DSP HW5 106061151 [illegible]

1. $A_s = -20\log_{10}(0.0001)$ $\quad W_c = \frac{W_p+W_s}{2} = 0.4\pi$

$= -20\times -4$ $\quad \Delta W = W_s - W_p = 0.2\pi$

$= 80$

2.

(a) $w[n] = \left(0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right)\right) w_R[n]$

$= \left(0.5 - 0.5\cdot\frac{1}{2}\left(e^{j\frac{2\pi n}{M}} + e^{-j\frac{2\pi n}{M}}\right)\right) w_R[n]$

$W(e^{jw}) = \frac{1}{2}W_R(e^{jw}) - \frac{1}{4}W_R(e^{j(w+\frac{2\pi}{M})}) - \frac{1}{4}W_R(e^{j(w-\frac{2\pi}{M})})$

(b) 因為Hann window 在mainlobe時，會因為$-\frac{1}{4}\delta(f\pm\frac{1}{M})$這項而使width變寬；但在sidelobe時，因為sidelobe的peak被乘以$\frac{1}{2}$還減掉周圍的amplitude，因此sidelobe的dB會大幅降低

3. $H(z) = (1-z^{-4})\frac{1}{1-z^{-1}} = 1+z^{-1}+z^{-2}+z^{-3}$

$H(e^{jw}) = 1+e^{-jw}+e^{-j2w}+e^{-j3w}$

$= \left(e^{j\frac{3}{2}w} + e^{j\frac{1}{2}w} + e^{-j\frac{1}{2}w} + e^{-j\frac{3}{2}w}\right)\cdot e^{-j\frac{3}{2}w}$

$= \left[2\cos(\frac{1}{2}w) + 2\cos(\frac{3}{2}w)\right]\cdot e^{-j\frac{3}{2}w}$

(a) $|H(e^{jw})| = \left|2\cos(\frac{1}{2}w) + 2\cos(\frac{3}{2}w)\right|$

圖見下頁

(b) $A(e^{jw}) = 2\cos(\frac{1}{2}w) + 2\cos(\frac{3}{2}w)$，圖見下頁

Magnitude response把負的值都變正了，所以看不出來何時為正，何時為負

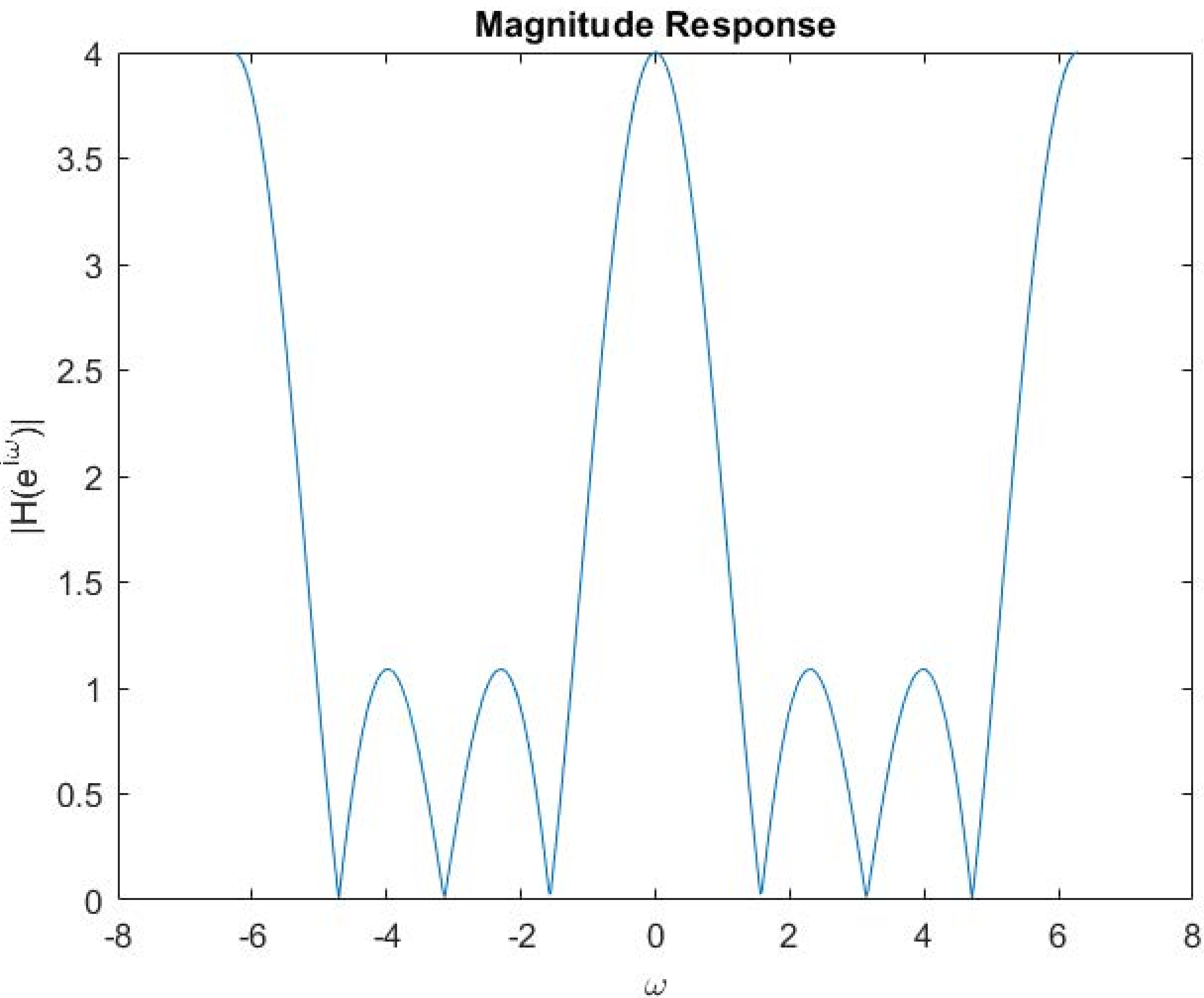
而把Amplitude responce取絕對值就是magnitude responce

(c) $\angle H(e^{jw}) = \angle e^{-j\frac{3}{2}w} = \angle(\cos\frac{3}{2}w + j\sin\frac{3}{2}w) = \tan^{-1}(\tan\frac{3}{2}w)$，圖見下頁

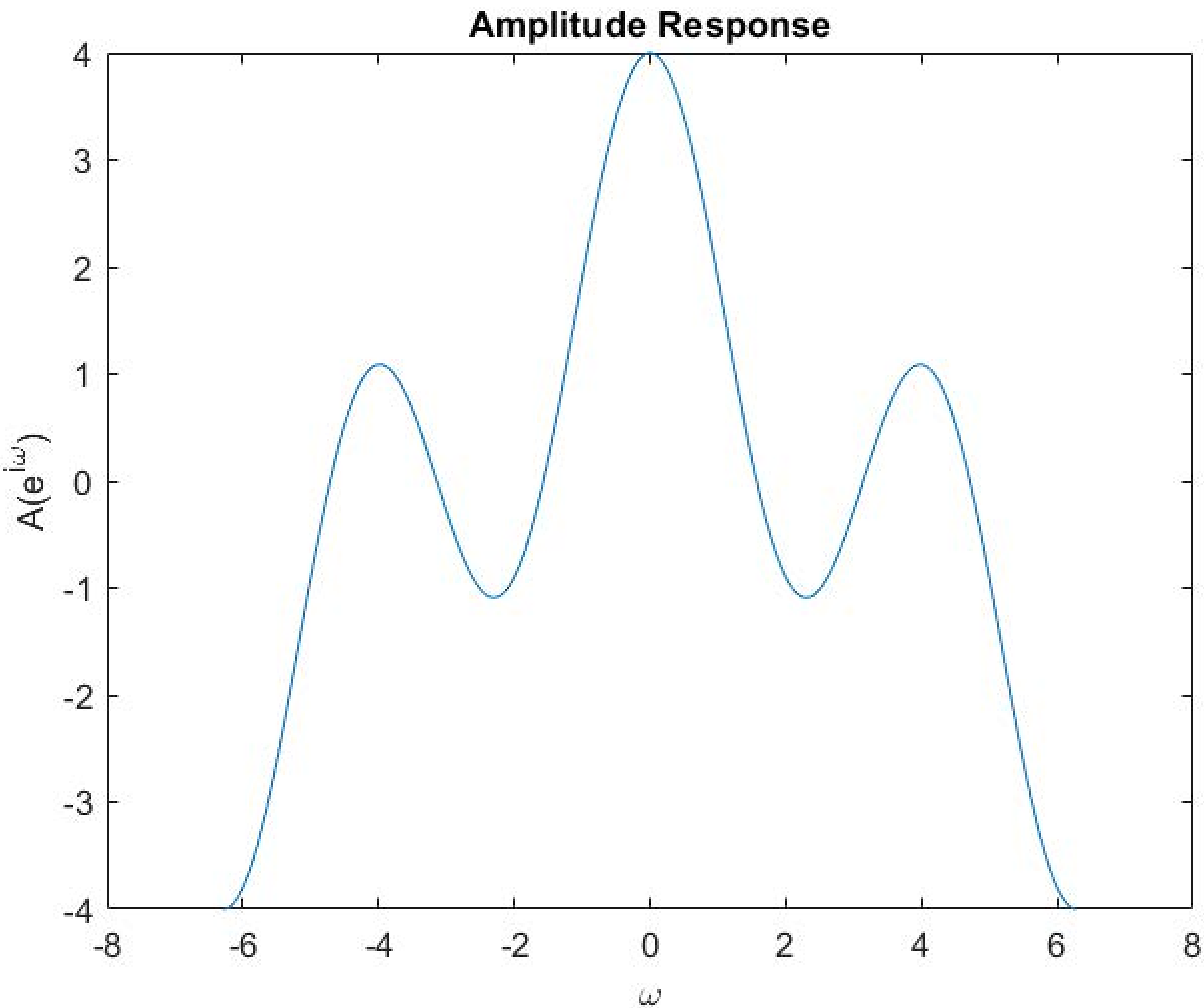
(d) $\varphi(e^{jw}) = -\frac{3}{2}w$，圖見下頁

可以看成是Angle responce每超过$\frac{\pi}{2}\sim-\frac{\pi}{2}$的範圍就$\pm\pi$而產生phase responce

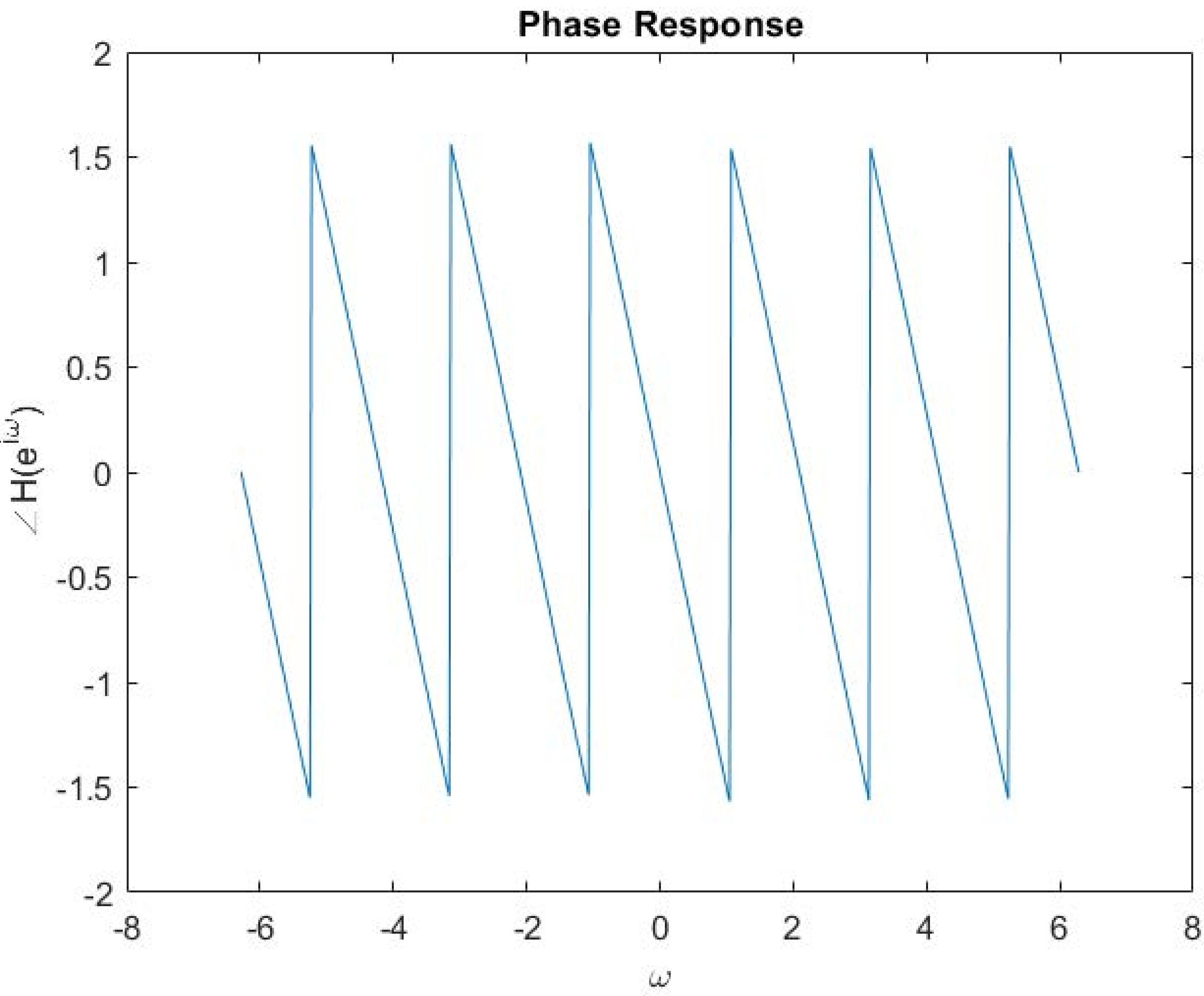
也可以看成是phase responce如果遇到不連續的点就繼續延伸而產生Angle responce



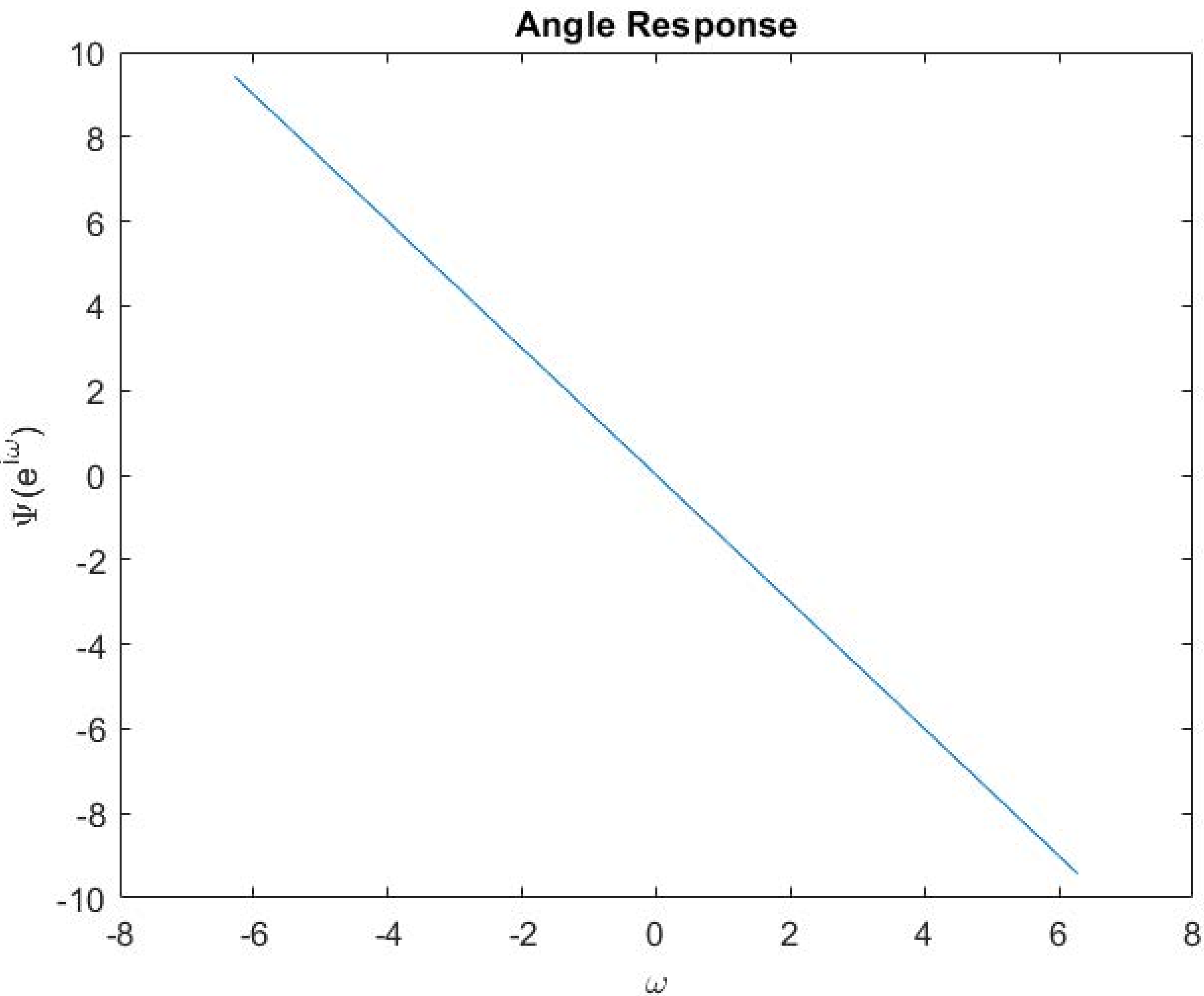
Magnitude Response
4
3.5
3
2.5
2
1.5
1
0.5
0
$|H(e^{j\omega})|$
-8
-6
-4
-2
0
2
4
6
8
ω



Amplitude Response
4
3
2
1
0
-1
-2
-3
-4
A(e^{jω})
-8
-6
-4
-2
0
2
4
6
8
ω



Phase Response
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-8
-6
-4
-2
0
2
4
6
8
ω
$\angle H(e^{j\omega})$


Angle Response
10
8
6
4
2
0
-2
-4
-6
-8
-10
-8
-6
-4
-2
0
2
4
6
8
Ψ(e^jω)
ω

4. $h[n] = -h[M-n],\ 0 \le n \le M$, odd M

(a)

$$H(e^{jw}) = \sum_{n=0}^{M} h[n] e^{-jw\cdot n}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n] e^{-jw\cdot n} + \sum_{n=\frac{M+1}{2}}^{M} h[n] e^{-jw\cdot n}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n] e^{-jw\cdot n} + \sum_{n=0}^{\frac{M-1}{2}} -h[n] e^{-jw(M-n)}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n] \left(e^{+jw(\frac{M}{2}-n)} - e^{-jw(\frac{M}{2}-n)}\right) e^{-jw\frac{M}{2}}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} 2h[n] \sin\left(w\left(\tfrac{M}{2}-n\right)\right) j \cdot e^{-jw\frac{M}{2}}$$

$$= \sum_{k=\frac{1}{2}}^{\frac{M}{2}} 2h\left[\tfrac{M}{2}-k\right] \sin(wk)\, j e^{-jw\frac{M}{2}}$$

$$= \sum_{k=1}^{\frac{M+1}{2}} 2h\left[\tfrac{M+1}{2}-k\right] \sin\left(w\left(k-\tfrac{1}{2}\right)\right) j e^{-jw\frac{M}{2}}$$

$$= \sum_{k=1}^{\frac{M+1}{2}} d[k] \sin\left(w\left(k-\tfrac{1}{2}\right)\right) j e^{-jw\frac{M}{2}}, \quad d[k] = 2h\left[\tfrac{M+1}{2}-k\right]$$

$$= A(e^{jw})\, j e^{-jw\frac{M}{2}}$$

(b)

$$A(e^{jw}) = \sum_{k=1}^{\frac{M+1}{2}} d[k] \sin\left(w\left(k-\tfrac{1}{2}\right)\right)$$

$$= \sum_{k=1}^{\frac{M+1}{2}} d[k]\left[2 \sin\tfrac{1}{2}w \cos w(k-1) + \sin w\left(k-\tfrac{3}{2}\right)\right]$$

$\sin(\alpha+\beta) = 2\sin\alpha\cos\beta + \sin(\beta-\alpha)$

$$= \sin\frac{w}{2}\left\{(d[1]+d[2]+d[3]+\cdots) + 2(d[2]+d[3]+\cdots) + 2(d[3]+\cdots)\cos w + \cdots\right\}$$

$$= \sin\frac{w}{2} \sum_{k=0}^{\frac{M-1}{2}} \hat{d}[k] \cos kw$$

where
$$d[k] = \begin{cases} \frac{1}{2}\left(2\hat{d}[k-1] - \hat{d}[k]\right), & k < \frac{M+1}{2} \\ \hat{d}\left[\frac{M-1}{2}\right], & k = \frac{M+1}{2} \end{cases}$$