2-Token Conjecture. For every parity automaton $\mathcal{A}$,

Bagnol, Kuperberg '18

$\mathcal{A}$ is history-deterministic

$\Longleftrightarrow$

Eve wins the 2-token game on $\mathcal{A}$

Aditya Prakash

University of Warwick, UK

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Lower bound:

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$NP \cap coNP$, $n^{O(\log d)}$

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Upper bound:

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Henzinger, Piterman '06

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2-token conjecture

$\Downarrow$

PSPACE, $2^{3d} \cdot \mathrm{Poly}(n)$

Upper bound:

$n^{\Theta(n)}$

Henzinger, Piterman'06

Joint work with

Karoliina Lehtinen

Aix-Marseille Université,
France

To appear in STOC 2025

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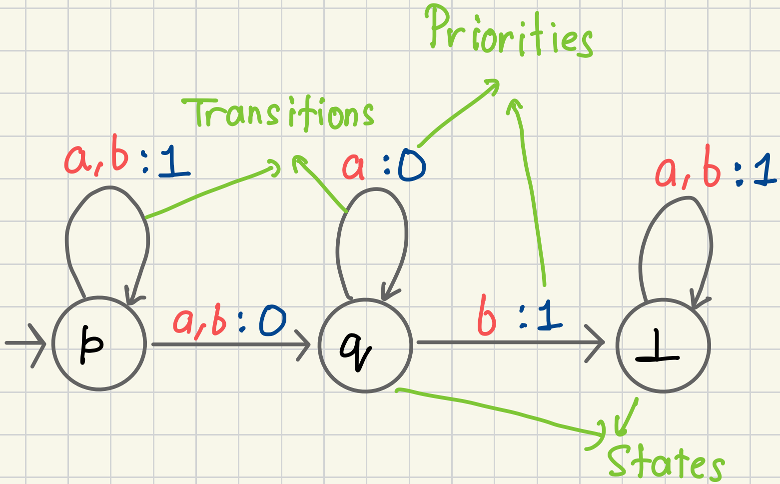
Parity Automaton

Alphabet = $\{a, b\}$



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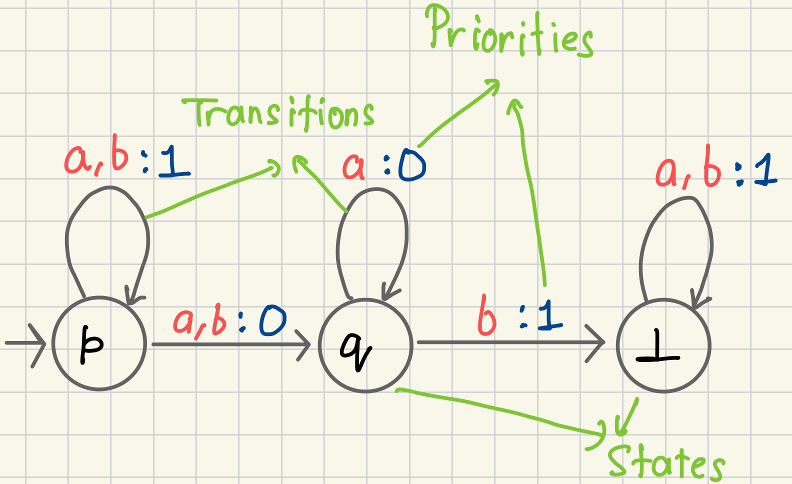

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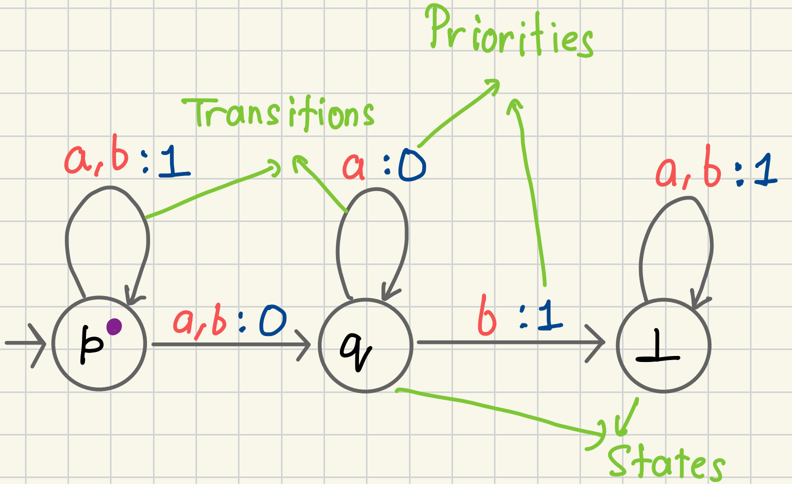

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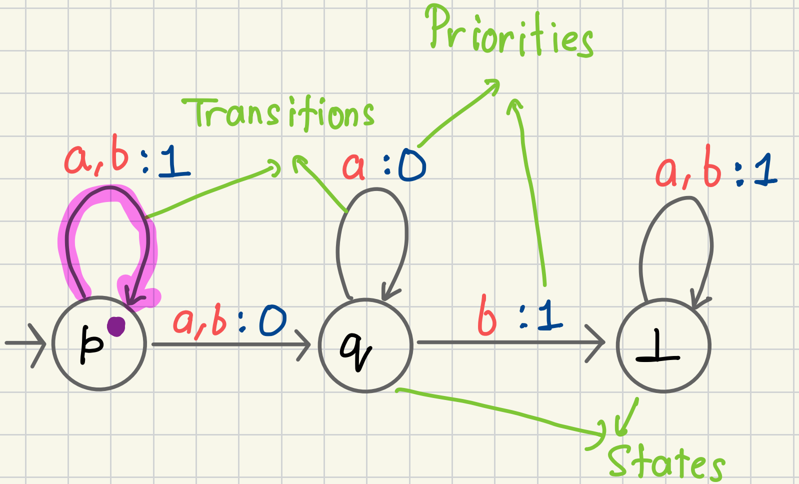
Run: $p \xrightarrow{b:1} p$



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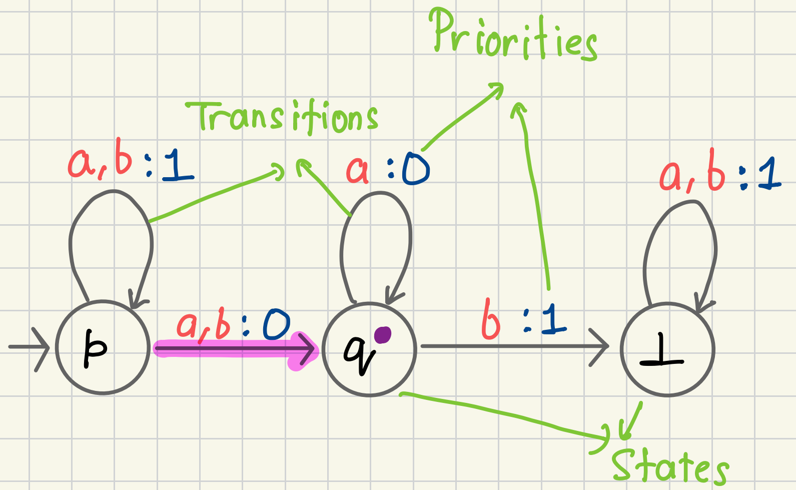
Run: $p \xrightarrow{b:1} p \xrightarrow{a:0} q$



Parity Automaton

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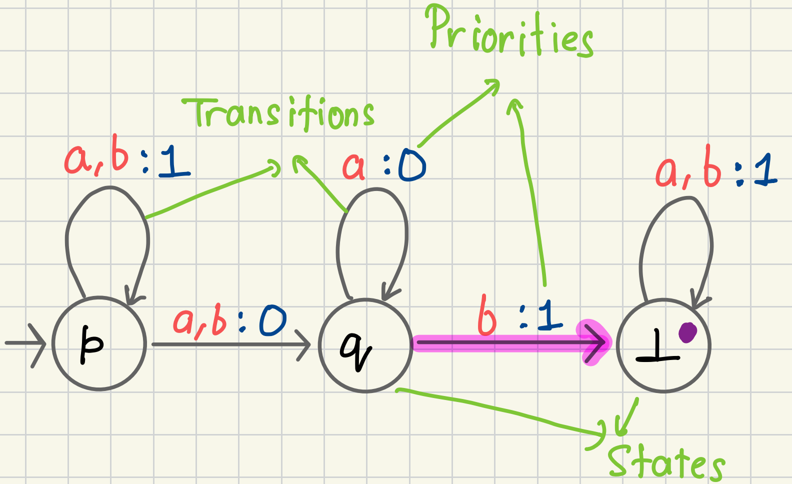
b a b a a a a a a ...

Run: $p \xrightarrow{b:1} p \xrightarrow{a:0} q \xrightarrow{b:1} \perp$



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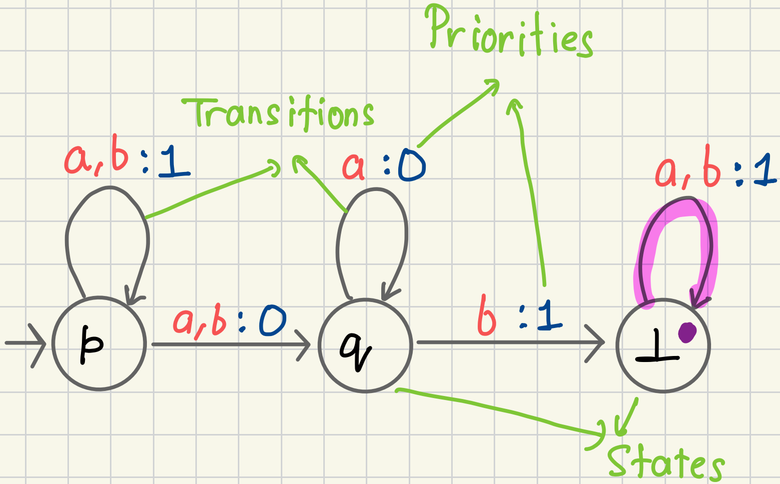

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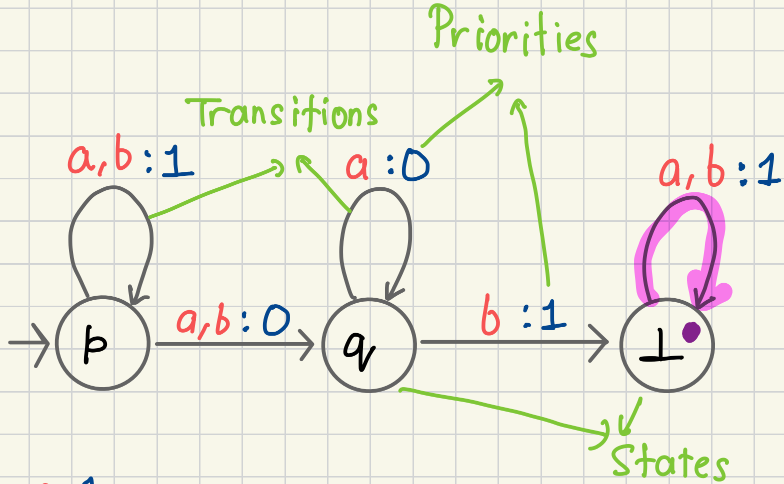

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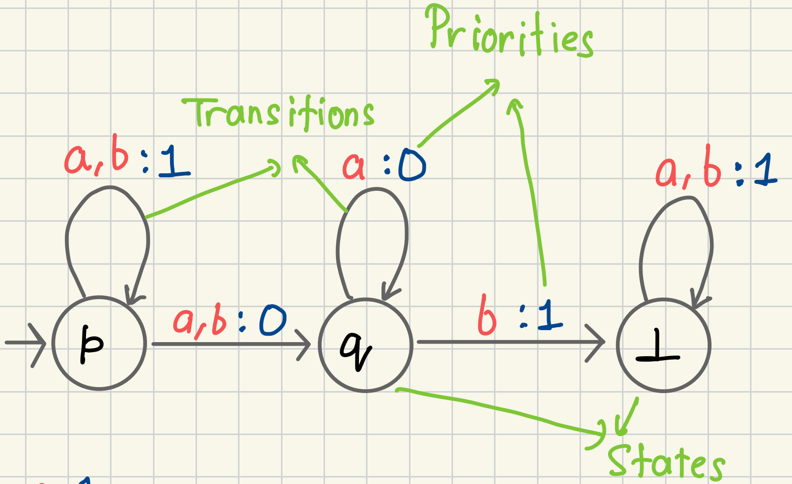

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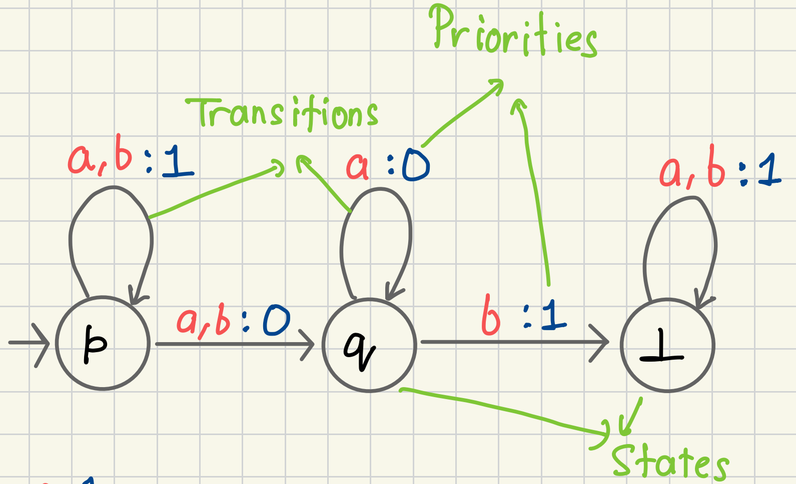

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A run is accepting if the least priority occurring ∞ly often is even

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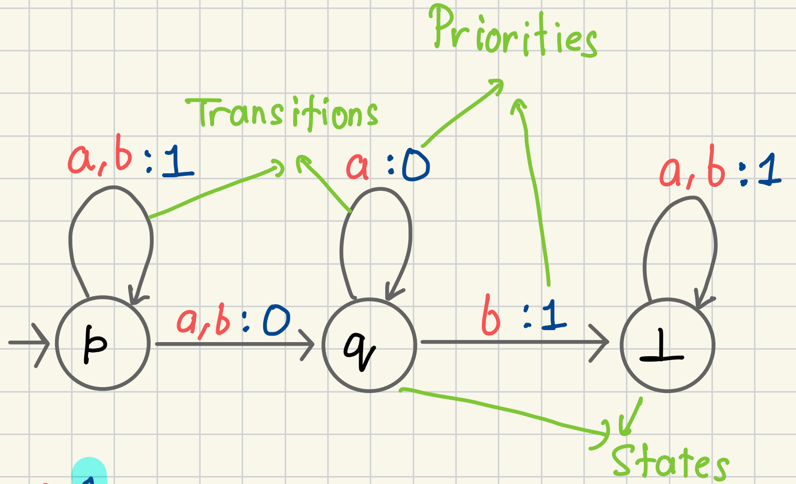

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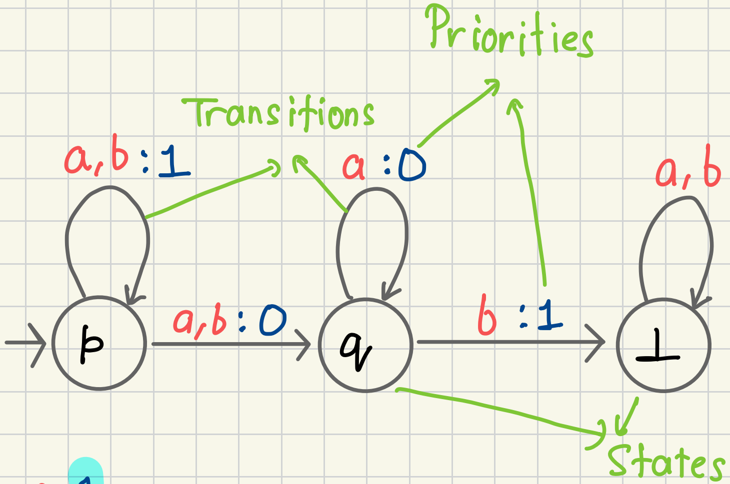

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A run is accepting if the least priority occurring ∞ly often is even

A word is accepting if there is an accepting run on it.

Language: Set of accepting words

Parity Automaton

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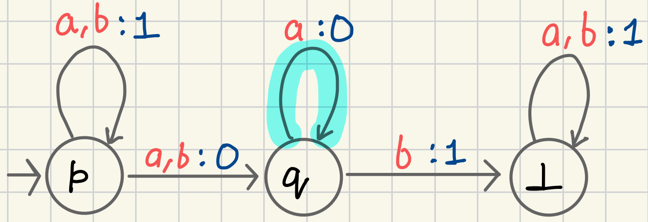

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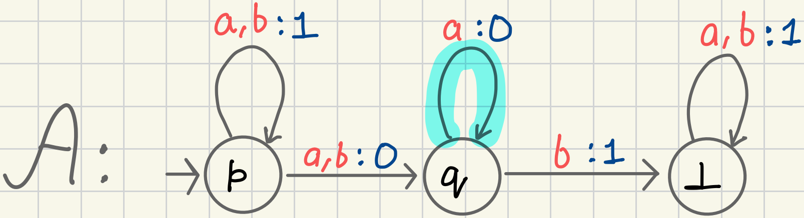

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⇓

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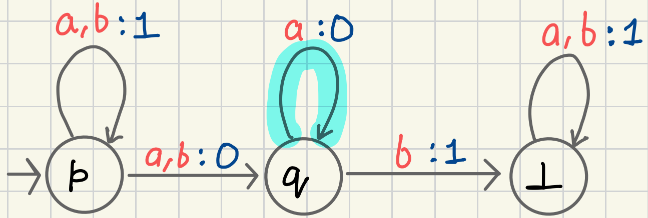

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$$L(\mathcal{A}) = \{ \text{Words that contain finitely many } b\text{'s} \}$$

Parity-Index Hierarchy

$[i,j]$ automata: priorities in $\{i, i+1, \dots, j\}$

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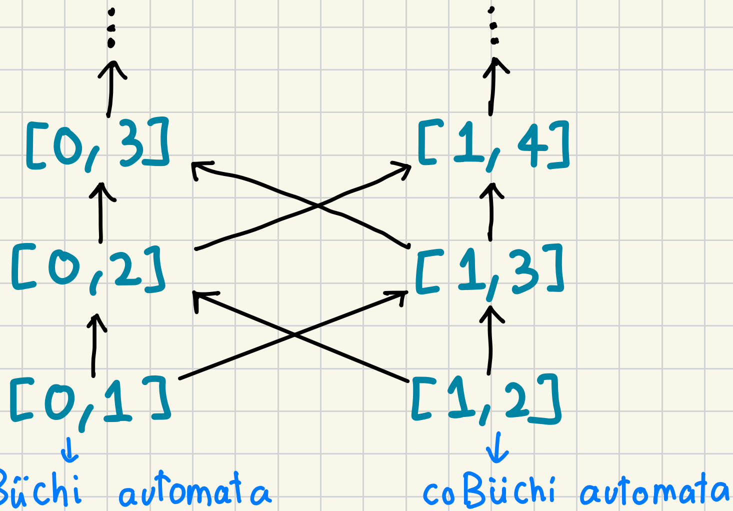

Parity-Index Hierarchy

$[i,j]$ automata: priorities in $\{i, i+1, \dots, j\}$

$$\begin{tikzcd}
\vdots & \vdots \\
{[0,3]} \arrow[u] & {[1,4]} \arrow[u] \\
{[0,2]} \arrow[u] \arrow[ur] & {[1,3]} \arrow[u] \arrow[ul] \\
{[0,1]} \arrow[u] \arrow[ur] \arrow[d] & {[1,2]} \arrow[u] \arrow[ul] \arrow[d] \\
\text{Büchi automata} & \text{coBüchi automata}
\end{tikzcd}$$

Priority 1 transitions occur finitely often

or

Eventually, only priority 2 transitions occur

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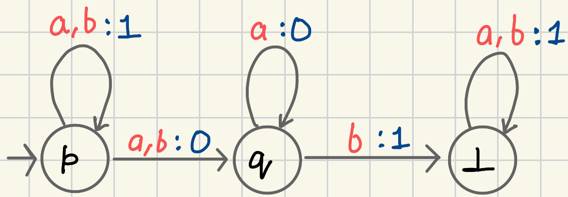
Non-example:

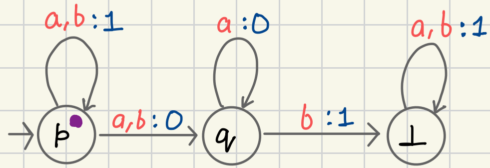
A:



L(A) = { Words that contain finitely many b's }

More precisely: Eve wins the HD game

Automaton $\mathcal{A}$: HD game on $\mathcal{A}$



HD game

Adam

Eve p

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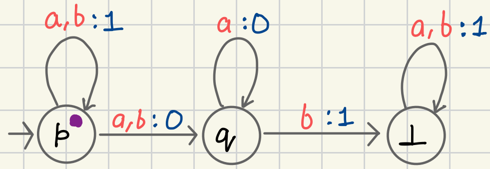
Players : Eve , Adam

Starts with an Eve's token at $\rightarrow p$



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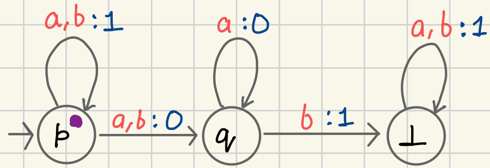

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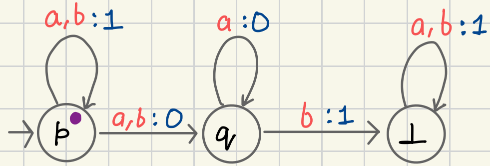

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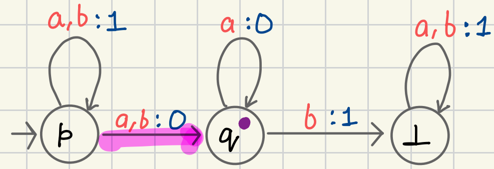

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Eve $p \xrightarrow{a:0} q$

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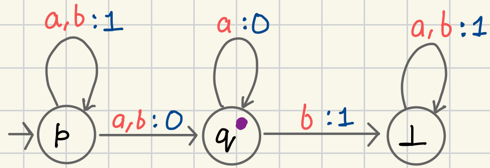

HD game

Adam	a	b
Eve	$p \xrightarrow{a:0} q$	

More precisely: Eve wins the HD game

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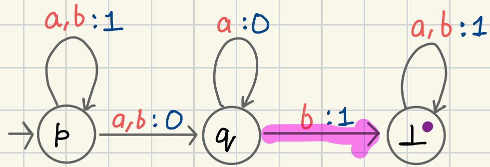

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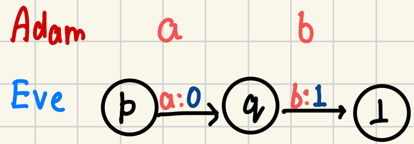

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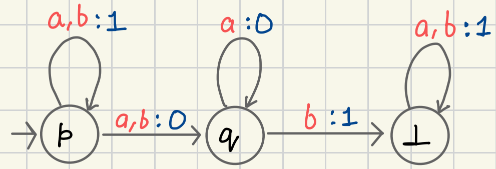

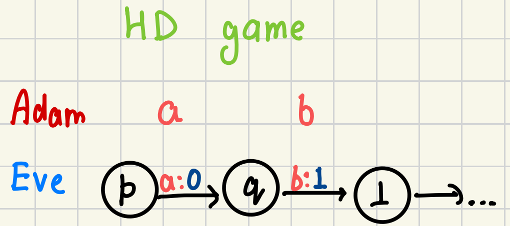

More precisely: Eve wins the HD game

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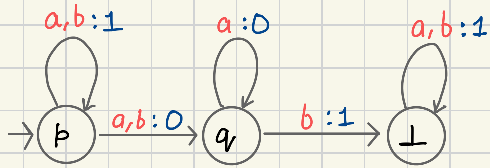

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HD game

Adam $\quad a \quad b$

Eve $\quad p \xrightarrow{a:0} q \xrightarrow{b:1} \bot \longrightarrow \ldots$

In the limit: Adam buids a word w, Eve builds a run ρ on that word.

More precisely: Eve wins the HD game

Automaton $\mathcal{A}$: HD game on $\mathcal{A}$

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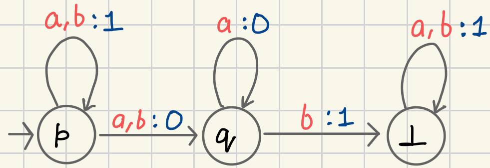

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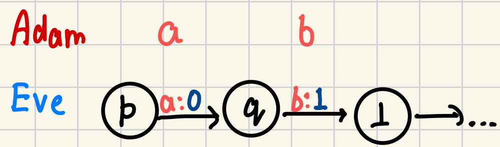

In the limit: Adam buids a word w, Eve builds a run ρ on that word.

Eve's winning condition: if $w \in L(\mathcal{A})$, then ρ is accepting.

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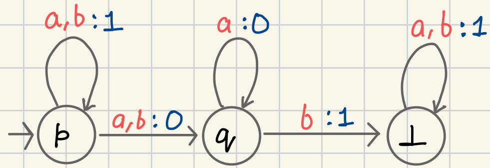

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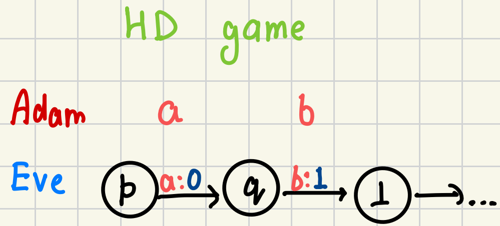

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A is HD $\Longleftrightarrow$ Eve has a winning strategy.

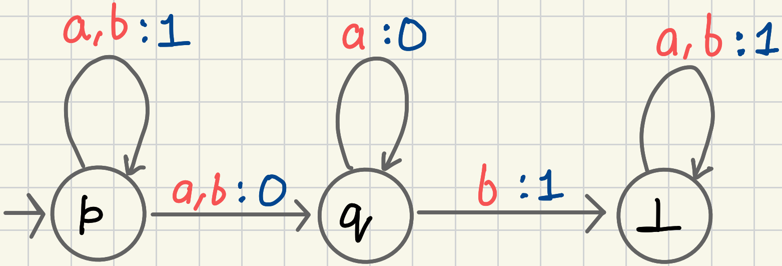
Quick Note on Games

For games in this talk: either Eve has a winning strategy
or Adam has a winning strategy.

If Player has a winning strategy in $\mathcal{G}$
then Player wins $\mathcal{G}$.

Non-example (of HD automata)

$\mathcal{A}$:



$L(\mathcal{A}) = \{$ words that contain finitely many b's $\}$

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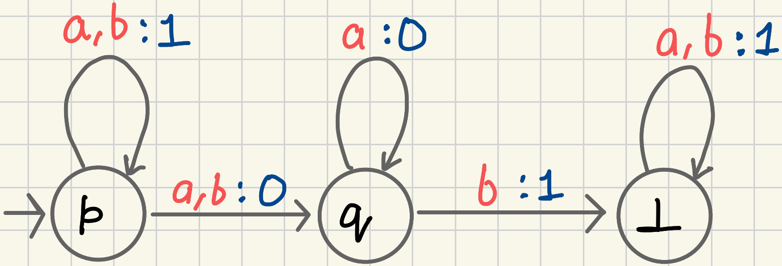

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1. Select a's until Eve's token moves to $\rightarrow q$

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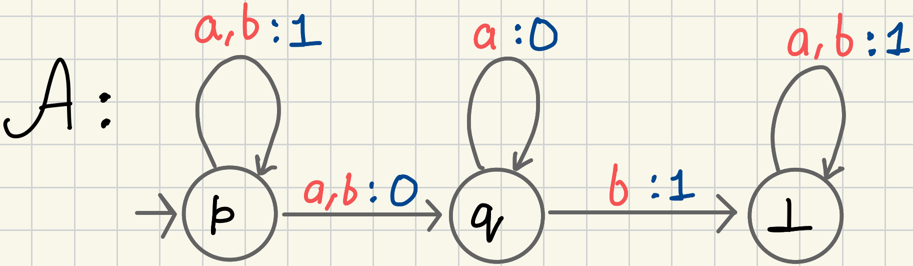

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Adam's winning strategy in the HD game:

1. Select a's until Eve's token moves to $\rightarrow q$

(if her token stays in p, then her run is rejecting but $aaa\ldots \in L(A)$)

2. When Eve's token moves to $\rightarrow q$, Adam plays $baaa\ldots$

Adam's word $aa\ldots abaaa\ldots \in L(A)$, and Eve's run is rejecting.

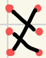
Example 0

Example 0

Deterministic parity automata

The example: coBüchi automaton

[Kuperberg, Skrzypczak '15]

Alphabet Σ = {  ,  ,  }

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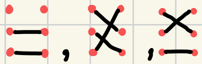
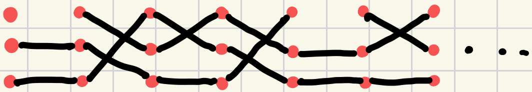
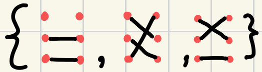
Alphabet $\Sigma = \{$  $\}$

Words are graphs:

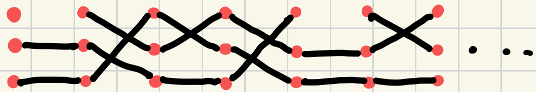


The example: coBüchi automaton

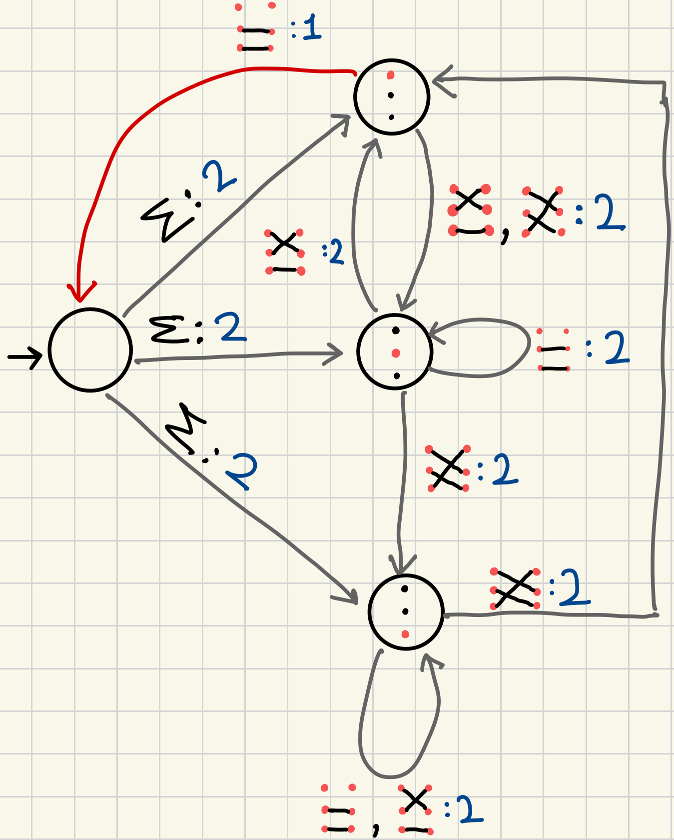
[Kuperberg, Skrzypczak '15]

Alphabet Σ = 

Words are graphs:



Run is accepting if it contains finitely many 1's.



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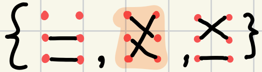

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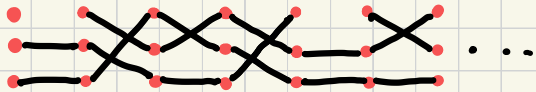

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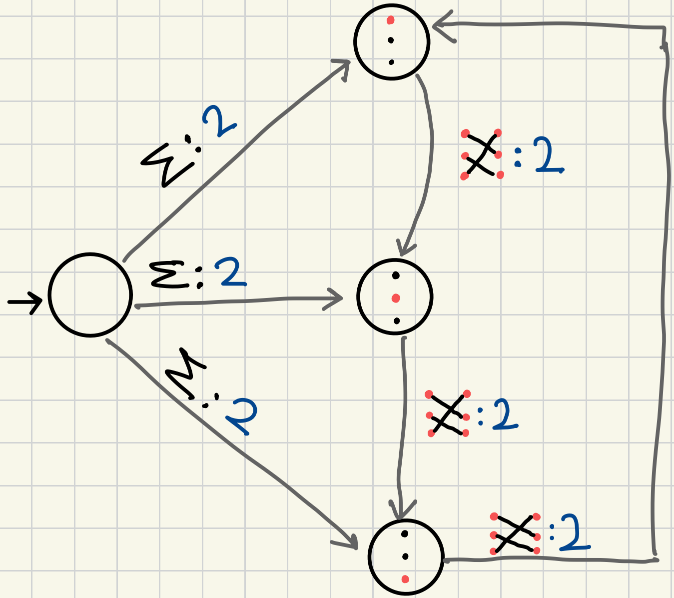

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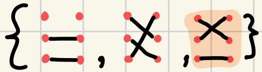
Alphabet Σ = 

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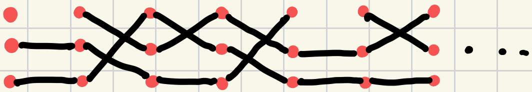

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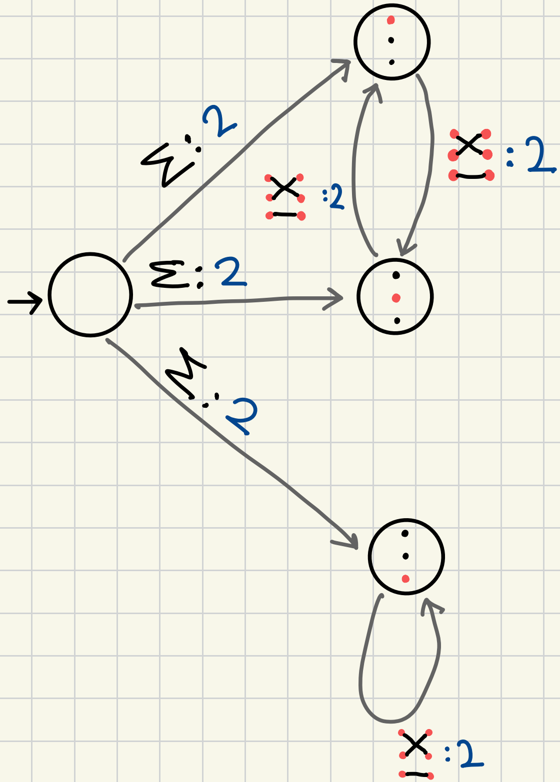

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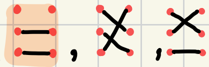
Alphabet Σ = {  }

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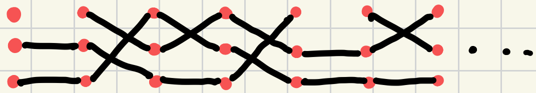

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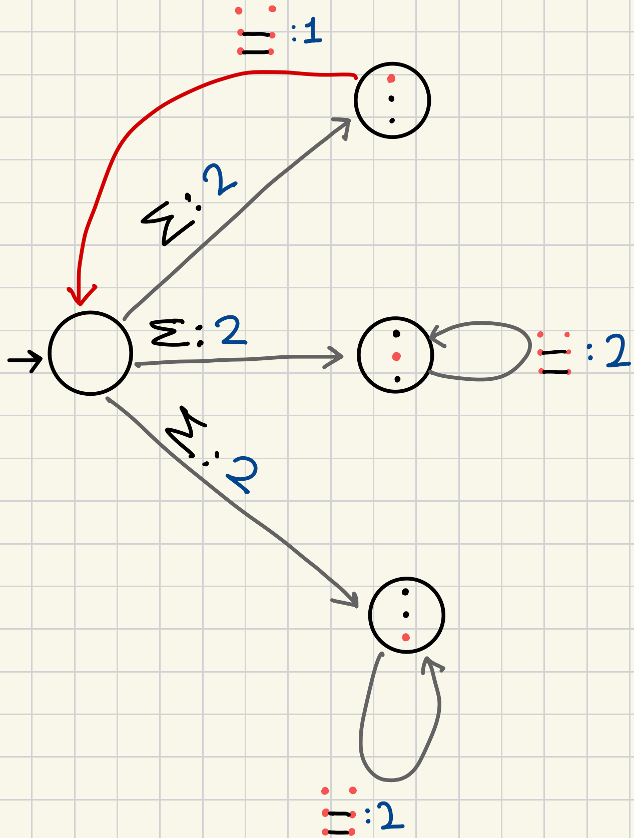

The example: coBüchi automaton

[Kuperberg, Skrzypczak '15]

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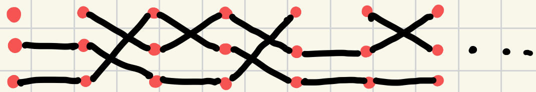

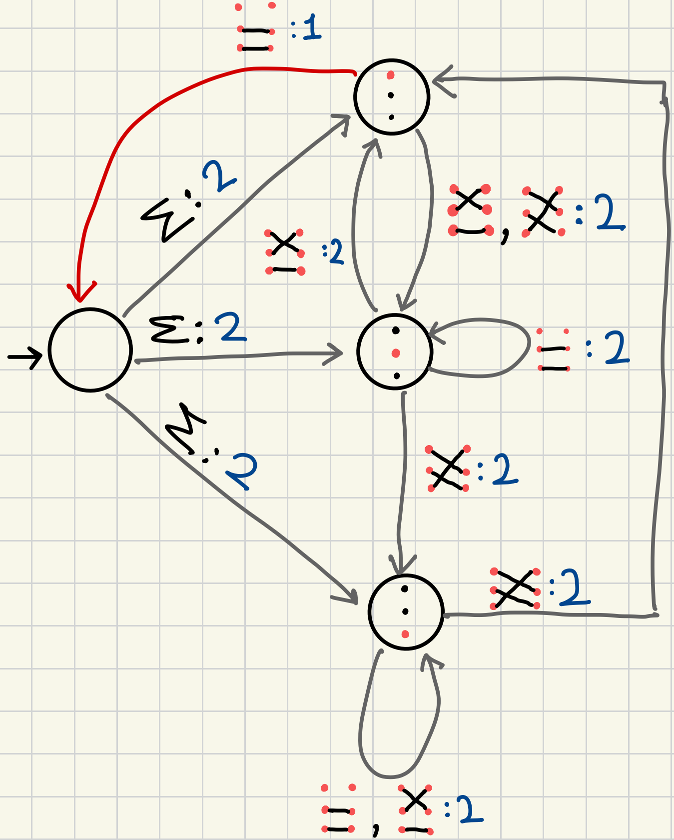

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Language = ?

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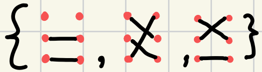

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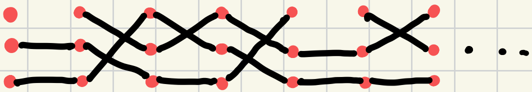

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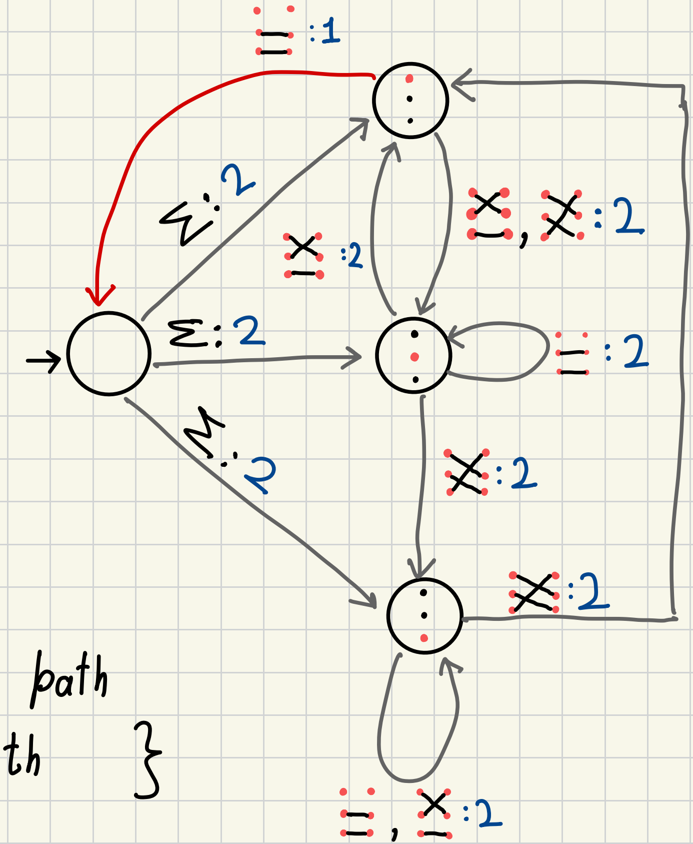
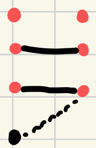
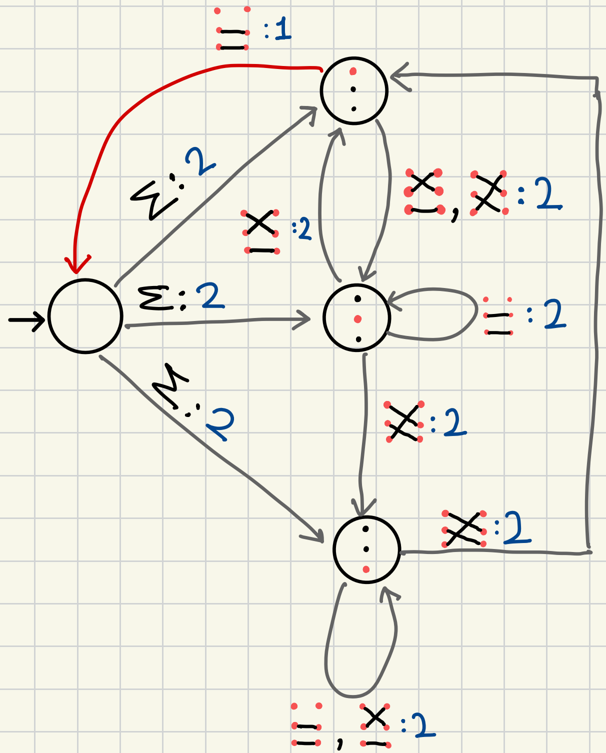


Language = { Graphs that contain a path of infinite length }

This automaton is HD!

Eve's strategy in the HD game:





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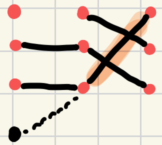

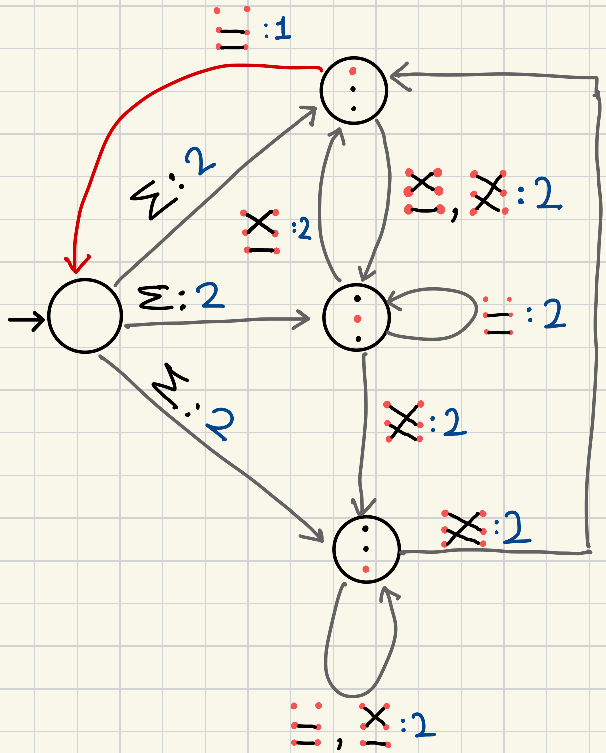

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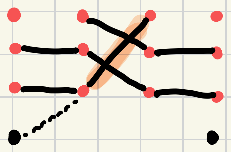

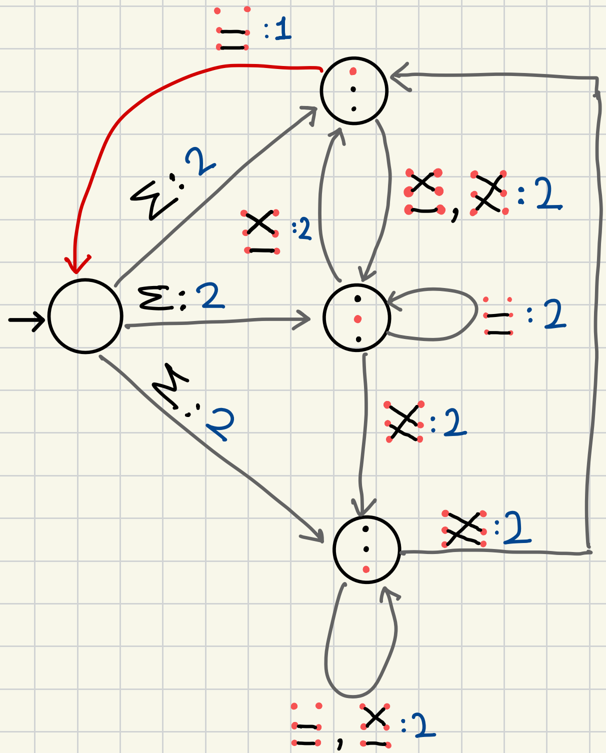

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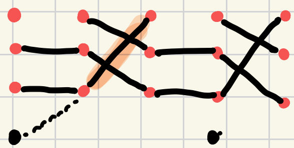
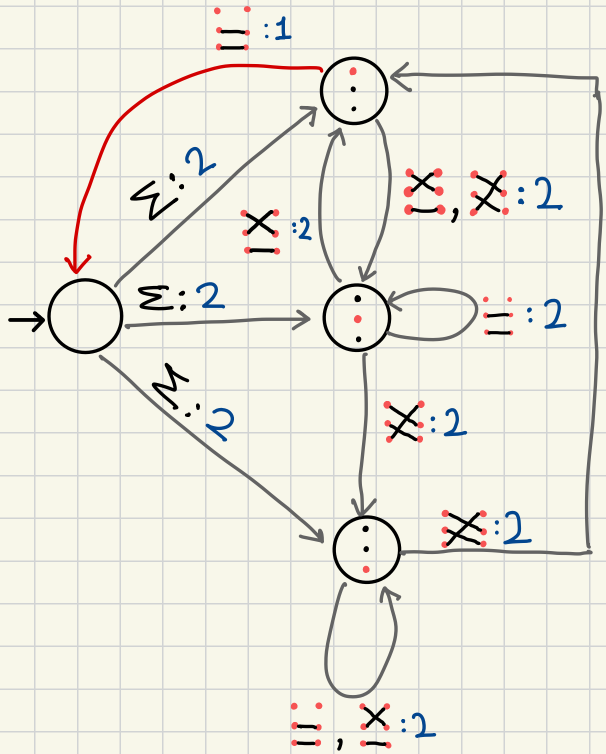


At →○ , pick the state with the longest unbroken path.



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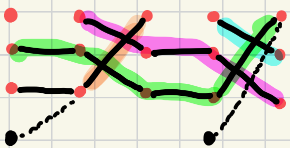

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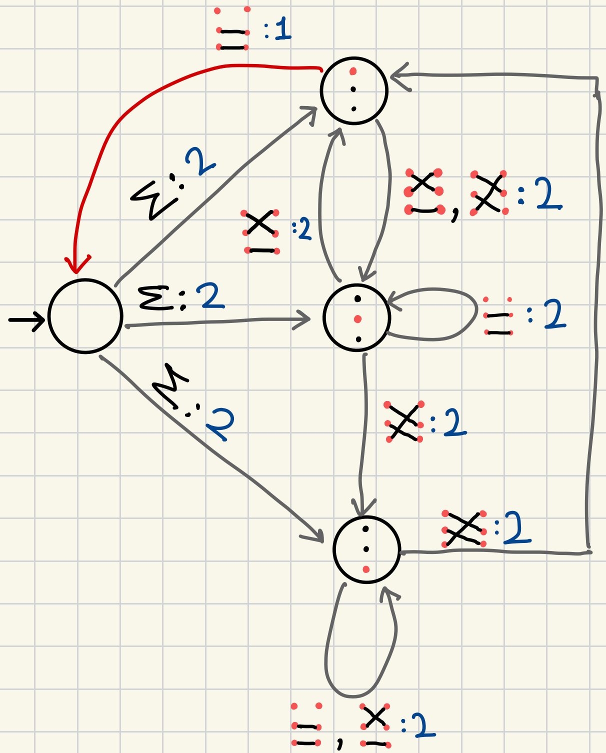

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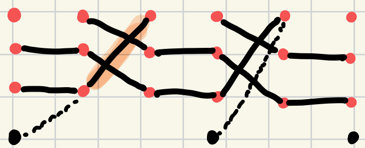

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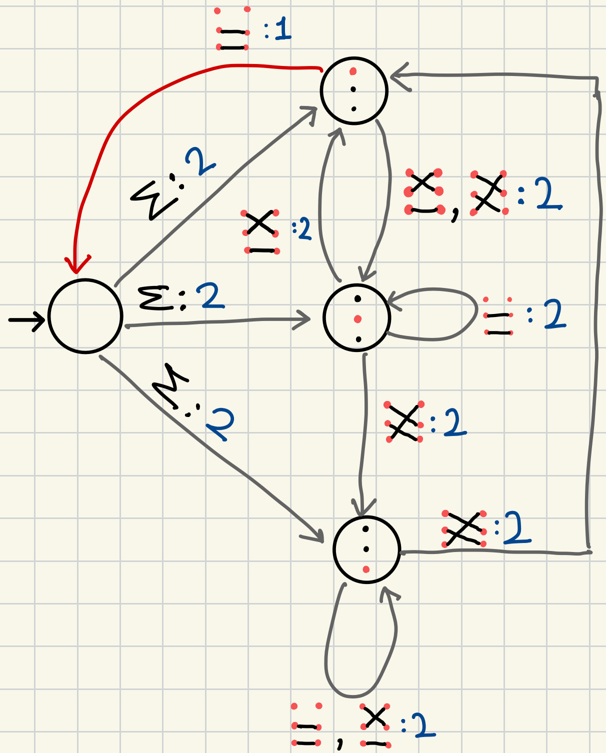

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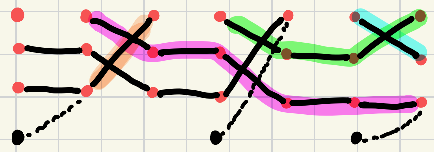

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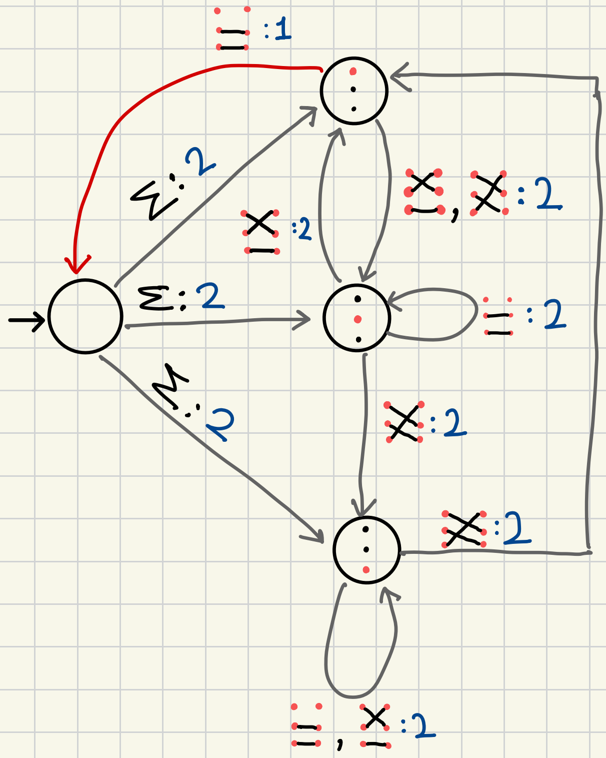

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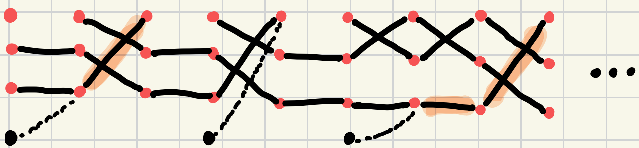

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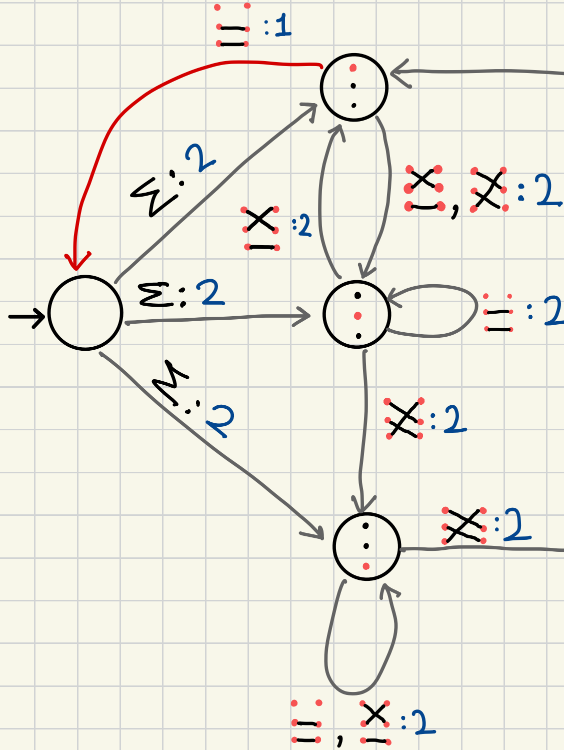

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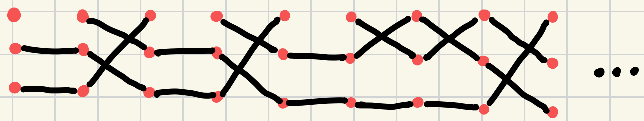

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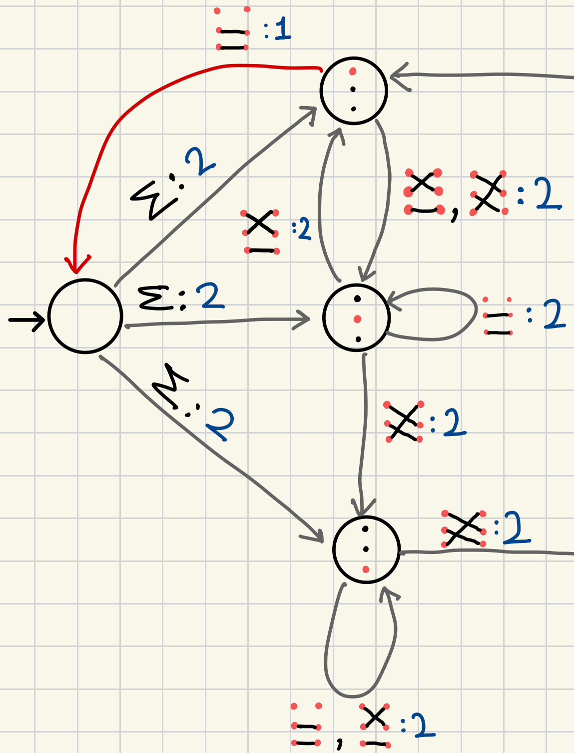

Language = { Graphs with an ∞ path }

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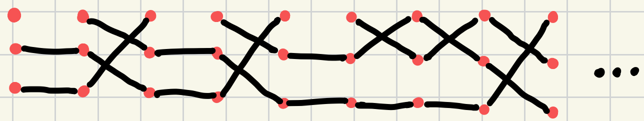
This idea will appear again!

Eve's strategy in the HD game:



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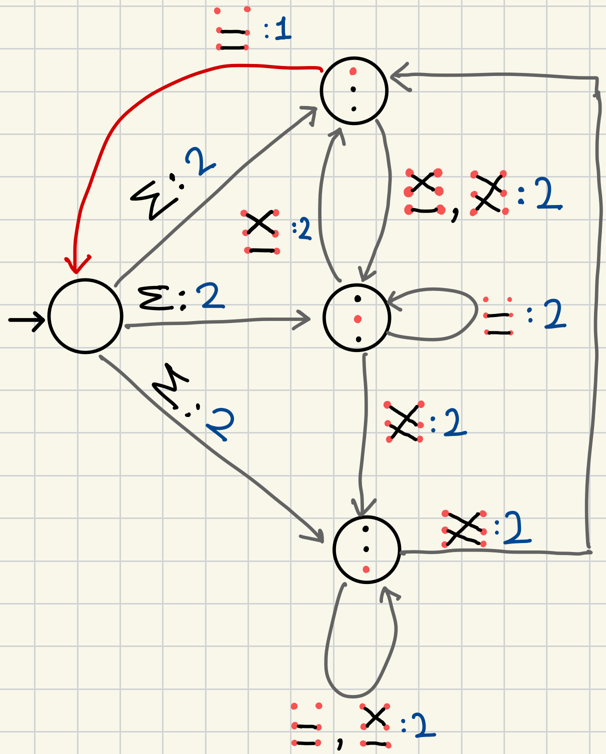

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Why HD automata?

* Good-for-games:

2-player game $\mathcal{G}$ with winning condition as $L(\mathcal{A})$.

If $\mathcal{A}$ is HD, then $\mathcal{G}$ and $\mathcal{G} \times \mathcal{A}$ have the same-winner.

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* Provides canonical representation for ω-regular languages

  [Ehlers, Schewe'22], [Abu Radi, Kupferman'20]

Recognising HD parity automaton

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Henzinger, Piterman'06

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Henzinger, Piterman'06

Alternate approach: Easily-solvable games with the same winner

History of recognising HD parity automata

$[i,j]$ automata: priorities in $\{i, i+1, \dots, j\}$

$$\begin{tikzcd}
\vdots & \vdots \\
{[0,3]} \arrow[u] & {[1,4]} \arrow[u] \\
{[0,2]} \arrow[u] \arrow[ur] & {[1,3]} \arrow[u] \arrow[ul] \\
{[0,1]} \arrow[u] \arrow[ur] & {[1,2]} \arrow[u] \arrow[ul] \\
\text{Büchi} & \text{coBüchi}
\end{tikzcd}$$

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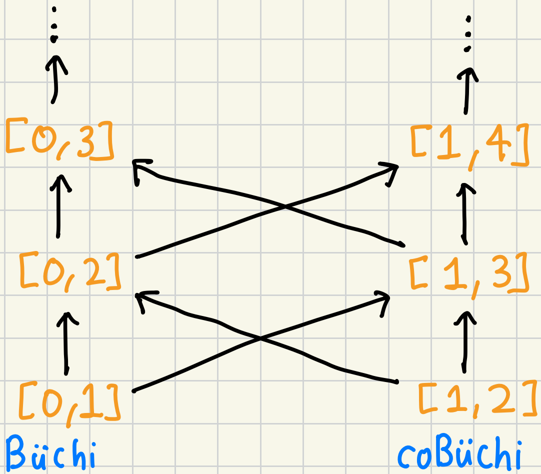

Kuperberg, Skrzypczak'15.

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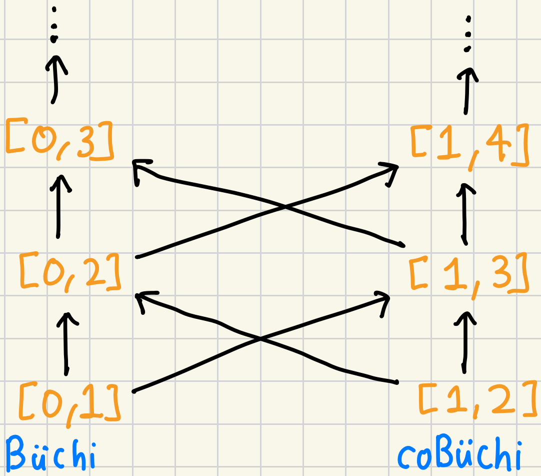

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Bagnol, Kuperberg'18. → Use 2-token game

Büchi automata.

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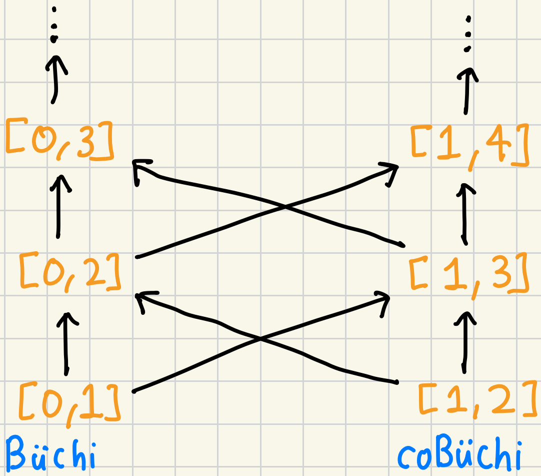

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Use 2-token game

Boker, Kuperberg, Lehtinen, Skrzypczak'20

coBüchi automata

2-Token Conjecture. For every parity automaton $\mathcal{A}$,

Bagnol, Kuperberg '18

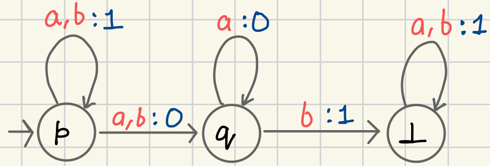
$\mathcal{A}$ is history-deterministic

$\Longleftrightarrow$

Eve wins the 2-token game on $\mathcal{A}$

The 2-token game

Automaton $\mathcal{A}$: the 2-token game on $\mathcal{A}$



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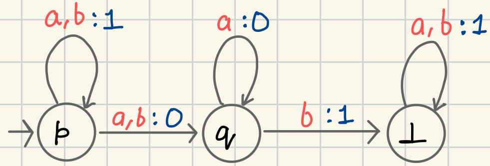
Starts with an Eve's token and two of Adam's tokens at $\to p$



2-token game

Adam

Eve p

Adam p

Adam p

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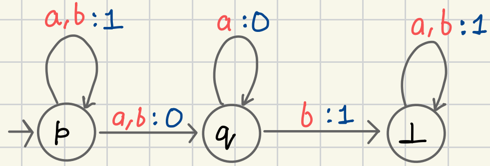

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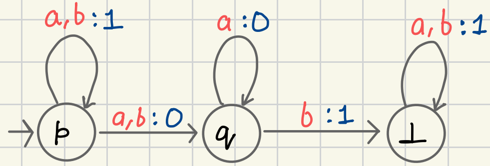

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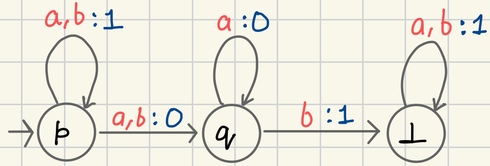

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Eve $p \xrightarrow{a:0} q$

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Adam p

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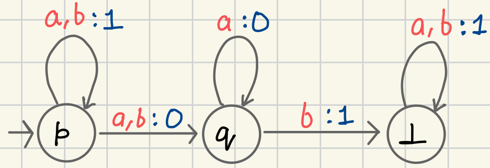

2-token game

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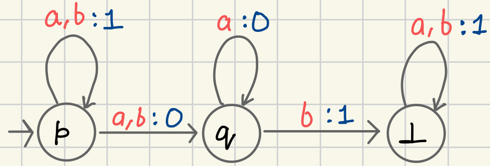

2-token game

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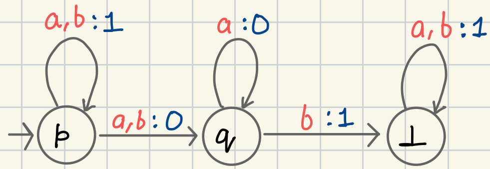

2-token game

Adam		a	b
Eve	$p \xrightarrow{a:0}$	$q \xrightarrow{b:1}$	$\perp$
Adam	$p \xrightarrow{a:0}$	q	
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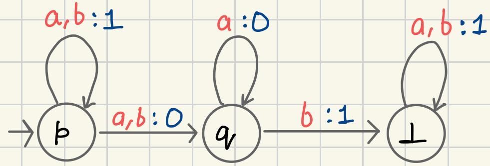

2-token game

Adam	a	b
Eve	$p \xrightarrow{a:0} q$	$\xrightarrow{b:1} \bot$
Adam	$p \xrightarrow{a:0} q$	$\xrightarrow{b:1} \bot$
Adam	$p \xrightarrow{a:1} p$	$\xrightarrow{b:0} q$

The 2-token game

Automaton $\mathcal{A}$: the 2-token game on $\mathcal{A}$

Starts with an Eve's token and two of Adam's tokens at $\to p$

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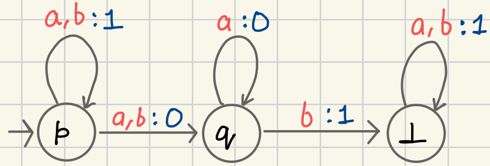

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Adam $\quad a \quad b$

Eve $\quad p \xrightarrow{a:0} q \xrightarrow{b:1} \bot \longrightarrow \ldots$

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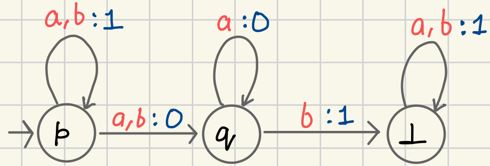

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Eve's winning condition: if either of Adam's runs are accepting then Eve's run is accepting.

HD game vs. 2-token game

Winning condition in HD game:

if Adam's word is in L(A) then Eve's run is accepting.

Winning condition in 2-token game:

if either of Adam's runs are accepting then Eve's run is accepting.

HD game vs. 2-token game

Winning condition in HD game:

if Adam's word is in $L(A)$ then Eve's run is accepting.

Winning condition in 2-token game:

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Observation.

If A is HD then Eve wins the 2-token game on A.

Proof. Use Eve's winning strategy in the HD game to play in the 2-token

2-Token Conjecture. For every parity automaton $\mathcal{A}$,

Bagnol, Kuperberg '18

$\mathcal{A}$ is history-deterministic

$\Longleftrightarrow$

Eve wins the 2-token game on $\mathcal{A}$

Why 2 tokens?

Why 2 tokens?

* 2 tokens are plenty!

Lemma. If Eve wins the 2-token game on A
Bagnol, Kuperberg'18
then Eve wins the k-token game on A for all $k \geq 1$.

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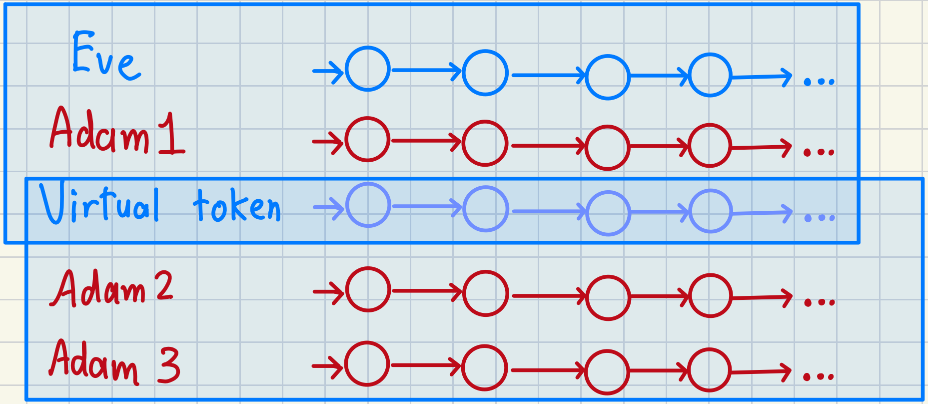

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Bagnol, Kuperberg'18

Theorem. For every Büchi or coBüchi automaton $\mathcal{A}$,
Bagnol, Kuperberg'18 Boker, Kuperberg, Lehtinen, Skrzypczak'20

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Adam wins the k-token game on $\mathcal{A}$,

where $k = 2^{2^n}$

Just 1 token

Theorem. For every Büchi or coBüchi automaton A, if Eve wins the 1-token game from everywhere on A then A is HD.

Just 1 token

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Corollary

Theorem. For every Büchi or coBüchi automaton A,

Bagnol, Kuperberg '18 — Boker, Kuperberg, Lehtinen, Skrzypczak '20

Eve wins the 2-token game on A $\Longrightarrow$ A is history-deterministic.

Theorem. For every Büchi or coBüchi automaton A, if Eve wins the 1-token game from everywhere on A then A is HD.

Proof sketch.

CoBüchi automata or [1,2] automata	Büchi automata or [0,1] automata
Automaton A	Automaton A

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CoBüchi automata or [1,2] automata	Büchi automata or [0,1] automata
Automaton $\mathcal{A}$	Automaton $\mathcal{A}$
$\Downarrow$ Modify	$\Downarrow$ Modify
$\mathcal{A}$ has safe-coverage.	$\mathcal{A}$ has reach-covering.

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CoBüchi automata or [1,2] automata

$$\text{Automaton } \mathcal{A} \xrightarrow{\text{Modify}} \mathcal{A} \text{ has safe-coverage.} \xrightarrow{\text{Prove}} \mathcal{A} \text{ is HD}$$

Büchi automata or [0,1] automata

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Automaton $\mathcal{A}$	Automaton $\mathcal{A}$
⇝ Modify	⇝ Modify
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↓ Prove	↓ Prove
$\mathcal{A}$ is HD	$\mathcal{A}$ is HD.

Safety automata: simpler coBüchi automata

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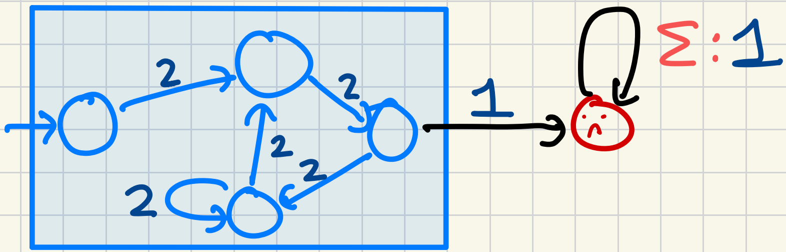
All transitions of priority 1 are on a rejecting sink state.



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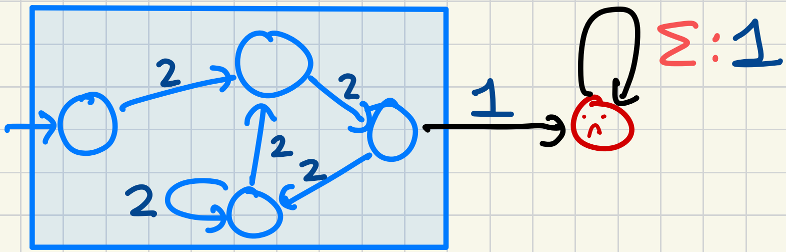

Theorem
Löding'12

For every safety automaton A,
Eve wins the 1-token game on A

$\Leftrightarrow$

A is determinisable-by-pruning $\Leftrightarrow$ A is HD.

Safe approximation

CoBüchi automaton $\mathcal{A}$ $\rightsquigarrow$ Safety automaton $\mathcal{A}_{>1}$

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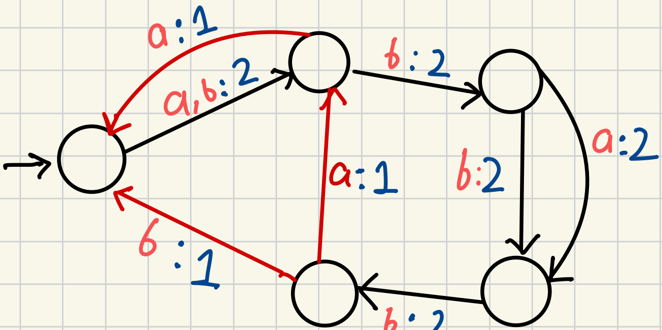

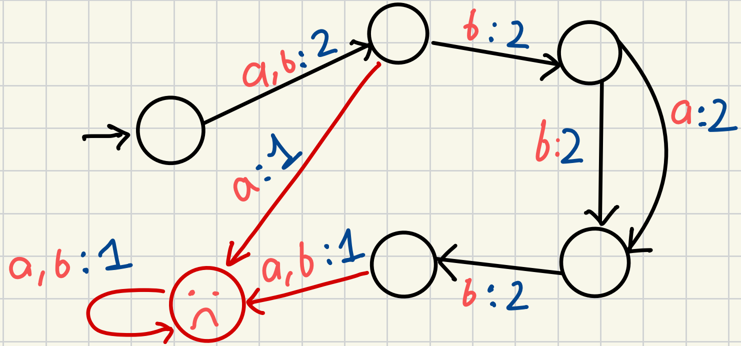

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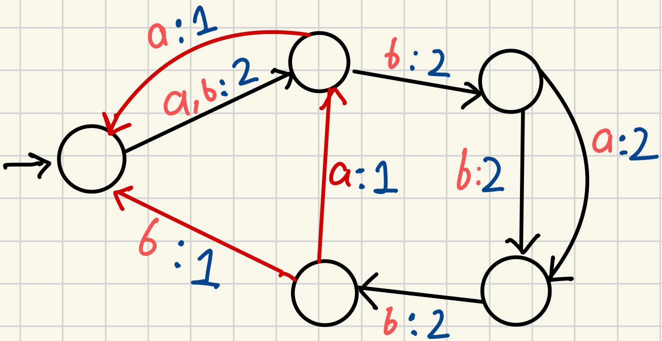

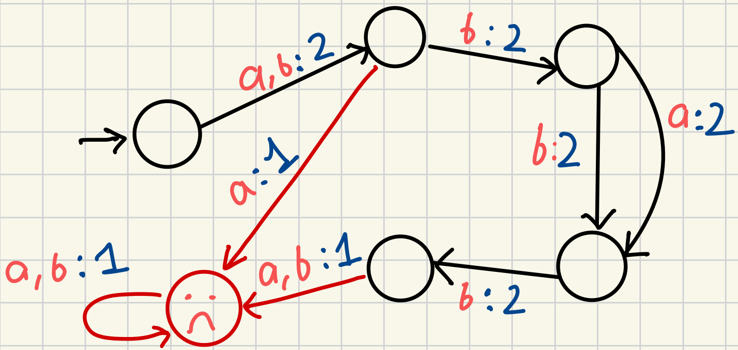

Safe coverage for a coBüchi automaton.*

There are states p, such that Eve wins 1-token game from p in $\mathcal{A}_{>1}$.

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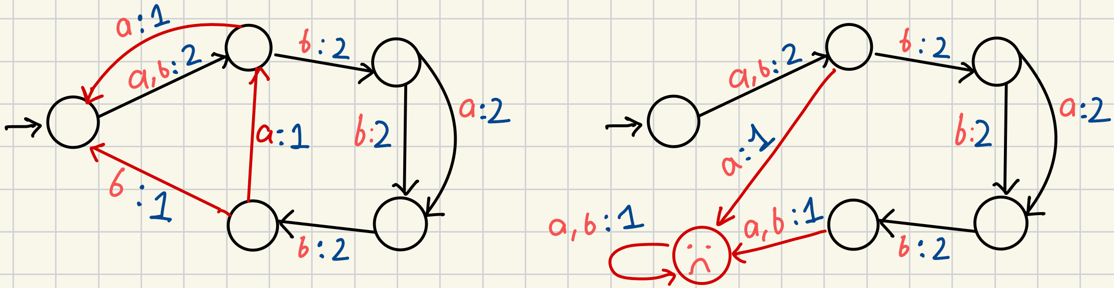

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$(A_{>1}, p)$ is HD $\Leftarrow$

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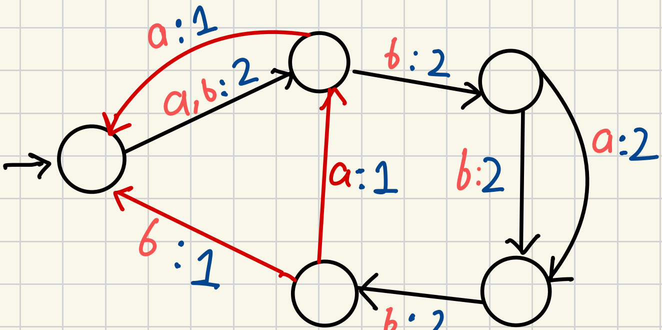

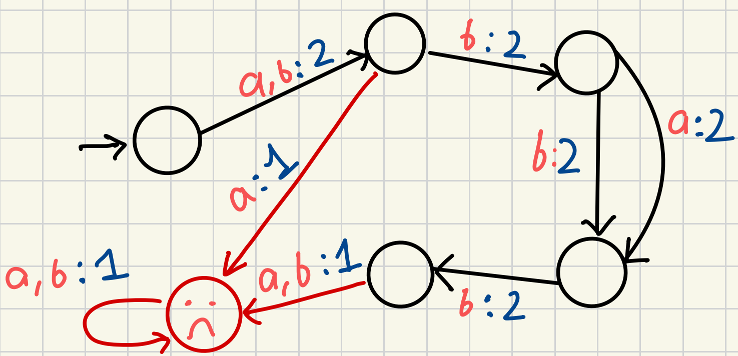

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Theorem. For every Büchi or coBüchi automaton $\mathcal{A}$, if Eve wins the 1-token game from everywhere on $\mathcal{A}$ then $\mathcal{A}$ is HD.

Proof.

CoBüchi automata
or [1,2] automata

Automaton $\mathcal{A}$

$\Big\downarrow$ Modify

$\mathcal{A}$ has safe-coverage.

$\Big\downarrow$ Prove

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$\Big\downarrow$ Modify

$\mathcal{A}$ has reach-covering.

$\Big\downarrow$ Prove

$\mathcal{A}$ is HD.

[Acharya, Jurdziński, me, ICALP'24]

Büchi coBüchi

Safety Reachability

⋮	⋮
[0,4]	[1,5]
[0,3]	[1,4]
[0,2]	[1,3]
[0,1]	[1,2]
Büchi	coBüchi
Safety	Reachability

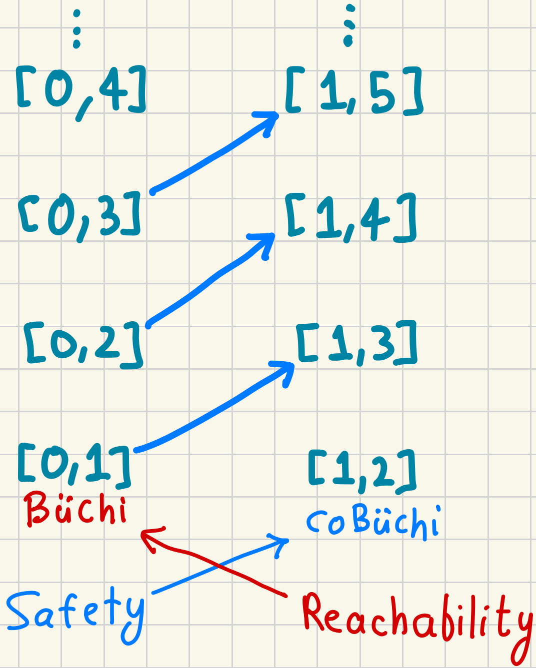
Even-to-odd

2-token conjecture for [0,K] automata

$\Downarrow$

2-token conjecture for [1,K+2] automata

Safe double coverage



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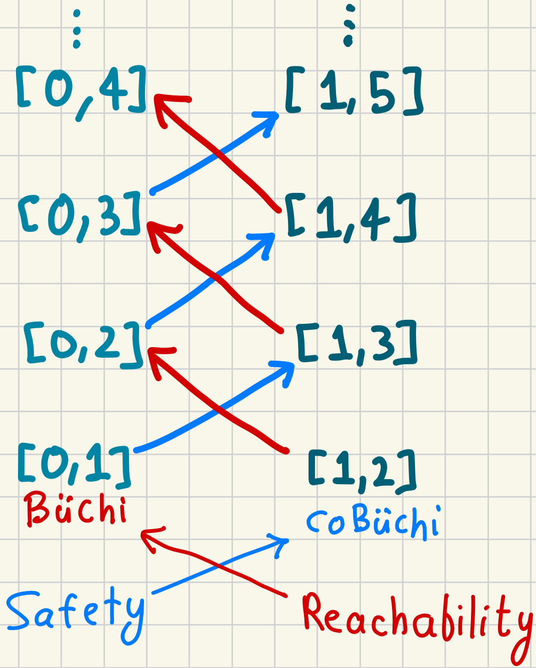

Odd-to-even

2-token conjecture for [1,K] automata

$\Downarrow$

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Reach double covering

2-Token Theorem. For every parity automaton $\mathcal{A}$,

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Upper bound:

PSPACE,

$2^{3d} \cdot \mathrm{Poly}(n)$

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Lower bound:

NP-hard

[P, FoSSaCS'24]

me :)

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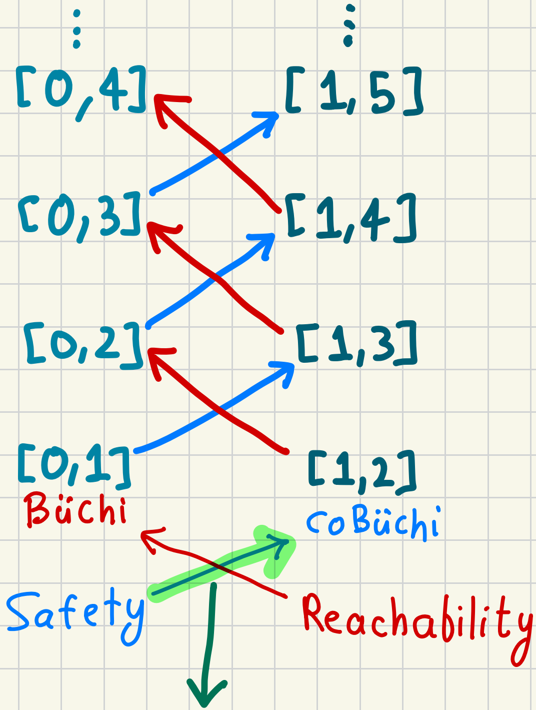

Rest of this talk!

Theorem: Every coBüchi automaton on which Eve wins the 1-token game from everywhere is HD.

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Proof. CoBüchi automaton $\mathcal{A}$

$$\Big\downarrow \text{Modify}$$

$\mathcal{A}$ has safe coverage

$$\Big\downarrow \text{Prove}$$

$\mathcal{A}$ is HD.

Token game from different states

For an automaton A and states q, p, r:

$G2(q; p, r)$ in A : 2-token game where Eve's token starts at q, Adam's tokens start at p and r.

Token game from different states

For an automaton $\mathcal{A}$ and states q, p, r:

$G2(q; p, r)$ in $\mathcal{A}$: 2-token game where Eve's token starts at q, Adam's tokens start at p and r.

$G1(q; p)$ in $\mathcal{A}$: 1-token game where Eve's token starts at q, Adam's token starts at p.

Token game from different states

For an automaton A and states q, p, r:

$G2(q; p, r)$ in A : 2-token game where Eve's token starts at q, Adam's tokens start at p and r.

$G1(q; p)$ in A : 1-token game where Eve's token starts at q, Adam's token starts at p.

Winning condition: If any of Adam's run is accepting then Eve's run is accepting.

Observation: $G1$ transitivity

Lemma: If Eve wins $G1(p;q)$ and $G1(q;r)$ in A, then Eve wins $G1(p;r)$ in A.

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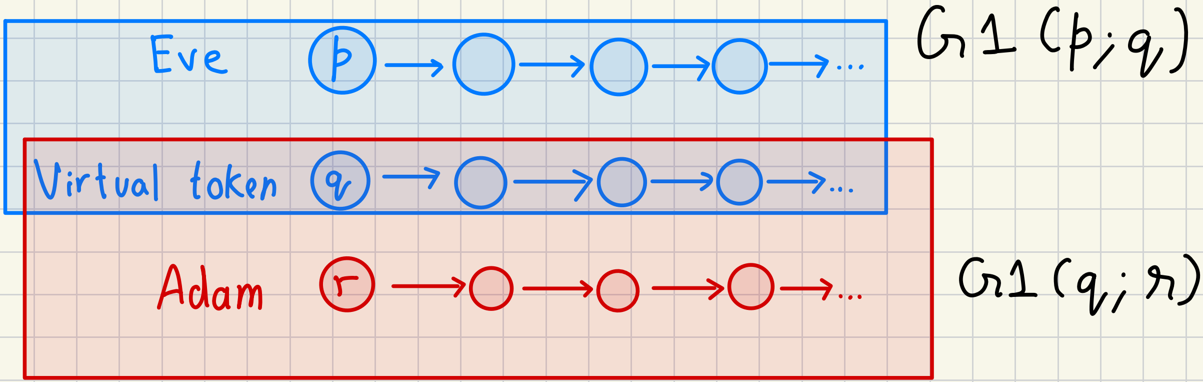
Proof.



Weak coreachability

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Automaton A: p and q are coreachable in A if there is a finite word u, such that

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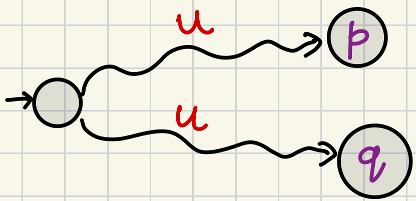

Weak coreachability: Transitive closure of coreachability.

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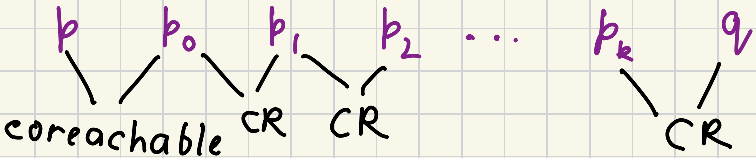
Automaton A: p and q are coreachable in A if there is a finite word u, such that

Weak coreachability: Transitive closure of coreachability.

p and q are weakly coreachable in A if $\exists\, p_0, p_1, \dots p_k$ s.t.



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Equivalence relation

↑

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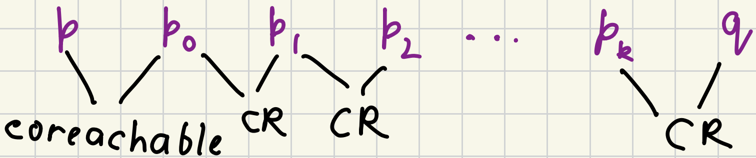

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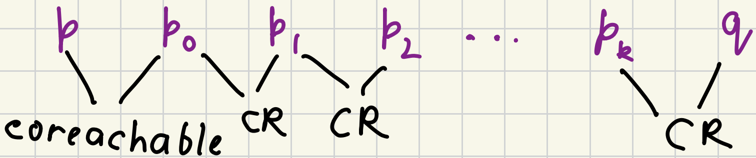
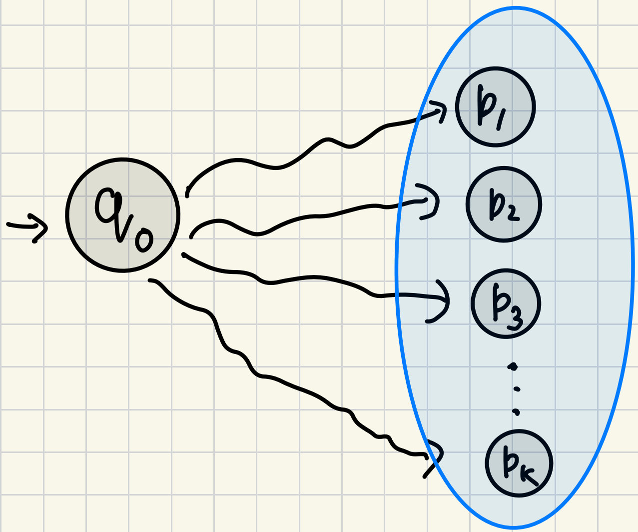


$\boxed{(p,q) \in WCR(\mathcal{A})}$

$\boxed{p \in WCR(\mathcal{A}, q)}$

Weak coreachability

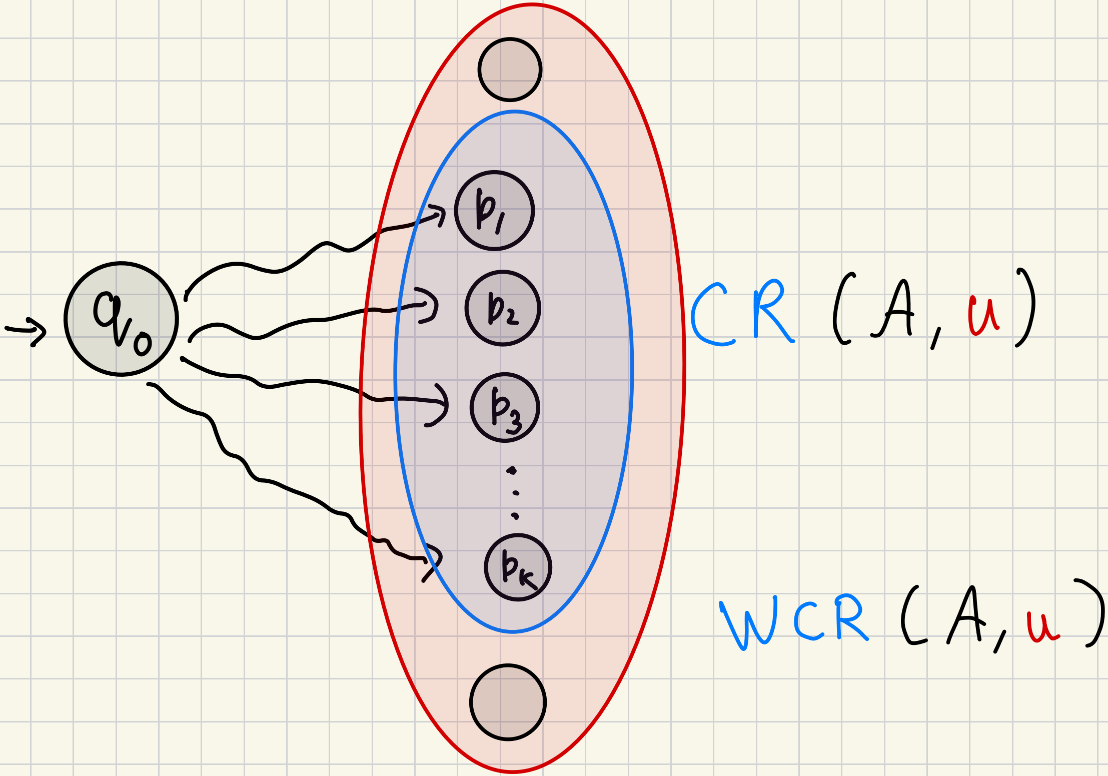
Automaton A and a finite word u.



$CR(A, u)$

Weak coreachability

Automaton $\mathcal{A}$ and a finite word u.



Winning token games from everywhere

Eve wins 1-token game from everywhere in A

if she wins $G1(p, q)$ for all $(p, q) \in CR(A)$

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Eve wins 1-token game from everywhere in A

if she wins $G1(p; q)$ for all $(p, q) \in$ ~~$CR(A)$~~
$WCR(A)$.

Recall: $G1$ is transitive

Safe coverage

Defn. A coBüchi automaton $\mathcal{A}$ has safe coverage if for all q, there is $p \in \mathrm{WCR}(A, q)$ such that,

$$\text{Eve wins } G1(p\,; q) \text{ in } A_{>1}.$$

Theorem: Every coBüchi automaton on which Eve wins the 1-token game from everywhere is HD.

Proof. CoBüchi automaton $\mathcal{A}$

$\rightsquigarrow$ Modify

$\mathcal{A}$ has safe coverage

$\downarrow$ Prove

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$$q \quad p_0 \quad \underbrace{p_1 \quad p_2}_{G1(p_2; p_1)} \quad \ldots$$

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$$q \quad p_0 \quad \underbrace{p_1 \quad p_2}_{G1(p_2; p_1)} \quad \cdots \; p \; \cdots \; p \; \cdots$$

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and Eve wins $G1(p; p)$ in $A_{>1}$.

$q \quad p_0 \quad p_1 \quad p_2 \quad \ldots \; p \; \ldots \; p \; \ldots$

$G1(p_2; p_1)$

Recall: $G1$ is transitive

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$$\Downarrow$$

$(A_{>1}, p)$ is determinisable-by-pruning.

Safe coverage

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$$\text{and Eve wins } G1(p; p) \text{ in } A_{>1}.$$

$$\Downarrow$$

$$(A_{\geq 1}, p) \text{ is determinisable-by-pruning.}$$

Safe-deterministic states:

States p, such that Eve wins $G1(p; p)$ in $A_{>1}$.

Lemma. If a coBüchi automaton $\mathcal{A}$ has safe coverage and Eve wins 1-token game from everywhere in $\mathcal{A}$ then $\mathcal{A}$ is HD.

Proof. In the HD game on $\mathcal{A}$, Adam builds a word w.

$$w = a_0 \quad a_1 \quad a_2 \quad a_3 \quad \ldots$$

Safe coverage

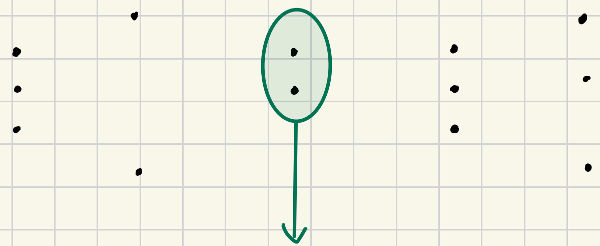
For each q $\exists p \in WCR(A, q)$ s.t. Eve wins $G1(p; q)$ in $A_{\geq 1}$ and $(A_{\geq 1}, p)$ is DBP.

Lemma. If a coBüchi automaton $\mathcal{A}$ has safe coverage and Eve wins 1-token game from everywhere in $\mathcal{A}$ then $\mathcal{A}$ is HD.

Proof. In the HD game on $\mathcal{A}$, Adam builds a word w.
Eve build a DAG D_w in her memory as follows:

$w = \; a_0 \quad a_1 \quad a_2 \quad a_3 \quad \ldots$

D_w



States p in $WCR(A, a_0 a_1)$ that are safe deterministic

Safe coverage

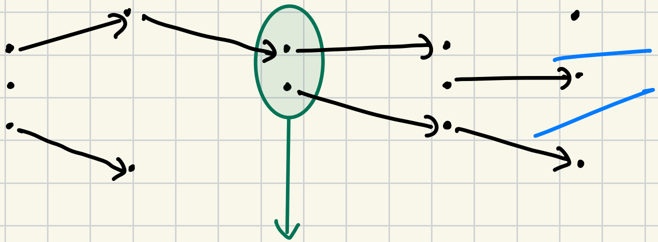
For each q $\exists p \in WCR(A, q)$ s.t. Eve wins $G1(p; q)$ in $A_{\geq 1}$ and $(A_{\geq 1}, p)$ is DBP.

Lemma. If a coBüchi automaton $\mathcal{A}$ has safe coverage and Eve wins 1-token game from everywhere in $\mathcal{A}$ then $\mathcal{A}$ is HD.

Proof. In the HD game on $\mathcal{A}$, Adam builds a word ω. Eve build a DAG D_ω in her memory as follows:

$\omega = \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad \ldots$

D_ω



deterministic transitions of priority 2 in $\mathcal{A}_{>1}$.

States p in $WCR(\mathcal{A}, a_0 a_1)$ that are safe deterministic

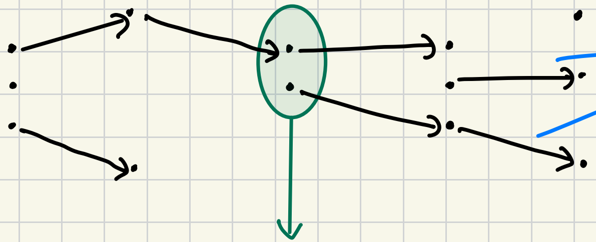
Safe coverage

For each q $\exists p \in WCR(A, q)$ s.t. Eve wins $G1(p;q)$ in $A_{>1}$ and $(A_{>1}, p)$ is DBP.

Proof. In the HD game on $\mathcal{A}$, Adam builds a word w.
Eve build a DAG D_w in her memory as follows:

$w = \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad \ldots$

D_w



deterministic transitions of priority 2 in $\mathcal{A}_{>1}$.

States p in $WCR(\mathcal{A}, a_0 a_1)$ that are safe deterministic

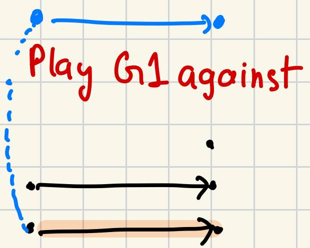
Claim: If w is in $L(\mathcal{A})$, then D_w contains an infinite path.

Proof. Eve builds a run in the HD game using D_w.

$w = a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \ldots$

Eve's token

D_w



Recall:

Eve wins 1-token game from everywhere

Proof. Eve builds a run in the HD game using D_w.

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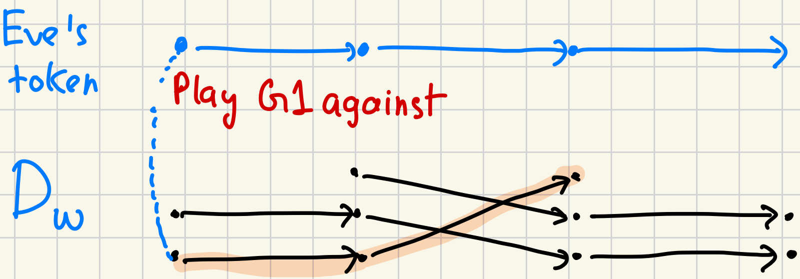

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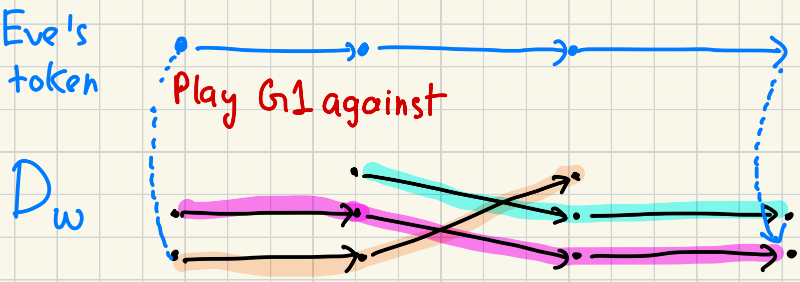
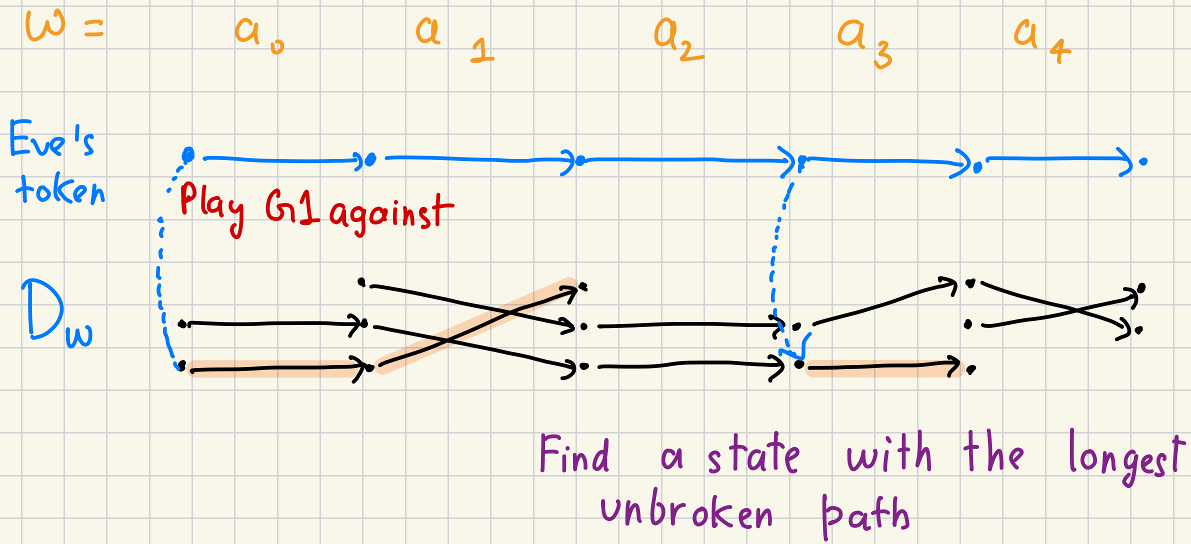


Find a state with the longest unbroken path

Recall:

Eve wins 1-token game from everywhere

Proof. Eve builds a run in the HD game using D_w.

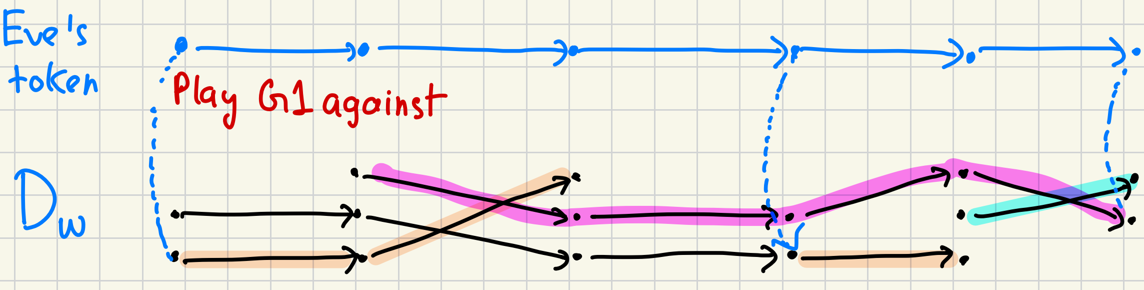


Recall:

Eve wins 1-token game from everywhere

Proof. Eve builds a run in the HD game using D_w.

$w = a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4$



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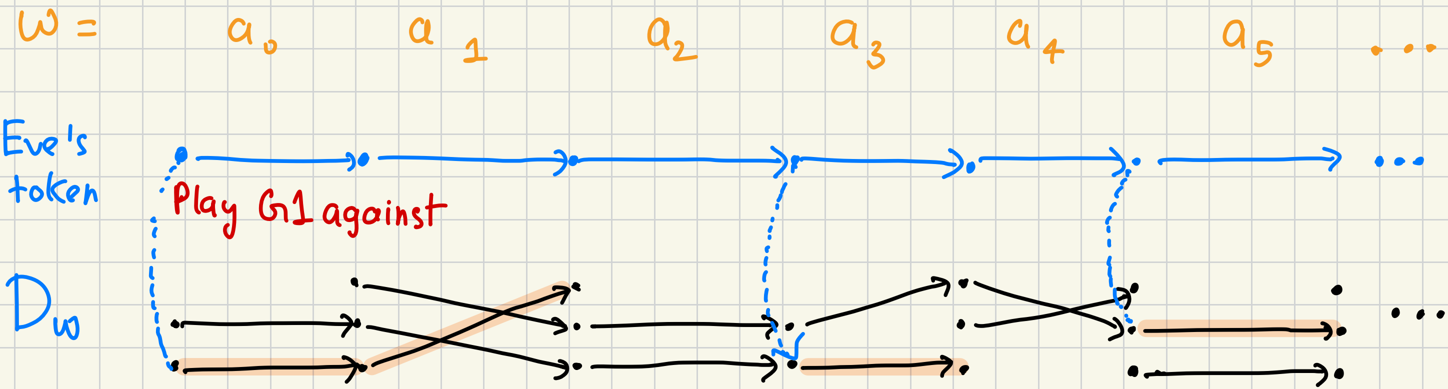

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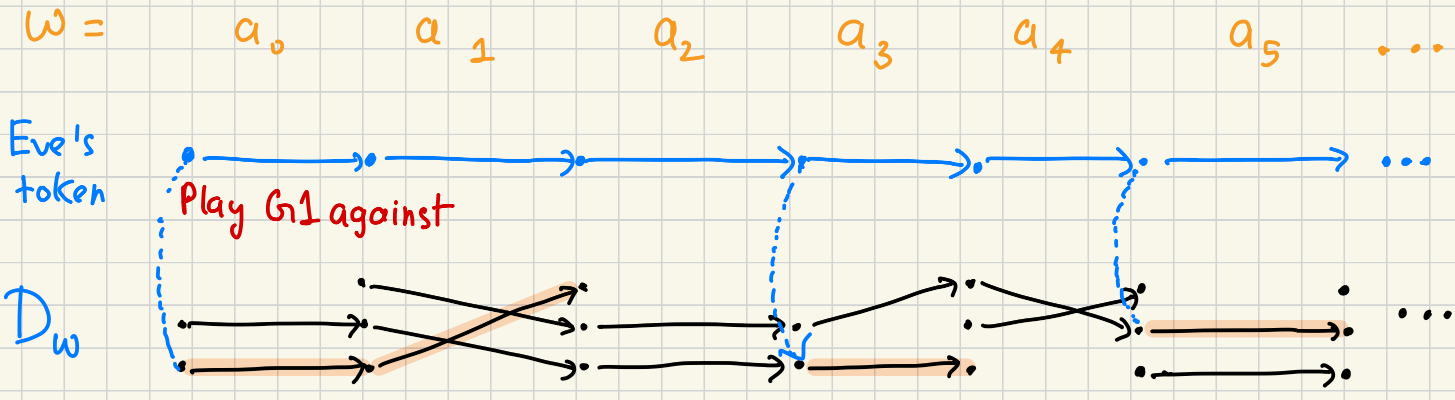
Proof. Eve builds a run in the HD game using D_w.



Find a state with the longest unbroken path

w in $L(A) \Rightarrow D_w$ contains an ∞ path

$\Downarrow$

Eve's run is accepting.

Recall:

Eve wins 1-token game from everywhere

Lemma. If a coBüchi automaton $\mathcal{A}$ has safe coverage and Eve wins 1-token game from everywhere in $\mathcal{A}$ then $\mathcal{A}$ is HD.

Theorem. For every Büchi or coBüchi automaton $\mathcal{A}$, if Eve wins the 1-token game from everywhere on $\mathcal{A}$ then $\mathcal{A}$ is HD.

Proof sketch.

CoBüchi automata
or [1,2] automata

Automaton $\mathcal{A}$

Modify

$\mathcal{A}$ has safe-coverage.

Prove

$\mathcal{A}$ is HD

Büchi automata
or [0,1] automata

Automaton $\mathcal{A}$

Modify

$\mathcal{A}$ has reach-covering.

Prove

$\mathcal{A}$ is HD.

Büchi automata: reach covering

Reach covering.

For each q $\exists p \in WCR(A, q)$ s.t.
Eve wins $G1(q; p)$ in $A_{>0}$.

Büchi automata: reach covering

For coBüchi automata: safe coverage

Reach covering.

For each q $\exists p \in WCR(A,q)$ s.t. Eve wins $G1(q;p)$ in $A_{>0}$.

Safe coverage

For each q $\exists p \in WCR(A,q)$ s.t. Eve wins $G1(p;q)$ in $A_{>1}$.

Büchi automata $\longrightarrow$ $[0,1]$ automata

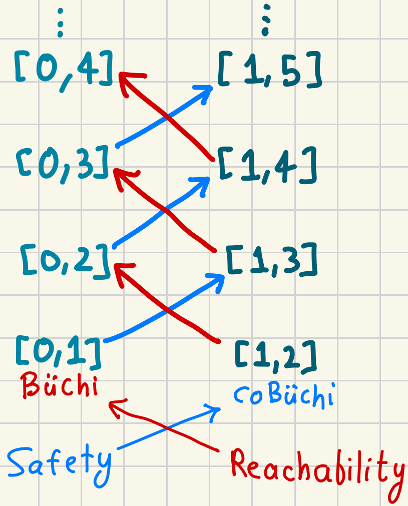
Even-to-odd

2-token conjecture for $[0,K]$ automata

$\Downarrow$

2-token conjecture for $[1,K+2]$ automata

Safe double coverage



Odd-to-even

2-token conjecture for $[1,K]$ automata

$\Downarrow$

2-token conjecture for $[0,K]$ automata

Even-to-odd

2-token conjecture for $[0,K]$ automata

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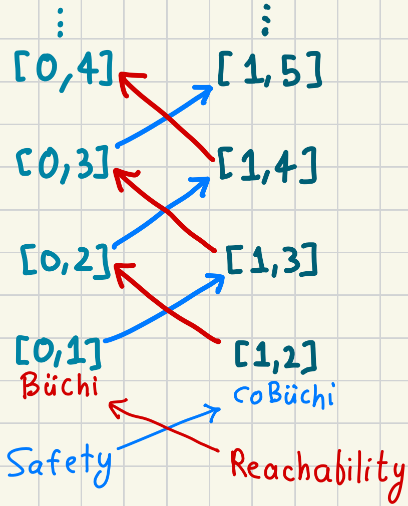
Safe double coverage



Odd-to-even

2-token conjecture for $[1,K]$ automata

$\Downarrow$

2-token conjecture for $[0,K]$ automata

For each q $\exists\, p \in WCR(A,q)$ s.t.

Eve wins $G2(p; q, q)$ in $A_{>1}$.

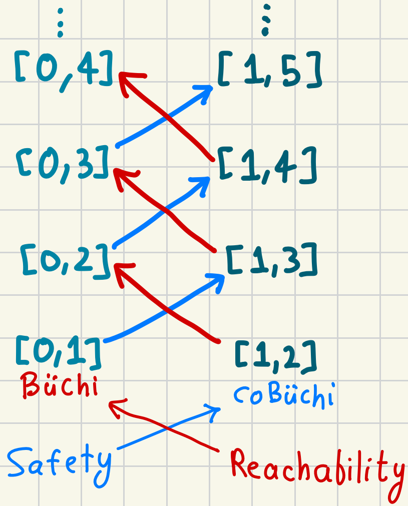
Even-to-odd

2-token conjecture for [0,K] automata

$\Downarrow$

2-token conjecture for [1,K+2] automata

Safe double coverage



Odd-to-even

2-token conjecture for [1,K] automata

$\Downarrow$

2-token conjecture for [0,K] automata

Reach double covering

For each q $\exists\, p \in WCR(A,q)$ s.t. Eve wins $G2(p;q,q)$ in $A_{>1}$.

For each q $\exists\, p \in WCR(A,q)$ s.t. Eve wins $G2(q;p,p)$ in $A_{>0}$.

2-Token Theorem. For every parity automaton $\mathcal{A}$,

$\mathcal{A}$ is history-deterministic

$\Longleftrightarrow$

Eve wins the 2-token game on $\mathcal{A}$

Problem. Given a parity automaton, decide if it is HD.

Lower bound:

NP-hard

Upper bound:

PSPACE,

$2^{3d} \cdot \mathrm{Poly}(n)$

Find these results in:

History-Deterministic Parity automata: Games, Comblexity, and the 2-Token Theorem, on arxiv

Open Problems

Given a parity automaton, decide if it is HD.

Lower bound:

NP-hard

Upper bound:

PSPACE

Open Topic

* Resolving Nondeterminism in Automata with Randomness, with Tom A. Henzinger and K. S. Thejaswini

Open Topic

* Resolving Nondeterminism in Automata with Randomness,
  with Tom A. Henzinger and K. S. Thejaswini

Conjecture: Every HD Parity automata can be efficiently transformed into one where Eve almost surely wins the HD game using positional memoryless strategy.

(True for coBüchi HD automata)

Open Problem.

* Does the 2-token conjecture hold for infinite-state systems with parity / ω-regular acceptance condition?

  E.g. timed automata, pushdown automata

Open Problem.

* Does the 2-token conjecture hold for infinite-state systems with parity / ω-regular acceptance condition?
  E.g. timed automata, pushdown automata

Theorem: There is an infinite state coBüchi automaton $\mathcal{A}$ s.t., Eve wins 2-token game on $\mathcal{A}$ but $\mathcal{A}$ is not HD.

2-Token Theorem. For every parity automaton A,

A is history-deterministic

$\Longleftrightarrow$

Eve wins the 2-token game on A

Thanks!

:)

Problem. Given a parity automaton, decide if it is HD.

Lower bound:

NP-hard

Upper bound:

PSPACE,

$2^{3d} \cdot \mathrm{Poly}(n)$