Control Theory (AE 308)

Course Project Report

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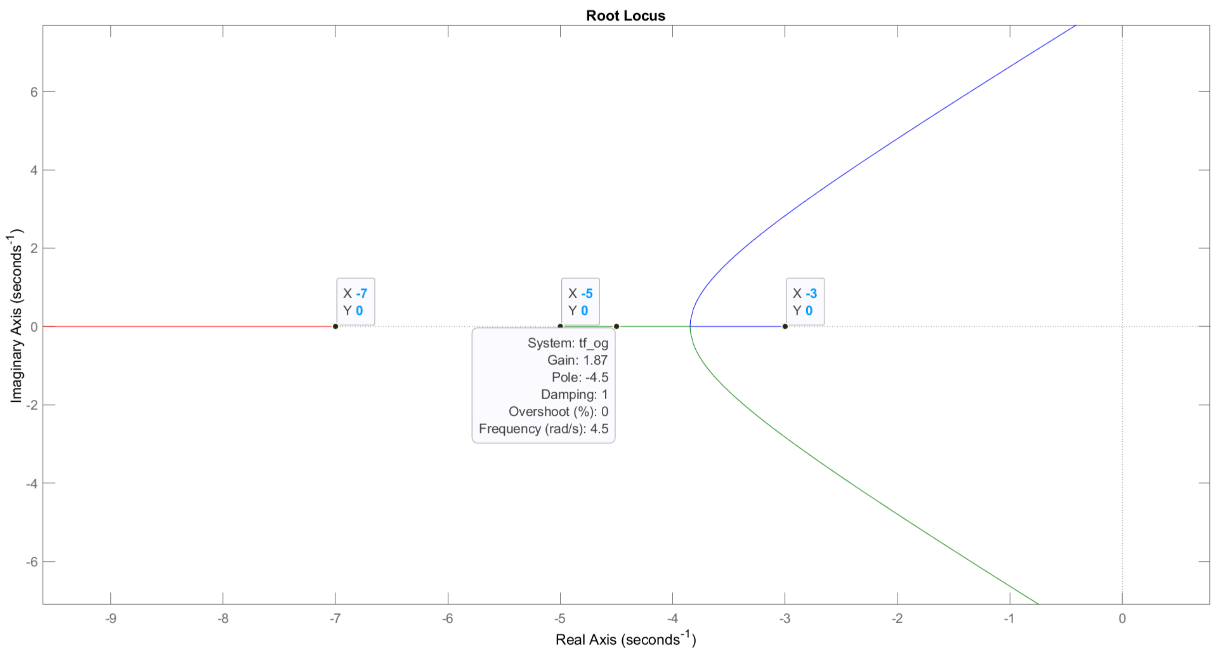
1. Introduction

The Open Loop transfer function given in the problem statement was :

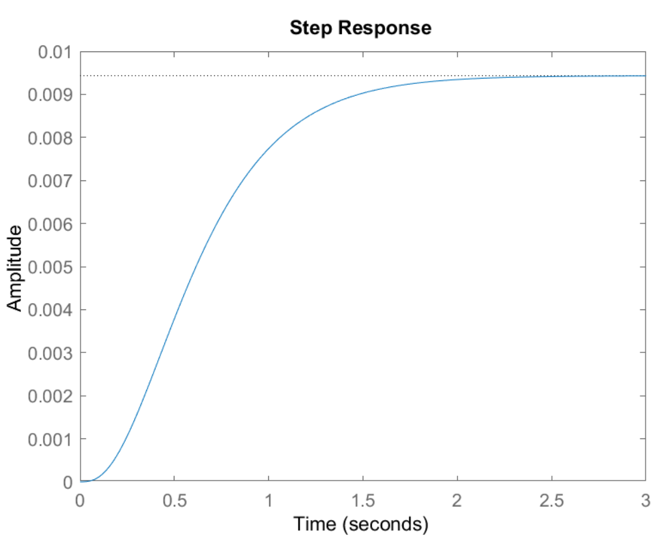
$$G(s) = \frac{K}{(s+3)(s+5)(s+7)}$$

The given Open Loop Transfer Function has poles at -3,-5,-7 and no zeroes.

The root locus for this Transfer Function is as follows :



Application of Step Input gives the following response :



The Performance Parameters of this Open Loop Transfer Function for Step Input are :

RiseTime: 0.9733
TransientTime: 1.7549
SettlingTime: 1.7549
SettlingMin: 0.0085
SettlingMax: 0.0094
Overshoot: 0
Undershoot: 0
Peak: 0.0094
PeakTime: 3.4822

It is evident that the Peak of response is quite less than the desired value, ie, 1.

2. Control Objectives

The Objective is to tune the Transfer function so as to fulfill 3 condition :

- The steady state error for a step input response should be zero.
- The settling time for a step input response should be less than 2 sec.
- The % overshoot for a step input response should be less than 25%.

Presently the Settling time (1.7549sec) and Overshoot(0) are well under the limits. But the Steady State Error is not zero.

3. Controller Design

In order to get zero steady state error, a PI controller has to be used.
The Transfer Function for a PI controller is :

$$G(s) = \frac{K(s+a)}{s}$$

For getting the values of K and a, we have to use the % overshoot and settling time conditions to find dominant poles from where we want the root locus to pass.
Using the % Overshoot condition,

$$exp(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}) = 0.25$$

$$\zeta = \sqrt{\frac{\ln^2(0.25)}{\ln^2(0.25)+\pi^2}}$$

Using the Settling Time condition,

$$\frac{4}{\sigma} = 2$$

$$\sigma = \omega_n\zeta$$

$$\sigma = 2$$

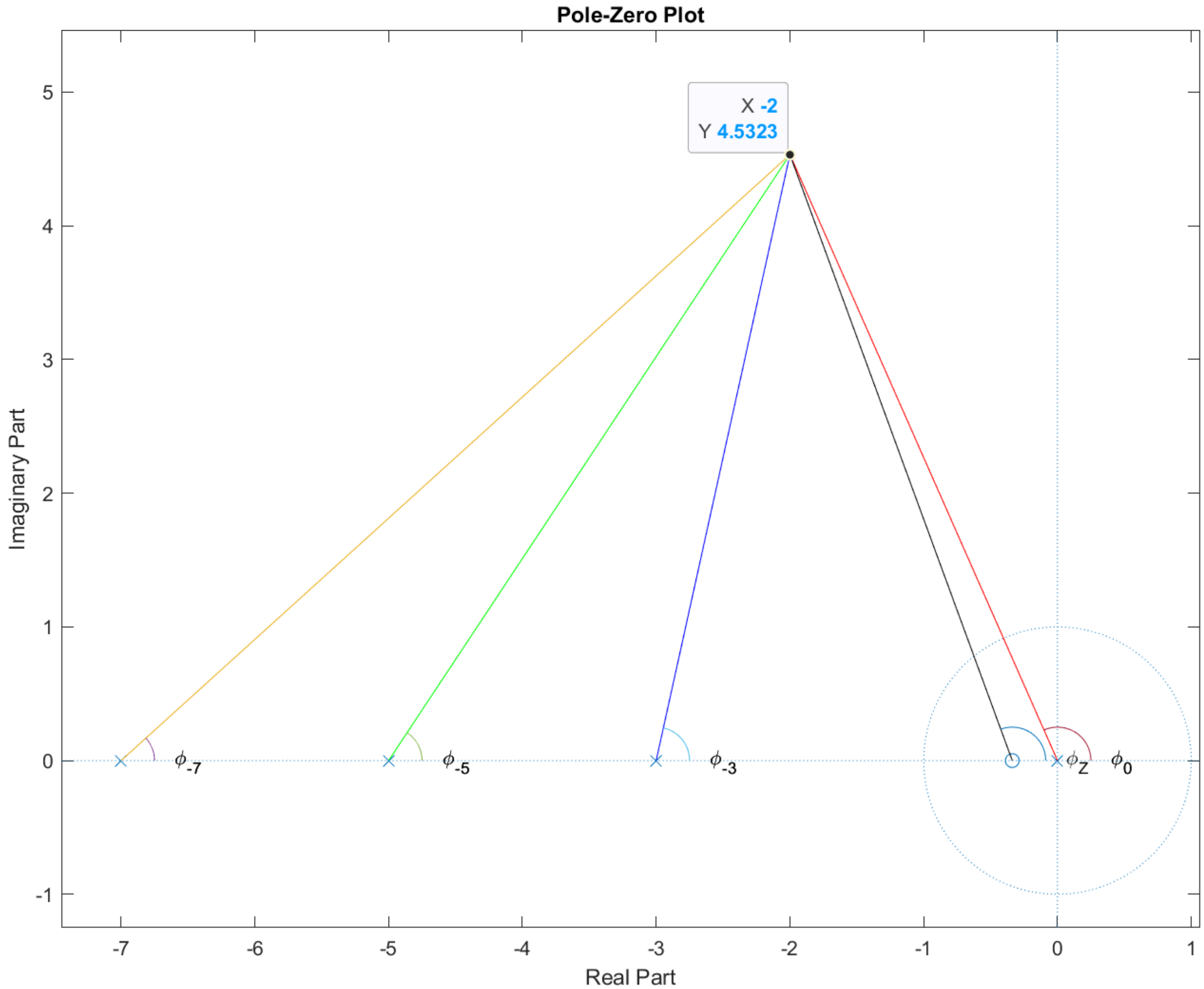
$$\omega_n = 4.9540$$

$$\omega_d = \sqrt{\omega_n^2 - \sigma^2}$$

$$\omega_d = 4.5323$$

The required dominant pole will be at : $P_{PI} = -\sigma \pm \omega_d$

Introduction of PI Controller also made a pole at origin and a zero somewhere in the s plane. To find that zero, the angle contribution from pole P_{PI} due to rest of the other poles and zeros will be equal to -180°.



$$-\phi_{-7} - \phi_{-5} - \phi_{-3} + \phi_{Z_{PI}} - \phi_{P_{PI}} = -180^\circ$$
$$-42.19^\circ - 56.59^\circ - 77.53^\circ + \phi_{Z_{PI}} - 113.8^\circ = -180^\circ$$
$$\phi_{Z_{PI}} = 110.13^\circ$$

$$\tan(\phi_{Z_{PI}}) = \frac{4.5323}{-2 - Z_{PI}}$$
$$Z_{PI} = -0.338$$

The final Open Loop Transfer Function will be :

$[G_c \cdot G_p = K_p(\frac{s+0.338}{s})\frac{1}{(s+3)(s+5)(s+7)}]$

Since $s = -2 + 4.5323j$ will satisfy the closed loop transfer function as a pole, this is a zero of the transfer function $G_c \cdot G_p + 1$

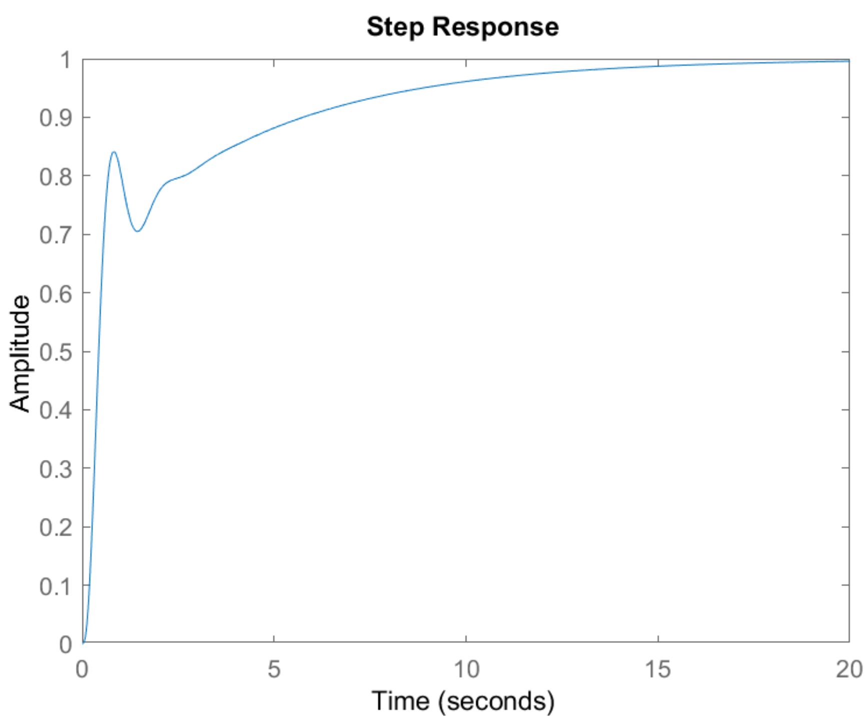
Putting s in the equation and taking the magnitude of both sides,

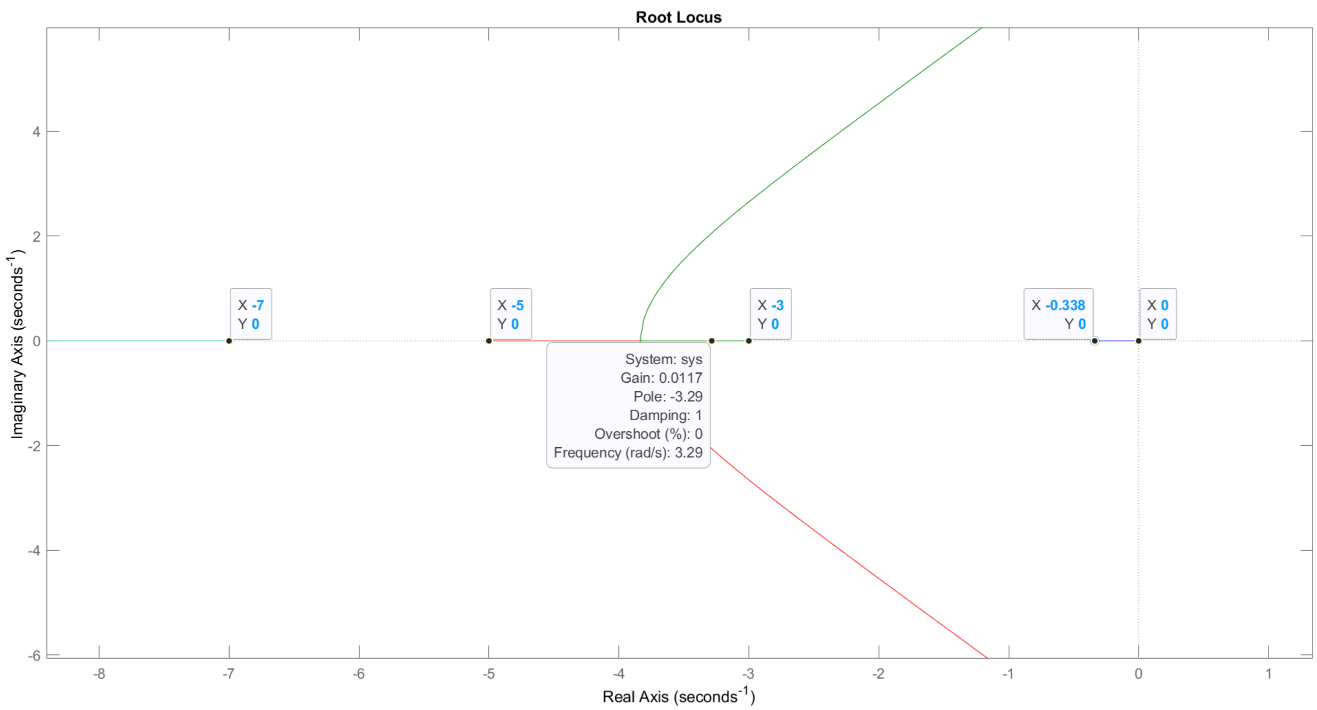
$|G_c \cdot G_p| = 1$

We get $K_p = 173.84$

4. Simulation Result

Using $K_p = 173.84$ and a = 0.338 in the final open loop transfer function and plotting the step input response of closed loop transfer function of the same.





The Performance Parameters of this Closed Loop Transfer Function for Step Input are :

RiseTime: 5.5677
TransientTime: 12.9812
SettlingTime: 12.9812
SettlingMin: 0.9000
SettlingMax: 0.9977
Overshoot: 0
Undershoot: 0
Peak: 0.9977
PeakTime: 22.7535

The steady state error for the system has drastically reduced but at the same time, the settling time has also gone up. Finetuning the Kp and a values manually, it was found that the Settling time was below 2 sec and %Overshoot was less than 25% when Kp = 186 and a = -1.57. Following performance parameters were observed.

RiseTime: 0.3792
TransientTime: 1.9949

SettlingTime: 1.9949

SettlingMin: 0.8926

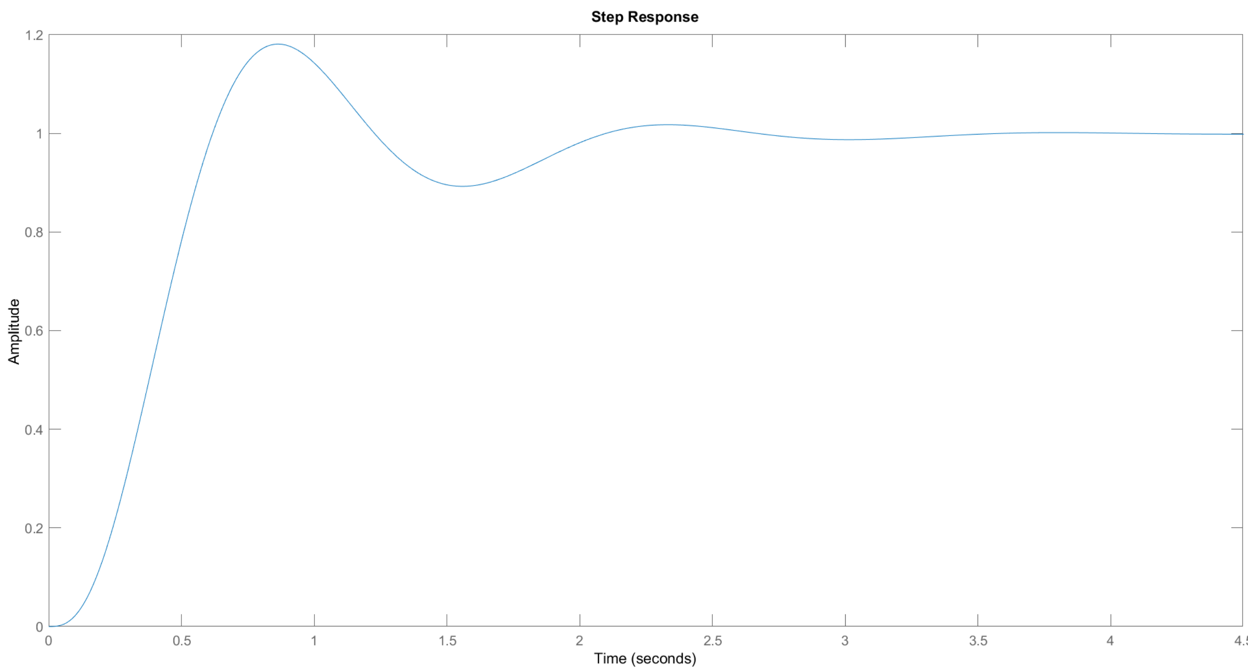
SettlingMax: 1.1811

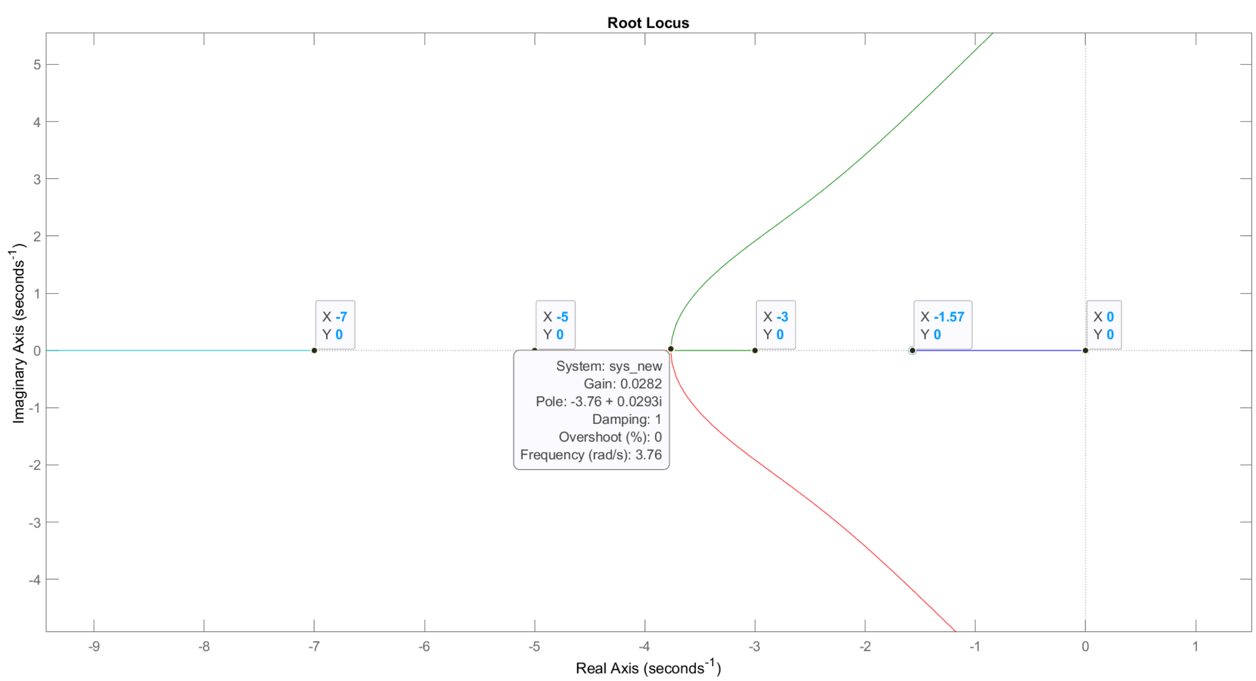
Overshoot: 18.1131

Undershoot: 0

Peak: 1.1811

PeakTime: 0.8641





5. Conclusion

The transfer function was successfully tuned and the results were well in line with the desired outputs.
The Final Open Loop Transfer Function is :

$$G_c \cdot G_p = 186(\frac{s+1.57}{s})\frac{1}{(s+3)(s+5)(s+7)}$$

The respective Closed Loop Transfer Function for a unity Feedback is :

$$\frac{186\, s\, (s+1.57)\, (s+3)\, (s+5)\, (s+7)}{s\, (s+3)\, (s+5)\, (s+7)\, (s+10.66)\, (s+1.321)\, (s^2 + 3.02s + 20.74)}$$