Master Theorem
CSCI 305

For recurrences in the form $T(n) = aT(\frac{n}{b}) + f(n)$, with a base case $T(n) \in \Theta(1)$.

Three cases:

1 . Work is dominated by subproblems.

a. **Condition:** $f(n) = O(n^c)$ where $c < c_{crit}$.

b. **Bound:** $T(n) = \Theta(n^{c_{crit}})$

2a.

a. **Condition:** $f(n) = \Theta(n^{c_{crit}} \log^k(n)$ for $k > -1$.

b. **Bound:** $T(n) = \Theta(n^{c_{crit}} \log^{k+1}(n))$

2b.

a. **Condition:** $f(n) = \Theta(n^{c_{crit}} \log^k(n)$ for $k = -1$.

b. **Bound:** $T(n) = \Theta(n^{c_{crit}} \log(\log(n)))$

2c.

a. **Condition:** $f(n) = \Theta(n^{c_{crit}} \log^k(n)$ for $k < -1$.

b. **Bound:** $T(n) = \Theta(n^{c_{crit}})$

3 . Work to split/recombine dominates the subproblems.

a. **Condition:** $f(n) = \Omega(n^c)$ where $c > c_{crit}$ **and** $af(\frac{n}{b}) \leq kf(n)$ for $k < 1$ and sufficiently large n. Second condition is called the *regularity condition*.

b. **Bound:** $T(n) = \Theta(f(n))$

Examples:

1. $T(n) = 8T(\frac{n}{2}) + 1000n^2$

2. $T(n) = 2T(n/2) + 10n$

3. $T(n) = 2T(n/2) + n^2$