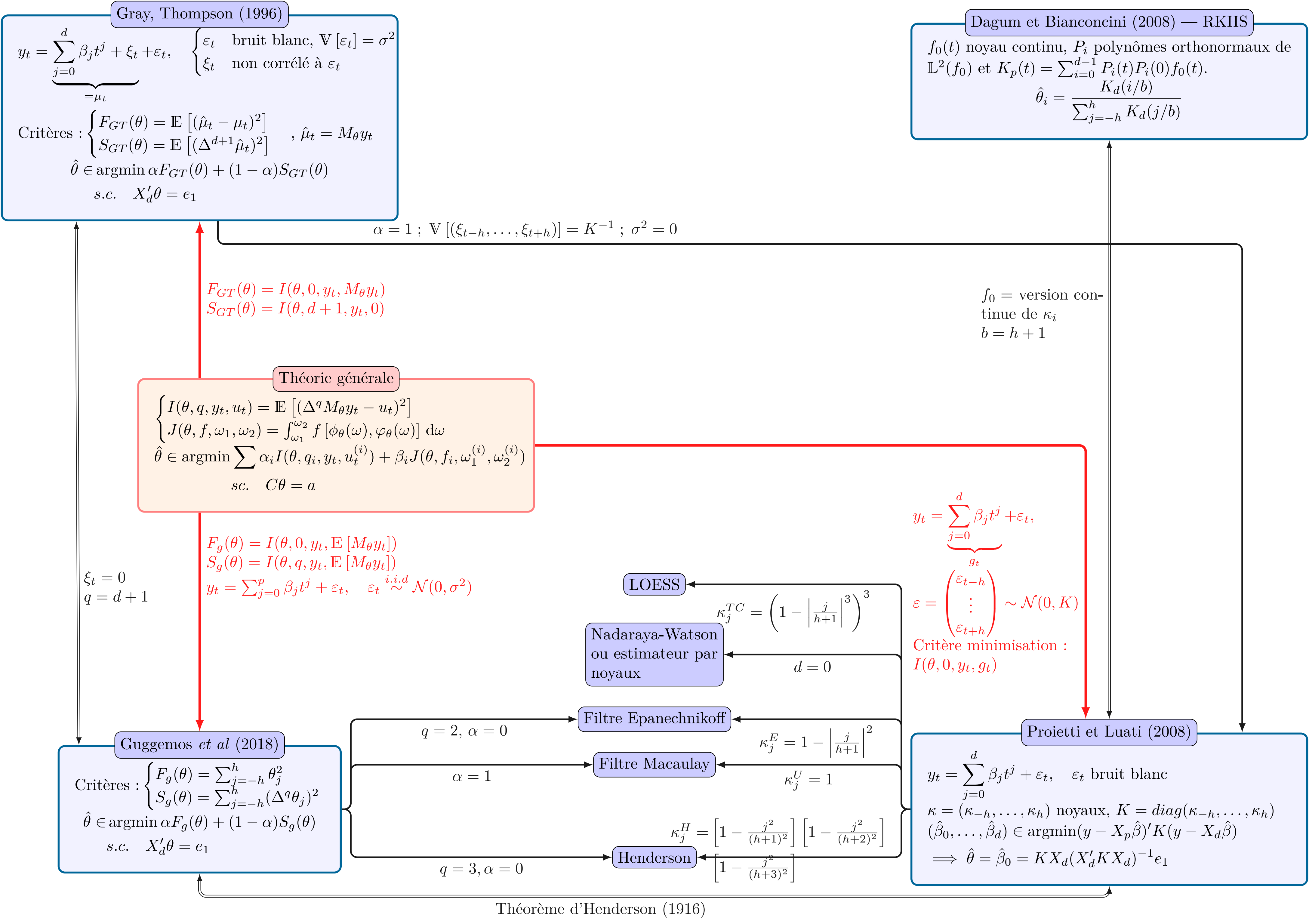

Gray, Thompson (1996)
$y_t = \underbrace{\sum_{j=0}^{d} \beta_j t^j + \xi_t}_{=\mu_t} + \varepsilon_t, \quad \begin{cases} \varepsilon_t & \text{bruit blanc, } \mathbb{V}[\varepsilon_t] = \sigma^2 \\ \xi_t & \text{non corrélé à } \varepsilon_t \end{cases}$
Critères : $\begin{cases} F_{GT}(\theta) = \mathbb{E}\left[(\hat{\mu}_t - \mu_t)^2\right] \\ S_{GT}(\theta) = \mathbb{E}\left[(\Delta^{d+1}\hat{\mu}_t)^2\right] \end{cases}, \ \hat{\mu}_t = M_\theta y_t$
$\hat{\theta} \in \text{argmin}\, \alpha F_{GT}(\theta) + (1-\alpha) S_{GT}(\theta)$
$s.c. \quad X_d'\theta = e_1$
Dagum et Bianconcini (2008) — RKHS
$f_0(t)$ noyau continu, P_i polynômes orthonormaux de $\mathbb{L}^2(f_0)$ et $K_p(t) = \sum_{i=0}^{d-1} P_i(t)P_i(0)f_0(t)$.
$\hat{\theta}_i = \frac{K_d(i/b)}{\sum_{j=-h}^{h} K_d(j/b)}$
$\alpha = 1$; $\mathbb{V}[(\xi_{t-h}, \dots, \xi_{t+h})] = K^{-1}$; $\sigma^2 = 0$
$F_{GT}(\theta) = I(\theta, 0, y_t, M_\theta y_t)$
$S_{GT}(\theta) = I(\theta, d+1, y_t, 0)$
$f_0 =$ version continue de κ_i
$b = h + 1$
Théorie générale
$\begin{cases} I(\theta, q, y_t, u_t) = \mathbb{E}\left[(\Delta^q M_\theta y_t - u_t)^2\right] \\ J(\theta, f, \omega_1, \omega_2) = \int_{\omega_1}^{\omega_2} f\left[\phi_\theta(\omega), \varphi_\theta(\omega)\right] \mathrm{d}\omega \end{cases}$
$\hat{\theta} \in \text{argmin} \sum \alpha_i I(\theta, q_i, y_t, u_t^{(i)}) + \beta_i J(\theta, f_i, \omega_1^{(i)}, \omega_2^{(i)})$
$sc. \quad C\theta = a$
$y_t = \underbrace{\sum_{j=0}^{d} \beta_j t^j}_{g_t} + \varepsilon_t,$
$\varepsilon = \begin{pmatrix} \varepsilon_{t-h} \\ \vdots \\ \varepsilon_{t+h} \end{pmatrix} \sim \mathcal{N}(0, K)$
Critère minimisation :
$I(\theta, 0, y_t, g_t)$
$F_g(\theta) = I(\theta, 0, y_t, \mathbb{E}[M_\theta y_t])$
$S_g(\theta) = I(\theta, q, y_t, \mathbb{E}[M_\theta y_t])$
$y_t = \sum_{j=0}^{p} \beta_j t^j + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d}{\sim} \mathcal{N}(0, \sigma^2)$
$\xi_t = 0$
$q = d + 1$
LOESS
$\kappa_j^{TC} = \left(1 - \left|\frac{j}{h+1}\right|^3\right)^3$
Nadaraya-Watson ou estimateur par noyaux
$d = 0$
Filtre Epanechnikoff
$q = 2, \ \alpha = 0$
$\kappa_j^E = 1 - \left|\frac{j}{h+1}\right|^2$
Filtre Macaulay
$\alpha = 1$
$\kappa_j^U = 1$
Henderson
$q = 3, \ \alpha = 0$
$\kappa_j^H = \left[1 - \frac{j^2}{(h+1)^2}\right]\left[1 - \frac{j^2}{(h+2)^2}\right]\left[1 - \frac{j^2}{(h+3)^2}\right]$
Guggemos *et al* (2018)
Critères : $\begin{cases} F_g(\theta) = \sum_{j=-h}^{h} \theta_j^2 \\ S_g(\theta) = \sum_{j=-h}^{h} (\Delta^q \theta_j)^2 \end{cases}$
$\hat{\theta} \in \text{argmin}\, \alpha F_g(\theta) + (1-\alpha) S_g(\theta)$
$s.c. \quad X_d'\theta = e_1$
Proietti et Luati (2008)
$y_t = \sum_{j=0}^{d} \beta_j t^j + \varepsilon_t, \quad \varepsilon_t$ bruit blanc
$\kappa = (\kappa_{-h}, \dots, \kappa_h)$ noyaux, $K = diag(\kappa_{-h}, \dots, \kappa_h)$
$(\hat{\beta}_0, \dots, \hat{\beta}_d) \in \text{argmin}(y - X_p\hat{\beta})' K (y - X_d\hat{\beta})$
$\implies \hat{\theta} = \hat{\beta}_0 = K X_d (X_d' K X_d)^{-1} e_1$
Théorème d'Henderson (1916)