad /^2$$

$$y^2=3\arccos x-\pi$$

$$y^2+\pi=3\arccos x \quad /:3$$

$$\frac{y^2+\pi}{3}=\arccos x \quad \Big| \cos \quad [0,\pi]$$

$$x=\cos\left(\frac{y^2+\pi}{3}\right)$$

$$f^{-1}=\cos\left(\frac{y^2+\pi}{3}\right)$$

Zad 4.) $f(x) = 3\arcsin(2x) + \pi$

a) Dokažite da je komp. dvije strogo rast. fija, strogo rast f.

b) Odredite D_f, $\operatorname{Im} f$ i skicirajte graf f.

c) Ima li $f: D(f) \to \operatorname{Im}(f)$ inverzna fja? Obrazložite.
Ako ima odredite f^{-1}, $D(f^{-1})$ te skicirajte f^{-1}.

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a) $f \circ g$ strogo rastuća : $x_1 > x_2 \Rightarrow (f \circ g)(x_1) > (f \circ g)(x_2)$

$$x_1 > x_2 \ / \ g \nearrow$$

$$g(x_1) > g(x_2) \nearrow \longrightarrow f(g(x_1)) > f(g(x_2))$$

b) $D_f = ?$

uvjet (arc sin x): $2x \in [-1, 1] \ /:2$

$$\boxed{x \in \left[-\frac{1}{2}, \frac{1}{2}\right] \Rightarrow D_f}$$

$\operatorname{Im}(f) = ?$

$$-\frac{1}{2} \leq x \leq \frac{1}{2} \ /\cdot 2$$

$$-1 \leq 2x \leq 1 \ | \arcsin \nearrow$$

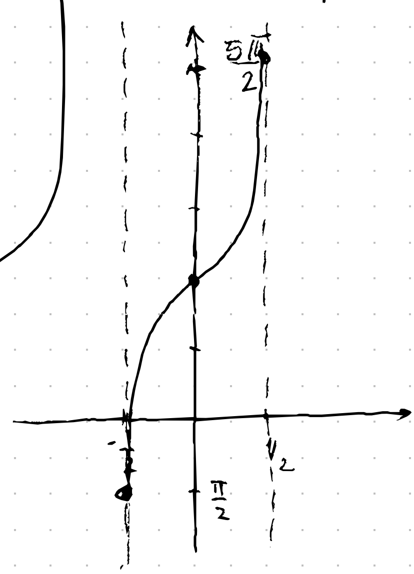
$$\arcsin(-1) \leq \arcsin(2x) \leq \arcsin(1)$$

$$\frac{3\pi}{2} \leq \arcsin(2x) \leq \frac{\pi}{2} \ /\cdot 3$$

$$-\frac{3\pi}{2} \leq 3\arcsin(2x) \leq \frac{3\pi}{2} \ /+\pi$$

$$-\frac{\pi}{2} \leq \underbrace{3\arcsin(2x) + \pi}_{f(x)} \leq \frac{5\pi}{2}$$

$$\boxed{\operatorname{Im}_f = \left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]}$$



c) $f: D_f \to Im(f)$ ima inverz (već nam kaže u zad.)

$\longrightarrow$ surjekcija

injekc. iz grafa se vidi (b) zad

$f^{-1} = ?$

$y = 3\arcsin(2\pi) + \pi$

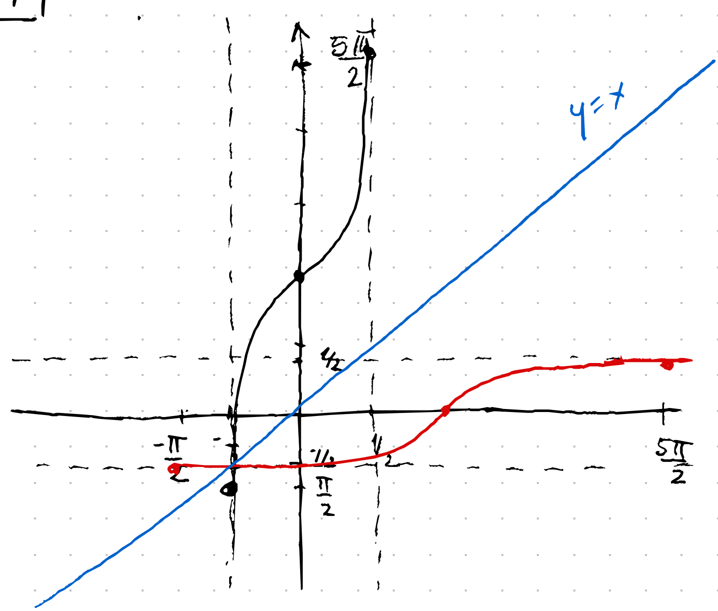
$$\arcsin(2x) = \frac{y-\pi}{3} \quad / \sin \quad \Big| \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$2x = \sin\left(\frac{y-\pi}{3}\right) \quad /:2$$

$$x = \frac{1}{2}\sin\left(\frac{y-\pi}{3}\right)$$

$$\boxed{f^{-1}(y) = \frac{1}{2}\sin\left(\frac{y-\pi}{3}\right)}$$

$$\boxed{D_{f^{-1}} = Im_f = \left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]}$$



Zad)

a) $\sin(2\arcsin x)=x$

b) $4\sin(\ln x)=x$

a) Domena

① $x\in[-1,1]$ (D arc sin)

② $x\in[-1,1]$ (Im(sin)

$\sin(2\arcsin x)=x$

$\underbrace{2\sin(\arcsin x)}\cdot\cos(\arcsin x)=x$

$2x\cos(\arcsin x)=x$

$x(2\cos(\arcsin x)-1)=0$

$x=0 \rightarrow x\in[-1,1]$ ✓

$2\cos(\arcsin x)-1=0$

$\cos(\arcsin x)=\frac{1}{2}$

$\arcsin x=\frac{\pi}{3} \quad / \sin|_{[-1,1]}$

$x=\sin\left(\frac{\pi}{3}\right)$

$\underline{\underline{x_1=\frac{\sqrt{3}}{2}}}$ $\quad$ $\underline{\underline{x_2=\frac{-\sqrt{3}}{2}}}$

$\in[-1,1]$ ✓

$4\sin(\ln x)=x$

b) Domena ;

$\boxed{x>0}$

pitat Bračić

šta nije da sin nije injekcija

jer

$\sin(x_1)=\sin x_2$

može bit različit x