Flip a fair coin three times. Let Xi = 1 if flip *i* is heads, or Xi=0 if flip *i* is tails.

Let Y = X1 + X2 + X3 be the total number of heads in three flips.

1. One possible outcome of three flips is “H, T, T”, also written as (X1, X2, X3) = (1, 0, 0). Enumerate all possible outcomes:

|  |  |
| --- | --- |
| Outcome | (X1, X2, X3) |
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1. What possible values can Y take? Make a table of values of Y, number of outcomes that lead to that value of Y, and the probability of each outcome.

|  |  |  |
| --- | --- | --- |
| Y | # of ways | P[Y-y] |

1. Compute the expected value of Y, E[Y] = Σy y P(Y = y), using your table.
2. For independent random variables, E[X1 and X2 and X3] = E[X1] E[X2] E[X3].

Are our random variables independent? Why or why not?

Compute the probability that all coins come up heads assuming independence, and check whether it matches your answer in (2).

Consider the following algorithm.

Hire-Assistant(n):

best = 0

for i=1 to n:

interview candidate i

if candidate i is better than candidate best:

best = i

hire candidate i

1. In Hire-Assistant, assuming candidates are presented in random order, what is the probability that you hire exactly one time?
2. What is the probability that we hire exactly n times?