

Gray, Thompson (1996)

$$y_t = \underbrace{\sum_{j=0}^p \beta_j t^j}_{=\mu_t} + \xi_t + \varepsilon_t, \quad \varepsilon_t \text{ bruit blanc}$$

Critères : $\begin{cases} F(\theta) = \mathbb{E} [(\hat{\mu}_t - \mu_t)^2] \\ S(\theta) = \mathbb{E} [(\Delta^{p+1} \hat{g}_t)^2] \end{cases}$

$$\hat{\theta} \in \operatorname{argmin} \alpha F(\theta) + (1 - \alpha) S(\theta)$$

s.c. $X_p' \theta = e_1$

$F(\theta) = G(\theta, 0, y_t, g_t)$
 $S(\theta) = G(\theta, p+1, y_t, 0)$

Théorie générale

$$\begin{cases} G(\theta, q, y_t, u_t) = \mathbb{E} [(\Delta^q L_\theta y_t - u_t)^2] \\ F(\theta) = \int_0^\pi f[\phi_\theta(\omega, \varphi_\theta(\omega))] d\omega \\ \hat{\theta} \in \operatorname{argmin} \sum \alpha_i G(\theta, q, y_t, u_t^{(i)}) + \beta_i F_i(\theta) \end{cases}$$

s.c. $C\theta = a$

$F(\theta) = G(\theta, 0, y_t, \mathbb{E}[L_\theta y_t])$
 $S(\theta) = G(\theta, q, y_t, \mathbb{E}[L_\theta y_t])$
 $y_t = \sum_{j=0}^p \beta_j t^j + \varepsilon_t, \quad \varepsilon_t \stackrel{i.i.d}{\sim} \mathcal{N}(0, \sigma^2)$

$\xi_t = 0$
 $q = p+1$

Guggemos *et al* (2018)

Critères : $\begin{cases} F(\theta) = \sum_{j=-h}^h \theta_j^2 \\ S(\theta) = \sum_{j=-h}^h (\Delta^q \theta_j)^2 \end{cases}$

$$\hat{\theta} \in \operatorname{argmin} \alpha F(\theta) + (1 - \alpha) S(\theta)$$

s.c. $X_p' \theta = e_1$

$q = p+1, \alpha = 1$

$q = p+1, \alpha = 1$

$p = 2, q = 3, \alpha = 0$

Nadaraya-Watson
ou estimateur par
noyaux

Filtre Epanechnikoff

Filtre Macaulay

Henderson

$p = 0$

$\kappa_j^E = 1 - \left| \frac{j}{h+1} \right|^2$

$\kappa_j^U = 1$

$\kappa_j^H = \left[1 - \frac{j^2}{(h+1)^2} \right] \left[1 - \frac{j^2}{(h+2)^2} \right]$
 $\left[1 - \frac{j^2}{(h+3)^2} \right]$

Dagum et Bianconcini (2008) — RKHS

$f_0(t)$ noyau continu, P_i polynômes orthonormaux de $\mathbb{L}^2(f_0)$ et $K_p(t) = \sum_{i=0}^{p-1} P_i(t) P_i(0) f_0(t)$.

$$\hat{\theta}_i = \frac{K_p(i/b)}{\sum_{j=-h}^h K_p(j/b)}$$

$f_0 =$ version con-
tinue de κ_i
 $b = h+1$

$y_t = \sum_{j=0}^p \beta_j t^j + \varepsilon_t,$
 $\underbrace{\hspace{1cm}}_{g_t}$
 $\varepsilon = \begin{pmatrix} \varepsilon_{t-h} \\ \vdots \\ \varepsilon_{t+h} \end{pmatrix} \sim \mathcal{N}(0, K)$
 Critère minimisation :
 $G(\theta, 0, y_t, g_t)$

Proietti et Luati (2008)

$y_t = \sum_{j=0}^p \beta_j t^j + \varepsilon_t, \quad \varepsilon_t \text{ bruit blanc}$
 $\kappa = (\kappa_{-h}, \dots, \kappa_h)$ noyaux, $K = \operatorname{diag}(\kappa_{-h}, \dots, \kappa_h)$
 $(\hat{\beta}_0, \dots, \hat{\beta}_p) \in \operatorname{argmin} (y - X_p \hat{\beta})' K (y - X_p \hat{\beta})$
 $\Rightarrow \hat{\theta} = K X (X' K X)^{-1} e_1$