Homework 1

Advanced Econometrics

Due on Wednesday, March 24 (before 11:59pm)

Answer the questions below in whatever format you prefer (paper or electronic) as long it contains the code and the answer to all questions. Submit your work by mail to Laura Magazzini (laura.magazzini@santannapisa.it), Sebastiano Michele Zema (sebastianomichele.zema@santannapisa.it), and Tommaso Rughi (tommaso.rughi@santannapisa.it). Be sure that we can **replicate your results** (set the seed equal to your University ID).

1. Use R to generate $n = 200$ observations from the following model ($i = 1, ..., 200$)

$$y_i = \beta_1 + \beta_2 x_i + u_i \tag{1}$$

   where x_i and u_i are independent, and (u_i, x_i) is independent of (u_j, x_j) for $i \neq j$; $u_i \sim (\chi^2_3 - 3)$ (chi-squared distributed with 3 degrees of freedoms *minus* 3), $x_i \sim N(0, 1)$, $\beta_1 = 0.5$ and $\beta_2 = 0.3$.

2. Using the data generated in the previous model to estimate β_1 and β_2 using OLS. Do not use the built-in command `lm`, but rather compute the coefficients using the formulas developed during classes.

3. Calculate the variance of the asymptotic distribution of $\hat{\beta}_2$ that is appropriate for the assumption of the model. Again, do not use R built-in commands to answer this question.

4. Check your results using built-in R commands.

5. Construct a 95% confidence interval for β_2. Would the confidence interval be wider or smaller in case the sample size is set to $n = 400$? Motivate your answer.

6. Since you generated the data yourself, you know which assumptions hold for the model above. Answer the following questions providing a brief (1 line max) justification:

   (a) Is OLS unbiased for β_1?

   (b) Is OLS unbiased for β_2?

   (c) Is the model conditionally homoskedastic?

   (d) Is $\hat{\beta}_2$ estimating the partial effect?

7. Now consider the model in (1) but $u_i = \sqrt{x_i} \times \varepsilon_i$ and $\varepsilon_i \sim (\chi^2_3 - 3)$. Discuss how the answers to the previous questions change (if you think something should be modified when considering this alternative set up).