106061151 劉宇得 訊號與系 HW4

Prob 1.

1. $x(t) = \sum_{-\infty}^{+\infty} \frac{1}{T} \boxed{\int_0^T x(t) e^{-j\frac{k2\pi t}{T}} dt} e^{+j\frac{k2\pi t}{T}}$

$X[k] = \int_0^T x(t) e^{-j\frac{k2\pi t}{T}} dt$

$x(t) = \sum_{-\infty}^{+\infty} \frac{1}{T} X[k] e^{+j\frac{k2\pi t}{T}}$

2. $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} \boxed{\sum_{n=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{T}}} e^{+j\frac{k2\pi n}{T}}$

$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{T}}$

$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{k2\pi n}{T}}$

Prob.2

$x_p[n] = x_p(nT_s)$

$x_{p\Delta}(t) = x_p(t) \times \sum_{n=-\infty}^{+\infty} \delta(t-nT_s)$

$$
\begin{aligned}
X_{p\Delta}[k] &= \int_0^T x_{p\Delta}(t) e^{-j\frac{k2\pi t}{T}} dt \\
&= \int_0^T x_p(t) \times \sum_{n=-\infty}^{+\infty} \delta(t-nT_s) e^{-j\frac{k2\pi t}{T}} dt \\
&= \int_0^T \sum_{n=-\infty}^{+\infty} x_p(nT_s) \delta(t-nT_s) e^{-j\frac{k2\pi nT_s}{T}} dt \\
&= \sum_{n=0}^{N-1} x_p(nT_s) e^{-j\frac{k2\pi n}{N}} \int_0^T \delta(t-nT_s) dt \\
&= \sum_{n=0}^{N-1} x_p(nT_s) e^{-j\frac{k2\pi n}{N}} \\
&= \sum_{n=0}^{N-1} x_p[n] e^{-j\frac{k2\pi n}{N}} \\
&= \tilde{X}_p(k)
\end{aligned}
$$

$x_\Delta(t) = x(t) \times \sum_{n=-\infty}^{+\infty} \delta(t-nT_s)$

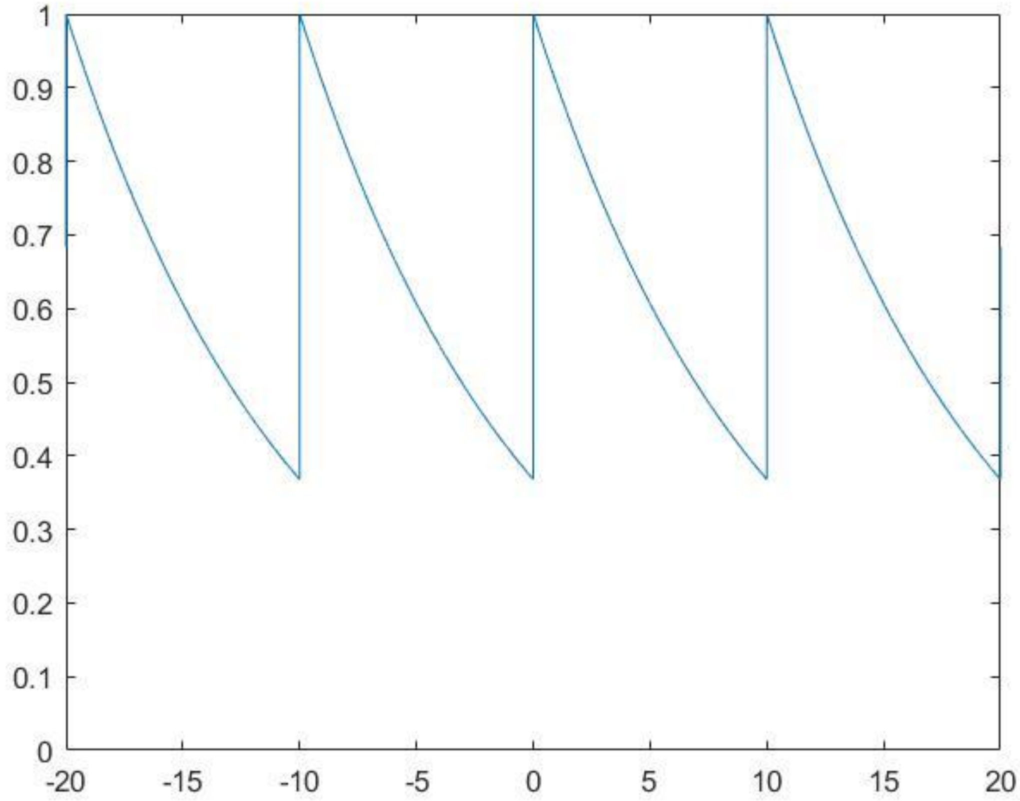
$\sum_{n=-\infty}^{+\infty} \delta(t-nT_s) = \frac{1}{T_s} \sum_{m=-\infty}^{+\infty} 1 \cdot e^{+j\frac{m2\pi t}{T_s}}$

$$
\begin{aligned}
X_{p\Delta}[k] &= \int_0^T x(t) \times \sum_{n=-\infty}^{+\infty} \delta(t-nT_s) e^{-j\frac{k2\pi t}{T}} dt \\
&= \int_0^T x(t) f_s \sum_{m=-\infty}^{+\infty} e^{+j\frac{m2\pi t}{T_s}} e^{-j\frac{k2\pi t}{T}} dt \\
&= f_s \sum_{m=-\infty}^{+\infty} \int_0^T x(t) e^{-j\frac{(k-mN)2\pi t}{T}} dt \\
&= f_s \sum_{m=-\infty}^{+\infty} X_p[k-mN]
\end{aligned}
$$

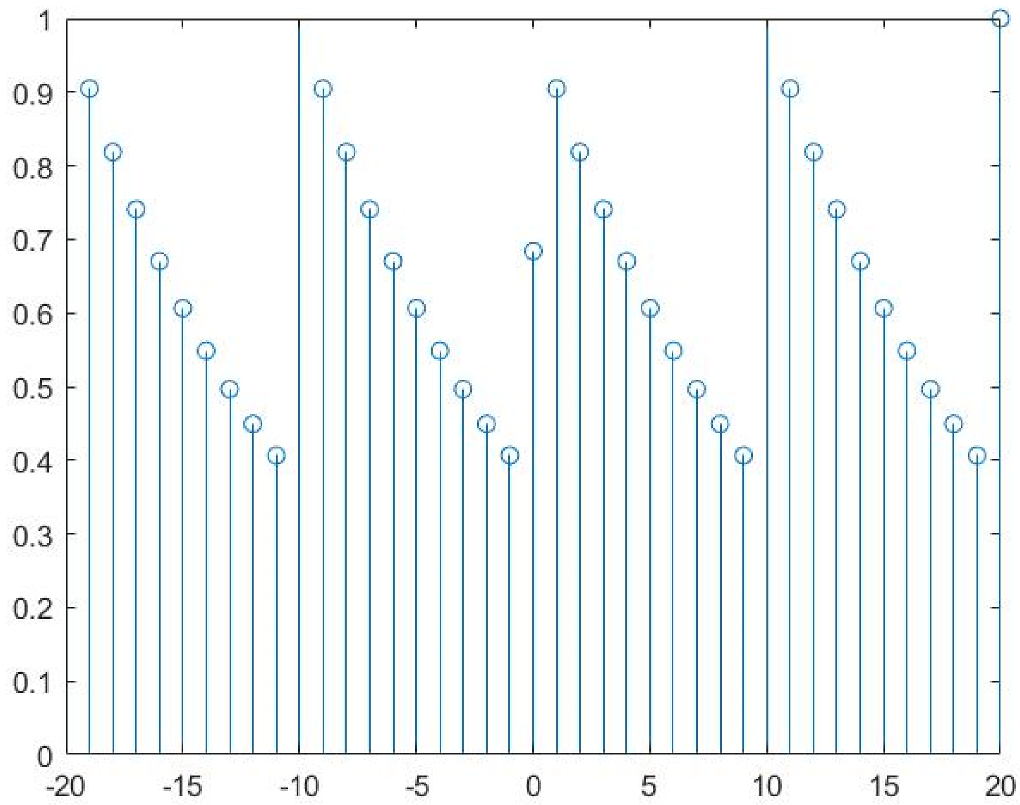
$\tilde{X}_p(k) = f_s \sum_{m=-\infty}^{+\infty} X_p[k-mN]$ #

3.

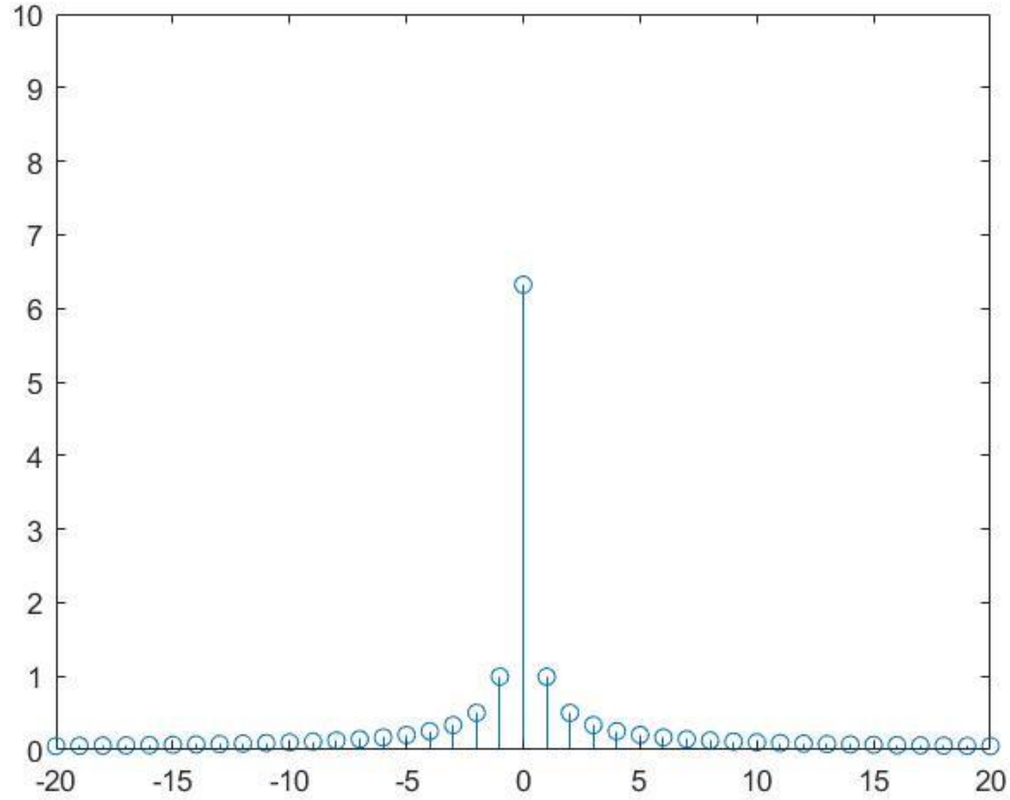
xp(t)



xp[n]



Xp[k]



~Xp[k]