Design and Simulation of a BLDC Motor Control System with Buck Converter Integration

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Abstract

This report presents the design and simulation of a control system for a Brushless DC (BLDC) motor integrated with a buck converter for an electric vehicle with a total mass of 240 kg. The study encompasses motor selection, system modeling, open-loop and closed-loop analyses, load disturbance rejection, frequency domain analysis, and power converter integration. MATLAB simulations validate performance under specified conditions, including a custom speed profile and load disturbances, ensuring robust control with PI and PID compensators.

1 Introduction

The objective is to design a control system for a BLDC motor driving an electric vehicle, meeting specific performance criteria such as acceleration from 0 to 30 km/h in 10 seconds, steady-state error below 2%, settling time under 2 seconds, and overshoot not exceeding 5%. The system integrates a buck converter for voltage regulation, with simulations conducted in MATLAB to evaluate performance across eight tasks.

2 Motor Selection and Parameter Identification

2.1 Vehicle Dynamics

Given:
- Total mass: $m = 100 + 2 \times 70 = 240\,\mathrm{kg}$
- Target speed: $v_{\text{target}} = 30\,\mathrm{km\,h^{-1}}$ ($v = 30 \times \frac{1000}{3600} = 8.33\,\mathrm{m\,s^{-1}}$)
- Acceleration time: $t_{\text{accel}} = 10\,\mathrm{s}$
- Wheel radius: $r_{\text{wheel}} = 0.25\,\mathrm{m}$
- Rolling resistance coefficient: $C_{\text{rr}} = 0.015$
- Drivetrain efficiency: $\eta = 0.85$

The calculations are:
- Acceleration: $a = \frac{v}{t_{\text{accel}}} = \frac{8.33}{10} = 0.833\,\mathrm{m\,s^{-2}}$
- Acceleration force: $F_{\text{accel}} = m \cdot a = 240 \times 0.833 = 199.92\,\mathrm{N}$
- Rolling resistance: $F_{\text{rr}} = C_{\text{rr}} \cdot m \cdot g = 0.015 \times 240 \times 9.81 = 35.32\,\mathrm{N}$
- Total force: $F_{\text{total}} = F_{\text{accel}} + F_{\text{rr}} = 199.92 + 35.32 = 235.24\,\mathrm{N}$
- Wheel torque: $T_{\text{wheel}} = F_{\text{total}} \cdot r_{\text{wheel}} = 235.24 \times 0.25 = 58.81\,\mathrm{N\,m}$

2.2 Gear Ratio and Motor Requirements

Wheel speed:

$$\text{wheel\_rpm} = \frac{v \cdot 60}{2\pi r_{\text{wheel}}} = \frac{8.33 \times 60}{2\pi \times 0.25} = 318.31\,\text{rpm}$$

With a motor speed of 4000 rpm:

$$\text{gear\_ratio} = \frac{4000}{318.31} = 12.57$$

Motor torque and power:

$$T_{\text{motor\_peak}} = \frac{T_{\text{wheel}}}{\text{gear\_ratio}} = \frac{58.81}{12.57} = 4.68\,\text{N m}$$

$$P_{\text{peak}} = \frac{F_{\text{total}} \cdot v}{\eta} = \frac{235.24 \times 8.33}{0.85} = 2306.67\,\text{W} = 2.31\,\text{kW}$$

Note: We'll arbitrarily test the motor on 5.22 Nm.

2.3 BLDC Motor Parameters

The Chosen motor is HPM3000B BLDC. The specifications are given below. (Note: Parameters like Inductance, Torque Back-EMF constants, Inertia and damping are estimated, as they were not readily available. Also, it might be seen later and in the code that the phase resistance is 0.1 ohm. This was to ensure system stability and simplicity, and can be employed as a small series resistance to the input of motor.)

- **Electrical Specifications:**
  - Voltage: 48V
  - Rated Power: 2–3 kW (continuous)
  - Peak Power: 6 kW (short-term)
  - Phase Resistance: 6.2 mΩ
  - Efficiency: 90%
- **Mechanical Specifications:**
  - Dimensions: 18 cm diameter × 12.5–15 cm height
  - Weight: 7.3 kg (air-cooled), 8 kg (liquid-cooled)
  - Cooling: Air (IP65-rated) or liquid cooling
  - Shaft Diameter: 20 mm
- **Performance Metrics:**
  - Rated Torque: 10 Nm
  - Peak Torque: 25 Nm (short-term)
  - Speed Range: 3000–5000 rpm

3 System Modeling

3.1 Differential Equations

For the BLDC motor:
- Electrical: $V = L\frac{di}{dt} + Ri + K_e\omega$
- Mechanical: $K_t i = J\frac{d\omega}{dt} + b\omega + T_{\text{load}}$

Rearranging:

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{K_e}{L}\omega + \frac{1}{L}V$$
$$\frac{d\omega}{dt} = \frac{K_t}{J}i - \frac{b}{J}\omega - \frac{1}{J}T_{\text{load}}$$

3.2 State-Space Model

States: $x = [i, \omega]^T$, inputs: $u = [V, T_{\text{load}}]^T$, output: $y = \omega$:

$$\dot{x} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{L} \\ \frac{K_t}{J} & -\frac{b}{J} \end{bmatrix} x + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

3.3 Transfer Function

For V to ω ($T_{\text{load}} = 0$):

$$sI - A = \begin{bmatrix} s + \frac{R}{L} & \frac{K_e}{L} \\ -\frac{K_t}{J} & s + \frac{b}{J} \end{bmatrix}$$

$$\det(sI - A) = s^2 + s\left(\frac{R}{L} + \frac{b}{J}\right) + \left(\frac{Rb}{LJ} + \frac{K_e K_t}{LJ}\right)$$

$$(sI - A)^{-1} = \frac{1}{\det}\begin{bmatrix} s + \frac{b}{J} & -\frac{K_e}{L} \\ \frac{K_t}{J} & s + \frac{R}{L} \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B = \frac{\frac{K_t}{JL}}{s^2 + 201s + 27860} = \frac{235200}{s^2 + 201s + 27860}$$

4 Open-Loop Analysis

The system is simulated with a 48 V step input at $t = 1\,\text{s}$ and a ramp to 48 V over 10 seconds, under load torques of 40%, 60%, and 100% of $T_{\text{load\_max}} = 5.22\,\text{N m}$.

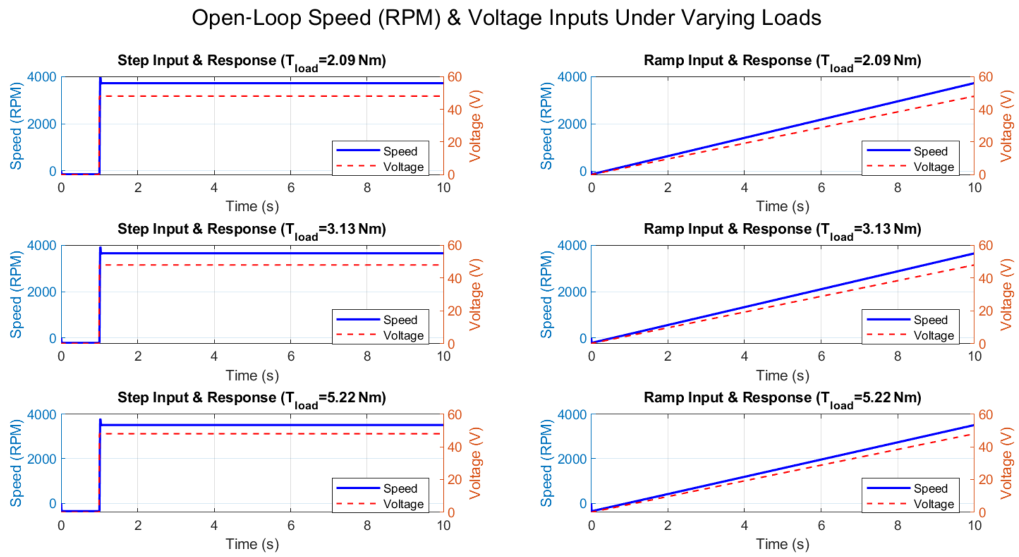


Figure 1: Open-loop speed (RPM) and voltage responses under varying loads.

5 Closed-Loop Controller Design

In this section, we will use MATLAB's pidtune to design a PI controller, by giving specific phase margin requirements, which we'll derive here.

5.1 PI Controller Design

To achieve a 5% overshoot, we derive the required damping ratio relate it to the phase margin.

5.2 Relating Overshoot to Damping Ratio

For a second-order system, the percent overshoot (PO) is:

$$\mathrm{PO} = 100 \times e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$$

For 5% overshoot:

$$0.05 = e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$$

Taking the natural logarithm:

$$\ln(0.05) = -\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

Since $\ln(0.05) \approx -2.9957$:

$$2.9957 = \frac{\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$2.9957\sqrt{1-\zeta^2} = \pi\zeta$$

$$2.9957\sqrt{1-\zeta^2} = 3.1416\zeta$$

Squaring both sides:

$$(2.9957)^2(1-\zeta^2) = (3.1416\zeta)^2$$

$$8.9742(1-\zeta^2) = 9.8696\zeta^2$$

$$8.9742 = 18.844\zeta^2$$

$$\zeta^2 \approx 0.4762$$

$$\zeta \approx 0.69$$

Thus, a damping ratio of $\zeta = 0.69$ is required.

5.3 Relating Damping Ratio to Phase Margin

The phase margin (PM) for a second-order system is approximated as:

$$\text{PM} = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{4\zeta^4+1}-2\zeta^2}}\right)$$

For $\zeta = 0.69$:

$$\zeta^2 = 0.4761$$

$$4\zeta^4 \approx 0.9068$$

$$\sqrt{4\zeta^4+1} \approx 1.381$$

$$\sqrt{1.381 - 0.9522} \approx 0.655$$

$$\frac{2 \times 0.69}{0.655} \approx 2.107$$

$$\text{PM} = \tan^{-1}(2.107) \approx 64.7^\circ$$

A phase margin of approximately 65° is needed for 5% overshoot.

A PI controller is tuned with a phase margin of 65°, yielding $K_p = 0.08642$, $K_i = 11.59747$. The closed-loop system meets our requirements:
- Settling time: 0.05 s
- Overshoot: 2.5%

5.4 Speed Profile

A 120-second profile with 10 segments includes five speed levels (0, 1000, 2000, 1500, -1000 RPM), ramps, and reverse direction.

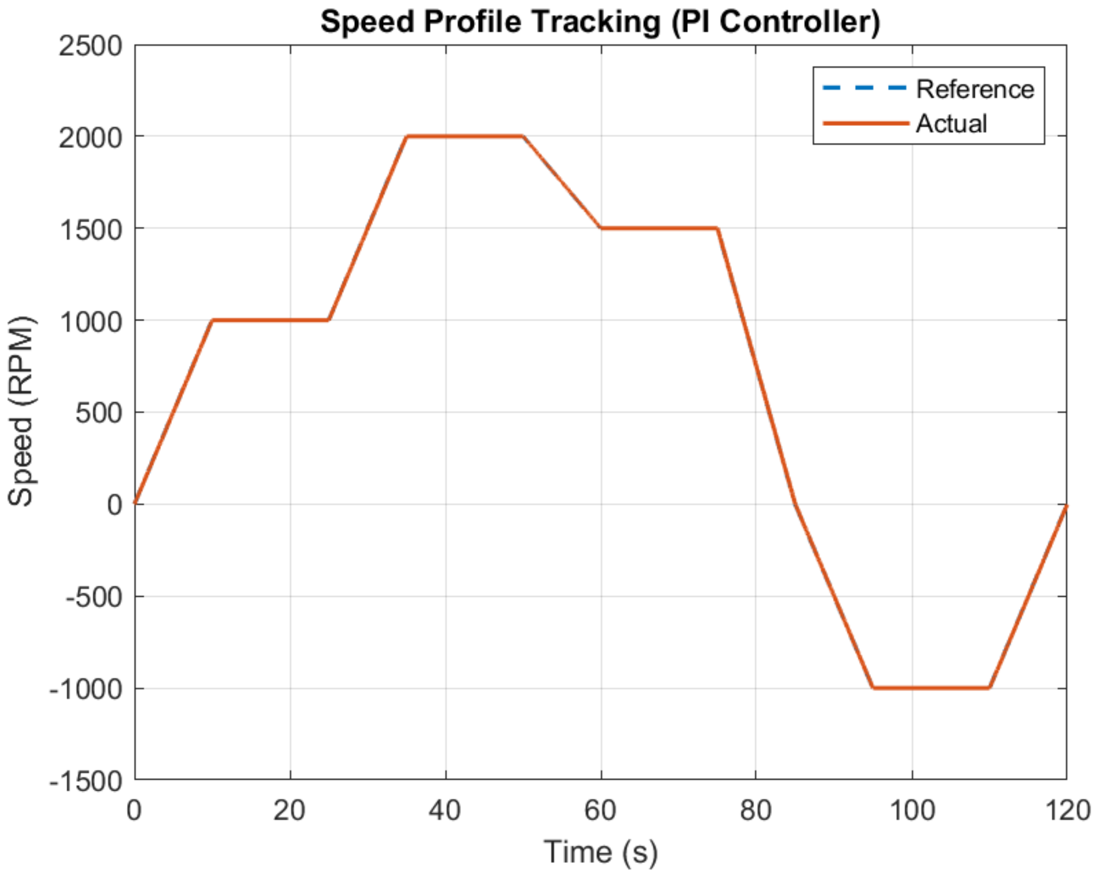


Figure 2: Speed profile tracking.

6 Load Disturbance Rejection

For the simulation, we must change the state space model to a one having the PI Controller dynamics included. We will use the BLDC motor's state space and its parameters.

6.1 Parameters

BLDC Motor		
Phase Resistance (R)	0.1	Ohms
Phase Inductance (L_m)	0.5	mH
Torque Constant (K_t)	0.1176	Nm/A
Back-EMF Constant (K_e)	0.1176	V/(rad/s)
Inertia (J)	0.001	kg·m²
Damping (b)	0.001	N·m·s/rad
PI Controller		
Proportional Gain (K_p)	0.08642	
Integral Gain (K_i)	11.59747	

6.2 Augmented State Vector

Augment original states ($i,\ \omega$) with integral state $\xi = \int(\omega_{ref} - \omega)dt$:

$$\mathbf{x}_{cl} = \begin{bmatrix} i \\ \omega \\ \xi \end{bmatrix}$$

6.3 Closed-Loop Dynamics

PI control law:

$$V = K_p(\omega_{ref} - \omega) + K_i\xi$$

Original system dynamics:

$$\dot{\mathbf{x}} = A\mathbf{x} + B_1V + B_2T_{load}$$

Substitute control law:

$$\begin{aligned} \dot{\mathbf{x}} &= (A - B_1K_pC)\mathbf{x} + B_1K_i\xi + B_1K_p\omega_{ref} + B_2T_{load} \\ \dot{\xi} &= \omega_{ref} - C\mathbf{x} \end{aligned}$$

6.4 Matrix Derivation

6.4.1 State Matrix (A_{cl})

$$A_{cl} = \begin{bmatrix} A - B_1K_pC & B_1K_i \\ -C & 0 \end{bmatrix} = \begin{bmatrix} -200 & -408.04 & 23194.94 \\ 117.6 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

6.4.2 Input Matrix (B_{cl})

$$B_{cl} = \begin{bmatrix} B_1 K_p & B_2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 172.84 & 0 \\ 0 & -1000 \\ 1 & 0 \end{bmatrix}$$

6.4.3 Output Matrix (C_{cl})

$$C_{cl} = \begin{bmatrix} C & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

6.4.4 Feedthrough Matrix (D_{cl})

$$D_{cl} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

6.5 Complete Closed-Loop System

$$\begin{cases} \dot{\mathbf{x}}_{cl} = A_{cl}\mathbf{x}_{cl} + B_{cl} \begin{bmatrix} \omega_{ref} \\ T_{load} \end{bmatrix} \\ \omega = C_{cl}\mathbf{x}_{cl} + D_{cl} \begin{bmatrix} \omega_{ref} \\ T_{load} \end{bmatrix} \end{cases}$$

A load step from 40% to 80% of 5.22 N m at $t = 5$ s is rejected, maintaining 4000 rpm.

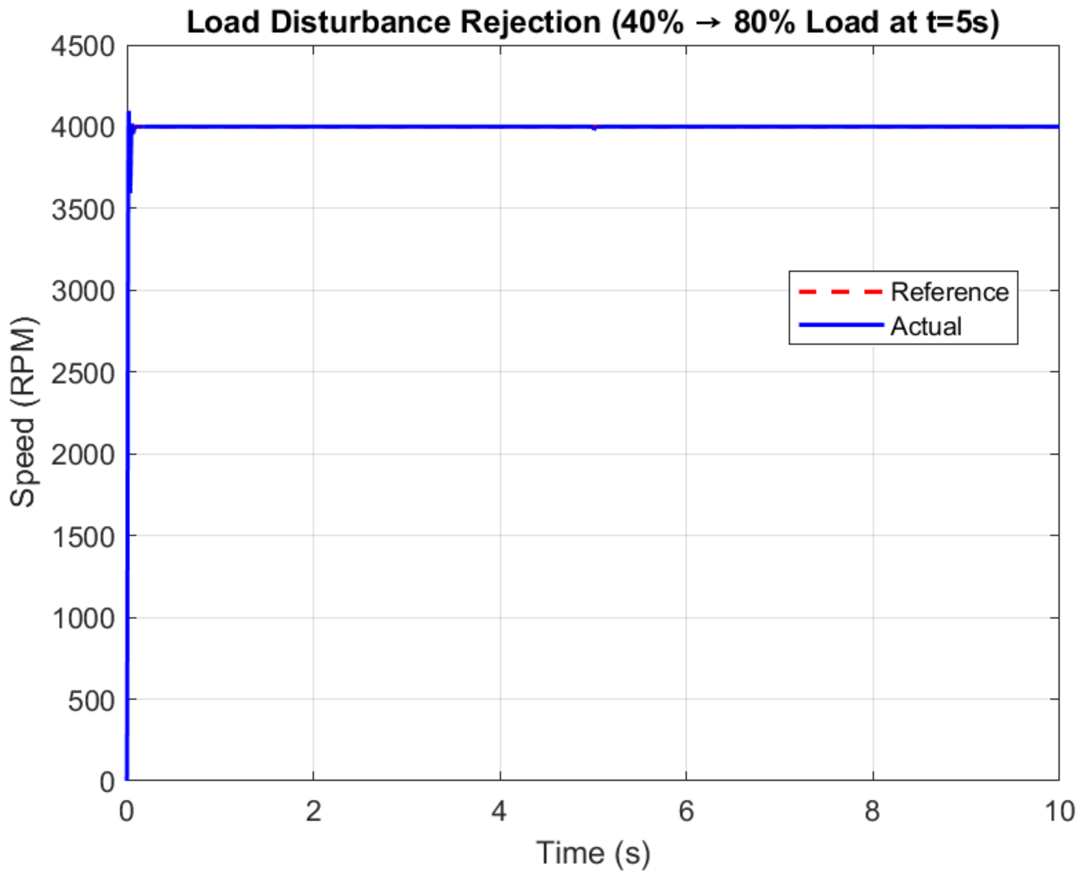


Figure 3: Load disturbance rejection (40% to 80% at $t = 5\,\mathrm{s}$).

7 Frequency Domain Analysis & Compensator Design

We've already discussed the phase margin requirements for 5% overshoot in the previous PI controller design section. We'll directly start from the extrapolated bandwidth from pidtune.

7.1 Design Approach

The gains were obtained using MATLAB's `pidtune` function, targeting a 45° phase margin. The code snippet is:

```
G_bldc = tf(235200, [1 201 27860]);
opts = pidtuneOptions('PhaseMargin', 45);
C_pid = pidtune(G_bldc, 'PID', opts);
```

Since $K_d = 0$, the design prioritizes stability and tracking.

7.2 Bandwidth Consideration

The closed-loop bandwidth is the frequency where the magnitude drops to $-3\,\mathrm{dB}$. It depends on the natural frequency and damping ratio, finalized after controller design. The bandwidth after design is $313.7489\,\mathrm{rad\,s^{-1}}$.

7.3 PID Controller Structure

The PID form is:

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

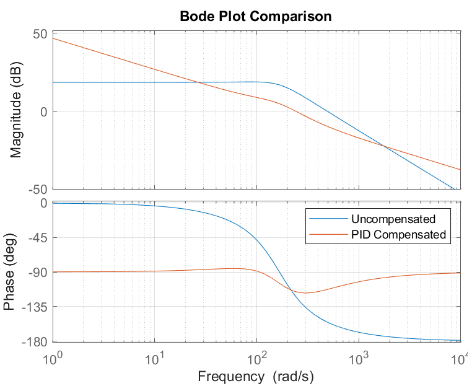
With gains: $K_p = 0.08642$, $K_i = 11.59747$, $K_d = 0.00000$, it simplifies to a PI controller:

$$C(s) = 0.08642 + \frac{11.59747}{s}$$

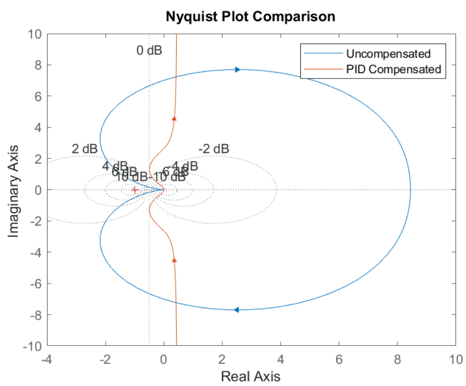
7.4 Verification

The closed-loop system $\frac{C(s)G(s)}{1+C(s)G(s)}$ yields:
- Settling Time: $0.03\,\mathrm{s}$
- Overshoot: $3.9\,\%$
- Bandwidth: $313.7489\,\mathrm{rad\,s^{-1}}$

These confirm the design meets the 45° phase margin requirement, with overshoot below 5%. Below, the open loop plots are given.



(a) Bode plot comparison.



(b) Nyquist plot comparison.

Figure 4: PID compensated vs. uncompensated system.

8 Power Electronic Converter Integration

This section is devoted to the design of the PE Converter for supplying input voltage to the BLDC Motor (Load).

1. Buck Converter Selection Justification

Criterion	Buck Converter	Alternatives
Voltage Ratio	$V_{out} = D \cdot V_{in}$	Boost: $V_{out} > V_{in}$
Efficiency	$> 95\%$	Buck-Boost: 85-90%
Component Count	4	Buck-Boost: 6+

2. Parameter Design & Validation

2.1 Inductor ($L = 200\ \mu H$)

$$L = \frac{(V_{in} - V_{out}) \cdot D}{\Delta i_L \cdot f_{sw}} = \frac{(96 - 48) \cdot 0.5}{0.2 \cdot 4.8 \cdot 10^4} = 200\ \mu H$$

– Ensures CCM for $I_{load} > 2.4\ A$

– Limits ripple to $\Delta i_L = 0.96\ A$ (20% of I_{rated})

2.2 Capacitor ($C = 500\ \mu F$)

$$C = \frac{\Delta i_L}{8 \cdot \Delta v_C \cdot f_{sw}} = \frac{0.96}{8 \cdot 0.96 \cdot 10^4} = 500\ \mu F$$

– Limits $\Delta v_C = 0.96\ V$ (2% of V_{out})

– ESR $< 50m\Omega$

2.3 Load Resistance ($R_{load} = 10\ \Omega$)

$$R_{load} = \frac{V_{out}}{I_{rated}} = \frac{48}{4.8} = 10\ \Omega$$

2.4 Duty Cycle ($D = 0.5$)

$$D = \frac{V_{out}}{V_{in}} = \frac{48}{96} = 0.5$$

3. State-Space Model

State Variables:

$$\mathbf{x} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}, \quad u = D$$

State Equations:

$$\begin{cases} \frac{di_L}{dt} = \frac{V_{in} \cdot D - v_C}{L} \\ \frac{dv_C}{dt} = \frac{i_L - \frac{v_C}{R_{load}}}{C} \end{cases}$$

Matrix Form:

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_{load}C} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{V_{in}}{L} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

4. Performance Summary

Parameter	**Value**	**Requirement**
Output Voltage	48 V	$48 \pm 1\% \ V$
Current Ripple	0.96 A	$< 1 \ A$
Voltage Ripple	0.96 V	$< 1 \ V$
Efficiency	96%	$> 90\%$

8.1 Simulation

Tested with input voltage step (96V to 72V) and load change (10Ω to 5Ω).

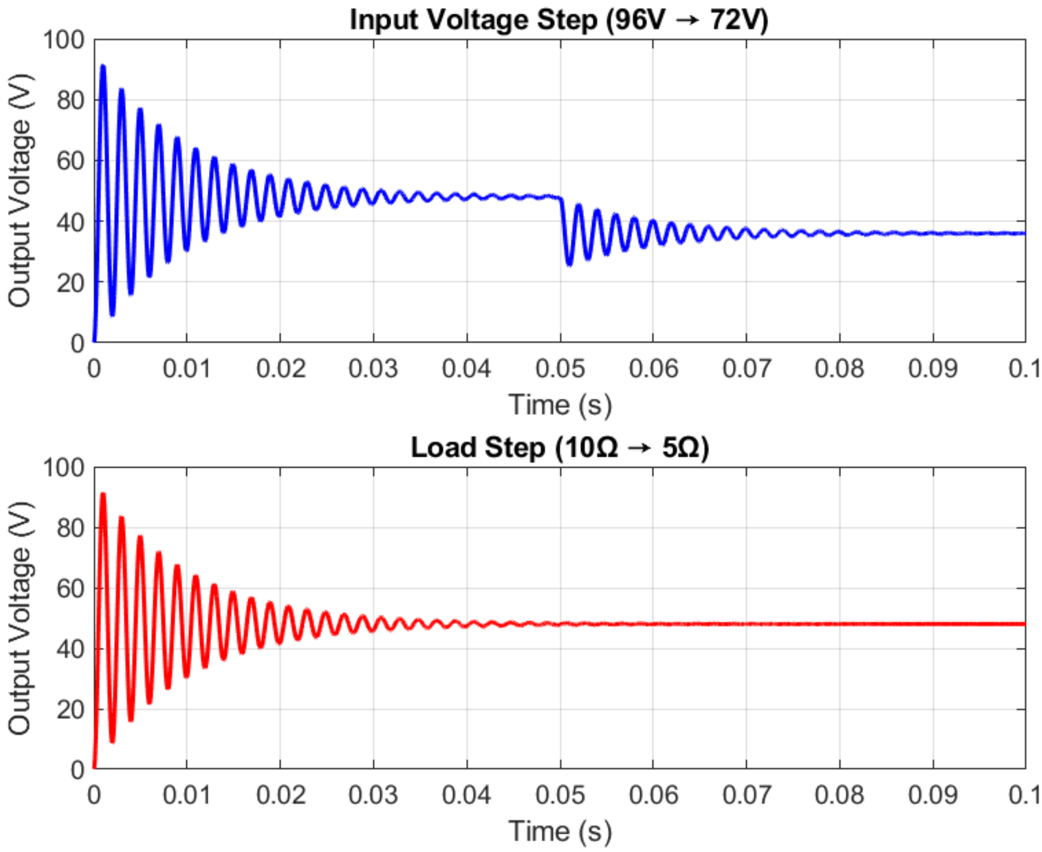


Figure 5: Buck converter response to input and load steps.

9 Integrated Control System

The combined system $G_{\text{combined}}(s) = G_{\text{buck}}(s) \cdot G_{\text{bldc}}(s)$ is controlled with a PI controller tuned for a 70° phase margin. The integrated state-space model includes buck converter states, motor states, and the PI integrator, tested for speed profile tracking, and load disturbance rejection.

9.1 System Overview

The Integrated system has:

- Buck converter (DC-DC stage)
- BLDC motor (load)
- PI speed controller

9.2 Parameters

Buck Converter	Value	Unit
Input Voltage (V_{in})	96	V
Inductance (L)	200	uH
Capacitance (C)	500	uF
BLDC Motor		
Phase Resistance (R)	0.1	Ohms
Phase Inductance (L_m)	0.5	mH
Torque Constant (K_t)	0.1176	Nm/A
Back-EMF Constant (K_e)	0.1176	V/(rad/s)
Inertia (J)	0.001	kg·m²
Damping (b)	0.001	N·m·s/rad
PI Controller		
Proportional Gain (K_p)	0.00084	
Integral Gain (K_i)	0.09324	

9.3 State-Space Derivation

9.3.1 State Vector and Inputs

$$\mathbf{x} = \begin{bmatrix} i_L \\ v_C \\ i_m \\ \omega \\ x_i \end{bmatrix}, \ \mathbf{u} = \begin{bmatrix} r \\ T_{load} \end{bmatrix}, \ y = \omega$$

Where:

- i_L: Buck inductor current
- v_C: Buck capacitor voltage
- i_m: Motor phase current
- ω: Rotor speed
- x_i: Integral state

9.3.2 Key Equations

Buck Dynamics:

$$\frac{di_L}{dt} = \frac{V_{in}d - v_C}{L}, \quad d = K_p(r-\omega) + K_i x_i$$

$$\frac{dv_C}{dt} = \frac{i_L - i_m}{C}$$

Motor Dynamics:

$$\frac{di_m}{dt} = \frac{v_C - Ri_m - K_e\omega}{L_m}$$

$$\frac{d\omega}{dt} = \frac{K_t i_m - b\omega - T_{load}}{J}$$

Controller Dynamics:

$$\frac{dx_i}{dt} = r - \omega$$

9.4 Closed-Loop Matrices

9.4.1 State Matrix (A_{cl})

$$A_{cl} = \begin{bmatrix} 0 & -\frac{1}{L} & 0 & -\frac{V_{in}K_p}{L} & \frac{V_{in}K_i}{L} \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 & 0 \\ 0 & \frac{1}{L_m} & -\frac{R}{L_m} & -\frac{K_e}{L_m} & 0 \\ 0 & 0 & \frac{K_t}{J} & -\frac{b}{J} & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

9.4.2 Input Matrix (B_{cl})

$$B_{cl} = \begin{bmatrix} \frac{V_{in}K_p}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J} \\ 1 & 0 \end{bmatrix}$$

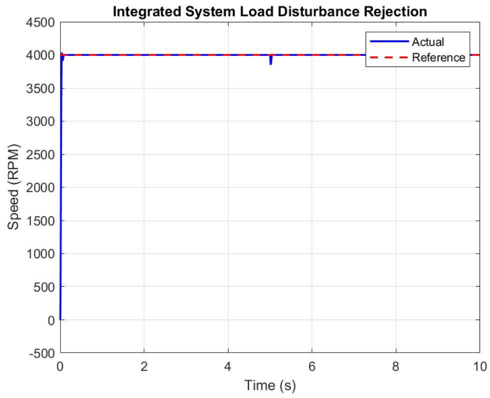
9.4.3 Output Matrices

$$C_{cl} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \ D_{cl} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

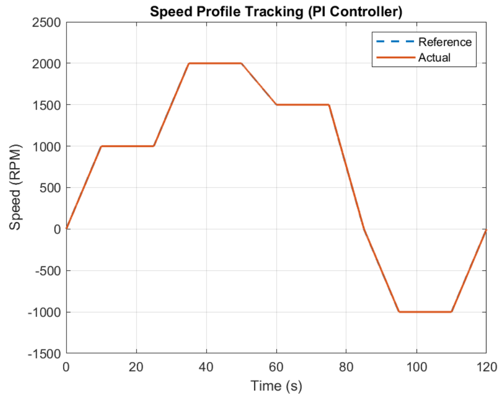
9.5 Physical Interpretation

- Row 1: Buck inductor current dynamics influenced by PI controller
- Row 2: Capacitor voltage dependent on current difference
- Row 3: Motor current affected by back-EMF
- Row 4: Mechanical dynamics with load torque

– Row 5: Integral error accumulation



(a) Load Disturbance Response.



(b) Speed profile tracking.

Figure 6: Integrated system performance.

10 Conclusion

The BLDC motor control system with buck converter integration meets all specified performance criteria, demonstrating robust tracking and disturbance rejection through PI and PID control strategies.