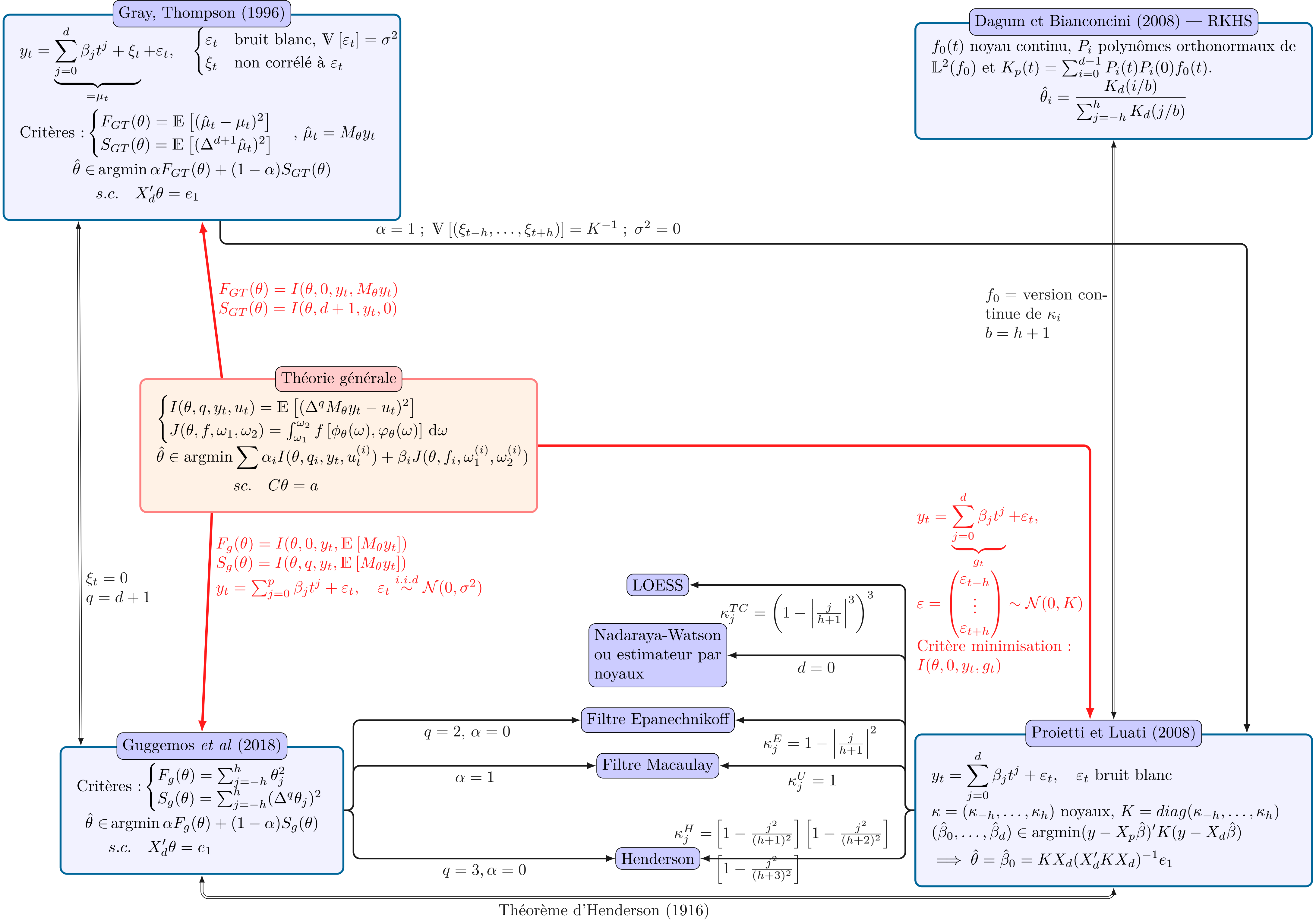
Gray, Thompson (1996)
$y_t = \underbrace{\sum_{j=0}^{d}\beta_j t^j + \xi_t}_{=\mu_t} + \varepsilon_t, \quad \begin{cases}\varepsilon_t & \text{bruit blanc, } \mathbb{V}[\varepsilon_t]=\sigma^2\\ \xi_t & \text{non corrélé à } \varepsilon_t\end{cases}$
Critères : $\begin{cases}F_{GT}(\theta) = \mathbb{E}\left[(\hat\mu_t-\mu_t)^2\right]\\ S_{GT}(\theta) = \mathbb{E}\left[(\Delta^{d+1}\hat\mu_t)^2\right]\end{cases}$ $, \hat\mu_t = M_\theta y_t$
$\hat\theta \in \operatorname{argmin} \alpha F_{GT}(\theta) + (1-\alpha)S_{GT}(\theta)$
$s.c. \quad X_d'\theta = e_1$
Dagum et Bianconcini (2008) — RKHS
$f_0(t)$ noyau continu, P_i polynômes orthonormaux de $\mathbb{L}^2(f_0)$ et $K_p(t) = \sum_{i=0}^{d-1} P_i(t)P_i(0)f_0(t)$.
$\hat\theta_i = \frac{K_d(i/b)}{\sum_{j=-h}^{h} K_d(j/b)}$
$\alpha = 1$; $\mathbb{V}\left[(\xi_{t-h},\dots,\xi_{t+h})\right] = K^{-1}$; $\sigma^2 = 0$
$F_{GT}(\theta) = I(\theta, 0, y_t, M_\theta y_t)$
$S_{GT}(\theta) = I(\theta, d+1, y_t, 0)$
$f_0 =$ version continue de κ_i
$b = h+1$
Théorie générale
$\begin{cases}I(\theta, q, y_t, u_t) = \mathbb{E}\left[(\Delta^q M_\theta y_t - u_t)^2\right]\\ J(\theta, f, \omega_1, \omega_2) = \int_{\omega_1}^{\omega_2} f\left[\phi_\theta(\omega), \varphi_\theta(\omega)\right] \mathrm{d}\omega\end{cases}$
$\hat\theta \in \operatorname{argmin} \sum \alpha_i I(\theta, q_i, y_t, u_t^{(i)}) + \beta_i J(\theta, f_i, \omega_1^{(i)}, \omega_2^{(i)})$
$sc. \quad C\theta = a$
$y_t = \underbrace{\sum_{j=0}^{d}\beta_j t^j}_{g_t} + \varepsilon_t,$
$\varepsilon = \begin{pmatrix}\varepsilon_{t-h}\\ \vdots \\ \varepsilon_{t+h}\end{pmatrix} \sim \mathcal{N}(0, K)$
Critère minimisation : $I(\theta, 0, y_t, g_t)$
$F_g(\theta) = I(\theta, 0, y_t, \mathbb{E}\left[M_\theta y_t\right])$
$S_g(\theta) = I(\theta, q, y_t, \mathbb{E}\left[M_\theta y_t\right])$
$y_t = \sum_{j=0}^{p}\beta_j t^j + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d}{\sim} \mathcal{N}(0,\sigma^2)$
$\xi_t = 0$
$q = d+1$
LOESS
$\kappa_j^{TC} = \left(1 - \left|\frac{j}{h+1}\right|^3\right)^3$
Nadaraya-Watson ou estimateur par noyaux
$d = 0$
Filtre Epanechnikoff
$q = 2, \alpha = 0$
$\kappa_j^E = 1 - \left|\frac{j}{h+1}\right|^2$
Filtre Macaulay
$\alpha = 1$
$\kappa_j^U = 1$
$\kappa_j^H = \left[1 - \frac{j^2}{(h+1)^2}\right]\left[1 - \frac{j^2}{(h+2)^2}\right]\left[1 - \frac{j^2}{(h+3)^2}\right]$
Henderson
$q = 3, \alpha = 0$
Guggemos *et al* (2018)
Critères : $\begin{cases}F_g(\theta) = \sum_{j=-h}^{h}\theta_j^2\\ S_g(\theta) = \sum_{j=-h}^{h}(\Delta^q\theta_j)^2\end{cases}$
$\hat\theta \in \operatorname{argmin} \alpha F_g(\theta) + (1-\alpha)S_g(\theta)$
$s.c. \quad X_d'\theta = e_1$
Proietti et Luati (2008)
$y_t = \sum_{j=0}^{d}\beta_j t^j + \varepsilon_t, \quad \varepsilon_t$ bruit blanc
$\kappa = (\kappa_{-h},\dots,\kappa_h)$ noyaux, $K = diag(\kappa_{-h},\dots,\kappa_h)$
$(\hat\beta_0,\dots,\hat\beta_d) \in \operatorname{argmin}(y - X_p\hat\beta)'K(y - X_d\hat\beta)$
$\implies \hat\theta = \hat\beta_0 = KX_d(X_d'KX_d)^{-1}e_1$
Théorème d'Henderson (1916)