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1.(a) $\tilde{x}[n] = 2\cos(\frac{\pi n}{4})$ $N=8$

$$= e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{k\cdot n}$$

$$= \sum_{n=0}^{7} (e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}) e^{-j\frac{\pi}{4}\cdot kn}$$

$$X[0] = \sum_{n=0}^{7} (e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}) = 0$$

$$X[1] = \sum_{n=0}^{7} 1 + e^{-j\frac{\pi n}{2}} = 8\times 1 = 8$$

$$X[2] = \sum_{n=0}^{7} e^{-j\frac{\pi n}{4}} + e^{-j\frac{3\pi n}{4}} = 0$$

$$X[3] = \sum_{n=0}^{7} e^{-j\frac{\pi n}{2}} + e^{j\pi n} = 0$$

$$X[4] = \sum_{n=0}^{7} e^{-j\frac{3\pi n}{4}} + e^{-j\frac{5\pi n}{4}} = 0$$

$$X[5] = \sum_{n=0}^{7} e^{-j\pi n} + e^{-j\frac{3\pi n}{2}} = 0$$

$$X[6] = \sum_{n=0}^{7} e^{-j\frac{5\pi n}{4}} + e^{-j\frac{7\pi n}{4}} = 0$$

$$X[7] = \sum_{n=0}^{7} e^{-j\frac{3\pi n}{2}} + 1 = 8\times 1 = 8$$

(b) $\tilde{x}[n] = 3\sin(\frac{\pi n}{4}) + 4\cos(\frac{3\pi n}{4})$, $N=8$

$$= \frac{3}{2j}(e^{j\frac{\pi n}{4}} - e^{-j\frac{\pi n}{4}}) + 2(e^{j\frac{3\pi n}{4}} + e^{-j\frac{3\pi n}{4}})$$

$$X[k] = \sum_{n=0}^{7} \tilde{x}[n]\cdot e^{-j\frac{\pi}{4}k\cdot n}$$

$X[0] = 0$

$X[1] = \frac{3}{2j}\times 8 = -12j$

$X[2] = 0$

$X[3] = 2\times 8 = 16$

$X[4] = 0$

$X[5] = 2\times 8 = 16$

$X[6] = 0$

$X[7] = \frac{-3}{2j}\times 8 = +12j$

2.(a)

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{k\cdot n}$$

$$= \sum_{n=0}^{N/2-1} x[n] W_N^{k\cdot n} + \sum_{n=N/2}^{N-1} x[n] W_N^{k\cdot n} \quad (\because N \text{ is even}, N/2 \in \mathbb{N})$$

$$= \sum_{n=0}^{N/2-1} x[n] W_N^{k\cdot n} + \sum_{n=N/2}^{N-1} -x[\langle n+N/2\rangle_N] W_N^{k\cdot n} \quad (\because x[n] = -x[\langle n+N/2\rangle_N])$$

$$= \sum_{n=0}^{N/2-1} x[n] W_N^{k\cdot n} + \sum_{n=0}^{N/2-1} -x[n] W_N^{k\cdot(n-N/2)}$$

$$= \sum_{n=0}^{N/2-1} x[n] W_N^{k\cdot n} + \sum_{n=0}^{N/2-1} -x[n]\cdot W_N^{k\cdot n} \quad (\because W_N^{k\cdot -N/2} = e^{j\frac{2\pi}{N}\cdot k\cdot\frac{-N}{2}}$$

$$= e^{j\pi k}$$

$$= 0$$

$= 1$) (because k is even, $\pi k = n\cdot 2\pi$, $n \in \mathbb{N}$)

2.(b)

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{k\cdot n}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} x[n] W_N^{k\cdot n} + \sum_{n=\frac{N}{4}}^{\frac{N}{2}-1} x[n] W_N^{k\cdot n} + \sum_{n=\frac{N}{2}}^{\frac{3N}{4}-1} x[n] W_N^{k\cdot n} + \sum_{n=\frac{3N}{4}}^{N-1} x[n] W_N^{k\cdot n} \quad (\because \tfrac{N}{4} \in \mathbb{N})$$

$$= \sum_{n=0}^{\frac{N}{4}-1} x[n] W_N^{k\cdot n} + \sum_{n=N/4}^{\frac{N}{2}-1} -x[\langle n+\tfrac{N}{4}\rangle_N] W_N^{k\cdot n} + \sum_{n=\frac{N}{2}}^{\frac{3N}{4}-1} x[n] W_N^{k\cdot n} + \sum_{n=\frac{3N}{4}}^{N-1} -x[\langle n+\tfrac{N}{4}\rangle_N] W_N^{k\cdot n}$$

$$(\because x[n] = -x[\langle n+\tfrac{N}{4}\rangle_N])$$

$$= \sum_{n=0}^{\frac{N}{4}-1} x[n] W_N^{k\cdot n} + \sum_{n=0}^{\frac{N}{4}-1} -x[n] W_N^{k\cdot(n-\frac{N}{4})} + \sum_{n=\frac{N}{2}}^{\frac{3N}{4}-1} x[n] W_N^{kn} + \sum_{n=\frac{N}{2}}^{\frac{3N}{4}-1} -x[n] W_N^{k(n-\frac{N}{4})}$$

$$= 0 \quad (\because W_N^{k\cdot\frac{N}{4}} = e^{-j\frac{2\pi}{N}\cdot k\cdot\frac{N}{4}} = e^{-j\frac{\pi k}{2}} = 1)(\because k = 4l,\ \tfrac{\pi k}{2} = 2\pi l)$$

3(a)

$$x_4[n] = x_1[n] \circledN x_2[n]$$

$$= \sum_{m=0}^{N-1} x_1[m]\, x_2[\langle n-m\rangle_N]$$

$$= \sum_{l=-\infty}^{+\infty} \sum_{m=0}^{N-1} x_1[m]\, x_2[n+lN-m]$$

$$= \sum_{l=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} x_1[m]\, x_2[n+lN-m] \quad (\because x_1[m]=0,\ \text{for } m<0 \text{ or } m\geq N)$$

$$= \sum_{l=-\infty}^{+\infty} x_1[n+lN] * x_2[n+lN]$$

$$= \sum_{l=-\infty}^{+\infty} x_3[n+lN] \ \# \quad (\because x_3[n] = x_1[n] * x_2[n])$$

(b) $x_3[n]=0$ for $n<0$ or $n \geq N_1+N_2-1 = L$

① If $N \geq L$,

$$x_4[n] = \sum_{l=-\infty}^{+\infty} x_3[n+lN] = x_3[n] \Rightarrow e[n] = x_4[n] - x_3[n] = x_3[n] - x_3[n] = 0$$

② If $\max(N_1,N_2) \leq N < L$ (Note $2N \geq 2\max(N_1,N_2) > L$)

$$x_4[n] = \sum_{l=0}^{1} x_3[n+lN] = x_3[n] + x_3[n+N] \Rightarrow e[n] = x_4[n] - x_3[n] = x_3[n+N]$$

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$$e[n] = \begin{cases} x_3[n+N], & \max(N_1, N_2) \le N < L \\ 0, & N \ge L \end{cases}$$

(c) ①

$N=5$

$x_1 = [1,2,3,4,0]$

$x_2 = [4,3,2,1,0]$

$x_4[n] = \sum_{m=0}^{N-1} x_1[m]\, x_2[\langle n-m \rangle_N]$ $\quad x_3[n] = [4,11,20,30,20,11,4,0,0]$

$$x_4[n] = \begin{bmatrix} 4 & 0 & 1 & 2 & 3 \\ 3 & 4 & 0 & 1 & 2 \\ 2 & 3 & 4 & 0 & 1 \\ 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 20 \\ 30 \\ 20 \end{bmatrix}$$

(a) $x_4[n] = \sum_{l=0}^{1} x_3[n+l\cdot 5] = [15,15,20,30,20]$

(b) $N=5 < L = 4+4-1 = 7$

$e[n] = x_3[n+N] = [11,4,0,0,0]$

②

$N=8$

$x_4[n] = [4,11,20,30,20,11,4,0]$ $\quad x_3[n] = [4,11,20,30,20,11,4,0,0,0,0,0,0,0,0,0]$

(a) $x_4[n] = \sum_{l=0}^{1} x_3[n+l\cdot 8] = [4,11,20,30,20,11,4,0]$

(b) $N=8 \ge L=7$

$e[n]=0$ #

4.

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{N}k\cdot n}$$

$$\tilde{X}_3[k] = \sum_{n=0}^{3N-1} \tilde{x}_3[n]\, e^{-j\frac{2\pi}{3N}k\cdot n}$$

$$= \sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{3N}k\cdot n} + \sum_{n=N}^{2N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{3N}k\cdot n} + \sum_{n=2N}^{3N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{3N}k\cdot n}$$

$$= \tilde{X}[\tfrac{k}{3}] + e^{-j2\pi\cdot\frac{k}{3}}\tilde{X}[\tfrac{k}{3}] + e^{-j2\pi\frac{2k}{3}}\tilde{X}[\tfrac{k}{3}]$$

$$= (1 + e^{-j2\pi\frac{k}{3}} + e^{-j2\pi\frac{2k}{3}})\,\tilde{X}[\tfrac{k}{3}]$$

5.

$\because$ $x[n]$ is real, $X[k] = \tilde{X}^*[\langle -k\rangle_N]$

$\Rightarrow X[k] = [4, 2-3j, 3+2j, -4+6j, 8-7j, 8+7j, -4-6j, 3-2j, 2+3j]$

(a) $x[\langle n-m\rangle_N] \xleftrightarrow{DFT} W_N^{km} X[k]$

$x_1[n] = x[\langle n+2\rangle_9] \xleftrightarrow{DFT} W_9^{-2k} X[k]$

$$X_1[k] = e^{+j\frac{2\pi}{9}\cdot 2k} X[k]$$

$$= [4, e^{+j\frac{4\pi}{9}}(2-3j), e^{+j\frac{8\pi}{9}}(3+2j), e^{+j\frac{4\pi}{3}}(-4+6j), e^{+j\frac{16\pi}{9}}(8-7j), e^{+j\frac{2\pi}{9}}(8+7j), e^{+j\frac{2\pi}{3}}(-4-6j), e^{+j\frac{10\pi}{9}}(3-2j), e^{+j\frac{14\pi}{9}}\cdot(2+3j)] \ \#$$

(b) $x[\langle -n\rangle_N] \xleftrightarrow{DFT} X[\langle -k\rangle_N]$

$x_2[n] = 2x[\langle 2-n\rangle_9] = 2x[\langle -(n-2)\rangle_9] \longleftrightarrow 2\cdot W_9^{2k}\cdot X[\langle -k\rangle_9]$

$$X_2[k] = 2\cdot e^{-j\frac{2\pi}{9}\cdot 2k} X[\langle -k\rangle_9]$$

$$= [8, e^{-j\frac{4\pi}{9}}(4+6j), e^{-j\frac{8\pi}{9}}(6-4j), e^{-j\frac{4\pi}{3}}(-8-12j), e^{-j\frac{16\pi}{9}}(16+14j), e^{-j\frac{2\pi}{9}}(16-14j), e^{-j\frac{2\pi}{3}}(-8+12j), e^{-j\frac{10\pi}{9}}(6+4j), e^{-j\frac{14\pi}{9}}\cdot(4-6j)] \ \#$$

5.(c)

$$h[n] \circledN x[n] \xleftrightarrow{DFT} H[k]X[k]$$

$$x_3[n] = x[n] ⑨ x[\langle -n \rangle_9] \xleftrightarrow{DFT} X[k]\,X[\langle -k \rangle_9]$$

$$X_3[k] = [16, 13, 13, 52, 113, 113, 52, 13, 13]$$ #

5.(d)

$$x_4[n] = x[n]x[n] \xleftrightarrow{DFT} X_4[k] = \frac{1}{9} X[k] ⑨ X[k]$$

$$X_4[k] = [\tfrac{2}{3}, \tfrac{125}{9}, \tfrac{-5}{3}, \tfrac{10}{3}, 11, 11, \tfrac{10}{3}, \tfrac{-5}{3}, \tfrac{125}{9}]$$

5.(e)

$$W_N^{-mn} x[n] \xleftrightarrow{DFT} X[\langle k-m \rangle_N]$$

$$x_5[n] = x[n]e^{i\frac{2\pi}{9}\cdot 2n} \xleftrightarrow{DFT} X_5[k] = X[\langle k+2 \rangle_N]$$

$$X_5[k] = [3+2i, -4+6i, 8-7i, 8+7i, -4-6i, 3-2i, 2+3i, 4, 2-3i]$$ #