

Gray, Thompson (1996)

Minimisation des révisions au filtre
symétrique sous contrainte

Contrainte

Filtre asymétrique
sans biais
 $\Rightarrow X'_d \hat{\theta}^a = e_1$

Filtre asymétrique
de biais constant
 $\Rightarrow X'_{d-1} \hat{\theta}^a = e_1$

$$y_t = \sum_{j=0}^d \beta_j t^j + \xi_t + \varepsilon_t, \text{ avec } \varepsilon_t \text{ bruit blanc } \mathbb{V}[\varepsilon_t] = \sigma^2 \text{ et } \xi_t \text{ non corrél     } \varepsilon_t, \\ \Omega = \mathbb{V}[(\xi_{t-h}, \dots, \xi_{t+h})]$$

$$\min R(\theta) = \mathbb{E} \left[(M_{\theta^s} y_t) - M_{\theta} y_t)^2 \right]$$

$$= I(\theta, 0, y_t, M_{\theta^s} y_t)$$

$$sc. \quad X'_d \theta = e_1 \text{ ou } X'_{d-1} \theta = e_1$$

Théorie générale

$$\begin{cases} I(\theta, q, y_t, u_t) = \mathbb{E} [(\Delta^q M_\theta y_t - u_t)^2] \\ J(\theta, f, \omega_1, \omega_2) = \int_{\omega_1}^{\omega_2} f[\phi_\theta(\omega), \varphi_\theta(\omega)] \, d\omega \end{cases}$$

$$\hat{\theta} \in \underset{sc. \quad C\theta = a}{\operatorname{argmin}} \sum \alpha_i I(\theta, q_i, y_t, u_t^{(i)}) + \beta_i J(\theta, f_i, \omega_1^{(i)}, \omega_2^{(i)})$$

$$\begin{aligned} F_g(\theta) &= I(\theta, 0, y_t, \mathbb{E}[L\theta y_t]) \\ S_g(\theta) &= I(\theta, q, y_t, \mathbb{E}[L\theta y_t]) \\ T_g(\theta) &= J(\theta, f: (\rho, \varphi) \mapsto \rho^2 \sin(\varphi)^2, 0, \omega_1) \\ y_t &= \sum_{i=0}^d \beta_j t^{ij} + \varepsilon_t, \quad \varepsilon_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2) \end{aligned}$$
Guggemos *et al* (2018)

$$\begin{aligned} F_g(\theta) &= \sum_j \theta_j^2 \\ S_g(\theta) &= \sum_j (\Delta^q \theta_j)^2 \\ T_g(\theta) &= \int_0^{\omega_1} \phi_\theta^2(\omega) \sin^2(\varphi_a(\omega)) \, d\omega \\ \hat{\theta} &\in \operatorname{argmin} \nu_1 F_g(\theta) + \nu_2 S_g(\theta) + (1 - \nu_1 - \nu_2) T_g(\theta) \\ \text{s.c.} \quad X'_q \theta &= e_1 \end{aligned}$$

Wildi, McElroy (2019)

$$\begin{aligned} A_w(\theta) &= 2 \int_0^{\omega_1} (\rho_s(\omega) - \rho_\theta(\omega))^2 h(\omega) \, d\omega \\ T_w(\theta) &= 8 \int_0^{\omega_1} \rho_s(\lambda) \rho_\theta(\lambda) \sin^2 \left(\frac{\varphi_\theta(\omega)}{2} \right) h(\omega) \, d\omega \\ S_w(\theta) &= 2 \int_{\omega_1}^{\pi} (\rho_s(\omega)^2 - \rho_\theta(\omega))^2 h(\omega) \, d\omega \\ \min \nu_1 A_w + \nu_2 T_w + (1 - \nu_1 - \nu_2) S_w \end{aligned}$$

$$\sigma^2 = 0, \Omega = K^{-1}$$
$$\begin{cases} f_1(\rho, \varphi, \omega) = 2(\rho_s(\omega) - \rho)^2 h(\omega) \\ f_2(\rho, \varphi, \omega) = 8\rho_s(\omega)\rho \sin^2\left(\frac{\varphi}{2}\right) h(\omega) \\ A_w(\theta) = J(\theta, f_1, 0, \omega_1) \\ T_w(\theta) = J(\theta, f_2, 0, \omega_1) \\ S_w(\theta) = J(\theta, f_1, \omega_1, \pi) \\ R_w(\theta) = J(\theta, f_2, \omega_1, \pi). \end{cases}$$

Dagum et Bianconcini (2008) — RKHS

$f_0(t)$ noyau continu, P_i polynômes orthonormaux de $\mathbb{L}^2(f_0)$ et $K_p(t) = \sum_{i=0}^{p-1} P_i(t)P_i(0)f_0(t)$.

$$\hat{\theta}_i = \frac{K_p(i/b)}{\sum_{j=-b}^q K_p(j/b)}$$

b choisit optimalement pour minimiser :

- l'erreur quadratique moyenne de révision ($b_{q,\gamma}$)
- l'accuracy $A_w(b_{q,G})$
- la smoothness $S_w(b_{q,s})$
- la timeliness $T_w(b_{q,\varphi})$

$$\begin{array}{ll} \min & R(\theta), A_w(\theta), S_w(\theta), \\ & \text{ou } T_w(\theta) \\ \text{s.c.} & \theta_i = K_p(i/b) \text{ et} \\ & \sum_i \theta_i = 1 \end{array}$$

$$\begin{array}{ll} \min & R(\theta) + \alpha_T T_g(\theta) \\ \text{s.c.} & \text{linéaires} \end{array}$$

Extension en intégrant la timeliness

Minimisation sous contrainte des révisions + $\alpha_T T_q(\theta)$

 $\alpha_T = 0$

Proietti et Luati (2008)

 $d = 1$

LC/Musgrave

$y_t = \gamma_0 + \delta t + \varepsilon_t$,
 ε_t bruit blanc et θ^a préserve
 constantes

$$\begin{cases} U = X_0 \\ Z = x_1 \\ D = \sigma^2 I \end{cases}$$

Modèle général

$$y = U\gamma + Z\delta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, D) \text{ et } [U \ Z] \subset X$$

Minimisation des révisions à θ^s sous contrainte :

$$U_p' \theta = U' \theta^s, \quad U = \begin{bmatrix} U_p \\ U_f \end{bmatrix} \text{ avec } U_p \text{ de } h+1+q \text{ lignes}$$
$$d := 2$$
 $d = 3$
$$K^{-1}$$
$$\sigma^2 = 0, \Omega = K^{-1},$$

$$d = 3$$

$$\begin{cases} U = X_1 \\ Z = x_2 \\ D = \sigma^2 I \end{cases}$$

$$\begin{cases} U = X_2 \\ Z = x_3 \\ D = \sigma^2 I \end{cases}$$

$$\begin{cases} D = K^- \\ U = X_3 \\ Z = 0 \end{cases}$$