INTRODUCTION TO MECHATRONICS AND MEASUREMENT SYSTEMS

2nd edition

SOLUTIONS MANUAL

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Solutions Manual

This manual contains solutions to the end-of-chapter problems in the second edition of "Introduction to Mechatronics and Measurement Systems." Only a few of the open-ended problems that do not have a unique answer are left for your creative solutions. More information, including an example course outline, a suggested laboratory syllabus, MathCAD files for examples in the book, and other supplemental material are provided on the Internet at:

http://www.engr.colostate.edu/~dga/mechatronics.html

We have class-tested the textbook for several years, and it should be relatively free from errors. However, if you notice any errors or have suggestions or advice concerning the textbook's content or approach, please feel free to contact us via e-mail at dga@engr.colostate.edu. We will post corrections for reported errors on our Web site.

Thank you for choosing our book. We hope it helps you provide your students with an enjoyable and fruitful learning experience in the cross-disciplinary subject of mechatronics.

2.1 D = 0.06408 in = 0.001628 m.

$$A = \frac{\pi D^2}{4} = 2.082 \times 10^{-6}$$

$\rho = 1.7 \times 10^{-8}\ \Omega m$, L = 1000 m

$$R = \frac{\rho L}{A} = 8.2\Omega$$

2.2

(a) $R_1 = 21 \times 10^4 \pm 20\%$ so $168k\Omega \le R_1 \le 252k\Omega$

(b) $R_2 = 07 \times 10^3 \pm 20\%$ so $5.6k\Omega \le R_2 \le 8.4k\Omega$

(c) $R_s = R_1 + R_2 = 217k\Omega \pm 20\%$ so $174k\Omega \le R_s \le 260k\Omega$

(d) $$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{p_{MIN}} = \frac{R_{1_{MIN}} R_{2_{MIN}}}{R_{1_{MIN}} + R_{2_{MIN}}} = 5.43k\Omega$$

$$R_{p_{MAX}} = \frac{R_{1_{MAX}} R_{2_{MAX}}}{R_{1_{MAX}} + R_{2_{MAX}}} = 8.14k\Omega$$

2.3 $R_1 = 10 \times 10^2$, $R_2 = 25 \times 10^1$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{(10 \times 10^2)(25 \times 10^1)}{10 \times 10^2 + 25 \times 10^1} = 20 \times 10^1$$

a = 2 = red, b = 0 = black, c = 1 = brown, d = gold

2.4 In series, the trim pot will add an adjustable value ranging from 0 to its maximum value to the original resistor value depending on the trim setting. When in parallel, the trim pot could be 0Ω perhaps causing a short. Furthermore, the trim value will not be additive with the fixed resistor.

2.5 No, as long as you are consistent in your application, you will obtain correct answers. If you assume the wrong current direction, the result will be negative.

2.6 Place two 100Ω resistors in parallel and you immediately have a 50Ω resistance.

2.7 From KCL, $I_s = I_1 + I_2 + I_3$

so from Ohm's Law $\frac{V_s}{R_{eq}} = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$

Therefore, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ so $R_{eq} = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$

2.8 From Ohm's Law and Question 2.7, $V = \frac{I_s}{R_{eq}} = \frac{I_s}{\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3}}$

and for one resistor, $V = I_1 R_1$

Therefore, $I_1 = \left(\frac{R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}\right) I_s$

2.9 $\lim_{R_1 \to \infty}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{R_1 R_2}{R_1} = R_2$

2.10 $I = C_{eq}\frac{dV}{dt} = C_1\frac{dV_1}{dt} = C_2\frac{dV_2}{dt}$

From KVL,

$$V = V_1 + V_2$$

so

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt}$$

Therefore,

$$\frac{I}{C_{eq}} = \frac{I}{C_1} + \frac{I}{C_2} \quad \text{so} \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

2.11 $V = V_1 = V_2$

$I_1 = C_1\frac{dV_1}{dt} = C_1\frac{dV}{dt}$ and $I_2 = C_2\frac{dV_2}{dt} = C_2\frac{dV}{dt}$

From KCL,

$$I = I_1 + I_2 = C_1\frac{dV}{dt} + C_2\frac{dV}{dt} = \frac{dV}{dt}(C_1 + C_2)$$

Since $I = C_{eq}\frac{dV}{dt}$

$$C_{eq} = C_1 + C_2$$

2.12 $I = I_1 = I_2$

From KVL,

$$V = V_1 + V_2 = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} = \frac{dI}{dt}(L_1 + L_2)$$

Since $V = L_{eq}\frac{dI}{dt}$

$$L_{eq} = L_1 + L_2$$

2.13 $V = L\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$

From KCL, $I = I_1 + I_2$ so $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$

Therefore, $\frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2}$ so $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1L_2}{L_1 + L_2}$

2.14 $V_o = 1V$, regardless of the resistance value.

2.15 From Voltage Division, $V_o = \frac{40}{10 + 40}(5 - 15) = -8V$

2.16 Combining R_2 and R_3 in parallel,

$$R_{23} = \frac{R_2R_3}{R_2 + R_3} = \frac{2(3)}{2 + 3} = 1.2k$$

and combining this with R_1 in series,

$$R_{123} = R_1 + R_{23} = 2.2k$$

(a) Using Ohm's Law,

$$I_1 = \frac{V_{in}}{R_{123}} = \frac{5V}{2.2k} = 2.27mA$$

(b) Using current division,

$$I_3 = \frac{R_2}{R_2 + R_3} I_1 = \frac{2}{5} 2.27\text{mA} = 0.909\text{mA}$$

(c) Since R_2 and R_3 are in parallel, and since V_{in} divides between R_1 and R_{23},

$$V_3 = V_{23} = \frac{R_{23}}{R_1 + R_{23}} V_{in} = \frac{1.2}{2.2} 5\text{V} = 2.73\text{V}$$

2.17

(a) From Ohm's Law,

$$I_4 = \frac{V_{out} - V_1}{R_{24}} = \frac{14.2\text{V} - 10\text{V}}{6\text{k}} = 0.7\text{mA}$$

(b) $V_5 = V_6 = V_{56} = V_{out} - V_2 = 14.2\text{V} - 20\text{V} = -5.8\text{V}$

2.18

(a) $R_{45} = R_4 + R_5 = 3\text{k}\Omega$

$$R_{345} = \frac{R_3 R_{45}}{R_3 + R_{45}} = 1.5\text{k}\Omega$$

$$R_{2345} = R_2 + R_{345} = 3.5\text{k}\Omega$$

$$R_{eq} = \frac{R_1 R_{2345}}{R_1 + R_{2345}} = 0.778\text{k}\Omega$$

(b) $V_A = \dfrac{R_{345}}{R_2 + R_{345}} V_s = 4.29\text{V}$

(c) $I_{345} = \dfrac{V_A}{R_{345}} = 2.86\text{mA}$

$$I_5 = \frac{R_3}{R_3 + R_{45}} I_{345} = 1.43\text{mA}$$

2.19 Using superposition,

$$V_{R2_1} = \frac{R_2}{R_1 + R_2} V_1 = 0.909\text{V}$$

$$V_{R2_2} = \frac{R_1}{R_1 + R_2} i_1 = 9.09V$$

$$V_{R2} = V_{R2_1} + V_{R2_2} = 10.0V$$

2.20 $R_{45} = \frac{R_4 R_5}{R_4 + R_5} = 0.5k\Omega$

$$I = \frac{V_1 - V_2}{R_1 + R_2} = -0.5mA$$

$$V_A = \frac{R_{45}}{R_3 + R_{45}}(V_1 - V_2) = -0.238V$$

2.21 It will depend on your instrumentation, but the oscilloscope typically has an input impedance of 1 MΩ.

2.22 Since the input impedance of the oscilloscope is 1 MΩ, the impedance of the source will be in parallel, and the oscilloscope impedance will affect the measured voltage. Draw a sketch of the equivalent circuit to convince yourself.

2.23 $R_{23} = \frac{R_2 R_3}{R_2 + R_3}$

$$V_{out} = \frac{R_{23}}{R_3 + R_{23}} V_{in}$$

(a) $R_{23} = 9.90k\Omega$, $V_{out} = 0.995V_{in}$

(b) $R_{23} = 333k\Omega$, $V_{out} = 1.00V_{in}$

When the impedance of the load is lower (10k vs. 500k), the accuracy is not as good.

2.24 It will depend on the supply; check the specifications before answering.

2.25 $V_{in} = 5\langle 45°\rangle$

Combining R_2 and L in series and the result in parallel with C gives:

$$Z_{R_2LC} = \frac{(R_2 + Z_L)Z_C}{(R_2 + Z_L) + Z_C} = 1860.52\langle -60.25°\rangle = 923.22 - 1615.30j$$

Using voltage division,

$$V_C = \frac{Z_{R_2LC}}{R_1 + Z_{R_2LC}} V_{in}$$

where

$$R_1 + Z_{R_2LC} = 1000 + 923.22 - 1615.30j = 2511.57\langle -40.02°\rangle$$

so

$$V_C = \frac{1860.52\langle -60.25°\rangle}{2511.57\langle -40.02°\rangle} 5\langle 45°\rangle = 3.70\langle 24.8°\rangle = 3.70\langle 0.433\text{rad}\rangle$$

Therefore,

$$V_C(t) = 3.70\cos(3000t + 0.433)V$$

2.26 With steady state dc V_s, C is open circuit. So

$$V_C = V_s = 10V \quad \text{so} \quad V_{R_1} = 0V \quad \text{and} \quad V_{R_2} = V_s = 10V$$

2.27

(a) In steady state dc, C is open circuit and L is short circuit. So

$$I = \frac{V_s}{R_1 + R_2} = 0.025\text{mA}$$

(b) $\omega = \pi$

$$Z_C = \frac{-j}{\omega C} = \frac{10^6}{\pi} j = \frac{10^6}{\pi} \angle -90° \Omega$$

$$Z_{LR_2} = Z_L + R_2 = j\omega L + R_2 = (10^5 + 20\pi j)\Omega = 10^5 \angle 0.036° \Omega$$

$$Z_{CLR_2} = \frac{Z_C Z_{LR_2}}{Z_C + Z_{LR_2}} = (91040 - 28550j)\Omega = 95410\angle -17.4° \Omega$$

$$Z_{eq} = R_1 + Z_{CLR_2} = (191040 - 28550j)\Omega = 193200\angle -8.50° \Omega$$

$$I_s = \frac{V_s}{Z_{eq}} = 0.0259\angle 8.50° \text{mA}$$

$$I = \frac{Z_C}{Z_C + Z_{LR_2}} I_s = (0.954\angle -17.44°) I_s = 0.0247\angle -8.94° \text{mA}$$

So

$$I(t) = 24.7\cos(\pi t - 0.156)\mu A$$

2.28

(a) $\omega = \pi\frac{\text{rad}}{\text{sec}}, \ f = \frac{\omega}{2\pi} = 0.5\text{Hz}$

$A_{pp} = 2A = 4.0, \ dc_{offset} = 0$

(b) $\omega = 2\pi\frac{\text{rad}}{\text{sec}}, \quad f = \frac{\omega}{2\pi} = 1\text{Hz}$

$A_{pp} = 2A = 2, \ dc_{offset} = 10.0$

(c) $\omega = 2\pi\frac{\text{rad}}{\text{sec}}, \quad f = \frac{\omega}{2\pi} = 1\text{Hz}$

$A_{pp} = 2A = 6.0, \ dc_{offset} = 0$

(d) $\omega = 0\frac{\text{rad}}{\text{sec}}, \quad f = \frac{\omega}{2\pi} = 0\text{Hz}$

$A_{pp} = 2A = 0, \ dc_{offset} = \sin(\pi) + \cos(\pi) = -1$

2.29 $P = \frac{V_{rms}^2}{R} = 100\text{W}$

2.30 $V_{rms} = \left(\frac{V_{pp}}{2}\right)/(\sqrt{2}) = 35.36\text{V}$

$P = \frac{V_{rms}^2}{R} = 12.5\text{W}$

2.31 $V_m = \sqrt{2}V_{rms} = 169.7\text{V}$

2.32 For $V_{rms} = 120\text{V}, \ V_m = \sqrt{2}V_{rms} = 169.7\text{V}$, and $f = 60$ Hz,

$$V(t) = V_m\sin(2\pi f + \phi) = 169.7\sin(120\pi t + \phi)$$

2.33 From Ohm's Law,

$$I = \frac{5V - 2V}{R} = \frac{3V}{R}$$

Since $10mA \le I \le 100mA$,

$$10mA \le \frac{3V}{R} \le 100mA$$

giving

$$\frac{3V}{100mA} \le R \le \frac{3V}{10mA} \quad \text{or} \quad 30\Omega \le R \le 300\Omega$$

For a resistor, $P = \frac{V^2}{R}$, so the smallest allowable resistance would need a power rating of at least:

$$P = \frac{(3V)^2}{30\Omega} = 0.3W$$

so a 1/2 W resistor should be specified.
The largest allowable resistance would need a power rating of at least:

$$P = \frac{(3V)^2}{300\Omega} = 0.03W$$

so a 1/4 W resistor would provide more than enough capacity.

2.34 Using KVL and KCL gives:

$$V_1 = I_{R_1}R_1$$

$$V_1 = (I_1 - I_{R_1})R_2 + (I_1 - I_{R_1} - I_2)R_3$$

$$V_3 - V_2 = (I_1 - I_{R_1} - I_2)R_3 - I_2R_4$$

The first loop equation gives:

$$I_{R_1} = \frac{V_1}{R_1} = 10mA$$

Using this in the other two loop equations gives:

$$10 = (I_1 - 10m)2k + (I_1 - 10m - I_2)3k$$

$$10 - 5 = (I_1 - 10m - I_2)3k - I_24k$$

or

$$(5k)I_1 - (3k)I_2 = 60$$

$$(3k)I_1 - (7k)I_2 = 35$$

Solving these equations gives:

$$I_1 = 12.12\text{mA} \text{ and } I_2 = 0.1923\text{mA}$$

(a) $V_{out} = I_2R_4 - V_2 = -4.23\text{V}$

(b) $P_1 = I_1V_1 = 121\text{mW}$, $P_2 = I_2V_2 = 0.962\text{mW}$, $P_3 = -I_2V_3 = -1.92\text{mW}$

2.35 Using KVL and KCL gives:

$$V_1 = I_{R_1}R_1$$

$$V_1 = (I_1 - I_{R_1})R_2 + (I_1 - I_{R_1} - I_2)R_3$$

$$V_3 - V_2 = (I_1 - I_{R_1} - I_2)R_3 - I_2R_4$$

The first loop equation gives:

$$I_{R_1} = \frac{V_1}{R_1} = 10\text{mA}$$

Using this in the other two loop equations gives:

$$10 = (I_1 - 10\text{m})2\text{k} + (I_1 - 10\text{m} - I_2)2\text{k}$$

$$10 - 5 = (I_1 - 10\text{m} - I_2)2\text{k} - I_2 1\text{k}$$

or

$$(4k)I_1 - (2k)I_2 = 50$$

$$(2k)I_1 - (3k)I_2 = 25$$

Solving these equations gives:

$$I_1 = 12.5\text{mA} \text{ and } I_2 = 0\text{mA}$$

(a) $V_{out} = I_2R_4 - V_2 = -5\text{V}$

(b) $P_1 = I_1V_1 = 125\text{mW}$, $P_2 = I_2V_2 = 0\text{mW}$, $P_3 = -I_2V_3 = 0\text{mW}$

2.36 $P_{avg} = \frac{1}{T}\int_0^T V(t)I(t)dt = \frac{V_mI_m}{T}\int_0^T \sin(\omega t + \phi_V)\sin(\omega t + \phi_I)dt$

Using the product formula trigonometric identity,

$$P_{avg} = \frac{V_m I_m}{2T}\int_0^T (\cos(\phi_V - \phi_I) - \cos(2\omega t + \phi_V + \phi_I))dt$$

Therefore,

$$P_{avg} = \frac{V_m I_m}{2}\cos(\phi_V - \phi_I) = \frac{V_m I_m}{2}\cos(\theta)$$

2.37 $I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \sin^2(\omega t + \phi_I)dt}$

Using the double angle trigonometric identity,

$$I_{rms} = \sqrt{\frac{I_m^2}{T}\int_0^T \left(\frac{1}{2} - \cos[2(\omega t + \phi_I)]\right)dt}$$

Therefore,

$$I_{rms} = \sqrt{\frac{I_m^2}{T}\left(\frac{T}{2}\right)} = \frac{I_m}{2}$$

2.38 $R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 5k\Omega$

$$V_o = \frac{R_{23}}{R_1 + R_{23}} V_i = \frac{1}{2}\sin(2\pi t)$$

This is a sin wave with half the amplitude of the input with a period of 1s.

2.39 $\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{120V}{24V} = 5$

2.40 $R_L = R_i = 100\Omega$ for maximum power

2.41 The BNC cable is far more effective in shielding the input signals from electromagnetic interference since no loops are formed.