DSP HW3 106061151 劉安得

1. (a) $Y(z)=\frac{1}{4}\left(X(z)+z^{-1}\cdot X(z)\right)-\frac{1}{4}\left(z^{-2}\cdot X(z)+z^{-3}\cdot X(z)\right)$

$$Y(z)=\left(\frac{1}{4}+\frac{1}{4}z^{-1}-\frac{1}{4}z^{-2}-\frac{1}{4}z^{-3}\right)X(z)$$

$$H(z)=\frac{1}{4}+\frac{1}{4}z^{-1}-\frac{1}{4}z^{-2}-\frac{1}{4}z^{-3}$$

$$H(e^{jw})=\frac{1}{4}+\frac{1}{4}e^{-jw}-\frac{1}{4}e^{-2jw}-\frac{1}{4}e^{-3jw}$$

$$=\frac{1}{4}+\frac{1}{4}(\cos(-w)+j\sin(-w))-\frac{1}{4}(\cos(-2w)+j\sin(-2w))-\frac{1}{4}(\cos(-3w)+j\sin(-3w))$$

$$=\left(\frac{1}{4}+\frac{1}{4}\cos(w)-\frac{1}{4}\cos(2w)-\frac{1}{4}\cos(3w)\right)+j\left(-\frac{1}{4}\sin(w)+\frac{1}{4}\sin(2w)+\frac{1}{4}\sin(3w)\right)$$

$$|H(e^{jw})|=\frac{1}{4}\left[(1+\cos(w)-\cos(2w)-\cos(3w))^2+(-\sin(w)+\sin(2w)+\sin(3w))^2\right]^{1/2}$$

$$\angle H(e^{jw})=\tan^{-1}\frac{-\sin(w)+\sin(2w)+\sin(3w)}{1+\cos(w)-\cos(2w)-\cos(3w)}\ \#$$

(b) $Y(z)=X(z)-z^{-4}X(z)+0.6561\cdot z^{-4}Y(z)$

$$H(z)=\frac{1-z^{-4}}{1-0.6561z^{-4}}$$

$$H(e^{jw})=\frac{1-e^{-4jw}}{1-0.6561e^{-4jw}}=\frac{1-\cos(4w)+j\sin(4w)}{1-0.6561\cos(4w)+j0.6561\sin(4w)}$$

$$|H(e^{jw})|=\frac{\sqrt{2-2\cos 4w}}{\sqrt{1+0.6561^2-2\cdot 0.6561\cos(4w)}}$$

$$\angle H(e^{jw})=\tan^{-1}\frac{\sin(4w)}{1-\cos(4w)}-\tan^{-1}\frac{0.6561\sin(4w)}{1-0.6561\cos(4w)}\ \#$$

2. (a) $H(e^{jw})=\sum_{k=-\infty}^{\infty}h[k]e^{-jwk}$

$$=1-2\cdot e^{-jw}+3e^{-2jw}-4e^{-3jw}+4e^{-5jw}-3e^{-6jw}+2e^{-7jw}-e^{-8jw}$$

$$=[1-2\cos w+3\cos(2w)-4\cos(3w)+4\cos(5w)-3\cos(6w)+2\cos(7w)-\cos(8w)]$$

$$+j[2\sin w-3\sin(2w)+4\sin(3w)-4\sin(5w)+3\sin(6w)-2\sin(7w)+\sin(8w)]$$

$H(e^{j\cdot 0.1\pi})=0.2657+j\cdot(-0.0863)$ $\quad C_1=0.28 \quad \phi_1=-0.9\pi$

$H(e^{j\cdot 0.3\pi})=1.8676+j\cdot(2.5706)$ $\quad C_2=3.18 \quad \phi_2=0.3\pi$

$H(e^{j\cdot 0.5\pi})=-4j$ $\quad C_3=4 \quad \phi_3=-0.5\pi$

$$y[n]=C_1\sin(0.1\pi n+\phi_1)+C_2\sin(0.3\pi n+\phi_2)+C_3\sin(0.5\pi n+\phi_3)$$

$\Rightarrow$ magnitude distortion and phase distortion

2.(b)

$$Y(z) = 10z^{-10}X(z)$$

$$H(z) = 10z^{-10}$$

$$H(e^{jw}) = 10e^{-10jw}$$

$$= 10(\cos(10w) - j\sin(10w))$$

$$|H(e^{jw})| = 10$$

$$\angle H(e^{jw}) = -10w = -wn_d \quad (n_d = 10) \text{ (linear phase)}$$

$\Rightarrow$ no distortion

3(a) $H_r(j\Omega) = \begin{cases} \dfrac{\Omega T/2}{\sin(\frac{\Omega T}{2})} e^{j\Omega T/2}, & |\Omega| < \pi/T \\ 0, & \text{O.W.} \end{cases}$

$w = \Omega T$

$\Rightarrow H_r(e^{jw}) = \begin{cases} \dfrac{w/2}{\sin\frac{w}{2}} e^{j\frac{w}{2}}, & |w| < \pi \\ 0, & \text{O.W.} \end{cases}$

(b) $H_{FIR}(e^{jw}) = -\frac{1}{16} + \frac{9}{8} e^{-jw} - \frac{1}{16} e^{-2jw}$

$= -\frac{1}{16} + \frac{9}{8}(\cos w - j\sin w) - \frac{1}{16}(\cos(2w) - j\sin(2w))$

$= \left(-\frac{1}{16} + \frac{9}{8}\cos w - \frac{1}{16}\cos(2w)\right) + j\left(-\frac{9}{8}\sin w + \frac{1}{16}\sin(2w)\right)$

$|H_{FIR}(e^{jw})| = \left[\left(-\frac{1}{16} + \frac{9}{8}\cos w - \frac{1}{16}\cos(2w)\right)^2 + \left(-\frac{9}{8}\sin w + \frac{1}{16}\sin(2w)\right)^2\right]^{1/2}$

$H_r(e^{jw}) = \frac{w/2}{\sin\frac{w}{2}}\left(\cos\frac{w}{2} + j\sin\frac{w}{2}\right)$

$|H_r(e^{jw})| = \frac{w/2}{\sin\frac{w}{2}}$

(c) $H_{IIR}(e^{jw}) = \frac{9}{8 + e^{-jw}}$

$= \frac{9}{8 + \cos w - j\sin w}$

$|H_{IIR}(e^{jw})| = \frac{9}{\sqrt{(8+\cos w)^2 + \sin^2 w}} = \frac{3}{\sqrt{65 + 16\cos w}}$

4.

(a) $H(s) = \frac{(s-3)(s+1)(s-(2+j))(s-(2-j))}{(s+5)(s-(-3+3j))(s-(-3-3j))(s-(-2+2j))(s-(-2-2j))}$

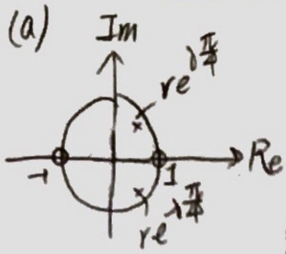
there exist zero in the left half plane of s, therefore H is a nonminimum phase system

(b) $H(s) = H_{min}(s) H_{ap}(s)$

$H_{min}(s) = \frac{(s+3)(s+1)(s-(-2+j))(s-(-2-j))}{(s+5)(s-(-3+3j))(s-(-3-3j))(s-(-2+2j))(s-(-2-2j))}$

$H_{ap}(s) = \frac{s-3}{s+3} \cdot \frac{s-(2+j)}{s-(-2+j)} \cdot \frac{s-(2-j)}{s-(-2-j)}$

5.(a)



zero = 1, -1

pole = $r\cdot e^{j\frac{\pi}{4}}, r\cdot e^{-j\frac{\pi}{4}}$

$$|H(e^{jw})| = |b_0|\,|1-e^{-jw}|\cdot|1+e^{-jw}| \div |1-r\cdot e^{j\frac{\pi}{4}}e^{-jw}| \div |1-re^{-j\frac{\pi}{4}}e^{-jw}|$$

when $w=\pm(\frac{\pi}{4}\pm 0.05)$, $|H(e^{jw})| = \frac{1}{\sqrt{2}}$

$\Rightarrow r = 0.95$ $\qquad e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}$

when $w=\frac{\pi}{4}+0.05$

$$|H(e^{jw})| = |b_0|\cdot\sqrt{2-\cos\theta}\cdot\sqrt{2+\cos\theta} \div \sqrt{1+0.95^2-2\times0.95\times\cos(0.05)} \div \sqrt{1+0.95^2-2\times0.95\times\cos(\frac{\pi}{2}+0.05)}$$

$= 1$

$b_0 = 0.58$ (Matlab 實作時根據結果調整為 0.66)

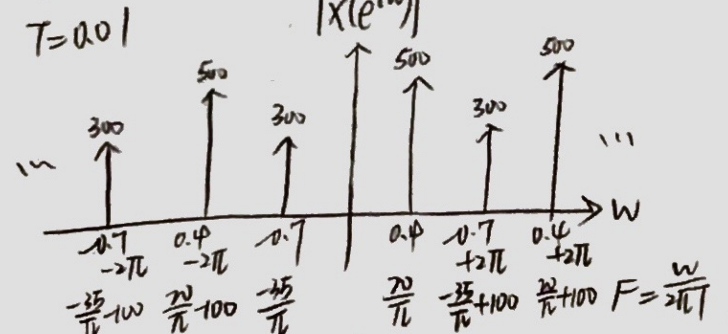
6.(a) (i) $X_c(j\Omega) = 5\delta(\Omega-40)+3\delta(\Omega+70)$

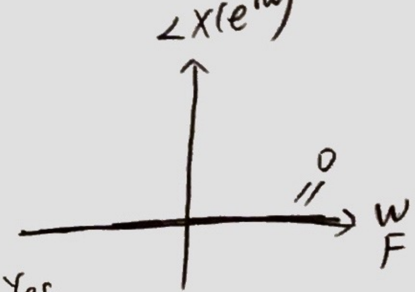
$$X(e^{jw}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\frac{w}{T}-j\frac{2\pi}{T}k\right)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} 5\delta\left(\frac{w}{T}-\frac{2\pi}{T}k-40\right)+3\delta\left(\frac{w}{T}-\frac{2\pi}{T}k+70\right)$$

(ii) (iii)

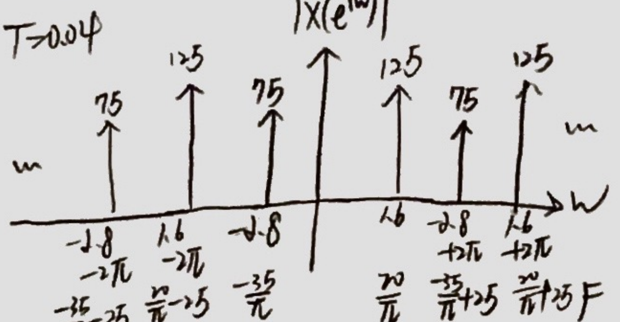
T=0.01

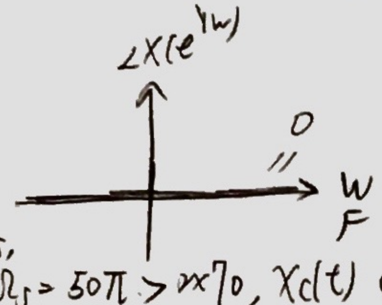




Yes, ∵ $\Omega_s = \frac{2\pi}{T} = 200\pi > 2\times70$, $x_c(t)$ can be recovered from $x[n]$

T=0.04

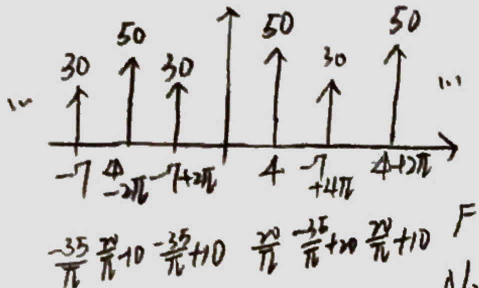


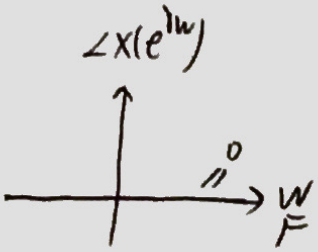


Yes,

∵ $\Omega_s = 50\pi > 2\times70$, $x_c(t)$ can be recovered from $x[n]$

$T = 0.1$





No, $\Omega_s = 20\pi < 2\times 70$. $x_c(t)$ can't be recovered from $x[n]$

(b)

$$x_c(t) = 3+2(e^{j16\pi t} - e^{-j16\pi t}) + 10(e^{j24\pi t} - e^{-j24\pi t})$$

$$X_c(j\Omega) = 3+ \quad (\delta(\Omega-16\pi)-\delta(\Omega+16\pi)) + 5(\delta(\Omega-24\pi)-\delta(\Omega+24\pi))$$

$$X(e^{jw}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j\frac{w}{T} - j\frac{2\pi}{T}k)$$

$$T = \frac{1}{F_s}$$

圖見 Matlab

7. (a) $\frac{10}{2^8}$ V or 2^8 steps

(b)

$$SQNR = 6.02B + 1.76$$
$$= 6.02\times 8 + 1.76$$
$$= 49.92 \#$$

(c) $F_s = \frac{2048}{8} = 256$

Folding freq $= 128$

Nyquist rate $= 2\times\Omega_H = 2\times 500\pi = 1000\pi$