CS3430 S26: Scientific Computing
My Observations on HW 5, Problem 3 – Ramanujan and Chudnovsky

Vladimir Kulyukin

Problem 3: Interpretation of Unit Test Output (Ramanujan and Chudnovsky)

In this problem, I compared two extremely fast algorithms for computing π: Ramanujan's series and the Chudnovsky series implemented using binary splitting. Both generators were implemented with `mpmath` using fixed working precision, and their convergence behavior was examined term by term.

Structural Expectations

The structural unit tests confirmed that both required generators exist and are callable:

- `pi_ramanujan_mp`
- `pi_chudnovsky_bs_mp`

They also confirmed that both generators yield successive approximations of π. The first three values produced by each generator were:

```
Ramanujan first 3 approximations:
[3.1415927300133056603...,
 3.1415926535897938780...,
 3.1415926535897932384...]

Chudnovsky(BS) first 3 approximations:
[3.1415926535897342076...,
 3.1415926535897932384...,
 3.1415926535897932384...]
```

Even from these raw values, it is clear that both methods converge very rapidly, with Chudnovsky approaching the true value of π almost immediately.

Ramanujan Series Convergence

The convergence test for Ramanujan's series examined the first three terms. The observed absolute errors were:

```
term 1: |err| = 7.642351242e-8
term 2: |err| = 6.395362624e-16
term 3: |err| = 5.682423256e-24
```

These results show that Ramanujan's series converges extremely fast. After just two terms, the approximation is already accurate to roughly 15 decimal places, and the third term pushes the error well below 10^{-20}. At this point, further improvements are increasingly limited by numerical representation rather than by the mathematics of the series.

Chudnovsky Series with Binary Splitting

The Chudnovsky series, implemented using binary splitting, converges even faster. The unit tests reported the following absolute errors:

```
term 1: |err| = 5.903079419e-14
term 2: |err| = 3.078478043e-28
term 3: |err| = 1.72052026e-42
```

After only two terms, the approximation already exceeds 25 correct decimal digits, and the third term pushes the error below 10^{-40}. This behavior justifies the description of Chudnovsky's method as *explosively fast*. At this stage, the limiting factor is no longer convergence, but the available working precision.

Key Lesson Reinforced by the Tests

These unit tests reinforce several important ideas from lecture:

- Both Ramanujan and Chudnovsky formulas are mathematically correct.
- Their convergence rates differ dramatically.
- With sufficiently fast algorithms, numerical representation becomes the dominant bottleneck.

In particular, the Chudnovsky series converges so quickly that additional terms provide no visible benefit unless the working precision is increased. This observation motivates the use of arbitrary-precision arithmetic and careful precision management in high-accuracy scientific computing.

Takeaway

Problem 3 demonstrates that modern π algorithms can converge far faster than floating-point representations can support. Once convergence becomes this rapid, correctness depends not only on the mathematical formula, but on how precision is chosen, managed, and interpreted.