

Joker Games for checking History-Determinism

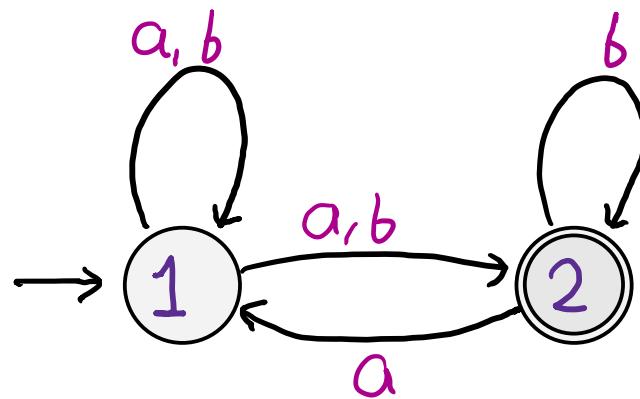
Aditya Prakash

University of Warwick, UK

Ongoing work with Udi Boker, Marcin Jurdziński, Karoliina Lehtinen

Running Example

Büchi Automata

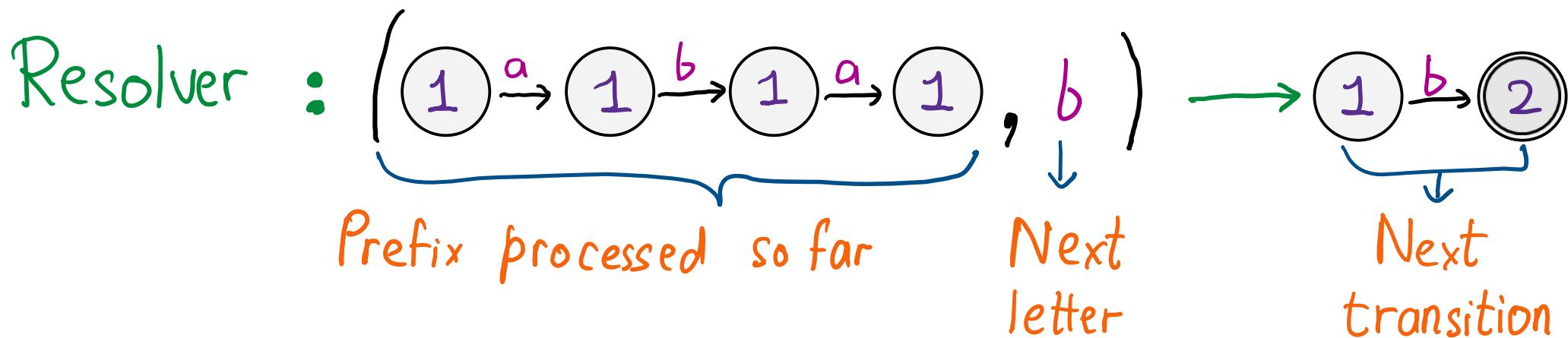


Language : Words that see b only often.

History-Deterministic Automata

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A non-deterministic automaton is history-deterministic if non-deterministic choices can be resolved 'on-the-fly' while processing the input.

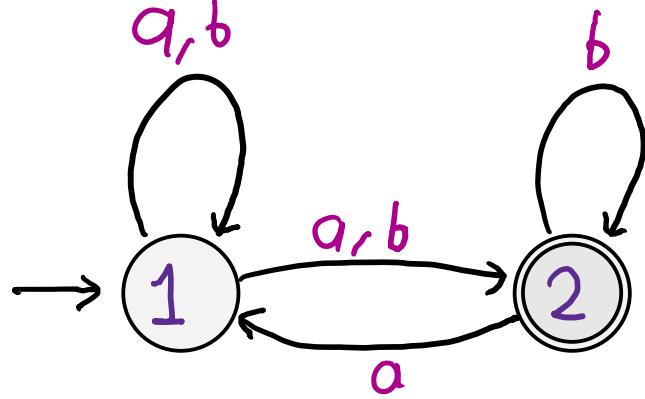


History-Determinism Game

Starts at $\rightarrow 1$

Adam selects letter a_i

Eve selects transition $q_i \xrightarrow{a_i} q_{i+1}$

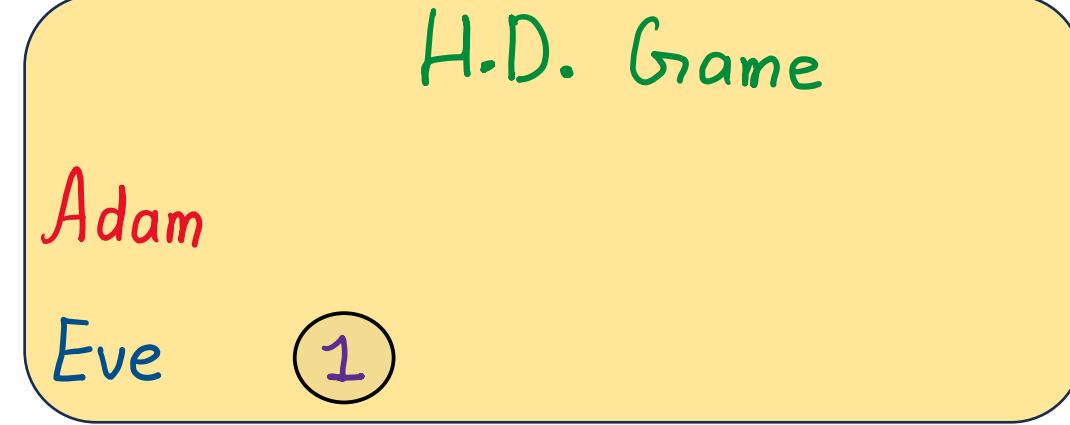
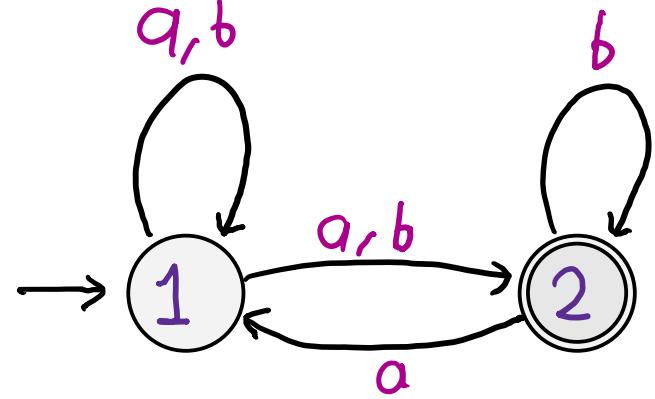


History-Determinism Game

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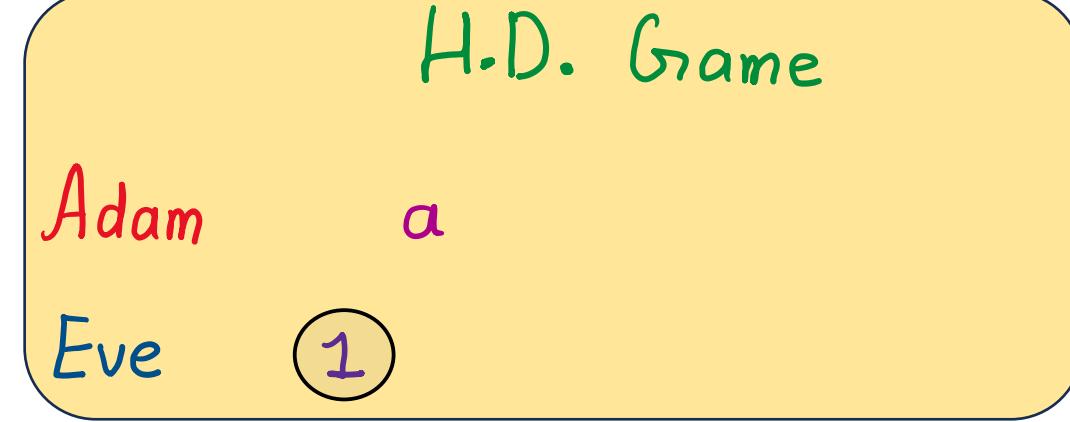
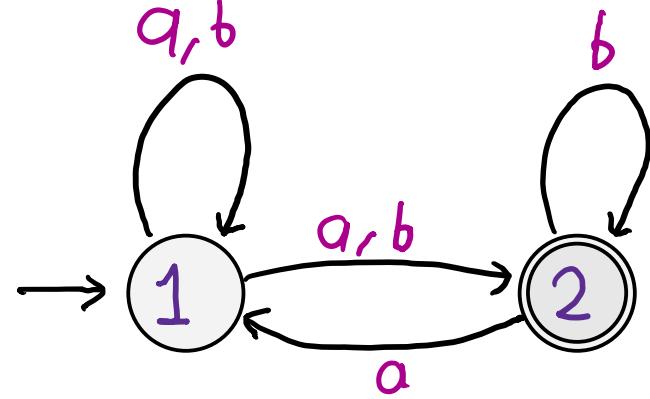


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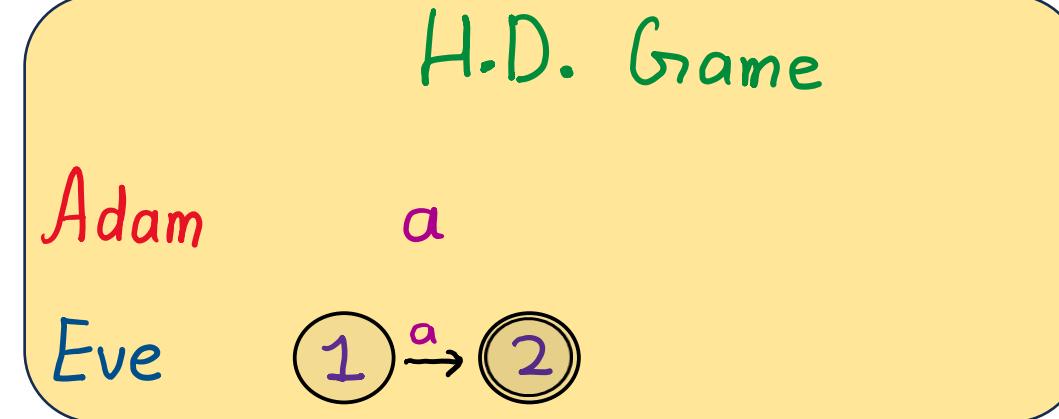
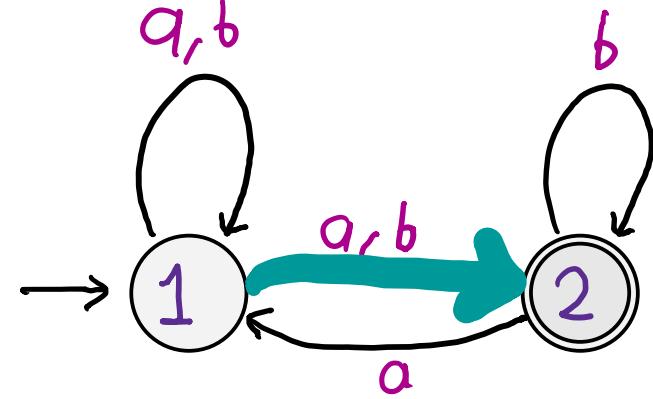


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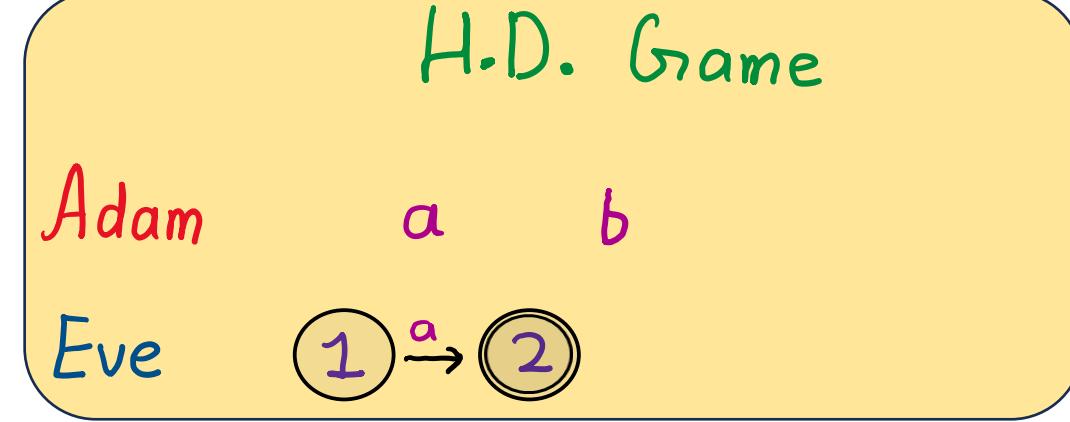
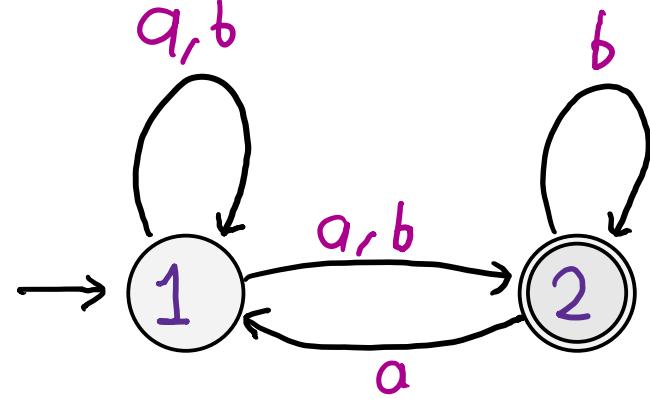


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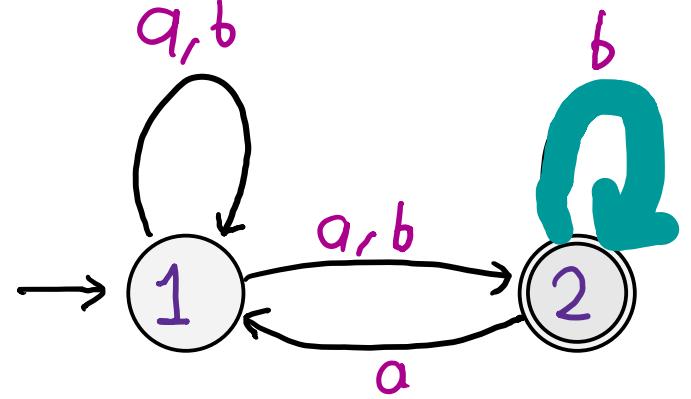


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H.D. Game

Adam

a b

Eve

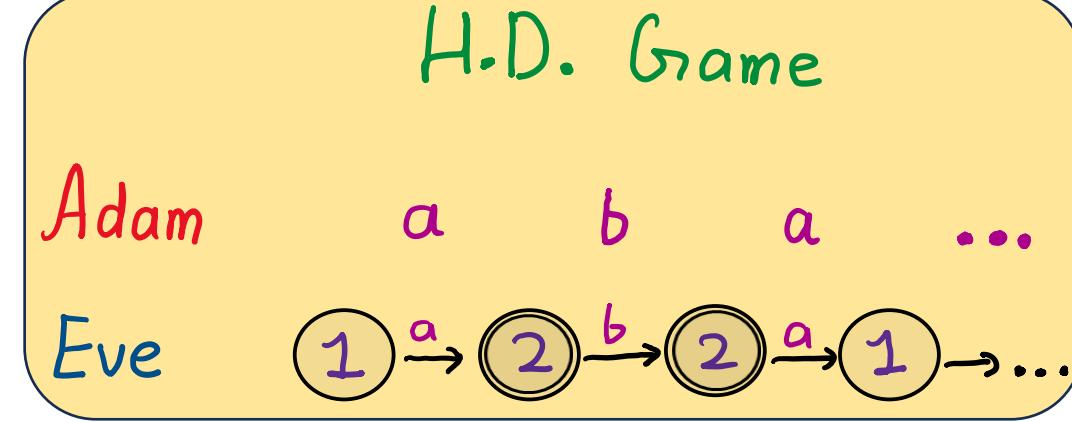
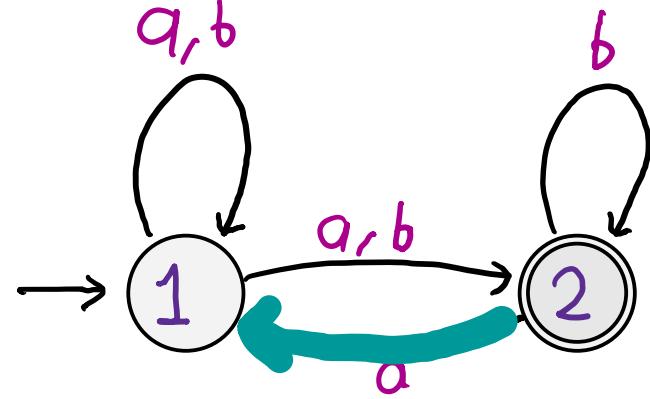


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History-Determinism Game

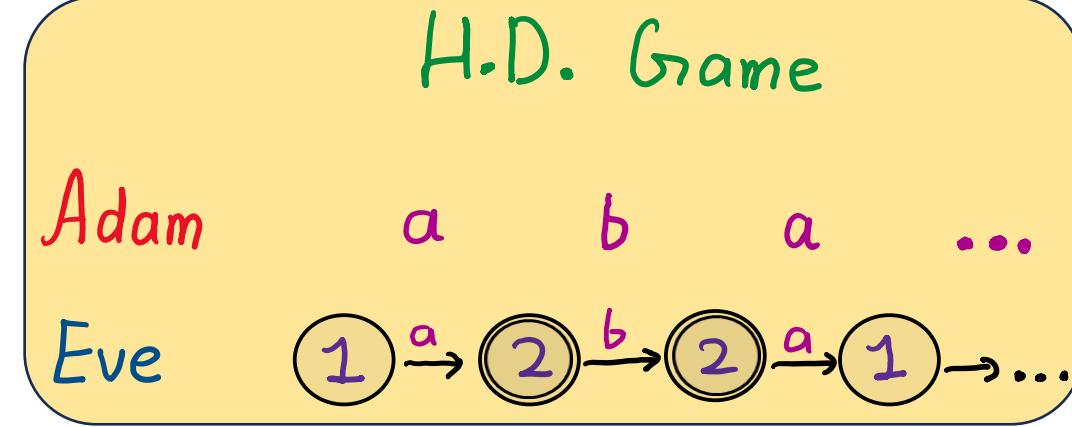
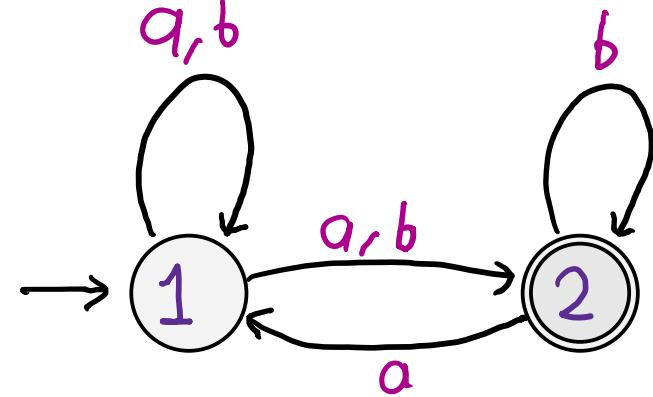
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Construct an accepting run whenever Adam's word is accepting.



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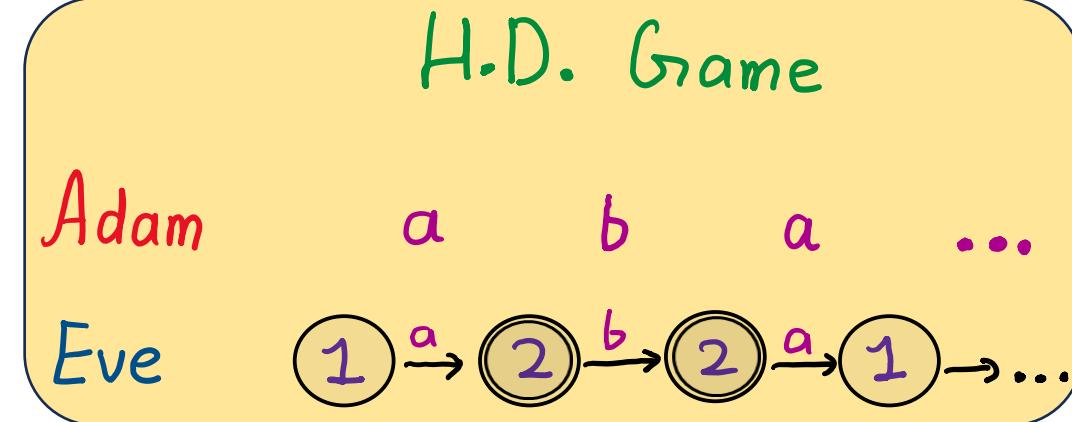
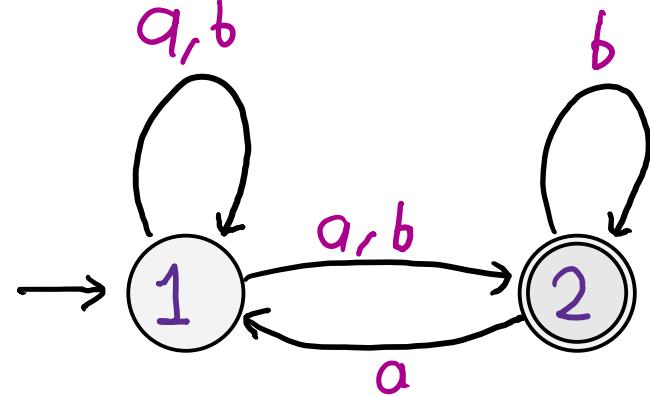
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HD Automata: Eve has a winning strategy



Determinisation of H.D. Büchi Automata

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Theorem [Kuperberg, Skrzypczak'15]

H.D. Büchi automaton with N states



Deterministic Büchi automaton with $\Theta(N^2)$ states

Determinisation of H.D. Büchi Automata

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H.D. Büchi automaton with N states

 ↓
Deterministic Büchi automaton with $\Theta(N^2)$ states

Requires non-deterministic polynomial time

Problem: Can H.D. Büchi automata be determinised in polynomial time?

Determinisation of H.D. Büchi Automata

Theorem [Kuperberg, Skrzypczak'15]

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Deterministic Büchi automaton with $\Theta(N^2)$ states

Requires non-deterministic polynomial time

Problem: Can H.D. Büchi automata be determinised in polynomial time?

Theorem: Yes

Complexity of checking History-Determinism

History-Determinism Game

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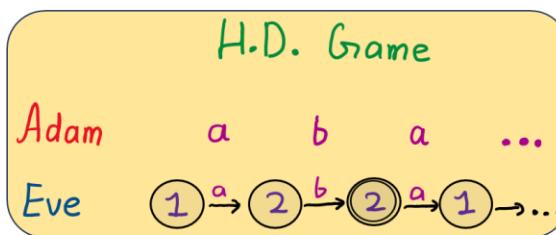
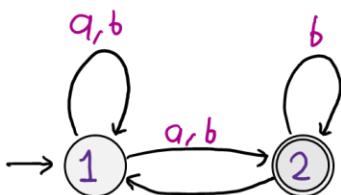
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Solving HD game: EXPTIME

[Henzinger, Piterman '06]

Complexity of checking History-Determinism

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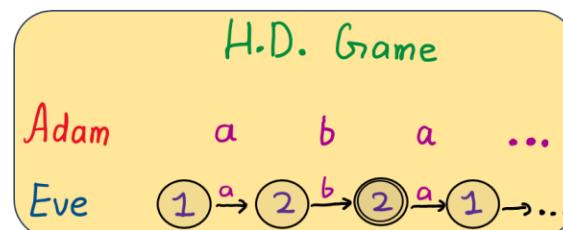
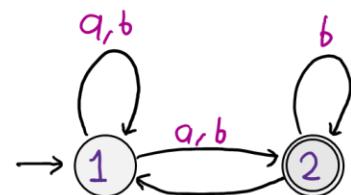
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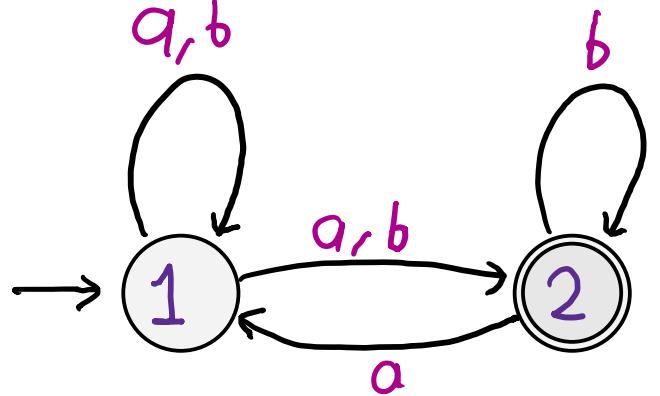


Solving HD game: EXPTIME

[Henzinger, Piterman '06]

Determinisation is expensive!

2-Token Games



2-Token Games

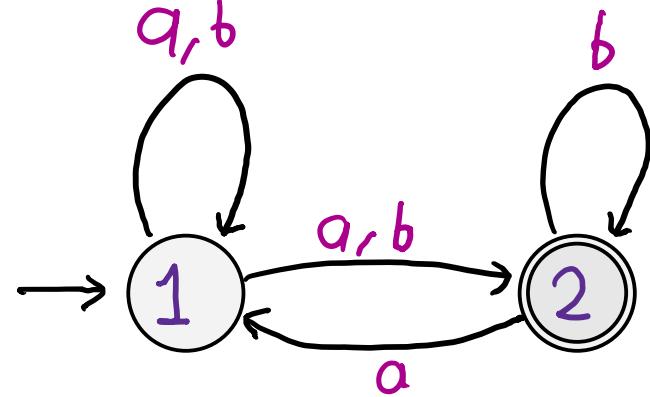
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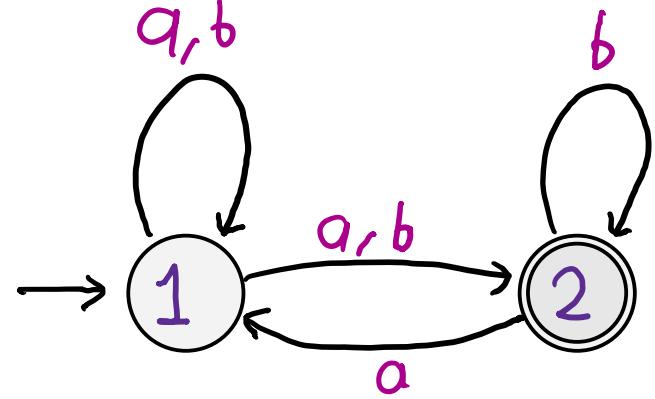
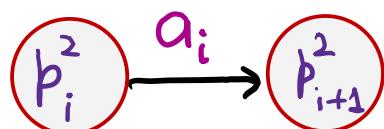
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2-Token Game

Adam

Eve

1

Adam

1

Adam

1

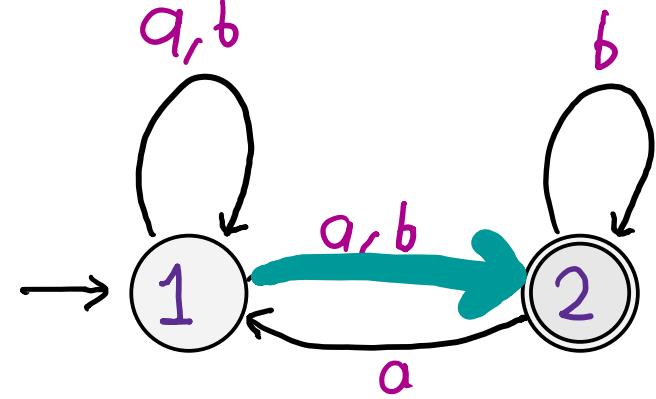
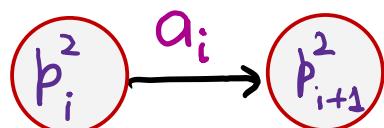
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2-Token Game

Adam a

Eve $1 \xrightarrow{a} 2$

Adam 1

Adam 1

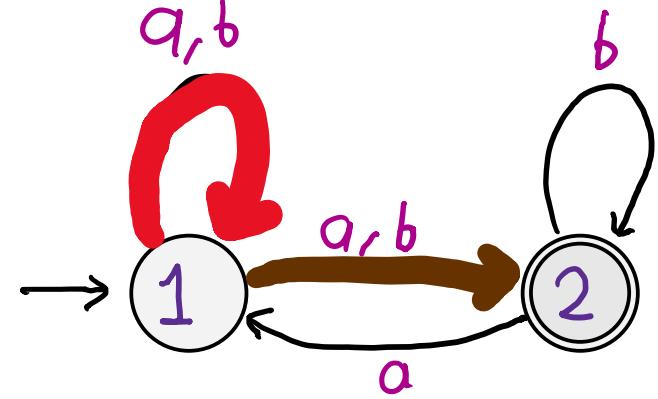
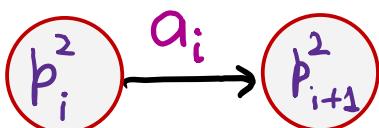
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2-Token Game

Adam a

Eve $1 \xrightarrow{a} 2$

Adam $1 \xrightarrow{a} 2$

Adam $1 \xrightarrow{a} 1$

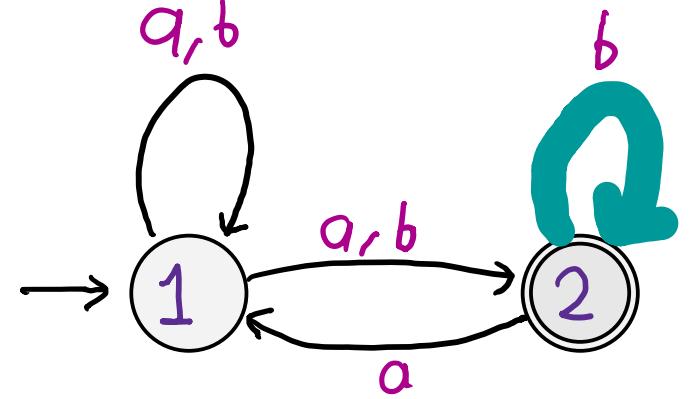
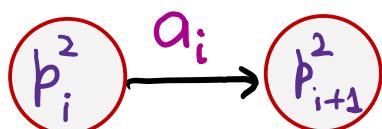
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2-Token Game

Adam

a b

Eve



Adam



Adam



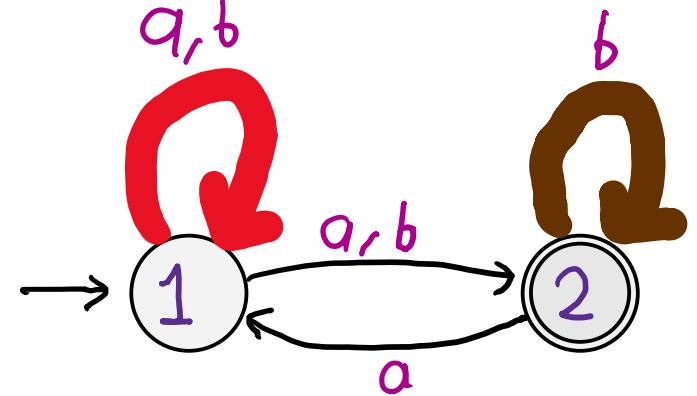
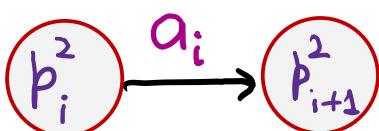
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2-Token Game

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Eve



Adam



Adam



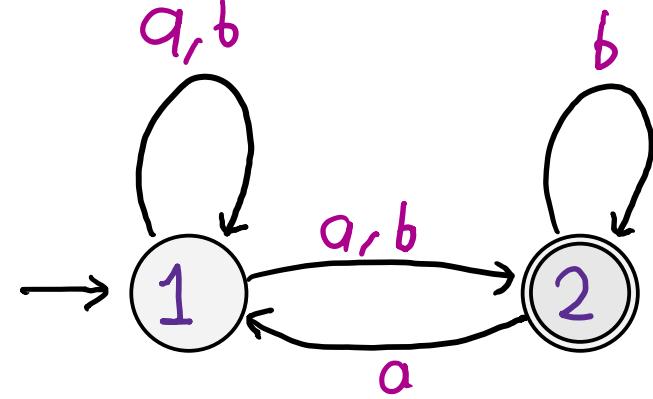
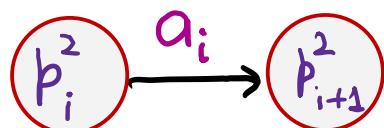
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2-Token Game

Adam

a b a ...

Eve

$1 \xrightarrow{a} 2 \xrightarrow{b} 2 \xrightarrow{a} 1 \rightarrow \dots$

Adam

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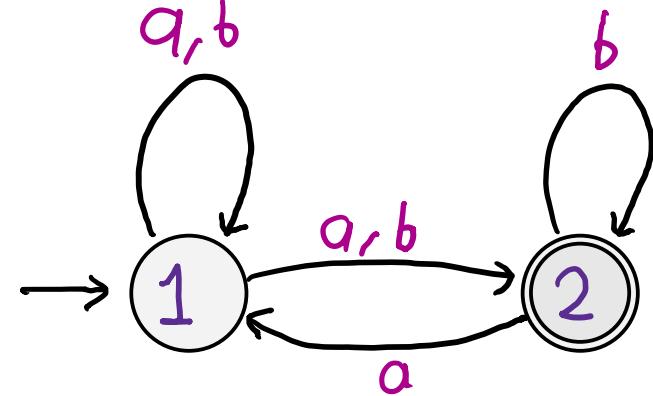
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Winning cond^{n.} for Eve: Construct an accepting run if one of Adam's run is accepting.



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2-Token Games vs. HD Game

2-Token Games

Starts at $\rightarrow \textcircled{1}, \rightarrow \textcircled{1}, \rightarrow \textcircled{1}$

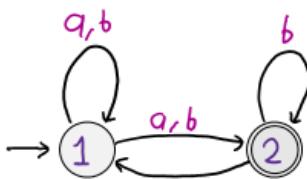
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2-Token Game	
Adam	a b a ...
Eve	1 \xrightarrow{a} 2 \xrightarrow{b} 2 \xrightarrow{a} 1 $\rightarrow \dots$
Adam	1 \xrightarrow{a} 2 \xrightarrow{b} 2 \xrightarrow{a} 1 $\rightarrow \dots$
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History-Determinism Game

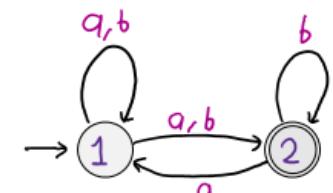
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Eve selects transition $q_i \xrightarrow{a_i} q_{i+1}$

Winning condⁿ for Eve:

Construct an accepting run whenever Adam's word is accepting.



H.D. Game	
Adam	a b a ...
Eve	1 \xrightarrow{a} 2 \xrightarrow{b} 2 \xrightarrow{a} 1 $\rightarrow \dots$

HD Automata: Eve has a winning strategy

2-Token Games vs. HD Game

2-Token Games

Starts at $\rightarrow \textcircled{1}, \rightarrow \textcircled{1}, \rightarrow \textcircled{1}$

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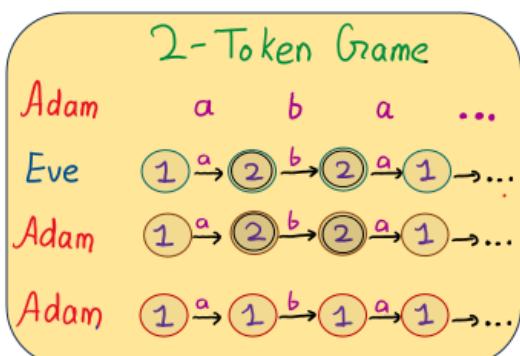
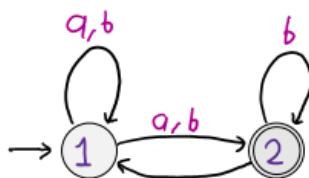
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History-Determinism Game

Starts at $\rightarrow \textcircled{1}$

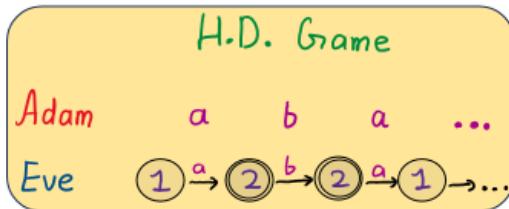
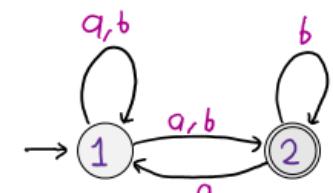
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2-Token Games vs. HD Game

2-Token Games

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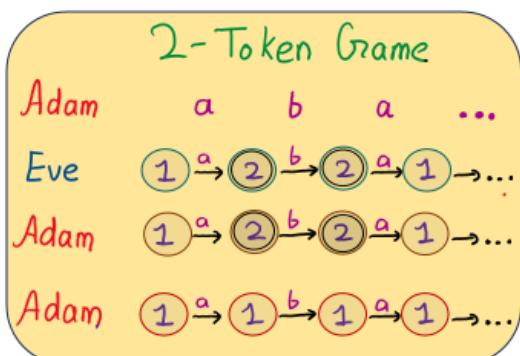
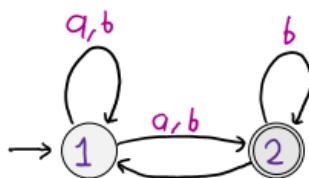
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Winning condⁿ for Eve: Construct an

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History-Determinism Game

Starts at $\rightarrow 1$

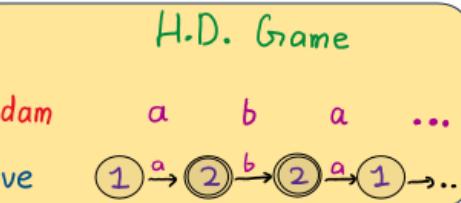
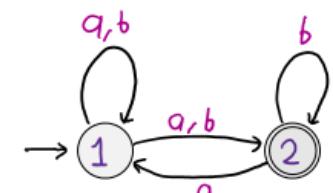
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HD Automata: Eve has a winning strategy



Eve wins H.D. game \Rightarrow Eve wins 2-token game

Token Games for checking History-Determinism

Lemma [Bagnol, Kuperberg '18]

Eve wins 2-token game \iff Eve wins k-token game for all $k \geq 0$.

Token Games for checking History-Determinism

Lemma [Bagnol, Kuþerberg '18]

Eve wins 2-token game \iff Eve wins k-token game for all $k \geq 0$.

Theorem [Bagnol, Kuþerberg'18]

For all Büchi automata A

Eve wins 2-token game on A \iff A is H.D.

Token Games for checking History-Determinism

Theorem [Boker, Kuperberg, Lehtinen, Skrzypczak '20]

For all co-Büchi automata A ,

Eve wins 2-token game on $A \Leftrightarrow A$ is H.D.

Token Games for checking History-Determinism

Theorem [Boker, Kuperberg, Lehtinen, Skrzypczak '18]

For all co-Büchi automata A ,

Eve wins 2-token game on $A \Leftrightarrow A$ is H.D.

2-Token Conjecture: For all parity automata A ,

Eve wins 2-token game on $A \Leftrightarrow A$ is H.D.

Token Games for checking History-Determinism

Theorem

Token Games for checking History-Determinism

Theorem

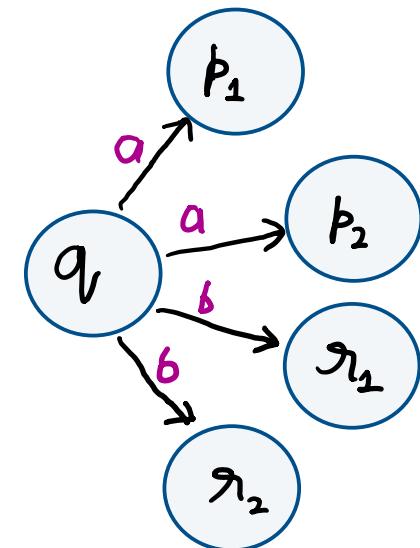
On a semantically deterministic Büchi automaton A ,
Eve wins 1-token game on $A \Leftrightarrow A$ is H.D.

Token Games for checking History-Determinism

Theorem

On a semantically deterministic Büchi automaton A ,
Eve wins 1-token game on $A \Leftrightarrow A$ is H.D.

Semantic determinism:



$$L(p_i) = a^{-1} \cdot L(q)$$

$$L(r_i) = b^{-1} \cdot L(q)$$

Token Games for checking History-Determinism

Theorem

On a semantically deterministic Büchi automaton A ,
Eve wins 1-token game on $A \Leftrightarrow A$ is H.D.

Corollary , also [Bagnol and Kuperberg' 18]

For all Büchi automata A
Eve wins 2-token game on $A \Leftrightarrow A$ is H. D.

Token Games for checking History-Determinism

One-Token Conjecture :

On a semantically deterministic parity automaton A ,
Eve wins 1-token game on $A \Leftrightarrow A$ is H.D.

Token Games for checking History-Determinism

One-Token Conjecture :

On a semantically deterministic parity automaton A ,

Eve wins 1-token game on $A \Leftrightarrow A$ is H.D.

Observation : One-token conjecture \Rightarrow Two-token conjecture

Conclusion

*

	Büchi	co-Büchi	Parity
1-token conjecture	[To appear]	Open	Open
2-token conjecture	[BK'18]	[BKLS'20]	Open

Conclusion

*

Büchi

co-Büchi

Parity

1-token conjecture

[To appear]

Open

Open

2-token conjecture

[BK'18]

[BKLS'20]

Open

*

For A Büchi, Eve wins Joker Game $\Leftrightarrow A$ is H.D.

Conclusion

*

	Büchi	co-Büchi	Parity
1-token conjecture	[To appear]	Open	Open
2-token conjecture	[BK'18]	[BKLS'20]	Open

- *
- For A Büchi, Eve wins **Joker Game** $\Leftrightarrow A$ is H.D.
- *
- H.D. Büchi automaton $\xrightarrow[\text{Poly. time}]{}$ Deterministic Büchi automaton
 N states $\leq N^2$ states

Conclusion

*

	Büchi	co-Büchi	Parity
1-token conjecture	[To appear]	Open	Open
2-token conjecture	[BK'18]	[BKLS'20]	Open

*

For A Büchi, Eve wins **Joker Game** $\Leftrightarrow A$ is H.D.

*

H.D. Büchi automaton $\xrightarrow[\text{Poly. time}]{}$ Deterministic Büchi automaton
 N states $\leq N^2$ states

Open: Can we do better than N^2 ?

Summary

Henzinger,
Piterman, 2006

→ 2-token conjecture for H.D. Büchi

→ Determinisation of H.D. Büchi in P
→ 1-token conjecture for H.D. Büchi

Bagnol,
Kuperberg, 2018

Kuperberg,
Skrzybczak, 2015

→ Determinisation of HD Büchi in NP
→ Deciding HDness of co-Büchi in P

Boker, Jurdziński,
Lehtinen & me!, to appear

Boker, Kuperberg,
Lehtinen, Skrzypczak, 2020

Bonus

Slides

Joker Games

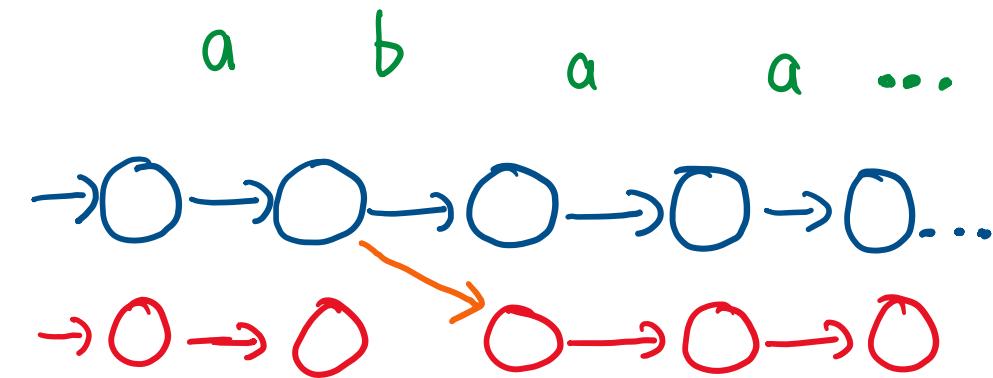
Starts at $\rightarrow q_0 \rightarrow q_{j_0}$

1. Adam selects letter a_i

2. Eve selects transition $q_i \xrightarrow{a_i} q_{i+1}$

3. Adam selects transition $p_i \quad p_{i+1}$

and selects transition $q_i \quad p_{i+1}$



, or plays **Joker**

Winning Condition for Eve : Eve constructs an accepting run if Adam's 'run' is accepting and Adam has played finite Jokers.

1-Joker Games

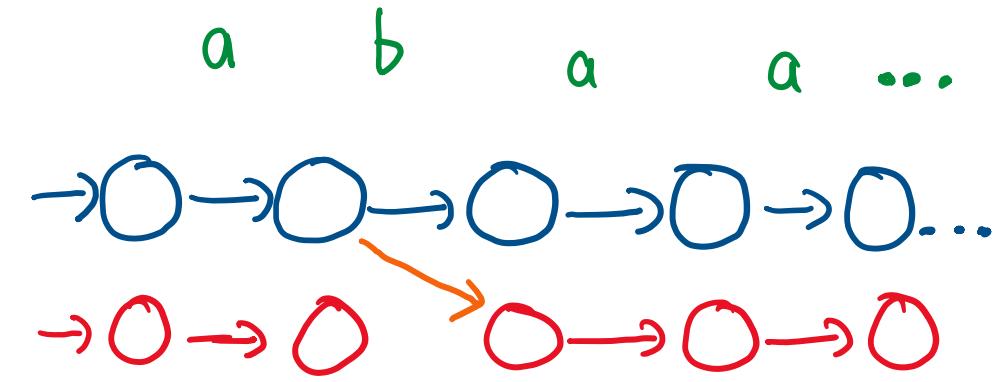
Starts at $q_0 \rightarrow q_0$

1. Adam selects letter a_i

2. Eve selects transition $q_i \xrightarrow{a_i} q_{i+1}$

3. Adam selects transition $p_i \rightarrow p_{i+1}$, or plays Joker

and selects transition $q_i \rightarrow p_{i+1}$



Winning Condition for Eve: Eve constructs an accepting run if Adam's 'run' is accepting and Adam has played ≤ 1 Joker.

For a Büchi Automaton:

- Running time for solving 1-Joker game: $\Theta(n^3 m |\Sigma|^2)$
- Running time for solving 2-Token game: $\Theta(n^4 m^2 |\Sigma|^2)$

1 - Token Conjecture



1 - Joker Conjecture



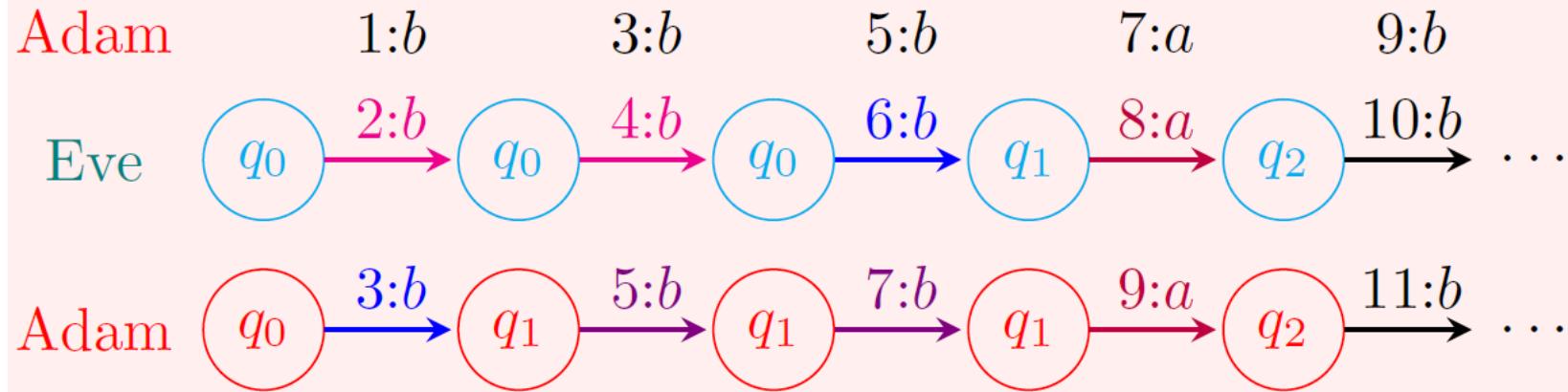
Joker Conjecture



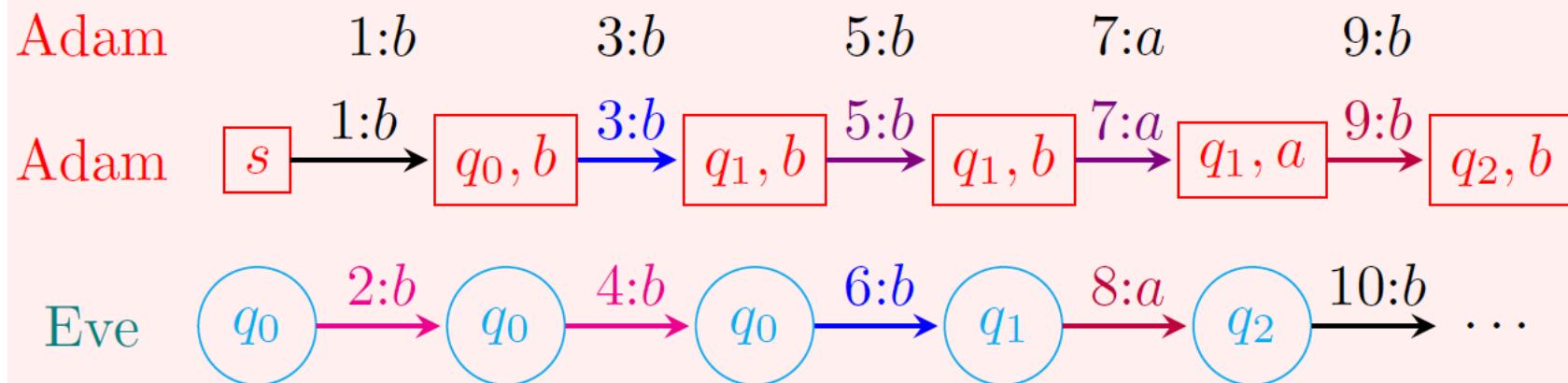
2 - Token Conjecture

Power of 1-Token Game

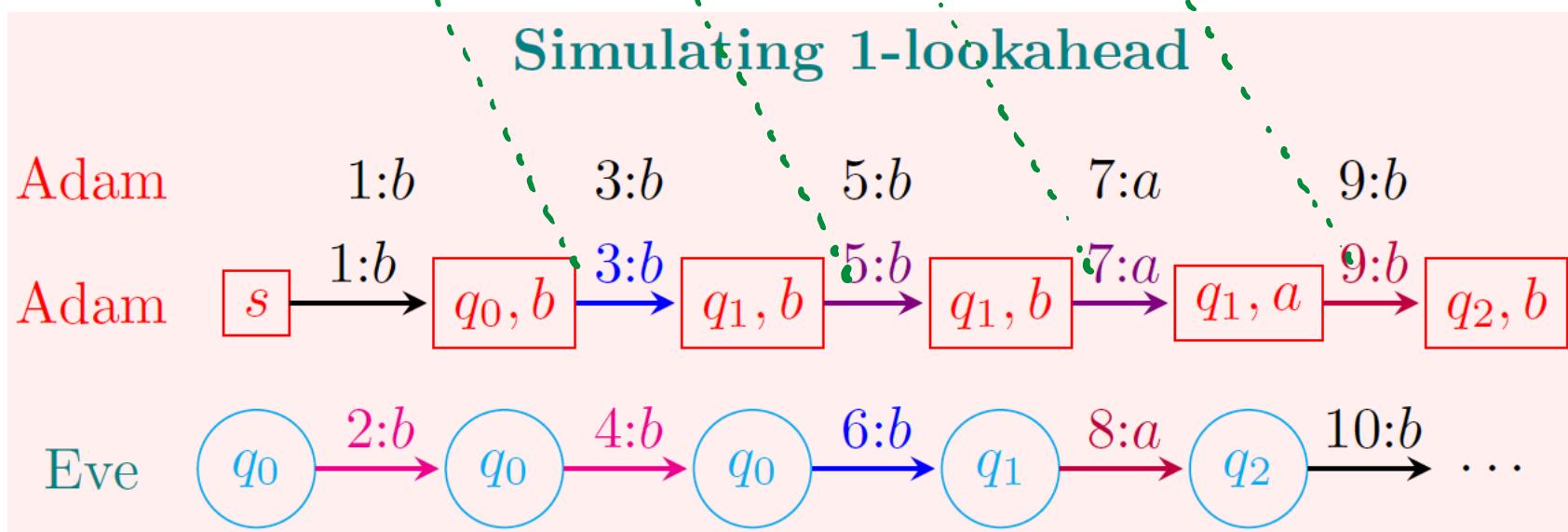
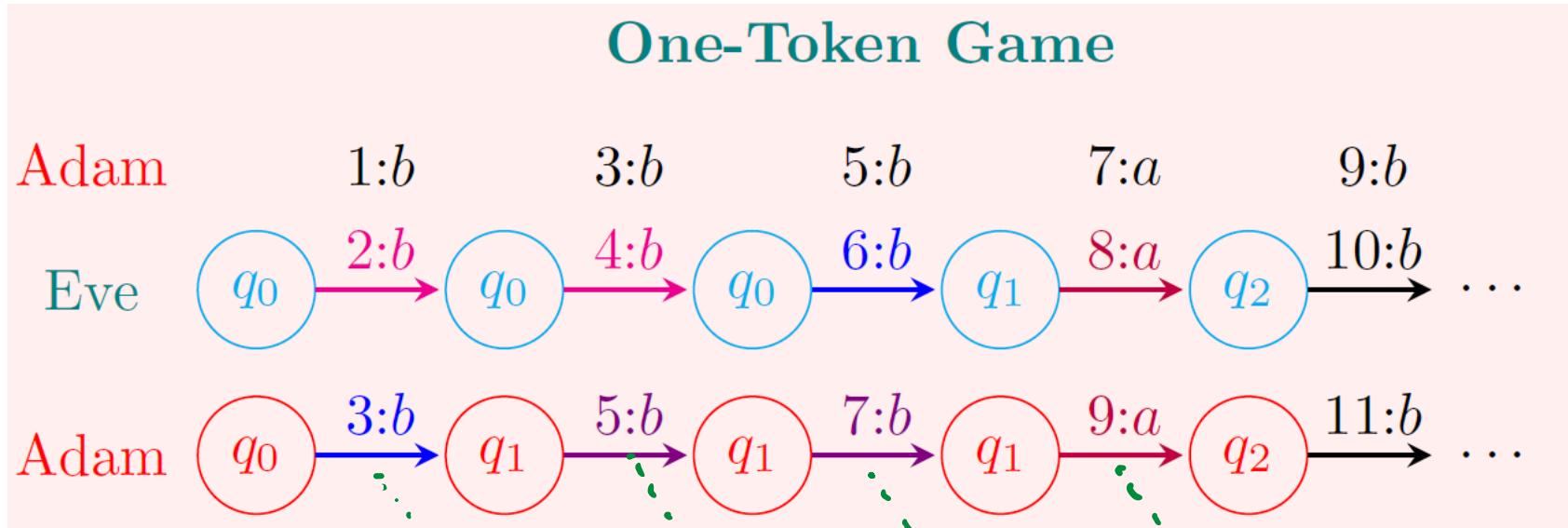
One-Token Game



Simulating 1-lookahead



Power of 1-Token Game



Eve wins

1-token game



A simulates
Delay(A)



A simulates
Delay^k(A) $\forall k \geq 0$.