

Induction

1. Define your logical predicate $P(n)$.
2. Prove that $P(0)$ is true.
3. Prove that $P(n)$ implies $P(n+1)$.
 - often done by proving $P(n)$ assuming all $P(1), P(2), \dots, P(n-1)$ are true
 - these are termed weak and strong induction, respectively

Problem 1

Prove that for all n in the nonnegative integers,

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

Problem 2

Prove that every integer greater than 1 has a prime divisor.

Hint: two cases to consider for a given integer n . Use strong induction.

Problem 3

Prove that all trees with n vertices contain $n - 1$ edges.

Problem 4

Any convex polygon P with $k \geq 3$ vertices can be decomposed into a set of $k - 2$ triangles whose interiors do not overlap.