Response to referee

Associated Editor

1. Page 3, line 46: you say that it is "equivalently defined" and then provide the conjugate, writing $e^{i\omega k}$ instead of $e^{-i\omega k}$. "Equivalence" implies something akin to equality, so I would soften this to "alternatively", since the two expressions are not equal.
2. Page 5, line 53: replace "have" with "design", I think, because you are constructing these filters, right?
3. Section 1.2.1: these results are well-known; the derivatives of the frequency response function at $\omega = 0$ yield the conditions. So the results could be stated with references, and the derivations omitted, and this subsection compactified.
4. Page 7, line 40: why say the series is extended by one year? The extensions are for as many years as needed by the particular filters (typically, half their length in years).
5. Page 8, line 10: omit the $\forall q$ since you have stated this qualification in the previous line.
6. Table 1: I repeat the referees here. The table needs more explication in the text. What is ω_1? Define ∇, Guggemos, etc.
7. Page 13, equation (4): it is hard to understand what θ_f and θ_p are, and what γ and δ are. Add more explanation to the text.
8. Page 15, line 36: should be "same length as v"
9. Page 15, line 40: I couldn't find the matrix T defined. How is it connected to timeliness? That is, when you display the entries of the matrix T, you need to make a connection between those values and the concept of timeliness.
10. Page 15, line 53: I don't see why v in this equation should be defined through the case bracket; everything starting with "with" could be moved to the following text.
11. Page 17, lines 53-55: This discussion needs more clarity; it seems a bit terse.

12. Appendix B: rather than typing out the R code, it would be better to have the files available as supplementary material. (Or possibly put in a github repo and provide the links in the text.)

Referee 1

The authors should provide a clear definition of trend-cycle, TC_t, and irregular, I_t. In other words, how is the decomposition $y_t = TC_t + I_t$ identified? In section 1 they seem to hint at the interpretation of TC_t as the low-pass component of yt with cutoff frequency $\pi/6$ (and correspondingly they interpret I_t as the high-pass component). This point should be clarified, I think. Related questions are: are y_t and its components stochastic or deterministic? The treatment in section 1.2.1 suggests that TC_t is an unknown possibly nonlinear deterministic function of time. The variance inflation factor $\sum_k \theta_k^2$ implies that I_t is white noise.

2. The idea of minimizing the noise component ($M_\theta(I_t) \simeq 0$) is not very formal. Do you mean that the unconditional variance is minimized? Or the long run variance?

3. A definition of turning point would also be needed. Are these 'regular' points at which the slope of the trend changes sign? Or changepoints?

4. The notation should be unified: why not writing $TC_t = \mu(t)$ and in section 2 using t instead of x? The observation index i hould be replaced by t, as in section 2.1. Similarly, why not using t directly and start section 1.2 by stating that you assume that yt is generated as $y_t = \mu(t) + \varepsilon_t$, etc. Secondly, you should clarify the meaning of (3), where you set $\mu(t) = u_t'\gamma + z_t'\delta$ (also you assume that the noise component has possibly a non-scalar covariance matrix, do you need the matrix D for later treatment?

5. You should state that you assume a fixed bandwidth. A nearest neighbour bandwidth should be interesting to consider: as such it would contribute more to the reduction of the variance at the expenses of the phase shift.

6. Is the phase shift related to the notion of bias in local polynomial smoothing?

Minor points

• Section 1.1. It would perhaps be useful to provide the expression of the phase angle, $\varphi(\omega)$ in terms of the real and imaginary part of the transfer function of the linear filter. • Section 1.1. The symbol p for the period is already in use from the previous section, where it denotes the bandwidth and number of past observations used by the filter. • Page 4. The definition of the high-pass component (oscillations in the frequency range from $\pi/6$ to π as undesirable oscillations is controversial. • Figure 1. In the caption, the ration I/C was never defined before. The description of the three panels should be better explained in the main text. • Section 1.2 makes a reference to X-11 and X-13-ARIMA. The author assumes that the reader is knowledgeable about seasonal adjustment methods. Perhaps the authors should state that

their intent is to separate the trend-cycle component from the 'noise' component. • Page 7, line 38. 'which extends' is repeated twice. • Page 8. What is the meaning of $y_1^*n \ldots, y_h^*$ being the implicit forecasts induced by w_0, etc.? The forecasts are obtained from an assumed polynomial trend model? • Table 1. Some criteria are labelled as Guggemos, not referenced in the main text. • Figures 6-7. The labels of the time axes and the months are in French.

Referee 2

The author simulates time series of length 60 years to illustrate the methodology at the end of the series. It may be worth considering (in addition) shorter series. Another thought is that the ARIMA model being used to extend the series appears to be modeled using the full series; what if the ARIMA model used to extend the series was instead estimated using just a short span toward the end of the series?

The author avoids numbering the Introduction and Conclusion; numbering both (and adjusting the numbering of the other sections accordingly) may be better.

Adding the appropriate items for both R and X-13ARIMA-SEATS to the References may be warranted. In addition, in the text, it would probably be preferable to use R (the letter itself) instead of the R icon.

Some of the important results appear in the tables and figures, but the accompanying discussion in the text should still be fleshed out more. In addition, in Figures 2-4 and 6-7 (and the figures in the supplement), the lines can be distinguished by color, but it may be helpful to find some way to keep them distinguishable without the benefit of color. It might be the case that there is too much information to display neatly in some of these.

It is somewhat unclear just how prominent the new functionality being added via the R package rjd3filters is intended to be in this paper.

Referee 3

1. Equivalence between Asymmetric and Symmetric Filters (Page 8, Line 9) I suggest that the authors provide a more explicit explanation on page 8, line 9, regarding the equivalence between asymmetric and symmetric filters. It would be beneficial to clarify the normalization difference and ensure a clear understanding of the relationship between the two types of filters.

2. Symbol [[ ]] Meaning (Page 11, Lines 53-54) The meaning of the symbol [[ ]] on page 11, lines 53-54, is not clear. I recommend that the authors explicitly define this symbol or provide a clear explanation to avoid any ambiguity in its interpretation.

3. Variance Determination in Local Parameterization (Page 17) The determination of variance in the local parameterization of asymmetric filters (page

17) could benefit from a more detailed explanation. I recommend that the authors clearly identify the quantities appearing in the denominator for better understanding.

4. Applicability to High-Frequency Data. It would be valuable if the authors could explicitly address the applicability of the proposed procedure to high-frequency data. If modifications or considerations are necessary for such data, it should be discussed to provide a comprehensive understanding of the methodology.

5. Clarity on Phase Shift Computation (Page 20) A clearer explanation of how the phase shift is computed on page 20 would enhance the paper's overall clarity. The authors should specify the method used for determining the phase shift, particularly in relation to the identification of true turning points based on symmetric filter estimates.

6. Commentary on Figure 5 I suggest that the authors include a detailed commentary on Figure 5 in the paper. Providing explanations for the results shown in the figure would be crucial for readers to better understand the performance of the filters/methods.

7. Better Description of Results in Table 3 The meaning of "second and third estimates" in Table 3 on page 22 is not clear. I recommend that the authors provide a more detailed description or explanation of the results to improve comprehension.

8. Consideration of Different Series Periodicity It would be beneficial if the authors could address the suggestion of considering series with different periodicities in the real data application. Discussing how the proposed methodology performs when applied to series with various frequencies would add depth to the study.

Referee 4

1) The paper lacks in describing the contribution from the beginning. I had to read the paper a couple of times to clearly identify the contribution. I found the contribution of the paper clearly stated on page 16, but I think I should come before in the text.
2) The paper omits to consider the Covid19 period and how this influences the trend and cycle decomposition. I understand that this is not the main focus of the paper, but at least the authors should address this problem in word and possible solution to address it, e.g. outlier correction, robust distributions, etc.
3) I think a quote from other works can be removed from the paper. For example, I would remove the quote in point 2) of page 7.